

Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/1.1.3.4-e-x-
 $\int \frac{e^{-x} (b x^n + c + d x^{n-p})}{(a + b x^n)^m} dx$

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [702]. This is test number [14].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. IntegrateAlgebraic under Mathematica 12.3.1 on windows 10. https://github.com/stblake/algebraic_integration. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (702)	0.00 (0)
Mathematica	100.00 (702)	0.00 (0)
Fricas	94.44 (663)	5.56 (39)
Giac	76.21 (535)	23.79 (167)
Mupad	75.64 (531)	24.36 (171)
Maple	74.36 (522)	25.64 (180)
IntegrateAlgebraic	73.36 (515)	26.64 (187)
Maxima	55.70 (391)	44.30 (311)
Sympy	42.88 (301)	% 57.12 (401)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

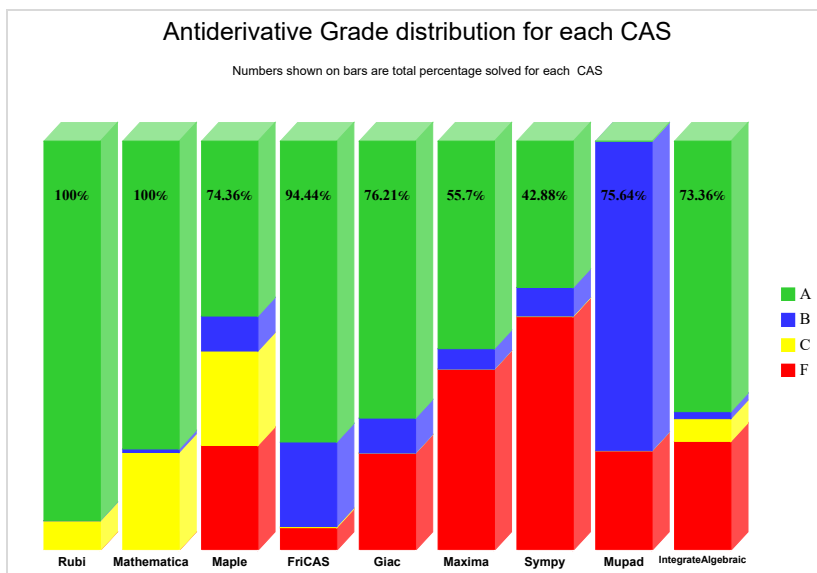
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

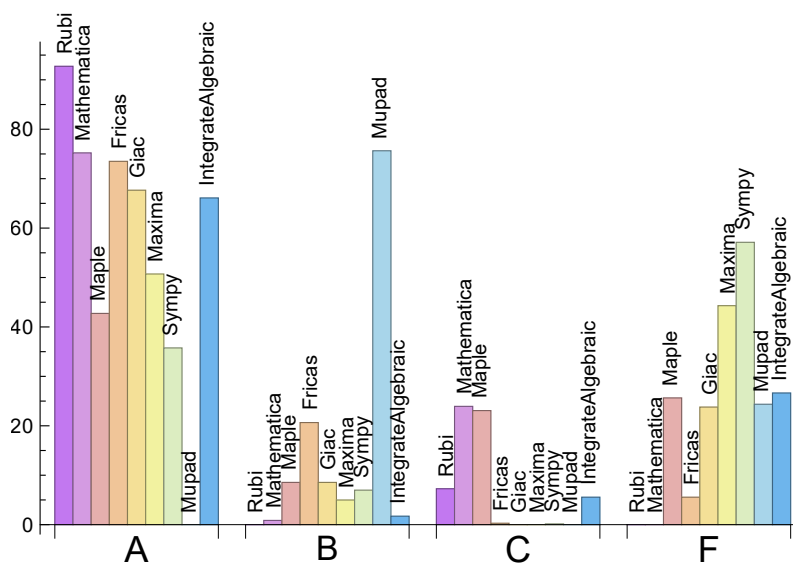
System	% A grade	% B grade	% C grade	% F grade
Rubi	92.74	0.00	7.26	0.00
Mathematica	75.21	0.85	23.93	0.00
Fricas	73.50	20.66	0.28	5.56
Giac	67.66	8.55	0.00	23.79
IntegrateAlgebraic	66.10	1.71	5.56	26.64
Maxima	50.71	4.99	0.00	44.30
Maple	42.74	8.55	23.08	25.64
Sympy	35.75	6.98	0.14	57.12
Mupad	N/A	75.64	0.00	24.36

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure F.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima

and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	180	100.00 %	0.00 %	0.00 %
Fricas	39	0.00 %	89.74 %	10.26 %
IntegrateAlgebraic	187	100.00 %	0.00 %	0.00 %
Giac	167	69.46 %	3.59 %	26.95 %
Maxima	311	76.21 %	0.00 %	23.79 %
Sympy	401	62.59 %	35.41 %	2.00 %
Mupad	171	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

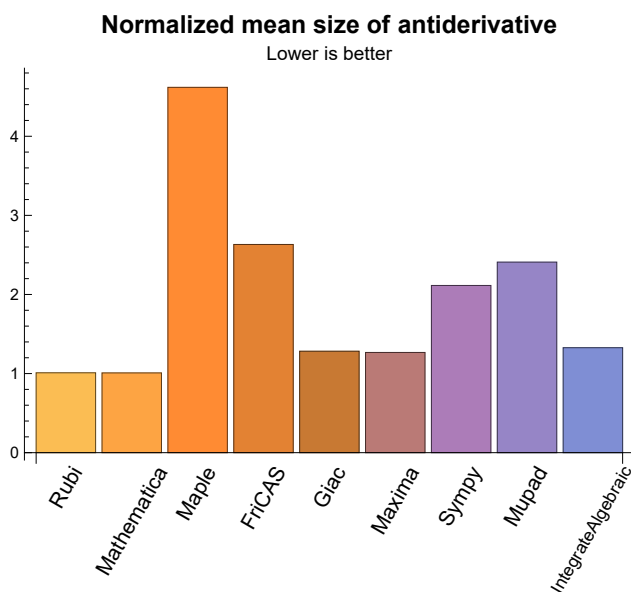
1.3 Performance

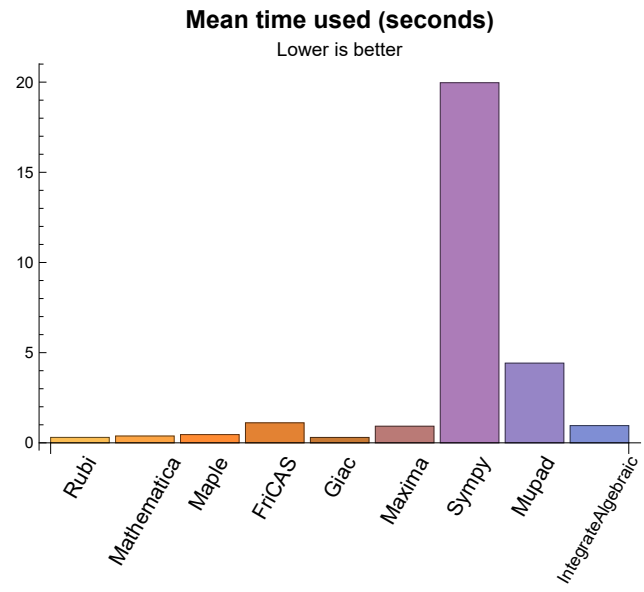
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.30	139.40	1.01	104.00	1.00
Mathematica	0.38	122.25	1.01	94.00	0.91
Maple	0.46	519.93	4.62	216.50	1.31
Maxima	0.92	119.81	1.27	97.00	1.02
Fricas	1.11	391.97	2.63	213.00	1.98
Sympy	19.97	194.23	2.11	114.00	1.17
Giac	0.30	151.79	1.28	114.00	1.04
Mupad	4.42	439.68	2.41	117.00	1.12
IntegrateAlgebraic	0.96	172.46	1.33	126.00	1.11

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {351, 352, 353, 354, 355, 356, 357, 358, 366, 367, 368, 369, 370, 371, 372, 386, 398, 399, 407, 408, 409, 411, 412, 413, 421, 422, 423, 424, 425, 426, 427, 435, 436, 437, 438, 439, 440, 441, 448, 449, 450, 451, 452, 453, 454, 486, 487, 488, 490, 491, 492}

Mathematica {214, 215, 237, 245, 246, 247, 248, 249, 250, 251, 252, 351, 352, 353, 355, 356, 357, 358, 366, 367, 368, 370, 371, 372, 380, 381, 383, 384, 385, 386, 393, 394, 395, 396, 397, 398, 399, 407, 408, 409, 410, 411, 412, 413, 421, 422, 423, 424, 425, 426, 427, 435, 436, 437, 438, 439, 440, 441, 448, 449, 450, 451, 452, 453, 454, 462, 463, 465, 466, 467, 474, 475, 476, 477, 478, 486, 487, 488, 489, 490, 491, 492, 520, 529, 530, 540, 541, 542, 551, 552, 560, 561, 562, 571, 572, 580, 581, 582, 649, 650}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by

failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```

from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')

```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

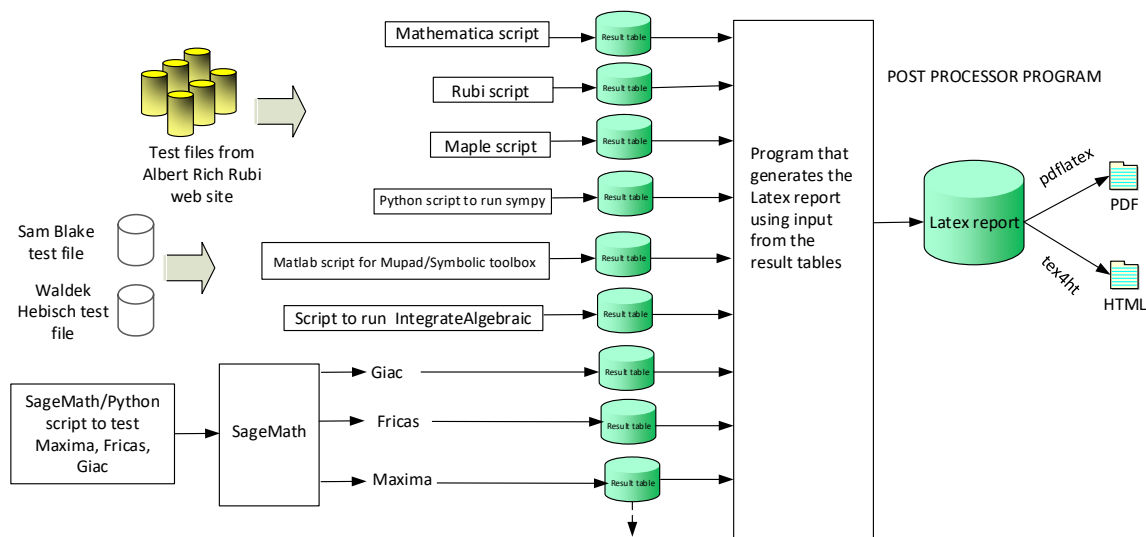
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. integer. 1 if result was verified or 0 if not verified.
The following field present only in Rubi and Mathematica Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 359, 360, 361, 362, 363, 364, 365, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 400, 401, 402, 403, 404, 405, 406, 410, 414, 415, 416, 417, 418, 419, 420, 428, 429, 430, 431, 432, 433, 434, 442, 443, 444, 445, 446, 447, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 489, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702 }

B grade: { }

C grade: { 351, 352, 353, 354, 355, 356, 357, 358, 366, 367, 368, 369, 370, 371, 372, 386, 398, 399, 407, 408, 409, 411, 412, 413, 421, 422, 423, 424, 425, 426, 427, 435, 436, 437, 438, 439, 440, 441, 448, 449, 450, 451, 452, 453, 454, 486, 487, 488, 490, 491, 492 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 154, 157, 159, 162, 165, 167, 170, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 240, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 301, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 359, 360, 361, 362, 363, 364, 365, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 382, 384, 385, 387, 388, 389, 390, 391, 392, 403, 412, 413, 414, 415, 416, 417, 418, 419, 420, 442, 443, 444, 445, 446, 447, 464, 468, 469, 470, 471, 472, 473, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 566, 567, 568, 569, 570, 573, 574, 575, 576, 577, 578, 579, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 599, 600, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 614, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 675, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699 }

B grade: { 30, 54, 652, 672, 673, 674 }

C grade: { 152, 153, 155, 156, 158, 160, 161, 163, 164, 166, 168, 169, 171, 172, 174, 186, 197, 198, 202, 203, 214, 215, 237, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 268, 270, 271, 272, 283, 294, 296, 297, 298, 299, 300, 308, 317, 318, 319, 320, 351, 352, 353, 354, 355, 356, 357, 358, 366, 367, 368, 369, 380, 381, 383, 386, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 411, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 465, 466, 467, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 518, 529, 540, 551, 552, 560, 561, 571, 572, 580, 581, 598, 601, 602, 613, 615, 638, 639, 657, 676, 700, 701, 702 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 340, 342, 343, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 583, 584, 585, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 599, 600, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 616, 617, 618, 619, 620, 621, 622, 623, 624, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 653, 654, 655, 658, 659, 661, 662, 663, 664, 665, 668, 669, 670, 671, 677, 678, 679, 680, 699, 700, 701 }

B grade: { 27, 30, 54, 124, 125, 126, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 586, 597, 598, 601, 602, 613, 614, 615, 625, 643, 652, 656, 660, 666, 667, 672, 673, 674, 675, 681, 682, 683, 684, 686 }

C grade: { 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 373, 374, 375, 376, 377, 382, 383, 384, 385, 387, 388, 389, 390, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 409, 410, 411, 412, 413, 685 }

F grade: { 321, 322, 323, 324, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 378, 379, 380, 381, 386, 391, 392, 393, 394, 405, 406, 407, 408, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 657, 676, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 702 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 209, 210, 211, 216, 217, 218, 219, 223, 224, 225, 226, 230, 231, 232, 233, 238, 239, 240, 241, 273, 274, 275, 276, 280, 281, 282, 283, 287, 288, 289, 290, 294, 295, 296, 297, 328, 344, 345, 346, 347, 359, 360, 361, 362, 373, 374, 375, 376, 377, 387, 388, 389, 390, 400, 401, 402, 403, 404, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 618, 619, 620, 621, 622, 623, 624, 629, 630, 631, 632, 633, 634, 635, 636, 637, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 670, 676, 677, 678, 679, 685, 699, 700, 701, 702 }

B grade: { 30, 54, 180, 191, 192, 197, 198, 583, 584, 597, 598, 599, 613, 614, 615, 616, 617, 625, 626, 627, 628, 638, 639, 652, 665, 671, 672, 673, 674, 675, 680, 681, 682, 683, 684 }

C grade: { }

F grade: { 207, 208, 212, 213, 214, 215, 220, 221, 222, 227, 228, 229, 234, 235, 236, 237, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 277, 278, 279, 284, 285, 286, 291, 292, 293, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 383, 384, 385, 386, 391, 392, 393, 394, 395, 396, 397, 398, 399, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 154, 157, 159, 162, 165, 167, 170, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 270, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 307, 308, 309, 310, 312, 314, 315, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 373, 374, 375, 376, 377, 379, 380, 387, 388, 389, 390, 391, 392, 393, 394, 398, 399, 400, 401, 402, 404, 405, 406, 414, 415, 416, 417, 418, 419, 420, 421, 423, 433, 434, 442, 443, 444, 445, 446, 447, 448, 449, 455, 456, 460, 461, 462, 463, 483, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 530, 531, 533, 535, 536, 537, 538, 542, 543, 544, 545, 546, 547, 548, 549, 552, 554, 556, 557, 558, 562, 563, 564, 565, 566, 567, 568, 569, 572, 574, 576, 577, 578, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 653, 654, 655, 656, 657, 658, 660, 661, 662, 663, 664, 666, 667, 668, 669, 670, 677, 678, 679, 680, 685, 687, 688, 689, 690, 692, 693, 694, 695, 696, 699, 700, 701, 702 }

B grade: { 27, 30, 54, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 124, 125, 126, 152, 153, 155, 156, 158, 160, 161, 163, 164, 166, 168, 169, 171, 172, 174, 214, 215, 237, 251, 252, 268, 269, 271, 272, 304, 311, 313, 316, 317, 318, 319, 320, 322, 323, 348, 353, 369, 382, 383, 384, 385, 386, 395, 396, 397, 403, 407, 408, 409, 410, 411, 412, 413, 422, 428, 429, 430, 431, 432, 435, 436, 437, 457, 458, 459, 468, 469, 470, 471, 472, 473, 474, 475, 479, 480, 481, 482, 484, 485, 486, 487, 505, 506, 507, 508, 509, 510, 511, 512, 528, 529, 532, 534, 539, 540, 541, 550, 551, 553, 555, 559, 560, 561, 570, 571, 573, 575, 579, 580, 581, 603, 629, 652, 659, 665, 671, 672, 673, 674, 675, 676, 681, 682, 683, 684, 686, 691, 697, 698 }

C grade: { 378, 381 }

F grade: { 245, 246, 247, 248, 249, 250, 354, 355, 356, 357, 358, 370, 371, 372, 424, 425, 426, 427, 438, 439, 440, 441, 450, 451, 452, 453, 454, 464, 465, 466, 467, 476, 477, 478, 488, 489, 490, 491, 492 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 112, 114, 115, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 156, 157, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 210, 211, 212, 216, 217, 218, 219, 220, 224, 225, 226, 227, 232, 233, 234, 240, 241, 242, 253, 254, 255, 256, 258, 259, 260, 261, 265, 266, 270, 271, 326, 327, 328, 335, 336, 337, 339, 513, 515, 517, 523, 524, 545, 546, 565, 566, 584, 585, 586, 587, 588, 589, 590, 595, 596, 597, 601, 602, 603, 612, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 629, 630, 631, 632, 633, 636, 639, 656, 658, 660, 661, 662, 668, 677, 678, 679, 680, 699 }

B grade: { 27, 30, 110, 111, 113, 180, 186, 192, 197, 325, 330, 331, 333, 334, 495, 496, 500, 503, 583, 591, 592, 593, 594, 598, 599, 604, 605, 606, 607, 608, 609, 610, 611, 613, 614, 615, 622, 628, 634, 635, 637, 638, 657, 672, 673, 674, 681, 682, 683 }

C grade: { 651 }

F grade: { 50, 51, 52, 53, 54, 55, 107, 108, 109, 116, 117, 118, 119, 120, 121, 122, 123, 151, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 203, 208, 209, 213, 214, 215, 221, 222, 223, 228, 229, 230, 231, 235, 236, 237, 238, 239, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 257, 262, 263, 264, 267, 268, 269, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 329, 332, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 497, 498, 499, 501, 502, 504, 505, 506, 507, 508, 509, 510, 511, 512, 514, 516, 518, 519, 520, 521, 522, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 600, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 652, 653, 654, 655, 659, 663, 664, 665, 666, 667, 669, 670, 671, 675, 676, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 700, 701, 702 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 325, 326, 328, 335, 336, 337, 338, 339, 341, 342, 373, 374, 375, 376, 377, 378, 379, 387, 388, 389, 390, 391, 392, 400, 401, 402, 403, 404, 405, 406, 414, 415, 416, 417, 418, 419, 420, 428, 429, 432, 433, 434, 442, 444, 445, 446, 447, 455, 456, 457, 458, 459, 460, 461, 468, 469, 470, 471, 472, 473, 479, 480, 481, 484, 485, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 515, 517, 519, 521, 522, 523, 524, 525, 528, 529, 531, 532, 533, 534, 535, 536, 543, 544, 545, 546, 547, 550, 553, 554, 555, 556, 557, 563, 564, 565, 566, 567, 570, 571, 573, 574, 575, 576, 577, 583, 584, 585, 591, 592, 593, 594, 596, 597, 598, 599, 600, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 630, 640, 641, 647, 648, 649, 650, 651, 653, 654, 655, 656, 657, 672, 677, 678, 679, 681, 682, 683, 685, 699 }

B grade: { 27, 30, 54, 124, 125, 126, 323, 324, 329, 332, 430, 431, 443, 482, 483, 518, 520, 530, 537, 538, 539, 540, 541, 542, 549, 558, 560, 572, 578, 579, 580, 581, 582, 586, 587, 588, 589, 590, 595, 601, 602, 603, 604, 605, 606, 619, 620, 621, 642, 643, 645, 646, 652, 673, 674, 675, 676, 700, 701, 702 }

C grade: { }

F grade: { 214, 215, 237, 245, 246, 247, 248, 249, 250, 251, 252, 327, 330, 331, 333, 334, 340, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 380, 381, 382, 383, 384, 385, 386, 393, 394, 395, 396, 397, 398, 399, 407, 408, 409, 410, 411, 412, 413, 421, 422, 423, 424, 425, 426, 427, 435, 436, 437, 438, 439, 440, 441, 448, 449, 450, 451, 452, 453, 454, 462, 463, 464, 465, 466, 467, 474, 475, 476, 477, 478, 486, 487, 488, 489, 490, 491, 492, 514, 516, 526, 527, 548, 551, 552, 559, 561, 562, 568, 569, 618, 622, 623, 624, 625, 626, 627, 628, 629, 631, 632, 633, 634, 635, 636, 637, 638, 639, 644, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 680, 684, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 359, 360, 361, 362, 363, 364, 365, 373, 374, 375, 376, 377, 378, 379, 387, 388, 389, 390, 391, 392, 400, 401, 402, 403, 404, 405, 406, 414, 415, 416, 417, 418, 419, 420, 428, 429, 430, 431, 432, 433, 434, 442, 443, 444, 445, 446, 447, 455, 456, 457, 458, 459, 460, 461, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 485, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 515, 517, 519, 521, 522, 523, 524, 525, 531, 532, 533, 534, 535, 536, 543, 544, 545, 546, 547, 553, 554, 555, 556, 557, 563, 564, 565, 566, 567, 573, 574, 575, 576, 577, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 613, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 640, 641, 643, 644, 645, 646, 647, 648, 649, 650, 652, 653, 654, 655, 656, 657, 658, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 699, 700, 701, 702 }

C grade: { }

F grade: { 245, 246, 247, 248, 249, 250, 251, 252, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 351, 352, 353, 354, 355, 356, 357, 358, 366, 367, 368, 369, 370, 371, 372, 380, 381, 382, 383, 384, 385, 386, 393, 394, 395, 396, 397, 398, 399, 407, 408, 409, 410, 411, 412, 413, 421, 422, 423, 424, 425, 426, 427, 435, 436, 437, 438, 439, 440, 441, 448, 449, 450, 451, 452, 453, 454, 462, 463, 464, 465, 466, 467, 474, 475, 476, 477, 478, 486, 487, 488, 489, 490, 491, 492, 514, 516, 518, 520, 526, 527, 528, 529, 530, 537, 538, 539, 540, 541, 542, 548, 549, 550, 551, 552, 558, 559, 560, 561, 562, 568, 569, 570, 571, 572, 578, 579, 580, 581, 582, 597, 598, 611, 612, 614, 615, 626, 638, 639, 642, 651, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698 }

2.1.9 Integrate Algebraic

A grade: { 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 428, 429, 430, 431, 432, 433, 434, 442, 443, 444, 445, 446, 447, 455, 456, 457, 458, 459, 460, 461, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 485, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 649, 650, 655, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 675, 680, 684, 685 }

B grade: { 642, 643, 644, 645, 646, 647, 648, 651, 652, 653, 654, 686 }

C grade: { 421, 422, 423, 424, 425, 426, 427, 435, 436, 437, 438, 439, 440, 441, 448, 449, 450, 451, 452, 453, 454, 462, 463, 464, 465, 466, 467, 474, 475, 476, 477, 478, 486, 487, 488, 489, 490, 491, 492 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 214, 215, 237, 245, 246, 247, 248, 249, 250, 251, 252, 398, 399, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 656, 657, 672, 673, 674, 676, 677, 678, 679, 681, 682, 683, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	28	27	29	29	29	28	0
N.S.	1	1.00	1.00	0.85	0.82	0.88	0.88	0.88	0.85	0.00
time (sec)	N/A	0.033	0.009	0.037	0.565	0.652	0.066	0.179	0.200	0.000
Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	28	27	29	29	29	28	0
N.S.	1	1.00	1.00	0.85	0.82	0.88	0.88	0.88	0.85	0.00
time (sec)	N/A	0.017	0.007	0.045	0.630	0.699	0.067	0.148	2.483	0.000
Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	25	24	26	26	26	25	0
N.S.	1	1.00	1.00	0.89	0.86	0.93	0.93	0.93	0.89	0.00
time (sec)	N/A	0.013	0.007	0.036	0.563	1.078	0.064	0.150	0.034	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	28	28	25	27	28	26	0
N.S.	1	1.00	1.00	0.97	0.97	0.86	0.93	0.97	0.90	0.00
time (sec)	N/A	0.021	0.009	0.038	0.571	0.763	0.113	0.150	0.035	0.001

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	30	27	29	26	29	28	0
N.S.	1	1.00	1.00	0.97	0.87	0.94	0.84	0.94	0.90	0.00
time (sec)	N/A	0.015	0.011	0.049	0.504	0.727	0.108	0.147	0.039	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	24	24	28	24	23	24	0
N.S.	1	1.00	1.00	0.86	0.86	1.00	0.86	0.82	0.86	0.00
time (sec)	N/A	0.016	0.009	0.044	0.478	0.794	0.115	0.159	2.337	0.001

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	26	28	30	26	40	25	0
N.S.	1	1.00	1.00	0.90	0.97	1.03	0.90	1.38	0.86	0.00
time (sec)	N/A	0.022	0.013	0.050	0.475	0.810	0.210	0.164	0.040	0.001

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	32	28	29	29	31	31	29	0
N.S.	1	1.00	1.03	0.90	0.94	0.94	1.00	1.00	0.94	0.00
time (sec)	N/A	0.017	0.013	0.046	0.632	1.291	0.250	0.184	0.033	0.001

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	30	25	27	29	29	29	28	0
N.S.	1	1.00	1.07	0.89	0.96	1.04	1.04	1.04	1.00	0.00
time (sec)	N/A	0.016	0.012	0.041	0.488	0.742	0.287	0.184	2.316	0.001
Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	31	28	30	31	29	37	29	0
N.S.	1	1.00	1.07	0.97	1.03	1.07	1.00	1.28	1.00	0.00
time (sec)	N/A	0.021	0.018	0.044	0.509	0.927	0.542	0.164	0.052	0.000
Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	51	52	51	53	54	53	51	0
N.S.	1	1.00	1.21	1.24	1.21	1.26	1.29	1.26	1.21	0.00
time (sec)	N/A	0.072	0.016	0.039	0.491	1.028	0.076	0.153	2.375	0.000
Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	52	51	53	54	53	51	0
N.S.	1	1.00	1.00	0.95	0.93	0.96	0.98	0.96	0.93	0.00
time (sec)	N/A	0.034	0.008	0.046	0.608	1.061	0.076	0.149	0.043	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	49	48	50	51	50	48	0
N.S.	1	1.00	1.00	0.98	0.96	1.00	1.02	1.00	0.96	0.00
time (sec)	N/A	0.025	0.039	0.035	0.663	0.642	0.076	0.168	0.043	0.000
Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	51	52	52	49	53	52	49	0
N.S.	1	1.00	1.11	1.13	1.13	1.07	1.15	1.13	1.07	0.00
time (sec)	N/A	0.033	0.038	0.042	0.538	0.772	0.145	0.149	0.038	0.000
Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	53	53	51	53	49	52	50	0
N.S.	1	1.00	1.00	1.00	0.96	1.00	0.92	0.98	0.94	0.00
time (sec)	N/A	0.028	0.022	0.043	0.591	0.736	0.153	0.151	0.050	0.001
Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	49	48	53	49	48	48	0
N.S.	1	1.00	1.00	0.98	0.96	1.06	0.98	0.96	0.96	0.00
time (sec)	N/A	0.027	0.016	0.045	0.487	0.826	0.152	0.169	0.046	0.001

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	49	51	52	54	51	69	49	0
N.S.	1	1.00	0.96	1.00	1.02	1.06	1.00	1.35	0.96	0.00
time (sec)	N/A	0.050	0.026	0.052	0.483	0.639	0.273	0.148	0.043	0.001

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	51	50	53	53	53	54	52	0
N.S.	1	1.00	0.96	0.94	1.00	1.00	1.00	1.02	0.98	0.00
time (sec)	N/A	0.031	0.017	0.049	0.724	0.717	0.310	0.149	0.049	0.001

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	46	51	53	53	51	50	0
N.S.	1	1.00	1.00	0.92	1.02	1.06	1.06	1.02	1.00	0.00
time (sec)	N/A	0.028	0.020	0.058	0.574	1.371	0.354	0.150	2.371	0.001

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	51	51	54	55	51	70	52	0
N.S.	1	1.00	1.00	1.00	1.06	1.08	1.00	1.37	1.02	0.00
time (sec)	N/A	0.038	0.022	0.067	0.666	0.747	0.756	0.207	2.359	0.001

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	54	48	54	53	58	56	53	0
N.S.	1	1.00	1.02	0.91	1.02	1.00	1.09	1.06	1.00	0.00
time (sec)	N/A	0.029	0.017	0.045	0.567	0.802	0.874	0.152	0.045	0.001

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	45	51	53	54	53	50	0
N.S.	1	1.00	1.00	0.90	1.02	1.06	1.08	1.06	1.00	0.00
time (sec)	N/A	0.029	0.031	0.046	0.493	0.826	0.986	0.148	2.342	0.001

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	124	119	125	136	125	107	0
N.S.	1	1.00	1.00	1.06	1.02	1.07	1.16	1.07	0.91	0.00
time (sec)	N/A	0.098	0.022	0.038	0.460	0.714	0.096	0.160	0.050	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	107	124	119	125	136	125	107	0
N.S.	1	1.00	1.13	1.31	1.25	1.32	1.43	1.32	1.13	0.00
time (sec)	N/A	0.237	0.029	0.038	0.473	0.734	0.096	0.173	2.338	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	124	119	125	134	125	107	0
N.S.	1	1.00	1.00	1.06	1.02	1.07	1.15	1.07	0.91	0.00
time (sec)	N/A	0.075	0.020	0.042	0.475	0.689	0.096	0.153	0.041	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	124	119	125	136	125	107	0
N.S.	1	1.00	1.00	1.06	1.02	1.07	1.16	1.07	0.91	0.00
time (sec)	N/A	0.068	0.020	0.033	0.588	0.517	0.096	0.152	0.041	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	107	124	119	125	138	125	107	0
N.S.	1	1.00	1.60	1.85	1.78	1.87	2.06	1.87	1.60	0.00
time (sec)	N/A	0.148	0.025	0.040	0.447	1.285	0.100	0.156	0.040	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	124	119	125	136	125	107	0
N.S.	1	1.00	1.00	1.06	1.02	1.07	1.16	1.07	0.91	0.00
time (sec)	N/A	0.070	0.021	0.042	0.644	0.820	0.098	0.227	0.041	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	124	119	124	133	124	107	0
N.S.	1	1.00	1.00	1.06	1.02	1.06	1.14	1.06	0.91	0.00
time (sec)	N/A	0.065	0.018	0.039	0.443	0.754	0.096	0.156	0.040	0.000
Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	107	124	119	125	136	125	107	0
N.S.	1	1.00	2.55	2.95	2.83	2.98	3.24	2.98	2.55	0.00
time (sec)	N/A	0.069	0.030	0.042	0.443	0.716	0.095	0.173	0.041	0.000
Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	124	119	124	134	124	106	0
N.S.	1	1.00	1.00	1.06	1.02	1.06	1.15	1.06	0.91	0.00
time (sec)	N/A	0.063	0.018	0.043	0.646	0.703	0.094	0.153	0.040	0.000
Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	109	121	115	120	128	120	103	0
N.S.	1	1.00	1.00	1.11	1.06	1.10	1.17	1.10	0.94	0.00
time (sec)	N/A	0.055	0.017	0.039	0.599	0.704	0.093	0.169	0.041	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	113	124	120	117	134	124	105	0
N.S.	1	1.00	1.28	1.41	1.36	1.33	1.52	1.41	1.19	0.00
time (sec)	N/A	0.067	0.029	0.042	0.449	0.816	0.242	0.155	0.046	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	112	125	118	121	129	124	106	0
N.S.	1	1.00	1.00	1.12	1.05	1.08	1.15	1.11	0.95	0.00
time (sec)	N/A	0.061	0.035	0.049	0.539	0.762	0.248	0.151	0.042	0.001

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	112	120	116	121	128	119	104	0
N.S.	1	1.00	1.00	1.07	1.04	1.08	1.14	1.06	0.93	0.00
time (sec)	N/A	0.061	0.032	0.040	0.607	1.031	0.244	0.155	0.043	0.001

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	115	123	120	123	133	143	105	0
N.S.	1	1.00	1.02	1.09	1.06	1.09	1.18	1.27	0.93	0.00
time (sec)	N/A	0.117	0.045	0.054	0.553	0.750	0.368	0.158	2.348	0.001

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	115	123	121	121	133	127	109	0
N.S.	1	1.00	1.02	1.09	1.07	1.07	1.18	1.12	0.96	0.00
time (sec)	N/A	0.067	0.063	0.047	0.489	0.732	0.422	0.157	0.042	0.001
Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	113	119	120	121	133	124	108	0
N.S.	1	1.00	1.00	1.05	1.06	1.07	1.18	1.10	0.96	0.00
time (sec)	N/A	0.063	0.057	0.043	0.733	0.527	0.478	0.179	0.043	0.001
Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	106	124	122	123	131	148	113	0
N.S.	1	1.00	0.93	1.09	1.07	1.08	1.15	1.30	0.99	0.00
time (sec)	N/A	0.106	0.057	0.049	0.649	0.774	0.941	0.156	0.049	0.001
Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	110	117	121	121	129	127	113	0
N.S.	1	1.00	1.00	1.06	1.10	1.10	1.17	1.15	1.03	0.00
time (sec)	N/A	0.067	0.038	0.047	0.513	1.109	1.118	0.173	2.341	0.001

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	113	114	120	121	133	124	111	0
N.S.	1	1.00	1.00	1.01	1.06	1.07	1.18	1.10	0.98	0.00
time (sec)	N/A	0.062	0.038	0.044	0.593	0.717	1.211	0.162	0.044	0.001

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	106	124	123	123	129	150	118	0
N.S.	1	1.00	0.93	1.09	1.08	1.08	1.13	1.32	1.04	0.00
time (sec)	N/A	0.104	0.055	0.049	0.469	0.645	2.244	0.154	0.050	0.001

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	118	111	122	121	131	127	118	0
N.S.	1	1.00	1.03	0.97	1.06	1.05	1.14	1.10	1.03	0.00
time (sec)	N/A	0.063	0.024	0.053	0.465	1.052	2.931	0.154	2.364	0.001

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	109	108	120	121	131	124	116	0
N.S.	1	1.00	1.00	0.99	1.10	1.11	1.20	1.14	1.06	0.00
time (sec)	N/A	0.064	0.042	0.054	0.534	0.793	3.727	0.183	0.067	0.001

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	118	124	123	123	129	149	122	0
N.S.	1	1.00	1.04	1.09	1.08	1.08	1.13	1.31	1.07	0.00
time (sec)	N/A	0.099	0.048	0.051	0.634	0.622	6.644	0.157	0.064	0.001
Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	117	107	122	121	134	128	123	0
N.S.	1	1.00	1.02	0.93	1.06	1.05	1.17	1.11	1.07	0.00
time (sec)	N/A	0.062	0.051	0.058	0.558	0.635	10.765	0.159	2.375	0.001
Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	110	102	119	121	129	123	120	0
N.S.	1	1.00	1.00	0.93	1.08	1.10	1.17	1.12	1.09	0.00
time (sec)	N/A	0.068	0.071	0.052	0.614	0.560	22.983	0.163	2.392	0.001
Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	116	123	123	123	129	145	121	0
N.S.	1	1.00	1.03	1.09	1.09	1.09	1.14	1.28	1.07	0.00
time (sec)	N/A	0.091	0.068	0.049	0.586	0.669	29.301	0.155	0.079	0.001

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	118	104	122	121	134	128	121	0
N.S.	1	1.00	1.03	0.90	1.06	1.05	1.17	1.11	1.05	0.00
time (sec)	N/A	0.062	0.033	0.043	0.498	0.559	66.092	0.160	2.363	0.001

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	110	101	119	121	0	125	119	0
N.S.	1	1.00	1.00	0.92	1.08	1.10	0.00	1.14	1.08	0.00
time (sec)	N/A	0.061	0.053	0.048	0.518	0.675	0.000	0.171	0.077	0.001

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	121	124	123	123	0	136	121	0
N.S.	1	1.00	1.33	1.36	1.35	1.35	0.00	1.49	1.33	0.00
time (sec)	N/A	0.054	0.045	0.047	0.508	0.539	0.000	0.158	0.093	0.001

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	119	104	121	121	0	127	119	0
N.S.	1	1.00	1.05	0.92	1.07	1.07	0.00	1.12	1.05	0.00
time (sec)	N/A	0.064	0.032	0.043	0.460	0.495	0.000	0.153	2.369	0.001

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	121	104	121	121	0	127	121	0
N.S.	1	1.00	1.03	0.89	1.03	1.03	0.00	1.09	1.03	0.00
time (sec)	N/A	0.059	0.037	0.048	0.552	0.529	0.000	0.153	2.347	0.001
Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	118	104	121	121	0	127	122	0
N.S.	1	1.00	2.46	2.17	2.52	2.52	0.00	2.65	2.54	0.00
time (sec)	N/A	0.032	0.036	0.046	0.612	0.614	0.000	0.183	2.370	0.001
Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	104	121	121	0	127	121	0
N.S.	1	1.00	1.00	0.89	1.03	1.03	0.00	1.09	1.03	0.00
time (sec)	N/A	0.065	0.050	0.049	0.697	0.511	0.000	0.165	0.064	0.001
Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	183	183	171	249	182	167	114	217	164	0
N.S.	1	1.00	0.93	1.36	0.99	0.91	0.62	1.19	0.90	0.00
time (sec)	N/A	0.149	0.135	0.044	1.431	0.681	1.372	0.237	0.268	0.001

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	47	62	50	51	46	52	52	0
N.S.	1	1.00	0.87	1.15	0.93	0.94	0.85	0.96	0.96	0.00
time (sec)	N/A	0.057	0.022	0.041	0.499	0.560	0.960	0.170	0.079	0.001

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	154	226	157	162	114	207	144	0
N.S.	1	1.00	0.92	1.35	0.94	0.97	0.68	1.24	0.86	0.00
time (sec)	N/A	0.120	0.093	0.045	1.096	0.528	0.872	0.186	2.545	0.001

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	152	221	154	145	87	186	162	0
N.S.	1	1.00	0.94	1.36	0.95	0.90	0.54	1.15	1.00	0.00
time (sec)	N/A	0.116	0.093	0.043	1.084	0.490	1.147	0.170	2.610	0.001

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	31	40	31	30	27	32	31	0
N.S.	1	1.00	0.89	1.14	0.89	0.86	0.77	0.91	0.89	0.00
time (sec)	N/A	0.033	0.029	0.040	0.540	0.580	0.883	0.217	0.063	0.001

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	152	198	131	382	92	161	126	0
N.S.	1	1.00	1.01	1.32	0.87	2.55	0.61	1.07	0.84	0.00
time (sec)	N/A	0.090	0.057	0.041	1.144	0.752	1.106	0.189	2.566	0.001
Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	129	195	128	369	71	133	123	0
N.S.	1	1.00	0.89	1.34	0.88	2.54	0.49	0.92	0.85	0.00
time (sec)	N/A	0.077	0.089	0.045	1.157	0.694	1.016	0.176	2.541	0.000
Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	37	35	32	26	34	36	0
N.S.	1	1.00	1.00	1.09	1.03	0.94	0.76	1.00	1.06	0.00
time (sec)	N/A	0.033	0.014	0.050	0.590	0.481	2.116	0.193	0.105	0.001
Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	134	195	140	372	90	155	126	0
N.S.	1	1.00	0.91	1.33	0.95	2.53	0.61	1.05	0.86	0.00
time (sec)	N/A	0.086	0.094	0.057	1.157	0.680	0.940	0.223	2.545	0.001

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	135	195	140	411	73	161	126	0
N.S.	1	1.00	0.91	1.31	0.94	2.76	0.49	1.08	0.85	0.00
time (sec)	N/A	0.093	0.103	0.048	1.223	0.499	1.232	0.176	0.244	0.001

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	49	56	48	47	41	69	46	0
N.S.	1	1.00	0.98	1.12	0.96	0.94	0.82	1.38	0.92	0.00
time (sec)	N/A	0.049	0.022	0.048	0.483	0.505	2.360	0.165	2.405	0.001

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	154	216	147	158	112	197	178	0
N.S.	1	1.00	0.93	1.31	0.89	0.96	0.68	1.19	1.08	0.00
time (sec)	N/A	0.115	0.141	0.046	1.137	0.631	0.961	0.225	2.591	0.001

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	168	154	217	148	176	99	176	145	0
N.S.	1	1.00	0.92	1.29	0.88	1.05	0.59	1.05	0.86	0.00
time (sec)	N/A	0.119	0.134	0.048	1.281	0.601	0.932	0.176	2.564	0.001

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	70	81	70	73	61	99	70	0
N.S.	1	1.00	1.01	1.17	1.01	1.06	0.88	1.43	1.01	0.00
time (sec)	N/A	0.064	0.033	0.050	0.492	0.526	2.688	0.186	0.127	0.001

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	184	173	247	178	180	139	216	161	0
N.S.	1	1.00	0.94	1.34	0.97	0.98	0.76	1.17	0.88	0.00
time (sec)	N/A	0.134	0.144	0.057	1.289	0.851	1.135	0.183	2.581	0.001

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	233	233	203	288	218	271	156	244	209	0
N.S.	1	1.00	0.87	1.24	0.94	1.16	0.67	1.05	0.90	0.00
time (sec)	N/A	0.138	0.169	0.057	1.233	0.717	2.116	0.201	2.620	0.001

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	72	97	82	121	82	106	86	0
N.S.	1	1.00	0.88	1.18	1.00	1.48	1.00	1.29	1.05	0.00
time (sec)	N/A	0.091	0.101	0.053	0.492	0.571	2.143	0.183	0.083	0.001

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	185	266	192	257	151	236	179	0
N.S.	1	1.00	0.86	1.24	0.89	1.20	0.70	1.10	0.83	0.00
time (sec)	N/A	0.140	0.169	0.048	1.102	0.669	2.120	0.183	0.268	0.001

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	213	181	257	187	240	126	211	193	0
N.S.	1	1.00	0.85	1.21	0.88	1.13	0.59	0.99	0.91	0.00
time (sec)	N/A	0.130	0.150	0.086	1.012	0.847	2.536	0.180	2.621	0.001

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	50	74	60	81	56	91	62	0
N.S.	1	1.00	0.83	1.23	1.00	1.35	0.93	1.52	1.03	0.00
time (sec)	N/A	0.058	0.063	0.052	0.458	0.687	1.608	0.182	0.081	0.001

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	196	165	235	162	578	126	189	158	0
N.S.	1	1.00	0.84	1.20	0.83	2.95	0.64	0.96	0.81	0.00
time (sec)	N/A	0.110	0.140	0.080	1.171	0.789	2.093	0.208	2.578	0.001

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	190	190	160	228	157	573	102	166	150	0
N.S.	1	1.00	0.84	1.20	0.83	3.02	0.54	0.87	0.79	0.00
time (sec)	N/A	0.106	0.170	0.044	1.216	0.554	1.668	0.185	2.577	0.001

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	47	40	44	36	65	37	0
N.S.	1	1.00	1.00	1.15	0.98	1.07	0.88	1.59	0.90	0.00
time (sec)	N/A	0.038	0.032	0.055	0.582	0.564	1.225	0.184	2.348	0.001

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	146	223	160	548	117	186	145	0
N.S.	1	1.00	0.85	1.30	0.94	3.20	0.68	1.09	0.85	0.00
time (sec)	N/A	0.088	0.134	0.049	1.210	0.548	1.482	0.181	0.250	0.001

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	145	221	158	537	97	160	143	0
N.S.	1	1.00	0.86	1.31	0.93	3.18	0.57	0.95	0.85	0.00
time (sec)	N/A	0.086	0.139	0.057	1.102	0.705	1.434	0.177	2.550	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	46	53	51	70	46	61	47	0
N.S.	1	1.00	0.90	1.04	1.00	1.37	0.90	1.20	0.92	0.00
time (sec)	N/A	0.046	0.034	0.054	0.483	0.699	1.162	0.214	0.141	0.001

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	196	164	241	166	570	122	180	156	0
N.S.	1	1.01	0.84	1.24	0.85	2.92	0.63	0.92	0.80	0.00
time (sec)	N/A	0.106	0.144	0.052	1.078	0.697	1.417	0.183	2.569	0.001

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	196	163	237	172	618	109	188	159	0
N.S.	1	1.00	0.83	1.21	0.88	3.15	0.56	0.96	0.81	0.00
time (sec)	N/A	0.103	0.156	0.048	1.232	0.823	1.927	0.174	2.565	0.001

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	64	87	76	118	70	80	78	0
N.S.	1	1.00	0.84	1.14	1.00	1.55	0.92	1.05	1.03	0.00
time (sec)	N/A	0.076	0.072	0.060	0.504	0.490	1.444	0.189	2.432	0.001

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	185	257	186	259	153	231	209	0
N.S.	1	1.00	0.86	1.20	0.87	1.20	0.71	1.07	0.97	0.00
time (sec)	N/A	0.127	0.238	0.057	1.421	0.601	2.324	0.221	2.615	0.001
Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	183	252	186	277	138	206	176	0
N.S.	1	1.00	0.85	1.17	0.87	1.29	0.64	0.96	0.82	0.00
time (sec)	N/A	0.127	0.180	0.053	1.319	0.532	1.843	0.197	2.574	0.001
Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	85	116	106	154	100	149	100	0
N.S.	1	1.00	0.88	1.20	1.09	1.59	1.03	1.54	1.03	0.00
time (sec)	N/A	0.099	0.115	0.075	0.652	0.463	2.397	0.165	0.144	0.001
Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	94	134	115	179	112	131	117	0
N.S.	1	1.00	0.88	1.25	1.07	1.67	1.05	1.22	1.09	0.00
time (sec)	N/A	0.140	0.072	0.054	0.538	0.639	4.941	0.183	0.097	0.001

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	92	110	94	142	94	93	94	0
N.S.	1	1.00	1.05	1.25	1.07	1.61	1.07	1.06	1.07	0.00
time (sec)	N/A	0.088	0.040	0.048	0.524	0.634	3.945	0.180	2.402	0.001

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	64	81	72	89	70	61	70	0
N.S.	1	1.00	0.97	1.23	1.09	1.35	1.06	0.92	1.06	0.00
time (sec)	N/A	0.068	0.025	0.051	0.474	0.749	3.692	0.188	2.385	0.001

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	30	39	42	42	42	28	44	0
N.S.	1	1.00	0.94	1.22	1.31	1.31	1.31	0.88	1.38	0.00
time (sec)	N/A	0.022	0.015	0.053	0.457	0.687	1.815	0.183	2.332	0.001

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	59	68	77	119	75	74	71	0
N.S.	1	1.00	0.87	1.00	1.13	1.75	1.10	1.09	1.04	0.00
time (sec)	N/A	0.058	0.051	0.059	0.454	0.746	1.481	0.203	0.161	0.001

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	87	117	109	197	107	136	107	0
N.S.	1	1.00	0.86	1.16	1.08	1.95	1.06	1.35	1.06	0.00
time (sec)	N/A	0.100	0.072	0.060	0.504	0.616	3.272	0.195	2.458	0.001
Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	108	147	136	229	133	131	130	0
N.S.	1	1.00	0.89	1.20	1.11	1.88	1.09	1.07	1.07	0.00
time (sec)	N/A	0.130	0.110	0.058	0.519	0.644	3.553	0.200	0.153	0.001
Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	216	308	228	364	192	259	213	0
N.S.	1	1.00	0.88	1.25	0.93	1.48	0.78	1.05	0.87	0.00
time (sec)	N/A	0.156	0.323	0.061	1.366	0.649	6.239	0.330	2.582	0.001
Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	244	244	210	299	223	347	163	234	227	0
N.S.	1	1.00	0.86	1.23	0.91	1.42	0.67	0.96	0.93	0.00
time (sec)	N/A	0.165	0.244	0.056	1.237	0.604	4.009	0.238	0.320	0.001

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	222	222	194	275	196	792	162	210	187	0
N.S.	1	1.00	0.87	1.24	0.88	3.57	0.73	0.95	0.84	0.00
time (sec)	N/A	0.140	0.237	0.062	1.322	0.917	5.854	0.211	2.558	0.001

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	220	220	188	268	191	789	141	187	183	0
N.S.	1	1.00	0.85	1.22	0.87	3.59	0.64	0.85	0.83	0.00
time (sec)	N/A	0.126	0.273	0.054	1.361	0.651	3.392	0.196	2.603	0.001

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	181	241	195	756	155	206	175	0
N.S.	1	1.00	0.90	1.20	0.97	3.76	0.77	1.02	0.87	0.00
time (sec)	N/A	0.107	0.290	0.052	1.266	0.541	4.749	0.199	0.267	0.001

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	178	239	193	743	136	187	173	0
N.S.	1	1.00	0.89	1.20	0.97	3.73	0.68	0.94	0.87	0.00
time (sec)	N/A	0.113	0.244	0.049	1.253	0.719	3.253	0.192	2.559	0.001

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	178	251	195	752	153	207	175	0
N.S.	1	1.00	0.89	1.25	0.97	3.74	0.76	1.03	0.87	0.00
time (sec)	N/A	0.115	0.241	0.053	1.399	0.786	2.032	0.215	0.268	0.001
Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	197	197	175	249	192	743	133	180	173	0
N.S.	1	1.00	0.89	1.26	0.97	3.77	0.68	0.91	0.88	0.00
time (sec)	N/A	0.103	0.185	0.056	1.370	0.637	1.513	0.195	0.256	0.000
Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	227	227	193	281	199	776	162	204	185	0
N.S.	1	1.00	0.85	1.24	0.88	3.42	0.71	0.90	0.81	0.00
time (sec)	N/A	0.123	0.280	0.063	1.409	0.694	1.902	0.196	2.598	0.001
Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	227	227	189	277	201	812	143	209	188	0
N.S.	1	1.00	0.83	1.22	0.89	3.58	0.63	0.92	0.83	0.00
time (sec)	N/A	0.131	0.261	0.054	1.198	0.520	1.731	0.191	2.583	0.001

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	214	299	221	366	189	254	240	0
N.S.	1	1.00	0.87	1.22	0.90	1.49	0.77	1.03	0.98	0.00
time (sec)	N/A	0.158	0.274	0.056	1.419	0.920	1.905	0.198	2.637	0.001

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	210	295	221	384	173	229	207	0
N.S.	1	1.00	0.85	1.20	0.90	1.56	0.70	0.93	0.84	0.00
time (sec)	N/A	0.142	0.317	0.062	1.465	0.664	2.176	0.193	2.584	0.001

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	66	65	68	72	0	70	68	0
N.S.	1	1.00	0.94	0.93	0.97	1.03	0.00	1.00	0.97	0.00
time (sec)	N/A	0.066	0.067	0.047	0.585	1.164	0.000	0.184	2.844	0.001

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	301	301	242	269	324	273	0	311	1751	0
N.S.	1	1.00	0.80	0.89	1.08	0.91	0.00	1.03	5.82	0.00
time (sec)	N/A	0.310	0.240	0.050	1.271	1.359	0.000	0.214	11.360	0.001

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	296	296	238	266	349	228	0	308	873	0
N.S.	1	1.00	0.80	0.90	1.18	0.77	0.00	1.04	2.95	0.00
time (sec)	N/A	0.267	0.165	0.052	1.355	0.937	0.000	0.212	1.829	0.001
Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	43	50	49	42	144	51	51	0
N.S.	1	1.00	0.81	0.94	0.92	0.79	2.72	0.96	0.96	0.00
time (sec)	N/A	0.050	0.061	0.051	0.463	0.986	6.748	0.186	0.313	0.001
Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	288	288	224	246	289	244	573	286	1364	0
N.S.	1	1.00	0.78	0.85	1.00	0.85	1.99	0.99	4.74	0.00
time (sec)	N/A	0.154	0.127	0.049	1.272	1.052	123.979	0.246	9.046	0.001
Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	288	288	224	246	317	199	342	278	1265	0
N.S.	1	1.00	0.78	0.85	1.10	0.69	1.19	0.97	4.39	0.00
time (sec)	N/A	0.148	0.124	0.054	1.206	0.797	20.201	0.204	8.117	0.001

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	31	42	41	31	138	51	602	0
N.S.	1	1.00	0.69	0.93	0.91	0.69	3.07	1.13	13.38	0.00
time (sec)	N/A	0.031	0.020	0.046	0.483	0.845	2.474	0.183	0.257	0.001

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	288	288	224	222	265	201	515	290	982	0
N.S.	1	1.00	0.78	0.77	0.92	0.70	1.79	1.01	3.41	0.00
time (sec)	N/A	0.143	0.144	0.047	1.287	0.841	14.506	0.247	5.415	0.001

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	288	288	224	222	293	254	447	278	1364	0
N.S.	1	1.00	0.78	0.77	1.02	0.88	1.55	0.97	4.74	0.00
time (sec)	N/A	0.142	0.182	0.050	1.343	0.982	133.334	0.213	9.007	0.001

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	54	59	61	54	0	71	58	0
N.S.	1	1.00	0.87	0.95	0.98	0.87	0.00	1.15	0.94	0.00
time (sec)	N/A	0.060	0.050	0.049	0.543	1.558	0.000	0.184	2.840	0.001

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	299	299	244	257	300	238	0	305	716	0
N.S.	1	1.00	0.82	0.86	1.00	0.80	0.00	1.02	2.39	0.00
time (sec)	N/A	0.272	0.213	0.059	1.200	0.893	0.000	0.252	3.854	0.001
Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	301	301	259	257	328	301	0	309	1829	0
N.S.	1	1.00	0.86	0.85	1.09	1.00	0.00	1.03	6.08	0.00
time (sec)	N/A	0.255	0.224	0.052	1.136	2.623	0.000	0.206	11.834	0.001
Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	88	87	87	99	0	111	87	0
N.S.	1	1.00	1.01	1.00	1.00	1.14	0.00	1.28	1.00	0.00
time (sec)	N/A	0.092	0.106	0.060	0.485	4.787	0.000	0.193	3.223	0.001
Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	318	318	282	291	341	305	0	328	1734	0
N.S.	1	1.00	0.89	0.92	1.07	0.96	0.00	1.03	5.45	0.00
time (sec)	N/A	0.380	0.265	0.052	1.126	3.409	0.000	0.243	11.370	0.001

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	321	321	282	293	369	356	0	336	1860	0
N.S.	1	1.00	0.88	0.91	1.15	1.11	0.00	1.05	5.79	0.00
time (sec)	N/A	0.456	0.221	0.054	1.231	1.042	0.000	0.206	11.573	0.001
Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	119	124	117	127	0	165	118	0
N.S.	1	1.00	1.00	1.04	0.98	1.07	0.00	1.39	0.99	0.00
time (sec)	N/A	0.129	0.063	0.056	0.520	11.767	0.000	0.261	3.212	0.001
Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	352	352	304	334	376	332	0	377	1814	0
N.S.	1	1.00	0.86	0.95	1.07	0.94	0.00	1.07	5.15	0.00
time (sec)	N/A	0.501	0.245	0.053	1.160	1.204	0.000	0.215	11.909	0.001
Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	137	1078	205	851	5418	1331	559	0
N.S.	1	1.00	0.93	7.28	1.39	5.75	36.61	8.99	3.78	0.00
time (sec)	N/A	0.099	0.264	0.052	0.518	0.915	23.936	0.285	3.213	0.355

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	66	262	91	215	1057	332	177	0
N.S.	1	1.00	0.93	3.69	1.28	3.03	14.89	4.68	2.49	0.00
time (sec)	N/A	0.040	0.058	0.049	0.495	0.892	6.312	0.201	2.718	0.071

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	42	110	53	92	410	143	95	0
N.S.	1	1.00	0.93	2.44	1.18	2.04	9.11	3.18	2.11	0.00
time (sec)	N/A	0.021	0.047	0.043	0.613	0.848	2.891	0.166	2.655	0.030

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	32	27	32	46	29	31	35
N.S.	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79	0.90
time (sec)	N/A	0.015	0.017	0.046	0.448	0.879	20.862	0.168	0.053	0.022

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	32	27	32	46	29	31	41
N.S.	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79	1.05
time (sec)	N/A	0.016	0.016	0.045	0.465	1.318	12.714	0.155	2.560	0.021

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	32	27	32	46	29	31	35
N.S.	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79	0.90
time (sec)	N/A	0.015	0.015	0.046	0.449	0.930	6.742	0.178	0.044	0.018

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	32	27	30	46	29	31	41
N.S.	1	1.00	0.85	0.82	0.69	0.77	1.18	0.74	0.79	1.05
time (sec)	N/A	0.016	0.023	0.045	0.467	0.889	3.370	0.150	0.041	0.020

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	33	32	27	29	44	29	31	41
N.S.	1	1.00	0.89	0.86	0.73	0.78	1.19	0.78	0.84	1.11
time (sec)	N/A	0.015	0.070	0.045	0.505	0.745	2.299	0.153	2.590	0.019

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	35	32	27	29	44	29	31	35
N.S.	1	1.00	0.95	0.86	0.73	0.78	1.19	0.78	0.84	0.95
time (sec)	N/A	0.015	0.011	0.044	0.461	0.783	2.772	0.149	2.599	0.024

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	34	32	27	28	46	29	31	34
N.S.	1	1.00	0.87	0.82	0.69	0.72	1.18	0.74	0.79	0.87
time (sec)	N/A	0.015	0.013	0.042	0.482	0.894	3.736	0.157	0.040	0.023

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	36	32	27	29	42	29	30	35
N.S.	1	1.00	0.97	0.86	0.73	0.78	1.14	0.78	0.81	0.95
time (sec)	N/A	0.016	0.012	0.041	0.453	0.749	4.205	0.261	0.042	0.025

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	56	51	56	80	53	51	59
N.S.	1	1.00	0.84	0.89	0.81	0.89	1.27	0.84	0.81	0.94
time (sec)	N/A	0.031	0.064	0.049	0.558	0.760	47.810	0.153	2.567	0.032

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	56	51	56	80	53	51	69
N.S.	1	1.00	1.00	0.89	0.81	0.89	1.27	0.84	0.81	1.10
time (sec)	N/A	0.031	0.046	0.043	0.544	0.761	29.392	0.175	0.047	0.035

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	56	51	56	80	53	51	69
N.S.	1	1.00	0.84	0.89	0.81	0.89	1.27	0.84	0.81	1.10
time (sec)	N/A	0.030	0.060	0.045	0.499	0.777	21.960	0.163	0.047	0.033

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	56	51	54	80	53	51	69
N.S.	1	1.00	0.84	0.89	0.81	0.86	1.27	0.84	0.81	1.10
time (sec)	N/A	0.029	0.110	0.043	0.537	0.625	5.314	0.155	0.049	0.037

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	53	56	51	53	78	53	51	69
N.S.	1	1.00	0.87	0.92	0.84	0.87	1.28	0.87	0.84	1.13
time (sec)	N/A	0.029	0.067	0.046	0.512	0.823	9.219	0.227	0.045	0.033

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	60	56	51	53	78	53	51	59
N.S.	1	1.00	0.98	0.92	0.84	0.87	1.28	0.87	0.84	0.97
time (sec)	N/A	0.030	0.070	0.049	0.539	0.714	7.796	0.154	0.050	0.042

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	57	56	51	53	80	53	51	59
N.S.	1	1.00	0.90	0.89	0.81	0.84	1.27	0.84	0.81	0.94
time (sec)	N/A	0.030	0.067	0.054	0.510	0.769	11.737	0.160	0.048	0.042

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	57	56	51	53	76	53	51	59
N.S.	1	1.00	0.93	0.92	0.84	0.87	1.25	0.87	0.84	0.97
time (sec)	N/A	0.031	0.018	0.047	0.458	0.884	18.414	0.155	0.048	0.039

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	71	80	73	78	114	77	69	97
N.S.	1	1.00	0.84	0.94	0.86	0.92	1.34	0.91	0.81	1.14
time (sec)	N/A	0.044	0.062	0.043	0.568	0.725	92.986	0.164	2.519	0.041

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	80	73	78	114	77	69	97
N.S.	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81	1.14
time (sec)	N/A	0.041	0.100	0.043	0.504	0.815	58.699	0.154	0.030	0.042

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	80	73	78	114	77	69	97
N.S.	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81	1.14
time (sec)	N/A	0.042	0.083	0.049	0.481	0.829	39.599	0.170	0.032	0.043

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	71	80	73	76	114	77	69	97
N.S.	1	1.00	0.84	0.94	0.86	0.89	1.34	0.91	0.81	1.14
time (sec)	N/A	0.041	0.066	0.046	0.673	0.798	6.341	0.155	0.033	0.047

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	83	80	73	75	112	77	69	97
N.S.	1	1.00	1.00	0.96	0.88	0.90	1.35	0.93	0.83	1.17
time (sec)	N/A	0.043	0.061	0.041	0.557	0.801	23.640	0.161	0.030	0.041

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	81	80	73	75	112	77	69	83
N.S.	1	1.00	0.98	0.96	0.88	0.90	1.35	0.93	0.83	1.00
time (sec)	N/A	0.041	0.044	0.045	0.561	1.000	19.488	0.156	0.033	0.053

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	77	80	73	75	112	77	69	83
N.S.	1	1.00	0.91	0.94	0.86	0.88	1.32	0.91	0.81	0.98
time (sec)	N/A	0.045	0.040	0.043	0.641	0.794	28.368	0.161	0.034	0.044
Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	78	80	73	75	110	77	69	83
N.S.	1	1.00	0.94	0.96	0.88	0.90	1.33	0.93	0.83	1.00
time (sec)	N/A	0.041	0.085	0.041	0.549	0.676	29.278	0.163	0.032	0.047
Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	67	78	58	143	0	64	111	71
N.S.	1	1.00	0.92	1.07	0.79	1.96	0.00	0.88	1.52	0.97
time (sec)	N/A	0.048	0.124	0.047	1.316	0.682	0.000	0.171	2.607	0.066
Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	288	288	54	377	295	2433	881	289	1933	193
N.S.	1	1.00	0.19	1.31	1.02	8.45	3.06	1.00	6.71	0.67
time (sec)	N/A	0.519	0.093	0.178	1.173	1.075	177.800	0.211	2.889	0.210

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	270	45	356	212	3635	857	280	1640	167
N.S.	1	1.00	0.17	1.32	0.79	13.46	3.17	1.04	6.07	0.62
time (sec)	N/A	0.551	0.071	0.164	1.139	1.081	64.299	1.351	2.853	0.231
Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	52	53	39	108	537	39	93	53
N.S.	1	1.00	0.98	1.00	0.74	2.04	10.13	0.74	1.75	1.00
time (sec)	N/A	0.036	0.080	0.054	1.206	0.807	22.390	0.166	2.600	0.046
Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	268	268	43	353	278	2424	833	280	1915	166
N.S.	1	1.00	0.16	1.32	1.04	9.04	3.11	1.04	7.15	0.62
time (sec)	N/A	0.473	0.029	0.151	1.424	0.995	23.158	0.213	2.885	0.255
Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	268	268	46	355	212	3663	836	280	1700	166
N.S.	1	1.00	0.17	1.32	0.79	13.67	3.12	1.04	6.34	0.62
time (sec)	N/A	0.552	0.014	0.157	1.343	1.149	59.226	0.408	2.856	0.236

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	53	53	39	120	527	39	102	53
N.S.	1	1.00	1.00	1.00	0.74	2.26	9.94	0.74	1.92	1.00
time (sec)	N/A	0.037	0.050	0.056	1.400	0.776	156.584	0.166	0.101	0.051

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	270	47	358	278	2424	0	280	2023	167
N.S.	1	1.00	0.17	1.33	1.03	8.98	0.00	1.04	7.49	0.62
time (sec)	N/A	0.485	0.019	0.175	1.424	0.953	0.000	0.215	2.913	0.194

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	77	93	68	222	0	68	116	77
N.S.	1	1.00	0.81	0.98	0.72	2.34	0.00	0.72	1.22	0.81
time (sec)	N/A	0.055	0.179	0.064	1.239	0.721	0.000	0.181	2.650	0.114

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	312	312	76	405	311	2566	0	313	1884	196
N.S.	1	1.00	0.24	1.30	1.00	8.22	0.00	1.00	6.04	0.63
time (sec)	N/A	0.502	0.087	0.166	1.355	0.840	0.000	0.330	2.888	0.653

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	289	62	387	235	3787	0	302	1578	190
N.S.	1	1.00	0.21	1.34	0.81	13.10	0.00	1.04	5.46	0.66
time (sec)	N/A	0.549	0.098	0.164	1.470	1.133	0.000	1.377	2.869	0.572

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	71	74	61	190	0	63	115	71
N.S.	1	1.00	1.00	1.04	0.86	2.68	0.00	0.89	1.62	1.00
time (sec)	N/A	0.042	0.055	0.056	1.399	0.803	0.000	0.284	0.141	0.099

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	289	68	387	301	2555	0	302	1922	190
N.S.	1	1.00	0.24	1.34	1.04	8.84	0.00	1.04	6.65	0.66
time (sec)	N/A	0.468	0.032	0.163	1.203	1.065	0.000	0.311	2.917	0.569

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	318	318	70	401	240	3798	0	307	1757	197
N.S.	1	1.00	0.22	1.26	0.75	11.94	0.00	0.97	5.53	0.62
time (sec)	N/A	0.686	0.137	0.162	1.159	0.966	0.000	0.468	2.912	0.610

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	97	79	93	67	232	0	66	139	79
N.S.	1	1.01	0.82	0.97	0.70	2.42	0.00	0.69	1.45	0.82
time (sec)	N/A	0.057	0.130	0.066	1.187	0.815	0.000	0.161	0.152	0.111
Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	318	318	74	395	312	2584	0	313	2080	200
N.S.	1	1.00	0.23	1.24	0.98	8.13	0.00	0.98	6.54	0.63
time (sec)	N/A	0.504	0.111	0.159	1.193	1.085	0.000	0.219	2.957	0.598
Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	93	96	96	314	0	84	133	92
N.S.	1	1.00	0.89	0.92	0.92	3.02	0.00	0.81	1.28	0.88
time (sec)	N/A	0.057	0.194	0.066	1.430	0.770	0.000	0.206	2.761	0.166
Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	327	327	92	416	341	2714	0	328	1944	213
N.S.	1	1.00	0.28	1.27	1.04	8.30	0.00	1.00	5.94	0.65
time (sec)	N/A	0.493	0.110	0.161	1.396	0.786	0.000	0.232	2.984	0.780

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	327	327	62	411	271	3951	0	328	1672	212
N.S.	1	1.00	0.19	1.26	0.83	12.08	0.00	1.00	5.11	0.65
time (sec)	N/A	0.597	0.065	0.161	1.226	0.815	0.000	0.368	2.893	0.738

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	94	97	96	313	0	84	136	92
N.S.	1	1.00	0.90	0.93	0.92	3.01	0.00	0.81	1.31	0.88
time (sec)	N/A	0.060	0.108	0.067	1.273	0.568	0.000	0.180	2.706	0.163

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	321	321	91	407	336	2674	0	322	1952	210
N.S.	1	1.00	0.28	1.27	1.05	8.33	0.00	1.00	6.08	0.65
time (sec)	N/A	0.522	0.048	0.169	1.215	1.002	0.000	0.235	2.950	0.793

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	351	351	113	441	273	3904	0	329	1786	219
N.S.	1	1.00	0.32	1.26	0.78	11.12	0.00	0.94	5.09	0.62
time (sec)	N/A	0.616	0.133	0.170	1.289	0.971	0.000	0.482	2.911	0.627

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	130	102	133	100	347	0	88	163	102
N.S.	1	1.01	0.79	1.03	0.78	2.69	0.00	0.68	1.26	0.79
time (sec)	N/A	0.072	0.185	0.072	1.346	0.832	0.000	0.182	2.728	0.171
Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	351	351	96	435	346	2690	0	334	2109	222
N.S.	1	1.00	0.27	1.24	0.99	7.66	0.00	0.95	6.01	0.63
time (sec)	N/A	0.541	0.122	0.171	1.342	1.332	0.000	0.241	2.959	0.643
Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	75	77	118	99	219	104	154	80
N.S.	1	1.00	0.73	0.75	1.15	0.96	2.13	1.01	1.50	0.78
time (sec)	N/A	0.084	0.062	0.047	0.618	0.791	4.166	0.159	2.717	0.052
Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	57	53	84	75	168	73	114	56
N.S.	1	1.00	0.78	0.73	1.15	1.03	2.30	1.00	1.56	0.77
time (sec)	N/A	0.060	0.044	0.041	0.507	0.835	1.818	0.175	2.662	0.040

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	34	31	49	50	117	44	44	34
N.S.	1	1.00	0.74	0.67	1.07	1.09	2.54	0.96	0.96	0.74
time (sec)	N/A	0.040	0.031	0.041	0.591	0.710	0.738	0.153	2.598	0.027

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	60	50	67	125	76	61	80	61
N.S.	1	1.00	0.94	0.78	1.05	1.95	1.19	0.95	1.25	0.95
time (sec)	N/A	0.043	0.051	0.046	1.204	0.767	25.832	0.153	2.714	0.049

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	63	72	107	143	134	68	76	65
N.S.	1	1.00	0.75	0.86	1.27	1.70	1.60	0.81	0.90	0.77
time (sec)	N/A	0.063	0.046	0.048	1.264	0.902	43.661	0.168	2.933	0.106

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	93	96	158	172	160	120	93	79
N.S.	1	1.00	1.06	1.09	1.80	1.95	1.82	1.36	1.06	0.90
time (sec)	N/A	0.070	0.075	0.049	1.317	0.758	129.138	0.200	3.119	0.118

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	78	77	118	124	267	104	206	80
N.S.	1	1.00	0.76	0.75	1.15	1.20	2.59	1.01	2.00	0.78
time (sec)	N/A	0.079	0.092	0.051	0.615	1.115	8.440	0.162	2.655	0.059
Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	57	53	84	99	216	73	211	56
N.S.	1	1.00	0.78	0.73	1.15	1.36	2.96	1.00	2.89	0.77
time (sec)	N/A	0.058	0.055	0.047	0.610	0.818	4.755	0.182	2.715	0.044
Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	34	31	49	73	165	44	150	34
N.S.	1	1.00	0.74	0.67	1.07	1.59	3.59	0.96	3.26	0.74
time (sec)	N/A	0.040	0.028	0.047	0.493	1.020	2.520	0.166	3.346	0.031
Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	80	66	80	172	82	80	131	85
N.S.	1	1.00	0.99	0.81	0.99	2.12	1.01	0.99	1.62	1.05
time (sec)	N/A	0.057	0.082	0.049	1.257	0.823	66.499	0.165	2.793	0.071

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	80	101	134	169	223	103	111	85
N.S.	1	1.00	0.73	0.92	1.22	1.54	2.03	0.94	1.01	0.77
time (sec)	N/A	0.084	0.090	0.049	1.438	0.566	58.401	0.202	3.379	0.113

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	59	107	171	191	243	131	110	84
N.S.	1	1.00	0.51	0.93	1.49	1.66	2.11	1.14	0.96	0.73
time (sec)	N/A	0.087	0.036	0.052	1.306	0.894	153.107	0.185	3.472	0.151

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	78	77	118	76	175	101	104	80
N.S.	1	1.00	0.76	0.75	1.15	0.74	1.70	0.98	1.01	0.78
time (sec)	N/A	0.074	0.079	0.043	0.588	0.924	3.441	0.160	2.676	0.052

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	56	53	83	52	124	70	52	56
N.S.	1	1.00	0.77	0.73	1.14	0.71	1.70	0.96	0.71	0.77
time (sec)	N/A	0.055	0.058	0.047	0.450	0.810	1.825	0.160	2.649	0.042

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	33	30	48	29	75	38	29	33
N.S.	1	1.00	0.72	0.65	1.04	0.63	1.63	0.83	0.63	0.72
time (sec)	N/A	0.038	0.035	0.043	0.462	1.114	0.976	0.157	2.604	0.028

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	37	54	105	65	40	57	48
N.S.	1	1.00	1.00	0.77	1.12	2.19	1.35	0.83	1.19	1.00
time (sec)	N/A	0.032	0.047	0.044	1.080	1.009	11.270	0.187	2.715	0.045

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	57	62	109	126	80	62	67	58
N.S.	1	1.00	0.98	1.07	1.88	2.17	1.38	1.07	1.16	1.00
time (sec)	N/A	0.046	0.074	0.053	1.171	1.506	31.833	0.161	2.894	0.066

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	81	102	178	173	163	121	95	80
N.S.	1	1.00	0.90	1.13	1.98	1.92	1.81	1.34	1.06	0.89
time (sec)	N/A	0.070	0.286	0.075	1.199	1.083	71.181	0.194	2.995	0.115

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	77	77	116	88	175	114	152	80
N.S.	1	1.00	0.75	0.75	1.13	0.85	1.70	1.11	1.48	0.78
time (sec)	N/A	0.077	0.053	0.045	0.590	1.036	3.902	0.175	2.771	0.055
Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	55	52	81	63	124	77	60	56
N.S.	1	1.00	0.75	0.71	1.11	0.86	1.70	1.05	0.82	0.77
time (sec)	N/A	0.055	0.044	0.049	0.648	0.979	1.844	0.176	2.680	0.048
Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	33	30	47	41	75	38	33	33
N.S.	1	1.00	0.72	0.65	1.02	0.89	1.63	0.83	0.72	0.72
time (sec)	N/A	0.037	0.027	0.046	0.556	0.552	1.082	0.177	2.615	0.035
Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	58	57	70	170	56	53	65	58
N.S.	1	1.00	1.00	0.98	1.21	2.93	0.97	0.91	1.12	1.00
time (sec)	N/A	0.040	0.066	0.047	1.352	1.065	20.823	0.162	2.769	0.073

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	57	100	144	233	264	99	131	77
N.S.	1	1.00	0.66	1.16	1.67	2.71	3.07	1.15	1.52	0.90
time (sec)	N/A	0.065	0.020	0.061	1.388	0.961	79.790	0.168	2.929	0.129

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	120	60	141	215	289	192	137	167	102
N.S.	1	1.02	0.51	1.19	1.82	2.45	1.63	1.16	1.42	0.86
time (sec)	N/A	0.090	0.023	0.048	1.343	0.928	179.010	0.178	3.180	0.163

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	73	76	116	98	338	104	145	80
N.S.	1	1.00	0.71	0.74	1.13	0.95	3.28	1.01	1.41	0.78
time (sec)	N/A	0.078	0.067	0.048	0.592	0.842	5.183	0.166	2.797	0.059

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	54	53	84	75	240	63	60	56
N.S.	1	1.00	0.74	0.73	1.15	1.03	3.29	0.86	0.82	0.77
time (sec)	N/A	0.056	0.047	0.050	0.488	0.714	2.424	0.199	2.761	0.049

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	33	30	49	52	144	32	33	33
N.S.	1	1.00	0.72	0.65	1.07	1.13	3.13	0.70	0.72	0.72
time (sec)	N/A	0.036	0.029	0.046	0.499	0.732	1.335	0.162	2.678	0.038

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	62	85	81	243	76	67	80	70
N.S.	1	1.00	0.81	1.10	1.05	3.16	0.99	0.87	1.04	0.91
time (sec)	N/A	0.050	0.047	0.086	1.169	0.559	38.427	0.168	2.782	0.087

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	57	157	170	351	0	101	198	99
N.S.	1	1.00	0.50	1.39	1.50	3.11	0.00	0.89	1.75	0.88
time (sec)	N/A	0.087	0.029	0.091	1.254	0.747	0.000	0.186	2.971	0.135

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	77	506	69	156	85	82	109	78
N.S.	1	1.00	0.79	5.22	0.71	1.61	0.88	0.85	1.12	0.80
time (sec)	N/A	0.094	0.087	4.458	1.153	1.155	38.268	0.169	4.539	0.069

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	65	446	53	129	68	64	88	67
N.S.	1	1.00	0.86	5.87	0.70	1.70	0.89	0.84	1.16	0.88
time (sec)	N/A	0.062	0.041	0.170	1.299	0.930	16.958	0.192	4.278	0.054
Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	54	425	42	110	51	44	71	57
N.S.	1	1.00	0.95	7.46	0.74	1.93	0.89	0.77	1.25	1.00
time (sec)	N/A	0.046	0.016	0.152	1.208	0.777	6.097	0.159	3.866	0.047
Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	59	468	0	147	66	50	93	65
N.S.	1	1.00	0.91	7.20	0.00	2.26	1.02	0.77	1.43	1.00
time (sec)	N/A	0.056	0.018	0.187	0.000	0.815	13.281	0.165	4.658	0.052
Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	88	511	0	194	0	72	113	88
N.S.	1	1.00	1.00	5.81	0.00	2.20	0.00	0.82	1.28	1.00
time (sec)	N/A	0.077	0.043	0.191	0.000	0.950	0.000	0.172	4.858	0.109

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	65	467	53	129	0	64	88	69
N.S.	1	1.00	0.83	5.99	0.68	1.65	0.00	0.82	1.13	0.88
time (sec)	N/A	0.074	0.070	0.245	1.345	0.814	0.000	0.158	5.383	0.056
Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	56	425	43	112	65	49	71	59
N.S.	1	1.00	0.95	7.20	0.73	1.90	1.10	0.83	1.20	1.00
time (sec)	N/A	0.048	0.022	0.180	1.455	0.846	15.581	0.156	4.859	0.049
Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	413	29	87	37	29	56	40
N.S.	1	1.00	1.00	10.32	0.72	2.18	0.92	0.72	1.40	1.00
time (sec)	N/A	0.036	0.010	0.175	1.244	0.722	10.089	0.183	5.210	0.040
Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	59	433	0	148	63	53	94	65
N.S.	1	1.00	0.91	6.66	0.00	2.28	0.97	0.82	1.45	1.00
time (sec)	N/A	0.055	0.021	0.192	0.000	0.838	12.247	0.167	5.507	0.053

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	88	477	0	194	0	72	112	88
N.S.	1	1.00	1.00	5.42	0.00	2.20	0.00	0.82	1.27	1.00
time (sec)	N/A	0.077	0.035	0.227	0.000	0.648	0.000	0.158	5.719	0.087
Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	206	206	67	416	0	2274	0	0	453	0
N.S.	1	1.00	0.33	2.02	0.00	11.04	0.00	0.00	2.20	0.00
time (sec)	N/A	0.032	0.026	0.178	0.000	2.366	0.000	0.000	25.804	15.359
Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	127	127	28	164	0	1191	0	0	653	0
N.S.	1	1.00	0.22	1.29	0.00	9.38	0.00	0.00	5.14	0.00
time (sec)	N/A	0.019	0.029	0.796	0.000	1.260	0.000	0.000	3.419	20.881
Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	81	582	96	169	99	100	118	82
N.S.	1	1.00	0.73	5.24	0.86	1.52	0.89	0.90	1.06	0.74
time (sec)	N/A	0.097	0.111	0.354	1.306	0.904	60.266	0.163	3.510	0.074

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	70	507	82	147	82	83	98	71
N.S.	1	1.00	0.78	5.63	0.91	1.63	0.91	0.92	1.09	0.79
time (sec)	N/A	0.085	0.064	0.161	1.209	0.752	30.233	0.171	3.404	0.063

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	58	446	66	121	65	65	78	59
N.S.	1	1.00	0.84	6.46	0.96	1.75	0.94	0.94	1.13	0.86
time (sec)	N/A	0.057	0.040	0.180	1.304	0.995	14.908	0.211	3.508	0.056

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	47	425	56	101	46	43	59	50
N.S.	1	1.00	0.94	8.50	1.12	2.02	0.92	0.86	1.18	1.00
time (sec)	N/A	0.043	0.027	0.152	1.154	0.816	5.109	0.160	3.496	0.043

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	53	468	0	138	60	48	125	58
N.S.	1	1.00	0.91	8.07	0.00	2.38	1.03	0.83	2.16	1.00
time (sec)	N/A	0.054	0.018	0.158	0.000	0.909	8.219	0.164	4.687	0.043

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	81	511	0	186	0	73	69	81
N.S.	1	1.00	1.00	6.31	0.00	2.30	0.00	0.90	0.85	1.00
time (sec)	N/A	0.072	0.029	0.176	0.000	0.760	0.000	0.170	3.748	0.081
Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	96	574	0	188	0	100	83	95
N.S.	1	1.00	0.90	5.36	0.00	1.76	0.00	0.93	0.78	0.89
time (sec)	N/A	0.094	0.060	0.186	0.000	0.795	0.000	0.176	3.910	0.136
Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	93	634	110	191	0	117	135	93
N.S.	1	1.00	0.72	4.88	0.85	1.47	0.00	0.90	1.04	0.72
time (sec)	N/A	0.113	0.133	0.252	1.285	0.553	0.000	0.165	3.525	0.075
Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	81	541	96	169	110	100	115	82
N.S.	1	1.00	0.74	4.96	0.88	1.55	1.01	0.92	1.06	0.75
time (sec)	N/A	0.102	0.087	0.157	1.335	0.962	108.786	0.163	3.495	0.076

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	70	462	82	147	90	83	95	71
N.S.	1	1.00	0.80	5.25	0.93	1.67	1.02	0.94	1.08	0.81
time (sec)	N/A	0.072	0.039	0.176	1.440	0.627	62.737	0.184	3.521	0.066
Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	58	441	68	121	65	65	75	59
N.S.	1	1.00	0.87	6.58	1.01	1.81	0.97	0.97	1.12	0.88
time (sec)	N/A	0.057	0.038	0.158	1.388	0.577	29.253	0.163	3.449	0.057
Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	500	0	152	73	61	89	73
N.S.	1	1.00	1.00	6.85	0.00	2.08	1.00	0.84	1.22	1.00
time (sec)	N/A	0.070	0.048	0.189	0.000	0.849	24.743	0.177	5.893	0.052
Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	78	556	0	186	0	64	56	78
N.S.	1	1.00	1.00	7.13	0.00	2.38	0.00	0.82	0.72	1.00
time (sec)	N/A	0.072	0.046	0.262	0.000	0.864	0.000	0.166	3.524	0.084

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	96	617	0	218	0	101	87	95
N.S.	1	1.00	0.92	5.93	0.00	2.10	0.00	0.97	0.84	0.91
time (sec)	N/A	0.095	0.106	0.235	0.000	0.874	0.000	0.190	3.737	0.108
Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	69	528	82	146	0	82	98	72
N.S.	1	1.00	0.77	5.87	0.91	1.62	0.00	0.91	1.09	0.80
time (sec)	N/A	0.081	0.106	0.281	1.264	0.918	0.000	0.158	3.221	0.065
Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	58	468	66	121	0	65	78	61
N.S.	1	1.00	0.82	6.59	0.93	1.70	0.00	0.92	1.10	0.86
time (sec)	N/A	0.069	0.069	0.158	1.312	1.098	0.000	0.187	3.393	0.059
Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	49	425	56	103	61	48	60	52
N.S.	1	1.00	0.94	8.17	1.08	1.98	1.17	0.92	1.15	1.00
time (sec)	N/A	0.044	0.020	0.168	1.400	0.646	16.347	0.155	3.271	0.046

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	413	42	78	32	27	45	33
N.S.	1	1.00	1.00	12.52	1.27	2.36	0.97	0.82	1.36	1.00
time (sec)	N/A	0.033	0.011	0.195	1.213	0.836	11.202	0.160	3.233	0.034
Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	51	433	0	139	58	54	47	58
N.S.	1	1.00	0.88	7.47	0.00	2.40	1.00	0.93	0.81	1.00
time (sec)	N/A	0.052	0.029	0.174	0.000	0.887	12.219	0.171	3.283	0.046
Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	81	477	0	184	0	73	73	81
N.S.	1	1.00	1.00	5.89	0.00	2.27	0.00	0.90	0.90	1.00
time (sec)	N/A	0.073	0.052	0.186	0.000	0.801	0.000	0.164	3.417	0.080
Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	95	540	0	217	0	101	94	95
N.S.	1	1.00	0.89	5.05	0.00	2.03	0.00	0.94	0.88	0.89
time (sec)	N/A	0.098	0.071	0.207	0.000	0.808	0.000	0.180	3.501	0.139

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	141	141	67	416	0	2459	0	0	272	0
N.S.	1	1.00	0.48	2.95	0.00	17.44	0.00	0.00	1.93	0.00
time (sec)	N/A	0.413	0.061	0.165	0.000	2.471	0.000	0.000	40.219	15.449

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	66	560	82	189	0	82	95	73
N.S.	1	1.00	0.73	6.22	0.91	2.10	0.00	0.91	1.06	0.81
time (sec)	N/A	0.104	0.057	0.297	1.251	0.725	0.000	0.197	3.782	0.077

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	53	501	68	161	0	58	75	62
N.S.	1	1.00	0.75	7.06	0.96	2.27	0.00	0.82	1.06	0.87
time (sec)	N/A	0.073	0.055	0.170	1.122	0.624	0.000	0.168	3.712	0.065

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	49	456	56	149	58	47	60	52
N.S.	1	1.00	0.94	8.77	1.08	2.87	1.12	0.90	1.15	1.00
time (sec)	N/A	0.046	0.035	0.184	1.209	0.508	27.571	0.165	3.677	0.053

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	43	435	58	147	51	48	63	55
N.S.	1	1.00	0.78	7.91	1.05	2.67	0.93	0.87	1.15	1.00
time (sec)	N/A	0.049	0.012	0.214	1.228	0.725	25.526	0.180	3.630	0.055

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	63	485	0	213	78	68	68	76
N.S.	1	1.00	0.83	6.38	0.00	2.80	1.03	0.89	0.89	1.00
time (sec)	N/A	0.070	0.028	0.189	0.000	0.657	17.016	0.162	3.662	0.064

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	77	549	0	272	0	100	88	91
N.S.	1	1.00	0.77	5.49	0.00	2.72	0.00	1.00	0.88	0.91
time (sec)	N/A	0.095	0.036	0.188	0.000	0.658	0.000	0.179	3.801	0.101

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	91	636	0	303	0	118	112	106
N.S.	1	1.00	0.71	4.97	0.00	2.37	0.00	0.92	0.88	0.83
time (sec)	N/A	0.124	0.079	0.217	0.000	0.791	0.000	0.165	4.025	0.140

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-2)	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	310	310	83	538	0	0	0	0	-1	0
N.S.	1	1.00	0.27	1.74	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.051	0.149	0.425	0.000	0.000	0.000	0.000	0.000	31.157

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-2)	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	316	316	83	509	0	0	0	0	-1	0
N.S.	1	1.00	0.26	1.61	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.054	0.099	0.372	0.000	0.000	0.000	0.000	0.000	31.171

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	320	320	84	510	0	5060	0	0	-1	0
N.S.	1	1.00	0.26	1.59	0.00	15.81	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.053	0.106	0.364	0.000	22.764	0.000	0.000	0.000	31.228

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	322	322	86	541	0	5060	0	0	-1	0
N.S.	1	1.00	0.27	1.68	0.00	15.71	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.054	0.132	0.392	0.000	21.049	0.000	0.000	0.000	31.154

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	121	514	0	280	128	139	176	139
N.S.	1	1.00	0.97	4.11	0.00	2.24	1.02	1.11	1.41	1.11
time (sec)	N/A	0.129	0.307	0.425	0.000	0.700	30.272	0.174	6.171	0.161

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	88	458	0	195	95	96	136	100
N.S.	1	1.00	0.95	4.92	0.00	2.10	1.02	1.03	1.46	1.08
time (sec)	N/A	0.078	0.093	0.244	0.000	0.552	14.674	0.162	6.058	0.119

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	434	0	156	68	66	82	80
N.S.	1	1.00	1.00	6.20	0.00	2.23	0.97	0.94	1.17	1.14
time (sec)	N/A	0.059	0.037	0.229	0.000	0.640	6.349	0.157	6.156	0.072

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	81	476	0	383	85	79	114	95
N.S.	1	1.00	0.95	5.60	0.00	4.51	1.00	0.93	1.34	1.12
time (sec)	N/A	0.074	0.052	0.262	0.000	0.539	12.248	0.172	7.938	0.089

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	107	518	0	513	0	107	137	125
N.S.	1	1.00	0.93	4.50	0.00	4.46	0.00	0.93	1.19	1.09
time (sec)	N/A	0.125	0.178	0.249	0.000	0.518	0.000	0.192	5.130	0.264

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	145	605	0	410	153	193	330	195
N.S.	1	1.00	0.94	3.93	0.00	2.66	0.99	1.25	2.14	1.27
time (sec)	N/A	0.155	0.236	0.359	0.000	0.918	129.923	0.174	6.122	0.287

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	111	531	0	297	116	151	215	138
N.S.	1	1.00	0.92	4.42	0.00	2.48	0.97	1.26	1.79	1.15
time (sec)	N/A	0.104	0.121	0.324	0.000	0.777	70.599	0.189	6.134	0.175

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	85	507	0	204	90	113	143	95
N.S.	1	1.00	0.89	5.28	0.00	2.12	0.94	1.18	1.49	0.99
time (sec)	N/A	0.081	0.086	0.246	0.000	0.690	35.293	0.170	5.908	0.168

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	105	565	0	486	102	112	155	114
N.S.	1	1.00	1.01	5.43	0.00	4.67	0.98	1.08	1.49	1.10
time (sec)	N/A	0.112	0.097	0.261	0.000	0.999	39.290	0.177	7.879	0.232
Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	108	620	0	538	0	121	167	130
N.S.	1	1.00	0.93	5.34	0.00	4.64	0.00	1.04	1.44	1.12
time (sec)	N/A	0.145	0.133	0.280	0.000	0.875	0.000	0.174	9.517	0.250
Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	91	488	0	289	0	106	121	102
N.S.	1	1.00	0.88	4.69	0.00	2.78	0.00	1.02	1.16	0.98
time (sec)	N/A	0.105	0.216	0.339	0.000	0.868	0.000	0.172	5.426	0.185
Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	448	0	205	0	64	86	84
N.S.	1	1.00	1.00	6.05	0.00	2.77	0.00	0.86	1.16	1.14
time (sec)	N/A	0.070	0.116	0.227	0.000	0.876	0.000	0.172	5.098	0.084

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	51	426	0	130	39	40	70	61
N.S.	1	1.00	1.00	8.35	0.00	2.55	0.76	0.78	1.37	1.20
time (sec)	N/A	0.050	0.017	0.275	0.000	0.851	10.261	0.157	5.888	0.053

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	81	453	0	431	70	71	114	95
N.S.	1	1.00	0.95	5.33	0.00	5.07	0.82	0.84	1.34	1.12
time (sec)	N/A	0.076	0.100	0.245	0.000	0.457	19.391	0.166	7.313	0.118

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	151	498	0	565	0	104	142	127
N.S.	1	1.00	1.29	4.26	0.00	4.83	0.00	0.89	1.21	1.09
time (sec)	N/A	0.118	0.131	0.253	0.000	0.485	0.000	0.192	8.424	0.230

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	100	527	0	440	0	103	115	124
N.S.	1	1.00	0.93	4.93	0.00	4.11	0.00	0.96	1.07	1.16
time (sec)	N/A	0.121	0.080	0.341	0.000	0.802	0.000	0.183	6.457	0.209

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	88	487	0	326	0	78	94	92
N.S.	1	1.00	1.07	5.94	0.00	3.98	0.00	0.95	1.15	1.12
time (sec)	N/A	0.074	0.121	0.271	0.000	0.812	0.000	0.191	5.990	0.150
Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	52	463	0	236	66	73	89	109
N.S.	1	1.00	0.68	6.01	0.00	3.06	0.86	0.95	1.16	1.42
time (sec)	N/A	0.066	0.026	0.229	0.000	0.699	32.782	0.189	5.848	0.095
Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	89	512	0	790	104	111	139	124
N.S.	1	1.00	0.78	4.49	0.00	6.93	0.91	0.97	1.22	1.09
time (sec)	N/A	0.113	0.048	0.286	0.000	0.575	24.626	0.166	8.444	0.290
Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	117	575	0	1120	0	173	597	165
N.S.	1	1.00	0.74	3.64	0.00	7.09	0.00	1.09	3.78	1.04
time (sec)	N/A	0.222	0.058	0.408	0.000	0.865	0.000	0.174	10.472	0.394

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	101	952	107	219	0	110	127	95
N.S.	1	1.00	0.86	8.14	0.91	1.87	0.00	0.94	1.09	0.81
time (sec)	N/A	0.094	0.106	0.276	1.238	0.640	0.000	0.164	4.087	0.122

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	90	892	91	191	0	93	107	84
N.S.	1	1.00	0.88	8.75	0.89	1.87	0.00	0.91	1.05	0.82
time (sec)	N/A	0.076	0.086	0.207	1.220	0.680	0.000	0.184	4.013	0.079

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	79	874	79	165	0	69	87	73
N.S.	1	1.00	0.96	10.66	0.96	2.01	0.00	0.84	1.06	0.89
time (sec)	N/A	0.059	0.042	0.166	1.255	0.465	0.000	0.159	3.985	0.073

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	61	439	66	149	0	53	72	63
N.S.	1	1.00	0.95	6.86	1.03	2.33	0.00	0.83	1.12	0.98
time (sec)	N/A	0.049	0.062	0.179	1.392	0.487	0.000	0.159	3.926	0.061

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	102	912	0	226	0	79	76	88
N.S.	1	1.00	1.16	10.36	0.00	2.57	0.00	0.90	0.86	1.00
time (sec)	N/A	0.072	0.049	0.179	0.000	0.534	0.000	0.160	3.981	0.080

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	97	957	0	278	0	113	117	102
N.S.	1	1.00	0.78	7.72	0.00	2.24	0.00	0.91	0.94	0.82
time (sec)	N/A	0.102	0.151	0.211	0.000	0.522	0.000	0.172	4.212	0.116

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	112	1020	0	310	0	105	154	118
N.S.	1	1.00	0.68	6.22	0.00	1.89	0.00	0.64	0.94	0.72
time (sec)	N/A	0.131	0.211	0.188	0.000	0.473	0.000	0.181	4.453	0.170

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	111	998	119	239	0	127	147	104
N.S.	1	1.00	0.83	7.45	0.89	1.78	0.00	0.95	1.10	0.78
time (sec)	N/A	0.110	0.082	0.275	1.183	0.442	0.000	0.173	4.104	0.101

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	102	920	107	219	0	111	127	93
N.S.	1	1.00	0.86	7.73	0.90	1.84	0.00	0.93	1.07	0.78
time (sec)	N/A	0.094	0.126	0.189	1.391	0.444	0.000	0.197	4.052	0.089
Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	90	902	93	192	0	93	107	82
N.S.	1	1.00	0.93	9.30	0.96	1.98	0.00	0.96	1.10	0.85
time (sec)	N/A	0.075	0.061	0.166	1.333	0.445	0.000	0.169	4.044	0.081
Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	43	451	79	162	0	69	87	71
N.S.	1	1.00	0.56	5.86	1.03	2.10	0.00	0.90	1.13	0.92
time (sec)	N/A	0.060	0.015	0.177	1.311	0.439	0.000	0.169	3.987	0.068
Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	100	956	0	220	0	70	101	85
N.S.	1	1.00	1.18	11.25	0.00	2.59	0.00	0.82	1.19	1.00
time (sec)	N/A	0.079	0.055	0.174	0.000	0.458	0.000	0.169	4.722	0.077

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	97	1014	0	280	0	114	110	103
N.S.	1	1.00	0.80	8.38	0.00	2.31	0.00	0.94	0.91	0.85
time (sec)	N/A	0.103	0.156	0.189	0.000	0.454	0.000	0.188	4.235	0.119
Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	112	1075	0	310	0	129	151	118
N.S.	1	1.00	0.70	6.68	0.00	1.93	0.00	0.80	0.94	0.73
time (sec)	N/A	0.133	0.240	0.189	0.000	0.474	0.000	0.178	4.618	0.144
Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	91	916	93	195	0	93	107	84
N.S.	1	1.00	0.96	9.64	0.98	2.05	0.00	0.98	1.13	0.88
time (sec)	N/A	0.073	0.048	0.296	1.344	0.440	0.000	0.194	4.061	0.089
Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	82	874	79	167	0	69	87	73
N.S.	1	1.00	0.99	10.53	0.95	2.01	0.00	0.83	1.05	0.88
time (sec)	N/A	0.064	0.053	0.199	1.561	0.431	0.000	0.174	3.997	0.074

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	63	861	67	155	0	58	72	63
N.S.	1	1.00	0.98	13.45	1.05	2.42	0.00	0.91	1.12	0.98
time (sec)	N/A	0.051	0.052	0.174	1.322	0.470	0.000	0.162	4.009	0.063
Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	64	442	72	153	0	59	75	67
N.S.	1	1.00	0.96	6.60	1.07	2.28	0.00	0.88	1.12	1.00
time (sec)	N/A	0.049	0.037	0.190	1.389	0.438	0.000	0.157	3.977	0.061
Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	83	880	0	226	0	79	80	88
N.S.	1	1.00	0.94	10.00	0.00	2.57	0.00	0.90	0.91	1.00
time (sec)	N/A	0.075	0.096	0.186	0.000	0.468	0.000	0.190	4.008	0.087
Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	97	926	0	280	0	114	117	103
N.S.	1	1.00	0.78	7.47	0.00	2.26	0.00	0.92	0.94	0.83
time (sec)	N/A	0.101	0.149	0.197	0.000	0.450	0.000	0.163	4.111	0.128

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	112	989	0	310	0	128	155	118
N.S.	1	1.00	0.68	6.03	0.00	1.89	0.00	0.78	0.95	0.72
time (sec)	N/A	0.129	0.208	0.197	0.000	0.650	0.000	0.181	4.368	0.151
Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	88	970	98	233	0	88	111	127
N.S.	1	1.00	0.93	10.21	1.03	2.45	0.00	0.93	1.17	1.34
time (sec)	N/A	0.076	0.046	0.288	1.212	0.758	0.000	0.189	4.383	0.089
Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	71	926	81	223	0	67	94	115
N.S.	1	1.00	0.86	11.16	0.98	2.69	0.00	0.81	1.13	1.39
time (sec)	N/A	0.065	0.105	0.178	1.412	0.712	0.000	0.175	4.300	0.080
Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	68	908	83	223	0	76	96	117
N.S.	1	1.00	0.80	10.68	0.98	2.62	0.00	0.89	1.13	1.38
time (sec)	N/A	0.062	0.025	0.183	1.234	0.712	0.000	0.189	4.264	0.082

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	43	463	85	219	0	72	97	76
N.S.	1	1.00	0.49	5.26	0.97	2.49	0.00	0.82	1.10	0.86
time (sec)	N/A	0.064	0.013	0.190	1.270	0.595	0.000	0.161	4.261	0.095
Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	97	953	0	316	0	93	101	163
N.S.	1	1.00	0.92	8.99	0.00	2.98	0.00	0.88	0.95	1.54
time (sec)	N/A	0.095	0.045	0.171	0.000	0.664	0.000	0.172	4.332	0.100
Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	117	1019	0	368	0	129	133	114
N.S.	1	1.00	0.82	7.13	0.00	2.57	0.00	0.90	0.93	0.80
time (sec)	N/A	0.125	0.064	0.195	0.000	0.779	0.000	0.165	4.562	0.148
Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	135	1106	0	398	0	149	171	208
N.S.	1	1.00	0.73	5.98	0.00	2.15	0.00	0.81	0.92	1.12
time (sec)	N/A	0.163	0.064	0.210	0.000	0.896	0.000	0.181	4.763	0.194

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	147	917	0	469	0	136	202	142
N.S.	1	1.00	0.91	5.70	0.00	2.91	0.00	0.84	1.25	0.88
time (sec)	N/A	0.192	0.291	0.359	0.000	0.917	0.000	0.179	6.842	0.271
Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	117	897	0	334	0	102	152	108
N.S.	1	1.00	0.86	6.60	0.00	2.46	0.00	0.75	1.12	0.79
time (sec)	N/A	0.109	0.154	0.263	0.000	0.626	0.000	0.172	6.087	0.199
Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	80	453	0	255	0	79	125	90
N.S.	1	1.00	1.00	5.66	0.00	3.19	0.00	0.99	1.56	1.12
time (sec)	N/A	0.065	0.114	0.270	0.000	0.809	0.000	0.202	5.686	0.176
Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	112	934	0	856	0	114	182	131
N.S.	1	1.00	0.93	7.72	0.00	7.07	0.00	0.94	1.50	1.08
time (sec)	N/A	0.118	0.294	0.285	0.000	0.731	0.000	0.171	8.281	0.511

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	190	978	0	870	0	183	438	157
N.S.	1	1.00	1.18	6.07	0.00	5.40	0.00	1.14	2.72	0.98
time (sec)	N/A	0.218	0.283	0.262	0.000	1.106	0.000	0.215	9.692	0.537
Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	162	1003	0	443	0	211	331	211
N.S.	1	1.00	0.86	5.31	0.00	2.34	0.00	1.12	1.75	1.12
time (sec)	N/A	0.237	0.185	0.367	0.000	0.688	0.000	0.177	7.752	0.292
Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	125	983	0	314	0	173	229	150
N.S.	1	1.00	0.77	6.03	0.00	1.93	0.00	1.06	1.40	0.92
time (sec)	N/A	0.137	0.136	0.269	0.000	1.104	0.000	0.187	7.379	0.291
Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	54	466	0	234	0	122	170	103
N.S.	1	1.00	0.57	4.96	0.00	2.49	0.00	1.30	1.81	1.10
time (sec)	N/A	0.081	0.024	0.266	0.000	0.904	0.000	0.169	7.354	0.193

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	122	1036	0	686	0	155	214	156
N.S.	1	1.00	0.93	7.91	0.00	5.24	0.00	1.18	1.63	1.19
time (sec)	N/A	0.140	0.186	0.267	0.000	1.070	0.000	0.174	9.143	0.308
Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	142	1093	0	838	0	216	531	179
N.S.	1	1.00	0.84	6.43	0.00	4.93	0.00	1.27	3.12	1.05
time (sec)	N/A	0.258	0.333	0.248	0.000	1.182	0.000	0.217	10.816	0.678
Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	107	911	0	475	0	134	160	143
N.S.	1	1.00	0.87	7.41	0.00	3.86	0.00	1.09	1.30	1.16
time (sec)	N/A	0.144	0.276	0.424	0.000	1.143	0.000	0.226	7.290	0.315
Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	98	892	0	348	0	116	111	109
N.S.	1	1.00	0.99	9.01	0.00	3.52	0.00	1.17	1.12	1.10
time (sec)	N/A	0.084	0.097	0.272	0.000	1.109	0.000	0.166	6.846	0.161

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	85	457	0	302	0	93	104	97
N.S.	1	1.00	0.98	5.25	0.00	3.47	0.00	1.07	1.20	1.11
time (sec)	N/A	0.073	0.104	0.256	0.000	0.894	0.000	0.164	6.373	0.114
Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	123	915	0	862	0	139	162	146
N.S.	1	1.00	0.93	6.93	0.00	6.53	0.00	1.05	1.23	1.11
time (sec)	N/A	0.139	0.265	0.276	0.000	0.927	0.000	0.163	9.647	0.307
Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	163	961	0	1236	0	257	355	187
N.S.	1	1.00	0.88	5.19	0.00	6.68	0.00	1.39	1.92	1.01
time (sec)	N/A	0.243	0.611	0.287	0.000	0.905	0.000	0.172	11.551	0.642
Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	149	134	978	0	746	0	195	367	151
N.S.	1	0.99	0.89	6.52	0.00	4.97	0.00	1.30	2.45	1.01
time (sec)	N/A	0.195	0.298	0.364	0.000	0.922	0.000	0.185	7.898	0.249

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	91	958	0	630	0	181	247	123
N.S.	1	1.00	0.68	7.15	0.00	4.70	0.00	1.35	1.84	0.92
time (sec)	N/A	0.114	0.033	0.274	0.000	1.420	0.000	0.196	7.704	0.261
Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	54	485	0	450	0	153	199	110
N.S.	1	1.00	0.50	4.49	0.00	4.17	0.00	1.42	1.84	1.02
time (sec)	N/A	0.091	0.018	0.249	0.000	1.465	0.000	0.207	7.429	0.236
Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	123	1002	0	1819	0	226	288	183
N.S.	1	1.00	0.72	5.83	0.00	10.58	0.00	1.31	1.67	1.06
time (sec)	N/A	0.223	0.127	0.273	0.000	1.662	0.000	0.185	12.182	0.724
Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	189	1067	0	2384	0	367	18847	272
N.S.	1	1.00	0.78	4.43	0.00	9.89	0.00	1.52	78.20	1.13
time (sec)	N/A	0.357	0.156	0.246	0.000	2.004	0.000	0.222	19.626	0.949

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	123	0	0	256	0	104	283	122
N.S.	1	1.00	1.40	0.00	0.00	2.91	0.00	1.18	3.22	1.39
time (sec)	N/A	0.093	0.266	0.177	0.000	0.577	0.000	0.211	9.251	1.304

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	85	0	0	194	0	54	49	48
N.S.	1	1.00	1.77	0.00	0.00	4.04	0.00	1.12	1.02	1.00
time (sec)	N/A	0.059	0.096	0.652	0.000	0.639	0.000	0.184	5.043	0.850

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	0	0	204	0	89	136	48
N.S.	1	1.00	1.00	0.00	0.00	4.25	0.00	1.85	2.83	1.00
time (sec)	N/A	0.048	0.019	0.585	0.000	0.649	0.000	0.190	7.572	0.885

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	91	0	0	278	0	413	481	119
N.S.	1	1.00	1.00	0.00	0.00	3.05	0.00	4.54	5.29	1.31
time (sec)	N/A	0.080	0.065	0.122	0.000	0.787	0.000	0.235	10.775	1.649

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	145	7293	0	295	292	251	-1	162
N.S.	1	1.00	0.90	45.30	0.00	1.83	1.81	1.56	-0.01	1.01
time (sec)	N/A	0.113	0.242	8.015	0.000	1.260	130.857	0.754	0.000	0.603

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	119	6858	0	221	201	137	-1	123
N.S.	1	1.00	0.98	56.68	0.00	1.83	1.66	1.13	-0.01	1.02
time (sec)	N/A	0.089	0.145	1.092	0.000	1.158	9.670	0.419	0.000	0.515

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	87	6668	0	207	160	0	-1	99
N.S.	1	1.00	0.74	56.51	0.00	1.75	1.36	0.00	-0.01	0.84
time (sec)	N/A	0.084	0.209	1.040	0.000	1.547	9.715	0.000	0.000	0.562

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	87	3759	81	180	131	109	-1	75
N.S.	1	1.00	1.10	47.58	1.03	2.28	1.66	1.38	-0.01	0.95
time (sec)	N/A	0.046	0.181	0.999	1.217	0.948	61.862	0.246	0.000	0.310

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	167	7705	0	355	0	387	-1	200
N.S.	1	1.00	0.83	38.33	0.00	1.77	0.00	1.93	-0.00	1.00
time (sec)	N/A	0.139	0.335	1.142	0.000	1.203	0.000	1.455	0.000	0.836
Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	143	7290	0	273	335	0	-1	162
N.S.	1	1.00	0.89	45.28	0.00	1.70	2.08	0.00	-0.01	1.01
time (sec)	N/A	0.113	0.182	1.064	0.000	1.559	25.464	0.000	0.000	0.712
Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	126	7108	0	255	289	0	-1	125
N.S.	1	1.00	0.83	46.76	0.00	1.68	1.90	0.00	-0.01	0.82
time (sec)	N/A	0.104	0.154	1.183	0.000	1.450	24.317	0.000	0.000	0.762
Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	188	8117	0	409	0	563	-1	238
N.S.	1	1.00	0.78	33.68	0.00	1.70	0.00	2.34	-0.00	0.99
time (sec)	N/A	0.159	0.314	1.065	0.000	1.332	0.000	1.488	0.000	0.946

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	146	7702	0	323	413	0	-1	200
N.S.	1	1.00	0.73	38.32	0.00	1.61	2.05	0.00	-0.00	1.00
time (sec)	N/A	0.127	0.426	1.104	0.000	1.386	56.284	0.000	0.000	0.769
Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	150	7544	0	309	403	0	-1	157
N.S.	1	1.00	0.80	40.13	0.00	1.64	2.14	0.00	-0.01	0.84
time (sec)	N/A	0.128	0.213	1.157	0.000	1.281	63.945	0.000	0.000	0.848
Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	97	6861	0	245	194	114	-1	124
N.S.	1	1.00	0.80	56.70	0.00	2.02	1.60	0.94	-0.01	1.02
time (sec)	N/A	0.086	0.169	1.010	0.000	1.441	95.303	0.365	0.000	1.055
Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	78	6424	0	184	107	72	-1	89
N.S.	1	1.00	0.94	77.40	0.00	2.22	1.29	0.87	-0.01	1.07
time (sec)	N/A	0.067	0.059	1.148	0.000	1.280	6.806	0.363	0.000	0.532

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	65	3397	0	183	60	110	-1	81
N.S.	1	1.00	0.87	45.29	0.00	2.44	0.80	1.47	-0.01	1.08
time (sec)	N/A	0.057	0.064	1.003	0.000	1.192	10.112	0.260	0.000	0.789

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	109	7016	0	307	0	107	-1	136
N.S.	1	1.00	0.91	58.47	0.00	2.56	0.00	0.89	-0.01	1.13
time (sec)	N/A	0.086	0.176	1.132	0.000	1.925	0.000	0.366	0.000	1.761

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	93	3654	0	234	95	75	-1	107
N.S.	1	1.00	1.09	42.99	0.00	2.75	1.12	0.88	-0.01	1.26
time (sec)	N/A	0.062	0.099	1.000	0.000	0.861	33.620	0.619	0.000	1.007

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	45	39	0	57	0	0	70	71
N.S.	1	1.00	0.67	0.58	0.00	0.85	0.00	0.00	1.04	1.06
time (sec)	N/A	0.029	0.023	0.047	0.000	0.653	0.000	0.000	4.747	1.037

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	119	7081	0	345	0	102	-1	137
N.S.	1	1.00	1.04	62.11	0.00	3.03	0.00	0.89	-0.01	1.20
time (sec)	N/A	0.075	0.303	1.053	0.000	0.835	0.000	0.476	0.000	1.909
Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	44	39	0	59	0	64	73	76
N.S.	1	1.00	0.56	0.49	0.00	0.75	0.00	0.81	0.92	0.96
time (sec)	N/A	0.031	0.039	0.049	0.000	0.615	0.000	0.324	4.628	1.263
Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	65	62	0	93	0	0	115	100
N.S.	1	1.00	0.62	0.60	0.00	0.89	0.00	0.00	1.11	0.96
time (sec)	N/A	0.046	0.036	0.049	0.000	0.629	0.000	0.000	4.804	1.112
Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	220	220	230	0	183	186	0	0	240	224
N.S.	1	1.00	1.05	0.00	0.83	0.85	0.00	0.00	1.09	1.02
time (sec)	N/A	0.236	0.259	0.654	1.554	0.743	0.000	0.000	4.809	0.250

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	207	0	155	174	0	0	219	213
N.S.	1	1.00	1.19	0.00	0.89	1.00	0.00	0.00	1.26	1.22
time (sec)	N/A	0.178	0.139	0.631	1.406	0.491	0.000	0.000	4.650	0.186
Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	186	0	153	157	0	0	200	202
N.S.	1	1.00	1.08	0.00	0.89	0.91	0.00	0.00	1.16	1.17
time (sec)	N/A	0.149	0.096	0.636	1.434	0.781	0.000	0.000	4.661	0.179
Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	167	0	139	144	0	0	194	190
N.S.	1	1.00	1.11	0.00	0.93	0.96	0.00	0.00	1.29	1.27
time (sec)	N/A	0.119	0.053	0.636	1.271	0.696	0.000	0.000	4.638	0.168
Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	214	214	233	0	0	1046	0	0	345	286
N.S.	1	1.00	1.09	0.00	0.00	4.89	0.00	0.00	1.61	1.34
time (sec)	N/A	0.174	0.107	0.598	0.000	0.720	0.000	0.000	5.097	0.351

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	268	268	280	0	0	321	0	0	455	318
N.S.	1	1.00	1.04	0.00	0.00	1.20	0.00	0.00	1.70	1.19
time (sec)	N/A	0.247	0.117	0.615	0.000	0.738	0.000	0.000	5.345	0.569
Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	283	283	314	0	0	345	0	0	490	340
N.S.	1	1.00	1.11	0.00	0.00	1.22	0.00	0.00	1.73	1.20
time (sec)	N/A	0.264	0.138	0.652	0.000	0.637	0.000	0.000	5.442	0.718
Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	268	66	177	0	0	362	0	0	-1	364
N.S.	1	0.25	0.66	0.00	0.00	1.35	0.00	0.00	-0.00	1.36
time (sec)	N/A	0.060	0.210	0.649	0.000	0.637	0.000	0.000	0.000	1.213
Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	233	66	160	0	0	338	0	0	-1	342
N.S.	1	0.28	0.69	0.00	0.00	1.45	0.00	0.00	-0.00	1.47
time (sec)	N/A	0.063	0.166	0.737	0.000	0.635	0.000	0.000	0.000	0.870

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	237	423	130	0	0	0	0	0	-1	239
N.S.	1	1.78	0.55	0.00	0.00	0.00	0.00	0.00	-0.00	1.01
time (sec)	N/A	29.209	5.201	0.625	0.000	0.000	0.000	0.000	0.000	0.571

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	416	61	154	0	0	0	0	0	-1	533
N.S.	1	0.15	0.37	0.00	0.00	0.00	0.00	0.00	-0.00	1.28
time (sec)	N/A	0.031	0.154	0.631	0.000	0.000	0.000	0.000	0.000	3.089

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	163	0	183	209	0	0	261	225
N.S.	1	1.00	0.73	0.00	0.82	0.94	0.00	0.00	1.17	1.01
time (sec)	N/A	0.250	0.276	0.576	1.252	0.648	0.000	0.000	4.846	0.269

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	153	0	155	197	0	0	206	214
N.S.	1	1.00	0.86	0.00	0.88	1.11	0.00	0.00	1.16	1.21
time (sec)	N/A	0.200	0.172	0.575	1.169	0.725	0.000	0.000	4.915	0.222

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	175	143	0	155	181	0	0	221	203
N.S.	1	1.00	0.82	0.00	0.89	1.03	0.00	0.00	1.26	1.16
time (sec)	N/A	0.156	0.147	0.605	1.120	0.940	0.000	0.000	4.839	0.207

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	130	0	140	167	0	0	186	193
N.S.	1	1.00	0.85	0.00	0.92	1.09	0.00	0.00	1.22	1.26
time (sec)	N/A	0.126	0.092	0.622	1.314	1.169	0.000	0.000	4.830	0.195

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	214	214	164	0	0	530	0	0	369	286
N.S.	1	1.00	0.77	0.00	0.00	2.48	0.00	0.00	1.72	1.34
time (sec)	N/A	0.182	0.080	0.682	0.000	0.739	0.000	0.000	5.855	0.350

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	213	0	0	612	0	0	490	319
N.S.	1	1.00	0.79	0.00	0.00	2.28	0.00	0.00	1.82	1.19
time (sec)	N/A	0.263	0.093	0.624	0.000	0.513	0.000	0.000	5.561	0.550

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	284	284	247	0	0	660	0	0	513	341
N.S.	1	1.00	0.87	0.00	0.00	2.32	0.00	0.00	1.81	1.20
time (sec)	N/A	0.279	0.156	0.574	0.000	0.499	0.000	0.000	5.455	0.677
Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	264	66	244	0	0	701	0	0	-1	361
N.S.	1	0.25	0.92	0.00	0.00	2.66	0.00	0.00	-0.00	1.37
time (sec)	N/A	0.063	0.305	0.595	0.000	0.518	0.000	0.000	0.000	1.136
Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	229	66	216	0	0	653	0	0	-1	339
N.S.	1	0.29	0.94	0.00	0.00	2.85	0.00	0.00	-0.00	1.48
time (sec)	N/A	0.063	0.217	0.583	0.000	0.509	0.000	0.000	0.000	0.838
Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	200	61	156	0	0	611	0	0	-1	309
N.S.	1	0.30	0.78	0.00	0.00	3.06	0.00	0.00	-0.00	1.54
time (sec)	N/A	0.033	0.163	0.630	0.000	0.541	0.000	0.000	0.000	0.604

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	113	792	119	118	0	134	133	148
N.S.	1	1.00	0.89	6.24	0.94	0.93	0.00	1.06	1.05	1.17
time (sec)	N/A	0.093	0.104	9.325	1.241	0.934	0.000	0.173	5.034	0.153
Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	128	683	119	137	0	127	133	144
N.S.	1	1.00	1.00	5.34	0.93	1.07	0.00	0.99	1.04	1.12
time (sec)	N/A	0.088	0.057	8.409	1.248	0.870	0.000	0.176	4.730	0.140
Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	96	673	97	106	0	98	111	131
N.S.	1	1.00	0.99	6.94	1.00	1.09	0.00	1.01	1.14	1.35
time (sec)	N/A	0.077	0.066	6.727	1.190	1.165	0.000	0.201	4.649	0.108
Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	95	671	97	125	0	98	111	132
N.S.	1	1.00	0.97	6.85	0.99	1.28	0.00	1.00	1.13	1.35
time (sec)	N/A	0.064	0.033	3.820	1.214	1.008	0.000	0.176	4.685	0.122

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	73	476	86	90	0	87	100	116
N.S.	1	1.00	0.89	5.80	1.05	1.10	0.00	1.06	1.22	1.41
time (sec)	N/A	0.059	0.025	5.594	1.300	0.575	0.000	0.181	4.894	0.092

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	C	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	133	0	0	410	0	149	256	195
N.S.	1	1.00	0.97	0.00	0.00	2.99	0.00	1.09	1.87	1.42
time (sec)	N/A	0.091	0.045	2.438	0.000	1.482	0.000	0.197	4.798	0.201

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	153	0	0	187	0	163	382	215
N.S.	1	1.00	0.97	0.00	0.00	1.19	0.00	1.04	2.43	1.37
time (sec)	N/A	0.103	0.218	2.822	0.000	0.476	0.000	0.188	4.860	0.266

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	154	226	144	0	0	201	0	0	-1	230
N.S.	1	1.47	0.94	0.00	0.00	1.31	0.00	0.00	-0.01	1.49
time (sec)	N/A	0.149	0.158	1.916	0.000	0.472	0.000	0.000	0.000	0.544

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	C	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	135	207	26	0	0	452	0	0	-1	212
N.S.	1	1.53	0.19	0.00	0.00	3.35	0.00	0.00	-0.01	1.57
time (sec)	N/A	0.105	0.020	1.737	0.000	1.430	0.000	0.000	0.000	0.370
Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	122	112	931	0	253	0	0	-1	126
N.S.	1	1.39	1.27	10.58	0.00	2.88	0.00	0.00	-0.01	1.43
time (sec)	N/A	0.057	0.080	4.935	0.000	2.602	0.000	0.000	0.000	0.271
Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	105	139	82	929	0	307	0	0	-1	145
N.S.	1	1.32	0.78	8.85	0.00	2.92	0.00	0.00	-0.01	1.38
time (sec)	N/A	0.072	0.597	4.490	0.000	2.578	0.000	0.000	0.000	0.290
Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	124	140	123	955	0	283	0	0	-1	149
N.S.	1	1.13	0.99	7.70	0.00	2.28	0.00	0.00	-0.01	1.20
time (sec)	N/A	0.088	5.246	2.747	0.000	2.551	0.000	0.000	0.000	0.309

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	141	175	133	963	0	320	0	0	-1	157
N.S.	1	1.24	0.94	6.83	0.00	2.27	0.00	0.00	-0.01	1.11
time (sec)	N/A	0.092	5.115	4.067	0.000	2.607	0.000	0.000	0.000	0.305
Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	233	26	26	0	0	373	0	0	-1	340
N.S.	1	0.11	0.11	0.00	0.00	1.60	0.00	0.00	-0.00	1.46
time (sec)	N/A	0.013	0.017	3.084	0.000	2.318	0.000	0.000	0.000	1.075
Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	151	1579	119	142	0	127	135	141
N.S.	1	1.00	1.21	12.63	0.95	1.14	0.00	1.02	1.08	1.13
time (sec)	N/A	0.095	0.107	7.448	1.522	0.451	0.000	0.179	5.185	0.139
Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	135	1584	97	114	0	98	113	132
N.S.	1	1.00	1.38	16.16	0.99	1.16	0.00	1.00	1.15	1.35
time (sec)	N/A	0.075	0.089	7.461	1.192	0.479	0.000	0.178	4.976	0.125

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	118	1582	97	130	0	98	113	129
N.S.	1	1.00	1.24	16.65	1.02	1.37	0.00	1.03	1.19	1.36
time (sec)	N/A	0.062	0.050	5.099	1.384	0.465	0.000	0.234	4.892	0.104

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	94	529	86	98	0	87	102	117
N.S.	1	1.00	1.13	6.37	1.04	1.18	0.00	1.05	1.23	1.41
time (sec)	N/A	0.056	0.035	6.280	1.176	0.492	0.000	0.197	5.052	0.098

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	179	0	0	182	0	149	344	195
N.S.	1	1.00	1.31	0.00	0.00	1.33	0.00	1.09	2.51	1.42
time (sec)	N/A	0.090	0.052	2.376	0.000	0.485	0.000	0.240	4.921	0.187

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	196	0	0	195	0	163	368	216
N.S.	1	1.00	1.24	0.00	0.00	1.23	0.00	1.03	2.33	1.37
time (sec)	N/A	0.103	0.100	3.079	0.000	0.486	0.000	0.215	5.073	0.253

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	160	228	78	0	0	232	0	0	-1	232
N.S.	1	1.42	0.49	0.00	0.00	1.45	0.00	0.00	-0.01	1.45
time (sec)	N/A	0.163	0.086	2.045	0.000	0.462	0.000	0.000	0.000	0.540

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	139	207	26	0	0	197	0	0	-1	212
N.S.	1	1.49	0.19	0.00	0.00	1.42	0.00	0.00	-0.01	1.53
time (sec)	N/A	0.103	0.051	1.759	0.000	0.459	0.000	0.000	0.000	0.383

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	88	122	38	938	0	283	0	0	-1	126
N.S.	1	1.39	0.43	10.66	0.00	3.22	0.00	0.00	-0.01	1.43
time (sec)	N/A	0.061	0.007	4.243	0.000	1.924	0.000	0.000	0.000	0.295

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	103	137	81	1387	0	272	0	0	-1	143
N.S.	1	1.33	0.79	13.47	0.00	2.64	0.00	0.00	-0.01	1.39
time (sec)	N/A	0.072	0.555	4.057	0.000	1.939	0.000	0.000	0.000	0.304

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	124	140	145	1386	0	312	0	0	-1	149
N.S.	1	1.13	1.17	11.18	0.00	2.52	0.00	0.00	-0.01	1.20
time (sec)	N/A	0.084	13.899	3.514	0.000	1.856	0.000	0.000	0.000	0.332
Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	291	26	115	696	0	356	0	0	-1	0
N.S.	1	0.09	0.40	2.39	0.00	1.22	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.019	0.163	14.511	0.000	2.518	0.000	0.000	0.000	11.003
Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	294	26	120	991	0	396	0	0	-1	0
N.S.	1	0.09	0.41	3.37	0.00	1.35	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.019	0.100	14.426	0.000	2.416	0.000	0.000	0.000	34.015
Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	53	682	128	140	0	136	148	156
N.S.	1	1.00	0.38	4.84	0.91	0.99	0.00	0.96	1.05	1.11
time (sec)	N/A	0.114	0.037	3.784	1.066	0.468	0.000	0.195	4.861	0.178

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	48	497	119	159	0	120	139	151
N.S.	1	1.00	0.37	3.82	0.92	1.22	0.00	0.92	1.07	1.16
time (sec)	N/A	0.101	0.019	3.791	1.274	0.454	0.000	0.180	5.150	0.159
Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	41	672	108	130	0	109	128	146
N.S.	1	1.00	0.36	5.84	0.94	1.13	0.00	0.95	1.11	1.27
time (sec)	N/A	0.090	0.014	3.782	1.271	0.472	0.000	0.175	4.890	0.147
Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	95	667	97	148	0	98	117	141
N.S.	1	1.00	0.95	6.67	0.97	1.48	0.00	0.98	1.17	1.41
time (sec)	N/A	0.068	0.093	3.763	1.526	0.465	0.000	0.184	4.846	0.137
Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	34	667	97	125	0	98	117	141
N.S.	1	1.00	0.34	6.67	0.97	1.25	0.00	0.98	1.17	1.41
time (sec)	N/A	0.068	0.010	3.867	1.338	0.467	0.000	0.193	4.903	0.122

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	54	0	0	226	0	160	253	219
N.S.	1	1.00	0.35	0.00	0.00	1.47	0.00	1.04	1.64	1.42
time (sec)	N/A	0.111	0.023	2.719	0.000	0.490	0.000	0.187	5.399	0.248
Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	175	64	0	0	238	0	181	399	232
N.S.	1	1.00	0.37	0.00	0.00	1.36	0.00	1.03	2.28	1.33
time (sec)	N/A	0.116	0.032	2.293	0.000	0.465	0.000	0.215	5.024	0.305
Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	174	26	152	0	0	271	0	0	-1	248
N.S.	1	0.15	0.87	0.00	0.00	1.56	0.00	0.00	-0.01	1.43
time (sec)	N/A	0.018	0.248	1.457	0.000	0.494	0.000	0.000	0.000	0.574
Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	153	26	142	0	0	239	0	0	-1	238
N.S.	1	0.17	0.93	0.00	0.00	1.56	0.00	0.00	-0.01	1.56
time (sec)	N/A	0.018	0.164	1.685	0.000	0.456	0.000	0.000	0.000	0.456

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	106	38	38	627	0	318	0	0	-1	152
N.S.	1	0.36	0.36	5.92	0.00	3.00	0.00	0.00	-0.01	1.43
time (sec)	N/A	0.018	0.009	4.228	0.000	2.584	0.000	0.000	0.000	0.355
Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	106	140	100	944	0	288	0	0	-1	152
N.S.	1	1.32	0.94	8.91	0.00	2.72	0.00	0.00	-0.01	1.43
time (sec)	N/A	0.076	0.048	3.009	0.000	2.599	0.000	0.000	0.000	0.343
Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	124	204	192	570	0	340	0	0	-1	161
N.S.	1	1.65	1.55	4.60	0.00	2.74	0.00	0.00	-0.01	1.30
time (sec)	N/A	8.121	3.108	2.916	0.000	2.599	0.000	0.000	0.000	0.369
Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	144	397	133	851	0	316	0	0	-1	164
N.S.	1	2.76	0.92	5.91	0.00	2.19	0.00	0.00	-0.01	1.14
time (sec)	N/A	8.318	5.158	4.018	0.000	2.708	0.000	0.000	0.000	0.386

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	162	643	136	649	0	351	0	0	-1	169
N.S.	1	3.97	0.84	4.01	0.00	2.17	0.00	0.00	-0.01	1.04
time (sec)	N/A	10.595	5.205	2.684	0.000	2.587	0.000	0.000	0.000	0.386

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	264	264	270	0	0	325	0	379	442	340
N.S.	1	1.00	1.02	0.00	0.00	1.23	0.00	1.44	1.67	1.29
time (sec)	N/A	0.386	0.683	0.652	0.000	0.448	0.000	0.292	4.984	0.502

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	220	220	230	0	0	282	0	320	336	283
N.S.	1	1.00	1.05	0.00	0.00	1.28	0.00	1.45	1.53	1.29
time (sec)	N/A	0.259	0.408	0.629	0.000	0.456	0.000	0.253	4.933	0.368

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	186	204	0	0	222	0	276	298	239
N.S.	1	1.00	1.10	0.00	0.00	1.19	0.00	1.48	1.60	1.28
time (sec)	N/A	0.202	0.466	0.620	0.000	0.447	0.000	0.246	4.617	0.222

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	205	0	0	206	0	223	249	215
N.S.	1	1.00	1.29	0.00	0.00	1.30	0.00	1.40	1.57	1.35
time (sec)	N/A	0.155	0.287	0.607	0.000	0.438	0.000	0.271	4.621	0.235

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	268	0	0	276	0	311	1607	334
N.S.	1	1.00	1.09	0.00	0.00	1.12	0.00	1.26	6.53	1.36
time (sec)	N/A	0.234	0.465	0.566	0.000	0.446	0.000	0.805	4.744	0.445

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	340	340	366	0	0	429	0	351	1917	383
N.S.	1	1.00	1.08	0.00	0.00	1.26	0.00	1.03	5.64	1.13
time (sec)	N/A	0.384	1.242	0.617	0.000	0.558	0.000	0.776	9.990	0.674

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	370	370	411	0	0	472	0	465	2767	448
N.S.	1	1.00	1.11	0.00	0.00	1.28	0.00	1.26	7.48	1.21
time (sec)	N/A	0.489	1.837	0.633	0.000	2.003	0.000	0.810	12.554	0.811

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	204	145	146	0	0	0	0	0	-1	394
N.S.	1	0.71	0.72	0.00	0.00	0.00	0.00	0.00	-0.00	1.93
time (sec)	N/A	0.133	0.776	0.621	0.000	0.000	0.000	0.000	0.000	2.297
Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	258	451	451	0	0	0	0	0	-1	444
N.S.	1	1.75	1.75	0.00	0.00	0.00	0.00	0.00	-0.00	1.72
time (sec)	N/A	0.940	2.753	0.602	0.000	0.000	0.000	0.000	0.000	2.618
Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	318	905	214	0	0	0	0	0	-1	503
N.S.	1	2.85	0.67	0.00	0.00	0.00	0.00	0.00	-0.00	1.58
time (sec)	N/A	2.642	5.237	0.634	0.000	0.000	0.000	0.000	0.000	2.987
Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	266	266	148	0	0	455	0	409	490	340
N.S.	1	1.00	0.56	0.00	0.00	1.71	0.00	1.54	1.84	1.28
time (sec)	N/A	0.322	0.124	0.605	0.000	0.687	0.000	0.287	5.127	0.648

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	104	0	0	398	0	350	385	284
N.S.	1	1.00	0.47	0.00	0.00	1.78	0.00	1.57	1.73	1.27
time (sec)	N/A	0.259	0.098	0.714	0.000	0.607	0.000	0.267	5.105	0.398
Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	68	0	0	353	0	306	302	241
N.S.	1	1.00	0.36	0.00	0.00	1.88	0.00	1.63	1.61	1.28
time (sec)	N/A	0.199	0.046	0.682	0.000	0.414	0.000	0.549	5.058	0.307
Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	47	0	0	323	0	259	238	218
N.S.	1	1.00	0.29	0.00	0.00	1.99	0.00	1.60	1.47	1.35
time (sec)	N/A	0.170	0.026	0.578	0.000	0.436	0.000	0.271	5.049	0.250
Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	245	245	115	0	0	425	0	341	1963	333
N.S.	1	1.00	0.47	0.00	0.00	1.73	0.00	1.39	8.01	1.36
time (sec)	N/A	0.229	0.121	0.622	0.000	0.461	0.000	0.802	4.770	0.470

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	347	347	202	0	0	1030	0	400	1908	384
N.S.	1	1.00	0.58	0.00	0.00	2.97	0.00	1.15	5.50	1.11
time (sec)	N/A	0.389	0.158	0.608	0.000	0.547	0.000	0.807	10.571	0.715
Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	370	370	240	0	0	1151	0	493	2788	448
N.S.	1	1.00	0.65	0.00	0.00	3.11	0.00	1.33	7.54	1.21
time (sec)	N/A	0.496	0.377	0.669	0.000	1.616	0.000	0.782	15.187	0.878
Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	334	64	525	0	0	1164	0	0	-1	608
N.S.	1	0.19	1.57	0.00	0.00	3.49	0.00	0.00	-0.00	1.82
time (sec)	N/A	0.056	0.757	0.584	0.000	1.600	0.000	0.000	0.000	6.446
Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	272	64	286	0	0	1091	0	0	-1	549
N.S.	1	0.24	1.05	0.00	0.00	4.01	0.00	0.00	-0.00	2.02
time (sec)	N/A	0.059	0.594	0.620	0.000	0.568	0.000	0.000	0.000	4.957

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F(-1)	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	320	819	819	0	0	0	0	0	-1	501
N.S.	1	2.56	2.56	0.00	0.00	0.00	0.00	0.00	-0.00	1.57
time (sec)	N/A	2.626	4.995	0.608	0.000	0.000	0.000	0.000	0.000	3.354
Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	258	0	0	369	0	394	477	340
N.S.	1	1.00	1.03	0.00	0.00	1.47	0.00	1.57	1.90	1.35
time (sec)	N/A	0.363	0.538	0.622	0.000	0.563	0.000	0.303	5.123	0.459
Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	255	0	0	298	0	348	348	278
N.S.	1	1.00	1.21	0.00	0.00	1.41	0.00	1.65	1.65	1.32
time (sec)	N/A	0.245	0.337	0.745	0.000	0.430	0.000	0.291	5.057	0.271
Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	187	232	0	0	246	0	297	304	234
N.S.	1	1.00	1.24	0.00	0.00	1.32	0.00	1.59	1.63	1.25
time (sec)	N/A	0.208	0.325	0.647	0.000	0.440	0.000	0.297	4.725	0.236

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	261	331	0	0	320	0	357	796	349
N.S.	1	1.00	1.27	0.00	0.00	1.23	0.00	1.37	3.05	1.34
time (sec)	N/A	0.304	0.669	0.779	0.000	0.450	0.000	0.840	6.080	0.655
Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	399	399	389	0	0	383	0	394	2047	398
N.S.	1	1.00	0.97	0.00	0.00	0.96	0.00	0.99	5.13	1.00
time (sec)	N/A	0.484	1.548	0.651	0.000	0.701	0.000	0.859	10.881	0.766
Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	440	440	429	0	0	503	0	481	2841	445
N.S.	1	1.00	0.98	0.00	0.00	1.14	0.00	1.09	6.46	1.01
time (sec)	N/A	0.623	1.254	0.678	0.000	2.020	0.000	0.859	13.254	1.074
Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	A	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	334	65	225	0	0	550	0	0	-1	611
N.S.	1	0.19	0.67	0.00	0.00	1.65	0.00	0.00	-0.00	1.83
time (sec)	N/A	0.059	0.312	0.590	0.000	2.052	0.000	0.000	0.000	9.965

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F(-1)	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	318	260	270	0	0	0	0	0	-1	501
N.S.	1	0.82	0.85	0.00	0.00	0.00	0.00	0.00	-0.00	1.58
time (sec)	N/A	1.801	2.701	0.638	0.000	0.000	0.000	0.000	0.000	3.689
Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F(-1)	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	392	1446	1446	0	0	0	0	0	-1	577
N.S.	1	3.69	3.69	0.00	0.00	0.00	0.00	0.00	-0.00	1.47
time (sec)	N/A	4.391	5.442	0.621	0.000	0.000	0.000	0.000	0.000	4.753
Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	290	290	157	0	0	1004	0	454	438	340
N.S.	1	1.00	0.54	0.00	0.00	3.46	0.00	1.57	1.51	1.17
time (sec)	N/A	0.317	0.258	0.601	0.000	0.760	0.000	0.326	5.115	0.625
Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	244	244	145	0	0	873	0	371	339	284
N.S.	1	1.00	0.59	0.00	0.00	3.58	0.00	1.52	1.39	1.16
time (sec)	N/A	0.243	0.138	0.596	0.000	0.736	0.000	0.279	5.090	0.415

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	103	0	0	768	0	313	267	247
N.S.	1	1.00	0.51	0.00	0.00	3.78	0.00	1.54	1.32	1.22
time (sec)	N/A	0.214	0.095	0.592	0.000	0.786	0.000	0.261	5.113	0.401
Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	168	69	0	0	667	0	257	219	224
N.S.	1	1.00	0.41	0.00	0.00	3.97	0.00	1.53	1.30	1.33
time (sec)	N/A	0.161	0.031	0.612	0.000	0.732	0.000	0.279	5.097	0.206
Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	50	0	0	592	0	226	208	201
N.S.	1	1.00	0.34	0.00	0.00	4.08	0.00	1.56	1.43	1.39
time (sec)	N/A	0.125	0.019	0.605	0.000	0.817	0.000	0.287	4.934	0.166
Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	244	244	140	0	0	628	0	326	702	332
N.S.	1	1.00	0.57	0.00	0.00	2.57	0.00	1.34	2.88	1.36
time (sec)	N/A	0.212	0.178	0.613	0.000	0.447	0.000	0.744	6.436	0.491

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	176	235	124	0	0	0	0	0	-1	349
N.S.	1	1.34	0.70	0.00	0.00	0.00	0.00	0.00	-0.01	1.98
time (sec)	N/A	0.226	0.066	0.621	0.000	0.000	0.000	0.000	0.000	1.955

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	214	271	207	0	0	0	0	0	-1	379
N.S.	1	1.27	0.97	0.00	0.00	0.00	0.00	0.00	-0.00	1.77
time (sec)	N/A	0.286	1.025	0.586	0.000	0.000	0.000	0.000	0.000	2.306

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	262	317	821	0	0	0	0	0	-1	419
N.S.	1	1.21	3.13	0.00	0.00	0.00	0.00	0.00	-0.00	1.60
time (sec)	N/A	0.325	2.546	0.585	0.000	0.000	0.000	0.000	0.000	3.109

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	251	0	0	1322	0	372	331	284
N.S.	1	1.00	1.04	0.00	0.00	5.49	0.00	1.54	1.37	1.18
time (sec)	N/A	0.263	0.664	0.612	0.000	0.833	0.000	0.263	5.012	0.496

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	211	0	0	1156	0	312	292	246
N.S.	1	1.00	1.05	0.00	0.00	5.75	0.00	1.55	1.45	1.22
time (sec)	N/A	0.212	0.425	0.610	0.000	0.472	0.000	0.297	4.688	0.421
Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	202	0	0	1060	0	253	232	221
N.S.	1	1.00	1.22	0.00	0.00	6.42	0.00	1.53	1.41	1.34
time (sec)	N/A	0.155	0.180	0.643	0.000	0.607	0.000	0.232	4.684	0.215
Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	164	0	0	927	0	221	213	201
N.S.	1	1.00	1.13	0.00	0.00	6.39	0.00	1.52	1.47	1.39
time (sec)	N/A	0.124	0.062	0.599	0.000	0.556	0.000	0.288	4.847	0.159
Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	245	245	308	0	0	472	0	321	1413	333
N.S.	1	1.00	1.26	0.00	0.00	1.93	0.00	1.31	5.77	1.36
time (sec)	N/A	0.211	0.446	0.611	0.000	0.501	0.000	0.756	4.942	0.591

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	173	232	128	0	0	0	0	0	-1	347
N.S.	1	1.34	0.74	0.00	0.00	0.00	0.00	0.00	-0.01	2.01
time (sec)	N/A	0.221	0.065	0.653	0.000	0.000	0.000	0.000	0.000	2.022
Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	215	269	267	0	0	0	0	0	-1	377
N.S.	1	1.25	1.24	0.00	0.00	0.00	0.00	0.00	-0.00	1.75
time (sec)	N/A	0.297	1.888	0.612	0.000	0.000	0.000	0.000	0.000	2.411
Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	347	347	157	0	0	1300	0	431	564	393
N.S.	1	1.00	0.45	0.00	0.00	3.75	0.00	1.24	1.63	1.13
time (sec)	N/A	0.441	0.265	0.458	0.000	0.839	0.000	0.257	5.189	0.712
Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	147	0	0	1141	0	372	493	322
N.S.	1	1.00	0.58	0.00	0.00	4.51	0.00	1.47	1.95	1.27
time (sec)	N/A	0.340	0.111	0.464	0.000	0.726	0.000	0.259	5.160	0.504

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	101	0	0	1004	0	325	449	268
N.S.	1	1.00	0.50	0.00	0.00	4.95	0.00	1.60	2.21	1.32
time (sec)	N/A	0.242	0.068	0.428	0.000	0.705	0.000	0.248	4.986	0.462

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	77	0	0	872	0	301	412	230
N.S.	1	1.00	0.44	0.00	0.00	5.01	0.00	1.73	2.37	1.32
time (sec)	N/A	0.171	0.034	0.674	0.000	0.821	0.000	0.254	4.978	0.256

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	50	0	0	262	0	285	389	223
N.S.	1	1.00	0.30	0.00	0.00	1.57	0.00	1.71	2.33	1.34
time (sec)	N/A	0.165	0.021	0.612	0.000	0.670	0.000	0.256	4.923	0.241

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	271	271	86	0	0	975	0	389	3804	360
N.S.	1	1.00	0.32	0.00	0.00	3.60	0.00	1.44	14.04	1.33
time (sec)	N/A	0.316	0.045	0.708	0.000	0.798	0.000	0.793	5.342	0.851

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	92	89	84	100	0	88	88	0
N.S.	1	1.00	1.02	0.99	0.93	1.11	0.00	0.98	0.98	0.00
time (sec)	N/A	0.096	0.066	0.059	0.467	6.244	0.000	0.174	5.880	0.001
Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	66	65	68	72	0	70	68	0
N.S.	1	1.00	0.94	0.93	0.97	1.03	0.00	1.00	0.97	0.00
time (sec)	N/A	0.064	0.049	0.053	0.570	1.683	0.000	0.202	5.633	0.001
Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	43	50	49	42	144	51	51	0
N.S.	1	1.00	0.81	0.94	0.92	0.79	2.72	0.96	0.96	0.00
time (sec)	N/A	0.050	0.026	0.052	0.621	0.746	60.853	0.188	5.096	0.001
Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	31	42	41	31	138	51	1012	0
N.S.	1	1.00	0.69	0.93	0.91	0.69	3.07	1.13	22.49	0.00
time (sec)	N/A	0.031	0.020	0.051	0.519	0.422	1.914	0.337	4.987	0.001

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	54	59	61	54	0	73	58	0
N.S.	1	1.00	0.87	0.95	0.98	0.87	0.00	1.18	0.94	0.00
time (sec)	N/A	0.064	0.033	0.057	0.710	1.997	0.000	0.181	5.492	0.001
Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	88	87	87	99	0	112	87	0
N.S.	1	1.00	1.01	1.00	1.00	1.14	0.00	1.29	1.00	0.00
time (sec)	N/A	0.090	0.045	0.059	0.554	8.265	0.000	0.188	6.214	0.001
Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	104	105	100	576	0	112	532	0
N.S.	1	1.00	0.93	0.94	0.89	5.14	0.00	1.00	4.75	0.00
time (sec)	N/A	0.273	0.170	0.061	1.325	2.635	0.000	0.190	5.705	0.001
Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	82	81	80	416	932	80	518	0
N.S.	1	1.00	0.89	0.88	0.87	4.52	10.13	0.87	5.63	0.00
time (sec)	N/A	0.118	0.141	0.056	1.189	0.793	29.695	0.175	5.724	0.001

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	66	60	59	325	0	59	379	0
N.S.	1	1.00	0.84	0.76	0.75	4.11	0.00	0.75	4.80	0.00
time (sec)	N/A	0.062	0.041	0.056	1.165	0.494	0.000	0.175	5.345	0.001
Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	66	60	59	325	0	59	399	0
N.S.	1	1.00	0.84	0.76	0.75	4.11	0.00	0.75	5.05	0.00
time (sec)	N/A	0.048	0.050	0.058	1.277	0.500	0.000	0.180	5.295	0.001
Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	169	81	80	432	1103	80	354	0
N.S.	1	1.00	1.84	0.88	0.87	4.70	11.99	0.87	3.85	0.00
time (sec)	N/A	0.122	0.230	0.056	1.202	0.882	84.889	0.207	5.348	0.001
Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	193	105	101	592	0	103	535	0
N.S.	1	1.00	1.72	0.94	0.90	5.29	0.00	0.92	4.78	0.00
time (sec)	N/A	0.218	0.262	0.062	1.513	3.447	0.000	0.184	5.530	0.001

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	457	457	377	328	375	1378	0	469	6361	0
N.S.	1	1.00	0.82	0.72	0.82	3.02	0.00	1.03	13.92	0.00
time (sec)	N/A	0.443	0.245	0.059	1.216	0.722	0.000	0.223	5.633	0.001
Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	449	449	340	320	363	1358	0	453	2553	0
N.S.	1	1.00	0.76	0.71	0.81	3.02	0.00	1.01	5.69	0.00
time (sec)	N/A	0.285	0.121	0.053	1.421	0.538	0.000	0.242	5.725	0.001
Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	449	449	340	296	361	1238	0	437	5889	0
N.S.	1	1.00	0.76	0.66	0.80	2.76	0.00	0.97	13.12	0.00
time (sec)	N/A	0.274	0.113	0.059	1.393	0.470	0.000	0.236	5.859	0.001
Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	449	449	340	296	363	1258	0	477	6633	0
N.S.	1	1.00	0.76	0.66	0.81	2.80	0.00	1.06	14.77	0.00
time (sec)	N/A	0.270	0.139	0.057	1.207	0.502	0.000	0.228	5.497	0.001

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	449	449	340	320	365	1356	0	437	6153	0
N.S.	1	1.00	0.76	0.71	0.81	3.02	0.00	0.97	13.70	0.00
time (sec)	N/A	0.259	0.196	0.056	1.384	0.627	0.000	0.215	5.853	0.001
Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	460	460	385	331	384	1395	0	488	5962	0
N.S.	1	1.00	0.84	0.72	0.83	3.03	0.00	1.06	12.96	0.00
time (sec)	N/A	0.445	0.220	0.058	1.266	0.894	0.000	0.207	6.084	0.001
Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	462	462	406	343	390	1415	0	472	7459	0
N.S.	1	1.00	0.88	0.74	0.84	3.06	0.00	1.02	16.15	0.00
time (sec)	N/A	0.432	0.263	0.059	1.360	5.399	0.000	0.194	6.190	0.001
Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	479	479	428	365	405	1456	0	483	4547	0
N.S.	1	1.00	0.89	0.76	0.85	3.04	0.00	1.01	9.49	0.00
time (sec)	N/A	0.597	0.378	0.066	1.235	9.197	0.000	0.235	6.012	0.001

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	88	1015	0	195	90	96	87	98
N.S.	1	1.00	0.95	10.91	0.00	2.10	0.97	1.03	0.94	1.05
time (sec)	N/A	0.082	0.109	0.272	0.000	0.433	17.444	0.165	4.688	0.109

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	114	1066	0	714	0	0	-1	173
N.S.	1	1.00	0.95	8.88	0.00	5.95	0.00	0.00	-0.01	1.44
time (sec)	N/A	0.154	0.192	0.278	0.000	0.557	0.000	0.000	0.000	0.474

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	69	988	0	156	65	66	54	80
N.S.	1	1.00	0.99	14.11	0.00	2.23	0.93	0.94	0.77	1.14
time (sec)	N/A	0.061	0.051	0.204	0.000	0.446	8.185	0.183	4.660	0.090

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	89	1000	0	612	0	0	-1	144
N.S.	1	1.00	0.98	10.99	0.00	6.73	0.00	0.00	-0.01	1.58
time (sec)	N/A	0.076	0.065	0.196	0.000	0.494	0.000	0.000	0.000	0.348

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	81	1037	0	383	82	79	199	95
N.S.	1	1.00	0.95	12.20	0.00	4.51	0.96	0.93	2.34	1.12
time (sec)	N/A	0.075	0.094	0.262	0.000	0.464	12.052	0.179	4.869	0.124
Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	53	1075	0	281	0	121	-1	128
N.S.	1	1.00	0.70	14.14	0.00	3.70	0.00	1.59	-0.01	1.68
time (sec)	N/A	0.086	0.064	0.259	0.000	0.455	0.000	1.285	0.000	0.388
Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	107	1107	0	513	0	107	269	125
N.S.	1	1.00	0.93	9.63	0.00	4.46	0.00	0.93	2.34	1.09
time (sec)	N/A	0.124	0.134	0.270	0.000	0.471	0.000	0.169	5.387	0.265
Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	110	110	130	1116	0	329	0	225	-1	151
N.S.	1	1.00	1.18	10.15	0.00	2.99	0.00	2.05	-0.01	1.37
time (sec)	N/A	0.160	5.330	0.263	0.000	0.521	0.000	1.749	0.000	0.535

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	91	378	0	289	0	106	102	101
N.S.	1	1.00	0.88	3.63	0.00	2.78	0.00	1.02	0.98	0.97
time (sec)	N/A	0.112	0.257	0.283	0.000	0.717	0.000	0.162	4.825	0.177
Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	72	335	0	205	0	64	58	84
N.S.	1	1.00	0.97	4.53	0.00	2.77	0.00	0.86	0.78	1.14
time (sec)	N/A	0.065	0.079	0.209	0.000	0.740	0.000	0.194	4.729	0.098
Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	51	316	0	130	37	40	40	61
N.S.	1	1.00	1.00	6.20	0.00	2.55	0.73	0.78	0.78	1.20
time (sec)	N/A	0.049	0.018	0.213	0.000	0.694	14.270	0.156	4.797	0.052
Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	81	347	0	431	66	71	652	95
N.S.	1	1.00	0.95	4.08	0.00	5.07	0.78	0.84	7.67	1.12
time (sec)	N/A	0.073	0.086	0.287	0.000	0.727	20.207	0.170	5.028	0.111

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	151	402	0	565	0	104	396	127
N.S.	1	1.00	1.29	3.44	0.00	4.83	0.00	0.89	3.38	1.09
time (sec)	N/A	0.120	0.170	0.292	0.000	0.702	0.000	0.167	5.351	0.252
Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	118	408	0	739	0	0	-1	177
N.S.	1	1.00	0.96	3.32	0.00	6.01	0.00	0.00	-0.01	1.44
time (sec)	N/A	0.149	0.210	0.316	0.000	0.752	0.000	0.000	0.000	0.699
Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	90	356	0	632	0	0	-1	144
N.S.	1	1.00	0.99	3.91	0.00	6.95	0.00	0.00	-0.01	1.58
time (sec)	N/A	0.087	0.069	0.333	0.000	0.674	0.000	0.000	0.000	0.455
Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	95	322	0	245	0	72	-1	106
N.S.	1	1.00	1.76	5.96	0.00	4.54	0.00	1.33	-0.02	1.96
time (sec)	N/A	0.044	0.063	0.324	0.000	0.505	0.000	0.179	0.000	0.316

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	80	80	179	350	0	332	0	116	-1	142
N.S.	1	1.00	2.24	4.38	0.00	4.15	0.00	1.45	-0.01	1.78
time (sec)	N/A	0.088	4.830	0.297	0.000	0.530	0.000	0.219	0.000	0.481
Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	115	115	137	383	0	418	0	205	-1	163
N.S.	1	1.00	1.19	3.33	0.00	3.63	0.00	1.78	-0.01	1.42
time (sec)	N/A	0.164	5.549	0.270	0.000	0.544	0.000	1.675	0.000	0.888
Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	175	175	923	0	622	0	180	186	198
N.S.	1	1.00	1.00	5.27	0.00	3.55	0.00	1.03	1.06	1.13
time (sec)	N/A	0.173	0.306	0.247	0.000	0.551	0.000	0.175	5.191	0.486
Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	107	876	0	475	0	134	144	143
N.S.	1	1.00	0.87	7.12	0.00	3.86	0.00	1.09	1.17	1.16
time (sec)	N/A	0.146	0.355	0.190	0.000	0.616	0.000	0.168	5.117	0.329

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	98	851	0	348	0	116	95	109
N.S.	1	1.00	0.99	8.60	0.00	3.52	0.00	1.17	0.96	1.10
time (sec)	N/A	0.087	0.113	0.197	0.000	0.708	0.000	0.170	4.932	0.167
Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	85	541	0	302	0	93	84	97
N.S.	1	1.00	0.98	6.22	0.00	3.47	0.00	1.07	0.97	1.11
time (sec)	N/A	0.074	0.102	0.206	0.000	0.721	0.000	0.164	4.852	0.108
Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	123	880	0	862	0	139	3017	146
N.S.	1	1.00	0.93	6.67	0.00	6.53	0.00	1.05	22.86	1.11
time (sec)	N/A	0.142	0.334	0.272	0.000	0.682	0.000	0.168	5.869	0.321
Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	163	938	0	1236	0	257	3822	187
N.S.	1	1.00	0.88	5.07	0.00	6.68	0.00	1.39	20.66	1.01
time (sec)	N/A	0.251	0.695	0.277	0.000	0.915	0.000	0.169	7.047	0.656

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	191	191	150	953	0	1386	0	337	-1	208
N.S.	1	1.00	0.79	4.99	0.00	7.26	0.00	1.76	-0.01	1.09
time (sec)	N/A	0.365	0.566	0.273	0.000	1.938	0.000	0.590	0.000	2.288
Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	135	893	0	1077	0	298	-1	166
N.S.	1	1.00	0.96	6.33	0.00	7.64	0.00	2.11	-0.01	1.18
time (sec)	N/A	0.162	0.224	0.238	0.000	0.838	0.000	0.545	0.000	1.594
Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	124	861	0	426	0	244	-1	145
N.S.	1	1.00	1.33	9.26	0.00	4.58	0.00	2.62	-0.01	1.56
time (sec)	N/A	0.092	0.583	0.240	0.000	0.537	0.000	1.630	0.000	1.043
Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	104	104	407	867	0	467	0	237	-1	124
N.S.	1	1.00	3.91	8.34	0.00	4.49	0.00	2.28	-0.01	1.19
time (sec)	N/A	0.094	6.013	0.194	0.000	0.599	0.000	0.376	0.000	0.643

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	149	149	155	885	0	612	0	418	-1	159
N.S.	1	1.00	1.04	5.94	0.00	4.11	0.00	2.81	-0.01	1.07
time (sec)	N/A	0.199	5.633	0.269	0.000	0.617	0.000	1.681	0.000	1.373
Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	208	208	175	923	0	760	0	395	-1	217
N.S.	1	1.00	0.84	4.44	0.00	3.65	0.00	1.90	-0.00	1.04
time (sec)	N/A	0.328	5.871	0.246	0.000	0.776	0.000	2.038	0.000	2.282
Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	91	0	0	288	0	106	103	101
N.S.	1	1.00	0.88	0.00	0.00	2.77	0.00	1.02	0.99	0.97
time (sec)	N/A	0.107	0.277	0.634	0.000	0.433	0.000	0.170	4.866	0.183
Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	72	0	0	205	0	64	58	84
N.S.	1	1.00	0.97	0.00	0.00	2.77	0.00	0.86	0.78	1.14
time (sec)	N/A	0.063	0.130	0.601	0.000	0.418	0.000	0.192	4.745	0.098

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	51	0	0	130	37	40	40	61
N.S.	1	1.00	1.00	0.00	0.00	2.55	0.73	0.78	0.78	1.20
time (sec)	N/A	0.046	0.020	0.572	0.000	0.412	23.762	0.167	4.725	0.054

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	81	0	0	431	66	71	652	95
N.S.	1	1.00	0.95	0.00	0.00	5.07	0.78	0.84	7.67	1.12
time (sec)	N/A	0.074	0.126	0.631	0.000	0.450	26.110	0.176	5.191	0.117

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	151	0	0	565	0	104	396	127
N.S.	1	1.00	1.29	0.00	0.00	4.83	0.00	0.89	3.38	1.09
time (sec)	N/A	0.119	0.181	0.725	0.000	0.467	0.000	0.177	5.615	0.265

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	118	0	0	739	0	0	-1	177
N.S.	1	1.00	0.96	0.00	0.00	6.01	0.00	0.00	-0.01	1.44
time (sec)	N/A	0.153	0.236	0.921	0.000	0.546	0.000	0.000	0.000	1.107

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	90	0	0	632	0	156	-1	144
N.S.	1	1.00	0.99	0.00	0.00	6.95	0.00	1.71	-0.01	1.58
time (sec)	N/A	0.087	0.091	0.610	0.000	0.525	0.000	0.234	0.000	0.622
Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	95	0	0	245	0	72	-1	106
N.S.	1	1.00	1.76	0.00	0.00	4.54	0.00	1.33	-0.02	1.96
time (sec)	N/A	0.049	0.090	0.605	0.000	0.495	0.000	0.247	0.000	0.399
Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	80	80	179	0	0	332	0	0	-1	142
N.S.	1	1.00	2.24	0.00	0.00	4.15	0.00	0.00	-0.01	1.78
time (sec)	N/A	0.089	1.169	0.644	0.000	0.511	0.000	0.000	0.000	0.657
Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	115	115	253	0	0	416	0	0	-1	163
N.S.	1	1.00	2.20	0.00	0.00	3.62	0.00	0.00	-0.01	1.42
time (sec)	N/A	0.163	4.650	0.658	0.000	0.530	0.000	0.000	0.000	1.296

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	107	0	0	475	0	134	144	143
N.S.	1	1.00	0.87	0.00	0.00	3.86	0.00	1.09	1.17	1.16
time (sec)	N/A	0.140	0.357	0.454	0.000	0.646	0.000	0.441	5.093	0.327

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	98	0	0	348	0	116	95	109
N.S.	1	1.00	0.99	0.00	0.00	3.52	0.00	1.17	0.96	1.10
time (sec)	N/A	0.080	0.096	0.632	0.000	0.688	0.000	0.289	4.989	0.172

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	85	0	0	302	0	93	84	97
N.S.	1	1.00	0.98	0.00	0.00	3.47	0.00	1.07	0.97	1.11
time (sec)	N/A	0.069	0.101	0.619	0.000	0.754	0.000	0.302	4.929	0.110

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	123	0	0	862	0	139	3025	146
N.S.	1	1.00	0.93	0.00	0.00	6.53	0.00	1.05	22.92	1.11
time (sec)	N/A	0.127	0.354	0.775	0.000	0.925	0.000	0.407	6.226	0.319

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	163	0	0	1236	0	257	3860	187
N.S.	1	1.00	0.88	0.00	0.00	6.68	0.00	1.39	20.86	1.01
time (sec)	N/A	0.228	0.683	0.559	0.000	0.919	0.000	0.410	7.290	0.656
Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	135	0	0	1077	0	343	-1	166
N.S.	1	1.00	0.96	0.00	0.00	7.64	0.00	2.43	-0.01	1.18
time (sec)	N/A	0.158	0.276	0.627	0.000	1.408	0.000	0.511	0.000	2.625
Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	124	0	0	426	0	0	-1	145
N.S.	1	1.00	1.33	0.00	0.00	4.58	0.00	0.00	-0.01	1.56
time (sec)	N/A	0.090	0.581	0.656	0.000	0.790	0.000	0.000	0.000	1.328
Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	104	104	407	0	0	467	0	237	-1	124
N.S.	1	1.00	3.91	0.00	0.00	4.49	0.00	2.28	-0.01	1.19
time (sec)	N/A	0.098	0.998	0.605	0.000	0.592	0.000	0.284	0.000	0.805

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	149	149	869	0	0	612	0	0	-1	159
N.S.	1	1.00	5.83	0.00	0.00	4.11	0.00	0.00	-0.01	1.07
time (sec)	N/A	0.194	5.401	0.483	0.000	0.608	0.000	0.000	0.000	1.634

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	208	208	175	0	0	760	0	0	-1	217
N.S.	1	1.00	0.84	0.00	0.00	3.65	0.00	0.00	-0.00	1.04
time (sec)	N/A	0.308	5.858	0.448	0.000	0.775	0.000	0.000	0.000	2.883

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	91	0	0	288	0	106	103	101
N.S.	1	1.00	0.88	0.00	0.00	2.77	0.00	1.02	0.99	0.97
time (sec)	N/A	0.099	0.216	0.593	0.000	0.428	0.000	0.160	4.678	0.159

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	72	0	0	205	0	64	58	84
N.S.	1	1.00	0.97	0.00	0.00	2.77	0.00	0.86	0.78	1.14
time (sec)	N/A	0.061	0.078	0.642	0.000	0.459	0.000	0.157	4.730	0.096

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	51	0	0	130	37	40	40	61
N.S.	1	1.00	1.00	0.00	0.00	2.55	0.73	0.78	0.78	1.20
time (sec)	N/A	0.047	0.017	0.625	0.000	0.421	44.568	0.166	4.586	0.053
Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	81	0	0	431	66	71	652	95
N.S.	1	1.00	0.95	0.00	0.00	5.07	0.78	0.84	7.67	1.12
time (sec)	N/A	0.079	0.099	0.683	0.000	0.478	37.858	0.159	4.813	0.115
Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	151	0	0	565	0	104	396	127
N.S.	1	1.00	1.29	0.00	0.00	4.83	0.00	0.89	3.38	1.09
time (sec)	N/A	0.113	0.139	0.637	0.000	0.469	0.000	0.176	5.507	0.262
Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	118	0	0	739	0	0	-1	177
N.S.	1	1.00	0.96	0.00	0.00	6.01	0.00	0.00	-0.01	1.44
time (sec)	N/A	0.131	0.240	0.581	0.000	0.558	0.000	0.000	0.000	1.902

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	90	0	0	632	0	0	-1	144
N.S.	1	1.00	0.99	0.00	0.00	6.95	0.00	0.00	-0.01	1.58
time (sec)	N/A	0.110	0.068	0.596	0.000	0.522	0.000	0.000	0.000	0.944
Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	95	0	0	245	0	72	-1	106
N.S.	1	1.00	1.76	0.00	0.00	4.54	0.00	1.33	-0.02	1.96
time (sec)	N/A	0.055	0.068	0.616	0.000	0.486	0.000	0.203	0.000	0.490
Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	80	80	179	0	0	332	0	116	-1	142
N.S.	1	1.00	2.24	0.00	0.00	4.15	0.00	1.45	-0.01	1.78
time (sec)	N/A	0.087	0.832	0.664	0.000	0.503	0.000	0.229	0.000	0.799
Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	115	115	253	0	0	416	0	205	-1	163
N.S.	1	1.00	2.20	0.00	0.00	3.62	0.00	1.78	-0.01	1.42
time (sec)	N/A	0.161	2.273	0.662	0.000	0.524	0.000	1.686	0.000	2.183

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	107	0	0	475	0	134	144	143
N.S.	1	1.00	0.87	0.00	0.00	3.86	0.00	1.09	1.17	1.16
time (sec)	N/A	0.142	0.257	0.469	0.000	0.474	0.000	0.177	5.021	0.326
Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	98	0	0	348	0	116	95	109
N.S.	1	1.00	0.99	0.00	0.00	3.52	0.00	1.17	0.96	1.10
time (sec)	N/A	0.083	0.092	0.594	0.000	0.434	0.000	0.169	4.837	0.169
Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	85	0	0	302	0	93	84	97
N.S.	1	1.00	0.98	0.00	0.00	3.47	0.00	1.07	0.97	1.11
time (sec)	N/A	0.073	0.096	0.603	0.000	0.438	0.000	0.160	4.797	0.107
Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	123	0	0	862	0	139	3017	146
N.S.	1	1.00	0.93	0.00	0.00	6.53	0.00	1.05	22.86	1.11
time (sec)	N/A	0.137	0.355	0.799	0.000	0.490	0.000	0.209	5.820	0.304

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	163	0	0	1236	0	257	3832	187
N.S.	1	1.00	0.88	0.00	0.00	6.68	0.00	1.39	20.71	1.01
time (sec)	N/A	0.229	0.602	0.658	0.000	0.556	0.000	0.172	7.851	0.647
Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	135	0	0	1077	0	298	-1	166
N.S.	1	1.00	0.96	0.00	0.00	7.64	0.00	2.11	-0.01	1.18
time (sec)	N/A	0.160	0.264	0.647	0.000	0.819	0.000	0.652	0.000	3.667
Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	124	0	0	426	0	244	-1	145
N.S.	1	1.00	1.33	0.00	0.00	4.58	0.00	2.62	-0.01	1.56
time (sec)	N/A	0.086	0.549	0.642	0.000	0.539	0.000	1.595	0.000	1.815
Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	104	104	407	0	0	467	0	237	-1	124
N.S.	1	1.00	3.91	0.00	0.00	4.49	0.00	2.28	-0.01	1.19
time (sec)	N/A	0.090	0.941	0.601	0.000	0.577	0.000	0.389	0.000	0.985

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	149	149	869	0	0	612	0	418	-1	159
N.S.	1	1.00	5.83	0.00	0.00	4.11	0.00	2.81	-0.01	1.07
time (sec)	N/A	0.185	2.034	0.439	0.000	0.610	0.000	1.764	0.000	1.972
Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	208	208	175	0	0	760	0	395	-1	217
N.S.	1	1.00	0.84	0.00	0.00	3.65	0.00	1.90	-0.00	1.04
time (sec)	N/A	0.297	5.847	0.455	0.000	0.763	0.000	2.277	0.000	4.942
Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	121	162	243	242	226	143	134	111
N.S.	1	1.00	0.98	1.32	1.98	1.97	1.84	1.16	1.09	0.90
time (sec)	N/A	0.094	0.257	0.057	1.361	0.437	72.291	0.193	5.713	0.153
Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	100	122	159	191	144	105	93	87
N.S.	1	1.00	1.11	1.36	1.77	2.12	1.60	1.17	1.03	0.97
time (sec)	N/A	0.068	0.136	0.055	1.240	0.438	59.054	0.257	5.305	0.154

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	71	129	108	155	107	92	68	68
N.S.	1	1.00	0.85	1.54	1.29	1.85	1.27	1.10	0.81	0.81
time (sec)	N/A	0.058	0.188	0.056	1.206	0.433	60.432	0.289	5.104	0.109
Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	82	109	67	166	75	163	57	74
N.S.	1	1.00	1.39	1.85	1.14	2.81	1.27	2.76	0.97	1.25
time (sec)	N/A	0.043	0.155	0.055	1.226	0.441	23.441	0.488	5.206	0.085
Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	47	48	49	60	58	250	91	66
N.S.	1	1.00	1.02	1.04	1.07	1.30	1.26	5.43	1.98	1.43
time (sec)	N/A	0.037	0.017	0.046	0.724	0.432	4.022	0.918	4.817	0.056
Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	69	70	84	85	78	310	126	90
N.S.	1	1.00	0.93	0.95	1.14	1.15	1.05	4.19	1.70	1.22
time (sec)	N/A	0.059	0.025	0.046	0.599	0.432	4.564	1.382	4.924	0.064

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	79	94	118	109	112	370	168	114
N.S.	1	1.00	0.76	0.90	1.13	1.05	1.08	3.56	1.62	1.10
time (sec)	N/A	0.079	0.064	0.048	0.565	0.482	5.210	2.061	5.224	0.080
Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	90	118	152	133	146	430	210	138
N.S.	1	1.00	0.67	0.88	1.13	0.99	1.09	3.21	1.57	1.03
time (sec)	N/A	0.102	0.078	0.048	0.466	0.526	5.863	2.982	5.607	0.081
Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	108	113	158	131	1386	175	117	112
N.S.	1	1.00	0.72	0.75	1.05	0.87	9.24	1.17	0.78	0.75
time (sec)	N/A	0.073	0.084	0.054	0.643	0.432	6.441	0.220	4.591	0.105
Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	86	89	124	107	910	140	97	88
N.S.	1	1.00	0.74	0.76	1.06	0.91	7.78	1.20	0.83	0.75
time (sec)	N/A	0.062	0.065	0.047	0.564	0.418	4.920	0.168	4.518	0.090

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	64	65	90	82	422	105	77	64
N.S.	1	1.00	0.76	0.77	1.07	0.98	5.02	1.25	0.92	0.76
time (sec)	N/A	0.040	0.046	0.058	0.528	0.417	3.992	0.166	4.488	0.086
Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	42	43	55	57	119	72	54	42
N.S.	1	1.00	0.79	0.81	1.04	1.08	2.25	1.36	1.02	0.79
time (sec)	N/A	0.025	0.027	0.050	0.661	0.417	3.040	0.213	4.442	0.067
Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	84	83	75	156	107	116	80	83
N.S.	1	1.00	1.27	1.26	1.14	2.36	1.62	1.76	1.21	1.26
time (sec)	N/A	0.037	0.057	0.056	1.376	0.439	3.266	0.190	4.720	0.094
Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	75	135	133	164	107	76	97	89
N.S.	1	1.00	0.88	1.59	1.56	1.93	1.26	0.89	1.14	1.05
time (sec)	N/A	0.046	0.090	0.053	1.224	0.446	4.484	0.249	5.108	0.152

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	100	175	193	194	144	130	-1	103
N.S.	1	1.00	1.10	1.92	2.12	2.13	1.58	1.43	-0.01	1.13
time (sec)	N/A	0.051	0.185	0.059	1.355	0.437	6.972	0.240	0.000	0.179
Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	68	220	277	244	226	153	-1	128
N.S.	1	1.00	0.55	1.79	2.25	1.98	1.84	1.24	-0.01	1.04
time (sec)	N/A	0.081	0.030	0.059	1.273	0.455	11.421	0.301	0.000	0.238
Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	123	162	240	243	253	144	130	112
N.S.	1	1.00	1.00	1.32	1.95	1.98	2.06	1.17	1.06	0.91
time (sec)	N/A	0.090	0.142	0.060	1.298	0.448	100.225	0.233	5.783	0.190
Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	89	174	171	203	0	126	105	88
N.S.	1	1.00	0.77	1.51	1.49	1.77	0.00	1.10	0.91	0.77
time (sec)	N/A	0.084	0.245	0.055	1.327	0.455	0.000	0.293	5.703	0.143

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	78	216	134	195	187	225	95	93
N.S.	1	1.00	0.71	1.96	1.22	1.77	1.70	2.05	0.86	0.85
time (sec)	N/A	0.073	0.095	0.063	1.233	0.442	56.156	0.541	5.651	0.167
Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	90	153	80	213	73	254	72	96
N.S.	1	1.00	1.18	2.01	1.05	2.80	0.96	3.34	0.95	1.26
time (sec)	N/A	0.054	0.071	0.066	1.310	0.435	54.144	1.068	5.831	0.113
Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	49	48	49	84	138	370	122	90
N.S.	1	1.00	1.07	1.04	1.07	1.83	3.00	8.04	2.65	1.96
time (sec)	N/A	0.036	0.022	0.049	0.593	0.424	14.156	1.866	5.333	0.065
Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	71	70	84	109	194	430	164	114
N.S.	1	1.00	0.96	0.95	1.14	1.47	2.62	5.81	2.22	1.54
time (sec)	N/A	0.056	0.033	0.049	0.642	0.481	15.608	2.691	5.759	0.074

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	94	94	118	134	262	490	206	138
N.S.	1	1.00	0.90	0.90	1.13	1.29	2.52	4.71	1.98	1.33
time (sec)	N/A	0.072	0.040	0.053	0.580	0.541	17.961	3.285	6.315	0.083
Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	115	118	152	157	326	550	248	162
N.S.	1	1.00	0.86	0.88	1.13	1.17	2.43	4.10	1.85	1.21
time (sec)	N/A	0.096	0.061	0.052	0.630	0.608	19.447	4.630	6.812	0.089
Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	110	115	158	155	3351	175	137	114
N.S.	1	1.00	0.73	0.77	1.05	1.03	22.34	1.17	0.91	0.76
time (sec)	N/A	0.072	0.110	0.049	0.635	0.418	12.596	0.183	4.662	0.130
Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	89	91	124	132	2304	140	118	90
N.S.	1	1.00	0.76	0.78	1.06	1.13	19.69	1.20	1.01	0.77
time (sec)	N/A	0.057	0.090	0.051	0.490	0.436	9.646	0.179	4.569	0.102

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	66	67	90	106	1340	105	97	66
N.S.	1	1.00	0.79	0.80	1.07	1.26	15.95	1.25	1.15	0.79
time (sec)	N/A	0.042	0.055	0.055	0.755	0.425	7.638	0.171	4.551	0.093
Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	44	45	55	80	498	72	77	44
N.S.	1	1.00	0.83	0.85	1.04	1.51	9.40	1.36	1.45	0.83
time (sec)	N/A	0.025	0.036	0.044	0.647	0.410	5.948	0.183	4.626	0.080
Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	81	99	91	203	184	140	-1	107
N.S.	1	1.00	0.94	1.15	1.06	2.36	2.14	1.63	-0.01	1.24
time (sec)	N/A	0.055	0.147	0.059	1.350	0.452	5.249	0.200	0.000	0.122
Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	105	170	163	190	202	115	-1	109
N.S.	1	1.00	0.87	1.40	1.35	1.57	1.67	0.95	-0.01	0.90
time (sec)	N/A	0.058	0.088	0.061	1.359	0.439	7.646	0.265	0.000	0.183

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	68	213	207	216	216	145	78	107
N.S.	1	1.00	0.61	1.90	1.85	1.93	1.93	1.29	0.70	0.96
time (sec)	N/A	0.062	0.034	0.058	1.424	0.439	11.776	0.257	5.858	0.217
Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	126	259	275	246	253	173	-1	127
N.S.	1	1.00	1.02	2.11	2.24	2.00	2.06	1.41	-0.01	1.03
time (sec)	N/A	0.064	0.244	0.066	1.383	0.463	18.949	0.299	0.000	0.254
Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	71	302	354	298	287	214	-1	152
N.S.	1	1.00	0.45	1.90	2.23	1.87	1.81	1.35	-0.01	0.96
time (sec)	N/A	0.089	0.049	0.073	1.573	0.474	29.163	0.374	0.000	0.306
Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	95	129	178	192	150	113	99	88
N.S.	1	1.00	1.06	1.43	1.98	2.13	1.67	1.26	1.10	0.98
time (sec)	N/A	0.067	0.112	0.059	1.171	0.436	71.064	0.256	5.348	0.140

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	79	90	109	146	66	88	59	67
N.S.	1	1.00	1.34	1.53	1.85	2.47	1.12	1.49	1.00	1.14
time (sec)	N/A	0.040	0.055	0.052	1.195	0.440	84.411	0.259	5.077	0.098

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	73	70	54	130	63	0	35	51
N.S.	1	1.00	1.70	1.63	1.26	3.02	1.47	0.00	0.81	1.19
time (sec)	N/A	0.033	0.049	0.054	1.105	0.439	22.339	0.000	4.851	0.068

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	39	47	48	39	138	88	35	44
N.S.	1	1.00	0.91	1.09	1.12	0.91	3.21	2.05	0.81	1.02
time (sec)	N/A	0.036	0.048	0.049	0.534	0.409	7.054	0.410	4.565	0.049

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	60	70	83	62	204	153	58	66
N.S.	1	1.00	0.83	0.97	1.15	0.86	2.83	2.12	0.81	0.92
time (sec)	N/A	0.052	0.039	0.047	0.489	0.430	13.172	0.498	4.676	0.065

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	91	94	118	86	269	219	102	90
N.S.	1	1.00	0.90	0.93	1.17	0.85	2.66	2.17	1.01	0.89
time (sec)	N/A	0.069	0.048	0.047	0.576	0.430	18.001	0.623	4.716	0.073
Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	56	67	85	59	338	0	53	57
N.S.	1	1.00	0.68	0.82	1.04	0.72	4.12	0.00	0.65	0.70
time (sec)	N/A	0.033	0.075	0.052	0.491	0.413	3.553	0.000	5.314	0.083
Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	34	44	49	36	70	0	67	34
N.S.	1	1.00	0.67	0.86	0.96	0.71	1.37	0.00	1.31	0.67
time (sec)	N/A	0.020	0.056	0.044	0.592	0.424	2.755	0.000	4.917	0.062
Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	71	73	58	131	39	0	65	67
N.S.	1	1.00	1.51	1.55	1.23	2.79	0.83	0.00	1.38	1.43
time (sec)	N/A	0.029	0.041	0.051	1.276	0.444	2.856	0.000	4.998	0.081

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	80	105	121	144	66	0	94	82
N.S.	1	1.00	1.31	1.72	1.98	2.36	1.08	0.00	1.54	1.34
time (sec)	N/A	0.038	0.064	0.057	1.177	0.423	4.921	0.000	5.529	0.116
Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	107	146	200	201	150	0	-1	104
N.S.	1	1.00	1.15	1.57	2.15	2.16	1.61	0.00	-0.01	1.12
time (sec)	N/A	0.050	0.179	0.063	1.234	0.454	8.461	0.000	0.000	0.201
Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	120	111	140	215	304	177	0	134	119
N.S.	1	1.02	0.94	1.19	1.82	2.58	1.50	0.00	1.14	1.01
time (sec)	N/A	0.085	0.192	0.060	1.385	0.462	103.904	0.000	6.339	0.183
Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	89	115	144	249	264	0	90	93
N.S.	1	1.00	1.03	1.34	1.67	2.90	3.07	0.00	1.05	1.08
time (sec)	N/A	0.058	0.217	0.058	1.204	0.438	52.419	0.000	5.609	0.178

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	73	75	69	200	49	0	54	73
N.S.	1	1.00	1.40	1.44	1.33	3.85	0.94	0.00	1.04	1.40
time (sec)	N/A	0.040	0.094	0.058	1.260	0.461	20.333	0.000	5.058	0.110
Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	36	46	46	46	68	37	46	46
N.S.	1	1.00	0.86	1.10	1.10	1.10	1.62	0.88	1.10	1.10
time (sec)	N/A	0.035	0.021	0.052	0.461	0.422	3.421	0.264	4.519	0.061
Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	60	69	81	73	61	0	66	75
N.S.	1	1.00	0.88	1.01	1.19	1.07	0.90	0.00	0.97	1.10
time (sec)	N/A	0.052	0.027	0.050	0.611	0.422	11.071	0.000	4.642	0.080
Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	81	94	116	98	90	0	91	99
N.S.	1	1.00	0.81	0.94	1.16	0.98	0.90	0.00	0.91	0.99
time (sec)	N/A	0.071	0.030	0.053	0.615	0.429	13.279	0.000	4.837	0.099

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	104	118	151	121	122	0	154	123
N.S.	1	1.00	0.83	0.94	1.20	0.96	0.97	0.00	1.22	0.98
time (sec)	N/A	0.089	0.035	0.054	0.549	0.454	16.143	0.000	4.920	0.108

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	80	91	128	95	561	0	79	90
N.S.	1	1.00	0.72	0.82	1.15	0.86	5.05	0.00	0.71	0.81
time (sec)	N/A	0.049	0.076	0.055	0.558	0.425	7.831	0.000	5.768	0.104

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	57	66	90	70	267	0	81	65
N.S.	1	1.00	0.72	0.84	1.14	0.89	3.38	0.00	1.03	0.82
time (sec)	N/A	0.030	0.050	0.061	0.533	0.429	7.306	0.000	5.195	0.090

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	33	43	53	47	65	0	38	40
N.S.	1	1.00	0.73	0.96	1.18	1.04	1.44	0.00	0.84	0.89
time (sec)	N/A	0.030	0.026	0.044	0.681	0.479	7.513	0.000	4.898	0.079

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	71	79	80	195	206	0	60	77
N.S.	1	1.00	1.20	1.34	1.36	3.31	3.49	0.00	1.02	1.31
time (sec)	N/A	0.034	0.039	0.059	1.215	0.463	11.714	0.000	5.106	0.117
Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	B	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	57	132	162	248	262	0	-1	101
N.S.	1	1.00	0.62	1.43	1.76	2.70	2.85	0.00	-0.01	1.10
time (sec)	N/A	0.050	0.029	0.059	1.218	0.439	18.437	0.000	0.000	0.242
Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	60	157	243	314	180	0	-1	126
N.S.	1	1.00	0.49	1.28	1.98	2.55	1.46	0.00	-0.01	1.02
time (sec)	N/A	0.069	0.030	0.063	1.284	0.446	29.807	0.000	0.000	0.249
Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	111	75	57	62	0	162	831	216
N.S.	1	1.00	0.82	0.56	0.42	0.46	0.00	1.20	6.16	1.60
time (sec)	N/A	0.061	0.069	0.054	0.546	0.394	0.000	0.290	52.028	2.437

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	99	65	47	57	0	127	632	176
N.S.	1	1.00	0.95	0.62	0.45	0.55	0.00	1.22	6.08	1.69
time (sec)	N/A	0.049	0.056	0.049	0.489	0.410	0.000	0.217	31.390	1.303
Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	87	52	37	52	0	92	-1	404
N.S.	1	1.00	1.19	0.71	0.51	0.71	0.00	1.26	-0.01	5.53
time (sec)	N/A	0.032	0.075	0.047	0.581	0.405	0.000	0.189	0.000	1.475
Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	72	72	26	46	0	57	41	268
N.S.	1	1.00	1.95	1.95	0.70	1.24	0.00	1.54	1.11	7.24
time (sec)	N/A	0.021	0.035	0.050	0.494	0.412	0.000	0.175	5.069	1.175
Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	74	47	27	55	0	0	129	184
N.S.	1	1.00	1.10	0.70	0.40	0.82	0.00	0.00	1.93	2.75
time (sec)	N/A	0.031	0.079	0.054	1.280	0.396	0.000	0.000	6.249	1.133

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	23	10	30	0	48	31	512
N.S.	1	1.00	1.00	0.74	0.32	0.97	0.00	1.55	1.00	16.52
time (sec)	N/A	0.009	0.016	0.052	1.320	0.421	0.000	0.229	5.257	2.382
Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	36	28	21	37	0	90	43	836
N.S.	1	1.00	0.57	0.44	0.33	0.59	0.00	1.43	0.68	13.27
time (sec)	N/A	0.020	0.021	0.052	1.312	0.408	0.000	0.280	5.075	10.092
Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	41	33	31	44	0	111	55	1160
N.S.	1	1.00	0.44	0.35	0.33	0.47	0.00	1.18	0.59	12.34
time (sec)	N/A	0.031	0.031	0.054	1.550	0.397	0.000	0.290	5.038	36.638
Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	46	38	41	49	0	132	67	1484
N.S.	1	1.00	0.37	0.30	0.33	0.39	0.00	1.06	0.54	11.87
time (sec)	N/A	0.042	0.033	0.061	1.217	0.405	0.000	0.426	5.023	111.252

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	104	104	67	65	47	57	0	76	632	176
N.S.	1	1.00	0.64	0.62	0.45	0.55	0.00	0.73	6.08	1.69
time (sec)	N/A	0.045	0.054	0.058	0.612	0.406	0.000	0.231	27.092	1.421
Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	73	73	62	55	37	52	0	59	429	136
N.S.	1	1.00	0.85	0.75	0.51	0.71	0.00	0.81	5.88	1.86
time (sec)	N/A	0.032	0.036	0.063	0.460	0.405	0.000	0.227	18.765	1.323
Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	55	41	24	46	83	39	-1	268
N.S.	1	1.00	1.57	1.17	0.69	1.31	2.37	1.11	-0.03	7.66
time (sec)	N/A	0.021	0.012	0.061	0.453	0.413	27.907	0.223	0.000	1.268
Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	26	40	16	27	0	20	6	38
N.S.	1	1.00	3.25	5.00	2.00	3.38	0.00	2.50	0.75	4.75
time (sec)	N/A	0.011	0.007	0.049	0.647	0.407	0.000	0.195	5.287	1.016

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	20	10	25	0	25	19	188
N.S.	1	1.00	1.00	0.69	0.34	0.86	0.00	0.86	0.66	6.48
time (sec)	N/A	0.010	0.010	0.056	1.208	0.416	0.000	0.222	5.561	1.137
Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	36	25	21	34	0	48	33	512
N.S.	1	1.00	0.57	0.40	0.33	0.54	0.00	0.76	0.52	8.13
time (sec)	N/A	0.020	0.014	0.058	1.200	0.400	0.000	0.222	5.504	2.746
Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	41	30	31	39	0	69	43	127
N.S.	1	1.00	0.44	0.32	0.33	0.41	0.00	0.73	0.46	1.35
time (sec)	N/A	0.033	0.029	0.064	1.297	0.407	0.000	0.257	5.658	1.371
Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	36	15	11	10	16	11	0
N.S.	1	1.00	1.00	2.40	1.00	0.73	0.67	1.07	0.73	0.00
time (sec)	N/A	0.014	0.006	0.052	0.455	0.399	0.122	0.185	0.087	0.001

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	110	0	36	40	39	38	31	0
N.S.	1	1.00	5.00	0.00	1.64	1.82	1.77	1.73	1.41	0.00
time (sec)	N/A	0.015	0.166	0.674	0.954	0.411	8.426	0.292	4.942	0.258
Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	56	69	69	58	332	0	162	67
N.S.	1	1.00	0.89	1.10	1.10	0.92	5.27	0.00	2.57	1.06
time (sec)	N/A	0.068	0.065	0.099	0.565	0.450	4.025	0.000	5.724	0.069
Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	97	131	151	224	0	0	-1	109
N.S.	1	1.00	0.96	1.30	1.50	2.22	0.00	0.00	-0.01	1.08
time (sec)	N/A	0.105	0.329	0.105	0.665	0.466	0.000	0.000	0.000	0.128
Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	115	284	231	177	320	0	-1	155
N.S.	1	1.00	0.88	2.18	1.78	1.36	2.46	0.00	-0.01	1.19
time (sec)	N/A	0.138	0.354	0.082	0.531	0.454	135.872	0.000	0.000	0.106

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	82	173	150	108	202	0	-1	97
N.S.	1	1.00	0.91	1.92	1.67	1.20	2.24	0.00	-0.01	1.08
time (sec)	N/A	0.089	0.138	0.077	0.698	0.441	63.508	0.000	0.000	0.079
Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	50	87	83	56	105	0	-1	55
N.S.	1	1.00	0.83	1.45	1.38	0.93	1.75	0.00	-0.02	0.92
time (sec)	N/A	0.054	0.058	0.071	0.699	0.431	26.972	0.000	0.000	0.052
Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	44	59	60	45	0	0	-1	55
N.S.	1	1.00	0.81	1.09	1.11	0.83	0.00	0.00	-0.02	1.02
time (sec)	N/A	0.054	0.045	0.070	0.586	0.444	0.000	0.000	0.000	0.060
Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	58	109	121	120	0	0	-1	75
N.S.	1	1.00	0.77	1.45	1.61	1.60	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.062	0.135	0.092	0.515	0.447	0.000	0.000	0.000	0.090

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	97	203	243	267	0	0	-1	100
N.S.	1	1.00	0.92	1.93	2.31	2.54	0.00	0.00	-0.01	0.95
time (sec)	N/A	0.090	0.168	0.112	0.663	0.462	0.000	0.000	0.000	0.125

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	138	342	286	230	0	0	-1	220
N.S.	1	1.00	0.87	2.16	1.81	1.46	0.00	0.00	-0.01	1.39
time (sec)	N/A	0.151	0.314	0.070	0.663	0.486	0.000	0.000	0.000	0.134

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	103	236	192	146	0	0	-1	141
N.S.	1	1.00	0.87	2.00	1.63	1.24	0.00	0.00	-0.01	1.19
time (sec)	N/A	0.115	0.269	0.072	0.541	0.477	0.000	0.000	0.000	0.102

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	78	125	112	82	139	0	-1	82
N.S.	1	1.00	0.91	1.45	1.30	0.95	1.62	0.00	-0.01	0.95
time (sec)	N/A	0.076	0.099	0.072	0.534	0.435	57.945	0.000	0.000	0.069

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	66	78	81	74	0	0	-1	70
N.S.	1	1.00	0.93	1.10	1.14	1.04	0.00	0.00	-0.01	0.99
time (sec)	N/A	0.069	0.105	0.096	0.626	0.463	0.000	0.000	0.000	0.073
Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	90	163	147	166	0	0	-1	95
N.S.	1	1.00	0.95	1.72	1.55	1.75	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.090	0.110	0.080	0.582	0.451	0.000	0.000	0.000	0.114
Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	112	214	262	301	0	0	-1	110
N.S.	1	1.00	0.93	1.78	2.18	2.51	0.00	0.00	-0.01	0.92
time (sec)	N/A	0.105	0.275	0.121	0.677	0.454	0.000	0.000	0.000	0.146
Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	172	155	154	154	175	13	154	0
N.S.	1	1.00	12.29	11.07	11.00	11.00	12.50	0.93	11.00	0.00
time (sec)	N/A	0.002	0.006	0.040	0.566	0.354	0.115	0.148	0.156	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	182	157	156	156	182	156	156	0
N.S.	1	1.00	11.38	9.81	9.75	9.75	11.38	9.75	9.75	0.00
time (sec)	N/A	0.041	0.006	0.038	0.731	0.359	0.122	0.188	4.925	0.000
Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	186	157	156	156	185	156	156	0
N.S.	1	1.00	11.62	9.81	9.75	9.75	11.56	9.75	9.75	0.00
time (sec)	N/A	0.036	0.021	0.038	0.624	0.360	0.121	0.163	4.772	0.000
Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	230	229	189	0	189	229	21
N.S.	1	1.00	1.00	10.95	10.90	9.00	0.00	9.00	10.90	1.00
time (sec)	N/A	0.016	0.190	0.079	0.644	0.457	0.000	0.572	5.209	0.041
Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	107	0	16	32	0	70	25	0
N.S.	1	1.00	8.23	0.00	1.23	2.46	0.00	5.38	1.92	0.00
time (sec)	N/A	0.014	0.184	0.675	0.842	0.433	0.000	0.279	4.810	0.096

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	9	9	9	9	10	8	11	8	0
N.S.	1	1.12	1.12	1.12	1.12	1.25	1.00	1.38	1.00	0.00
time (sec)	N/A	0.006	0.018	0.042	0.478	0.400	0.124	0.152	4.662	0.001
Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	17	13	12	18	13	0
N.S.	1	1.00	1.00	0.93	1.13	0.87	0.80	1.20	0.87	0.00
time (sec)	N/A	0.015	0.009	0.053	0.488	0.397	0.178	0.151	0.060	0.001
Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	17	13	12	15	13	0
N.S.	1	1.00	1.00	0.93	1.13	0.87	0.80	1.00	0.87	0.00
time (sec)	N/A	0.014	0.020	0.048	0.554	0.403	0.193	0.162	4.628	0.001
Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	17	47	17	29	0	15	24
N.S.	1	1.00	1.00	1.13	3.13	1.13	1.93	0.00	1.00	1.60
time (sec)	N/A	0.015	0.016	0.047	0.489	0.435	0.619	0.000	4.723	0.032

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	177	81	81	87	13	12	0
N.S.	1	1.00	1.00	12.64	5.79	5.79	6.21	0.93	0.86	0.00
time (sec)	N/A	0.002	0.024	0.061	0.612	0.423	0.886	0.170	7.087	0.001

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	197	81	81	87	15	14	0
N.S.	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88	0.00
time (sec)	N/A	0.014	0.047	0.063	0.638	0.408	1.356	0.155	2.315	0.001

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	197	81	81	87	15	14	0
N.S.	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88	0.00
time (sec)	N/A	0.013	0.054	0.062	0.614	0.394	1.908	0.169	9.981	0.001

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	203	612	105	0	0	105	21
N.S.	1	1.00	1.00	9.67	29.14	5.00	0.00	0.00	5.00	1.00
time (sec)	N/A	0.013	0.251	0.098	0.787	0.476	0.000	0.000	4.987	0.096

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	44	86	53	46	0	56	37	46
N.S.	1	1.00	0.85	1.65	1.02	0.88	0.00	1.08	0.71	0.88
time (sec)	N/A	0.028	0.043	1.147	1.252	0.423	0.000	0.155	4.817	0.071
Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	142	143	0	757	0	0	-1	442
N.S.	1	1.00	1.53	1.54	0.00	8.14	0.00	0.00	-0.01	4.75
time (sec)	N/A	0.086	0.682	0.102	0.000	0.706	0.000	0.000	0.000	2.670
Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	223	0	0	607	0	0	-1	0
N.S.	1	1.00	0.88	0.00	0.00	2.41	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.234	1.093	1.016	0.000	0.542	0.000	0.000	0.000	0.952
Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	178	0	0	469	0	0	-1	0
N.S.	1	1.00	0.89	0.00	0.00	2.36	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.164	0.672	0.961	0.000	0.496	0.000	0.000	0.000	0.428

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	141	0	0	359	0	0	-1	0
N.S.	1	1.00	0.97	0.00	0.00	2.46	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.121	0.513	1.012	0.000	0.461	0.000	0.000	0.000	0.141

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	123	0	0	281	0	0	-1	0
N.S.	1	1.00	1.38	0.00	0.00	3.16	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.088	0.321	0.974	0.000	0.524	0.000	0.000	0.000	0.144

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	122	0	0	408	0	0	-1	0
N.S.	1	1.00	1.34	0.00	0.00	4.48	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.089	0.901	0.947	0.000	0.597	0.000	0.000	0.000	0.315

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	57	0	0	135	0	0	-1	0
N.S.	1	1.00	0.60	0.00	0.00	1.42	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.065	0.075	0.918	0.000	0.739	0.000	0.000	0.000	0.859

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	358	358	274	0	0	771	0	0	-1	0
N.S.	1	1.00	0.77	0.00	0.00	2.15	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.405	2.024	0.965	0.000	0.584	0.000	0.000	0.000	0.959
Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	241	0	0	607	0	0	-1	0
N.S.	1	1.00	0.83	0.00	0.00	2.09	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.316	1.007	0.934	0.000	0.542	0.000	0.000	0.000	0.423
Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	221	221	191	0	0	471	0	0	-1	0
N.S.	1	1.00	0.86	0.00	0.00	2.13	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.245	0.624	0.993	0.000	0.513	0.000	0.000	0.000	0.143
Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	157	0	0	361	0	0	-1	0
N.S.	1	1.00	1.05	0.00	0.00	2.41	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.152	0.464	0.997	0.000	0.500	0.000	0.000	0.000	0.145

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	185	0	0	540	0	0	-1	0
N.S.	1	1.00	1.39	0.00	0.00	4.06	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.161	0.522	1.003	0.000	0.574	0.000	0.000	0.000	0.312

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	217	0	0	769	0	0	-1	0
N.S.	1	1.00	1.48	0.00	0.00	5.23	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.142	0.812	1.091	0.000	0.794	0.000	0.000	0.000	0.851

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	21	29	25	46	35	22	0
N.S.	1	1.00	1.00	1.05	1.45	1.25	2.30	1.75	1.10	0.00
time (sec)	N/A	0.004	0.018	0.046	0.824	0.420	3.098	0.164	4.795	0.036

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	97	26	35	32	0	52	47	0
N.S.	1	1.00	3.59	0.96	1.30	1.19	0.00	1.93	1.74	0.00
time (sec)	N/A	0.009	0.132	0.044	0.889	0.433	0.000	0.191	4.882	0.040

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	97	26	35	32	0	56	47	0
N.S.	1	1.00	3.59	0.96	1.30	1.19	0.00	2.07	1.74	0.00
time (sec)	N/A	0.009	0.096	0.045	0.692	0.434	0.000	0.262	4.902	0.045

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	101	0	39	35	0	66	54	0
N.S.	1	1.00	3.74	0.00	1.44	1.30	0.00	2.44	2.00	0.00
time (sec)	N/A	0.012	0.162	0.688	0.957	0.451	0.000	0.253	4.863	0.109

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [102] had the largest ratio of [.4706]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	18	0.111
2	A	2	1	1.00	16	0.062
3	A	2	1	1.00	15	0.067
4	A	3	2	1.00	18	0.111
5	A	2	1	1.00	18	0.056
6	A	2	1	1.00	18	0.056
7	A	3	2	1.00	18	0.111
8	A	2	1	1.00	18	0.056
9	A	2	1	1.00	18	0.056
10	A	3	2	1.00	18	0.111
11	A	3	2	1.00	20	0.100
12	A	2	1	1.00	18	0.056
13	A	2	1	1.00	17	0.059
14	A	4	3	1.00	20	0.150
15	A	2	1	1.00	20	0.050
16	A	2	1	1.00	20	0.050
17	A	3	2	1.00	20	0.100
18	A	2	1	1.00	20	0.050
19	A	2	1	1.00	20	0.050
20	A	3	2	1.00	20	0.100
21	A	2	1	1.00	20	0.050
22	A	2	1	1.00	20	0.050
23	A	2	1	1.00	20	0.050
24	A	3	2	1.00	20	0.100
25	A	2	1	1.00	20	0.050
26	A	2	1	1.00	20	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	3	2	1.00	20	0.100
28	A	2	1	1.00	20	0.050
29	A	2	1	1.00	20	0.050
30	A	3	2	1.00	20	0.100
31	A	2	1	1.00	18	0.056
32	A	2	1	1.00	17	0.059
33	A	4	3	1.00	20	0.150
34	A	2	1	1.00	20	0.050
35	A	2	1	1.00	20	0.050
36	A	3	2	1.00	20	0.100
37	A	2	1	1.00	20	0.050
38	A	2	1	1.00	20	0.050
39	A	3	2	1.00	20	0.100
40	A	2	1	1.00	20	0.050
41	A	2	1	1.00	20	0.050
42	A	3	2	1.00	20	0.100
43	A	2	1	1.00	20	0.050
44	A	2	1	1.00	20	0.050
45	A	3	2	1.00	20	0.100
46	A	2	1	1.00	20	0.050
47	A	2	1	1.00	20	0.050
48	A	3	2	1.00	20	0.100
49	A	2	1	1.00	20	0.050
50	A	2	1	1.00	20	0.050
51	A	4	3	1.00	20	0.150
52	A	2	1	1.00	20	0.050
53	A	2	1	1.00	20	0.050
54	A	3	3	1.00	20	0.150
55	A	2	1	1.00	20	0.050
56	A	9	8	1.00	20	0.400
57	A	3	2	1.00	20	0.100
58	A	8	8	1.00	20	0.400
59	A	8	8	1.00	20	0.400
60	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	7	1.00	18	0.389
62	A	7	7	1.00	17	0.412
63	A	3	2	1.00	20	0.100
64	A	7	7	1.00	20	0.350
65	A	7	7	1.00	20	0.350
66	A	3	2	1.00	20	0.100
67	A	8	8	1.00	20	0.400
68	A	8	8	1.00	20	0.400
69	A	3	2	1.00	20	0.100
70	A	9	8	1.00	20	0.400
71	A	9	8	1.00	20	0.400
72	A	3	2	1.00	20	0.100
73	A	9	8	1.00	20	0.400
74	A	9	8	1.00	20	0.400
75	A	3	2	1.00	20	0.100
76	A	8	8	1.00	20	0.400
77	A	8	8	1.00	20	0.400
78	A	3	2	1.00	20	0.100
79	A	7	7	1.00	18	0.389
80	A	7	7	1.00	17	0.412
81	A	3	2	1.00	20	0.100
82	A	8	8	1.01	20	0.400
83	A	8	8	1.00	20	0.400
84	A	3	2	1.00	20	0.100
85	A	9	8	1.00	20	0.400
86	A	9	8	1.00	20	0.400
87	A	3	2	1.00	20	0.100
88	A	3	2	1.00	20	0.100
89	A	3	2	1.00	20	0.100
90	A	3	2	1.00	20	0.100
91	A	2	2	1.00	20	0.100
92	A	3	2	1.00	20	0.100
93	A	3	2	1.00	20	0.100
94	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	10	9	1.00	20	0.450
96	A	10	9	1.00	20	0.450
97	A	9	9	1.00	20	0.450
98	A	9	9	1.00	20	0.450
99	A	8	8	1.00	20	0.400
100	A	8	8	1.00	20	0.400
101	A	8	8	1.00	18	0.444
102	A	8	8	1.00	17	0.471
103	A	9	9	1.00	20	0.450
104	A	9	9	1.00	20	0.450
105	A	10	9	1.00	20	0.450
106	A	10	9	1.00	20	0.450
107	A	3	2	1.00	22	0.091
108	A	15	8	1.00	22	0.364
109	A	14	8	1.00	22	0.364
110	A	3	2	1.00	22	0.091
111	A	13	7	1.00	22	0.318
112	A	13	7	1.00	22	0.318
113	A	4	3	1.00	22	0.136
114	A	13	7	1.00	20	0.350
115	A	13	7	1.00	19	0.368
116	A	3	2	1.00	22	0.091
117	A	15	8	1.00	22	0.364
118	A	14	8	1.00	22	0.364
119	A	3	2	1.00	22	0.091
120	A	16	9	1.00	22	0.409
121	A	15	9	1.00	22	0.409
122	A	3	2	1.00	22	0.091
123	A	17	9	1.00	22	0.409
124	A	2	1	1.00	20	0.050
125	A	2	1	1.00	20	0.050
126	A	2	1	1.00	18	0.056
127	A	2	1	1.00	20	0.050
128	A	2	1	1.00	20	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	2	1	1.00	20	0.050
130	A	2	1	1.00	20	0.050
131	A	2	1	1.00	20	0.050
132	A	2	1	1.00	20	0.050
133	A	2	1	1.00	20	0.050
134	A	2	1	1.00	20	0.050
135	A	2	1	1.00	22	0.045
136	A	2	1	1.00	22	0.045
137	A	2	1	1.00	22	0.045
138	A	2	1	1.00	22	0.045
139	A	2	1	1.00	22	0.045
140	A	2	1	1.00	22	0.045
141	A	2	1	1.00	22	0.045
142	A	2	1	1.00	22	0.045
143	A	2	1	1.00	22	0.045
144	A	2	1	1.00	22	0.045
145	A	2	1	1.00	22	0.045
146	A	2	1	1.00	22	0.045
147	A	2	1	1.00	22	0.045
148	A	2	1	1.00	22	0.045
149	A	2	1	1.00	22	0.045
150	A	2	1	1.00	22	0.045
151	A	5	5	1.00	22	0.227
152	A	13	9	1.00	22	0.409
153	A	12	8	1.00	22	0.364
154	A	4	4	1.00	22	0.182
155	A	12	8	1.00	22	0.364
156	A	12	8	1.00	22	0.364
157	A	4	4	1.00	22	0.182
158	A	12	8	1.00	22	0.364
159	A	5	5	1.00	22	0.227
160	A	13	9	1.00	22	0.409
161	A	12	8	1.00	22	0.364
162	A	4	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
163	A	12	8	1.00	22	0.364
164	A	13	9	1.00	22	0.409
165	A	5	5	1.01	22	0.227
166	A	13	9	1.00	22	0.409
167	A	5	5	1.00	22	0.227
168	A	13	9	1.00	22	0.409
169	A	13	9	1.00	22	0.409
170	A	5	5	1.00	22	0.227
171	A	13	9	1.00	22	0.409
172	A	14	10	1.00	22	0.454
173	A	6	6	1.01	22	0.273
174	A	14	10	1.00	22	0.454
175	A	3	2	1.00	22	0.091
176	A	3	2	1.00	22	0.091
177	A	3	2	1.00	22	0.091
178	A	5	5	1.00	22	0.227
179	A	5	5	1.00	22	0.227
180	A	5	5	1.00	22	0.227
181	A	3	2	1.00	22	0.091
182	A	3	2	1.00	22	0.091
183	A	3	2	1.00	22	0.091
184	A	6	5	1.00	22	0.227
185	A	6	5	1.00	22	0.227
186	A	6	6	1.00	22	0.273
187	A	3	2	1.00	22	0.091
188	A	3	2	1.00	22	0.091
189	A	3	2	1.00	22	0.091
190	A	4	4	1.00	22	0.182
191	A	4	4	1.00	22	0.182
192	A	5	5	1.00	22	0.227
193	A	3	2	1.00	22	0.091
194	A	3	2	1.00	22	0.091
195	A	3	2	1.00	22	0.091
196	A	4	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	5	5	1.00	22	0.227
198	A	6	5	1.02	22	0.227
199	A	3	2	1.00	22	0.091
200	A	3	2	1.00	22	0.091
201	A	3	2	1.00	22	0.091
202	A	5	5	1.00	22	0.227
203	A	6	5	1.00	22	0.227
204	A	6	5	1.00	26	0.192
205	A	5	5	1.00	26	0.192
206	A	4	4	1.00	26	0.154
207	A	6	5	1.00	26	0.192
208	A	7	6	1.00	26	0.231
209	A	5	4	1.00	26	0.154
210	A	4	4	1.00	26	0.154
211	A	3	3	1.00	26	0.115
212	A	6	5	1.00	26	0.192
213	A	7	6	1.00	26	0.231
214	A	1	1	1.00	24	0.042
215	A	1	1	1.00	22	0.045
216	A	6	5	1.00	27	0.185
217	A	6	5	1.00	27	0.185
218	A	5	5	1.00	27	0.185
219	A	4	4	1.00	27	0.148
220	A	6	5	1.00	27	0.185
221	A	7	6	1.00	27	0.222
222	A	8	7	1.00	27	0.259
223	A	7	5	1.00	27	0.185
224	A	7	5	1.00	27	0.185
225	A	6	5	1.00	27	0.185
226	A	5	4	1.00	27	0.148
227	A	7	6	1.00	27	0.222
228	A	7	6	1.00	27	0.222
229	A	8	7	1.00	27	0.259
230	A	5	4	1.00	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
231	A	5	4	1.00	27	0.148
232	A	4	4	1.00	27	0.148
233	A	3	3	1.00	27	0.111
234	A	6	5	1.00	27	0.185
235	A	7	6	1.00	27	0.222
236	A	8	7	1.00	27	0.259
237	A	8	7	1.00	25	0.280
238	A	7	5	1.00	27	0.185
239	A	5	4	1.00	27	0.148
240	A	4	4	1.00	27	0.148
241	A	4	4	1.00	27	0.148
242	A	7	6	1.00	27	0.222
243	A	8	7	1.00	27	0.259
244	A	9	8	1.00	27	0.296
245	A	1	1	1.00	33	0.030
246	A	1	1	1.00	35	0.029
247	A	1	1	1.00	35	0.029
248	A	1	1	1.00	37	0.027
249	A	1	1	1.00	33	0.030
250	A	1	1	1.00	35	0.029
251	A	1	1	1.00	36	0.028
252	A	1	1	1.00	36	0.028
253	A	6	5	1.00	24	0.208
254	A	5	5	1.00	24	0.208
255	A	4	4	1.00	24	0.167
256	A	6	4	1.00	24	0.167
257	A	7	5	1.00	24	0.208
258	A	7	5	1.00	24	0.208
259	A	6	5	1.00	24	0.208
260	A	5	4	1.00	24	0.167
261	A	7	5	1.00	24	0.208
262	A	7	5	1.00	24	0.208
263	A	5	4	1.00	24	0.167
264	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	3	3	1.00	24	0.125
266	A	6	4	1.00	24	0.167
267	A	7	5	1.00	24	0.208
268	A	5	4	1.00	24	0.167
269	A	4	4	1.00	24	0.167
270	A	4	4	1.00	24	0.167
271	A	7	5	1.00	24	0.208
272	A	8	6	1.00	24	0.250
273	A	6	6	1.00	27	0.222
274	A	6	6	1.00	27	0.222
275	A	5	5	1.00	27	0.185
276	A	4	4	1.00	27	0.148
277	A	7	6	1.00	27	0.222
278	A	8	7	1.00	27	0.259
279	A	9	7	1.00	27	0.259
280	A	7	7	1.00	27	0.259
281	A	7	6	1.00	27	0.222
282	A	6	5	1.00	27	0.185
283	A	5	5	1.00	27	0.185
284	A	7	6	1.00	27	0.222
285	A	8	7	1.00	27	0.259
286	A	9	7	1.00	27	0.259
287	A	5	5	1.00	27	0.185
288	A	5	5	1.00	27	0.185
289	A	4	4	1.00	27	0.148
290	A	4	4	1.00	27	0.148
291	A	7	6	1.00	27	0.222
292	A	8	7	1.00	27	0.259
293	A	9	7	1.00	27	0.259
294	A	5	5	1.00	27	0.185
295	A	5	5	1.00	27	0.185
296	A	5	5	1.00	27	0.185
297	A	5	4	1.00	27	0.148
298	A	8	7	1.00	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
299	A	9	8	1.00	27	0.296
300	A	10	8	1.00	27	0.296
301	A	6	6	1.00	24	0.250
302	A	5	5	1.00	24	0.208
303	A	4	4	1.00	24	0.167
304	A	7	5	1.00	24	0.208
305	A	8	6	1.00	24	0.250
306	A	7	6	1.00	24	0.250
307	A	6	5	1.00	24	0.208
308	A	5	5	1.00	24	0.208
309	A	7	5	1.00	24	0.208
310	A	8	6	1.00	24	0.250
311	A	5	5	1.00	24	0.208
312	A	4	4	1.00	24	0.167
313	A	4	4	1.00	24	0.167
314	A	7	5	1.00	24	0.208
315	A	8	6	1.00	24	0.250
316	A	5	5	0.99	24	0.208
317	A	5	5	1.00	24	0.208
318	A	5	4	1.00	24	0.167
319	A	8	6	1.00	24	0.250
320	A	9	7	1.00	24	0.292
321	A	5	5	1.00	26	0.192
322	A	4	4	1.00	26	0.154
323	A	3	3	1.00	26	0.115
324	A	4	4	1.00	26	0.154
325	A	7	7	1.00	26	0.269
326	A	6	6	1.00	26	0.231
327	A	6	6	1.00	26	0.231
328	A	6	6	1.00	24	0.250
329	A	8	7	1.00	26	0.269
330	A	7	6	1.00	26	0.231
331	A	7	6	1.00	26	0.231
332	A	9	7	1.00	26	0.269

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
333	A	8	6	1.00	26	0.231
334	A	8	6	1.00	26	0.231
335	A	6	6	1.00	26	0.231
336	A	5	5	1.00	26	0.192
337	A	5	5	1.00	26	0.192
338	A	6	6	1.00	26	0.231
339	A	5	5	1.00	26	0.192
340	A	2	2	1.00	26	0.077
341	A	6	6	1.00	26	0.231
342	A	2	2	1.00	26	0.077
343	A	3	3	1.00	26	0.115
344	A	8	7	1.00	28	0.250
345	A	8	7	1.00	28	0.250
346	A	7	7	1.00	28	0.250
347	A	6	6	1.00	28	0.214
348	A	10	6	1.00	28	0.214
349	A	13	8	1.00	28	0.286
350	A	12	8	1.00	28	0.286
351	C	2	2	0.25	28	0.071
352	C	2	2	0.28	28	0.071
353	C	2	2	0.33	26	0.077
354	C	2	2	0.49	28	0.071
355	C	2	2	0.64	28	0.071
356	C	2	2	1.16	28	0.071
357	C	2	2	1.78	28	0.071
358	C	2	2	0.15	25	0.080
359	A	8	7	1.00	28	0.250
360	A	8	7	1.00	28	0.250
361	A	7	7	1.00	28	0.250
362	A	6	6	1.00	28	0.214
363	A	10	6	1.00	28	0.214
364	A	13	8	1.00	28	0.286
365	A	12	8	1.00	28	0.286
366	C	2	2	0.25	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
367	C	2	2	0.29	28	0.071
368	C	2	2	0.30	25	0.080
369	C	2	2	0.50	28	0.071
370	C	2	2	0.66	28	0.071
371	C	2	2	1.17	28	0.071
372	C	2	2	1.66	28	0.071
373	A	7	6	1.00	22	0.273
374	A	7	6	1.00	22	0.273
375	A	7	6	1.00	22	0.273
376	A	6	6	1.00	22	0.273
377	A	5	5	1.00	22	0.227
378	A	10	7	1.00	22	0.318
379	A	11	8	1.00	22	0.364
380	A	15	10	1.47	22	0.454
381	A	14	9	1.53	22	0.409
382	A	7	7	1.39	19	0.368
383	A	8	8	1.32	22	0.364
384	A	9	8	1.13	22	0.364
385	A	9	8	1.24	22	0.364
386	C	1	1	0.11	20	0.050
387	A	7	6	1.00	22	0.273
388	A	7	6	1.00	22	0.273
389	A	6	6	1.00	22	0.273
390	A	5	5	1.00	22	0.227
391	A	10	7	1.00	22	0.318
392	A	11	8	1.00	22	0.364
393	A	14	10	1.42	22	0.454
394	A	14	9	1.49	22	0.409
395	A	7	7	1.39	20	0.350
396	A	8	8	1.33	22	0.364
397	A	9	8	1.13	22	0.364
398	C	1	1	0.09	22	0.045
399	C	1	1	0.09	22	0.045
400	A	11	9	1.00	22	0.409

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
401	A	11	9	1.00	22	0.409
402	A	9	8	1.00	22	0.364
403	A	6	6	1.00	22	0.273
404	A	6	6	1.00	22	0.273
405	A	11	8	1.00	22	0.364
406	A	13	9	1.00	22	0.409
407	C	1	1	0.15	22	0.045
408	C	1	1	0.17	22	0.045
409	C	1	1	0.36	22	0.045
410	A	8	8	1.32	19	0.421
411	C	1	1	1.65	22	0.045
412	C	1	1	2.76	22	0.045
413	C	1	1	3.97	22	0.045
414	A	8	7	1.00	24	0.292
415	A	8	7	1.00	24	0.292
416	A	7	7	1.00	24	0.292
417	A	6	6	1.00	24	0.250
418	A	10	7	1.00	24	0.292
419	A	13	9	1.00	24	0.375
420	A	12	9	1.00	24	0.375
421	C	2	2	0.19	24	0.083
422	C	2	2	0.23	24	0.083
423	C	2	2	0.27	22	0.091
424	C	2	2	0.52	24	0.083
425	C	2	2	0.71	24	0.083
426	C	2	2	1.75	24	0.083
427	C	2	2	2.85	24	0.083
428	A	8	7	1.00	24	0.292
429	A	8	7	1.00	24	0.292
430	A	7	7	1.00	24	0.292
431	A	6	6	1.00	24	0.250
432	A	10	7	1.00	24	0.292
433	A	13	9	1.00	24	0.375
434	A	12	9	1.00	24	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
435	C	2	2	0.19	24	0.083
436	C	2	2	0.24	24	0.083
437	C	2	2	0.25	21	0.095
438	C	2	2	0.53	24	0.083
439	C	2	2	0.72	24	0.083
440	C	2	2	1.75	24	0.083
441	C	2	2	2.56	24	0.083
442	A	9	7	1.00	24	0.292
443	A	8	7	1.00	24	0.292
444	A	7	6	1.00	24	0.250
445	A	11	8	1.00	24	0.333
446	A	15	9	1.00	24	0.375
447	A	14	10	1.00	24	0.417
448	C	2	2	0.19	24	0.083
449	C	2	2	0.23	22	0.091
450	C	2	2	0.25	24	0.083
451	C	2	2	0.45	24	0.083
452	C	2	2	0.68	24	0.083
453	C	2	2	0.82	24	0.083
454	C	2	2	3.69	24	0.083
455	A	7	6	1.00	24	0.250
456	A	7	6	1.00	24	0.250
457	A	7	6	1.00	24	0.250
458	A	6	6	1.00	24	0.250
459	A	5	5	1.00	24	0.208
460	A	10	7	1.00	24	0.292
461	A	11	8	1.00	24	0.333
462	A	15	9	1.44	24	0.375
463	A	14	8	1.48	24	0.333
464	A	7	7	1.40	21	0.333
465	A	8	8	1.34	24	0.333
466	A	9	8	1.27	24	0.333
467	A	9	8	1.21	24	0.333
468	A	7	6	1.00	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
469	A	7	6	1.00	24	0.250
470	A	6	6	1.00	24	0.250
471	A	5	5	1.00	24	0.208
472	A	10	7	1.00	24	0.292
473	A	11	8	1.00	24	0.333
474	A	16	9	1.43	24	0.375
475	A	14	8	1.48	24	0.333
476	A	7	7	1.40	22	0.318
477	A	8	8	1.34	24	0.333
478	A	9	8	1.25	24	0.333
479	A	11	7	1.00	24	0.292
480	A	9	7	1.00	24	0.292
481	A	7	6	1.00	24	0.250
482	A	6	6	1.00	24	0.250
483	A	6	6	1.00	24	0.250
484	A	11	8	1.00	24	0.333
485	A	13	9	1.00	24	0.375
486	C	2	2	0.21	24	0.083
487	C	2	2	0.26	24	0.083
488	C	2	2	0.53	24	0.083
489	A	8	8	1.33	21	0.381
490	C	2	2	2.37	24	0.083
491	C	2	2	3.31	24	0.083
492	C	2	2	4.23	24	0.083
493	A	3	2	1.00	22	0.091
494	A	3	2	1.00	22	0.091
495	A	3	2	1.00	22	0.091
496	A	4	3	1.00	22	0.136
497	A	3	2	1.00	22	0.091
498	A	3	2	1.00	22	0.091
499	A	6	5	1.00	22	0.227
500	A	5	4	1.00	22	0.182
501	A	4	3	1.00	22	0.136
502	A	4	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
503	A	5	4	1.00	22	0.182
504	A	6	5	1.00	22	0.227
505	A	20	8	1.00	22	0.364
506	A	19	7	1.00	22	0.318
507	A	19	7	1.00	22	0.318
508	A	19	7	1.00	22	0.318
509	A	19	7	1.00	19	0.368
510	A	21	8	1.00	22	0.364
511	A	20	8	1.00	22	0.364
512	A	22	9	1.00	22	0.409
513	A	5	5	1.00	24	0.208
514	A	7	7	1.00	24	0.292
515	A	4	4	1.00	24	0.167
516	A	6	6	1.00	22	0.273
517	A	6	4	1.00	24	0.167
518	A	5	5	1.00	24	0.208
519	A	7	5	1.00	24	0.208
520	A	6	6	1.00	24	0.250
521	A	5	4	1.00	24	0.167
522	A	4	4	1.00	24	0.167
523	A	3	3	1.00	24	0.125
524	A	6	4	1.00	24	0.167
525	A	7	5	1.00	24	0.208
526	A	7	7	1.00	24	0.292
527	A	6	6	1.00	24	0.250
528	A	3	3	1.00	22	0.136
529	A	5	5	1.00	24	0.208
530	A	6	6	1.00	24	0.250
531	A	5	5	1.00	24	0.208
532	A	5	5	1.00	24	0.208
533	A	4	4	1.00	24	0.167
534	A	4	4	1.00	24	0.167
535	A	7	5	1.00	24	0.208
536	A	8	6	1.00	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
537	A	8	8	1.00	24	0.333
538	A	7	7	1.00	24	0.292
539	A	5	5	1.00	24	0.208
540	A	4	4	1.00	22	0.182
541	A	6	6	1.00	24	0.250
542	A	7	6	1.00	24	0.250
543	A	5	4	1.00	24	0.167
544	A	4	4	1.00	24	0.167
545	A	3	3	1.00	24	0.125
546	A	6	4	1.00	24	0.167
547	A	7	5	1.00	24	0.208
548	A	7	7	1.00	24	0.292
549	A	6	6	1.00	24	0.250
550	A	3	3	1.00	24	0.125
551	A	5	5	1.00	24	0.208
552	A	6	6	1.00	24	0.250
553	A	5	5	1.00	24	0.208
554	A	4	4	1.00	24	0.167
555	A	4	4	1.00	24	0.167
556	A	7	5	1.00	24	0.208
557	A	8	6	1.00	24	0.250
558	A	7	7	1.00	24	0.292
559	A	5	5	1.00	24	0.208
560	A	4	4	1.00	24	0.167
561	A	6	6	1.00	24	0.250
562	A	7	6	1.00	24	0.250
563	A	5	4	1.00	24	0.167
564	A	4	4	1.00	24	0.167
565	A	3	3	1.00	24	0.125
566	A	6	4	1.00	24	0.167
567	A	7	5	1.00	24	0.208
568	A	7	7	1.00	24	0.292
569	A	6	6	1.00	24	0.250
570	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
571	A	5	5	1.00	24	0.208
572	A	6	6	1.00	24	0.250
573	A	5	5	1.00	24	0.208
574	A	4	4	1.00	24	0.167
575	A	4	4	1.00	24	0.167
576	A	7	5	1.00	24	0.208
577	A	8	6	1.00	24	0.250
578	A	7	7	1.00	24	0.292
579	A	5	5	1.00	24	0.208
580	A	4	4	1.00	24	0.167
581	A	6	6	1.00	24	0.250
582	A	7	6	1.00	24	0.250
583	A	6	6	1.00	22	0.273
584	A	5	5	1.00	22	0.227
585	A	5	5	1.00	20	0.250
586	A	5	5	1.00	22	0.227
587	A	3	2	1.00	22	0.091
588	A	3	2	1.00	22	0.091
589	A	3	2	1.00	22	0.091
590	A	3	2	1.00	22	0.091
591	A	5	3	1.00	22	0.136
592	A	4	3	1.00	22	0.136
593	A	3	3	1.00	22	0.136
594	A	2	2	1.00	22	0.091
595	A	5	5	1.00	22	0.227
596	A	5	5	1.00	19	0.263
597	A	5	5	1.00	22	0.227
598	A	6	6	1.00	22	0.273
599	A	6	5	1.00	22	0.227
600	A	6	6	1.00	22	0.273
601	A	6	5	1.00	20	0.250
602	A	6	5	1.00	22	0.227
603	A	3	2	1.00	22	0.091
604	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
605	A	3	2	1.00	22	0.091
606	A	3	2	1.00	22	0.091
607	A	5	3	1.00	22	0.136
608	A	4	3	1.00	22	0.136
609	A	3	3	1.00	22	0.136
610	A	2	2	1.00	22	0.091
611	A	6	5	1.00	22	0.227
612	A	6	6	1.00	22	0.273
613	A	6	5	1.00	19	0.263
614	A	6	5	1.00	22	0.227
615	A	7	6	1.00	22	0.273
616	A	5	5	1.00	22	0.227
617	A	4	4	1.00	20	0.200
618	A	4	4	1.00	22	0.182
619	A	3	2	1.00	22	0.091
620	A	3	2	1.00	22	0.091
621	A	3	2	1.00	22	0.091
622	A	3	3	1.00	22	0.136
623	A	2	2	1.00	22	0.091
624	A	4	4	1.00	19	0.210
625	A	4	4	1.00	22	0.182
626	A	5	5	1.00	22	0.227
627	A	6	5	1.02	22	0.227
628	A	5	5	1.00	20	0.250
629	A	4	4	1.00	22	0.182
630	A	3	2	1.00	22	0.091
631	A	3	2	1.00	22	0.091
632	A	3	2	1.00	22	0.091
633	A	3	2	1.00	22	0.091
634	A	4	4	1.00	22	0.182
635	A	3	3	1.00	22	0.136
636	A	3	3	1.00	19	0.158
637	A	4	4	1.00	22	0.182
638	A	5	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
639	A	6	6	1.00	22	0.273
640	A	6	4	1.00	28	0.143
641	A	5	4	1.00	28	0.143
642	A	4	4	1.00	28	0.143
643	A	3	3	1.00	28	0.107
644	A	4	4	1.00	28	0.143
645	A	1	1	1.00	28	0.036
646	A	2	2	1.00	28	0.071
647	A	3	2	1.00	28	0.071
648	A	4	2	1.00	28	0.071
649	A	5	3	1.00	28	0.107
650	A	4	3	1.00	28	0.107
651	A	3	3	1.00	28	0.107
652	A	2	2	1.00	28	0.071
653	A	1	1	1.00	28	0.036
654	A	2	2	1.00	28	0.071
655	A	3	2	1.00	28	0.071
656	A	3	2	1.00	18	0.111
657	A	1	1	1.00	33	0.030
658	A	3	2	1.00	22	0.091
659	A	3	2	1.00	22	0.091
660	A	3	2	1.00	26	0.077
661	A	3	2	1.00	26	0.077
662	A	3	2	1.00	24	0.083
663	A	3	2	1.00	26	0.077
664	A	3	2	1.00	26	0.077
665	A	3	2	1.00	26	0.077
666	A	3	2	1.00	26	0.077
667	A	3	2	1.00	26	0.077
668	A	3	2	1.00	24	0.083
669	A	3	2	1.00	26	0.077
670	A	3	2	1.00	26	0.077
671	A	3	2	1.00	26	0.077
672	A	1	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
673	A	2	2	1.00	21	0.095
674	A	2	2	1.00	21	0.095
675	A	2	2	1.00	25	0.080
676	A	1	1	1.00	31	0.032
677	A	2	1	1.12	17	0.059
678	A	3	2	1.00	21	0.095
679	A	3	2	1.00	21	0.095
680	A	3	2	1.00	21	0.095
681	A	1	1	1.00	17	0.059
682	A	2	2	1.00	21	0.095
683	A	2	2	1.00	21	0.095
684	A	2	2	1.00	25	0.080
685	A	5	5	1.00	22	0.227
686	A	7	7	1.00	26	0.269
687	A	8	6	1.00	30	0.200
688	A	7	6	1.00	30	0.200
689	A	6	6	1.00	30	0.200
690	A	5	5	1.00	30	0.167
691	A	5	5	1.00	30	0.167
692	A	3	3	1.00	30	0.100
693	A	9	7	1.00	30	0.233
694	A	8	7	1.00	30	0.233
695	A	7	7	1.00	30	0.233
696	A	6	6	1.00	30	0.200
697	A	6	6	1.00	30	0.200
698	A	6	6	1.00	30	0.200
699	A	1	1	1.00	17	0.059
700	A	1	1	1.00	27	0.037
701	A	1	1	1.00	27	0.037
702	A	1	1	1.00	27	0.037

Chapter 3

Listing of integrals

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3.12	$\int x (a + bx^3)^2 (A + Bx^3) dx$	292
3.13	$\int (a + bx^3)^2 (A + Bx^3) dx$	295
3.14	$\int \frac{(a+bx^3)^2(A+Bx^3)}{x} dx$	298
3.15	$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^2} dx$	302
3.16	$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3} dx$	305
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3.19	$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6} dx$	315
3.20	$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7} dx$	318
3.21	$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8} dx$	322
3.22	$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9} dx$	325
3.23	$\int x^9 (a + bx^3)^5 (A + Bx^3) dx$	328
3.24	$\int x^8 (a + bx^3)^5 (A + Bx^3) dx$	331
3.25	$\int x^7 (a + bx^3)^5 (A + Bx^3) dx$	335
3.26	$\int x^6 (a + bx^3)^5 (A + Bx^3) dx$	338
3.27	$\int x^5 (a + bx^3)^5 (A + Bx^3) dx$	341
3.28	$\int x^4 (a + bx^3)^5 (A + Bx^3) dx$	345
3.29	$\int x^3 (a + bx^3)^5 (A + Bx^3) dx$	348
3.30	$\int x^2 (a + bx^3)^5 (A + Bx^3) dx$	351
3.31	$\int x (a + bx^3)^5 (A + Bx^3) dx$	354
3.32	$\int (a + bx^3)^5 (A + Bx^3) dx$	357
3.33	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x} dx$	360
3.34	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2} dx$	364
3.35	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^3} dx$	367
3.36	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^4} dx$	370
3.37	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5} dx$	374
3.38	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^6} dx$	377
3.39	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7} dx$	380
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3.42	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{10}} dx$	390

3.43	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}} dx$	394
3.44	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}} dx$	397
3.45	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}} dx$	400
3.46	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}} dx$	404
3.47	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{15}} dx$	407
3.48	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}} dx$	410
3.49	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}} dx$	414
3.50	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{18}} dx$	417
3.51	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{19}} dx$	420
3.52	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{20}} dx$	424
3.53	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}} dx$	427
3.54	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}} dx$	430
3.55	$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}} dx$	434
3.56	$\int \frac{x^6(A+Bx^3)}{a+bx^3} dx$	437
3.57	$\int \frac{x^5(A+Bx^3)}{a+bx^3} dx$	442
3.58	$\int \frac{x^4(A+Bx^3)}{a+bx^3} dx$	446
3.59	$\int \frac{x^3(A+Bx^3)}{a+bx^3} dx$	451
3.60	$\int \frac{x^2(A+Bx^3)}{a+bx^3} dx$	456
3.61	$\int \frac{x(A+Bx^3)}{a+bx^3} dx$	459
3.62	$\int \frac{A+Bx^3}{a+bx^3} dx$	464
3.63	$\int \frac{A+Bx^3}{x(a+bx^3)} dx$	469
3.64	$\int \frac{A+Bx^3}{x^2(a+bx^3)} dx$	472
3.65	$\int \frac{A+Bx^3}{x^3(a+bx^3)} dx$	477
3.66	$\int \frac{A+Bx^3}{x^4(a+bx^3)} dx$	482

3.67	$\int \frac{A+Bx^3}{x^5(a+bx^3)} dx$	486
3.68	$\int \frac{A+Bx^3}{x^6(a+bx^3)} dx$	491
3.69	$\int \frac{A+Bx^3}{x^7(a+bx^3)} dx$	496
3.70	$\int \frac{A+Bx^3}{x^8(a+bx^3)} dx$	500
3.71	$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2} dx$	506
3.72	$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2} dx$	512
3.73	$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2} dx$	516
3.74	$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2} dx$	522
3.75	$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2} dx$	528
3.76	$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2} dx$	532
3.77	$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2} dx$	538
3.78	$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2} dx$	544
3.79	$\int \frac{x(A+Bx^3)}{(a+bx^3)^2} dx$	548
3.80	$\int \frac{A+Bx^3}{(a+bx^3)^2} dx$	554
3.81	$\int \frac{A+Bx^3}{x(a+bx^3)^2} dx$	560
3.82	$\int \frac{A+Bx^3}{x^2(a+bx^3)^2} dx$	564
3.83	$\int \frac{A+Bx^3}{x^3(a+bx^3)^2} dx$	570
3.84	$\int \frac{A+Bx^3}{x^4(a+bx^3)^2} dx$	576
3.85	$\int \frac{A+Bx^3}{x^5(a+bx^3)^2} dx$	580
3.86	$\int \frac{A+Bx^3}{x^6(a+bx^3)^2} dx$	586
3.87	$\int \frac{A+Bx^3}{x^7(a+bx^3)^2} dx$	592

3.88	$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3} dx$	596
3.89	$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3} dx$	600
3.90	$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3} dx$	604
3.91	$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3} dx$	608
3.92	$\int \frac{A+Bx^3}{x(a+bx^3)^3} dx$	612
3.93	$\int \frac{A+Bx^3}{x^4(a+bx^3)^3} dx$	616
3.94	$\int \frac{A+Bx^3}{x^7(a+bx^3)^3} dx$	620
3.95	$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3} dx$	624
3.96	$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3} dx$	630
3.97	$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3} dx$	636
3.98	$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3} dx$	642
3.99	$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3} dx$	648
3.100	$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3} dx$	654
3.101	$\int \frac{x(A+Bx^3)}{(a+bx^3)^3} dx$	660
3.102	$\int \frac{A+Bx^3}{(a+bx^3)^3} dx$	666
3.103	$\int \frac{A+Bx^3}{x^2(a+bx^3)^3} dx$	672
3.104	$\int \frac{A+Bx^3}{x^3(a+bx^3)^3} dx$	678
3.105	$\int \frac{A+Bx^3}{x^5(a+bx^3)^3} dx$	684
3.106	$\int \frac{A+Bx^3}{x^6(a+bx^3)^3} dx$	690
3.107	$\int \frac{x^8}{(a+bx^3)(c+dx^3)} dx$	696

3.108	$\int \frac{x^7}{(a+bx^3)(c+dx^3)} dx$	699
3.109	$\int \frac{x^6}{(a+bx^3)(c+dx^3)} dx$	705
3.110	$\int \frac{x^5}{(a+bx^3)(c+dx^3)} dx$	711
3.111	$\int \frac{x^4}{(a+bx^3)(c+dx^3)} dx$	714
3.112	$\int \frac{x^3}{(a+bx^3)(c+dx^3)} dx$	720
3.113	$\int \frac{x^2}{(a+bx^3)(c+dx^3)} dx$	726
3.114	$\int \frac{x}{(a+bx^3)(c+dx^3)} dx$	730
3.115	$\int \frac{1}{(a+bx^3)(c+dx^3)} dx$	736
3.116	$\int \frac{1}{x(a+bx^3)(c+dx^3)} dx$	742
3.117	$\int \frac{1}{x^2(a+bx^3)(c+dx^3)} dx$	745
3.118	$\int \frac{1}{x^3(a+bx^3)(c+dx^3)} dx$	751
3.119	$\int \frac{1}{x^4(a+bx^3)(c+dx^3)} dx$	757
3.120	$\int \frac{1}{x^5(a+bx^3)(c+dx^3)} dx$	760
3.121	$\int \frac{1}{x^6(a+bx^3)(c+dx^3)} dx$	767
3.122	$\int \frac{1}{x^7(a+bx^3)(c+dx^3)} dx$	774
3.123	$\int \frac{1}{x^8(a+bx^3)(c+dx^3)} dx$	778
3.124	$\int x^m (a + bx^3)^5 (A + Bx^3) dx$	785
3.125	$\int x^m (a + bx^3)^2 (A + Bx^3) dx$	794
3.126	$\int x^m (a + bx^3) (A + Bx^3) dx$	798
3.127	$\int x^{7/2} (a + bx^3) (A + Bx^3) dx$	802
3.128	$\int x^{5/2} (a + bx^3) (A + Bx^3) dx$	805
3.129	$\int x^{3/2} (a + bx^3) (A + Bx^3) dx$	808
3.130	$\int \sqrt{x} (a + bx^3) (A + Bx^3) dx$	811
3.131	$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}} dx$	814
3.132	$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}} dx$	817
3.133	$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}} dx$	820

3.134	$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}} dx$	823
3.135	$\int x^{7/2} (a+bx^3)^2 (A+Bx^3) dx$	826
3.136	$\int x^{5/2} (a+bx^3)^2 (A+Bx^3) dx$	829
3.137	$\int x^{3/2} (a+bx^3)^2 (A+Bx^3) dx$	832
3.138	$\int \sqrt{x} (a+bx^3)^2 (A+Bx^3) dx$	835
3.139	$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}} dx$	838
3.140	$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{3/2}} dx$	841
3.141	$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}} dx$	844
3.142	$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}} dx$	847
3.143	$\int x^{7/2} (a+bx^3)^3 (A+Bx^3) dx$	850
3.144	$\int x^{5/2} (a+bx^3)^3 (A+Bx^3) dx$	853
3.145	$\int x^{3/2} (a+bx^3)^3 (A+Bx^3) dx$	856
3.146	$\int \sqrt{x} (a+bx^3)^3 (A+Bx^3) dx$	859
3.147	$\int \frac{(a+bx^3)^3 (A+Bx^3)}{\sqrt{x}} dx$	862
3.148	$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}} dx$	865
3.149	$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}} dx$	868
3.150	$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}} dx$	871
3.151	$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3} dx$	874
3.152	$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3} dx$	878
3.153	$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3} dx$	886
3.154	$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3} dx$	894
3.155	$\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)} dx$	898
3.156	$\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)} dx$	906
3.157	$\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)} dx$	915
3.158	$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)} dx$	920

3.159	$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2} dx$	928
3.160	$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2} dx$	933
3.161	$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2} dx$	941
3.162	$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2} dx$	949
3.163	$\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)^2} dx$	953
3.164	$\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^2} dx$	961
3.165	$\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)^2} dx$	970
3.166	$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^2} dx$	975
3.167	$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3} dx$	983
3.168	$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3} dx$	988
3.169	$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3} dx$	997
3.170	$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3} dx$	1006
3.171	$\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)^3} dx$	1011
3.172	$\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^3} dx$	1019
3.173	$\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)^3} dx$	1029
3.174	$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^3} dx$	1034
3.175	$\int x^8 \sqrt{a+bx^3} (A+Bx^3) dx$	1044
3.176	$\int x^5 \sqrt{a+bx^3} (A+Bx^3) dx$	1048
3.177	$\int x^2 \sqrt{a+bx^3} (A+Bx^3) dx$	1052
3.178	$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x} dx$	1056
3.179	$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4} dx$	1060
3.180	$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7} dx$	1065

3.181	$\int x^8 (a + bx^3)^{3/2} (A + Bx^3) dx$.1070
3.182	$\int x^5 (a + bx^3)^{3/2} (A + Bx^3) dx$.1074
3.183	$\int x^2 (a + bx^3)^{3/2} (A + Bx^3) dx$.1078
3.184	$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x} dx$.1082
3.185	$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4} dx$.1087
3.186	$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7} dx$.1092
3.187	$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}} dx$.1097
3.188	$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}} dx$.1101
3.189	$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}} dx$.1105
3.190	$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}} dx$.1109
3.191	$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}} dx$.1113
3.192	$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}} dx$.1117
3.193	$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}} dx$.1122
3.194	$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}} dx$.1126
3.195	$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}} dx$.1130
3.196	$\int \frac{A+Bx^3}{x(a+bx^3)^{3/2}} dx$.1134
3.197	$\int \frac{A+Bx^3}{x^4(a+bx^3)^{3/2}} dx$.1138
3.198	$\int \frac{A+Bx^3}{x^7(a+bx^3)^{3/2}} dx$.1143
3.199	$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}} dx$.1148
3.200	$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}} dx$.1152
3.201	$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}} dx$.1156
3.202	$\int \frac{A+Bx^3}{x(a+bx^3)^{5/2}} dx$.1160

3.203	$\int \frac{A+Bx^3}{x^4(a+bx^3)^{5/2}} dx$1165
3.204	$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3} dx$1170
3.205	$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3} dx$1175
3.206	$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3} dx$1180
3.207	$\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)} dx$1184
3.208	$\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)} dx$1189
3.209	$\int \frac{x^8}{\sqrt{c+dx^3}(4c+dx^3)} dx$1194
3.210	$\int \frac{x^5}{\sqrt{c+dx^3}(4c+dx^3)} dx$1198
3.211	$\int \frac{x^2}{\sqrt{c+dx^3}(4c+dx^3)} dx$1202
3.212	$\int \frac{1}{x\sqrt{c+dx^3}(4c+dx^3)} dx$1206
3.213	$\int \frac{1}{x^4\sqrt{c+dx^3}(4c+dx^3)} dx$1211
3.214	$\int \frac{x}{\sqrt{c+dx^3}(4c+dx^3)} dx$1216
3.215	$\int \frac{x}{\sqrt{1-x^3}(4-x^3)} dx$1221
3.216	$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3} dx$1225
3.217	$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3} dx$1230
3.218	$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3} dx$1235
3.219	$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3} dx$1240
3.220	$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)} dx$1244
3.221	$\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)} dx$1249
3.222	$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)} dx$1254
3.223	$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3} dx$1260
3.224	$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3} dx$1265
3.225	$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3} dx$1270
3.226	$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3} dx$1275

3.227	$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)} dx$.1280
3.228	$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)} dx$.1285
3.229	$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)} dx$.1290
3.230	$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}} dx$.1296
3.231	$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}} dx$.1301
3.232	$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}} dx$.1305
3.233	$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}} dx$.1309
3.234	$\int \frac{1}{x(8c-dx^3)\sqrt{c+dx^3}} dx$.1313
3.235	$\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}} dx$.1318
3.236	$\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}} dx$.1323
3.237	$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}} dx$.1328
3.238	$\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}} dx$.1334
3.239	$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}} dx$.1339
3.240	$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}} dx$.1344
3.241	$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}} dx$.1349
3.242	$\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}} dx$.1353
3.243	$\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}} dx$.1358
3.244	$\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}} dx$.1364
3.245	$\int \frac{x}{\sqrt{a+bx^3} (2(5+3\sqrt{3})a+bx^3)} dx$.1370
3.246	$\int \frac{x}{\sqrt{a-bx^3} (2(5+3\sqrt{3})a-bx^3)} dx$.1374
3.247	$\int \frac{x}{\sqrt{-a+bx^3} (-2(5+3\sqrt{3})a+bx^3)} dx$.1378
3.248	$\int \frac{x}{\sqrt{-a-bx^3} (-2(5+3\sqrt{3})a-bx^3)} dx$.1382
3.249	$\int \frac{x}{\sqrt{a+bx^3} (2(5-3\sqrt{3})a+bx^3)} dx$.1386

3.250	$\int \frac{x}{\sqrt{a-bx^3}(2(5-3\sqrt{3})a-bx^3)} dx$.1390
3.251	$\int \frac{x}{(2(5-3\sqrt{3})a-bx^3)\sqrt{-a+bx^3}} dx$.1394
3.252	$\int \frac{x}{\sqrt{-a-bx^3}(2(5-3\sqrt{3})a+bx^3)} dx$.1401
3.253	$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3} dx$.1408
3.254	$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3} dx$.1413
3.255	$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3} dx$.1418
3.256	$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)} dx$.1423
3.257	$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)} dx$.1428
3.258	$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3} dx$.1433
3.259	$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3} dx$.1438
3.260	$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3} dx$.1443
3.261	$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)} dx$.1448
3.262	$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)} dx$.1453
3.263	$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}} dx$.1458
3.264	$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}} dx$.1463
3.265	$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}} dx$.1467
3.266	$\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}} dx$.1471
3.267	$\int \frac{1}{x^4(a+bx^3)\sqrt{c+dx^3}} dx$.1476
3.268	$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}} dx$.1481
3.269	$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}} dx$.1486
3.270	$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}} dx$.1491
3.271	$\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}} dx$.1496
3.272	$\int \frac{1}{x^4(a+bx^3)(c+dx^3)^{3/2}} dx$.1501

3.273	$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2} dx$1507
3.274	$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2} dx$1513
3.275	$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2} dx$1519
3.276	$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2} dx$1524
3.277	$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2} dx$1528
3.278	$\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)^2} dx$1534
3.279	$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)^2} dx$1540
3.280	$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$1546
3.281	$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$1552
3.282	$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$1558
3.283	$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$1564
3.284	$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2} dx$1569
3.285	$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2} dx$1575
3.286	$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2} dx$1581
3.287	$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}} dx$1587
3.288	$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}} dx$1593
3.289	$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}} dx$1598
3.290	$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}} dx$1603
3.291	$\int \frac{1}{x(8c-dx^3)^2\sqrt{c+dx^3}} dx$1607

3.292	$\int \frac{1}{x^4(8c-dx^3)^2\sqrt{c+dx^3}} dx$.1613
3.293	$\int \frac{1}{x^7(8c-dx^3)^2\sqrt{c+dx^3}} dx$.1619
3.294	$\int \frac{x^{11}}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx$.1625
3.295	$\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx$.1631
3.296	$\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx$.1636
3.297	$\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx$.1641
3.298	$\int \frac{1}{x(8c-dx^3)^2(c+dx^3)^{3/2}} dx$.1646
3.299	$\int \frac{1}{x^4(8c-dx^3)^2(c+dx^3)^{3/2}} dx$.1652
3.300	$\int \frac{1}{x^7(8c-dx^3)^2(c+dx^3)^{3/2}} dx$.1658
3.301	$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2} dx$.1664
3.302	$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2} dx$.1670
3.303	$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2} dx$.1675
3.304	$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2} dx$.1680
3.305	$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2} dx$.1685
3.306	$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2} dx$.1691
3.307	$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2} dx$.1697
3.308	$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2} dx$.1703
3.309	$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2} dx$.1708
3.310	$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2} dx$.1713
3.311	$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}} dx$.1719

3.312	$\int \frac{x^5}{(a+bx^3)^2 \sqrt{c+dx^3}} dx$.1724
3.313	$\int \frac{x^2}{(a+bx^3)^2 \sqrt{c+dx^3}} dx$.1729
3.314	$\int \frac{1}{x(a+bx^3)^2 \sqrt{c+dx^3}} dx$.1734
3.315	$\int \frac{1}{x^4(a+bx^3)^2 \sqrt{c+dx^3}} dx$.1739
3.316	$\int \frac{x^8}{(a+bx^3)^2 (c+dx^3)^{3/2}} dx$.1745
3.317	$\int \frac{x^5}{(a+bx^3)^2 (c+dx^3)^{3/2}} dx$.1750
3.318	$\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{3/2}} dx$.1755
3.319	$\int \frac{1}{x(a+bx^3)^2 (c+dx^3)^{3/2}} dx$.1760
3.320	$\int \frac{1}{x^4(a+bx^3)^2 (c+dx^3)^{3/2}} dx$.1767
3.321	$\int \frac{x^5}{\sqrt{a+bx^3} \sqrt{c+dx^3}} dx$.1782
3.322	$\int \frac{x^2}{\sqrt{a+bx^3} \sqrt{c+dx^3}} dx$.1787
3.323	$\int \frac{1}{x \sqrt{a+bx^3} \sqrt{c+dx^3}} dx$.1791
3.324	$\int \frac{1}{x^4 \sqrt{a+bx^3} \sqrt{c+dx^3}} dx$.1795
3.325	$\int (ex)^{7/2} \sqrt{a+bx^3} (A+Bx^3) dx$.1800
3.326	$\int \sqrt{ex} \sqrt{a+bx^3} (A+Bx^3) dx$.1805
3.327	$\int \frac{\sqrt{a+bx^3} (A+Bx^3)}{(ex)^{5/2}} dx$.1810
3.328	$\int \frac{\sqrt{a+bx^3} (A+Bx^3)}{x^{11/2}} dx$.1815
3.329	$\int (ex)^{7/2} (a+bx^3)^{3/2} (A+Bx^3) dx$.1822
3.330	$\int \sqrt{ex} (a+bx^3)^{3/2} (A+Bx^3) dx$.1827
3.331	$\int \frac{(a+bx^3)^{3/2} (A+Bx^3)}{(ex)^{5/2}} dx$.1833
3.332	$\int (ex)^{7/2} (a+bx^3)^{5/2} (A+Bx^3) dx$.1838
3.333	$\int \sqrt{ex} (a+bx^3)^{5/2} (A+Bx^3) dx$.1844
3.334	$\int \frac{(a+bx^3)^{5/2} (A+Bx^3)}{(ex)^{5/2}} dx$.1850
3.335	$\int \frac{(ex)^{7/2} (A+Bx^3)}{\sqrt{a+bx^3}} dx$.1855

3.336	$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}} dx$	1860
3.337	$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}} dx$	1864
3.338	$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}} dx$	1870
3.339	$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}} dx$	1875
3.340	$\int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{3/2}} dx$	1881
3.341	$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}} dx$	1885
3.342	$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}} dx$	1890
3.343	$\int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{5/2}} dx$	1894
3.344	$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3} dx$	1898
3.345	$\int \frac{x^8\sqrt[3]{a+bx^3}}{ad-bdx^3} dx$	1904
3.346	$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3} dx$	1910
3.347	$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3} dx$	1916
3.348	$\int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)} dx$	1921
3.349	$\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)} dx$	1927
3.350	$\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)} dx$	1933
3.351	$\int \frac{x^7\sqrt[3]{a+bx^3}}{ad-bdx^3} dx$	1939
3.352	$\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3} dx$	1943
3.353	$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3} dx$	1947
3.354	$\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)} dx$	1951
3.355	$\int \frac{\sqrt[3]{a+bx^3}}{x^5(ad-bdx^3)} dx$	1955
3.356	$\int \frac{\sqrt[3]{a+bx^3}}{x^8(ad-bdx^3)} dx$	1959
3.357	$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)} dx$	1963

3.358	$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3} dx$1967
3.359	$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3} dx$1971
3.360	$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3} dx$1977
3.361	$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3} dx$1983
3.362	$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3} dx$1989
3.363	$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)} dx$1995
3.364	$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)} dx$2001
3.365	$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)} dx$2008
3.366	$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3} dx$2015
3.367	$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3} dx$2019
3.368	$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3} dx$2023
3.369	$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)} dx$2027
3.370	$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)} dx$2031
3.371	$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)} dx$2035
3.372	$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)} dx$2039
3.373	$\int \frac{x^{14}}{\sqrt[3]{1-x^3}(1+x^3)} dx$2043
3.374	$\int \frac{x^{11}}{\sqrt[3]{1-x^3}(1+x^3)} dx$2048
3.375	$\int \frac{x^8}{\sqrt[3]{1-x^3}(1+x^3)} dx$2053
3.376	$\int \frac{x^5}{\sqrt[3]{1-x^3}(1+x^3)} dx$2058
3.377	$\int \frac{x^2}{\sqrt[3]{1-x^3}(1+x^3)} dx$2063
3.378	$\int \frac{1}{x\sqrt[3]{1-x^3}(1+x^3)} dx$2068

3.379	$\int \frac{1}{x^4 \sqrt[3]{1-x^3} (1+x^3)} dx$2073
3.380	$\int \frac{x^6}{\sqrt[3]{1-x^3} (1+x^3)} dx$2078
3.381	$\int \frac{x^3}{\sqrt[3]{1-x^3} (1+x^3)} dx$2083
3.382	$\int \frac{1}{\sqrt[3]{1-x^3} (1+x^3)} dx$2088
3.383	$\int \frac{1}{x^3 \sqrt[3]{1-x^3} (1+x^3)} dx$2094
3.384	$\int \frac{1}{x^6 \sqrt[3]{1-x^3} (1+x^3)} dx$2100
3.385	$\int \frac{1}{x^9 \sqrt[3]{1-x^3} (1+x^3)} dx$2106
3.386	$\int \frac{x}{\sqrt[3]{1-x^3} (1+x^3)} dx$2112
3.387	$\int \frac{x^{11}}{(1-x^3)^{2/3} (1+x^3)} dx$2115
3.388	$\int \frac{x^8}{(1-x^3)^{2/3} (1+x^3)} dx$2121
3.389	$\int \frac{x^5}{(1-x^3)^{2/3} (1+x^3)} dx$2126
3.390	$\int \frac{x^2}{(1-x^3)^{2/3} (1+x^3)} dx$2131
3.391	$\int \frac{1}{x(1-x^3)^{2/3} (1+x^3)} dx$2136
3.392	$\int \frac{1}{x^4(1-x^3)^{2/3} (1+x^3)} dx$2141
3.393	$\int \frac{x^7}{(1-x^3)^{2/3} (1+x^3)} dx$2146
3.394	$\int \frac{x^4}{(1-x^3)^{2/3} (1+x^3)} dx$2152
3.395	$\int \frac{x}{(1-x^3)^{2/3} (1+x^3)} dx$2157
3.396	$\int \frac{1}{x^2(1-x^3)^{2/3} (1+x^3)} dx$2163
3.397	$\int \frac{1}{x^5(1-x^3)^{2/3} (1+x^3)} dx$2169
3.398	$\int \frac{x^6}{(1-x^3)^{2/3} (1+x^3)} dx$2175
3.399	$\int \frac{1}{x^3(1-x^3)^{2/3} (1+x^3)} dx$2179
3.400	$\int \frac{x^{14}}{(1-x^3)^{4/3} (1+x^3)} dx$2183

3.401	$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)} dx$	2189
3.402	$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)} dx$	2195
3.403	$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)} dx$	2201
3.404	$\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)} dx$	2206
3.405	$\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)} dx$	2211
3.406	$\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)} dx$	2216
3.407	$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)} dx$	2222
3.408	$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)} dx$	2226
3.409	$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)} dx$	2229
3.410	$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)} dx$	2233
3.411	$\int \frac{1}{x^3(1-x^3)^{4/3}(1+x^3)} dx$	2239
3.412	$\int \frac{1}{x^6(1-x^3)^{4/3}(1+x^3)} dx$	2243
3.413	$\int \frac{1}{x^9(1-x^3)^{4/3}(1+x^3)} dx$	2247
3.414	$\int \frac{x^{11} \sqrt[3]{a+bx^3}}{c+dx^3} dx$	2251
3.415	$\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3} dx$	2257
3.416	$\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} dx$	2263
3.417	$\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} dx$	2268
3.418	$\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)} dx$	2273
3.419	$\int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)} dx$	2279
3.420	$\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)} dx$	2286
3.421	$\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3} dx$	2293
3.422	$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3} dx$	2297
3.423	$\int \frac{x \sqrt[3]{a+bx^3}}{c+dx^3} dx$	2301

3.424	$\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)} dx$2305
3.425	$\int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)} dx$2309
3.426	$\int \frac{\sqrt[3]{a+bx^3}}{x^8(c+dx^3)} dx$2313
3.427	$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(c+dx^3)} dx$2317
3.428	$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3} dx$2321
3.429	$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3} dx$2327
3.430	$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3} dx$2333
3.431	$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3} dx$2339
3.432	$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)} dx$2344
3.433	$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)} dx$2350
3.434	$\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)} dx$2357
3.435	$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3} dx$2365
3.436	$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3} dx$2370
3.437	$\int \frac{(a+bx^3)^{2/3}}{c+dx^3} dx$2374
3.438	$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)} dx$2378
3.439	$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)} dx$2382
3.440	$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)} dx$2386
3.441	$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)} dx$2390
3.442	$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3} dx$2395
3.443	$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3} dx$2401
3.444	$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3} dx$2407

3.445	$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)} dx$.2412
3.446	$\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)} dx$.2418
3.447	$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)} dx$.2425
3.448	$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3} dx$.2433
3.449	$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3} dx$.2437
3.450	$\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)} dx$.2441
3.451	$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)} dx$.2445
3.452	$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)} dx$.2449
3.453	$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)} dx$.2453
3.454	$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)} dx$.2457
3.455	$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$.2462
3.456	$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$.2468
3.457	$\int \frac{x^8}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$.2474
3.458	$\int \frac{x^5}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$.2480
3.459	$\int \frac{x^2}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$.2486
3.460	$\int \frac{1}{x\sqrt[3]{a+bx^3}(c+dx^3)} dx$.2491
3.461	$\int \frac{1}{x^4\sqrt[3]{a+bx^3}(c+dx^3)} dx$.2497
3.462	$\int \frac{x^6}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$.2504
3.463	$\int \frac{x^3}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$.2510
3.464	$\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$.2515
3.465	$\int \frac{1}{x^3\sqrt[3]{a+bx^3}(c+dx^3)} dx$.2520

3.466	$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}(c+dx^3)} dx$.2525
3.467	$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}(c+dx^3)} dx$.2530
3.468	$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)} dx$.2535
3.469	$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)} dx$.2541
3.470	$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)} dx$.2547
3.471	$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)} dx$.2553
3.472	$\int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)} dx$.2558
3.473	$\int \frac{1}{x^4(a+bx^3)^{2/3}(c+dx^3)} dx$.2564
3.474	$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)} dx$.2571
3.475	$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)} dx$.2577
3.476	$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)} dx$.2582
3.477	$\int \frac{1}{x^2(a+bx^3)^{2/3}(c+dx^3)} dx$.2587
3.478	$\int \frac{1}{x^5(a+bx^3)^{2/3}(c+dx^3)} dx$.2592
3.479	$\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)} dx$.2597
3.480	$\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)} dx$.2603
3.481	$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)} dx$.2609
3.482	$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)} dx$.2615
3.483	$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)} dx$.2621
3.484	$\int \frac{1}{x(a+bx^3)^{4/3}(c+dx^3)} dx$.2626
3.485	$\int \frac{1}{x^4(a+bx^3)^{4/3}(c+dx^3)} dx$.2634
3.486	$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)} dx$.2644
3.487	$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)} dx$.2649

3.488	$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)} dx$.2653
3.489	$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)} dx$.2657
3.490	$\int \frac{1}{x^3(a+bx^3)^{4/3}(c+dx^3)} dx$.2662
3.491	$\int \frac{1}{x^6(a+bx^3)^{4/3}(c+dx^3)} dx$.2666
3.492	$\int \frac{1}{x^9(a+bx^3)^{4/3}(c+dx^3)} dx$.2671
3.493	$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)} dx$.2676
3.494	$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)} dx$.2679
3.495	$\int \frac{x^7}{(a+bx^4)(c+dx^4)} dx$.2682
3.496	$\int \frac{x^3}{(a+bx^4)(c+dx^4)} dx$.2685
3.497	$\int \frac{1}{x(a+bx^4)(c+dx^4)} dx$.2689
3.498	$\int \frac{1}{x^5(a+bx^4)(c+dx^4)} dx$.2692
3.499	$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)} dx$.2695
3.500	$\int \frac{x^9}{(a+bx^4)(c+dx^4)} dx$.2700
3.501	$\int \frac{x^5}{(a+bx^4)(c+dx^4)} dx$.2705
3.502	$\int \frac{x}{(a+bx^4)(c+dx^4)} dx$.2709
3.503	$\int \frac{1}{x^3(a+bx^4)(c+dx^4)} dx$.2713
3.504	$\int \frac{1}{x^7(a+bx^4)(c+dx^4)} dx$.2718
3.505	$\int \frac{x^8}{(a+bx^4)(c+dx^4)} dx$.2723
3.506	$\int \frac{x^6}{(a+bx^4)(c+dx^4)} dx$.2732
3.507	$\int \frac{x^4}{(a+bx^4)(c+dx^4)} dx$.2739
3.508	$\int \frac{x^2}{(a+bx^4)(c+dx^4)} dx$.2748
3.509	$\int \frac{1}{(a+bx^4)(c+dx^4)} dx$.2757
3.510	$\int \frac{1}{x^2(a+bx^4)(c+dx^4)} dx$.2766
3.511	$\int \frac{1}{x^4(a+bx^4)(c+dx^4)} dx$.2775

3.512	$\int \frac{1}{x^6(a+bx^4)(c+dx^4)} dx$.2785
3.513	$\int \frac{x^7 \sqrt{c+dx^4}}{a+bx^4} dx$.2794
3.514	$\int \frac{x^5 \sqrt{c+dx^4}}{a+bx^4} dx$.2799
3.515	$\int \frac{x^3 \sqrt{c+dx^4}}{a+bx^4} dx$.2805
3.516	$\int \frac{x \sqrt{c+dx^4}}{a+bx^4} dx$.2810
3.517	$\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)} dx$.2815
3.518	$\int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)} dx$.2820
3.519	$\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)} dx$.2825
3.520	$\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)} dx$.2830
3.521	$\int \frac{x^{11}}{(a+bx^4) \sqrt{c+dx^4}} dx$.2836
3.522	$\int \frac{x^7}{(a+bx^4) \sqrt{c+dx^4}} dx$.2840
3.523	$\int \frac{x^3}{(a+bx^4) \sqrt{c+dx^4}} dx$.2844
3.524	$\int \frac{1}{x(a+bx^4) \sqrt{c+dx^4}} dx$.2848
3.525	$\int \frac{1}{x^5(a+bx^4) \sqrt{c+dx^4}} dx$.2853
3.526	$\int \frac{x^9}{(a+bx^4) \sqrt{c+dx^4}} dx$.2858
3.527	$\int \frac{x^5}{(a+bx^4) \sqrt{c+dx^4}} dx$.2863
3.528	$\int \frac{x}{(a+bx^4) \sqrt{c+dx^4}} dx$.2868
3.529	$\int \frac{1}{x^3(a+bx^4) \sqrt{c+dx^4}} dx$.2872
3.530	$\int \frac{1}{x^7(a+bx^4) \sqrt{c+dx^4}} dx$.2877
3.531	$\int \frac{x^{15}}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$.2883
3.532	$\int \frac{x^{11}}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$.2889
3.533	$\int \frac{x^7}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$.2894
3.534	$\int \frac{x^3}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$.2899

3.535	$\int \frac{1}{x(a+bx^4)^2 \sqrt{c+dx^4}} dx$2904
3.536	$\int \frac{1}{x^5(a+bx^4)^2 \sqrt{c+dx^4}} dx$2911
3.537	$\int \frac{x^{13}}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$2919
3.538	$\int \frac{x^9}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$2925
3.539	$\int \frac{x^5}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$2931
3.540	$\int \frac{x}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$2936
3.541	$\int \frac{1}{x^3(a+bx^4)^2 \sqrt{c+dx^4}} dx$2941
3.542	$\int \frac{1}{x^7(a+bx^4)^2 \sqrt{c+dx^4}} dx$2947
3.543	$\int \frac{x^{17}}{(a+bx^6) \sqrt{c+dx^6}} dx$2953
3.544	$\int \frac{x^{11}}{(a+bx^6) \sqrt{c+dx^6}} dx$2957
3.545	$\int \frac{x^5}{(a+bx^6) \sqrt{c+dx^6}} dx$2961
3.546	$\int \frac{1}{x(a+bx^6) \sqrt{c+dx^6}} dx$2965
3.547	$\int \frac{1}{x^7(a+bx^6) \sqrt{c+dx^6}} dx$2970
3.548	$\int \frac{x^{14}}{(a+bx^6) \sqrt{c+dx^6}} dx$2975
3.549	$\int \frac{x^8}{(a+bx^6) \sqrt{c+dx^6}} dx$2980
3.550	$\int \frac{x^2}{(a+bx^6) \sqrt{c+dx^6}} dx$2985
3.551	$\int \frac{1}{x^4(a+bx^6) \sqrt{c+dx^6}} dx$2989
3.552	$\int \frac{1}{x^{10}(a+bx^6) \sqrt{c+dx^6}} dx$2993
3.553	$\int \frac{x^{17}}{(a+bx^6)^2 \sqrt{c+dx^6}} dx$2998
3.554	$\int \frac{x^{11}}{(a+bx^6)^2 \sqrt{c+dx^6}} dx$3003
3.555	$\int \frac{x^5}{(a+bx^6)^2 \sqrt{c+dx^6}} dx$3007
3.556	$\int \frac{1}{x(a+bx^6)^2 \sqrt{c+dx^6}} dx$3011

3.557	$\int \frac{1}{x^7(a+bx^6)^2\sqrt{c+dx^6}} dx$3017
3.558	$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}} dx$3024
3.559	$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}} dx$3030
3.560	$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}} dx$3034
3.561	$\int \frac{1}{x^4(a+bx^6)^2\sqrt{c+dx^6}} dx$3039
3.562	$\int \frac{1}{x^{10}(a+bx^6)^2\sqrt{c+dx^6}} dx$3045
3.563	$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}} dx$3050
3.564	$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}} dx$3054
3.565	$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}} dx$3058
3.566	$\int \frac{1}{x(a+bx^8)\sqrt{c+dx^8}} dx$3062
3.567	$\int \frac{1}{x^9(a+bx^8)\sqrt{c+dx^8}} dx$3067
3.568	$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}} dx$3072
3.569	$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}} dx$3077
3.570	$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}} dx$3081
3.571	$\int \frac{1}{x^5(a+bx^8)\sqrt{c+dx^8}} dx$3085
3.572	$\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}} dx$3090
3.573	$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}} dx$3095
3.574	$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}} dx$3100
3.575	$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}} dx$3104
3.576	$\int \frac{1}{x(a+bx^8)^2\sqrt{c+dx^8}} dx$3108
3.577	$\int \frac{1}{x^9(a+bx^8)^2\sqrt{c+dx^8}} dx$3114
3.578	$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}} dx$3121

- 3.579 $\int \frac{x^{11}}{(a+bx^8)^2 \sqrt{c+dx^8}} dx \dots\dots\dots .3127$
- 3.580 $\int \frac{x^3}{(a+bx^8)^2 \sqrt{c+dx^8}} dx \dots\dots\dots .3132$
- 3.581 $\int \frac{1}{x^5(a+bx^8)^2 \sqrt{c+dx^8}} dx \dots\dots\dots .3137$
- 3.582 $\int \frac{1}{x^{13}(a+bx^8)^2 \sqrt{c+dx^8}} dx \dots\dots\dots .3143$
- 3.583 $\int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x^5 dx \dots\dots\dots .3149$
- 3.584 $\int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x^3 dx \dots\dots\dots .3155$
- 3.585 $\int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x dx \dots\dots\dots .3160$
- 3.586 $\int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x} dx \dots\dots\dots .3165$
- 3.587 $\int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^3} dx \dots\dots\dots .3170$
- 3.588 $\int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^5} dx \dots\dots\dots .3174$
- 3.589 $\int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^7} dx \dots\dots\dots .3178$
- 3.590 $\int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^9} dx \dots\dots\dots .3182$
- 3.591 $\int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x^{10} dx \dots\dots\dots .3186$
- 3.592 $\int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x^8 dx \dots\dots\dots .3191$
- 3.593 $\int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x^6 dx \dots\dots\dots .3195$
- 3.594 $\int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x^4 dx \dots\dots\dots .3199$
- 3.595 $\int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x^2 dx \dots\dots\dots .3203$
- 3.596 $\int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} dx \dots\dots\dots .3208$
- 3.597 $\int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^2} dx \dots\dots\dots .3213$
- 3.598 $\int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^4} dx \dots\dots\dots .3218$
- 3.599 $\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^5 dx \dots\dots\dots .3224$

3.600	$\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^3 dx$.3229
3.601	$\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x dx$.3234
3.602	$\int \frac{\left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2}}{x} dx$.3239
3.603	$\int \frac{\left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2}}{x^3} dx$.3244
3.604	$\int \frac{\left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2}}{x^5} dx$.3248
3.605	$\int \frac{\left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2}}{x^7} dx$.3252
3.606	$\int \frac{\left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2}}{x^9} dx$.3256
3.607	$\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^{12} dx$.3260
3.608	$\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^{10} dx$.3266
3.609	$\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^8 dx$.3271
3.610	$\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^6 dx$.3276
3.611	$\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^4 dx$.3280
3.612	$\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^2 dx$.3285
3.613	$\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} dx$.3290
3.614	$\int \frac{\left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2}}{x^2} dx$.3295
3.615	$\int \frac{\left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2}}{x^4} dx$.3300
3.616	$\int \frac{\left(a + \frac{b}{x^2}\right) x^3}{\sqrt{c + \frac{d}{x^2}}} dx$.3306
3.617	$\int \frac{\left(a + \frac{b}{x^2}\right) x}{\sqrt{c + \frac{d}{x^2}}} dx$.3311
3.618	$\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}}} x dx$.3316

3.619	$\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^3} dx$ 3321
3.620	$\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^5} dx$ 3325
3.621	$\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^7} dx$ 3329
3.622	$\int \frac{\left(a + \frac{b}{x^2}\right) x^4}{\sqrt{c + \frac{d}{x^2}}} dx$ 3333
3.623	$\int \frac{\left(a + \frac{b}{x^2}\right) x^2}{\sqrt{c + \frac{d}{x^2}}} dx$ 3337
3.624	$\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}}} dx$ 3341
3.625	$\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^2} dx$ 3346
3.626	$\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^4} dx$ 3351
3.627	$\int \frac{\left(a + \frac{b}{x^2}\right) x^3}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$ 3356
3.628	$\int \frac{\left(a + \frac{b}{x^2}\right) x}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$ 3362
3.629	$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x} dx$ 3368
3.630	$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^3} dx$ 3373
3.631	$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^5} dx$ 3377
3.632	$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^7} dx$ 3381
3.633	$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^9} dx$ 3385

3.634	$\int \frac{\left(a + \frac{b}{x^2}\right)x^4}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$3389
3.635	$\int \frac{\left(a + \frac{b}{x^2}\right)x^2}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$3394
3.636	$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$3398
3.637	$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^2} dx$3402
3.638	$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^4} dx$3407
3.639	$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^6} dx$3413
3.640	$\int \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2} dx$3419
3.641	$\int \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} dx$3424
3.642	$\int \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} dx$3429
3.643	$\int \frac{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}}{\sqrt{x}} dx$3433
3.644	$\int \frac{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}}{x^{3/2}} dx$3437
3.645	$\int \frac{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}}{x^{5/2}} dx$3442
3.646	$\int \frac{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}}{x^{7/2}} dx$3446
3.647	$\int \frac{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}}{x^{9/2}} dx$3450
3.648	$\int \frac{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}}{x^{11/2}} dx$3454
3.649	$\int \frac{1}{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2}} dx$3459
3.650	$\int \frac{1}{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2}} dx$3464
3.651	$\int \frac{\sqrt{x}}{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}} dx$3468
3.652	$\int \frac{1}{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x}} dx$3472

3.653	$\int \frac{1}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{3/2}} dx$.3475
3.654	$\int \frac{1}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{5/2}} dx$.3478
3.655	$\int \frac{1}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{7/2}} dx$.3482
3.656	$\int \frac{1+x^6}{x(1-x^6)} dx$.3486
3.657	$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n) dx$.3489
3.658	$\int \frac{1}{x(a+bx^n)(c+dx^n)} dx$.3492
3.659	$\int \frac{1}{x(a+bx^n)^2(c+dx^n)} dx$.3496
3.660	$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n} dx$.3499
3.661	$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n} dx$.3503
3.662	$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n} dx$.3507
3.663	$\int \frac{1}{(a+bx^n)(c+dx^n) x^{-1+2n}} dx$.3511
3.664	$\int \frac{1}{(a+bx^n)^2(c+dx^n) x^{-1+2n}} dx$.3514
3.665	$\int \frac{1}{(a+bx^n)^3(c+dx^n) x^{-1+2n}} dx$.3518
3.666	$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n} dx$.3522
3.667	$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n} dx$.3526
3.668	$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n} dx$.3530
3.669	$\int \frac{1}{(a+bx^n)(c+dx^n) x^{-1+3n}} dx$.3534
3.670	$\int \frac{1}{(a+bx^n)^2(c+dx^n) x^{-1+3n}} dx$.3537
3.671	$\int \frac{1}{(a+bx^n)^3(c+dx^n) x^{-1+3n}} dx$.3541
3.672	$\int x^{13}(b+cx)^{13}(b+2cx) dx$.3545
3.673	$\int x^{27}(b+cx^2)^{13}(b+2cx^2) dx$.3548
3.674	$\int x^{41}(b+cx^3)^{13}(b+2cx^3) dx$.3551
3.675	$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n) dx$.3554
3.676	$\int x^{-1+m}(a+bx^n)^{-1+p}(am+b(m+np)x^n) dx$.3557
3.677	$\int \frac{b+2cx}{x(b+cx)} dx$.3560
3.678	$\int \frac{b+2cx^2}{x(b+cx^2)} dx$.3563
3.679	$\int \frac{b+2cx^3}{x(b+cx^3)} dx$.3566

3.680	$\int \frac{b+2cx^n}{x(b+cx^n)} dx$3569
3.681	$\int \frac{b+2cx}{x^8(b+cx)^8} dx$3572
3.682	$\int \frac{b+2cx^2}{x^{15}(b+cx^2)^8} dx$3575
3.683	$\int \frac{b+2cx^3}{x^{22}(b+cx^3)^8} dx$3578
3.684	$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8} dx$3581
3.685	$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}} dx$3585
3.686	$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x} dx$3589
3.687	$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}} dx$3595
3.688	$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}} dx$3600
3.689	$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}} dx$3605
3.690	$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}} dx$3610
3.691	$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}} dx$3614
3.692	$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}} dx$3618
3.693	$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}} dx$3622
3.694	$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}} dx$3628
3.695	$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}} dx$3633
3.696	$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}} dx$3638
3.697	$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}} dx$3643
3.698	$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}} dx$3648
3.699	$\int x^p(b+cx)^p(b+2cx) dx$3653
3.700	$\int x^{-1+2(1+p)}(b+cx^2)^p(b+2cx^2) dx$3656
3.701	$\int x^{-1+3(1+p)}(b+cx^3)^p(b+2cx^3) dx$3659
3.702	$\int x^{-1+n(1+p)}(b+cx^n)^p(b+2cx^n) dx$3662

3.1 $\int x^2 (a + bx^3) (A + Bx^3) dx$

Optimal. Leaf size=33

$$\frac{1}{6}x^6(aB + Ab) + \frac{1}{3}aAx^3 + \frac{1}{9}bBx^9$$

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {444, 43}

$$\frac{1}{6}x^6(aB + Ab) + \frac{1}{3}aAx^3 + \frac{1}{9}bBx^9$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x^3)*(A + B*x^3),x]

[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int x^2 (a + bx^3) (A + Bx^3) dx &= \frac{1}{3} \text{Subst} \left(\int (a + bx)(A + Bx) dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int (aA + (Ab + aB)x + bBx^2) dx, x, x^3 \right) \\ &= \frac{1}{3}aAx^3 + \frac{1}{6}(Ab + aB)x^6 + \frac{1}{9}bBx^9 \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{6}x^6(aB + Ab) + \frac{1}{3}aAx^3 + \frac{1}{9}bBx^9$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x^3)*(A + B*x^3), x]

[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + bx^3) (A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a + b*x^3)*(A + B*x^3), x]

[Out] IntegrateAlgebraic[x^2*(a + b*x^3)*(A + B*x^3), x]

fricas [A] time = 0.65, size = 29, normalized size = 0.88

$$\frac{1}{9}x^9bB + \frac{1}{6}x^6aB + \frac{1}{6}x^6bA + \frac{1}{3}x^3aA$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)*(B*x^3+A), x, algorithm="fricas")

[Out] 1/9*x^9*b*B + 1/6*x^6*a*B + 1/6*x^6*b*A + 1/3*x^3*a*A

giac [A] time = 0.18, size = 29, normalized size = 0.88

$$\frac{1}{9}Bbx^9 + \frac{1}{6}Bax^6 + \frac{1}{6}Abx^6 + \frac{1}{3}Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)*(B*x^3+A), x, algorithm="giac")

[Out] 1/9*B*b*x^9 + 1/6*B*a*x^6 + 1/6*A*b*x^6 + 1/3*A*a*x^3

maple [A] time = 0.04, size = 28, normalized size = 0.85

$$\frac{Bbx^9}{9} + \frac{(Ab + Ba)x^6}{6} + \frac{Aax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^3+a)*(B*x^3+A),x)`

[Out] $1/3*A*a*x^3+1/6*(A*b+B*a)*x^6+1/9*b*B*x^9$

maxima [A] time = 0.57, size = 27, normalized size = 0.82

$$\frac{1}{9} B b x^9 + \frac{1}{6} (B a + A b) x^6 + \frac{1}{3} A a x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")`

[Out] $1/9*B*b*x^9 + 1/6*(B*a + A*b)*x^6 + 1/3*A*a*x^3$

mupad [B] time = 0.20, size = 28, normalized size = 0.85

$$\frac{B b x^9}{9} + \left(\frac{A b}{6} + \frac{B a}{6} \right) x^6 + \frac{A a x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(A + B*x^3)*(a + b*x^3),x)`

[Out] $x^6*((A*b)/6 + (B*a)/6) + (A*a*x^3)/3 + (B*b*x^9)/9$

sympy [A] time = 0.07, size = 29, normalized size = 0.88

$$\frac{A a x^3}{3} + \frac{B b x^9}{9} + x^6 \left(\frac{A b}{6} + \frac{B a}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**3+a)*(B*x**3+A),x)`

[Out] $A*a*x**3/3 + B*b*x**9/9 + x**6*(A*b/6 + B*a/6)$

3.2 $\int x(a + bx^3)(A + Bx^3) dx$

Optimal. Leaf size=33

$$\frac{1}{5}x^5(aB + Ab) + \frac{1}{2}aAx^2 + \frac{1}{8}bBx^8$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {448}

$$\frac{1}{5}x^5(aB + Ab) + \frac{1}{2}aAx^2 + \frac{1}{8}bBx^8$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^3)*(A + B*x^3), x]

[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x(a + bx^3)(A + Bx^3) dx &= \int (aAx + (Ab + aB)x^4 + bBx^7) dx \\ &= \frac{1}{2}aAx^2 + \frac{1}{5}(Ab + aB)x^5 + \frac{1}{8}bBx^8 \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{5}x^5(aB + Ab) + \frac{1}{2}aAx^2 + \frac{1}{8}bBx^8$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^3)*(A + B*x^3), x]

[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + bx^3)(A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a + b*x^3)*(A + B*x^3), x]

[Out] IntegrateAlgebraic[x*(a + b*x^3)*(A + B*x^3), x]

fricas [A] time = 0.70, size = 29, normalized size = 0.88

$$\frac{1}{8}x^8bB + \frac{1}{5}x^5aB + \frac{1}{5}x^5bA + \frac{1}{2}x^2aA$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)*(B*x^3+A), x, algorithm="fricas")

[Out] 1/8*x^8*b*B + 1/5*x^5*a*B + 1/5*x^5*b*A + 1/2*x^2*a*A

giac [A] time = 0.15, size = 29, normalized size = 0.88

$$\frac{1}{8}Bbx^8 + \frac{1}{5}Bax^5 + \frac{1}{5}Abx^5 + \frac{1}{2}Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)*(B*x^3+A), x, algorithm="giac")

[Out] 1/8*B*b*x^8 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/2*A*a*x^2

maple [A] time = 0.04, size = 28, normalized size = 0.85

$$\frac{Bbx^8}{8} + \frac{(Ab + Ba)x^5}{5} + \frac{Aax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a)*(B*x^3+A), x)

[Out] 1/2*A*a*x^2+1/5*(A*b+B*a)*x^5+1/8*b*B*x^8

maxima [A] time = 0.63, size = 27, normalized size = 0.82

$$\frac{1}{8}Bbx^8 + \frac{1}{5}(Ba + Ab)x^5 + \frac{1}{2}Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")

[Out] 1/8*B*b*x^8 + 1/5*(B*a + A*b)*x^5 + 1/2*A*a*x^2

mupad [B] time = 2.48, size = 28, normalized size = 0.85

$$\frac{Bbx^8}{8} + \left(\frac{Ab}{5} + \frac{Ba}{5}\right)x^5 + \frac{Aax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(A + B*x^3)*(a + b*x^3),x)

[Out] x^5*((A*b)/5 + (B*a)/5) + (A*a*x^2)/2 + (B*b*x^8)/8

sympy [A] time = 0.07, size = 29, normalized size = 0.88

$$\frac{Aax^2}{2} + \frac{Bbx^8}{8} + x^5\left(\frac{Ab}{5} + \frac{Ba}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**3+a)*(B*x**3+A),x)

[Out] A*a*x**2/2 + B*b*x**8/8 + x**5*(A*b/5 + B*a/5)

3.3 $\int (a + bx^3)(A + Bx^3) dx$

Optimal. Leaf size=28

$$\frac{1}{4}x^4(aB + Ab) + aAx + \frac{1}{7}bBx^7$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {373}

$$\frac{1}{4}x^4(aB + Ab) + aAx + \frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)*(A + B*x^3), x]

[Out] a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7

Rule 373

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int (a + bx^3)(A + Bx^3) dx &= \int (aA + (Ab + aB)x^3 + bBx^6) dx \\ &= aAx + \frac{1}{4}(Ab + aB)x^4 + \frac{1}{7}bBx^7 \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{1}{4}x^4(aB + Ab) + aAx + \frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)*(A + B*x^3), x]

[Out] a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx^3)(A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^3)*(A + B*x^3), x]

[Out] IntegrateAlgebraic[(a + b*x^3)*(A + B*x^3), x]

fricas [A] time = 1.08, size = 26, normalized size = 0.93

$$\frac{1}{7}x^7bB + \frac{1}{4}x^4aB + \frac{1}{4}x^4bA + xaA$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A), x, algorithm="fricas")

[Out] 1/7*x^7*b*B + 1/4*x^4*a*B + 1/4*x^4*b*A + x*a*A

giac [A] time = 0.15, size = 26, normalized size = 0.93

$$\frac{1}{7}Bbx^7 + \frac{1}{4}Bax^4 + \frac{1}{4}Abx^4 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A), x, algorithm="giac")

[Out] 1/7*B*b*x^7 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + A*a*x

maple [A] time = 0.04, size = 25, normalized size = 0.89

$$\frac{Bbx^7}{7} + \frac{(Ab + Ba)x^4}{4} + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(B*x^3+A), x)

[Out] A*a*x+1/4*(A*b+B*a)*x^4+1/7*b*B*x^7

maxima [A] time = 0.56, size = 24, normalized size = 0.86

$$\frac{1}{7}Bbx^7 + \frac{1}{4}(Ba + Ab)x^4 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)*(B*x^3+A),x, algorithm="maxima")`

[Out] `1/7*B*b*x^7 + 1/4*(B*a + A*b)*x^4 + A*a*x`

mupad [B] time = 0.03, size = 25, normalized size = 0.89

$$\frac{B b x^7}{7} + \left(\frac{A b}{4} + \frac{B a}{4} \right) x^4 + A a x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)*(a + b*x^3),x)`

[Out] `x^4*((A*b)/4 + (B*a)/4) + A*a*x + (B*b*x^7)/7`

sympy [A] time = 0.06, size = 26, normalized size = 0.93

$$A a x + \frac{B b x^7}{7} + x^4 \left(\frac{A b}{4} + \frac{B a}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)*(B*x**3+A),x)`

[Out] `A*a*x + B*b*x**7/7 + x**4*(A*b/4 + B*a/4)`

$$3.4 \quad \int \frac{(a+bx^3)(A+Bx^3)}{x} dx$$

Optimal. Leaf size=29

$$\frac{1}{3}x^3(aB + Ab) + aA \log(x) + \frac{1}{6}bBx^6$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {446, 76}

$$\frac{1}{3}x^3(aB + Ab) + aA \log(x) + \frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*(A + B*x^3))/x,x]

[Out] ((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]

Rule 76

Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)(A + Bx^3)}{x} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)(A + Bx)}{x} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(Ab + aB + \frac{aA}{x} + bBx \right) dx, x, x^3 \right) \\ &= \frac{1}{3} (Ab + aB)x^3 + \frac{1}{6} bBx^6 + aA \log(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{1}{3}x^3(aB + Ab) + aA \log(x) + \frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(A + B*x^3))/x,x]

[Out] ((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)(A + Bx^3)}{x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x,x]

[Out] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x, x]

fricas [A] time = 0.76, size = 25, normalized size = 0.86

$$\frac{1}{6}Bbx^6 + \frac{1}{3}(Ba + Ab)x^3 + Aa \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="fricas")

[Out] 1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + A*a*log(x)

giac [A] time = 0.15, size = 28, normalized size = 0.97

$$\frac{1}{6}Bbx^6 + \frac{1}{3}Bax^3 + \frac{1}{3}Abx^3 + Aa \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="giac")

[Out] 1/6*B*b*x^6 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*log(abs(x))

maple [A] time = 0.04, size = 28, normalized size = 0.97

$$\frac{Bbx^6}{6} + \frac{Abx^3}{3} + \frac{Bax^3}{3} + Aa \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)*(B*x^3+A)/x,x)`

[Out] `1/6*b*B*x^6+1/3*A*x^3*b+1/3*B*a*x^3+A*a*log(x)`

maxima [A] time = 0.57, size = 28, normalized size = 0.97

$$\frac{1}{6} B b x^6 + \frac{1}{3} (B a + A b) x^3 + \frac{1}{3} A a \log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="maxima")`

[Out] `1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + 1/3*A*a*log(x^3)`

mupad [B] time = 0.03, size = 26, normalized size = 0.90

$$x^3 \left(\frac{A b}{3} + \frac{B a}{3} \right) + \frac{B b x^6}{6} + A a \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3))/x,x)`

[Out] `x^3*((A*b)/3 + (B*a)/3) + (B*b*x^6)/6 + A*a*log(x)`

sympy [A] time = 0.11, size = 27, normalized size = 0.93

$$A a \log(x) + \frac{B b x^6}{6} + x^3 \left(\frac{A b}{3} + \frac{B a}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)*(B*x**3+A)/x,x)`

[Out] `A*a*log(x) + B*b*x**6/6 + x**3*(A*b/3 + B*a/3)`

$$3.5 \quad \int \frac{(a+bx^3)(A+Bx^3)}{x^2} dx$$

Optimal. Leaf size=31

$$\frac{1}{2}x^2(aB + Ab) - \frac{aA}{x} + \frac{1}{5}bBx^5$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {448}

$$\frac{1}{2}x^2(aB + Ab) - \frac{aA}{x} + \frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*(A + B*x^3))/x^2,x]

[Out] -((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)(A + Bx^3)}{x^2} dx &= \int \left(\frac{aA}{x^2} + (Ab + aB)x + bBx^4 \right) dx \\ &= -\frac{aA}{x} + \frac{1}{2}(Ab + aB)x^2 + \frac{1}{5}bBx^5 \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{1}{2}x^2(aB + Ab) - \frac{aA}{x} + \frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^2,x]

[Out] -((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)(A + Bx^3)}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^2,x]

[Out] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^2, x]

fricas [A] time = 0.73, size = 29, normalized size = 0.94

$$\frac{2 Bbx^6 + 5 (Ba + Ab)x^3 - 10 Aa}{10x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="fricas")

[Out] 1/10*(2*B*b*x^6 + 5*(B*a + A*b)*x^3 - 10*A*a)/x

giac [A] time = 0.15, size = 29, normalized size = 0.94

$$\frac{1}{5} Bbx^5 + \frac{1}{2} Bax^2 + \frac{1}{2} Abx^2 - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="giac")

[Out] 1/5*B*b*x^5 + 1/2*B*a*x^2 + 1/2*A*b*x^2 - A*a/x

maple [A] time = 0.05, size = 30, normalized size = 0.97

$$\frac{Bbx^5}{5} + \frac{Abx^2}{2} + \frac{Bax^2}{2} - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(B*x^3+A)/x^2,x)

[Out] 1/5*b*B*x^5+1/2*A*b*x^2+1/2*B*a*x^2-A*a/x

maxima [A] time = 0.50, size = 27, normalized size = 0.87

$$\frac{1}{5} Bbx^5 + \frac{1}{2} (Ba + Ab)x^2 - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="maxima")

[Out] 1/5*B*b*x^5 + 1/2*(B*a + A*b)*x^2 - A*a/x

mupad [B] time = 0.04, size = 28, normalized size = 0.90

$$x^2 \left(\frac{Ab}{2} + \frac{Ba}{2} \right) - \frac{Aa}{x} + \frac{Bbx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3))/x^2,x)

[Out] x^2*((A*b)/2 + (B*a)/2) - (A*a)/x + (B*b*x^5)/5

sympy [A] time = 0.11, size = 26, normalized size = 0.84

$$-\frac{Aa}{x} + \frac{Bbx^5}{5} + x^2 \left(\frac{Ab}{2} + \frac{Ba}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(B*x**3+A)/x**2,x)

[Out] -A*a/x + B*b*x**5/5 + x**2*(A*b/2 + B*a/2)

$$3.6 \quad \int \frac{(a+bx^3)(A+Bx^3)}{x^3} dx$$

Optimal. Leaf size=28

$$x(aB + Ab) - \frac{aA}{2x^2} + \frac{1}{4}bBx^4$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {448}

$$x(aB + Ab) - \frac{aA}{2x^2} + \frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*(A + B*x^3))/x^3,x]

[Out] -(a*A)/(2*x^2) + (A*b + a*B)*x + (b*B*x^4)/4

Rule 448

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)(A + Bx^3)}{x^3} dx &= \int \left(Ab \left(1 + \frac{aB}{Ab} \right) + \frac{aA}{x^3} + bBx^3 \right) dx \\ &= -\frac{aA}{2x^2} + (Ab + aB)x + \frac{1}{4}bBx^4 \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$x(aB + Ab) - \frac{aA}{2x^2} + \frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^3,x]

[Out] -1/2*(a*A)/x^2 + (A*b + a*B)*x + (b*B*x^4)/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)(A + Bx^3)}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^3,x]

[Out] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^3, x]

fricas [A] time = 0.79, size = 28, normalized size = 1.00

$$\frac{Bbx^6 + 4(Ba + Ab)x^3 - 2Aa}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="fricas")

[Out] 1/4*(B*b*x^6 + 4*(B*a + A*b)*x^3 - 2*A*a)/x^2

giac [A] time = 0.16, size = 23, normalized size = 0.82

$$\frac{1}{4}Bbx^4 + Bax + Abx - \frac{Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="giac")

[Out] 1/4*B*b*x^4 + B*a*x + A*b*x - 1/2*A*a/x^2

maple [A] time = 0.04, size = 24, normalized size = 0.86

$$\frac{Bbx^4}{4} + Abx + Bax - \frac{Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(B*x^3+A)/x^3,x)

[Out] 1/4*b*B*x^4+A*b*x+B*a*x-1/2*A*a/x^2

maxima [A] time = 0.48, size = 24, normalized size = 0.86

$$\frac{1}{4}Bbx^4 + (Ba + Ab)x - \frac{Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="maxima")

[Out] 1/4*B*b*x^4 + (B*a + A*b)*x - 1/2*A*a/x^2

mupad [B] time = 2.34, size = 24, normalized size = 0.86

$$x (A b + B a) - \frac{A a}{2 x^2} + \frac{B b x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3))/x^3,x)

[Out] x*(A*b + B*a) - (A*a)/(2*x^2) + (B*b*x^4)/4

sympy [A] time = 0.12, size = 24, normalized size = 0.86

$$-\frac{A a}{2 x^2} + \frac{B b x^4}{4} + x (A b + B a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(B*x**3+A)/x**3,x)

[Out] -A*a/(2*x**2) + B*b*x**4/4 + x*(A*b + B*a)

$$3.7 \quad \int \frac{(a+bx^3)(A+Bx^3)}{x^4} dx$$

Optimal. Leaf size=29

$$\log(x)(aB + Ab) - \frac{aA}{3x^3} + \frac{1}{3}bBx^3$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {446, 76}

$$\log(x)(aB + Ab) - \frac{aA}{3x^3} + \frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*(A + B*x^3))/x^4, x]

[Out] -(a*A)/(3*x^3) + (b*B*x^3)/3 + (A*b + a*B)*Log[x]

Rule 76

Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)(A + Bx^3)}{x^4} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)(A + Bx)}{x^2} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(bB + \frac{aA}{x^2} + \frac{Ab + aB}{x} \right) dx, x, x^3 \right) \\ &= -\frac{aA}{3x^3} + \frac{1}{3}bBx^3 + (Ab + aB)\log(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\log(x)(aB + Ab) - \frac{aA}{3x^3} + \frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^4,x]

[Out] -1/3*(a*A)/x^3 + (b*B*x^3)/3 + (A*b + a*B)*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)(A + Bx^3)}{x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^4,x]

[Out] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^4, x]

fricas [A] time = 0.81, size = 30, normalized size = 1.03

$$\frac{Bbx^6 + 3(Ba + Ab)x^3 \log(x) - Aa}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="fricas")

[Out] 1/3*(B*b*x^6 + 3*(B*a + A*b)*x^3*log(x) - A*a)/x^3

giac [A] time = 0.16, size = 40, normalized size = 1.38

$$\frac{1}{3}Bbx^3 + (Ba + Ab) \log(|x|) - \frac{Bax^3 + Abx^3 + Aa}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="giac")

[Out] 1/3*B*b*x^3 + (B*a + A*b)*log(abs(x)) - 1/3*(B*a*x^3 + A*b*x^3 + A*a)/x^3

maple [A] time = 0.05, size = 26, normalized size = 0.90

$$\frac{Bbx^3}{3} + Ab \ln(x) + Ba \ln(x) - \frac{Aa}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)*(B*x^3+A)/x^4,x)`

[Out] $1/3*B*b*x^3-1/3*A*a/x^3+A*b*\ln(x)+B*a*\ln(x)$

maxima [A] time = 0.48, size = 28, normalized size = 0.97

$$\frac{1}{3}Bbx^3 + \frac{1}{3}(Ba + Ab)\log(x^3) - \frac{Aa}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="maxima")`

[Out] $1/3*B*b*x^3 + 1/3*(B*a + A*b)*\log(x^3) - 1/3*A*a/x^3$

mupad [B] time = 0.04, size = 25, normalized size = 0.86

$$\ln(x) (A b + B a) - \frac{A a}{3 x^3} + \frac{B b x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3))/x^4,x)`

[Out] $\log(x)*(A*b + B*a) - (A*a)/(3*x^3) + (B*b*x^3)/3$

sympy [A] time = 0.21, size = 26, normalized size = 0.90

$$-\frac{Aa}{3x^3} + \frac{Bbx^3}{3} + (Ab + Ba)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)*(B*x**3+A)/x**4,x)`

[Out] $-A*a/(3*x**3) + B*b*x**3/3 + (A*b + B*a)*\log(x)$

$$3.8 \quad \int \frac{(a+bx^3)(A+Bx^3)}{x^5} dx$$

Optimal. Leaf size=31

$$-\frac{aB + Ab}{x} - \frac{aA}{4x^4} + \frac{1}{2}bBx^2$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {448}

$$-\frac{aB + Ab}{x} - \frac{aA}{4x^4} + \frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*(A + B*x^3))/x^5, x]

[Out] -(a*A)/(4*x^4) - (A*b + a*B)/x + (b*B*x^2)/2

Rule 448

Int[((e_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)(A + Bx^3)}{x^5} dx &= \int \left(\frac{aA}{x^5} + \frac{Ab + aB}{x^2} + bBx \right) dx \\ &= -\frac{aA}{4x^4} - \frac{Ab + aB}{x} + \frac{1}{2}bBx^2 \end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.03

$$\frac{-aB - Ab}{x} - \frac{aA}{4x^4} + \frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^5, x]

[Out] -1/4*(a*A)/x^4 + (- (A*b) - a*B)/x + (b*B*x^2)/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)(A + Bx^3)}{x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^5,x]

[Out] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^5, x]

fricas [A] time = 1.29, size = 29, normalized size = 0.94

$$\frac{2 Bbx^6 - 4 (Ba + Ab)x^3 - Aa}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="fricas")

[Out] 1/4*(2*B*b*x^6 - 4*(B*a + A*b)*x^3 - A*a)/x^4

giac [A] time = 0.18, size = 31, normalized size = 1.00

$$\frac{1}{2} Bbx^2 - \frac{4 Bax^3 + 4 Abx^3 + Aa}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="giac")

[Out] 1/2*B*b*x^2 - 1/4*(4*B*a*x^3 + 4*A*b*x^3 + A*a)/x^4

maple [A] time = 0.05, size = 28, normalized size = 0.90

$$\frac{Bbx^2}{2} - \frac{Ab + Ba}{x} - \frac{Aa}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(B*x^3+A)/x^5,x)

[Out] 1/2*b*B*x^2-1/4*a*A/x^4-(A*b+B*a)/x

maxima [A] time = 0.63, size = 29, normalized size = 0.94

$$\frac{1}{2} Bbx^2 - \frac{4 (Ba + Ab)x^3 + Aa}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="maxima")

[Out] 1/2*B*b*x^2 - 1/4*(4*(B*a + A*b)*x^3 + A*a)/x^4

mupad [B] time = 0.03, size = 29, normalized size = 0.94

$$\frac{Bbx^2}{2} - \frac{(Ab + Ba)x^3 + \frac{Aa}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3))/x^5,x)

[Out] (B*b*x^2)/2 - ((A*a)/4 + x^3*(A*b + B*a))/x^4

sympy [A] time = 0.25, size = 31, normalized size = 1.00

$$\frac{Bbx^2}{2} + \frac{-Aa + x^3(-4Ab - 4Ba)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(B*x**3+A)/x**5,x)

[Out] B*b*x**2/2 + (-A*a + x**3*(-4*A*b - 4*B*a))/(4*x**4)

$$3.9 \quad \int \frac{(a+bx^3)(A+Bx^3)}{x^6} dx$$

Optimal. Leaf size=28

$$-\frac{aB + Ab}{2x^2} - \frac{aA}{5x^5} + bBx$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {448}

$$-\frac{aB + Ab}{2x^2} - \frac{aA}{5x^5} + bBx$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*(A + B*x^3))/x^6, x]

[Out] -(a*A)/(5*x^5) - (A*b + a*B)/(2*x^2) + b*B*x

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)(A + Bx^3)}{x^6} dx &= \int \left(bB + \frac{aA}{x^6} + \frac{Ab + aB}{x^3} \right) dx \\ &= -\frac{aA}{5x^5} - \frac{Ab + aB}{2x^2} + bBx \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.07

$$\frac{-aB - Ab}{2x^2} - \frac{aA}{5x^5} + bBx$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^6, x]

[Out] -1/5*(a*A)/x^5 + (- (A*b) - a*B)/(2*x^2) + b*B*x

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)(A + Bx^3)}{x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^6,x]

[Out] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^6, x]

fricas [A] time = 0.74, size = 29, normalized size = 1.04

$$\frac{10 Bbx^6 - 5(Ba + Ab)x^3 - 2 Aa}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="fricas")

[Out] 1/10*(10*B*b*x^6 - 5*(B*a + A*b)*x^3 - 2*A*a)/x^5

giac [A] time = 0.18, size = 29, normalized size = 1.04

$$Bbx - \frac{5 Bax^3 + 5 Abx^3 + 2 Aa}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="giac")

[Out] B*b*x - 1/10*(5*B*a*x^3 + 5*A*b*x^3 + 2*A*a)/x^5

maple [A] time = 0.04, size = 25, normalized size = 0.89

$$Bbx - \frac{Ab + Ba}{2x^2} - \frac{Aa}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(B*x^3+A)/x^6,x)

[Out] B*b*x-1/5*a*A/x^5-1/2*(A*b+B*a)/x^2

maxima [A] time = 0.49, size = 27, normalized size = 0.96

$$Bbx - \frac{5(Ba + Ab)x^3 + 2 Aa}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="maxima")

[Out] B*b*x - 1/10*(5*(B*a + A*b)*x^3 + 2*A*a)/x^5

mupad [B] time = 2.32, size = 28, normalized size = 1.00

$$Bbx - \frac{\left(\frac{Ab}{2} + \frac{Ba}{2}\right)x^3 + \frac{Aa}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3))/x^6,x)

[Out] B*b*x - ((A*a)/5 + x^3*((A*b)/2 + (B*a)/2))/x^5

sympy [A] time = 0.29, size = 29, normalized size = 1.04

$$Bbx + \frac{-2Aa + x^3(-5Ab - 5Ba)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(B*x**3+A)/x**6,x)

[Out] B*b*x + (-2*A*a + x**3*(-5*A*b - 5*B*a))/(10*x**5)

$$3.10 \quad \int \frac{(a+bx^3)(A+Bx^3)}{x^7} dx$$

Optimal. Leaf size=29

$$-\frac{aB + Ab}{3x^3} - \frac{aA}{6x^6} + bB \log(x)$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {446, 76}

$$-\frac{aB + Ab}{3x^3} - \frac{aA}{6x^6} + bB \log(x)$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*(A + B*x^3))/x^7, x]

[Out] -(a*A)/(6*x^6) - (A*b + a*B)/(3*x^3) + b*B*Log[x]

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)(A+Bx^3)}{x^7} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a+bx)(A+Bx)}{x^3} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{aA}{x^3} + \frac{Ab+aB}{x^2} + \frac{bB}{x} \right) dx, x, x^3 \right) \\ &= -\frac{aA}{6x^6} - \frac{Ab+aB}{3x^3} + bB \log(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.07

$$\frac{-aB - Ab}{3x^3} - \frac{aA}{6x^6} + bB \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^7, x]

[Out] -1/6*(a*A)/x^6 + (-(A*b) - a*B)/(3*x^3) + b*B*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)(A + Bx^3)}{x^7} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^7, x]

[Out] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^7, x]

fricas [A] time = 0.93, size = 31, normalized size = 1.07

$$\frac{6 Bbx^6 \log(x) - 2 (Ba + Ab)x^3 - Aa}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^7, x, algorithm="fricas")

[Out] 1/6*(6*B*b*x^6*log(x) - 2*(B*a + A*b)*x^3 - A*a)/x^6

giac [A] time = 0.16, size = 37, normalized size = 1.28

$$Bb \log(|x|) - \frac{3 Bbx^6 + 2 Bax^3 + 2 Abx^3 + Aa}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^7, x, algorithm="giac")

[Out] B*b*log(abs(x)) - 1/6*(3*B*b*x^6 + 2*B*a*x^3 + 2*A*b*x^3 + A*a)/x^6

maple [A] time = 0.04, size = 28, normalized size = 0.97

$$Bb \ln(x) - \frac{Ab}{3x^3} - \frac{Ba}{3x^3} - \frac{Aa}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)*(B*x^3+A)/x^7,x)`

[Out] `-1/3/x^3*A*b-1/3/x^3*B*a-1/6*a*A/x^6+B*b*ln(x)`

maxima [A] time = 0.51, size = 30, normalized size = 1.03

$$\frac{1}{3} B b \log(x^3) - \frac{2(Ba + Ab)x^3 + Aa}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="maxima")`

[Out] `1/3*B*b*log(x^3) - 1/6*(2*(B*a + A*b)*x^3 + A*a)/x^6`

mupad [B] time = 0.05, size = 29, normalized size = 1.00

$$B b \ln(x) - \frac{\left(\frac{A b}{3} + \frac{B a}{3}\right) x^3 + \frac{A a}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3))/x^7,x)`

[Out] `B*b*log(x) - ((A*a)/6 + x^3*((A*b)/3 + (B*a)/3))/x^6`

sympy [A] time = 0.54, size = 29, normalized size = 1.00

$$B b \log(x) + \frac{-A a + x^3(-2 A b - 2 B a)}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)*(B*x**3+A)/x**7,x)`

[Out] `B*b*log(x) + (-A*a + x**3*(-2*A*b - 2*B*a))/(6*x**6)`

3.11 $\int x^2 (a + bx^3)^2 (A + Bx^3) dx$

Optimal. Leaf size=42

$$\frac{(a + bx^3)^3 (Ab - aB)}{9b^2} + \frac{B(a + bx^3)^4}{12b^2}$$

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {444, 43}

$$\frac{(a + bx^3)^3 (Ab - aB)}{9b^2} + \frac{B(a + bx^3)^4}{12b^2}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x^3)^2*(A + B*x^3),x]

[Out] ((A*b - a*B)*(a + b*x^3)^3)/(9*b^2) + (B*(a + b*x^3)^4)/(12*b^2)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int x^2 (a + bx^3)^2 (A + Bx^3) dx &= \frac{1}{3} \text{Subst} \left(\int (a + bx)^2 (A + Bx) dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{(Ab - aB)(a + bx)^2}{b} + \frac{B(a + bx)^3}{b} \right) dx, x, x^3 \right) \\ &= \frac{(Ab - aB)(a + bx^3)^3}{9b^2} + \frac{B(a + bx^3)^4}{12b^2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.21

$$\frac{1}{36}x^3(12a^2A + 4bx^6(2aB + Ab) + 6ax^3(aB + 2Ab) + 3b^2Bx^9)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x^3)^2*(A + B*x^3), x]

[Out] (x^3*(12*a^2*A + 6*a*(2*A*b + a*B)*x^3 + 4*b*(A*b + 2*a*B)*x^6 + 3*b^2*B*x^9))/36

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + bx^3)^2 (A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a + b*x^3)^2*(A + B*x^3), x]

[Out] IntegrateAlgebraic[x^2*(a + b*x^3)^2*(A + B*x^3), x]

fricas [A] time = 1.03, size = 53, normalized size = 1.26

$$\frac{1}{12}x^{12}b^2B + \frac{2}{9}x^9baB + \frac{1}{9}x^9b^2A + \frac{1}{6}x^6a^2B + \frac{1}{3}x^6baA + \frac{1}{3}x^3a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A), x, algorithm="fricas")

[Out] 1/12*x^12*b^2*B + 2/9*x^9*b*a*B + 1/9*x^9*b^2*A + 1/6*x^6*a^2*B + 1/3*x^6*b*a*A + 1/3*x^3*a^2*A

giac [A] time = 0.15, size = 53, normalized size = 1.26

$$\frac{1}{12}Bb^2x^{12} + \frac{2}{9}Babx^9 + \frac{1}{9}Ab^2x^9 + \frac{1}{6}Ba^2x^6 + \frac{1}{3}Aabx^6 + \frac{1}{3}Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A), x, algorithm="giac")

[Out] 1/12*B*b^2*x^12 + 2/9*B*a*b*x^9 + 1/9*A*b^2*x^9 + 1/6*B*a^2*x^6 + 1/3*A*a*b*x^6 + 1/3*A*a^2*x^3

maple [A] time = 0.04, size = 52, normalized size = 1.24

$$\frac{Bb^2x^{12}}{12} + \frac{(b^2A + 2abB)x^9}{9} + \frac{Aa^2x^3}{3} + \frac{(2abA + a^2B)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^3+a)^2*(B*x^3+A),x)`

[Out] $1/12*b^2*B*x^{12}+1/9*(A*b^2+2*B*a*b)*x^9+1/6*(2*A*a*b+B*a^2)*x^6+1/3*A*a^2*x^3$

maxima [A] time = 0.49, size = 51, normalized size = 1.21

$$\frac{1}{12} B b^2 x^{12} + \frac{1}{9} (2 B a b + A b^2) x^9 + \frac{1}{6} (B a^2 + 2 A a b) x^6 + \frac{1}{3} A a^2 x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")`

[Out] $1/12*B*b^2*x^{12} + 1/9*(2*B*a*b + A*b^2)*x^9 + 1/6*(B*a^2 + 2*A*a*b)*x^6 + 1/3*A*a^2*x^3$

mupad [B] time = 2.38, size = 51, normalized size = 1.21

$$x^6 \left(\frac{B a^2}{6} + \frac{A b a}{3} \right) + x^9 \left(\frac{A b^2}{9} + \frac{2 B a b}{9} \right) + \frac{A a^2 x^3}{3} + \frac{B b^2 x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(A + B*x^3)*(a + b*x^3)^2,x)`

[Out] $x^6*((B*a^2)/6 + (A*a*b)/3) + x^9*((A*b^2)/9 + (2*B*a*b)/9) + (A*a^2*x^3)/3 + (B*b^2*x^{12})/12$

sympy [A] time = 0.08, size = 54, normalized size = 1.29

$$\frac{A a^2 x^3}{3} + \frac{B b^2 x^{12}}{12} + x^9 \left(\frac{A b^2}{9} + \frac{2 B a b}{9} \right) + x^6 \left(\frac{A a b}{3} + \frac{B a^2}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**3+a)**2*(B*x**3+A),x)`

[Out] $A*a**2*x**3/3 + B*b**2*x**12/12 + x**9*(A*b**2/9 + 2*B*a*b/9) + x**6*(A*a*b/3 + B*a**2/6)$

$$3.12 \quad \int x (a + bx^3)^2 (A + Bx^3) dx$$

Optimal. Leaf size=55

$$\frac{1}{2}a^2Ax^2 + \frac{1}{8}bx^8(2aB + Ab) + \frac{1}{5}ax^5(aB + 2Ab) + \frac{1}{11}b^2Bx^{11}$$

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {448}

$$\frac{1}{2}a^2Ax^2 + \frac{1}{8}bx^8(2aB + Ab) + \frac{1}{5}ax^5(aB + 2Ab) + \frac{1}{11}b^2Bx^{11}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^3)^2*(A + B*x^3),x]

[Out] (a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B*x^11)/11

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x (a + bx^3)^2 (A + Bx^3) dx &= \int (a^2Ax + a(2Ab + aB)x^4 + b(Ab + 2aB)x^7 + b^2Bx^{10}) dx \\ &= \frac{1}{2}a^2Ax^2 + \frac{1}{5}a(2Ab + aB)x^5 + \frac{1}{8}b(Ab + 2aB)x^8 + \frac{1}{11}b^2Bx^{11} \end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{2}a^2Ax^2 + \frac{1}{8}bx^8(2aB + Ab) + \frac{1}{5}ax^5(aB + 2Ab) + \frac{1}{11}b^2Bx^{11}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^3)^2*(A + B*x^3),x]

[Out] $(a^2Ax^2)/2 + (a(2Ab + aB)x^5)/5 + (b(Ab + 2aB)x^8)/8 + (b^2Bx^{11})/11$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + bx^3)^2(A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a + b*x^3)^2*(A + B*x^3),x]

[Out] IntegrateAlgebraic[x*(a + b*x^3)^2*(A + B*x^3), x]

fricas [A] time = 1.06, size = 53, normalized size = 0.96

$$\frac{1}{11}x^{11}b^2B + \frac{1}{4}x^8baB + \frac{1}{8}x^8b^2A + \frac{1}{5}x^5a^2B + \frac{2}{5}x^5baA + \frac{1}{2}x^2a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")

[Out] $1/11*x^{11}*b^2*B + 1/4*x^8*b*a*B + 1/8*x^8*b^2*A + 1/5*x^5*a^2*B + 2/5*x^5*b*a*A + 1/2*x^2*a^2*A$

giac [A] time = 0.15, size = 53, normalized size = 0.96

$$\frac{1}{11}Bb^2x^{11} + \frac{1}{4}Babx^8 + \frac{1}{8}Ab^2x^8 + \frac{1}{5}Ba^2x^5 + \frac{2}{5}Aabx^5 + \frac{1}{2}Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")

[Out] $1/11*B*b^2*x^{11} + 1/4*B*a*b*x^8 + 1/8*A*b^2*x^8 + 1/5*B*a^2*x^5 + 2/5*A*a*b*x^5 + 1/2*A*a^2*x^2$

maple [A] time = 0.05, size = 52, normalized size = 0.95

$$\frac{Bb^2x^{11}}{11} + \frac{(b^2A + 2abB)x^8}{8} + \frac{Aa^2x^2}{2} + \frac{(2abA + a^2B)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a)^2*(B*x^3+A),x)

[Out] $1/11*b^2*B*x^{11} + 1/8*(A*b^2 + 2*B*a*b)*x^8 + 1/5*(2*A*a*b + B*a^2)*x^5 + 1/2*a^2*A*x^2$

maxima [A] time = 0.61, size = 51, normalized size = 0.93

$$\frac{1}{11} B b^2 x^{11} + \frac{1}{8} (2 B a b + A b^2) x^8 + \frac{1}{5} (B a^2 + 2 A a b) x^5 + \frac{1}{2} A a^2 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")

[Out] 1/11*B*b^2*x^11 + 1/8*(2*B*a*b + A*b^2)*x^8 + 1/5*(B*a^2 + 2*A*a*b)*x^5 + 1/2*A*a^2*x^2

mupad [B] time = 0.04, size = 51, normalized size = 0.93

$$x^5 \left(\frac{B a^2}{5} + \frac{2 A b a}{5} \right) + x^8 \left(\frac{A b^2}{8} + \frac{B a b}{4} \right) + \frac{A a^2 x^2}{2} + \frac{B b^2 x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(A + B*x^3)*(a + b*x^3)^2,x)

[Out] x^5*((B*a^2)/5 + (2*A*a*b)/5) + x^8*((A*b^2)/8 + (B*a*b)/4) + (A*a^2*x^2)/2 + (B*b^2*x^11)/11

sympy [A] time = 0.08, size = 54, normalized size = 0.98

$$\frac{A a^2 x^2}{2} + \frac{B b^2 x^{11}}{11} + x^8 \left(\frac{A b^2}{8} + \frac{B a b}{4} \right) + x^5 \left(\frac{2 A a b}{5} + \frac{B a^2}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**3+a)**2*(B*x**3+A),x)

[Out] A*a**2*x**2/2 + B*b**2*x**11/11 + x**8*(A*b**2/8 + B*a*b/4) + x**5*(2*A*a*b/5 + B*a**2/5)

3.13 $\int (a + bx^3)^2 (A + Bx^3) dx$

Optimal. Leaf size=50

$$a^2 Ax + \frac{1}{7}bx^7(2aB + Ab) + \frac{1}{4}ax^4(aB + 2Ab) + \frac{1}{10}b^2Bx^{10}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {373}

$$a^2 Ax + \frac{1}{7}bx^7(2aB + Ab) + \frac{1}{4}ax^4(aB + 2Ab) + \frac{1}{10}b^2Bx^{10}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^2*(A + B*x^3), x]

[Out] a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)/10

Rule 373

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int (a + bx^3)^2 (A + Bx^3) dx &= \int (a^2A + a(2Ab + aB)x^3 + b(Ab + 2aB)x^6 + b^2Bx^9) dx \\ &= a^2Ax + \frac{1}{4}a(2Ab + aB)x^4 + \frac{1}{7}b(Ab + 2aB)x^7 + \frac{1}{10}b^2Bx^{10} \end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 1.00

$$a^2 Ax + \frac{1}{7}bx^7(2aB + Ab) + \frac{1}{4}ax^4(aB + 2Ab) + \frac{1}{10}b^2Bx^{10}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^2*(A + B*x^3), x]

[Out] $a^2Ax + (a(2Ab + aB)x^4)/4 + (b(Ab + 2aB)x^7)/7 + (b^2Bx^{10})/10$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx^3)^2 (A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^3)^2*(A + B*x^3), x]

[Out] IntegrateAlgebraic[(a + b*x^3)^2*(A + B*x^3), x]

fricas [A] time = 0.64, size = 50, normalized size = 1.00

$$\frac{1}{10}x^{10}b^2B + \frac{2}{7}x^7baB + \frac{1}{7}x^7b^2A + \frac{1}{4}x^4a^2B + \frac{1}{2}x^4baA + xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A), x, algorithm="fricas")

[Out] $1/10*x^{10}*b^2*B + 2/7*x^7*b*a*B + 1/7*x^7*b^2*A + 1/4*x^4*a^2*B + 1/2*x^4*b*a*A + x*a^2*A$

giac [A] time = 0.17, size = 50, normalized size = 1.00

$$\frac{1}{10}Bb^2x^{10} + \frac{2}{7}Babx^7 + \frac{1}{7}Ab^2x^7 + \frac{1}{4}Ba^2x^4 + \frac{1}{2}Aabx^4 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A), x, algorithm="giac")

[Out] $1/10*B*b^2*x^{10} + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/4*B*a^2*x^4 + 1/2*A*a*b*x^4 + A*a^2*x$

maple [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{Bb^2x^{10}}{10} + \frac{(b^2A + 2abB)x^7}{7} + Aa^2x + \frac{(2abA + a^2B)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(B*x^3+A), x)

[Out] $1/10*b^2*B*x^{10} + 1/7*(A*b^2 + 2*B*a*b)*x^7 + 1/4*(2*A*a*b + B*a^2)*x^4 + A*a^2*x$

maxima [A] time = 0.66, size = 48, normalized size = 0.96

$$\frac{1}{10} B b^2 x^{10} + \frac{1}{7} (2 B a b + A b^2) x^7 + \frac{1}{4} (B a^2 + 2 A a b) x^4 + A a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")

[Out] 1/10*B*b^2*x^10 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/4*(B*a^2 + 2*A*a*b)*x^4 + A*a^2*x

mupad [B] time = 0.04, size = 48, normalized size = 0.96

$$x^4 \left(\frac{B a^2}{4} + \frac{A b a}{2} \right) + x^7 \left(\frac{A b^2}{7} + \frac{2 B a b}{7} \right) + \frac{B b^2 x^{10}}{10} + A a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)*(a + b*x^3)^2,x)

[Out] x^4*((B*a^2)/4 + (A*a*b)/2) + x^7*((A*b^2)/7 + (2*B*a*b)/7) + (B*b^2*x^10)/10 + A*a^2*x

sympy [A] time = 0.08, size = 51, normalized size = 1.02

$$A a^2 x + \frac{B b^2 x^{10}}{10} + x^7 \left(\frac{A b^2}{7} + \frac{2 B a b}{7} \right) + x^4 \left(\frac{A a b}{2} + \frac{B a^2}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*(B*x**3+A),x)

[Out] A*a**2*x + B*b**2*x**10/10 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**4*(A*a*b/2 + B*a**2/4)

$$3.14 \quad \int \frac{(a+bx^3)^2(A+Bx^3)}{x} dx$$

Optimal. Leaf size=46

$$a^2 A \log(x) + \frac{2}{3} a A b x^3 + \frac{B(a+bx^3)^3}{9b} + \frac{1}{6} A b^2 x^6$$

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {446, 80, 43}

$$a^2 A \log(x) + \frac{2}{3} a A b x^3 + \frac{B(a+bx^3)^3}{9b} + \frac{1}{6} A b^2 x^6$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(A + B*x^3))/x,x]

[Out] (2*a*A*b*x^3)/3 + (A*b^2*x^6)/6 + (B*(a + b*x^3)^3)/(9*b) + a^2*A*Log[x]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^2 (A + Bx^3)}{x} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^2 (A + Bx)}{x} dx, x, x^3 \right) \\
&= \frac{B(a + bx^3)^3}{9b} + \frac{1}{3} A \text{Subst} \left(\int \frac{(a + bx)^2}{x} dx, x, x^3 \right) \\
&= \frac{B(a + bx^3)^3}{9b} + \frac{1}{3} A \text{Subst} \left(\int \left(2ab + \frac{a^2}{x} + b^2x \right) dx, x, x^3 \right) \\
&= \frac{2}{3} aAbx^3 + \frac{1}{6} Ab^2x^6 + \frac{B(a + bx^3)^3}{9b} + a^2 A \log(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 1.11

$$a^2 A \log(x) + \frac{1}{6} bx^6 (2aB + Ab) + \frac{1}{3} ax^3 (aB + 2Ab) + \frac{1}{9} b^2 Bx^9$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x,x]

[Out] (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^6)/6 + (b^2*B*x^9)/9 + a^2*A*L
og[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^2 (A + Bx^3)}{x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x, x]

fricas [A] time = 0.77, size = 49, normalized size = 1.07

$$\frac{1}{9} Bb^2x^9 + \frac{1}{6} (2Bab + Ab^2)x^6 + \frac{1}{3} (Ba^2 + 2Aab)x^3 + Aa^2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="fricas")

[Out] $\frac{1}{9}Bb^2x^9 + \frac{1}{6}(2B^*a*b + A*b^2)*x^6 + \frac{1}{3}(B^*a^2 + 2A^*a*b)*x^3 + A^*a^2*\log(x)$

giac [A] time = 0.15, size = 52, normalized size = 1.13

$$\frac{1}{9}Bb^2x^9 + \frac{1}{3}Babx^6 + \frac{1}{6}Ab^2x^6 + \frac{1}{3}Ba^2x^3 + \frac{2}{3}Aabx^3 + Aa^2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="giac")`

[Out] $\frac{1}{9}B^*b^2*x^9 + \frac{1}{3}B^*a*b*x^6 + \frac{1}{6}A^*b^2*x^6 + \frac{1}{3}B^*a^2*x^3 + \frac{2}{3}A^*a*b*x^3 + A^*a^2*\log(\text{abs}(x))$

maple [A] time = 0.04, size = 52, normalized size = 1.13

$$\frac{Bb^2x^9}{9} + \frac{Ab^2x^6}{6} + \frac{Babx^6}{3} + \frac{2Aabx^3}{3} + \frac{Ba^2x^3}{3} + Aa^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^2*(B*x^3+A)/x,x)`

[Out] $\frac{1}{9}B^*b^2*x^9 + \frac{1}{6}A^*b^2*x^6 + \frac{1}{3}B^*x^6*a*b + \frac{2}{3}a^*A^*b*x^3 + \frac{1}{3}B^*a^2*x^3 + A^*a^2*\ln(x)$

maxima [A] time = 0.54, size = 52, normalized size = 1.13

$$\frac{1}{9}Bb^2x^9 + \frac{1}{6}(2Bab + Ab^2)x^6 + \frac{1}{3}(Ba^2 + 2Aab)x^3 + \frac{1}{3}Aa^2\log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="maxima")`

[Out] $\frac{1}{9}B^*b^2*x^9 + \frac{1}{6}(2B^*a*b + A^*b^2)*x^6 + \frac{1}{3}(B^*a^2 + 2A^*a*b)*x^3 + \frac{1}{3}A^*a^2*\log(x^3)$

mupad [B] time = 0.04, size = 49, normalized size = 1.07

$$x^3 \left(\frac{Ba^2}{3} + \frac{2Aba}{3} \right) + x^6 \left(\frac{Ab^2}{6} + \frac{Bab}{3} \right) + \frac{Bb^2x^9}{9} + Aa^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^2)/x,x)`

[Out] $x^3 \left(\frac{B a^2}{3} + \frac{2 A a b}{3} \right) + x^6 \left(\frac{A b^2}{6} + \frac{B a b}{3} \right) + \frac{B b^2 x^9}{9} + A a^2 \log(x)$

sympy [A] time = 0.15, size = 53, normalized size = 1.15

$$A a^2 \log(x) + \frac{B b^2 x^9}{9} + x^6 \left(\frac{A b^2}{6} + \frac{B a b}{3} \right) + x^3 \left(\frac{2 A a b}{3} + \frac{B a^2}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*(B*x**3+A)/x,x)`

[Out] $A a^2 \log(x) + \frac{B b^2 x^9}{9} + x^6 \left(\frac{A b^2}{6} + \frac{B a b}{3} \right) + x^3 \left(\frac{2 A a b}{3} + \frac{B a^2}{3} \right)$

$$3.15 \quad \int \frac{(a+bx^3)^2(A+Bx^3)}{x^2} dx$$

Optimal. Leaf size=53

$$-\frac{a^2A}{x} + \frac{1}{5}bx^5(2aB + Ab) + \frac{1}{2}ax^2(aB + 2Ab) + \frac{1}{8}b^2Bx^8$$

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{a^2A}{x} + \frac{1}{5}bx^5(2aB + Ab) + \frac{1}{2}ax^2(aB + 2Ab) + \frac{1}{8}b^2Bx^8$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^2,x]

[Out] -((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^8)/8

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^2(A+Bx^3)}{x^2} dx &= \int \left(\frac{a^2A}{x^2} + a(2Ab + aB)x + b(Ab + 2aB)x^4 + b^2Bx^7 \right) dx \\ &= -\frac{a^2A}{x} + \frac{1}{2}a(2Ab + aB)x^2 + \frac{1}{5}b(Ab + 2aB)x^5 + \frac{1}{8}b^2Bx^8 \end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.00

$$-\frac{a^2A}{x} + \frac{1}{5}bx^5(2aB + Ab) + \frac{1}{2}ax^2(aB + 2Ab) + \frac{1}{8}b^2Bx^8$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^2,x]

[Out] -((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^8)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^2 (A + Bx^3)}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^2,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^2, x]

fricas [A] time = 0.74, size = 53, normalized size = 1.00

$$\frac{5 B b^2 x^9 + 8 (2 B a b + A b^2) x^6 + 20 (B a^2 + 2 A a b) x^3 - 40 A a^2}{40 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="fricas")

[Out] 1/40*(5*B*b^2*x^9 + 8*(2*B*a*b + A*b^2)*x^6 + 20*(B*a^2 + 2*A*a*b)*x^3 - 40*A*a^2)/x

giac [A] time = 0.15, size = 52, normalized size = 0.98

$$\frac{1}{8} B b^2 x^8 + \frac{2}{5} B a b x^5 + \frac{1}{5} A b^2 x^5 + \frac{1}{2} B a^2 x^2 + A a b x^2 - \frac{A a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="giac")

[Out] 1/8*B*b^2*x^8 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/2*B*a^2*x^2 + A*a*b*x^2 - A*a^2/x

maple [A] time = 0.04, size = 53, normalized size = 1.00

$$\frac{B b^2 x^8}{8} + \frac{A b^2 x^5}{5} + \frac{2 B a b x^5}{5} + A a b x^2 + \frac{B a^2 x^2}{2} - \frac{A a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(B*x^3+A)/x^2,x)

[Out] $\frac{1}{8}b^2Bx^8 + \frac{1}{5}Ax^5b^2 + \frac{2}{5}Bx^5ab + Aabx^2 + \frac{1}{2}Ba^2x^2 - \frac{Aa^2}{x}$

maxima [A] time = 0.59, size = 51, normalized size = 0.96

$$\frac{1}{8}Bb^2x^8 + \frac{1}{5}(2Bab + Ab^2)x^5 + \frac{1}{2}(Ba^2 + 2Aab)x^2 - \frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="maxima")

[Out] $\frac{1}{8}Bb^2x^8 + \frac{1}{5}(2Bab + Ab^2)x^5 + \frac{1}{2}(Ba^2 + 2Aab)x^2 - \frac{Aa^2}{x}$

mupad [B] time = 0.05, size = 50, normalized size = 0.94

$$x^2 \left(\frac{Ba^2}{2} + Aba \right) + x^5 \left(\frac{Ab^2}{5} + \frac{2Bab}{5} \right) - \frac{Aa^2}{x} + \frac{Bb^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^2,x)

[Out] $x^2 * ((Ba^2)/2 + Aab) + x^5 * ((Ab^2)/5 + (2*Bab)/5) - (Aa^2)/x + (Bb^2 * x^8)/8$

sympy [A] time = 0.15, size = 49, normalized size = 0.92

$$-\frac{Aa^2}{x} + \frac{Bb^2x^8}{8} + x^5 \left(\frac{Ab^2}{5} + \frac{2Bab}{5} \right) + x^2 \left(Aab + \frac{Ba^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**2,x)

[Out] $-Aa**2/x + B*b**2*x**8/8 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**2*(A*a*b + B*a**2/2)$

$$3.16 \quad \int \frac{(a+bx^3)^2(A+Bx^3)}{x^3} dx$$

Optimal. Leaf size=50

$$-\frac{a^2A}{2x^2} + \frac{1}{4}bx^4(2aB + Ab) + ax(aB + 2Ab) + \frac{1}{7}b^2Bx^7$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{a^2A}{2x^2} + \frac{1}{4}bx^4(2aB + Ab) + ax(aB + 2Ab) + \frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^3, x]

[Out] -(a^2*A)/(2*x^2) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^7)/7

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^2(A+Bx^3)}{x^3} dx &= \int \left(a(2Ab + aB) + \frac{a^2A}{x^3} + b(Ab + 2aB)x^3 + b^2Bx^6 \right) dx \\ &= -\frac{a^2A}{2x^2} + a(2Ab + aB)x + \frac{1}{4}b(Ab + 2aB)x^4 + \frac{1}{7}b^2Bx^7 \end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.00

$$-\frac{a^2A}{2x^2} + \frac{1}{4}bx^4(2aB + Ab) + ax(aB + 2Ab) + \frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^3,x]

[Out] $-1/2*(a^2*A)/x^2 + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^7)/7$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^2 (A + Bx^3)}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^3,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^3, x]

fricas [A] time = 0.83, size = 53, normalized size = 1.06

$$\frac{4 B b^2 x^9 + 7 (2 B a b + A b^2) x^6 + 28 (B a^2 + 2 A a b) x^3 - 14 A a^2}{28 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="fricas")

[Out] $1/28*(4*B*b^2*x^9 + 7*(2*B*a*b + A*b^2)*x^6 + 28*(B*a^2 + 2*A*a*b)*x^3 - 14*A*a^2)/x^2$

giac [A] time = 0.17, size = 48, normalized size = 0.96

$$\frac{1}{7} B b^2 x^7 + \frac{1}{2} B a b x^4 + \frac{1}{4} A b^2 x^4 + B a^2 x + 2 A a b x - \frac{A a^2}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="giac")

[Out] $1/7*B*b^2*x^7 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + B*a^2*x + 2*A*a*b*x - 1/2*A*a^2/x^2$

maple [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{B b^2 x^7}{7} + \frac{A b^2 x^4}{4} + \frac{B a b x^4}{2} + 2 A a b x + B a^2 x - \frac{A a^2}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(B*x^3+A)/x^3,x)

[Out] $1/7*b^2*B*x^7+1/4*A*b^2*x^4+1/2*B*x^4*a*b+2*a*b*A*x+B*a^2*x-1/2*A*a^2/x^2$

maxima [A] time = 0.49, size = 48, normalized size = 0.96

$$\frac{1}{7} B b^2 x^7 + \frac{1}{4} (2 B a b + A b^2) x^4 + (B a^2 + 2 A a b) x - \frac{A a^2}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="maxima")`

[Out] $1/7*B*b^2*x^7 + 1/4*(2*B*a*b + A*b^2)*x^4 + (B*a^2 + 2*A*a*b)*x - 1/2*A*a^2/x^2$

mupad [B] time = 0.05, size = 48, normalized size = 0.96

$$x^4 \left(\frac{A b^2}{4} + \frac{B a b}{2} \right) + x (B a^2 + 2 A b a) - \frac{A a^2}{2 x^2} + \frac{B b^2 x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^2)/x^3,x)`

[Out] $x^4*((A*b^2)/4 + (B*a*b)/2) + x*(B*a^2 + 2*A*a*b) - (A*a^2)/(2*x^2) + (B*b^2*x^7)/7$

sympy [A] time = 0.15, size = 49, normalized size = 0.98

$$-\frac{A a^2}{2 x^2} + \frac{B b^2 x^7}{7} + x^4 \left(\frac{A b^2}{4} + \frac{B a b}{2} \right) + x (2 A a b + B a^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*(B*x**3+A)/x**3,x)`

[Out] $-A*a**2/(2*x**2) + B*b**2*x**7/7 + x**4*(A*b**2/4 + B*a*b/2) + x*(2*A*a*b + B*a**2)$

$$3.17 \quad \int \frac{(a+bx^3)^2(A+Bx^3)}{x^4} dx$$

Optimal. Leaf size=51

$$-\frac{a^2A}{3x^3} + \frac{1}{3}bx^3(2aB + Ab) + a \log(x)(aB + 2Ab) + \frac{1}{6}b^2Bx^6$$

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 76}

$$-\frac{a^2A}{3x^3} + \frac{1}{3}bx^3(2aB + Ab) + a \log(x)(aB + 2Ab) + \frac{1}{6}b^2Bx^6$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^4,x]

[Out] -(a^2*A)/(3*x^3) + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^6)/6 + a*(2*A*b + a*B)*Log[x]

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^2 (A + Bx^3)}{x^4} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^2 (A + Bx)}{x^2} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(b(Ab + 2aB) + \frac{a^2 A}{x^2} + \frac{a(2Ab + aB)}{x} + b^2 Bx \right) dx, x, x^3 \right) \\
&= -\frac{a^2 A}{3x^3} + \frac{1}{3} b(Ab + 2aB)x^3 + \frac{1}{6} b^2 Bx^6 + a(2Ab + aB) \log(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.96

$$\frac{1}{6} \left(-\frac{2a^2 A}{x^3} + 2bx^3(2aB + Ab) + 6a \log(x)(aB + 2Ab) + b^2 Bx^6 \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^4,x]

[Out] ((-2*a^2*A)/x^3 + 2*b*(A*b + 2*a*B)*x^3 + b^2*B*x^6 + 6*a*(2*A*b + a*B)*Log[x])/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^2 (A + Bx^3)}{x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^4,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^4, x]

fricas [A] time = 0.64, size = 54, normalized size = 1.06

$$\frac{Bb^2x^9 + 2(2Bab + Ab^2)x^6 + 6(Ba^2 + 2Aab)x^3 \log(x) - 2Aa^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="fricas")

[Out] 1/6*(B*b^2*x^9 + 2*(2*B*a*b + A*b^2)*x^6 + 6*(B*a^2 + 2*A*a*b)*x^3*log(x) - 2*A*a^2)/x^3

giac [A] time = 0.15, size = 69, normalized size = 1.35

$$\frac{1}{6} B b^2 x^6 + \frac{2}{3} B a b x^3 + \frac{1}{3} A b^2 x^3 + (B a^2 + 2 A a b) \log(|x|) - \frac{B a^2 x^3 + 2 A a b x^3 + A a^2}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="giac")

[Out] 1/6*B*b^2*x^6 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + (B*a^2 + 2*A*a*b)*log(abs(x)) - 1/3*(B*a^2*x^3 + 2*A*a*b*x^3 + A*a^2)/x^3

maple [A] time = 0.05, size = 51, normalized size = 1.00

$$\frac{B b^2 x^6}{6} + \frac{A b^2 x^3}{3} + \frac{2 B a b x^3}{3} + 2 A a b \ln(x) + B a^2 \ln(x) - \frac{A a^2}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(B*x^3+A)/x^4,x)

[Out] 1/6*b^2*B*x^6+1/3*A*x^3*b^2+2/3*B*x^3*a*b-1/3*A*a^2/x^3+2*A*ln(x)*a*b+B*a^2*ln(x)

maxima [A] time = 0.48, size = 52, normalized size = 1.02

$$\frac{1}{6} B b^2 x^6 + \frac{1}{3} (2 B a b + A b^2) x^3 + \frac{1}{3} (B a^2 + 2 A a b) \log(x^3) - \frac{A a^2}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="maxima")

[Out] 1/6*B*b^2*x^6 + 1/3*(2*B*a*b + A*b^2)*x^3 + 1/3*(B*a^2 + 2*A*a*b)*log(x^3) - 1/3*A*a^2/x^3

mupad [B] time = 0.04, size = 49, normalized size = 0.96

$$x^3 \left(\frac{A b^2}{3} + \frac{2 B a b}{3} \right) + \ln(x) (B a^2 + 2 A b a) - \frac{A a^2}{3 x^3} + \frac{B b^2 x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^4,x)

[Out] x^3*((A*b^2)/3 + (2*B*a*b)/3) + log(x)*(B*a^2 + 2*A*a*b) - (A*a^2)/(3*x^3) + (B*b^2*x^6)/6

sympy [A] time = 0.27, size = 51, normalized size = 1.00

$$-\frac{Aa^2}{3x^3} + \frac{Bb^2x^6}{6} + a(2Ab + Ba)\log(x) + x^3\left(\frac{Ab^2}{3} + \frac{2Bab}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**4,x)

[Out] -A*a**2/(3*x**3) + B*b**2*x**6/6 + a*(2*A*b + B*a)*log(x) + x**3*(A*b**2/3 + 2*B*a*b/3)

$$3.18 \quad \int \frac{(a+bx^3)^2(A+Bx^3)}{x^5} dx$$

Optimal. Leaf size=53

$$-\frac{a^2A}{4x^4} + \frac{1}{2}bx^2(2aB + Ab) - \frac{a(aB + 2Ab)}{x} + \frac{1}{5}b^2Bx^5$$

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{a^2A}{4x^4} + \frac{1}{2}bx^2(2aB + Ab) - \frac{a(aB + 2Ab)}{x} + \frac{1}{5}b^2Bx^5$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^5,x]

[Out] -(a^2*A)/(4*x^4) - (a*(2*A*b + a*B))/x + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*x^5)/5

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^2(A + Bx^3)}{x^5} dx &= \int \left(\frac{a^2A}{x^5} + \frac{a(2Ab + aB)}{x^2} + b(Ab + 2aB)x + b^2Bx^4 \right) dx \\ &= -\frac{a^2A}{4x^4} - \frac{a(2Ab + aB)}{x} + \frac{1}{2}b(Ab + 2aB)x^2 + \frac{1}{5}b^2Bx^5 \end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.96

$$\frac{-5a^2A + 10bx^6(2aB + Ab) - 20ax^3(aB + 2Ab) + 4b^2Bx^9}{20x^4}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^5,x]

[Out] $(-5*a^2*A - 20*a*(2*A*b + a*B))*x^3 + 10*b*(A*b + 2*a*B)*x^6 + 4*b^2*B*x^9)/(20*x^4)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^2 (A + Bx^3)}{x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^5,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^5, x]

fricas [A] time = 0.72, size = 53, normalized size = 1.00

$$\frac{4 B b^2 x^9 + 10 (2 B a b + A b^2) x^6 - 20 (B a^2 + 2 A a b) x^3 - 5 A a^2}{20 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="fricas")

[Out] $1/20*(4*B*b^2*x^9 + 10*(2*B*a*b + A*b^2)*x^6 - 20*(B*a^2 + 2*A*a*b)*x^3 - 5*A*a^2)/x^4$

giac [A] time = 0.15, size = 54, normalized size = 1.02

$$\frac{1}{5} B b^2 x^5 + B a b x^2 + \frac{1}{2} A b^2 x^2 - \frac{4 B a^2 x^3 + 8 A a b x^3 + A a^2}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="giac")

[Out] $1/5*B*b^2*x^5 + B*a*b*x^2 + 1/2*A*b^2*x^2 - 1/4*(4*B*a^2*x^3 + 8*A*a*b*x^3 + A*a^2)/x^4$

maple [A] time = 0.05, size = 50, normalized size = 0.94

$$\frac{B b^2 x^5}{5} + \frac{A b^2 x^2}{2} + B a b x^2 - \frac{(2 A b + B a) a}{x} - \frac{A a^2}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(B*x^3+A)/x^5,x)

[Out] $\frac{1}{5}b^2Bx^5 + \frac{1}{2}Ax^2b^2 + B^2abx^2 - \frac{1}{4}a^2A/x^4 - a(2Ab + B^2a)/x$

maxima [A] time = 0.72, size = 53, normalized size = 1.00

$$\frac{1}{5}Bb^2x^5 + \frac{1}{2}(2Bab + Ab^2)x^2 - \frac{4(Ba^2 + 2Aab)x^3 + Aa^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="maxima")`

[Out] $\frac{1}{5}B^2b^2x^5 + \frac{1}{2}(2B^2ab + Ab^2)x^2 - \frac{1}{4}(4(B^2a^2 + 2A^2ab)x^3 + A^2a^2)/x^4$

mupad [B] time = 0.05, size = 52, normalized size = 0.98

$$x^2 \left(\frac{Ab^2}{2} + Bab \right) - \frac{x^3 (Ba^2 + 2Aba) + \frac{Aa^2}{4}}{x^4} + \frac{Bb^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^2)/x^5,x)`

[Out] $x^2 * ((Ab^2)/2 + B^2ab) - (x^3 * (Ba^2 + 2A^2ab) + (A^2a^2)/4) / x^4 + (B^2b^2 * x^5) / 5$

sympy [A] time = 0.31, size = 53, normalized size = 1.00

$$\frac{Bb^2x^5}{5} + x^2 \left(\frac{Ab^2}{2} + Bab \right) + \frac{-Aa^2 + x^3(-8Aab - 4Ba^2)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*(B*x**3+A)/x**5,x)`

[Out] $B^2b^2x^5/5 + x^2*(Ab^2/2 + B^2ab) + (-A^2a^2 + x^3*(-8A^2ab - 4B^2a^2))/(4x^4)$

$$3.19 \quad \int \frac{(a+bx^3)^2(A+Bx^3)}{x^6} dx$$

Optimal. Leaf size=50

$$-\frac{a^2A}{5x^5} - \frac{a(aB+2Ab)}{2x^2} + bx(2aB+Ab) + \frac{1}{4}b^2Bx^4$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{a^2A}{5x^5} - \frac{a(aB+2Ab)}{2x^2} + bx(2aB+Ab) + \frac{1}{4}b^2Bx^4$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^6,x]

[Out] -(a^2*A)/(5*x^5) - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*x^4)/4

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^2(A+Bx^3)}{x^6} dx &= \int \left(b(Ab+2aB) + \frac{a^2A}{x^6} + \frac{a(2Ab+aB)}{x^3} + b^2Bx^3 \right) dx \\ &= -\frac{a^2A}{5x^5} - \frac{a(2Ab+aB)}{2x^2} + b(Ab+2aB)x + \frac{1}{4}b^2Bx^4 \end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.00

$$-\frac{a^2A}{5x^5} - \frac{a(aB+2Ab)}{2x^2} + bx(2aB+Ab) + \frac{1}{4}b^2Bx^4$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^6,x]

[Out] $-1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*x^4)/4$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^2 (A + Bx^3)}{x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^6,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^6, x]

fricas [A] time = 1.37, size = 53, normalized size = 1.06

$$\frac{5 B b^2 x^9 + 20 (2 B a b + A b^2) x^6 - 10 (B a^2 + 2 A a b) x^3 - 4 A a^2}{20 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="fricas")

[Out] $1/20*(5*B*b^2*x^9 + 20*(2*B*a*b + A*b^2)*x^6 - 10*(B*a^2 + 2*A*a*b)*x^3 - 4*A*a^2)/x^5$

giac [A] time = 0.15, size = 51, normalized size = 1.02

$$\frac{1}{4} B b^2 x^4 + 2 B a b x + A b^2 x - \frac{5 B a^2 x^3 + 10 A a b x^3 + 2 A a^2}{10 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="giac")

[Out] $1/4*B*b^2*x^4 + 2*B*a*b*x + A*b^2*x - 1/10*(5*B*a^2*x^3 + 10*A*a*b*x^3 + 2*A*a^2)/x^5$

maple [A] time = 0.06, size = 46, normalized size = 0.92

$$\frac{B b^2 x^4}{4} + A b^2 x + 2 B a b x - \frac{(2 A b + B a) a}{2 x^2} - \frac{A a^2}{5 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(B*x^3+A)/x^6,x)

[Out] $1/4*B*b^2*x^4+b^2*A*x+2*a*b*B*x-1/5*a^2*A/x^5-1/2*a*(2*A*b+B*a)/x^2$

maxima [A] time = 0.57, size = 51, normalized size = 1.02

$$\frac{1}{4} B b^2 x^4 + (2 B a b + A b^2) x - \frac{5 (B a^2 + 2 A a b) x^3 + 2 A a^2}{10 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="maxima")`

[Out] $1/4*B*b^2*x^4 + (2*B*a*b + A*b^2)*x - 1/10*(5*(B*a^2 + 2*A*a*b)*x^3 + 2*A*a^2)/x^5$

mupad [B] time = 2.37, size = 50, normalized size = 1.00

$$x (A b^2 + 2 B a b) - \frac{x^3 \left(\frac{B a^2}{2} + A b a \right) + \frac{A a^2}{5}}{x^5} + \frac{B b^2 x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^2)/x^6,x)`

[Out] $x*(A*b^2 + 2*B*a*b) - (x^3*((B*a^2)/2 + A*a*b) + (A*a^2)/5)/x^5 + (B*b^2*x^4)/4$

sympy [A] time = 0.35, size = 53, normalized size = 1.06

$$\frac{B b^2 x^4}{4} + x (A b^2 + 2 B a b) + \frac{-2 A a^2 + x^3 (-10 A a b - 5 B a^2)}{10 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*(B*x**3+A)/x**6,x)`

[Out] $B*b**2*x**4/4 + x*(A*b**2 + 2*B*a*b) + (-2*A*a**2 + x**3*(-10*A*a*b - 5*B*a**2))/(10*x**5)$

$$3.20 \quad \int \frac{(a+bx^3)^2(A+Bx^3)}{x^7} dx$$

Optimal. Leaf size=51

$$-\frac{a^2A}{6x^6} - \frac{a(aB+2Ab)}{3x^3} + b \log(x)(2aB+Ab) + \frac{1}{3}b^2Bx^3$$

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 76}

$$-\frac{a^2A}{6x^6} - \frac{a(aB+2Ab)}{3x^3} + b \log(x)(2aB+Ab) + \frac{1}{3}b^2Bx^3$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^7,x]

[Out] -(a^2*A)/(6*x^6) - (a*(2*A*b + a*B))/(3*x^3) + (b^2*B*x^3)/3 + b*(A*b + 2*a*B)*Log[x]

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^2 (A + Bx^3)}{x^7} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^2 (A + Bx)}{x^3} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(b^2 B + \frac{a^2 A}{x^3} + \frac{a(2Ab + aB)}{x^2} + \frac{b(Ab + 2aB)}{x} \right) dx, x, x^3 \right) \\
&= -\frac{a^2 A}{6x^6} - \frac{a(2Ab + aB)}{3x^3} + \frac{1}{3} b^2 B x^3 + b(Ab + 2aB) \log(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$\frac{1}{6} \left(-\frac{a^2 (A + 2Bx^3)}{x^6} + 6b \log(x)(2aB + Ab) - \frac{4aAb}{x^3} + 2b^2 Bx^3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^7, x]

[Out] ((-4*a*A*b)/x^3 + 2*b^2*B*x^3 - (a^2*(A + 2*B*x^3))/x^6 + 6*b*(A*b + 2*a*B)*Log[x])/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^2 (A + Bx^3)}{x^7} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^7, x]

[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^7, x]

fricas [A] time = 0.75, size = 55, normalized size = 1.08

$$\frac{2 B b^2 x^9 + 6 (2 B a b + A b^2) x^6 \log(x) - 2 (B a^2 + 2 A a b) x^3 - A a^2}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7, x, algorithm="fricas")

[Out] 1/6*(2*B*b^2*x^9 + 6*(2*B*a*b + A*b^2)*x^6*log(x) - 2*(B*a^2 + 2*A*a*b)*x^3 - A*a^2)/x^6

giac [A] time = 0.21, size = 70, normalized size = 1.37

$$\frac{1}{3} B b^2 x^3 + (2 B a b + A b^2) \log(|x|) - \frac{6 B a b x^6 + 3 A b^2 x^6 + 2 B a^2 x^3 + 4 A a b x^3 + A a^2}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="giac")

[Out] 1/3*B*b^2*x^3 + (2*B*a*b + A*b^2)*log(abs(x)) - 1/6*(6*B*a*b*x^6 + 3*A*b^2*x^6 + 2*B*a^2*x^3 + 4*A*a*b*x^3 + A*a^2)/x^6

maple [A] time = 0.07, size = 51, normalized size = 1.00

$$\frac{B b^2 x^3}{3} + A b^2 \ln(x) + 2 B a b \ln(x) - \frac{2 A a b}{3 x^3} - \frac{B a^2}{3 x^3} - \frac{A a^2}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(B*x^3+A)/x^7,x)

[Out] 1/3*b^2*B*x^3-2/3*a/x^3*A*b-1/3*a^2/x^3*B-1/6*a^2*A/x^6+A*ln(x)*b^2+2*B*ln(x)*a*b

maxima [A] time = 0.67, size = 54, normalized size = 1.06

$$\frac{1}{3} B b^2 x^3 + \frac{1}{3} (2 B a b + A b^2) \log(x^3) - \frac{2 (B a^2 + 2 A a b) x^3 + A a^2}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="maxima")

[Out] 1/3*B*b^2*x^3 + 1/3*(2*B*a*b + A*b^2)*log(x^3) - 1/6*(2*(B*a^2 + 2*A*a*b)*x^3 + A*a^2)/x^6

mupad [B] time = 2.36, size = 52, normalized size = 1.02

$$\ln(x) (A b^2 + 2 B a b) - \frac{x^3 \left(\frac{B a^2}{3} + \frac{2 A b a}{3} \right) + \frac{A a^2}{6}}{x^6} + \frac{B b^2 x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^7,x)

[Out] log(x)*(A*b^2 + 2*B*a*b) - (x^3*((B*a^2)/3 + (2*A*a*b)/3) + (A*a^2)/6)/x^6 + (B*b^2*x^3)/3

sympy [A] time = 0.76, size = 51, normalized size = 1.00

$$\frac{Bb^2x^3}{3} + b(Ab + 2Ba)\log(x) + \frac{-Aa^2 + x^3(-4Aab - 2Ba^2)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**7,x)

[Out] B*b**2*x**3/3 + b*(A*b + 2*B*a)*log(x) + (-A*a**2 + x**3*(-4*A*a*b - 2*B*a*
*2))/(6*x**6)

$$3.21 \quad \int \frac{(a+bx^3)^2(A+Bx^3)}{x^8} dx$$

Optimal. Leaf size=53

$$-\frac{a^2A}{7x^7} - \frac{a(aB+2Ab)}{4x^4} - \frac{b(2aB+Ab)}{x} + \frac{1}{2}b^2Bx^2$$

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{a^2A}{7x^7} - \frac{a(aB+2Ab)}{4x^4} - \frac{b(2aB+Ab)}{x} + \frac{1}{2}b^2Bx^2$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^8, x]

[Out] -(a^2*A)/(7*x^7) - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/x + (b^2*B*x^2)/2

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^2(A+Bx^3)}{x^8} dx &= \int \left(\frac{a^2A}{x^8} + \frac{a(2Ab+aB)}{x^5} + \frac{b(Ab+2aB)}{x^2} + b^2Bx \right) dx \\ &= -\frac{a^2A}{7x^7} - \frac{a(2Ab+aB)}{4x^4} - \frac{b(Ab+2aB)}{x} + \frac{1}{2}b^2Bx^2 \end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.02

$$\frac{a^2(4A+7Bx^3) + 14abx^3(A+4Bx^3) - 14b^2x^6(Bx^3-2A)}{28x^7}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^8,x]

[Out] $-1/28*(-14*b^2*x^6*(-2*A + B*x^3) + 14*a*b*x^3*(A + 4*B*x^3) + a^2*(4*A + 7*B*x^3))/x^7$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^2 (A + Bx^3)}{x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^8,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^8, x]

fricas [A] time = 0.80, size = 53, normalized size = 1.00

$$\frac{14 B b^2 x^9 - 28 (2 B a b + A b^2) x^6 - 7 (B a^2 + 2 A a b) x^3 - 4 A a^2}{28 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="fricas")

[Out] $1/28*(14*B*b^2*x^9 - 28*(2*B*a*b + A*b^2)*x^6 - 7*(B*a^2 + 2*A*a*b)*x^3 - 4*A*a^2)/x^7$

giac [A] time = 0.15, size = 56, normalized size = 1.06

$$\frac{1}{2} B b^2 x^2 - \frac{56 B a b x^6 + 28 A b^2 x^6 + 7 B a^2 x^3 + 14 A a b x^3 + 4 A a^2}{28 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="giac")

[Out] $1/2*B*b^2*x^2 - 1/28*(56*B*a*b*x^6 + 28*A*b^2*x^6 + 7*B*a^2*x^3 + 14*A*a*b*x^3 + 4*A*a^2)/x^7$

maple [A] time = 0.04, size = 48, normalized size = 0.91

$$\frac{B b^2 x^2}{2} - \frac{(A b + 2 B a) b}{x} - \frac{(2 A b + B a) a}{4 x^4} - \frac{A a^2}{7 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(B*x^3+A)/x^8,x)

[Out] $-1/7*a^2*A/x^7-1/4*a*(2*A*b+B*a)/x^4-b*(A*b+2*B*a)/x+1/2*b^2*B*x^2$

maxima [A] time = 0.57, size = 54, normalized size = 1.02

$$\frac{1}{2} B b^2 x^2 - \frac{28 (2 B a b + A b^2) x^6 + 7 (B a^2 + 2 A a b) x^3 + 4 A a^2}{28 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="maxima")`

[Out] $1/2*B*b^2*x^2 - 1/28*(28*(2*B*a*b + A*b^2)*x^6 + 7*(B*a^2 + 2*A*a*b)*x^3 + 4*A*a^2)/x^7$

mupad [B] time = 0.05, size = 53, normalized size = 1.00

$$\frac{B b^2 x^2}{2} - \frac{x^3 \left(\frac{B a^2}{4} + \frac{A b a}{2} \right) + x^6 (A b^2 + 2 B a b) + \frac{A a^2}{7}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^2)/x^8,x)`

[Out] $(B*b^2*x^2)/2 - (x^3*((B*a^2)/4 + (A*a*b)/2) + x^6*(A*b^2 + 2*B*a*b) + (A*a^2)/7)/x^7$

sympy [A] time = 0.87, size = 58, normalized size = 1.09

$$\frac{B b^2 x^2}{2} + \frac{-4 A a^2 + x^6 (-28 A b^2 - 56 B a b) + x^3 (-14 A a b - 7 B a^2)}{28 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*(B*x**3+A)/x**8,x)`

[Out] $B*b**2*x**2/2 + (-4*A*a**2 + x**6*(-28*A*b**2 - 56*B*a*b) + x**3*(-14*A*a*b - 7*B*a**2))/(28*x**7)$

$$3.22 \quad \int \frac{(a+bx^3)^2(A+Bx^3)}{x^9} dx$$

Optimal. Leaf size=50

$$-\frac{a^2A}{8x^8} - \frac{a(aB+2Ab)}{5x^5} - \frac{b(2aB+Ab)}{2x^2} + b^2Bx$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{a^2A}{8x^8} - \frac{a(aB+2Ab)}{5x^5} - \frac{b(2aB+Ab)}{2x^2} + b^2Bx$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^9,x]

[Out] -(a^2*A)/(8*x^8) - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) + b^2*B*x

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^2(A+Bx^3)}{x^9} dx &= \int \left(b^2B + \frac{a^2A}{x^9} + \frac{a(2Ab+aB)}{x^6} + \frac{b(Ab+2aB)}{x^3} \right) dx \\ &= -\frac{a^2A}{8x^8} - \frac{a(2Ab+aB)}{5x^5} - \frac{b(Ab+2aB)}{2x^2} + b^2Bx \end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 1.00

$$-\frac{a^2A}{8x^8} - \frac{a(aB+2Ab)}{5x^5} - \frac{b(2aB+Ab)}{2x^2} + b^2Bx$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^9,x]

[Out] $-1/8*(a^2*A)/x^8 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) + b^2*B*x$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^2 (A + Bx^3)}{x^9} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^9,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^9, x]

fricas [A] time = 0.83, size = 53, normalized size = 1.06

$$\frac{40 Bb^2x^9 - 20 (2 Bab + Ab^2)x^6 - 8 (Ba^2 + 2 Aab)x^3 - 5 Aa^2}{40 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="fricas")

[Out] $1/40*(40*B*b^2*x^9 - 20*(2*B*a*b + A*b^2)*x^6 - 8*(B*a^2 + 2*A*a*b)*x^3 - 5*A*a^2)/x^8$

giac [A] time = 0.15, size = 53, normalized size = 1.06

$$Bb^2x - \frac{40 Babx^6 + 20 Ab^2x^6 + 8 Ba^2x^3 + 16 Aabx^3 + 5 Aa^2}{40 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="giac")

[Out] $B*b^2*x - 1/40*(40*B*a*b*x^6 + 20*A*b^2*x^6 + 8*B*a^2*x^3 + 16*A*a*b*x^3 + 5*A*a^2)/x^8$

maple [A] time = 0.05, size = 45, normalized size = 0.90

$$B b^2 x - \frac{(A b + 2 B a) b}{2 x^2} - \frac{(2 A b + B a) a}{5 x^5} - \frac{A a^2}{8 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(B*x^3+A)/x^9,x)

[Out] $-1/8*a^2*A/x^8-1/5*a*(2*A*b+B*a)/x^5-1/2*b*(A*b+2*B*a)/x^2+b^2*B*x$

maxima [A] time = 0.49, size = 51, normalized size = 1.02

$$Bb^2x - \frac{20(2Bab + Ab^2)x^6 + 8(Ba^2 + 2Aab)x^3 + 5Aa^2}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="maxima")`

[Out] $B*b^2*x - 1/40*(20*(2*B*a*b + A*b^2)*x^6 + 8*(B*a^2 + 2*A*a*b)*x^3 + 5*A*a^2)/x^8$

mupad [B] time = 2.34, size = 50, normalized size = 1.00

$$Bb^2x - \frac{x^3\left(\frac{Ba^2}{5} + \frac{2Aba}{5}\right) + x^6\left(\frac{Ab^2}{2} + B a b\right) + \frac{Aa^2}{8}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^2)/x^9,x)`

[Out] $B*b^2*x - (x^3*((B*a^2)/5 + (2*A*a*b)/5) + x^6*((A*b^2)/2 + B*a*b) + (A*a^2)/8)/x^8$

sympy [A] time = 0.99, size = 54, normalized size = 1.08

$$Bb^2x + \frac{-5Aa^2 + x^6(-20Ab^2 - 40Bab) + x^3(-16Aab - 8Ba^2)}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*(B*x**3+A)/x**9,x)`

[Out] $B*b**2*x + (-5*A*a**2 + x**6*(-20*A*b**2 - 40*B*a*b) + x**3*(-16*A*a*b - 8*B*a**2))/(40*x**8)$

3.23 $\int x^9 (a + bx^3)^5 (A + Bx^3) dx$

Optimal. Leaf size=117

$$\frac{1}{10}a^5Ax^{10} + \frac{1}{13}a^4x^{13}(aB+5Ab) + \frac{5}{16}a^3bx^{16}(aB+2Ab) + \frac{10}{19}a^2b^2x^{19}(aB+Ab) + \frac{1}{25}b^4x^{25}(5aB+Ab) + \frac{5}{22}ab^3x^{22}(2aB+Ab)$$

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{10}{19}a^2b^2x^{19}(aB+Ab) + \frac{5}{16}a^3bx^{16}(aB+2Ab) + \frac{1}{13}a^4x^{13}(aB+5Ab) + \frac{1}{10}a^5Ax^{10} + \frac{1}{25}b^4x^{25}(5aB+Ab) + \frac{5}{22}ab^3x^{22}(2aB+Ab) + \frac{1}{28}b^5Bx^{28}$$

Antiderivative was successfully verified.

[In] Int[x^9*(a + b*x^3)^5*(A + B*x^3),x]

[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^16)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22 + (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^9 (a + bx^3)^5 (A + Bx^3) dx &= \int (a^5 Ax^9 + a^4(5Ab + aB)x^{12} + 5a^3b(2Ab + aB)x^{15} + 10a^2b^2(Ab + aB)x^{18} + 5a^2b^3x^{21} + 5a^2b^4x^{24} + 5a^2b^5x^{27} + 5a^3b^2x^{20} + 5a^3b^3x^{23} + 5a^3b^4x^{26} + 5a^3b^5x^{29} + 5a^4b^2x^{17} + 5a^4b^3x^{20} + 5a^4b^4x^{23} + 5a^4b^5x^{26} + 5a^5b^2x^{19} + 5a^5b^3x^{22} + 5a^5b^4x^{25} + 5a^5b^5x^{28}) (A + Bx^3) dx \\ &= \frac{1}{10}a^5Ax^{10} + \frac{1}{13}a^4(5Ab + aB)x^{13} + \frac{5}{16}a^3b(2Ab + aB)x^{16} + \frac{10}{19}a^2b^2(Ab + aB)x^{19} + \frac{5}{22}a^2b^3x^{22} + \frac{5}{25}a^2b^4x^{25} + \frac{5}{28}a^2b^5x^{28} + \frac{1}{10}a^3b^2x^{20} + \frac{1}{13}a^3b^3x^{23} + \frac{1}{16}a^3b^4x^{26} + \frac{1}{19}a^3b^5x^{29} + \frac{1}{22}a^4b^2x^{17} + \frac{1}{25}a^4b^3x^{20} + \frac{1}{28}a^4b^4x^{23} + \frac{1}{31}a^4b^5x^{26} + \frac{1}{34}a^5b^2x^{19} + \frac{1}{37}a^5b^3x^{22} + \frac{1}{40}a^5b^4x^{25} + \frac{1}{43}a^5b^5x^{28} \end{aligned}$$

Mathematica [A] time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{10}a^5Ax^{10} + \frac{1}{13}a^4x^{13}(aB+5Ab) + \frac{5}{16}a^3bx^{16}(aB+2Ab) + \frac{10}{19}a^2b^2x^{19}(aB+Ab) + \frac{1}{25}b^4x^{25}(5aB+Ab) + \frac{5}{22}ab^3x^{22}(2aB+Ab) + \frac{1}{28}b^5Bx^{28}$$

Antiderivative was successfully verified.

[In] Integrate[x^9*(a + b*x^3)^5*(A + B*x^3),x]

[Out] $(a^5 A x^{10})/10 + (a^4 (5 A b + a B) x^{13})/13 + (5 a^3 b (2 A b + a B) x^{16})/16 + (10 a^2 b^2 (A b + a B) x^{19})/19 + (5 a b^3 (A b + 2 a B) x^{22})/22 + (b^4 (A b + 5 a B) x^{25})/25 + (b^5 B x^{28})/28$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9 (a + b x^3)^5 (A + B x^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^9*(a + b*x^3)^5*(A + B*x^3), x]

[Out] IntegrateAlgebraic[x^9*(a + b*x^3)^5*(A + B*x^3), x]

fricas [A] time = 0.71, size = 125, normalized size = 1.07

$\frac{1}{28} x^{28} b^5 B + \frac{1}{5} x^{25} b^4 a B + \frac{1}{25} x^{25} b^5 A + \frac{5}{11} x^{22} b^3 a^2 B + \frac{5}{22} x^{22} b^4 a A + \frac{10}{19} x^{19} b^2 a^3 B + \frac{10}{19} x^{19} b^3 a^2 A + \frac{5}{16} x^{16} b a^4 B + \frac{5}{8} x^{16} b^2 a^3 A + \frac{1}{13} x^{13} a^5 B + \frac{5}{13} x^{13} b a^4 A + \frac{1}{10} x^{10} a^5 A$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A), x, algorithm="fricas")

[Out] $\frac{1}{28} x^{28} b^5 B + \frac{1}{5} x^{25} b^4 a B + \frac{1}{25} x^{25} b^5 A + \frac{5}{11} x^{22} b^3 a^2 B + \frac{5}{22} x^{22} b^4 a A + \frac{10}{19} x^{19} b^2 a^3 B + \frac{10}{19} x^{19} b^3 a^2 A + \frac{5}{16} x^{16} b a^4 B + \frac{5}{8} x^{16} b^2 a^3 A + \frac{1}{13} x^{13} a^5 B + \frac{5}{13} x^{13} b a^4 A + \frac{1}{10} x^{10} a^5 A$

giac [A] time = 0.16, size = 125, normalized size = 1.07

$\frac{1}{28} B b^5 x^{28} + \frac{1}{5} B a b^4 x^{25} + \frac{1}{25} A b^5 x^{25} + \frac{5}{11} B a^2 b^3 x^{22} + \frac{5}{22} A a b^4 x^{22} + \frac{10}{19} B a^3 b^2 x^{19} + \frac{10}{19} A a^2 b^3 x^{19} + \frac{5}{16} B a^4 b x^{16} + \frac{5}{8} A a^3 b^2 x^{16} + \frac{1}{13} B a^5 x^{13} + \frac{5}{13} A a^4 b x^{13} + \frac{1}{10} A a^5 x^{10}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A), x, algorithm="giac")

[Out] $\frac{1}{28} B b^5 x^{28} + \frac{1}{5} B a b^4 x^{25} + \frac{1}{25} A b^5 x^{25} + \frac{5}{11} B a^2 b^3 x^{22} + \frac{5}{22} A a b^4 x^{22} + \frac{10}{19} B a^3 b^2 x^{19} + \frac{10}{19} A a^2 b^3 x^{19} + \frac{5}{16} B a^4 b x^{16} + \frac{5}{8} A a^3 b^2 x^{16} + \frac{1}{13} B a^5 x^{13} + \frac{5}{13} A a^4 b x^{13} + \frac{1}{10} A a^5 x^{10}$

maple [A] time = 0.04, size = 124, normalized size = 1.06

$\frac{B b^5 x^{28}}{28} + \frac{(b^5 A + 5 a b^4 B) x^{25}}{25} + \frac{(5 a b^4 A + 10 a^2 b^3 B) x^{22}}{22} + \frac{(10 a^2 b^3 A + 10 a^3 b^2 B) x^{19}}{19} + \frac{A a^5 x^{10}}{10} + \frac{(10 a^3 b^2 A + 5 a^4 b B) x^{16}}{16} + \frac{(5 a^4 b A + a^5 B) x^{13}}{13}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9*(b*x^3+a)^5*(B*x^3+A), x)

[Out] $1/28*b^5*B*x^{28}+1/25*(A*b^5+5*B*a*b^4)*x^{25}+1/22*(5*A*a*b^4+10*B*a^2*b^3)*x^{22}+1/19*(10*A*a^2*b^3+10*B*a^3*b^2)*x^{19}+1/16*(10*A*a^3*b^2+5*B*a^4*b)*x^{16}+1/13*(5*A*a^4*b+B*a^5)*x^{13}+1/10*a^5*A*x^{10}$

maxima [A] time = 0.46, size = 119, normalized size = 1.02

$$\frac{1}{28} B b^5 x^{28} + \frac{1}{25} (5 B a b^4 + A b^5) x^{25} + \frac{5}{22} (2 B a^2 b^3 + A a b^4) x^{22} + \frac{10}{19} (B a^3 b^2 + A a^2 b^3) x^{19} + \frac{5}{16} (B a^4 b + 2 A a^3 b^2) x^{16} + \frac{1}{10} A a^5 x^{10} + \frac{1}{13} (B a^5 + 5 A a^4 b) x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")`

[Out] $1/28*B*b^5*x^{28} + 1/25*(5*B*a*b^4 + A*b^5)*x^{25} + 5/22*(2*B*a^2*b^3 + A*a*b^4)*x^{22} + 10/19*(B*a^3*b^2 + A*a^2*b^3)*x^{19} + 5/16*(B*a^4*b + 2*A*a^3*b^2)*x^{16} + 1/10*A*a^5*x^{10} + 1/13*(B*a^5 + 5*A*a^4*b)*x^{13}$

mupad [B] time = 0.05, size = 107, normalized size = 0.91

$$x^{13} \left(\frac{B a^5}{13} + \frac{5 A a b^4}{13} \right) + x^{25} \left(\frac{A b^5}{25} + \frac{B a b^4}{5} \right) + \frac{A a^5 x^{10}}{10} + \frac{B b^5 x^{28}}{28} + \frac{10 a^2 b^2 x^{19} (A b + B a)}{19} + \frac{5 a^3 b x^{16} (2 A b + B a)}{16} + \frac{5 a b^3 x^{22} (A b + 2 B a)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9*(A + B*x^3)*(a + b*x^3)^5,x)`

[Out] $x^{13}*((B*a^5)/13 + (5*A*a^4*b)/13) + x^{25}*((A*b^5)/25 + (B*a*b^4)/5) + (A*a^5*x^{10})/10 + (B*b^5*x^{28})/28 + (10*a^2*b^2*x^{19}*(A*b + B*a))/19 + (5*a^3*b*x^{16}*(2*A*b + B*a))/16 + (5*a*b^3*x^{22}*(A*b + 2*B*a))/22$

sympy [A] time = 0.10, size = 136, normalized size = 1.16

$$\frac{A a^5 x^{10}}{10} + \frac{B b^5 x^{28}}{28} + x^{25} \left(\frac{A b^5}{25} + \frac{B a b^4}{5} \right) + x^{22} \left(\frac{5 A a b^4}{22} + \frac{5 B a^2 b^3}{11} \right) + x^{19} \left(\frac{10 A a^2 b^3}{19} + \frac{10 B a^3 b^2}{19} \right) + x^{16} \left(\frac{5 A a^3 b^2}{8} + \frac{5 B a^4 b}{16} \right) + x^{13} \left(\frac{5 A a^4 b}{13} + \frac{B a^5}{13} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9*(b*x**3+a)**5*(B*x**3+A),x)`

[Out] $A*a**5*x**10/10 + B*b**5*x**28/28 + x**25*(A*b**5/25 + B*a*b**4/5) + x**22*(5*A*a*b**4/22 + 5*B*a**2*b**3/11) + x**19*(10*A*a**2*b**3/19 + 10*B*a**3*b**2/19) + x**16*(5*A*a**3*b**2/8 + 5*B*a**4*b/16) + x**13*(5*A*a**4*b/13 + B*a**5/13)$

3.24 $\int x^8 (a + bx^3)^5 (A + Bx^3) dx$

Optimal. Leaf size=95

$$\frac{a^2 (a + bx^3)^6 (Ab - aB)}{18b^4} + \frac{(a + bx^3)^8 (Ab - 3aB)}{24b^4} - \frac{a (a + bx^3)^7 (2Ab - 3aB)}{21b^4} + \frac{B (a + bx^3)^9}{27b^4}$$

Rubi [A] time = 0.24, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 76}

$$\frac{a^2 (a + bx^3)^6 (Ab - aB)}{18b^4} + \frac{(a + bx^3)^8 (Ab - 3aB)}{24b^4} - \frac{a (a + bx^3)^7 (2Ab - 3aB)}{21b^4} + \frac{B (a + bx^3)^9}{27b^4}$$

Antiderivative was successfully verified.

[In] Int[x^8*(a + b*x^3)^5*(A + B*x^3),x]

[Out] (a^2*(A*b - a*B)*(a + b*x^3)^6)/(18*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^3)^7)/(21*b^4) + ((A*b - 3*a*B)*(a + b*x^3)^8)/(24*b^4) + (B*(a + b*x^3)^9)/(27*b^4)

Rule 76

Int[((d_)*(x_))^(n_)*((a_) + (b_)*(x_))*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int x^8 (a + bx^3)^5 (A + Bx^3) dx &= \frac{1}{3} \text{Subst} \left(\int x^2 (a + bx)^5 (A + Bx) dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{a^2(-Ab + aB)(a + bx)^5}{b^3} + \frac{a(-2Ab + 3aB)(a + bx)^6}{b^3} + \frac{(Ab - 3aB)(a + bx)^7}{b^3} \right) dx, x, x^3 \right) \\
&= \frac{a^2(Ab - aB)(a + bx^3)^6}{18b^4} - \frac{a(2Ab - 3aB)(a + bx^3)^7}{21b^4} + \frac{(Ab - 3aB)(a + bx^3)^8}{24b^4} + \dots
\end{aligned}$$

Mathematica [A] time = 0.03, size = 107, normalized size = 1.13

$$\frac{x^9 (168a^5A + 126a^4x^3(aB + 5Ab) + 504a^3bx^6(aB + 2Ab) + 840a^2b^2x^9(aB + Ab) + 63b^4x^{15}(5aB + Ab) + 360ab^3x^{12}(2aB + Ab) + 56b^5Bx^{18})}{1512}$$

Antiderivative was successfully verified.

[In] Integrate[x^8*(a + b*x^3)^5*(A + B*x^3), x]

[Out] (x^9*(168*a^5*A + 126*a^4*(5*A*b + a*B)*x^3 + 504*a^3*b*(2*A*b + a*B)*x^6 + 840*a^2*b^2*(A*b + a*B)*x^9 + 360*a*b^3*(A*b + 2*a*B)*x^12 + 63*b^4*(A*b + 5*a*B)*x^15 + 56*b^5*B*x^18))/1512

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 (a + bx^3)^5 (A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8*(a + b*x^3)^5*(A + B*x^3), x]

[Out] IntegrateAlgebraic[x^8*(a + b*x^3)^5*(A + B*x^3), x]

fricas [A] time = 0.73, size = 125, normalized size = 1.32

$$\frac{1}{27}x^{27}b^5B + \frac{5}{24}x^{24}b^4aB + \frac{1}{24}x^{24}b^5A + \frac{10}{21}x^{21}b^3a^2B + \frac{5}{21}x^{21}b^4aA + \frac{5}{9}x^{18}b^2a^3B + \frac{5}{9}x^{18}b^3a^2A + \frac{1}{3}x^{15}ba^4B + \frac{2}{3}x^{15}b^2a^3A + \frac{1}{12}x^{12}a^5B + \frac{5}{12}x^{12}ba^4A + \frac{1}{9}x^9a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A), x, algorithm="fricas")

[Out] 1/27*x^27*b^5*B + 5/24*x^24*b^4*a*B + 1/24*x^24*b^5*A + 10/21*x^21*b^3*a^2*B + 5/21*x^21*b^4*a*A + 5/9*x^18*b^2*a^3*B + 5/9*x^18*b^3*a^2*A + 1/3*x^15*b*a^4*B + 2/3*x^15*b^2*a^3*A + 1/12*x^12*a^5*B + 5/12*x^12*b*a^4*A + 1/9*x^9*a^5*A

giac [A] time = 0.17, size = 125, normalized size = 1.32

$$\frac{1}{27} B b^5 x^{27} + \frac{5}{24} B a b^4 x^{24} + \frac{1}{24} A b^5 x^{24} + \frac{10}{21} B a^2 b^3 x^{21} + \frac{5}{21} A a b^4 x^{21} + \frac{5}{9} B a^3 b^2 x^{18} + \frac{5}{9} A a^2 b^3 x^{18} + \frac{1}{3} B a^4 b x^{15} + \frac{2}{3} A a^3 b^2 x^{15} + \frac{1}{12} B a^5 x^{12} + \frac{5}{12} A a^4 b x^{12} + \frac{1}{9} A a^5 x^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")

$$[Out] \frac{1}{27} B b^5 x^{27} + \frac{5}{24} B a b^4 x^{24} + \frac{1}{24} A b^5 x^{24} + \frac{10}{21} B a^2 b^3 x^{21} + \frac{5}{21} A a b^4 x^{21} + \frac{5}{9} B a^3 b^2 x^{18} + \frac{5}{9} A a^2 b^3 x^{18} + \frac{1}{3} B a^4 b x^{15} + \frac{2}{3} A a^3 b^2 x^{15} + \frac{1}{12} B a^5 x^{12} + \frac{5}{12} A a^4 b x^{12} + \frac{1}{9} A a^5 x^9$$

maple [A] time = 0.04, size = 124, normalized size = 1.31

$$\frac{B b^5 x^{27}}{27} + \frac{(b^5 A + 5 a b^4 B) x^{24}}{24} + \frac{(5 a b^4 A + 10 a^2 b^3 B) x^{21}}{21} + \frac{(10 a^2 b^3 A + 10 a^3 b^2 B) x^{18}}{18} + \frac{A a^5 x^9}{9} + \frac{(10 a^3 b^2 A + 5 a^4 b B) x^{15}}{15} + \frac{(5 a^4 b A + a^5 B) x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(b*x^3+a)^5*(B*x^3+A),x)

$$[Out] \frac{1}{27} b^5 B x^{27} + \frac{1}{24} (A b^5 + 5 B a b^4) x^{24} + \frac{1}{21} (5 A a b^4 + 10 B a^2 b^3) x^{21} + \frac{1}{18} (10 A a^2 b^3 + 10 B a^3 b^2) x^{18} + \frac{1}{15} (10 A a^3 b^2 + 5 B a^4 b) x^{15} + \frac{1}{9} A a^5 x^9 + \frac{1}{12} (5 A a^4 b + B a^5) x^{12} + \frac{1}{9} a^5 A x^9$$

maxima [A] time = 0.47, size = 119, normalized size = 1.25

$$\frac{1}{27} B b^5 x^{27} + \frac{1}{24} (5 B a b^4 + A b^5) x^{24} + \frac{5}{21} (2 B a^2 b^3 + A a b^4) x^{21} + \frac{5}{9} (B a^3 b^2 + A a^2 b^3) x^{18} + \frac{1}{3} (B a^4 b + 2 A a^3 b^2) x^{15} + \frac{1}{9} A a^5 x^9 + \frac{1}{12} (B a^5 + 5 A a^4 b) x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")

$$[Out] \frac{1}{27} B b^5 x^{27} + \frac{1}{24} (5 B a b^4 + A b^5) x^{24} + \frac{5}{21} (2 B a^2 b^3 + A a b^4) x^{21} + \frac{5}{9} (B a^3 b^2 + A a^2 b^3) x^{18} + \frac{1}{3} (B a^4 b + 2 A a^3 b^2) x^{15} + \frac{1}{9} A a^5 x^9 + \frac{1}{12} (B a^5 + 5 A a^4 b) x^{12}$$

mupad [B] time = 2.34, size = 107, normalized size = 1.13

$$x^{12} \left(\frac{B a^5}{12} + \frac{5 A a b^4}{12} \right) + x^{24} \left(\frac{A b^5}{24} + \frac{5 B a b^4}{24} \right) + \frac{A a^5 x^9}{9} + \frac{B b^5 x^{27}}{27} + \frac{5 a^2 b^2 x^{18} (A b + B a)}{9} + \frac{a^3 b x^{15} (2 A b + B a)}{3} + \frac{5 a b^3 x^{21} (A b + 2 B a)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(A + B*x^3)*(a + b*x^3)^5,x)

$$[Out] x^{12} \left(\frac{B a^5}{12} + \frac{5 A a b^4}{12} \right) + x^{24} \left(\frac{A b^5}{24} + \frac{5 B a b^4}{24} \right) + \frac{A a^5 x^9}{9} + \frac{B b^5 x^{27}}{27} + \frac{5 a^2 b^2 x^{18} (A b + B a)}{9} + \frac{a^3 b x^{15} (2 A b + B a)}{3} + \frac{5 a b^3 x^{21} (A b + 2 B a)}{21}$$

sympy [A] time = 0.10, size = 136, normalized size = 1.43

$$\frac{Aa^5x^9}{9} + \frac{Bb^5x^{27}}{27} + x^{24} \left(\frac{Ab^5}{24} + \frac{5Bab^4}{24} \right) + x^{21} \left(\frac{5Aab^4}{21} + \frac{10Ba^2b^3}{21} \right) + x^{18} \left(\frac{5Aa^2b^3}{9} + \frac{5Ba^3b^2}{9} \right) + x^{15} \left(\frac{2Aa^3b^2}{3} + \frac{Ba^4b}{3} \right) + x^{12} \left(\frac{5Aa^4b}{12} + \frac{Ba^5}{12} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(b*x**3+a)**5*(B*x**3+A),x)

[Out] A*a**5*x**9/9 + B*b**5*x**27/27 + x**24*(A*b**5/24 + 5*B*a*b**4/24) + x**21*(5*A*a*b**4/21 + 10*B*a**2*b**3/21) + x**18*(5*A*a**2*b**3/9 + 5*B*a**3*b**2/9) + x**15*(2*A*a**3*b**2/3 + B*a**4*b/3) + x**12*(5*A*a**4*b/12 + B*a**5/12)

3.25 $\int x^7 (a + bx^3)^5 (A + Bx^3) dx$

Optimal. Leaf size=117

$$\frac{1}{8}a^5Ax^8 + \frac{1}{11}a^4x^{11}(aB+5Ab) + \frac{5}{14}a^3bx^{14}(aB+2Ab) + \frac{10}{17}a^2b^2x^{17}(aB+Ab) + \frac{1}{23}b^4x^{23}(5aB+Ab) + \frac{1}{4}ab^3x^{20}(2aB+Ab)$$

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{10}{17}a^2b^2x^{17}(aB+Ab) + \frac{5}{14}a^3bx^{14}(aB+2Ab) + \frac{1}{11}a^4x^{11}(aB+5Ab) + \frac{1}{8}a^5Ax^8 + \frac{1}{23}b^4x^{23}(5aB+Ab) + \frac{1}{4}ab^3x^{20}(2aB+Ab) + \frac{1}{26}b^5Bx^{26}$$

Antiderivative was successfully verified.

[In] Int[x^7*(a + b*x^3)^5*(A + B*x^3), x]

[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)/14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b^4*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^7 (a + bx^3)^5 (A + Bx^3) dx &= \int (a^5 Ax^7 + a^4(5Ab + aB)x^{10} + 5a^3b(2Ab + aB)x^{13} + 10a^2b^2(Ab + aB)x^{16} + 5a^2b^3(Ab + aB)x^{19} + 5a^2b^4(Ab + aB)x^{22} + 5a^2b^5(Ab + aB)x^{25} + Bx^{26}) dx \\ &= \frac{1}{8}a^5Ax^8 + \frac{1}{11}a^4(5Ab + aB)x^{11} + \frac{5}{14}a^3b(2Ab + aB)x^{14} + \frac{10}{17}a^2b^2(Ab + aB)x^{17} + \frac{5}{4}a^2b^3(Ab + aB)x^{20} + \frac{5}{23}a^2b^4(Ab + aB)x^{23} + \frac{1}{26}a^2b^5(Ab + aB)x^{26} + \frac{1}{26}Bx^{27} \end{aligned}$$

Mathematica [A] time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{8}a^5Ax^8 + \frac{1}{11}a^4x^{11}(aB+5Ab) + \frac{5}{14}a^3bx^{14}(aB+2Ab) + \frac{10}{17}a^2b^2x^{17}(aB+Ab) + \frac{1}{23}b^4x^{23}(5aB+Ab) + \frac{1}{4}ab^3x^{20}(2aB+Ab) + \frac{1}{26}b^5Bx^{26}$$

Antiderivative was successfully verified.

[In] Integrate[x^7*(a + b*x^3)^5*(A + B*x^3), x]

[Out] $(a^5Ax^8)/8 + (a^4(5A*b + a*B)*x^{11})/11 + (5a^3*b*(2A*b + a*B)*x^{14})/14 + (10a^2*b^2*(A*b + a*B)*x^{17})/17 + (a*b^3*(A*b + 2a*B)*x^{20})/4 + (b^4*(A*b + 5a*B)*x^{23})/23 + (b^5B*x^{26})/26$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 (a + bx^3)^5 (A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7*(a + b*x^3)^5*(A + B*x^3), x]

[Out] IntegrateAlgebraic[x^7*(a + b*x^3)^5*(A + B*x^3), x]

fricas [A] time = 0.69, size = 125, normalized size = 1.07

$$\frac{1}{26}x^{26}b^5B + \frac{5}{23}x^{23}b^4aB + \frac{1}{23}x^{23}b^5A + \frac{1}{2}x^{20}b^3a^2B + \frac{1}{4}x^{20}b^4aA + \frac{10}{17}x^{17}b^2a^3B + \frac{10}{17}x^{17}b^3a^2A + \frac{5}{14}x^{14}ba^4B + \frac{5}{7}x^{14}b^2a^3A + \frac{1}{11}x^{11}a^5B + \frac{5}{11}x^{11}ba^4A + \frac{1}{8}x^8a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A), x, algorithm="fricas")

[Out] $1/26*x^{26}*b^5*B + 5/23*x^{23}*b^4*a*B + 1/23*x^{23}*b^5*A + 1/2*x^{20}*b^3*a^2*B + 1/4*x^{20}*b^4*a*A + 10/17*x^{17}*b^2*a^3*B + 10/17*x^{17}*b^3*a^2*A + 5/14*x^{14}*b*a^4*B + 5/7*x^{14}*b^2*a^3*A + 1/11*x^{11}*a^5*B + 5/11*x^{11}*b*a^4*A + 1/8*x^8*a^5*A$

giac [A] time = 0.15, size = 125, normalized size = 1.07

$$\frac{1}{26}Bb^5x^{26} + \frac{5}{23}Bab^4x^{23} + \frac{1}{23}Ab^5x^{23} + \frac{1}{2}Ba^2b^3x^{20} + \frac{1}{4}Aab^4x^{20} + \frac{10}{17}Ba^3b^2x^{17} + \frac{10}{17}Aa^2b^3x^{17} + \frac{5}{14}Ba^4bx^{14} + \frac{5}{7}Aa^3b^2x^{14} + \frac{1}{11}Ba^5x^{11} + \frac{5}{11}Aa^4bx^{11} + \frac{1}{8}Aa^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A), x, algorithm="giac")

[Out] $1/26*B*b^5*x^{26} + 5/23*B*a*b^4*x^{23} + 1/23*A*b^5*x^{23} + 1/2*B*a^2*b^3*x^{20} + 1/4*A*a*b^4*x^{20} + 10/17*B*a^3*b^2*x^{17} + 10/17*A*a^2*b^3*x^{17} + 5/14*B*a^4*b*x^{14} + 5/7*A*a^3*b^2*x^{14} + 1/11*B*a^5*x^{11} + 5/11*A*a^4*b*x^{11} + 1/8*A*a^5*x^8$

maple [A] time = 0.04, size = 124, normalized size = 1.06

$$\frac{Bb^5x^{26}}{26} + \frac{(b^5A + 5ab^4B)x^{23}}{23} + \frac{(5ab^4A + 10a^2b^3B)x^{20}}{20} + \frac{(10a^2b^3A + 10a^3b^2B)x^{17}}{17} + \frac{Aa^5x^8}{8} + \frac{(10a^3b^2A + 5a^4bB)x^{14}}{14} + \frac{(5a^4bA + a^5B)x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(b*x^3+a)^5*(B*x^3+A), x)

[Out] $\frac{1}{26}b^5Bx^{26} + \frac{1}{23}(Ab^5 + 5B^2a^2b^4)x^{23} + \frac{1}{20}(5A^2a^2b^4 + 10B^2a^2b^3)x^{20} + \frac{1}{17}(10A^2a^2b^3 + 10B^2a^3b^2)x^{17} + \frac{1}{14}(10A^2a^3b^2 + 5B^2a^4b)x^{14} + \frac{1}{11}(5A^2a^4b + B^2a^5)x^{11} + \frac{1}{8}a^5A^2x^8$

maxima [A] time = 0.47, size = 119, normalized size = 1.02

$$\frac{1}{26}Bb^5x^{26} + \frac{1}{23}(5Bab^4 + Ab^5)x^{23} + \frac{1}{4}(2Ba^2b^3 + Aab^4)x^{20} + \frac{10}{17}(Ba^3b^2 + Aa^2b^3)x^{17} + \frac{5}{14}(Ba^4b + 2Aa^3b^2)x^{14} + \frac{1}{8}Aa^5x^8 + \frac{1}{11}(Ba^5 + 5Aa^4b)x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A), x, algorithm="maxima")

[Out] $\frac{1}{26}Bb^5x^{26} + \frac{1}{23}(5B^2a^2b^4 + A^2b^5)x^{23} + \frac{1}{4}(2B^2a^2b^3 + A^2a^2b^4)x^{20} + \frac{10}{17}(B^2a^3b^2 + A^2a^2b^3)x^{17} + \frac{5}{14}(B^2a^4b + 2A^2a^3b^2)x^{14} + \frac{1}{8}A^2a^5x^8 + \frac{1}{11}(B^2a^5 + 5A^2a^4b)x^{11}$

mupad [B] time = 0.04, size = 107, normalized size = 0.91

$$x^{11} \left(\frac{Ba^5}{11} + \frac{5Aba^4}{11} \right) + x^{23} \left(\frac{Ab^5}{23} + \frac{5Bab^4}{23} \right) + \frac{Aa^5x^8}{8} + \frac{Bb^5x^{26}}{26} + \frac{10a^2b^2x^{17}(Ab+Ba)}{17} + \frac{5a^3bx^{14}(2Ab+Ba)}{14} + \frac{ab^3x^{20}(Ab+2Ba)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(A + B*x^3)*(a + b*x^3)^5, x)

[Out] $x^{11} \left(\frac{B^2a^5}{11} + \frac{5A^2a^4b}{11} \right) + x^{23} \left(\frac{A^2b^5}{23} + \frac{5B^2a^2b^4}{23} \right) + \left(\frac{A^2a^5x^8}{8} + \frac{B^2b^5x^{26}}{26} + \frac{10A^2a^2b^2x^{17}(A^2b + B^2a)}{17} + \frac{5A^2a^3bx^{14}(2A^2b + B^2a)}{14} + \frac{a^2b^3x^{20}(A^2b + 2B^2a)}{4} \right)$

sympy [A] time = 0.10, size = 134, normalized size = 1.15

$$\frac{Aa^5x^8}{8} + \frac{Bb^5x^{26}}{26} + x^{23} \left(\frac{Ab^5}{23} + \frac{5Bab^4}{23} \right) + x^{20} \left(\frac{Aab^4}{4} + \frac{Ba^2b^3}{2} \right) + x^{17} \left(\frac{10Aa^2b^3}{17} + \frac{10Ba^3b^2}{17} \right) + x^{14} \left(\frac{5Aa^3b^2}{7} + \frac{5Ba^4b}{14} \right) + x^{11} \left(\frac{5Aa^4b}{11} + \frac{Ba^5}{11} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(b*x**3+a)**5*(B*x**3+A), x)

[Out] $A^2a^5x^8/8 + B^2b^5x^{26}/26 + x^{23}(A^2b^5/23 + 5B^2a^2b^4/23) + x^{20}(A^2a^2b^4/4 + B^2a^2b^3/2) + x^{17}(10A^2a^2b^3/17 + 10B^2a^3b^2/17) + x^{14}(5A^2a^3b^2/7 + 5B^2a^4b/14) + x^{11}(5A^2a^4b/11 + B^2a^5/11)$

3.26 $\int x^6 (a + bx^3)^5 (A + Bx^3) dx$

Optimal. Leaf size=117

$$\frac{1}{7}a^5Ax^7 + \frac{1}{10}a^4x^{10}(aB+5Ab) + \frac{5}{13}a^3bx^{13}(aB+2Ab) + \frac{5}{8}a^2b^2x^{16}(aB+Ab) + \frac{1}{22}b^4x^{22}(5aB+Ab) + \frac{5}{19}ab^3x^{19}(2aB+Ab) + \frac{1}{25}b^5Bx^{25}$$

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{5}{8}a^2b^2x^{16}(aB+Ab) + \frac{5}{13}a^3bx^{13}(aB+2Ab) + \frac{1}{10}a^4x^{10}(aB+5Ab) + \frac{1}{7}a^5Ax^7 + \frac{1}{22}b^4x^{22}(5aB+Ab) + \frac{5}{19}ab^3x^{19}(2aB+Ab) + \frac{1}{25}b^5Bx^{25}$$

Antiderivative was successfully verified.

[In] Int[x^6*(a + b*x^3)^5*(A + B*x^3),x]

[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)/13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (b^4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^6 (a + bx^3)^5 (A + Bx^3) dx &= \int (a^5 Ax^6 + a^4(5Ab + aB)x^9 + 5a^3b(2Ab + aB)x^{12} + 10a^2b^2(Ab + aB)x^{15} + 5ab^3(Ab + aB)x^{18} + b^4Bx^{21}) dx \\ &= \frac{1}{7}a^5Ax^7 + \frac{1}{10}a^4(5Ab + aB)x^{10} + \frac{5}{13}a^3b(2Ab + aB)x^{13} + \frac{5}{8}a^2b^2(Ab + aB)x^{16} + \frac{5}{19}ab^3(Ab + aB)x^{19} + \frac{1}{25}b^4Bx^{22} \end{aligned}$$

Mathematica [A] time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{7}a^5Ax^7 + \frac{1}{10}a^4x^{10}(aB+5Ab) + \frac{5}{13}a^3bx^{13}(aB+2Ab) + \frac{5}{8}a^2b^2x^{16}(aB+Ab) + \frac{1}{22}b^4x^{22}(5aB+Ab) + \frac{5}{19}ab^3x^{19}(2aB+Ab) + \frac{1}{25}b^5Bx^{25}$$

Antiderivative was successfully verified.

[In] Integrate[x^6*(a + b*x^3)^5*(A + B*x^3),x]

[Out] $(a^5Ax^7)/7 + (a^4(5A*b + a*B)*x^{10})/10 + (5*a^3*b*(2A*b + a*B)*x^{13})/13 + (5*a^2*b^2*(A*b + a*B)*x^{16})/8 + (5*a*b^3*(A*b + 2*a*B)*x^{19})/19 + (b^4*(A*b + 5*a*B)*x^{22})/22 + (b^5*B*x^{25})/25$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 (a + bx^3)^5 (A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6*(a + b*x^3)^5*(A + B*x^3), x]

[Out] IntegrateAlgebraic[x^6*(a + b*x^3)^5*(A + B*x^3), x]

fricas [A] time = 0.52, size = 125, normalized size = 1.07

$$\frac{1}{25}x^{25}b^5B + \frac{5}{22}x^{22}b^4aB + \frac{1}{22}x^{22}b^5A + \frac{10}{19}x^{19}b^3a^2B + \frac{5}{19}x^{19}b^4aA + \frac{5}{8}x^{16}b^2a^3B + \frac{5}{8}x^{16}b^3a^2A + \frac{5}{13}x^{13}ba^4B + \frac{10}{13}x^{13}b^2a^3A + \frac{1}{10}x^{10}a^5B + \frac{1}{2}x^{10}ba^4A + \frac{1}{7}x^7a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A), x, algorithm="fricas")

[Out] $1/25*x^{25}*b^5*B + 5/22*x^{22}*b^4*a*B + 1/22*x^{22}*b^5*A + 10/19*x^{19}*b^3*a^2*B + 5/19*x^{19}*b^4*a*A + 5/8*x^{16}*b^2*a^3*B + 5/8*x^{16}*b^3*a^2*A + 5/13*x^{13}*b*a^4*B + 10/13*x^{13}*b^2*a^3*A + 1/10*x^{10}*a^5*B + 1/2*x^{10}*b*a^4*A + 1/7*x^7*a^5*A$

giac [A] time = 0.15, size = 125, normalized size = 1.07

$$\frac{1}{25}Bb^5x^{25} + \frac{5}{22}Bab^4x^{22} + \frac{1}{22}Ab^5x^{22} + \frac{10}{19}Ba^2b^3x^{19} + \frac{5}{19}Aab^4x^{19} + \frac{5}{8}Ba^3b^2x^{16} + \frac{5}{8}Aa^2b^3x^{16} + \frac{5}{13}Ba^4bx^{13} + \frac{10}{13}Aa^3b^2x^{13} + \frac{1}{10}Ba^5x^{10} + \frac{1}{2}Aa^4bx^{10} + \frac{1}{7}Aa^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A), x, algorithm="giac")

[Out] $1/25*B*b^5*x^{25} + 5/22*B*a*b^4*x^{22} + 1/22*A*b^5*x^{22} + 10/19*B*a^2*b^3*x^{19} + 5/19*A*a*b^4*x^{19} + 5/8*B*a^3*b^2*x^{16} + 5/8*A*a^2*b^3*x^{16} + 5/13*B*a^4*b*x^{13} + 10/13*A*a^3*b^2*x^{13} + 1/10*B*a^5*x^{10} + 1/2*A*a^4*b*x^{10} + 1/7*A*a^5*x^7$

maple [A] time = 0.03, size = 124, normalized size = 1.06

$$\frac{Bb^5x^{25}}{25} + \frac{(b^5A + 5ab^4B)x^{22}}{22} + \frac{(5ab^4A + 10a^2b^3B)x^{19}}{19} + \frac{(10a^2b^3A + 10a^3b^2B)x^{16}}{16} + \frac{Aa^5x^7}{7} + \frac{(10a^3b^2A + 5a^4bB)x^{13}}{13} + \frac{(5a^4bA + a^5B)x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(b*x^3+a)^5*(B*x^3+A), x)

[Out] $\frac{1}{25}b^5Bx^{25} + \frac{1}{22}(Ab^5 + 5BAb^4)x^{22} + \frac{1}{19}(5A^2b^4 + 10BAb^3)x^{19} + \frac{1}{16}(10A^2b^3 + 10BAb^2)x^{16} + \frac{1}{13}(10A^3b^2 + 5BAb^4)x^{13} + \frac{1}{10}(5A^4b + BAb^5)x^{10} + \frac{1}{7}A^5x^7$

maxima [A] time = 0.59, size = 119, normalized size = 1.02

$$\frac{1}{25}Bb^5x^{25} + \frac{1}{22}(5BAb^4 + Ab^5)x^{22} + \frac{5}{19}(2Ba^2b^3 + Aab^4)x^{19} + \frac{5}{8}(Ba^3b^2 + Aa^2b^3)x^{16} + \frac{5}{13}(Ba^4b + 2Aa^3b^2)x^{13} + \frac{1}{7}Aa^5x^7 + \frac{1}{10}(Ba^5 + 5Aa^4b)x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(b*x^3+a)^5*(B*x^3+A), x, algorithm="maxima")`

[Out] $\frac{1}{25}Bb^5x^{25} + \frac{1}{22}(5BAb^4 + Ab^5)x^{22} + \frac{5}{19}(2BAb^3 + A^2b^4)x^{19} + \frac{5}{8}(Ba^3b^2 + A^2b^3)x^{16} + \frac{5}{13}(Ba^4b + 2A^3b^2)x^{13} + \frac{1}{7}A^5x^7 + \frac{1}{10}(BAb^5 + 5A^4b)x^{10}$

mupad [B] time = 0.04, size = 107, normalized size = 0.91

$$x^{10} \left(\frac{Ba^5}{10} + \frac{Ab^4}{2} \right) + x^{22} \left(\frac{Ab^5}{22} + \frac{5BAb^4}{22} \right) + \frac{Aa^5x^7}{7} + \frac{Bb^5x^{25}}{25} + \frac{5a^2b^2x^{16}(Ab+Ba)}{8} + \frac{5a^3bx^{13}(2Ab+Ba)}{13} + \frac{5ab^3x^{19}(Ab+2Ba)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(A + B*x^3)*(a + b*x^3)^5, x)`

[Out] $x^{10} \left(\frac{Ba^5}{10} + \frac{A^4b}{2} \right) + x^{22} \left(\frac{Ab^5}{22} + \frac{5BAb^4}{22} \right) + \frac{A^5x^7}{7} + \frac{Bb^5x^{25}}{25} + \frac{5a^2b^2x^{16}(Ab+Ba)}{8} + \frac{5a^3bx^{13}(2Ab+Ba)}{13} + \frac{5ab^3x^{19}(Ab+2Ba)}{19}$

sympy [A] time = 0.10, size = 136, normalized size = 1.16

$$\frac{Aa^5x^7}{7} + \frac{Bb^5x^{25}}{25} + x^{22} \left(\frac{Ab^5}{22} + \frac{5BAb^4}{22} \right) + x^{19} \left(\frac{5Aab^4}{19} + \frac{10Ba^2b^3}{19} \right) + x^{16} \left(\frac{5Aa^2b^3}{8} + \frac{5Ba^3b^2}{8} \right) + x^{13} \left(\frac{10Aa^3b^2}{13} + \frac{5Ba^4b}{13} \right) + x^{10} \left(\frac{Aa^4b}{2} + \frac{Ba^5}{10} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*(b*x**3+a)**5*(B*x**3+A), x)`

[Out] $A^5x^7/7 + Bb^5x^{25}/25 + x^{22}(Ab^5/22 + 5BAb^4/22) + x^{19}(5Aab^4/19 + 10Ba^2b^3/19) + x^{16}(5Aa^2b^3/8 + 5Ba^3b^2/8) + x^{13}(10Aa^3b^2/13 + 5Ba^4b/13) + x^{10}(Aa^4b/2 + Ba^5/10)$

$$3.27 \quad \int x^5 (a + bx^3)^5 (A + Bx^3) dx$$

Optimal. Leaf size=67

$$\frac{(a + bx^3)^7 (Ab - 2aB)}{21b^3} - \frac{a(a + bx^3)^6 (Ab - aB)}{18b^3} + \frac{B(a + bx^3)^8}{24b^3}$$

Rubi [A] time = 0.15, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 76}

$$\frac{(a + bx^3)^7 (Ab - 2aB)}{21b^3} - \frac{a(a + bx^3)^6 (Ab - aB)}{18b^3} + \frac{B(a + bx^3)^8}{24b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5*(a + b*x^3)^5*(A + B*x^3),x]

[Out] -(a*(A*b - a*B)*(a + b*x^3)^6)/(18*b^3) + ((A*b - 2*a*B)*(a + b*x^3)^7)/(21*b^3) + (B*(a + b*x^3)^8)/(24*b^3)

Rule 76

Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int x^5 (a + bx^3)^5 (A + Bx^3) dx &= \frac{1}{3} \text{Subst} \left(\int x(a + bx)^5 (A + Bx) dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{a(-Ab + aB)(a + bx)^5}{b^2} + \frac{(Ab - 2aB)(a + bx)^6}{b^2} + \frac{B(a + bx)^7}{b^2} \right) dx, x, x^3 \right) \\ &= -\frac{a(Ab - aB)(a + bx^3)^6}{18b^3} + \frac{(Ab - 2aB)(a + bx^3)^7}{21b^3} + \frac{B(a + bx^3)^8}{24b^3} \end{aligned}$$

Mathematica [A] time = 0.03, size = 107, normalized size = 1.60

$$\frac{1}{504} x^6 (84a^5 A + 56a^4 x^3 (aB + 5Ab) + 210a^3 b x^6 (aB + 2Ab) + 336a^2 b^2 x^9 (aB + Ab) + 24b^4 x^{15} (5aB + Ab) + 140ab^3 x^{12} (2aB + Ab) + 21b^5 B x^{18})$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(a + b*x^3)^5*(A + B*x^3), x]

[Out] (x^6*(84*a^5*A + 56*a^4*(5*A*b + a*B))*x^3 + 210*a^3*b*(2*A*b + a*B)*x^6 + 336*a^2*b^2*(A*b + a*B)*x^9 + 140*a*b^3*(A*b + 2*a*B)*x^12 + 24*b^4*(A*b + 5*a*B)*x^15 + 21*b^5*B*x^18)/504

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (a + bx^3)^5 (A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5*(a + b*x^3)^5*(A + B*x^3), x]

[Out] IntegrateAlgebraic[x^5*(a + b*x^3)^5*(A + B*x^3), x]

fricas [B] time = 1.28, size = 125, normalized size = 1.87

$$\frac{1}{24} x^{24} b^5 B + \frac{5}{21} x^{21} b^4 a B + \frac{1}{21} x^{21} b^5 A + \frac{5}{9} x^{18} b^3 a^2 B + \frac{5}{18} x^{18} b^4 a A + \frac{2}{3} x^{15} b^2 a^3 B + \frac{2}{3} x^{15} b^3 a^2 A + \frac{5}{12} x^{12} b a^4 B + \frac{5}{6} x^{12} b^2 a^3 A + \frac{1}{9} x^9 a^5 B + \frac{5}{9} x^9 b a^4 A + \frac{1}{6} x^6 a^5 A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A), x, algorithm="fricas")

[Out] 1/24*x^24*b^5*B + 5/21*x^21*b^4*a*B + 1/21*x^21*b^5*A + 5/9*x^18*b^3*a^2*B + 5/18*x^18*b^4*a*A + 2/3*x^15*b^2*a^3*B + 2/3*x^15*b^3*a^2*A + 5/12*x^12*b*a^4*B + 5/6*x^12*b^2*a^3*A + 1/9*x^9*a^5*B + 5/9*x^9*b*a^4*A + 1/6*x^6*a^5*A

giac [B] time = 0.16, size = 125, normalized size = 1.87

$$\frac{1}{24} B b^5 x^{24} + \frac{5}{21} B a b^4 x^{21} + \frac{1}{21} A b^5 x^{21} + \frac{5}{9} B a^2 b^3 x^{18} + \frac{5}{18} A a b^4 x^{18} + \frac{2}{3} B a^3 b^2 x^{15} + \frac{2}{3} A a^2 b^3 x^{15} + \frac{5}{12} B a^4 b x^{12} + \frac{5}{6} A a^3 b^2 x^{12} + \frac{1}{9} B a^5 x^9 + \frac{5}{9} A a^4 b x^9 + \frac{1}{6} A a^5 x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")

$$[Out] \frac{1}{24} B b^5 x^{24} + \frac{5}{21} B a b^4 x^{21} + \frac{1}{21} A b^5 x^{21} + \frac{5}{9} B a^2 b^3 x^{18} + \frac{5}{18} A a b^4 x^{18} + \frac{2}{3} B a^3 b^2 x^{15} + \frac{2}{3} A a^2 b^3 x^{15} + \frac{5}{12} B a^4 b x^{12} + \frac{5}{6} A a^3 b^2 x^{12} + \frac{1}{9} B a^5 x^9 + \frac{5}{9} A a^4 b x^9 + \frac{1}{6} A a^5 x^6$$

maple [B] time = 0.04, size = 124, normalized size = 1.85

$$\frac{B b^5 x^{24}}{24} + \frac{(b^5 A + 5 a b^4 B) x^{21}}{21} + \frac{(5 a b^4 A + 10 a^2 b^3 B) x^{18}}{18} + \frac{(10 a^2 b^3 A + 10 a^3 b^2 B) x^{15}}{15} + \frac{A a^5 x^6}{6} + \frac{(10 a^3 b^2 A + 5 a^4 b B) x^{12}}{12} + \frac{(5 a^4 b A + a^5 B) x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(b*x^3+a)^5*(B*x^3+A),x)

$$[Out] \frac{1}{24} b^5 B x^{24} + \frac{1}{21} (A b^5 + 5 B a b^4) x^{21} + \frac{1}{18} (5 A a b^4 + 10 B a^2 b^3) x^{18} + \frac{1}{15} (10 A a^2 b^3 + 10 B a^3 b^2) x^{15} + \frac{1}{12} (10 A a^3 b^2 + 5 B a^4 b) x^{12} + \frac{1}{9} (5 A a^4 b + B a^5) x^9 + \frac{1}{6} a^5 A x^6$$

maxima [A] time = 0.45, size = 119, normalized size = 1.78

$$\frac{1}{24} B b^5 x^{24} + \frac{1}{21} (5 B a b^4 + A b^5) x^{21} + \frac{5}{18} (2 B a^2 b^3 + A a b^4) x^{18} + \frac{2}{3} (B a^3 b^2 + A a^2 b^3) x^{15} + \frac{5}{12} (B a^4 b + 2 A a^3 b^2) x^{12} + \frac{1}{6} A a^5 x^6 + \frac{1}{9} (B a^5 + 5 A a^4 b) x^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")

$$[Out] \frac{1}{24} B b^5 x^{24} + \frac{1}{21} (5 B a b^4 + A b^5) x^{21} + \frac{5}{18} (2 B a^2 b^3 + A a b^4) x^{18} + \frac{2}{3} (B a^3 b^2 + A a^2 b^3) x^{15} + \frac{5}{12} (B a^4 b + 2 A a^3 b^2) x^{12} + \frac{1}{6} A a^5 x^6 + \frac{1}{9} (B a^5 + 5 A a^4 b) x^9$$

mupad [B] time = 0.04, size = 107, normalized size = 1.60

$$x^9 \left(\frac{B a^5}{9} + \frac{5 A b a^4}{9} \right) + x^{21} \left(\frac{A b^5}{21} + \frac{5 B a b^4}{21} \right) + \frac{A a^5 x^6}{6} + \frac{B b^5 x^{24}}{24} + \frac{2 a^2 b^2 x^{15} (A b + B a)}{3} + \frac{5 a^3 b x^{12} (2 A b + B a)}{12} + \frac{5 a b^3 x^{18} (A b + 2 B a)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(A + B*x^3)*(a + b*x^3)^5,x)

$$[Out] x^9 \left(\frac{(B a^5)/9 + (5 A a^4 b)/9}{1} \right) + x^{21} \left(\frac{(A b^5)/21 + (5 B a b^4)/21}{1} \right) + \frac{(A a^5 x^6)/6 + (B b^5 x^{24})/24 + (2 a^2 b^2 x^{15} (A b + B a))/3 + (5 a^3 b x^{12} (2 A b + B a))}{12} + \frac{5 a b^3 x^{18} (A b + 2 B a)}{18}$$

sympy [B] time = 0.10, size = 138, normalized size = 2.06

$$\frac{Aa^5x^6}{6} + \frac{Bb^5x^{24}}{24} + x^{21} \left(\frac{Ab^5}{21} + \frac{5Bab^4}{21} \right) + x^{18} \left(\frac{5Aab^4}{18} + \frac{5Ba^2b^3}{9} \right) + x^{15} \left(\frac{2Aa^2b^3}{3} + \frac{2Ba^3b^2}{3} \right) + x^{12} \left(\frac{5Aa^3b^2}{6} + \frac{5Ba^4b}{12} \right) + x^9 \left(\frac{5Aa^4b}{9} + \frac{Ba^5}{9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(b*x**3+a)**5*(B*x**3+A),x)

[Out] A*a**5*x**6/6 + B*b**5*x**24/24 + x**21*(A*b**5/21 + 5*B*a*b**4/21) + x**18*(5*A*a*b**4/18 + 5*B*a**2*b**3/9) + x**15*(2*A*a**2*b**3/3 + 2*B*a**3*b**2/3) + x**12*(5*A*a**3*b**2/6 + 5*B*a**4*b/12) + x**9*(5*A*a**4*b/9 + B*a**5/9)

[Out] $(a^5 A x^5)/5 + (a^4 (5 A b + a B) x^8)/8 + (5 a^3 b (2 A b + a B) x^{11})/11 + (5 a^2 b^2 (A b + a B) x^{14})/7 + (5 a b^3 (A b + 2 a B) x^{17})/17 + (b^4 (A b + 5 a B) x^{20})/20 + (b^5 B x^{23})/23$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 (a + b x^3)^5 (A + B x^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4*(a + b*x^3)^5*(A + B*x^3), x]

[Out] IntegrateAlgebraic[x^4*(a + b*x^3)^5*(A + B*x^3), x]

fricas [A] time = 0.82, size = 125, normalized size = 1.07

$$\frac{1}{23} x^{23} b^5 B + \frac{1}{4} x^{20} b^4 a B + \frac{1}{20} x^{20} b^5 A + \frac{10}{17} x^{17} b^3 a^2 B + \frac{5}{17} x^{17} b^4 a A + \frac{5}{7} x^{14} b^2 a^3 B + \frac{5}{7} x^{14} b^3 a^2 A + \frac{5}{11} x^{11} b a^4 B + \frac{10}{11} x^{11} b^2 a^3 A + \frac{1}{8} x^8 a^5 B + \frac{5}{8} x^8 b a^4 A + \frac{1}{5} x^5 a^5 A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A), x, algorithm="fricas")

[Out] $1/23 * x^{23} * b^5 * B + 1/4 * x^{20} * b^4 * a * B + 1/20 * x^{20} * b^5 * A + 10/17 * x^{17} * b^3 * a^2 * B + 5/17 * x^{17} * b^4 * a * A + 5/7 * x^{14} * b^2 * a^3 * B + 5/7 * x^{14} * b^3 * a^2 * A + 5/11 * x^{11} * b * a^4 * B + 10/11 * x^{11} * b^2 * a^3 * A + 1/8 * x^8 * a^5 * B + 5/8 * x^8 * b * a^4 * A + 1/5 * x^5 * a^5 * A$

giac [A] time = 0.23, size = 125, normalized size = 1.07

$$\frac{1}{23} B b^5 x^{23} + \frac{1}{4} B a b^4 x^{20} + \frac{1}{20} A b^5 x^{20} + \frac{10}{17} B a^2 b^3 x^{17} + \frac{5}{17} A a b^4 x^{17} + \frac{5}{7} B a^3 b^2 x^{14} + \frac{5}{7} A a^2 b^3 x^{14} + \frac{5}{11} B a^4 b x^{11} + \frac{10}{11} A a^3 b^2 x^{11} + \frac{1}{8} B a^5 x^8 + \frac{5}{8} A a^4 b x^8 + \frac{1}{5} A a^5 x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A), x, algorithm="giac")

[Out] $1/23 * B * b^5 * x^{23} + 1/4 * B * a * b^4 * x^{20} + 1/20 * A * b^5 * x^{20} + 10/17 * B * a^2 * b^3 * x^{17} + 5/17 * A * a * b^4 * x^{17} + 5/7 * B * a^3 * b^2 * x^{14} + 5/7 * A * a^2 * b^3 * x^{14} + 5/11 * B * a^4 * b * x^{11} + 10/11 * A * a^3 * b^2 * x^{11} + 1/8 * B * a^5 * x^8 + 5/8 * A * a^4 * b * x^8 + 1/5 * A * a^5 * x^5$

maple [A] time = 0.04, size = 124, normalized size = 1.06

$$\frac{B b^5 x^{23}}{23} + \frac{(b^5 A + 5 a b^4 B) x^{20}}{20} + \frac{(5 a b^4 A + 10 a^2 b^3 B) x^{17}}{17} + \frac{(10 a^2 b^3 A + 10 a^3 b^2 B) x^{14}}{14} + \frac{A a^5 x^5}{5} + \frac{(10 a^3 b^2 A + 5 a^4 b B) x^{11}}{11} + \frac{(5 a^4 b A + a^5 B) x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(b*x^3+a)^5*(B*x^3+A), x)

[Out] $\frac{1}{23}b^5Bx^{23} + \frac{1}{20}(A^5b^5 + 5B^4a^5)x^{20} + \frac{1}{17}(5A^4a^5b^4 + 10B^3a^5b^3)x^{17} + \frac{1}{14}(10A^3a^5b^3 + 10B^2a^5b^2)x^{14} + \frac{1}{11}(10A^2a^5b^2 + 5B^4a^5b)x^{11} + \frac{1}{8}(5A^4a^5b + B^5a^5)x^8 + \frac{1}{5}a^5A^5x^5$

maxima [A] time = 0.64, size = 119, normalized size = 1.02

$$\frac{1}{23}Bb^5x^{23} + \frac{1}{20}(5Bab^4 + Ab^5)x^{20} + \frac{5}{17}(2Ba^2b^3 + Aab^4)x^{17} + \frac{5}{7}(Ba^3b^2 + Aa^2b^3)x^{14} + \frac{5}{11}(Ba^4b + 2Aa^3b^2)x^{11} + \frac{1}{5}Aa^5x^5 + \frac{1}{8}(Ba^5 + 5Aa^4b)x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")

[Out] $\frac{1}{23}Bb^5x^{23} + \frac{1}{20}(5B^4a^5 + Ab^5)x^{20} + \frac{5}{17}(2B^3a^5b^3 + A^4a^5b^4)x^{17} + \frac{5}{7}(B^2a^5b^2 + A^3a^5b^3)x^{14} + \frac{5}{11}(B^4a^5b + 2A^3a^5b^2)x^{11} + \frac{1}{5}A^5a^5x^5 + \frac{1}{8}(B^5a^5 + 5A^4a^5b)x^8$

mupad [B] time = 0.04, size = 107, normalized size = 0.91

$$x^8 \left(\frac{Ba^5}{8} + \frac{5Aba^4}{8} \right) + x^{20} \left(\frac{Ab^5}{20} + \frac{Bab^4}{4} \right) + \frac{Aa^5x^5}{5} + \frac{Bb^5x^{23}}{23} + \frac{5a^2b^2x^{14}(Ab+Ba)}{7} + \frac{5a^3bx^{11}(2Ab+Ba)}{11} + \frac{5ab^3x^{17}(Ab+2Ba)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(A + B*x^3)*(a + b*x^3)^5,x)

[Out] $x^8 \left(\frac{B^5a^5}{8} + \frac{5A^4a^5b}{8} \right) + x^{20} \left(\frac{A^5b^5}{20} + \frac{B^4a^5b^4}{4} \right) + \frac{A^5a^5x^5}{5} + \frac{B^5b^5x^{23}}{23} + \frac{5A^2a^5b^2x^{14}(A^2b + B^2a)}{7} + \frac{5A^3a^5bx^{11}(2A^2b + B^2a)}{11} + \frac{5A^4a^5b^3x^{17}(A^2b + 2B^2a)}{17}$

sympy [A] time = 0.10, size = 136, normalized size = 1.16

$$\frac{Aa^5x^5}{5} + \frac{Bb^5x^{23}}{23} + x^{20} \left(\frac{Ab^5}{20} + \frac{Bab^4}{4} \right) + x^{17} \left(\frac{5Aab^4}{17} + \frac{10Ba^2b^3}{17} \right) + x^{14} \left(\frac{5Aa^2b^3}{7} + \frac{5Ba^3b^2}{7} \right) + x^{11} \left(\frac{10Aa^3b^2}{11} + \frac{5Ba^4b}{11} \right) + x^8 \left(\frac{5Aa^4b}{8} + \frac{Ba^5}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(b*x**3+a)**5*(B*x**3+A),x)

[Out] $A^5a^5x^5/5 + B^5b^5x^{23}/23 + x^{20}(A^5b^5/20 + B^4a^5b^4/4) + x^{17}(5A^4a^5b^4/17 + 10B^3a^5b^3/17) + x^{14}(5A^3a^5b^3/7 + 5B^2a^5b^2/7) + x^{11}(10A^2a^5b^2/11 + 5B^4a^5b/11) + x^8(5A^4a^5b/8 + B^5a^5/8)$

$$3.29 \quad \int x^3 (a + bx^3)^5 (A + Bx^3) dx$$

Optimal. Leaf size=117

$$\frac{1}{4}a^5Ax^4 + \frac{1}{7}a^4x^7(aB+5Ab) + \frac{1}{2}a^3bx^{10}(aB+2Ab) + \frac{10}{13}a^2b^2x^{13}(aB+Ab) + \frac{1}{19}b^4x^{19}(5aB+Ab) + \frac{5}{16}ab^3x^{16}(2aB+Ab) + \frac{1}{22}b^5Bx^{22}$$

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{10}{13}a^2b^2x^{13}(aB+Ab) + \frac{1}{2}a^3bx^{10}(aB+2Ab) + \frac{1}{7}a^4x^7(aB+5Ab) + \frac{1}{4}a^5Ax^4 + \frac{1}{19}b^4x^{19}(5aB+Ab) + \frac{5}{16}ab^3x^{16}(2aB+Ab) + \frac{1}{22}b^5Bx^{22}$$

Antiderivative was successfully verified.

[In] Int[x^3*(a + b*x^3)^5*(A + B*x^3),x]

[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 + (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^3 (a + bx^3)^5 (A + Bx^3) dx &= \int (a^5 Ax^3 + a^4(5Ab + aB)x^6 + 5a^3b(2Ab + aB)x^9 + 10a^2b^2(Ab + aB)x^{12} + 5ab^3(Ab + aB)x^{15} + b^4Bx^{18}) dx \\ &= \frac{1}{4}a^5Ax^4 + \frac{1}{7}a^4(5Ab + aB)x^7 + \frac{1}{2}a^3b(2Ab + aB)x^{10} + \frac{10}{13}a^2b^2(Ab + aB)x^{13} + \frac{5}{16}ab^3(Ab + aB)x^{16} + \frac{1}{22}b^4Bx^{19} \end{aligned}$$

Mathematica [A] time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{4}a^5Ax^4 + \frac{1}{7}a^4x^7(aB+5Ab) + \frac{1}{2}a^3bx^{10}(aB+2Ab) + \frac{10}{13}a^2b^2x^{13}(aB+Ab) + \frac{1}{19}b^4x^{19}(5aB+Ab) + \frac{5}{16}ab^3x^{16}(2aB+Ab) + \frac{1}{22}b^5Bx^{22}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a + b*x^3)^5*(A + B*x^3),x]

[Out] $(a^5 A x^4)/4 + (a^4 (5 A b + a B) x^7)/7 + (a^3 b (2 A b + a B) x^{10})/2 + (10 a^2 b^2 (A b + a B) x^{13})/13 + (5 a b^3 (A b + 2 a B) x^{16})/16 + (b^4 (A b + 5 a B) x^{19})/19 + (b^5 B x^{22})/22$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b x^3)^5 (A + B x^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3*(a + b*x^3)^5*(A + B*x^3), x]

[Out] IntegrateAlgebraic[x^3*(a + b*x^3)^5*(A + B*x^3), x]

fricas [A] time = 0.75, size = 124, normalized size = 1.06

$$\frac{1}{22} x^{22} b^5 B + \frac{5}{19} x^{19} b^4 a B + \frac{1}{19} x^{19} b^5 A + \frac{5}{8} x^{16} b^3 a^2 B + \frac{5}{16} x^{16} b^4 a A + \frac{10}{13} x^{13} b^2 a^3 B + \frac{10}{13} x^{13} b^3 a^2 A + \frac{1}{2} x^{10} b a^4 B + x^{10} b^2 a^3 A + \frac{1}{7} x^7 a^5 B + \frac{5}{7} x^7 b a^4 A + \frac{1}{4} x^4 a^5 A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A), x, algorithm="fricas")

[Out] $1/22 * x^{22} * b^5 * B + 5/19 * x^{19} * b^4 * a * B + 1/19 * x^{19} * b^5 * A + 5/8 * x^{16} * b^3 * a^2 * B + 5/16 * x^{16} * b^4 * a * A + 10/13 * x^{13} * b^2 * a^3 * B + 10/13 * x^{13} * b^3 * a^2 * A + 1/2 * x^{10} * b * a^4 * B + x^{10} * b^2 * a^3 * A + 1/7 * x^7 * a^5 * B + 5/7 * x^7 * b * a^4 * A + 1/4 * x^4 * a^5 * A$

giac [A] time = 0.16, size = 124, normalized size = 1.06

$$\frac{1}{22} B b^5 x^{22} + \frac{5}{19} B a b^4 x^{19} + \frac{1}{19} A b^5 x^{19} + \frac{5}{8} B a^2 b^3 x^{16} + \frac{5}{16} A a b^4 x^{16} + \frac{10}{13} B a^3 b^2 x^{13} + \frac{10}{13} A a^2 b^3 x^{13} + \frac{1}{2} B a^4 b x^{10} + A a^3 b^2 x^{10} + \frac{1}{7} B a^5 x^7 + \frac{5}{7} A a^4 b x^7 + \frac{1}{4} A a^5 x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A), x, algorithm="giac")

[Out] $1/22 * B * b^5 * x^{22} + 5/19 * B * a * b^4 * x^{19} + 1/19 * A * b^5 * x^{19} + 5/8 * B * a^2 * b^3 * x^{16} + 5/16 * A * a * b^4 * x^{16} + 10/13 * B * a^3 * b^2 * x^{13} + 10/13 * A * a^2 * b^3 * x^{13} + 1/2 * B * a^4 * b * x^{10} + A * a^3 * b^2 * x^{10} + 1/7 * B * a^5 * x^7 + 5/7 * A * a^4 * b * x^7 + 1/4 * A * a^5 * x^4$

maple [A] time = 0.04, size = 124, normalized size = 1.06

$$\frac{B b^5 x^{22}}{22} + \frac{(b^5 A + 5 a b^4 B) x^{19}}{19} + \frac{(5 a b^4 A + 10 a^2 b^3 B) x^{16}}{16} + \frac{(10 a^2 b^3 A + 10 a^3 b^2 B) x^{13}}{13} + \frac{A a^5 x^4}{4} + \frac{(10 a^3 b^2 A + 5 a^4 b B) x^{10}}{10} + \frac{(5 a^4 b A + a^5 B) x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x^3+a)^5*(B*x^3+A), x)

[Out] $1/22*b^5*B*x^{22}+1/19*(A*b^5+5*B*a*b^4)*x^{19}+1/16*(5*A*a*b^4+10*B*a^2*b^3)*x^{16}+1/13*(10*A*a^2*b^3+10*B*a^3*b^2)*x^{13}+1/10*(10*A*a^3*b^2+5*B*a^4*b)*x^{10}+1/7*(5*A*a^4*b+B*a^5)*x^7+1/4*a^5*A*x^4$

maxima [A] time = 0.44, size = 119, normalized size = 1.02

$$\frac{1}{22} B b^5 x^{22} + \frac{1}{19} (5 B a b^4 + A b^5) x^{19} + \frac{5}{16} (2 B a^2 b^3 + A a b^4) x^{16} + \frac{10}{13} (B a^3 b^2 + A a^2 b^3) x^{13} + \frac{1}{2} (B a^4 b + 2 A a^3 b^2) x^{10} + \frac{1}{4} A a^5 x^4 + \frac{1}{7} (B a^5 + 5 A a^4 b) x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")`

[Out] $1/22*B*b^5*x^{22} + 1/19*(5*B*a*b^4 + A*b^5)*x^{19} + 5/16*(2*B*a^2*b^3 + A*a*b^4)*x^{16} + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^{13} + 1/2*(B*a^4*b + 2*A*a^3*b^2)*x^{10} + 1/4*A*a^5*x^4 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7$

mupad [B] time = 0.04, size = 107, normalized size = 0.91

$$x^7 \left(\frac{B a^5}{7} + \frac{5 A b a^4}{7} \right) + x^{19} \left(\frac{A b^5}{19} + \frac{5 B a b^4}{19} \right) + \frac{A a^5 x^4}{4} + \frac{B b^5 x^{22}}{22} + \frac{10 a^2 b^2 x^{13} (A b + B a)}{13} + \frac{a^3 b x^{10} (2 A b + B a)}{2} + \frac{5 a b^3 x^{16} (A b + 2 B a)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(A + B*x^3)*(a + b*x^3)^5,x)`

[Out] $x^7*((B*a^5)/7 + (5*A*a^4*b)/7) + x^{19}*((A*b^5)/19 + (5*B*a*b^4)/19) + (A*a^5*x^4)/4 + (B*b^5*x^{22})/22 + (10*a^2*b^2*x^{13}*(A*b + B*a))/13 + (a^3*b*x^{10}*(2*A*b + B*a))/2 + (5*a*b^3*x^{16}*(A*b + 2*B*a))/16$

sympy [A] time = 0.10, size = 133, normalized size = 1.14

$$\frac{A a^5 x^4}{4} + \frac{B b^5 x^{22}}{22} + x^{19} \left(\frac{A b^5}{19} + \frac{5 B a b^4}{19} \right) + x^{16} \left(\frac{5 A a b^4}{16} + \frac{5 B a^2 b^3}{8} \right) + x^{13} \left(\frac{10 A a^2 b^3}{13} + \frac{10 B a^3 b^2}{13} \right) + x^{10} \left(A a^3 b^2 + \frac{B a^4 b}{2} \right) + x^7 \left(\frac{5 A a^4 b}{7} + \frac{B a^5}{7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**3+a)**5*(B*x**3+A),x)`

[Out] $A*a**5*x**4/4 + B*b**5*x**22/22 + x**19*(A*b**5/19 + 5*B*a*b**4/19) + x**16*(5*A*a*b**4/16 + 5*B*a**2*b**3/8) + x**13*(10*A*a**2*b**3/13 + 10*B*a**3*b**2/13) + x**10*(A*a**3*b**2 + B*a**4*b/2) + x**7*(5*A*a**4*b/7 + B*a**5/7)$

3.30 $\int x^2 (a + bx^3)^5 (A + Bx^3) dx$

Optimal. Leaf size=42

$$\frac{(a + bx^3)^6 (Ab - aB)}{18b^2} + \frac{B(a + bx^3)^7}{21b^2}$$

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {444, 43}

$$\frac{(a + bx^3)^6 (Ab - aB)}{18b^2} + \frac{B(a + bx^3)^7}{21b^2}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x^3)^5*(A + B*x^3),x]

[Out] ((A*b - a*B)*(a + b*x^3)^6)/(18*b^2) + (B*(a + b*x^3)^7)/(21*b^2)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int x^2 (a + bx^3)^5 (A + Bx^3) dx &= \frac{1}{3} \text{Subst} \left(\int (a + bx)^5 (A + Bx) dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{(Ab - aB)(a + bx)^5}{b} + \frac{B(a + bx)^6}{b} \right) dx, x, x^3 \right) \\ &= \frac{(Ab - aB)(a + bx^3)^6}{18b^2} + \frac{B(a + bx^3)^7}{21b^2} \end{aligned}$$

Mathematica [B] time = 0.03, size = 107, normalized size = 2.55

$$\frac{1}{126}x^3(42a^5A + 21a^4x^3(aB + 5Ab) + 70a^3bx^6(aB + 2Ab) + 105a^2b^2x^9(aB + Ab) + 7b^4x^{15}(5aB + Ab) + 42ab^3x^{12}(2aB + Ab) + 6b^5Bx^{18})$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x^3)^5*(A + B*x^3), x]

[Out] (x^3*(42*a^5*A + 21*a^4*(5*A*b + a*B)*x^3 + 70*a^3*b*(2*A*b + a*B)*x^6 + 10*5*a^2*b^2*(A*b + a*B)*x^9 + 42*a*b^3*(A*b + 2*a*B)*x^12 + 7*b^4*(A*b + 5*a*B)*x^15 + 6*b^5*B*x^18))/126

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + bx^3)^5 (A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a + b*x^3)^5*(A + B*x^3), x]

[Out] IntegrateAlgebraic[x^2*(a + b*x^3)^5*(A + B*x^3), x]

fricas [B] time = 0.72, size = 125, normalized size = 2.98

$$\frac{1}{21}x^{21}b^5B + \frac{5}{18}x^{18}b^4aB + \frac{1}{18}x^{18}b^5A + \frac{2}{3}x^{15}b^3a^2B + \frac{1}{3}x^{15}b^4aA + \frac{5}{6}x^{12}b^2a^3B + \frac{5}{6}x^{12}b^3a^2A + \frac{5}{9}x^9ba^4B + \frac{10}{9}x^9b^2a^3A + \frac{1}{6}x^6a^5B + \frac{5}{6}x^6ba^4A + \frac{1}{3}x^3a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A), x, algorithm="fricas")

[Out] 1/21*x^21*b^5*B + 5/18*x^18*b^4*a*B + 1/18*x^18*b^5*A + 2/3*x^15*b^3*a^2*B + 1/3*x^15*b^4*a*A + 5/6*x^12*b^2*a^3*B + 5/6*x^12*b^3*a^2*A + 5/9*x^9*b*a^4*B + 10/9*x^9*b^2*a^3*A + 1/6*x^6*a^5*B + 5/6*x^6*b*a^4*A + 1/3*x^3*a^5*A

giac [B] time = 0.17, size = 125, normalized size = 2.98

$$\frac{1}{21}Bb^5x^{21} + \frac{5}{18}Bab^4x^{18} + \frac{1}{18}Ab^5x^{18} + \frac{2}{3}Ba^2b^3x^{15} + \frac{1}{3}Aab^4x^{15} + \frac{5}{6}Ba^3b^2x^{12} + \frac{5}{6}Aa^2b^3x^{12} + \frac{5}{9}Ba^4bx^9 + \frac{10}{9}Aa^3b^2x^9 + \frac{1}{6}Ba^5x^6 + \frac{5}{6}Aa^4bx^6 + \frac{1}{3}Aa^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A), x, algorithm="giac")

[Out] 1/21*B*b^5*x^21 + 5/18*B*a*b^4*x^18 + 1/18*A*b^5*x^18 + 2/3*B*a^2*b^3*x^15 + 1/3*A*a*b^4*x^15 + 5/6*B*a^3*b^2*x^12 + 5/6*A*a^2*b^3*x^12 + 5/9*B*a^4*b*x^9 + 10/9*A*a^3*b^2*x^9 + 1/6*B*a^5*x^6 + 5/6*A*a^4*b*x^6 + 1/3*A*a^5*x^3

maple [B] time = 0.04, size = 124, normalized size = 2.95

$$\frac{B b^5 x^{21}}{21} + \frac{(b^5 A + 5 a b^4 B) x^{18}}{18} + \frac{(5 a b^4 A + 10 a^2 b^3 B) x^{15}}{15} + \frac{(10 a^2 b^3 A + 10 a^3 b^2 B) x^{12}}{12} + \frac{A a^5 x^3}{3} + \frac{(10 a^3 b^2 A + 5 a^4 b B) x^9}{9} + \frac{(5 a^4 b A + a^5 B) x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^3+a)^5*(B*x^3+A), x)

[Out] 1/21*b^5*B*x^21+1/18*(A*b^5+5*B*a*b^4)*x^18+1/15*(5*A*a*b^4+10*B*a^2*b^3)*x^15+1/12*(10*A*a^2*b^3+10*B*a^3*b^2)*x^12+1/9*(10*A*a^3*b^2+5*B*a^4*b)*x^9+1/6*(5*A*a^4*b+B*a^5)*x^6+1/3*a^5*A*x^3

maxima [B] time = 0.44, size = 119, normalized size = 2.83

$$\frac{1}{21} B b^5 x^{21} + \frac{1}{18} (5 B a b^4 + A b^5) x^{18} + \frac{1}{3} (2 B a^2 b^3 + A a b^4) x^{15} + \frac{5}{6} (B a^3 b^2 + A a^2 b^3) x^{12} + \frac{5}{9} (B a^4 b + 2 A a^3 b^2) x^9 + \frac{1}{3} A a^5 x^3 + \frac{1}{6} (B a^5 + 5 A a^4 b) x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A), x, algorithm="maxima")

[Out] 1/21*B*b^5*x^21 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 1/3*(2*B*a^2*b^3 + A*a*b^4)*x^15 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 5/9*(B*a^4*b + 2*A*a^3*b^2)*x^9 + 1/3*A*a^5*x^3 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6

mupad [B] time = 0.04, size = 107, normalized size = 2.55

$$x^6 \left(\frac{B a^5}{6} + \frac{5 A b a^4}{6} \right) + x^{18} \left(\frac{A b^5}{18} + \frac{5 B a b^4}{18} \right) + \frac{A a^5 x^3}{3} + \frac{B b^5 x^{21}}{21} + \frac{5 a^2 b^2 x^{12} (A b + B a)}{6} + \frac{5 a^3 b x^9 (2 A b + B a)}{9} + \frac{a b^3 x^{15} (A b + 2 B a)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(A + B*x^3)*(a + b*x^3)^5, x)

[Out] x^6*((B*a^5)/6 + (5*A*a^4*b)/6) + x^18*((A*b^5)/18 + (5*B*a*b^4)/18) + (A*a^5*x^3)/3 + (B*b^5*x^21)/21 + (5*a^2*b^2*x^12*(A*b + B*a))/6 + (5*a^3*b*x^9*(2*A*b + B*a))/9 + (a*b^3*x^15*(A*b + 2*B*a))/3

sympy [B] time = 0.10, size = 136, normalized size = 3.24

$$\frac{A a^5 x^3}{3} + \frac{B b^5 x^{21}}{21} + x^{18} \left(\frac{A b^5}{18} + \frac{5 B a b^4}{18} \right) + x^{15} \left(\frac{A a b^4}{3} + \frac{2 B a^2 b^3}{3} \right) + x^{12} \left(\frac{5 A a^2 b^3}{6} + \frac{5 B a^3 b^2}{6} \right) + x^9 \left(\frac{10 A a^3 b^2}{9} + \frac{5 B a^4 b}{9} \right) + x^6 \left(\frac{5 A a^4 b}{6} + \frac{B a^5}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**3+a)**5*(B*x**3+A), x)

[Out] A*a**5*x**3/3 + B*b**5*x**21/21 + x**18*(A*b**5/18 + 5*B*a*b**4/18) + x**15*(A*a*b**4/3 + 2*B*a**2*b**3/3) + x**12*(5*A*a**2*b**3/6 + 5*B*a**3*b**2/6) + x**9*(10*A*a**3*b**2/9 + 5*B*a**4*b/9) + x**6*(5*A*a**4*b/6 + B*a**5/6)

3.31 $\int x (a + bx^3)^5 (A + Bx^3) dx$

Optimal. Leaf size=117

$$\frac{1}{2}a^5Ax^2 + \frac{1}{5}a^4x^5(aB+5Ab) + \frac{5}{8}a^3bx^8(aB+2Ab) + \frac{10}{11}a^2b^2x^{11}(aB+Ab) + \frac{1}{17}b^4x^{17}(5aB+Ab) + \frac{5}{14}ab^3x^{14}(2aB+Ab) + \frac{1}{20}b^5Bx^{20}$$

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {448}

$$\frac{10}{11}a^2b^2x^{11}(aB+Ab) + \frac{5}{8}a^3bx^8(aB+2Ab) + \frac{1}{5}a^4x^5(aB+5Ab) + \frac{1}{2}a^5Ax^2 + \frac{1}{17}b^4x^{17}(5aB+Ab) + \frac{5}{14}ab^3x^{14}(2aB+Ab) + \frac{1}{20}b^5Bx^{20}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^3)^5*(A + B*x^3),x]

[Out] (a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 + (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^4*(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x (a + bx^3)^5 (A + Bx^3) dx &= \int (a^5Ax + a^4(5Ab + aB)x^4 + 5a^3b(2Ab + aB)x^7 + 10a^2b^2(Ab + aB)x^{10} + 5ab^3(Ab + aB)x^{13} + b^4Bx^{16}) dx \\ &= \frac{1}{2}a^5Ax^2 + \frac{1}{5}a^4(5Ab + aB)x^5 + \frac{5}{8}a^3b(2Ab + aB)x^8 + \frac{10}{11}a^2b^2(Ab + aB)x^{11} + \frac{5}{14}ab^3(Ab + aB)x^{14} + \frac{1}{20}b^4Bx^{17} \end{aligned}$$

Mathematica [A] time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{2}a^5Ax^2 + \frac{1}{5}a^4x^5(aB+5Ab) + \frac{5}{8}a^3bx^8(aB+2Ab) + \frac{10}{11}a^2b^2x^{11}(aB+Ab) + \frac{1}{17}b^4x^{17}(5aB+Ab) + \frac{5}{14}ab^3x^{14}(2aB+Ab) + \frac{1}{20}b^5Bx^{20}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^3)^5*(A + B*x^3),x]

[Out] $(a^5Ax^2)/2 + (a^4(5A*b + a*B)*x^5)/5 + (5*a^3*b*(2A*b + a*B)*x^8)/8 + (10*a^2*b^2*(A*b + a*B)*x^{11})/11 + (5*a*b^3*(A*b + 2*a*B)*x^{14})/14 + (b^4*(A*b + 5*a*B)*x^{17})/17 + (b^5*B*x^{20})/20$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + bx^3)^5(A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a + b*x^3)^5*(A + B*x^3), x]

[Out] IntegrateAlgebraic[x*(a + b*x^3)^5*(A + B*x^3), x]

fricas [A] time = 0.70, size = 124, normalized size = 1.06

$$\frac{1}{20}x^{20}b^5B + \frac{5}{17}x^{17}b^4aB + \frac{1}{17}x^{17}b^5A + \frac{5}{7}x^{14}b^3a^2B + \frac{5}{14}x^{14}b^4aA + \frac{10}{11}x^{11}b^2a^3B + \frac{10}{11}x^{11}b^3a^2A + \frac{5}{8}x^8ba^4B + \frac{5}{4}x^8b^2a^3A + \frac{1}{5}x^5a^5B + x^5ba^4A + \frac{1}{2}x^2a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^5*(B*x^3+A), x, algorithm="fricas")

[Out] $1/20*x^{20}*b^5*B + 5/17*x^{17}*b^4*a*B + 1/17*x^{17}*b^5*A + 5/7*x^{14}*b^3*a^2*B + 5/14*x^{14}*b^4*a*A + 10/11*x^{11}*b^2*a^3*B + 10/11*x^{11}*b^3*a^2*A + 5/8*x^8*b*a^4*B + 5/4*x^8*b^2*a^3*A + 1/5*x^5*a^5*B + x^5*b*a^4*A + 1/2*x^2*a^5*A$

giac [A] time = 0.15, size = 124, normalized size = 1.06

$$\frac{1}{20}Bb^5x^{20} + \frac{5}{17}Bab^4x^{17} + \frac{1}{17}Ab^5x^{17} + \frac{5}{7}Ba^2b^3x^{14} + \frac{5}{14}Aab^4x^{14} + \frac{10}{11}Ba^3b^2x^{11} + \frac{10}{11}Aa^2b^3x^{11} + \frac{5}{8}Ba^4bx^8 + \frac{5}{4}Aa^3b^2x^8 + \frac{1}{5}Ba^5x^5 + Aa^4bx^5 + \frac{1}{2}Aa^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^5*(B*x^3+A), x, algorithm="giac")

[Out] $1/20*B*b^5*x^{20} + 5/17*B*a*b^4*x^{17} + 1/17*A*b^5*x^{17} + 5/7*B*a^2*b^3*x^{14} + 5/14*A*a*b^4*x^{14} + 10/11*B*a^3*b^2*x^{11} + 10/11*A*a^2*b^3*x^{11} + 5/8*B*a^4*b*x^8 + 5/4*A*a^3*b^2*x^8 + 1/5*B*a^5*x^5 + A*a^4*b*x^5 + 1/2*A*a^5*x^2$

maple [A] time = 0.04, size = 124, normalized size = 1.06

$$\frac{Bb^5x^{20}}{20} + \frac{(b^5A + 5ab^4B)x^{17}}{17} + \frac{(5ab^4A + 10a^2b^3B)x^{14}}{14} + \frac{(10a^2b^3A + 10a^3b^2B)x^{11}}{11} + \frac{Aa^5x^2}{2} + \frac{(10a^3b^2A + 5a^4bB)x^8}{8} + \frac{(5a^4bA + a^5B)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a)^5*(B*x^3+A), x)

[Out] $1/20*b^5*B*x^{20}+1/17*(A*b^5+5*B*a*b^4)*x^{17}+1/14*(5*A*a*b^4+10*B*a^2*b^3)*x^{14}+1/11*(10*A*a^2*b^3+10*B*a^3*b^2)*x^{11}+1/8*(10*A*a^3*b^2+5*B*a^4*b)*x^8+1/5*(5*A*a^4*b+B*a^5)*x^5+1/2*a^5*A*x^2$

maxima [A] time = 0.65, size = 119, normalized size = 1.02

$$\frac{1}{20} B b^5 x^{20} + \frac{1}{17} (5 B a b^4 + A b^5) x^{17} + \frac{5}{14} (2 B a^2 b^3 + A a b^4) x^{14} + \frac{10}{11} (B a^3 b^2 + A a^2 b^3) x^{11} + \frac{5}{8} (B a^4 b + 2 A a^3 b^2) x^8 + \frac{1}{2} A a^5 x^2 + \frac{1}{5} (B a^5 + 5 A a^4 b) x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")`

[Out] $1/20*B*b^5*x^{20} + 1/17*(5*B*a*b^4 + A*b^5)*x^{17} + 5/14*(2*B*a^2*b^3 + A*a*b^4)*x^{14} + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^{11} + 5/8*(B*a^4*b + 2*A*a^3*b^2)*x^8 + 1/2*A*a^5*x^2 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5$

mupad [B] time = 0.04, size = 106, normalized size = 0.91

$$x^5 \left(\frac{B a^5}{5} + A b a^4 \right) + x^{17} \left(\frac{A b^5}{17} + \frac{5 B a b^4}{17} \right) + \frac{A a^5 x^2}{2} + \frac{B b^5 x^{20}}{20} + \frac{10 a^2 b^2 x^{11} (A b + B a)}{11} + \frac{5 a^3 b x^8 (2 A b + B a)}{8} + \frac{5 a b^3 x^{14} (A b + 2 B a)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(A + B*x^3)*(a + b*x^3)^5,x)`

[Out] $x^5*((B*a^5)/5 + A*a^4*b) + x^{17}*((A*b^5)/17 + (5*B*a*b^4)/17) + (A*a^5*x^2)/2 + (B*b^5*x^{20})/20 + (10*a^2*b^2*x^{11}*(A*b + B*a))/11 + (5*a^3*b*x^8*(2*A*b + B*a))/8 + (5*a*b^3*x^{14}*(A*b + 2*B*a))/14$

sympy [A] time = 0.09, size = 134, normalized size = 1.15

$$\frac{A a^5 x^2}{2} + \frac{B b^5 x^{20}}{20} + x^{17} \left(\frac{A b^5}{17} + \frac{5 B a b^4}{17} \right) + x^{14} \left(\frac{5 A a b^4}{14} + \frac{5 B a^2 b^3}{7} \right) + x^{11} \left(\frac{10 A a^2 b^3}{11} + \frac{10 B a^3 b^2}{11} \right) + x^8 \left(\frac{5 A a^3 b^2}{4} + \frac{5 B a^4 b}{8} \right) + x^5 \left(A a^4 b + \frac{B a^5}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**3+a)**5*(B*x**3+A),x)`

[Out] $A*a**5*x**2/2 + B*b**5*x**20/20 + x**17*(A*b**5/17 + 5*B*a*b**4/17) + x**14*(5*A*a*b**4/14 + 5*B*a**2*b**3/7) + x**11*(10*A*a**2*b**3/11 + 10*B*a**3*b**2/11) + x**8*(5*A*a**3*b**2/4 + 5*B*a**4*b/8) + x**5*(A*a**4*b + B*a**5/5)$

$$3.32 \quad \int (a + bx^3)^5 (A + Bx^3) dx$$

Optimal. Leaf size=109

$$a^5 Ax + \frac{1}{4} a^4 x^4 (aB + 5Ab) + \frac{5}{7} a^3 bx^7 (aB + 2Ab) + a^2 b^2 x^{10} (aB + Ab) + \frac{1}{16} b^4 x^{16} (5aB + Ab) + \frac{5}{13} ab^3 x^{13} (2aB + Ab) + \frac{1}{19} b^5 Bx^{19}$$

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {373}

$$a^2 b^2 x^{10} (aB + Ab) + \frac{5}{7} a^3 bx^7 (aB + 2Ab) + \frac{1}{4} a^4 x^4 (aB + 5Ab) + a^5 Ax + \frac{1}{16} b^4 x^{16} (5aB + Ab) + \frac{5}{13} ab^3 x^{13} (2aB + Ab) + \frac{1}{19} b^5 Bx^{19}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^5*(A + B*x^3), x]

[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*b^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*x^16)/16 + (b^5*B*x^19)/19

Rule 373

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int (a + bx^3)^5 (A + Bx^3) dx &= \int (a^5 A + a^4(5Ab + aB)x^3 + 5a^3b(2Ab + aB)x^6 + 10a^2b^2(Ab + aB)x^9 + 5ab^3(Ab + aB)x^{12} + b^4Bx^{15}) dx \\ &= a^5 Ax + \frac{1}{4} a^4 (5Ab + aB) x^4 + \frac{5}{7} a^3 b (2Ab + aB) x^7 + a^2 b^2 (Ab + aB) x^{10} + \frac{5}{13} ab^3 (Ab + aB) x^{13} + \frac{1}{19} b^4 B x^{16} \end{aligned}$$

Mathematica [A] time = 0.02, size = 109, normalized size = 1.00

$$a^5 Ax + \frac{1}{4} a^4 x^4 (aB + 5Ab) + \frac{5}{7} a^3 bx^7 (aB + 2Ab) + a^2 b^2 x^{10} (aB + Ab) + \frac{1}{16} b^4 x^{16} (5aB + Ab) + \frac{5}{13} ab^3 x^{13} (2aB + Ab) + \frac{1}{19} b^5 Bx^{19}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^5*(A + B*x^3), x]

[Out] $a^5 A x + (a^4 (5 A b + a B) x^4) / 4 + (5 a^3 b (2 A b + a B) x^7) / 7 + a^2 b^2 (A b + a B) x^{10} + (5 a b^3 (A b + 2 a B) x^{13}) / 13 + (b^4 (A b + 5 a B) x^{16}) / 16 + (b^5 B x^{19}) / 19$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b x^3)^5 (A + B x^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^3)^5*(A + B*x^3), x]

[Out] IntegrateAlgebraic[(a + b*x^3)^5*(A + B*x^3), x]

fricas [A] time = 0.70, size = 120, normalized size = 1.10

$$\frac{1}{19} x^{19} b^5 B + \frac{5}{16} x^{16} b^4 a B + \frac{1}{16} x^{16} b^5 A + \frac{10}{13} x^{13} b^3 a^2 B + \frac{5}{13} x^{13} b^4 a A + x^{10} b^2 a^3 B + x^{10} b^3 a^2 A + \frac{5}{7} x^7 b a^4 B + \frac{10}{7} x^7 b^2 a^3 A + \frac{1}{4} x^4 a^5 B + \frac{5}{4} x^4 b a^4 A + x a^5 A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A), x, algorithm="fricas")

[Out] $1/19 * x^{19} * b^5 * B + 5/16 * x^{16} * b^4 * a * B + 1/16 * x^{16} * b^5 * A + 10/13 * x^{13} * b^3 * a^2 * B + 5/13 * x^{13} * b^4 * a * A + x^{10} * b^2 * a^3 * B + x^{10} * b^3 * a^2 * A + 5/7 * x^7 * b * a^4 * B + 10/7 * x^7 * b^2 * a^3 * A + 1/4 * x^4 * a^5 * B + 5/4 * x^4 * b * a^4 * A + x * a^5 * A$

giac [A] time = 0.17, size = 120, normalized size = 1.10

$$\frac{1}{19} B b^5 x^{19} + \frac{5}{16} B a b^4 x^{16} + \frac{1}{16} A b^5 x^{16} + \frac{10}{13} B a^2 b^3 x^{13} + \frac{5}{13} A a b^4 x^{13} + B a^3 b^2 x^{10} + A a^2 b^3 x^{10} + \frac{5}{7} B a^4 b x^7 + \frac{10}{7} A a^3 b^2 x^7 + \frac{1}{4} B a^5 x^4 + \frac{5}{4} A a^4 b x^4 + A a^5 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A), x, algorithm="giac")

[Out] $1/19 * B * b^5 * x^{19} + 5/16 * B * a * b^4 * x^{16} + 1/16 * A * b^5 * x^{16} + 10/13 * B * a^2 * b^3 * x^{13} + 5/13 * A * a * b^4 * x^{13} + B * a^3 * b^2 * x^{10} + A * a^2 * b^3 * x^{10} + 5/7 * B * a^4 * b * x^7 + 10/7 * A * a^3 * b^2 * x^7 + 1/4 * B * a^5 * x^4 + 5/4 * A * a^4 * b * x^4 + A * a^5 * x$

maple [A] time = 0.04, size = 121, normalized size = 1.11

$$\frac{B b^5 x^{19}}{19} + \frac{(b^5 A + 5 a b^4 B) x^{16}}{16} + \frac{(5 a b^4 A + 10 a^2 b^3 B) x^{13}}{13} + \frac{(10 a^2 b^3 A + 10 a^3 b^2 B) x^{10}}{10} + A a^5 x + \frac{(10 a^3 b^2 A + 5 a^4 b B) x^7}{7} + \frac{(5 a^4 b A + a^5 B) x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A), x)

[Out] $\frac{1}{19}b^5Bx^{19} + \frac{1}{16}(A^5 + 5B^4a^4)x^{16} + \frac{1}{13}(5A^4a^4 + 10B^3a^3b^3)x^{13} + \frac{1}{10}(10A^3a^3b^3 + 10B^2a^2b^2)x^{10} + \frac{1}{7}(10A^2a^2b^2 + 5B^4a^4b)x^7 + \frac{1}{4}(5A^4a^4b + B^5a^5)x^4 + a^5A^5x$

maxima [A] time = 0.60, size = 115, normalized size = 1.06

$$\frac{1}{19}Bb^5x^{19} + \frac{1}{16}(5Bab^4 + Ab^5)x^{16} + \frac{5}{13}(2Ba^2b^3 + Aab^4)x^{13} + (Ba^3b^2 + Aa^2b^3)x^{10} + \frac{5}{7}(Ba^4b + 2Aa^3b^2)x^7 + Aa^5x + \frac{1}{4}(Ba^5 + 5Aa^4b)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")`

[Out] $\frac{1}{19}Bb^5x^{19} + \frac{1}{16}(5B^4a^4 + A^5b^5)x^{16} + \frac{5}{13}(2B^3a^3b^3 + A^4a^4b)x^{13} + (B^2a^2b^2 + A^3a^3b^2)x^{10} + \frac{5}{7}(B^4a^4b + 2A^3a^3b^2)x^7 + A^5a^5x + \frac{1}{4}(B^5a^5 + 5A^4a^4b)x^4$

mupad [B] time = 0.04, size = 103, normalized size = 0.94

$$x^4 \left(\frac{Ba^5}{4} + \frac{5Aba^4}{4} \right) + x^{16} \left(\frac{Ab^5}{16} + \frac{5Bab^4}{16} \right) + \frac{Bb^5x^{19}}{19} + Aa^5x + a^2b^2x^{10}(Ab + Ba) + \frac{5a^3bx^7(2Ab + Ba)}{7} + \frac{5ab^3x^{13}(Ab + 2Ba)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)*(a + b*x^3)^5,x)`

[Out] $x^4 \left(\frac{B^5a^5}{4} + \frac{5A^4a^4b}{4} \right) + x^{16} \left(\frac{A^5b^5}{16} + \frac{5B^4a^4b}{16} \right) + \frac{Bb^5x^{19}}{19} + A^5a^5x + a^2b^2x^{10}(A^4b + B^3a^3) + \frac{5a^3bx^7(2A^4b + B^3a^3)}{7} + \frac{5a^4b^3x^{13}(A^4b + 2B^3a^3)}{13}$

sympy [A] time = 0.09, size = 128, normalized size = 1.17

$$Aa^5x + \frac{Bb^5x^{19}}{19} + x^{16} \left(\frac{Ab^5}{16} + \frac{5Bab^4}{16} \right) + x^{13} \left(\frac{5Aab^4}{13} + \frac{10Ba^2b^3}{13} \right) + x^{10} (Aa^2b^3 + Ba^3b^2) + x^7 \left(\frac{10Aa^3b^2}{7} + \frac{5Ba^4b}{7} \right) + x^4 \left(\frac{5Aa^4b}{4} + \frac{Ba^5}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**5*(B*x**3+A),x)`

[Out] $A^5a^5x + \frac{Bb^5x^{19}}{19} + x^{16} \left(\frac{A^5b^5}{16} + \frac{5B^4a^4b}{16} \right) + x^{13} \left(\frac{5A^4a^4b}{13} + \frac{10B^3a^3b^3}{13} \right) + x^{10} \left(\frac{A^4a^4b}{10} + \frac{B^3a^3b^2}{10} \right) + x^7 \left(\frac{10A^3a^3b^2}{7} + \frac{5B^2a^2b^2}{7} \right) + x^4 \left(\frac{5A^4a^4b}{4} + \frac{B^5a^5}{4} \right)$

$$3.33 \quad \int \frac{(a+bx^3)^5(A+Bx^3)}{x} dx$$

Optimal. Leaf size=88

$$a^5 A \log(x) + \frac{5}{3} a^4 A b x^3 + \frac{5}{3} a^3 A b^2 x^6 + \frac{10}{9} a^2 A b^3 x^9 + \frac{5}{12} a A b^4 x^{12} + \frac{B(a+bx^3)^6}{18b} + \frac{1}{15} A b^5 x^{15}$$

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {446, 80, 43}

$$\frac{10}{9} a^2 A b^3 x^9 + \frac{5}{3} a^3 A b^2 x^6 + \frac{5}{3} a^4 A b x^3 + a^5 A \log(x) + \frac{5}{12} a A b^4 x^{12} + \frac{B(a+bx^3)^6}{18b} + \frac{1}{15} A b^5 x^{15}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x,x]

[Out] (5*a^4*A*b*x^3)/3 + (5*a^3*A*b^2*x^6)/3 + (10*a^2*A*b^3*x^9)/9 + (5*a*A*b^4*x^12)/12 + (A*b^5*x^15)/15 + (B*(a + b*x^3)^6)/(18*b) + a^5*A*Log[x]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^5 (A + Bx^3)}{x} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^5 (A + Bx)}{x} dx, x, x^3 \right) \\
&= \frac{B(a + bx^3)^6}{18b} + \frac{1}{3} A \text{Subst} \left(\int \frac{(a + bx)^5}{x} dx, x, x^3 \right) \\
&= \frac{B(a + bx^3)^6}{18b} + \frac{1}{3} A \text{Subst} \left(\int \left(5a^4b + \frac{a^5}{x} + 10a^3b^2x + 10a^2b^3x^2 + 5ab^4x^3 + b^5x^4 \right) dx, x, x^3 \right) \\
&= \frac{5}{3} a^4 A b x^3 + \frac{5}{3} a^3 A b^2 x^6 + \frac{10}{9} a^2 A b^3 x^9 + \frac{5}{12} a A b^4 x^{12} + \frac{1}{15} A b^5 x^{15} + \frac{B(a + bx^3)^6}{18b} +
\end{aligned}$$

Mathematica [A] time = 0.03, size = 113, normalized size = 1.28

$$a^5 A \log(x) + \frac{1}{3} a^4 x^3 (aB + 5Ab) + \frac{5}{6} a^3 b x^6 (aB + 2Ab) + \frac{10}{9} a^2 b^2 x^9 (aB + Ab) + \frac{1}{15} b^4 x^{15} (5aB + Ab) + \frac{5}{12} a b^3 x^{12} (2aB + Ab) + \frac{1}{18} b^5 B x^{18}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x,x]

[Out] (a^4*(5*A*b + a*B)*x^3)/3 + (5*a^3*b*(2*A*b + a*B)*x^6)/6 + (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^12)/12 + (b^4*(A*b + 5*a*B)*x^15)/15 + (b^5*B*x^18)/18 + a^5*A*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x, x]

fricas [A] time = 0.82, size = 117, normalized size = 1.33

$$\frac{1}{18} B b^5 x^{18} + \frac{1}{15} (5 B a b^4 + A b^5) x^{15} + \frac{5}{12} (2 B a^2 b^3 + A a b^4) x^{12} + \frac{10}{9} (B a^3 b^2 + A a^2 b^3) x^9 + \frac{5}{6} (B a^4 b + 2 A a^3 b^2) x^6 + A a^5 \log(x) + \frac{1}{3} (B a^5 + 5 A a^4 b) x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="fricas")

[Out] $\frac{1}{18}Bb^5x^{18} + \frac{1}{15}(5B^*a^*b^4 + A^*b^5)x^{15} + \frac{5}{12}(2B^*a^2b^3 + A^*a^*b^4)x^{12} + \frac{10}{9}(B^*a^3b^2 + A^*a^2b^3)x^9 + \frac{5}{6}(B^*a^4b + 2A^*a^3b^2)x^6 + A^*a^5\log(x) + \frac{1}{3}(B^*a^5 + 5A^*a^4b)x^3$

giac [A] time = 0.16, size = 124, normalized size = 1.41

$$\frac{1}{18}Bb^5x^{18} + \frac{1}{3}Bab^4x^{15} + \frac{1}{15}Ab^5x^{15} + \frac{5}{6}Ba^2b^3x^{12} + \frac{5}{12}Aab^4x^{12} + \frac{10}{9}Ba^3b^2x^9 + \frac{10}{9}Aa^2b^3x^9 + \frac{5}{6}Ba^4bx^6 + \frac{5}{3}Aa^3b^2x^6 + \frac{1}{3}Ba^5x^3 + \frac{5}{3}Aa^4bx^3 + Aa^5\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="giac")`

[Out] $\frac{1}{18}Bb^5x^{18} + \frac{1}{3}B^*a^*b^4x^{15} + \frac{1}{15}A^*b^5x^{15} + \frac{5}{6}B^*a^2b^3x^{12} + \frac{5}{12}A^*a^*b^4x^{12} + \frac{10}{9}B^*a^3b^2x^9 + \frac{10}{9}A^*a^2b^3x^9 + \frac{5}{6}B^*a^4bx^6 + \frac{5}{3}A^*a^3b^2x^6 + \frac{1}{3}B^*a^5x^3 + \frac{5}{3}A^*a^4bx^3 + A^*a^5\log(\text{abs}(x))$

maple [A] time = 0.04, size = 124, normalized size = 1.41

$$\frac{Bb^5x^{18}}{18} + \frac{Ab^5x^{15}}{15} + \frac{Bab^4x^{15}}{3} + \frac{5Aab^4x^{12}}{12} + \frac{5Ba^2b^3x^{12}}{6} + \frac{10Aa^2b^3x^9}{9} + \frac{10Ba^3b^2x^9}{9} + \frac{5Aa^3b^2x^6}{3} + \frac{5Ba^4bx^6}{6} + \frac{5Aa^4bx^3}{3} + \frac{Ba^5x^3}{3} + Aa^5\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^5*(B*x^3+A)/x,x)`

[Out] $\frac{1}{18}Bb^5x^{18} + \frac{1}{15}A^*b^5x^{15} + \frac{1}{3}B^*x^{15}a^*b^4 + \frac{5}{12}a^*A^*b^4x^{12} + \frac{5}{6}B^*x^{12}a^2b^3 + \frac{10}{9}a^2A^*b^3x^9 + \frac{10}{9}B^*x^9a^3b^2 + \frac{5}{3}a^3A^*b^2x^6 + \frac{5}{6}B^*x^6a^4b + \frac{5}{3}a^4A^*bx^3 + \frac{1}{3}B^*x^3a^5 + a^5A^*\ln(x)$

maxima [A] time = 0.45, size = 120, normalized size = 1.36

$$\frac{1}{18}Bb^5x^{18} + \frac{1}{15}(5Bab^4 + Ab^5)x^{15} + \frac{5}{12}(2Ba^2b^3 + Aab^4)x^{12} + \frac{10}{9}(Ba^3b^2 + Aa^2b^3)x^9 + \frac{5}{6}(Ba^4b + 2Aa^3b^2)x^6 + \frac{1}{3}Aa^5\log(x^3) + \frac{1}{3}(Ba^5 + 5Aa^4b)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="maxima")`

[Out] $\frac{1}{18}Bb^5x^{18} + \frac{1}{15}(5B^*a^*b^4 + A^*b^5)x^{15} + \frac{5}{12}(2B^*a^2b^3 + A^*a^*b^4)x^{12} + \frac{10}{9}(B^*a^3b^2 + A^*a^2b^3)x^9 + \frac{5}{6}(B^*a^4b + 2A^*a^3b^2)x^6 + \frac{1}{3}A^*a^5\log(x^3) + \frac{1}{3}(B^*a^5 + 5A^*a^4b)x^3$

mupad [B] time = 0.05, size = 105, normalized size = 1.19

$$x^3 \left(\frac{Ba^5}{3} + \frac{5Aba^4}{3} \right) + x^{15} \left(\frac{Ab^5}{15} + \frac{Bab^4}{3} \right) + \frac{Bb^5x^{18}}{18} + Aa^5\ln(x) + \frac{10a^2b^2x^9(Ab + Ba)}{9} + \frac{5a^3bx^6(2Ab + Ba)}{6} + \frac{5ab^3x^{12}(Ab + 2Ba)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^5)/x,x)`

[Out] $x^3 \cdot ((B \cdot a^5)/3 + (5 \cdot A \cdot a^4 \cdot b)/3) + x^{15} \cdot ((A \cdot b^5)/15 + (B \cdot a \cdot b^4)/3) + (B \cdot b^5 \cdot x^{18})/18 + A \cdot a^5 \cdot \log(x) + (10 \cdot a^2 \cdot b^2 \cdot x^9 \cdot (A \cdot b + B \cdot a))/9 + (5 \cdot a^3 \cdot b \cdot x^6 \cdot (2 \cdot A \cdot b + B \cdot a))/6 + (5 \cdot a \cdot b^3 \cdot x^{12} \cdot (A \cdot b + 2 \cdot B \cdot a))/12$

sympy [A] time = 0.24, size = 134, normalized size = 1.52

$$Aa^5 \log(x) + \frac{Bb^5x^{18}}{18} + x^{15} \left(\frac{Ab^5}{15} + \frac{Bab^4}{3} \right) + x^{12} \left(\frac{5Aab^4}{12} + \frac{5Ba^2b^3}{6} \right) + x^9 \left(\frac{10Aa^2b^3}{9} + \frac{10Ba^3b^2}{9} \right) + x^6 \left(\frac{5Aa^3b^2}{3} + \frac{5Ba^4b}{6} \right) + x^3 \left(\frac{5Aa^4b}{3} + \frac{Ba^5}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**5*(B*x**3+A)/x,x)`

[Out] $A \cdot a^{**5} \cdot \log(x) + B \cdot b^{**5} \cdot x^{**18}/18 + x^{**15} \cdot (A \cdot b^{**5}/15 + B \cdot a \cdot b^{**4}/3) + x^{**12} \cdot (5 \cdot A \cdot a \cdot b^{**4}/12 + 5 \cdot B \cdot a^{**2} \cdot b^{**3}/6) + x^{**9} \cdot (10 \cdot A \cdot a^{**2} \cdot b^{**3}/9 + 10 \cdot B \cdot a^{**3} \cdot b^{**2}/9) + x^{**6} \cdot (5 \cdot A \cdot a^{**3} \cdot b^{**2}/3 + 5 \cdot B \cdot a^{**4} \cdot b/6) + x^{**3} \cdot (5 \cdot A \cdot a^{**4} \cdot b/3 + B \cdot a^{**5}/3)$

$$3.34 \quad \int \frac{(a+bx^3)^5(A+Bx^3)}{x^2} dx$$

Optimal. Leaf size=112

$$-\frac{a^5A}{x} + \frac{1}{2}a^4x^2(aB+5Ab) + a^3bx^5(aB+2Ab) + \frac{5}{4}a^2b^2x^8(aB+Ab) + \frac{1}{14}b^4x^{14}(5aB+Ab) + \frac{5}{11}ab^3x^{11}(2aB+Ab) + \frac{1}{17}b^5Bx^{17}$$

Rubi [A] time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{5}{4}a^2b^2x^8(aB+Ab) + a^3bx^5(aB+2Ab) + \frac{1}{2}a^4x^2(aB+5Ab) - \frac{a^5A}{x} + \frac{1}{14}b^4x^{14}(5aB+Ab) + \frac{5}{11}ab^3x^{11}(2aB+Ab) + \frac{1}{17}b^5Bx^{17}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^2,x]

[Out] -((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^2*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5*a*B)*x^14)/14 + (b^5*B*x^17)/17

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^5(A+Bx^3)}{x^2} dx &= \int \left(\frac{a^5A}{x^2} + a^4(5Ab+aB)x + 5a^3b(2Ab+aB)x^4 + 10a^2b^2(Ab+aB)x^7 + 5ab^3(Ab+ \right. \\ &= \left. -\frac{a^5A}{x} + \frac{1}{2}a^4(5Ab+aB)x^2 + a^3b(2Ab+aB)x^5 + \frac{5}{4}a^2b^2(Ab+aB)x^8 + \frac{5}{11}ab^3(Ab+ \right. \end{aligned}$$

Mathematica [A] time = 0.03, size = 112, normalized size = 1.00

$$-\frac{a^5A}{x} + \frac{1}{2}a^4x^2(aB+5Ab) + a^3bx^5(aB+2Ab) + \frac{5}{4}a^2b^2x^8(aB+Ab) + \frac{1}{14}b^4x^{14}(5aB+Ab) + \frac{5}{11}ab^3x^{11}(2aB+Ab) + \frac{1}{17}b^5Bx^{17}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^2,x]

[Out] $-\frac{(a^5 A)}{x} + \frac{a^4(5Ab + aB)x^2}{2} + \frac{a^3 b(2Ab + aB)x^5}{2} + \frac{5a^2 b^2(Ab + aB)x^8}{4} + \frac{5a b^3(Ab + 2aB)x^{11}}{11} + \frac{b^4(Ab + 5aB)x^{14}}{14} + \frac{b^5 B x^{17}}{17}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^2,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^2, x]

fricas [A] time = 0.76, size = 121, normalized size = 1.08

$$\frac{308 B b^5 x^{18} + 374 (5 B a b^4 + A b^5) x^{15} + 2380 (2 B a^2 b^3 + A a b^4) x^{12} + 6545 (B a^3 b^2 + A a^2 b^3) x^9 + 5236 (B a^4 b + 2 A a^3 b^2) x^6 - 5236 A a^5 + 2618 (B a^5 + 5 A a^4 b) x^3}{5236 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="fricas")

[Out] $\frac{1}{5236} (308 B b^5 x^{18} + 374 (5 B a b^4 + A b^5) x^{15} + 2380 (2 B a^2 b^3 + A a b^4) x^{12} + 6545 (B a^3 b^2 + A a^2 b^3) x^9 + 5236 (B a^4 b + 2 A a^3 b^2) x^6 - 5236 A a^5 + 2618 (B a^5 + 5 A a^4 b) x^3) / x$

giac [A] time = 0.15, size = 124, normalized size = 1.11

$$\frac{1}{17} B b^5 x^{17} + \frac{5}{14} B a b^4 x^{14} + \frac{1}{14} A b^5 x^{14} + \frac{10}{11} B a^2 b^3 x^{11} + \frac{5}{11} A a b^4 x^{11} + \frac{5}{4} B a^3 b^2 x^8 + \frac{5}{4} A a^2 b^3 x^8 + B a^4 b x^5 + 2 A a^3 b^2 x^5 + \frac{1}{2} B a^5 x^2 + \frac{5}{2} A a^4 b x^2 - \frac{A a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="giac")

[Out] $\frac{1}{17} B b^5 x^{17} + \frac{5}{14} B a b^4 x^{14} + \frac{1}{14} A b^5 x^{14} + \frac{10}{11} B a^2 b^3 x^{11} + \frac{5}{11} A a b^4 x^{11} + \frac{5}{4} B a^3 b^2 x^8 + \frac{5}{4} A a^2 b^3 x^8 + B a^4 b x^5 + 2 A a^3 b^2 x^5 + \frac{1}{2} B a^5 x^2 + \frac{5}{2} A a^4 b x^2 - \frac{A a^5}{x}$

maple [A] time = 0.05, size = 125, normalized size = 1.12

$$\frac{B b^5 x^{17}}{17} + \frac{A b^5 x^{14}}{14} + \frac{5 B a b^4 x^{14}}{14} + \frac{5 A a b^4 x^{11}}{11} + \frac{10 B a^2 b^3 x^{11}}{11} + \frac{5 A a^2 b^3 x^8}{4} + \frac{5 B a^3 b^2 x^8}{4} + 2 A a^3 b^2 x^5 + B a^4 b x^5 + \frac{5 A a^4 b x^2}{2} + \frac{B a^5 x^2}{2} - \frac{A a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^2,x)

[Out] 1/17*b^5*B*x^17+1/14*A*x^14*b^5+5/14*B*x^14*a*b^4+5/11*A*x^11*a*b^4+10/11*B*x^11*a^2*b^3+5/4*A*x^8*a^2*b^3+5/4*B*x^8*a^3*b^2+2*A*x^5*a^3*b^2+B*x^5*a^4*b+5/2*A*x^2*a^4*b+1/2*B*x^2*a^5-a^5*A/x

maxima [A] time = 0.54, size = 118, normalized size = 1.05

$$\frac{1}{17} B b^5 x^{17} + \frac{1}{14} (5 B a b^4 + A b^5) x^{14} + \frac{5}{11} (2 B a^2 b^3 + A a b^4) x^{11} + \frac{5}{4} (B a^3 b^2 + A a^2 b^3) x^8 + (B a^4 b + 2 A a^3 b^2) x^5 - \frac{A a^5}{x} + \frac{1}{2} (B a^5 + 5 A a^4 b) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="maxima")

[Out] 1/17*B*b^5*x^17 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/11*(2*B*a^2*b^3 + A*a*b^4)*x^11 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + (B*a^4*b + 2*A*a^3*b^2)*x^5 - A*a^5/x + 1/2*(B*a^5 + 5*A*a^4*b)*x^2

mupad [B] time = 0.04, size = 106, normalized size = 0.95

$$x^2 \left(\frac{B a^5}{2} + \frac{5 A b a^4}{2} \right) + x^{14} \left(\frac{A b^5}{14} + \frac{5 B a b^4}{14} \right) - \frac{A a^5}{x} + \frac{B b^5 x^{17}}{17} + \frac{5 a^2 b^2 x^8 (A b + B a)}{4} + a^3 b x^5 (2 A b + B a) + \frac{5 a b^3 x^{11} (A b + 2 B a)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^2,x)

[Out] x^2*((B*a^5)/2 + (5*A*a^4*b)/2) + x^14*((A*b^5)/14 + (5*B*a*b^4)/14) - (A*a^5)/x + (B*b^5*x^17)/17 + (5*a^2*b^2*x^8*(A*b + B*a))/4 + a^3*b*x^5*(2*A*b + B*a) + (5*a*b^3*x^11*(A*b + 2*B*a))/11

sympy [A] time = 0.25, size = 129, normalized size = 1.15

$$-\frac{A a^5}{x} + \frac{B b^5 x^{17}}{17} + x^{14} \left(\frac{A b^5}{14} + \frac{5 B a b^4}{14} \right) + x^{11} \left(\frac{5 A a b^4}{11} + \frac{10 B a^2 b^3}{11} \right) + x^8 \left(\frac{5 A a^2 b^3}{4} + \frac{5 B a^3 b^2}{4} \right) + x^5 (2 A a^3 b^2 + B a^4 b) + x^2 \left(\frac{5 A a^4 b}{2} + \frac{B a^5}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**2,x)

[Out] -A*a**5/x + B*b**5*x**17/17 + x**14*(A*b**5/14 + 5*B*a*b**4/14) + x**11*(5*A*a*b**4/11 + 10*B*a**2*b**3/11) + x**8*(5*A*a**2*b**3/4 + 5*B*a**3*b**2/4) + x**5*(2*A*a**3*b**2 + B*a**4*b) + x**2*(5*A*a**4*b/2 + B*a**5/2)

$$3.35 \quad \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} dx$$

Optimal. Leaf size=112

$$-\frac{a^5 A}{2x^2} + a^4 x(aB+5Ab) + \frac{5}{4} a^3 bx^4(aB+2Ab) + \frac{10}{7} a^2 b^2 x^7(aB+Ab) + \frac{1}{13} b^4 x^{13}(5aB+Ab) + \frac{1}{2} ab^3 x^{10}(2aB+Ab) + \frac{1}{16} b^5 Bx^{16}$$

Rubi [A] time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{10}{7} a^2 b^2 x^7(aB+Ab) + \frac{5}{4} a^3 bx^4(aB+2Ab) + a^4 x(aB+5Ab) - \frac{a^5 A}{2x^2} + \frac{1}{13} b^4 x^{13}(5aB+Ab) + \frac{1}{2} ab^3 x^{10}(2aB+Ab) + \frac{1}{16} b^5 Bx^{16}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^3, x]

[Out] $-(a^5 A)/(2*x^2) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (10*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^{10})/2 + (b^4*(A*b + 5*a*B)*x^{13})/13 + (b^5*B*x^{16})/16$

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} dx &= \int \left(a^4(5Ab+aB) + \frac{a^5 A}{x^3} + 5a^3 b(2Ab+aB)x^3 + 10a^2 b^2(Ab+aB)x^6 + 5ab^3(Ab+ \right. \\ &= -\frac{a^5 A}{2x^2} + a^4(5Ab+aB)x + \frac{5}{4} a^3 b(2Ab+aB)x^4 + \frac{10}{7} a^2 b^2(Ab+aB)x^7 + \frac{1}{2} ab^3(Ab+ \end{aligned}$$

Mathematica [A] time = 0.03, size = 112, normalized size = 1.00

$$-\frac{a^5 A}{2x^2} + a^4 x(aB+5Ab) + \frac{5}{4} a^3 bx^4(aB+2Ab) + \frac{10}{7} a^2 b^2 x^7(aB+Ab) + \frac{1}{13} b^4 x^{13}(5aB+Ab) + \frac{1}{2} ab^3 x^{10}(2aB+Ab) + \frac{1}{16} b^5 Bx^{16}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^3,x]

[Out] $-\frac{1}{2} \frac{a^5 A}{x^2} + a^4 (5 A b + a B) x + \frac{5 a^3 b (2 A b + a B) x^4}{4} + (10 a^2 b^2 (A b + a B) x^7) / 7 + \frac{a b^3 (A b + 2 a B) x^{10}}{2} + \frac{b^4 (A b + 5 a B) x^{13}}{13} + \frac{b^5 B x^{16}}{16}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^3,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^3, x]

fricas [A] time = 1.03, size = 121, normalized size = 1.08

$$\frac{91 B b^5 x^{18} + 112 (5 B a b^4 + A b^5) x^{15} + 728 (2 B a^2 b^3 + A a b^4) x^{12} + 2080 (B a^3 b^2 + A a^2 b^3) x^9 + 1820 (B a^4 b + 2 A a^3 b^2) x^6 - 728 A a^5 + 1456 (B a^5 + 5 A a^4 b) x^3}{1456 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="fricas")

[Out] $\frac{1}{1456} (91 B b^5 x^{18} + 112 (5 B a b^4 + A b^5) x^{15} + 728 (2 B a^2 b^3 + A a b^4) x^{12} + 2080 (B a^3 b^2 + A a^2 b^3) x^9 + 1820 (B a^4 b + 2 A a^3 b^2) x^6 - 728 A a^5 + 1456 (B a^5 + 5 A a^4 b) x^3) / x^2$

giac [A] time = 0.16, size = 119, normalized size = 1.06

$$\frac{1}{16} B b^5 x^{16} + \frac{5}{13} B a b^4 x^{13} + \frac{1}{13} A b^5 x^{13} + B a^2 b^3 x^{10} + \frac{1}{2} A a b^4 x^{10} + \frac{10}{7} B a^3 b^2 x^7 + \frac{10}{7} A a^2 b^3 x^7 + \frac{5}{4} B a^4 b x^4 + \frac{5}{2} A a^3 b^2 x^4 + B a^5 x + 5 A a^4 b x - \frac{A a^5}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="giac")

[Out] $\frac{1}{16} B b^5 x^{16} + \frac{5}{13} B a b^4 x^{13} + \frac{1}{13} A b^5 x^{13} + B a^2 b^3 x^{10} + \frac{1}{2} A a b^4 x^{10} + \frac{10}{7} B a^3 b^2 x^7 + \frac{10}{7} A a^2 b^3 x^7 + \frac{5}{4} B a^4 b x^4 + \frac{5}{2} A a^3 b^2 x^4 + B a^5 x + 5 A a^4 b x - \frac{1}{2} \frac{A a^5}{x^2}$

maple [A] time = 0.04, size = 120, normalized size = 1.07

$$\frac{B b^5 x^{16}}{16} + \frac{A b^5 x^{13}}{13} + \frac{5 B a b^4 x^{13}}{13} + \frac{A a b^4 x^{10}}{2} + B a^2 b^3 x^{10} + \frac{10 A a^2 b^3 x^7}{7} + \frac{10 B a^3 b^2 x^7}{7} + \frac{5 A a^3 b^2 x^4}{2} + \frac{5 B a^4 b x^4}{4} + 5 A a^4 b x + B a^5 x - \frac{A a^5}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^5*(B*x^3+A)/x^3,x)`

[Out] $\frac{1}{16}b^5Bx^{16} + \frac{1}{13}Axx^{13}b^5 + \frac{5}{13}Bx^{13}ab^4 + \frac{1}{2}Axx^{10}ab^4 + Bx^{10}a^2b^3 + \frac{10}{7}Axx^7a^2b^3 + \frac{10}{7}Bx^7a^3b^2 + \frac{5}{2}Axx^4a^3b^2 + \frac{5}{4}Bx^4a^4b + 5a^4bAx + Ba^5x - \frac{1}{2}a^5A/x^2$

maxima [A] time = 0.61, size = 116, normalized size = 1.04

$$\frac{1}{16}Bb^5x^{16} + \frac{1}{13}(5Bab^4 + Ab^5)x^{13} + \frac{1}{2}(2Ba^2b^3 + Aab^4)x^{10} + \frac{10}{7}(Ba^3b^2 + Aa^2b^3)x^7 + \frac{5}{4}(Ba^4b + 2Aa^3b^2)x^4 - \frac{Aa^5}{2x^2} + (Ba^5 + 5Aa^4b)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="maxima")`

[Out] $\frac{1}{16}Bb^5x^{16} + \frac{1}{13}(5Bab^4 + Ab^5)x^{13} + \frac{1}{2}(2Bab^3 + Aab^4)x^{10} + \frac{10}{7}(Ba^3b^2 + Aa^2b^3)x^7 + \frac{5}{4}(Ba^4b + 2Aa^3b^2)x^4 - \frac{1}{2}Aa^5/x^2 + (Ba^5 + 5Aa^4b)x$

mupad [B] time = 0.04, size = 104, normalized size = 0.93

$$x(Ba^5 + 5Aab^4) + x^{13}\left(\frac{Ab^5}{13} + \frac{5Bab^4}{13}\right) - \frac{Aa^5}{2x^2} + \frac{Bb^5x^{16}}{16} + \frac{10a^2b^2x^7(Ab + Ba)}{7} + \frac{5a^3bx^4(2Ab + Ba)}{4} + \frac{ab^3x^{10}(Ab + 2Ba)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^5)/x^3,x)`

[Out] $x(Ba^5 + 5Aa^4b) + x^{13}\left(\frac{Ab^5}{13} + \frac{5Bab^4}{13}\right) - \frac{Aa^5}{(2x^2)} + \frac{Bb^5x^{16}}{16} + \frac{(10a^2b^2x^7(Ab + Ba))}{7} + \frac{(5a^3bx^4(2Ab + Ba + Ba))}{4} + \frac{(ab^3x^{10}(Ab + 2Ba))}{2}$

sympy [A] time = 0.24, size = 128, normalized size = 1.14

$$-\frac{Aa^5}{2x^2} + \frac{Bb^5x^{16}}{16} + x^{13}\left(\frac{Ab^5}{13} + \frac{5Bab^4}{13}\right) + x^{10}\left(\frac{Aab^4}{2} + Ba^2b^3\right) + x^7\left(\frac{10Aa^2b^3}{7} + \frac{10Ba^3b^2}{7}\right) + x^4\left(\frac{5Aa^3b^2}{2} + \frac{5Ba^4b}{4}\right) + x(5Aa^4b + Ba^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**5*(B*x**3+A)/x**3,x)`

[Out] $-Aa^{**5}/(2x^{**2}) + Bb^{**5}x^{**16}/16 + x^{**13}(Ab^{**5}/13 + 5Bab^{**4}/13) + x^{**10}(Aab^{**4}/2 + Baa^{**2}b^{**3}) + x^{**7}(10Aa^{**2}b^{**3}/7 + 10Baa^{**3}b^{**2}/7) + x^{**4}(5Aa^{**3}b^{**2}/2 + 5Baa^{**4}b/4) + x(5Aa^{**4}b + Baa^{**5})$

$$3.36 \quad \int \frac{(a+bx^3)^5(A+Bx^3)}{x^4} dx$$

Optimal. Leaf size=113

$$-\frac{a^5A}{3x^3} + a^4 \log(x)(aB+5Ab) + \frac{5}{3}a^3bx^3(aB+2Ab) + \frac{5}{3}a^2b^2x^6(aB+Ab) + \frac{1}{12}b^4x^{12}(5aB+Ab) + \frac{5}{9}ab^3x^9(2aB+Ab) + \frac{1}{15}b^5Bx^{15}$$

Rubi [A] time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 76}

$$\frac{5}{3}a^2b^2x^6(aB+Ab) + \frac{5}{3}a^3bx^3(aB+2Ab) + a^4 \log(x)(aB+5Ab) - \frac{a^5A}{3x^3} + \frac{1}{12}b^4x^{12}(5aB+Ab) + \frac{5}{9}ab^3x^9(2aB+Ab) + \frac{1}{15}b^5Bx^{15}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^4, x]

[Out] -(a^5*A)/(3*x^3) + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*x^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^5*B*x^15)/15 + a^4*(5*A*b + a*B)*Log[x]

Rule 76

Int[((d_)*(x_))^(n_)*((a_) + (b_)*(x_))*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^5 (A + Bx^3)}{x^4} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^5 (A + Bx)}{x^2} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(5a^3b(2Ab + aB) + \frac{a^5A}{x^2} + \frac{a^4(5Ab + aB)}{x} + 10a^2b^2(Ab + aB)x + 5ab^3(Ab + aB)x^2 \right) dx, x, x^3 \right) \\ &= -\frac{a^5A}{3x^3} + \frac{5}{3}a^3b(2Ab + aB)x^3 + \frac{5}{3}a^2b^2(Ab + aB)x^6 + \frac{5}{9}ab^3(Ab + 2aB)x^9 + \frac{1}{12}b^4(Ab + aB)x^{12} + \frac{1}{15}b^5Bx^{15} \end{aligned}$$

Mathematica [A] time = 0.04, size = 115, normalized size = 1.02

$$-\frac{a^5A}{3x^3} + \frac{5}{3}a^3bx^3(aB + 2Ab) + \frac{5}{3}a^2b^2x^6(aB + Ab) + \log(x)(a^5B + 5a^4Ab) + \frac{1}{12}b^4x^{12}(5aB + Ab) + \frac{5}{9}ab^3x^9(2aB + Ab) + \frac{1}{15}b^5Bx^{15}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^4, x]

[Out] -1/3*(a^5*A)/x^3 + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*x^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^5*B*x^15)/15 + (5*a^4*A*b + a^5*B)*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^4, x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^4, x]

fricas [A] time = 0.75, size = 123, normalized size = 1.09

$$\frac{12 Bb^5x^{18} + 15(5 Bab^4 + Ab^5)x^{15} + 100(2 Ba^2b^3 + Aab^4)x^{12} + 300(Ba^3b^2 + Aa^2b^3)x^9 + 300(Ba^4b + 2 Aa^3b^2)x^6 - 60 Aa^5 + 180(Ba^5 + 5 Aa^4b)x^3 \log(x)}{180x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4, x, algorithm="fricas")

[Out] 1/180*(12*B*b^5*x^18 + 15*(5*B*a*b^4 + A*b^5)*x^15 + 100*(2*B*a^2*b^3 + A*a*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 300*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 60*A*a^5 + 180*(B*a^5 + 5*A*a^4*b)*x^3*log(x))/x^3

giac [A] time = 0.16, size = 143, normalized size = 1.27

$$\frac{1}{15} B b^5 x^{15} + \frac{5}{12} B a b^4 x^{12} + \frac{1}{12} A b^5 x^{12} + \frac{10}{9} B a^2 b^3 x^9 + \frac{5}{9} A a b^4 x^9 + \frac{5}{3} B a^3 b^2 x^6 + \frac{5}{3} A a^2 b^3 x^6 + \frac{5}{3} B a^4 b x^3 + \frac{10}{3} A a^3 b^2 x^3 + (B a^5 + 5 A a^4 b) \log(|x|) - \frac{B a^5 x^3 + 5 A a^4 b x^3 + A a^5}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="giac")

$$\begin{aligned} \text{[Out]} & \frac{1}{15} B b^5 x^{15} + \frac{5}{12} B a b^4 x^{12} + \frac{1}{12} A b^5 x^{12} + \frac{10}{9} B a^2 b^3 x^9 \\ & + \frac{5}{9} A a b^4 x^9 + \frac{5}{3} B a^3 b^2 x^6 + \frac{5}{3} A a^2 b^3 x^6 + \frac{5}{3} B a^4 b x^3 \\ & + \frac{10}{3} A a^3 b^2 x^3 + (B a^5 + 5 A a^4 b) \log(\text{abs}(x)) - \frac{1}{3} (B a^5 x^3 + 5 A a^4 b x^3 + A a^5) / x^3 \end{aligned}$$

maple [A] time = 0.05, size = 123, normalized size = 1.09

$$\frac{B b^5 x^{15}}{15} + \frac{A b^5 x^{12}}{12} + \frac{5 B a b^4 x^{12}}{12} + \frac{5 A a b^4 x^9}{9} + \frac{10 B a^2 b^3 x^9}{9} + \frac{5 A a^2 b^3 x^6}{3} + \frac{5 B a^3 b^2 x^6}{3} + \frac{10 A a^3 b^2 x^3}{3} + \frac{5 B a^4 b x^3}{3} + 5 A a^4 b \ln(x) + B a^5 \ln(x) - \frac{A a^5}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^4,x)

$$\begin{aligned} \text{[Out]} & \frac{1}{15} b^5 B x^{15} + \frac{1}{12} A x^{12} b^5 + \frac{5}{12} B x^{12} a b^4 + \frac{5}{9} A x^9 a b^4 + \frac{10}{9} B x^9 \\ & a^2 b^3 + \frac{5}{3} A x^6 a^2 b^3 + \frac{5}{3} B x^6 a^3 b^2 + \frac{10}{3} A x^3 a^3 b^2 + \frac{5}{3} B x^3 a^4 b \\ & - \frac{1}{3} a^5 B / x^3 + 5 A \ln(x) a^4 b + B \ln(x) a^5 \end{aligned}$$

maxima [A] time = 0.55, size = 120, normalized size = 1.06

$$\frac{1}{15} B b^5 x^{15} + \frac{1}{12} (5 B a b^4 + A b^5) x^{12} + \frac{5}{9} (2 B a^2 b^3 + A a b^4) x^9 + \frac{5}{3} (B a^3 b^2 + A a^2 b^3) x^6 + \frac{5}{3} (B a^4 b + 2 A a^3 b^2) x^3 - \frac{A a^5}{3 x^3} + \frac{1}{3} (B a^5 + 5 A a^4 b) \log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="maxima")

$$\begin{aligned} \text{[Out]} & \frac{1}{15} B b^5 x^{15} + \frac{1}{12} (5 B a b^4 + A b^5) x^{12} + \frac{5}{9} (2 B a^2 b^3 + A a b^4) x^9 \\ & + \frac{5}{3} (B a^3 b^2 + A a^2 b^3) x^6 + \frac{5}{3} (B a^4 b + 2 A a^3 b^2) x^3 \\ & - \frac{1}{3} A a^5 / x^3 + \frac{1}{3} (B a^5 + 5 A a^4 b) \log(x^3) \end{aligned}$$

mupad [B] time = 2.35, size = 105, normalized size = 0.93

$$x^{12} \left(\frac{A b^5}{12} + \frac{5 B a b^4}{12} \right) + \ln(x) (B a^5 + 5 A a b^4) - \frac{A a^5}{3 x^3} + \frac{B b^5 x^{15}}{15} + \frac{5 a^2 b^2 x^6 (A b + B a)}{3} + \frac{5 a^3 b x^3 (2 A b + B a)}{3} + \frac{5 a b^3 x^9 (A b + 2 B a)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^4,x)

$$\begin{aligned} \text{[Out]} & x^{12} \left(\frac{A b^5}{12} + \frac{5 B a b^4}{12} \right) + \log(x) (B a^5 + 5 A a^4 b) - \frac{A a^5}{3 x^3} \\ & + \frac{B b^5 x^{15}}{15} + \frac{5 a^2 b^2 x^6 (A b + B a)}{3} + \frac{5 a^3 b x^3 (2 A b + B a)}{3} + \frac{5 a b^3 x^9 (A b + 2 B a)}{9} \end{aligned}$$

sympy [A] time = 0.37, size = 133, normalized size = 1.18

$$-\frac{Aa^5}{3x^3} + \frac{Bb^5x^{15}}{15} + a^4(5Ab + Ba)\log(x) + x^{12}\left(\frac{Ab^5}{12} + \frac{5Bab^4}{12}\right) + x^9\left(\frac{5Aab^4}{9} + \frac{10Ba^2b^3}{9}\right) + x^6\left(\frac{5Aa^2b^3}{3} + \frac{5Ba^3b^2}{3}\right) + x^3\left(\frac{10Aa^3b^2}{3} + \frac{5Ba^4b}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**4,x)

[Out] $-A*a**5/(3*x**3) + B*b**5*x**15/15 + a**4*(5*A*b + B*a)*\log(x) + x**12*(A*b**5/12 + 5*B*a*b**4/12) + x**9*(5*A*a*b**4/9 + 10*B*a**2*b**3/9) + x**6*(5*A*a**2*b**3/3 + 5*B*a**3*b**2/3) + x**3*(10*A*a**3*b**2/3 + 5*B*a**4*b/3)$

$$3.37 \quad \int \frac{(a+bx^3)^5(A+Bx^3)}{x^5} dx$$

Optimal. Leaf size=113

$$-\frac{a^5A}{4x^4} - \frac{a^4(aB+5Ab)}{x} + \frac{5}{2}a^3bx^2(aB+2Ab) + 2a^2b^2x^5(aB+Ab) + \frac{1}{11}b^4x^{11}(5aB+Ab) + \frac{5}{8}ab^3x^8(2aB+Ab) + \frac{1}{14}b^5Bx^{14}$$

Rubi [A] time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$2a^2b^2x^5(aB+Ab) + \frac{5}{2}a^3bx^2(aB+2Ab) - \frac{a^4(aB+5Ab)}{x} - \frac{a^5A}{4x^4} + \frac{1}{11}b^4x^{11}(5aB+Ab) + \frac{5}{8}ab^3x^8(2aB+Ab) + \frac{1}{14}b^5Bx^{14}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^5,x]

[Out] -(a^5*A)/(4*x^4) - (a^4*(5*A*b + a*B))/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 + 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*a*B)*x^11)/11 + (b^5*B*x^14)/14

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^5(A+Bx^3)}{x^5} dx &= \int \left(\frac{a^5A}{x^5} + \frac{a^4(5Ab+aB)}{x^2} + 5a^3b(2Ab+aB)x + 10a^2b^2(Ab+aB)x^4 + 5ab^3(Ab+2aB)x^7 + \frac{1}{11}b^4x^{11}(5aB+Ab) + \frac{5}{8}ab^3x^8(2aB+Ab) + \frac{1}{14}b^5Bx^{14} \right) dx \\ &= -\frac{a^5A}{4x^4} - \frac{a^4(5Ab+aB)}{x} + \frac{5}{2}a^3b(2Ab+aB)x^2 + 2a^2b^2(Ab+aB)x^5 + \frac{5}{8}ab^3(Ab+2aB)x^8 + \frac{1}{11}b^4x^{11}(5aB+Ab) + \frac{1}{14}b^5Bx^{14} \end{aligned}$$

Mathematica [A] time = 0.06, size = 115, normalized size = 1.02

$$-\frac{a^5A}{4x^4} + \frac{5}{2}a^3bx^2(aB+2Ab) + 2a^2b^2x^5(aB+Ab) + \frac{a^5(-B) - 5a^4Ab}{x} + \frac{1}{11}b^4x^{11}(5aB+Ab) + \frac{5}{8}ab^3x^8(2aB+Ab) + \frac{1}{14}b^5Bx^{14}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^5,x]

[Out] $-1/4*(a^5*A)/x^4 + (-5*a^4*A*b - a^5*B)/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 + 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*a*B)*x^{11})/11 + (b^5*B*x^{14})/14$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^5,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^5, x]

fricas [A] time = 0.73, size = 121, normalized size = 1.07

$$\frac{44 B b^5 x^{18} + 56 (5 B a b^4 + A b^5) x^{15} + 385 (2 B a^2 b^3 + A a b^4) x^{12} + 1232 (B a^3 b^2 + A a^2 b^3) x^9 + 1540 (B a^4 b + 2 A a^3 b^2) x^6 - 154 A a^5 - 616 (B a^5 + 5 A a^4 b) x^3}{616 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="fricas")

[Out] $1/616*(44*B*b^5*x^{18} + 56*(5*B*a*b^4 + A*b^5)*x^{15} + 385*(2*B*a^2*b^3 + A*a*b^4)*x^{12} + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1540*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 154*A*a^5 - 616*(B*a^5 + 5*A*a^4*b)*x^3)/x^4$

giac [A] time = 0.16, size = 127, normalized size = 1.12

$$\frac{1}{14} B b^5 x^{14} + \frac{5}{11} B a b^4 x^{11} + \frac{1}{11} A b^5 x^{11} + \frac{5}{4} B a^2 b^3 x^8 + \frac{5}{8} A a b^4 x^8 + 2 B a^3 b^2 x^5 + 2 A a^2 b^3 x^5 + \frac{5}{2} B a^4 b x^2 + 5 A a^3 b^2 x^2 - \frac{4 B a^5 x^3 + 20 A a^4 b x^3 + A a^5}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="giac")

[Out] $1/14*B*b^5*x^{14} + 5/11*B*a*b^4*x^{11} + 1/11*A*b^5*x^{11} + 5/4*B*a^2*b^3*x^8 + 5/8*A*a*b^4*x^8 + 2*B*a^3*b^2*x^5 + 2*A*a^2*b^3*x^5 + 5/2*B*a^4*b*x^2 + 5*A*a^3*b^2*x^2 - 1/4*(4*B*a^5*x^3 + 20*A*a^4*b*x^3 + A*a^5)/x^4$

maple [A] time = 0.05, size = 123, normalized size = 1.09

$$\frac{B b^5 x^{14}}{14} + \frac{A b^5 x^{11}}{11} + \frac{5 B a b^4 x^{11}}{11} + \frac{5 A a b^4 x^8}{8} + \frac{5 B a^2 b^3 x^8}{4} + 2 A a^2 b^3 x^5 + 2 B a^3 b^2 x^5 + 5 A a^3 b^2 x^2 + \frac{5 B a^4 b x^2}{2} - \frac{(5 A b + B a) a^4}{x} - \frac{A a^5}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^5,x)

[Out] 1/14*b^5*B*x^14+1/11*A*x^11*b^5+5/11*B*x^11*a*b^4+5/8*A*x^8*a*b^4+5/4*B*x^8*a^2*b^3+2*A*x^5*a^2*b^3+2*B*x^5*a^3*b^2+5*A*x^2*a^3*b^2+5/2*B*x^2*a^4*b-1/4*a^5*A/x^4-a^4*(5*A*b+B*a)/x

maxima [A] time = 0.49, size = 121, normalized size = 1.07

$$\frac{1}{14} B b^5 x^{14} + \frac{1}{11} (5 B a b^4 + A b^5) x^{11} + \frac{5}{8} (2 B a^2 b^3 + A a b^4) x^8 + 2 (B a^3 b^2 + A a^2 b^3) x^5 + \frac{5}{2} (B a^4 b + 2 A a^3 b^2) x^2 - \frac{A a^5 + 4 (B a^5 + 5 A a^4 b) x^3}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="maxima")

[Out] 1/14*B*b^5*x^14 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/8*(2*B*a^2*b^3 + A*a*b^4)*x^8 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 + 5/2*(B*a^4*b + 2*A*a^3*b^2)*x^2 - 1/4*(A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^4

mupad [B] time = 0.04, size = 109, normalized size = 0.96

$$x^{11} \left(\frac{A b^5}{11} + \frac{5 B a b^4}{11} \right) - \frac{A a^5 + x^3 (B a^5 + 5 A a b^4)}{x^4} + \frac{B b^5 x^{14}}{14} + 2 a^2 b^2 x^5 (A b + B a) + \frac{5 a^3 b x^2 (2 A b + B a)}{2} + \frac{5 a b^3 x^8 (A b + 2 B a)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^5,x)

[Out] x^11*((A*b^5)/11 + (5*B*a*b^4)/11) - ((A*a^5)/4 + x^3*(B*a^5 + 5*A*a^4*b))/x^4 + (B*b^5*x^14)/14 + 2*a^2*b^2*x^5*(A*b + B*a) + (5*a^3*b*x^2*(2*A*b + B*a))/2 + (5*a*b^3*x^8*(A*b + 2*B*a))/8

sympy [A] time = 0.42, size = 133, normalized size = 1.18

$$\frac{B b^5 x^{14}}{14} + x^{11} \left(\frac{A b^5}{11} + \frac{5 B a b^4}{11} \right) + x^8 \left(\frac{5 A a b^4}{8} + \frac{5 B a^2 b^3}{4} \right) + x^5 (2 A a^2 b^3 + 2 B a^3 b^2) + x^2 \left(5 A a^3 b^2 + \frac{5 B a^4 b}{2} \right) + \frac{-A a^5 + x^3 (-20 A a^4 b - 4 B a^5)}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**5,x)

[Out] B*b**5*x**14/14 + x**11*(A*b**5/11 + 5*B*a*b**4/11) + x**8*(5*A*a*b**4/8 + 5*B*a**2*b**3/4) + x**5*(2*A*a**2*b**3 + 2*B*a**3*b**2) + x**2*(5*A*a**3*b**2 + 5*B*a**4*b/2) + (-A*a**5 + x**3*(-20*A*a**4*b - 4*B*a**5))/(4*x**4)

$$3.38 \quad \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} dx$$

Optimal. Leaf size=113

$$-\frac{a^5 A}{5x^5} - \frac{a^4(aB+5Ab)}{2x^2} + 5a^3bx(aB+2Ab) + \frac{5}{2}a^2b^2x^4(aB+Ab) + \frac{1}{10}b^4x^{10}(5aB+Ab) + \frac{5}{7}ab^3x^7(2aB+Ab) + \frac{1}{13}b^5Bx^{13}$$

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{5}{2}a^2b^2x^4(aB+Ab) - \frac{a^4(aB+5Ab)}{2x^2} + 5a^3bx(aB+2Ab) - \frac{a^5A}{5x^5} + \frac{1}{10}b^4x^{10}(5aB+Ab) + \frac{5}{7}ab^3x^7(2aB+Ab) + \frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^6, x]

[Out] -(a^5*A)/(5*x^5) - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x + (5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b + 5*a*B)*x^10)/10 + (b^5*B*x^13)/13

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} dx &= \int \left(5a^3b(2Ab+aB) + \frac{a^5A}{x^6} + \frac{a^4(5Ab+aB)}{x^3} + 10a^2b^2(Ab+aB)x^3 + 5ab^3(Ab+2aB)x^7 + \frac{1}{13}b^5Bx^{13} \right) dx \\ &= -\frac{a^5A}{5x^5} - \frac{a^4(5Ab+aB)}{2x^2} + 5a^3b(2Ab+aB)x + \frac{5}{2}a^2b^2(Ab+aB)x^4 + \frac{5}{7}ab^3(Ab+2aB)x^7 + \frac{1}{13}b^5Bx^{13} \end{aligned}$$

Mathematica [A] time = 0.06, size = 113, normalized size = 1.00

$$-\frac{a^5A}{5x^5} - \frac{a^4(aB+5Ab)}{2x^2} + 5a^3bx(aB+2Ab) + \frac{5}{2}a^2b^2x^4(aB+Ab) + \frac{1}{10}b^4x^{10}(5aB+Ab) + \frac{5}{7}ab^3x^7(2aB+Ab) + \frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^6,x]

[Out] $-\frac{1}{5} \frac{a^5 A}{x^5} - \frac{a^4 (5 A b + a B)}{(2 x^2)} + 5 a^3 b (2 A b + a B) x + \frac{5 a^2 b^2 (A b + a B) x^4}{2} + \frac{5 a b^3 (A b + 2 a B) x^7}{7} + \frac{b^4 (A b + 5 a B) x^{10}}{10} + \frac{b^5 B x^{13}}{13}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^6,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^6, x]

fricas [A] time = 0.53, size = 121, normalized size = 1.07

$$\frac{70 B b^5 x^{18} + 91 (5 B a b^4 + A b^5) x^{15} + 650 (2 B a^2 b^3 + A a b^4) x^{12} + 2275 (B a^3 b^2 + A a^2 b^3) x^9 + 4550 (B a^4 b + 2 A a^3 b^2) x^6 - 182 A a^5 - 455 (B a^5 + 5 A a^4 b) x^3}{910 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="fricas")

[Out] $\frac{1}{910} (70 B b^5 x^{18} + 91 (5 B a b^4 + A b^5) x^{15} + 650 (2 B a^2 b^3 + A a b^4) x^{12} + 2275 (B a^3 b^2 + A a^2 b^3) x^9 + 4550 (B a^4 b + 2 A a^3 b^2) x^6 - 182 A a^5 - 455 (B a^5 + 5 A a^4 b) x^3) / x^5$

giac [A] time = 0.18, size = 124, normalized size = 1.10

$$\frac{1}{13} B b^5 x^{13} + \frac{1}{2} B a b^4 x^{10} + \frac{1}{10} A b^5 x^{10} + \frac{10}{7} B a^2 b^3 x^7 + \frac{5}{7} A a b^4 x^7 + \frac{5}{2} B a^3 b^2 x^4 + \frac{5}{2} A a^2 b^3 x^4 + 5 B a^4 b x + 10 A a^3 b^2 x - \frac{5 B a^5 x^3 + 25 A a^4 b x^3 + 2 A a^5}{10 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="giac")

[Out] $\frac{1}{13} B b^5 x^{13} + \frac{1}{2} B a b^4 x^{10} + \frac{1}{10} A b^5 x^{10} + \frac{10}{7} B a^2 b^3 x^7 + \frac{5}{7} A a b^4 x^7 + \frac{5}{2} B a^3 b^2 x^4 + \frac{5}{2} A a^2 b^3 x^4 + 5 B a^4 b x + 10 A a^3 b^2 x - \frac{1}{10} (5 B a^5 x^3 + 25 A a^4 b x^3 + 2 A a^5) / x^5$

maple [A] time = 0.04, size = 119, normalized size = 1.05

$$\frac{B b^5 x^{13}}{13} + \frac{A b^5 x^{10}}{10} + \frac{B a b^4 x^{10}}{2} + \frac{5 A a b^4 x^7}{7} + \frac{10 B a^2 b^3 x^7}{7} + \frac{5 A a^2 b^3 x^4}{2} + \frac{5 B a^3 b^2 x^4}{2} + 10 A a^3 b^2 x + 5 B a^4 b x - \frac{(5 A b + B a) a^4}{2 x^2} - \frac{A a^5}{5 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^6,x)

[Out] 1/13*b^5*B*x^13+1/10*A*x^10*b^5+1/2*B*x^10*a*b^4+5/7*A*x^7*a*b^4+10/7*B*x^7*a^2*b^3+5/2*A*x^4*a^2*b^3+5/2*B*x^4*a^3*b^2+10*A*a^3*b^2*x+5*B*a^4*b*x-1/5*a^5*A/x^5-1/2*a^4*(5*A*b+B*a)/x^2

maxima [A] time = 0.73, size = 120, normalized size = 1.06

$$\frac{1}{13}Bb^5x^{13} + \frac{1}{10}(5Bab^4 + Ab^5)x^{10} + \frac{5}{7}(2Ba^2b^3 + Aab^4)x^7 + \frac{5}{2}(Ba^3b^2 + Aa^2b^3)x^4 + 5(Ba^4b + 2Aa^3b^2)x - \frac{2Aa^5 + 5(Ba^5 + 5Aa^4b)x^3}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="maxima")

[Out] 1/13*B*b^5*x^13 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/7*(2*B*a^2*b^3 + A*a*b^4)*x^7 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 + 5*(B*a^4*b + 2*A*a^3*b^2)*x - 1/10*(2*A*a^5 + 5*(B*a^5 + 5*A*a^4*b))*x^3/x^5

mupad [B] time = 0.04, size = 108, normalized size = 0.96

$$x^{10} \left(\frac{Ab^5}{10} + \frac{Bab^4}{2} \right) - \frac{Aa^5}{5} + x^3 \left(\frac{Ba^5}{2} + \frac{5Aab^4}{2} \right) + \frac{Bb^5x^{13}}{13} + \frac{5a^2b^2x^4(Ab+Ba)}{2} + 5a^3bx(2Ab+Ba) + \frac{5ab^3x^7(Ab+2Ba)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^6,x)

[Out] x^10*((A*b^5)/10 + (B*a*b^4)/2) - ((A*a^5)/5 + x^3*((B*a^5)/2 + (5*A*a^4*b)/2))/x^5 + (B*b^5*x^13)/13 + (5*a^2*b^2*x^4*(A*b + B*a))/2 + 5*a^3*b*x*(2*A*b + B*a) + (5*a*b^3*x^7*(A*b + 2*B*a))/7

sympy [A] time = 0.48, size = 133, normalized size = 1.18

$$\frac{Bb^5x^{13}}{13} + x^{10} \left(\frac{Ab^5}{10} + \frac{Bab^4}{2} \right) + x^7 \left(\frac{5Aab^4}{7} + \frac{10Ba^2b^3}{7} \right) + x^4 \left(\frac{5Aa^2b^3}{2} + \frac{5Ba^3b^2}{2} \right) + x(10Aa^3b^2 + 5Ba^4b) + \frac{-2Aa^5 + x^3(-25Aa^4b - 5Ba^5)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**6,x)

[Out] B*b**5*x**13/13 + x**10*(A*b**5/10 + B*a*b**4/2) + x**7*(5*A*a*b**4/7 + 10*B*a**2*b**3/7) + x**4*(5*A*a**2*b**3/2 + 5*B*a**3*b**2/2) + x*(10*A*a**3*b**2 + 5*B*a**4*b) + (-2*A*a**5 + x**3*(-25*A*a**4*b - 5*B*a**5))/(10*x**5)

$$3.39 \quad \int \frac{(a+bx^3)^5(A+Bx^3)}{x^7} dx$$

Optimal. Leaf size=114

$$-\frac{a^5A}{6x^6} - \frac{a^4(aB+5Ab)}{3x^3} + 5a^3b \log(x)(aB+2Ab) + \frac{10}{3}a^2b^2x^3(aB+Ab) + \frac{1}{9}b^4x^9(5aB+Ab) + \frac{5}{6}ab^3x^6(2aB+Ab) + \frac{1}{12}b^5Bx^{12}$$

Rubi [A] time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 76}

$$\frac{10}{3}a^2b^2x^3(aB+Ab) - \frac{a^4(aB+5Ab)}{3x^3} + 5a^3b \log(x)(aB+2Ab) - \frac{a^5A}{6x^6} + \frac{1}{9}b^4x^9(5aB+Ab) + \frac{5}{6}ab^3x^6(2aB+Ab) + \frac{1}{12}b^5Bx^{12}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^7, x]

[Out] -(a^5*A)/(6*x^6) - (a^4*(5*A*b + a*B))/(3*x^3) + (10*a^2*b^2*(A*b + a*B)*x^3)/3 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^9)/9 + (b^5*B*x^12)/12 + 5*a^3*b*(2*A*b + a*B)*Log[x]

Rule 76

Int[((d_)*(x_))^(n_)*((a_) + (b_)*(x_))*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^5 (A + Bx^3)}{x^7} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^5 (A + Bx)}{x^3} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(10a^2b^2(Ab + aB) + \frac{a^5A}{x^3} + \frac{a^4(5Ab + aB)}{x^2} + \frac{5a^3b(2Ab + aB)}{x} + 5ab^2 \right) dx, x, x^3 \right) \\ &= -\frac{a^5A}{6x^6} - \frac{a^4(5Ab + aB)}{3x^3} + \frac{10}{3}a^2b^2(Ab + aB)x^3 + \frac{5}{6}ab^3(Ab + 2aB)x^6 + \frac{1}{9}b^4(Ab + 5a^2) \end{aligned}$$

Mathematica [A] time = 0.06, size = 106, normalized size = 0.93

$$\frac{1}{36} \left(-\frac{6a^5A}{x^6} - \frac{12a^4(aB + 5Ab)}{x^3} + 180a^3b \log(x)(aB + 2Ab) + 120a^2b^2x^3(aB + Ab) + 4b^4x^9(5aB + Ab) + 30ab^3x^6(2aB + Ab) + 3b^5Bx^{12} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^7, x]

[Out] ((-6*a^5*A)/x^6 - (12*a^4*(5*A*b + a*B))/x^3 + 120*a^2*b^2*(A*b + a*B)*x^3 + 30*a*b^3*(A*b + 2*a*B)*x^6 + 4*b^4*(A*b + 5*a*B)*x^9 + 3*b^5*B*x^12 + 180*a^3*b*(2*A*b + a*B)*Log[x])/36

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^7} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^7, x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^7, x]

fricas [A] time = 0.77, size = 123, normalized size = 1.08

$$\frac{3Bb^5x^{18} + 4(5Bab^4 + Ab^5)x^{15} + 30(2Ba^2b^3 + Aab^4)x^{12} + 120(Ba^3b^2 + Aa^2b^3)x^9 + 180(Ba^4b + 2Aa^3b^2)x^6 \log(x) - 6Aa^5 - 12(Ba^5 + 5Aa^4b)x^3}{36x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7, x, algorithm="fricas")

[Out] 1/36*(3*B*b^5*x^18 + 4*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^4)*x^12 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 180*(B*a^4*b + 2*A*a^3*b^2)*x^6*log(x) - 6*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^6

giac [A] time = 0.16, size = 148, normalized size = 1.30

$$\frac{1}{12} B b^5 x^{12} + \frac{5}{9} B a b^4 x^9 + \frac{1}{9} A b^5 x^9 + \frac{5}{3} B a^2 b^3 x^6 + \frac{5}{6} A a b^4 x^6 + \frac{10}{3} B a^3 b^2 x^3 + \frac{10}{3} A a^2 b^3 x^3 + 5 (B a^4 b + 2 A a^3 b^2) \log(|x|) - \frac{15 B a^4 b x^6 + 30 A a^3 b^2 x^6 + 2 B a^5 x^3 + 10 A a^4 b x^3 + A a^5}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="giac")

[Out] $\frac{1}{12} B b^5 x^{12} + \frac{5}{9} B a b^4 x^9 + \frac{1}{9} A b^5 x^9 + \frac{5}{3} B a^2 b^3 x^6 + \frac{5}{6} A a b^4 x^6 + \frac{10}{3} B a^3 b^2 x^3 + \frac{10}{3} A a^2 b^3 x^3 + 5 (B a^4 b + 2 A a^3 b^2) \log(\text{abs}(x)) - \frac{1}{6} (15 B a^4 b x^6 + 30 A a^3 b^2 x^6 + 2 B a^5 x^3 + 10 A a^4 b x^3 + A a^5) / x^6$

maple [A] time = 0.05, size = 124, normalized size = 1.09

$$\frac{B b^5 x^{12}}{12} + \frac{A b^5 x^9}{9} + \frac{5 B a b^4 x^9}{9} + \frac{5 A a b^4 x^6}{6} + \frac{5 B a^2 b^3 x^6}{3} + \frac{10 A a^2 b^3 x^3}{3} + \frac{10 B a^3 b^2 x^3}{3} + 10 A a^3 b^2 \ln(x) + 5 B a^4 b \ln(x) - \frac{5 A a^4 b}{3 x^3} - \frac{B a^5}{3 x^3} - \frac{A a^5}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^7,x)

[Out] $\frac{1}{12} b^5 B x^{12} + \frac{1}{9} A x^9 b^5 + \frac{5}{9} B x^9 a b^4 + \frac{5}{6} A x^6 a b^4 + \frac{5}{3} B x^6 a^2 b^3 + \frac{10}{3} A x^3 a^2 b^3 + \frac{10}{3} B x^3 a^3 b^2 - \frac{5}{3} a^4 \ln(x) - \frac{1}{3} a^5 \ln(x) - \frac{1}{6} a^5 B x^6 + 10 A \ln(x) a^3 b^2 + 5 B \ln(x) a^4 b$

maxima [A] time = 0.65, size = 122, normalized size = 1.07

$$\frac{1}{12} B b^5 x^{12} + \frac{1}{9} (5 B a b^4 + A b^5) x^9 + \frac{5}{6} (2 B a^2 b^3 + A a b^4) x^6 + \frac{10}{3} (B a^3 b^2 + A a^2 b^3) x^3 + \frac{5}{3} (B a^4 b + 2 A a^3 b^2) \log(x^3) - \frac{A a^5 + 2 (B a^5 + 5 A a^4 b) x^3}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="maxima")

[Out] $\frac{1}{12} B b^5 x^{12} + \frac{1}{9} (5 B a b^4 + A b^5) x^9 + \frac{5}{6} (2 B a^2 b^3 + A a b^4) x^6 + \frac{10}{3} (B a^3 b^2 + A a^2 b^3) x^3 + \frac{5}{3} (B a^4 b + 2 A a^3 b^2) \log(x^3) - \frac{1}{6} (A a^5 + 2 (B a^5 + 5 A a^4 b) x^3) / x^6$

mupad [B] time = 0.05, size = 113, normalized size = 0.99

$$\ln(x) (5 B a^4 b + 10 A a^3 b^2) - \frac{\frac{A a^5}{6} + x^3 \left(\frac{B a^5}{3} + \frac{5 A a b^4}{3} \right)}{x^6} + x^9 \left(\frac{A b^5}{9} + \frac{5 B a b^4}{9} \right) + \frac{B b^5 x^{12}}{12} + \frac{10 a^2 b^2 x^3 (A b + B a)}{3} + \frac{5 a b^3 x^6 (A b + 2 B a)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^7,x)

[Out] $\log(x) \cdot (10Aa^3b^2 + 5Ba^4b) - ((Aa^5)/6 + x^3((Ba^5)/3 + (5Aa^4b)/3))/x^6 + x^9((Ab^5)/9 + (5Ba^4b^4)/9) + (Bb^5x^{12})/12 + (10a^2b^2x^3(Ab + Ba))/3 + (5ab^3x^6(Ab + 2Ba))/6$

sympy [A] time = 0.94, size = 131, normalized size = 1.15

$$\frac{Bb^5x^{12}}{12} + 5a^3b(2Ab + Ba)\log(x) + x^9\left(\frac{Ab^5}{9} + \frac{5Bab^4}{9}\right) + x^6\left(\frac{5Aab^4}{6} + \frac{5Ba^2b^3}{3}\right) + x^3\left(\frac{10Aa^2b^3}{3} + \frac{10Ba^3b^2}{3}\right) + \frac{-Aa^5 + x^3(-10Aa^4b - 2Ba^5)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**7, x)

[Out] $Bb^5x^{12}/12 + 5a^3b(2Ab + Ba)\log(x) + x^9(Ab^5/9 + 5Ba^4b^4/9) + x^6(5Aa^2b^3/3 + 5Ba^3b^2/3) + x^3(10Aa^2b^3/3 + 10Ba^3b^2/3) + (-Aa^5 + x^3(-10Aa^4b - 2Ba^5))/(6x^6)$

$$3.40 \quad \int \frac{(a+bx^3)^5(A+Bx^3)}{x^8} dx$$

Optimal. Leaf size=110

$$-\frac{a^5 A}{7x^7} - \frac{a^4(aB + 5Ab)}{4x^4} - \frac{5a^3b(aB + 2Ab)}{x} + 5a^2b^2x^2(aB + Ab) + \frac{1}{8}b^4x^8(5aB + Ab) + ab^3x^5(2aB + Ab) + \frac{1}{11}b^5Bx^{11}$$

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$5a^2b^2x^2(aB + Ab) - \frac{a^4(aB + 5Ab)}{4x^4} - \frac{5a^3b(aB + 2Ab)}{x} - \frac{a^5A}{7x^7} + \frac{1}{8}b^4x^8(5aB + Ab) + ab^3x^5(2aB + Ab) + \frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^8,x]

[Out] $-(a^5A)/(7*x^7) - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x + 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*x^8)/8 + (b^5*B*x^{11})/11$

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^5(A+Bx^3)}{x^8} dx &= \int \left(\frac{a^5A}{x^8} + \frac{a^4(5Ab+aB)}{x^5} + \frac{5a^3b(2Ab+aB)}{x^2} + 10a^2b^2(Ab+aB)x + 5ab^3(Ab+2aB)x^2 \right. \\ &= \left. -\frac{a^5A}{7x^7} - \frac{a^4(5Ab+aB)}{4x^4} - \frac{5a^3b(2Ab+aB)}{x} + 5a^2b^2(Ab+aB)x^2 + ab^3(Ab+2aB)x^5 \right) dx \end{aligned}$$

Mathematica [A] time = 0.04, size = 110, normalized size = 1.00

$$-\frac{a^5 A}{7x^7} - \frac{a^4(aB + 5Ab)}{4x^4} - \frac{5a^3b(aB + 2Ab)}{x} + 5a^2b^2x^2(aB + Ab) + \frac{1}{8}b^4x^8(5aB + Ab) + ab^3x^5(2aB + Ab) + \frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^8,x]

[Out] $-1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x + 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*x^8)/8 + (b^5*B*x^{11})/11$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^8,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^8, x]

fricas [A] time = 1.11, size = 121, normalized size = 1.10

$$\frac{56 B b^5 x^{18} + 77 (5 B a b^4 + A b^5) x^{15} + 616 (2 B a^2 b^3 + A a b^4) x^{12} + 3080 (B a^3 b^2 + A a^2 b^3) x^9 - 3080 (B a^4 b + 2 A a^3 b^2) x^6 - 88 A a^5 - 154 (B a^5 + 5 A a^4 b) x^3}{616 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="fricas")

[Out] $1/616*(56*B*b^5*x^{18} + 77*(5*B*a*b^4 + A*b^5)*x^{15} + 616*(2*B*a^2*b^3 + A*a*b^4)*x^{12} + 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 3080*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 88*A*a^5 - 154*(B*a^5 + 5*A*a^4*b)*x^3)/x^7$

giac [A] time = 0.17, size = 127, normalized size = 1.15

$$\frac{1}{11} B b^5 x^{11} + \frac{5}{8} B a b^4 x^8 + \frac{1}{8} A b^5 x^8 + 2 B a^2 b^3 x^5 + A a b^4 x^5 + 5 B a^3 b^2 x^2 + 5 A a^2 b^3 x^2 - \frac{140 B a^4 b x^6 + 280 A a^3 b^2 x^6 + 7 B a^5 x^3 + 35 A a^4 b x^3 + 4 A a^5}{28 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="giac")

[Out] $1/11*B*b^5*x^{11} + 5/8*B*a*b^4*x^8 + 1/8*A*b^5*x^8 + 2*B*a^2*b^3*x^5 + A*a*b^4*x^5 + 5*B*a^3*b^2*x^2 + 5*A*a^2*b^3*x^2 - 1/28*(140*B*a^4*b*x^6 + 280*A*a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 4*A*a^5)/x^7$

maple [A] time = 0.05, size = 117, normalized size = 1.06

$$\frac{B b^5 x^{11}}{11} + \frac{A b^5 x^8}{8} + \frac{5 B a b^4 x^8}{8} + A a b^4 x^5 + 2 B a^2 b^3 x^5 + 5 A a^2 b^3 x^2 + 5 B a^3 b^2 x^2 - \frac{5 (2 A b + B a) a^3 b}{x} - \frac{(5 A b + B a) a^4}{4 x^4} - \frac{A a^5}{7 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^8,x)

[Out] 1/11*b^5*B*x^11+1/8*A*x^8*b^5+5/8*B*x^8*a*b^4+A*a*b^4*x^5+2*B*a^2*b^3*x^5+5*A*x^2*a^2*b^3+5*B*x^2*a^3*b^2-1/4*a^4*(5*A*b+B*a)/x^4-1/7*a^5*A/x^7-5*a^3*b*(2*A*b+B*a)/x

maxima [A] time = 0.51, size = 121, normalized size = 1.10

$$\frac{1}{11} B b^5 x^{11} + \frac{1}{8} (5 B a b^4 + A b^5) x^8 + (2 B a^2 b^3 + A a b^4) x^5 + 5 (B a^3 b^2 + A a^2 b^3) x^2 - \frac{140 (B a^4 b + 2 A a^3 b^2) x^6 + 4 A a^5 + 7 (B a^5 + 5 A a^4 b) x^3}{28 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="maxima")

[Out] 1/11*B*b^5*x^11 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + (2*B*a^2*b^3 + A*a*b^4)*x^5 + 5*(B*a^3*b^2 + A*a^2*b^3)*x^2 - 1/28*(140*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 4*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^7

mupad [B] time = 2.34, size = 113, normalized size = 1.03

$$x^8 \left(\frac{A b^5}{8} + \frac{5 B a b^4}{8} \right) - \frac{\frac{A a^5}{7} + x^6 (5 B a^4 b + 10 A a^3 b^2) + x^3 \left(\frac{B a^5}{4} + \frac{5 A b a^4}{4} \right)}{x^7} + \frac{B b^5 x^{11}}{11} + 5 a^2 b^2 x^2 (A b + B a) + a b^3 x^5 (A b + 2 B a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^8,x)

[Out] x^8*((A*b^5)/8 + (5*B*a*b^4)/8) - ((A*a^5)/7 + x^6*(10*A*a^3*b^2 + 5*B*a^4*b) + x^3*((B*a^5)/4 + (5*A*a^4*b)/4))/x^7 + (B*b^5*x^11)/11 + 5*a^2*b^2*x^2*(A*b + B*a) + a*b^3*x^5*(A*b + 2*B*a)

sympy [A] time = 1.12, size = 129, normalized size = 1.17

$$\frac{B b^5 x^{11}}{11} + x^8 \left(\frac{A b^5}{8} + \frac{5 B a b^4}{8} \right) + x^5 (A a b^4 + 2 B a^2 b^3) + x^2 (5 A a^2 b^3 + 5 B a^3 b^2) + \frac{-4 A a^5 + x^6 (-280 A a^3 b^2 - 140 B a^4 b) + x^3 (-35 A a^4 b - 7 B a^5)}{28 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**8,x)

[Out] B*b**5*x**11/11 + x**8*(A*b**5/8 + 5*B*a*b**4/8) + x**5*(A*a*b**4 + 2*B*a**2*b**3) + x**2*(5*A*a**2*b**3 + 5*B*a**3*b**2) + (-4*A*a**5 + x**6*(-280*A*a**3*b**2 - 140*B*a**4*b) + x**3*(-35*A*a**4*b - 7*B*a**5))/(28*x**7)

$$3.41 \quad \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9} dx$$

Optimal. Leaf size=113

$$\frac{a^5 A}{8x^8} - \frac{a^4(aB+5Ab)}{5x^5} - \frac{5a^3b(aB+2Ab)}{2x^2} + 10a^2b^2x(aB+Ab) + \frac{1}{7}b^4x^7(5aB+Ab) + \frac{5}{4}ab^3x^4(2aB+Ab) + \frac{1}{10}b^5Bx^{10}$$

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$10a^2b^2x(aB+Ab) - \frac{5a^3b(aB+2Ab)}{2x^2} - \frac{a^4(aB+5Ab)}{5x^5} - \frac{a^5A}{8x^8} + \frac{1}{7}b^4x^7(5aB+Ab) + \frac{5}{4}ab^3x^4(2aB+Ab) + \frac{1}{10}b^5Bx^{10}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^9, x]

[Out] $-(a^5A)/(8*x^8) - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(2*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*b + 5*a*B)*x^7)/7 + (b^5*B*x^{10})/10$

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9} dx &= \int \left(10a^2b^2(Ab+aB) + \frac{a^5A}{x^9} + \frac{a^4(5Ab+aB)}{x^6} + \frac{5a^3b(2Ab+aB)}{x^3} + 5ab^3(Ab+2aB) \right) dx \\ &= -\frac{a^5A}{8x^8} - \frac{a^4(5Ab+aB)}{5x^5} - \frac{5a^3b(2Ab+aB)}{2x^2} + 10a^2b^2(Ab+aB)x + \frac{5}{4}ab^3(Ab+2aB)x^4 + \frac{1}{10}b^5Bx^{10} \end{aligned}$$

Mathematica [A] time = 0.04, size = 113, normalized size = 1.00

$$-\frac{a^5A}{8x^8} - \frac{a^4(aB+5Ab)}{5x^5} - \frac{5a^3b(aB+2Ab)}{2x^2} + 10a^2b^2x(aB+Ab) + \frac{1}{7}b^4x^7(5aB+Ab) + \frac{5}{4}ab^3x^4(2aB+Ab) + \frac{1}{10}b^5Bx^{10}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^9,x]

[Out] $-\frac{1}{8} \frac{a^5 A}{x^8} - \frac{a^4 (5 A b + a B)}{(5 x^5)} - \frac{(5 a^3 b (2 A b + a B))}{(2 x^2)} + 10 a^2 b^2 (A b + a B) x + \frac{(5 a b^3 (A b + 2 a B) x^4)}{4} + \frac{(b^4 (A b + 5 a B) x^7)}{7} + \frac{(b^5 B x^{10})}{10}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^9} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^9,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^9, x]

fricas [A] time = 0.72, size = 121, normalized size = 1.07

$$\frac{28 B b^5 x^{18} + 40 (5 B a b^4 + A b^5) x^{15} + 350 (2 B a^2 b^3 + A a b^4) x^{12} + 2800 (B a^3 b^2 + A a^2 b^3) x^9 - 700 (B a^4 b + 2 A a^3 b^2) x^6 - 35 A a^5 - 56 (B a^5 + 5 A a^4 b) x^3}{280 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="fricas")

[Out] $\frac{1}{280} (28 B b^5 x^{18} + 40 (5 B a b^4 + A b^5) x^{15} + 350 (2 B a^2 b^3 + A a b^4) x^{12} + 2800 (B a^3 b^2 + A a^2 b^3) x^9 - 700 (B a^4 b + 2 A a^3 b^2) x^6 - 35 A a^5 - 56 (B a^5 + 5 A a^4 b) x^3) / x^8$

giac [A] time = 0.16, size = 124, normalized size = 1.10

$$\frac{1}{10} B b^5 x^{10} + \frac{5}{7} B a b^4 x^7 + \frac{1}{7} A b^5 x^7 + \frac{5}{2} B a^2 b^3 x^4 + \frac{5}{4} A a b^4 x^4 + 10 B a^3 b^2 x + 10 A a^2 b^3 x - \frac{100 B a^4 b x^6 + 200 A a^3 b^2 x^6 + 8 B a^5 x^3 + 40 A a^4 b x^3 + 5 A a^5}{40 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="giac")

[Out] $\frac{1}{10} B b^5 x^{10} + \frac{5}{7} B a b^4 x^7 + \frac{1}{7} A b^5 x^7 + \frac{5}{2} B a^2 b^3 x^4 + \frac{5}{4} A a b^4 x^4 + 10 B a^3 b^2 x + 10 A a^2 b^3 x - \frac{1}{40} (100 B a^4 b x^6 + 200 A a^3 b^2 x^6 + 8 B a^5 x^3 + 40 A a^4 b x^3 + 5 A a^5) / x^8$

maple [A] time = 0.04, size = 114, normalized size = 1.01

$$\frac{B b^5 x^{10}}{10} + \frac{A b^5 x^7}{7} + \frac{5 B a b^4 x^7}{7} + \frac{5 A a b^4 x^4}{4} + \frac{5 B a^2 b^3 x^4}{2} + 10 A a^2 b^3 x + 10 B a^3 b^2 x - \frac{5 (2 A b + B a) a^3 b}{2 x^2} - \frac{(5 A b + B a) a^4}{5 x^5} - \frac{A a^5}{8 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^9,x)

[Out] $1/10*b^5*B*x^{10}+1/7*A*b^5*x^7+5/7*B*a*b^4*x^7+5/4*A*x^4*a*b^4+5/2*B*x^4*a^2*b^3+10*a^2*b^3*A*x+10*a^3*b^2*B*x-1/5*a^4*(5*A*b+B*a)/x^5-1/8*a^5*A/x^8-5/2*a^3*b*(2*A*b+B*a)/x^2$

maxima [A] time = 0.59, size = 120, normalized size = 1.06

$$\frac{1}{10}Bb^5x^{10} + \frac{1}{7}(5Bab^4 + Ab^5)x^7 + \frac{5}{4}(2Ba^2b^3 + Aab^4)x^4 + 10(Ba^3b^2 + Aa^2b^3)x - \frac{100(Ba^4b + 2Aa^3b^2)x^6 + 5Aa^5 + 8(Ba^5 + 5Aa^4b)x^3}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="maxima")

[Out] $1/10*B*b^5*x^{10} + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + 5/4*(2*B*a^2*b^3 + A*a*b^4)*x^4 + 10*(B*a^3*b^2 + A*a^2*b^3)*x - 1/40*(100*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 5*A*a^5 + 8*(B*a^5 + 5*A*a^4*b)*x^3)/x^8$

mupad [B] time = 0.04, size = 111, normalized size = 0.98

$$x^7 \left(\frac{Ab^5}{7} + \frac{5Bab^4}{7} \right) - \frac{\frac{Aa^5}{8} + x^6 \left(\frac{5Ba^4b}{2} + 5Aa^3b^2 \right) + x^3 \left(\frac{Ba^5}{5} + Aba^4 \right)}{x^8} + \frac{Bb^5x^{10}}{10} + 10a^2b^2x(Ab + Ba) + \frac{5ab^3x^4(Ab + 2Ba)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^9,x)

[Out] $x^7*((A*b^5)/7 + (5*B*a*b^4)/7) - ((A*a^5)/8 + x^6*(5*A*a^3*b^2 + (5*B*a^4*b)/2) + x^3*((B*a^5)/5 + A*a^4*b))/x^8 + (B*b^5*x^{10})/10 + 10*a^2*b^2*x*(A*b + B*a) + (5*a*b^3*x^4*(A*b + 2*B*a))/4$

sympy [A] time = 1.21, size = 133, normalized size = 1.18

$$\frac{Bb^5x^{10}}{10} + x^7 \left(\frac{Ab^5}{7} + \frac{5Bab^4}{7} \right) + x^4 \left(\frac{5Aab^4}{4} + \frac{5Ba^2b^3}{2} \right) + x(10Aa^2b^3 + 10Ba^3b^2) + \frac{-5Aa^5 + x^6(-200Aa^3b^2 - 100Ba^4b) + x^3(-40Aa^4b - 8Ba^5)}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**9,x)

[Out] $B*b^{**5}*x^{**10}/10 + x^{**7}*(A*b^{**5}/7 + 5*B*a*b^{**4}/7) + x^{**4}*(5*A*a*b^{**4}/4 + 5*B*a^{**2}*b^{**3}/2) + x*(10*A*a^{**2}*b^{**3} + 10*B*a^{**3}*b^{**2}) + (-5*A*a^{**5} + x^{**6}*(-200*A*a^{**3}*b^{**2} - 100*B*a^{**4}*b) + x^{**3}*(-40*A*a^{**4}*b - 8*B*a^{**5}))/ (40*x^{**8})$

$$3.42 \quad \int \frac{(a+bx^3)^5(A+Bx^3)}{x^{10}} dx$$

Optimal. Leaf size=114

$$\frac{a^5 A}{9x^9} - \frac{a^4(aB + 5Ab)}{6x^6} - \frac{5a^3b(aB + 2Ab)}{3x^3} + 10a^2b^2 \log(x)(aB + Ab) + \frac{1}{6}b^4x^6(5aB + Ab) + \frac{5}{3}ab^3x^3(2aB + Ab) + \frac{1}{9}b^5Bx^9$$

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 76}

$$10a^2b^2 \log(x)(aB + Ab) - \frac{5a^3b(aB + 2Ab)}{3x^3} - \frac{a^4(aB + 5Ab)}{6x^6} - \frac{a^5 A}{9x^9} + \frac{1}{6}b^4x^6(5aB + Ab) + \frac{5}{3}ab^3x^3(2aB + Ab) + \frac{1}{9}b^5Bx^9$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^10,x]

[Out] -(a^5*A)/(9*x^9) - (a^4*(5*A*b + a*B))/(6*x^6) - (5*a^3*b*(2*A*b + a*B))/(3*x^3) + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*x^9)/9 + 10*a^2*b^2*(A*b + a*B)*Log[x]

Rule 76

Int[((d_)*(x_)^(n_))*((a_) + (b_)*(x_)^p)*((e_) + (f_)*(x_)^q)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^n)^(p_)*((c_) + (d_)*(x_)^n)^(q_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{10}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^5 (A + Bx)}{x^4} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(5ab^3(Ab + 2aB) + \frac{a^5 A}{x^4} + \frac{a^4(5Ab + aB)}{x^3} + \frac{5a^3b(2Ab + aB)}{x^2} + \frac{10a^2b^2}{x} \right) dx, x, x^3 \right) \\ &= -\frac{a^5 A}{9x^9} - \frac{a^4(5Ab + aB)}{6x^6} - \frac{5a^3b(2Ab + aB)}{3x^3} + \frac{5}{3}ab^3(Ab + 2aB)x^3 + \frac{1}{6}b^4(Ab + 5aB)x^6 \end{aligned}$$

Mathematica [A] time = 0.06, size = 106, normalized size = 0.93

$$\frac{1}{18} \left(-\frac{2a^5 A}{x^9} - \frac{3a^4(aB + 5Ab)}{x^6} - \frac{30a^3b(aB + 2Ab)}{x^3} + 180a^2b^2 \log(x)(aB + Ab) + 3b^4x^6(5aB + Ab) + 30ab^3x^3(2aB + Ab) + 2b^5Bx^9 \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^10, x]

[Out] ((-2*a^5*A)/x^9 - (3*a^4*(5*A*b + a*B))/x^6 - (30*a^3*b*(2*A*b + a*B))/x^3 + 30*a*b^3*(A*b + 2*a*B)*x^3 + 3*b^4*(A*b + 5*a*B)*x^6 + 2*b^5*B*x^9 + 180*a^2*b^2*(A*b + a*B)*Log[x])/18

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{10}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^10, x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^10, x]

fricas [A] time = 0.64, size = 123, normalized size = 1.08

$$\frac{2Bb^5x^{18} + 3(5Bab^4 + Ab^5)x^{15} + 30(2Ba^2b^3 + Aab^4)x^{12} + 180(Ba^3b^2 + Aa^2b^3)x^9 \log(x) - 30(Ba^4b + 2Aa^3b^2)x^6 - 2Aa^5 - 3(Ba^5 + 5Aa^4b)x^3}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10, x, algorithm="fricas")

[Out] 1/18*(2*B*b^5*x^18 + 3*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^4)*x^12 + 180*(B*a^3*b^2 + A*a^2*b^3)*x^9*log(x) - 30*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 2*A*a^5 - 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9

giac [A] time = 0.15, size = 150, normalized size = 1.32

$$\frac{1}{9} B b^5 x^9 + \frac{5}{6} B a b^4 x^6 + \frac{1}{6} A b^5 x^6 + \frac{10}{3} B a^2 b^3 x^3 + \frac{5}{3} A a b^4 x^3 + 10 (B a^3 b^2 + A a^2 b^3) \log(|x|) - \frac{110 B a^3 b^2 x^9 + 110 A a^2 b^3 x^9 + 30 B a^4 b x^6 + 60 A a^3 b^2 x^6 + 3 B a^5 x^3 + 15 A a^4 b x^3 + 2 A a^5}{18 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="giac")

[Out] $\frac{1}{9} B b^5 x^9 + \frac{5}{6} B a b^4 x^6 + \frac{1}{6} A b^5 x^6 + \frac{10}{3} B a^2 b^3 x^3 + \frac{5}{3} A a b^4 x^3 + 10 (B a^3 b^2 + A a^2 b^3) \log(\text{abs}(x)) - \frac{1}{18} (110 B a^3 b^2 x^9 + 110 A a^2 b^3 x^9 + 30 B a^4 b x^6 + 60 A a^3 b^2 x^6 + 3 B a^5 x^3 + 15 A a^4 b x^3 + 2 A a^5) / x^9$

maple [A] time = 0.05, size = 124, normalized size = 1.09

$$\frac{B b^5 x^9}{9} + \frac{A b^5 x^6}{6} + \frac{5 B a b^4 x^6}{6} + \frac{5 A a b^4 x^3}{3} + \frac{10 B a^2 b^3 x^3}{3} + 10 A a^2 b^3 \ln(x) + 10 B a^3 b^2 \ln(x) - \frac{10 A a^3 b^2}{3 x^3} - \frac{5 B a^4 b}{3 x^3} - \frac{5 A a^4 b}{6 x^6} - \frac{B a^5}{6 x^6} - \frac{A a^5}{9 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^10,x)

[Out] $\frac{1}{9} B b^5 x^9 + \frac{1}{6} A b^5 x^6 + \frac{5}{6} B a b^4 x^6 + \frac{5}{3} A a b^4 x^3 + \frac{10}{3} B a^2 b^3 x^3 + \frac{5}{3} A a^2 b^3 x^3 - \frac{10}{3} B a^3 b^2 \ln(x) - \frac{5}{3} A a^3 b^2 \ln(x) - \frac{1}{9} B a^4 b \ln(x) - \frac{5}{6} A a^4 b \ln(x) - \frac{1}{6} B a^5 \ln(x) - \frac{5}{9} A a^5 \ln(x) + \frac{10}{3} B a^2 b^3 \ln(x) + \frac{10}{3} A a^2 b^3 \ln(x)$

maxima [A] time = 0.47, size = 123, normalized size = 1.08

$$\frac{1}{9} B b^5 x^9 + \frac{1}{6} (5 B a b^4 + A b^5) x^6 + \frac{5}{3} (2 B a^2 b^3 + A a b^4) x^3 + \frac{10}{3} (B a^3 b^2 + A a^2 b^3) \log(x^3) - \frac{30 (B a^4 b + 2 A a^3 b^2) x^6 + 2 A a^5 + 3 (B a^5 + 5 A a^4 b) x^3}{18 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="maxima")

[Out] $\frac{1}{9} B b^5 x^9 + \frac{1}{6} (5 B a b^4 + A b^5) x^6 + \frac{5}{3} (2 B a^2 b^3 + A a b^4) x^3 + \frac{10}{3} (B a^3 b^2 + A a^2 b^3) \log(x^3) - \frac{1}{18} (30 (B a^4 b + 2 A a^3 b^2) x^6 + 2 A a^5 + 3 (B a^5 + 5 A a^4 b) x^3) / x^9$

mupad [B] time = 0.05, size = 118, normalized size = 1.04

$$x^6 \left(\frac{A b^5}{6} + \frac{5 B a b^4}{6} \right) - \frac{\frac{A a^5}{9} + x^6 \left(\frac{5 B a^4 b}{3} + \frac{10 A a^3 b^2}{3} \right) + x^3 \left(\frac{B a^5}{6} + \frac{5 A a^4 b}{6} \right)}{x^9} + \ln(x) (10 B a^3 b^2 + 10 A a^2 b^3) + \frac{B b^5 x^9}{9} + \frac{5 a b^3 x^3 (A b + 2 B a)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^10,x)

[Out] $x^6 \left(\frac{A b^5}{6} + \frac{5 B a^4 b}{6} \right) - \left(\frac{A a^5}{9} + x^6 \left(\frac{10 A a^3 b^2}{3} + \frac{5 B a^4 b}{3} \right) + x^3 \left(\frac{B a^5}{6} + \frac{5 A a^4 b}{6} \right) \right) / x^9 + \log(x) \left(\frac{10 A a^2 b^3}{3} + \frac{10 B a^3 b^2}{3} + \frac{B b^5 x^9}{9} + \frac{5 a^3 b^3 x^3 (A b + 2 B a)}{3} \right)$

sympy [A] time = 2.24, size = 129, normalized size = 1.13

$$\frac{B b^5 x^9}{9} + 10 a^2 b^2 (A b + B a) \log(x) + x^6 \left(\frac{A b^5}{6} + \frac{5 B a b^4}{6} \right) + x^3 \left(\frac{5 A a b^4}{3} + \frac{10 B a^2 b^3}{3} \right) + \frac{-2 A a^5 + x^6 (-60 A a^3 b^2 - 30 B a^4 b) + x^3 (-15 A a^4 b - 3 B a^5)}{18 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**10,x)

[Out] $B b^5 x^9 / 9 + 10 a^2 b^2 (A b + B a) \log(x) + x^6 \left(\frac{A b^5}{6} + \frac{5 B a^4 b}{6} \right) + x^3 \left(\frac{5 A a^3 b^2}{3} + \frac{10 B a^2 b^3}{3} \right) + \frac{(-2 A a^5 + x^6 (-60 A a^3 b^2 - 30 B a^4 b) + x^3 (-15 A a^4 b - 3 B a^5))}{(18 x^9)}$

$$3.43 \quad \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}} dx$$

Optimal. Leaf size=115

$$-\frac{a^5 A}{10x^{10}} - \frac{a^4(aB + 5Ab)}{7x^7} - \frac{5a^3b(aB + 2Ab)}{4x^4} - \frac{10a^2b^2(aB + Ab)}{x} + \frac{1}{5}b^4x^5(5aB + Ab) + \frac{5}{2}ab^3x^2(2aB + Ab) + \frac{1}{8}b^5Bx^8$$

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{10a^2b^2(aB + Ab)}{x} - \frac{5a^3b(aB + 2Ab)}{4x^4} - \frac{a^4(aB + 5Ab)}{7x^7} - \frac{a^5 A}{10x^{10}} + \frac{1}{5}b^4x^5(5aB + Ab) + \frac{5}{2}ab^3x^2(2aB + Ab) + \frac{1}{8}b^5Bx^8$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^11,x]

[Out] -(a^5*A)/(10*x^10) - (a^4*(5*A*b + a*B))/(7*x^7) - (5*a^3*b*(2*A*b + a*B))/(4*x^4) - (10*a^2*b^2*(A*b + a*B))/x + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4*(A*b + 5*a*B)*x^5)/5 + (b^5*B*x^8)/8

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}} dx &= \int \left(\frac{a^5 A}{x^{11}} + \frac{a^4(5Ab + aB)}{x^8} + \frac{5a^3b(2Ab + aB)}{x^5} + \frac{10a^2b^2(Ab + aB)}{x^2} + 5ab^3(Ab + 2aB) \right. \\ &\quad \left. - \frac{a^5 A}{10x^{10}} - \frac{a^4(5Ab + aB)}{7x^7} - \frac{5a^3b(2Ab + aB)}{4x^4} - \frac{10a^2b^2(Ab + aB)}{x} + \frac{5}{2}ab^3(Ab + 2aB) \right) dx \end{aligned}$$

Mathematica [A] time = 0.02, size = 118, normalized size = 1.03

$$\frac{-4a^5(7A + 10Bx^3) - 50a^4bx^3(4A + 7Bx^3) - 700a^3b^2x^6(A + 4Bx^3) + 1400a^2b^3x^9(Bx^3 - 2A) + 140ab^4x^{12}(5A + 2Bx^3) + 7b^5x^{15}(8A + 5Bx^3)}{280x^{10}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^11,x]

[Out] (1400*a^2*b^3*x^9*(-2*A + B*x^3) + 140*a*b^4*x^12*(5*A + 2*B*x^3) - 700*a^3*b^2*x^6*(A + 4*B*x^3) + 7*b^5*x^15*(8*A + 5*B*x^3) - 50*a^4*b*x^3*(4*A + 7*B*x^3) - 4*a^5*(7*A + 10*B*x^3))/(280*x^10)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{11}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^11,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^11, x]

fricas [A] time = 1.05, size = 121, normalized size = 1.05

$$\frac{35 B b^5 x^{18} + 56 (5 B a b^4 + A b^5) x^{15} + 700 (2 B a^2 b^3 + A a b^4) x^{12} - 2800 (B a^3 b^2 + A a^2 b^3) x^9 - 350 (B a^4 b + 2 A a^3 b^2) x^6 - 28 A a^5 - 40 (B a^5 + 5 A a^4 b) x^3}{280 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="fricas")

[Out] 1/280*(35*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 700*(2*B*a^2*b^3 + A*a*b^4)*x^12 - 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 350*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 28*A*a^5 - 40*(B*a^5 + 5*A*a^4*b)*x^3)/x^10

giac [A] time = 0.15, size = 127, normalized size = 1.10

$$\frac{1}{8} B b^5 x^8 + B a b^4 x^5 + \frac{1}{5} A b^5 x^5 + 5 B a^2 b^3 x^2 + \frac{5}{2} A a b^4 x^2 - \frac{1400 B a^3 b^2 x^9 + 1400 A a^2 b^3 x^9 + 175 B a^4 b x^6 + 350 A a^3 b^2 x^6 + 20 B a^5 x^3 + 100 A a^4 b x^3 + 14 A a^5}{140 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="giac")

[Out] 1/8*B*b^5*x^8 + B*a*b^4*x^5 + 1/5*A*b^5*x^5 + 5*B*a^2*b^3*x^2 + 5/2*A*a*b^4*x^2 - 1/140*(1400*B*a^3*b^2*x^9 + 1400*A*a^2*b^3*x^9 + 175*B*a^4*b*x^6 + 350*A*a^3*b^2*x^6 + 20*B*a^5*x^3 + 100*A*a^4*b*x^3 + 14*A*a^5)/x^10

maple [A] time = 0.05, size = 111, normalized size = 0.97

$$\frac{B b^5 x^8}{8} + \frac{A b^5 x^5}{5} + B a b^4 x^5 + \frac{5 A a b^4 x^2}{2} + 5 B a^2 b^3 x^2 - \frac{10 (A b + B a) a^2 b^2}{x} - \frac{5 (2 A b + B a) a^3 b}{4 x^4} - \frac{(5 A b + B a) a^4}{7 x^7} - \frac{A a^5}{10 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^5*(B*x^3+A)/x^11,x)`

[Out] $\frac{1}{8}b^5Bx^8 + \frac{1}{5}A^5x^5 + Bx^5 + 5Ab^4 + \frac{5}{2}A^2x^2 + 5Bx^2 + a^2b^3 - \frac{5}{4}a^3b(2Ab + Ba)/x^4 - \frac{1}{7}a^4(5Ab + Ba)/x^7 - 10a^2b^2(Ab + Ba)/x - \frac{1}{10}a^5A/x^{10}$

maxima [A] time = 0.47, size = 122, normalized size = 1.06

$$\frac{1}{8}Bb^5x^8 + \frac{1}{5}(5Bab^4 + Ab^5)x^5 + \frac{5}{2}(2Ba^2b^3 + Aab^4)x^2 - \frac{1400(Ba^3b^2 + Aa^2b^3)x^9 + 175(Ba^4b + 2Aa^3b^2)x^6 + 14Aa^5 + 20(Ba^5 + 5Aa^4b)x^3}{140x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="maxima")`

[Out] $\frac{1}{8}Bb^5x^8 + \frac{1}{5}(5BAb^4 + Ab^5)x^5 + \frac{5}{2}(2BAb^2 + AAb^4)x^2 - \frac{1}{140}(1400(BAb^3 + AAb^2)x^9 + 175(BA^4b + 2AA^3b^2)x^6 + 14AA^5 + 20(BA^5 + 5AA^4b)x^3)/x^{10}$

mupad [B] time = 2.36, size = 118, normalized size = 1.03

$$x^5 \left(\frac{Ab^5}{5} + Bab^4 \right) - \frac{\frac{Aa^5}{10} + x^6 \left(\frac{5Ba^4b}{4} + \frac{5Aa^3b^2}{2} \right) + x^3 \left(\frac{Ba^5}{7} + \frac{5Aba^4}{7} \right) + x^9 (10Ba^3b^2 + 10Aa^2b^3)}{x^{10}} + \frac{Bb^5x^8}{8} + \frac{5ab^3x^2(Ab + 2Ba)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^5)/x^11,x)`

[Out] $x^5 \left(\frac{Ab^5}{5} + BAb^4 \right) - \left(\frac{Aa^5}{10} + x^6 \left(\frac{5Aa^3b^2}{2} + \frac{5BAb^4}{4} \right) + x^3 \left(\frac{Ba^5}{7} + \frac{5Aa^4b}{7} \right) + x^9 (10Aa^2b^3 + 10BAb^3) \right) / x^{10} + \frac{Bb^5x^8}{8} + \frac{5a^3b^3x^2(Ab + 2Ba)}{2}$

sympy [A] time = 2.93, size = 131, normalized size = 1.14

$$\frac{Bb^5x^8}{8} + x^5 \left(\frac{Ab^5}{5} + Bab^4 \right) + x^2 \left(\frac{5Aab^4}{2} + 5Ba^2b^3 \right) + \frac{-14Aa^5 + x^9(-1400Aa^2b^3 - 1400Ba^3b^2) + x^6(-350Aa^3b^2 - 175Ba^4b) + x^3(-100Aa^4b - 20Ba^5)}{140x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**5*(B*x**3+A)/x**11,x)`

[Out] $Bb^5x^8/8 + x^5(Ab^5/5 + BAb^4) + x^2(5Aa^3b^2/2 + 5BAb^4) + (-14Aa^5 + x^9(-1400Aa^2b^3 - 1400BAb^3) + x^6(-350Aa^3b^2 - 175BAb^4b) + x^3(-100Aa^4b - 20BAb^5))/(140x^{10})$

$$3.44 \quad \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}} dx$$

Optimal. Leaf size=109

$$\frac{a^5 A}{11x^{11}} - \frac{a^4(aB + 5Ab)}{8x^8} - \frac{a^3b(aB + 2Ab)}{x^5} - \frac{5a^2b^2(aB + Ab)}{x^2} + \frac{1}{4}b^4x^4(5aB + Ab) + 5ab^3x(2aB + Ab) + \frac{1}{7}b^5Bx^7$$

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{5a^2b^2(aB + Ab)}{x^2} - \frac{a^3b(aB + 2Ab)}{x^5} - \frac{a^4(aB + 5Ab)}{8x^8} - \frac{a^5 A}{11x^{11}} + \frac{1}{4}b^4x^4(5aB + Ab) + 5ab^3x(2aB + Ab) + \frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^12, x]

[Out] -(a^5*A)/(11*x^11) - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x^5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5*a*B)*x^4)/4 + (b^5*B*x^7)/7

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}} dx &= \int \left(5ab^3(Ab + 2aB) + \frac{a^5 A}{x^{12}} + \frac{a^4(5Ab + aB)}{x^9} + \frac{5a^3b(2Ab + aB)}{x^6} + \frac{10a^2b^2(Ab + aB)}{x^3} \right. \\ &\quad \left. + \frac{a^5 A}{11x^{11}} - \frac{a^4(5Ab + aB)}{8x^8} - \frac{a^3b(2Ab + aB)}{x^5} - \frac{5a^2b^2(Ab + aB)}{x^2} + 5ab^3(Ab + 2aB)x \right) dx \end{aligned}$$

Mathematica [A] time = 0.04, size = 109, normalized size = 1.00

$$-\frac{a^5 A}{11x^{11}} - \frac{a^4(aB + 5Ab)}{8x^8} - \frac{a^3b(aB + 2Ab)}{x^5} - \frac{5a^2b^2(aB + Ab)}{x^2} + \frac{1}{4}b^4x^4(5aB + Ab) + 5ab^3x(2aB + Ab) + \frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^12,x]

[Out] $-1/11*(a^5*A)/x^{11} - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x^5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5*a*B)*x^4)/4 + (b^5*B*x^7)/7$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{12}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^12,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^12, x]

fricas [A] time = 0.79, size = 121, normalized size = 1.11

$$\frac{88 B b^5 x^{18} + 154 (5 B a b^4 + A b^5) x^{15} + 3080 (2 B a^2 b^3 + A a b^4) x^{12} - 3080 (B a^3 b^2 + A a^2 b^3) x^9 - 616 (B a^4 b + 2 A a^3 b^2) x^6 - 56 A a^5 - 77 (B a^5 + 5 A a^4 b) x^3}{616 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="fricas")

[Out] $1/616*(88*B*b^5*x^{18} + 154*(5*B*a*b^4 + A*b^5)*x^{15} + 3080*(2*B*a^2*b^3 + A*a*b^4)*x^{12} - 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 616*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 56*A*a^5 - 77*(B*a^5 + 5*A*a^4*b)*x^3)/x^{11}$

giac [A] time = 0.18, size = 124, normalized size = 1.14

$$\frac{1}{7} B b^5 x^7 + \frac{5}{4} B a b^4 x^4 + \frac{1}{4} A b^5 x^4 + 10 B a^2 b^3 x + 5 A a b^4 x - \frac{440 B a^3 b^2 x^9 + 440 A a^2 b^3 x^9 + 88 B a^4 b x^6 + 176 A a^3 b^2 x^6 + 11 B a^5 x^3 + 55 A a^4 b x^3 + 8 A a^5}{88 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="giac")

[Out] $1/7*B*b^5*x^7 + 5/4*B*a*b^4*x^4 + 1/4*A*b^5*x^4 + 10*B*a^2*b^3*x + 5*A*a*b^4*x - 1/88*(440*B*a^3*b^2*x^9 + 440*A*a^2*b^3*x^9 + 88*B*a^4*b*x^6 + 176*A*a^3*b^2*x^6 + 11*B*a^5*x^3 + 55*A*a^4*b*x^3 + 8*A*a^5)/x^{11}$

maple [A] time = 0.05, size = 108, normalized size = 0.99

$$\frac{B b^5 x^7}{7} + \frac{A b^5 x^4}{4} + \frac{5 B a b^4 x^4}{4} + 5 A a b^4 x + 10 B a^2 b^3 x - \frac{5 (A b + B a) a^2 b^2}{x^2} - \frac{(2 A b + B a) a^3 b}{x^5} - \frac{(5 A b + B a) a^4}{8 x^8} - \frac{A a^5}{11 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^12,x)

[Out] $1/7*b^5*B*x^7+1/4*A*x^4*b^5+5/4*B*x^4*a*b^4+5*a*b^4*A*x+10*a^2*b^3*B*x-a^3*b*(2*A*b+B*a)/x^5-1/8*a^4*(5*A*b+B*a)/x^8-5*a^2*b^2*(A*b+B*a)/x^2-1/11*a^5*A/x^11$

maxima [A] time = 0.53, size = 120, normalized size = 1.10

$$\frac{1}{7}Bb^5x^7 + \frac{1}{4}(5Bab^4 + Ab^5)x^4 + 5(2Ba^2b^3 + Aab^4)x - \frac{440(Ba^3b^2 + Aa^2b^3)x^9 + 88(Ba^4b + 2Aa^3b^2)x^6 + 8Aa^5 + 11(Ba^5 + 5Aa^4b)x^3}{88x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="maxima")

[Out] $1/7*B*b^5*x^7 + 1/4*(5*B*a*b^4 + A*b^5)*x^4 + 5*(2*B*a^2*b^3 + A*a*b^4)*x - 1/88*(440*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 88*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 8*A*a^5 + 11*(B*a^5 + 5*A*a^4*b)*x^3)/x^{11}$

mupad [B] time = 0.07, size = 116, normalized size = 1.06

$$x^4 \left(\frac{Ab^5}{4} + \frac{5Bab^4}{4} \right) - \frac{\frac{Aa^5}{11} + x^6 (Ba^4b + 2Aa^3b^2) + x^3 \left(\frac{Ba^5}{8} + \frac{5Aba^4}{8} \right) + x^9 (5Ba^3b^2 + 5Aa^2b^3)}{x^{11}} + \frac{Bb^5x^7}{7} + 5ab^3x(Ab + 2Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^12,x)

[Out] $x^4*((A*b^5)/4 + (5*B*a*b^4)/4) - ((A*a^5)/11 + x^6*(2*A*a^3*b^2 + B*a^4*b) + x^3*((B*a^5)/8 + (5*A*a^4*b)/8) + x^9*(5*A*a^2*b^3 + 5*B*a^3*b^2))/x^{11} + (B*b^5*x^7)/7 + 5*a*b^3*x*(A*b + 2*B*a)$

sympy [A] time = 3.73, size = 131, normalized size = 1.20

$$\frac{Bb^5x^7}{7} + x^4 \left(\frac{Ab^5}{4} + \frac{5Bab^4}{4} \right) + x(5Aab^4 + 10Ba^2b^3) + \frac{-8Aa^5 + x^9(-440Aa^2b^3 - 440Ba^3b^2) + x^6(-176Aa^3b^2 - 88Ba^4b) + x^3(-55Aa^4b - 11Ba^5)}{88x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**12,x)

[Out] $B*b**5*x**7/7 + x**4*(A*b**5/4 + 5*B*a*b**4/4) + x*(5*A*a*b**4 + 10*B*a**2*b**3) + (-8*A*a**5 + x**9*(-440*A*a**2*b**3 - 440*B*a**3*b**2) + x**6*(-176*A*a**3*b**2 - 88*B*a**4*b) + x**3*(-55*A*a**4*b - 11*B*a**5))/(88*x**11)$

$$3.45 \quad \int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}} dx$$

Optimal. Leaf size=114

$$\frac{a^5 A}{12x^{12}} - \frac{a^4(aB + 5Ab)}{9x^9} - \frac{5a^3b(aB + 2Ab)}{6x^6} - \frac{10a^2b^2(aB + Ab)}{3x^3} + \frac{1}{3}b^4x^3(5aB + Ab) + 5ab^3 \log(x)(2aB + Ab) + \frac{1}{6}b^5Bx^6$$

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 76}

$$-\frac{10a^2b^2(aB + Ab)}{3x^3} - \frac{5a^3b(aB + 2Ab)}{6x^6} - \frac{a^4(aB + 5Ab)}{9x^9} - \frac{a^5A}{12x^{12}} + \frac{1}{3}b^4x^3(5aB + Ab) + 5ab^3 \log(x)(2aB + Ab) + \frac{1}{6}b^5Bx^6$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^13,x]

[Out] -(a^5*A)/(12*x^12) - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*b + a*B))/(6*x^6) - (10*a^2*b^2*(A*b + a*B))/(3*x^3) + (b^4*(A*b + 5*a*B)*x^3)/3 + (b^5*B*x^6)/6 + 5*a*b^3*(A*b + 2*a*B)*Log[x]

Rule 76

Int[((d_)*(x_))^(n_)*((a_) + (b_)*(x_))*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{13}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^5 (A + Bx)}{x^5} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(b^4 (Ab + 5aB) + \frac{a^5 A}{x^5} + \frac{a^4 (5Ab + aB)}{x^4} + \frac{5a^3 b (2Ab + aB)}{x^3} + \frac{10a^2 b^2}{x^2} \right) dx, x, x^3 \right) \\
&= -\frac{a^5 A}{12x^{12}} - \frac{a^4 (5Ab + aB)}{9x^9} - \frac{5a^3 b (2Ab + aB)}{6x^6} - \frac{10a^2 b^2 (Ab + aB)}{3x^3} + \frac{1}{3} b^4 (Ab + 5aB)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 118, normalized size = 1.04

$$-\frac{a^5 (3A + 4Bx^3) + 10a^4 bx^3 (2A + 3Bx^3) + 60a^3 b^2 x^6 (A + 2Bx^3) + 120a^2 Ab^3 x^9 - 180ab^3 x^{12} \log(x)(2aB + Ab) - 60ab^4 Bx^{15} - 6b^5 x^{15} (2A + Bx^3)}{36x^{12}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^13,x]

[Out] -1/36*(120*a^2*A*b^3*x^9 - 60*a*b^4*B*x^15 - 6*b^5*x^15*(2*A + B*x^3) + 60*a^3*b^2*x^6*(A + 2*B*x^3) + 10*a^4*b*b*x^3*(2*A + 3*B*x^3) + a^5*(3*A + 4*B*x^3) - 180*a*b^3*(A*b + 2*a*B)*x^12*Log[x])/x^12

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{13}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^13,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^13, x]

fricas [A] time = 0.62, size = 123, normalized size = 1.08

$$\frac{6Bb^5x^{18} + 12(5Bab^4 + Ab^5)x^{15} + 180(2Ba^2b^3 + Aab^4)x^{12} \log(x) - 120(Ba^3b^2 + Aa^2b^3)x^9 - 30(Ba^4b + 2Aa^3b^2)x^6 - 3Aa^5 - 4(Ba^5 + 5Aa^4b)x^3}{36x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="fricas")

[Out] 1/36*(6*B*b^5*x^18 + 12*(5*B*a*b^4 + A*b^5)*x^15 + 180*(2*B*a^2*b^3 + A*a*b^4)*x^12*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 30*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 3*A*a^5 - 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12

giac [A] time = 0.16, size = 149, normalized size = 1.31

$$\frac{1}{6} B b^5 x^6 + \frac{5}{3} B a b^4 x^3 + \frac{1}{3} A b^5 x^3 + 5 (2 B a^2 b^3 + A a b^4) \log(|x|) - \frac{250 B a^2 b^3 x^{12} + 125 A a b^4 x^{12} + 120 B a^3 b^2 x^9 + 120 A a^2 b^3 x^9 + 30 B a^4 b x^6 + 60 A a^3 b^2 x^6 + 4 B a^5 x^3 + 20 A a^4 b x^3 + 3 A a^5}{36 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="giac")

[Out] $\frac{1}{6} B b^5 x^6 + \frac{5}{3} B a b^4 x^3 + \frac{1}{3} A b^5 x^3 + 5 (2 B a^2 b^3 + A a b^4) \log(\text{abs}(x)) - \frac{1}{36} (250 B a^2 b^3 x^{12} + 125 A a b^4 x^{12} + 120 B a^3 b^2 x^9 + 120 A a^2 b^3 x^9 + 30 B a^4 b x^6 + 60 A a^3 b^2 x^6 + 4 B a^5 x^3 + 20 A a^4 b x^3 + 3 A a^5) / x^{12}$

maple [A] time = 0.05, size = 124, normalized size = 1.09

$$\frac{B b^5 x^6}{6} + \frac{A b^5 x^3}{3} + \frac{5 B a b^4 x^3}{3} + 5 A a b^4 \ln(x) + 10 B a^2 b^3 \ln(x) - \frac{10 A a^2 b^3}{3 x^3} - \frac{10 B a^3 b^2}{3 x^3} - \frac{5 A a^3 b^2}{3 x^6} - \frac{5 B a^4 b}{6 x^6} - \frac{5 A a^4 b}{9 x^9} - \frac{B a^5}{9 x^9} - \frac{A a^5}{12 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^13,x)

[Out] $\frac{1}{6} b^5 B x^6 + \frac{1}{3} A x^3 b^5 + \frac{5}{3} B x^3 a b^4 - \frac{1}{12} a^5 A / x^{12} - \frac{10}{3} b^3 a^2 / x^9 - \frac{3}{10} A - \frac{10}{3} b^2 a^3 / x^6 - \frac{5}{9} a^4 / x^9 - \frac{1}{9} a^5 / x^9 - \frac{5}{3} a^3 b^2 / x^6 - \frac{5}{6} a^4 b / x^6 - \frac{5}{6} A \ln(x) + 10 B \ln(x) + a^2 b^3$

maxima [A] time = 0.63, size = 123, normalized size = 1.08

$$\frac{1}{6} B b^5 x^6 + \frac{1}{3} (5 B a b^4 + A b^5) x^3 + \frac{5}{3} (2 B a^2 b^3 + A a b^4) \log(x^3) - \frac{120 (B a^3 b^2 + A a^2 b^3) x^9 + 30 (B a^4 b + 2 A a^3 b^2) x^6 + 3 A a^5 + 4 (B a^5 + 5 A a^4 b) x^3}{36 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="maxima")

[Out] $\frac{1}{6} B b^5 x^6 + \frac{1}{3} (5 B a b^4 + A b^5) x^3 + \frac{5}{3} (2 B a^2 b^3 + A a b^4) \log(x^3) - \frac{1}{36} (120 (B a^3 b^2 + A a^2 b^3) x^9 + 30 (B a^4 b + 2 A a^3 b^2) x^6 + 3 A a^5 + 4 (B a^5 + 5 A a^4 b) x^3) / x^{12}$

mupad [B] time = 0.06, size = 122, normalized size = 1.07

$$\ln(x) (10 B a^2 b^3 + 5 A a b^4) - \frac{\frac{A a^5}{12} + x^6 \left(\frac{5 B a^4 b}{6} + \frac{5 A a^3 b^2}{3} \right) + x^3 \left(\frac{B a^5}{9} + \frac{5 A b a^4}{9} \right) + x^9 \left(\frac{10 B a^3 b^2}{3} + \frac{10 A a^2 b^3}{3} \right)}{x^{12}} + x^3 \left(\frac{A b^5}{3} + \frac{5 B a b^4}{3} \right) + \frac{B b^5 x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^13,x)

[Out] $\log(x) \cdot (10Ba^2b^3 + 5Aab^4) - ((Aa^5)/12 + x^6 \cdot ((5Aa^3b^2)/3 + (5Ba^4b)/6) + x^3 \cdot ((Ba^5)/9 + (5Aa^4b)/9) + x^9 \cdot ((10Aa^2b^3)/3 + (10Ba^3b^2)/3) / x^{12} + x^3 \cdot ((Ab^5)/3 + (5Bab^4)/3) + (Bb^5x^6)/6$

sympy [A] time = 6.64, size = 129, normalized size = 1.13

$$\frac{Bb^5x^6}{6} + 5ab^3(Ab + 2Ba)\log(x) + x^3\left(\frac{Ab^5}{3} + \frac{5Bab^4}{3}\right) + \frac{-3Aa^5 + x^9(-120Aa^2b^3 - 120Ba^3b^2) + x^6(-60Aa^3b^2 - 30Ba^4b) + x^3(-20Aa^4b - 4Ba^5)}{36x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**13,x)

[Out] $Bb^5x^6/6 + 5a^3b^3(Ab + 2Ba)\log(x) + x^3(Ab^5/3 + 5Bab^4/3) + (-3Aa^5 + x^9(-120Aa^2b^3 - 120Ba^3b^2) + x^6(-60Aa^3b^2 - 30Ba^4b) + x^3(-20Aa^4b - 4Ba^5))/(36x^{12})$

$$3.46 \quad \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} dx$$

Optimal. Leaf size=115

$$-\frac{a^5 A}{13x^{13}} - \frac{a^4(aB+5Ab)}{10x^{10}} - \frac{5a^3b(aB+2Ab)}{7x^7} - \frac{5a^2b^2(aB+Ab)}{2x^4} + \frac{1}{2}b^4x^2(5aB+Ab) - \frac{5ab^3(2aB+Ab)}{x} + \frac{1}{5}b^5Bx^5$$

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{5a^2b^2(aB+Ab)}{2x^4} - \frac{a^4(aB+5Ab)}{10x^{10}} - \frac{5a^3b(aB+2Ab)}{7x^7} - \frac{a^5 A}{13x^{13}} + \frac{1}{2}b^4x^2(5aB+Ab) - \frac{5ab^3(2aB+Ab)}{x} + \frac{1}{5}b^5Bx^5$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^14,x]

[Out] -(a^5*A)/(13*x^13) - (a^4*(5*A*b + a*B))/(10*x^10) - (5*a^3*b*(2*A*b + a*B))/(7*x^7) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A*b + 2*a*B))/x + (b^4*(A*b + 5*a*B)*x^2)/2 + (b^5*B*x^5)/5

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^(m*(a + b*x^n)^p*(c + d*x^n)^q], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} dx &= \int \left(\frac{a^5 A}{x^{14}} + \frac{a^4(5Ab+aB)}{x^{11}} + \frac{5a^3b(2Ab+aB)}{x^8} + \frac{10a^2b^2(Ab+aB)}{x^5} + \frac{5ab^3(Ab+2aB)}{x^2} \right) dx \\ &= -\frac{a^5 A}{13x^{13}} - \frac{a^4(5Ab+aB)}{10x^{10}} - \frac{5a^3b(2Ab+aB)}{7x^7} - \frac{5a^2b^2(Ab+aB)}{2x^4} - \frac{5ab^3(Ab+2aB)}{x} \end{aligned}$$

Mathematica [A] time = 0.05, size = 117, normalized size = 1.02

$$\frac{a^5(70A+91Bx^3)+65a^4bx^3(7A+10Bx^3)+325a^3b^2x^6(4A+7Bx^3)+2275a^2b^3x^9(A+4Bx^3)-2275ab^4x^{12}(Bx^3-2A)-91b^5x^{15}(5A+2Bx^3)}{910x^{13}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^14,x]

[Out] $-1/910*(-2275*a*b^4*x^{12}*(-2*A + B*x^3) - 91*b^5*x^{15}*(5*A + 2*B*x^3) + 2275*a^2*b^3*x^9*(A + 4*B*x^3) + 325*a^3*b^2*x^6*(4*A + 7*B*x^3) + 65*a^4*b*x^3*(7*A + 10*B*x^3) + a^5*(70*A + 91*B*x^3))/x^{13}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{14}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^14,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^14, x]

fricas [A] time = 0.64, size = 121, normalized size = 1.05

$$\frac{182 B b^5 x^{18} + 455 (5 B a b^4 + A b^5) x^{15} - 4550 (2 B a^2 b^3 + A a b^4) x^{12} - 2275 (B a^3 b^2 + A a^2 b^3) x^9 - 650 (B a^4 b + 2 A a^3 b^2) x^6 - 70 A a^5 - 91 (B a^5 + 5 A a^4 b) x^3}{910 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="fricas")

[Out] $1/910*(182*B*b^5*x^{18} + 455*(5*B*a*b^4 + A*b^5)*x^{15} - 4550*(2*B*a^2*b^3 + A*a*b^4)*x^{12} - 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 650*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 70*A*a^5 - 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^{13}$

giac [A] time = 0.16, size = 128, normalized size = 1.11

$$\frac{1}{5} B b^5 x^5 + \frac{5}{2} B a b^4 x^2 + \frac{1}{2} A b^5 x^2 - \frac{9100 B a^2 b^3 x^{12} + 4550 A a b^4 x^{12} + 2275 B a^3 b^2 x^9 + 2275 A a^2 b^3 x^9 + 650 B a^4 b x^6 + 1300 A a^3 b^2 x^6 + 91 B a^5 x^3 + 455 A a^4 b x^3 + 70 A a^5}{910 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="giac")

[Out] $1/5*B*b^5*x^5 + 5/2*B*a*b^4*x^2 + 1/2*A*b^5*x^2 - 1/910*(9100*B*a^2*b^3*x^{12} + 4550*A*a*b^4*x^{12} + 2275*B*a^3*b^2*x^9 + 2275*A*a^2*b^3*x^9 + 650*B*a^4*b*x^6 + 1300*A*a^3*b^2*x^6 + 91*B*a^5*x^3 + 455*A*a^4*b*x^3 + 70*A*a^5)/x^{13}$

maple [A] time = 0.06, size = 107, normalized size = 0.93

$$\frac{B b^5 x^5}{5} + \frac{A b^5 x^2}{2} + \frac{5 B a b^4 x^2}{2} - \frac{5 (A b + 2 B a) a b^3}{x} - \frac{5 (A b + B a) a^2 b^2}{2 x^4} - \frac{5 (2 A b + B a) a^3 b}{7 x^7} - \frac{(5 A b + B a) a^4}{10 x^{10}} - \frac{A a^5}{13 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^5*(B*x^3+A)/x^14,x)`

[Out] $\frac{1}{5}b^5x^5 + \frac{1}{2}A^2x^2b^5 + \frac{5}{2}Bx^2ab^4 - \frac{5}{2}a^2b^2(Ab+Ba)/x^4 - \frac{1}{13}a^5A/x^{13} - \frac{5}{7}a^3b(2Ab+Ba)/x^7 - 5a^2b^3(Ab+2Ba)/x - \frac{1}{10}a^4(5Ab+Ba)/x^{10}$

maxima [A] time = 0.56, size = 122, normalized size = 1.06

$$\frac{1}{5}Bb^5x^5 + \frac{1}{2}(5Bab^4 + Ab^5)x^2 - \frac{4550(2Ba^2b^3 + Aab^4)x^{12} + 2275(Ba^3b^2 + Aa^2b^3)x^9 + 650(Ba^4b + 2Aa^3b^2)x^6 + 70Aa^5 + 91(Ba^5 + 5Aa^4b)x^3}{910x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="maxima")`

[Out] $\frac{1}{5}Bb^5x^5 + \frac{1}{2}(5Bab^4 + Ab^5)x^2 - \frac{1}{910}(4550(2Ba^2b^3 + Aa^2b^4)x^{12} + 2275(Ba^3b^2 + Aa^2b^3)x^9 + 650(Ba^4b + 2Aa^3b^2)x^6 + 70Aa^5 + 91(Ba^5 + 5Aa^4b)x^3)/x^{13}$

mupad [B] time = 2.37, size = 123, normalized size = 1.07

$$x^2 \left(\frac{Ab^5}{2} + \frac{5Bab^4}{2} \right) - \frac{\frac{Aa^5}{13} + x^{12}(10Ba^2b^3 + 5Aab^4) + x^6 \left(\frac{5Ba^4b}{7} + \frac{10Aa^3b^2}{7} \right) + x^3 \left(\frac{Ba^5}{10} + \frac{Ab^4}{2} \right) + x^9 \left(\frac{5Ba^3b^2}{2} + \frac{5Aa^2b^3}{2} \right)}{x^{13}} + \frac{Bb^5x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^5)/x^14,x)`

[Out] $x^2 * ((Ab^5)/2 + (5Bab^4)/2) - ((Aa^5)/13 + x^{12} * (10Ba^2b^3 + 5Aa^2b^4) + x^6 * ((10Aa^3b^2)/7 + (5Ba^4b)/7) + x^3 * ((Ba^5)/10 + (Aa^4b)/2) + x^9 * ((5Aa^2b^3)/2 + (5Ba^3b^2)/2) / x^{13} + (Bb^5x^5)/5$

sympy [A] time = 10.76, size = 134, normalized size = 1.17

$$\frac{Bb^5x^5}{5} + x^2 \left(\frac{Ab^5}{2} + \frac{5Bab^4}{2} \right) + \frac{-70Aa^5 + x^{12}(-4550Aab^4 - 9100Ba^2b^3) + x^9(-2275Aa^2b^3 - 2275Ba^3b^2) + x^6(-1300Aa^3b^2 - 650Ba^4b) + x^3(-455Aa^4b - 91Ba^5)}{910x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**5*(B*x**3+A)/x**14,x)`

[Out] $Bb^5x^5/5 + x^2 * (Ab^5/2 + 5Bab^4/2) + (-70Aa^5 + x^{12} * (-4550Aa^2b^3 - 9100Ba^2b^3) + x^9 * (-2275Aa^2b^3 - 2275Ba^3b^2) + x^6 * (-1300Aa^3b^2 - 650Ba^4b) + x^3 * (-455Aa^4b - 91Ba^5)) / (910 * x^{13})$

$$3.47 \quad \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}} dx$$

Optimal. Leaf size=110

$$\frac{a^5 A}{14x^{14}} - \frac{a^4(aB + 5Ab)}{11x^{11}} - \frac{5a^3b(aB + 2Ab)}{8x^8} - \frac{2a^2b^2(aB + Ab)}{x^5} + b^4x(5aB + Ab) - \frac{5ab^3(2aB + Ab)}{2x^2} + \frac{1}{4}b^5Bx^4$$

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{2a^2b^2(aB + Ab)}{x^5} - \frac{a^4(aB + 5Ab)}{11x^{11}} - \frac{5a^3b(aB + 2Ab)}{8x^8} - \frac{a^5 A}{14x^{14}} - \frac{5ab^3(2aB + Ab)}{2x^2} + b^4x(5aB + Ab) + \frac{1}{4}b^5Bx^4$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^15,x]

[Out] -(a^5*A)/(14*x^14) - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B))/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2) + b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}} dx &= \int \left(b^4(Ab + 5aB) + \frac{a^5 A}{x^{15}} + \frac{a^4(5Ab + aB)}{x^{12}} + \frac{5a^3b(2Ab + aB)}{x^9} + \frac{10a^2b^2(Ab + aB)}{x^6} \right. \\ &\quad \left. - \frac{a^5 A}{14x^{14}} - \frac{a^4(5Ab + aB)}{11x^{11}} - \frac{5a^3b(2Ab + aB)}{8x^8} - \frac{2a^2b^2(Ab + aB)}{x^5} - \frac{5ab^3(Ab + 2aB)}{2x^2} \right) dx \end{aligned}$$

Mathematica [A] time = 0.07, size = 110, normalized size = 1.00

$$-\frac{a^5 A}{14x^{14}} - \frac{a^4(aB + 5Ab)}{11x^{11}} - \frac{5a^3b(aB + 2Ab)}{8x^8} - \frac{2a^2b^2(aB + Ab)}{x^5} + b^4x(5aB + Ab) - \frac{5ab^3(2aB + Ab)}{2x^2} + \frac{1}{4}b^5Bx^4$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^15,x]

[Out] $-\frac{1}{14} \frac{a^5 A}{x^{14}} - \frac{a^4 (5 A b + a B)}{(11 x^{11})} - \frac{(5 a^3 b (2 A b + a B))}{(8 x^8)} - \frac{(2 a^2 b^2 (A b + a B))}{x^5} - \frac{(5 a b^3 (A b + 2 a B))}{(2 x^2)} + b^4 (A b + 5 a B) x + \frac{b^5 B x^4}{4}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{15}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^15,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^15, x]

fricas [A] time = 0.56, size = 121, normalized size = 1.10

$$\frac{154 B b^5 x^{18} + 616 (5 B a b^4 + A b^5) x^{15} - 1540 (2 B a^2 b^3 + A a b^4) x^{12} - 1232 (B a^3 b^2 + A a^2 b^3) x^9 - 385 (B a^4 b + 2 A a^3 b^2) x^6 - 44 A a^5 - 56 (B a^5 + 5 A a^4 b) x^3}{616 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="fricas")

[Out] $\frac{1}{616} (154 B b^5 x^{18} + 616 (5 B a b^4 + A b^5) x^{15} - 1540 (2 B a^2 b^3 + A a b^4) x^{12} - 1232 (B a^3 b^2 + A a^2 b^3) x^9 - 385 (B a^4 b + 2 A a^3 b^2) x^6 - 44 A a^5 - 56 (B a^5 + 5 A a^4 b) x^3) / x^{14}$

giac [A] time = 0.16, size = 123, normalized size = 1.12

$$\frac{\frac{1}{4} B b^5 x^4 + 5 B a b^4 x + A b^5 x - \frac{3080 B a^2 b^3 x^{12} + 1540 A a b^4 x^{12} + 1232 B a^3 b^2 x^9 + 1232 A a^2 b^3 x^9 + 385 B a^4 b x^6 + 770 A a^3 b^2 x^6 + 56 B a^5 x^3 + 280 A a^4 b x^3 + 44 A a^5}{616 x^{14}}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="giac")

[Out] $\frac{1}{4} B b^5 x^4 + 5 B a b^4 x + A b^5 x - \frac{1}{616} (3080 B a^2 b^3 x^{12} + 1540 A a b^4 x^{12} + 1232 B a^3 b^2 x^9 + 1232 A a^2 b^3 x^9 + 385 B a^4 b x^6 + 770 A a^3 b^2 x^6 + 56 B a^5 x^3 + 280 A a^4 b x^3 + 44 A a^5) / x^{14}$

maple [A] time = 0.05, size = 102, normalized size = 0.93

$$\frac{B b^5 x^4}{4} + A b^5 x + 5 B a b^4 x - \frac{5 (A b + 2 B a) a b^3}{2 x^2} - \frac{2 (A b + B a) a^2 b^2}{x^5} - \frac{5 (2 A b + B a) a^3 b}{8 x^8} - \frac{(5 A b + B a) a^4}{11 x^{11}} - \frac{A a^5}{14 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

$$3.48 \quad \int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}} dx$$

Optimal. Leaf size=113

$$\frac{a^5 A}{15x^{15}} - \frac{a^4(aB + 5Ab)}{12x^{12}} - \frac{5a^3b(aB + 2Ab)}{9x^9} - \frac{5a^2b^2(aB + Ab)}{3x^6} + b^4 \log(x)(5aB + Ab) - \frac{5ab^3(2aB + Ab)}{3x^3} + \frac{1}{3}b^5 Bx^3$$

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 76}

$$-\frac{5a^2b^2(aB + Ab)}{3x^6} - \frac{a^4(aB + 5Ab)}{12x^{12}} - \frac{5a^3b(aB + 2Ab)}{9x^9} - \frac{a^5 A}{15x^{15}} - \frac{5ab^3(2aB + Ab)}{3x^3} + b^4 \log(x)(5aB + Ab) + \frac{1}{3}b^5 Bx^3$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^16,x]

[Out] -(a^5*A)/(15*x^15) - (a^4*(5*A*b + a*B))/(12*x^12) - (5*a^3*b*(2*A*b + a*B))/(9*x^9) - (5*a^2*b^2*(A*b + a*B))/(3*x^6) - (5*a*b^3*(A*b + 2*a*B))/(3*x^3) + (b^5*B*x^3)/3 + b^4*(A*b + 5*a*B)*Log[x]

Rule 76

Int[((d_)*(x_))^(n_)*((a_) + (b_)*(x_))*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{16}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^5 (A + Bx)}{x^6} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(b^5 B + \frac{a^5 A}{x^6} + \frac{a^4 (5Ab + aB)}{x^5} + \frac{5a^3 b (2Ab + aB)}{x^4} + \frac{10a^2 b^2 (Ab + aB)}{x^3} \right) dx, x, x^3 \right) \\ &= -\frac{a^5 A}{15x^{15}} - \frac{a^4 (5Ab + aB)}{12x^{12}} - \frac{5a^3 b (2Ab + aB)}{9x^9} - \frac{5a^2 b^2 (Ab + aB)}{3x^6} - \frac{5ab^3 (Ab + 2aB)}{3x^3} \end{aligned}$$

Mathematica [A] time = 0.07, size = 116, normalized size = 1.03

$$b^4 \log(x)(5aB + Ab) - \frac{3a^5 (4A + 5Bx^3) + 25a^4 bx^3 (3A + 4Bx^3) + 100a^3 b^2 x^6 (2A + 3Bx^3) + 300a^2 b^3 x^9 (A + 2Bx^3) + 300aAb^4 x^{12} - 60b^5 Bx^{18}}{180x^{15}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^16,x]

[Out] -1/180*(300*a*A*b^4*x^12 - 60*b^5*B*x^18 + 300*a^2*b^3*x^9*(A + 2*B*x^3) + 100*a^3*b^2*x^6*(2*A + 3*B*x^3) + 25*a^4*b*x^3*(3*A + 4*B*x^3) + 3*a^5*(4*A + 5*B*x^3))/x^15 + b^4*(A*b + 5*a*B)*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{16}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^16,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^16, x]

fricas [A] time = 0.67, size = 123, normalized size = 1.09

$$\frac{60 Bb^5 x^{18} + 180 (5 Bab^4 + Ab^5) x^{15} \log(x) - 300 (2 Ba^2 b^3 + Aab^4) x^{12} - 300 (Ba^3 b^2 + Aa^2 b^3) x^9 - 100 (Ba^4 b + 2 Aa^3 b^2) x^6 - 12 Aa^5 - 15 (Ba^5 + 5 Aa^4 b) x^3}{180 x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="fricas")

[Out] 1/180*(60*B*b^5*x^18 + 180*(5*B*a*b^4 + A*b^5)*x^15*log(x) - 300*(2*B*a^2*b^3 + A*a*b^4)*x^12 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 100*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 12*A*a^5 - 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15

giac [A] time = 0.15, size = 145, normalized size = 1.28

$$\frac{1}{3} B b^5 x^3 + (5 B a b^4 + A b^5) \log(|x|) - \frac{685 B a b^4 x^{15} + 137 A b^5 x^{15} + 600 B a^2 b^3 x^{12} + 300 A a b^4 x^{12} + 300 B a^3 b^2 x^9 + 300 A a^2 b^3 x^9 + 100 B a^4 b x^6 + 200 A a^3 b^2 x^6 + 15 B a^5 x^3 + 75 A a^4 b x^3 + 12 A a^5}{180 x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="giac")

[Out] $\frac{1}{3} B b^5 x^3 + (5 B a b^4 + A b^5) \log(\text{abs}(x)) - \frac{1}{180} (685 B a b^4 x^{15} + 137 A b^5 x^{15} + 600 B a^2 b^3 x^{12} + 300 A a b^4 x^{12} + 300 B a^3 b^2 x^9 + 300 A a^2 b^3 x^9 + 100 B a^4 b x^6 + 200 A a^3 b^2 x^6 + 15 B a^5 x^3 + 75 A a^4 b x^3 + 12 A a^5) / x^{15}$

maple [A] time = 0.05, size = 123, normalized size = 1.09

$$\frac{B b^5 x^3}{3} + A b^5 \ln(x) + 5 B a b^4 \ln(x) - \frac{5 A a b^4}{3 x^3} - \frac{10 B a^2 b^3}{3 x^3} - \frac{5 A a^2 b^3}{3 x^6} - \frac{5 B a^3 b^2}{3 x^6} - \frac{10 A a^3 b^2}{9 x^9} - \frac{5 B a^4 b}{9 x^9} - \frac{5 A a^4 b}{12 x^{12}} - \frac{B a^5}{12 x^{12}} - \frac{A a^5}{15 x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^16,x)

[Out] $\frac{1}{3} b^5 B x^3 - \frac{5}{12} a^4 / x^{12} A b - \frac{1}{12} a^5 / x^{12} B - \frac{5}{3} a b^4 / x^3 A - \frac{10}{3} a^2 b^3 / x^3 B - \frac{1}{15} a^5 A / x^{15} - \frac{10}{9} a^3 b^2 / x^9 A - \frac{5}{9} a^4 b / x^9 B - \frac{5}{3} b^3 a^2 / x^6 A - \frac{5}{3} b^2 a^3 / x^6 B + A \ln(x) b^5 + 5 B \ln(x) a b^4$

maxima [A] time = 0.59, size = 123, normalized size = 1.09

$$\frac{1}{3} B b^5 x^3 + \frac{1}{3} (5 B a b^4 + A b^5) \log(x^3) - \frac{300 (2 B a^2 b^3 + A a b^4) x^{12} + 300 (B a^3 b^2 + A a^2 b^3) x^9 + 100 (B a^4 b + 2 A a^3 b^2) x^6 + 12 A a^5 + 15 (B a^5 + 5 A a^4 b) x^3}{180 x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="maxima")

[Out] $\frac{1}{3} B b^5 x^3 + \frac{1}{3} (5 B a b^4 + A b^5) \log(x^3) - \frac{1}{180} (300 (2 B a^2 b^3 + A a b^4) x^{12} + 300 (B a^3 b^2 + A a^2 b^3) x^9 + 100 (B a^4 b + 2 A a^3 b^2) x^6 + 12 A a^5 + 15 (B a^5 + 5 A a^4 b) x^3) / x^{15}$

mupad [B] time = 0.08, size = 121, normalized size = 1.07

$$\ln(x) (A b^5 + 5 B a b^4) - \frac{\frac{A a^5}{15} + x^{12} \left(\frac{10 B a^2 b^3}{3} + \frac{5 A a b^4}{3} \right) + x^6 \left(\frac{5 B a^4 b}{9} + \frac{10 A a^3 b^2}{9} \right) + x^3 \left(\frac{B a^5}{12} + \frac{5 A a b^4}{12} \right) + x^9 \left(\frac{5 B a^3 b^2}{3} + \frac{5 A a^2 b^3}{3} \right)}{x^{15}} + \frac{B b^5 x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^16,x)

[Out] $\log(x) \cdot (A \cdot b^5 + 5 \cdot B \cdot a \cdot b^4) - \left(\frac{A \cdot a^5}{15} + x^{12} \cdot \left(\frac{10 \cdot B \cdot a^2 \cdot b^3}{3} + \frac{5 \cdot A \cdot a \cdot b^4}{3} \right) + x^6 \cdot \left(\frac{10 \cdot A \cdot a^3 \cdot b^2}{9} + \frac{5 \cdot B \cdot a^4 \cdot b}{9} \right) + x^3 \cdot \left(\frac{B \cdot a^5}{12} + \frac{5 \cdot A \cdot a^4 \cdot b}{12} \right) + x^9 \cdot \left(\frac{5 \cdot A \cdot a^2 \cdot b^3}{3} + \frac{5 \cdot B \cdot a^3 \cdot b^2}{3} \right) \right) / x^{15} + \frac{B \cdot b^5 \cdot x^3}{3}$

sympy [A] time = 29.30, size = 129, normalized size = 1.14

$$\frac{Bb^5x^3}{3} + b^4(Ab + 5Ba)\log(x) + \frac{-12Aa^5 + x^{12}(-300Aab^4 - 600Ba^2b^3) + x^9(-300Aa^2b^3 - 300Ba^3b^2) + x^6(-200Aa^3b^2 - 100Ba^4b) + x^3(-75Aa^4b - 15Ba^5)}{180x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**16,x)

[Out] $B \cdot b^5 \cdot x^3 / 3 + b^4 \cdot (A \cdot b + 5 \cdot B \cdot a) \cdot \log(x) + (-12 \cdot A \cdot a^5 + x^{12} \cdot (-300 \cdot A \cdot a \cdot b^4 - 600 \cdot B \cdot a^2 \cdot b^3) + x^9 \cdot (-300 \cdot A \cdot a^2 \cdot b^3 - 300 \cdot B \cdot a^3 \cdot b^2) + x^6 \cdot (-200 \cdot A \cdot a^3 \cdot b^2 - 100 \cdot B \cdot a^4 \cdot b) + x^3 \cdot (-75 \cdot A \cdot a^4 \cdot b - 15 \cdot B \cdot a^5)) / (180 \cdot x^{15})$

$$3.49 \quad \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}} dx$$

Optimal. Leaf size=115

$$-\frac{a^5 A}{16x^{16}} - \frac{a^4(aB + 5Ab)}{13x^{13}} - \frac{a^3b(aB + 2Ab)}{2x^{10}} - \frac{10a^2b^2(aB + Ab)}{7x^7} - \frac{b^4(5aB + Ab)}{x} - \frac{5ab^3(2aB + Ab)}{4x^4} + \frac{1}{2}b^5Bx^2$$

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{10a^2b^2(aB + Ab)}{7x^7} - \frac{a^4(aB + 5Ab)}{13x^{13}} - \frac{a^3b(aB + 2Ab)}{2x^{10}} - \frac{a^5 A}{16x^{16}} - \frac{5ab^3(2aB + Ab)}{4x^4} - \frac{b^4(5aB + Ab)}{x} + \frac{1}{2}b^5Bx^2$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^17,x]

[Out] -(a^5*A)/(16*x^16) - (a^4*(5*A*b + a*B))/(13*x^13) - (a^3*b*(2*A*b + a*B))/(2*x^10) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (5*a*b^3*(A*b + 2*a*B))/(4*x^4) - (b^4*(A*b + 5*a*B))/x + (b^5*B*x^2)/2

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}} dx &= \int \left(\frac{a^5 A}{x^{17}} + \frac{a^4(5Ab + aB)}{x^{14}} + \frac{5a^3b(2Ab + aB)}{x^{11}} + \frac{10a^2b^2(Ab + aB)}{x^8} + \frac{5ab^3(Ab + 2aB)}{x^5} \right. \\ &= \left. -\frac{a^5 A}{16x^{16}} - \frac{a^4(5Ab + aB)}{13x^{13}} - \frac{a^3b(2Ab + aB)}{2x^{10}} - \frac{10a^2b^2(Ab + aB)}{7x^7} - \frac{5ab^3(Ab + 2aB)}{4x^4} \right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 118, normalized size = 1.03

$$\frac{7a^5(13A + 16Bx^3) + 56a^4bx^3(10A + 13Bx^3) + 208a^3b^2x^6(7A + 10Bx^3) + 520a^2b^3x^9(4A + 7Bx^3) + 1820ab^4x^{12}(A + 4Bx^3) - 728b^5x^{15}(Bx^3 - 2A)}{1456x^{16}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^17, x]

[Out] $-1/1456*(-728*b^5*x^{15}*(-2*A + B*x^3) + 1820*a*b^4*x^{12}*(A + 4*B*x^3) + 520*a^2*b^3*x^9*(4*A + 7*B*x^3) + 208*a^3*b^2*x^6*(7*A + 10*B*x^3) + 56*a^4*b*x^3*(10*A + 13*B*x^3) + 7*a^5*(13*A + 16*B*x^3))/x^{16}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{17}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^17, x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^17, x]

fricas [A] time = 0.56, size = 121, normalized size = 1.05

$$\frac{728 B b^5 x^{18} - 1456 (5 B a b^4 + A b^5) x^{15} - 1820 (2 B a^2 b^3 + A a b^4) x^{12} - 2080 (B a^3 b^2 + A a^2 b^3) x^9 - 728 (B a^4 b + 2 A a^3 b^2) x^6 - 91 A a^5 - 112 (B a^5 + 5 A a^4 b) x^3}{1456 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17, x, algorithm="fricas")

[Out] $1/1456*(728*B*b^5*x^{18} - 1456*(5*B*a*b^4 + A*b^5)*x^{15} - 1820*(2*B*a^2*b^3 + A*a*b^4)*x^{12} - 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 728*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 91*A*a^5 - 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^{16}$

giac [A] time = 0.16, size = 128, normalized size = 1.11

$$\frac{1}{2} B b^5 x^2 - \frac{7280 B a b^4 x^{15} + 1456 A b^5 x^{15} + 3640 B a^2 b^3 x^{12} + 1820 A a b^4 x^{12} + 2080 B a^3 b^2 x^9 + 2080 A a^2 b^3 x^9 + 728 B a^4 b x^6 + 1456 A a^3 b^2 x^6 + 112 B a^5 x^3 + 560 A a^4 b x^3 + 91 A a^5}{1456 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17, x, algorithm="giac")

[Out] $1/2*B*b^5*x^2 - 1/1456*(7280*B*a*b^4*x^{15} + 1456*A*b^5*x^{15} + 3640*B*a^2*b^3*x^{12} + 1820*A*a*b^4*x^{12} + 2080*B*a^3*b^2*x^9 + 2080*A*a^2*b^3*x^9 + 728*B*a^4*b*x^6 + 1456*A*a^3*b^2*x^6 + 112*B*a^5*x^3 + 560*A*a^4*b*x^3 + 91*A*a^5)/x^{16}$

maple [A] time = 0.04, size = 104, normalized size = 0.90

$$\frac{B b^5 x^2}{2} - \frac{(A b + 5 B a) b^4}{x} - \frac{5 (A b + 2 B a) a b^3}{4 x^4} - \frac{10 (A b + B a) a^2 b^2}{7 x^7} - \frac{(2 A b + B a) a^3 b}{2 x^{10}} - \frac{(5 A b + B a) a^4}{13 x^{13}} - \frac{A a^5}{16 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^17,x)

[Out] $-1/16*a^5*A/x^{16}-1/13*a^4*(5*A*b+B*a)/x^{13}-1/2*a^3*b*(2*A*b+B*a)/x^{10}-10/7*a^2*b^2*(A*b+B*a)/x^7-5/4*a*b^3*(A*b+2*B*a)/x^4-b^4*(A*b+5*B*a)/x+1/2*b^5*B*x^2$

maxima [A] time = 0.50, size = 122, normalized size = 1.06

$$\frac{1}{2} B b^5 x^2 - \frac{1456(5 B a b^4 + A b^5) x^{15} + 1820(2 B a^2 b^3 + A a b^4) x^{12} + 2080(B a^3 b^2 + A a^2 b^3) x^9 + 728(B a^4 b + 2 A a^3 b^2) x^6 + 91 A a^5 + 112(B a^5 + 5 A a^4 b) x^3}{1456 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="maxima")

[Out] $1/2*B*b^5*x^2 - 1/1456*(1456*(5*B*a*b^4 + A*b^5)*x^{15} + 1820*(2*B*a^2*b^3 + A*a*b^4)*x^{12} + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 728*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 91*A*a^5 + 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^{16}$

mupad [B] time = 2.36, size = 121, normalized size = 1.05

$$\frac{B b^5 x^2}{2} - \frac{A a^5}{16} + x^6 \left(\frac{B a^4 b}{2} + A a^3 b^2 \right) + x^{12} \left(\frac{5 B a^2 b^3}{2} + \frac{5 A a b^4}{4} \right) + x^3 \left(\frac{B a^5}{13} + \frac{5 A b a^4}{13} \right) + x^{15} (A b^5 + 5 B a b^4) + x^9 \left(\frac{10 B a^3 b^2}{7} + \frac{10 A a^2 b^3}{7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^17,x)

[Out] $(B*b^5*x^2)/2 - ((A*a^5)/16 + x^6*(A*a^3*b^2 + (B*a^4*b)/2) + x^{12}*((5*B*a^2*b^3)/2 + (5*A*a*b^4)/4) + x^3*((B*a^5)/13 + (5*A*a^4*b)/13) + x^{15}*(A*b^5 + 5*B*a*b^4) + x^9*((10*A*a^2*b^3)/7 + (10*B*a^3*b^2)/7))/x^{16}$

sympy [A] time = 66.09, size = 134, normalized size = 1.17

$$\frac{B b^5 x^2}{2} + \frac{-91 A a^5 + x^{15}(-1456 A b^5 - 7280 B a b^4) + x^{12}(-1820 A a b^4 - 3640 B a^2 b^3) + x^9(-2080 A a^2 b^3 - 2080 B a^3 b^2) + x^6(-1456 A a^3 b^2 - 728 B a^4 b) + x^3(-560 A a^4 b - 112 B a^5)}{1456 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**17,x)

[Out] $B*b**5*x**2/2 + (-91*A*a**5 + x**15*(-1456*A*b**5 - 7280*B*a*b**4) + x**12*(-1820*A*a*b**4 - 3640*B*a**2*b**3) + x**9*(-2080*A*a**2*b**3 - 2080*B*a**3*b**2) + x**6*(-1456*A*a**3*b**2 - 728*B*a**4*b) + x**3*(-560*A*a**4*b - 112*B*a**5))/(1456*x**16)$

$$3.50 \quad \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}} dx$$

Optimal. Leaf size=110

$$\frac{a^5 A}{17x^{17}} - \frac{a^4(aB + 5Ab)}{14x^{14}} - \frac{5a^3b(aB + 2Ab)}{11x^{11}} - \frac{5a^2b^2(aB + Ab)}{4x^8} - \frac{b^4(5aB + Ab)}{2x^2} - \frac{ab^3(2aB + Ab)}{x^5} + b^5 Bx$$

Rubi [A] time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{5a^2b^2(aB + Ab)}{4x^8} - \frac{a^4(aB + 5Ab)}{14x^{14}} - \frac{5a^3b(aB + 2Ab)}{11x^{11}} - \frac{a^5 A}{17x^{17}} - \frac{ab^3(2aB + Ab)}{x^5} - \frac{b^4(5aB + Ab)}{2x^2} + b^5 Bx$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^18,x]

[Out] $-(a^5 A)/(17*x^{17}) - (a^4*(5*A*b + a*B))/(14*x^{14}) - (5*a^3*b*(2*A*b + a*B))/(11*x^{11}) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5 - (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x$

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}} dx &= \int \left(b^5 B + \frac{a^5 A}{x^{18}} + \frac{a^4(5Ab + aB)}{x^{15}} + \frac{5a^3b(2Ab + aB)}{x^{12}} + \frac{10a^2b^2(Ab + aB)}{x^9} + \frac{5ab^3(Ab + 2aB)}{x^6} \right. \\ &\quad \left. - \frac{a^5 A}{17x^{17}} - \frac{a^4(5Ab + aB)}{14x^{14}} - \frac{5a^3b(2Ab + aB)}{11x^{11}} - \frac{5a^2b^2(Ab + aB)}{4x^8} - \frac{ab^3(Ab + 2aB)}{x^5} \right) dx \end{aligned}$$

Mathematica [A] time = 0.05, size = 110, normalized size = 1.00

$$-\frac{a^5 A}{17x^{17}} - \frac{a^4(aB + 5Ab)}{14x^{14}} - \frac{5a^3b(aB + 2Ab)}{11x^{11}} - \frac{5a^2b^2(aB + Ab)}{4x^8} - \frac{b^4(5aB + Ab)}{2x^2} - \frac{ab^3(2aB + Ab)}{x^5} + b^5 Bx$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^18,x]

[Out] $-1/17*(a^5*A)/x^{17} - (a^4*(5*A*b + a*B))/(14*x^{14}) - (5*a^3*b*(2*A*b + a*B))/(11*x^{11}) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5 - (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{18}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^18,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^18, x]

fricas [A] time = 0.67, size = 121, normalized size = 1.10

$$\frac{5236 B b^5 x^{18} - 2618 (5 B a b^4 + A b^5) x^{15} - 5236 (2 B a^2 b^3 + A a b^4) x^{12} - 6545 (B a^3 b^2 + A a^2 b^3) x^9 - 2380 (B a^4 b + 2 A a^3 b^2) x^6 - 308 A a^5 - 374 (B a^5 + 5 A a^4 b) x^3}{5236 x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="fricas")

[Out] $1/5236*(5236*B*b^5*x^{18} - 2618*(5*B*a*b^4 + A*b^5)*x^{15} - 5236*(2*B*a^2*b^3 + A*a*b^4)*x^{12} - 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 2380*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 308*A*a^5 - 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^{17}$

giac [A] time = 0.17, size = 125, normalized size = 1.14

$$B b^5 x - \frac{13090 B a b^4 x^{15} + 2618 A b^5 x^{15} + 10472 B a^2 b^3 x^{12} + 5236 A a b^4 x^{12} + 6545 B a^3 b^2 x^9 + 6545 A a^2 b^3 x^9 + 2380 B a^4 b x^6 + 4760 A a^3 b^2 x^6 + 374 B a^5 x^3 + 1870 A a^4 b x^3 + 308 A a^5}{5236 x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="giac")

[Out] $B*b^5*x - 1/5236*(13090*B*a*b^4*x^{15} + 2618*A*b^5*x^{15} + 10472*B*a^2*b^3*x^{12} + 5236*A*a*b^4*x^{12} + 6545*B*a^3*b^2*x^9 + 6545*A*a^2*b^3*x^9 + 2380*B*a^4*b*x^6 + 4760*A*a^3*b^2*x^6 + 374*B*a^5*x^3 + 1870*A*a^4*b*x^3 + 308*A*a^5)/x^{17}$

maple [A] time = 0.05, size = 101, normalized size = 0.92

$$B b^5 x - \frac{(A b + 5 B a) b^4}{2 x^2} - \frac{(A b + 2 B a) a b^3}{x^5} - \frac{5 (A b + B a) a^2 b^2}{4 x^8} - \frac{5 (2 A b + B a) a^3 b}{11 x^{11}} - \frac{(5 A b + B a) a^4}{14 x^{14}} - \frac{A a^5}{17 x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^5*(B*x^3+A)/x^18,x)`

[Out] $-1/17*a^5*A/x^{17}-1/14*a^4*(5*A*b+B*a)/x^{14}-5/11*a^3*b*(2*A*b+B*a)/x^{11}-5/4*a^2*b^2*(A*b+B*a)/x^8-a*b^3*(A*b+2*B*a)/x^5-1/2*b^4*(A*b+5*B*a)/x^2+b^5*B*x$

maxima [A] time = 0.52, size = 119, normalized size = 1.08

$$Bb^5x - \frac{2618(5Bab^4 + Ab^5)x^{15} + 5236(2Ba^2b^3 + Aab^4)x^{12} + 6545(Ba^3b^2 + Aa^2b^3)x^9 + 2380(Ba^4b + 2Aa^3b^2)x^6 + 308Aa^5 + 374(Ba^5 + 5Aa^4b)x^3}{5236x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="maxima")`

[Out] $B*b^5*x - 1/5236*(2618*(5*B*a*b^4 + A*b^5)*x^{15} + 5236*(2*B*a^2*b^3 + A*a*b^4)*x^{12} + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 2380*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 308*A*a^5 + 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^{17}$

mupad [B] time = 0.08, size = 119, normalized size = 1.08

$$Bb^5x - \frac{\frac{Aa^5}{17} + x^{12}(2Ba^2b^3 + Aab^4) + x^6\left(\frac{5Ba^4b}{11} + \frac{10Aa^3b^2}{11}\right) + x^3\left(\frac{Ba^5}{14} + \frac{5Aab^4}{14}\right) + x^{15}\left(\frac{Ab^5}{2} + \frac{5Bab^4}{2}\right) + x^9\left(\frac{5Ba^3b^2}{4} + \frac{5Aa^2b^3}{4}\right)}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^5)/x^18,x)`

[Out] $B*b^5*x - ((A*a^5)/17 + x^{12}*(2*B*a^2*b^3 + A*a*b^4) + x^6*((10*A*a^3*b^2)/11 + (5*B*a^4*b)/11) + x^3*((B*a^5)/14 + (5*A*a^4*b)/14) + x^{15}*((A*b^5)/2 + (5*B*a*b^4)/2) + x^9*((5*A*a^2*b^3)/4 + (5*B*a^3*b^2)/4))/x^{17}$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**5*(B*x**3+A)/x**18,x)`

[Out] Timed out

$$3.51 \quad \int \frac{(a+bx^3)^5(A+Bx^3)}{x^{19}} dx$$

Optimal. Leaf size=91

$$-\frac{a^5B}{15x^{15}} - \frac{5a^4bB}{12x^{12}} - \frac{10a^3b^2B}{9x^9} - \frac{5a^2b^3B}{3x^6} - \frac{A(a+bx^3)^6}{18ax^{18}} - \frac{5ab^4B}{3x^3} + b^5B \log(x)$$

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {446, 78, 43}

$$-\frac{5a^2b^3B}{3x^6} - \frac{10a^3b^2B}{9x^9} - \frac{5a^4bB}{12x^{12}} - \frac{a^5B}{15x^{15}} - \frac{A(a+bx^3)^6}{18ax^{18}} - \frac{5ab^4B}{3x^3} + b^5B \log(x)$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^19,x]

[Out] -(a^5*B)/(15*x^15) - (5*a^4*b*B)/(12*x^12) - (10*a^3*b^2*B)/(9*x^9) - (5*a^2*b^3*B)/(3*x^6) - (5*a*b^4*B)/(3*x^3) - (A*(a + b*x^3)^6)/(18*a*x^18) + b^5*B*Log[x]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1))]/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[

b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{19}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^5 (A + Bx)}{x^7} dx, x, x^3 \right) \\
 &= -\frac{A(a + bx^3)^6}{18ax^{18}} + \frac{1}{3} B \text{Subst} \left(\int \frac{(a + bx)^5}{x^6} dx, x, x^3 \right) \\
 &= -\frac{A(a + bx^3)^6}{18ax^{18}} + \frac{1}{3} B \text{Subst} \left(\int \left(\frac{a^5}{x^6} + \frac{5a^4b}{x^5} + \frac{10a^3b^2}{x^4} + \frac{10a^2b^3}{x^3} + \frac{5ab^4}{x^2} + \frac{b^5}{x} \right) dx, x, x^3 \right) \\
 &= -\frac{a^5B}{15x^{15}} - \frac{5a^4bB}{12x^{12}} - \frac{10a^3b^2B}{9x^9} - \frac{5a^2b^3B}{3x^6} - \frac{5ab^4B}{3x^3} - \frac{A(a + bx^3)^6}{18ax^{18}} + b^5B \log(x)
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 121, normalized size = 1.33

$$\frac{2a^5(5A + 6Bx^3) + 15a^4bx^3(4A + 5Bx^3) + 50a^3b^2x^6(3A + 4Bx^3) + 100a^2b^3x^9(2A + 3Bx^3) + 150ab^4x^{12}(A + 2Bx^3) + 60Ab^5x^{15} - 180b^5Bx^{18} \log(x)}{180x^{18}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^19, x]

[Out] -1/180*(60*A*b^5*x^15 + 150*a*b^4*x^12*(A + 2*B*x^3) + 100*a^2*b^3*x^9*(2*A + 3*B*x^3) + 50*a^3*b^2*x^6*(3*A + 4*B*x^3) + 15*a^4*b*x^3*(4*A + 5*B*x^3) + 2*a^5*(5*A + 6*B*x^3) - 180*b^5*B*x^18*Log[x])/x^18

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{19}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^19, x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^19, x]

fricas [A] time = 0.54, size = 123, normalized size = 1.35

$$\frac{180Bb^5x^{18} \log(x) - 60(5Bab^4 + Ab^5)x^{15} - 150(2Ba^2b^3 + Aab^4)x^{12} - 200(Ba^3b^2 + Aa^2b^3)x^9 - 75(Ba^4b + 2Aa^3b^2)x^6 - 10Aa^5 - 12(Ba^5 + 5Aa^4b)x^3}{180x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="fricas")

[Out] 1/180*(180*B*b^5*x^18*log(x) - 60*(5*B*a*b^4 + A*b^5)*x^15 - 150*(2*B*a^2*b^3 + A*a*b^4)*x^12 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 75*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 10*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18

giac [A] time = 0.16, size = 136, normalized size = 1.49

$$Bb^5 \log(|x|) - \frac{147Bb^5x^{18} + 300Bab^4x^{15} + 60Ab^5x^{15} + 300Ba^2b^3x^{12} + 150Aab^4x^{12} + 200Ba^3b^2x^9 + 200Aa^2b^3x^9 + 75Ba^4bx^6 + 150Aa^3b^2x^6 + 12Ba^5x^3 + 60Aa^4bx^3 + 10Aa^5}{180x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="giac")

[Out] B*b^5*log(abs(x)) - 1/180*(147*B*b^5*x^18 + 300*B*a*b^4*x^15 + 60*A*b^5*x^15 + 300*B*a^2*b^3*x^12 + 150*A*a*b^4*x^12 + 200*B*a^3*b^2*x^9 + 200*A*a^2*b^3*x^9 + 75*B*a^4*b*x^6 + 150*A*a^3*b^2*x^6 + 12*B*a^5*x^3 + 60*A*a^4*b*x^3 + 10*A*a^5)/x^18

maple [A] time = 0.05, size = 124, normalized size = 1.36

$$Bb^5 \ln(x) - \frac{Ab^5}{3x^3} - \frac{5Bab^4}{3x^3} - \frac{5Aab^4}{6x^6} - \frac{5Ba^2b^3}{3x^6} - \frac{10Aa^2b^3}{9x^9} - \frac{10Ba^3b^2}{9x^9} - \frac{5Aa^3b^2}{6x^{12}} - \frac{5Ba^4b}{12x^{12}} - \frac{Aa^4b}{3x^{15}} - \frac{Ba^5}{15x^{15}} - \frac{Aa^5}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^19,x)

[Out] -5/6*a^3*b^2/x^12*A-5/12*a^4*b*B/x^12-1/3*b^5/x^3*A-5/3*a*b^4*B/x^3-1/18*A*a^5/x^18-1/3*a^4/x^15*A*b-1/15*a^5*B/x^15-10/9*b^3*a^2/x^9*A-10/9*a^3*b^2*B/x^9-5/6*a*b^4/x^6*A-5/3*a^2*b^3*B/x^6+b^5*B*ln(x)

maxima [A] time = 0.51, size = 123, normalized size = 1.35

$$\frac{1}{3}Bb^5 \log(x^3) - \frac{60(5Bab^4 + Ab^5)x^{15} + 150(2Ba^2b^3 + Aab^4)x^{12} + 200(Ba^3b^2 + Aa^2b^3)x^9 + 75(Ba^4b + 2Aa^3b^2)x^6 + 10Aa^5 + 12(Ba^5 + 5Aa^4b)x^3}{180x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="maxima")

[Out] 1/3*B*b^5*log(x^3) - 1/180*(60*(5*B*a*b^4 + A*b^5)*x^15 + 150*(2*B*a^2*b^3 + A*a*b^4)*x^12 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 75*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 10*A*a^5 + 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18

mupad [B] time = 0.09, size = 121, normalized size = 1.33

$$Bb^5 \ln(x) - \frac{\frac{Aa^5}{18} + x^{12} \left(\frac{5Ba^2b^3}{3} + \frac{5Aab^4}{6} \right) + x^6 \left(\frac{5Ba^4b}{12} + \frac{5Aa^3b^2}{6} \right) + x^3 \left(\frac{Ba^5}{15} + \frac{Aa^4b}{3} \right) + x^{15} \left(\frac{Ab^5}{3} + \frac{5Bab^4}{3} \right) + x^9 \left(\frac{10Ba^3b^2}{9} + \frac{10Aa^2b^3}{9} \right)}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^19,x)
```

```
[Out] B*b^5*log(x) - ((A*a^5)/18 + x^12*((5*B*a^2*b^3)/3 + (5*A*a*b^4)/6) + x^6*(
(5*A*a^3*b^2)/6 + (5*B*a^4*b)/12) + x^3*((B*a^5)/15 + (A*a^4*b)/3) + x^15*(
(A*b^5)/3 + (5*B*a*b^4)/3) + x^9*((10*A*a^2*b^3)/9 + (10*B*a^3*b^2)/9))/x^1
8
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**19,x)
```

```
[Out] Timed out
```

$$3.52 \quad \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} dx$$

Optimal. Leaf size=113

$$\frac{a^5 A}{19x^{19}} - \frac{a^4(aB + 5Ab)}{16x^{16}} - \frac{5a^3b(aB + 2Ab)}{13x^{13}} - \frac{a^2b^2(aB + Ab)}{x^{10}} - \frac{b^4(5aB + Ab)}{4x^4} - \frac{5ab^3(2aB + Ab)}{7x^7} - \frac{b^5 B}{x}$$

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{a^2b^2(aB + Ab)}{x^{10}} - \frac{a^4(aB + 5Ab)}{16x^{16}} - \frac{5a^3b(aB + 2Ab)}{13x^{13}} - \frac{a^5 A}{19x^{19}} - \frac{5ab^3(2aB + Ab)}{7x^7} - \frac{b^4(5aB + Ab)}{4x^4} - \frac{b^5 B}{x}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^20,x]

[Out] -(a^5*A)/(19*x^19) - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*A*b + a*B))/(13*x^13) - (a^2*b^2*(A*b + a*B))/x^10 - (5*a*b^3*(A*b + 2*a*B))/(7*x^7) - (b^4*(A*b + 5*a*B))/(4*x^4) - (b^5*B)/x

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^(m)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{20}} dx &= \int \left(\frac{a^5 A}{x^{20}} + \frac{a^4(5Ab + aB)}{x^{17}} + \frac{5a^3b(2Ab + aB)}{x^{14}} + \frac{10a^2b^2(Ab + aB)}{x^{11}} + \frac{5ab^3(Ab + 2aB)}{x^8} \right. \\ &\quad \left. + \frac{a^2b^2(Ab + aB)}{x^5} + \frac{b^4(5aB + Ab)}{x^2} + \frac{b^5 B}{x} \right) dx \\ &= \frac{a^5 A}{19x^{19}} - \frac{a^4(5Ab + aB)}{16x^{16}} - \frac{5a^3b(2Ab + aB)}{13x^{13}} - \frac{a^2b^2(Ab + aB)}{x^{10}} - \frac{5ab^3(Ab + 2aB)}{7x^7} \\ &\quad - \frac{a^2b^2(Ab + aB)}{5x^5} - \frac{b^4(5aB + Ab)}{4x^2} - \frac{b^5 B}{x} \end{aligned}$$

Mathematica [A] time = 0.03, size = 119, normalized size = 1.05

$$\frac{91a^5(16A + 19Bx^3) + 665a^4bx^3(13A + 16Bx^3) + 2128a^3b^2x^6(10A + 13Bx^3) + 3952a^2b^3x^9(7A + 10Bx^3) + 4940ab^4x^{12}(4A + 7Bx^3) + 6916b^5x^{15}(A + 4Bx^3)}{27664x^{19}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^20,x]

[Out] $-1/27664*(6916*b^5*x^{15}*(A + 4*B*x^3) + 4940*a*b^4*x^{12}*(4*A + 7*B*x^3) + 3952*a^2*b^3*x^9*(7*A + 10*B*x^3) + 2128*a^3*b^2*x^6*(10*A + 13*B*x^3) + 665*a^4*b*x^3*(13*A + 16*B*x^3) + 91*a^5*(16*A + 19*B*x^3))/x^{19}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{20}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^20,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^20, x]

fricas [A] time = 0.49, size = 121, normalized size = 1.07

$$\frac{27664 B b^5 x^{18} + 6916 (5 B a b^4 + A b^5) x^{15} + 19760 (2 B a^2 b^3 + A a b^4) x^{12} + 27664 (B a^3 b^2 + A a^2 b^3) x^9 + 10640 (B a^4 b + 2 A a^3 b^2) x^6 + 1456 A a^5 + 1729 (B a^5 + 5 A a^4 b) x^3}{27664 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="fricas")

[Out] $-1/27664*(27664*B*b^5*x^{18} + 6916*(5*B*a*b^4 + A*b^5)*x^{15} + 19760*(2*B*a^2*b^3 + A*a*b^4)*x^{12} + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^{19}$

giac [A] time = 0.15, size = 127, normalized size = 1.12

$$\frac{27664 B b^5 x^{18} + 34580 B a b^4 x^{15} + 6916 A b^5 x^{15} + 39520 B a^2 b^3 x^{12} + 19760 A a b^4 x^{12} + 27664 B a^3 b^2 x^9 + 27664 A a^2 b^3 x^9 + 10640 B a^4 b x^6 + 21280 A a^3 b^2 x^6 + 1729 B a^5 x^3 + 8645 A a^4 b x^3 + 1456 A a^5}{27664 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="giac")

[Out] $-1/27664*(27664*B*b^5*x^{18} + 34580*B*a*b^4*x^{15} + 6916*A*b^5*x^{15} + 39520*B*a^2*b^3*x^{12} + 19760*A*a*b^4*x^{12} + 27664*B*a^3*b^2*x^9 + 27664*A*a^2*b^3*x^9 + 10640*B*a^4*b*x^6 + 21280*A*a^3*b^2*x^6 + 1729*B*a^5*x^3 + 8645*A*a^4*b*x^3 + 1456*A*a^5)/x^{19}$

maple [A] time = 0.04, size = 104, normalized size = 0.92

$$\frac{B b^5}{x} - \frac{(A b + 5 B a) b^4}{4 x^4} - \frac{5 (A b + 2 B a) a b^3}{7 x^7} - \frac{(A b + B a) a^2 b^2}{x^{10}} - \frac{5 (2 A b + B a) a^3 b}{13 x^{13}} - \frac{(5 A b + B a) a^4}{16 x^{16}} - \frac{A a^5}{19 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^5*(B*x^3+A)/x^20,x)`

[Out] $-1/19*a^5*A/x^{19}-1/16*a^4*(5*A*b+B*a)/x^{16}-5/13*a^3*b*(2*A*b+B*a)/x^{13}-a^2*b^2*(A*b+B*a)/x^{10}-5/7*a*b^3*(A*b+2*B*a)/x^7-1/4*b^4*(A*b+5*B*a)/x^4-b^5*B/x$

maxima [A] time = 0.46, size = 121, normalized size = 1.07

$$\frac{27664 B b^5 x^{18} + 6916 (5 B a b^4 + A b^5) x^{15} + 19760 (2 B a^2 b^3 + A a b^4) x^{12} + 27664 (B a^3 b^2 + A a^2 b^3) x^9 + 10640 (B a^4 b + 2 A a^3 b^2) x^6 + 1456 A a^5 + 1729 (B a^5 + 5 A a^4 b) x^3}{27664 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="maxima")`

[Out] $-1/27664*(27664*B*b^5*x^{18} + 6916*(5*B*a*b^4 + A*b^5)*x^{15} + 19760*(2*B*a^2*b^3 + A*a*b^4)*x^{12} + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^{19}$

mupad [B] time = 2.37, size = 119, normalized size = 1.05

$$\frac{\frac{A a^5}{19} + x^{12} \left(\frac{10 B a^2 b^3}{7} + \frac{5 A a b^4}{7} \right) + x^6 \left(\frac{5 B a^4 b}{13} + \frac{10 A a^3 b^2}{13} \right) + x^3 \left(\frac{B a^5}{16} + \frac{5 A b a^4}{16} \right) + x^{15} \left(\frac{A b^5}{4} + \frac{5 B a b^4}{4} \right) + x^9 (B a^3 b^2 + A a^2 b^3) + B b^5 x^{18}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^5)/x^20,x)`

[Out] $-((A*a^5)/19 + x^{12}*((10*B*a^2*b^3)/7 + (5*A*a*b^4)/7) + x^6*((10*A*a^3*b^2)/13 + (5*B*a^4*b)/13) + x^3*((B*a^5)/16 + (5*A*a^4*b)/16) + x^{15}*((A*b^5)/4 + (5*B*a*b^4)/4) + x^9*(A*a^2*b^3 + B*a^3*b^2) + B*b^5*x^{18})/x^{19}$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**5*(B*x**3+A)/x**20,x)`

[Out] Timed out

$$3.53 \quad \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}} dx$$

Optimal. Leaf size=117

$$\frac{a^5 A}{20x^{20}} - \frac{a^4(aB + 5Ab)}{17x^{17}} - \frac{5a^3b(aB + 2Ab)}{14x^{14}} - \frac{10a^2b^2(aB + Ab)}{11x^{11}} - \frac{b^4(5aB + Ab)}{5x^5} - \frac{5ab^3(2aB + Ab)}{8x^8} - \frac{b^5 B}{2x^2}$$

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{10a^2b^2(aB + Ab)}{11x^{11}} - \frac{a^4(aB + 5Ab)}{17x^{17}} - \frac{5a^3b(aB + 2Ab)}{14x^{14}} - \frac{a^5 A}{20x^{20}} - \frac{5ab^3(2aB + Ab)}{8x^8} - \frac{b^4(5aB + Ab)}{5x^5} - \frac{b^5 B}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^21,x]

[Out] $-(a^5 A)/(20*x^{20}) - (a^4*(5*A*b + a*B))/(17*x^{17}) - (5*a^3*b*(2*A*b + a*B))/(14*x^{14}) - (10*a^2*b^2*(A*b + a*B))/(11*x^{11}) - (5*a*b^3*(A*b + 2*a*B))/(8*x^8) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(2*x^2)$

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{21}} dx &= \int \left(\frac{a^5 A}{x^{21}} + \frac{a^4(5Ab + aB)}{x^{18}} + \frac{5a^3b(2Ab + aB)}{x^{15}} + \frac{10a^2b^2(Ab + aB)}{x^{12}} + \frac{5ab^3(Ab + 2aB)}{x^9} \right. \\ &\quad \left. - \frac{a^4(5Ab + aB)}{17x^{17}} - \frac{5a^3b(2Ab + aB)}{14x^{14}} - \frac{10a^2b^2(Ab + aB)}{11x^{11}} - \frac{5ab^3(Ab + 2aB)}{8x^8} \right) dx \end{aligned}$$

Mathematica [A] time = 0.04, size = 121, normalized size = 1.03

$$\frac{154a^5(17A + 20Bx^3) + 1100a^4bx^3(14A + 17Bx^3) + 3400a^3b^2x^6(11A + 14Bx^3) + 5950a^2b^3x^9(8A + 11Bx^3) + 6545ab^4x^{12}(5A + 8Bx^3) + 5236b^5x^{15}(2A + 5Bx^3)}{52360x^{20}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^21,x]

[Out] $-1/52360*(5236*b^5*x^{15}*(2*A + 5*B*x^3) + 6545*a*b^4*x^{12}*(5*A + 8*B*x^3) + 5950*a^2*b^3*x^9*(8*A + 11*B*x^3) + 3400*a^3*b^2*x^6*(11*A + 14*B*x^3) + 100*a^4*b*x^3*(14*A + 17*B*x^3) + 154*a^5*(17*A + 20*B*x^3))/x^{20}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{21}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^21,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^21, x]

fricas [A] time = 0.53, size = 121, normalized size = 1.03

$$\frac{26180 B b^5 x^{18} + 10472 (5 B a b^4 + A b^5) x^{15} + 32725 (2 B a^2 b^3 + A a b^4) x^{12} + 47600 (B a^3 b^2 + A a^2 b^3) x^9 + 18700 (B a^4 b + 2 A a^3 b^2) x^6 + 2618 A a^5 + 3080 (B a^5 + 5 A a^4 b) x^3}{52360 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="fricas")

[Out] $-1/52360*(26180*B*b^5*x^{18} + 10472*(5*B*a*b^4 + A*b^5)*x^{15} + 32725*(2*B*a^2*b^3 + A*a*b^4)*x^{12} + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^{20}$

giac [A] time = 0.15, size = 127, normalized size = 1.09

$$\frac{26180 B b^5 x^{18} + 52360 B a b^4 x^{15} + 10472 A b^5 x^{15} + 65450 B a^2 b^3 x^{12} + 32725 A a b^4 x^{12} + 47600 B a^3 b^2 x^9 + 47600 A a^2 b^3 x^9 + 18700 B a^4 b x^6 + 37400 A a^3 b^2 x^6 + 3080 B a^5 x^3 + 15400 A a^4 b x^3 + 2618 A a^5}{52360 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="giac")

[Out] $-1/52360*(26180*B*b^5*x^{18} + 52360*B*a*b^4*x^{15} + 10472*A*b^5*x^{15} + 65450*B*a^2*b^3*x^{12} + 32725*A*a*b^4*x^{12} + 47600*B*a^3*b^2*x^9 + 47600*A*a^2*b^3*x^9 + 18700*B*a^4*b*x^6 + 37400*A*a^3*b^2*x^6 + 3080*B*a^5*x^3 + 15400*A*a^4*b*x^3 + 2618*A*a^5)/x^{20}$

maple [A] time = 0.05, size = 104, normalized size = 0.89

$$-\frac{B b^5}{2x^2} - \frac{(Ab + 5Ba) b^4}{5x^5} - \frac{5(Ab + 2Ba) a b^3}{8x^8} - \frac{10(Ab + Ba) a^2 b^2}{11x^{11}} - \frac{5(2Ab + Ba) a^3 b}{14x^{14}} - \frac{(5Ab + Ba) a^4}{17x^{17}} - \frac{A a^5}{20x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b*x^3+a)^5*(B*x^3+A)/x^{21},x)$

[Out] $-1/20*a^5*A/x^{20}-1/17*a^4*(5*A*b+B*a)/x^{17}-5/14*a^3*b*(2*A*b+B*a)/x^{14}-10/11*a^2*b^2*(A*b+B*a)/x^{11}-5/8*a*b^3*(A*b+2*B*a)/x^8-1/5*b^4*(A*b+5*B*a)/x^5-1/2*b^5*B/x^2$

maxima [A] time = 0.55, size = 121, normalized size = 1.03

$$\frac{26180 B b^5 x^{18} + 10472 (5 B a b^4 + A b^5) x^{15} + 32725 (2 B a^2 b^3 + A a b^4) x^{12} + 47600 (B a^3 b^2 + A a^2 b^3) x^9 + 18700 (B a^4 b + 2 A a^3 b^2) x^6 + 2618 A a^5 + 3080 (B a^5 + 5 A a^4 b) x^3}{52360 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b*x^3+a)^5*(B*x^3+A)/x^{21},x, \text{algorithm}="maxima")$

[Out] $-1/52360*(26180*B*b^5*x^{18} + 10472*(5*B*a*b^4 + A*b^5)*x^{15} + 32725*(2*B*a^2*b^3 + A*a*b^4)*x^{12} + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^{20}$

mupad [B] time = 2.35, size = 121, normalized size = 1.03

$$\frac{\frac{A a^5}{20} + x^{12} \left(\frac{5 B a^2 b^3}{4} + \frac{5 A a b^4}{8} \right) + x^6 \left(\frac{5 B a^4 b}{14} + \frac{5 A a^3 b^2}{7} \right) + x^3 \left(\frac{B a^5}{17} + \frac{5 A a b^4}{17} \right) + x^{15} \left(\frac{A b^5}{5} + B a b^4 \right) + x^9 \left(\frac{10 B a^3 b^2}{11} + \frac{10 A a^2 b^3}{11} \right) + \frac{B b^5 x^{18}}{2}}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(((A + B*x^3)*(a + b*x^3)^5)/x^{21},x)$

[Out] $-((A*a^5)/20 + x^{12}*((5*B*a^2*b^3)/4 + (5*A*a*b^4)/8) + x^6*((5*A*a^3*b^2)/7 + (5*B*a^4*b)/14) + x^3*((B*a^5)/17 + (5*A*a^4*b)/17) + x^{15}*((A*b^5)/5 + B*a*b^4) + x^9*((10*A*a^2*b^3)/11 + (10*B*a^3*b^2)/11) + (B*b^5*x^{18})/2)/x^{20}$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b*x**3+a)**5*(B*x**3+A)/x**21,x)$

[Out] Timed out

$$3.54 \quad \int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}} dx$$

Optimal. Leaf size=48

$$\frac{(a+bx^3)^6(Ab-7aB)}{126a^2x^{18}} - \frac{A(a+bx^3)^6}{21ax^{21}}$$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {446, 78, 37}

$$\frac{(a+bx^3)^6(Ab-7aB)}{126a^2x^{18}} - \frac{A(a+bx^3)^6}{21ax^{21}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^22,x]

[Out] -(A*(a + b*x^3)^6)/(21*a*x^21) + ((A*b - 7*a*B)*(a + b*x^3)^6)/(126*a^2*x^18)

Rule 37

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp [((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{22}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a+bx)^5 (A+Bx)}{x^8} dx, x, x^3 \right) \\ &= -\frac{A(a+bx^3)^6}{21ax^{21}} + \frac{(-Ab+7aB) \text{Subst} \left(\int \frac{(a+bx)^5}{x^7} dx, x, x^3 \right)}{21a} \\ &= -\frac{A(a+bx^3)^6}{21ax^{21}} + \frac{(Ab-7aB)(a+bx^3)^6}{126a^2x^{18}} \end{aligned}$$

Mathematica [B] time = 0.04, size = 118, normalized size = 2.46

$$\frac{a^5(6A+7Bx^3)+7a^4bx^3(5A+6Bx^3)+21a^3b^2x^6(4A+5Bx^3)+35a^2b^3x^9(3A+4Bx^3)+35ab^4x^{12}(2A+3Bx^3)+21b^5x^{15}(A+2Bx^3)}{126x^{21}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^22, x]

[Out] -1/126*(21*b^5*x^15*(A + 2*B*x^3) + 35*a*b^4*x^12*(2*A + 3*B*x^3) + 35*a^2*b^3*x^9*(3*A + 4*B*x^3) + 21*a^3*b^2*x^6*(4*A + 5*B*x^3) + 7*a^4*b*x^3*(5*A + 6*B*x^3) + a^5*(6*A + 7*B*x^3))/x^21

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{22}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^22, x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^22, x]

fricas [B] time = 0.61, size = 121, normalized size = 2.52

$$\frac{42Bb^5x^{18}+21(5Bab^4+Ab^5)x^{15}+70(2Ba^2b^3+Aab^4)x^{12}+105(Ba^3b^2+Aa^2b^3)x^9+42(Ba^4b+2Aa^3b^2)x^6+6Aa^5+7(Ba^5+5Aa^4b)x^3}{126x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22, x, algorithm="fricas")

[Out] $-1/126*(42*B*b^5*x^{18} + 21*(5*B*a*b^4 + A*b^5)*x^{15} + 70*(2*B*a^2*b^3 + A*a*b^4)*x^{12} + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^{21}$

giac [B] time = 0.18, size = 127, normalized size = 2.65

$$\frac{42 B b^5 x^{18} + 105 B a b^4 x^{15} + 21 A b^5 x^{15} + 140 B a^2 b^3 x^{12} + 70 A a b^4 x^{12} + 105 B a^3 b^2 x^9 + 105 A a^2 b^3 x^9 + 42 B a^4 b x^6 + 84 A a^3 b^2 x^6 + 7 B a^5 x^3 + 35 A a^4 b x^3 + 6 A a^5}{126 x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="giac")

[Out] $-1/126*(42*B*b^5*x^{18} + 105*B*a*b^4*x^{15} + 21*A*b^5*x^{15} + 140*B*a^2*b^3*x^{12} + 70*A*a*b^4*x^{12} + 105*B*a^3*b^2*x^9 + 105*A*a^2*b^3*x^9 + 42*B*a^4*b*x^6 + 84*A*a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 6*A*a^5)/x^{21}$

maple [B] time = 0.05, size = 104, normalized size = 2.17

$$\frac{B b^5}{3 x^3} - \frac{(A b + 5 B a) b^4}{6 x^6} - \frac{5 (A b + 2 B a) a b^3}{9 x^9} - \frac{5 (A b + B a) a^2 b^2}{6 x^{12}} - \frac{(2 A b + B a) a^3 b}{3 x^{15}} - \frac{(5 A b + B a) a^4}{18 x^{18}} - \frac{A a^5}{21 x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^22,x)

[Out] $-5/6*b^2*a^2*(A*b+B*a)/x^{12}-1/3*B*b^5/x^3-1/18*a^4*(5*A*b+B*a)/x^{18}-1/3*a^3*b*(2*A*b+B*a)/x^{15}-5/9*a*b^3*(A*b+2*B*a)/x^9-1/6*b^4*(A*b+5*B*a)/x^6-1/21*A*a^5/x^{21}$

maxima [B] time = 0.61, size = 121, normalized size = 2.52

$$\frac{42 B b^5 x^{18} + 21 (5 B a b^4 + A b^5) x^{15} + 70 (2 B a^2 b^3 + A a b^4) x^{12} + 105 (B a^3 b^2 + A a^2 b^3) x^9 + 42 (B a^4 b + 2 A a^3 b^2) x^6 + 6 A a^5 + 7 (B a^5 + 5 A a^4 b) x^3}{126 x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="maxima")

[Out] $-1/126*(42*B*b^5*x^{18} + 21*(5*B*a*b^4 + A*b^5)*x^{15} + 70*(2*B*a^2*b^3 + A*a*b^4)*x^{12} + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^{21}$

mupad [B] time = 2.37, size = 122, normalized size = 2.54

$$\frac{\frac{A a^5}{21} + x^6 \left(\frac{B a^4 b}{3} + \frac{2 A a^3 b^2}{3} \right) + x^{12} \left(\frac{10 B a^2 b^3}{9} + \frac{5 A a b^4}{9} \right) + x^3 \left(\frac{B a^5}{18} + \frac{5 A b a^4}{18} \right) + x^{15} \left(\frac{A b^5}{6} + \frac{5 B a b^4}{6} \right) + x^9 \left(\frac{5 B a^3 b^2}{6} + \frac{5 A a^2 b^3}{6} \right) + \frac{B b^5 x^{18}}{3}}{x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^5)/x^22,x)`

[Out] $-\frac{(Aa^5)}{21} + x^6\left(\frac{2Aa^3b^2}{3} + \frac{Ba^4b}{3}\right) + x^{12}\left(\frac{10Ba^2b^3}{9} + \frac{5Aa^4b}{9}\right) + x^3\left(\frac{Ba^5}{18} + \frac{5Aa^4b}{18}\right) + x^{15}\left(\frac{Ab^5}{6} + \frac{5Ba^3b^4}{6}\right) + x^9\left(\frac{5Aa^2b^3}{6} + \frac{5Ba^3b^2}{6}\right) + \frac{Bb^5x^{18}}{3}/x^{21}$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**5*(B*x**3+A)/x**22,x)`

[Out] Timed out

$$3.55 \quad \int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} dx$$

Optimal. Leaf size=117

$$\frac{a^5 A}{22x^{22}} - \frac{a^4(aB + 5Ab)}{19x^{19}} - \frac{5a^3b(aB + 2Ab)}{16x^{16}} - \frac{10a^2b^2(aB + Ab)}{13x^{13}} - \frac{b^4(5aB + Ab)}{7x^7} - \frac{ab^3(2aB + Ab)}{2x^{10}} - \frac{b^5B}{4x^4}$$

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{10a^2b^2(aB + Ab)}{13x^{13}} - \frac{a^4(aB + 5Ab)}{19x^{19}} - \frac{5a^3b(aB + 2Ab)}{16x^{16}} - \frac{a^5 A}{22x^{22}} - \frac{ab^3(2aB + Ab)}{2x^{10}} - \frac{b^4(5aB + Ab)}{7x^7} - \frac{b^5B}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^23,x]

[Out] -(a^5*A)/(22*x^22) - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B))/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/(2*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{23}} dx &= \int \left(\frac{a^5 A}{x^{23}} + \frac{a^4(5Ab + aB)}{x^{20}} + \frac{5a^3b(2Ab + aB)}{x^{17}} + \frac{10a^2b^2(Ab + aB)}{x^{14}} + \frac{5ab^3(Ab + 2aB)}{x^{11}} \right. \\ &\quad \left. - \frac{a^5 A}{22x^{22}} - \frac{a^4(5Ab + aB)}{19x^{19}} - \frac{5a^3b(2Ab + aB)}{16x^{16}} - \frac{10a^2b^2(Ab + aB)}{13x^{13}} - \frac{ab^3(Ab + 2aB)}{2x^{10}} \right) dx \end{aligned}$$

Mathematica [A] time = 0.05, size = 117, normalized size = 1.00

$$\frac{a^5 A}{22x^{22}} - \frac{a^4(aB + 5Ab)}{19x^{19}} - \frac{5a^3b(aB + 2Ab)}{16x^{16}} - \frac{10a^2b^2(aB + Ab)}{13x^{13}} - \frac{b^4(5aB + Ab)}{7x^7} - \frac{ab^3(2aB + Ab)}{2x^{10}} - \frac{b^5B}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^23,x]

[Out] $-1/22*(a^5*A)/x^{22} - (a^4*(5*A*b + a*B))/(19*x^{19}) - (5*a^3*b*(2*A*b + a*B))/(16*x^{16}) - (10*a^2*b^2*(A*b + a*B))/(13*x^{13}) - (a*b^3*(A*b + 2*a*B))/(2*x^{10}) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{23}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^23,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^23, x]

fricas [A] time = 0.51, size = 121, normalized size = 1.03

$$\frac{76076 B b^5 x^{18} + 43472 (5 B a b^4 + A b^5) x^{15} + 152152 (2 B a^2 b^3 + A a b^4) x^{12} + 234080 (B a^3 b^2 + A a^2 b^3) x^9 + 95095 (B a^4 b + 2 A a^3 b^2) x^6 + 13832 A a^5 + 16016 (B a^5 + 5 A a^4 b) x^3}{304304 x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="fricas")

[Out] $-1/304304*(76076*B*b^5*x^{18} + 43472*(5*B*a*b^4 + A*b^5)*x^{15} + 152152*(2*B*a^2*b^3 + A*a*b^4)*x^{12} + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^{22}$

giac [A] time = 0.16, size = 127, normalized size = 1.09

$$\frac{76076 B b^5 x^{18} + 217360 B a b^4 x^{15} + 43472 A b^5 x^{15} + 304304 B a^2 b^3 x^{12} + 152152 A a b^4 x^{12} + 234080 B a^3 b^2 x^9 + 234080 A a^2 b^3 x^9 + 95095 B a^4 b x^6 + 190190 A a^3 b^2 x^6 + 16016 B a^5 x^3 + 80080 A a^4 b x^3 + 13832 A a^5}{304304 x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="giac")

[Out] $-1/304304*(76076*B*b^5*x^{18} + 217360*B*a*b^4*x^{15} + 43472*A*b^5*x^{15} + 304304*B*a^2*b^3*x^{12} + 152152*A*a*b^4*x^{12} + 234080*B*a^3*b^2*x^9 + 234080*A*a^2*b^3*x^9 + 95095*B*a^4*b*x^6 + 190190*A*a^3*b^2*x^6 + 16016*B*a^5*x^3 + 80080*A*a^4*b*x^3 + 13832*A*a^5)/x^{22}$

maple [A] time = 0.05, size = 104, normalized size = 0.89

$$\frac{B b^5}{4x^4} - \frac{(Ab + 5Ba) b^4}{7x^7} - \frac{(Ab + 2Ba) a b^3}{2x^{10}} - \frac{10 (Ab + Ba) a^2 b^2}{13x^{13}} - \frac{5 (2Ab + Ba) a^3 b}{16x^{16}} - \frac{(5Ab + Ba) a^4}{19x^{19}} - \frac{A a^5}{22x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^5*(B*x^3+A)/x^23,x)`

[Out] $-1/22*a^5*A/x^{22}-1/19*a^4*(5*A*b+B*a)/x^{19}-5/16*a^3*b*(2*A*b+B*a)/x^{16}-10/13*a^2*b^2*(A*b+B*a)/x^{13}-1/2*a*b^3*(A*b+2*B*a)/x^{10}-1/7*b^4*(A*b+5*B*a)/x^7-1/4*b^5*B/x^4$

maxima [A] time = 0.70, size = 121, normalized size = 1.03

$$\frac{76076 B b^5 x^{18} + 43472 (5 B a b^4 + A b^5) x^{15} + 152152 (2 B a^2 b^3 + A a b^4) x^{12} + 234080 (B a^3 b^2 + A a^2 b^3) x^9 + 95095 (B a^4 b + 2 A a^3 b^2) x^6 + 13832 A a^5 + 16016 (B a^5 + 5 A a^4 b) x^3}{304304 x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="maxima")`

[Out] $-1/304304*(76076*B*b^5*x^{18} + 43472*(5*B*a*b^4 + A*b^5)*x^{15} + 152152*(2*B*a^2*b^3 + A*a*b^4)*x^{12} + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^{22}$

mupad [B] time = 0.06, size = 121, normalized size = 1.03

$$\frac{\frac{A a^5}{22} + x^{12} \left(B a^2 b^3 + \frac{A a b^4}{2} \right) + x^6 \left(\frac{5 B a^4 b}{16} + \frac{5 A a^3 b^2}{8} \right) + x^3 \left(\frac{B a^5}{19} + \frac{5 A b a^4}{19} \right) + x^{15} \left(\frac{A b^5}{7} + \frac{5 B a b^4}{7} \right) + x^9 \left(\frac{10 B a^3 b^2}{13} + \frac{10 A a^2 b^3}{13} \right) + \frac{B b^5 x^{18}}{4}}{x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^5)/x^23,x)`

[Out] $-((A*a^5)/22 + x^{12}*(B*a^2*b^3 + (A*a*b^4)/2) + x^6*((5*A*a^3*b^2)/8 + (5*B*a^4*b)/16) + x^3*((B*a^5)/19 + (5*A*a^4*b)/19) + x^{15}*((A*b^5)/7 + (5*B*a*b^4)/7) + x^9*((10*A*a^2*b^3)/13 + (10*B*a^3*b^2)/13) + (B*b^5*x^{18})/4)/x^2$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**5*(B*x**3+A)/x**23,x)`

[Out] Timed out

$$3.56 \quad \int \frac{x^6(A+Bx^3)}{a+bx^3} dx$$

Optimal. Leaf size=183

$$\frac{a^{4/3}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{10/3}} + \frac{a^{4/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{10/3}} - \frac{a^{4/3}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^{10/3}}$$

Rubi [A] time = 0.15, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {459, 302, 200, 31, 634, 617, 204, 628}

$$-\frac{a^{4/3}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{10/3}} + \frac{a^{4/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{10/3}} - \frac{a^{4/3}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^{10/3}} + \frac{x^4(Ab - aB)}{4b^2} - \frac{ax(Ab - aB)}{b^3} + \frac{Bx^7}{7b}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(A + B*x^3))/(a + b*x^3), x]

[Out] -((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^7)/(7*b) - (a^(4/3)*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(10/3)) + (a^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(10/3)) - (a^(4/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3))

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6 (A + Bx^3)}{a + bx^3} dx &= \frac{Bx^7}{7b} - \frac{(-7Ab + 7aB)}{7b} \int \frac{x^6}{a+bx^3} dx \\
&= \frac{Bx^7}{7b} - \frac{(-7Ab + 7aB) \int \left(-\frac{a}{b^2} + \frac{x^3}{b} + \frac{a^2}{b^2(a+bx^3)} \right) dx}{7b} \\
&= -\frac{a(Ab - aB)x}{b^3} + \frac{(Ab - aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{(a^2(Ab - aB)) \int \frac{1}{a+bx^3} dx}{b^3} \\
&= -\frac{a(Ab - aB)x}{b^3} + \frac{(Ab - aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{(a^{4/3}(Ab - aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3b^3} + \frac{(a^{4/3}(Ab - aB))}{3b^3} \\
&= -\frac{a(Ab - aB)x}{b^3} + \frac{(Ab - aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{a^{4/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{10/3}} - \frac{(a^{4/3}(Ab - aB))}{3b^{10/3}} \\
&= -\frac{a(Ab - aB)x}{b^3} + \frac{(Ab - aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{a^{4/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{10/3}} - \frac{a^{4/3}(Ab - aB)}{3b^{10/3}} \\
&= -\frac{a(Ab - aB)x}{b^3} + \frac{(Ab - aB)x^4}{4b^2} + \frac{Bx^7}{7b} - \frac{a^{4/3}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}} + \frac{a^{4/3}(Ab - aB)}{3b^{10/3}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 171, normalized size = 0.93

$$\frac{14a^{4/3}(aB - Ab) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) - 28a^{4/3}(aB - Ab) \log(\sqrt[3]{a} + \sqrt[3]{b}x) + 28\sqrt{3}a^{4/3}(aB - Ab) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right) + 21b^{4/3}x^4(Ab - aB) + 84a\sqrt[3]{b}x(aB - Ab) + 12b^{7/3}Bx^7}{84b^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3), x]

[Out] (84*a*b^(1/3)*(-(A*b) + a*B)*x + 21*b^(4/3)*(A*b - a*B)*x^4 + 12*b^(7/3)*B*x^7 + 28*sqrt[3]*a^(4/3)*(-(A*b) + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] - 28*a^(4/3)*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(84*b^(10/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 (A + Bx^3)}{a + bx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^6*(A + B*x^3))/(a + b*x^3), x]

[Out] IntegrateAlgebraic[(x^6*(A + B*x^3))/(a + b*x^3), x]

fricas [A] time = 0.68, size = 167, normalized size = 0.91

$$\frac{12 B b^2 x^7 - 21 (B a b - A b^2) x^4 - 28 \sqrt{3} (B a^2 - A a b) \left(\frac{a}{b}\right)^{\frac{1}{3}} \arctan\left(\frac{2 \sqrt{3} b x \left(\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3} a}{3 a}\right) + 14 (B a^2 - A a b) \left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x^2 - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 28 (B a^2 - A a b) \left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + 84 (B a^2 - A a b) x}{84 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(B*x^3+A)/(b*x^3+a), x, algorithm="fricas")

[Out] 1/84*(12*B*b^2*x^7 - 21*(B*a*b - A*b^2)*x^4 - 28*sqrt(3)*(B*a^2 - A*a*b)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 14*(B*a^2 - A*a*b)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(B*a^2 - A*a*b)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(B*a^2 - A*a*b)*x/b^3

giac [A] time = 0.24, size = 217, normalized size = 1.19

$$\frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} Ba^2 - (-ab^2)^{\frac{1}{3}} Aab \right) \arctan\left(\frac{\sqrt{3} \left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 b^4} - \frac{\left((-ab^2)^{\frac{1}{3}} Ba^2 - (-ab^2)^{\frac{1}{3}} Aab \right) \log\left(x^2 + x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b^4} + \frac{\left(Ba^3 b^4 - A a^2 b^5 \right) \left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 a b^7} + \frac{4 B b^6 x^7 - 7 B a b^5 x^4 + 7 A b^6 x^4 + 28 B a^2 b^4 x - 28 A a b^5 x}{28 b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(B*x^3+A)/(b*x^3+a), x, algorithm="giac")

[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a^2 - (-a*b^2)^(1/3)*A*a*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*B*a^2 - (-a*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/3*(B*a^3*b^4 - A*a^2*b^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/28*(4*B*b^6*x^7 - 7*B*a*b^5*x^4 + 7*A*b^6*x^4 + 28*B*a^2*b^4*x - 28*A*a*b^5*x)/b^7

maple [A] time = 0.04, size = 249, normalized size = 1.36

$$\frac{B x^7}{7 b} + \frac{A x^4}{4 b} - \frac{B a x^4}{4 b^2} + \frac{\sqrt{3} A a^2 \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1 \right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^3} + \frac{A a^2 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^3} - \frac{A a^2 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^3} - \frac{A a x}{b^2} - \frac{\sqrt{3} B a^3 \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1 \right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^4} - \frac{B a^3 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^4} + \frac{B a^3 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^4} + \frac{B a^2 x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(B*x^3+A)/(b*x^3+a), x)

[Out] $\frac{1}{7}Bx^7/b + 1/4/bA^*x^4 - 1/4/b^2B^*x^4a - 1/b^2a^*A^*x + 1/b^3a^2B^*x + 1/3a^2/b^3(a/b)^{(2/3)} \ln(x + (a/b)^{(1/3)}) * A - 1/3a^3/b^4(a/b)^{(2/3)} \ln(x + (a/b)^{(1/3)}) * B - 1/6a^2/b^3(a/b)^{(2/3)} \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) * A + 1/6a^3/b^4(a/b)^{(2/3)} \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) * B + 1/3a^2/b^3(a/b)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(a/b)^{(1/3)} * x - 1)) * A - 1/3a^3/b^4(a/b)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(a/b)^{(1/3)} * x - 1)) * B$

maxima [A] time = 1.43, size = 182, normalized size = 0.99

$$\frac{4Bb^2x^7 - 7(Bab - Ab^2)x^4 + 28(Ba^2 - Aab)x}{28b^3} - \frac{\sqrt{3}(Ba^3 - Aa^2b) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba^3 - Aa^2b) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba^3 - Aa^2b) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(B*x^3+A)/(b*x^3+a), x, algorithm="maxima")

[Out] $\frac{1}{28} * (4 * B * b^2 * x^7 - 7 * (B * a * b - A * b^2) * x^4 + 28 * (B * a^2 - A * a * b) * x) / b^3 - 1/3 * \sqrt{3} * (B * a^3 - A * a^2 * b) * \arctan(1/3 * \sqrt{3} * (2 * x - (a/b)^{(1/3)}) / (a/b)^{(1/3)}) / (b^4 * (a/b)^{(2/3)}) + 1/6 * (B * a^3 - A * a^2 * b) * \log(x^2 - x * (a/b)^{(1/3)} + (a/b)^{(2/3)}) / (b^4 * (a/b)^{(2/3)}) - 1/3 * (B * a^3 - A * a^2 * b) * \log(x + (a/b)^{(1/3)}) / (b^4 * (a/b)^{(2/3)})$

mupad [B] time = 0.27, size = 164, normalized size = 0.90

$$x^4 \left(\frac{A}{4b} - \frac{Ba}{4b^2} \right) + \frac{Bx^7}{7b} + \frac{a^{4/3} \ln(b^{1/3}x + a^{1/3}) (Ab - Ba)}{3b^{10/3}} - \frac{ax \left(\frac{A}{b} - \frac{Ba}{b^2} \right)}{b} - \frac{a^{4/3} \ln(a^{1/3} - 2b^{1/3}x + \sqrt{3} a^{1/3} 1i) \left(\frac{1}{2} + \frac{\sqrt{3} 1i}{2} \right) (Ab - Ba)}{3b^{10/3}} + \frac{a^{4/3} \ln(2b^{1/3}x - a^{1/3} + \sqrt{3} a^{1/3} 1i) \left(\frac{1}{2} + \frac{\sqrt{3} 1i}{2} \right) (Ab - Ba)}{3b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(A + B*x^3))/(a + b*x^3), x)

[Out] $x^4 * (A / (4 * b) - (B * a) / (4 * b^2)) + (B * x^7) / (7 * b) + (a^{(4/3)} * \log(b^{(1/3)} * x + a^{(1/3)}) * (A * b - B * a)) / (3 * b^{(10/3)}) - (a * x * (A / b - (B * a) / b^2)) / b - (a^{(4/3)} * \log(3^{(1/2)} * a^{(1/3)} * 1i - 2 * b^{(1/3)} * x + a^{(1/3)}) * ((3^{(1/2)} * 1i) / 2 + 1/2) * (A * b - B * a)) / (3 * b^{(10/3)}) + (a^{(4/3)} * \log(3^{(1/2)} * a^{(1/3)} * 1i + 2 * b^{(1/3)} * x - a^{(1/3)}) * ((3^{(1/2)} * 1i) / 2 - 1/2) * (A * b - B * a)) / (3 * b^{(10/3)})$

sympy [A] time = 1.37, size = 114, normalized size = 0.62

$$\frac{Bx^7}{7b} + x^4 \left(\frac{A}{4b} - \frac{Ba}{4b^2} \right) + x \left(-\frac{Aa}{b^2} + \frac{Ba^2}{b^3} \right) + \text{RootSum} \left(27t^3b^{10} - A^3a^4b^3 + 3A^2Ba^5b^2 - 3AB^2a^6b + B^3a^7, \left(t \mapsto t \log \left(-\frac{3tb^3}{-Aab + Ba^2} + x \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(B*x**3+A)/(b*x**3+a), x)

[Out] $B * x^{**7} / (7 * b) + x^{**4} * (A / (4 * b) - B * a / (4 * b^{**2})) + x * (-A * a / b^{**2} + B * a^{**2} / b^{**3}) + \text{RootSum}(27 * _t^{**3} * b^{**10} - A^{**3} * a^{**4} * b^{**3} + 3 * A^{**2} * B * a^{**5} * b^{**2} - 3 * A * B^{**2} * a^{**6} * b + B^{**3} * a^{**7}, \text{Lambda}(_t, _t * \log(-3 * _t * b^{**3} / (-A * a * b + B * a^{**2}) + x))$

$$3.57 \quad \int \frac{x^5(A+Bx^3)}{a+bx^3} dx$$

Optimal. Leaf size=54

$$-\frac{a(Ab - aB) \log(a + bx^3)}{3b^3} + \frac{x^3(Ab - aB)}{3b^2} + \frac{Bx^6}{6b}$$

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 77}

$$\frac{x^3(Ab - aB)}{3b^2} - \frac{a(Ab - aB) \log(a + bx^3)}{3b^3} + \frac{Bx^6}{6b}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(A + B*x^3))/(a + b*x^3),x]

[Out] ((A*b - a*B)*x^3)/(3*b^2) + (B*x^6)/(6*b) - (a*(A*b - a*B)*Log[a + b*x^3])/(3*b^3)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5(A+Bx^3)}{a+bx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(A+Bx)}{a+bx} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{Ab-aB}{b^2} + \frac{Bx}{b} + \frac{a(-Ab+aB)}{b^2(a+bx)} \right) dx, x, x^3 \right) \\
&= \frac{(Ab-aB)x^3}{3b^2} + \frac{Bx^6}{6b} - \frac{a(Ab-aB) \log(a+bx^3)}{3b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.87

$$\frac{bx^3(-2aB + 2Ab + bBx^3) + 2a(aB - Ab) \log(a + bx^3)}{6b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3), x]

[Out] (b*x^3*(2*A*b - 2*a*B + b*B*x^3) + 2*a*(-(A*b) + a*B)*Log[a + b*x^3])/(6*b^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(A+Bx^3)}{a+bx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3), x]

[Out] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3), x]

fricas [A] time = 0.56, size = 51, normalized size = 0.94

$$\frac{Bb^2x^6 - 2(Bab - Ab^2)x^3 + 2(Ba^2 - Aab) \log(bx^3 + a)}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^3+A)/(b*x^3+a), x, algorithm="fricas")

[Out] 1/6*(B*b^2*x^6 - 2*(B*a*b - A*b^2)*x^3 + 2*(B*a^2 - A*a*b)*log(b*x^3 + a))/b^3

giac [A] time = 0.17, size = 52, normalized size = 0.96

$$\frac{Bbx^6 - 2Bax^3 + 2Abx^3}{6b^2} + \frac{(Ba^2 - Aab) \log(|bx^3 + a|)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")

[Out] 1/6*(B*b*x^6 - 2*B*a*x^3 + 2*A*b*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(abs(b*x^3 + a))/b^3

maple [A] time = 0.04, size = 62, normalized size = 1.15

$$\frac{Bx^6}{6b} + \frac{Ax^3}{3b} - \frac{Bax^3}{3b^2} - \frac{Aa \ln(bx^3 + a)}{3b^2} + \frac{Ba^2 \ln(bx^3 + a)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(B*x^3+A)/(b*x^3+a),x)

[Out] 1/6*B*x^6/b+1/3/b*A*x^3-1/3/b^2*B*a*x^3-1/3*a/b^2*ln(b*x^3+a)*A+1/3*a^2/b^3*ln(b*x^3+a)*B

maxima [A] time = 0.50, size = 50, normalized size = 0.93

$$\frac{Bbx^6 - 2(Ba - Ab)x^3}{6b^2} + \frac{(Ba^2 - Aab) \log(bx^3 + a)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")

[Out] 1/6*(B*b*x^6 - 2*(B*a - A*b)*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(b*x^3 + a)/b^3

mupad [B] time = 0.08, size = 52, normalized size = 0.96

$$x^3 \left(\frac{A}{3b} - \frac{Ba}{3b^2} \right) + \frac{\ln(bx^3 + a) (Ba^2 - Aab)}{3b^3} + \frac{Bx^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*(A + B*x^3))/(a + b*x^3),x)

[Out] x^3*(A/(3*b) - (B*a)/(3*b^2)) + (log(a + b*x^3)*(B*a^2 - A*a*b))/(3*b^3) + (B*x^6)/(6*b)

sympy [A] time = 0.96, size = 46, normalized size = 0.85

$$\frac{Bx^6}{6b} + \frac{a(-Ab + Ba) \log(a + bx^3)}{3b^3} + x^3 \left(\frac{A}{3b} - \frac{Ba}{3b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(B*x**3+A)/(b*x**3+a),x)

[Out] B*x**6/(6*b) + a*(-A*b + B*a)*log(a + b*x**3)/(3*b**3) + x**3*(A/(3*b) - B*a/(3*b**2))

$$3.58 \quad \int \frac{x^4(A+Bx^3)}{a+bx^3} dx$$

Optimal. Leaf size=167

$$-\frac{a^{2/3}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{8/3}} + \frac{a^{2/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{8/3}} + \frac{a^{2/3}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^{8/3}}$$

Rubi [A] time = 0.12, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {459, 321, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{2/3}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{8/3}} + \frac{a^{2/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{8/3}} + \frac{a^{2/3}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^{8/3}} + \frac{x^2(Ab - aB)}{2b^2} + \frac{Bx^5}{5b}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(A + B*x^3))/(a + b*x^3), x]

[Out] ((A*b - a*B)*x^2)/(2*b^2) + (B*x^5)/(5*b) + (a^(2/3)*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^(8/3)) + (a^(2/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x]/(3*b^(8/3)) - (a^(2/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(8/3)))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(n-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
+ 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4 (A + Bx^3)}{a + bx^3} dx &= \frac{Bx^5}{5b} - \frac{(-5Ab + 5aB)}{5b} \int \frac{x^4}{a+bx^3} dx \\
&= \frac{(Ab - aB)x^2}{2b^2} + \frac{Bx^5}{5b} - \frac{(a(Ab - aB)) \int \frac{x}{a+bx^3} dx}{b^2} \\
&= \frac{(Ab - aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{(a^{2/3}(Ab - aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3b^{7/3}} - \frac{(a^{2/3}(Ab - aB)) \int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2} dx}{3b^{7/3}} \\
&= \frac{(Ab - aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{8/3}} - \frac{(a^{2/3}(Ab - aB)) \int \frac{-\sqrt[3]{a} \sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2} dx}{6b^{8/3}} \\
&= \frac{(Ab - aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{8/3}} - \frac{a^{2/3}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x)}{6b^{8/3}} \\
&= \frac{(Ab - aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^{8/3}} + \frac{a^{2/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{8/3}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 154, normalized size = 0.92

$$\frac{5a^{2/3}(aB - Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2) - 10a^{2/3}(aB - Ab) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 10\sqrt{3} a^{2/3}(aB - Ab) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3} \sqrt[3]{a}}\right) + 15b^{2/3}x^2(Ab - aB) + 6b^{5/3}Bx^5}{30b^{8/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3), x]

[Out] (15*b^(2/3)*(A*b - a*B)*x^2 + 6*b^(5/3)*B*x^5 - 10*Sqrt[3]*a^(2/3)*(-(A*b) + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x] + 5*a^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(30*b^(8/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (A + Bx^3)}{a + bx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(A + B*x^3))/(a + b*x^3), x]

[Out] IntegrateAlgebraic[(x^4*(A + B*x^3))/(a + b*x^3), x]

fricas [A] time = 0.53, size = 162, normalized size = 0.97

$$\frac{6Bbx^5 - 15(Ba - Ab)x^2 + 10\sqrt{3}(Ba - Ab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} - \sqrt{3}a}{3a}\right) + 5(Ba - Ab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} \log\left(ax^2 - bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}} + a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right) - 10(Ba - Ab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} \log\left(ax + b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)}{30b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")

[Out] 1/30*(6*B*b*x^5 - 15*(B*a - A*b)*x^2 + 10*sqrt(3)*(B*a - A*b)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*(B*a - A*b)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) - 10*(B*a - A*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3))/b^2

giac [A] time = 0.19, size = 207, normalized size = 1.24

$$\frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}Ba - (-ab^2)^{\frac{2}{3}}Ab\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^4} + \frac{\left((-ab^2)^{\frac{2}{3}}Ba - (-ab^2)^{\frac{2}{3}}Ab\right) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^4} - \frac{\left(Ba^2b^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Aab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab^5} + \frac{2Bb^4x^5 - 5Bab^3x^2 + 5Ab^4x^2}{10b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")

[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/6*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B*a^2*b^3*(-a/b)^(1/3) - A*a*b^4*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) + 1/10*(2*B*b^4*x^5 - 5*B*a*b^3*x^2 + 5*A*b^4*x^2)/b^5

maple [A] time = 0.04, size = 226, normalized size = 1.35

$$\frac{Bx^5}{5b} + \frac{Ax^2}{2b} - \frac{Bax^2}{2b^2} - \frac{\sqrt{3}Aa \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} + \frac{Aa \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} - \frac{Aa \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} + \frac{\sqrt{3}Ba^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3} - \frac{Ba^2 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3} + \frac{Ba^2 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(B*x^3+A)/(b*x^3+a),x)

[Out] 1/5*B*x^5/b+1/2/b*A*x^2-1/2/b^2*B*x^2*a+1/3*a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-1/3*a^2/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B-1/6*a/b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6*a^2/b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B-1/3*a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b

$\sqrt[3]{(1/3)*x-1}) * A + 1/3 * a^2 / b^3 * 3^{(1/2)} / (a/b)^{(1/3)} * \arctan(1/3 * 3^{(1/2)} * (2 / (a/b)^{(1/3)} * x - 1)) * B$

maxima [A] time = 1.10, size = 157, normalized size = 0.94

$$\frac{\sqrt{3}(Ba^2 - Aab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2Bbx^5 - 5(Ba - Ab)x^2}{10b^2} + \frac{(Ba^2 - Aab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba^2 - Aab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")

[Out] $\frac{1}{3} * \sqrt{3} * (B * a^2 - A * a * b) * \arctan(1/3 * \sqrt{3} * (2 * x - (a/b)^{(1/3)}) / (a/b)^{(1/3)}) / (b^3 * (a/b)^{(1/3)}) + 1/10 * (2 * B * b * x^5 - 5 * (B * a - A * b) * x^2) / b^2 + 1/6 * (B * a^2 - A * a * b) * \log(x^2 - x * (a/b)^{(1/3)} + (a/b)^{(2/3)}) / (b^3 * (a/b)^{(1/3)}) - 1/3 * (B * a^2 - A * a * b) * \log(x + (a/b)^{(1/3)}) / (b^3 * (a/b)^{(1/3)})$

mupad [B] time = 2.55, size = 144, normalized size = 0.86

$$x^2 \left(\frac{A}{2b} - \frac{Ba}{2b^2} \right) + \frac{Bx^5}{5b} + \frac{a^{2/3} \ln(b^{1/3}x + a^{1/3})(Ab - Ba)}{3b^{8/3}} + \frac{a^{2/3} \ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i) \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2} \right) (Ab - Ba)}{3b^{8/3}} - \frac{a^{2/3} \ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i) \left(\frac{1}{2} + \frac{\sqrt{3}i}{2} \right) (Ab - Ba)}{3b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(A + B*x^3))/(a + b*x^3),x)

[Out] $x^2 * (A / (2 * b) - (B * a) / (2 * b^2)) + (B * x^5) / (5 * b) + (a^{(2/3)} * \log(b^{(1/3)} * x + a^{(1/3)}) * (A * b - B * a)) / (3 * b^{(8/3)}) + (a^{(2/3)} * \log(3^{(1/2)} * a^{(1/3)} * 1i - 2 * b^{(1/3)} * x + a^{(1/3)}) * ((3^{(1/2)} * 1i) / 2 - 1/2) * (A * b - B * a)) / (3 * b^{(8/3)}) - (a^{(2/3)} * \log(3^{(1/2)} * a^{(1/3)} * 1i + 2 * b^{(1/3)} * x - a^{(1/3)}) * ((3^{(1/2)} * 1i) / 2 + 1/2) * (A * b - B * a)) / (3 * b^{(8/3)})$

sympy [A] time = 0.87, size = 114, normalized size = 0.68

$$\frac{Bx^5}{5b} + x^2 \left(\frac{A}{2b} - \frac{Ba}{2b^2} \right) + \text{RootSum} \left(27t^3b^8 - A^3a^2b^3 + 3A^2Ba^3b^2 - 3AB^2a^4b + B^3a^5, \left(t \mapsto t \log \left(\frac{9t^2b^5}{A^2ab^2 - 2ABa^2b + B^2a^3} + x \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(B*x**3+A)/(b*x**3+a),x)

[Out] $B * x^{**5} / (5 * b) + x^{**2} * (A / (2 * b) - B * a / (2 * b^{**2})) + \text{RootSum}(27 * _t^{**3} * b^{**8} - A^{**3} * a^{**2} * b^{**3} + 3 * A^{**2} * B * a^{**3} * b^{**2} - 3 * A * B^{**2} * a^{**4} * b + B^{**3} * a^{**5}, \text{Lambda}(_t, _t * \log(9 * _t^{**2} * b^{**5} / (A^{**2} * a * b^{**2} - 2 * A * B * a^{**2} * b + B^{**2} * a^{**3}) + x)))$

$$3.59 \quad \int \frac{x^3(A+Bx^3)}{a+bx^3} dx$$

Optimal. Leaf size=162

$$\frac{\sqrt[3]{a}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{7/3}} - \frac{\sqrt[3]{a}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{7/3}} + \frac{\sqrt[3]{a}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^{7/3}}$$

Rubi [A] time = 0.12, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {459, 321, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{7/3}} + \frac{x(Ab - aB)}{b^2} - \frac{\sqrt[3]{a}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{7/3}} + \frac{\sqrt[3]{a}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^{7/3}} + \frac{Bx^4}{4b}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(A + B*x^3))/(a + b*x^3), x]

[Out] ((A*b - a*B)*x)/b^2 + (B*x^4)/(4*b) + (a^(1/3)*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^(7/3)) - (a^(1/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x]/(3*b^(7/3)) + (a^(1/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(7/3)))

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^-1, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
+ 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (A + Bx^3)}{a + bx^3} dx &= \frac{Bx^4}{4b} - \frac{(-4Ab + 4aB)}{4b} \int \frac{x^3}{a+bx^3} dx \\
&= \frac{(Ab - aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{(a(Ab - aB))}{b^2} \int \frac{1}{a+bx^3} dx \\
&= \frac{(Ab - aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{(\sqrt[3]{a}(Ab - aB))}{3b^2} \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx - \frac{(\sqrt[3]{a}(Ab - aB))}{3b^2} \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx \\
&= \frac{(Ab - aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{\sqrt[3]{a}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{7/3}} + \frac{(\sqrt[3]{a}(Ab - aB))}{6b^{7/3}} \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx \\
&= \frac{(Ab - aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{\sqrt[3]{a}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{7/3}} + \frac{\sqrt[3]{a}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x)}{6b^{7/3}} \\
&= \frac{(Ab - aB)x}{b^2} + \frac{Bx^4}{4b} + \frac{\sqrt[3]{a}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}} - \frac{\sqrt[3]{a}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{7/3}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 152, normalized size = 0.94

$$\frac{-2\sqrt[3]{a}(aB - Ab) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) + 12\sqrt[3]{b}x(Ab - aB) + 4\sqrt[3]{a}(aB - Ab) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 4\sqrt{3}\sqrt[3]{a}(aB - Ab) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right) + 3b^{4/3}Bx^4}{12b^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3), x]

[Out] (12*b^(1/3)*(A*b - a*B)*x + 3*b^(4/3)*B*x^4 - 4*Sqrt[3]*a^(1/3)*(-(A*b) + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x] - 2*a^(1/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(12*b^(7/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (A + Bx^3)}{a + bx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(A + B*x^3))/(a + b*x^3), x]

[Out] IntegrateAlgebraic[(x^3*(A + B*x^3))/(a + b*x^3), x]

fricas [A] time = 0.49, size = 145, normalized size = 0.90

$$\frac{3Bbx^4 - 4\sqrt{3}(Ba - Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}a}{3a}\right) + 2(Ba - Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - 4(Ba - Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) - 12(Ba - Ab)x}{12b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")

[Out] $\frac{1}{12} * (3 * B * b * x^4 - 4 * \text{sqrt}(3) * (B * a - A * b) * (-a/b)^{(1/3)} * \arctan(1/3 * (2 * \text{sqrt}(3) * b * x * (-a/b)^{(2/3)} - \text{sqrt}(3) * a) / a) + 2 * (B * a - A * b) * (-a/b)^{(1/3)} * \log(x^2 + x * (-a/b)^{(1/3)} + (-a/b)^{(2/3)}) - 4 * (B * a - A * b) * (-a/b)^{(1/3)} * \log(x - (-a/b)^{(1/3)}) - 12 * (B * a - A * b) * x) / b^2$

giac [A] time = 0.17, size = 186, normalized size = 1.15

$$\frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} Ba - (-ab^2)^{\frac{1}{3}} Ab \right) \arctan\left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3} + \frac{\left((-ab^2)^{\frac{1}{3}} Ba - (-ab^2)^{\frac{1}{3}} Ab \right) \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^3} - \frac{(Ba^2b^2 - Aab^3) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab^4} + \frac{Bb^3x^4 - 4Bab^2x + 4Ab^3x}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{1}{3} * \text{sqrt}(3) * ((-a*b^2)^{(1/3)} * B * a - (-a*b^2)^{(1/3)} * A * b) * \arctan(1/3 * \text{sqrt}(3) * (2 * x + (-a/b)^{(1/3)}) / (-a/b)^{(1/3)}) / b^3 + 1/6 * ((-a*b^2)^{(1/3)} * B * a - (-a*b^2)^{(1/3)} * A * b) * \log(x^2 + x * (-a/b)^{(1/3)} + (-a/b)^{(2/3)}) / b^3 - 1/3 * (B * a^2 * b^2 - A * a * b^3) * (-a/b)^{(1/3)} * \log(\text{abs}(x - (-a/b)^{(1/3)}) / (a * b^4) + 1/4 * (B * b^3 * x^4 - 4 * B * a * b^2 * x + 4 * A * b^3 * x) / b^4$

maple [A] time = 0.04, size = 221, normalized size = 1.36

$$\frac{Bx^4}{4b} - \frac{\sqrt{3} Aa \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} - \frac{Aa \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} + \frac{Aa \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} + \frac{Ax}{b} + \frac{\sqrt{3} Ba^2 \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} + \frac{Ba^2 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} - \frac{Ba^2 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} - \frac{Bax}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(B*x^3+A)/(b*x^3+a),x)

[Out] $\frac{1}{4} * B * x^4 / b + 1/b * A * x - 1/b^2 * B * a * x - 1/3 * a/b^2 / (a/b)^{(2/3)} * \ln(x + (a/b)^{(1/3)}) * A + 1/3 * a^2/b^3 / (a/b)^{(2/3)} * \ln(x + (a/b)^{(1/3)}) * B + 1/6 * a/b^2 / (a/b)^{(2/3)} * \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) * A - 1/6 * a^2/b^3 / (a/b)^{(2/3)} * \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) * B - 1/3 * a/b^2 / (a/b)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(a/b)^{(1/3)}) *$

$x-1)) * A + 1/3 * a^2/b^3 / (a/b)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(a/b)^{(1/3)} * x - 1)) * B$

maxima [A] time = 1.08, size = 154, normalized size = 0.95

$$\frac{Bbx^4 - 4(Ba - Ab)x}{4b^2} + \frac{\sqrt{3}(Ba^2 - Aab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba^2 - Aab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba^2 - Aab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^3+A)/(b*x^3+a), x, algorithm="maxima")

[Out] $1/4 * (B * b * x^4 - 4 * (B * a - A * b) * x) / b^2 + 1/3 * \sqrt{3} * (B * a^2 - A * a * b) * \arctan(1/3 * \sqrt{3} * (2 * x - (a/b)^{(1/3)}) / (a/b)^{(1/3)}) / (b^3 * (a/b)^{(2/3)}) - 1/6 * (B * a^2 - A * a * b) * \log(x^2 - x * (a/b)^{(1/3)} + (a/b)^{(2/3)}) / (b^3 * (a/b)^{(2/3)}) + 1/3 * (B * a^2 - A * a * b) * \log(x + (a/b)^{(1/3)}) / (b^3 * (a/b)^{(2/3)})$

mupad [B] time = 2.61, size = 162, normalized size = 1.00

$$x \left(\frac{A}{b} - \frac{Ba}{b^2} \right) + \frac{Bx^4}{4b} + \frac{(-a)^{1/3} \ln((-a)^{4/3} + ab^{1/3}x)(Ab - Ba)}{3b^{7/3}} - \frac{(-a)^{1/3} \ln(2ab^{1/3}x - (-a)^{4/3} - \sqrt{3}(-a)^{4/3}i) \left(\frac{1}{2} + \frac{\sqrt{3}i}{2} \right) (Ab - Ba)}{3b^{7/3}} + \frac{(-a)^{1/3} \ln(2ab^{1/3}x - (-a)^{4/3} + \sqrt{3}(-a)^{4/3}i) \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2} \right) (Ab - Ba)}{3b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(A + B*x^3))/(a + b*x^3), x)

[Out] $x * (A/b - (B * a) / b^2) + (B * x^4) / (4 * b) + ((-a)^{(1/3)} * \log((-a)^{(4/3)} + a * b^{(1/3)} * x) * (A * b - B * a)) / (3 * b^{(7/3)}) - ((-a)^{(1/3)} * \log(2 * a * b^{(1/3)} * x - 3^{(1/2)} * (-a)^{(4/3)} * 1i - (-a)^{(4/3)} * ((3^{(1/2)} * 1i) / 2 + 1/2) * (A * b - B * a)) / (3 * b^{(7/3)}) + ((-a)^{(1/3)} * \log(3^{(1/2)} * (-a)^{(4/3)} * 1i - (-a)^{(4/3)} + 2 * a * b^{(1/3)} * x) * ((3^{(1/2)} * 1i) / 2 - 1/2) * (A * b - B * a)) / (3 * b^{(7/3)})$

sympy [A] time = 1.15, size = 87, normalized size = 0.54

$$\frac{Bx^4}{4b} + x \left(\frac{A}{b} - \frac{Ba}{b^2} \right) + \text{RootSum} \left(27t^3b^7 + A^3ab^3 - 3A^2Ba^2b^2 + 3AB^2a^3b - B^3a^4, \left(t \mapsto t \log \left(\frac{3tb^2}{-Ab + Ba} + x \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(B*x**3+A)/(b*x**3+a), x)

[Out] $B * x^{**4} / (4 * b) + x * (A / b - B * a / b^{**2}) + \text{RootSum}(27 * t^{**3} * b^{**7} + A^{**3} * a * b^{**3} - 3 * A^{**2} * B * a^{**2} * b^{**2} + 3 * A * B^{**2} * a^{**3} * b - B^{**3} * a^{**4}, \text{Lambda}(t, t * \log(3 * t * b^{**2} / (-A * b + B * a) + x)))$

$$3.60 \quad \int \frac{x^2(A+Bx^3)}{a+bx^3} dx$$

Optimal. Leaf size=35

$$\frac{(Ab - aB) \log(a + bx^3)}{3b^2} + \frac{Bx^3}{3b}$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {444, 43}

$$\frac{(Ab - aB) \log(a + bx^3)}{3b^2} + \frac{Bx^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(A + B*x^3))/(a + b*x^3),x]

[Out] (B*x^3)/(3*b) + ((A*b - a*B)*Log[a + b*x^3])/(3*b^2)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2(A+Bx^3)}{a+bx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A+Bx}{a+bx} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{B}{b} + \frac{Ab-aB}{b(a+bx)} \right) dx, x, x^3 \right) \\ &= \frac{Bx^3}{3b} + \frac{(Ab-aB) \log(a+bx^3)}{3b^2} \end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.89

$$\frac{(Ab - aB) \log(a + bx^3) + bBx^3}{3b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3), x]

[Out] (b*B*x^3 + (A*b - a*B)*Log[a + b*x^3])/(3*b^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (A + Bx^3)}{a + bx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3), x]

[Out] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3), x]

fricas [A] time = 0.58, size = 30, normalized size = 0.86

$$\frac{Bbx^3 - (Ba - Ab) \log(bx^3 + a)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a), x, algorithm="fricas")

[Out] 1/3*(B*b*x^3 - (B*a - A*b)*log(b*x^3 + a))/b^2

giac [A] time = 0.22, size = 32, normalized size = 0.91

$$\frac{Bx^3}{3b} - \frac{(Ba - Ab) \log(|bx^3 + a|)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a), x, algorithm="giac")

[Out] 1/3*B*x^3/b - 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/b^2

maple [A] time = 0.04, size = 40, normalized size = 1.14

$$\frac{Bx^3}{3b} + \frac{A \ln(bx^3 + a)}{3b} - \frac{Ba \ln(bx^3 + a)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(B*x^3+A)/(b*x^3+a),x)`

[Out] $1/3*B*x^3/b+1/3/b*\ln(b*x^3+a)*A-1/3/b^2*\ln(b*x^3+a)*B*a$

maxima [A] time = 0.54, size = 31, normalized size = 0.89

$$\frac{Bx^3}{3b} - \frac{(Ba - Ab) \log(bx^3 + a)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")`

[Out] $1/3*B*x^3/b - 1/3*(B*a - A*b)*\log(b*x^3 + a)/b^2$

mupad [B] time = 0.06, size = 31, normalized size = 0.89

$$\frac{Bx^3}{3b} + \frac{\ln(bx^3 + a)(Ab - Ba)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(A + B*x^3))/(a + b*x^3),x)`

[Out] $(B*x^3)/(3*b) + (\log(a + b*x^3)*(A*b - B*a))/(3*b^2)$

sympy [A] time = 0.88, size = 27, normalized size = 0.77

$$\frac{Bx^3}{3b} - \frac{(-Ab + Ba) \log(a + bx^3)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(B*x**3+A)/(b*x**3+a),x)`

[Out] $B*x**3/(3*b) - (-A*b + B*a)*\log(a + b*x**3)/(3*b**2)$

$$3.61 \quad \int \frac{x(A+Bx^3)}{a+bx^3} dx$$

Optimal. Leaf size=150

$$\frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6 \sqrt[3]{a} b^{5/3}} - \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3 \sqrt[3]{a} b^{5/3}} - \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} \sqrt[3]{a} b^{5/3}} + \frac{Bx^2}{2b}$$

Rubi [A] time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {459, 292, 31, 634, 617, 204, 628}

$$\frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6 \sqrt[3]{a} b^{5/3}} - \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3 \sqrt[3]{a} b^{5/3}} - \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} \sqrt[3]{a} b^{5/3}} + \frac{Bx^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[(x*(A + B*x^3))/(a + b*x^3), x]

[Out] (B*x^2)/(2*b) - ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) - ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(5/3)) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(5/3))

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(A + Bx^3)}{a + bx^3} dx &= \frac{Bx^2}{2b} - \frac{(-2Ab + 2aB)}{2b} \int \frac{x}{a+bx^3} dx \\
&= \frac{Bx^2}{2b} - \frac{(Ab - aB) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3\sqrt[3]{a}b^{4/3}} + \frac{(Ab - aB) \int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3\sqrt[3]{a}b^{4/3}} \\
&= \frac{Bx^2}{2b} - \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3\sqrt[3]{a}b^{5/3}} + \frac{(Ab - aB) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6\sqrt[3]{a}b^{5/3}} + \frac{(Ab - aB) \int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6\sqrt[3]{a}b^{5/3}} \\
&= \frac{Bx^2}{2b} - \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3\sqrt[3]{a}b^{5/3}} + \frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6\sqrt[3]{a}b^{5/3}} + \frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6\sqrt[3]{a}b^{5/3}} \\
&= \frac{Bx^2}{2b} - \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}} - \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3\sqrt[3]{a}b^{5/3}} + \frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6\sqrt[3]{a}b^{5/3}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 152, normalized size = 1.01

$$-\frac{(aB - Ab) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6\sqrt[3]{a}b^{5/3}} + \frac{(aB - Ab) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3\sqrt[3]{a}b^{5/3}} - \frac{(aB - Ab) \tan^{-1}\left(\frac{2\sqrt[3]{b}x - \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}} + \frac{Bx^2}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(A + B*x^3))/(a + b*x^3), x]

[Out] (B*x^2)/(2*b) - ((-(A*b) + a*B)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*b^(5/3)) + ((-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x]/(3*a^(1/3)*b^(5/3)) - ((-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(1/3)*b^(5/3)))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A + Bx^3)}{a + bx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(A + B*x^3))/(a + b*x^3), x]

[Out] IntegrateAlgebraic[(x*(A + B*x^3))/(a + b*x^3), x]

fricas [A] time = 0.75, size = 382, normalized size = 2.55

$$\frac{3Ba^2x^2 - 3\sqrt{3}(Ba^2b - Ab^3)\sqrt{\frac{a^2}{b^2}} \log\left(\frac{2x^2 + 2\sqrt{3}\frac{a}{b}x + \frac{a^2}{b^2}}{3\frac{a^2}{b^2}}\right) - (-ab)^{\frac{2}{3}}(Ba - Ab)\log\left(\frac{2x^2 + (-ab)^{\frac{1}{3}}bx + (-ab)^{\frac{2}{3}}}{3\frac{a^2}{b^2}}\right) + 2(-ab)^{\frac{2}{3}}(Ba - Ab)\log\left(\frac{bx - (-ab)^{\frac{1}{3}}}{3\frac{a^2}{b^2}}\right)}{6ab^2} + \frac{3Ba^2x^2 - 6\sqrt{3}(Ba^2b - Ab^3)\sqrt{\frac{a^2}{b^2}} \arctan\left(\frac{\sqrt{3}\frac{2x + (-\frac{a}{b})^{\frac{1}{3}}}}{3\frac{a^2}{b^2}}\right) - (-ab)^{\frac{2}{3}}(Ba - Ab)\log\left(\frac{2x^2 + (-ab)^{\frac{1}{3}}bx + (-ab)^{\frac{2}{3}}}{3\frac{a^2}{b^2}}\right) + 2(-ab)^{\frac{2}{3}}(Ba - Ab)\log\left(\frac{bx - (-ab)^{\frac{1}{3}}}{3\frac{a^2}{b^2}}\right)}{6ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")

[Out] [1/6*(3*B*a*b^2*x^2 - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - (-a*b^2)^(2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*(-a*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3), 1/6*(3*B*a*b^2*x^2 - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (-a*b^2)^(2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*(-a*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3)]

giac [A] time = 0.19, size = 161, normalized size = 1.07

$$\frac{Bx^2}{2b} - \frac{\sqrt{3}(Ba - Ab) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(-ab^2)^{\frac{1}{3}}b} + \frac{(Ba - Ab) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(-ab^2)^{\frac{1}{3}}b} + \frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")

[Out] 1/2*B*x^2/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b) + 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b) + 1/3*(B*a*b*(-a/b)^(1/3) - A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)

maple [A] time = 0.04, size = 198, normalized size = 1.32

$$\frac{Bx^2}{2b} + \frac{\sqrt{3}A \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b} - \frac{A \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b} + \frac{A \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b} - \frac{\sqrt{3}Ba \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} + \frac{Ba \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} - \frac{Ba \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(B*x^3+A)/(b*x^3+a),x)

[Out] $\frac{1}{2}Bx^2/b - 1/3b/(a/b)^{1/3} \ln(x + (a/b)^{1/3}) * A + 1/3/b^2/(a/b)^{1/3} \ln(x + (a/b)^{1/3}) * B * a + 1/6/b/(a/b)^{1/3} \ln(x^2 - (a/b)^{1/3} * x + (a/b)^{2/3}) * A - 1/6/b^2/(a/b)^{1/3} \ln(x^2 - (a/b)^{1/3} * x + (a/b)^{2/3}) * B * a + 1/3/b * 3^{1/2}/(a/b)^{1/3} * \arctan(1/3 * 3^{1/2} * (2/(a/b)^{1/3} * x - 1)) * A - 1/3/b^2 * 3^{1/2}/(a/b)^{1/3} * \arctan(1/3 * 3^{1/2} * (2/(a/b)^{1/3} * x - 1)) * B * a$

maxima [A] time = 1.14, size = 131, normalized size = 0.87

$$\frac{Bx^2}{2b} - \frac{\sqrt{3}(Ba - Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba - Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba - Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")`

[Out] $\frac{1}{2}Bx^2/b - 1/3\sqrt{3}*(B*a - A*b)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{1/3}))/((a/b)^{1/3})/(b^2*(a/b)^{1/3}) - 1/6*(B*a - A*b)*\log(x^2 - x*(a/b)^{1/3} + (a/b)^{2/3})/(b^2*(a/b)^{1/3}) + 1/3*(B*a - A*b)*\log(x + (a/b)^{1/3})/(b^2*(a/b)^{1/3})$

mupad [B] time = 2.57, size = 126, normalized size = 0.84

$$\frac{Bx^2}{2b} - \frac{\ln(b^{1/3}x + a^{1/3})(Ab - Ba)}{3a^{1/3}b^{5/3}} - \frac{\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab - Ba)}{3a^{1/3}b^{5/3}} + \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab - Ba)}{3a^{1/3}b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(A + B*x^3))/(a + b*x^3),x)`

[Out] $\frac{Bx^2}{2b} - \frac{(\log(b^{1/3}x + a^{1/3})*(A*b - B*a))/(3*a^{1/3}*b^{5/3}) - (\log(3^{1/2}*a^{1/3}*i - 2*b^{1/3}*x + a^{1/3}))*((3^{1/2}*i)/2 - 1/2)*(A*b - B*a))/(3*a^{1/3}*b^{5/3}) + (\log(3^{1/2}*a^{1/3}*i + 2*b^{1/3}*x - a^{1/3}))*((3^{1/2}*i)/2 + 1/2)*(A*b - B*a))/(3*a^{1/3}*b^{5/3})}$

sympy [A] time = 1.11, size = 92, normalized size = 0.61

$$\frac{Bx^2}{2b} + \text{RootSum}\left(27t^3ab^5 + A^3b^3 - 3A^2Bab^2 + 3AB^2a^2b - B^3a^3, \left(t \mapsto t \log\left(\frac{9t^2ab^3}{A^2b^2 - 2ABab + B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(B*x**3+A)/(b*x**3+a),x)`

[Out] $Bx^2/(2b) + \text{RootSum}(27*_t**3*a*b**5 + A**3*b**3 - 3*A**2*B*a*b**2 + 3*A*B**2*a**2*b - B**3*a**3, \text{Lambda}(_t, _t*\log(9*_t**2*a*b**3/(A**2*b**2 - 2*A*B*a*b + B**2*a**2) + x)))$

$$3.62 \quad \int \frac{A+Bx^3}{a+bx^3} dx$$

Optimal. Leaf size=145

$$\frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3} b^{4/3}} + \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3} b^{4/3}} - \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} b^{4/3}} + \frac{Bx}{b}$$

Rubi [A] time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {388, 200, 31, 634, 617, 204, 628}

$$-\frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3} b^{4/3}} + \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3} b^{4/3}} - \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} b^{4/3}} + \frac{Bx}{b}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(a + b*x^3), x]

[Out] (B*x)/b - ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(4/3)) + ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(4/3)) - ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(2/3)*b^(4/3)))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{a + bx^3} dx &= \frac{Bx}{b} - \frac{(-Ab + aB) \int \frac{1}{a+bx^3} dx}{b} \\
&= \frac{Bx}{b} + \frac{(Ab - aB) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{2/3}b} + \frac{(Ab - aB) \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{2/3}b} \\
&= \frac{Bx}{b} + \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}b^{4/3}} - \frac{(Ab - aB) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{2/3}b^{4/3}} + \frac{(Ab - aB) \int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x}}{2\sqrt[3]{a}b} \\
&= \frac{Bx}{b} + \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}b^{4/3}} - \frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{2/3}b^{4/3}} + \frac{(Ab - aB) \text{Subs}}{2\sqrt[3]{a}b} \\
&= \frac{Bx}{b} - \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}} + \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}b^{4/3}} - \frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{2/3}b^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 129, normalized size = 0.89

$$\frac{-(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) + 6a^{2/3}\sqrt[3]{b}Bx + 2(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 2\sqrt{3}(Ab - aB) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(a + b*x^3), x]

[Out] (6*a^(2/3)*b^(1/3)*B*x - 2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{a + bx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(a + b*x^3), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(a + b*x^3), x]

fricas [A] time = 0.69, size = 369, normalized size = 2.54

$$\frac{6Ba^2bx - 3\sqrt{3}(Ba^2b - Ab^2)\sqrt{\frac{(a^2)^2}{3}} \log\left(\frac{2a^2x - (a^2)^{1/3}x + (a^2)^{2/3}}{3(a^2)^{1/3}}\right) + (a^2)^{1/3}(Ba - Ab)\log\left(\frac{a^2x^2 - (a^2)^{1/3}x + (a^2)^{2/3}}{3}\right) - 2(a^2)^{1/3}(Ba - Ab)\log\left(\frac{a^2x + (a^2)^{1/3}}{3}\right)}{6a^2b^2} + \frac{6Ba^2bx - 6\sqrt{3}(Ba^2b - Ab^2)\sqrt{\frac{(a^2)^2}{3}} \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{1/3}\right)}{3\left(\frac{a}{b}\right)^{1/3}}\right) + (a^2)^{1/3}(Ba - Ab)\log\left(\frac{a^2x^2 - (a^2)^{1/3}x + (a^2)^{2/3}}{3}\right) - 2(a^2)^{1/3}(Ba - Ab)\log\left(\frac{a^2x + (a^2)^{1/3}}{3}\right)}{6a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="fricas")

[Out] $\frac{1}{6}*(6*B*a^2*b*x - 3*\sqrt{1/3}*(B*a^2*b - A*a*b^2)*\sqrt{-a^2*b}^{(1/3)}/b)*\log((2*a*b*x^3 - 3*(a^2*b)^{(1/3)}*a*x - a^2 + 3*\sqrt{1/3}*(2*a*b*x^2 + (a^2*b)^{(2/3)}*x - (a^2*b)^{(1/3)}*a)*\sqrt{-a^2*b}^{(1/3)}/b))/(b*x^3 + a) + (a^2*b)^{(2/3)}*(B*a - A*b)*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) - 2*(a^2*b)^{(2/3)}*(B*a - A*b)*\log(a*b*x + (a^2*b)^{(2/3)})/(a^2*b^2), \frac{1}{6}*(6*B*a^2*b*x - 6*\sqrt{1/3}*(B*a^2*b - A*a*b^2)*\sqrt{(a^2*b)^{(1/3)}/b})*\arctan(\sqrt{1/3}*(2*(a^2*b)^{(2/3)}*x - (a^2*b)^{(1/3)}*a)*\sqrt{(a^2*b)^{(1/3)}/b}/a^2) + (a^2*b)^{(2/3)}*(B*a - A*b)*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) - 2*(a^2*b)^{(2/3)}*(B*a - A*b)*\log(a*b*x + (a^2*b)^{(2/3)})/(a^2*b^2)]$

giac [A] time = 0.18, size = 133, normalized size = 0.92

$$\frac{\sqrt{3}(Ba - Ab) \arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{a}{b}\right)^{1/3}\right)}{3\left(\frac{a}{b}\right)^{1/3}}\right)}{3(-ab^2)^{2/3}} + \frac{(Ba - Ab) \log\left(x^2 + x\left(\frac{a}{b}\right)^{1/3} + \left(\frac{a}{b}\right)^{2/3}\right)}{6(-ab^2)^{2/3}} + \frac{Bx}{b} + \frac{(Ba - Ab)\left(\frac{a}{b}\right)^{1/3} \log\left(x - \left(\frac{a}{b}\right)^{1/3}\right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{1}{3}*\sqrt{3}*(B*a - A*b)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(-a*b^2)^{(2/3)} + 1/6*(B*a - A*b)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(-a*b^2)^{(2/3)} + B*x/b + 1/3*(B*a - A*b)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a*b)$

maple [A] time = 0.04, size = 195, normalized size = 1.34

$$\frac{\sqrt{3} A \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{1/3}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{2/3} b} + \frac{A \ln\left(x + \left(\frac{a}{b}\right)^{1/3}\right)}{3\left(\frac{a}{b}\right)^{2/3} b} - \frac{A \ln\left(x^2 - \left(\frac{a}{b}\right)^{1/3} x + \left(\frac{a}{b}\right)^{2/3}\right)}{6\left(\frac{a}{b}\right)^{2/3} b} - \frac{\sqrt{3} B a \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{1/3}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{2/3} b^2} - \frac{B a \ln\left(x + \left(\frac{a}{b}\right)^{1/3}\right)}{3\left(\frac{a}{b}\right)^{2/3} b^2} + \frac{B a \ln\left(x^2 - \left(\frac{a}{b}\right)^{1/3} x + \left(\frac{a}{b}\right)^{2/3}\right)}{6\left(\frac{a}{b}\right)^{2/3} b^2} + \frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/(b*x^3+a),x)

[Out] $B*x/b + 1/3/b/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*A - 1/3/b^2/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*B*a - 1/6/b/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*A + 1/6/b^2/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*B*a + 1/3/b/(a/b)^{(2/3)}*3^{(1/2)}*arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*A - 1/3/b^2/(a/b)^{(2/3)}*3^{(1/2)}*arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*B*a$

maxima [A] time = 1.16, size = 128, normalized size = 0.88

$$\frac{Bx}{b} - \frac{\sqrt{3}(Ba - Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba - Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba - Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/(b*x^3+a),x, algorithm="maxima")`

[Out] $B*x/b - 1/3*\sqrt{3}*(B*a - A*b)*arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(b^2*(a/b)^{(2/3)}) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^2*(a/b)^{(2/3)}) - 1/3*(B*a - A*b)*log(x + (a/b)^{(1/3)})/(b^2*(a/b)^{(2/3)})$

mupad [B] time = 2.54, size = 123, normalized size = 0.85

$$\frac{Bx}{b} + \frac{\ln(b^{1/3}x + a^{1/3})(Ab - Ba)}{3a^{2/3}b^{4/3}} - \frac{\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}1i)\left(\frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)(Ab - Ba)}{3a^{2/3}b^{4/3}} + \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}1i)\left(-\frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)(Ab - Ba)}{3a^{2/3}b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)/(a + b*x^3),x)`

[Out] $(B*x)/b + (\log(b^{(1/3)}*x + a^{(1/3)})*(A*b - B*a))/(3*a^{(2/3)}*b^{(4/3)}) - (\log(3^{(1/2)}*a^{(1/3)}*1i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a))/(3*a^{(2/3)}*b^{(4/3)}) + (\log(3^{(1/2)}*a^{(1/3)}*1i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a))/(3*a^{(2/3)}*b^{(4/3)})$

sympy [A] time = 1.02, size = 71, normalized size = 0.49

$$\frac{Bx}{b} + \text{RootSum}\left(27t^3a^2b^4 - A^3b^3 + 3A^2Bab^2 - 3AB^2a^2b + B^3a^3, \left(t \mapsto t \log\left(-\frac{3tab}{-Ab + Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)/(b*x**3+a),x)`

[Out] $B*x/b + \text{RootSum}(27*_t**3*a**2*b**4 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B**2*a**2*b + B**3*a**3, \text{Lambda}(_t, _t*\log(-3*_t*a*b/(-A*b + B*a) + x))$

$$3.63 \quad \int \frac{A+Bx^3}{x(a+bx^3)} dx$$

Optimal. Leaf size=34

$$\frac{A \log(x)}{a} - \frac{(Ab - aB) \log(a + bx^3)}{3ab}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 72}

$$\frac{A \log(x)}{a} - \frac{(Ab - aB) \log(a + bx^3)}{3ab}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x*(a + b*x^3)), x]

[Out] (A*Log[x])/a - ((A*b - a*B)*Log[a + b*x^3])/(3*a*b)

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^3}{x(a + bx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x(a + bx)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{A}{ax} + \frac{-Ab + aB}{a(a + bx)} \right) dx, x, x^3 \right) \\ &= \frac{A \log(x)}{a} - \frac{(Ab - aB) \log(a + bx^3)}{3ab} \end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{(aB - Ab) \log(a + bx^3)}{3ab} + \frac{A \log(x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)),x]

[Out] (A*Log[x])/a + ((-(A*b) + a*B)*Log[a + b*x^3])/(3*a*b)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x(a + bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)),x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)), x]

fricas [A] time = 0.48, size = 32, normalized size = 0.94

$$\frac{3Ab \log(x) + (Ba - Ab) \log(bx^3 + a)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="fricas")

[Out] 1/3*(3*A*b*log(x) + (B*a - A*b)*log(b*x^3 + a))/(a*b)

giac [A] time = 0.19, size = 34, normalized size = 1.00

$$\frac{A \log(|x|)}{a} + \frac{(Ba - Ab) \log(|bx^3 + a|)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="giac")

[Out] A*log(abs(x))/a + 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/(a*b)

maple [A] time = 0.05, size = 37, normalized size = 1.09

$$\frac{A \ln(x)}{a} - \frac{A \ln(bx^3 + a)}{3a} + \frac{B \ln(bx^3 + a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^3+A)/x/(b*x^3+a),x)`

[Out] $-1/3/a*\ln(b*x^3+a)*A+1/3/b*\ln(b*x^3+a)*B+A/a*\ln(x)$

maxima [A] time = 0.59, size = 35, normalized size = 1.03

$$\frac{A \log(x^3)}{3a} + \frac{(Ba - Ab) \log(bx^3 + a)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="maxima")`

[Out] $1/3*A*\log(x^3)/a + 1/3*(B*a - A*b)*\log(b*x^3 + a)/(a*b)$

mupad [B] time = 0.10, size = 36, normalized size = 1.06

$$\frac{B \ln(bx^3 + a)}{3b} - \frac{A \ln(bx^3 + a)}{3a} + \frac{A \ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)/(x*(a + b*x^3)),x)`

[Out] $(B*\log(a + b*x^3))/(3*b) - (A*\log(a + b*x^3))/(3*a) + (A*\log(x))/a$

sympy [A] time = 2.12, size = 26, normalized size = 0.76

$$\frac{A \log(x)}{a} + \frac{(-Ab + Ba) \log\left(\frac{a}{b} + x^3\right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)/x/(b*x**3+a),x)`

[Out] $A*\log(x)/a + (-A*b + B*a)*\log(a/b + x**3)/(3*a*b)$

$$3.64 \quad \int \frac{A+Bx^3}{x^2(a+bx^3)} dx$$

Optimal. Leaf size=147

$$-\frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{4/3} b^{2/3}} + \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{4/3} b^{2/3}} + \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{4/3} b^{2/3}} - \frac{A}{ax}$$

Rubi [A] time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {453, 292, 31, 634, 617, 204, 628}

$$-\frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{4/3} b^{2/3}} + \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{4/3} b^{2/3}} + \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{4/3} b^{2/3}} - \frac{A}{ax}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)), x]

[Out] -(A/(a*x)) + ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(4/3)*b^(2/3)) + ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*b^(2/3)) - ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(n-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^2(a + bx^3)} dx &= -\frac{A}{ax} - \frac{(Ab - aB) \int \frac{x}{a+bx^3} dx}{a} \\
&= -\frac{A}{ax} + \frac{(Ab - aB) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{4/3}\sqrt[3]{b}} - \frac{(Ab - aB) \int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{4/3}\sqrt[3]{b}} \\
&= -\frac{A}{ax} + \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{4/3}b^{2/3}} - \frac{(Ab - aB) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{4/3}b^{2/3}} - \frac{(Ab - aB) \int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{2a\sqrt[3]{b}} \\
&= -\frac{A}{ax} + \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{4/3}b^{2/3}} - \frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{4/3}b^{2/3}} - \frac{(Ab - aB)}{2a\sqrt[3]{b}} \\
&= -\frac{A}{ax} + \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}} + \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{4/3}b^{2/3}} - \frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{4/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 134, normalized size = 0.91

$$\frac{-x(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) - 6\sqrt[3]{a}Ab^{2/3} + 2x(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x) + 2\sqrt{3}x(Ab - aB) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{6a^{4/3}b^{2/3}x}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)), x]

[Out] (-6*a^(1/3)*A*b^(2/3) + 2*Sqrt[3]*(A*b - a*B)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(A*b - a*B)*x*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)*x*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(2/3)*x)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^2(a + bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^2*(a + b*x^3)), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^2*(a + b*x^3)), x]

fricas [A] time = 0.68, size = 372, normalized size = 2.53

$$\frac{6 \sqrt{3} (Ba^2 - Ab^2) \sqrt{\frac{(a^2)^2}{a}} \log\left(\frac{2 \sqrt{2} \sqrt{a^2 + (a^2)^2} \sqrt{\frac{(a^2)^2}{a}} - \sqrt{\frac{(a^2)^2}{a}}}{2 \sqrt{2} \sqrt{a^2 + (a^2)^2} \sqrt{\frac{(a^2)^2}{a}} + \sqrt{\frac{(a^2)^2}{a}}}\right) - (a^2)^{\frac{1}{3}} (Ba - Ab) \log\left(\frac{(a^2)^2}{a} + (a^2)^{\frac{1}{3}}\right) + 2 (a^2)^{\frac{1}{3}} (Ba - Ab) \log\left(\frac{(a^2)^2}{a} + (a^2)^{\frac{1}{3}}\right)}{6 a^2 b^2 x} + \frac{6 \sqrt{3} (Ba^2 - Ab^2) \sqrt{\frac{(a^2)^2}{a}} \arctan\left(\frac{\sqrt{2} \sqrt{a^2 + (a^2)^2} \sqrt{\frac{(a^2)^2}{a}}}{\sqrt{\frac{(a^2)^2}{a}}}\right) - (a^2)^{\frac{1}{3}} (Ba - Ab) \log\left(\frac{(a^2)^2}{a} + (a^2)^{\frac{1}{3}}\right) + 2 (a^2)^{\frac{1}{3}} (Ba - Ab) \log\left(\frac{(a^2)^2}{a} + (a^2)^{\frac{1}{3}}\right)}{6 a^2 b^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/6*(6*A*a*b^2 + 3*\sqrt{1/3}*(B*a^2*b - A*a*b^2)*x*\sqrt{-(a*b^2)^{(1/3)}/a}) \\ & * \log((2*b^2*x^3 - a*b - 3*\sqrt{1/3}*(a*b*x + 2*(a*b^2)^{(2/3)}*x^2 - (a*b^2)^{(1/3)*a})*\sqrt{-(a*b^2)^{(1/3)}/a} - 3*(a*b^2)^{(2/3)*x}/(b*x^3 + a)) - (a*b^2)^{(2/3)} \\ & *(B*a - A*b)*x*\log(b^2*x^2 - (a*b^2)^{(1/3)*b*x + (a*b^2)^{(2/3)}) + 2*(a*b^2)^{(2/3)} \\ & *(B*a - A*b)*x*\log(b*x + (a*b^2)^{(1/3)})]/(a^2*b^2*x), -1/6*(6*A*a*b^2 + 6*\sqrt{1/3}*(B*a^2*b - A*a*b^2)*x*\sqrt{(a*b^2)^{(1/3)}/a}*\arctan(-\sqrt{1/3} \\ & *(2*b*x - (a*b^2)^{(1/3)})*\sqrt{(a*b^2)^{(1/3)}/a}/b - (a*b^2)^{(2/3)}*(B*a - A*b)*x*\log(b^2*x^2 - (a*b^2)^{(1/3)*b*x + (a*b^2)^{(2/3)}) + 2*(a*b^2)^{(2/3)} \\ & *(B*a - A*b)*x*\log(b*x + (a*b^2)^{(1/3)})]/(a^2*b^2*x)] \end{aligned}$$

giac [A] time = 0.22, size = 155, normalized size = 1.05

$$\frac{\sqrt{3} (Ba - Ab) \arctan\left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 \left(-ab^2\right)^{\frac{1}{3}} a} - \frac{(Ba - Ab) \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(-ab^2\right)^{\frac{1}{3}} a} - \frac{\left(Ba \left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 a^2} - \frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/3*\sqrt{3}*(B*a - A*b)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)}) \\ &)/(((a*b^2)^{(1/3)*a} - 1/6*(B*a - A*b)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/((a*b^2)^{(1/3)*a} - 1/3*(B*a*(-a/b)^{(1/3)} - A*b*(-a/b)^{(1/3))*(-a/b)^{(1/3)} \\ & * \log(\text{abs}(x - (-a/b)^{(1/3)})))/a^2 - A/(a*x) \end{aligned}$$

maple [A] time = 0.06, size = 195, normalized size = 1.33

$$\frac{\sqrt{3} A \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} a} + \frac{A \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - A \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} a} + \frac{\sqrt{3} B \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b} - \frac{B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b} - \frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^2/(b*x^3+a),x)

[Out] $\frac{1}{3} \frac{a}{b} \left(\frac{a}{b}\right)^{1/3} \ln(x + \left(\frac{a}{b}\right)^{1/3}) * A - \frac{1}{3} \frac{b}{a} \left(\frac{a}{b}\right)^{1/3} \ln(x + \left(\frac{a}{b}\right)^{1/3}) * B - \frac{1}{6} \frac{a}{b} \left(\frac{a}{b}\right)^{1/3} \ln(x^2 - \left(\frac{a}{b}\right)^{1/3} * x + \left(\frac{a}{b}\right)^{2/3}) * A + \frac{1}{6} \frac{b}{a} \left(\frac{a}{b}\right)^{1/3} \ln(x^2 - \left(\frac{a}{b}\right)^{1/3} * x + \left(\frac{a}{b}\right)^{2/3}) * B - \frac{1}{3} \frac{a^3}{b} \left(\frac{a}{b}\right)^{1/3} \arctan\left(\frac{1}{3} \sqrt{3} \left(\frac{1}{2}\right) * \left(\frac{2}{\left(\frac{a}{b}\right)^{1/3}} * x - 1\right)\right) * A + \frac{1}{3} \frac{b^3}{a} \sqrt{3} \left(\frac{1}{2}\right) / \left(\frac{a}{b}\right)^{1/3} \arctan\left(\frac{1}{3} \sqrt{3} \left(\frac{1}{2}\right) * \left(\frac{2}{\left(\frac{a}{b}\right)^{1/3}} * x - 1\right)\right) * B - A/a/x$

maxima [A] time = 1.16, size = 140, normalized size = 0.95

$$\frac{\sqrt{3}(Ba - Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{1/3}\right)}{3\left(\frac{a}{b}\right)^{1/3}}\right)}{3ab\left(\frac{a}{b}\right)^{1/3}} + \frac{(Ba - Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{1/3} + \left(\frac{a}{b}\right)^{2/3}\right)}{6ab\left(\frac{a}{b}\right)^{1/3}} - \frac{(Ba - Ab) \log\left(x + \left(\frac{a}{b}\right)^{1/3}\right)}{3ab\left(\frac{a}{b}\right)^{1/3}} - \frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="maxima")

[Out] $\frac{1}{3} \sqrt{3} * (B*a - A*b) * \arctan\left(\frac{1}{3} \sqrt{3} * (2*x - \left(\frac{a}{b}\right)^{1/3}) / \left(\frac{a}{b}\right)^{1/3}\right) / (a*b * \left(\frac{a}{b}\right)^{1/3}) + \frac{1}{6} * (B*a - A*b) * \log(x^2 - x * \left(\frac{a}{b}\right)^{1/3} + \left(\frac{a}{b}\right)^{2/3}) / (a*b * \left(\frac{a}{b}\right)^{1/3}) - \frac{1}{3} * (B*a - A*b) * \log(x + \left(\frac{a}{b}\right)^{1/3}) / (a*b * \left(\frac{a}{b}\right)^{1/3}) - A/(a*x)$

mupad [B] time = 2.54, size = 126, normalized size = 0.86

$$\frac{\ln(b^{1/3}x + a^{1/3})(Ab - Ba)}{3a^{4/3}b^{2/3}} - \frac{A}{ax} + \frac{\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab - Ba)}{3a^{4/3}b^{2/3}} - \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab - Ba)}{3a^{4/3}b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^2*(a + b*x^3)),x)

[Out] $(\log(b^{1/3} * x + a^{1/3}) * (A * b - B * a)) / (3 * a^{4/3} * b^{2/3}) - A / (a * x) + (\log(3^{1/2} * a^{1/3} * i - 2 * b^{1/3} * x + a^{1/3}) * ((3^{1/2} * i) / 2 - 1/2) * (A * b - B * a)) / (3 * a^{4/3} * b^{2/3}) - (\log(3^{1/2} * a^{1/3} * i + 2 * b^{1/3} * x - a^{1/3}) * ((3^{1/2} * i) / 2 + 1/2) * (A * b - B * a)) / (3 * a^{4/3} * b^{2/3})$

sympy [A] time = 0.94, size = 90, normalized size = 0.61

$$-\frac{A}{ax} + \text{RootSum}\left(27t^3a^4b^2 - A^3b^3 + 3A^2Bab^2 - 3AB^2a^2b + B^3a^3, \left(t \mapsto t \log\left(\frac{9t^2a^3b}{A^2b^2 - 2ABab + B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**2/(b*x**3+a),x)

[Out] $-A/(a*x) + \text{RootSum}(27*_t**3*a**4*b**2 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B**2*a**2*b + B**3*a**3, \text{Lambda}(_t, _t * \log(9*_t**2*a**3*b/(A**2*b**2 - 2*A*B*a*b + B**2*a**2) + x)))$

$$3.65 \quad \int \frac{A+Bx^3}{x^3(a+bx^3)} dx$$

Optimal. Leaf size=149

$$\frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{5/3} \sqrt[3]{b}} - \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{5/3} \sqrt[3]{b}} + \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{5/3} \sqrt[3]{b}} - \frac{A}{2ax^2}$$

Rubi [A] time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {453, 200, 31, 634, 617, 204, 628}

$$\frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{5/3} \sqrt[3]{b}} - \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{5/3} \sqrt[3]{b}} + \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{5/3} \sqrt[3]{b}} - \frac{A}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)), x]

[Out] -A/(2*a*x^2) + ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*b^(1/3)) - ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(1/3)) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^3(a + bx^3)} dx &= -\frac{A}{2ax^2} - \frac{(2Ab - 2aB) \int \frac{1}{a+bx^3} dx}{2a} \\
&= -\frac{A}{2ax^2} - \frac{(Ab - aB) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{5/3}} - \frac{(Ab - aB) \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{5/3}} \\
&= -\frac{A}{2ax^2} - \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{5/3}\sqrt[3]{b}} - \frac{(Ab - aB) \int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{2a^{4/3}} + \frac{(Ab - aB) \int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{5/3}} \\
&= -\frac{A}{2ax^2} - \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{5/3}\sqrt[3]{b}} + \frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{5/3}\sqrt[3]{b}} - \frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{5/3}} \\
&= -\frac{A}{2ax^2} + \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}} - \frac{(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{5/3}\sqrt[3]{b}} + \frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{5/3}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 135, normalized size = 0.91

$$\frac{\frac{(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{\sqrt[3]{b}} - \frac{3a^{2/3}A}{x^2} + \frac{2(aB - Ab) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{\sqrt[3]{b}} + \frac{2\sqrt{3}(Ab - aB) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{6a^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)), x]

[Out] ((-3*a^(2/3)*A)/x^2 + (2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (2*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(6*a^(5/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^3(a + bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^3*(a + b*x^3)), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^3*(a + b*x^3)), x]

fricas [A] time = 0.50, size = 411, normalized size = 2.76

$$\frac{3\sqrt{3}\sqrt{(Ba^2b - Ab^3)\sqrt{\frac{a^2}{3}}\arctan\left(\frac{\sqrt{3}\sqrt{2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + (-a^2)^{\frac{1}{3}}(Ba - Ab^2)\log\left(\frac{a^2 - (-a^2)^{\frac{1}{3}}x - (-a^2)^{\frac{2}{3}}}{(-a^2)^{\frac{1}{3}}(Ba - Ab^2)\log\left(\frac{a^2 - (-a^2)^{\frac{1}{3}}x - (-a^2)^{\frac{2}{3}}}{(-a^2)^{\frac{1}{3}}}\right) + 2(-a^2)^{\frac{1}{3}}(Ba - Ab^2)\log\left(\frac{a^2 - (-a^2)^{\frac{1}{3}}x - (-a^2)^{\frac{2}{3}}}{(-a^2)^{\frac{1}{3}}}\right) - 3Aa^2}\right)}{6a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="fricas")

[Out] $[-1/6*(3*\sqrt{3}*(B*a^2*b - A*a*b^2)*x^2*\sqrt{(-a^2*b)^{(1/3)}/b}*\log((2*a*b*x^3 + 3*(-a^2*b)^{(1/3)}*a*x - a^2 - 3*\sqrt{3}*(2*a*b*x^2 + (-a^2*b)^{(2/3)})*x + (-a^2*b)^{(1/3)}*a)*\sqrt{(-a^2*b)^{(1/3)}/b}))/b + (-a^2*b)^{(2/3)}*(B*a - A*b)*x^2*\log(a*b*x^2 - (-a^2*b)^{(2/3)}*x - (-a^2*b)^{(1/3)}*a) - 2*(-a^2*b)^{(2/3)}*(B*a - A*b)*x^2*\log(a*b*x + (-a^2*b)^{(2/3)}) + 3*A*a^2*b)/(a^3*b*x^2), 1/6*(6*\sqrt{3}*(B*a^2*b - A*a*b^2)*x^2*\sqrt{-(-a^2*b)^{(1/3)}/b}*arctan(\sqrt{3}*(2*(-a^2*b)^{(2/3)}*x + (-a^2*b)^{(1/3)}*a)*\sqrt{-(-a^2*b)^{(1/3)}/b})/a^2) - (-a^2*b)^{(2/3)}*(B*a - A*b)*x^2*\log(a*b*x^2 - (-a^2*b)^{(2/3)}*x - (-a^2*b)^{(1/3)}*a) + 2*(-a^2*b)^{(2/3)}*(B*a - A*b)*x^2*\log(a*b*x + (-a^2*b)^{(2/3)}) - 3*A*a^2*b)/(a^3*b*x^2)]$

giac [A] time = 0.18, size = 161, normalized size = 1.08

$$-\frac{(Ba - Ab)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^2} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba - \left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba - \left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b} - \frac{A}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="giac")

[Out] $-1/3*(B*a - A*b)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^2 + 1/3*\sqrt{3}*((-a*b^2)^{(1/3)}*B*a - (-a*b^2)^{(1/3)}*A*b)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)}))/(-a/b)^{(1/3)}/(a^2*b) + 1/6*((-a*b^2)^{(1/3)}*B*a - (-a*b^2)^{(1/3)}*A*b)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^2*b) - 1/2*A/(a*x^2)$

maple [A] time = 0.05, size = 195, normalized size = 1.31

$$-\frac{\sqrt{3}A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a} - \frac{A\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a} + \frac{A\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a} + \frac{\sqrt{3}B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b} + \frac{B\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b} - \frac{B\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b} - \frac{A}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^3+A)/x^3/(b*x^3+a),x)`

[Out]
$$-1/3/a/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*A+1/3/b/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*B+1/6/a/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*A-1/6/b/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*B-1/3/a/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*A+1/3/b/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*B-1/2*A/a/x^2$$

maxima [A] time = 1.22, size = 140, normalized size = 0.94

$$\frac{\sqrt{3}(Ba - Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba - Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba - Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{A}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="maxima")`

[Out]
$$1/3*\sqrt{3}*(B*a - A*b)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a*b*(a/b)^{(2/3)}) - 1/6*(B*a - A*b)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a*b*(a/b)^{(2/3)}) + 1/3*(B*a - A*b)*\log(x + (a/b)^{(1/3)})/(a*b*(a/b)^{(2/3)}) - 1/2*A/(a*x^2)$$

mupad [B] time = 0.24, size = 126, normalized size = 0.85

$$-\frac{A}{2ax^2} - \frac{\ln(b^{1/3}x + a^{1/3})(Ab - Ba)}{3a^{5/3}b^{1/3}} + \frac{\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab - Ba)}{3a^{5/3}b^{1/3}} - \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab - Ba)}{3a^{5/3}b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)/(x^3*(a + b*x^3)),x)`

[Out]
$$(\log(3^{(1/2)}*a^{(1/3)}*i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*i)/2 + 1/2)*(A*b - B*a))/(3*a^{(5/3)}*b^{(1/3)}) - (\log(b^{(1/3)}*x + a^{(1/3)})*(A*b - B*a))/(3*a^{(5/3)}*b^{(1/3)}) - A/(2*a*x^2) - (\log(3^{(1/2)}*a^{(1/3)}*i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*i)/2 - 1/2)*(A*b - B*a))/(3*a^{(5/3)}*b^{(1/3)})$$

sympy [A] time = 1.23, size = 73, normalized size = 0.49

$$-\frac{A}{2ax^2} + \text{RootSum}\left(27t^3a^5b + A^3b^3 - 3A^2Bab^2 + 3AB^2a^2b - B^3a^3, \left(t \mapsto t \log\left(\frac{3ta^2}{-Ab + Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)/x**3/(b*x**3+a),x)`

[Out]
$$-A/(2*a*x**2) + \text{RootSum}(27*_t**3*a**5*b + A**3*b**3 - 3*A**2*B*a*b**2 + 3*A**B**2*a**2*b - B**3*a**3, \text{Lambda}(_t, _t*\log(3*_t*a**2/(-A*b + B*a) + x)))$$

$$3.66 \quad \int \frac{A+Bx^3}{x^4(a+bx^3)} dx$$

Optimal. Leaf size=50

$$\frac{(Ab - aB) \log(a + bx^3)}{3a^2} - \frac{\log(x)(Ab - aB)}{a^2} - \frac{A}{3ax^3}$$

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 77}

$$\frac{(Ab - aB) \log(a + bx^3)}{3a^2} - \frac{\log(x)(Ab - aB)}{a^2} - \frac{A}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)),x]

[Out] -A/(3*a*x^3) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3])/(3*a^2)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^3}{x^4(a + bx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x^2(a + bx)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{A}{ax^2} + \frac{-Ab + aB}{a^2x} - \frac{b(-Ab + aB)}{a^2(a + bx)} \right) dx, x, x^3 \right) \\ &= -\frac{A}{3ax^3} - \frac{(Ab - aB) \log(x)}{a^2} + \frac{(Ab - aB) \log(a + bx^3)}{3a^2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.98

$$\frac{(Ab - aB) \log(a + bx^3)}{3a^2} + \frac{\log(x)(aB - Ab)}{a^2} - \frac{A}{3ax^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)), x]

[Out] -1/3*A/(a*x^3) + ((-(A*b) + a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3])/(3*a^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^4(a + bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)), x]

fricas [A] time = 0.50, size = 47, normalized size = 0.94

$$-\frac{(Ba - Ab)x^3 \log(bx^3 + a) - 3(Ba - Ab)x^3 \log(x) + Aa}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a), x, algorithm="fricas")

[Out] -1/3*((B*a - A*b)*x^3*log(b*x^3 + a) - 3*(B*a - A*b)*x^3*log(x) + A*a)/(a^2*x^3)

giac [A] time = 0.17, size = 69, normalized size = 1.38

$$\frac{(Ba - Ab) \log(|x|)}{a^2} - \frac{(Bab - Ab^2) \log(|bx^3 + a|)}{3a^2b} - \frac{Bax^3 - Abx^3 + Aa}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="giac")

[Out] (B*a - A*b)*log(abs(x))/a^2 - 1/3*(B*a*b - A*b^2)*log(abs(b*x^3 + a))/(a^2*b) - 1/3*(B*a*x^3 - A*b*x^3 + A*a)/(a^2*x^3)

maple [A] time = 0.05, size = 56, normalized size = 1.12

$$-\frac{Ab \ln(x)}{a^2} + \frac{Ab \ln(bx^3 + a)}{3a^2} + \frac{B \ln(x)}{a} - \frac{B \ln(bx^3 + a)}{3a} - \frac{A}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^4/(b*x^3+a),x)

[Out] 1/3/a^2*ln(b*x^3+a)*A*b-1/3/a*ln(b*x^3+a)*B-1/3*A/a/x^3-1/a^2*ln(x)*A*b+B/a*ln(x)

maxima [A] time = 0.48, size = 48, normalized size = 0.96

$$-\frac{(Ba - Ab) \log(bx^3 + a)}{3a^2} + \frac{(Ba - Ab) \log(x^3)}{3a^2} - \frac{A}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="maxima")

[Out] -1/3*(B*a - A*b)*log(b*x^3 + a)/a^2 + 1/3*(B*a - A*b)*log(x^3)/a^2 - 1/3*A/(a*x^3)

mupad [B] time = 2.40, size = 46, normalized size = 0.92

$$\frac{\ln(bx^3 + a) (Ab - Ba)}{3a^2} - \frac{A}{3ax^3} - \frac{\ln(x) (Ab - Ba)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^4*(a + b*x^3)),x)

[Out] (log(a + b*x^3)*(A*b - B*a))/(3*a^2) - A/(3*a*x^3) - (log(x)*(A*b - B*a))/a^2

sympy [A] time = 2.36, size = 41, normalized size = 0.82

$$-\frac{A}{3ax^3} + \frac{(-Ab + Ba) \log(x)}{a^2} - \frac{(-Ab + Ba) \log\left(\frac{a}{b} + x^3\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**4/(b*x**3+a), x)

[Out] -A/(3*a*x**3) + (-A*b + B*a)*log(x)/a**2 - (-A*b + B*a)*log(a/b + x**3)/(3*a**2)

$$3.67 \quad \int \frac{A+Bx^3}{x^5(a+bx^3)} dx$$

Optimal. Leaf size=165

$$\frac{\sqrt[3]{b}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{7/3}} - \frac{\sqrt[3]{b}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{7/3}} - \frac{\sqrt[3]{b}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{7/3}} + \frac{A}{4ax^4}$$

Rubi [A] time = 0.11, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {453, 325, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{7/3}} + \frac{Ab - aB}{a^2 x} - \frac{\sqrt[3]{b}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{7/3}} - \frac{\sqrt[3]{b}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{7/3}} - \frac{A}{4ax^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)),x]

[Out] -A/(4*a*x^4) + (A*b - a*B)/(a^2*x) - (b^(1/3)*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(7/3)) - (b^(1/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x]/(3*a^(7/3)) + (b^(1/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(7/3)))

Rule 31

Int[((a_) + (b_.)*(x_))^(−1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(−1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(−1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m+1)*(a + b*x^n)^(p+1))/(a*e*(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^5(a + bx^3)} dx &= -\frac{A}{4ax^4} - \frac{(4Ab - 4aB) \int \frac{1}{x^2(a+bx^3)} dx}{4a} \\
&= -\frac{A}{4ax^4} + \frac{Ab - aB}{a^2x} + \frac{(b(Ab - aB)) \int \frac{x}{a+bx^3} dx}{a^2} \\
&= -\frac{A}{4ax^4} + \frac{Ab - aB}{a^2x} - \frac{(b^{2/3}(Ab - aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{7/3}} + \frac{(b^{2/3}(Ab - aB)) \int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{7/3}} \\
&= -\frac{A}{4ax^4} + \frac{Ab - aB}{a^2x} - \frac{\sqrt[3]{b}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{7/3}} + \frac{(\sqrt[3]{b}(Ab - aB)) \int \frac{-\sqrt[3]{a} \sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{7/3}} \\
&= -\frac{A}{4ax^4} + \frac{Ab - aB}{a^2x} - \frac{\sqrt[3]{b}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{7/3}} + \frac{\sqrt[3]{b}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{6a^{7/3}} \\
&= -\frac{A}{4ax^4} + \frac{Ab - aB}{a^2x} - \frac{\sqrt[3]{b}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{7/3}} - \frac{\sqrt[3]{b}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{7/3}} + \frac{\sqrt[3]{b}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{6a^{7/3}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 154, normalized size = 0.93

$$\frac{2\sqrt[3]{b}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2) - \frac{3a^{4/3}A}{x^4} + \frac{12\sqrt[3]{a}(Ab - aB)}{x} + 4\sqrt[3]{b}(aB - Ab) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 4\sqrt{3} \sqrt[3]{b}(Ab - aB) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3} \sqrt[3]{a}}\right)}{12a^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)), x]

[Out] ((-3*a^(4/3)*A)/x^4 + (12*a^(1/3)*(A*b - a*B))/x - 4*Sqrt[3]*b^(1/3)*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x] + 2*b^(1/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(12*a^(7/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^5(a + bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^5*(a + b*x^3)), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^5*(a + b*x^3)), x]

fricas [A] time = 0.63, size = 158, normalized size = 0.96

$$\frac{4\sqrt{3}(Ba - Ab)x^4\left(\frac{-b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{-b}{a}\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) - 2(Ba - Ab)x^4\left(\frac{-b}{a}\right)^{\frac{1}{3}}\log\left(bx^2 - ax\left(\frac{-b}{a}\right)^{\frac{2}{3}} - a\left(\frac{-b}{a}\right)^{\frac{1}{3}}\right) + 4(Ba - Ab)x^4\left(\frac{-b}{a}\right)^{\frac{1}{3}}\log\left(bx + a\left(\frac{-b}{a}\right)^{\frac{2}{3}}\right) + 12(Ba - Ab)x^3 + 3Aa}{12a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="fricas")

[Out] $-1/12*(4*\sqrt{3}*(B*a - A*b)*x^4*(-b/a)^{(1/3)}*\arctan(2/3*\sqrt{3}*x*(-b/a)^{(1/3)} + 1/3*\sqrt{3}) - 2*(B*a - A*b)*x^4*(-b/a)^{(1/3)}*\log(b*x^2 - a*x*(-b/a)^{(2/3)} - a*(-b/a)^{(1/3)}) + 4*(B*a - A*b)*x^4*(-b/a)^{(1/3)}*\log(b*x + a*(-b/a)^{(2/3)}) + 12*(B*a - A*b)*x^3 + 3*A*a)/(a^2*x^4)$

giac [A] time = 0.22, size = 197, normalized size = 1.19

$$\frac{\left(Bab\left(\frac{-a}{b}\right)^{\frac{1}{3}} - Ab^2\left(\frac{-a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{-a}{b}\right)^{\frac{1}{3}}\log\left(x - \left(\frac{-a}{b}\right)^{\frac{1}{3}}\right) + \sqrt{3}\left((-ab^2)^{\frac{2}{3}}Ba - (-ab^2)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{-a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{-a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3} - \frac{\left((-ab^2)^{\frac{2}{3}}Ba - (-ab^2)^{\frac{2}{3}}Ab\right)\log\left(x^2 + x\left(\frac{-a}{b}\right)^{\frac{1}{3}} + \left(\frac{-a}{b}\right)^{\frac{2}{3}}\right)}{6a^3b} - \frac{4Bax^3 - 4Abx^3 + Aa}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="giac")

[Out] $1/3*(B*a*b*(-a/b)^{(1/3)} - A*b^2*(-a/b)^{(1/3)})*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)})/a^3 + 1/3*\sqrt{3}*((-a*b^2)^{(2/3)}*B*a - (-a*b^2)^{(2/3)}*A*b)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^3*b) - 1/6*((-a*b^2)^{(2/3)}*B*a - (-a*b^2)^{(2/3)}*A*b)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^3*b) - 1/4*(4*B*a*x^3 - 4*A*b*x^3 + A*a)/(a^2*x^4)$

maple [A] time = 0.05, size = 216, normalized size = 1.31

$$\frac{\sqrt{3} Ab \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2} - \frac{Ab \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2} + \frac{Ab \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2} - \frac{\sqrt{3} B \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a} + \frac{B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a} - \frac{B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a} + \frac{Ab}{a^2x} - \frac{B}{ax} - \frac{A}{4ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^5/(b*x^3+a),x)

[Out] $-1/3/a^2*b/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)})*A+1/3/a/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)})*B+1/6/a^2*b/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*A-1/6/a/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*B+1/3/a^2*b*3^{(1/2)}/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*A-1/3/a*3^{(1/2)}/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*B-1/4*A/a/x^4+1/a^2/x*A*b-B/a/x$

maxima [A] time = 1.14, size = 147, normalized size = 0.89

$$\frac{\sqrt{3}(Ba - Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - (Ba - Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + (Ba - Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - \frac{4(Ba - Ab)x^3 + Aa}{4a^2x^4}}{3a^2\left(\frac{a}{b}\right)^{\frac{1}{3}} - 6a^2\left(\frac{a}{b}\right)^{\frac{1}{3}} + 3a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="maxima")

[Out] $-\frac{1}{3}\sqrt{3}(B*a - A*b)*\arctan\left(\frac{1}{3}\sqrt{3}\left(2*x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)/\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)/\left(a^2*\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - \frac{1}{6}(B*a - A*b)*\log\left(x^2 - x*\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)/\left(a^2*\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + \frac{1}{3}(B*a - A*b)*\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)/\left(a^2*\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - \frac{1}{4}(4*(B*a - A*b)*x^3 + A*a)/(a^2*x^4)$

mupad [B] time = 2.59, size = 178, normalized size = 1.08

$$\frac{(-b)^{1/3} \ln(a^{1/3}(-b)^{8/3} + b^3x)(Ab - Ba) - \frac{A}{3a} - \frac{x^3(Ab - Ba)}{x^4} + \frac{(-b)^{1/3} \ln(a^{1/3}(-b)^{8/3} - 2b^3x + \sqrt{3}a^{1/3}(-b)^{8/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab - Ba)}{3a^{7/3}} - \frac{(-b)^{1/3} \ln(2b^3x - a^{1/3}(-b)^{8/3} + \sqrt{3}a^{1/3}(-b)^{8/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab - Ba)}{3a^{7/3}}}{3a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^5*(a + b*x^3)),x)

[Out] $\frac{((-b)^{1/3}*\log(a^{1/3}*(-b)^{8/3} + b^3*x)*(A*b - B*a))/(3*a^{7/3}) - (A/(4*a) - (x^3*(A*b - B*a))/a^2)/x^4 + ((-b)^{1/3}*\log(a^{1/3}*(-b)^{8/3} - 2*b^3*x + 3^{1/2}*a^{1/3}*(-b)^{8/3}*i)*((3^{1/2}*i)/2 - 1/2)*(A*b - B*a))/(3*a^{7/3}) - ((-b)^{1/3}*\log(2*b^3*x - a^{1/3}*(-b)^{8/3} + 3^{1/2}*a^{1/3}*(-b)^{8/3}*i)*((3^{1/2}*i)/2 + 1/2)*(A*b - B*a))/(3*a^{7/3})}{3a^{7/3}}$

sympy [A] time = 0.96, size = 112, normalized size = 0.68

$$\text{RootSum}\left(27t^3a^7 + A^3b^4 - 3A^2Bab^3 + 3AB^2a^2b^2 - B^3a^3b, \left(t \mapsto t \log\left(\frac{9t^2a^5}{A^2b^3 - 2ABab^2 + B^2a^2b} + x\right)\right)\right) + \frac{-Aa + x^3(4Ab - 4Ba)}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**5/(b*x**3+a),x)

[Out] $\text{RootSum}(27*_t**3*a**7 + A**3*b**4 - 3*A**2*B*a*b**3 + 3*A*B**2*a**2*b**2 - B**3*a**3*b, \text{Lambda}(_t, _t*\log(9*_t**2*a**5/(A**2*b**3 - 2*A*B*a*b**2 + B**2*a**2*b) + x))) + (-A*a + x**3*(4*A*b - 4*B*a))/(4*a**2*x**4)$

$$3.68 \quad \int \frac{A+Bx^3}{x^6(a+bx^3)} dx$$

Optimal. Leaf size=168

$$\frac{b^{2/3}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{8/3}} + \frac{b^{2/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{8/3}} - \frac{b^{2/3}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{8/3}}$$

Rubi [A] time = 0.12, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {453, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{8/3}} + \frac{b^{2/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{8/3}} - \frac{b^{2/3}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{8/3}} + \frac{Ab - aB}{2a^2 x^2} - \frac{A}{5ax^5}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)), x]

[Out] $-A/(5*a*x^5) + (A*b - a*B)/(2*a^2*x^2) - (b^{(2/3)}*(A*b - a*B)*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)}*x)/(\text{Sqrt}[3]*a^{(1/3)})]) / (\text{Sqrt}[3]*a^{(8/3)}) + (b^{(2/3)}*(A*b - a*B)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x]) / (3*a^{(8/3)}) - (b^{(2/3)}*(A*b - a*B)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]) / (6*a^{(8/3)})$

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 325

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m+1)*(a + b*x^n)^(p+1))/(a*e*(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^6(a + bx^3)} dx &= -\frac{A}{5ax^5} - \frac{(5Ab - 5aB) \int \frac{1}{x^3(a+bx^3)} dx}{5a} \\
&= -\frac{A}{5ax^5} + \frac{Ab - aB}{2a^2x^2} + \frac{(b(Ab - aB)) \int \frac{1}{a+bx^3} dx}{a^2} \\
&= -\frac{A}{5ax^5} + \frac{Ab - aB}{2a^2x^2} + \frac{(b(Ab - aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{8/3}} + \frac{(b(Ab - aB)) \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{8/3}} \\
&= -\frac{A}{5ax^5} + \frac{Ab - aB}{2a^2x^2} + \frac{b^{2/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{8/3}} - \frac{(b^{2/3}(Ab - aB)) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{8/3}} \\
&= -\frac{A}{5ax^5} + \frac{Ab - aB}{2a^2x^2} + \frac{b^{2/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{8/3}} - \frac{b^{2/3}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x)}{6a^{8/3}} \\
&= -\frac{A}{5ax^5} + \frac{Ab - aB}{2a^2x^2} - \frac{b^{2/3}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}} + \frac{b^{2/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{8/3}} -
\end{aligned}$$

Mathematica [A] time = 0.13, size = 154, normalized size = 0.92

$$\frac{5b^{2/3}(aB - Ab) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) + \frac{15a^{2/3}(Ab - aB)}{x^2} - \frac{6a^{5/3}A}{x^5} + 10b^{2/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 10\sqrt{3}b^{2/3}(Ab - aB) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{30a^{8/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)), x]

[Out] ((-6*a^(5/3)*A)/x^5 + (15*a^(2/3)*(A*b - a*B))/x^2 - 10*Sqrt[3]*b^(2/3)*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] + 5*b^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(30*a^(8/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^6(a + bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^6*(a + b*x^3)), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^6*(a + b*x^3)), x]

fricas [A] time = 0.60, size = 176, normalized size = 1.05

$$\frac{10\sqrt{3}(Ba - Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}} - \sqrt{3}b}{3b}\right) - 5(Ba - Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} \log\left(b^2x^2 - abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} + a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right) + 10(Ba - Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} \log\left(bx + a\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right) + 15(Ba - Ab)x^3 + 6Aa}{30a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^6/(b*x^3+a), x, algorithm="fricas")

[Out]
$$-1/30*(10*\sqrt{3}*(B*a - A*b)*x^5*(b^2/a^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3}*a*x*(b^2/a^2)^{(2/3)} - \sqrt{3}*b)/b) - 5*(B*a - A*b)*x^5*(b^2/a^2)^{(1/3)}*\log(b^2*x^2 - a*b*x*(b^2/a^2)^{(1/3)} + a^2*(b^2/a^2)^{(2/3)}) + 10*(B*a - A*b)*x^5*(b^2/a^2)^{(1/3)}*\log(b*x + a*(b^2/a^2)^{(1/3)}) + 15*(B*a - A*b)*x^3 + 6*A*a)/(a^2*x^5)$$

giac [A] time = 0.18, size = 176, normalized size = 1.05

$$\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}Ba - (-ab^2)^{\frac{1}{3}}Ab\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3} + \frac{(Bab - Ab^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^3} - \frac{\left((-ab^2)^{\frac{1}{3}}Ba - (-ab^2)^{\frac{1}{3}}Ab\right) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^3} - \frac{5Bax^3 - 5Abx^3 + 2Aa}{10a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^6/(b*x^3+a), x, algorithm="giac")

[Out]
$$-1/3*\sqrt{3}*\left((-a*b^2)^{(1/3)}*B*a - (-a*b^2)^{(1/3)}*A*b\right)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/a^3 + 1/3*(B*a*b - A*b^2)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^3 - 1/6*\left((-a*b^2)^{(1/3)}*B*a - (-a*b^2)^{(1/3)}*A*b\right)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/a^3 - 1/10*(5*B*a*x^3 - 5*A*b*x^3 + 2*A*a)/(a^2*x^5)$$

maple [A] time = 0.05, size = 217, normalized size = 1.29

$$\frac{\sqrt{3} Ab \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2} + \frac{Ab \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2} - \frac{Ab \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2} - \frac{\sqrt{3} B \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a} - \frac{B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a} + \frac{B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a} + \frac{Ab}{2a^2x^2} - \frac{B}{2ax^2} - \frac{A}{5ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^6/(b*x^3+a), x)

[Out]
$$1/3/a^2*b/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*A - 1/3/a/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*B - 1/6/a^2*b/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*A + 1/6/a/(a/b)^{(2/3)}*$$

$(2/3) \cdot \ln(x^2 - (a/b)^{1/3} \cdot x + (a/b)^{2/3}) \cdot B + 1/3 \cdot a^2 \cdot b / (a/b)^{2/3} \cdot 3^{1/2} \cdot \arctan(1/3 \cdot 3^{1/2} \cdot (2/(a/b)^{1/3} \cdot x - 1)) \cdot A - 1/3 \cdot a / (a/b)^{2/3} \cdot 3^{1/2} \cdot \arctan(1/3 \cdot 3^{1/2} \cdot (2/(a/b)^{1/3} \cdot x - 1)) \cdot B - 1/5 \cdot A/a/x^5 + 1/2 \cdot a^2/x^2 \cdot A \cdot b - 1/2 \cdot a/x^2 \cdot B$

maxima [A] time = 1.28, size = 148, normalized size = 0.88

$$\frac{\sqrt{3}(Ba - Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba - Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba - Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5(Ba - Ab)x^3 + 2Aa}{10a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^6/(b*x^3+a), x, algorithm="maxima")

[Out] $-1/3 \cdot \sqrt{3} \cdot (B \cdot a - A \cdot b) \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2 \cdot x - (a/b)^{1/3})) / (a/b)^{1/3} / (a^2 \cdot (a/b)^{2/3}) + 1/6 \cdot (B \cdot a - A \cdot b) \cdot \log(x^2 - x \cdot (a/b)^{1/3} + (a/b)^{2/3}) / (a^2 \cdot (a/b)^{2/3}) - 1/3 \cdot (B \cdot a - A \cdot b) \cdot \log(x + (a/b)^{1/3}) / (a^2 \cdot (a/b)^{2/3}) - 1/10 \cdot (5 \cdot (B \cdot a - A \cdot b) \cdot x^3 + 2 \cdot A \cdot a) / (a^2 \cdot x^5)$

mupad [B] time = 2.56, size = 145, normalized size = 0.86

$$\frac{b^{2/3} \ln(b^{1/3} x + a^{1/3}) (Ab - Ba)}{3a^{8/3}} - \frac{A}{5a} - \frac{x^3(Ab - Ba)}{2a^2} - \frac{b^{2/3} \ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)}{3a^{8/3}} \left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) (Ab - Ba) + \frac{b^{2/3} \ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)}{3a^{8/3}} \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) (Ab - Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^6*(a + b*x^3)), x)

[Out] $(b^{2/3} \cdot \log(b^{1/3} \cdot x + a^{1/3})) \cdot (A \cdot b - B \cdot a) / (3 \cdot a^{8/3}) - (A / (5 \cdot a) - (x^3 \cdot (A \cdot b - B \cdot a)) / (2 \cdot a^2)) / x^5 - (b^{2/3} \cdot \log(3^{1/2} \cdot a^{1/3} \cdot i - 2 \cdot b^{1/3} \cdot x + a^{1/3})) \cdot ((3^{1/2} \cdot i) / 2 + 1/2) \cdot (A \cdot b - B \cdot a) / (3 \cdot a^{8/3}) + (b^{2/3} \cdot \log(3^{1/2} \cdot a^{1/3} \cdot i + 2 \cdot b^{1/3} \cdot x - a^{1/3})) \cdot ((3^{1/2} \cdot i) / 2 - 1/2) \cdot (A \cdot b - B \cdot a) / (3 \cdot a^{8/3})$

sympy [A] time = 0.93, size = 99, normalized size = 0.59

$$\text{RootSum}\left(27t^3a^8 - A^3b^5 + 3A^2Bab^4 - 3AB^2a^2b^3 + B^3a^3b^2, \left(t \mapsto t \log\left(-\frac{3ta^3}{-Ab^2 + Bab} + x\right)\right)\right) + \frac{-2Aa + x^3(5Ab - 5Ba)}{10a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**6/(b*x**3+a), x)

[Out] $\text{RootSum}(27 \cdot t^3 \cdot a^{**8} - A^{**3} \cdot b^{**5} + 3 \cdot A^{**2} \cdot B \cdot a \cdot b^{**4} - 3 \cdot A \cdot B^{**2} \cdot a^{**2} \cdot b^{**3} + B^{**3} \cdot a^{**3} \cdot b^{**2}, \text{Lambda}(t, t \cdot \log(-3 \cdot t \cdot a^{**3} / (-A \cdot b^{**2} + B \cdot a \cdot b) + x))) + (-2 \cdot A \cdot a + x^{**3} \cdot (5 \cdot A \cdot b - 5 \cdot B \cdot a)) / (10 \cdot a^{**2} \cdot x^{**5})$

$$3.69 \quad \int \frac{A+Bx^3}{x^7(a+bx^3)} dx$$

Optimal. Leaf size=69

$$-\frac{b(Ab - aB) \log(a + bx^3)}{3a^3} + \frac{b \log(x)(Ab - aB)}{a^3} + \frac{Ab - aB}{3a^2x^3} - \frac{A}{6ax^6}$$

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 77}

$$\frac{Ab - aB}{3a^2x^3} - \frac{b(Ab - aB) \log(a + bx^3)}{3a^3} + \frac{b \log(x)(Ab - aB)}{a^3} - \frac{A}{6ax^6}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)),x]

[Out] -A/(6*a*x^6) + (A*b - a*B)/(3*a^2*x^3) + (b*(A*b - a*B)*Log[x])/a^3 - (b*(A*b - a*B)*Log[a + b*x^3])/(3*a^3)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0])
|| EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x]
/; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^3}{x^7(a + bx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x^3(a + bx)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{A}{ax^3} + \frac{-Ab + aB}{a^2x^2} - \frac{b(-Ab + aB)}{a^3x} + \frac{b^2(-Ab + aB)}{a^3(a + bx)} \right) dx, x, x^3 \right) \\ &= -\frac{A}{6ax^6} + \frac{Ab - aB}{3a^2x^3} + \frac{b(Ab - aB) \log(x)}{a^3} - \frac{b(Ab - aB) \log(a + bx^3)}{3a^3} \end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 1.01

$$\frac{6bx^6 \log(x)(Ab - aB) - a(aA + 2aBx^3 - 2Abx^3) + 2bx^6(aB - Ab) \log(a + bx^3)}{6a^3x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)), x]

[Out] $(-(a*(a*A - 2*A*b*x^3 + 2*a*B*x^3)) + 6*b*(A*b - a*B)*x^6*\text{Log}[x] + 2*b*(-(A*b) + a*B)*x^6*\text{Log}[a + b*x^3])/(6*a^3*x^6)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^7(a + bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^7*(a + b*x^3)), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^7*(a + b*x^3)), x]

fricas [A] time = 0.53, size = 73, normalized size = 1.06

$$\frac{2(Bab - Ab^2)x^6 \log(bx^3 + a) - 6(Bab - Ab^2)x^6 \log(x) - 2(Ba^2 - Aab)x^3 - Aa^2}{6a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a), x, algorithm="fricas")

[Out] $1/6*(2*(B*a*b - A*b^2)*x^6*\log(b*x^3 + a) - 6*(B*a*b - A*b^2)*x^6*\log(x) - 2*(B*a^2 - A*a*b)*x^3 - A*a^2)/(a^3*x^6)$

giac [A] time = 0.19, size = 99, normalized size = 1.43

$$-\frac{(Bab - Ab^2)\log(|x|)}{a^3} + \frac{(Bab^2 - Ab^3)\log(|bx^3 + a|)}{3a^3b} + \frac{3Babx^6 - 3Ab^2x^6 - 2Ba^2x^3 + 2Aabx^3 - Aa^2}{6a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="giac")

[Out] $-(B*a*b - A*b^2)*\log(\text{abs}(x))/a^3 + 1/3*(B*a*b^2 - A*b^3)*\log(\text{abs}(b*x^3 + a))/(a^3*b) + 1/6*(3*B*a*b*x^6 - 3*A*b^2*x^6 - 2*B*a^2*x^3 + 2*A*a*b*x^3 - A*a^2)/(a^3*x^6)$

maple [A] time = 0.05, size = 81, normalized size = 1.17

$$\frac{A b^2 \ln(x)}{a^3} - \frac{A b^2 \ln(b x^3 + a)}{3 a^3} - \frac{B b \ln(x)}{a^2} + \frac{B b \ln(b x^3 + a)}{3 a^2} + \frac{A b}{3 a^2 x^3} - \frac{B}{3 a x^3} - \frac{A}{6 a x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^7/(b*x^3+a),x)

[Out] $-1/3*b^2/a^3*\ln(b*x^3+a)*A+1/3*b/a^2*\ln(b*x^3+a)*B-1/6*A/a/x^6+1/3/a^2/x^3*A*b-1/3/a/x^3*B+b^2/a^3*\ln(x)*A-b/a^2*\ln(x)*B$

maxima [A] time = 0.49, size = 70, normalized size = 1.01

$$\frac{(Bab - Ab^2)\log(bx^3 + a)}{3a^3} - \frac{(Bab - Ab^2)\log(x^3)}{3a^3} - \frac{2(Ba - Ab)x^3 + Aa}{6a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="maxima")

[Out] $1/3*(B*a*b - A*b^2)*\log(b*x^3 + a)/a^3 - 1/3*(B*a*b - A*b^2)*\log(x^3)/a^3 - 1/6*(2*(B*a - A*b)*x^3 + A*a)/(a^2*x^6)$

mupad [B] time = 0.13, size = 70, normalized size = 1.01

$$\frac{\ln(x) (A b^2 - B a b)}{a^3} - \frac{\ln(b x^3 + a) (A b^2 - B a b)}{3 a^3} - \frac{\frac{A}{6 a} - \frac{x^3 (A b - B a)}{3 a^2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^7*(a + b*x^3)),x)

[Out] $(\log(x) \cdot (A \cdot b^2 - B \cdot a \cdot b)) / a^3 - (\log(a + b \cdot x^3) \cdot (A \cdot b^2 - B \cdot a \cdot b)) / (3 \cdot a^3) - (A / (6 \cdot a) - (x^3 \cdot (A \cdot b - B \cdot a)) / (3 \cdot a^2)) / x^6$

sympy [A] time = 2.69, size = 61, normalized size = 0.88

$$\frac{-Aa + x^3(2Ab - 2Ba)}{6a^2x^6} - \frac{b(-Ab + Ba)\log(x)}{a^3} + \frac{b(-Ab + Ba)\log\left(\frac{a}{b} + x^3\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**7/(b*x**3+a),x)

[Out] $(-A \cdot a + x^3 \cdot (2 \cdot A \cdot b - 2 \cdot B \cdot a)) / (6 \cdot a^2 \cdot x^6) - b \cdot (-A \cdot b + B \cdot a) \cdot \log(x) / a^3 + b \cdot (-A \cdot b + B \cdot a) \cdot \log(a/b + x^3) / (3 \cdot a^3)$

$$3.70 \quad \int \frac{A+Bx^3}{x^8(a+bx^3)} dx$$

Optimal. Leaf size=184

$$\frac{b^{4/3}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{10/3}} + \frac{b^{4/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{10/3}} + \frac{b^{4/3}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{10/3}}$$

Rubi [A] time = 0.13, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {453, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}(Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{10/3}} + \frac{b^{4/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{10/3}} + \frac{b^{4/3}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{10/3}} + \frac{Ab - aB}{4a^2 x^4} - \frac{b(Ab - aB)}{a^3 x} - \frac{A}{7ax^7}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^8*(a + b*x^3)), x]

[Out] -A/(7*a*x^7) + (A*b - a*B)/(4*a^2*x^4) - (b*(A*b - a*B))/(a^3*x) + (b^(4/3)*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(10/3)) + (b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x]/(3*a^(10/3)) - (b^(4/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(10/3)))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(n-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 325


```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m+1)*(a + b*x^n)^(p+1))/(a*e*(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^8(a + bx^3)} dx &= -\frac{A}{7ax^7} - \frac{(7Ab - 7aB) \int \frac{1}{x^5(a+bx^3)} dx}{7a} \\
&= -\frac{A}{7ax^7} + \frac{Ab - aB}{4a^2x^4} + \frac{(b(Ab - aB)) \int \frac{1}{x^2(a+bx^3)} dx}{a^2} \\
&= -\frac{A}{7ax^7} + \frac{Ab - aB}{4a^2x^4} - \frac{b(Ab - aB)}{a^3x} - \frac{(b^2(Ab - aB)) \int \frac{x}{a+bx^3} dx}{a^3} \\
&= -\frac{A}{7ax^7} + \frac{Ab - aB}{4a^2x^4} - \frac{b(Ab - aB)}{a^3x} + \frac{(b^{5/3}(Ab - aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{10/3}} - \frac{(b^{5/3}(Ab - aB)) \int \frac{1}{a^{2/3} - \sqrt[3]{b}x} dx}{3a^{10/3}} \\
&= -\frac{A}{7ax^7} + \frac{Ab - aB}{4a^2x^4} - \frac{b(Ab - aB)}{a^3x} + \frac{b^{4/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{10/3}} - \frac{(b^{4/3}(Ab - aB)) \int \frac{1}{a^{2/3} - \sqrt[3]{b}x} dx}{6a^{10/3}} \\
&= -\frac{A}{7ax^7} + \frac{Ab - aB}{4a^2x^4} - \frac{b(Ab - aB)}{a^3x} + \frac{b^{4/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{10/3}} - \frac{b^{4/3}(Ab - aB) \log(a^{2/3} - \sqrt[3]{b}x)}{6a^{10/3}} \\
&= -\frac{A}{7ax^7} + \frac{Ab - aB}{4a^2x^4} - \frac{b(Ab - aB)}{a^3x} + \frac{b^{4/3}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}} + \frac{b^{4/3}(Ab - aB) \log(\sqrt[3]{a} - \sqrt[3]{b}x)}{3a^{10/3}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 173, normalized size = 0.94

$$\frac{14b^{4/3}(aB - Ab) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) + \frac{21a^{4/3}(Ab - aB)}{x^4} - \frac{12a^{7/3}A}{x^7} + 28b^{4/3}(Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x) + 28\sqrt{3}b^{4/3}(Ab - aB) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right) + \frac{84\sqrt[3]{a}b(aB - Ab)}{x}}{84a^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^8*(a + b*x^3)), x]

[Out] ((-12*a^(7/3)*A)/x^7 + (21*a^(4/3)*(A*b - a*B))/x^4 + (84*a^(1/3)*b*(-(A*b) + a*B))/x + 28*sqrt[3]*b^(4/3)*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] + 28*b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b^(4/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*a^(10/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^8(a + bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^8*(a + b*x^3)), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^8*(a + b*x^3)), x]

fricas [A] time = 0.85, size = 180, normalized size = 0.98

$$\frac{28\sqrt{3}(Bab - Ab^2)x^7\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) + 14(Bab - Ab^2)x^7\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2 - ax\left(\frac{b}{a}\right)^{\frac{2}{3}} + a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right) - 28(Bab - Ab^2)x^7\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx + a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right) + 84(Bab - Ab^2)x^6 - 21(Ba^2 - Aab)x^3 - 12Aa^2}{84a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^8/(b*x^3+a), x, algorithm="fricas")

[Out] $\frac{1}{84}*(28*\sqrt{3}*(B*a*b - A*b^2)*x^7*(b/a)^{(1/3)}*\arctan(2/3*\sqrt{3}*x*(b/a)^{(1/3)} - 1/3*\sqrt{3}) + 14*(B*a*b - A*b^2)*x^7*(b/a)^{(1/3)}*\log(b*x^2 - a*x*(b/a)^{(2/3)} + a*(b/a)^{(1/3)}) - 28*(B*a*b - A*b^2)*x^7*(b/a)^{(1/3)}*\log(b*x + a*(b/a)^{(2/3)}) + 84*(B*a*b - A*b^2)*x^6 - 21*(B*a^2 - A*a*b)*x^3 - 12*A*a^2)/(a^3*x^7)$

giac [A] time = 0.18, size = 216, normalized size = 1.17

$$\frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}Ba - (-ab^2)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - \left(Bab^2\left(\frac{a}{b}\right)^{\frac{1}{3}} - Ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + \frac{\left((-ab^2)^{\frac{2}{3}}Ba - (-ab^2)^{\frac{2}{3}}Ab\right)\log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^4} + \frac{28Babx^6 - 28Ab^2x^6 - 7Ba^2x^3 + 7Aabx^3 - 4Aa^2}{28a^3x^7}}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^8/(b*x^3+a), x, algorithm="giac")

[Out] $-1/3*\sqrt{3}*((-a*b^2)^{(2/3)}*B*a - (-a*b^2)^{(2/3)}*A*b)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/a^4 - 1/3*(B*a*b^2*(-a/b)^{(1/3)} - A*b^3*(-a/b)^{(1/3)})*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^4 + 1/6*((-a*b^2)^{(2/3)}*B*a - (-a*b^2)^{(2/3)}*A*b)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/a^4 + 1/28*(28*B*a*b*x^6 - 28*A*b^2*x^6 - 7*B*a^2*x^3 + 7*A*a*b*x^3 - 4*A*a^2)/(a^3*x^7)$

maple [A] time = 0.06, size = 247, normalized size = 1.34

$$\frac{\sqrt{3}Ab^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3} + \frac{Ab^2\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3} - \frac{Ab^2\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3} + \frac{\sqrt{3}Bb\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2} - \frac{Bb\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2} + \frac{Bb\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2} - \frac{Ab^2}{a^3x} + \frac{Bb}{a^2x} + \frac{Ab}{4a^2x^4} - \frac{B}{4a^4} - \frac{A}{7ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^8/(b*x^3+a), x)

[Out] $1/3/a^3*b^2/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)})*A-1/3/a^2*b/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)})*B-1/6/a^3*b^2/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*A+1/6/a^2*b/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*B-1/3/a^3*b^2*3^{(1/2)}/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*A+1/3/a^2*b*3^{(1/2)}/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*B-1/7*A/a/x^7+1/4/a^2/x^4*A*b-1/4/a/x^4*B-b^2/a^3/x*A+b/a^2/x*B$

maxima [A] time = 1.29, size = 178, normalized size = 0.97

$$\frac{\sqrt{3}(Bab - Ab^2) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Bab - Ab^2) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Bab - Ab^2) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{28(Bab - Ab^2)x^6 - 7(Ba^2 - Aab)x^3 - 4Aa^2}{28a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="maxima")

[Out] $1/3*\sqrt{3}*(B*a*b - A*b^2)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a^3*(a/b)^{(1/3)}) + 1/6*(B*a*b - A*b^2)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a^3*(a/b)^{(1/3)}) - 1/3*(B*a*b - A*b^2)*\log(x + (a/b)^{(1/3)})/(a^3*(a/b)^{(1/3)}) + 1/28*(28*(B*a*b - A*b^2)*x^6 - 7*(B*a^2 - A*a*b)*x^3 - 4*A*a^2)/(a^3*x^7)$

mupad [B] time = 2.58, size = 161, normalized size = 0.88

$$\frac{b^{4/3} \ln(b^{1/3}x + a^{1/3})(Ab - Ba)}{3a^{10/3}} - \frac{A}{7a} - \frac{x^3(Ab - Ba)}{4a^2} + \frac{bx^6(Ab - Ba)}{a^3} + \frac{b^{4/3} \ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)}{3a^{10/3}} \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab - Ba) - \frac{b^{4/3} \ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)}{3a^{10/3}} \left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab - Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^8*(a + b*x^3)),x)

[Out] $(b^{(4/3)}*\log(b^{(1/3)}*x + a^{(1/3)})*(A*b - B*a))/(3*a^{(10/3)}) - (A/(7*a) - (x^3*(A*b - B*a))/(4*a^2) + (b*x^6*(A*b - B*a))/a^3)/x^7 + (b^{(4/3)}*\log(3^{(1/2)}*a^{(1/3)}*1i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a))/(3*a^{(10/3)}) - (b^{(4/3)}*\log(3^{(1/2)}*a^{(1/3)}*1i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a))/(3*a^{(10/3)})$

sympy [A] time = 1.13, size = 139, normalized size = 0.76

$$\text{RootSum}\left(27t^3a^{10} - A^3b^7 + 3A^2Bab^6 - 3AB^2a^2b^5 + B^3a^3b^4, \left(t \mapsto t \log\left(\frac{9t^2a^7}{A^2b^5 - 2ABab^4 + B^2a^2b^3} + x\right)\right)\right) + \frac{-4Aa^2 + x^6(-28Ab^2 + 28Bab) + x^3(7Aab - 7Ba^2)}{28a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**8/(b*x**3+a),x)

[Out] $\text{RootSum}(27*_t**3*a**10 - A**3*b**7 + 3*A**2*B*a*b**6 - 3*A*B**2*a**2*b**5 + B**3*a**3*b**4, \text{Lambda}(_t, _t*\log(9*_t**2*a**7/(A**2*b**5 - 2*A*B*a*b**4 +$

$$\frac{B^{**2}a^{**2}b^{**3} + x))) + (-4Aa^{**2} + x^{**6}(-28Ab^{**2} + 28Bab) + x^{**3}(7Aab - 7Ba^{**2}))}{(28a^{**3}x^{**7})}$$

$$3.71 \quad \int \frac{x^9(A+Bx^3)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=233

$$\frac{a^{4/3}(7Ab - 10aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18b^{13/3}} + \frac{a^{4/3}(7Ab - 10aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9b^{13/3}} - \frac{a^{4/3}(7Ab - 10aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{13/3}}$$

Rubi [A] time = 0.14, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 302, 200, 31, 634, 617, 204, 628}

$$\frac{a^{4/3}(7Ab - 10aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18b^{13/3}} + \frac{a^{4/3}(7Ab - 10aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9b^{13/3}} - \frac{a^{4/3}(7Ab - 10aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{13/3}} - \frac{x^7(7Ab - 10aB)}{21ab^2} + \frac{x^4(7Ab - 10aB)}{12b^3} - \frac{ax(7Ab - 10aB)}{3b^4} + \frac{x^{10}(Ab - aB)}{3ab(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] -(a*(7*A*b - 10*a*B)*x)/(3*b^4) + ((7*A*b - 10*a*B)*x^4)/(12*b^3) - ((7*A*b - 10*a*B)*x^7)/(21*a*b^2) + ((A*b - a*B)*x^10)/(3*a*b*(a + b*x^3)) - (a^(4/3)*(7*A*b - 10*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(13/3)) + (a^(4/3)*(7*A*b - 10*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(13/3)) - (a^(4/3)*(7*A*b - 10*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(13/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(−1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(−1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(−1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9 (A + Bx^3)}{(a + bx^3)^2} dx &= \frac{(Ab - aB)x^{10}}{3ab(a + bx^3)} + \frac{(-7Ab + 10aB) \int \frac{x^9}{a+bx^3} dx}{3ab} \\
&= \frac{(Ab - aB)x^{10}}{3ab(a + bx^3)} + \frac{(-7Ab + 10aB) \int \left(\frac{a^2}{b^3} - \frac{ax^3}{b^2} + \frac{x^6}{b} - \frac{a^3}{b^3(a+bx^3)} \right) dx}{3ab} \\
&= -\frac{a(7Ab - 10aB)x}{3b^4} + \frac{(7Ab - 10aB)x^4}{12b^3} - \frac{(7Ab - 10aB)x^7}{21ab^2} + \frac{(Ab - aB)x^{10}}{3ab(a + bx^3)} + \frac{(a^2(7Ab - 10aB))}{3b^4} \\
&= -\frac{a(7Ab - 10aB)x}{3b^4} + \frac{(7Ab - 10aB)x^4}{12b^3} - \frac{(7Ab - 10aB)x^7}{21ab^2} + \frac{(Ab - aB)x^{10}}{3ab(a + bx^3)} + \frac{(a^{4/3}(7Ab - 10aB))}{3b^4} \\
&= -\frac{a(7Ab - 10aB)x}{3b^4} + \frac{(7Ab - 10aB)x^4}{12b^3} - \frac{(7Ab - 10aB)x^7}{21ab^2} + \frac{(Ab - aB)x^{10}}{3ab(a + bx^3)} + \frac{a^{4/3}(7Ab - 10aB)}{3b^4} \\
&= -\frac{a(7Ab - 10aB)x}{3b^4} + \frac{(7Ab - 10aB)x^4}{12b^3} - \frac{(7Ab - 10aB)x^7}{21ab^2} + \frac{(Ab - aB)x^{10}}{3ab(a + bx^3)} + \frac{a^{4/3}(7Ab - 10aB)}{3b^4} \\
&= -\frac{a(7Ab - 10aB)x}{3b^4} + \frac{(7Ab - 10aB)x^4}{12b^3} - \frac{(7Ab - 10aB)x^7}{21ab^2} + \frac{(Ab - aB)x^{10}}{3ab(a + bx^3)} + \frac{a^{4/3}(7Ab - 10aB)}{3b^4}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 203, normalized size = 0.87

$$\frac{14a^{4/3}(10aB - 7Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx^3 + b^{2/3}x^2}) - 28a^{4/3}(10aB - 7Ab) \log(\sqrt[3]{a} + \sqrt[3]{bx^3}) + 28\sqrt{3} a^{4/3}(10aB - 7Ab) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx^3}}{\sqrt{3}}\right) + \frac{84a^2 \sqrt[3]{bx^3(aB - Ab)}}{a + bx^3} + 63b^{4/3}x^4(Ab - 2aB) + 252a\sqrt[3]{bx^3}(3aB - 2Ab) + 36b^{7/3}Bx^7}{252b^{13/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] (252*a*b^(1/3)*(-2*A*b + 3*a*B)*x + 63*b^(4/3)*(A*b - 2*a*B)*x^4 + 36*b^(7/3)*B*x^7 + (84*a^2*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + 28*sqrt[3]*a^(4/3)*(-7*A*b + 10*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] - 28*a^(4/3)*(-7*A*b + 10*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/3)*(-7*A*b + 10*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(252*b^(13/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9 (A + Bx^3)}{(a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] IntegrateAlgebraic[(x^9*(A + B*x^3))/(a + b*x^3)^2, x]

fricas [A] time = 0.72, size = 271, normalized size = 1.16

$$\frac{36 B b^5 x^{10} - 9 (10 B a b^2 - 7 A b^3) x^7 + 63 (10 B a^2 b - 7 A a^2 b^2) x^4 - 28 \sqrt{3} (10 B a^3 - 7 A a^2 b + (10 B a^2 b - 7 A a b^2) x^3) \left(\frac{x}{b}\right)^{\frac{1}{3}} \arctan\left(\frac{x \sqrt{3} \left(\frac{x}{b}\right)^{\frac{1}{3}} - \sqrt{3} x}{-3x}\right) + 14 (10 B a^3 - 7 A a^2 b + (10 B a^2 b - 7 A a b^2) x^3) \left(\frac{x}{b}\right)^{\frac{1}{3}} \log\left(x^2 - x \left(\frac{x}{b}\right)^{\frac{1}{3}} + \left(\frac{x}{b}\right)^{\frac{2}{3}}\right) - 28 (10 B a^3 - 7 A a^2 b + (10 B a^2 b - 7 A a b^2) x^3) \left(\frac{x}{b}\right)^{\frac{1}{3}} \log\left(x + \left(\frac{x}{b}\right)^{\frac{1}{3}}\right) + 84 (10 B a^3 - 7 A a^2 b) x}{252 (b^5 x^3 + a b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/252*(36*B*b^3*x^10 - 9*(10*B*a*b^2 - 7*A*b^3)*x^7 + 63*(10*B*a^2*b - 7*A*a*b^2)*x^4 - 28*sqrt(3)*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 14*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(10*B*a^3 - 7*A*a^2*b)*x)/(b^5*x^3 + a*b^4)

giac [A] time = 0.20, size = 244, normalized size = 1.05

$$\frac{\sqrt{3} (10 (-ab^2)^{\frac{1}{3}} Ba^2 - 7 (-ab^2)^{\frac{1}{3}} Ab^3) \arctan\left(\frac{\sqrt{3} (2x + \left(\frac{x}{b}\right)^{\frac{1}{3}})}{3 \left(\frac{x}{b}\right)^{\frac{1}{3}}}\right) + (10 Ba^3 - 7 Aa^2b) \left(\frac{x}{b}\right)^{\frac{1}{3}} \log\left(x - \left(\frac{x}{b}\right)^{\frac{1}{3}}\right) - (10 (-ab^2)^{\frac{1}{3}} Ba^2 - 7 (-ab^2)^{\frac{1}{3}} Ab^3) \log\left(x^2 + x \left(\frac{x}{b}\right)^{\frac{1}{3}} + \left(\frac{x}{b}\right)^{\frac{2}{3}}\right) + Ba^2x - Aa^2bx}{9 b^5} + \frac{4 B b^{12} x^7 - 14 B a b^{11} x^4 + 7 A b^{12} x^4 + 84 B a^2 b^{10} x - 56 A a b^{11} x}{3 (b x^3 + a) b^4} + \frac{10 \sqrt{3} B a^3 \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{b} - 1\right)}{\left(\frac{x}{b}\right)^{\frac{1}{3}}}\right) + 7 A a^2 \ln\left(x + \left(\frac{x}{b}\right)^{\frac{1}{3}}\right) - 7 A a^2 \ln\left(x^2 - \left(\frac{x}{b}\right)^{\frac{1}{3}} x + \left(\frac{x}{b}\right)^{\frac{2}{3}}\right) - 2 A a x}{9 \left(\frac{x}{b}\right)^{\frac{2}{3}} b^4} - \frac{10 B a^3 \ln\left(x + \left(\frac{x}{b}\right)^{\frac{1}{3}}\right) + 5 B a^3 \ln\left(x^2 - \left(\frac{x}{b}\right)^{\frac{1}{3}} x + \left(\frac{x}{b}\right)^{\frac{2}{3}}\right) + 3 B a^2 x}{9 \left(\frac{x}{b}\right)^{\frac{2}{3}} b^5} - \frac{10 B a^3 \ln\left(x + \left(\frac{x}{b}\right)^{\frac{1}{3}}\right) + 5 B a^3 \ln\left(x^2 - \left(\frac{x}{b}\right)^{\frac{1}{3}} x + \left(\frac{x}{b}\right)^{\frac{2}{3}}\right) + 3 B a^2 x}{9 \left(\frac{x}{b}\right)^{\frac{2}{3}} b^5} + \frac{3 B a^2 x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/9*sqrt(3)*(10*(-a*b^2)^(1/3)*B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 + 1/9*(10*B*a^3 - 7*A*a^2*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 1/18*(10*(-a*b^2)^(1/3)*B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 + 1/3*(B*a^3*x - A*a^2*b*x)/((b*x^3 + a)*b^4) + 1/28*(4*B*b^12*x^7 - 14*B*a*b^11*x^4 + 7*A*b^12*x^4 + 84*B*a^2*b^10*x - 56*A*a*b^11*x)/b^14

maple [A] time = 0.06, size = 288, normalized size = 1.24

$$\frac{B x^7}{7 b^2} - \frac{A x^4}{4 b^2} - \frac{B a x^4}{2 b^3} - \frac{A a^2 x}{3 (b x^3 + a) b^3} + \frac{B a^2 x}{3 (b x^3 + a) b^4} + \frac{7 \sqrt{3} A a^2 \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{b} - 1\right)}{\left(\frac{x}{b}\right)^{\frac{1}{3}}}\right)}{9 \left(\frac{x}{b}\right)^{\frac{2}{3}} b^4} + \frac{7 A a^2 \ln\left(x + \left(\frac{x}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{x}{b}\right)^{\frac{2}{3}} b^4} - \frac{7 A a^2 \ln\left(x^2 - \left(\frac{x}{b}\right)^{\frac{1}{3}} x + \left(\frac{x}{b}\right)^{\frac{2}{3}}\right)}{18 \left(\frac{x}{b}\right)^{\frac{2}{3}} b^4} - \frac{2 A a x}{b^3} - \frac{10 \sqrt{3} B a^3 \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{b} - 1\right)}{\left(\frac{x}{b}\right)^{\frac{1}{3}}}\right)}{9 \left(\frac{x}{b}\right)^{\frac{2}{3}} b^5} - \frac{10 B a^3 \ln\left(x + \left(\frac{x}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{x}{b}\right)^{\frac{2}{3}} b^5} + \frac{5 B a^3 \ln\left(x^2 - \left(\frac{x}{b}\right)^{\frac{1}{3}} x + \left(\frac{x}{b}\right)^{\frac{2}{3}}\right)}{9 \left(\frac{x}{b}\right)^{\frac{2}{3}} b^5} + \frac{3 B a^2 x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^9*(B*x^3+A)/(b*x^3+a)^2, x)$

[Out] $\frac{1}{7}b^{-2}Bx^7 + \frac{1}{4}b^{-2}Ax^4 - \frac{1}{2}b^{-3}Bx^4a - \frac{2}{b^3}aAx + \frac{3}{b^4}a^2Bx - \frac{1}{3}a^2b^{-3}x/(b*x^3+a) * A + \frac{1}{3}a^3/b^4x/(b*x^3+a) * B + \frac{7}{9}a^2/b^4A/(a/b)^{(2/3)} * \ln(x + (a/b)^{(1/3)}) - \frac{7}{18}a^2/b^4A/(a/b)^{(2/3)} * \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) + \frac{7}{9}a^2/b^4A/(a/b)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(a/b)^{(1/3)} * x - 1)) - \frac{10}{9}a^3/b^5B/(a/b)^{(2/3)} * \ln(x + (a/b)^{(1/3)}) + \frac{5}{9}a^3/b^5B/(a/b)^{(2/3)} * \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) - \frac{10}{9}a^3/b^5B/(a/b)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(a/b)^{(1/3)} * x - 1))$

maxima [A] time = 1.23, size = 218, normalized size = 0.94

$$\frac{(Ba^3 - Aa^2b)x}{3(b^5x^3 + ab^4)} + \frac{4Bb^2x^7 - 7(2Bab - Ab^2)x^4 + 28(3Ba^2 - 2Aab)x}{28b^4} - \frac{\sqrt{3}(10Ba^3 - 7Aa^2b) \arctan\left(\frac{\sqrt{3}\left(2x\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(10Ba^3 - 7Aa^2b) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(10Ba^3 - 7Aa^2b) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^9*(B*x^3+A)/(b*x^3+a)^2, x, \text{algorithm}="maxima")$

[Out] $\frac{1}{3}*(B*a^3 - A*a^2*b)*x/(b^5*x^3 + a*b^4) + \frac{1}{28}*(4*B*b^2*x^7 - 7*(2*B*a*b - A*b^2)*x^4 + 28*(3*B*a^2 - 2*A*a*b)*x)/b^4 - \frac{1}{9}*\sqrt{3}*(10*B*a^3 - 7*A*a^2*b)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(b^5*(a/b)^{(2/3)}) + \frac{1}{18}*(10*B*a^3 - 7*A*a^2*b)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^5*(a/b)^{(2/3)}) - \frac{1}{9}*(10*B*a^3 - 7*A*a^2*b)*\log(x + (a/b)^{(1/3)})/(b^5*(a/b)^{(2/3)})$

mupad [B] time = 2.62, size = 209, normalized size = 0.90

$$x^4 \left(\frac{A}{4b^2} - \frac{Ba}{2b^3} \right) - x \left(\frac{2a \left(\frac{a}{b} - \frac{2Ba}{b^2} \right)}{b} + \frac{Ba^2}{b^4} \right) + \frac{Bx^7}{7b^2} + \frac{x \left(\frac{Ba^3 - Aa^2b}{3} \right)}{b^5x^3 + ab^4} + \frac{a^{4/3} \ln(b^{1/3}x + a^{1/3})}{9b^{13/3}} (7Ab - 10Ba) - \frac{a^{4/3} \ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)}{9b^{13/3}} \left(\frac{1}{2} + \frac{\sqrt{3}i}{2} \right) (7Ab - 10Ba) + \frac{a^{4/3} \ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)}{9b^{13/3}} \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2} \right) (7Ab - 10Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^9*(A + B*x^3))/(a + b*x^3)^2, x)$

[Out] $x^4*(A/(4*b^2) - (B*a)/(2*b^3)) - x*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4) + (B*x^7)/(7*b^2) + (x*((B*a^3)/3 - (A*a^2*b)/3))/(a*b^4 + b^5*x^3) + (a^{(4/3)}*\log(b^{(1/3)}*x + a^{(1/3)})*(7*A*b - 10*B*a))/(9*b^{(13/3)}) - (a^{(4/3)}*\log(3^{(1/2)}*a^{(1/3)}*1i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*1i)/2 + 1/2)*(7*A*b - 10*B*a))/(9*b^{(13/3)}) + (a^{(4/3)}*\log(3^{(1/2)}*a^{(1/3)}*1i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*1i)/2 - 1/2)*(7*A*b - 10*B*a))/(9*b^{(13/3)})$

sympy [A] time = 2.12, size = 156, normalized size = 0.67

$$\frac{Bx^7}{7b^2} + x^4 \left(\frac{A}{4b^2} - \frac{Ba}{2b^3} \right) + x \left(-\frac{2Aa}{b^3} + \frac{3Ba^2}{b^4} \right) + \frac{x(-Aa^2b + Ba^3)}{3ab^4 + 3b^5x^3} + \text{RootSum} \left(729t^3b^{13} - 343A^3a^4b^3 + 1470A^2Ba^5b^2 - 2100AB^2a^6b + 1000B^3a^7, \left(t \mapsto t \log \left(-\frac{9tb^4}{-7Aab + 10Ba^2} + x \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**7/(7*b**2) + x**4*(A/(4*b**2) - B*a/(2*b**3)) + x*(-2*A*a/b**3 + 3*B*a
**2/b**4) + x*(-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3) + RootSum(729*_
t**3*b**13 - 343*A**3*a**4*b**3 + 1470*A**2*B*a**5*b**2 - 2100*A*B**2*a**6*
b + 1000*B**3*a**7, Lambda(_t, _t*log(-9*_t*b**4/(-7*A*a*b + 10*B*a**2) + x
)))
```

$$3.72 \quad \int \frac{x^8(A+Bx^3)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=82

$$-\frac{a^2(Ab - aB)}{3b^4(a + bx^3)} - \frac{a(2Ab - 3aB) \log(a + bx^3)}{3b^4} + \frac{x^3(Ab - 2aB)}{3b^3} + \frac{Bx^6}{6b^2}$$

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 77}

$$-\frac{a^2(Ab - aB)}{3b^4(a + bx^3)} + \frac{x^3(Ab - 2aB)}{3b^3} - \frac{a(2Ab - 3aB) \log(a + bx^3)}{3b^4} + \frac{Bx^6}{6b^2}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^6)/(6*b^2) - (a^2*(A*b - a*B))/(3*b^4*(a + b*x^3)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{x^8 (A + Bx^3)}{(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (A + Bx)}{(a + bx)^2} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{Ab - 2aB}{b^3} + \frac{Bx}{b^2} - \frac{a^2(-Ab + aB)}{b^3(a + bx)^2} + \frac{a(-2Ab + 3aB)}{b^3(a + bx)} \right) dx, x, x^3 \right) \\ &= \frac{(Ab - 2aB)x^3}{3b^3} + \frac{Bx^6}{6b^2} - \frac{a^2(Ab - aB)}{3b^4(a + bx^3)} - \frac{a(2Ab - 3aB) \log(a + bx^3)}{3b^4} \end{aligned}$$

Mathematica [A] time = 0.10, size = 72, normalized size = 0.88

$$\frac{\frac{2a^2(aB - Ab)}{a + bx^3} + 2bx^3(Ab - 2aB) + 2a(3aB - 2Ab) \log(a + bx^3) + b^2Bx^6}{6b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] (2*b*(A*b - 2*a*B)*x^3 + b^2*B*x^6 + (2*a^2*(-(A*b) + a*B))/(a + b*x^3) + 2*a*(-2*A*b + 3*a*B)*Log[a + b*x^3])/(6*b^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 (A + Bx^3)}{(a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] IntegrateAlgebraic[(x^8*(A + B*x^3))/(a + b*x^3)^2, x]

fricas [A] time = 0.57, size = 121, normalized size = 1.48

$$\frac{Bb^3x^9 - (3Bab^2 - 2Ab^3)x^6 + 2Ba^3 - 2Aa^2b - 2(2Ba^2b - Aab^2)x^3 + 2(3Ba^3 - 2Aa^2b + (3Ba^2b - 2Aab^2)x^3) \log(bx^3 + a)}{6(b^5x^3 + ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{6}(Bb^3x^9 - (3B^2a^2b^2 - 2A^2b^3)x^6 + 2B^2a^3 - 2A^2a^2b - 2(2B^2a^2b - A^2a^2b^2)x^3 + 2(3B^2a^3 - 2A^2a^2b + (3B^2a^2b - 2A^2a^2b^2)x^3) \log(bx^3 + a))/(b^5x^3 + ab^4)$

giac [A] time = 0.18, size = 106, normalized size = 1.29

$$\frac{(3Ba^2 - 2Aab) \log(|bx^3 + a|)}{3b^4} + \frac{Bb^2x^6 - 4Babx^3 + 2Ab^2x^3}{6b^4} - \frac{3Ba^2bx^3 - 2Aab^2x^3 + 2Ba^3 - Aa^2b}{3(bx^3 + a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")`

[Out] $\frac{1}{3}(3B^2a^2 - 2A^2a^2b) \log(\text{abs}(bx^3 + a))/b^4 + \frac{1}{6}(Bb^2x^6 - 4B^2a^2bx^3 + 2A^2b^2x^3)/b^4 - \frac{1}{3}(3B^2a^2bx^3 - 2A^2a^2b^2x^3 + 2B^2a^3 - A^2a^2b)/((bx^3 + a)b^4)$

maple [A] time = 0.05, size = 97, normalized size = 1.18

$$\frac{Bx^6}{6b^2} + \frac{Ax^3}{3b^2} - \frac{2Ba^3}{3b^3} - \frac{Aa^2}{3(bx^3 + a)b^3} - \frac{2Aa \ln(bx^3 + a)}{3b^3} + \frac{Ba^3}{3(bx^3 + a)b^4} + \frac{Ba^2 \ln(bx^3 + a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*(B*x^3+A)/(b*x^3+a)^2,x)`

[Out] $\frac{1}{6}Bx^6/b^2 + \frac{1}{3}x^3/b^2 + \frac{2}{3}B^2a^2x^3/b^3 - \frac{2}{3}Aa^2/b^3 \ln(bx^3+a) + \frac{Aa^2}{b^4} \ln(bx^3+a) + \frac{B}{3} - \frac{1}{3}Aa^2/b^3 \ln(bx^3+a) + \frac{1}{3}Aa^3/b^4 \ln(bx^3+a) + B$

maxima [A] time = 0.49, size = 82, normalized size = 1.00

$$\frac{Ba^3 - Aa^2b}{3(b^5x^3 + ab^4)} + \frac{Bbx^6 - 2(2Ba - Ab)x^3}{6b^3} + \frac{(3Ba^2 - 2Aab) \log(bx^3 + a)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] $\frac{1}{3}(B^2a^3 - A^2a^2b)/(b^5x^3 + ab^4) + \frac{1}{6}(Bb^2x^6 - 2(2B^2a^2b - A^2a^2b)x^3)/b^3 + \frac{1}{3}(3B^2a^2 - 2A^2a^2b) \log(bx^3 + a)/b^4$

mupad [B] time = 0.08, size = 86, normalized size = 1.05

$$x^3 \left(\frac{A}{3b^2} - \frac{2Ba}{3b^3} \right) + \frac{\ln(bx^3 + a) (3Ba^2 - 2Aab)}{3b^4} + \frac{Bx^6}{6b^2} + \frac{Ba^3 - Aa^2b}{3b(b^4x^3 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^8*(A + B*x^3))/(a + b*x^3)^2,x)`

[Out] $x^3*(A/(3*b^2) - (2*B*a)/(3*b^3)) + (\log(a + b*x^3)*(3*B*a^2 - 2*A*a*b))/(3*b^4) + (B*x^6)/(6*b^2) + (B*a^3 - A*a^2*b)/(3*b*(a*b^3 + b^4*x^3))$

sympy [A] time = 2.14, size = 82, normalized size = 1.00

$$\frac{Bx^6}{6b^2} + \frac{a(-2Ab + 3Ba) \log(a + bx^3)}{3b^4} + x^3 \left(\frac{A}{3b^2} - \frac{2Ba}{3b^3} \right) + \frac{-Aa^2b + Ba^3}{3ab^4 + 3b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(B*x**3+A)/(b*x**3+a)**2,x)`

[Out] $B*x**6/(6*b**2) + a*(-2*A*b + 3*B*a)*\log(a + b*x**3)/(3*b**4) + x**3*(A/(3*b**2) - 2*B*a/(3*b**3)) + (-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3)$

$$3.73 \quad \int \frac{x^7(A+Bx^3)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=215

$$-\frac{a^{2/3}(5Ab - 8aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18b^{11/3}} + \frac{a^{2/3}(5Ab - 8aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9b^{11/3}} + \frac{a^{2/3}(5Ab - 8aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b} x}{\sqrt[3]{a}}\right)}{3\sqrt{3} b^{11/3}}$$

Rubi [A] time = 0.14, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 302, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{2/3}(5Ab - 8aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18b^{11/3}} + \frac{a^{2/3}(5Ab - 8aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9b^{11/3}} + \frac{a^{2/3}(5Ab - 8aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt[3]{a}}\right)}{3\sqrt{3} b^{11/3}} - \frac{x^5(5Ab - 8aB)}{15ab^2} + \frac{x^2(5Ab - 8aB)}{6b^3} + \frac{x^8(Ab - aB)}{3ab(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] ((5*A*b - 8*a*B)*x^2)/(6*b^3) - ((5*A*b - 8*a*B)*x^5)/(15*a*b^2) + ((A*b - a*B)*x^8)/(3*a*b*(a + b*x^3)) + (a^(2/3)*(5*A*b - 8*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(11/3)) + (a^(2/3)*(5*A*b - 8*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(11/3)) - (a^(2/3)*(5*A*b - 8*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(11/3))

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^-1, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7 (A + Bx^3)}{(a + bx^3)^2} dx &= \frac{(Ab - aB)x^8}{3ab(a + bx^3)} + \frac{(-5Ab + 8aB) \int \frac{x^7}{a+bx^3} dx}{3ab} \\
&= \frac{(Ab - aB)x^8}{3ab(a + bx^3)} + \frac{(-5Ab + 8aB) \int \left(-\frac{ax}{b^2} + \frac{x^4}{b} + \frac{a^2x}{b^2(a+bx^3)} \right) dx}{3ab} \\
&= \frac{(5Ab - 8aB)x^2}{6b^3} - \frac{(5Ab - 8aB)x^5}{15ab^2} + \frac{(Ab - aB)x^8}{3ab(a + bx^3)} - \frac{(a(5Ab - 8aB)) \int \frac{x}{a+bx^3} dx}{3b^3} \\
&= \frac{(5Ab - 8aB)x^2}{6b^3} - \frac{(5Ab - 8aB)x^5}{15ab^2} + \frac{(Ab - aB)x^8}{3ab(a + bx^3)} + \frac{(a^{2/3}(5Ab - 8aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9b^{10/3}} - \frac{(a^{2/3}(5Ab - 8aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9b^{10/3}} \\
&= \frac{(5Ab - 8aB)x^2}{6b^3} - \frac{(5Ab - 8aB)x^5}{15ab^2} + \frac{(Ab - aB)x^8}{3ab(a + bx^3)} + \frac{a^{2/3}(5Ab - 8aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9b^{11/3}} - \frac{(a^{2/3}(5Ab - 8aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9b^{10/3}} \\
&= \frac{(5Ab - 8aB)x^2}{6b^3} - \frac{(5Ab - 8aB)x^5}{15ab^2} + \frac{(Ab - aB)x^8}{3ab(a + bx^3)} + \frac{a^{2/3}(5Ab - 8aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9b^{11/3}} - \frac{(a^{2/3}(5Ab - 8aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9b^{10/3}} \\
&= \frac{(5Ab - 8aB)x^2}{6b^3} - \frac{(5Ab - 8aB)x^5}{15ab^2} + \frac{(Ab - aB)x^8}{3ab(a + bx^3)} + \frac{a^{2/3}(5Ab - 8aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9b^{11/3}} - \frac{(a^{2/3}(5Ab - 8aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9b^{10/3}} \\
&= \frac{(5Ab - 8aB)x^2}{6b^3} - \frac{(5Ab - 8aB)x^5}{15ab^2} + \frac{(Ab - aB)x^8}{3ab(a + bx^3)} + \frac{a^{2/3}(5Ab - 8aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{11/3}} + \frac{(a^{2/3}(5Ab - 8aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9b^{10/3}}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 185, normalized size = 0.86

$$\frac{5a^{2/3}(8aB - 5Ab) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) - 10a^{2/3}(8aB - 5Ab) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 10\sqrt{3}a^{2/3}(8aB - 5Ab) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right) + 45b^{2/3}x^2(Ab - 2aB) + \frac{30ab^{2/3}x^2(Ab - aB)}{a + bx^3} + 18b^{5/3}Bx^5}{90b^{11/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] (45*b^(2/3)*(A*b - 2*a*B)*x^2 + 18*b^(5/3)*B*x^5 + (30*a*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3) - 10*Sqrt[3]*a^(2/3)*(-5*A*b + 8*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(1/3) + b^(1/3)*x] + 5*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(90*b^(11/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 (A + Bx^3)}{(a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] IntegrateAlgebraic[(x^7*(A + B*x^3))/(a + b*x^3)^2, x]

fricas [A] time = 0.67, size = 257, normalized size = 1.20

$$\frac{18 B a^2 x^8 - 9 (8 B a b - 5 A b^2) x^5 - 15 (8 B a^2 - 5 A a b) x^2 + 10 \sqrt{3} ((8 B a b - 5 A b^2) x^3 + 8 B a^2 - 5 A a b) \left(\frac{a}{b}\right)^{\frac{1}{3}} \arctan\left(\frac{2 \sqrt{3} \ln\left(\frac{a}{b}\right)^{\frac{1}{3}} - \sqrt{3} a}{3 a}\right) + 5 ((8 B a b - 5 A b^2) x^3 + 8 B a^2 - 5 A a b) \left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(a x^2 - b x \left(\frac{a}{b}\right)^{\frac{1}{3}} + a \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 10 ((8 B a b - 5 A b^2) x^3 + 8 B a^2 - 5 A a b) \left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(a x + b \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{90 (b^2 x^3 + a b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/90*(18*B*b^2*x^8 - 9*(8*B*a*b - 5*A*b^2)*x^5 - 15*(8*B*a^2 - 5*A*a*b)*x^2 + 10*sqrt(3)*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3) *arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(1/3) + a*(a^2/b^2)^(1/3)) - 10*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(1/3)))/(b^4*x^3 + a*b^3)

giac [A] time = 0.18, size = 236, normalized size = 1.10

$$\frac{\left(8 B a^2 \left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5 A a b \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9 a b^3} - \frac{\sqrt{3} \left(8 \left(-a b^2\right)^{\frac{2}{3}} B a - 5 \left(-a b^2\right)^{\frac{2}{3}} A b\right) \arctan\left(\frac{\sqrt{3} \left(2 x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 b^5} - \frac{B a^2 x^2 - A a b x^2}{3 \left(b x^3 + a\right) b^3} + \frac{\left(8 \left(-a b^2\right)^{\frac{2}{3}} B a - 5 \left(-a b^2\right)^{\frac{2}{3}} A b\right) \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 b^5} + \frac{2 B b^8 x^5 - 10 B a b^7 x^2 + 5 A b^6 x^2}{10 b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/9*(8*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) - 1/9*sqrt(3)*(8*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 1/3*(B*a^2*x^2 - A*a*b*x^2)/((b*x^3 + a)*b^3) + 1/18*(8*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 + 1/10*(2*B*b^8*x^5 - 10*B*a*b^7*x^2 + 5*A*b^8*x^2)/b^10

maple [A] time = 0.05, size = 266, normalized size = 1.24

$$\frac{B x^5}{5 b^2} + \frac{A a x^2}{3 (b x^3 + a) b^2} - \frac{B a^2 x^2}{3 (b x^3 + a) b^3} + \frac{A x^2}{2 b^2} - \frac{B a x^2}{b^3} - \frac{5 \sqrt{3} A a \arctan\left(\frac{\sqrt{3} \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^3} + \frac{5 A a \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^3} - \frac{5 A a \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^3} + \frac{8 \sqrt{3} B a^2 \arctan\left(\frac{\sqrt{3} \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^4} - \frac{8 B a^2 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^4} + \frac{4 B a^2 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^7*(B*x^3+A)/(b*x^3+a)^2,x)$

[Out] $\frac{1}{5}b^{-2}Bx^5 + \frac{1}{2}b^{-2}Ax^2 - \frac{1}{b^3}Bx^2*a + \frac{1}{3}a/b^2*x^2/(b*x^3+a)*A - \frac{1}{3}a^2/b^3*x^2/(b*x^3+a)*B + \frac{5}{9}a/b^3*A/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)}) - \frac{5}{18}a/b^3*A/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)}) - \frac{5}{9}a/b^3*A*3^{(1/2)}/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1)) - \frac{8}{9}a^2/b^4*B/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)}) + \frac{4}{9}a^2/b^4*B/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)}) + \frac{8}{9}a^2/b^4*B*3^{(1/2)}/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))$

maxima [A] time = 1.10, size = 192, normalized size = 0.89

$$\frac{(Ba^2 - Aab)x^2}{3(b^4x^3 + ab^3)} + \frac{\sqrt{3}(8Ba^2 - 5Aab)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2Bbx^5 - 5(2Ba - Ab)x^2}{10b^3} + \frac{(8Ba^2 - 5Aab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(8Ba^2 - 5Aab)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^7*(B*x^3+A)/(b*x^3+a)^2,x, \text{algorithm}="maxima")$

[Out] $-\frac{1}{3}*(B*a^2 - A*a*b)*x^2/(b^4*x^3 + a*b^3) + \frac{1}{9}*\text{sqrt}(3)*(8*B*a^2 - 5*A*a*b)*\arctan(1/3*\text{sqrt}(3)*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(b^4*(a/b)^{(1/3)}) + \frac{1}{10}*(2*B*b*x^5 - 5*(2*B*a - A*b)*x^2)/b^3 + \frac{1}{18}*(8*B*a^2 - 5*A*a*b)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^4*(a/b)^{(1/3)}) - \frac{1}{9}*(8*B*a^2 - 5*A*a*b)*\log(x + (a/b)^{(1/3)})/(b^4*(a/b)^{(1/3)})$

mupad [B] time = 0.27, size = 179, normalized size = 0.83

$$x^2\left(\frac{A}{2b^2} - \frac{Ba}{b^3}\right) + \frac{Bx^5}{5b^2} - \frac{x^2\left(\frac{Ba^2}{3} - \frac{Aab}{3}\right)}{b^4x^3 + ab^3} + \frac{a^{2/3}\ln(b^{1/3}x + a^{1/3})(5Ab - 8Ba)}{9b^{11/3}} + \frac{a^{2/3}\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(5Ab - 8Ba)}{9b^{11/3}} - \frac{a^{2/3}\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(5Ab - 8Ba)}{9b^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^7*(A + B*x^3))/(a + b*x^3)^2,x)$

[Out] $x^2*(A/(2*b^2) - (B*a)/b^3) + (B*x^5)/(5*b^2) - (x^2*((B*a^2)/3 - (A*a*b)/3))/(a*b^3 + b^4*x^3) + (a^{(2/3)}*\log(b^{(1/3)}*x + a^{(1/3)})*(5*A*b - 8*B*a))/(9*b^{(11/3)}) + (a^{(2/3)}*\log(3^{(1/2)}*a^{(1/3)}*i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*i)/2 - 1/2)*(5*A*b - 8*B*a))/(9*b^{(11/3)}) - (a^{(2/3)}*\log(3^{(1/2)}*a^{(1/3)}*i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*i)/2 + 1/2)*(5*A*b - 8*B*a))/(9*b^{(11/3)})$

sympy [A] time = 2.12, size = 151, normalized size = 0.70

$$\frac{Bx^5}{5b^2} + x^2\left(\frac{A}{2b^2} - \frac{Ba}{b^3}\right) + \frac{x^2(Aab - Ba^2)}{3ab^3 + 3b^4x^3} + \text{RootSum}\left(729t^3b^{11} - 125A^3a^2b^3 + 600A^2Ba^3b^2 - 960AB^2a^4b + 512B^3a^5, \left(t \mapsto t \log\left(\frac{81t^2b^7}{25A^2ab^2 - 80ABa^2b + 64B^2a^3 + x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**2,x)

[Out] $Bx^5/(5b^2) + x^2(A/(2b^2) - Ba/b^3) + x^2(Aab - Ba^2)/(3ab^3 + 3b^4x^3) + \text{RootSum}(729_t^3b^{11} - 125A^3a^2b^3 + 600A^2Ba^3b^2 - 960AB^2a^4b + 512B^3a^5, \text{Lambda}(_t, _t \log(81_t^2b^7/(25A^2ab^2 - 80ABa^2b + 64B^2a^3) + x)))$

$$3.74 \quad \int \frac{x^6(A+Bx^3)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=213

$$\frac{\sqrt[3]{a}(4Ab-7aB)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{18b^{10/3}} - \frac{\sqrt[3]{a}(4Ab-7aB)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{9b^{10/3}} + \frac{\sqrt[3]{a}(4Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{10/3}}$$

Rubi [A] time = 0.13, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 302, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}(4Ab-7aB)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{18b^{10/3}} - \frac{x^4(4Ab-7aB)}{12ab^2} + \frac{x(4Ab-7aB)}{3b^3} - \frac{\sqrt[3]{a}(4Ab-7aB)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{9b^{10/3}} + \frac{\sqrt[3]{a}(4Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{10/3}} + \frac{x^7(Ab-aB)}{3ab(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] ((4*A*b - 7*a*B)*x)/(3*b^3) - ((4*A*b - 7*a*B)*x^4)/(12*a*b^2) + ((A*b - a*B)*x^7)/(3*a*b*(a + b*x^3)) + (a^(1/3)*(4*A*b - 7*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(10/3)) - (a^(1/3)*(4*A*b - 7*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(10/3)) + (a^(1/3)*(4*A*b - 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(10/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6 (A + Bx^3)}{(a + bx^3)^2} dx &= \frac{(Ab - aB)x^7}{3ab(a + bx^3)} + \frac{(-4Ab + 7aB) \int \frac{x^6}{a+bx^3} dx}{3ab} \\
&= \frac{(Ab - aB)x^7}{3ab(a + bx^3)} + \frac{(-4Ab + 7aB) \int \left(-\frac{a}{b^2} + \frac{x^3}{b} + \frac{a^2}{b^2(a+bx^3)} \right) dx}{3ab} \\
&= \frac{(4Ab - 7aB)x}{3b^3} - \frac{(4Ab - 7aB)x^4}{12ab^2} + \frac{(Ab - aB)x^7}{3ab(a + bx^3)} - \frac{(a(4Ab - 7aB)) \int \frac{1}{a+bx^3} dx}{3b^3} \\
&= \frac{(4Ab - 7aB)x}{3b^3} - \frac{(4Ab - 7aB)x^4}{12ab^2} + \frac{(Ab - aB)x^7}{3ab(a + bx^3)} - \frac{(\sqrt[3]{a}(4Ab - 7aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9b^3} - \frac{(\sqrt[3]{a})}{3b^3} \\
&= \frac{(4Ab - 7aB)x}{3b^3} - \frac{(4Ab - 7aB)x^4}{12ab^2} + \frac{(Ab - aB)x^7}{3ab(a + bx^3)} - \frac{\sqrt[3]{a}(4Ab - 7aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9b^{10/3}} + \frac{(\sqrt[3]{a})}{3b^3} \\
&= \frac{(4Ab - 7aB)x}{3b^3} - \frac{(4Ab - 7aB)x^4}{12ab^2} + \frac{(Ab - aB)x^7}{3ab(a + bx^3)} - \frac{\sqrt[3]{a}(4Ab - 7aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9b^{10/3}} + \frac{\sqrt[3]{a}}{3b^3} \\
&= \frac{(4Ab - 7aB)x}{3b^3} - \frac{(4Ab - 7aB)x^4}{12ab^2} + \frac{(Ab - aB)x^7}{3ab(a + bx^3)} + \frac{\sqrt[3]{a}(4Ab - 7aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{10/3}} - \frac{\sqrt[3]{a}}{3b^3}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 181, normalized size = 0.85

$$\frac{-2\sqrt[3]{a}(7aB - 4Ab) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) + \frac{12a\sqrt[3]{b}x(Ab - aB)}{a+bx^3} + 36\sqrt[3]{b}x(Ab - 2aB) + 4\sqrt[3]{a}(7aB - 4Ab) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 4\sqrt{3}\sqrt[3]{a}(7aB - 4Ab) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right) + 9b^{4/3}Bx^4}{36b^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] (36*b^(1/3)*(A*b - 2*a*B)*x + 9*b^(4/3)*B*x^4 + (12*a*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3) - 4*Sqrt[3]*a^(1/3)*(-4*A*b + 7*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(1/3) + b^(1/3)*x] - 2*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(36*b^(10/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 (A + Bx^3)}{(a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] IntegrateAlgebraic[(x^6*(A + B*x^3))/(a + b*x^3)^2, x]

fricas [A] time = 0.85, size = 240, normalized size = 1.13

$$\frac{9 B b^2 x^7 - 9 (7 B a b - 4 A b^2) x^4 - 4 \sqrt{3} (7 B a b - 4 A b^2) x^3 + 7 B a^2 - 4 A a b \left(-\frac{a}{b}\right)^{\frac{1}{3}} \arctan\left(\frac{2 \sqrt{3} \ln\left(\frac{x}{b}\right) - \sqrt{3} a}{3 x}\right) + 2 \left((7 B a b - 4 A b^2) x^3 + 7 B a^2 - 4 A a b\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - 4 \left((7 B a b - 4 A b^2) x^3 + 7 B a^2 - 4 A a b\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) - 12 (7 B a^2 - 4 A a b) x}{36 (b^4 x^3 + a b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{36} (9 B b^2 x^7 - 9 (7 B a b - 4 A b^2) x^4 - 4 \sqrt{3} (7 B a b - 4 A b^2) x^3 + 7 B a^2 - 4 A a b) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \arctan\left(\frac{1}{3} \sqrt{3} (2 \sqrt{3} b x \left(-\frac{a}{b}\right)^{\frac{1}{3}} - \sqrt{3} a) / a\right) + 2 \left((7 B a b - 4 A b^2) x^3 + 7 B a^2 - 4 A a b\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - 4 \left((7 B a b - 4 A b^2) x^3 + 7 B a^2 - 4 A a b\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) - 12 (7 B a^2 - 4 A a b) x / (b^4 x^3 + a b^3)$

giac [A] time = 0.18, size = 211, normalized size = 0.99

$$\frac{\sqrt{3} \left(7 (-ab^2)^{\frac{1}{3}} Ba - 4 (-ab^2)^{\frac{1}{3}} Ab\right) \arctan\left(\frac{\sqrt{3} \left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 b^4} - \frac{(7 B a^2 - 4 A a b) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9 a b^3} + \frac{(7 (-ab^2)^{\frac{1}{3}} Ba - 4 (-ab^2)^{\frac{1}{3}} Ab) \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 b^4} - \frac{B a^2 x - A a b x}{3 (b x^3 + a) b^3} + \frac{B b^6 x^4 - 8 B a b^5 x + 4 A b^6 x}{4 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")

[Out] $\frac{1}{9} \sqrt{3} (7 (-a b^2)^{\frac{1}{3}} B a - 4 (-a b^2)^{\frac{1}{3}} A b) \arctan\left(\frac{1}{3} \sqrt{3} (2 x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}) / \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) / b^4 - \frac{1}{9} (7 B a^2 - 4 A a b) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\text{abs}\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\right) / (a b^3) + \frac{1}{18} (7 (-a b^2)^{\frac{1}{3}} B a - 4 (-a b^2)^{\frac{1}{3}} A b) \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) / b^4 - \frac{1}{3} (B a^2 x - A a b x) / ((b x^3 + a) b^3) + \frac{1}{4} (B b^6 x^4 - 8 B a b^5 x + 4 A b^6 x) / b^8$

maple [A] time = 0.09, size = 257, normalized size = 1.21

$$\frac{B x^4}{4 b^2} + \frac{A a x}{3 (b x^3 + a) b^2} - \frac{B a^2 x}{3 (b x^3 + a) b^3} - \frac{4 \sqrt{3} A a \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{b} - 1\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^3} - \frac{4 A a \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^3} + \frac{2 A a \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^3} + \frac{A x}{b^2} + \frac{7 \sqrt{3} B a^2 \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{b} - 1\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^4} + \frac{7 B a^2 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^4} - \frac{7 B a^2 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^4} - \frac{2 B a x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^6*(B*x^3+A)/(b*x^3+a)^2, x)$

[Out] $\frac{1}{4}b^{-2}Bx^4 + \frac{1}{b^2}Ax - \frac{2}{b^3}B^2a^2x + \frac{1}{3}a/b^2 * x / (b*x^3+a) * A - \frac{1}{3}a^2/b^3 * x / (b*x^3+a) * B - \frac{4}{9}a/b^3 * A / (a/b)^{(2/3)} * \ln(x + (a/b)^{(1/3)}) + \frac{2}{9}a/b^3 * A / (a/b)^{(2/3)} * \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) - \frac{4}{9}a/b^3 * A / (a/b)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(a/b)^{(1/3)} * x - 1)) + \frac{7}{9}a^2/b^4 * B / (a/b)^{(2/3)} * \ln(x + (a/b)^{(1/3)}) - \frac{7}{18}a^2/b^4 * B / (a/b)^{(2/3)} * \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) + \frac{7}{9}a^2/b^4 * B / (a/b)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(a/b)^{(1/3)} * x - 1))$

maxima [A] time = 1.01, size = 187, normalized size = 0.88

$$\frac{(Ba^2 - Aab)x}{3(b^4x^3 + ab^3)} + \frac{Bbx^4 - 4(2Ba - Ab)x}{4b^3} + \frac{\sqrt{3}(7Ba^2 - 4Aab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(7Ba^2 - 4Aab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(7Ba^2 - 4Aab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^6*(B*x^3+A)/(b*x^3+a)^2, x, \text{algorithm}="maxima")$

[Out] $-\frac{1}{3}(B*a^2 - A*a*b)*x/(b^4*x^3 + a*b^3) + \frac{1}{4}(B*b*x^4 - 4*(2*B*a - A*b)*x)/b^3 + \frac{1}{9}\sqrt{3}*(7*B*a^2 - 4*A*a*b)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(b^4*(a/b)^{(2/3)}) - \frac{1}{18}*(7*B*a^2 - 4*A*a*b)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^4*(a/b)^{(2/3)}) + \frac{1}{9}*(7*B*a^2 - 4*A*a*b)*\log(x + (a/b)^{(1/3)})/(b^4*(a/b)^{(2/3)})$

mupad [B] time = 2.62, size = 193, normalized size = 0.91

$$x \left(\frac{A}{b^2} - \frac{2Ba}{b^3} \right) - \frac{x \left(\frac{Bx^2}{3} - \frac{Aab}{3} \right)}{b^4x^3 + ab^3} + \frac{Bx^4}{4b^2} + \frac{(-a)^{1/3} \ln((-a)^{4/3} + ab^{1/3}x)}{9b^{10/3}} (4Ab - 7Ba) - \frac{(-a)^{1/3} \ln((-a)^{4/3} - 2ab^{1/3}x + \sqrt{3}(-a)^{4/3}1i)}{9b^{10/3}} \left(\frac{1}{2} + \frac{\sqrt{3}1i}{2} \right) (4Ab - 7Ba) + \frac{(-a)^{1/3} \ln(2ab^{1/3}x - (-a)^{4/3} + \sqrt{3}(-a)^{4/3}1i)}{9b^{10/3}} \left(-\frac{1}{2} + \frac{\sqrt{3}1i}{2} \right) (4Ab - 7Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^6*(A + B*x^3))/(a + b*x^3)^2, x)$

[Out] $x*(A/b^2 - (2*B*a)/b^3) - (x*((B*a^2)/3 - (A*a*b)/3))/(a*b^3 + b^4*x^3) + (B*x^4)/(4*b^2) + ((-a)^{(1/3)}*\log((-a)^{(4/3)} + a*b^{(1/3)}*x)*(4*A*b - 7*B*a))/(9*b^{(10/3)}) - ((-a)^{(1/3)}*\log((-a)^{(4/3)} + 3^{(1/2)}*(-a)^{(4/3)}*1i - 2*a*b^{(1/3)}*x)*((3^{(1/2)}*1i)/2 + 1/2)*(4*A*b - 7*B*a))/(9*b^{(10/3)}) + ((-a)^{(1/3)}*\log(3^{(1/2)}*(-a)^{(4/3)}*1i - (-a)^{(4/3)} + 2*a*b^{(1/3)}*x)*((3^{(1/2)}*1i)/2 - 1/2)*(4*A*b - 7*B*a))/(9*b^{(10/3)})$

sympy [A] time = 2.54, size = 126, normalized size = 0.59

$$\frac{Bx^4}{4b^2} + x \left(\frac{A}{b^2} - \frac{2Ba}{b^3} \right) + \frac{x(Aab - Ba^2)}{3ab^3 + 3b^4x^3} + \text{RootSum}\left(729t^3b^{10} + 64A^3ab^3 - 336A^2Ba^2b^2 + 588AB^2a^3b - 343B^3a^4, \left(t \mapsto t \log\left(\frac{9tb^3}{-4Ab + 7Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**4/(4*b**2) + x*(A/b**2 - 2*B*a/b**3) + x*(A*a*b - B*a**2)/(3*a*b**3 +  
3*b**4*x**3) + RootSum(729*_t**3*b**10 + 64*A**3*a*b**3 - 336*A**2*B*a**2*b  
**2 + 588*A*B**2*a**3*b - 343*B**3*a**4, Lambda(_t, _t*log(9*_t*b**3/(-4*A*  
b + 7*B*a) + x)))
```

$$3.75 \quad \int \frac{x^5(A+Bx^3)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=60

$$\frac{a(Ab - aB)}{3b^3(a + bx^3)} + \frac{(Ab - 2aB) \log(a + bx^3)}{3b^3} + \frac{Bx^3}{3b^2}$$

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 77}

$$\frac{a(Ab - aB)}{3b^3(a + bx^3)} + \frac{(Ab - 2aB) \log(a + bx^3)}{3b^3} + \frac{Bx^3}{3b^2}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] (B*x^3)/(3*b^2) + (a*(A*b - a*B))/(3*b^3*(a + b*x^3)) + ((A*b - 2*a*B)*Log[a + b*x^3])/(3*b^3)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (A + Bx^3)}{(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(A + Bx)}{(a + bx)^2} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{B}{b^2} + \frac{a(-Ab + aB)}{b^2(a + bx)^2} + \frac{Ab - 2aB}{b^2(a + bx)} \right) dx, x, x^3 \right) \\
&= \frac{Bx^3}{3b^2} + \frac{a(Ab - aB)}{3b^3(a + bx^3)} + \frac{(Ab - 2aB) \log(a + bx^3)}{3b^3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.83

$$\frac{\frac{a(Ab - aB)}{a + bx^3} + (Ab - 2aB) \log(a + bx^3) + bBx^3}{3b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] (b*B*x^3 + (a*(A*b - a*B))/(a + b*x^3) + (A*b - 2*a*B)*Log[a + b*x^3])/(3*b^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (A + Bx^3)}{(a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3)^2, x]

fricas [A] time = 0.69, size = 81, normalized size = 1.35

$$\frac{Bb^2x^6 + Babx^3 - Ba^2 + Aab - ((2Bab - Ab^2)x^3 + 2Ba^2 - Aab) \log(bx^3 + a)}{3(b^4x^3 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{3} * (B * b^2 * x^6 + B * a * b * x^3 - B * a^2 + A * a * b - ((2 * B * a * b - A * b^2) * x^3 + 2 * B * a^2 - A * a * b) * \log(b * x^3 + a)) / (b^4 * x^3 + a * b^3)$

giac [A] time = 0.18, size = 91, normalized size = 1.52

$$\frac{\frac{(bx^3+a)B}{b^2} + \frac{(2Ba-Ab)\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right) - \frac{Ba^2b}{bx^3+a} - \frac{Aab^2}{bx^3+a}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")`

[Out] $\frac{1}{3} * ((b * x^3 + a) * B / b^2 + (2 * B * a - A * b) * \log(\text{abs}(b * x^3 + a) / ((b * x^3 + a)^2 * \text{abs}(b)))) / b^2 - (B * a^2 * b / (b * x^3 + a) - A * a * b^2 / (b * x^3 + a)) / b^3 / b$

maple [A] time = 0.05, size = 74, normalized size = 1.23

$$\frac{Bx^3}{3b^2} + \frac{Aa}{3(bx^3+a)b^2} + \frac{A \ln(bx^3+a)}{3b^2} - \frac{Ba^2}{3(bx^3+a)b^3} - \frac{2Ba \ln(bx^3+a)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(B*x^3+A)/(b*x^3+a)^2,x)`

[Out] $\frac{1}{3} * B * x^3 / b^2 + \frac{1}{3} / b^2 * \ln(b * x^3 + a) * A - \frac{2}{3} / b^3 * \ln(b * x^3 + a) * B * a + \frac{1}{3} / b^2 * a / (b * x^3 + a) * A - \frac{1}{3} / b^3 * a^2 / (b * x^3 + a) * B$

maxima [A] time = 0.46, size = 60, normalized size = 1.00

$$\frac{Bx^3}{3b^2} - \frac{Ba^2 - Aab}{3(b^4x^3 + ab^3)} - \frac{(2Ba - Ab)\log(bx^3 + a)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] $\frac{1}{3} * B * x^3 / b^2 - \frac{1}{3} * (B * a^2 - A * a * b) / (b^4 * x^3 + a * b^3) - \frac{1}{3} * (2 * B * a - A * b) * \log(b * x^3 + a) / b^3$

mupad [B] time = 0.08, size = 62, normalized size = 1.03

$$\frac{Bx^3}{3b^2} + \frac{\ln(bx^3+a)(Ab-2Ba)}{3b^3} - \frac{Ba^2-Aab}{3b(b^3x^3+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(A + B*x^3))/(a + b*x^3)^2,x)`

[Out] $(B*x^3)/(3*b^2) + (\log(a + b*x^3)*(A*b - 2*B*a))/(3*b^3) - (B*a^2 - A*a*b)/(3*b*(a*b^2 + b^3*x^3))$

sympy [A] time = 1.61, size = 56, normalized size = 0.93

$$\frac{Bx^3}{3b^2} + \frac{Aab - Ba^2}{3ab^3 + 3b^4x^3} - \frac{(-Ab + 2Ba) \log(a + bx^3)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(B*x**3+A)/(b*x**3+a)**2,x)`

[Out] $B*x**3/(3*b**2) + (A*a*b - B*a**2)/(3*a*b**3 + 3*b**4*x**3) - (-A*b + 2*B*a)*\log(a + b*x**3)/(3*b**3)$

$$3.76 \quad \int \frac{x^4(A+Bx^3)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=196

$$\frac{(2Ab - 5aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18 \sqrt[3]{a} b^{8/3}} - \frac{(2Ab - 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9 \sqrt[3]{a} b^{8/3}} - \frac{(2Ab - 5aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} \sqrt[3]{a} b^{8/3}} - x^2(2$$

Rubi [A] time = 0.11, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 321, 292, 31, 634, 617, 204, 628}

$$\frac{(2Ab - 5aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18 \sqrt[3]{a} b^{8/3}} - \frac{x^2(2Ab - 5aB)}{6ab^2} - \frac{(2Ab - 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9 \sqrt[3]{a} b^{8/3}} - \frac{(2Ab - 5aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} \sqrt[3]{a} b^{8/3}} + \frac{x^5(Ab - aB)}{3ab(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^2, x]

[Out] -((2*A*b - 5*a*B)*x^2)/(6*a*b^2) + ((A*b - a*B)*x^5)/(3*a*b*(a + b*x^3)) - ((2*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(8/3)) - ((2*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b^(8/3)) + ((2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(8/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(−1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(−1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(−1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4 (A + Bx^3)}{(a + bx^3)^2} dx &= \frac{(Ab - aB)x^5}{3ab(a + bx^3)} + \frac{(-2Ab + 5aB) \int \frac{x^4}{a+bx^3} dx}{3ab} \\
&= -\frac{(2Ab - 5aB)x^2}{6ab^2} + \frac{(Ab - aB)x^5}{3ab(a + bx^3)} + \frac{(2Ab - 5aB) \int \frac{x}{a+bx^3} dx}{3b^2} \\
&= -\frac{(2Ab - 5aB)x^2}{6ab^2} + \frac{(Ab - aB)x^5}{3ab(a + bx^3)} - \frac{(2Ab - 5aB) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9\sqrt[3]{a} b^{7/3}} + \frac{(2Ab - 5aB) \int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}} dx}{9\sqrt[3]{a} b^{7/3}} \\
&= -\frac{(2Ab - 5aB)x^2}{6ab^2} + \frac{(Ab - aB)x^5}{3ab(a + bx^3)} - \frac{(2Ab - 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9\sqrt[3]{a} b^{8/3}} + \frac{(2Ab - 5aB) \int \frac{-\sqrt[3]{a}}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}} dx}{18\sqrt[3]{a} b^{8/3}} \\
&= -\frac{(2Ab - 5aB)x^2}{6ab^2} + \frac{(Ab - aB)x^5}{3ab(a + bx^3)} - \frac{(2Ab - 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9\sqrt[3]{a} b^{8/3}} + \frac{(2Ab - 5aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b})}{18\sqrt[3]{a} b^{8/3}} \\
&= -\frac{(2Ab - 5aB)x^2}{6ab^2} + \frac{(Ab - aB)x^5}{3ab(a + bx^3)} - \frac{(2Ab - 5aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a} b^{8/3}} - \frac{(2Ab - 5aB) \log(\sqrt[3]{a} - \sqrt[3]{b})}{9\sqrt[3]{a} b^{8/3}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 165, normalized size = 0.84

$$\frac{(2Ab-5aB) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{\sqrt[3]{a}} - \frac{6b^{2/3}x^2(Ab-aB)}{a+bx^3} + \frac{2(5aB-2Ab) \log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{2\sqrt{3}(5aB-2Ab) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}} + 9b^{2/3}Bx^2$$

$$18b^{8/3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] (9*b^(2/3)*B*x^2 - (6*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3) + (2*Sqrt[3]*(-2*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (2*(-2*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + ((2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(18*b^(8/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (A + Bx^3)}{(a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] IntegrateAlgebraic[(x^4*(A + B*x^3))/(a + b*x^3)^2, x]

fricas [A] time = 0.79, size = 578, normalized size = 2.95

$$\frac{\sqrt{3} \left(5Ba - 2Ab \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{9 \left(-ab^2 \right)^{\frac{1}{3}} b^2} + \frac{\left(5Ba - 2Ab \right) \log \left(x^2 + x \left(-\frac{a}{b} \right)^{\frac{1}{3}} + \left(-\frac{a}{b} \right)^{\frac{2}{3}} \right)}{18 \left(-ab^2 \right)^{\frac{1}{3}} b^2} + \frac{\left(5Ba \left(-\frac{a}{b} \right)^{\frac{1}{3}} - 2Ab \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right) \left(-\frac{a}{b} \right)^{\frac{1}{3}} \log \left(\left| x - \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right| \right)}{9 ab^2} + \frac{Bax^2 - Abx^2}{3 \left(bx^3 + a \right) b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [1/18*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 3*sqrt(1/3)*(5*B*a^3*b - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^2*b^4), 1/18*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 6*sqrt(1/3)*(5*B*a^3*b - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - ((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^2*b^4)]

giac [A] time = 0.21, size = 189, normalized size = 0.96

$$\frac{Bx^2}{2b^2} - \frac{\sqrt{3} \left(5Ba - 2Ab \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{9 \left(-ab^2 \right)^{\frac{1}{3}} b^2} + \frac{\left(5Ba - 2Ab \right) \log \left(x^2 + x \left(-\frac{a}{b} \right)^{\frac{1}{3}} + \left(-\frac{a}{b} \right)^{\frac{2}{3}} \right)}{18 \left(-ab^2 \right)^{\frac{1}{3}} b^2} + \frac{\left(5Ba \left(-\frac{a}{b} \right)^{\frac{1}{3}} - 2Ab \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right) \left(-\frac{a}{b} \right)^{\frac{1}{3}} \log \left(\left| x - \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right| \right)}{9 ab^2} + \frac{Bax^2 - Abx^2}{3 \left(bx^3 + a \right) b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/2*B*x^2/b^2 - 1/9*sqrt(3)*(5*B*a - 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^2) + 1/18*(5*B*a - 2*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^2) + 1/9*(5*B*a*(-a/b)^(1/3) - 2*A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 1/3*(B*a*x^2 - A*b*x^2)/((b*x^3 + a)*b^2)

maple [A] time = 0.08, size = 235, normalized size = 1.20

$$\frac{Ax^2}{3(bx^3+a)b} + \frac{Bax^2}{3(bx^3+a)b^2} + \frac{Bx^2}{2b^2} + \frac{2\sqrt{3}A \arctan\left(\frac{\sqrt{3}\left(\frac{2x-\frac{a}{b}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} - \frac{2A \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} + \frac{A \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} - \frac{5\sqrt{3}Ba \arctan\left(\frac{\sqrt{3}\left(\frac{2x-\frac{a}{b}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3} + \frac{5Ba \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3} - \frac{5Ba \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(B*x^3+A)/(b*x^3+a)^2,x)

[Out] $\frac{1}{2}Bx^2/b^2 - \frac{1}{3}Bx^2/(b^3x^3+a) + \frac{1}{3}Bx^2/(b^3x^3+a) + \frac{5}{9}Bx^2/b^3 + \frac{A}{(a/b)^{1/3}} \ln(x + (a/b)^{1/3}) - \frac{5}{18}Bx^2/(a/b)^{1/3} \ln(x^2 - (a/b)^{1/3}x + (a/b)^{2/3}) - \frac{5}{9}Bx^2/(a/b)^{1/3} \arctan(1/3 \sqrt{3} \ln(x^2 - (a/b)^{1/3}x + (a/b)^{2/3})) - \frac{2}{9}Bx^2/(a/b)^{1/3} \ln(x + (a/b)^{1/3}) + \frac{1}{9}Bx^2/(a/b)^{1/3} \ln(x^2 - (a/b)^{1/3}x + (a/b)^{2/3}) + \frac{2}{9}Bx^2/(a/b)^{1/3} \arctan(1/3 \sqrt{3} \ln(x^2 - (a/b)^{1/3}x + (a/b)^{2/3}))$

maxima [A] time = 1.17, size = 162, normalized size = 0.83

$$\frac{(Ba - Ab)x^2}{3(b^3x^3 + ab^2)} + \frac{Bx^2}{2b^2} - \frac{\sqrt{3}(5Ba - 2Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(5Ba - 2Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(5Ba - 2Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{3}(Bx^2 - Ax^2)/(b^3x^3 + ab^2) + \frac{1}{2}Bx^2/b^2 - \frac{1}{9}\sqrt{3}(5Bx^2 - 2Ax^2) \arctan(1/3\sqrt{3}(2x - (a/b)^{1/3})/(a/b)^{1/3})/(b^3(a/b)^{1/3}) - \frac{1}{18}(5Bx^2 - 2Ax^2) \log(x^2 - x(a/b)^{1/3} + (a/b)^{2/3})/(b^3(a/b)^{1/3}) + \frac{1}{9}(5Bx^2 - 2Ax^2) \log(x + (a/b)^{1/3})/(b^3(a/b)^{1/3})$

mupad [B] time = 2.58, size = 158, normalized size = 0.81

$$\frac{Bx^2}{2b^2} - \frac{x^2\left(\frac{Ab}{3} - \frac{Ba}{3}\right)}{b^3x^3 + ab^2} - \frac{\ln(b^{1/3}x + a^{1/3})(2Ab - 5Ba)}{9a^{1/3}b^{8/3}} - \frac{\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}b^{1/3})\left(-\frac{1}{2} + \frac{\sqrt{3}11}{2}\right)(2Ab - 5Ba)}{9a^{1/3}b^{8/3}} + \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}b^{1/3})\left(\frac{1}{2} + \frac{\sqrt{3}11}{2}\right)(2Ab - 5Ba)}{9a^{1/3}b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(A + B*x^3))/(a + b*x^3)^2,x)

[Out] $\frac{Bx^2}{2b^2} - \frac{x^2((Ab)/3 - (Ba)/3)}{(a + b^3x^3)} - \frac{\log(b^{1/3}x + a^{1/3})(2Ab - 5Ba)}{(9a^{1/3}b^{8/3})} - \frac{\log(3^{1/2}a^{1/3} - 1 - 2b^{1/3}x + a^{1/3})(3^{1/2} - 1)}{(9a^{1/3}b^{8/3})} + \frac{\log(3^{1/2}a^{1/3} + 1 + 2b^{1/3}x - a^{1/3})(3^{1/2} + 1)}{(9a^{1/3}b^{8/3})}$

sympy [A] time = 2.09, size = 126, normalized size = 0.64

$$\frac{Bx^2}{2b^2} + \frac{x^2(-Ab + Ba)}{3ab^2 + 3b^3x^3} + \text{RootSum}\left(729t^3ab^8 + 8A^3b^3 - 60A^2Bab^2 + 150AB^2a^2b - 125B^3a^3, \left(t \mapsto t \log\left(\frac{81t^2ab^5}{4A^2b^2 - 20ABab + 25B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**2,x)

[Out] B*x**2/(2*b**2) + x**2*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*_t**3*a*b**8 + 8*A**3*b**3 - 60*A**2*B*a*b**2 + 150*A*B**2*a**2*b - 125*B**3*a**3, Lambda(_t, _t*log(81*_t**2*a*b**5/(4*A**2*b**2 - 20*A*B*a*b + 25*B**2*a**2) + x)))

$$3.77 \quad \int \frac{x^3(A+Bx^3)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=190

$$\frac{(Ab - 4aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{2/3} b^{7/3}} + \frac{(Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{2/3} b^{7/3}} - \frac{(Ab - 4aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} a^{2/3} b^{7/3}} - \frac{x(Ab - 4aB)}{3ab}$$

Rubi [A] time = 0.11, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 321, 200, 31, 634, 617, 204, 628}

$$\frac{(Ab - 4aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{2/3} b^{7/3}} + \frac{(Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{2/3} b^{7/3}} - \frac{(Ab - 4aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} a^{2/3} b^{7/3}} - \frac{x(Ab - 4aB)}{3ab^2} + \frac{x^4(Ab - aB)}{3ab(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] -((A*b - 4*a*B)*x)/(3*a*b^2) + ((A*b - a*B)*x^4)/(3*a*b*(a + b*x^3)) - ((A*b - 4*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(2/3)*b^(7/3)) + ((A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(2/3)*b^(7/3))) - ((A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(2/3)*b^(7/3)))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (A + Bx^3)}{(a + bx^3)^2} dx &= \frac{(Ab - aB)x^4}{3ab(a + bx^3)} + \frac{(-Ab + 4aB) \int \frac{x^3}{a+bx^3} dx}{3ab} \\
&= -\frac{(Ab - 4aB)x}{3ab^2} + \frac{(Ab - aB)x^4}{3ab(a + bx^3)} + \frac{(Ab - 4aB) \int \frac{1}{a+bx^3} dx}{3b^2} \\
&= -\frac{(Ab - 4aB)x}{3ab^2} + \frac{(Ab - aB)x^4}{3ab(a + bx^3)} + \frac{(Ab - 4aB) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9a^{2/3}b^2} + \frac{(Ab - 4aB) \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{9a^{2/3}b^2} \\
&= -\frac{(Ab - 4aB)x}{3ab^2} + \frac{(Ab - aB)x^4}{3ab(a + bx^3)} + \frac{(Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{2/3}b^{7/3}} - \frac{(Ab - 4aB) \int \frac{-\sqrt[3]{a}\sqrt[3]{b}x + 2\sqrt[3]{a}}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{18a^{2/3}b^{7/3}} \\
&= -\frac{(Ab - 4aB)x}{3ab^2} + \frac{(Ab - aB)x^4}{3ab(a + bx^3)} + \frac{(Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{2/3}b^{7/3}} - \frac{(Ab - 4aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x)}{18a^{2/3}b^{7/3}} \\
&= -\frac{(Ab - 4aB)x}{3ab^2} + \frac{(Ab - aB)x^4}{3ab(a + bx^3)} - \frac{(Ab - 4aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}} + \frac{(Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{2/3}b^{7/3}}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 160, normalized size = 0.84

$$\frac{(4aB - Ab) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{a^{2/3}} + \frac{2(Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{a^{2/3}} + \frac{2\sqrt{3}(4aB - Ab) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}} - \frac{6\sqrt[3]{b}x(Ab - aB)}{a + bx^3} + 18\sqrt[3]{b}Bx}{18b^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] (18*b^(1/3)*B*x - (6*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3) + (2*Sqrt[3]*(-(A*b) + 4*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + ((-(A*b) + 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^(7/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (A + Bx^3)}{(a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] IntegrateAlgebraic[(x^3*(A + B*x^3))/(a + b*x^3)^2, x]

fricas [A] time = 0.55, size = 573, normalized size = 3.02

$$\frac{\sqrt{3} \left(4Ba - Ab \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right) + (4Ba - Ab) \log \left(x^2 + x \left(-\frac{a}{b} \right)^{\frac{1}{3}} + \left(-\frac{a}{b} \right)^{\frac{2}{3}} \right) + \frac{Bx}{b^2} + \frac{(4Ba - Ab) \left(-\frac{a}{b} \right)^{\frac{1}{3}} \log \left(x - \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{9ab^2} + \frac{Bax - Abx}{3(bx^3 + a)b^2}}{9(-ab^2)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [1/18*(18*B*a^2*b^2*x^4 - 3*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^2 - A*a*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3), 1/18*(18*B*a^2*b^2*x^4 - 6*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^2 - A*a*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3)]

giac [A] time = 0.19, size = 166, normalized size = 0.87

$$\frac{\sqrt{3} (4Ba - Ab) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right) + (4Ba - Ab) \log \left(x^2 + x \left(-\frac{a}{b} \right)^{\frac{1}{3}} + \left(-\frac{a}{b} \right)^{\frac{2}{3}} \right) + \frac{Bx}{b^2} + \frac{(4Ba - Ab) \left(-\frac{a}{b} \right)^{\frac{1}{3}} \log \left(x - \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{9ab^2} + \frac{Bax - Abx}{3(bx^3 + a)b^2}}{9(-ab^2)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/9*sqrt(3)*(4*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) + 1/18*(4*B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) + B*x/b^2 + 1/9*(4*B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*b^2)

maple [A] time = 0.04, size = 228, normalized size = 1.20

$$\frac{Ax}{3(bx^3+a)b} + \frac{Bax}{3(bx^3+a)b^2} + \frac{\sqrt{3} A \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} + \frac{A \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - A \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} - \frac{4\sqrt{3} Ba \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} - \frac{4Ba \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - 2Ba \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} + \frac{Bx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(B*x^3+A)/(b*x^3+a)^2,x)

[Out] B/b^2*x-1/3/b*x/(b*x^3+a)*A+1/3/b^2*x/(b*x^3+a)*B*a-4/9/b^3*B*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+2/9/b^3*B*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9/b^3*B*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/9/b^2*A/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/18/b^2*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/9/b^2*A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))

maxima [A] time = 1.22, size = 157, normalized size = 0.83

$$\frac{(Ba - Ab)x}{3(b^3x^3 + ab^2)} + \frac{Bx}{b^2} - \frac{\sqrt{3}(4Ba - Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(4Ba - Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(4Ba - Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] 1/3*(B*a - A*b)*x/(b^3*x^3 + a*b^2) + B*x/b^2 - 1/9*sqrt(3)*(4*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 1/18*(4*B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 1/9*(4*B*a - A*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))

mupad [B] time = 2.58, size = 150, normalized size = 0.79

$$\frac{Bx}{b^2} - \frac{x\left(\frac{Ab}{3} - \frac{Ba}{3}\right)}{b^3x^3 + ab^2} + \frac{\ln(b^{1/3}x + a^{1/3})(Ab - 4Ba)}{9a^{2/3}b^{7/3}} - \frac{\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab - 4Ba)}{9a^{2/3}b^{7/3}} + \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab - 4Ba)}{9a^{2/3}b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(A + B*x^3))/(a + b*x^3)^2,x)

[Out] (B*x)/b^2 - (x*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) + (log(b^(1/3)*x + a^(1/3))*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3))

$$+ (\log(3^{(1/2)} * a^{(1/3)} * 1i + 2 * b^{(1/3)} * x - a^{(1/3)}) * ((3^{(1/2)} * 1i) / 2 - 1/2) * (A * b - 4 * B * a)) / (9 * a^{(2/3)} * b^{(7/3)})$$

sympy [A] time = 1.67, size = 102, normalized size = 0.54

$$\frac{Bx}{b^2} + \frac{x(-Ab + Ba)}{3ab^2 + 3b^3x^3} + \text{RootSum}\left(729t^3a^2b^7 - A^3b^3 + 12A^2Bab^2 - 48AB^2a^2b + 64B^3a^3, \left(t \mapsto t \log\left(-\frac{9tab^2}{-Ab + 4Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**2,x)

[Out] B*x/b**2 + x*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*_t**3*a**2*b**7 - A**3*b**3 + 12*A**2*B*a*b**2 - 48*A*B**2*a**2*b + 64*B**3*a**3, Lambda(_t, _t*log(-9*_t*a*b**2/(-A*b + 4*B*a) + x)))

$$3.78 \quad \int \frac{x^2(A+Bx^3)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=41

$$\frac{aB - Ab}{3b^2(a + bx^3)} + \frac{B \log(a + bx^3)}{3b^2}$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {444, 43}

$$\frac{B \log(a + bx^3)}{3b^2} - \frac{Ab - aB}{3b^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] -(A*b - a*B)/(3*b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2 (A + Bx^3)}{(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{(a + bx)^2} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{Ab - aB}{b(a + bx)^2} + \frac{B}{b(a + bx)} \right) dx, x, x^3 \right) \\ &= -\frac{Ab - aB}{3b^2 (a + bx^3)} + \frac{B \log(a + bx^3)}{3b^2} \end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.00

$$\frac{aB - Ab}{3b^2 (a + bx^3)} + \frac{B \log(a + bx^3)}{3b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] (- (A*b) + a*B)/(3*b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (A + Bx^3)}{(a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3)^2, x]

fricas [A] time = 0.56, size = 44, normalized size = 1.07

$$\frac{Ba - Ab + (Bbx^3 + Ba) \log(bx^3 + a)}{3(b^3x^3 + ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/3*(B*a - A*b + (B*b*x^3 + B*a)*log(b*x^3 + a))/(b^3*x^3 + a*b^2)

giac [A] time = 0.18, size = 65, normalized size = 1.59

$$-\frac{B \left(\frac{\log \left(\frac{|bx^3+a|}{(bx^3+a)^2|b|} \right)}{b} - \frac{a}{(bx^3+a)b} \right)}{3b} - \frac{A}{3(bx^3+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/3*B*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 + a)*b))/b - 1/3*A/((b*x^3 + a)*b)

maple [A] time = 0.06, size = 47, normalized size = 1.15

$$-\frac{A}{3(bx^3+a)b} + \frac{Ba}{3(bx^3+a)b^2} + \frac{B \ln(bx^3+a)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(B*x^3+A)/(b*x^3+a)^2,x)

[Out] 1/3*B*ln(b*x^3+a)/b^2-1/3/b/(b*x^3+a)*A+1/3/b^2/(b*x^3+a)*B*a

maxima [A] time = 0.58, size = 40, normalized size = 0.98

$$\frac{Ba - Ab}{3(b^3x^3 + ab^2)} + \frac{B \log(bx^3 + a)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] 1/3*(B*a - A*b)/(b^3*x^3 + a*b^2) + 1/3*B*log(b*x^3 + a)/b^2

mupad [B] time = 2.35, size = 37, normalized size = 0.90

$$\frac{B \ln(bx^3 + a)}{3b^2} - \frac{Ab - Ba}{3b^2(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(A + B*x^3))/(a + b*x^3)^2,x)

[Out] $(B \log(a + b x^3)) / (3 b^2) - (A b - B a) / (3 b^2 (a + b x^3))$

sympy [A] time = 1.23, size = 36, normalized size = 0.88

$$\frac{B \log(a + b x^3)}{3 b^2} + \frac{-A b + B a}{3 a b^2 + 3 b^3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(B*x**3+A)/(b*x**3+a)**2,x)`

[Out] $B \log(a + b x^3) / (3 b^2) + (-A b + B a) / (3 a b^2 + 3 b^3 x^3)$

$$3.79 \quad \int \frac{x(A+Bx^3)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=171

$$\frac{(2aB + Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{4/3} b^{5/3}} - \frac{(2aB + Ab) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{4/3} b^{5/3}} - \frac{(2aB + Ab) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} a^{4/3} b^{5/3}} + \frac{x^2(Ab - aB)}{3ab(a + bx^3)}$$

Rubi [A] time = 0.09, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {457, 292, 31, 634, 617, 204, 628}

$$\frac{(2aB + Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{4/3} b^{5/3}} - \frac{(2aB + Ab) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{4/3} b^{5/3}} - \frac{(2aB + Ab) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} a^{4/3} b^{5/3}} + \frac{x^2(Ab - aB)}{3ab(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x*(A + B*x^3))/(a + b*x^3)^2, x]

[Out] ((A*b - a*B)*x^2)/(3*a*b*(a + b*x^3)) - ((A*b + 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - ((A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(4/3)*b^(5/3)) + ((A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(4/3)*b^(5/3)))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(n-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 457


```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(A+Bx^3)}{(a+bx^3)^2} dx &= \frac{(Ab-aB)x^2}{3ab(a+bx^3)} + \frac{(Ab+2aB) \int \frac{x}{a+bx^3} dx}{3ab} \\
&= \frac{(Ab-aB)x^2}{3ab(a+bx^3)} - \frac{(Ab+2aB) \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x} dx}{9a^{4/3}b^{4/3}} + \frac{(Ab+2aB) \int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx}{9a^{4/3}b^{4/3}} \\
&= \frac{(Ab-aB)x^2}{3ab(a+bx^3)} - \frac{(Ab+2aB) \log(\sqrt[3]{a}+\sqrt[3]{b}x)}{9a^{4/3}b^{5/3}} + \frac{(Ab+2aB) \int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx}{18a^{4/3}b^{5/3}} + \frac{(Ab+2aB) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{18a^{4/3}b^{5/3}} \\
&= \frac{(Ab-aB)x^2}{3ab(a+bx^3)} - \frac{(Ab+2aB) \log(\sqrt[3]{a}+\sqrt[3]{b}x)}{9a^{4/3}b^{5/3}} + \frac{(Ab+2aB) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{18a^{4/3}b^{5/3}} \\
&= \frac{(Ab-aB)x^2}{3ab(a+bx^3)} - \frac{(Ab+2aB) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}} - \frac{(Ab+2aB) \log(\sqrt[3]{a}+\sqrt[3]{b}x)}{9a^{4/3}b^{5/3}} + \frac{(Ab+2aB) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{18a^{4/3}b^{5/3}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 146, normalized size = 0.85

$$\frac{(2aB+Ab) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2) - \frac{6\sqrt[3]{a}b^{2/3}x^2(aB-Ab)}{a+bx^3} - 2(2aB+Ab) \log(\sqrt[3]{a}+\sqrt[3]{b}x) - 2\sqrt{3}(2aB+Ab) \tan^{-1}\left(\frac{1-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{18a^{4/3}b^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] ((-6*a^(1/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(A*b + 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x] + (A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(4/3)*b^(5/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] IntegrateAlgebraic[(x*(A + B*x^3))/(a + b*x^3)^2, x]

fricas [A] time = 0.55, size = 548, normalized size = 3.20

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3} \left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - \frac{2Ba + Ab \log\left(x^2 + x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 (-ab^2)^{\frac{1}{3}} ab} - \frac{\left(2Ba \left(\frac{a}{b}\right)^{\frac{1}{3}} + Ab \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) \left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 a^2 b}}{9 (-ab^2)^{\frac{1}{3}} ab} - \frac{2B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^2} + \frac{(Ab - Ba)x^2}{3 (bx^3 + a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/18*(6*(B*a^2*b^2 - A*a*b^3)*x^2 - 3*\sqrt{1/3}*(2*B*a^3*b + A*a^2*b^2 + \\ & (2*B*a^2*b^2 + A*a*b^3)*x^3)*\sqrt{(-a*b^2)^{(1/3)/a}}*\log((2*b^2*x^3 - a*b + \\ & 3*\sqrt{1/3}*(a*b*x + 2*(-a*b^2)^{(2/3)}*x^2 + (-a*b^2)^{(1/3)*a})*\sqrt{(-a*b^2)^{(1/3)/a}} \\ & - 3*(-a*b^2)^{(2/3)*x}/(b*x^3 + a)) - ((2*B*a*b + A*b^2)*x^3 + 2*B \\ & *a^2 + A*a*b)*(-a*b^2)^{(2/3)}*\log(b^2*x^2 + (-a*b^2)^{(1/3)*b*x + (-a*b^2)^{(2/3)} \\ & + 2*((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^{(2/3)}*\log(b*x - \\ & (-a*b^2)^{(1/3)})))/(a^2*b^4*x^3 + a^3*b^3), -1/18*(6*(B*a^2*b^2 - A*a*b^3)*x \\ & ^2 - 6*\sqrt{1/3}*(2*B*a^3*b + A*a^2*b^2 + (2*B*a^2*b^2 + A*a*b^3)*x^3)*\sqrt{ \\ & (-a*b^2)^{(1/3)/a}}*\arctan(\sqrt{1/3}*(2*b*x + (-a*b^2)^{(1/3)})*\sqrt{(-a*b^2)^{(1/3)/a}}/b) \\ & - ((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^{(2/3)}*\log(b^2*x^2 + \\ & (-a*b^2)^{(1/3)*b*x + (-a*b^2)^{(2/3)} + 2*((2*B*a*b + A*b^2)*x^3 \\ & + 2*B*a^2 + A*a*b)*(-a*b^2)^{(2/3)}*\log(b*x - (-a*b^2)^{(1/3)})))/(a^2*b^4*x^3 \\ & + a^3*b^3)] \end{aligned}$$

giac [A] time = 0.18, size = 186, normalized size = 1.09

$$\frac{\sqrt{3} (2Ba + Ab) \arctan\left(\frac{\sqrt{3} \left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - \frac{2Ba + Ab \log\left(x^2 + x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 (-ab^2)^{\frac{1}{3}} ab} - \frac{\left(2Ba \left(\frac{a}{b}\right)^{\frac{1}{3}} + Ab \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) \left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 a^2 b}}{9 (-ab^2)^{\frac{1}{3}} ab} - \frac{2B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^2} + \frac{(Bax^2 - Abx^2)}{3 (bx^3 + a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/9*\sqrt{3}*(2*B*a + A*b)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)}) \\ & /((-a*b^2)^{(1/3)*a*b} - 1/18*(2*B*a + A*b)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)}) \\ & /((-a*b^2)^{(1/3)*a*b} - 1/9*(2*B*a*(-a/b)^{(1/3)} + A*b*(-a/b)^{(1/3)})*(-a/b)^{(1/3)} \\ & *\log(\text{abs}(x - (-a/b)^{(1/3)})))/(a^2*b) - 1/3*(B*a*x^2 - A*b*x^2)/((b*x^3 + a)*a*b) \end{aligned}$$

maple [A] time = 0.05, size = 223, normalized size = 1.30

$$\frac{\sqrt{3} A \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{1} - 1\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - \frac{A \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + A \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \left(\frac{a}{b}\right)^{\frac{1}{3}} ab} + \frac{2\sqrt{3} B \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{1} - 1\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - \frac{2B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^2} + \frac{(Ab - Ba)x^2}{3 (bx^3 + a)ab}}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} ab} - \frac{2B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^2} + \frac{(Ab - Ba)x^2}{3 (bx^3 + a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x*(B*x^3+A)/(b*x^3+a)^2, x)$

[Out] $\frac{1}{3}*(A*b-B*a)*x^2/a/b/(b*x^3+a)-1/9/b/a/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)})*A-2/9/b^2/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)})*B+1/18/b/a/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)})*x+(a/b)^{(2/3)})*A+1/9/b^2/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)})*x+(a/b)^{(2/3)})*B+1/9/b/a*3^{(1/2)}/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*A+2/9/b^2*3^{(1/2)}/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*B$

maxima [A] time = 1.21, size = 160, normalized size = 0.94

$$-\frac{(Ba - Ab)x^2}{3(ab^2x^3 + a^2b)} + \frac{\sqrt{3}(2Ba + Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(2Ba + Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(2Ba + Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x*(B*x^3+A)/(b*x^3+a)^2, x, \text{algorithm}="maxima")$

[Out] $-1/3*(B*a - A*b)*x^2/(a*b^2*x^3 + a^2*b) + 1/9*\text{sqrt}(3)*(2*B*a + A*b)*\arctan(1/3*\text{sqrt}(3)*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a*b^2*(a/b)^{(1/3)}) + 1/18*(2*B*a + A*b)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a*b^2*(a/b)^{(1/3)}) - 1/9*(2*B*a + A*b)*\log(x + (a/b)^{(1/3)})/(a*b^2*(a/b)^{(1/3)})$

mupad [B] time = 0.25, size = 145, normalized size = 0.85

$$\frac{x^2 (Ab - Ba)}{3ab(bx^3 + a)} - \frac{\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab + 2Ba)}{9a^{4/3}b^{5/3}} + \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab + 2Ba)}{9a^{4/3}b^{5/3}} - \frac{\ln(b^{1/3}x + a^{1/3})(Ab + 2Ba)}{9a^{4/3}b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x*(A + B*x^3))/(a + b*x^3)^2, x)$

[Out] $(\log(3^{(1/2)}*a^{(1/3)}*1i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*1i)/2 + 1/2)*(A*b + 2*B*a))/(9*a^{(4/3)}*b^{(5/3)}) - (\log(3^{(1/2)}*a^{(1/3)}*1i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*1i)/2 - 1/2)*(A*b + 2*B*a))/(9*a^{(4/3)}*b^{(5/3)}) - (\log(b^{(1/3)}*x + a^{(1/3)})*(A*b + 2*B*a))/(9*a^{(4/3)}*b^{(5/3)}) + (x^2*(A*b - B*a))/(3*a*b*(a + b*x^3))$

sympy [A] time = 1.48, size = 117, normalized size = 0.68

$$\frac{x^2 (Ab - Ba)}{3a^2b + 3ab^2x^3} + \text{RootSum}\left(729t^3a^4b^5 + A^3b^3 + 6A^2Bab^2 + 12AB^2a^2b + 8B^3a^3, \left(t \mapsto t \log\left(\frac{81t^2a^3b^3}{A^2b^2 + 4ABab + 4B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] x**2*(A*b - B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**4*b**5 +  
A**3*b**3 + 6*A**2*B*a*b**2 + 12*A*B**2*a**2*b + 8*B**3*a**3, Lambda(_t, _  
t*log(81*_t**2*a**3*b**3/(A**2*b**2 + 4*A*B*a*b + 4*B**2*a**2) + x)))
```

$$3.80 \quad \int \frac{A+Bx^3}{(a+bx^3)^2} dx$$

Optimal. Leaf size=169

$$-\frac{(aB + 2Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{5/3} b^{4/3}} + \frac{(aB + 2Ab) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{5/3} b^{4/3}} - \frac{(aB + 2Ab) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} a^{5/3} b^{4/3}} + \frac{x(AB - aB)}{3ab(a + bx^3)}$$

Rubi [A] time = 0.09, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {385, 200, 31, 634, 617, 204, 628}

$$-\frac{(aB + 2Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{5/3} b^{4/3}} + \frac{(aB + 2Ab) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{5/3} b^{4/3}} - \frac{(aB + 2Ab) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} a^{5/3} b^{4/3}} + \frac{x(AB - aB)}{3ab(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(a + b*x^3)^2, x]

[Out] ((A*b - a*B)*x)/(3*a*b*(a + b*x^3)) - ((2*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + ((2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(5/3)*b^(4/3)) - ((2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(5/3)*b^(4/3)))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{(a + bx^3)^2} dx &= \frac{(Ab - aB)x}{3ab(a + bx^3)} + \frac{(2Ab + aB) \int \frac{1}{a+bx^3} dx}{3ab} \\
&= \frac{(Ab - aB)x}{3ab(a + bx^3)} + \frac{(2Ab + aB) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9a^{5/3}b} + \frac{(2Ab + aB) \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{9a^{5/3}b} \\
&= \frac{(Ab - aB)x}{3ab(a + bx^3)} + \frac{(2Ab + aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{5/3}b^{4/3}} - \frac{(2Ab + aB) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{18a^{5/3}b^{4/3}} + \frac{(2Ab + aB) \int \frac{1}{a+bx^3} dx}{3ab} \\
&= \frac{(Ab - aB)x}{3ab(a + bx^3)} + \frac{(2Ab + aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{5/3}b^{4/3}} - \frac{(2Ab + aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{18a^{5/3}b^{4/3}} + \frac{(2Ab + aB) \int \frac{1}{a+bx^3} dx}{3ab} \\
&= \frac{(Ab - aB)x}{3ab(a + bx^3)} - \frac{(2Ab + aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} + \frac{(2Ab + aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{5/3}b^{4/3}} - \frac{(2Ab + aB) \int \frac{1}{a+bx^3} dx}{3ab}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 145, normalized size = 0.86

$$\frac{-(aB + 2Ab) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) - \frac{6a^{2/3}\sqrt[3]{b}x(aB - Ab)}{a + bx^3} + 2(aB + 2Ab) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 2\sqrt{3}(aB + 2Ab) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{18a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(a + b*x^3)^2,x]

[Out] ((-6*a^(2/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) - 2*sqrt[3]*(2*A*b + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] + 2*(2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] - (2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(5/3)*b^(4/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{(a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(a + b*x^3)^2,x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(a + b*x^3)^2, x]

fricas [A] time = 0.70, size = 537, normalized size = 3.18

$$\frac{\sqrt{3} \left((Ba + 2Ab) \arctan\left(\frac{\sqrt{3} \left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + (Ba + 2Ab) \log\left(x^2 + x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + (Ba + 2Ab) \left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right) \right)}{9 \left(-ab^2\right)^{\frac{2}{3}} a} - \frac{(Ba + 2Ab) \log\left(x^2 + x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \left(-ab^2\right)^{\frac{2}{3}} a} - \frac{(Ba + 2Ab) \left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9 a^2 b} - \frac{Bax - Abx}{3 (bx^3 + a) ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [1/18*(3*sqrt(1/3)*(B*a^3*b + 2*A*a^2*b^2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a) - ((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b - A*a^2*b^2)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(B*a^3*b + 2*A*a^2*b^2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b - A*a^2*b^2)*x)/(a^3*b^3*x^3 + a^4*b^2)]

giac [A] time = 0.18, size = 160, normalized size = 0.95

$$\frac{\sqrt{3} (Ba + 2Ab) \arctan\left(\frac{\sqrt{3} \left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 \left(-ab^2\right)^{\frac{2}{3}} a} - \frac{(Ba + 2Ab) \log\left(x^2 + x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \left(-ab^2\right)^{\frac{2}{3}} a} - \frac{(Ba + 2Ab) \left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9 a^2 b} - \frac{Bax - Abx}{3 (bx^3 + a) ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/9*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(B*a + 2*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(B*a + 2*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) - 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*a*b)

maple [A] time = 0.06, size = 221, normalized size = 1.31

$$\frac{2\sqrt{3} A \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1 \right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} ab} + \frac{2A \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} ab} - \frac{A \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} ab} + \frac{\sqrt{3} B \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1 \right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} + \frac{B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} - \frac{B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} + \frac{(Ab - Ba)x}{3 (bx^3 + a) ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^3+A)/(b*x^3+a)^2,x)`

[Out] $\frac{1}{3}*(A*b-B*a)*x/a/b/(b*x^3+a)+2/9/b/a/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*A+1/9/b^{2/3}*\ln(x+(a/b)^{(1/3)})*B-1/9/b/a/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*A-1/18/b^{2/3}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*B+2/9/b/a/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*A+1/9/b^{2/3}*(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*B$

maxima [A] time = 1.10, size = 158, normalized size = 0.93

$$-\frac{(Ba - Ab)x}{3(ab^2x^3 + a^2b)} + \frac{\sqrt{3}(Ba + 2Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba + 2Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba + 2Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] $-1/3*(B*a - A*b)*x/(a*b^2*x^3 + a^2*b) + 1/9*\sqrt{3}*(B*a + 2*A*b)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a*b^2*(a/b)^{(2/3)}) - 1/18*(B*a + 2*A*b)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a*b^2*(a/b)^{(2/3)}) + 1/9*(B*a + 2*A*b)*\log(x + (a/b)^{(1/3)})/(a*b^2*(a/b)^{(2/3)})$

mupad [B] time = 2.55, size = 143, normalized size = 0.85

$$\frac{\ln(b^{1/3}x + a^{1/3})(2Ab + Ba)}{9a^{5/3}b^{4/3}} - \frac{\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(2Ab + Ba)}{9a^{5/3}b^{4/3}} + \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(2Ab + Ba)}{9a^{5/3}b^{4/3}} + \frac{x(Ab - Ba)}{3ab(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)/(a + b*x^3)^2,x)`

[Out] $(\log(b^{(1/3)}*x + a^{(1/3)})*(2*A*b + B*a))/(9*a^{(5/3)}*b^{(4/3)}) - (\log(3^{(1/2)}*a^{(1/3)}*i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*i)/2 + 1/2)*(2*A*b + B*a))/(9*a^{(5/3)}*b^{(4/3)}) + (\log(3^{(1/2)}*a^{(1/3)}*i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*i)/2 - 1/2)*(2*A*b + B*a))/(9*a^{(5/3)}*b^{(4/3)}) + (x*(A*b - B*a))/(3*a*b*(a + b*x^3))$

sympy [A] time = 1.43, size = 97, normalized size = 0.57

$$\frac{x(Ab - Ba)}{3a^2b + 3ab^2x^3} + \text{RootSum}\left(729t^3a^5b^4 - 8A^3b^3 - 12A^2Bab^2 - 6AB^2a^2b - B^3a^3, \left(t \mapsto t \log\left(\frac{9ta^2b}{2Ab + Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] x*(A*b - B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 - 8*  
A**3*b**3 - 12*A**2*B*a*b**2 - 6*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*  
log(9*_t*a**2*b/(2*A*b + B*a) + x)))
```

$$3.81 \quad \int \frac{A+Bx^3}{x(a+bx^3)^2} dx$$

Optimal. Leaf size=51

$$-\frac{A \log(a+bx^3)}{3a^2} + \frac{A \log(x)}{a^2} + \frac{Ab-aB}{3ab(a+bx^3)}$$

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 77}

$$-\frac{A \log(a+bx^3)}{3a^2} + \frac{A \log(x)}{a^2} + \frac{Ab-aB}{3ab(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x*(a + b*x^3)^2), x]

[Out] (A*b - a*B)/(3*a*b*(a + b*x^3)) + (A*Log[x])/a^2 - (A*Log[a + b*x^3])/(3*a^2)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0])
|| EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x]
/; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x(a + bx)^2} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{A}{a^2x} + \frac{-Ab + aB}{a(a + bx)^2} - \frac{Ab}{a^2(a + bx)} \right) dx, x, x^3 \right) \\
&= \frac{Ab - aB}{3ab(a + bx^3)} + \frac{A \log(x)}{a^2} - \frac{A \log(a + bx^3)}{3a^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.90

$$\frac{\frac{a(Ab-aB)}{b(a+bx^3)} - A \log(a + bx^3) + 3A \log(x)}{3a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^2), x]

[Out] ((a*(A*b - a*B))/(b*(a + b*x^3)) + 3*A*Log[x] - A*Log[a + b*x^3])/(3*a^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x(a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)^2), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)^2), x]

fricas [A] time = 0.70, size = 70, normalized size = 1.37

$$\frac{Ba^2 - Aab + (Ab^2x^3 + Aab) \log(bx^3 + a) - 3(Ab^2x^3 + Aab) \log(x)}{3(a^2b^2x^3 + a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="fricas")

[Out] -1/3*(B*a^2 - A*a*b + (A*b^2*x^3 + A*a*b)*log(b*x^3 + a) - 3*(A*b^2*x^3 + A*a*b)*log(x))/(a^2*b^2*x^3 + a^3*b)

giac [A] time = 0.21, size = 61, normalized size = 1.20

$$-\frac{A \log(|bx^3 + a|)}{3a^2} + \frac{A \log(|x|)}{a^2} + \frac{Ab^2x^3 - Ba^2 + 2Aab}{3(bx^3 + a)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/3*A*log(abs(b*x^3 + a))/a^2 + A*log(abs(x))/a^2 + 1/3*(A*b^2*x^3 - B*a^2 + 2*A*a*b)/((b*x^3 + a)*a^2*b)

maple [A] time = 0.05, size = 53, normalized size = 1.04

$$\frac{A}{3(bx^3 + a)a} + \frac{A \ln(x)}{a^2} - \frac{A \ln(bx^3 + a)}{3a^2} - \frac{B}{3(bx^3 + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x/(b*x^3+a)^2,x)

[Out] -1/3*A*ln(b*x^3+a)/a^2+1/3/a/(b*x^3+a)*A-1/3/b/(b*x^3+a)*B+A/a^2*ln(x)

maxima [A] time = 0.48, size = 51, normalized size = 1.00

$$-\frac{Ba - Ab}{3(ab^2x^3 + a^2b)} - \frac{A \log(bx^3 + a)}{3a^2} + \frac{A \log(x^3)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="maxima")

[Out] -1/3*(B*a - A*b)/(a*b^2*x^3 + a^2*b) - 1/3*A*log(b*x^3 + a)/a^2 + 1/3*A*log(x^3)/a^2

mupad [B] time = 0.14, size = 47, normalized size = 0.92

$$\frac{A \ln(x)}{a^2} - \frac{A \ln(bx^3 + a)}{3a^2} + \frac{Ab - Ba}{3ab(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x*(a + b*x^3)^2),x)

[Out] $(A \log(x))/a^2 - (A \log(a + b x^3))/(3 a^2) + (A b - B a)/(3 a b (a + b x^3))$

sympy [A] time = 1.16, size = 46, normalized size = 0.90

$$\frac{A \log(x)}{a^2} - \frac{A \log\left(\frac{a}{b} + x^3\right)}{3 a^2} + \frac{A b - B a}{3 a^2 b + 3 a b^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x/(b*x**3+a)**2,x)

[Out] $A \log(x)/a^2 - A \log(a/b + x^3)/(3 a^2) + (A b - B a)/(3 a^2 b + 3 a b^2 x^3)$

$$3.82 \quad \int \frac{A+Bx^3}{x^2(a+bx^3)^2} dx$$

Optimal. Leaf size=195

$$-\frac{(4Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{7/3} b^{2/3}} + \frac{(4Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{7/3} b^{2/3}} + \frac{(4Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} a^{7/3} b^{2/3}} + \frac{aB - 4a^2 b}{3a^2 b}$$

Rubi [A] time = 0.11, antiderivative size = 196, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{(4Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{7/3} b^{2/3}} + \frac{(4Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{7/3} b^{2/3}} + \frac{(4Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} a^{7/3} b^{2/3}} - \frac{4Ab - aB}{3a^2 bx} + \frac{Ab - aB}{3abx(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^2), x]

[Out] $-(4A*b - a*B)/(3*a^2*b*x) + (A*b - a*B)/(3*a*b*x*(a + b*x^3)) + ((4A*b - a*B)*\text{ArcTan}[a^{(1/3)} - 2*b^{(1/3)}*x]/(\text{Sqrt}[3]*a^{(1/3)}))/ (3*\text{Sqrt}[3]*a^{(7/3)}*b^{(2/3)}) + ((4A*b - a*B)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x])/ (9*a^{(7/3)}*b^{(2/3)}) - ((4A*b - a*B)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/ (18*a^{(7/3)}*b^{(2/3)})$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] => Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] => -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] => -Dist[(3*Rt[a, 3]*Rt[b, 3])⁽⁻¹⁾, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^2(a + bx^3)^2} dx &= \frac{Ab - aB}{3abx(a + bx^3)} + \frac{(4Ab - aB) \int \frac{1}{x^2(a + bx^3)} dx}{3ab} \\
&= -\frac{4Ab - aB}{3a^2bx} + \frac{Ab - aB}{3abx(a + bx^3)} - \frac{(4Ab - aB) \int \frac{x}{a + bx^3} dx}{3a^2} \\
&= -\frac{4Ab - aB}{3a^2bx} + \frac{Ab - aB}{3abx(a + bx^3)} + \frac{(4Ab - aB) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9a^{7/3}\sqrt[3]{b}} - \frac{(4Ab - aB) \int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{9a^{7/3}\sqrt[3]{b}} \\
&= -\frac{4Ab - aB}{3a^2bx} + \frac{Ab - aB}{3abx(a + bx^3)} + \frac{(4Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{7/3}b^{2/3}} - \frac{(4Ab - aB) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^2}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{18a^{7/3}b^{2/3}} \\
&= -\frac{4Ab - aB}{3a^2bx} + \frac{Ab - aB}{3abx(a + bx^3)} + \frac{(4Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{7/3}b^{2/3}} - \frac{(4Ab - aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x)}{18a^{7/3}b^{2/3}} \\
&= -\frac{4Ab - aB}{3a^2bx} + \frac{Ab - aB}{3abx(a + bx^3)} + \frac{(4Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}} + \frac{(4Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{7/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 164, normalized size = 0.84

$$\frac{(aB - 4Ab) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{b^{2/3}} + \frac{2(4Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{b^{2/3}} + \frac{2\sqrt{3}(4Ab - aB) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} + \frac{6\sqrt[3]{a}x^2(aB - Ab)}{a + bx^3} - \frac{18\sqrt[3]{a}A}{x}$$

$18a^{7/3}$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^2), x]

[Out] ((-18*a^(1/3)*A)/x + (6*a^(1/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) + (2*sqrt[3]*(4*A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/b^(2/3) + (2*(4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-4*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(18*a^(7/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^2(a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^2*(a + b*x^3)^2), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^2*(a + b*x^3)^2), x]

fricas [A] time = 0.70, size = 570, normalized size = 2.92

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right) - (Ba - 4Ab) \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - \left(Ba \left(-\frac{a}{b}\right)^{\frac{1}{3}} - 4Ab \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(-ab^2\right)^{\frac{1}{3}} a^2} - \frac{(Ba - 4Ab) \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - \left(Ba \left(-\frac{a}{b}\right)^{\frac{1}{3}} - 4Ab \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{18 \left(-ab^2\right)^{\frac{1}{3}} a^2} - \frac{\left(Ba \left(-\frac{a}{b}\right)^{\frac{1}{3}} - 4Ab \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 a^3} + \frac{Bax^3 - 4Abx^3 - 3Aa}{3 \left(bx^4 + ax\right) a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/18*(18*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 3*\sqrt{1/3}*((B*a^2*b^2 - 4*A*a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*\sqrt{-(a*b^2)^{(1/3)}/a} * \log((2*b^2*x^3 - a*b - 3*\sqrt{1/3}*(a*b*x + 2*(a*b^2)^{(2/3)}*x^2 - (a*b^2)^{(1/3)*a})*\sqrt{-(a*b^2)^{(1/3)}/a} - 3*(a*b^2)^{(2/3)*x}/(b*x^3 + a)) - ((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^{(2/3)} * \log(b^2*x^2 - (a*b^2)^{(1/3)*b*x + (a*b^2)^{(2/3)}) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^{(2/3)} * \log(b*x + (a*b^2)^{(1/3)})]/(a^3*b^3*x^4 + a^4*b^2*x), \\ & -1/18*(18*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 6*\sqrt{1/3}*((B*a^2*b^2 - 4*A*a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*\sqrt{(a*b^2)^{(1/3)}/a} * \arctan(-\sqrt{1/3}*(2*b*x - (a*b^2)^{(1/3)}) * \sqrt{(a*b^2)^{(1/3)}/a}/b) - ((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^{(2/3)} * \log(b^2*x^2 - (a*b^2)^{(1/3)*b*x + (a*b^2)^{(2/3)}) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^{(2/3)} * \log(b*x + (a*b^2)^{(1/3)})]/(a^3*b^3*x^4 + a^4*b^2*x)] \end{aligned}$$

giac [A] time = 0.18, size = 180, normalized size = 0.92

$$\frac{\sqrt{3}(Ba - 4Ab) \arctan\left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 \left(-ab^2\right)^{\frac{1}{3}} a^2} - \frac{(Ba - 4Ab) \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - \left(Ba \left(-\frac{a}{b}\right)^{\frac{1}{3}} - 4Ab \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{18 \left(-ab^2\right)^{\frac{1}{3}} a^2} - \frac{\left(Ba \left(-\frac{a}{b}\right)^{\frac{1}{3}} - 4Ab \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 a^3} + \frac{Bax^3 - 4Abx^3 - 3Aa}{3 \left(bx^4 + ax\right) a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/9*\sqrt{3}*(B*a - 4*A*b)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/((-a*b^2)^{(1/3)*a^2} - 1/18*(B*a - 4*A*b)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/((-a*b^2)^{(1/3)*a^2} - 1/9*(B*a*(-a/b)^{(1/3)} - 4*A*b*(-a/b)^{(1/3)})*(-a/b)^{(1/3)} * \log(\text{abs}(x - (-a/b)^{(1/3)})))/a^3 + 1/3*(B*a*x^3 - 4*A*b*x^3 - 3*A*a)/((b*x^4 + a*x)*a^2) \end{aligned}$$

maple [A] time = 0.05, size = 241, normalized size = 1.24

$$\frac{Abx^2}{3(bx^3+a)a^2} + \frac{Bx^2}{3(bx^3+a)a} - \frac{4\sqrt{3}A \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2} + \frac{4A \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2} - \frac{2A \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2} + \frac{\sqrt{3}B \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab} - \frac{B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab} + \frac{B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab} - \frac{A}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^2/(b*x^3+a)^2,x)

[Out] $-1/3/a^2*x^2/(b*x^3+a)*A*b+1/3/a*x^2/(b*x^3+a)*B+4/9/a^2*A/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)})-2/9/a^2*A/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})-4/9/a^2*A*3^{(1/2)}/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))-1/9/a*B/b/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)})+1/18/a*B/b/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+1/9/a*B*3^{(1/2)}/b/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))-A/a^2/x$

maxima [A] time = 1.08, size = 166, normalized size = 0.85

$$\frac{(Ba-4Ab)x^3-3Aa}{3(a^2bx^4+a^3x)} + \frac{\sqrt{3}(Ba-4Ab) \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba-4Ab) \log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba-4Ab) \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="maxima")

[Out] $1/3*((B*a-4*A*b)*x^3-3*A*a)/(a^2*b*x^4+a^3*x)+1/9*\sqrt{3}*(B*a-4*A*b)*\arctan(1/3*\sqrt{3}*(2*x-(a/b)^{(1/3)})/(a/b)^{(1/3)})/(a^2*b*(a/b)^{(1/3)})+1/18*(B*a-4*A*b)*\log(x^2-x*(a/b)^{(1/3)}+(a/b)^{(2/3)})/(a^2*b*(a/b)^{(1/3)})-1/9*(B*a-4*A*b)*\log(x+(a/b)^{(1/3)})/(a^2*b*(a/b)^{(1/3)})$

mupad [B] time = 2.57, size = 156, normalized size = 0.80

$$\frac{\ln(b^{1/3}x+a^{1/3})(4Ab-Ba)}{9a^{7/3}b^{2/3}} - \frac{\frac{A}{a} + \frac{x^3(4Ab-Ba)}{3a^2}}{bx^4+ax} + \frac{\ln(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(4Ab-Ba)}{9a^{7/3}b^{2/3}} - \frac{\ln(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(4Ab-Ba)}{9a^{7/3}b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^2*(a + b*x^3)^2),x)

[Out] $(\log(b^{(1/3)}*x + a^{(1/3)})*(4*A*b - B*a))/(9*a^{(7/3)}*b^{(2/3)}) - (A/a + (x^3*(4*A*b - B*a))/(3*a^2))/(a*x + b*x^4) + (\log(3^{(1/2)}*a^{(1/3)}*i - 2*b^{(1/3)}*x + a^{(1/3)}))*((3^{(1/2)}*i)/2 - 1/2)*(4*A*b - B*a))/(9*a^{(7/3)}*b^{(2/3)}) - ($

$\log(3^{(1/2)}*a^{(1/3)}*1i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*1i)/2 + 1/2)*(4*A*b - B*a))/(9*a^{(7/3)}*b^{(2/3)})$

sympy [A] time = 1.42, size = 122, normalized size = 0.63

$$\frac{-3Aa + x^3(-4Ab + Ba)}{3a^3x + 3a^2bx^4} + \text{RootSum}\left(729t^3a^7b^2 - 64A^3b^3 + 48A^2Bab^2 - 12AB^2a^2b + B^3a^3, \left(t \mapsto t \log\left(\frac{81t^2a^5b}{16A^2b^2 - 8ABab + B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**2,x)

[Out] $(-3*A*a + x**3*(-4*A*b + B*a))/(3*a**3*x + 3*a**2*b*x**4) + \text{RootSum}(729*_t*3*a**7*b**2 - 64*A**3*b**3 + 48*A**2*B*a*b**2 - 12*A*B**2*a**2*b + B**3*a**3, \text{Lambda}(_t, _t*\log(81*_t**2*a**5*b/(16*A**2*b**2 - 8*A*B*a*b + B**2*a**2) + x)))$

$$3.83 \quad \int \frac{A+Bx^3}{x^3(a+bx^3)^2} dx$$

Optimal. Leaf size=196

$$\frac{(5Ab - 2aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{8/3} \sqrt[3]{b}} - \frac{(5Ab - 2aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{8/3} \sqrt[3]{b}} + \frac{(5Ab - 2aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} a^{8/3} \sqrt[3]{b}} + \frac{2aB}{6a}$$

Rubi [A] time = 0.10, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 325, 200, 31, 634, 617, 204, 628}

$$\frac{(5Ab - 2aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{8/3} \sqrt[3]{b}} - \frac{5Ab - 2aB}{6a^2 b x^2} - \frac{(5Ab - 2aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{8/3} \sqrt[3]{b}} + \frac{(5Ab - 2aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} a^{8/3} \sqrt[3]{b}} + \frac{Ab - aB}{3abx^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^2), x]

[Out] $-(5*A*b - 2*a*B)/(6*a^2*b*x^2) + (A*b - a*B)/(3*a*b*x^2*(a + b*x^3)) + ((5*A*b - 2*a*B)*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)}*x)/(\text{Sqrt}[3]*a^{(1/3)})]/(3*\text{Sqrt}[3]*a^{(8/3)}*b^{(1/3)}) - ((5*A*b - 2*a*B)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x]/(9*a^{(8/3)}*b^{(1/3)}) + ((5*A*b - 2*a*B)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]/(18*a^{(8/3)}*b^{(1/3)}))$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] => Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)⁽⁻¹⁾, x_Symbol] => Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] => -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^3(a + bx^3)^2} dx &= \frac{Ab - aB}{3abx^2(a + bx^3)} + \frac{(5Ab - 2aB) \int \frac{1}{x^3(a+bx^3)} dx}{3ab} \\
&= -\frac{5Ab - 2aB}{6a^2bx^2} + \frac{Ab - aB}{3abx^2(a + bx^3)} - \frac{(5Ab - 2aB) \int \frac{1}{a+bx^3} dx}{3a^2} \\
&= -\frac{5Ab - 2aB}{6a^2bx^2} + \frac{Ab - aB}{3abx^2(a + bx^3)} - \frac{(5Ab - 2aB) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9a^{8/3}} - \frac{(5Ab - 2aB) \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x} dx}{9a^{8/3}} \\
&= -\frac{5Ab - 2aB}{6a^2bx^2} + \frac{Ab - aB}{3abx^2(a + bx^3)} - \frac{(5Ab - 2aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{8/3}\sqrt[3]{b}} - \frac{(5Ab - 2aB) \int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x} dx}{6a^{7/3}} \\
&= -\frac{5Ab - 2aB}{6a^2bx^2} + \frac{Ab - aB}{3abx^2(a + bx^3)} - \frac{(5Ab - 2aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{8/3}\sqrt[3]{b}} + \frac{(5Ab - 2aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x)}{18a^{8/3}} \\
&= -\frac{5Ab - 2aB}{6a^2bx^2} + \frac{Ab - aB}{3abx^2(a + bx^3)} + \frac{(5Ab - 2aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}} - \frac{(5Ab - 2aB) \log(\sqrt[3]{a} - \sqrt[3]{b}x)}{9a^{8/3}\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 163, normalized size = 0.83

$$\frac{(5Ab-2aB) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{\sqrt[3]{b}} + \frac{6a^{2/3}x(aB-Ab)}{a+bx^3} - \frac{9a^{2/3}A}{x^2} + \frac{2(2aB-5Ab) \log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{2\sqrt{3}(5Ab-2aB) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{18a^{8/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^2), x]

[Out] ((-9*a^(2/3)*A)/x^2 + (6*a^(2/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + (2*sqrt[3]*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/b^(1/3) + (2*(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(18*a^(8/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

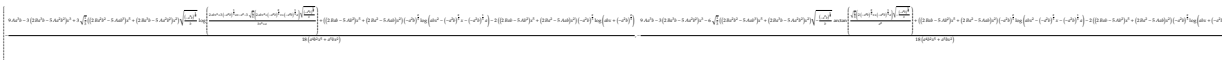
$$\int \frac{A + Bx^3}{x^3(a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^3*(a + b*x^3)^2), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^3*(a + b*x^3)^2), x]

fricas [A] time = 0.82, size = 618, normalized size = 3.15



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*x^3 + 3*\sqrt{1/3}*((2*B*a^2*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^2)*x^2)*\sqrt{(-a^2*b)^{(1/3)}/b}) \\ & * \log((2*a*b*x^3 + 3*(-a^2*b)^{(1/3)}*a*x - a^2 - 3*\sqrt{1/3}*(2*a*b*x^2 + (-a^2*b)^{(2/3)}*x + (-a^2*b)^{(1/3)}*a)*\sqrt{(-a^2*b)^{(1/3)}/b}))/ (b*x^3 + a)) + \\ & ((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^{(2/3)}*\log(a*b*x^2 - (-a^2*b)^{(2/3)}*x - (-a^2*b)^{(1/3)}*a) - 2*((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^{(2/3)}*\log(a*b*x + (-a^2*b)^{(2/3)}) \\ & / (a^4*b^2*x^5 + a^5*b*x^2), -1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*x^3 - 6*\sqrt{1/3}*((2*B*a^2*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^2)*x^2)*\sqrt{(-a^2*b)^{(1/3)}/b}) \\ & * \arctan(\sqrt{1/3}*(2*(-a^2*b)^{(2/3)}*x + (-a^2*b)^{(1/3)}*a)*\sqrt{(-a^2*b)^{(1/3)}/b})/a^2) + ((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^{(2/3)}*\log(a*b*x^2 - (-a^2*b)^{(2/3)}*x - (-a^2*b)^{(1/3)}*a) - 2*((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^{(2/3)}*\log(a*b*x + (-a^2*b)^{(2/3)}) \\ & / (a^4*b^2*x^5 + a^5*b*x^2)] \end{aligned}$$

giac [A] time = 0.17, size = 188, normalized size = 0.96

$$\frac{(2Ba - 5Ab)\left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^3} + \frac{\sqrt{3}\left(2(-ab^2)^{\frac{1}{3}}Ba - 5(-ab^2)^{\frac{1}{3}}Ab\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^3b} + \frac{Bax - Abx}{3(bx^3 + a)a^2} + \frac{\left(2(-ab^2)^{\frac{1}{3}}Ba - 5(-ab^2)^{\frac{1}{3}}Ab\right) \log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^3b} - \frac{A}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/9*(2*B*a - 5*A*b)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^3 + 1/9*\sqrt{3} \\ & *(2*(-a*b^2)^{(1/3)}*B*a - 5*(-a*b^2)^{(1/3)}*A*b)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)}) \\ & / (-a/b)^{(1/3)})/(-a/b)^{(1/3)}/(a^3*b) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*a^2) \\ & + 1/18*(2*(-a*b^2)^{(1/3)}*B*a - 5*(-a*b^2)^{(1/3)}*A*b)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)}) \\ & / (a^3*b) - 1/2*A/(a^2*x^2) \end{aligned}$$

maple [A] time = 0.05, size = 237, normalized size = 1.21

$$\frac{\frac{Abx}{3(bx^3+a)a^2} + \frac{Bx}{3(bx^3+a)a} - \frac{5\sqrt{3}A \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2} - \frac{5A \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2} + \frac{5A \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2} + \frac{2\sqrt{3}B \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab} + \frac{2B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab} - \frac{B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab} - \frac{A}{2a^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^3/(b*x^3+a)^2,x)

[Out] $-1/3/a^2*x/(b*x^3+a)*A*b+1/3/a*x/(b*x^3+a)*B-5/9/a^2*A/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})+5/18/a^2*A/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})-5/9/a^2*A/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))+2/9/a*B/b/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})-1/9/a*B/b/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+2/9/a*B/b/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))-1/2*A/a^2/x^2$

maxima [A] time = 1.23, size = 172, normalized size = 0.88

$$\frac{(2Ba-5Ab)x^3-3Aa}{6(a^2bx^5+a^3x^2)} + \frac{\sqrt{3}(2Ba-5Ab) \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2Ba-5Ab) \log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2Ba-5Ab) \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="maxima")

[Out] $1/6*((2*B*a - 5*A*b)*x^3 - 3*A*a)/(a^2*b*x^5 + a^3*x^2) + 1/9*\sqrt{3}*(2*B*a - 5*A*b)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a^2*b*(a/b)^{(2/3)}) - 1/18*(2*B*a - 5*A*b)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a^2*b*(a/b)^{(2/3)}) + 1/9*(2*B*a - 5*A*b)*\log(x + (a/b)^{(1/3)})/(a^2*b*(a/b)^{(2/3)})$

mupad [B] time = 2.57, size = 159, normalized size = 0.81

$$\frac{\frac{A}{2a} + \frac{x^3(5Ab-2Ba)}{6a^2}}{bx^5+ax^2} - \frac{\ln(b^{1/3}x+a^{1/3})(5Ab-2Ba)}{9a^{8/3}b^{1/3}} + \frac{\ln(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}i)\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)(5Ab-2Ba)}{9a^{8/3}b^{1/3}} - \frac{\ln(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}i)\left(-\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)(5Ab-2Ba)}{9a^{8/3}b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^3*(a + b*x^3)^2),x)

[Out] $(\log(3^{(1/2)}*a^{(1/3)}*i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*i)/2 + 1/2)*(5*A*b - 2*B*a))/(9*a^{(8/3)}*b^{(1/3)}) - (\log(b^{(1/3)}*x + a^{(1/3)})*(5*A*b - 2*B*a))/(9*a^{(8/3)}*b^{(1/3)}) - (A/(2*a) + (x^3*(5*A*b - 2*B*a))/(6*a^2))/(a*x^2$

$$+ b*x^5) - (\log(3^{(1/2)}*a^{(1/3)}*1i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*1i)/2 - 1/2)*(5*A*b - 2*B*a))/(9*a^{(8/3)}*b^{(1/3)})$$

sympy [A] time = 1.93, size = 109, normalized size = 0.56

$$\frac{-3Aa + x^3(-5Ab + 2Ba)}{6a^3x^2 + 6a^2bx^5} + \text{RootSum}\left(729t^3a^8b + 125A^3b^3 - 150A^2Bab^2 + 60AB^2a^2b - 8B^3a^3, \left(t \mapsto t \log\left(\frac{9ta^3}{-5Ab + 2Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**2,x)

[Out] (-3*A*a + x**3*(-5*A*b + 2*B*a))/(6*a**3*x**2 + 6*a**2*b*x**5) + RootSum(729*_t**3*a**8*b + 125*A**3*b**3 - 150*A**2*B*a*b**2 + 60*A*B**2*a**2*b - 8*B**3*a**3, Lambda(_t, _t*log(9*_t*a**3/(-5*A*b + 2*B*a) + x)))

$$3.84 \quad \int \frac{A+Bx^3}{x^4(a+bx^3)^2} dx$$

Optimal. Leaf size=76

$$\frac{(2Ab - aB) \log(a + bx^3)}{3a^3} - \frac{\log(x)(2Ab - aB)}{a^3} - \frac{Ab - aB}{3a^2(a + bx^3)} - \frac{A}{3a^2x^3}$$

Rubi [A] time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 77}

$$-\frac{Ab - aB}{3a^2(a + bx^3)} + \frac{(2Ab - aB) \log(a + bx^3)}{3a^3} - \frac{\log(x)(2Ab - aB)}{a^3} - \frac{A}{3a^2x^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^2), x]

[Out] -A/(3*a^2*x^3) - (A*b - a*B)/(3*a^2*(a + b*x^3)) - ((2*A*b - a*B)*Log[x])/a^3 + ((2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0])
|| EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x]
/; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^3}{x^4(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x^2(a + bx)^2} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{A}{a^2x^2} + \frac{-2Ab + aB}{a^3x} - \frac{b(-Ab + aB)}{a^2(a + bx)^2} - \frac{b(-2Ab + aB)}{a^3(a + bx)} \right) dx, x, x^3 \right) \\ &= -\frac{A}{3a^2x^3} - \frac{Ab - aB}{3a^2(a + bx^3)} - \frac{(2Ab - aB) \log(x)}{a^3} + \frac{(2Ab - aB) \log(a + bx^3)}{3a^3} \end{aligned}$$

Mathematica [A] time = 0.07, size = 64, normalized size = 0.84

$$\frac{\frac{a(aB - Ab)}{a + bx^3} + (2Ab - aB) \log(a + bx^3) + 3 \log(x)(aB - 2Ab) - \frac{aA}{x^3}}{3a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^2), x]

[Out] (-(a*A)/x^3) + (a*(-(A*b) + a*B))/(a + b*x^3) + 3*(-2*A*b + a*B)*Log[x] + (2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^4(a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)^2), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)^2), x]

fricas [A] time = 0.49, size = 118, normalized size = 1.55

$$\frac{(Ba^2 - 2Aab)x^3 - Aa^2 - ((Bab - 2Ab^2)x^6 + (Ba^2 - 2Aab)x^3) \log(bx^3 + a) + 3((Bab - 2Ab^2)x^6 + (Ba^2 - 2Aab)x^3) \log(x)}{3(a^3bx^6 + a^4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/3*((B*a^2 - 2*A*a*b)*x^3 - A*a^2 - ((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A*a*b)*x^3)*log(b*x^3 + a) + 3*((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A*a*b)*x^3)*log(x))/(a^3*b*x^6 + a^4*x^3)

giac [A] time = 0.19, size = 80, normalized size = 1.05

$$\frac{(Ba - 2Ab) \log(|x|)}{a^3} + \frac{Bax^3 - 2Abx^3 - Aa}{3(bx^6 + ax^3)a^2} - \frac{(Bab - 2Ab^2) \log(|bx^3 + a|)}{3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="giac")

[Out] (B*a - 2*A*b)*log(abs(x))/a^3 + 1/3*(B*a*x^3 - 2*A*b*x^3 - A*a)/((b*x^6 + a*x^3)*a^2) - 1/3*(B*a*b - 2*A*b^2)*log(abs(b*x^3 + a))/(a^3*b)

maple [A] time = 0.06, size = 87, normalized size = 1.14

$$-\frac{Ab}{3(bx^3 + a)a^2} - \frac{2Ab \ln(x)}{a^3} + \frac{2Ab \ln(bx^3 + a)}{3a^3} + \frac{B}{3(bx^3 + a)a} + \frac{B \ln(x)}{a^2} - \frac{B \ln(bx^3 + a)}{3a^2} - \frac{A}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^4/(b*x^3+a)^2,x)

[Out] 2/3*b/a^3*ln(b*x^3+a)*A-1/3/a^2*ln(b*x^3+a)*B-1/3*b/a^2/(b*x^3+a)*A+1/3/a/(b*x^3+a)*B-1/3*A/a^2/x^3-2/a^3*ln(x)*A*b+B/a^2*ln(x)

maxima [A] time = 0.50, size = 76, normalized size = 1.00

$$\frac{(Ba - 2Ab)x^3 - Aa}{3(a^2bx^6 + a^3x^3)} - \frac{(Ba - 2Ab) \log(bx^3 + a)}{3a^3} + \frac{(Ba - 2Ab) \log(x^3)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="maxima")

[Out] 1/3*((B*a - 2*A*b)*x^3 - A*a)/(a^2*b*x^6 + a^3*x^3) - 1/3*(B*a - 2*A*b)*log(b*x^3 + a)/a^3 + 1/3*(B*a - 2*A*b)*log(x^3)/a^3

mupad [B] time = 2.43, size = 78, normalized size = 1.03

$$\frac{\ln(bx^3 + a)(2Ab - Ba)}{3a^3} - \frac{\frac{A}{3a} + \frac{x^3(2Ab - Ba)}{3a^2}}{bx^6 + ax^3} - \frac{\ln(x)(2Ab - Ba)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^4*(a + b*x^3)^2),x)

[Out] $(\log(a + b*x^3)*(2*A*b - B*a))/(3*a^3) - (A/(3*a) + (x^3*(2*A*b - B*a))/(3*a^2))/(a*x^3 + b*x^6) - (\log(x)*(2*A*b - B*a))/a^3$

sympy [A] time = 1.44, size = 70, normalized size = 0.92

$$\frac{-Aa + x^3(-2Ab + Ba)}{3a^3x^3 + 3a^2bx^6} + \frac{(-2Ab + Ba)\log(x)}{a^3} - \frac{(-2Ab + Ba)\log\left(\frac{a}{b} + x^3\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**2,x)

[Out] $(-A*a + x**3*(-2*A*b + B*a))/(3*a**3*x**3 + 3*a**2*b*x**6) + (-2*A*b + B*a)*\log(x)/a**3 - (-2*A*b + B*a)*\log(a/b + x**3)/(3*a**3)$

$$3.85 \quad \int \frac{A+Bx^3}{x^5(a+bx^3)^2} dx$$

Optimal. Leaf size=215

$$\frac{\sqrt[3]{b}(7Ab - 4aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{10/3}} - \frac{\sqrt[3]{b}(7Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{10/3}} - \frac{\sqrt[3]{b}(7Ab - 4aB) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}$$

Rubi [A] time = 0.13, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 325, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}(7Ab - 4aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{10/3}} - \frac{7Ab - 4aB}{12a^2 b x^4} + \frac{7Ab - 4aB}{3a^3 x} - \frac{\sqrt[3]{b}(7Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{10/3}} - \frac{\sqrt[3]{b}(7Ab - 4aB) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}} + \frac{Ab - aB}{3abx^4(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^2), x]

[Out] $-(7A*b - 4a*B)/(12*a^2*b*x^4) + (7A*b - 4a*B)/(3*a^3*x) + (A*b - a*B)/(3*a*b*x^4*(a + b*x^3)) - (b^{(1/3)}*(7A*b - 4a*B)*\text{ArcTan}[a^{(1/3)} - 2*b^{(1/3)}*x]/(\text{Sqrt}[3]*a^{(1/3)}))/ (3*\text{Sqrt}[3]*a^{(10/3)}) - (b^{(1/3)}*(7A*b - 4a*B)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x]/(9*a^{(10/3)}) + (b^{(1/3)}*(7A*b - 4a*B)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(18*a^{(10/3)})$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])⁽⁻¹⁾, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^5 (a + bx^3)^2} dx &= \frac{Ab - aB}{3abx^4 (a + bx^3)} + \frac{(7Ab - 4aB) \int \frac{1}{x^5 (a + bx^3)} dx}{3ab} \\
&= -\frac{7Ab - 4aB}{12a^2bx^4} + \frac{Ab - aB}{3abx^4 (a + bx^3)} - \frac{(7Ab - 4aB) \int \frac{1}{x^2 (a + bx^3)} dx}{3a^2} \\
&= -\frac{7Ab - 4aB}{12a^2bx^4} + \frac{7Ab - 4aB}{3a^3x} + \frac{Ab - aB}{3abx^4 (a + bx^3)} + \frac{(b(7Ab - 4aB)) \int \frac{x}{a + bx^3} dx}{3a^3} \\
&= -\frac{7Ab - 4aB}{12a^2bx^4} + \frac{7Ab - 4aB}{3a^3x} + \frac{Ab - aB}{3abx^4 (a + bx^3)} - \frac{(b^{2/3}(7Ab - 4aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9a^{10/3}} + \frac{(b^{2/3}(7Ab - 4aB)) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{10/3}} \\
&= -\frac{7Ab - 4aB}{12a^2bx^4} + \frac{7Ab - 4aB}{3a^3x} + \frac{Ab - aB}{3abx^4 (a + bx^3)} - \frac{\sqrt[3]{b}(7Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{10/3}} + \frac{\sqrt[3]{b}(7Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{10/3}} \\
&= -\frac{7Ab - 4aB}{12a^2bx^4} + \frac{7Ab - 4aB}{3a^3x} + \frac{Ab - aB}{3abx^4 (a + bx^3)} - \frac{\sqrt[3]{b}(7Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{10/3}} + \frac{\sqrt[3]{b}(7Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{10/3}} \\
&= -\frac{7Ab - 4aB}{12a^2bx^4} + \frac{7Ab - 4aB}{3a^3x} + \frac{Ab - aB}{3abx^4 (a + bx^3)} - \frac{\sqrt[3]{b}(7Ab - 4aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}} - \frac{\sqrt[3]{b}(7Ab - 4aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 185, normalized size = 0.86

$$\frac{2\sqrt[3]{b}(7Ab - 4aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) - \frac{9a^{4/3}A}{x^4} - \frac{12\sqrt[3]{a}bx^2(aB - Ab)}{a + bx^3} - \frac{36\sqrt[3]{a}(aB - 2Ab)}{x} + 4\sqrt[3]{b}(4aB - 7Ab) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 4\sqrt{3}\sqrt[3]{b}(7Ab - 4aB) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{36a^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^2), x]

[Out] ((-9*a^(4/3)*A)/x^4 - (36*a^(1/3)*(-2*A*b + a*B))/x - (12*a^(1/3)*b*(-(A*b) + a*B)*x^2)/(a + b*x^3) - 4*sqrt[3]*b^(1/3)*(7*A*b - 4*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] + 4*b^(1/3)*(-7*A*b + 4*a*B)*Log[a^(1/3) + b^(1/3)*x] + 2*b^(1/3)*(7*A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(36*a^(10/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^5 (a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^5*(a + b*x^3)^2), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^5*(a + b*x^3)^2), x]

fricas [A] time = 0.60, size = 259, normalized size = 1.20

$$\frac{12(4Bab - 7Ab^2)x^6 + 9(4Ba^2 - 7Ab^2)x^3 + 9Aa^2 + 4\sqrt{3}((4Bab - 7Ab^2)x^7 + (4Ba^2 - 7Ab^2)x^4)\left(-\frac{1}{3}\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\sqrt{x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}}}{\frac{1}{3}\sqrt{3}}\right) - 2((4Bab - 7Ab^2)x^7 + (4Ba^2 - 7Ab^2)x^4)\left(-\frac{1}{3}\right)^{\frac{1}{3}} \log\left(bx^2 - ax\left(-\frac{1}{3}\right)^{\frac{1}{3}} - a\left(-\frac{1}{3}\right)^{\frac{1}{3}}\right) + 4((4Bab - 7Ab^2)x^7 + (4Ba^2 - 7Ab^2)x^4)\left(-\frac{1}{3}\right)^{\frac{1}{3}} \log\left(bx + a\left(-\frac{1}{3}\right)^{\frac{1}{3}}\right)}{36(a^2bx^2 + a^2x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="fricas")

[Out] $-1/36*(12*(4*B*a*b - 7*A*b^2)*x^6 + 9*(4*B*a^2 - 7*A*a*b)*x^3 + 9*A*a^2 + 4*\sqrt{3}*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^{(1/3)}*a$
 $\text{rctan}(2/3*\sqrt{3}*x*(-b/a)^{(1/3)} + 1/3*\sqrt{3}) - 2*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^{(1/3)}*\log(b*x^2 - a*x*(-b/a)^{(2/3)} - a*$
 $(-b/a)^{(1/3)}) + 4*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^{(1/3)}*\log(b*x + a*(-b/a)^{(2/3)})/(a^3*b*x^7 + a^4*x^4)$

giac [A] time = 0.22, size = 231, normalized size = 1.07

$$\frac{\left(4Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 7Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) + \sqrt{3}\left(4(-ab^2)^{\frac{2}{3}}Ba - 7(-ab^2)^{\frac{2}{3}}Ab\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^4} - \frac{Babx^2 - Ab^2x^2}{3(bx^3 + a)a^2} - \frac{\left(4(-ab^2)^{\frac{2}{3}}Ba - 7(-ab^2)^{\frac{2}{3}}Ab\right) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^4b} - \frac{4Bax^3 - 8Abx^3 + Aa}{4a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="giac")

[Out] $1/9*(4*B*a*b*(-a/b)^{(1/3)} - 7*A*b^2*(-a/b)^{(1/3)})*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^4 + 1/9*\sqrt{3}*(4*(-a*b^2)^{(2/3)}*B*a - 7*(-a*b^2)^{(2/3)}*A*b)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^4*b) - 1/3*(B*a*b*x^2 - A*b^2*x^2)/((b*x^3 + a)*a^3) - 1/18*(4*(-a*b^2)^{(2/3)}*B*a - 7*(-a*b^2)^{(2/3)}*A*b)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^4*b) - 1/4*(4*B*a*x^3 - 8*A*b*x^3 + A*a)/(a^3*x^4)$

maple [A] time = 0.06, size = 257, normalized size = 1.20

$$\frac{A b^2 x^2}{3(b x^3 + a) a^3} - \frac{B b x^2}{3(b x^3 + a) a^2} + \frac{7 \sqrt{3} A b \arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^3} - \frac{7 A b \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^3} + \frac{7 A b \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}} a^3} - \frac{4 \sqrt{3} B \arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^2} + \frac{4 B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^2} - \frac{2 B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^2} + \frac{2 A b}{a^2 x} - \frac{B}{a^2 x} - \frac{A}{4 a^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^3+A)/x^5/(b*x^3+a)^2,x)`

[Out] $\frac{1}{3}b^2/a^3x^2/(b^3x^3+a) - \frac{1}{3}b/a^2x^2/(b^3x^3+a) - \frac{7}{9}b/a^3A/(a/b)^{1/3} \ln(x+(a/b)^{1/3}) + \frac{7}{18}b/a^3A/(a/b)^{1/3} \ln(x^2-(a/b)^{1/3}x+(a/b)^{2/3}) + \frac{7}{9}b/a^3A^{3/2}/(a/b)^{1/3} \arctan(1/3 \cdot 3^{1/2} \cdot (2/(a/b)^{1/3}x-1)) + \frac{4}{9}a^2B/(a/b)^{1/3} \ln(x+(a/b)^{1/3}) - \frac{2}{9}a^2B/(a/b)^{1/3} \ln(x^2-(a/b)^{1/3}x+(a/b)^{2/3}) - \frac{4}{9}a^2B \cdot 3^{1/2}/(a/b)^{1/3} \arctan(1/3 \cdot 3^{1/2} \cdot (2/(a/b)^{1/3}x-1)) - \frac{1}{4}a^2A/x^4 + \frac{2}{a^3}b/x - B/a^2/x$

maxima [A] time = 1.42, size = 186, normalized size = 0.87

$$\frac{4(4Bab - 7Ab^2)x^6 + 3(4Ba^2 - 7Aab)x^3 + 3Aa^2}{12(a^3bx^7 + a^4x^4)} - \frac{\sqrt{3}(4Ba - 7Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(4Ba - 7Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(4Ba - 7Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] $-\frac{1}{12}(4(4B^2a^2b - 7A^2b^2)x^6 + 3(4B^2a^2 - 7A^2ab)x^3 + 3A^2a^2)/(a^3bx^7 + a^4x^4) - \frac{1}{9}\sqrt{3}(4B^2a^2 - 7A^2ab) \arctan(1/3\sqrt{3}(2x - (a/b)^{1/3})/(a/b)^{1/3})/(a^3(a/b)^{1/3}) - \frac{1}{18}(4B^2a^2 - 7A^2ab) \log(x^2 - x(a/b)^{1/3} + (a/b)^{2/3})/(a^3(a/b)^{1/3}) + \frac{1}{9}(4B^2a^2 - 7A^2ab) \log(x + (a/b)^{1/3})/(a^3(a/b)^{1/3})$

mupad [B] time = 2.62, size = 209, normalized size = 0.97

$$\frac{\frac{3(7Ab-4Ba)}{4a^2} - \frac{A}{4a} + \frac{b^2(7Ab-4Ba)}{3a^2}}{bx^7+a^4} + \frac{(-b)^{1/3} \ln(a^{1/3}(-b)^{8/3} + b^3x)}{9a^{10/3}} + \frac{(-b)^{1/3} \ln(a^{1/3}(-b)^{8/3} - 2b^3x + \sqrt{3}a^{1/3}(-b)^{8/3}i)}{9a^{10/3}} + \frac{\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(7Ab-4Ba)}{9a^{10/3}} - \frac{(-b)^{1/3} \ln(2b^3x - a^{1/3}(-b)^{8/3} + \sqrt{3}a^{1/3}(-b)^{8/3}i)}{9a^{10/3}} + \frac{\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(7Ab-4Ba)}{9a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)/(x^5*(a + b*x^3)^2),x)`

[Out] $\frac{(x^3(7Ab - 4B^2a))}{(4a^2) - A/(4a) + (bx^6(7Ab - 4B^2a))}{(3a^3)} / (ax^4 + bx^7) + \frac{((-b)^{1/3} \log(a^{1/3}(-b)^{8/3} + b^3x)(7Ab - 4B^2a))}{(9a^{10/3})} + \frac{((-b)^{1/3} \log(a^{1/3}(-b)^{8/3} - 2b^3x + 3^{1/2}a^{1/3}(-b)^{8/3}i))}{(9a^{10/3})} - \frac{((-b)^{1/3} \log(2b^3x - a^{1/3}(-b)^{8/3} + 3^{1/2}a^{1/3}(-b)^{8/3}i))}{(9a^{10/3})} + \frac{((-b)^{1/3} \log(a^{1/3}(-b)^{8/3} + b^3x)(7Ab - 4B^2a))}{(9a^{10/3})}$

sympy [A] time = 2.32, size = 153, normalized size = 0.71

$$\text{RootSum}\left(729t^3a^{10} + 343A^3b^4 - 588A^2Bab^3 + 336AB^2a^2b^2 - 64B^3a^3b, \left(t \mapsto t \log\left(\frac{81t^2a^7}{49A^2b^3 - 56ABab^2 + 16B^2a^2b} + x\right)\right)\right) + \frac{-3Aa^2 + x^6(28Ab^2 - 16Bab) + x^3(21Aab - 12Ba^2)}{12a^4x^4 + 12a^3bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**2,x)

[Out] RootSum(729*_t**3*a**10 + 343*A**3*b**4 - 588*A**2*B*a*b**3 + 336*A*B**2*a*
 *2*b**2 - 64*B**3*a**3*b, Lambda(_t, _t*log(81*_t**2*a**7/(49*A**2*b**3 - 5
 6*A*B*a*b**2 + 16*B**2*a**2*b) + x))) + (-3*A*a**2 + x**6*(28*A*b**2 - 16*B
 *a*b) + x**3*(21*A*a*b - 12*B*a**2))/(12*a**4*x**4 + 12*a**3*b*x**7)

$$3.86 \quad \int \frac{A+Bx^3}{x^6(a+bx^3)^2} dx$$

Optimal. Leaf size=215

$$\frac{b^{2/3}(8Ab - 5aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{11/3}} + \frac{b^{2/3}(8Ab - 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{11/3}} - \frac{b^{2/3}(8Ab - 5aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b} x}{\sqrt{3}}\right)}{3\sqrt{3} a^{11/3}}$$

Rubi [A] time = 0.13, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}(8Ab - 5aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{11/3}} + \frac{b^{2/3}(8Ab - 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{11/3}} - \frac{b^{2/3}(8Ab - 5aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b} x}{\sqrt{3}}\right)}{3\sqrt{3} a^{11/3}} + \frac{8Ab - 5aB}{6a^3 x^2} - \frac{8Ab - 5aB}{15a^2 b x^5} + \frac{Ab - aB}{3abx^5(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^2), x]

[Out] $-(8A*b - 5a*B)/(15*a^2*b*x^5) + (8A*b - 5a*B)/(6*a^3*x^2) + (A*b - a*B)/(3*a*b*x^5*(a + b*x^3)) - (b^{(2/3)}*(8A*b - 5a*B)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})]/(3*Sqrt[3]*a^{(11/3)}) + (b^{(2/3)}*(8A*b - 5a*B)*Log[a^{(1/3)} + b^{(1/3)}*x]/(9*a^{(11/3)}) - (b^{(2/3)}*(8A*b - 5a*B)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]/(18*a^{(11/3)})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^6(a + bx^3)^2} dx &= \frac{Ab - aB}{3abx^5(a + bx^3)} + \frac{(8Ab - 5aB) \int \frac{1}{x^6(a+bx^3)} dx}{3ab} \\
&= -\frac{8Ab - 5aB}{15a^2bx^5} + \frac{Ab - aB}{3abx^5(a + bx^3)} - \frac{(8Ab - 5aB) \int \frac{1}{x^3(a+bx^3)} dx}{3a^2} \\
&= -\frac{8Ab - 5aB}{15a^2bx^5} + \frac{8Ab - 5aB}{6a^3x^2} + \frac{Ab - aB}{3abx^5(a + bx^3)} + \frac{(b(8Ab - 5aB)) \int \frac{1}{a+bx^3} dx}{3a^3} \\
&= -\frac{8Ab - 5aB}{15a^2bx^5} + \frac{8Ab - 5aB}{6a^3x^2} + \frac{Ab - aB}{3abx^5(a + bx^3)} + \frac{(b(8Ab - 5aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9a^{11/3}} + \frac{(b(8Ab - 5aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9a^{11/3}} \\
&= -\frac{8Ab - 5aB}{15a^2bx^5} + \frac{8Ab - 5aB}{6a^3x^2} + \frac{Ab - aB}{3abx^5(a + bx^3)} + \frac{b^{2/3}(8Ab - 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{11/3}} - \frac{(b^{2/3}(8Ab - 5aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9a^{11/3}} \\
&= -\frac{8Ab - 5aB}{15a^2bx^5} + \frac{8Ab - 5aB}{6a^3x^2} + \frac{Ab - aB}{3abx^5(a + bx^3)} + \frac{b^{2/3}(8Ab - 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{11/3}} - \frac{b^{2/3}(8Ab - 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{11/3}} \\
&= -\frac{8Ab - 5aB}{15a^2bx^5} + \frac{8Ab - 5aB}{6a^3x^2} + \frac{Ab - aB}{3abx^5(a + bx^3)} - \frac{b^{2/3}(8Ab - 5aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}} + \frac{b^{2/3}(8Ab - 5aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 183, normalized size = 0.85

$$\frac{5b^{2/3}(5aB - 8Ab) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) - \frac{30a^{2/3}bx(aB - Ab)}{a+bx^3} - \frac{45a^{2/3}(aB - 2Ab)}{x^2} - \frac{18a^{5/3}A}{x^5} + 10b^{2/3}(8Ab - 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 10\sqrt{3}b^{2/3}(8Ab - 5aB) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{90a^{11/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^2), x]

[Out] ((-18*a^(5/3)*A)/x^5 - (45*a^(2/3)*(-2*A*b + a*B))/x^2 - (30*a^(2/3)*b*(-(A*b) + a*B)*x)/(a + b*x^3) - 10*sqrt[3]*b^(2/3)*(8*A*b - 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] + 10*b^(2/3)*(8*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x] + 5*b^(2/3)*(-8*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(90*a^(11/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^6 (a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^6*(a + b*x^3)^2), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^6*(a + b*x^3)^2), x]

fricas [A] time = 0.53, size = 277, normalized size = 1.29

$$\frac{15(5Bab - 8A^2)x^6 + 9(5Ba^2 - 8Aab)x^3 + 18Aa^2 + 10\sqrt{3}((5Bab - 8A^2)x^6 + (5Ba^2 - 8Aab)x^3) \left(\frac{x^3}{a}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}a\left(\frac{x^3}{a}\right)^{\frac{1}{3}} - \sqrt{3}a}{3a}\right) - 5((5Bab - 8A^2)x^6 + (5Ba^2 - 8Aab)x^3) \left(\frac{x^3}{a}\right)^{\frac{1}{3}} \log\left(b^2x^2 - abx\left(\frac{x^3}{a}\right)^{\frac{1}{3}} + a^2\left(\frac{x^3}{a}\right)^{\frac{2}{3}}\right) + 10((5Bab - 8A^2)x^6 + (5Ba^2 - 8Aab)x^3) \left(\frac{x^3}{a}\right)^{\frac{1}{3}} \log\left(bx + a\left(\frac{x^3}{a}\right)^{\frac{1}{3}}\right)}{90(a^3bx^3 + a^4x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="fricas")

[Out] $-1/90*(15*(5*B*a*b - 8*A*b^2)*x^6 + 9*(5*B*a^2 - 8*A*a*b)*x^3 + 18*A*a^2 + 10*\sqrt{3}*((5*B*a*b - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3}*a*x*(b^2/a^2)^{(2/3)} - \sqrt{3}*b)/b) - 5*((5*B*a*b - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^{(1/3)}*\log(b^2*x^2 - a*b*x*(b^2/a^2)^{(1/3)} + a^2*(b^2/a^2)^{(2/3})) + 10*((5*B*a*b - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^{(1/3)}*\log(b*x + a*(b^2/a^2)^{(1/3})))/(a^3*b*x^8 + a^4*x^5)$

giac [A] time = 0.20, size = 206, normalized size = 0.96

$$\frac{\sqrt{3}\left(5(-ab^2)^{\frac{1}{3}}Ba - 8(-ab^2)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^4} + \frac{(5Bab - 8A^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^4} - \frac{\left(5(-ab^2)^{\frac{1}{3}}Ba - 8(-ab^2)^{\frac{1}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^4} - \frac{Babx - Ab^2x - \frac{5Bax^3 - 10Abx^3 + 2Aa}{10a^3x^5}}{3(bx^3 + a)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="giac")

[Out] $-1/9*\sqrt{3}*(5*(-a*b^2)^{(1/3)}*B*a - 8*(-a*b^2)^{(1/3)}*A*b)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/a^4 + 1/9*(5*B*a*b - 8*A*b^2)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^4 - 1/18*(5*(-a*b^2)^{(1/3)}*B*a - 8*(-a*b^2)^{(1/3)}*A*b)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/a^4 - 1/3*(B*a*b*x - A*b^2*x)/((b*x^3 + a)*a^3) - 1/10*(5*B*a*x^3 - 10*A*b*x^3 + 2*A*a)/(a^3*x^5)$

maple [A] time = 0.05, size = 252, normalized size = 1.17

$$\frac{Ab^2x}{3(bx^3+a)a^3} - \frac{Bbx}{3(bx^3+a)a^2} + \frac{8\sqrt{3}Ab \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3} + \frac{8Ab \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3} - \frac{4Ab \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3} - \frac{5\sqrt{3}B \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2} - \frac{5B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2} + \frac{5B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2} + \frac{Ab}{a^3x^2} - \frac{B}{2a^2x^2} - \frac{A}{5a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^6/(b*x^3+a)^2,x)

[Out] $\frac{1}{3}b^2/a^3*x/(b*x^3+a)*A - \frac{1}{3}b/a^2*x/(b*x^3+a)*B + \frac{8}{9}b/a^3*A/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)}) - \frac{4}{9}b/a^3*A/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)}) + \frac{8}{9}b/a^3*A/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1)) - \frac{5}{9}b/a^3*A/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)}) + \frac{5}{18}b/a^3*A/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)}) - \frac{5}{9}b/a^3*A/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1)) - \frac{1}{5}b/a^2*A/x^5 + \frac{1}{a^3}B/x^2 - \frac{1}{2}b/a^2/x^2*B$

maxima [A] time = 1.32, size = 186, normalized size = 0.87

$$\frac{5(5Bab-8Ab^2)x^6+3(5Ba^2-8Aab)x^3+6Aa^2}{30(a^3bx^8+a^4x^5)} - \frac{\sqrt{3}(5Ba-8Ab) \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(5Ba-8Ab) \log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(5Ba-8Ab) \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="maxima")

[Out] $-\frac{1}{30}*(5*(5*B*a*b - 8*A*b^2)*x^6 + 3*(5*B*a^2 - 8*A*a*b)*x^3 + 6*A*a^2)/(a^3*b*x^8 + a^4*x^5) - \frac{1}{9}*\sqrt{3}*(5*B*a - 8*A*b)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a^3*(a/b)^{(2/3)}) + \frac{1}{18}*(5*B*a - 8*A*b)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a^3*(a/b)^{(2/3)}) - \frac{1}{9}*(5*B*a - 8*A*b)*\log(x + (a/b)^{(1/3)})/(a^3*(a/b)^{(2/3)})$

mupad [B] time = 2.57, size = 176, normalized size = 0.82

$$\frac{x^3(8Ab-5Ba) - \frac{A}{5a} + \frac{bx^6(8Ab-5Ba)}{6a^3}}{bx^8+ax^5} + \frac{b^{2/3} \ln(b^{1/3}x+a^{1/3})(8Ab-5Ba)}{9a^{11/3}} - \frac{b^{2/3} \ln(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}i)\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)(8Ab-5Ba)}{9a^{11/3}} + \frac{b^{2/3} \ln(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}i)\left(-\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)(8Ab-5Ba)}{9a^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^6*(a + b*x^3)^2),x)

[Out] $((x^3*(8*A*b - 5*B*a))/(10*a^2) - A/(5*a) + (b*x^6*(8*A*b - 5*B*a))/(6*a^3))/(a*x^5 + b*x^8) + (b^{(2/3)}*\log(b^{(1/3)}*x + a^{(1/3)})*(8*A*b - 5*B*a))/(9*a^{(11/3)}) - (b^{(2/3)}*\log(3^{(1/2)}*a^{(1/3)}*1i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*1i)/2 + 1/2)*(8*A*b - 5*B*a))/(9*a^{(11/3)}) + (b^{(2/3)}*\log(3^{(1/2)}*a^{(1/3)}$

) $\cdot i + 2\cdot b^{(1/3)}\cdot x - a^{(1/3)}\cdot((3^{(1/2)}\cdot i)/2 - 1/2)\cdot(8\cdot A\cdot b - 5\cdot B\cdot a))/(9\cdot a^{(11/3)})$

sympy [A] time = 1.84, size = 138, normalized size = 0.64

RootSum($729t^3a^{11} - 512A^3b^5 + 960A^2Bab^4 - 600AB^2a^2b^3 + 125B^3a^3b^2, (t \mapsto t \log(-\frac{9ta^4}{-8Ab^2 + 5Bab} + x))$) + $\frac{-6Aa^2 + x^6(40Ab^2 - 25Bab) + x^3(24Aab - 15Ba^2)}{30a^4x^5 + 30a^3bx^8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**2,x)

[Out] RootSum(729*_t**3*a**11 - 512*A**3*b**5 + 960*A**2*B*a*b**4 - 600*A*B**2*a*
*2*b**3 + 125*B**3*a**3*b**2, Lambda(_t, _t*log(-9*_t*a**4/(-8*A*b**2 + 5*B
*a*b) + x))) + (-6*A*a**2 + x**6*(40*A*b**2 - 25*B*a*b) + x**3*(24*A*a*b -
15*B*a**2))/(30*a**4*x**5 + 30*a**3*b*x**8)

$$3.87 \quad \int \frac{A+Bx^3}{x^7(a+bx^3)^2} dx$$

Optimal. Leaf size=97

$$-\frac{b(3Ab - 2aB) \log(a + bx^3)}{3a^4} + \frac{b \log(x)(3Ab - 2aB)}{a^4} + \frac{b(Ab - aB)}{3a^3(a + bx^3)} + \frac{2Ab - aB}{3a^3x^3} - \frac{A}{6a^2x^6}$$

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 77}

$$\frac{b(Ab - aB)}{3a^3(a + bx^3)} + \frac{2Ab - aB}{3a^3x^3} - \frac{b(3Ab - 2aB) \log(a + bx^3)}{3a^4} + \frac{b \log(x)(3Ab - 2aB)}{a^4} - \frac{A}{6a^2x^6}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^2), x]

[Out] -A/(6*a^2*x^6) + (2*A*b - a*B)/(3*a^3*x^3) + (b*(A*b - a*B))/(3*a^3*(a + b*x^3)) + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x^3])/(3*a^4)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A + Bx^3}{x^7(a + bx^3)^2} dx = \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x^3(a + bx)^2} dx, x, x^3 \right)$$

$$= \frac{1}{3} \text{Subst} \left(\int \left(\frac{A}{a^2x^3} + \frac{-2Ab + aB}{a^3x^2} - \frac{b(-3Ab + 2aB)}{a^4x} + \frac{b^2(-Ab + aB)}{a^3(a + bx)^2} + \frac{b^2(-3Ab + 2aB)}{a^4(a + bx)} \right) dx, x, x^3 \right)$$

$$= -\frac{A}{6a^2x^6} + \frac{2Ab - aB}{3a^3x^3} + \frac{b(Ab - aB)}{3a^3(a + bx^3)} + \frac{b(3Ab - 2aB) \log(x)}{a^4} - \frac{b(3Ab - 2aB) \log(a + bx^3)}{3a^4}$$

Mathematica [A] time = 0.11, size = 85, normalized size = 0.88

$$\frac{\frac{a^2A}{x^6} + \frac{2ab(aB - Ab)}{a + bx^3} + \frac{2a(aB - 2Ab)}{x^3} + 2b(3Ab - 2aB) \log(a + bx^3) - 6b \log(x)(3Ab - 2aB)}{6a^4}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^2), x]

[Out] -1/6*((a^2*A)/x^6 + (2*a*(-2*A*b + a*B))/x^3 + (2*a*b*(-(A*b) + a*B))/(a + b*x^3) - 6*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(3*A*b - 2*a*B)*Log[a + b*x^3])/a^4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^7(a + bx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^7*(a + b*x^3)^2), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^7*(a + b*x^3)^2), x]

fricas [A] time = 0.46, size = 154, normalized size = 1.59

$$\frac{2(2Ba^2b - 3Aab^2)x^6 + Aa^3 + (2Ba^3 - 3Aa^2b)x^3 - 2((2Bab^2 - 3Ab^3)x^9 + (2Ba^2b - 3Aab^2)x^6) \log(bx^3 + a) + 6((2Bab^2 - 3Ab^3)x^9 + (2Ba^2b - 3Aab^2)x^6) \log(x)}{6(a^4bx^9 + a^5x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="fricas")

[Out] -1/6*(2*(2*B*a^2*b - 3*A*a*b^2)*x^6 + A*a^3 + (2*B*a^3 - 3*A*a^2*b)*x^3 - 2*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(b*x^3 + a) +

$$6*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*\log(x)/(a^4*b*x^9 + a^5*x^6)$$

giac [A] time = 0.17, size = 149, normalized size = 1.54

$$-\frac{(2Bab - 3Ab^2)\log(|x|)}{a^4} + \frac{(2Bab^2 - 3Ab^3)\log(|bx^3 + a|)}{3a^4b} - \frac{2Ba^2x^3 - 3Ab^3x^3 + 3Ba^2b - 4Aab^2}{3(bx^3 + a)a^4} + \frac{6Babx^6 - 9Ab^2x^6 - 2Ba^2x^3 + 4Aabx^3 - Aa^2}{6a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="giac")

[Out] $-(2*B*a*b - 3*A*b^2)*\log(\text{abs}(x))/a^4 + 1/3*(2*B*a*b^2 - 3*A*b^3)*\log(\text{abs}(b*x^3 + a))/(a^4*b) - 1/3*(2*B*a*b^2*x^3 - 3*A*b^3*x^3 + 3*B*a^2*b - 4*A*a*b^2)/((b*x^3 + a)*a^4) + 1/6*(6*B*a*b*x^6 - 9*A*b^2*x^6 - 2*B*a^2*x^3 + 4*A*a*b*x^3 - A*a^2)/(a^4*x^6)$

maple [A] time = 0.08, size = 116, normalized size = 1.20

$$\frac{A b^2}{3(b x^3 + a) a^3} + \frac{3 A b^2 \ln(x)}{a^4} - \frac{A b^2 \ln(b x^3 + a)}{a^4} - \frac{B b}{3(b x^3 + a) a^2} - \frac{2 B b \ln(x)}{a^3} + \frac{2 B b \ln(b x^3 + a)}{3 a^3} + \frac{2 A b}{3 a^3 x^3} - \frac{B}{3 a^2 x^3} - \frac{A}{6 a^2 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^7/(b*x^3+a)^2,x)

[Out] $-1/a^4*b^2*\ln(b*x^3+a)*A+2/3/a^3*b*\ln(b*x^3+a)*B+1/3/a^3*b^2/(b*x^3+a)*A-1/3/a^2*b/(b*x^3+a)*B-1/6*A/a^2/x^6+2/3/a^3/x^3*A*b-1/3/a^2/x^3*B+3*b^2/a^4*\ln(x)*A-2*b/a^3*\ln(x)*B$

maxima [A] time = 0.65, size = 106, normalized size = 1.09

$$-\frac{2(2Bab - 3Ab^2)x^6 + (2Ba^2 - 3Aab)x^3 + Aa^2}{6(a^3bx^9 + a^4x^6)} + \frac{(2Bab - 3Ab^2)\log(bx^3 + a)}{3a^4} - \frac{(2Bab - 3Ab^2)\log(x^3)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="maxima")

[Out] $-1/6*(2*(2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3 + A*a^2)/(a^3*b*x^9 + a^4*x^6) + 1/3*(2*B*a*b - 3*A*b^2)*\log(b*x^3 + a)/a^4 - 1/3*(2*B*a*b - 3*A*b^2)*\log(x^3)/a^4$

mupad [B] time = 0.14, size = 100, normalized size = 1.03

$$\frac{\frac{x^3(3Ab-2Ba)}{6a^2} - \frac{A}{6a} + \frac{bx^6(3Ab-2Ba)}{3a^3}}{bx^9 + ax^6} - \frac{\ln(bx^3 + a)(3Ab^2 - 2Bab)}{3a^4} + \frac{\ln(x)(3Ab^2 - 2Bab)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)/(x^7*(a + b*x^3)^2), x)`

[Out]
$$\frac{(x^3(3Ab - 2Ba))/(6a^2) - A/(6a) + (b*x^6(3Ab - 2Ba))/(3a^3)}{(a*x^6 + b*x^9) - (\log(a + b*x^3)*(3Ab^2 - 2Bab)) / (3a^4) + (\log(x)*(3Ab^2 - 2Bab)) / a^4}$$

sympy [A] time = 2.40, size = 100, normalized size = 1.03

$$\frac{-Aa^2 + x^6(6Ab^2 - 4Bab) + x^3(3Aab - 2Ba^2)}{6a^4x^6 + 6a^3bx^9} - \frac{b(-3Ab + 2Ba)\log(x)}{a^4} + \frac{b(-3Ab + 2Ba)\log\left(\frac{a}{b} + x^3\right)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)/x**7/(b*x**3+a)**2, x)`

[Out]
$$\frac{(-A*a**2 + x**6*(6*A*b**2 - 4*B*a*b) + x**3*(3*A*a*b - 2*B*a**2))/(6*a**4*x**6 + 6*a**3*b*x**9) - b*(-3*A*b + 2*B*a)*\log(x)/a**4 + b*(-3*A*b + 2*B*a)*\log(a/b + x**3)/(3*a**4)}$$

$$3.88 \quad \int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=107

$$\frac{a^3(Ab - aB)}{6b^5(a + bx^3)^2} - \frac{a^2(3Ab - 4aB)}{3b^5(a + bx^3)} - \frac{a(Ab - 2aB) \log(a + bx^3)}{b^5} + \frac{x^3(Ab - 3aB)}{3b^4} + \frac{Bx^6}{6b^3}$$

Rubi [A] time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 77}

$$-\frac{a^2(3Ab - 4aB)}{3b^5(a + bx^3)} + \frac{a^3(Ab - aB)}{6b^5(a + bx^3)^2} + \frac{x^3(Ab - 3aB)}{3b^4} - \frac{a(Ab - 2aB) \log(a + bx^3)}{b^5} + \frac{Bx^6}{6b^3}$$

Antiderivative was successfully verified.

[In] Int[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] ((A*b - 3*a*B)*x^3)/(3*b^4) + (B*x^6)/(6*b^3) + (a^3*(A*b - a*B))/(6*b^5*(a + b*x^3)^2) - (a^2*(3*A*b - 4*a*B))/(3*b^5*(a + b*x^3)) - (a*(A*b - 2*a*B)*Log[a + b*x^3])/b^5

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{x^{11} (A + Bx^3)}{(a + bx^3)^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3 (A + Bx)}{(a + bx)^3} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{Ab - 3aB}{b^4} + \frac{Bx}{b^3} + \frac{a^3(-Ab + aB)}{b^4(a + bx)^3} - \frac{a^2(-3Ab + 4aB)}{b^4(a + bx)^2} + \frac{3a(-Ab + 2aB)}{b^4(a + bx)} \right) dx, x, x^3 \right) \\
&= \frac{(Ab - 3aB)x^3}{3b^4} + \frac{Bx^6}{6b^3} + \frac{a^3(Ab - aB)}{6b^5(a + bx^3)^2} - \frac{a^2(3Ab - 4aB)}{3b^5(a + bx^3)} - \frac{a(Ab - 2aB) \log(a + bx^3)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 94, normalized size = 0.88

$$\frac{\frac{a^3(Ab - aB)}{(a + bx^3)^2} + \frac{2a^2(4aB - 3Ab)}{a + bx^3} + 2bx^3(Ab - 3aB) + 6a(2aB - Ab) \log(a + bx^3) + b^2Bx^6}{6b^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11*(A + B*x^3))/(a + b*x^3)^3, x]

[Out] (2*b*(A*b - 3*a*B)*x^3 + b^2*B*x^6 + (a^3*(A*b - a*B))/(a + b*x^3)^2 + (2*a^2*(-3*A*b + 4*a*B))/(a + b*x^3) + 6*a*(-(A*b) + 2*a*B)*Log[a + b*x^3])/(6*b^5)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11} (A + Bx^3)}{(a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^11*(A + B*x^3))/(a + b*x^3)^3, x]

[Out] IntegrateAlgebraic[(x^11*(A + B*x^3))/(a + b*x^3)^3, x]

fricas [A] time = 0.64, size = 179, normalized size = 1.67

$$\frac{Bb^4x^{12} - 2(2Bab^3 - Ab^4)x^9 - (11Ba^2b^2 - 4Aab^3)x^6 + 7Ba^4 - 5Aa^3b + 2(Ba^3b - 2Aa^2b^2)x^3 + 6((2Ba^2b^2 - Aab^3)x^6 + 2Ba^4 - Aa^3b + 2(2Ba^3b - Aa^2b^2)x^3) \log(bx^3 + a)}{6(b^7x^6 + 2ab^6x^3 + a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3, x, algorithm="fricas")

[Out] $\frac{1}{6}(Bb^4x^{12} - 2(2B^2a^3b - Ab^4)x^9 - (11B^2a^2b^2 - 4A^2a^3b^3)x^6 + 7B^2a^4 - 5A^2a^3b + 2(B^2a^3b - 2A^2a^2b^2)x^3 + 6((2B^2a^2b^2 - A^2a^3b^3)x^6 + 2B^2a^4 - A^2a^3b + 2(2B^2a^3b - A^2a^2b^2)x^3) \log(bx^3 + a)) / (b^7x^6 + 2a^2b^6x^3 + a^2b^5)$

giac [A] time = 0.18, size = 131, normalized size = 1.22

$$\frac{(2Ba^2 - Aab) \log(|bx^3 + a|)}{b^5} + \frac{Bb^3x^6 - 6Bab^2x^3 + 2Ab^3x^3}{6b^6} - \frac{18Ba^2b^2x^6 - 9Aab^3x^6 + 28Ba^3bx^3 - 12Aa^2b^2x^3 + 11Ba^4 - 4Aa^3b}{6(bx^3 + a)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")`

[Out] $(2B^2a^2 - A^2a^3b) \log(\text{abs}(bx^3 + a)) / b^5 + \frac{1}{6}(Bb^3x^6 - 6B^2a^3b^2x^3 + 2A^2a^3b^3x^3) / b^6 - \frac{1}{6}(18B^2a^2b^2x^6 - 9A^2a^3b^3x^6 + 28B^2a^3bx^3 - 12A^2a^2b^2x^3 + 11B^2a^4 - 4A^2a^3b) / ((bx^3 + a)^2b^5)$

maple [A] time = 0.05, size = 134, normalized size = 1.25

$$\frac{Bx^6}{6b^3} + \frac{Ax^3}{3b^3} - \frac{Bax^3}{b^4} + \frac{Aa^3}{6(bx^3 + a)^2b^4} - \frac{Ba^4}{6(bx^3 + a)^2b^5} - \frac{Aa^2}{(bx^3 + a)b^4} - \frac{Aa \ln(bx^3 + a)}{b^4} + \frac{4Ba^3}{3(bx^3 + a)b^5} + \frac{2Ba^2 \ln(bx^3 + a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11*(B*x^3+A)/(b*x^3+a)^3,x)`

[Out] $\frac{1}{6}Bx^6/b^3 + \frac{1}{3}x^3/b^3 + \frac{1}{b^4}B^2a^3x^3 + \frac{1}{6}a^3/b^4 + \frac{1}{(bx^3+a)^2}A - \frac{1}{6}a^4/b^5 + \frac{1}{(bx^3+a)^2}B - \frac{a}{b^4} \ln(bx^3+a) + \frac{2a^2}{b^5} \ln(bx^3+a) + \frac{B-a^2}{b^4} + \frac{1}{(bx^3+a)}A + \frac{4}{3}a^3/b^5 + \frac{1}{(bx^3+a)}B$

maxima [A] time = 0.54, size = 115, normalized size = 1.07

$$\frac{7Ba^4 - 5Aa^3b + 2(4Ba^3b - 3Aa^2b^2)x^3}{6(b^7x^6 + 2ab^6x^3 + a^2b^5)} + \frac{Bbx^6 - 2(3Ba - Ab)x^3}{6b^4} + \frac{(2Ba^2 - Aab) \log(bx^3 + a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")`

[Out] $\frac{1}{6}(7B^2a^4 - 5A^2a^3b + 2(4B^2a^3b - 3A^2a^2b^2)x^3) / (b^7x^6 + 2a^2b^6x^3 + a^2b^5) + \frac{1}{6}(B^2bx^6 - 2(3B^2a - A^2b)x^3) / b^4 + (2B^2a^2 - A^2a^3b) \log(bx^3 + a) / b^5$

mupad [B] time = 0.10, size = 117, normalized size = 1.09

$$\frac{\frac{7Ba^4 - 5Aa^3b}{6b} + x^3 \left(\frac{4Ba^3}{3} - Aa^2b \right)}{a^2b^4 + 2ab^5x^3 + b^6x^6} + x^3 \left(\frac{A}{3b^3} - \frac{Ba}{b^4} \right) + \frac{\ln(bx^3 + a) (2Ba^2 - Aab)}{b^5} + \frac{Bx^6}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^11*(A + B*x^3))/(a + b*x^3)^3,x)`

[Out] $((7*B*a^4 - 5*A*a^3*b)/(6*b) + x^3*((4*B*a^3)/3 - A*a^2*b))/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x^3*(A/(3*b^3) - (B*a)/b^4) + (\log(a + b*x^3)*(2*B*a^2 - A*a*b))/b^5 + (B*x^6)/(6*b^3)$

sympy [A] time = 4.94, size = 112, normalized size = 1.05

$$\frac{Bx^6}{6b^3} + \frac{a(-Ab + 2Ba) \log(a + bx^3)}{b^5} + x^3 \left(\frac{A}{3b^3} - \frac{Ba}{b^4} \right) + \frac{-5Aa^3b + 7Ba^4 + x^3(-6Aa^2b^2 + 8Ba^3b)}{6a^2b^5 + 12ab^6x^3 + 6b^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11*(B*x**3+A)/(b*x**3+a)**3,x)`

[Out] $B*x**6/(6*b**3) + a*(-A*b + 2*B*a)*\log(a + b*x**3)/b**5 + x**3*(A/(3*b**3) - B*a/b**4) + (-5*A*a**3*b + 7*B*a**4 + x**3*(-6*A*a**2*b**2 + 8*B*a**3*b))/(6*a**2*b**5 + 12*a*b**6*x**3 + 6*b**7*x**6)$

$$3.89 \quad \int \frac{x^8(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=88

$$-\frac{a^2(Ab - aB)}{6b^4(a + bx^3)^2} + \frac{a(2Ab - 3aB)}{3b^4(a + bx^3)} + \frac{(Ab - 3aB)\log(a + bx^3)}{3b^4} + \frac{Bx^3}{3b^3}$$

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 77}

$$-\frac{a^2(Ab - aB)}{6b^4(a + bx^3)^2} + \frac{a(2Ab - 3aB)}{3b^4(a + bx^3)} + \frac{(Ab - 3aB)\log(a + bx^3)}{3b^4} + \frac{Bx^3}{3b^3}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] (B*x^3)/(3*b^3) - (a^2*(A*b - a*B))/(6*b^4*(a + b*x^3)^2) + (a*(2*A*b - 3*a*B))/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^8 (A + Bx^3)}{(a + bx^3)^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (A + Bx)}{(a + bx)^3} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{B}{b^3} - \frac{a^2(-Ab + aB)}{b^3(a + bx)^3} + \frac{a(-2Ab + 3aB)}{b^3(a + bx)^2} + \frac{Ab - 3aB}{b^3(a + bx)} \right) dx, x, x^3 \right) \\ &= \frac{Bx^3}{3b^3} - \frac{a^2(Ab - aB)}{6b^4 (a + bx^3)^2} + \frac{a(2Ab - 3aB)}{3b^4 (a + bx^3)} + \frac{(Ab - 3aB) \log(a + bx^3)}{3b^4} \end{aligned}$$

Mathematica [A] time = 0.04, size = 92, normalized size = 1.05

$$\frac{2aAb - 3a^2B}{3b^4 (a + bx^3)} + \frac{a^3B - a^2Ab}{6b^4 (a + bx^3)^2} + \frac{(Ab - 3aB) \log(a + bx^3)}{3b^4} + \frac{Bx^3}{3b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] (B*x^3)/(3*b^3) + (-a^2*A*b) + a^3*B)/(6*b^4*(a + b*x^3)^2) + (2*a*A*b - 3*a^2*B)/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 (A + Bx^3)}{(a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] IntegrateAlgebraic[(x^8*(A + B*x^3))/(a + b*x^3)^3, x]

fricas [A] time = 0.63, size = 142, normalized size = 1.61

$$\frac{2Bb^3x^9 + 4Bab^2x^6 - 5Ba^3 + 3Aa^2b - 4(Ba^2b - Aab^2)x^3 - 2((3Bab^2 - Ab^3)x^6 + 3Ba^3 - Aa^2b + 2(3Ba^2b - Aab^2)x^3) \log(bx^3 + a)}{6(b^6x^6 + 2ab^5x^3 + a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{6}*(2*B*b^3*x^9 + 4*B*a*b^2*x^6 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2*b - A*a*b^2)*x^3 - 2*((3*B*a*b^2 - A*b^3)*x^6 + 3*B*a^3 - A*a^2*b + 2*(3*B*a^2*b - A*a*b^2)*x^3)*\log(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)$

giac [A] time = 0.18, size = 93, normalized size = 1.06

$$\frac{Bx^3}{3b^3} - \frac{(3Ba - Ab)\log(|bx^3 + a|)}{3b^4} + \frac{9Bab^2x^6 - 3Ab^3x^6 + 12Ba^2bx^3 - 2Aab^2x^3 + 4Ba^3}{6(bx^3 + a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")`

[Out] $\frac{1}{3}B*x^3/b^3 - \frac{1}{3}*(3*B*a - A*b)*\log(\text{abs}(b*x^3 + a))/b^4 + \frac{1}{6}*(9*B*a*b^2*x^6 - 3*A*b^3*x^6 + 12*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 4*B*a^3)/((b*x^3 + a)^2*b^4)$

maple [A] time = 0.05, size = 110, normalized size = 1.25

$$\frac{Bx^3}{3b^3} - \frac{Aa^2}{6(bx^3 + a)^2b^3} + \frac{Ba^3}{6(bx^3 + a)^2b^4} + \frac{2Aa}{3(bx^3 + a)b^3} + \frac{A\ln(bx^3 + a)}{3b^3} - \frac{Ba^2}{(bx^3 + a)b^4} - \frac{Ba\ln(bx^3 + a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*(B*x^3+A)/(b*x^3+a)^3,x)`

[Out] $\frac{1}{3}B*x^3/b^3 - \frac{1}{6}/b^3*a^2/(b*x^3+a)^2*A + \frac{1}{6}/b^4*a^3/(b*x^3+a)^2*B + \frac{1}{3}/b^3*\ln(b*x^3+a)*A - \frac{1}{b^4}*\ln(b*x^3+a)*B*a + \frac{2}{3}/b^3*a/(b*x^3+a)*A - \frac{1}{b^4}a^2/(b*x^3+a)*B$

maxima [A] time = 0.52, size = 94, normalized size = 1.07

$$\frac{Bx^3}{3b^3} - \frac{5Ba^3 - 3Aa^2b + 2(3Ba^2b - 2Aab^2)x^3}{6(b^6x^6 + 2ab^5x^3 + a^2b^4)} - \frac{(3Ba - Ab)\log(bx^3 + a)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")`

[Out] $\frac{1}{3}B*x^3/b^3 - \frac{1}{6}*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x^3)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) - \frac{1}{3}*(3*B*a - A*b)*\log(b*x^3 + a)/b^4$

mupad [B] time = 2.40, size = 94, normalized size = 1.07

$$\frac{Bx^3}{3b^3} - \frac{x^3\left(Ba^2 - \frac{2Aab}{3}\right) + \frac{5Ba^3 - 3Aa^2b}{6b}}{a^2b^3 + 2ab^4x^3 + b^5x^6} + \frac{\ln(bx^3 + a)(Ab - 3Ba)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^8*(A + B*x^3))/(a + b*x^3)^3,x)`

[Out] $(B*x^3)/(3*b^3) - (x^3*(B*a^2 - (2*A*a*b)/3) + (5*B*a^3 - 3*A*a^2*b)/(6*b)) / (a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (\log(a + b*x^3)*(A*b - 3*B*a))/(3*b^4)$

sympy [A] time = 3.94, size = 94, normalized size = 1.07

$$\frac{Bx^3}{3b^3} + \frac{3Aa^2b - 5Ba^3 + x^3(4Aab^2 - 6Ba^2b)}{6a^2b^4 + 12ab^5x^3 + 6b^6x^6} - \frac{(-Ab + 3Ba)\log(a + bx^3)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(B*x**3+A)/(b*x**3+a)**3,x)`

[Out] $B*x**3/(3*b**3) + (3*A*a**2*b - 5*B*a**3 + x**3*(4*A*a*b**2 - 6*B*a**2*b)) / (6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) - (-A*b + 3*B*a)*\log(a + b*x**3)/(3*b**4)$

$$3.90 \quad \int \frac{x^5(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=66

$$-\frac{Ab-2aB}{3b^3(a+bx^3)} + \frac{a(Ab-aB)}{6b^3(a+bx^3)^2} + \frac{B \log(a+bx^3)}{3b^3}$$

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 77}

$$-\frac{Ab-2aB}{3b^3(a+bx^3)} + \frac{a(Ab-aB)}{6b^3(a+bx^3)^2} + \frac{B \log(a+bx^3)}{3b^3}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] (a*(A*b - a*B))/(6*b^3*(a + b*x^3)^2) - (A*b - 2*a*B)/(3*b^3*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^3)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (A + Bx^3)}{(a + bx^3)^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(A + Bx)}{(a + bx)^3} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{a(-Ab + aB)}{b^2(a + bx)^3} + \frac{Ab - 2aB}{b^2(a + bx)^2} + \frac{B}{b^2(a + bx)} \right) dx, x, x^3 \right) \\
&= \frac{a(Ab - aB)}{6b^3 (a + bx^3)^2} - \frac{Ab - 2aB}{3b^3 (a + bx^3)} + \frac{B \log(a + bx^3)}{3b^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.97

$$\frac{3a^2B - ab(A - 4Bx^3) + 2B(a + bx^3)^2 \log(a + bx^3) - 2Ab^2x^3}{6b^3(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] (3*a^2*B - 2*A*b^2*x^3 - a*b*(A - 4*B*x^3) + 2*B*(a + b*x^3)^2*Log[a + b*x^3])/(6*b^3*(a + b*x^3)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (A + Bx^3)}{(a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3)^3, x]

fricas [A] time = 0.75, size = 89, normalized size = 1.35

$$\frac{2(2Bab - Ab^2)x^3 + 3Ba^2 - Aab + 2(Bb^2x^6 + 2Babx^3 + Ba^2) \log(bx^3 + a)}{6(b^5x^6 + 2ab^4x^3 + a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{6}*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b + 2*(B*b^2*x^6 + 2*B*a*b*x^3 + B*a^2)*\log(b*x^3 + a))/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)$

giac [A] time = 0.19, size = 61, normalized size = 0.92

$$\frac{B \log(|bx^3 + a|)}{3b^3} + \frac{2(2Ba - Ab)x^3 + \frac{3Ba^2 - Aab}{b}}{6(bx^3 + a)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")`

[Out] $\frac{1}{3}*B*\log(\text{abs}(b*x^3 + a))/b^3 + \frac{1}{6}*(2*(2*B*a - A*b)*x^3 + (3*B*a^2 - A*a*b)/b)/((b*x^3 + a)^2*b^2)$

maple [A] time = 0.05, size = 81, normalized size = 1.23

$$\frac{Aa}{6(bx^3 + a)^2 b^2} - \frac{Ba^2}{6(bx^3 + a)^2 b^3} - \frac{A}{3(bx^3 + a)b^2} + \frac{2Ba}{3(bx^3 + a)b^3} + \frac{B \ln(bx^3 + a)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(B*x^3+A)/(b*x^3+a)^3,x)`

[Out] $\frac{1}{6}*a/b^2/(b*x^3+a)^2*A - \frac{1}{6}*a^2/b^3/(b*x^3+a)^2*B + \frac{1}{3}*B*\ln(b*x^3+a)/b^3 - \frac{1}{3}/b^2/(b*x^3+a)*A + \frac{2}{3}/b^3/(b*x^3+a)*B*a$

maxima [A] time = 0.47, size = 72, normalized size = 1.09

$$\frac{2(2Bab - Ab^2)x^3 + 3Ba^2 - Aab}{6(b^5x^6 + 2ab^4x^3 + a^2b^3)} + \frac{B \log(bx^3 + a)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")`

[Out] $\frac{1}{6}*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3) + \frac{1}{3}*B*\log(b*x^3 + a)/b^3$

mupad [B] time = 2.38, size = 70, normalized size = 1.06

$$\frac{\frac{3Ba^2 - Aab}{6b^3} - \frac{x^3(Ab - 2Ba)}{3b^2}}{a^2 + 2abx^3 + b^2x^6} + \frac{B \ln(bx^3 + a)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(A + B*x^3))/(a + b*x^3)^3,x)`

[Out] $((3Ba^2 - Aab)/(6b^3) - (x^3(Ab - 2Ba))/(3b^2))/(a^2 + b^2x^6 + 2abx^3) + (B\log(a + bx^3))/(3b^3)$

sympy [A] time = 3.69, size = 70, normalized size = 1.06

$$\frac{B \log(a + bx^3)}{3b^3} + \frac{-Aab + 3Ba^2 + x^3(-2Ab^2 + 4Bab)}{6a^2b^3 + 12ab^4x^3 + 6b^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(B*x**3+A)/(b*x**3+a)**3,x)`

[Out] $B\log(a + bx^3)/(3b^3) + (-Aab + 3Ba^2 + x^3(-2Ab^2 + 4Bab))/(6a^2b^3 + 12ab^4x^3 + 6b^5x^6)$

$$3.91 \quad \int \frac{x^2(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=32

$$-\frac{(A+Bx^3)^2}{6(a+bx^3)^2(Ab-aB)}$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {444, 37}

$$-\frac{(A+Bx^3)^2}{6(a+bx^3)^2(Ab-aB)}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] -(A + B*x^3)^2/(6*(A*b - a*B)*(a + b*x^3)^2)

Rule 37

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\int \frac{x^2 (A + Bx^3)}{(a + bx^3)^3} dx = \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{(a + bx)^3} dx, x, x^3 \right)$$

$$= -\frac{(A + Bx^3)^2}{6(Ab - aB)(a + bx^3)^2}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.94

$$\frac{B(a + 2bx^3) + Ab}{6b^2(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] -1/6*(A*b + B*(a + 2*b*x^3))/(b^2*(a + b*x^3)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (A + Bx^3)}{(a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3)^3, x]

fricas [A] time = 0.69, size = 42, normalized size = 1.31

$$-\frac{2Bbx^3 + Ba + Ab}{6(b^4x^6 + 2ab^3x^3 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)

giac [A] time = 0.18, size = 28, normalized size = 0.88

$$\frac{2Bbx^3 + Ba + Ab}{6(bx^3 + a)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")

[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/((b*x^3 + a)^2*b^2)

maple [A] time = 0.05, size = 39, normalized size = 1.22

$$-\frac{B}{3(bx^3 + a)b^2} - \frac{Ab - Ba}{6(bx^3 + a)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(B*x^3+A)/(b*x^3+a)^3,x)

[Out] -1/6*(A*b-B*a)/b^2/(b*x^3+a)^2-1/3*B/b^2/(b*x^3+a)

maxima [A] time = 0.46, size = 42, normalized size = 1.31

$$\frac{2Bbx^3 + Ba + Ab}{6(b^4x^6 + 2ab^3x^3 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)

mupad [B] time = 2.33, size = 44, normalized size = 1.38

$$-\frac{\frac{Ab+Ba}{6b^2} + \frac{Bx^3}{3b}}{a^2 + 2abx^3 + b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(A + B*x^3))/(a + b*x^3)^3,x)

[Out] -((A*b + B*a)/(6*b^2) + (B*x^3)/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)

sympy [A] time = 1.82, size = 42, normalized size = 1.31

$$\frac{-Ab - Ba - 2Bbx^3}{6a^2b^2 + 12ab^3x^3 + 6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**3,x)

[Out] (-A*b - B*a - 2*B*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)

$$3.92 \quad \int \frac{A+Bx^3}{x(a+bx^3)^3} dx$$

Optimal. Leaf size=68

$$-\frac{A \log(a+bx^3)}{3a^3} + \frac{A \log(x)}{a^3} + \frac{A}{3a^2(a+bx^3)} + \frac{Ab-aB}{6ab(a+bx^3)^2}$$

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 77}

$$\frac{A}{3a^2(a+bx^3)} - \frac{A \log(a+bx^3)}{3a^3} + \frac{A \log(x)}{a^3} + \frac{Ab-aB}{6ab(a+bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x*(a + b*x^3)^3), x]

[Out] (A*b - a*B)/(6*a*b*(a + b*x^3)^2) + A/(3*a^2*(a + b*x^3)) + (A*Log[x])/a^3 - (A*Log[a + b*x^3])/(3*a^3)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x(a + bx^3)^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x(a + bx)^3} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{A}{a^3x} + \frac{-Ab + aB}{a(a + bx)^3} - \frac{Ab}{a^2(a + bx)^2} - \frac{Ab}{a^3(a + bx)} \right) dx, x, x^3 \right) \\
&= \frac{Ab - aB}{6ab(a + bx^3)^2} + \frac{A}{3a^2(a + bx^3)} + \frac{A \log(x)}{a^3} - \frac{A \log(a + bx^3)}{3a^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.87

$$\frac{\frac{a(-a^2B + 3aAb + 2Ab^2x^3)}{b(a + bx^3)^2} - 2A \log(a + bx^3) + 6A \log(x)}{6a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^3), x]

[Out] ((a*(3*a*A*b - a^2*B + 2*A*b^2*x^3))/(b*(a + b*x^3)^2) + 6*A*Log[x] - 2*A*Log[a + b*x^3])/(6*a^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x(a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)^3), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)^3), x]

fricas [A] time = 0.75, size = 119, normalized size = 1.75

$$\frac{2Ab^2x^3 - Ba^3 + 3Aa^2b - 2(Ab^3x^6 + 2Aab^2x^3 + Aa^2b) \log(bx^3 + a) + 6(Ab^3x^6 + 2Aab^2x^3 + Aa^2b) \log(x)}{6(a^3b^3x^6 + 2a^4b^2x^3 + a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{6}*(2*A*a*b^2*x^3 - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*\log(b*x^3 + a) + 6*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*\log(x))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)$

giac [A] time = 0.20, size = 74, normalized size = 1.09

$$-\frac{A \log(|bx^3 + a|)}{3a^3} + \frac{A \log(|x|)}{a^3} + \frac{3Ab^3x^6 + 8Aab^2x^3 - Ba^3 + 6Aa^2b}{6(bx^3 + a)^2 a^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="giac")`

[Out] $-\frac{1}{3}*A*\log(\text{abs}(b*x^3 + a))/a^3 + A*\log(\text{abs}(x))/a^3 + \frac{1}{6}*(3*A*b^3*x^6 + 8*A*a*b^2*x^3 - B*a^3 + 6*A*a^2*b)/((b*x^3 + a)^2*a^3*b)$

maple [A] time = 0.06, size = 68, normalized size = 1.00

$$\frac{A}{6(bx^3 + a)^2 a} - \frac{B}{6(bx^3 + a)^2 b} + \frac{A}{3(bx^3 + a)a^2} + \frac{A \ln(x)}{a^3} - \frac{A \ln(bx^3 + a)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^3+A)/x/(b*x^3+a)^3,x)`

[Out] $\frac{1}{6}/a/(b*x^3+a)^2*A - \frac{1}{6}/b/(b*x^3+a)^2*B - \frac{1}{3}*A*\ln(b*x^3+a)/a^3 + \frac{1}{3}*A/a^2/(b*x^3+a) + A/a^3*\ln(x)$

maxima [A] time = 0.45, size = 77, normalized size = 1.13

$$\frac{2Ab^2x^3 - Ba^2 + 3Aab}{6(a^2b^3x^6 + 2a^3b^2x^3 + a^4b)} - \frac{A \log(bx^3 + a)}{3a^3} + \frac{A \log(x^3)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="maxima")`

[Out] $\frac{1}{6}*(2*A*b^2*x^3 - B*a^2 + 3*A*a*b)/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) - \frac{1}{3}*A*\log(b*x^3 + a)/a^3 + \frac{1}{3}*A*\log(x^3)/a^3$

mupad [B] time = 0.16, size = 71, normalized size = 1.04

$$\frac{\frac{3Ab-Ba}{6ab} + \frac{Abx^3}{3a^2}}{a^2 + 2abx^3 + b^2x^6} - \frac{A \ln(bx^3 + a)}{3a^3} + \frac{A \ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)/(x*(a + b*x^3)^3),x)`

[Out] $((3Ab - Ba)/(6ab) + (Abx^3)/(3a^2))/(a^2 + b^2x^6 + 2abx^3) - (A \log(a + bx^3))/(3a^3) + (A \log(x))/a^3$

sympy [A] time = 1.48, size = 75, normalized size = 1.10

$$\frac{A \log(x)}{a^3} - \frac{A \log\left(\frac{a}{b} + x^3\right)}{3a^3} + \frac{3Aab + 2Ab^2x^3 - Ba^2}{6a^4b + 12a^3b^2x^3 + 6a^2b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)/x/(b*x**3+a)**3,x)`

[Out] $A \log(x)/a^3 - A \log(a/b + x^3)/(3a^3) + (3Aab + 2Ab^2x^3 - Ba^2)/(6a^4b + 12a^3b^2x^3 + 6a^2b^3x^6)$

$$3.93 \quad \int \frac{A+Bx^3}{x^4(a+bx^3)^3} dx$$

Optimal. Leaf size=101

$$\frac{(3Ab - aB) \log(a + bx^3)}{3a^4} - \frac{\log(x)(3Ab - aB)}{a^4} - \frac{2Ab - aB}{3a^3(a + bx^3)} - \frac{A}{3a^3x^3} - \frac{Ab - aB}{6a^2(a + bx^3)^2}$$

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 77}

$$-\frac{2Ab - aB}{3a^3(a + bx^3)} - \frac{Ab - aB}{6a^2(a + bx^3)^2} + \frac{(3Ab - aB) \log(a + bx^3)}{3a^4} - \frac{\log(x)(3Ab - aB)}{a^4} - \frac{A}{3a^3x^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^3), x]

[Out] -A/(3*a^3*x^3) - (A*b - a*B)/(6*a^2*(a + b*x^3)^2) - (2*A*b - a*B)/(3*a^3*(a + b*x^3)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^3])/(3*a^4)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^3}{x^4 (a + bx^3)^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x^2 (a + bx)^3} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{A}{a^3 x^2} + \frac{-3Ab + aB}{a^4 x} - \frac{b(-Ab + aB)}{a^2 (a + bx)^3} - \frac{b(-2Ab + aB)}{a^3 (a + bx)^2} - \frac{b(-3Ab + aB)}{a^4 (a + bx)} \right) dx, \right. \\ &= -\frac{A}{3a^3 x^3} - \frac{Ab - aB}{6a^2 (a + bx^3)^2} - \frac{2Ab - aB}{3a^3 (a + bx^3)} - \frac{(3Ab - aB) \log(x)}{a^4} + \frac{(3Ab - aB) \log(a + bx^3)}{3a^4} \end{aligned}$$

Mathematica [A] time = 0.07, size = 87, normalized size = 0.86

$$\frac{\frac{a^2(aB-Ab)}{(a+bx^3)^2} + \frac{2a(aB-2Ab)}{a+bx^3} + 2(3Ab - aB) \log(a + bx^3) + 6 \log(x)(aB - 3Ab) - \frac{2aA}{x^3}}{6a^4}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^3), x]

[Out] ((-2*a*A)/x^3 + (a^2*(-(A*b) + a*B))/(a + b*x^3)^2 + (2*a*(-2*A*b + a*B))/(a + b*x^3) + 6*(-3*A*b + a*B)*Log[x] + 2*(3*A*b - a*B)*Log[a + b*x^3])/(6*a^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^4 (a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)^3), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)^3), x]

fricas [B] time = 0.62, size = 197, normalized size = 1.95

$$\frac{2(Ba^2b - 3Aab^2)x^6 - 2Aa^3 + 3(Ba^3 - 3Aa^2b)x^3 - 2((Bab^2 - 3Ab^3)x^9 + 2(Ba^2b - 3Aab^2)x^6 + (Ba^3 - 3Aa^2b)x^3) \log(bx^3 + a) + 6((Bab^2 - 3Ab^3)x^9 + 2(Ba^2b - 3Aab^2)x^6 + (Ba^3 - 3Aa^2b)x^3) \log(x)}{6(a^4b^2x^9 + 2a^5bx^6 + a^6x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{6}*(2*(B*a^2*b - 3*A*a*b^2)*x^6 - 2*A*a^3 + 3*(B*a^3 - 3*A*a^2*b)*x^3 - 2*((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*b^2)*x^6 + (B*a^3 - 3*A*a^2*b)*x^3)*\log(b*x^3 + a) + 6*((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*b^2)*x^6 + (B*a^3 - 3*A*a^2*b)*x^3)*\log(x)/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)$

giac [A] time = 0.20, size = 136, normalized size = 1.35

$$\frac{(Ba - 3Ab)\log(|x|)}{a^4} - \frac{(Bab - 3Ab^2)\log(|bx^3 + a|)}{3a^4b} + \frac{3Bab^2x^6 - 9Ab^3x^6 + 8Ba^2bx^3 - 22Aab^2x^3 + 6Ba^3 - 14Aa^2b}{6(bx^3 + a)^2a^4} - \frac{Bax^3 - 3Abx^3 + Aa}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="giac")

[Out] $(B*a - 3*A*b)*\log(\text{abs}(x))/a^4 - 1/3*(B*a*b - 3*A*b^2)*\log(\text{abs}(b*x^3 + a))/(a^4*b) + 1/6*(3*B*a*b^2*x^6 - 9*A*b^3*x^6 + 8*B*a^2*b*x^3 - 22*A*a*b^2*x^3 + 6*B*a^3 - 14*A*a^2*b)/((b*x^3 + a)^2*a^4) - 1/3*(B*a*x^3 - 3*A*b*x^3 + A*a)/(a^4*x^3)$

maple [A] time = 0.06, size = 117, normalized size = 1.16

$$-\frac{Ab}{6(bx^3 + a)^2a^2} + \frac{B}{6(bx^3 + a)^2a} - \frac{2Ab}{3(bx^3 + a)a^3} - \frac{3Ab\ln(x)}{a^4} + \frac{Ab\ln(bx^3 + a)}{a^4} + \frac{B}{3(bx^3 + a)a^2} + \frac{B\ln(x)}{a^3} - \frac{B\ln(bx^3 + a)}{3a^3} - \frac{A}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^4/(b*x^3+a)^3,x)

[Out] $-1/6/a^2*b/(b*x^3+a)^2*A + 1/6/a/(b*x^3+a)^2*B + 1/a^4*b*\ln(b*x^3+a)*A - 1/3/a^3*\ln(b*x^3+a)*B - 2/3/a^3*b/(b*x^3+a)*A + 1/3/a^2/(b*x^3+a)*B - 1/3*A/a^3/x^3 - 3*A/a^4*b*\ln(x) + B/a^3*\ln(x)$

maxima [A] time = 0.50, size = 109, normalized size = 1.08

$$\frac{2(Bab - 3Ab^2)x^6 + 3(Ba^2 - 3Aab)x^3 - 2Aa^2}{6(a^3b^2x^9 + 2a^4bx^6 + a^5x^3)} - \frac{(Ba - 3Ab)\log(bx^3 + a)}{3a^4} + \frac{(Ba - 3Ab)\log(x^3)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="maxima")

[Out] $1/6*(2*(B*a*b - 3*A*b^2)*x^6 + 3*(B*a^2 - 3*A*a*b)*x^3 - 2*A*a^2)/(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3) - 1/3*(B*a - 3*A*b)*\log(b*x^3 + a)/a^4 + 1/3*(B*a - 3*A*b)*\log(x^3)/a^4$

mupad [B] time = 2.46, size = 107, normalized size = 1.06

$$\frac{\ln(bx^3 + a)(3Ab - Ba)}{3a^4} - \frac{\frac{A}{3a} + \frac{x^3(3Ab - Ba)}{2a^2} + \frac{bx^6(3Ab - Ba)}{3a^3}}{a^2x^3 + 2abx^6 + b^2x^9} - \frac{\ln(x)(3Ab - Ba)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^4*(a + b*x^3)^3), x)

[Out] (log(a + b*x^3)*(3*A*b - B*a))/(3*a^4) - (A/(3*a) + (x^3*(3*A*b - B*a))/(2*a^2) + (b*x^6*(3*A*b - B*a))/(3*a^3))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6) - (log(x)*(3*A*b - B*a))/a^4

sympy [A] time = 3.27, size = 107, normalized size = 1.06

$$\frac{-2Aa^2 + x^6(-6Ab^2 + 2Bab) + x^3(-9Aab + 3Ba^2)}{6a^5x^3 + 12a^4bx^6 + 6a^3b^2x^9} + \frac{(-3Ab + Ba)\log(x)}{a^4} - \frac{(-3Ab + Ba)\log\left(\frac{a}{b} + x^3\right)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**3, x)

[Out] (-2*A*a**2 + x**6*(-6*A*b**2 + 2*B*a*b) + x**3*(-9*A*a*b + 3*B*a**2))/(6*a**5*x**3 + 12*a**4*b*x**6 + 6*a**3*b**2*x**9) + (-3*A*b + B*a)*log(x)/a**4 - (-3*A*b + B*a)*log(a/b + x**3)/(3*a**4)

$$3.94 \quad \int \frac{A+Bx^3}{x^7(a+bx^3)^3} dx$$

Optimal. Leaf size=122

$$-\frac{b(2Ab - aB) \log(a + bx^3)}{a^5} + \frac{3b \log(x)(2Ab - aB)}{a^5} + \frac{b(3Ab - 2aB)}{3a^4(a + bx^3)} + \frac{3Ab - aB}{3a^4x^3} + \frac{b(Ab - aB)}{6a^3(a + bx^3)^2} - \frac{A}{6a^3x^6}$$

Rubi [A] time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 77}

$$\frac{b(3Ab - 2aB)}{3a^4(a + bx^3)} + \frac{b(Ab - aB)}{6a^3(a + bx^3)^2} + \frac{3Ab - aB}{3a^4x^3} - \frac{b(2Ab - aB) \log(a + bx^3)}{a^5} + \frac{3b \log(x)(2Ab - aB)}{a^5} - \frac{A}{6a^3x^6}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^3), x]

[Out] -A/(6*a^3*x^6) + (3*A*b - a*B)/(3*a^4*x^3) + (b*(A*b - a*B))/(6*a^3*(a + b*x^3)^2) + (b*(3*A*b - 2*a*B))/(3*a^4*(a + b*x^3)) + (3*b*(2*A*b - a*B)*Log[x])/a^5 - (b*(2*A*b - a*B)*Log[a + b*x^3])/a^5

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^3}{x^7 (a + bx^3)^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x^3 (a + bx)^3} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{A}{a^3 x^3} + \frac{-3Ab + aB}{a^4 x^2} - \frac{3b(-2Ab + aB)}{a^5 x} + \frac{b^2(-Ab + aB)}{a^3 (a + bx)^3} + \frac{b^2(-3Ab + 2aB)}{a^4 (a + bx)^2} \right) dx, x, x^3 \right) \\ &= -\frac{A}{6a^3 x^6} + \frac{3Ab - aB}{3a^4 x^3} + \frac{b(Ab - aB)}{6a^3 (a + bx^3)^2} + \frac{b(3Ab - 2aB)}{3a^4 (a + bx^3)} + \frac{3b(2Ab - aB) \log(x)}{a^5} - \frac{b(2Ab - aB)}{a^4 (a + bx^3)} \end{aligned}$$

Mathematica [A] time = 0.11, size = 108, normalized size = 0.89

$$\frac{\frac{a^2 b (Ab - aB)}{(a + bx^3)^2} - \frac{a^2 A}{x^6} + \frac{2ab(3Ab - 2aB)}{a + bx^3} - \frac{2a(aB - 3Ab)}{x^3} + 6b(aB - 2Ab) \log(a + bx^3) + 18b \log(x)(2Ab - aB)}{6a^5}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^3), x]

[Out] $-\left(\frac{a^2 A}{x^6}\right) - \frac{(2a(-3Ab + aB))}{x^3} + \frac{(a^2 b(Ab - aB))}{(a + b x^3)^2} + \frac{(2a b(3Ab - 2aB))}{(a + b x^3)} + 18 b(2Ab - aB) \log(x) + 6 b(-2Ab + aB) \log(a + b x^3) / (6 a^5)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^7 (a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^7*(a + b*x^3)^3), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^7*(a + b*x^3)^3), x]

fricas [B] time = 0.64, size = 229, normalized size = 1.88

$$\frac{6(Ba^2b^2 - 2Aab^2)x^9 + 9(Ba^2b - 2Aa^2b^2)x^6 + Aa^4 + 2(Ba^4 - 2Aa^3b)x^3 - 6((Bab^3 - 2Ab^4)x^{12} + 2(Ba^2b^2 - 2Aab^3)x^9 + (Ba^3b - 2Aa^2b^2)x^6) \log(bx^3 + a) + 18((Bab^3 - 2Ab^4)x^{12} + 2(Ba^2b^2 - 2Aab^3)x^9 + (Ba^3b - 2Aa^2b^2)x^6) \log(x)}{6(a^2b^2x^{12} + 2a^6bx^9 + a^7x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $-1/6*(6*(B*a^2*b^2 - 2*A*a*b^3))*x^9 + 9*(B*a^3*b - 2*A*a^2*b^2)*x^6 + A*a^4 + 2*(B*a^4 - 2*A*a^3*b)*x^3 - 6*((B*a*b^3 - 2*A*b^4))*x^{12} + 2*(B*a^2*b^2 -$

$$2* A * a * b^3 * x^9 + (B * a^3 * b - 2 * A * a^2 * b^2) * x^6 * \log(b * x^3 + a) + 18 * ((B * a * b^3 - 2 * A * b^4) * x^{12} + 2 * (B * a^2 * b^2 - 2 * A * a * b^3) * x^9 + (B * a^3 * b - 2 * A * a^2 * b^2) * x^6) * \log(x) / (a^5 * b^2 * x^{12} + 2 * a^6 * b * x^9 + a^7 * x^6)$$

giac [A] time = 0.20, size = 131, normalized size = 1.07

$$\frac{3(Bab - 2Ab^2)\log(|x|)}{a^5} + \frac{(Bab^2 - 2Ab^3)\log(|bx^3 + a|)}{a^5b} - \frac{6Bab^2x^9 - 12Ab^3x^9 + 9Ba^2bx^6 - 18Aab^2x^6 + 2Ba^3x^3 - 4Aa^2bx^3 + Aa^3}{6(bx^6 + ax^3)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="giac")

$$[Out] -3*(B*a*b - 2*A*b^2)*\log(\text{abs}(x))/a^5 + (B*a*b^2 - 2*A*b^3)*\log(\text{abs}(b*x^3 + a))/(a^5*b) - 1/6*(6*B*a*b^2*x^9 - 12*A*b^3*x^9 + 9*B*a^2*b*x^6 - 18*A*a*b^2*x^6 + 2*B*a^3*x^3 - 4*A*a^2*b*x^3 + A*a^3)/((b*x^6 + a*x^3)^2*a^4)$$

maple [A] time = 0.06, size = 147, normalized size = 1.20

$$\frac{Ab^2}{6(bx^3+a)^2a^3} - \frac{Bb}{6(bx^3+a)^2a^2} + \frac{Ab^2}{(bx^3+a)a^4} + \frac{6Ab^2\ln(x)}{a^5} - \frac{2Ab^2\ln(bx^3+a)}{a^5} - \frac{2Bb}{3(bx^3+a)a^3} - \frac{3Bb\ln(x)}{a^4} + \frac{Bb\ln(bx^3+a)}{a^4} + \frac{Ab}{a^4x^3} - \frac{B}{3a^3x^3} - \frac{A}{6a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^7/(b*x^3+a)^3,x)

$$[Out] 1/6/a^3*b^2/(b*x^3+a)^2*A - 1/6/a^2*b/(b*x^3+a)^2*B - 2/a^5*b^2*\ln(b*x^3+a)*A + 1/a^4*b*\ln(b*x^3+a)*B + 1/a^4*b^2/(b*x^3+a)*A - 2/3/a^3*b/(b*x^3+a)*B - 1/6*A/a^3/x^6 + 1/a^4/x^3*A*b - 1/3/a^3/x^3*B + 6*b^2/a^5*\ln(x)*A - 3*b/a^4*\ln(x)*B$$

maxima [A] time = 0.52, size = 136, normalized size = 1.11

$$-\frac{6(Bab^2 - 2Ab^3)x^9 + 9(Ba^2b - 2Aab^2)x^6 + Aa^3 + 2(Ba^3 - 2Aa^2b)x^3}{6(a^4b^2x^{12} + 2a^5bx^9 + a^6x^6)} + \frac{(Bab - 2Ab^2)\log(bx^3 + a)}{a^5} - \frac{(Bab - 2Ab^2)\log(x^3)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="maxima")

$$[Out] -1/6*(6*(B*a*b^2 - 2*A*b^3)*x^9 + 9*(B*a^2*b - 2*A*a*b^2)*x^6 + A*a^3 + 2*(B*a^3 - 2*A*a^2*b)*x^3)/(a^4*b^2*x^{12} + 2*a^5*b*x^9 + a^6*x^6) + (B*a*b - 2*A*b^2)*\log(b*x^3 + a)/a^5 - (B*a*b - 2*A*b^2)*\log(x^3)/a^5$$

mupad [B] time = 0.15, size = 130, normalized size = 1.07

$$\frac{x^3(2Ab - Ba)}{3a^2} - \frac{A}{6a} + \frac{b^2x^9(2Ab - Ba)}{a^4} + \frac{3bx^6(2Ab - Ba)}{2a^3} - \frac{\ln(bx^3 + a)(2Ab^2 - Bab)}{a^5} + \frac{\ln(x)(6Ab^2 - 3Bab)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)/(x^7*(a + b*x^3)^3), x)`

[Out] $((x^3(2Ab - Ba))/(3a^2) - A/(6a) + (b^2x^9(2Ab - Ba))/a^4 + (3bx^6(2Ab - Ba))/(2a^3))/(a^2x^6 + b^2x^{12} + 2abx^9) - (\log(a + bx^3)(2Ab^2 - Babb))/a^5 + (\log(x)(6Ab^2 - 3Babb))/a^5$

sympy [A] time = 3.55, size = 133, normalized size = 1.09

$$\frac{-Aa^3 + x^9(12Ab^3 - 6Bab^2) + x^6(18Aab^2 - 9Ba^2b) + x^3(4Aa^2b - 2Ba^3)}{6a^6x^6 + 12a^5bx^9 + 6a^4b^2x^{12}} - \frac{3b(-2Ab + Ba)\log(x)}{a^5} + \frac{b(-2Ab + Ba)\log\left(\frac{a}{b} + x^3\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)/x**7/(b*x**3+a)**3, x)`

[Out] $(-Aa^3 + x^9(12Ab^3 - 6Babb^2) + x^6(18Aa^2b - 9Baa^2b) + x^3(4Aa^2b - 2Ba^3))/(6a^6x^6 + 12a^5bx^9 + 6a^4b^2x^{12}) - 3b(-2Ab + Ba)\log(x)/a^5 + b(-2Ab + Ba)\log(a/b + x^3)/a^5$

$$3.95 \quad \int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=246

$$-\frac{2a^{2/3}(5Ab-11aB)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{27b^{14/3}} + \frac{4a^{2/3}(5Ab-11aB)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27b^{14/3}} + \frac{4a^{2/3}(5Ab-11aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{14/3}}$$

Rubi [A] time = 0.16, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, number of rules / integrand size = 0.450, Rules used = {457, 288, 302, 292, 31, 634, 617, 204, 628}

$$\frac{2a^{2/3}(5Ab-11aB)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{27b^{14/3}} + \frac{4a^{2/3}(5Ab-11aB)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27b^{14/3}} + \frac{4a^{2/3}(5Ab-11aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{14/3}} + \frac{x^8(5Ab-11aB)}{18ab^2(a+bx^3)} - \frac{4x^5(5Ab-11aB)}{45ab^3} + \frac{2x^2(5Ab-11aB)}{9b^4} + \frac{x^{11}(Ab-aB)}{6ab(a+bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] (2*(5*A*b - 11*a*B)*x^2)/(9*b^4) - (4*(5*A*b - 11*a*B)*x^5)/(45*a*b^3) + ((A*b - a*B)*x^11)/(6*a*b*(a + b*x^3)^2) + ((5*A*b - 11*a*B)*x^8)/(18*a*b^2*(a + b*x^3)) + (4*a^(2/3)*(5*A*b - 11*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*b^(14/3)) + (4*a^(2/3)*(5*A*b - 11*a*B)*Log[a^(1/3) + b^(1/3)*x]/(27*b^(14/3)) - (2*a^(2/3)*(5*A*b - 11*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(27*b^(14/3)))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a + b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !I

LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 457

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ((!IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{x^{10} (A + Bx^3)}{(a + bx^3)^3} dx &= \frac{(Ab - aB)x^{11}}{6ab(a + bx^3)^2} + \frac{(-5Ab + 11aB) \int \frac{x^{10}}{(a+bx^3)^2} dx}{6ab} \\
&= \frac{(Ab - aB)x^{11}}{6ab(a + bx^3)^2} + \frac{(5Ab - 11aB)x^8}{18ab^2(a + bx^3)} - \frac{(4(5Ab - 11aB)) \int \frac{x^7}{a+bx^3} dx}{9ab^2} \\
&= \frac{(Ab - aB)x^{11}}{6ab(a + bx^3)^2} + \frac{(5Ab - 11aB)x^8}{18ab^2(a + bx^3)} - \frac{(4(5Ab - 11aB)) \int \left(-\frac{ax}{b^2} + \frac{x^4}{b} + \frac{a^2x}{b^2(a+bx^3)} \right) dx}{9ab^2} \\
&= \frac{2(5Ab - 11aB)x^2}{9b^4} - \frac{4(5Ab - 11aB)x^5}{45ab^3} + \frac{(Ab - aB)x^{11}}{6ab(a + bx^3)^2} + \frac{(5Ab - 11aB)x^8}{18ab^2(a + bx^3)} - \frac{(4a(5Ab - 11aB))}{9ab^2} \\
&= \frac{2(5Ab - 11aB)x^2}{9b^4} - \frac{4(5Ab - 11aB)x^5}{45ab^3} + \frac{(Ab - aB)x^{11}}{6ab(a + bx^3)^2} + \frac{(5Ab - 11aB)x^8}{18ab^2(a + bx^3)} + \frac{(4a^{2/3}(5Ab - 11aB))}{9ab^2} \\
&= \frac{2(5Ab - 11aB)x^2}{9b^4} - \frac{4(5Ab - 11aB)x^5}{45ab^3} + \frac{(Ab - aB)x^{11}}{6ab(a + bx^3)^2} + \frac{(5Ab - 11aB)x^8}{18ab^2(a + bx^3)} + \frac{4a^{2/3}(5Ab - 11aB)}{9ab^2} \\
&= \frac{2(5Ab - 11aB)x^2}{9b^4} - \frac{4(5Ab - 11aB)x^5}{45ab^3} + \frac{(Ab - aB)x^{11}}{6ab(a + bx^3)^2} + \frac{(5Ab - 11aB)x^8}{18ab^2(a + bx^3)} + \frac{4a^{2/3}(5Ab - 11aB)}{9ab^2} \\
&= \frac{2(5Ab - 11aB)x^2}{9b^4} - \frac{4(5Ab - 11aB)x^5}{45ab^3} + \frac{(Ab - aB)x^{11}}{6ab(a + bx^3)^2} + \frac{(5Ab - 11aB)x^8}{18ab^2(a + bx^3)} + \frac{4a^{2/3}(5Ab - 11aB)}{9ab^2}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 216, normalized size = 0.88

$$\frac{20a^{2/3}(11aB - 5Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2) - 40a^{2/3}(11aB - 5Ab) \log(\sqrt[3]{a} + \sqrt[3]{b} x) - 40\sqrt{3} a^{2/3}(11aB - 5Ab) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b} x}{\sqrt[3]{a}}\right) + \frac{45a^2 b^{2/3} x^2 (aB - Ab)}{(a + bx^3)^2} + 135b^{2/3} x^2 (Ab - 3aB) + \frac{30a^{2/3} x^2 (7Ab - 10aB)}{a + bx^3} + 54b^{5/3} Bx^5}{270b^{14/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^10*(A + B*x^3))/(a + b*x^3)^3, x]

[Out] $(135*b^{(2/3)}*(A*b - 3*a*B)*x^2 + 54*b^{(5/3)}*B*x^5 + (45*a^2*b^{(2/3)}*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (30*a*b^{(2/3)}*(7*A*b - 10*a*B)*x^2)/(a + b*x^3) - 40*\text{Sqrt}[3]*a^{(2/3)}*(-5*A*b + 11*a*B)*\text{ArcTan}[(1 - (2*b^{(1/3)}*x)/a^{(1/3)})/\text{Sqrt}[3]] - 40*a^{(2/3)}*(-5*A*b + 11*a*B)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x] + 20*a^{(2/3)}*(-5*A*b + 11*a*B)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(270*b^{(14/3)})$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10} (A + Bx^3)}{(a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^10*(A + B*x^3))/(a + b*x^3)^3, x]

[Out] IntegrateAlgebraic[(x^10*(A + B*x^3))/(a + b*x^3)^3, x]

fricas [A] time = 0.65, size = 364, normalized size = 1.48

$$\frac{54 B b^3 x^{11} - 27 (11 B a^2 - 5 A b^2) x^8 - 96 (11 B a^2 b - 5 A a b^2) x^5 - 60 (11 B a^3 - 5 A a^2 b) x^2 + 40 \sqrt{3} ((11 B a^2 b^2 - 5 A a^2 b^2) x^6 + 11 B a^3 - 5 A a^2 b + 2 (11 B a^2 b - 5 A a b^2) x^3) \arctan\left(\frac{x \sqrt{3} (a^{1/3} + b^{1/3} x)}{a^{1/3} + b^{1/3} x}\right) + 20 ((11 B a^2 b^2 - 5 A a^2 b^2) x^6 + 11 B a^3 - 5 A a^2 b + 2 (11 B a^2 b - 5 A a b^2) x^3) \log\left(x^2 - b x (a^{2/3} + b^{2/3} x) + a (a^{2/3} + b^{2/3} x)\right) - 40 ((11 B a^2 b^2 - 5 A a^2 b^2) x^6 + 11 B a^3 - 5 A a^2 b + 2 (11 B a^2 b - 5 A a b^2) x^3) \log\left(x + a^{1/3}\right)}{270 (b^3 x^3 + a^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $1/270*(54*B*b^3*x^{11} - 27*(11*B*a*b^2 - 5*A*b^3)*x^8 - 96*(11*B*a^2*b - 5*A*a*b^2)*x^5 - 60*(11*B*a^3 - 5*A*a^2*b)*x^2 + 40*\text{sqrt}(3)*((11*B*a*b^2 - 5*A*a*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)*x^3)*(a^2/b^2)^{(1/3)}*\text{arctan}(1/3*(2*\text{sqrt}(3)*b*x*(a^2/b^2)^{(1/3)} - \text{sqrt}(3)*a)/a) + 20*((11*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)*x^3)*(a^2/b^2)^{(1/3)}*\text{log}(a*x^2 - b*x*(a^2/b^2)^{(2/3)} + a*(a^2/b^2)^{(1/3)}) - 40*((11*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)*x^3)*(a^2/b^2)^{(1/3)}*\text{log}(a*x + b*(a^2/b^2)^{(2/3)}))/ (b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)$

giac [A] time = 0.33, size = 259, normalized size = 1.05

$$\frac{4 \left(11 B a^2 \left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5 A a b \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) + 4 \sqrt{3} \left(11 \left(-a b^2\right)^{\frac{2}{3}} B a - 5 \left(-a b^2\right)^{\frac{2}{3}} A b\right) \arctan\left(\frac{\sqrt{3} \left(2 x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 2 \left(11 \left(-a b^2\right)^{\frac{2}{3}} B a - 5 \left(-a b^2\right)^{\frac{2}{3}} A b\right) \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27 a b^4} + \frac{20 B a^2 b x^5 - 14 A a b^2 x^5 + 17 B a^3 x^2 - 11 A a^2 b x^2 + 2 B b^3 x^2 - 15 B a b^3 x^2 + 5 A b^3 x^2}{18 (b x^3 + a)^3 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $-4/27*(11*B*a^2*(-a/b)^{(1/3)} - 5*A*a*b*(-a/b)^{(1/3)}*(-a/b)^{(1/3)}*\text{log}(\text{abs}(x - (-a/b)^{(1/3)})))/(a*b^4) - 4/27*\text{sqrt}(3)*(11*(-a*b^2)^{(2/3)}*B*a - 5*(-a*b^2$

$$\begin{aligned} &)^{(2/3)} * A * b) * \arctan(1/3 * \sqrt{3} * (2 * x + (-a/b)^{(1/3)}) / (-a/b)^{(1/3)}) / b^6 + 2/ \\ &27 * (11 * (-a * b^2)^{(2/3)} * B * a - 5 * (-a * b^2)^{(2/3)} * A * b) * \log(x^2 + x * (-a/b)^{(1/3)} \\ &+ (-a/b)^{(2/3)}) / b^6 - 1/18 * (20 * B * a^2 * b * x^5 - 14 * A * a * b^2 * x^5 + 17 * B * a^3 * x^2 \\ &- 11 * A * a^2 * b * x^2) / ((b * x^3 + a)^2 * b^4) + 1/10 * (2 * B * b^12 * x^5 - 15 * B * a * b^11 * x^ \\ &2 + 5 * A * b^12 * x^2) / b^15 \end{aligned}$$

maple [A] time = 0.06, size = 308, normalized size = 1.25

$$\frac{7Ax^5}{9(bx^3+a)^2b^2} - \frac{10B a^2 x^5}{9(bx^3+a)^2b^5} + \frac{Bx^5}{5b^5} + \frac{11A a^2 x^2}{18(bx^3+a)^2b^3} - \frac{17B a^3 x^2}{18(bx^3+a)^2b^4} + \frac{Ax^2}{2b^3} - \frac{3Ba x^2}{2b^4} - \frac{20\sqrt{3} A b \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{b} - 1\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4} + \frac{20A b \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4} - \frac{10A b \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4} + \frac{44\sqrt{3} B a^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{b} - 1\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^5} - \frac{44B a^2 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^5} + \frac{22B a^2 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^10*(B*x^3+A)/(b*x^3+a)^3,x)

[Out] $1/5/b^3*B*x^5 + 1/2/b^3*A*x^2 - 3/2/b^4*B*x^2*a + 7/9*a/b^2/(b*x^3+a)^2*A*x^5 - 10/9*a^2/b^3/(b*x^3+a)^2*B*x^5 + 11/18*a^2/b^3/(b*x^3+a)^2*A*x^2 - 17/18*a^3/b^4/(b*x^3+a)^2*B*x^2 + 20/27*a/b^4*A/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)}) - 10/27*a/b^4*A/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)}) - 20/27*a/b^4*A*B^3^{(1/2)}/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1)) - 44/27*a^2/b^5*B/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)}) + 22/27*a^2/b^5*B/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)}) + 44/27*a^2/b^5*B*B^3^{(1/2)}/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))$

maxima [A] time = 1.37, size = 228, normalized size = 0.93

$$\frac{2(10Ba^2b - 7Aab^2)x^5 + (17Ba^3 - 11Aa^2b)x^2}{18(b^6x^6 + 2ab^5x^3 + a^2b^4)} + \frac{4\sqrt{3}(11Ba^2 - 5Aab)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2Bbx^5 - 5(3Ba - Ab)x^2}{10b^4} + \frac{2(11Ba^2 - 5Aab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4(11Ba^2 - 5Aab)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] $-1/18*(2*(10*B*a^2*b - 7*A*a*b^2)*x^5 + (17*B*a^3 - 11*A*a^2*b)*x^2)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) + 4/27*\sqrt{3}*(11*B*a^2 - 5*A*a*b)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(b^5*(a/b)^{(1/3)}) + 1/10*(2*B*b*x^5 - 5*(3*B*a - A*b)*x^2)/b^4 + 2/27*(11*B*a^2 - 5*A*a*b)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^5*(a/b)^{(1/3)}) - 4/27*(11*B*a^2 - 5*A*a*b)*\log(x + (a/b)^{(1/3)})/(b^5*(a/b)^{(1/3)})$

mupad [B] time = 2.58, size = 213, normalized size = 0.87

$$\frac{x^5\left(\frac{7Aa^2}{9} - \frac{10B a^2 b}{9}\right) - x^2\left(\frac{17Ba^3}{18} - \frac{11Aa^2b}{18}\right)}{a^2b^4 + 2ab^5x^3 + b^6x^6} + x^2\left(\frac{A}{2b^3} - \frac{3Ba}{2b^4}\right) + \frac{Bx^5}{5b^5} + \frac{4a^{2/3}\ln(b^{1/3}x + a^{1/3})(5Ab - 11Ba)}{27b^{14/3}} + \frac{4a^{2/3}\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}b)}{27b^{14/3}}\left(-\frac{1}{2} + \frac{\sqrt{3}b}{2}\right)(5Ab - 11Ba) - \frac{4a^{2/3}\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}b)}{27b^{14/3}}\left(\frac{1}{2} + \frac{\sqrt{3}b}{2}\right)(5Ab - 11Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^10*(A + B*x^3))/(a + b*x^3)^3,x)`

[Out] $(x^5*((7*A*a*b^2)/9 - (10*B*a^2*b)/9) - x^2*((17*B*a^3)/18 - (11*A*a^2*b)/18))/((a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x^2*(A/(2*b^3) - (3*B*a)/(2*b^4)) + (B*x^5)/(5*b^3) + (4*a^{2/3}*\log(b^{1/3}*x + a^{1/3})*(5*A*b - 11*B*a))/(27*b^{14/3}) + (4*a^{2/3}*\log(3^{1/2}*a^{1/3}*1i - 2*b^{1/3}*x + a^{1/3}))*((3^{1/2}*1i)/2 - 1/2)*(5*A*b - 11*B*a))/(27*b^{14/3}) - (4*a^{2/3}*\log(3^{1/2}*a^{1/3}*1i + 2*b^{1/3}*x - a^{1/3}))*((3^{1/2}*1i)/2 + 1/2)*(5*A*b - 11*B*a))/(27*b^{14/3}))$

sympy [A] time = 6.24, size = 192, normalized size = 0.78

$$\frac{Bx^5}{5b^3} + x^2 \left(\frac{A}{2b^3} - \frac{3Ba}{2b^4} \right) + \frac{x^5 (14Aab^2 - 20Ba^2b) + x^2 (11Aa^2b - 17Ba^3)}{18a^2b^4 + 36ab^5x^3 + 18b^6x^6} + \text{RootSum} \left(19683t^3b^{14} - 8000A^3a^2b^3 + 52800A^2Ba^2b^2 - 116160AB^2a^4b + 85184B^3a^5, \left(t \mapsto t \log \left(\frac{729t^2b^9}{400A^2ab^2 - 1760ABa^2b + 1936B^2a^3 + x} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**10*(B*x**3+A)/(b*x**3+a)**3,x)`

[Out] $B*x**5/(5*b**3) + x**2*(A/(2*b**3) - 3*B*a/(2*b**4)) + (x**5*(14*A*a*b**2 - 20*B*a**2*b) + x**2*(11*A*a**2*b - 17*B*a**3))/(18*a**2*b**4 + 36*a*b**5*x**3 + 18*b**6*x**6) + \text{RootSum}(19683*_t**3*b**14 - 8000*A**3*a**2*b**3 + 52800*A**2*B*a**3*b**2 - 116160*A*B**2*a**4*b + 85184*B**3*a**5, \text{Lambda}(_t, _t * \log(729*_t**2*b**9/(400*A**2*a*b**2 - 1760*A*B*a**2*b + 1936*B**2*a**3) + x)))$

$$3.96 \quad \int \frac{x^9(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=244

$$\frac{7\sqrt[3]{a}(2Ab-5aB)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{54b^{13/3}} - \frac{7\sqrt[3]{a}(2Ab-5aB)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27b^{13/3}} + \frac{7\sqrt[3]{a}(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{13/3}}$$

Rubi [A] time = 0.17, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {457, 288, 302, 200, 31, 634, 617, 204, 628}

$$\frac{7\sqrt[3]{a}(2Ab-5aB)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{54b^{13/3}} + \frac{x^2(2Ab-5aB)}{9ab^2(a+bx^3)} - \frac{7x^4(2Ab-5aB)}{36ab^3} + \frac{7x(2Ab-5aB)}{9b^4} - \frac{7\sqrt[3]{a}(2Ab-5aB)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27b^{13/3}} + \frac{7\sqrt[3]{a}(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{13/3}} + \frac{x^{10}(Ab-aB)}{6ab(a+bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] (7*(2*A*b - 5*a*B)*x)/(9*b^4) - (7*(2*A*b - 5*a*B)*x^4)/(36*a*b^3) + ((A*b - a*B)*x^10)/(6*a*b*(a + b*x^3)^2) + ((2*A*b - 5*a*B)*x^7)/(9*a*b^2*(a + b*x^3)) + (7*a^(1/3)*(2*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(13/3)) - (7*a^(1/3)*(2*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(13/3)) + (7*a^(1/3)*(2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(13/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9 (A + Bx^3)}{(a + bx^3)^3} dx &= \frac{(Ab - aB)x^{10}}{6ab(a + bx^3)^2} + \frac{(-4Ab + 10aB) \int \frac{x^9}{(a+bx^3)^2} dx}{6ab} \\
&= \frac{(Ab - aB)x^{10}}{6ab(a + bx^3)^2} + \frac{(2Ab - 5aB)x^7}{9ab^2(a + bx^3)} - \frac{(7(2Ab - 5aB)) \int \frac{x^6}{a+bx^3} dx}{9ab^2} \\
&= \frac{(Ab - aB)x^{10}}{6ab(a + bx^3)^2} + \frac{(2Ab - 5aB)x^7}{9ab^2(a + bx^3)} - \frac{(7(2Ab - 5aB)) \int \left(-\frac{a}{b^2} + \frac{x^3}{b} + \frac{a^2}{b^2(a+bx^3)} \right) dx}{9ab^2} \\
&= \frac{7(2Ab - 5aB)x}{9b^4} - \frac{7(2Ab - 5aB)x^4}{36ab^3} + \frac{(Ab - aB)x^{10}}{6ab(a + bx^3)^2} + \frac{(2Ab - 5aB)x^7}{9ab^2(a + bx^3)} - \frac{(7a(2Ab - 5aB))}{9b^4} \\
&= \frac{7(2Ab - 5aB)x}{9b^4} - \frac{7(2Ab - 5aB)x^4}{36ab^3} + \frac{(Ab - aB)x^{10}}{6ab(a + bx^3)^2} + \frac{(2Ab - 5aB)x^7}{9ab^2(a + bx^3)} - \frac{(7\sqrt[3]{a}(2Ab - 5aB))}{27b^4} \\
&= \frac{7(2Ab - 5aB)x}{9b^4} - \frac{7(2Ab - 5aB)x^4}{36ab^3} + \frac{(Ab - aB)x^{10}}{6ab(a + bx^3)^2} + \frac{(2Ab - 5aB)x^7}{9ab^2(a + bx^3)} - \frac{7\sqrt[3]{a}(2Ab - 5aB)}{27b^4} \\
&= \frac{7(2Ab - 5aB)x}{9b^4} - \frac{7(2Ab - 5aB)x^4}{36ab^3} + \frac{(Ab - aB)x^{10}}{6ab(a + bx^3)^2} + \frac{(2Ab - 5aB)x^7}{9ab^2(a + bx^3)} - \frac{7\sqrt[3]{a}(2Ab - 5aB)}{27b^4} \\
&= \frac{7(2Ab - 5aB)x}{9b^4} - \frac{7(2Ab - 5aB)x^4}{36ab^3} + \frac{(Ab - aB)x^{10}}{6ab(a + bx^3)^2} + \frac{(2Ab - 5aB)x^7}{9ab^2(a + bx^3)} + \frac{7\sqrt[3]{a}(2Ab - 5aB)}{9\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 210, normalized size = 0.86

$$\frac{-14\sqrt[3]{a}(5aB - 2Ab) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) + \frac{18a^2\sqrt[3]{b}x(aB - Ab)}{(a+bx^3)^2} + \frac{6a\sqrt[3]{b}x(13Ab - 19aB)}{a+bx^3} + 108\sqrt[3]{b}x(Ab - 3aB) + 28\sqrt[3]{a}(5aB - 2Ab) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 28\sqrt{3}\sqrt[3]{a}(5aB - 2Ab) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{a}x}{\sqrt{3}}\right) + 27b^{4/3}Bx^4}{108b^{13/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] $(108*b^{1/3}*(A*b - 3*a*B)*x + 27*b^{4/3}*B*x^4 + (18*a^2*b^{1/3})*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (6*a*b^{1/3}*(13*A*b - 19*a*B)*x)/(a + b*x^3) - 28*\text{Sqrt}[3]*a^{1/3}*(-2*A*b + 5*a*B)*\text{ArcTan}[(1 - (2*b^{1/3}*x)/a^{1/3})/\text{Sqrt}[3]] + 28*a^{1/3}*(-2*A*b + 5*a*B)*\text{Log}[a^{1/3} + b^{1/3}*x] - 14*a^{1/3}*(-2*A*b + 5*a*B)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2]/(108*b^{13/3})$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9 (A + Bx^3)}{(a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] IntegrateAlgebraic[(x^9*(A + B*x^3))/(a + b*x^3)^3, x]

fricas [A] time = 0.60, size = 347, normalized size = 1.42

$$\frac{27 B b^2 x^{10} - 54 (5 B a b^2 - 2 A b^3) x^7 - 147 (5 B a^2 b - 2 A a b^2) x^4 - 28 \sqrt{3} ((5 B a^2 b - 2 A b^3) x^6 + 5 B a^3 - 2 A a^2 b + 2 (5 B a^2 b - 2 A a b^2) x^3) (-a/b)^{1/3} \arctan\left(\frac{1/\sqrt{3} (2 \sqrt{3} b x - a)^{1/3}}{a^{1/3}}\right) + 14 ((5 B a^2 b - 2 A b^3) x^6 + 5 B a^3 - 2 A a^2 b + 2 (5 B a^2 b - 2 A a b^2) x^3) (-a/b)^{1/3} \log\left(x + x (-a/b)^{1/3} + (-a/b)^{2/3}\right) - 28 ((5 B a^2 b - 2 A b^3) x^6 + 5 B a^3 - 2 A a^2 b + 2 (5 B a^2 b - 2 A a b^2) x^3) (-a/b)^{1/3} \log\left(x - (-a/b)^{1/3}\right) - 84 (5 B a^2 b - 2 A a b^2) x}{108 (b x^3 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $1/108*(27*B*b^3*x^{10} - 54*(5*B*a*b^2 - 2*A*b^3)*x^7 - 147*(5*B*a^2*b - 2*A*a*b^2)*x^4 - 28*\text{sqrt}(3)*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^{1/3}*\text{arctan}(1/3*(2*\text{sqrt}(3)*b*x*(-a/b))^{2/3} - \text{sqrt}(3)*a)/a) + 14*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^{1/3}*\text{log}(x^2 + x*(-a/b)^{1/3} + (-a/b)^{2/3}) - 28*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^{1/3}*\text{log}(x - (-a/b)^{1/3}) - 84*(5*B*a^2*b - 2*A*a*b^2)*x)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)$

giac [A] time = 0.24, size = 234, normalized size = 0.96

$$\frac{7\sqrt{3}\left(5(-ab^2)^{\frac{1}{3}}Ba - 2(-ab^2)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x + (-\frac{a}{b})^{\frac{1}{3}}\right)^{\frac{1}{3}}}{3(-\frac{a}{b})^{\frac{1}{3}}}\right) - 7(5Ba^2 - 2Aab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right) + 7\left(5(-ab^2)^{\frac{1}{3}}Ba - 2(-ab^2)^{\frac{1}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - 19Ba^2bx^4 - 13Aab^2x^4 + 16Ba^3x - 10Aa^2bx + \frac{Bb^4x^4 - 12Bab^3x + 4Ab^4x}{4b^2}}{27b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $7/27*\text{sqrt}(3)*(5*(-a*b^2)^{1/3}*B*a - 2*(-a*b^2)^{1/3}*A*b)*\text{arctan}(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{1/3})/(-a/b)^{1/3})/b^5 - 7/27*(5*B*a^2 - 2*A*a*b)*(-a/b)^{1/3}*\text{log}(\text{abs}(x - (-a/b)^{1/3}))/a*b^4 + 7/54*(5*(-a*b^2)^{1/3}*B*a - 2*$

$(-a*b^2)^{(1/3)}*A*b*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/b^5 - 1/18*(19*B*a^2*b*x^4 - 13*A*a*b^2*x^4 + 16*B*a^3*x - 10*A*a^2*b*x)/((b*x^3 + a)^2*b^4) + 1/4*(B*b^9*x^4 - 12*B*a*b^8*x + 4*A*b^9*x)/b^{12}$

maple [A] time = 0.06, size = 299, normalized size = 1.23

$$\frac{13Aax^4}{18(bx^3+a)^2b^2} - \frac{19Ba^2x^4}{18(bx^3+a)^2b^2} + \frac{Bx^4}{4b^3} + \frac{5Aa^2x}{9(bx^3+a)^2b^3} - \frac{8Ba^3x}{9(bx^3+a)^2b^3} - \frac{14\sqrt{3}Aa \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{b}+1\right)}{\left(\frac{b}{b}\right)^{\frac{2}{3}}}\right)}{27\left(\frac{b}{b}\right)^{\frac{2}{3}}b^4} - \frac{14Aa \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{b}{b}\right)^{\frac{2}{3}}b^4} + \frac{7Aa \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{b}{b}\right)^{\frac{2}{3}}b^4} + \frac{Ax}{b^3} + \frac{35\sqrt{3}Ba^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{b}+1\right)}{\left(\frac{b}{b}\right)^{\frac{2}{3}}}\right)}{27\left(\frac{b}{b}\right)^{\frac{2}{3}}b^5} + \frac{35Ba^2 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{b}{b}\right)^{\frac{2}{3}}b^5} - \frac{35Ba^2 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{b}{b}\right)^{\frac{2}{3}}b^5} - \frac{3Bax}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9*(B*x^3+A)/(b*x^3+a)^3, x)

[Out] $1/4/b^3*B*x^4+1/b^3*A*x-3/b^4*B*a*x+13/18*a/b^2/(b*x^3+a)^2*A*x^4-19/18*a^2/b^3/(b*x^3+a)^2*B*x^4+5/9*a^2/b^3/(b*x^3+a)^2*A*x-8/9*a^3/b^4/(b*x^3+a)^2*B*x-14/27*a/b^4*A/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})+7/27*a/b^4*A/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})-14/27*a/b^4*A/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))+35/27*a^2/b^5*B/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})-35/54*a^2/b^5*B/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+35/27*a^2/b^5*B/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))$

maxima [A] time = 1.24, size = 223, normalized size = 0.91

$$\frac{(19Ba^2b-13Aab^2)x^4+2(8Ba^3-5Aa^2b)x}{18(b^6x^6+2ab^5x^3+a^2b^4)} + \frac{Bbx^4-4(3Ba-Ab)x}{4b^4} + \frac{7\sqrt{3}(5Ba^2-2Aab) \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{7(5Ba^2-2Aab) \log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{7(5Ba^2-2Aab) \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3, x, algorithm="maxima")

[Out] $-1/18*((19*B*a^2*b - 13*A*a*b^2)*x^4 + 2*(8*B*a^3 - 5*A*a^2*b)*x)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) + 1/4*(B*b*x^4 - 4*(3*B*a - A*b)*x)/b^4 + 7/27*\sqrt{3}*(5*B*a^2 - 2*A*a*b)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(b^5*(a/b)^{(2/3)}) - 7/54*(5*B*a^2 - 2*A*a*b)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^5*(a/b)^{(2/3)}) + 7/27*(5*B*a^2 - 2*A*a*b)*\log(x + (a/b)^{(1/3)})/(b^5*(a/b)^{(2/3)})$

mupad [B] time = 0.32, size = 227, normalized size = 0.93

$$\frac{x^4\left(\frac{13Aa^2}{18} - \frac{19Ba^2b}{18}\right) - x\left(\frac{5Ba^3}{9} - \frac{5Aa^2b}{9}\right)}{a^2b^4 + 2ab^5x^3 + b^6x^6} + x\left(\frac{A}{b^3} - \frac{3Ba}{b^4}\right) + \frac{Bx^4}{4b^3} + \frac{7(-a)^{1/3} \ln((-a)^{4/3} + ab^{1/3}x)}{27b^{1/3}} + \frac{7(-a)^{1/3} \ln((-a)^{4/3} - 2ab^{1/3}x + \sqrt{3}(-a)^{4/3}1)\left(\frac{1}{2} + \frac{\sqrt{3}1}{2}\right)}{27b^{1/3}} + \frac{7(-a)^{1/3} \ln(2ab^{1/3}x - (-a)^{4/3} + \sqrt{3}(-a)^{4/3}1)\left(\frac{1}{2} + \frac{\sqrt{3}1}{2}\right)}{27b^{1/3}} + \frac{7(-a)^{1/3} \ln(2ab^{1/3}x - (-a)^{4/3} + \sqrt{3}(-a)^{4/3}1)\left(\frac{1}{2} + \frac{\sqrt{3}1}{2}\right)}{27b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^9*(A + B*x^3))/(a + b*x^3)^3, x)

```
[Out] (x^4*((13*A*a*b^2)/18 - (19*B*a^2*b)/18) - x*((8*B*a^3)/9 - (5*A*a^2*b)/9))
/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x*(A/b^3 - (3*B*a)/b^4) + (B*x^4)/(4*b
^3) + (7*(-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x)*(2*A*b - 5*B*a))/(27*b^(1
3/3)) - (7*(-a)^(1/3)*log((-a)^(4/3) + 3^(1/2)*(-a)^(4/3)*1i - 2*a*b^(1/3)*
x)*((3^(1/2)*1i)/2 + 1/2)*(2*A*b - 5*B*a))/(27*b^(13/3)) + (7*(-a)^(1/3)*lo
g(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 - 1/2
)*(2*A*b - 5*B*a))/(27*b^(13/3))
```

sympy [A] time = 4.01, size = 163, normalized size = 0.67

$$\frac{Bx^4}{4b^3} + x\left(\frac{A}{b^3} - \frac{3Ba}{b^4}\right) + \frac{x^4(13Aab^2 - 19Ba^2b) + x(10Aa^2b - 16Ba^3)}{18a^2b^4 + 36ab^5x^3 + 18b^6x^6} + \text{RootSum}\left(19683t^3b^{13} + 2744A^3ab^3 - 20580A^2Bb^2b^2 + 51450AB^2a^3b - 42875B^3a^4, \left(t \mapsto t \log\left(\frac{27tb^4}{-14Ab + 35Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**4/(4*b**3) + x*(A/b**3 - 3*B*a/b**4) + (x**4*(13*A*a*b**2 - 19*B*a**2*
b) + x*(10*A*a**2*b - 16*B*a**3))/(18*a**2*b**4 + 36*a*b**5*x**3 + 18*b**6*
x**6) + RootSum(19683*_t**3*b**13 + 2744*A**3*a*b**3 - 20580*A**2*B*a**2*b*
**2 + 51450*A*B**2*a**3*b - 42875*B**3*a**4, Lambda(_t, _t*log(27*_t*b**4/(-
14*A*b + 35*B*a) + x)))
```

$$3.97 \quad \int \frac{x^7(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=222

$$\frac{5(Ab - 4aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54 \sqrt[3]{a} b^{11/3}} - \frac{5(Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27 \sqrt[3]{a} b^{11/3}} - \frac{5(Ab - 4aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{9\sqrt{3} \sqrt[3]{a} b^{11/3}} - 5x^2$$

Rubi [A] time = 0.14, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {457, 288, 321, 292, 31, 634, 617, 204, 628}

$$\frac{5(Ab - 4aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54 \sqrt[3]{a} b^{11/3}} + \frac{x^5(Ab - 4aB)}{9ab^2(a + bx^3)} - \frac{5x^2(Ab - 4aB)}{18ab^3} - \frac{5(Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27 \sqrt[3]{a} b^{11/3}} - \frac{5(Ab - 4aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{9\sqrt{3} \sqrt[3]{a} b^{11/3}} + \frac{x^8(Ab - aB)}{6ab(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] (-5*(A*b - 4*a*B)*x^2)/(18*a*b^3) + ((A*b - a*B)*x^8)/(6*a*b*(a + b*x^3)^2) + ((A*b - 4*a*B)*x^5)/(9*a*b^2*(a + b*x^3)) - (5*(A*b - 4*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(1/3)*b^(11/3)) - (5*(A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1/3)*b^(11/3)) + (5*(A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(11/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7 (A + Bx^3)}{(a + bx^3)^3} dx &= \frac{(Ab - aB)x^8}{6ab(a + bx^3)^2} + \frac{(-2Ab + 8aB) \int \frac{x^7}{(a+bx^3)^2} dx}{6ab} \\
&= \frac{(Ab - aB)x^8}{6ab(a + bx^3)^2} + \frac{(Ab - 4aB)x^5}{9ab^2(a + bx^3)} - \frac{(5(Ab - 4aB)) \int \frac{x^4}{a+bx^3} dx}{9ab^2} \\
&= -\frac{5(Ab - 4aB)x^2}{18ab^3} + \frac{(Ab - aB)x^8}{6ab(a + bx^3)^2} + \frac{(Ab - 4aB)x^5}{9ab^2(a + bx^3)} + \frac{(5(Ab - 4aB)) \int \frac{x}{a+bx^3} dx}{9b^3} \\
&= -\frac{5(Ab - 4aB)x^2}{18ab^3} + \frac{(Ab - aB)x^8}{6ab(a + bx^3)^2} + \frac{(Ab - 4aB)x^5}{9ab^2(a + bx^3)} - \frac{(5(Ab - 4aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{27\sqrt[3]{a} b^{10/3}} + \frac{(5(Ab - 4aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{27\sqrt[3]{a} b^{10/3}} \\
&= -\frac{5(Ab - 4aB)x^2}{18ab^3} + \frac{(Ab - aB)x^8}{6ab(a + bx^3)^2} + \frac{(Ab - 4aB)x^5}{9ab^2(a + bx^3)} - \frac{5(Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27\sqrt[3]{a} b^{11/3}} + \frac{5(Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27\sqrt[3]{a} b^{11/3}} \\
&= -\frac{5(Ab - 4aB)x^2}{18ab^3} + \frac{(Ab - aB)x^8}{6ab(a + bx^3)^2} + \frac{(Ab - 4aB)x^5}{9ab^2(a + bx^3)} - \frac{5(Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27\sqrt[3]{a} b^{11/3}} + \frac{5(Ab - 4aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27\sqrt[3]{a} b^{11/3}} \\
&= -\frac{5(Ab - 4aB)x^2}{18ab^3} + \frac{(Ab - aB)x^8}{6ab(a + bx^3)^2} + \frac{(Ab - 4aB)x^5}{9ab^2(a + bx^3)} - \frac{5(Ab - 4aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a} b^{11/3}} - \frac{5(Ab - 4aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a} b^{11/3}}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 194, normalized size = 0.87

$$\frac{5(Ab - 4aB) \log\left(\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2}{\sqrt[3]{a}}\right) - \frac{6b^{2/3}x^2(4Ab - 7aB)}{a + bx^3} + \frac{9ab^{2/3}x^2(Ab - aB)}{(a + bx^3)^2} + \frac{10(4aB - Ab) \log\left(\frac{\sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}} + \frac{10\sqrt{3}(4aB - Ab) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}} + 27b^{2/3}Bx^2}{54b^{11/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^3, x]

[Out] (27*b^(2/3)*B*x^2 + (9*a*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 - (6*b^(2/3)*(4*A*b - 7*a*B)*x^2)/(a + b*x^3) + (10*sqrt[3]*(-(A*b) + 4*a*B)*ArcTan[(1

$$- (2*b^{(1/3)*x}/a^{(1/3)})/\text{Sqrt}[3]]/a^{(1/3)} + (10*(-(A*b) + 4*a*B)*\text{Log}[a^{(1/3)} + b^{(1/3)*x}]/a^{(1/3)} + (5*(A*b - 4*a*B)*\text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x}} + b^{(2/3)*x^2}]/a^{(1/3)})/(54*b^{(11/3)})$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 (A + Bx^3)}{(a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] IntegrateAlgebraic[(x^7*(A + B*x^3))/(a + b*x^3)^3, x]

fricas [B] time = 0.92, size = 792, normalized size = 3.57

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/54*(27*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 - A*a^2*b^3)*x^2 - 15*\text{sqrt}(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b - A*a^3*b^2 + 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*\text{sqrt}((-a*b^2)^{(1/3)}/a)*\text{log}((2*b^2*x^3 - a*b + 3*\text{sqrt}(1/3)*(a*b*x + 2*(-a*b^2)^{(2/3)}*x^2 + (-a*b^2)^{(1/3)*a})*\text{sqrt}((-a*b^2)^{(1/3)}/a) - 3*(-a*b^2)^{(2/3)*x}/(b*x^3 + a)) - 5*((4*B*a*b^2 - A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^{(2/3)}*\text{log}(b^2*x^2 + (-a*b^2)^{(1/3)*b*x} + (-a*b^2)^{(2/3)}) + 10*((4*B*a*b^2 - A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^{(2/3)}*\text{log}(b*x - (-a*b^2)^{(1/3)})]/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5), 1/54*(27*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 - A*a^2*b^3)*x^2 - 30*\text{sqrt}(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b - A*a^3*b^2 + 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*\text{sqrt}(-(-a*b^2)^{(1/3)}/a)*\text{arctan}(\text{sqrt}(1/3)*(2*b*x + (-a*b^2)^{(1/3)})*\text{sqrt}(-(-a*b^2)^{(1/3)}/a)/b) - 5*((4*B*a*b^2 - A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^{(2/3)}*\text{log}(b^2*x^2 + (-a*b^2)^{(1/3)*b*x} + (-a*b^2)^{(2/3)}) + 10*((4*B*a*b^2 - A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^{(2/3)}*\text{log}(b*x - (-a*b^2)^{(1/3)})]/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5)] \end{aligned}$$

giac [A] time = 0.21, size = 210, normalized size = 0.95

$$\frac{Bx^2}{2b^3} - \frac{5\sqrt{3}(4Ba - Ab) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27(-ab^2)^{\frac{1}{3}}b^3} + \frac{5(4Ba - Ab) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54(-ab^2)^{\frac{1}{3}}b^3} + \frac{5\left(4Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27ab^3} + \frac{14Babx^5 - 8Ab^2x^5 + 11Ba^2x^2 - 5Aabx^2}{18(bx^3 + a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $\frac{1}{2}Bx^2/b^3 - \frac{5}{27}\sqrt{3}(4Ba - Ab)\arctan(1/3\sqrt{3}(2x + (-a/b)^{1/3})/(-a/b)^{1/3})/((-a/b)^{1/3})/((-a/b^2)^{1/3}b^3) + \frac{5}{54}(4Ba - Ab)\log(x^2 + x(-a/b)^{1/3} + (-a/b)^{2/3})/((-a/b^2)^{1/3}b^3) + \frac{5}{27}(4Ba(-a/b)^{1/3} - Ab(-a/b)^{1/3})(-a/b)^{1/3}\log(\text{abs}(x - (-a/b)^{1/3}))/a^3 + \frac{1}{18}(14Babx^5 - 8A^2bx^5 + 11B^2ax^2 - 5A^2bx^2)/((bx^3 + a)^2b^3)$

maple [A] time = 0.06, size = 275, normalized size = 1.24

$$-\frac{4Ax^5}{9(bx^3+a)^2b} + \frac{7Ba^5}{9(bx^3+a)^2b^2} - \frac{5Aax^2}{18(bx^3+a)^2b^2} + \frac{11Ba^2x^2}{18(bx^3+a)^2b^2} + \frac{Bx^2}{2b^3} + \frac{5\sqrt{3}A\arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{b}\right)^{1/3}}{3}\right)}{27\left(\frac{a}{b}\right)^{1/3}b^3} - \frac{5A\ln\left(x + \left(\frac{a}{b}\right)^{1/3}\right)}{27\left(\frac{a}{b}\right)^{1/3}b^3} + \frac{5A\ln\left(x^2 - \left(\frac{a}{b}\right)^{1/3}x + \left(\frac{a}{b}\right)^{2/3}\right)}{54\left(\frac{a}{b}\right)^{1/3}b^3} - \frac{20\sqrt{3}Ba\arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{b}\right)^{1/3}}{3}\right)}{27\left(\frac{a}{b}\right)^{1/3}b^4} + \frac{20Ba\ln\left(x + \left(\frac{a}{b}\right)^{1/3}\right)}{27\left(\frac{a}{b}\right)^{1/3}b^4} - \frac{10Ba\ln\left(x^2 - \left(\frac{a}{b}\right)^{1/3}x + \left(\frac{a}{b}\right)^{2/3}\right)}{27\left(\frac{a}{b}\right)^{1/3}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(B*x^3+A)/(b*x^3+a)^3,x)

[Out] $\frac{1}{2}B/b^3x^2 - 4/9b/(bx^3+a)^2A^2x^5 + 7/9b^2/(bx^3+a)^2B^2x^5a - 5/18b^2/(bx^3+a)^2A^2x^2a + 11/18b^3/(bx^3+a)^2B^2x^2a^2 - 5/27b^3A/(a/b)^{1/3}\ln(x + (a/b)^{1/3}) + 5/54b^3A/(a/b)^{1/3}\ln(x^2 - (a/b)^{1/3}x + (a/b)^{2/3}) + 5/27b^3A^3^{1/2}/(a/b)^{1/3}\arctan(1/3\sqrt{3}^{1/2}(2/(a/b)^{1/3}x - 1)) + 20/27b^4B^2a/(a/b)^{1/3}\ln(x + (a/b)^{1/3}) - 10/27b^4B^2a/(a/b)^{1/3}\ln(x^2 - (a/b)^{1/3}x + (a/b)^{2/3}) - 20/27b^4B^2a^3^{1/2}/(a/b)^{1/3}\arctan(1/3\sqrt{3}^{1/2}(2/(a/b)^{1/3}x - 1))$

maxima [A] time = 1.32, size = 196, normalized size = 0.88

$$\frac{2(7Bab - 4Ab^2)x^5 + (11Ba^2 - 5Aab)x^2}{18(b^5x^6 + 2ab^4x^3 + a^2b^3)} + \frac{Bx^2}{2b^3} - \frac{5\sqrt{3}(4Ba - Ab)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{1/3}\right)}{3\left(\frac{a}{b}\right)^{1/3}}\right)}{27b^4\left(\frac{a}{b}\right)^{1/3}} - \frac{5(4Ba - Ab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{1/3} + \left(\frac{a}{b}\right)^{2/3}\right)}{54b^4\left(\frac{a}{b}\right)^{1/3}} + \frac{5(4Ba - Ab)\log\left(x + \left(\frac{a}{b}\right)^{1/3}\right)}{27b^4\left(\frac{a}{b}\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] $\frac{1}{18}(2(7B^2ab - 4A^2b^2)x^5 + (11B^2a^2 - 5A^2ab)x^2)/(b^5x^6 + 2a^2b^4x^3 + a^2b^2b^3) + \frac{1}{2}Bx^2/b^3 - \frac{5}{27}\sqrt{3}(4Ba - Ab)\arctan(1/3\sqrt{3}(2x - (a/b)^{1/3})/(a/b)^{1/3})/(b^4(a/b)^{1/3}) - \frac{5}{54}(4Ba - Ab)\log(x^2 - x(a/b)^{1/3} + (a/b)^{2/3})/(b^4(a/b)^{1/3}) + \frac{5}{27}(4Ba - Ab)\log(x + (a/b)^{1/3})/(b^4(a/b)^{1/3})$

mupad [B] time = 2.56, size = 187, normalized size = 0.84

$$\frac{x^2\left(\frac{11Ba^2}{18} - \frac{5Aab}{18}\right) - x^5\left(\frac{4Ab^2}{9} - \frac{7Bab}{9}\right)}{a^2b^3 + 2ab^4x^3 + b^5x^6} + \frac{Bx^2}{2b^3} - \frac{5\ln(b^{1/3}x + a^{1/3})(Ab - 4Ba)}{27a^{1/3}b^{11/3}} - \frac{5\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}1i)\left(-\frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)(Ab - 4Ba)}{27a^{1/3}b^{11/3}} + \frac{5\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}1i)\left(\frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)(Ab - 4Ba)}{27a^{1/3}b^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^7*(A + B*x^3))/(a + b*x^3)^3,x)`

[Out] $(x^2*((11*B*a^2)/18 - (5*A*a*b)/18) - x^5*((4*A*b^2)/9 - (7*B*a*b)/9))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (B*x^2)/(2*b^3) - (5*\log(b^{(1/3)}*x + a^{(1/3)}))*(A*b - 4*B*a)/(27*a^{(1/3)}*b^{(11/3)}) - (5*\log(3^{(1/2)}*a^{(1/3)}*1i - 2*b^{(1/3)}*x + a^{(1/3)}))*((3^{(1/2)}*1i)/2 - 1/2)*(A*b - 4*B*a)/(27*a^{(1/3)}*b^{(11/3)}) + (5*\log(3^{(1/2)}*a^{(1/3)}*1i + 2*b^{(1/3)}*x - a^{(1/3)}))*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - 4*B*a)/(27*a^{(1/3)}*b^{(11/3)})$

sympy [A] time = 5.85, size = 162, normalized size = 0.73

$$\frac{Bx^2}{2b^3} + \frac{x^5(-8Ab^2 + 14Bab) + x^2(-5Aab + 11Ba^2)}{18a^2b^3 + 36ab^4x^3 + 18b^5x^6} + \text{RootSum}\left(19683t^3ab^{11} + 125A^3b^3 - 1500A^2Bab^2 + 6000AB^2a^2b - 8000B^3a^3, \left(t \mapsto t \log\left(\frac{729t^2ab^7}{25A^2b^2 - 200ABab + 400B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(B*x**3+A)/(b*x**3+a)**3,x)`

[Out] $B*x**2/(2*b**3) + (x**5*(-8*A*b**2 + 14*B*a*b) + x**2*(-5*A*a*b + 11*B*a**2))/(18*a**2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + \text{RootSum}(19683*_t**3*a*b**11 + 125*A**3*b**3 - 1500*A**2*B*a*b**2 + 6000*A*B**2*a**2*b - 8000*B**3*a**3, \text{Lambda}(_t, _t*\log(729*_t**2*a*b**7/(25*A**2*b**2 - 200*A*B*a*b + 400*B**2*a**2) + x)))$

$$3.98 \quad \int \frac{x^6(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=220

$$\frac{(Ab - 7aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{27a^{2/3} b^{10/3}} + \frac{2(Ab - 7aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{2/3} b^{10/3}} - \frac{2(Ab - 7aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{9\sqrt{3} a^{2/3} b^{10/3}} - \frac{2x(Ab - 7aB)}{18ab^2(a + bx^3)} + \frac{x^7(Ab - aB)}{6ab(a + bx^3)^2}$$

Rubi [A] time = 0.13, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {457, 288, 321, 200, 31, 634, 617, 204, 628}

$$\frac{(Ab - 7aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{27a^{2/3} b^{10/3}} + \frac{2(Ab - 7aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{2/3} b^{10/3}} - \frac{2(Ab - 7aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{9\sqrt{3} a^{2/3} b^{10/3}} + \frac{x^4(Ab - 7aB)}{18ab^2(a + bx^3)} - \frac{2x(Ab - 7aB)}{9ab^3} + \frac{x^7(Ab - aB)}{6ab(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] (-2*(A*b - 7*a*B)*x)/(9*a*b^3) + ((A*b - a*B)*x^7)/(6*a*b*(a + b*x^3)^2) + ((A*b - 7*a*B)*x^4)/(18*a*b^2*(a + b*x^3)) - (2*(A*b - 7*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(10/3)) + (2*(A*b - 7*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(10/3)) - ((A*b - 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(2/3)*b^(10/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
 \int \frac{x^6 (A + Bx^3)}{(a + bx^3)^3} dx &= \frac{(Ab - aB)x^7}{6ab(a + bx^3)^2} + \frac{(-Ab + 7aB) \int \frac{x^6}{(a+bx^3)^2} dx}{6ab} \\
 &= \frac{(Ab - aB)x^7}{6ab(a + bx^3)^2} + \frac{(Ab - 7aB)x^4}{18ab^2(a + bx^3)} - \frac{(2(Ab - 7aB)) \int \frac{x^3}{a+bx^3} dx}{9ab^2} \\
 &= -\frac{2(Ab - 7aB)x}{9ab^3} + \frac{(Ab - aB)x^7}{6ab(a + bx^3)^2} + \frac{(Ab - 7aB)x^4}{18ab^2(a + bx^3)} + \frac{(2(Ab - 7aB)) \int \frac{1}{a+bx^3} dx}{9b^3} \\
 &= -\frac{2(Ab - 7aB)x}{9ab^3} + \frac{(Ab - aB)x^7}{6ab(a + bx^3)^2} + \frac{(Ab - 7aB)x^4}{18ab^2(a + bx^3)} + \frac{(2(Ab - 7aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{27a^{2/3}b^3} + \frac{(2(Ab - 7aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{27a^{2/3}b^3} \\
 &= -\frac{2(Ab - 7aB)x}{9ab^3} + \frac{(Ab - aB)x^7}{6ab(a + bx^3)^2} + \frac{(Ab - 7aB)x^4}{18ab^2(a + bx^3)} + \frac{2(Ab - 7aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{2/3}b^{10/3}} - \frac{2(Ab - 7aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{2/3}b^{10/3}} \\
 &= -\frac{2(Ab - 7aB)x}{9ab^3} + \frac{(Ab - aB)x^7}{6ab(a + bx^3)^2} + \frac{(Ab - 7aB)x^4}{18ab^2(a + bx^3)} + \frac{2(Ab - 7aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{2/3}b^{10/3}} - \frac{2(Ab - 7aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{2/3}b^{10/3}} \\
 &= -\frac{2(Ab - 7aB)x}{9ab^3} + \frac{(Ab - aB)x^7}{6ab(a + bx^3)^2} + \frac{(Ab - 7aB)x^4}{18ab^2(a + bx^3)} - \frac{2(Ab - 7aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}} + \frac{2(Ab - 7aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}}
 \end{aligned}$$

Mathematica [A] time = 0.27, size = 188, normalized size = 0.85

$$\frac{2(7aB - Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{a^{2/3}} + \frac{4(Ab - 7aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{a^{2/3}} + \frac{4\sqrt{3}(7aB - Ab) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}} - \frac{3\sqrt[3]{b}x(7Ab - 13aB)}{a + bx^3} + \frac{9a\sqrt[3]{b}x(Ab - aB)}{(a + bx^3)^2} + 54\sqrt[3]{b}Bx$$

54b^{10/3}

Antiderivative was successfully verified.

[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^3, x]


```
[Out] (54*b^(1/3)*B*x + (9*a*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 - (3*b^(1/3)*(7
*A*b - 13*a*B)*x)/(a + b*x^3) + (4*Sqrt[3]*(-(A*b) + 7*a*B)*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*(A*b - 7*a*B)*Log[a^(1/3) + b^(1
/3)*x])/a^(2/3) + (2*(-(A*b) + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/a^(2/3))/(54*b^(10/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 (A + Bx^3)}{(a + bx^3)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^6*(A + B*x^3))/(a + b*x^3)^3, x]
```

fricas [B] time = 0.65, size = 789, normalized size = 3.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(54*B*a^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 6*sqrt(1/3)*((
7*B*a^2*b^3 - A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a^2
*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b
+ 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b + 2*(7
B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(7*B
a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), 1/54*(54*B*a
^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 12*sqrt(1/3)*((7*B*a^2*b^3
- A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a^2*b^3)*x^3)*s
qrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*
b + 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*
x + (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b + 2*(
7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(7*
B*a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4)]
```


$(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)}/(b^4*(a/b)^{(2/3)}) + 1/27*(7*B*a - A*b)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^4*(a/b)^{(2/3)}) - 2/27*(7*B*a - A*b)*\log(x + (a/b)^{(1/3)})/(b^4*(a/b)^{(2/3)})$

mupad [B] time = 2.60, size = 183, normalized size = 0.83

$$\frac{Bx}{b^3} - \frac{x^4 \left(\frac{7A^2}{18} - \frac{13Bab}{18} \right) - x \left(\frac{5Ba^2}{9} - \frac{2Aab}{9} \right)}{a^2 b^3 + 2a b^4 x^3 + b^5 x^6} + \frac{2 \ln(b^{1/3} x + a^{1/3}) (Ab - 7Ba)}{27 a^{2/3} b^{10/3}} - \frac{2 \ln(a^{1/3} - 2b^{1/3} x + \sqrt{3} a^{1/3} i) \left(\frac{1}{2} + \frac{\sqrt{3} i}{2} \right) (Ab - 7Ba)}{27 a^{2/3} b^{10/3}} + \frac{2 \ln(2b^{1/3} x - a^{1/3} + \sqrt{3} a^{1/3} i) \left(-\frac{1}{2} + \frac{\sqrt{3} i}{2} \right) (Ab - 7Ba)}{27 a^{2/3} b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(A + B*x^3))/(a + b*x^3)^3,x)

[Out] $(B*x)/b^3 - (x^4*((7*A*b^2)/18 - (13*B*a*b)/18) - x*((5*B*a^2)/9 - (2*A*a*b)/9))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (2*\log(b^{(1/3)}*x + a^{(1/3)})*(A*b - 7*B*a))/(27*a^{(2/3)}*b^{(10/3)}) - (2*\log(3^{(1/2)}*a^{(1/3)}*i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*i)/2 + 1/2)*(A*b - 7*B*a))/(27*a^{(2/3)}*b^{(10/3)}) + (2*\log(3^{(1/2)}*a^{(1/3)}*i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*i)/2 - 1/2)*(A*b - 7*B*a))/(27*a^{(2/3)}*b^{(10/3)})$

sympy [A] time = 3.39, size = 141, normalized size = 0.64

$$\frac{Bx}{b^3} + \frac{x^4(-7Ab^2 + 13Bab) + x(-4Aab + 10Ba^2)}{18a^2b^3 + 36ab^4x^3 + 18b^5x^6} + \text{RootSum}\left(19683t^3a^2b^{10} - 8A^3b^3 + 168A^2Bab^2 - 1176AB^2a^2b + 2744B^3a^3, \left(t \mapsto t \log\left(-\frac{27tab^3}{-2Ab + 14Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**3,x)

[Out] $B*x/b**3 + (x**4*(-7*A*b**2 + 13*B*a*b) + x*(-4*A*a*b + 10*B*a**2))/(18*a**2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + \text{RootSum}(19683*_t**3*a**2*b**10 - 8*A**3*b**3 + 168*A**2*B*a*b**2 - 1176*A*B**2*a**2*b + 2744*B**3*a**3, \text{Lamb da}(_t, _t*\log(-27*_t*a*b**3/(-2*A*b + 14*B*a) + x)))$

$$3.99 \quad \int \frac{x^4(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=201

$$\frac{(5aB + Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54a^{4/3} b^{8/3}} - \frac{(5aB + Ab) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{4/3} b^{8/3}} - \frac{(5aB + Ab) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3} a^{4/3} b^{8/3}} - \frac{x^2(5aB + Ab)}{18ab^2 (a + bx^3)^2}$$

Rubi [A] time = 0.11, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 288, 292, 31, 634, 617, 204, 628}

$$\frac{(5aB + Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54a^{4/3} b^{8/3}} - \frac{(5aB + Ab) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{4/3} b^{8/3}} - \frac{(5aB + Ab) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3} a^{4/3} b^{8/3}} - \frac{x^2(5aB + Ab)}{18ab^2 (a + bx^3)^2} + \frac{x^5(Ab - aB)}{6ab (a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^3, x]

[Out] ((A*b - a*B)*x^5)/(6*a*b*(a + b*x^3)^2) - ((A*b + 5*a*B)*x^2)/(18*a*b^2*(a + b*x^3)) - ((A*b + 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(8/3)) - ((A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(8/3)) + ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(8/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a + b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !I

LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 292

Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 457

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ((!IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{x^4 (A + Bx^3)}{(a + bx^3)^3} dx &= \frac{(Ab - aB)x^5}{6ab(a + bx^3)^2} + \frac{(Ab + 5aB) \int \frac{x^4}{(a+bx^3)^2} dx}{6ab} \\
&= \frac{(Ab - aB)x^5}{6ab(a + bx^3)^2} - \frac{(Ab + 5aB)x^2}{18ab^2(a + bx^3)} + \frac{(Ab + 5aB) \int \frac{x}{a+bx^3} dx}{9ab^2} \\
&= \frac{(Ab - aB)x^5}{6ab(a + bx^3)^2} - \frac{(Ab + 5aB)x^2}{18ab^2(a + bx^3)} - \frac{(Ab + 5aB) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{27a^{4/3}b^{7/3}} + \frac{(Ab + 5aB) \int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{27a^{4/3}b^{7/3}} \\
&= \frac{(Ab - aB)x^5}{6ab(a + bx^3)^2} - \frac{(Ab + 5aB)x^2}{18ab^2(a + bx^3)} - \frac{(Ab + 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{4/3}b^{8/3}} + \frac{(Ab + 5aB) \int \frac{-\sqrt[3]{a}\sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{54a^{4/3}b^{8/3}} \\
&= \frac{(Ab - aB)x^5}{6ab(a + bx^3)^2} - \frac{(Ab + 5aB)x^2}{18ab^2(a + bx^3)} - \frac{(Ab + 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{4/3}b^{8/3}} + \frac{(Ab + 5aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3})}{54a^{4/3}b^{8/3}} \\
&= \frac{(Ab - aB)x^5}{6ab(a + bx^3)^2} - \frac{(Ab + 5aB)x^2}{18ab^2(a + bx^3)} - \frac{(Ab + 5aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{8/3}} - \frac{(Ab + 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{4/3}b^{8/3}}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 181, normalized size = 0.90

$$\frac{(5aB + Ab) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{a^{4/3}} - \frac{2(5aB + Ab) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{a^{4/3}} - \frac{2\sqrt{3}(5aB + Ab) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}} + \frac{6b^{2/3}x^2(Ab - 4aB)}{a(a + bx^3)} - \frac{9b^{2/3}x^2(Ab - aB)}{(a + bx^3)^2}$$

$$54b^{8/3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] ((-9*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 + (6*b^(2/3)*(A*b - 4*a*B)*x^2)/(a*(a + b*x^3)) - (2*sqrt[3]*(A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/a^(4/3) - (2*(A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(54*b^(8/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (A + Bx^3)}{(a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] IntegrateAlgebraic[(x^4*(A + B*x^3))/(a + b*x^3)^3, x]

fricas [B] time = 0.54, size = 756, normalized size = 3.76

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] [-1/54*(6*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 - 3*sqrt(1/3)*((5*B*a^2*b^3 + A*a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), -1/54*(6*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 - 6*sqrt(1/3)*((5*B*a^2*b^3 + A*a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - ((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4)]

giac [A] time = 0.20, size = 206, normalized size = 1.02

$$\frac{\sqrt{3}(5Ba + Ab) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27(-ab^2)^{\frac{1}{3}}ab^2} - \frac{(5Ba + Ab) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54(-ab^2)^{\frac{1}{3}}ab^2} - \frac{\left(5Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} + Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^2b^2} - \frac{8Babx^5 - 2Ab^2x^5 + 5Ba^2x^2 + Aabx^2}{18(bx^3 + a)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $\frac{1}{27}\sqrt{3}(5Ba + Ab)\arctan\left(\frac{1}{3}\sqrt{3}(2x + (-a/b)^{1/3})/(-a/b)^{1/3}\right)/((-a/b)^{1/3})/((-a/b)^{1/3})^2 - \frac{1}{54}(5Ba + Ab)\log(x^2 + x(-a/b)^{1/3} + (-a/b)^{2/3})/((-a/b)^{1/3})^2 - \frac{1}{27}(5Ba(-a/b)^{1/3} + Ab(-a/b)^{1/3})^2(-a/b)^{1/3}\log(\text{abs}(x - (-a/b)^{1/3}))/a^2b^2 - \frac{1}{18}(8Ba^2bx^5 - 2A^2b^2x^5 + 5Ba^2x^2 + A^2bx^2)/(b^3x^3 + a)^2$

maple [A] time = 0.05, size = 241, normalized size = 1.20

$$\frac{\sqrt{3} A \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a b^2} - \frac{A \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a b^2} + \frac{A \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}} a b^2} + \frac{5\sqrt{3} B \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^3} - \frac{5B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^3} + \frac{5B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}} b^3} + \frac{(Ab-4Ba)x^5 - (Ab+5Ba)x^2}{9ab \cdot \frac{18b^2}{(bx^3+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(B*x^3+A)/(b*x^3+a)^3,x)

[Out] $\frac{1}{9}(Ab-4Ba)/abx^5 - \frac{1}{18}(Ab+5Ba)/b^2x^2/(b^3x^3+a)^2 - \frac{1}{27}b^2/a/(a/b)^{1/3}\ln(x+(a/b)^{1/3}) * A - \frac{5}{27}b^3/(a/b)^{1/3}\ln(x+(a/b)^{1/3}) * B + \frac{1}{5}4/b^2/a/(a/b)^{1/3}\ln(x^2-(a/b)^{1/3}x+(a/b)^{2/3}) * A + \frac{5}{54}b^3/(a/b)^{1/3}\ln(x^2-(a/b)^{1/3}x+(a/b)^{2/3}) * B + \frac{1}{27}b^2/a^3(1/2)/(a/b)^{1/3}\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1)) * A + \frac{5}{27}b^3*3^{1/2}/(a/b)^{1/3}\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1)) * B$

maxima [A] time = 1.27, size = 195, normalized size = 0.97

$$-\frac{2(4Bab - Ab^2)x^5 + (5Ba^2 + Aab)x^2}{18(ab^4x^6 + 2a^2b^3x^3 + a^3b^2)} + \frac{\sqrt{3}(5Ba + Ab)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(5Ba + Ab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(5Ba + Ab)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] $-\frac{1}{18}(2(4Ba^2b - Ab^2)x^5 + (5Ba^2 + A^2b)x^2)/(a^4b^3x^6 + 2a^2b^3x^3 + a^3b^2) + \frac{1}{27}\sqrt{3}(5Ba + Ab)\arctan\left(\frac{1}{3}\sqrt{3}(2x - (a/b)^{1/3})/(a/b)^{1/3}\right)/(a^4b^3(a/b)^{1/3}) + \frac{1}{54}(5Ba + Ab)\log(x^2 - x(a/b)^{1/3} + (a/b)^{2/3})/(a^4b^3(a/b)^{1/3}) - \frac{1}{27}(5Ba + Ab)\log(x + (a/b)^{1/3})/(a^4b^3(a/b)^{1/3})$

mupad [B] time = 0.27, size = 175, normalized size = 0.87

$$-\frac{x^2(Ab+5Ba) - x^5(Ab-4Ba)}{18b^2} - \frac{\ln(b^{1/3}x + a^{1/3})(Ab + 5Ba)}{27a^{4/3}b^{8/3}} - \frac{\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab + 5Ba)}{27a^{4/3}b^{8/3}} + \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab + 5Ba)}{27a^{4/3}b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4*(A + B*x^3))/(a + b*x^3)^3,x)`

[Out] $(\log(3^{1/2}*a^{1/3}*1i + 2*b^{1/3}*x - a^{1/3})*((3^{1/2}*1i)/2 + 1/2)*(A*b + 5*B*a))/(27*a^{4/3}*b^{8/3}) - (\log(b^{1/3}*x + a^{1/3})*(A*b + 5*B*a))/(27*a^{4/3}*b^{8/3}) - (\log(3^{1/2}*a^{1/3}*1i - 2*b^{1/3}*x + a^{1/3})*((3^{1/2}*1i)/2 - 1/2)*(A*b + 5*B*a))/(27*a^{4/3}*b^{8/3}) - ((x^2*(A*b + 5*B*a))/(18*b^2) - (x^5*(A*b - 4*B*a))/(9*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)$

sympy [A] time = 4.75, size = 155, normalized size = 0.77

$$\frac{x^5(2Ab^2 - 8Bab) + x^2(-Aab - 5Ba^2)}{18a^3b^2 + 36a^2b^3x^3 + 18ab^4x^6} + \text{RootSum}\left(19683t^3a^4b^8 + A^3b^3 + 15A^2Bab^2 + 75AB^2a^2b + 125B^3a^3, \left(t \mapsto t \log\left(\frac{729t^2a^3b^5}{A^2b^2 + 10ABab + 25B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(B*x**3+A)/(b*x**3+a)**3,x)`

[Out] $(x**5*(2*A*b**2 - 8*B*a*b) + x**2*(-A*a*b - 5*B*a**2))/(18*a**3*b**2 + 36*a**2*b**3*x**3 + 18*a*b**4*x**6) + \text{RootSum}(19683*_t**3*a**4*b**8 + A**3*b**3 + 15*A**2*B*a*b**2 + 75*A*B**2*a**2*b + 125*B**3*a**3, \text{Lambda}(_t, _t*\log(729*_t**2*a**3*b**5/(A**2*b**2 + 10*A*B*a*b + 25*B**2*a**2) + x)))$

$$3.100 \quad \int \frac{x^3(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=199

$$-\frac{(2aB + Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54a^{5/3}b^{7/3}} + \frac{(2aB + Ab) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{5/3}b^{7/3}} - \frac{(2aB + Ab) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}} - \frac{x(2aB + Ab)}{9ab^2}$$

Rubi [A] time = 0.11, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 288, 200, 31, 634, 617, 204, 628}

$$-\frac{(2aB + Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54a^{5/3}b^{7/3}} + \frac{(2aB + Ab) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{5/3}b^{7/3}} - \frac{(2aB + Ab) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}} - \frac{x(2aB + Ab)}{9ab^2(a + bx^3)} + \frac{x^4(Ab - aB)}{6ab(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] ((A*b - a*B)*x^4)/(6*a*b*(a + b*x^3)^2) - ((A*b + 2*a*B)*x)/(9*a*b^2*(a + b*x^3)) - ((A*b + 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(7/3)) + ((A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(7/3)) - ((A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(7/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (A + Bx^3)}{(a + bx^3)^3} dx &= \frac{(Ab - aB)x^4}{6ab(a + bx^3)^2} + \frac{(2Ab + 4aB) \int \frac{x^3}{(a+bx^3)^2} dx}{6ab} \\
&= \frac{(Ab - aB)x^4}{6ab(a + bx^3)^2} - \frac{(Ab + 2aB)x}{9ab^2(a + bx^3)} + \frac{(Ab + 2aB) \int \frac{1}{a+bx^3} dx}{9ab^2} \\
&= \frac{(Ab - aB)x^4}{6ab(a + bx^3)^2} - \frac{(Ab + 2aB)x}{9ab^2(a + bx^3)} + \frac{(Ab + 2aB) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{27a^{5/3}b^2} + \frac{(Ab + 2aB) \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{27a^{5/3}b^2} \\
&= \frac{(Ab - aB)x^4}{6ab(a + bx^3)^2} - \frac{(Ab + 2aB)x}{9ab^2(a + bx^3)} + \frac{(Ab + 2aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{5/3}b^{7/3}} - \frac{(Ab + 2aB) \int \frac{-\sqrt[3]{a}\sqrt[3]{b}}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{54a^{5/3}b^{7/3}} \\
&= \frac{(Ab - aB)x^4}{6ab(a + bx^3)^2} - \frac{(Ab + 2aB)x}{9ab^2(a + bx^3)} + \frac{(Ab + 2aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{5/3}b^{7/3}} - \frac{(Ab + 2aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3})}{54a^{5/3}b^{7/3}} \\
&= \frac{(Ab - aB)x^4}{6ab(a + bx^3)^2} - \frac{(Ab + 2aB)x}{9ab^2(a + bx^3)} - \frac{(Ab + 2aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}} + \frac{(Ab + 2aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{5/3}b^{7/3}}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 178, normalized size = 0.89

$$\frac{-\frac{(2aB+Ab) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{a^{5/3}} + \frac{2(2aB+Ab) \log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{a^{5/3}} - \frac{2\sqrt{3}(2aB+Ab) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{3\sqrt[3]{b}x(Ab-7aB)}{a(a+bx^3)} - \frac{9\sqrt[3]{b}x(Ab-aB)}{(a+bx^3)^2}}{54b^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^3, x]

[Out] ((-9*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 + (3*b^(1/3)*(A*b - 7*a*B)*x)/(a*(a + b*x^3)) - (2*sqrt[3]*(A*b + 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/a^(5/3) + (2*(A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - ((A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(54*b^(7/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (A + Bx^3)}{(a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] IntegrateAlgebraic[(x^3*(A + B*x^3))/(a + b*x^3)^3, x]

fricas [B] time = 0.72, size = 743, normalized size = 3.73

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/54*(3*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 3*\sqrt{1/3}*((2*B*a^2*b^3 + A*a*b^4)*x^6 + 2*B*a^4*b + A*a^3*b^2 + 2*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*\sqrt{-(a^2*b)^{(1/3)}/b}*\log((2*a*b*x^3 - 3*(a^2*b)^{(1/3)}*a*x - a^2 + 3*\sqrt{1/3}*(2*a*b*x^2 + (a^2*b)^{(2/3)}*x - (a^2*b)^{(1/3)}*a)*\sqrt{-(a^2*b)^{(1/3)}/b}))/ (b*x^3 + a) + ((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^2*b)^{(2/3)}*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) - 2*((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^2*b)^{(2/3)}*\log(a*b*x + (a^2*b)^{(2/3)}) + 6*(2*B*a^4*b + A*a^3*b^2)*x]/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), -1/54*(3*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 6*\sqrt{1/3}*((2*B*a^2*b^3 + A*a*b^4)*x^6 + 2*B*a^4*b + A*a^3*b^2 + 2*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*\sqrt{(a^2*b)^{(1/3)}/b}*\arctan(\sqrt{1/3}*(2*(a^2*b)^{(2/3)}*x - (a^2*b)^{(1/3)}*a)*\sqrt{(a^2*b)^{(1/3)}/b}/a^2) + ((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^2*b)^{(2/3)}*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) - 2*((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^2*b)^{(2/3)}*\log(a*b*x + (a^2*b)^{(2/3)}) + 6*(2*B*a^4*b + A*a^3*b^2)*x]/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3)] \end{aligned}$$

giac [A] time = 0.19, size = 187, normalized size = 0.94

$$\frac{\sqrt{3}(2Ba + Ab) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27(-ab^2)^{\frac{2}{3}}ab} - \frac{(2Ba + Ab) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54(-ab^2)^{\frac{2}{3}}ab} - \frac{(2Ba + Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^2b^2} - \frac{7Babx^4 - Ab^2x^4 + 4Ba^2x + 2Aabx}{18(bx^3 + a)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $-\frac{1}{27}\sqrt{3}(2B^*a + A^*b)^*\arctan\left(\frac{1}{3}\sqrt{3}(2x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)}\right)/((-a^*b^2)^{(2/3)}^*a^*b) - \frac{1}{54}(2B^*a + A^*b)^*\log(x^2 + x(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/((-a^*b^2)^{(2/3)}^*a^*b) - \frac{1}{27}(2B^*a + A^*b)^*(-a/b)^{(1/3)}^*\log(a^*b^2(x - (-a/b)^{(1/3)}))/(a^2b^2) - \frac{1}{18}(7B^*a^*b^*x^4 - A^*b^2x^4 + 4B^*a^2x^2 + 2A^*a^*b^*x)/((b^*x^3 + a)^2a^*b^2)$

maple [A] time = 0.05, size = 239, normalized size = 1.20

$$\frac{\sqrt{3} A \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} a b^2} + \frac{A \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - A \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} a b^2} + \frac{2\sqrt{3} B \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^3} + \frac{2B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^3} + \frac{(Ab-7Ba)x^4 - (Ab+2Ba)x}{18ab} \frac{1}{(bx^3+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(B*x^3+A)/(b*x^3+a)^3,x)

[Out] $\frac{1}{18}(A^*b-7B^*a)/a/b^*x^4 - \frac{1}{9}(A^*b+2B^*a)/b^2x^2/(b^*x^3+a)^2 + \frac{1}{27}b^2/a/(a/b)^{(2/3)}^*\ln(x+(a/b)^{(1/3)})^*A + \frac{2}{27}b^3/(a/b)^{(2/3)}^*\ln(x+(a/b)^{(1/3)})^*B - \frac{1}{54}b^2/a/(a/b)^{(2/3)}^*\ln(x^2-(a/b)^{(1/3)}x+(a/b)^{(2/3)})^*A - \frac{1}{27}b^3/(a/b)^{(2/3)}^*\ln(x^2-(a/b)^{(1/3)}x+(a/b)^{(2/3)})^*B + \frac{1}{27}b^2/a/(a/b)^{(2/3)}^*3^{(1/2)}^*\arctan(1/3^*3^{(1/2)}^*(2/(a/b)^{(1/3)}x-1))^*A + \frac{2}{27}b^3/(a/b)^{(2/3)}^*3^{(1/2)}^*\arctan(1/3^*3^{(1/2)}^*(2/(a/b)^{(1/3)}x-1))^*B$

maxima [A] time = 1.25, size = 193, normalized size = 0.97

$$-\frac{(7Bab - Ab^2)x^4 + 2(2Ba^2 + Aab)x}{18(ab^4x^6 + 2a^2b^3x^3 + a^3b^2)} + \frac{\sqrt{3}(2Ba + Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2Ba + Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2Ba + Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] $-\frac{1}{18}((7B^*a^*b - A^*b^2)^*x^4 + 2^*(2B^*a^2 + A^*a^*b)^*x)/(a^*b^4x^6 + 2^*a^2b^3x^3 + a^3b^2) + \frac{1}{27}\sqrt{3}(2B^*a + A^*b)^*\arctan\left(\frac{1}{3}\sqrt{3}(2x - (a/b)^{(1/3)})/(a/b)^{(1/3)}\right)/(a^*b^3(a/b)^{(2/3)}) - \frac{1}{54}(2B^*a + A^*b)^*\log(x^2 - x(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a^*b^3(a/b)^{(2/3)}) + \frac{1}{27}(2B^*a + A^*b)^*\log(x + (a/b)^{(1/3)})/(a^*b^3(a/b)^{(2/3)})$

mupad [B] time = 2.56, size = 173, normalized size = 0.87

$$\frac{\ln(b^{1/3}x + a^{1/3})(Ab + 2Ba)}{27a^{5/3}b^{7/3}} - \frac{x(Ab+2Ba) - x^4(Ab-7Ba)}{9b^2} \frac{1}{a^2 + 2abx^3 + b^2x^6} - \frac{\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab + 2Ba)}{27a^{5/3}b^{7/3}} + \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(Ab + 2Ba)}{27a^{5/3}b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(A + B*x^3))/(a + b*x^3)^3,x)`

[Out] $(\log(b^{1/3}*x + a^{1/3})*(A*b + 2*B*a))/(27*a^{5/3}*b^{7/3}) - ((x*(A*b + 2*B*a))/(9*b^2) - (x^4*(A*b - 7*B*a))/(18*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3) - (\log(3^{1/2}*a^{1/3}*1i - 2*b^{1/3}*x + a^{1/3})*((3^{1/2}*1i)/2 + 1/2)*(A*b + 2*B*a))/(27*a^{5/3}*b^{7/3}) + (\log(3^{1/2}*a^{1/3}*1i + 2*b^{1/3}*x - a^{1/3})*((3^{1/2}*1i)/2 - 1/2)*(A*b + 2*B*a))/(27*a^{5/3}*b^{7/3})$

sympy [A] time = 3.25, size = 136, normalized size = 0.68

$$\frac{x^4(Ab^2 - 7Bab) + x(-2Aab - 4Ba^2)}{18a^3b^2 + 36a^2b^3x^3 + 18ab^4x^6} + \text{RootSum}\left(19683t^3a^5b^7 - A^3b^3 - 6A^2Bab^2 - 12AB^2a^2b - 8B^3a^3, \left(t \mapsto t \log\left(\frac{27ta^2b^2}{Ab + 2Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(B*x**3+A)/(b*x**3+a)**3,x)`

[Out] $(x**4*(A*b**2 - 7*B*a*b) + x*(-2*A*a*b - 4*B*a**2))/(18*a**3*b**2 + 36*a**2*b**3*x**3 + 18*a*b**4*x**6) + \text{RootSum}(19683*_t**3*a**5*b**7 - A**3*b**3 - 6*A**2*B*a*b**2 - 12*A*B**2*a**2*b - 8*B**3*a**3, \text{Lambda}(_t, _t*\log(27*_t*a**2*b**2/(A*b + 2*B*a) + x)))$

$$3.101 \quad \int \frac{x(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=201

$$\frac{(aB + 2Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54a^{7/3}b^{5/3}} - \frac{(aB + 2Ab) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{7/3}b^{5/3}} - \frac{(aB + 2Ab) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{5/3}} + \frac{x^2(aB + 2Ab)}{9a^2b(a + bx^3)}$$

Rubi [A] time = 0.12, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {457, 290, 292, 31, 634, 617, 204, 628}

$$\frac{(aB + 2Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54a^{7/3}b^{5/3}} - \frac{(aB + 2Ab) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{7/3}b^{5/3}} - \frac{(aB + 2Ab) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{5/3}} + \frac{x^2(aB + 2Ab)}{9a^2b(a + bx^3)} + \frac{x^2(Ab - aB)}{6ab(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(x*(A + B*x^3))/(a + b*x^3)^3, x]

[Out] ((A*b - a*B)*x^2)/(6*a*b*(a + b*x^3)^2) + ((2*A*b + a*B)*x^2)/(9*a^2*b*(a + b*x^3)) - ((2*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(5/3)) - ((2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/(2*7*a^(7/3)*b^(5/3)) + ((2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(5/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

]

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(A+Bx^3)}{(a+bx^3)^3} dx &= \frac{(Ab-aB)x^2}{6ab(a+bx^3)^2} + \frac{(4Ab+2aB) \int \frac{x}{(a+bx^3)^2} dx}{6ab} \\
&= \frac{(Ab-aB)x^2}{6ab(a+bx^3)^2} + \frac{(2Ab+aB)x^2}{9a^2b(a+bx^3)} + \frac{(2Ab+aB) \int \frac{x}{a+bx^3} dx}{9a^2b} \\
&= \frac{(Ab-aB)x^2}{6ab(a+bx^3)^2} + \frac{(2Ab+aB)x^2}{9a^2b(a+bx^3)} - \frac{(2Ab+aB) \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x} dx}{27a^{7/3}b^{4/3}} + \frac{(2Ab+aB) \int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}}{27a^{7/3}b^{4/3}} \\
&= \frac{(Ab-aB)x^2}{6ab(a+bx^3)^2} + \frac{(2Ab+aB)x^2}{9a^2b(a+bx^3)} - \frac{(2Ab+aB) \log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27a^{7/3}b^{5/3}} + \frac{(2Ab+aB) \int \frac{-\sqrt[3]{a}\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}}{54a^{7/3}b^{5/3}} \\
&= \frac{(Ab-aB)x^2}{6ab(a+bx^3)^2} + \frac{(2Ab+aB)x^2}{9a^2b(a+bx^3)} - \frac{(2Ab+aB) \log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27a^{7/3}b^{5/3}} + \frac{(2Ab+aB) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x)}{54a^{7/3}b^{5/3}} \\
&= \frac{(Ab-aB)x^2}{6ab(a+bx^3)^2} + \frac{(2Ab+aB)x^2}{9a^2b(a+bx^3)} - \frac{(2Ab+aB) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{5/3}} - \frac{(2Ab+aB) \log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27a^{7/3}b^{5/3}}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 178, normalized size = 0.89

$$\frac{(aB+2Ab) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2) - \frac{9a^{4/3}b^{2/3}x^2(aB-Ab)}{(a+bx^3)^2} + \frac{6\sqrt[3]{a}b^{2/3}x^2(aB+2Ab)}{a+bx^3} - 2(aB+2Ab) \log(\sqrt[3]{a}+\sqrt[3]{b}x) - 2\sqrt{3}(aB+2Ab) \tan^{-1}\left(\frac{1-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{54a^{7/3}b^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^3, x]

[Out] ((-9*a^(4/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(1/3)*b^(2/3)*(2*A*b + a*B)*x^2)/(a + b*x^3) - 2*sqrt(3)*(2*A*b + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)] - 2*(2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] + (2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(7/3)*b^(5/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] IntegrateAlgebraic[(x*(A + B*x^3))/(a + b*x^3)^3, x]

fricas [B] time = 0.79, size = 752, normalized size = 3.74

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/54*(6*(B*a^2*b^3 + 2*A*a*b^4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 3* \\ & \text{sqrt}(1/3)*((B*a^2*b^3 + 2*A*a*b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3*b \\ & ^2 + 2*A*a^2*b^3)*x^3)*\text{sqrt}((-a*b^2)^{(1/3)}/a)*\log((2*b^2*x^3 - a*b + 3*\text{sqrt} \\ & (1/3)*(a*b*x + 2*(-a*b^2)^{(2/3)}*x^2 + (-a*b^2)^{(1/3)}*a)*\text{sqrt}((-a*b^2)^{(1/3)} \\ & /a) - 3*(-a*b^2)^{(2/3)}*x)/(b*x^3 + a)) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + \\ & 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^{(2/3)}*\log(b^2*x^2 + (-a* \\ & b^2)^{(1/3)}*b*x + (-a*b^2)^{(2/3)}) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A \\ & *a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^{(2/3)}*\log(b*x - (-a*b^2)^{(1/3)}) \\ &)]/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), 1/54*(6*(B*a^2*b^3 + 2*A*a*b^4)*x^5 - \\ & 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 6*\text{sqrt}(1/3)*((B*a^2*b^3 + 2*A*a*b^4)*x^6 + \\ & B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3*b^2 + 2*A*a^2*b^3)*x^3)*\text{sqrt}(-(-a*b^2)^{(1/3)}/a) \\ & *\arctan(\text{sqrt}(1/3)*(2*b*x + (-a*b^2)^{(1/3)})*\text{sqrt}(-(-a*b^2)^{(1/3)}/a)/b) + ((B*a*b^2 + 2*A*b^3)*x^6 + \\ & B*a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^{(2/3)}*\log(b^2*x^2 + (-a*b^2)^{(1/3)}*b*x + \\ & (-a*b^2)^{(2/3)}) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3) \\ & *(-a*b^2)^{(2/3)}*\log(b*x - (-a*b^2)^{(1/3)})]/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3)] \end{aligned}$$

giac [A] time = 0.21, size = 207, normalized size = 1.03

$$\frac{\sqrt{3}(Ba + 2Ab)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27(-ab^2)^{\frac{1}{3}}a^2b} - \frac{(Ba + 2Ab)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54(-ab^2)^{\frac{1}{3}}a^2b} - \frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^3b} + \frac{2Babx^5 + 4Ab^2x^5 - Ba^2x^2 + 7Aabx^2}{18(bx^3 + a)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/27*\text{sqrt}(3)*(B*a + 2*A*b)*\arctan(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)}) \\ & /((-a*b^2)^{(1/3)}*a^2*b) - 1/54*(B*a + 2*A*b)*\log(x^2 + x*(-a/b)^{(1/3)} \\ & + (-a/b)^{(2/3)})/((-a*b^2)^{(1/3)}*a^2*b) - 1/27*(B*a*(-a/b)^{(1/3)} + 2*A*b*(-a \\ & /b)^{(1/3)})*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^3*b + 1/18*(2*B*a*b*x^5 \\ & + 4*A*b^2*x^5 - B*a^2*x^2 + 7*A*a*b*x^2)/((b*x^3 + a)^2*a^2*b) \end{aligned}$$

maple [A] time = 0.05, size = 251, normalized size = 1.25

$$\frac{2\sqrt{3} A \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b} - \frac{2A \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b} + \frac{A \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b} + \frac{\sqrt{3} B \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^2} - \frac{B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^2} + \frac{B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^2} + \frac{\frac{(2Ab+Ba)x^5}{9a^2} + \frac{(7Ab-Ba)x^2}{18ab}}{(bx^3+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(B*x^3+A)/(b*x^3+a)^3,x)

[Out] (1/9*(2*A*b+B*a)/a^2*x^5+1/18*(7*A*b-B*a)/a/b*x^2)/(b*x^3+a)^2-2/27/a^2/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-1/27/a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B+1/27/a^2/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/54/a/b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+2/27/a^2/b^3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/27/a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B

maxima [A] time = 1.40, size = 195, normalized size = 0.97

$$\frac{2(Bab + 2Ab^2)x^5 - (Ba^2 - 7Aab)x^2}{18(a^2b^3x^6 + 2a^3b^2x^3 + a^4b)} + \frac{\sqrt{3}(Ba + 2Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba + 2Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba + 2Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] 1/18*(2*(B*a*b + 2*A*b^2)*x^5 - (B*a^2 - 7*A*a*b)*x^2)/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3)) + 1/54*(B*a + 2*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(1/3)) - 1/27*(B*a + 2*A*b)*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3))

mupad [B] time = 0.27, size = 175, normalized size = 0.87

$$\frac{\frac{x^5(2Ab+Ba)}{9a^2} + \frac{x^2(7Ab-Ba)}{18ab}}{a^2 + 2abx^3 + b^2x^6} - \frac{\ln(b^{1/3}x + a^{1/3})(2Ab + Ba)}{27a^{7/3}b^{5/3}} - \frac{\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(2Ab + Ba)}{27a^{7/3}b^{5/3}} + \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(2Ab + Ba)}{27a^{7/3}b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(A + B*x^3))/(a + b*x^3)^3,x)

[Out] ((x^5*(2*A*b + B*a))/(9*a^2) + (x^2*(7*A*b - B*a))/(18*a*b))/(a^2 + b^2*x^3 + 2*a*b*x^3) - (log(b^(1/3)*x + a^(1/3))*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3)) - (log(3^(1/2)*a^(1/3)*i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*i)/2 - 1/2))

)*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3))

sympy [A] time = 2.03, size = 153, normalized size = 0.76

$$\frac{x^5(4Ab^2 + 2Bab) + x^2(7Aab - Ba^2)}{18a^4b + 36a^3b^2x^3 + 18a^2b^3x^6} + \text{RootSum}\left(19683t^3a^7b^5 + 8A^3b^3 + 12A^2Bab^2 + 6AB^2a^2b + B^3a^3, \left(t \mapsto t \log\left(\frac{729t^2a^5b^3}{4A^2b^2 + 4ABab + B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x**3+A)/(b*x**3+a)**3,x)

[Out] (x**5*(4*A*b**2 + 2*B*a*b) + x**2*(7*A*a*b - B*a**2))/(18*a**4*b + 36*a**3*b**2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**7*b**5 + 8*A**3*b**3 + 12*A**2*B*a*b**2 + 6*A*B**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(729*_t**2*a**5*b**3/(4*A**2*b**2 + 4*A*B*a*b + B**2*a**2) + x)))

$$3.102 \quad \int \frac{A+Bx^3}{(a+bx^3)^3} dx$$

Optimal. Leaf size=197

$$\frac{(aB + 5Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54a^{8/3}b^{4/3}} + \frac{(aB + 5Ab) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{8/3}b^{4/3}} - \frac{(aB + 5Ab) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{4/3}} + \frac{x(aB + 5Ab)}{18a^2b}$$

Rubi [A] time = 0.10, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {385, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{(aB + 5Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54a^{8/3}b^{4/3}} + \frac{(aB + 5Ab) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{8/3}b^{4/3}} - \frac{(aB + 5Ab) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{4/3}} + \frac{x(aB + 5Ab)}{18a^2b(a + bx^3)} + \frac{x(aB - aB)}{6ab(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(a + b*x^3)^3, x]

[Out] ((A*b - a*B)*x)/(6*a*b*(a + b*x^3)^2) + ((5*A*b + a*B)*x)/(18*a^2*b*(a + b*x^3)) - ((5*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(8/3)*b^(4/3)) + ((5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(8/3)*b^(4/3)) - ((5*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(8/3)*b^(4/3)))

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F

reeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{(a + bx^3)^3} dx &= \frac{(Ab - aB)x}{6ab(a + bx^3)^2} + \frac{(5Ab + aB) \int \frac{1}{(a+bx^3)^2} dx}{6ab} \\
&= \frac{(Ab - aB)x}{6ab(a + bx^3)^2} + \frac{(5Ab + aB)x}{18a^2b(a + bx^3)} + \frac{(5Ab + aB) \int \frac{1}{a+bx^3} dx}{9a^2b} \\
&= \frac{(Ab - aB)x}{6ab(a + bx^3)^2} + \frac{(5Ab + aB)x}{18a^2b(a + bx^3)} + \frac{(5Ab + aB) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{27a^{8/3}b} + \frac{(5Ab + aB) \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{27a^{8/3}b} \\
&= \frac{(Ab - aB)x}{6ab(a + bx^3)^2} + \frac{(5Ab + aB)x}{18a^2b(a + bx^3)} + \frac{(5Ab + aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{8/3}b^{4/3}} - \frac{(5Ab + aB) \int \frac{-\sqrt[3]{a}\sqrt[3]{b}x + 2\sqrt[3]{a}}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{54a^{8/3}b^{4/3}} \\
&= \frac{(Ab - aB)x}{6ab(a + bx^3)^2} + \frac{(5Ab + aB)x}{18a^2b(a + bx^3)} + \frac{(5Ab + aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{8/3}b^{4/3}} - \frac{(5Ab + aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3})}{54a^{8/3}b^{4/3}} \\
&= \frac{(Ab - aB)x}{6ab(a + bx^3)^2} + \frac{(5Ab + aB)x}{18a^2b(a + bx^3)} - \frac{(5Ab + aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{4/3}} + \frac{(5Ab + aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{8/3}b^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 175, normalized size = 0.89

$$\frac{-(aB + 5Ab) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) - \frac{9a^{5/3}\sqrt[3]{b}x(aB - Ab)}{(a+bx^3)^2} + \frac{3a^{2/3}\sqrt[3]{b}x(aB + 5Ab)}{a+bx^3} + 2(aB + 5Ab) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 2\sqrt{3}(aB + 5Ab) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{54a^{8/3}b^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(a + b*x^3)^3, x]

[Out] ((-9*a^(5/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(2/3)*b^(1/3)*(5*A*b + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(5*A*b + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] - (5*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(8/3)*b^(4/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{(a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(a + b*x^3)^3,x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(a + b*x^3)^3, x]

fricas [B] time = 0.64, size = 743, normalized size = 3.77

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] [1/54*(3*(B*a^3*b^2 + 5*A*a^2*b^3)*x^4 + 3*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b^4)*x^6 + B*a^4*b + 5*A*a^3*b^2 + 2*(B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a) - ((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b^2)*x/(a^4*b^4*x^6 + 2*a^5*b^3*x^3 + a^6*b^2), 1/54*(3*(B*a^3*b^2 + 5*A*a^2*b^3)*x^4 + 6*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b^4)*x^6 + B*a^4*b + 5*A*a^3*b^2 + 2*(B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b^2)*x/(a^4*b^4*x^6 + 2*a^5*b^3*x^3 + a^6*b^2)]

giac [A] time = 0.20, size = 180, normalized size = 0.91

$$\frac{\sqrt{3}(Ba + 5Ab) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27(-ab^2)^{\frac{2}{3}}a^2} - \frac{(Ba + 5Ab) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54(-ab^2)^{\frac{2}{3}}a^2} - \frac{(Ba + 5Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^3b} + \frac{Babx^4 + 5Ab^2x^4 - 2Ba^2x + 8Aabx}{18(bx^3 + a)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")

[Out] -1/27*sqrt(3)*(B*a + 5*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(B*a + 5*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(B*a + 5*A*b)*(-a/b)^(1/3)*log(a*b*x - (-a/b)^(1/3))/(a^3*b) + 1/18*(B*a*b*x^4 + 5*A*b^2*x^4 - 2*B*a^2*x + 8*A*a*b*x)/((b*x^3 + a)^2*a^2*b)

maple [A] time = 0.06, size = 249, normalized size = 1.26

$$\frac{5\sqrt{3} A \arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b} + \frac{5A \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b} - \frac{5A \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b} + \frac{\sqrt{3} B \arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}ab^2} + \frac{B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}ab^2} - \frac{B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}ab^2} + \frac{\frac{(5Ab+Ba)x^4}{18a^2} + \frac{(4Ab-Ba)x}{9ab}}{(bx^3+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/(b*x^3+a)^3,x)

[Out] (1/18*(5*A*b+B*a)/a^2*x^4+1/9*(4*A*b-B*a)/a/b*x)/(b*x^3+a)^2+5/27/a^2/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A+1/27/a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*B-5/54/a^2/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/54/a/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+5/27/a^2/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/27/a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B

maxima [A] time = 1.37, size = 192, normalized size = 0.97

$$\frac{(Bab + 5Ab^2)x^4 - 2(Ba^2 - 4Aab)x}{18(a^2b^3x^6 + 2a^3b^2x^3 + a^4b)} + \frac{\sqrt{3}(Ba + 5Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba + 5Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba + 5Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] 1/18*((B*a*b + 5*A*b^2)*x^4 - 2*(B*a^2 - 4*A*a*b)*x)/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(B*a + 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) - 1/54*(B*a + 5*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) + 1/27*(B*a + 5*A*b)*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))

mupad [B] time = 0.26, size = 173, normalized size = 0.88

$$\frac{x^4 \frac{(5Ab+Ba)}{18a^2} + \frac{x(4Ab-Ba)}{9ab}}{a^2 + 2abx^3 + b^2x^6} + \frac{\ln(b^{1/3}x + a^{1/3})(5Ab+Ba)}{27a^{8/3}b^{4/3}} - \frac{\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(5Ab+Ba)}{27a^{8/3}b^{4/3}} + \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(5Ab+Ba)}{27a^{8/3}b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(a + b*x^3)^3,x)

[Out] ((x^4*(5*A*b + B*a))/(18*a^2) + (x*(4*A*b - B*a))/(9*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3) + (log(b^(1/3)*x + a^(1/3))*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3)) - (log(3^(1/2)*a^(1/3)*i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*i)/2 + 1/2)*

$$\frac{(5Ab + Ba)}{(27a^{8/3}b^{4/3})} + (\log(3^{1/2}a^{1/3}1i + 2b^{1/3}x - a^{1/3})) \cdot ((3^{1/2}1i)/2 - 1/2) \cdot \frac{(5Ab + Ba)}{(27a^{8/3}b^{4/3})}$$

sympy [A] time = 1.51, size = 133, normalized size = 0.68

$$\frac{x^4(5Ab^2 + Bab) + x(8Aab - 2Ba^2)}{18a^4b + 36a^3b^2x^3 + 18a^2b^3x^6} + \text{RootSum}\left(19683t^3a^8b^4 - 125A^3b^3 - 75A^2Bab^2 - 15AB^2a^2b - B^3a^3, \left(t \mapsto t \log\left(\frac{27ta^3b}{5Ab + Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/(b*x**3+a)**3,x)

[Out] (x**4*(5*A*b**2 + B*a*b) + x*(8*A*a*b - 2*B*a**2))/(18*a**4*b + 36*a**3*b**2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**8*b**4 - 125*A**3*b**3 - 75*A**2*B*a*b**2 - 15*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(27*_t*a**3*b/(5*A*b + B*a) + x)))

$$3.103 \quad \int \frac{A+Bx^3}{x^2(a+bx^3)^3} dx$$

Optimal. Leaf size=227

$$-\frac{(7Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{27a^{10/3} b^{2/3}} + \frac{2(7Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{10/3} b^{2/3}} + \frac{2(7Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{9\sqrt{3} a^{10/3} b^{2/3}} - \frac{2(7Ab - aB)}{9\sqrt{3} a^{10/3} b^{2/3}}$$

Rubi [A] time = 0.12, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {457, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{(7Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{27a^{10/3} b^{2/3}} + \frac{2(7Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{10/3} b^{2/3}} + \frac{2(7Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{9\sqrt{3} a^{10/3} b^{2/3}} + \frac{7Ab - aB}{18a^2 bx (a + bx^3)} - \frac{2(7Ab - aB)}{9a^3 bx} + \frac{Ab - aB}{6abx (a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^3), x]

[Out] (-2*(7*A*b - a*B))/(9*a^3*b*x) + (A*b - a*B)/(6*a*b*x*(a + b*x^3)^2) + (7*A*b - a*B)/(18*a^2*b*x*(a + b*x^3)) + (2*(7*A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(2/3)) + (2*(7*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(2/3)) - ((7*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(10/3)*b^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m + n*(p+1) + 1)/(a*n*(p+1)), Int[(c*x)^m*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^2 (a + bx^3)^3} dx &= \frac{Ab - aB}{6abx (a + bx^3)^2} + \frac{(7Ab - aB) \int \frac{1}{x^2(a+bx^3)^2} dx}{6ab} \\
&= \frac{Ab - aB}{6abx (a + bx^3)^2} + \frac{7Ab - aB}{18a^2bx (a + bx^3)} + \frac{(2(7Ab - aB)) \int \frac{1}{x^2(a+bx^3)} dx}{9a^2b} \\
&= -\frac{2(7Ab - aB)}{9a^3bx} + \frac{Ab - aB}{6abx (a + bx^3)^2} + \frac{7Ab - aB}{18a^2bx (a + bx^3)} - \frac{(2(7Ab - aB)) \int \frac{x}{a+bx^3} dx}{9a^3} \\
&= -\frac{2(7Ab - aB)}{9a^3bx} + \frac{Ab - aB}{6abx (a + bx^3)^2} + \frac{7Ab - aB}{18a^2bx (a + bx^3)} + \frac{(2(7Ab - aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{27a^{10/3}\sqrt[3]{b}} - \frac{(2(7Ab - aB)) \int \frac{x}{a+bx^3} dx}{9a^3} \\
&= -\frac{2(7Ab - aB)}{9a^3bx} + \frac{Ab - aB}{6abx (a + bx^3)^2} + \frac{7Ab - aB}{18a^2bx (a + bx^3)} + \frac{2(7Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{10/3}b^{2/3}} - \frac{(2(7Ab - aB)) \int \frac{x}{a+bx^3} dx}{9a^3} \\
&= -\frac{2(7Ab - aB)}{9a^3bx} + \frac{Ab - aB}{6abx (a + bx^3)^2} + \frac{7Ab - aB}{18a^2bx (a + bx^3)} + \frac{2(7Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{10/3}b^{2/3}} - \frac{(2(7Ab - aB)) \int \frac{x}{a+bx^3} dx}{9a^3} \\
&= -\frac{2(7Ab - aB)}{9a^3bx} + \frac{Ab - aB}{6abx (a + bx^3)^2} + \frac{7Ab - aB}{18a^2bx (a + bx^3)} + \frac{2(7Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{2/3}} + \frac{(2(7Ab - aB)) \int \frac{x}{a+bx^3} dx}{9a^3}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 193, normalized size = 0.85

$$\frac{2(aB-7Ab) \log\left(\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2}{b^{2/3}}\right) + \frac{9a^{4/3} x^2 (aB - Ab)}{(a + bx^3)^2} + \frac{4(7Ab - aB) \log\left(\frac{\sqrt[3]{a} + \sqrt[3]{b} x}{b^{2/3}}\right)}{b^{2/3}} + \frac{4\sqrt{3}(7Ab - aB) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} + \frac{6\sqrt[3]{a} x^2 (2aB - 5Ab)}{a + bx^3} - \frac{54\sqrt[3]{a} A}{x}}{54a^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^3), x]

[Out] ((-54*a^(1/3)*A)/x + (9*a^(4/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(1/3)*(-5*A*b + 2*a*B)*x^2)/(a + b*x^3) + (4*sqrt[3]*(7*A*b - a*B)*ArcTan[(1

$$- (2*b^{(1/3)*x}/a^{(1/3)})/\text{Sqrt}[3]]/b^{(2/3)} + (4*(7*A*b - a*B)*\text{Log}[a^{(1/3)} + b^{(1/3)*x}]/b^{(2/3)} + (2*(-7*A*b + a*B)*\text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}]/b^{(2/3)})/(54*a^{(10/3)})$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^2 (a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^2*(a + b*x^3)^3), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^2*(a + b*x^3)^3), x]

fricas [B] time = 0.69, size = 776, normalized size = 3.42

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="fricas")

[Out] [1/54*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 - 7*A*a^2*b^3)*x^3 - 6*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b^2 - 7*A*a^2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*((B*a*b^2 - 7*A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a*b^2 - 7*A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^6*b^2*x), 1/54*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 - 7*A*a^2*b^3)*x^3 - 12*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b^2 - 7*A*a^2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + 2*((B*a*b^2 - 7*A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a*b^2 - 7*A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^6*b^2*x)]

giac [A] time = 0.20, size = 204, normalized size = 0.90

$$\frac{2\sqrt{3}(Ba - 7Ab) \arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27(-ab^2)^{\frac{1}{3}}a^3} - \frac{(Ba - 7Ab) \log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27(-ab^2)^{\frac{1}{3}}a^3} - \frac{2\left(Ba\left(\frac{a}{b}\right)^{\frac{1}{3}} - 7Ab\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^4} - \frac{A}{a^3x} + \frac{4Babx^5 - 10Ab^2x^5 + 7Ba^2x^2 - 13Aabx^2}{18(bx^3 + a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="giac")

[Out] $\frac{2\sqrt{3}\sqrt{3}(B*a - 7*A*b)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{1/3})/(-a/b)^{1/3})}{((-a*b^2)^{1/3}*a^3) - 1/27*(B*a - 7*A*b)*\log(x^2 + x*(-a/b)^{1/3} + (-a/b)^{2/3})} + \frac{1/27*(B*a - 7*A*b)*\log(x^2 + x*(-a/b)^{1/3} + (-a/b)^{2/3})}{((-a*b^2)^{1/3}*a^3) - 2/27*(B*a*(-a/b)^{1/3} - 7*A*b*(-a/b)^{1/3})} + \frac{(-a/b)^{1/3}*\log(\text{abs}(x - (-a/b)^{1/3}))}{a^4 - A/(a^3*x) + 1/18*(4*B*a*b*x^5 - 10*A*b^2*x^5 + 7*B*a^2*x^2 - 13*A*a*b*x^2)/((b*x^3 + a)^2*a^3)}$

maple [A] time = 0.06, size = 281, normalized size = 1.24

$$\frac{5Ab^2x^5}{9(bx^3+a)^2a^3} + \frac{2Bbx^5}{9(bx^3+a)^2a^2} - \frac{13Abx^2}{18(bx^3+a)^2a^2} + \frac{7Bx^2}{18(bx^3+a)^2a} - \frac{14\sqrt{3}A\arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{b}\right)^{\frac{1}{3}}}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3} + \frac{14A\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - 7A\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3} + \frac{2\sqrt{3}B\arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{b}\right)^{\frac{1}{3}}}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b} - \frac{2B\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + B\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b} - \frac{A}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^2/(b*x^3+a)^3,x)

[Out] $-\frac{5}{9}/a^3/(b*x^3+a)^2*A*x^5*b^2+2/9/a^2/(b*x^3+a)^2*B*x^5*b-13/18/a^2/(b*x^3+a)^2*A*x^2*b+7/18/a/(b*x^3+a)^2*B*x^2+14/27/a^3*A/(a/b)^{1/3}*\ln(x+(a/b)^{1/3})-7/27/a^3*A/(a/b)^{1/3}*\ln(x^2-(a/b)^{1/3}*x+(a/b)^{2/3})-14/27/a^3*A*3^{1/2}/(a/b)^{1/3}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))-2/27/a^2*B/b/(a/b)^{1/3}*\ln(x+(a/b)^{1/3})+1/27/a^2*B/b/(a/b)^{1/3}*\ln(x^2-(a/b)^{1/3}*x+(a/b)^{2/3})+2/27/a^2*B*3^{1/2}/b/(a/b)^{1/3}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))-A/a^3/x$

maxima [A] time = 1.41, size = 199, normalized size = 0.88

$$\frac{4(Bab - 7Ab^2)x^6 + 7(Ba^2 - 7Aab)x^3 - 18Aa^2}{18(a^3b^2x^7 + 2a^4bx^4 + a^5x)} + \frac{2\sqrt{3}(Ba - 7Ab)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba - 7Ab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2(Ba - 7Ab)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="maxima")

[Out] $\frac{1}{18}*(4*(B*a*b - 7*A*b^2)*x^6 + 7*(B*a^2 - 7*A*a*b)*x^3 - 18*A*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x) + \frac{2\sqrt{3}\sqrt{3}(B*a - 7*A*b)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{1/3})/(a/b)^{1/3})}{(a^3*b*(a/b)^{1/3})} + \frac{1}{27}*(B*a - 7*A*b)*\log(x^2 - x*(a/b)^{1/3} + (a/b)^{2/3})/(a^3*b*(a/b)^{1/3}) - \frac{2}{27}*(B*a - 7*A*b)*\log(x + (a/b)^{1/3})/(a^3*b*(a/b)^{1/3})$

mupad [B] time = 2.60, size = 185, normalized size = 0.81

$$\frac{2\ln(b^{1/3}x + a^{1/3})(7Ab - Ba)}{27a^{10/3}b^{2/3}} - \frac{\frac{A}{a} + \frac{7x^3(7Ab - Ba)}{18a^2} + \frac{2bx^6(7Ab - Ba)}{9a^3}}{a^2x + 2abx^4 + b^2x^7} + \frac{2\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(7Ab - Ba)}{27a^{10/3}b^{2/3}} - \frac{2\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(7Ab - Ba)}{27a^{10/3}b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)/(x^2*(a + b*x^3)^3), x)`

[Out] $(2*\log(b^{(1/3)}*x + a^{(1/3)})*(7*A*b - B*a))/(27*a^{(10/3)}*b^{(2/3)}) - (A/a + (7*x^3*(7*A*b - B*a))/(18*a^2) + (2*b*x^6*(7*A*b - B*a))/(9*a^3))/(a^2*x + b^2*x^7 + 2*a*b*x^4) + (2*\log(3^{(1/2)}*a^{(1/3)}*1i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*1i)/2 - 1/2)*(7*A*b - B*a))/(27*a^{(10/3)}*b^{(2/3)}) - (2*\log(3^{(1/2)}*a^{(1/3)}*1i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*1i)/2 + 1/2)*(7*A*b - B*a))/(27*a^{(10/3)}*b^{(2/3)})$

sympy [A] time = 1.90, size = 162, normalized size = 0.71

$$\frac{-18Aa^2 + x^6(-28Ab^2 + 4Bab) + x^3(-49Aab + 7Ba^2)}{18a^5x + 36a^4bx^4 + 18a^3b^2x^7} + \text{RootSum}\left(19683t^3a^{10}b^2 - 2744A^3b^3 + 1176A^2Bab^2 - 168AB^2a^2b + 8B^3a^3, \left(t \mapsto t \log\left(\frac{729t^2a^7b}{196A^2b^2 - 56ABab + 4B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)/x**2/(b*x**3+a)**3, x)`

[Out] $(-18*A*a**2 + x**6*(-28*A*b**2 + 4*B*a*b) + x**3*(-49*A*a*b + 7*B*a**2))/(18*a**5*x + 36*a**4*b*x**4 + 18*a**3*b**2*x**7) + \text{RootSum}(19683*_t**3*a**10*b**2 - 2744*A**3*b**3 + 1176*A**2*B*a*b**2 - 168*A*B**2*a**2*b + 8*B**3*a**3, \text{Lambda}(_t, _t*\log(729*_t**2*a**7*b/(196*A**2*b**2 - 56*A*B*a*b + 4*B**2*a**2) + x)))$

$$3.104 \quad \int \frac{A+Bx^3}{x^3(a+bx^3)^3} dx$$

Optimal. Leaf size=227

$$\frac{5(4Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54a^{11/3} \sqrt[3]{b}} - \frac{5(4Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{11/3} \sqrt[3]{b}} + \frac{5(4Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{9\sqrt{3} a^{11/3} \sqrt[3]{b}} - \frac{5(4A}{18$$

Rubi [A] time = 0.13, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {457, 290, 325, 200, 31, 634, 617, 204, 628}

$$\frac{5(4Ab - aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54a^{11/3} \sqrt[3]{b}} + \frac{4Ab - aB}{9a^2 b x^2 (a + b x^3)} - \frac{5(4Ab - aB)}{18a^3 b x^2} - \frac{5(4Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{11/3} \sqrt[3]{b}} + \frac{5(4Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{9\sqrt{3} a^{11/3} \sqrt[3]{b}} + \frac{Ab - aB}{6abx^2 (a + b x^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^3), x]

[Out] (-5*(4*A*b - a*B))/(18*a^3*b*x^2) + (A*b - a*B)/(6*a*b*x^2*(a + b*x^3)^2) + (4*A*b - a*B)/(9*a^2*b*x^2*(a + b*x^3)) + (5*(4*A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(1/3)) - (5*(4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(1/3)) + (5*(4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx^3}{x^3(a + bx^3)^3} dx &= \frac{Ab - aB}{6abx^2(a + bx^3)^2} + \frac{(8Ab - 2aB) \int \frac{1}{x^3(a + bx^3)^2} dx}{6ab} \\
 &= \frac{Ab - aB}{6abx^2(a + bx^3)^2} + \frac{4Ab - aB}{9a^2bx^2(a + bx^3)} + \frac{(5(4Ab - aB)) \int \frac{1}{x^3(a + bx^3)} dx}{9a^2b} \\
 &= -\frac{5(4Ab - aB)}{18a^3bx^2} + \frac{Ab - aB}{6abx^2(a + bx^3)^2} + \frac{4Ab - aB}{9a^2bx^2(a + bx^3)} - \frac{(5(4Ab - aB)) \int \frac{1}{a + bx^3} dx}{9a^3} \\
 &= -\frac{5(4Ab - aB)}{18a^3bx^2} + \frac{Ab - aB}{6abx^2(a + bx^3)^2} + \frac{4Ab - aB}{9a^2bx^2(a + bx^3)} - \frac{(5(4Ab - aB)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{27a^{11/3}} \\
 &= -\frac{5(4Ab - aB)}{18a^3bx^2} + \frac{Ab - aB}{6abx^2(a + bx^3)^2} + \frac{4Ab - aB}{9a^2bx^2(a + bx^3)} - \frac{5(4Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{11/3}\sqrt[3]{b}} \\
 &= -\frac{5(4Ab - aB)}{18a^3bx^2} + \frac{Ab - aB}{6abx^2(a + bx^3)^2} + \frac{4Ab - aB}{9a^2bx^2(a + bx^3)} - \frac{5(4Ab - aB) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{11/3}\sqrt[3]{b}} + \\
 &= -\frac{5(4Ab - aB)}{18a^3bx^2} + \frac{Ab - aB}{6abx^2(a + bx^3)^2} + \frac{4Ab - aB}{9a^2bx^2(a + bx^3)} + \frac{5(4Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt[3]{b}}
 \end{aligned}$$

Mathematica [A] time = 0.26, size = 189, normalized size = 0.83

$$\frac{5(4Ab - aB) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{\sqrt[3]{b}} + \frac{9a^{5/3}x(aB - Ab)}{(a + bx^3)^2} + \frac{3a^{2/3}x(5aB - 11Ab)}{a + bx^3} - \frac{27a^{2/3}A}{x^2} + \frac{10(aB - 4Ab) \log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{10\sqrt{3}(4Ab - aB) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}$$

$$54a^{11/3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^3), x]

```
[Out] ((-27*a^(2/3)*A)/x^2 + (9*a^(5/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(2/3)*(-11*A*b + 5*a*B)*x)/(a + b*x^3) + (10*Sqrt[3]*(4*A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (10*(-4*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (5*(4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(54*a^(11/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^3(a + bx^3)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^3*(a + b*x^3)^3), x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^3*(a + b*x^3)^3), x]
```

fricas [B] time = 0.52, size = 812, normalized size = 3.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b + 24*(B*a^4*b - 4*A*a^3*b^2)*x^3 - 15*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*x^8 + 2*(B*a^3*b^2 - 4*A*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a) - 5*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*((-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*((-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^5*b^3*x^8 + 2*a^6*b^2*x^5 + a^7*b*x^2), 1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b + 24*(B*a^4*b - 4*A*a^3*b^2)*x^3 + 30*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*x^8 + 2*(B*a^3*b^2 - 4*A*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 5*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*((-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*((-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^5*b^3*x^8 + 2*a^6*b^2*x^5 + a^7*b*x^2)]
```

giac [A] time = 0.19, size = 209, normalized size = 0.92

$$\frac{5(Ba - 4Ab)\left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^4} + \frac{5\sqrt{3}\left((-ab^2)^{\frac{1}{3}}Ba - 4(-ab^2)^{\frac{1}{3}}Ab\right) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^4b} + \frac{5\left((-ab^2)^{\frac{1}{3}}Ba - 4(-ab^2)^{\frac{1}{3}}Ab\right) \log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^4b} + \frac{5Babx^6 - 20Ab^2x^6 + 8Ba^2x^3 - 32Aabx^3 - 9Aa^2}{18(bx^4 + ax)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="giac")

[Out] $-5/27*(B*a - 4*A*b)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^4 + 5/27*\text{sqrt}(3)*((-a*b^2)^{(1/3)}*B*a - 4*(-a*b^2)^{(1/3)}*A*b)*\arctan(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^4*b) + 5/54*((-a*b^2)^{(1/3)}*B*a - 4*(-a*b^2)^{(1/3)}*A*b)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^4*b) + 1/18*(5*B*a*b*x^6 - 20*A*b^2*x^6 + 8*B*a^2*x^3 - 32*A*a*b*x^3 - 9*A*a^2)/((b*x^4 + a*x)^2*a^3)$

maple [A] time = 0.05, size = 277, normalized size = 1.22

$$\frac{11Ab^2x^4}{18(bx^3+a)^2a^3} + \frac{5Bbx^4}{18(bx^3+a)^2a^2} - \frac{7Abx}{9(bx^3+a)^2a^2} + \frac{4Bx}{9(bx^3+a)^2a} - \frac{20\sqrt{3}A \arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3} - \frac{20A \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3} + \frac{10A \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3} + \frac{5\sqrt{3}B \arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b} + \frac{5B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b} - \frac{5B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b} - \frac{A}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^3/(b*x^3+a)^3,x)

[Out] $-11/18/a^3/(b*x^3+a)^2*A*x^4*b^2+5/18/a^2/(b*x^3+a)^2*B*x^4*b-7/9/a^2/(b*x^3+a)^2*b*A*x+4/9/a/(b*x^3+a)^2*B*x-20/27/a^3*A/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})+10/27/a^3*A/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})-20/27/a^3*A/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))+5/27/a^2*B/b/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})-5/54/a^2*B/b/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+5/27/a^2*B/b/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))-1/2*A/a^3/x^2$

maxima [A] time = 1.20, size = 201, normalized size = 0.89

$$\frac{5(Bab - 4Ab^2)x^6 + 8(Ba^2 - 4Aab)x^3 - 9Aa^2}{18(a^3b^2x^8 + 2a^4bx^5 + a^5x^2)} + \frac{5\sqrt{3}(Ba - 4Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5(Ba - 4Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5(Ba - 4Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="maxima")

[Out] $1/18*(5*(B*a*b - 4*A*b^2)*x^6 + 8*(B*a^2 - 4*A*a*b)*x^3 - 9*A*a^2)/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2) + 5/27*\text{sqrt}(3)*(B*a - 4*A*b)*\arctan(1/3*\text{sqrt}($

$$3) * (2 * x - (a/b)^{(1/3)}) / (a/b)^{(1/3)} / (a^3 * b * (a/b)^{(2/3)}) - 5/54 * (B * a - 4 * A * b) * \log(x^2 - x * (a/b)^{(1/3)} + (a/b)^{(2/3)}) / (a^3 * b * (a/b)^{(2/3)}) + 5/27 * (B * a - 4 * A * b) * \log(x + (a/b)^{(1/3)}) / (a^3 * b * (a/b)^{(2/3)})$$

mupad [B] time = 2.58, size = 188, normalized size = 0.83

$$-\frac{A}{2a} + \frac{4x^3(4Ab-Ba)}{9a^2} + \frac{5bx^6(4Ab-Ba)}{18a^3} - \frac{5 \ln(b^{1/3}x + a^{1/3})(4Ab-Ba)}{27a^{11/3}b^{1/3}} + \frac{5 \ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(4Ab-Ba)}{27a^{11/3}b^{1/3}} - \frac{5 \ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(4Ab-Ba)}{27a^{11/3}b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^3*(a + b*x^3)^3), x)

[Out] (5*log(3^(1/2)*a^(1/3)*i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*i)/2 + 1/2)*(4*A*b - B*a)/(27*a^(11/3)*b^(1/3)) - (5*log(b^(1/3)*x + a^(1/3))*(4*A*b - B*a))/(27*a^(11/3)*b^(1/3)) - (A/(2*a) + (4*x^3*(4*A*b - B*a))/(9*a^2) + (5*b*x^6*(4*A*b - B*a))/(18*a^3))/(a^2*x^2 + b^2*x^8 + 2*a*b*x^5) - (5*log(3^(1/2)*a^(1/3)*i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*i)/2 - 1/2)*(4*A*b - B*a))/(27*a^(11/3)*b^(1/3))

sympy [A] time = 1.73, size = 143, normalized size = 0.63

$$\frac{-9Aa^2 + x^6(-20Ab^2 + 5Bab) + x^3(-32Aab + 8Ba^2)}{18a^5x^2 + 36a^4bx^5 + 18a^3b^2x^8} + \text{RootSum}\left(19683t^3a^{11}b + 8000A^3b^3 - 6000A^2Bab^2 + 1500AB^2a^2b - 125B^3a^3, \left(t \mapsto t \log\left(\frac{27ta^4}{-20Ab + 5Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**3, x)

[Out] (-9*A*a**2 + x**6*(-20*A*b**2 + 5*B*a*b) + x**3*(-32*A*a*b + 8*B*a**2))/(18*a**5*x**2 + 36*a**4*b*x**5 + 18*a**3*b**2*x**8) + RootSum(19683*_t**3*a**11*b + 8000*A**3*b**3 - 6000*A**2*B*a*b**2 + 1500*A*B**2*a**2*b - 125*B**3*a**3, Lambda(_t, _t*log(27*_t*a**4/(-20*A*b + 5*B*a) + x)))

$$3.105 \quad \int \frac{A+Bx^3}{x^5(a+bx^3)^3} dx$$

Optimal. Leaf size=246

$$\frac{7\sqrt[3]{b}(5Ab-2aB)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{54a^{13/3}} - \frac{7\sqrt[3]{b}(5Ab-2aB)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27a^{13/3}} - \frac{7\sqrt[3]{b}(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{13/3}}$$

Rubi [A] time = 0.16, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {457, 290, 325, 292, 31, 634, 617, 204, 628}

$$\frac{7\sqrt[3]{b}(5Ab-2aB)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{54a^{13/3}} + \frac{5Ab-2aB}{9a^2bx^4(a+bx^3)} - \frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} - \frac{7\sqrt[3]{b}(5Ab-2aB)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27a^{13/3}} - \frac{7\sqrt[3]{b}(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{13/3}} + \frac{Ab-aB}{6abx^4(a+bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^3), x]

[Out] $(-7*(5*A*b - 2*a*B))/(36*a^3*b*x^4) + (7*(5*A*b - 2*a*B))/(9*a^4*x) + (A*b - a*B)/(6*a*b*x^4*(a + b*x^3)^2) + (5*A*b - 2*a*B)/(9*a^2*b*x^4*(a + b*x^3)) - (7*b^{(1/3)}*(5*A*b - 2*a*B)*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)}*x)/(\text{Sqrt}[3]*a^{(1/3)})])/(9*\text{Sqrt}[3]*a^{(13/3)}) - (7*b^{(1/3)}*(5*A*b - 2*a*B)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x])/(27*a^{(13/3)}) + (7*b^{(1/3)}*(5*A*b - 2*a*B)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(54*a^{(13/3)})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m+n*(p+1)+1)/(a*n*(p+1)), Int[(c*x)^m*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

]

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx^3}{x^5 (a + bx^3)^3} dx &= \frac{Ab - aB}{6abx^4 (a + bx^3)^2} + \frac{(10Ab - 4aB) \int \frac{1}{x^5 (a + bx^3)^2} dx}{6ab} \\
 &= \frac{Ab - aB}{6abx^4 (a + bx^3)^2} + \frac{5Ab - 2aB}{9a^2bx^4 (a + bx^3)} + \frac{(7(5Ab - 2aB)) \int \frac{1}{x^5 (a + bx^3)} dx}{9a^2b} \\
 &= -\frac{7(5Ab - 2aB)}{36a^3bx^4} + \frac{Ab - aB}{6abx^4 (a + bx^3)^2} + \frac{5Ab - 2aB}{9a^2bx^4 (a + bx^3)} - \frac{(7(5Ab - 2aB)) \int \frac{1}{x^2 (a + bx^3)} dx}{9a^3} \\
 &= -\frac{7(5Ab - 2aB)}{36a^3bx^4} + \frac{7(5Ab - 2aB)}{9a^4x} + \frac{Ab - aB}{6abx^4 (a + bx^3)^2} + \frac{5Ab - 2aB}{9a^2bx^4 (a + bx^3)} + \frac{(7b(5Ab - 2aB)) \int \frac{1}{x^2 (a + bx^3)} dx}{9a^3} \\
 &= -\frac{7(5Ab - 2aB)}{36a^3bx^4} + \frac{7(5Ab - 2aB)}{9a^4x} + \frac{Ab - aB}{6abx^4 (a + bx^3)^2} + \frac{5Ab - 2aB}{9a^2bx^4 (a + bx^3)} - \frac{(7b^{2/3}(5Ab - 2aB)) \int \frac{1}{x^2 (a + bx^3)} dx}{27} \\
 &= -\frac{7(5Ab - 2aB)}{36a^3bx^4} + \frac{7(5Ab - 2aB)}{9a^4x} + \frac{Ab - aB}{6abx^4 (a + bx^3)^2} + \frac{5Ab - 2aB}{9a^2bx^4 (a + bx^3)} - \frac{7\sqrt[3]{b}(5Ab - 2aB)}{27} \\
 &= -\frac{7(5Ab - 2aB)}{36a^3bx^4} + \frac{7(5Ab - 2aB)}{9a^4x} + \frac{Ab - aB}{6abx^4 (a + bx^3)^2} + \frac{5Ab - 2aB}{9a^2bx^4 (a + bx^3)} - \frac{7\sqrt[3]{b}(5Ab - 2aB)}{27} \\
 &= -\frac{7(5Ab - 2aB)}{36a^3bx^4} + \frac{7(5Ab - 2aB)}{9a^4x} + \frac{Ab - aB}{6abx^4 (a + bx^3)^2} + \frac{5Ab - 2aB}{9a^2bx^4 (a + bx^3)} - \frac{7\sqrt[3]{b}(5Ab - 2aB)}{27}
 \end{aligned}$$

Mathematica [A] time = 0.27, size = 214, normalized size = 0.87

$$\frac{14\sqrt[3]{b}(5Ab - 2aB) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) - \frac{18a^4b^2(aB - Ab)}{(a + bx^3)^2} - \frac{27a^4bA}{x^4} - \frac{12\sqrt[3]{a}bx^2(5aB - 8Ab)}{a + bx^3} - \frac{108\sqrt[3]{a}(aB - 3Ab)}{x} + 28\sqrt[3]{b}(2aB - 5Ab) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 28\sqrt{3}\sqrt[3]{b}(5Ab - 2aB) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{a}x}{\sqrt{3}}}{\sqrt{3}}\right)}{108a^{13/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^3), x]

[Out] $\left(\frac{-27a^{4/3}A}{x^4} - \frac{(108a^{1/3})(-3Ab + a^2B)}{x} - \frac{18a^{4/3}b(-Ab + a^2B)x^2}{(a + bx^3)^2} - \frac{12a^{1/3}b(-8Ab + 5a^2B)x^2}{(a + bx^3)^3} - 28\sqrt{3}b^{1/3}(5Ab - 2a^2B)\text{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt{3}}\right] + 28b^{1/3}(-5Ab + 2a^2B)\text{Log}\left[a^{1/3} + b^{1/3}x\right] + 14b^{1/3}(5Ab - 2a^2B)\text{Log}\left[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2\right]\right)/(108a^{13/3})$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^5(a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^5*(a + b*x^3)^3), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^5*(a + b*x^3)^3), x]

fricas [A] time = 0.92, size = 366, normalized size = 1.49

$$\frac{84(2Ba^2 - 5Aa^2)^2 + 147(2Ba^2 - 5Aa^2)^2 + 27Aa^4 + 54(2Ba^2 - 5Aa^2)^2 + 28\sqrt{3}(2Ba^2 - 5Aa^2)^2 + 2(2Ba^2 - 5Aa^2)^2 + (2Ba^2 - 5Aa^2)^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{a} + \frac{1}{\sqrt{3}}\right)}{\frac{108(a^{2/3}x^2 + 2a^{1/3}x + a^{1/3})}{108(a^{2/3}x^2 + 2a^{1/3}x + a^{1/3})}}\right) - 14(2Ba^2 - 5Aa^2)^2 + 2(2Ba^2 - 5Aa^2)^2 + (2Ba^2 - 5Aa^2)^2 \log\left(\frac{a^2 - a\left(\frac{2x}{a} + \frac{1}{\sqrt{3}}\right) - a\left(\frac{2x}{a} + \frac{1}{\sqrt{3}}\right)^2}{108(a^{2/3}x^2 + 2a^{1/3}x + a^{1/3})}\right) + 28(2Ba^2 - 5Aa^2)^2 + 2(2Ba^2 - 5Aa^2)^2 + (2Ba^2 - 5Aa^2)^2 \log\left(\frac{bx + a\left(\frac{2x}{a} + \frac{1}{\sqrt{3}}\right)}{108(a^{2/3}x^2 + 2a^{1/3}x + a^{1/3})}\right)}{108(a^{2/3}x^2 + 2a^{1/3}x + a^{1/3})^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $\frac{-1/108(84(2B*Ab^2 - 5A*Ab^3)*x^9 + 147(2B*Ab^2 - 5A*Ab^3)*x^6 + 27A*Ab^3 + 54(2B*Ab^2 - 5A*Ab^3)*x^3 + 28\sqrt{3}((2B*Ab^2 - 5A*Ab^3)*x^{10} + 2(2B*Ab^2 - 5A*Ab^3)*x^7 + (2B*Ab^3 - 5A*Ab^2)*x^4)*(-b/a)^{1/3} \arctan(2/3\sqrt{3}*x*(-b/a)^{1/3} + 1/3\sqrt{3}) - 14((2B*Ab^2 - 5A*Ab^3)*x^{10} + 2(2B*Ab^2 - 5A*Ab^3)*x^7 + (2B*Ab^3 - 5A*Ab^2)*x^4)*(-b/a)^{1/3} \log(b*x^2 - a*x*(-b/a)^{2/3} - a*(-b/a)^{1/3}) + 28((2B*Ab^2 - 5A*Ab^3)*x^{10} + 2(2B*Ab^2 - 5A*Ab^3)*x^7 + (2B*Ab^3 - 5A*Ab^2)*x^4)*(-b/a)^{1/3} \log(b*x + a*(-b/a)^{2/3})}{(a^4*b^2*x^{10} + 2*a^5*b*x^7 + a^6*x^4)}$

giac [A] time = 0.20, size = 254, normalized size = 1.03

$$\frac{7(2Ba^2(-\frac{a}{b})^{\frac{1}{3}} - 5Aa^2(-\frac{a}{b})^{\frac{1}{3}})(-\frac{a}{b})^{\frac{1}{3}} \log\left(\left|x - (-\frac{a}{b})^{\frac{1}{3}}\right|\right) + 7\sqrt{3}\left(2(-ab^2)^{\frac{1}{3}}Ba - 5(-ab^2)^{\frac{1}{3}}Ab\right) \arctan\left(\frac{\sqrt{3}\left(2x + (-\frac{a}{b})^{\frac{1}{3}}\right)}{3(-\frac{a}{b})^{\frac{1}{3}}}\right) - 7\left(2(-ab^2)^{\frac{1}{3}}Ba - 5(-ab^2)^{\frac{1}{3}}Ab\right) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - \frac{10Ba^2x^5 - 16Ab^2x^5 + 13Ba^2bx^2 - 19Aab^2x^2}{18(bx^3 + a)^2 a^4} - \frac{4Bax^3 - 12Abx^3 + Aa}{4a^4 x^4}}{27a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="giac")

[Out] $\frac{7}{27} \cdot (2B \cdot a \cdot b \cdot (-a/b)^{1/3} - 5A \cdot b^2 \cdot (-a/b)^{1/3}) \cdot (-a/b)^{1/3} \cdot \log(\text{abs}(x - (-a/b)^{1/3})) / a^5 + \frac{7}{27} \cdot \sqrt{3} \cdot (2 \cdot (-a \cdot b^2)^{2/3} \cdot B \cdot a - 5 \cdot (-a \cdot b^2)^{2/3}) \cdot A \cdot b \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2x + (-a/b)^{1/3}) / (-a/b)^{1/3}) / (a^5 \cdot b) - \frac{7}{54} \cdot (2 \cdot (-a \cdot b^2)^{2/3} \cdot B \cdot a - 5 \cdot (-a \cdot b^2)^{2/3}) \cdot A \cdot b \cdot \log(x^2 + x \cdot (-a/b)^{1/3} + (-a/b)^{2/3}) / (a^5 \cdot b) - \frac{1}{18} \cdot (10 \cdot B \cdot a \cdot b^2 \cdot x^5 - 16 \cdot A \cdot b^3 \cdot x^5 + 13 \cdot B \cdot a^2 \cdot b \cdot x^2 - 19 \cdot A \cdot a \cdot b^2 \cdot x^2) / ((b \cdot x^3 + a)^2 \cdot a^4) - \frac{1}{4} \cdot (4 \cdot B \cdot a \cdot x^3 - 12 \cdot A \cdot b \cdot x^3 + A \cdot a) / (a^4 \cdot x^4)$

maple [A] time = 0.06, size = 299, normalized size = 1.22

$$\frac{8A b^3 x^5}{9(b^3 + a)^2 a^4} - \frac{5B b^2 x^5}{9(b^3 + a)^2 a^3} + \frac{19A b^2 x^2}{18(b^3 + a)^2 a^3} - \frac{13B b x^2}{18(b^3 + a)^2 a^2} + \frac{35\sqrt{3} Ab \arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{b}\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^4} - \frac{35Ab \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^4} + \frac{35Ab \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}} a^4} - \frac{14\sqrt{3} B \arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{b}\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^3} + \frac{14B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^3} - \frac{7B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^3} + \frac{3Ab}{a^4 x} - \frac{B}{a^3 x} - \frac{A}{4a^3 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((B \cdot x^3 + A) / x^5 / (b \cdot x^3 + a)^3, x)$

[Out] $\frac{8}{9} \cdot \frac{1}{a^4} \cdot \frac{b^3}{(b \cdot x^3 + a)^2} \cdot x^5 \cdot A - \frac{5}{9} \cdot \frac{1}{a^3} \cdot \frac{b^2}{(b \cdot x^3 + a)^2} \cdot x^5 \cdot B + \frac{19}{18} \cdot \frac{1}{a^3} \cdot \frac{b^2}{(b \cdot x^3 + a)^2} \cdot x^5 \cdot A - \frac{13}{18} \cdot \frac{1}{a^2} \cdot \frac{b}{(b \cdot x^3 + a)^2} \cdot x^5 \cdot B - \frac{35}{27} \cdot \frac{1}{a^4} \cdot \frac{b \cdot A}{(a/b)^{1/3}} \cdot \ln(x + (a/b)^{1/3}) + \frac{35}{54} \cdot \frac{1}{a^4} \cdot \frac{b \cdot A}{(a/b)^{1/3}} \cdot \ln(x^2 - (a/b)^{1/3} \cdot x + (a/b)^{2/3}) + \frac{35}{27} \cdot \frac{1}{a^4} \cdot \frac{b \cdot A \cdot 3^{1/2}}{(a/b)^{1/3}} \cdot \arctan(1/3 \cdot 3^{1/2} \cdot (2/(a/b)^{1/3} \cdot x - 1)) + \frac{14}{27} \cdot \frac{1}{a^3} \cdot \frac{B}{(a/b)^{1/3}} \cdot \ln(x + (a/b)^{1/3}) - \frac{7}{27} \cdot \frac{1}{a^3} \cdot \frac{B}{(a/b)^{1/3}} \cdot \ln(x^2 - (a/b)^{1/3} \cdot x + (a/b)^{2/3}) - \frac{14}{27} \cdot \frac{1}{a^3} \cdot \frac{B \cdot 3^{1/2}}{(a/b)^{1/3}} \cdot \arctan(1/3 \cdot 3^{1/2} \cdot (2/(a/b)^{1/3} \cdot x - 1)) - \frac{1}{4} \cdot \frac{1}{a^3} \cdot \frac{A}{x^4} + \frac{3}{a^4} \cdot \frac{1}{x} \cdot A \cdot b - \frac{B}{a^3} \cdot \frac{1}{x}$

maxima [A] time = 1.42, size = 221, normalized size = 0.90

$$\frac{28(2Bab^2 - 5Ab^3)x^9 + 49(2Ba^2b - 5Aab^2)x^6 + 9Aa^3 + 18(2Ba^3 - 5Aa^2b)x^3}{36(a^4b^2x^{10} + 2a^5bx^7 + a^6x^4)} - \frac{7\sqrt{3}(2Ba - 5Ab) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{7(2Ba - 5Ab) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{7(2Ba - 5Ab) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((B \cdot x^3 + A) / x^5 / (b \cdot x^3 + a)^3, x, \text{algorithm}=\text{"maxima"})$

[Out] $-1/36 \cdot (28 \cdot (2B \cdot a \cdot b^2 - 5A \cdot a \cdot b^3) \cdot x^9 + 49 \cdot (2B \cdot a^2 \cdot b - 5A \cdot a \cdot b^2) \cdot x^6 + 9 \cdot A \cdot a^3 + 18 \cdot (2B \cdot a^3 - 5A \cdot a^2 \cdot b) \cdot x^3) / (a^4 \cdot b^2 \cdot x^{10} + 2 \cdot a^5 \cdot b \cdot x^7 + a^6 \cdot x^4) - \frac{7}{27} \cdot \sqrt{3} \cdot (2B \cdot a - 5A \cdot a \cdot b) \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2x - (a/b)^{1/3}) / (a/b)^{1/3}) / (a^4 \cdot (a/b)^{1/3}) - \frac{7}{54} \cdot (2B \cdot a - 5A \cdot a \cdot b) \cdot \log(x^2 - x \cdot (a/b)^{1/3} + (a/b)^{2/3}) / (a^4 \cdot (a/b)^{1/3}) + \frac{7}{27} \cdot (2B \cdot a - 5A \cdot a \cdot b) \cdot \log(x + (a/b)^{1/3}) / (a^4 \cdot (a/b)^{1/3})$

mupad [B] time = 2.64, size = 240, normalized size = 0.98

$$\frac{a^3(5Ab-2Ba) - \frac{A}{4a} + \frac{7b^2 \cdot 3(5A-2Ba)}{2a^2} + \frac{49b^6(5A-2Ba)}{36a^6}}{a^2x^4 + 2abx^7 + b^2x^{10}} + \frac{7(-b)^{1/3} \ln(a^{1/3}(-b)^{8/3} + b^3x)(5Ab-2Ba)}{27a^{13/3}} + \frac{7(-b)^{1/3} \ln(a^{1/3}(-b)^{8/3} - 2b^3x + \sqrt{3}a^{1/3}(-b)^{8/3})\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\right)(5Ab-2Ba)}{27a^{13/3}} - \frac{7(-b)^{1/3} \ln(2b^3x - a^{1/3}(-b)^{8/3} + \sqrt{3}a^{1/3}(-b)^{8/3})\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)(5Ab-2Ba)}{27a^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)/(x^5*(a + b*x^3)^3), x)`

[Out]
$$\begin{aligned} & ((x^3(5A*b - 2B*a))/(2*a^2) - A/(4*a) + (7*b^2*x^9(5A*b - 2B*a))/(9*a^4) \\ & + (49*b*x^6(5A*b - 2B*a))/(36*a^3))/(a^2*x^4 + b^2*x^{10} + 2*a*b*x^7) \\ & + (7*(-b)^{1/3}*\log(a^{1/3}*(-b)^{8/3} + b^3*x)*(5A*b - 2B*a))/(27*a^{13/3}) \\ & + (7*(-b)^{1/3}*\log(a^{1/3}*(-b)^{8/3} - 2*b^3*x + 3^{1/2}*a^{1/3}*(-b)^{8/3}*i) * ((3^{1/2}*i)/2 - 1/2)*(5A*b - 2B*a))/(27*a^{13/3}) \\ & - (7*(-b)^{1/3}*\log(2*b^3*x - a^{1/3}*(-b)^{8/3} + 3^{1/2}*a^{1/3}*(-b)^{8/3}*i) * ((3^{1/2}*i)/2 + 1/2)*(5A*b - 2B*a))/(27*a^{13/3}) \end{aligned}$$

sympy [A] time = 1.91, size = 189, normalized size = 0.77

$$\text{RootSum}\left(19683a^{13} + 42875A^3b^4 - 51450A^2Bab^3 + 20580AB^2a^2b^2 - 2744B^3a^3b, \left(t \mapsto t \log\left(\frac{729t^2a^9}{1225A^2b^3 - 980ABab^2 + 196B^2a^2b} + x\right)\right)\right) + \frac{-9Aa^3 + x^9(140Ab^2 - 56Bab^2) + x^6(245Aab^2 - 98Ba^2b) + x^3(90Aa^2b - 36Ba^3)}{36a^6x^4 + 72a^5bx^7 + 36a^4b^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)/x**5/(b*x**3+a)**3, x)`

[Out]
$$\begin{aligned} & \text{RootSum}(19683*_t**3*a**13 + 42875*A**3*b**4 - 51450*A**2*B*a*b**3 + 20580*A \\ & *B**2*a**2*b**2 - 2744*B**3*a**3*b, \text{Lambda}(_t, _t*\log(729*_t**2*a**9/(1225* \\ & A**2*b**3 - 980*A*B*a*b**2 + 196*B**2*a**2*b) + x))) + (-9*A*a**3 + x**9*(1 \\ & 40*A*b**3 - 56*B*a*b**2) + x**6*(245*A*a*b**2 - 98*B*a**2*b) + x**3*(90*A*a \\ & **2*b - 36*B*a**3))/(36*a**6*x**4 + 72*a**5*b*x**7 + 36*a**4*b**2*x**10) \end{aligned}$$

$$3.106 \quad \int \frac{A+Bx^3}{x^6(a+bx^3)^3} dx$$

Optimal. Leaf size=246

$$\frac{2b^{2/3}(11Ab - 5aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{27a^{14/3}} + \frac{4b^{2/3}(11Ab - 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{14/3}} - \frac{4b^{2/3}(11Ab - 5aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{14/3}}$$

Rubi [A] time = 0.14, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {457, 290, 325, 200, 31, 634, 617, 204, 628}

$$\frac{2b^{2/3}(11Ab - 5aB) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{27a^{14/3}} + \frac{4b^{2/3}(11Ab - 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{14/3}} - \frac{4b^{2/3}(11Ab - 5aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{14/3}} + \frac{11Ab - 5aB}{18a^2bx^5(a + bx^3)} + \frac{2(11Ab - 5aB)}{9a^4x^2} - \frac{4(11Ab - 5aB)}{45a^3bx^5} + \frac{Ab - aB}{6abx^5(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^3), x]

[Out] (-4*(11*A*b - 5*a*B))/(45*a^3*b*x^5) + (2*(11*A*b - 5*a*B))/(9*a^4*x^2) + (A*b - a*B)/(6*a*b*x^5*(a + b*x^3)^2) + (11*A*b - 5*a*B)/(18*a^2*b*x^5*(a + b*x^3)) - (4*b^(2/3)*(11*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(14/3)) + (4*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(14/3)) - (2*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(27*a^(14/3)))

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^-1, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx^3}{x^6(a + bx^3)^3} dx &= \frac{Ab - aB}{6abx^5(a + bx^3)^2} + \frac{(11Ab - 5aB) \int \frac{1}{x^6(a + bx^3)^2} dx}{6ab} \\
 &= \frac{Ab - aB}{6abx^5(a + bx^3)^2} + \frac{11Ab - 5aB}{18a^2bx^5(a + bx^3)} + \frac{(4(11Ab - 5aB)) \int \frac{1}{x^6(a + bx^3)} dx}{9a^2b} \\
 &= -\frac{4(11Ab - 5aB)}{45a^3bx^5} + \frac{Ab - aB}{6abx^5(a + bx^3)^2} + \frac{11Ab - 5aB}{18a^2bx^5(a + bx^3)} - \frac{(4(11Ab - 5aB)) \int \frac{1}{x^3(a + bx^3)} dx}{9a^3} \\
 &= -\frac{4(11Ab - 5aB)}{45a^3bx^5} + \frac{2(11Ab - 5aB)}{9a^4x^2} + \frac{Ab - aB}{6abx^5(a + bx^3)^2} + \frac{11Ab - 5aB}{18a^2bx^5(a + bx^3)} + \frac{(4b(11Ab - 5aB)) \int \frac{1}{x^3(a + bx^3)} dx}{9a^3} \\
 &= -\frac{4(11Ab - 5aB)}{45a^3bx^5} + \frac{2(11Ab - 5aB)}{9a^4x^2} + \frac{Ab - aB}{6abx^5(a + bx^3)^2} + \frac{11Ab - 5aB}{18a^2bx^5(a + bx^3)} + \frac{(4b(11Ab - 5aB)) \int \frac{1}{x^3(a + bx^3)} dx}{9a^3} \\
 &= -\frac{4(11Ab - 5aB)}{45a^3bx^5} + \frac{2(11Ab - 5aB)}{9a^4x^2} + \frac{Ab - aB}{6abx^5(a + bx^3)^2} + \frac{11Ab - 5aB}{18a^2bx^5(a + bx^3)} + \frac{4b^{2/3}(11Ab - 5aB) \int \frac{1}{x^3(a + bx^3)} dx}{9a^3} \\
 &= -\frac{4(11Ab - 5aB)}{45a^3bx^5} + \frac{2(11Ab - 5aB)}{9a^4x^2} + \frac{Ab - aB}{6abx^5(a + bx^3)^2} + \frac{11Ab - 5aB}{18a^2bx^5(a + bx^3)} + \frac{4b^{2/3}(11Ab - 5aB) \int \frac{1}{x^3(a + bx^3)} dx}{9a^3} \\
 &= -\frac{4(11Ab - 5aB)}{45a^3bx^5} + \frac{2(11Ab - 5aB)}{9a^4x^2} + \frac{Ab - aB}{6abx^5(a + bx^3)^2} + \frac{11Ab - 5aB}{18a^2bx^5(a + bx^3)} - \frac{4b^{2/3}(11Ab - 5aB) \int \frac{1}{x^3(a + bx^3)} dx}{9a^3}
 \end{aligned}$$

Mathematica [A] time = 0.32, size = 210, normalized size = 0.85

$$\frac{20b^{2/3}(5aB - 11Ab) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2) - \frac{45a^{5/3}bx(aB - Ab)}{(a + bx^3)^2} - \frac{15a^{2/3}bx(11aB - 17Ab)}{a + bx^3} - \frac{135a^{2/3}(aB - 3Ab)}{x^2} - \frac{54a^{5/3}A}{x^5} + 40b^{2/3}(11Ab - 5aB) \log(\sqrt[3]{a} + \sqrt[3]{b} x) - 40\sqrt{3} b^{2/3}(11Ab - 5aB) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}}\right)}{270a^{14/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^3), x]

[Out] $\left(\frac{-54a^{5/3}A}{x^5} - \frac{(135a^{2/3})(-3Ab + aB)}{x^2} - \frac{45a^{5/3}b(-Ab + aB)x}{(a + bx^3)^2} - \frac{(15a^{2/3})b(-17Ab + 11aB)x}{(a + bx^3)} - 40\sqrt{3}b^{2/3}(11Ab - 5aB)\text{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt{3}}\right] + 40b^{2/3}(11Ab - 5aB)\text{Log}[a^{1/3} + b^{1/3}x] + 20b^{2/3}(-11Ab + 5aB)\text{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2]\right)/(270a^{14/3})$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^6 (a + bx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^6*(a + b*x^3)^3), x]

[Out] IntegrateAlgebraic[(A + B*x^3)/(x^6*(a + b*x^3)^3), x]

fricas [A] time = 0.66, size = 384, normalized size = 1.56

$$\frac{60(5Ba^2 - 11Ab^2)^3 + 96(5Ba^2b - 11Aab^2)^3 + 54A^2 + 27(5Ba^2 - 11Ab^2)^2 + 40\sqrt{3}((5Ba^2 - 11Ab^2)^{3/2} + 2(5Ba^2b - 11Aab^2)^{3/2})\arctan\left(\frac{1 + \sqrt{3}\frac{b}{a}}{\sqrt{3}}\right) - 20((5Ba^2 - 11Ab^2)^{3/2} + 2(5Ba^2b - 11Aab^2)^{3/2})\log\left(\frac{b^2x^2 - ab\left(\frac{b}{a}\right)^{1/3} + a^2\left(\frac{b}{a}\right)^{2/3}}{a}\right) + 40((5Ba^2 - 11Ab^2)^{3/2} + 2(5Ba^2b - 11Aab^2)^{3/2})\log\left(\frac{bx + a\left(\frac{b}{a}\right)^{1/3}}{a}\right)}{270(a^{10}b^{11} + 2a^5b^8 + b^{14})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $\frac{-1/270(60(5B*a*b^2 - 11*A*b^3)*x^9 + 96(5*B*a^2*b - 11*A*a*b^2)*x^6 + 54*A*a^3 + 27*(5*B*a^3 - 11*A*a^2*b)*x^3 + 40*\sqrt{3}*((5*B*a*b^2 - 11*A*b^3)*x^{11} + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/a^2)^{1/3}*\arctan(1/3*(2*\sqrt{3})*a*x*(b^2/a^2)^{2/3} - \sqrt{3}*b)/b - 20*((5*B*a*b^2 - 11*A*b^3)*x^{11} + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/a^2)^{1/3}*\log(b^2*x^2 - a*b*x*(b^2/a^2)^{1/3} + a^2*(b^2/a^2)^{2/3}) + 40*((5*B*a*b^2 - 11*A*b^3)*x^{11} + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/a^2)^{1/3}*\log(b*x + a*(b^2/a^2)^{1/3}))}{(a^4*b^2*x^{11} + 2*a^5*b*x^8 + a^6*x^5)}$

giac [A] time = 0.19, size = 229, normalized size = 0.93

$$\frac{4\sqrt{3}\left(5(-ab^2)^{\frac{1}{2}}Ba - 11(-ab^2)^{\frac{1}{2}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 4(5Bab - 11Ab^2)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right) - 2\left(5(-ab^2)^{\frac{1}{2}}Ba - 11(-ab^2)^{\frac{1}{2}}Ab\right)\log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27a^5} + \frac{11Bab^2x^4 - 17Ab^3x^4 + 14Ba^2bx - 20Aab^2x - 5Bax^3 - 15Abx^3 + 2Aa}{18(bx^3 + a)^2a^4} - \frac{5Bax^3 - 15Abx^3 + 2Aa}{10a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="giac")

[Out] $-4/27*\sqrt{3}*(5*(-a*b^2)^{(1/3)}*B*a - 11*(-a*b^2)^{(1/3)}*A*b)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/a^5 + 4/27*(5*B*a*b - 11*A*b^2)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^5 - 2/27*(5*(-a*b^2)^{(1/3)}*B*a - 11*(-a*b^2)^{(1/3)}*A*b)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/a^5 - 1/18*(11*B*a*b^2*x^4 - 17*A*b^3*x^4 + 14*B*a^2*b*x - 20*A*a*b^2*x)/(b*x^3 + a)^2*a^4 - 1/10*(5*B*a*x^3 - 15*A*b*x^3 + 2*A*a)/(a^4*x^5)$

maple [A] time = 0.06, size = 295, normalized size = 1.20

$$\frac{17A b^3 x^4}{18(b x^3 + a)^2 a^4} - \frac{11B b^2 x^4}{18(b x^3 + a)^2 a^2} + \frac{10A b^2 x}{9(b x^3 + a)^2 a^3} - \frac{7B b x}{9(b x^3 + a)^2 a^2} + \frac{44\sqrt{3} A b \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{b}\right)^{\frac{1}{3}}}{3}\right)}{27\left(\frac{b}{a}\right)^{\frac{1}{3}} a^4} + \frac{44A b \ln\left(x + \left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{27\left(\frac{b}{a}\right)^{\frac{1}{3}} a^4} - \frac{22A b \ln\left(x^2 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{27\left(\frac{b}{a}\right)^{\frac{1}{3}} a^4} - \frac{20\sqrt{3} B \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{b}\right)^{\frac{1}{3}}}{3}\right)}{27\left(\frac{b}{a}\right)^{\frac{1}{3}} a^3} - \frac{20B \ln\left(x + \left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{27\left(\frac{b}{a}\right)^{\frac{1}{3}} a^3} + \frac{10B \ln\left(x^2 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{27\left(\frac{b}{a}\right)^{\frac{1}{3}} a^3} + \frac{3A b}{2a^2 x^2} - \frac{B}{2a^3 x^2} - \frac{A}{5a^3 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((B*x^3+A)/x^6/(b*x^3+a)^3, x)$

[Out] $17/18/a^4*b^3/(b*x^3+a)^2*A*x^4 - 11/18/a^3*b^2/(b*x^3+a)^2*B*x^4 + 10/9/a^3*b^2/(b*x^3+a)^2*A*x - 7/9/a^2*b/(b*x^3+a)^2*B*x + 44/27/a^4*b*A/(a/b)^{(2/3)}*\ln(x + (a/b)^{(1/3)}) - 22/27/a^4*b*A/(a/b)^{(2/3)}*\ln(x^2 - (a/b)^{(1/3)}*x + (a/b)^{(2/3)}) + 44/27/a^4*b*A/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x - 1)) - 20/27/a^3*B/(a/b)^{(2/3)}*\ln(x + (a/b)^{(1/3)}) + 10/27/a^3*B/(a/b)^{(2/3)}*\ln(x^2 - (a/b)^{(1/3)}*x + (a/b)^{(2/3)}) - 20/27/a^3*B/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x - 1)) - 1/5/a^3*A/x^5 + 3/2/a^4/x^2*A*b - 1/2/a^3/x^2*B$

maxima [A] time = 1.46, size = 221, normalized size = 0.90

$$\frac{20(5Bab^2 - 11Ab^3)x^9 + 32(5Ba^2b - 11Aa^2b^2)x^6 + 18Aa^3 + 9(5Ba^3 - 11Aa^2b)x^3}{90(a^4b^2x^{11} + 2a^5bx^8 + a^6x^5)} - \frac{4\sqrt{3}(5Ba - 11Ab)\arctan\left(\frac{\sqrt{3}\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2(5Ba - 11Ab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{4(5Ba - 11Ab)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((B*x^3+A)/x^6/(b*x^3+a)^3, x, \text{algorithm}="maxima")$

[Out] $-1/90*(20*(5*B*a*b^2 - 11*A*b^3)*x^9 + 32*(5*B*a^2*b - 11*A*a*b^2)*x^6 + 18*A*a^3 + 9*(5*B*a^3 - 11*A*a^2*b)*x^3)/(a^4*b^2*x^11 + 2*a^5*b*x^8 + a^6*x^5) - 4/27*\sqrt{3}*(5*B*a - 11*A*b)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a^4*(a/b)^{(2/3)}) + 2/27*(5*B*a - 11*A*b)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a^4*(a/b)^{(2/3)}) - 4/27*(5*B*a - 11*A*b)*\log(x + (a/b)^{(1/3)})/(a^4*(a/b)^{(2/3)})$

mupad [B] time = 2.58, size = 207, normalized size = 0.84

$$\frac{x^3(11Ab - 5Ba) - \frac{A}{10x^2} + \frac{2b^2x^2(11Ab - 5Ba)}{5a} + \frac{16b^4(11Ab - 5Ba)}{45a^3}}{a^2x^5 + 2abx^8 + b^2x^{11}} + \frac{4b^{2/3}\ln(b^{1/3}x + a^{1/3})(11Ab - 5Ba)}{27a^{14/3}} - \frac{4b^{2/3}\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(11Ab - 5Ba)}{27a^{14/3}} + \frac{4b^{2/3}\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(11Ab - 5Ba)}{27a^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)/(x^6*(a + b*x^3)^3),x)`

[Out]
$$\begin{aligned} & ((x^3(11A*b - 5B*a))/(10*a^2) - A/(5*a) + (2*b^2*x^9(11A*b - 5B*a)))/(9*a^4) \\ & + (16*b*x^6(11A*b - 5B*a))/(45*a^3)/(a^2*x^5 + b^2*x^11 + 2*a*b*x^8) \\ & + (4*b^{(2/3)}*\log(b^{(1/3)}*x + a^{(1/3)})*(11A*b - 5B*a))/(27*a^{(14/3)}) \\ & - (4*b^{(2/3)}*\log(3^{(1/2)}*a^{(1/3)}*i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*i)/2 + 1/2)*(11A*b - 5B*a))/(27*a^{(14/3)}) \\ & + (4*b^{(2/3)}*\log(3^{(1/2)}*a^{(1/3)}*i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*i)/2 - 1/2)*(11A*b - 5B*a))/(27*a^{(14/3)}) \end{aligned}$$

sympy [A] time = 2.18, size = 173, normalized size = 0.70

$$\text{RootSum}\left(19683t^3a^{14} - 85184A^3b^5 + 116160A^2Bab^4 - 52800AB^2a^2b^3 + 8000B^3a^3b^2, \left(t \mapsto t \log\left(-\frac{27ta^5}{-44Ab^2 + 20Bab} + x\right)\right) + \frac{-18Aa^3 + x^9(220Ab^3 - 100Bab^2) + x^6(352Aab^2 - 160Ba^2b) + x^3(99Aa^2b - 45Ba^3)}{90a^6x^5 + 180a^5bx^8 + 90a^4b^2x^{11}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)/x**6/(b*x**3+a)**3,x)`

[Out]
$$\begin{aligned} & \text{RootSum}(19683*_t**3*a**14 - 85184*A**3*b**5 + 116160*A**2*B*a*b**4 - 52800* \\ & A*B**2*a**2*b**3 + 8000*B**3*a**3*b**2, \text{Lambda}(_t, _t*\log(-27*_t*a**5/(-44* \\ & A*b**2 + 20*B*a*b) + x))) + (-18*A*a**3 + x**9*(220*A*b**3 - 100*B*a*b**2) \\ & + x**6*(352*A*a*b**2 - 160*B*a**2*b) + x**3*(99*A*a**2*b - 45*B*a**3))/(90* \\ & a**6*x**5 + 180*a**5*b*x**8 + 90*a**4*b**2*x**11) \end{aligned}$$

$$3.107 \quad \int \frac{x^8}{(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=70

$$\frac{a^2 \log(a+bx^3)}{3b^2(bc-ad)} - \frac{c^2 \log(c+dx^3)}{3d^2(bc-ad)} + \frac{x^3}{3bd}$$

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 72}

$$\frac{a^2 \log(a+bx^3)}{3b^2(bc-ad)} - \frac{c^2 \log(c+dx^3)}{3d^2(bc-ad)} + \frac{x^3}{3bd}$$

Antiderivative was successfully verified.

[In] Int[x^8/((a + b*x^3)*(c + d*x^3)),x]

[Out] x^3/(3*b*d) + (a^2*Log[a + b*x^3])/(3*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^3])/(3*d^2*(b*c - a*d))

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^8}{(a+bx^3)(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(a+bx)(c+dx)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{bd} + \frac{a^2}{b(bc-ad)(a+bx)} + \frac{c^2}{d(-bc+ad)(c+dx)} \right) dx, x, x^3 \right) \\ &= \frac{x^3}{3bd} + \frac{a^2 \log(a+bx^3)}{3b^2(bc-ad)} - \frac{c^2 \log(c+dx^3)}{3d^2(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 0.94

$$\frac{a^2 d^2 \log(a + bx^3) - b(dx^3(ad - bc) + bc^2 \log(c + dx^3))}{3b^2 d^2(bc - ad)}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((a + b*x^3)*(c + d*x^3)),x]

[Out] (a^2*d^2*Log[a + b*x^3] - b*(d*(-(b*c) + a*d)*x^3 + b*c^2*Log[c + d*x^3]))/(3*b^2*d^2*(b*c - a*d))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a + bx^3)(c + dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/((a + b*x^3)*(c + d*x^3)),x]

[Out] IntegrateAlgebraic[x^8/((a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 1.16, size = 72, normalized size = 1.03

$$\frac{a^2 d^2 \log(bx^3 + a) - b^2 c^2 \log(dx^3 + c) + (b^2 cd - abd^2)x^3}{3(b^3 cd^2 - ab^2 d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")

[Out] 1/3*(a^2*d^2*log(b*x^3 + a) - b^2*c^2*log(d*x^3 + c) + (b^2*c*d - a*b*d^2)*x^3)/(b^3*c*d^2 - a*b^2*d^3)

giac [A] time = 0.18, size = 70, normalized size = 1.00

$$\frac{a^2 \log(|bx^3 + a|)}{3(b^3 c - ab^2 d)} - \frac{c^2 \log(|dx^3 + c|)}{3(bcd^2 - ad^3)} + \frac{x^3}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")

[Out] 1/3*a^2*log(abs(b*x^3 + a))/(b^3*c - a*b^2*d) - 1/3*c^2*log(abs(d*x^3 + c))/(b*c*d^2 - a*d^3) + 1/3*x^3/(b*d)

maple [A] time = 0.05, size = 65, normalized size = 0.93

$$-\frac{a^2 \ln(bx^3 + a)}{3(ad - bc)b^2} + \frac{x^3}{3bd} + \frac{c^2 \ln(dx^3 + c)}{3(ad - bc)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(b*x^3+a)/(d*x^3+c),x)

[Out] 1/3*x^3/b/d-1/3*a^2/b^2/(a*d-b*c)*ln(b*x^3+a)+1/3*c^2/d^2/(a*d-b*c)*ln(d*x^3+c)

maxima [A] time = 0.59, size = 68, normalized size = 0.97

$$\frac{a^2 \log(bx^3 + a)}{3(b^3c - ab^2d)} - \frac{c^2 \log(dx^3 + c)}{3(bcd^2 - ad^3)} + \frac{x^3}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")

[Out] 1/3*a^2*log(b*x^3 + a)/(b^3*c - a*b^2*d) - 1/3*c^2*log(d*x^3 + c)/(b*c*d^2 - a*d^3) + 1/3*x^3/(b*d)

mupad [B] time = 2.84, size = 68, normalized size = 0.97

$$\frac{a^2 \ln(bx^3 + a)}{3b^3c - 3ab^2d} + \frac{c^2 \ln(dx^3 + c)}{3ad^3 - 3bcd^2} + \frac{x^3}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((a + b*x^3)*(c + d*x^3)),x)

[Out] (a^2*log(a + b*x^3))/(3*b^3*c - 3*a*b^2*d) + (c^2*log(c + d*x^3))/(3*a*d^3 - 3*b*c*d^2) + x^3/(3*b*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(b*x**3+a)/(d*x**3+c),x)

[Out] Timed out

$$3.108 \quad \int \frac{x^7}{(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=301

$$\frac{a^{5/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{5/3}(bc-ad)} - \frac{a^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{5/3}(bc-ad)} - \frac{a^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}(bc-ad)} - \frac{c^{5/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6d^{5/3}(bc-ad)}$$

Rubi [A] time = 0.31, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {479, 584, 292, 31, 634, 617, 204, 628}

$$\frac{a^{5/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{5/3}(bc-ad)} - \frac{a^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{5/3}(bc-ad)} - \frac{a^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}(bc-ad)} - \frac{c^{5/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6d^{5/3}(bc-ad)} + \frac{c^{5/3} \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3d^{5/3}(bc-ad)} + \frac{c^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{5/3}(bc-ad)} + \frac{x^2}{2bd}$$

Antiderivative was successfully verified.

[In] Int[x^7/((a + b*x^3)*(c + d*x^3)),x]

[Out] $x^2/(2*b*d) - (a^{(5/3)}*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*b^{(5/3)}*(b*c - a*d)) + (c^{(5/3)}*ArcTan[(c^{(1/3)} - 2*d^{(1/3)}*x)/(Sqrt[3]*c^{(1/3)})])/(Sqrt[3]*d^{(5/3)}*(b*c - a*d)) - (a^{(5/3)}*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*b^{(5/3)}*(b*c - a*d)) + (c^{(5/3)}*Log[c^{(1/3)} + d^{(1/3)}*x])/(3*d^{(5/3)}*(b*c - a*d)) + (a^{(5/3)}*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*b^{(5/3)}*(b*c - a*d)) - (c^{(5/3)}*Log[c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x + d^{(2/3)}*x^2])/(6*d^{(5/3)}*(b*c - a*d))$

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((g*x)^m*(a + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(a+bx^3)(c+dx^3)} dx &= \frac{x^2}{2bd} - \frac{\int \frac{x(2ac+2(bc+ad)x^3)}{(a+bx^3)(c+dx^3)} dx}{2bd} \\
&= \frac{x^2}{2bd} - \frac{\int \left(\frac{2a^2 dx}{(-bc+ad)(a+bx^3)} + \frac{2bc^2 x}{(bc-ad)(c+dx^3)} \right) dx}{2bd} \\
&= \frac{x^2}{2bd} + \frac{a^2 \int \frac{x}{a+bx^3} dx}{b(bc-ad)} - \frac{c^2 \int \frac{x}{c+dx^3} dx}{d(bc-ad)} \\
&= \frac{x^2}{2bd} - \frac{a^{5/3} \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3b^{4/3}(bc-ad)} + \frac{a^{5/3} \int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2} dx}{3b^{4/3}(bc-ad)} + \frac{c^{5/3} \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{3d^{4/3}(bc-ad)} - \frac{c^{5/3} \int \frac{1}{\sqrt[3]{c} - \sqrt[3]{d}x} dx}{3d^{4/3}(bc-ad)} \\
&= \frac{x^2}{2bd} - \frac{a^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3d^{5/3}(bc-ad)} + \frac{a^{5/3} \int \frac{-\sqrt[3]{a} \sqrt[3]{b}x + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2} dx}{6b^{5/3}(bc-ad)} - \frac{c^{5/3} \int \frac{-\sqrt[3]{c} \sqrt[3]{d}x + 2d^{2/3}x}{c^{2/3} - \sqrt[3]{c} \sqrt[3]{d}x + d^{2/3}x^2} dx}{6d^{5/3}(bc-ad)} \\
&= \frac{x^2}{2bd} - \frac{a^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3d^{5/3}(bc-ad)} + \frac{a^{5/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{6b^{5/3}(bc-ad)} - \frac{c^{5/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d}x + d^{2/3}x^2)}{6d^{5/3}(bc-ad)} \\
&= \frac{x^2}{2bd} - \frac{a^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^{5/3}(bc-ad)} + \frac{c^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3} \sqrt[3]{c}}\right)}{\sqrt{3} d^{5/3}(bc-ad)} - \frac{a^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3d^{5/3}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 242, normalized size = 0.80

$$\frac{\frac{a^{5/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{b^{5/3}} - \frac{2a^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{b^{5/3}} - \frac{2\sqrt{3} a^{5/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3} \sqrt[3]{a}}\right)}{b^{5/3}} - \frac{3ax^2}{b} - \frac{c^{5/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d}x + d^{2/3}x^2)}{d^{5/3}} + \frac{2c^{5/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{d^{5/3}} + \frac{2\sqrt{3} c^{5/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d}x}{\sqrt{3} \sqrt[3]{c}}\right)}{d^{5/3}} + \frac{3cx^2}{d}}{6bc - 6ad}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/((a + b*x^3)*(c + d*x^3)),x]

[Out] ((-3*a*x^2)/b + (3*c*x^2)/d - (2*sqrt[3]*a^(5/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/b^(5/3) + (2*sqrt[3]*c^(5/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/sqrt[3]])/d^(5/3) - (2*a^(5/3)*Log[a^(1/3) + b^(1/3)*x])/b^(5/3) + (2*c^(5/3)*Log[c^(1/3) + d^(1/3)*x])/d^(5/3) + (a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - (c^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(5/3)/(6*b*c - 6*a*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a + bx^3)(c + dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/((a + b*x^3)*(c + d*x^3)), x]

[Out] IntegrateAlgebraic[x^7/((a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 1.36, size = 273, normalized size = 0.91

$$\frac{2\sqrt{3}ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)-2\sqrt{3}bc\left(-\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ad\left(\frac{c}{d}\right)^{\frac{1}{3}}+\sqrt{3}c}{3c}\right)+ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{c}{d}\right)^{\frac{1}{3}}+a\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)+bc\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(cx^2-dx\left(-\frac{c}{d}\right)^{\frac{1}{3}}-c\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)-2ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)-2bc\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(cx+d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)+3(bc-ad)x^2}{6(b^2cd-abd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^3+a)/(d*x^3+c), x, algorithm="fricas")

[Out] $\frac{1}{6}*(2*\sqrt{3}*a*d*(a^2/b^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3}*b*x*(a^2/b^2)^{(1/3)} - \sqrt{3}*a)/a) - 2*\sqrt{3}*b*c*(-c^2/d^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3}*d*x*(-c^2/d^2)^{(1/3)} + \sqrt{3}*c)/c) + a*d*(a^2/b^2)^{(1/3)}*\log(a*x^2 - b*x*(a^2/b^2)^{(2/3)} + a*(a^2/b^2)^{(1/3)}) + b*c*(-c^2/d^2)^{(1/3)}*\log(c*x^2 - d*x*(-c^2/d^2)^{(2/3)} - c*(-c^2/d^2)^{(1/3)}) - 2*a*d*(a^2/b^2)^{(1/3)}*\log(a*x + b*(a^2/b^2)^{(2/3)}) - 2*b*c*(-c^2/d^2)^{(1/3)}*\log(c*x + d*(-c^2/d^2)^{(2/3)}) + 3*(b*c - a*d)*x^2/(b^2*c*d - a*b*d^2)$

giac [A] time = 0.21, size = 311, normalized size = 1.03

$$\frac{a^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ab^2c-a^2bd)} + \frac{c^2\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(bc^2d-acd^2)} - \frac{(-ab^2)^{\frac{2}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^4c-\sqrt{3}ab^2d} + \frac{(-cd^2)^{\frac{2}{3}}c\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4} + \frac{(-ab^2)^{\frac{2}{3}}a\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(b^4c-ab^2d)} - \frac{(-cd^2)^{\frac{2}{3}}c\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bcd^3-ad^4)} + \frac{x^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^3+a)/(d*x^3+c), x, algorithm="giac")

[Out] $-\frac{1}{3}*a^2*(-a/b)^{(2/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a*b^2*c - a^2*b*d) + \frac{1}{3}*c^2*(-c/d)^{(2/3)}*\log(\text{abs}(x - (-c/d)^{(1/3)}))/(b*c^2*d - a*c*d^2) - (-a*b^2)^{(2/3)}*a*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(\sqrt{3}*b^4*c - \sqrt{3}*a*b^2*d) + (-c*d^2)^{(2/3)}*c*\arctan(1/3*\sqrt{3}*(2*x + (-c/d)^{(1/3)})/(-c/d)^{(1/3)})/(\sqrt{3}*b*c*d^3 - \sqrt{3}*a*d^4) + \frac{1}{6}*(-a*b^2)^{(2/3)}*a*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(b^4*c - a*b^2*d) - \frac{1}{6}*(-c*d^2)^{(2/3)}*c*\log(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)})/(b*c*d^3 - a*d^4) + \frac{1}{2}*x^2/(b*d)$

maple [A] time = 0.05, size = 269, normalized size = 0.89

$$\frac{\sqrt{3} a^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\frac{a}{b}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} + \frac{a^2 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - a^2 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} - \frac{a^2 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} + \frac{\sqrt{3} c^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\frac{c}{d}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}d^2} - \frac{c^2 \ln\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right) - c^2 \ln\left(x^2 - \left(\frac{c}{d}\right)^{\frac{1}{3}}x + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}d^2} + \frac{c^2 \ln\left(x^2 - \left(\frac{c}{d}\right)^{\frac{1}{3}}x + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}d^2} + \frac{x^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(b*x^3+a)/(d*x^3+c), x)

[Out] $\frac{1}{2}x^2/b/d + \frac{1}{3}a^2/b^2/(ad-bc)/(a/b)^{1/3} \ln(x+(a/b)^{1/3}) - \frac{1}{6}a^2/b^2/(ad-bc)/(a/b)^{1/3} \ln(x^2-(a/b)^{1/3}x+(a/b)^{2/3}) - \frac{1}{3}a^2/b^2/(ad-bc) \cdot 3^{1/2}/(a/b)^{1/3} \arctan(1/3 \cdot 3^{1/2} \cdot (2/(a/b)^{1/3}x-1)) - \frac{1}{3}c^2/d^2/(ad-bc)/(c/d)^{1/3} \ln(x+(c/d)^{1/3}) + \frac{1}{6}c^2/d^2/(ad-bc)/(c/d)^{1/3} \ln(x^2-(c/d)^{1/3}x+(c/d)^{2/3}) + \frac{1}{3}c^2/d^2/(ad-bc) \cdot 3^{1/2}/(c/d)^{1/3} \arctan(1/3 \cdot 3^{1/2} \cdot (2/(c/d)^{1/3}x-1))$

maxima [A] time = 1.27, size = 324, normalized size = 1.08

$$\frac{\sqrt{3} a^2 \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(b^3c - ab^2d)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3} c^2 \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3(bcd^2 - ad^3)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{a^2 \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{c^2 \log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}} - ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} - \frac{a^2 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{c^2 \log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}} - ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{x^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^3+a)/(d*x^3+c), x, algorithm="maxima")

[Out] $\frac{1}{3}\sqrt{3}a^2 \arctan(1/3\sqrt{3}(2x - (a/b)^{1/3})/(a/b)^{1/3})/((b^3c - ab^2d)(a/b)^{1/3}) - \frac{1}{3}\sqrt{3}c^2 \arctan(1/3\sqrt{3}(2x - (c/d)^{1/3})/(c/d)^{1/3})/((bcd^2 - ad^3)(c/d)^{1/3}) + \frac{1}{6}a^2 \log(x^2 - x(a/b)^{1/3} + (a/b)^{2/3})/((b^3c(a/b)^{1/3} - ab^2d(a/b)^{1/3}) - \frac{1}{6}c^2 \log(x^2 - x(c/d)^{1/3} + (c/d)^{2/3})/((bcd^2(c/d)^{1/3} - ad^3(c/d)^{1/3}) - \frac{1}{3}a^2 \log(x + (a/b)^{1/3})/((b^3c(a/b)^{1/3} - ab^2d(a/b)^{1/3}) + \frac{1}{3}c^2 \log(x + (c/d)^{1/3})/((bcd^2(c/d)^{1/3} - ad^3(c/d)^{1/3}) + \frac{1}{2}x^2/(b*d)$

mupad [B] time = 11.36, size = 1751, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/((a + b*x^3)*(c + d*x^3)), x)

[Out] $\log((((27a^2b^3c^2d^3x^3 + b^2c^2d^3x^2 + a^2d^3x + b^3c^2d^3)(ad - b^3c)^2 + 27a^2b^3c^2d^3x^3 + b^2c^2d^3x^2 + a^2d^3x + b^3c^2d^3)(ad - b^3c)^4(a^5/(b^5(ad - b^3c)^3))^{2/3})(a^5/(b^5(ad - b^3c)^3))^{2/3})$

$$\begin{aligned}
& - b*c)^3))^{\frac{1}{3}})/3 - (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2 \\
& *d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^{\frac{2}{3}})/9 - (a^4*c^4*x*(a^2*d^2 \\
& + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^ \\
& 2*b^6*c*d^2 - 81*a*b^7*c^2*d))^{\frac{1}{3}} + \log((((27*a^2*b*c^2*d*x*(a^2*d^2 + \\
& b^2*c^2)*(a*d - b*c)^2 + 27*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(-c^5/(d^ \\
& 5*(a*d - b*c)^3))^{\frac{2}{3}})*(-c^5/(d^5*(a*d - b*c)^3))^{\frac{1}{3}})/3 - (9*(a*b^7*c^8 \\
& + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(-c^5/(d^5*(a*d \\
& - b*c)^3))^{\frac{2}{3}})/9 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))* \\
& (-c^5/(27*a^3*d^8 - 27*b^3*c^3*d^5 + 81*a*b^2*c^2*d^6 - 81*a^2*b*c*d^7))^{\frac{1}{3}} - (\log(((3^{\frac{1}{2}}*i \\
& + 1)^2*((3^{\frac{1}{2}}*i + 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^{\frac{1}{2}}*i \\
& + 1)^2*(a*d + b*c) \\
& *(a*d - b*c)^4*(a^5/(b^5*(a*d - b*c)^3))^{\frac{2}{3}})/4)*(a^5/(b^5*(a*d - b*c)^3) \\
&)^{\frac{1}{3}})/6 + (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b \\
& ^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^{\frac{2}{3}})/36 + (a^4*c^4*x*(a^2*d^2 + b^2*c^ \\
& 2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^2*b^6*c* \\
& d^2 - 81*a*b^7*c^2*d))^{\frac{1}{3}}*(3^{\frac{1}{2}}*i + 1))/2 + (\log(((3^{\frac{1}{2}}*i - 1)^2 \\
& *((3^{\frac{1}{2}}*i - 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (\\
& 27*a*b^3*c*d^3*(3^{\frac{1}{2}}*i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(a^5/(b^5*(a*d \\
& - b*c)^3))^{\frac{2}{3}})/4)*(a^5/(b^5*(a*d - b*c)^3))^{\frac{1}{3}})/6 - (9*(a*b^7*c^8 + a \\
& ^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c) \\
& ^3))^{\frac{2}{3}})/36 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5 \\
& /(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^2*b^6*c*d^2 - 81*a*b^7*c^2*d))^{\frac{1}{3}}*(\\
& 3^{\frac{1}{2}}*i - 1))/2 - (\log(((3^{\frac{1}{2}}*i + 1)^2*((3^{\frac{1}{2}}*i + 1)*(27*a^2*b* \\
& c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^{\frac{1}{2}}*i + 1) \\
&)^2*(a*d + b*c)*(a*d - b*c)^4*(-c^5/(d^5*(a*d - b*c)^3))^{\frac{2}{3}})/4)*(-c^5/(d \\
& ^5*(a*d - b*c)^3))^{\frac{1}{3}})/6 + (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a \\
& ^7*b*c^2*d^6))/(b^2*d^2))*(-c^5/(d^5*(a*d - b*c)^3))^{\frac{2}{3}})/36 + (a^4*c^4*x \\
& *(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d \\
& ^5 + 81*a*b^2*c^2*d^6 - 81*a^2*b*c*d^7))^{\frac{1}{3}}*(3^{\frac{1}{2}}*i + 1))/2 + (\log(((\\
& 3^{\frac{1}{2}}*i - 1)^2*((3^{\frac{1}{2}}*i - 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2) \\
& *(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^{\frac{1}{2}}*i - 1)^2*(a*d + b*c)*(a*d - b*c) \\
& ^4*(-c^5/(d^5*(a*d - b*c)^3))^{\frac{2}{3}})/4)*(-c^5/(d^5*(a*d - b*c)^3))^{\frac{1}{3}})/6 \\
& - (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))* \\
& (-c^5/(d^5*(a*d - b*c)^3))^{\frac{2}{3}})/36 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c \\
& *d))/(b^2*d^2))*(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d^5 + 81*a*b^2*c^2*d^6 - 81* \\
& a^2*b*c*d^7))^{\frac{1}{3}}*(3^{\frac{1}{2}}*i - 1))/2 + x^2/(2*b*d)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(b*x**3+a)/(d*x**3+c),x)

[Out] Timed out

$$3.109 \quad \int \frac{x^6}{(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=296

$$\frac{a^{4/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{4/3}(bc - ad)} + \frac{a^{4/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{4/3}(bc - ad)} - \frac{a^{4/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^{4/3}(bc - ad)} + \frac{c^{4/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6d^{4/3}(bc - ad)}$$

Rubi [A] time = 0.27, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {479, 522, 200, 31, 634, 617, 204, 628}

$$\frac{a^{4/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{4/3}(bc - ad)} + \frac{a^{4/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{4/3}(bc - ad)} - \frac{a^{4/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^{4/3}(bc - ad)} + \frac{c^{4/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6d^{4/3}(bc - ad)} - \frac{c^{4/3} \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3d^{4/3}(bc - ad)} + \frac{c^{4/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d} x}{\sqrt{3} \sqrt[3]{c}}\right)}{\sqrt{3} d^{4/3}(bc - ad)} + \frac{x}{bd}$$

Antiderivative was successfully verified.

[In] Int[x^6/((a + b*x^3)*(c + d*x^3)), x]

[Out] x/(b*d) - (a^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^(4/3)*(b*c - a*d)) + (c^(4/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]/(Sqrt[3]*d^(4/3)*(b*c - a*d)) + (a^(4/3)*Log[a^(1/3) + b^(1/3)*x]/(3*b^(4/3)*(b*c - a*d)) - (c^(4/3)*Log[c^(1/3) + d^(1/3)*x]/(3*d^(4/3)*(b*c - a*d)) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(4/3)*(b*c - a*d)) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(6*d^(4/3)*(b*c - a*d))

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(a+bx^3)(c+dx^3)} dx &= \frac{x}{bd} - \frac{\int \frac{ac+(bc+ad)x^3}{(a+bx^3)(c+dx^3)} dx}{bd} \\
&= \frac{x}{bd} + \frac{a^2 \int \frac{1}{a+bx^3} dx}{b(bc-ad)} - \frac{c^2 \int \frac{1}{c+dx^3} dx}{d(bc-ad)} \\
&= \frac{x}{bd} + \frac{a^{4/3} \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3b(bc-ad)} + \frac{a^{4/3} \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3b(bc-ad)} - \frac{c^{4/3} \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{3d(bc-ad)} - \frac{c^{4/3} \int \frac{1}{c+dx^3} dx}{3d(bc-ad)} \\
&= \frac{x}{bd} + \frac{a^{4/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3} \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6b^{4/3}(bc-ad)} + \frac{c^{4/3} \int \frac{1}{c+dx^3} dx}{3d(bc-ad)} \\
&= \frac{x}{bd} + \frac{a^{4/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6b^{4/3}(bc-ad)} - \frac{c^{4/3} \log(c+dx^3)}{3d(bc-ad)} \\
&= \frac{x}{bd} - \frac{a^{4/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}(bc-ad)} + \frac{c^{4/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{4/3}(bc-ad)} + \frac{a^{4/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3} \log(c+dx^3)}{3d(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 238, normalized size = 0.80

$$\frac{\frac{a^{4/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{b^{4/3}} + \frac{2a^{4/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{b^{4/3}} - \frac{2\sqrt{3}a^{4/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{4/3}} - \frac{6ax}{b} + \frac{c^{4/3} \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2)}{d^{4/3}} - \frac{2c^{4/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{d^{4/3}} + \frac{2\sqrt{3}c^{4/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{d^{4/3}} + \frac{6cx}{d}}{6bc - 6ad}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/((a + b*x^3)*(c + d*x^3)), x]

[Out] ((-6*a*x)/b + (6*c*x)/d - (2*Sqrt[3]*a^(4/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(4/3) + (2*Sqrt[3]*c^(4/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(4/3) + (2*a^(4/3)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) - (2*c^(4/3)*Log[c^(1/3) + d^(1/3)*x])/d^(4/3) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(4/3)/(6*b*c - 6*a*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/((a + b*x^3)*(c + d*x^3)),x]

[Out] IntegrateAlgebraic[x^6/((a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 0.94, size = 228, normalized size = 0.77

$$\frac{2\sqrt{3}ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\ln\left(\frac{x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+2\sqrt{3}bc\left(\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}d\ln\left(\frac{x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}-\sqrt{3}c}{3c}\right)}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)-ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)-bc\left(\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)+2ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+2bc\left(\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)-6(bc-ad)x}{6(b^2cd-abd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")

[Out] $-1/6*(2*\sqrt{3}*a*d*(-a/b)^{(1/3)}*\arctan(1/3*(2*\sqrt{3}*b*x*(-a/b)^{(2/3)} - \sqrt{3}*a)/a) + 2*\sqrt{3}*b*c*(c/d)^{(1/3)}*\arctan(1/3*(2*\sqrt{3}*d*x*(c/d)^{(2/3)} - \sqrt{3}*c)/c) - a*d*(-a/b)^{(1/3)}*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)}) - b*c*(c/d)^{(1/3)}*\log(x^2 - x*(c/d)^{(1/3)} + (c/d)^{(2/3)}) + 2*a*d*(-a/b)^{(1/3)}*\log(x - (-a/b)^{(1/3)}) + 2*b*c*(c/d)^{(1/3)}*\log(x + (c/d)^{(1/3)}) - 6*(b*c - a*d)*x/(b^2*c*d - a*b*d^2)$

giac [A] time = 0.21, size = 308, normalized size = 1.04

$$\frac{a^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ab^2c-a^2bd)}+\frac{c^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(bc^2d-acd^2)}+\frac{(-ab^2)^{\frac{1}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c-\sqrt{3}ab^2d}-\frac{(-cd^2)^{\frac{1}{3}}c\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3}+\frac{(-ab^2)^{\frac{1}{3}}a\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(b^3c-ab^2d)}-\frac{(-cd^2)^{\frac{1}{3}}c\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bc^2d-ad^3)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")

[Out] $-1/3*a^2*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a*b^2*c - a^2*b*d) + 1/3*c^2*(-c/d)^{(1/3)}*\log(\text{abs}(x - (-c/d)^{(1/3)}))/(b*c^2*d - a*c*d^2) + (-a*b^2)^{(1/3)}*a*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(sqrt{3}*b^3*c - sqrt{3}*a*b^2*d) - (-c*d^2)^{(1/3)}*c*\arctan(1/3*\sqrt{3}*(2*x + (-c/d)^{(1/3)})/(-c/d)^{(1/3)})/(sqrt{3}*b*c*d^2 - sqrt{3}*a*d^3) + 1/6*(-a*b^2)^{(1/3)}*a*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(b^3*c - a*b^2*d) - 1/6*(-c*d^2)^{(1/3)}*c*\log(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)})/(b*c*d^2 - a*d^3) + x/(b*d)$

maple [A] time = 0.05, size = 266, normalized size = 0.90

$$\frac{\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{1}-1\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{a^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{a^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{\sqrt{3}c^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{1}-1\right)}{\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}+\frac{c^2\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}-\frac{c^2\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(b*x^3+a)/(d*x^3+c),x)`

[Out] $x/b/d - 1/3/b^2*a^2/(a*d - b*c)/(a/b)^{(2/3)}*\ln(x + (a/b)^{(1/3)}) + 1/6/b^2*a^2/(a*d - b*c)/(a/b)^{(2/3)}*\ln(x^2 - (a/b)^{(1/3)}*x + (a/b)^{(2/3)}) - 1/3/b^2*a^2/(a*d - b*c)/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x - 1)) + 1/3/d^2*c^2/(a*d - b*c)/(c/d)^{(2/3)}*\ln(x + (c/d)^{(1/3)}) - 1/6/d^2*c^2/(a*d - b*c)/(c/d)^{(2/3)}*\ln(x^2 - (c/d)^{(1/3)}*x + (c/d)^{(2/3)}) + 1/3/d^2*c^2/(a*d - b*c)/(c/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x - 1))$

maxima [A] time = 1.35, size = 349, normalized size = 1.18

$$\frac{\sqrt{3}a^2 \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}}}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}c^2 \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}}}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}} - ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{a^2 \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c\left(\frac{a}{b}\right)^{\frac{2}{3}} - ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{c^2 \log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2\left(\frac{c}{d}\right)^{\frac{2}{3}} - ad^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{a^2 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{c^2 \log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}} - ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")`

[Out] $1/3*\sqrt{3}*a^2*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/((b^3*c*(a/b)^{(1/3)} - a*b^2*d*(a/b)^{(1/3)})*(a/b)^{(1/3)}) - 1/3*\sqrt{3}*c^2*\arctan(1/3*\sqrt{3}*(2*x - (c/d)^{(1/3)})/(c/d)^{(1/3)})/((b*c*d^2*(c/d)^{(1/3)} - a*d^3*(c/d)^{(1/3)})*(c/d)^{(1/3)}) - 1/6*a^2*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^3*c*(a/b)^{(2/3)} - a*b^2*d*(a/b)^{(2/3)}) + 1/6*c^2*\log(x^2 - x*(c/d)^{(1/3)} + (c/d)^{(2/3)})/(b*c*d^2*(c/d)^{(2/3)} - a*d^3*(c/d)^{(2/3)}) + 1/3*a^2*\log(x + (a/b)^{(1/3)})/(b^3*c*(a/b)^{(2/3)} - a*b^2*d*(a/b)^{(2/3)}) - 1/3*c^2*\log(x + (c/d)^{(1/3)})/(b*c*d^2*(c/d)^{(2/3)} - a*d^3*(c/d)^{(2/3)}) + x/(b*d)$

mupad [B] time = 1.83, size = 873, normalized size = 2.95

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/((a + b*x^3)*(c + d*x^3)),x)`

[Out] $\log(a*x + b^2*c*(-a^4/(b^4*(a*d - b*c)^3))^{(1/3)} - a*b*d*(-a^4/(b^4*(a*d - b*c)^3))^{(1/3)})*(a^4/(27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 - 81*a*b^6*c^2*d))^{(1/3)} + \log(c*x + a*d^2*(c^4/(d^4*(a*d - b*c)^3))^{(1/3)} - b*c*d*(c^4/(d^4*(a*d - b*c)^3))^{(1/3)})*(c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 + 81*a*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^{(1/3)} + x/(b*d) + (\log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4))/(b*d) - (3*a*c^2*(3^{(1/2)}*i - 1)*(-a^4/(b^4*(a*d - b*c)^3))^{(1/3)}*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*d))*(a^4/(27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 - 81*a*b^6*c^2*d))^{(1/3)}*(3^{(1/2)}*i - 1))/2 - (\log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4))/(b*d) + (3*a*c^2*(3^{(1/2)}*i - 1)))/2)$

$$\begin{aligned}
&) * 1i + 1) * (-a^4 / (b^4 * (a*d - b*c)^3))^{(1/3)} * (a^5*d^5 - b^5*c^5 + a*b^4*c^4*d \\
& - a^4*b*c*d^4) / (2*d)) * (a^4 / (27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 \\
& - 81*a*b^6*c^2*d))^{(1/3)} * (3^{(1/2)} * 1i + 1)) / 2 + (\log((3*x*(a^2*b^4*c^6 + a \\
& ^6*c^2*d^4)) / (b*d) + (3*a^2*c*(3^{(1/2)} * 1i - 1) * (c^4 / (d^4 * (a*d - b*c)^3))^{(1 \\
& /3)} * (a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - a^4*b*c*d^4)) / (2*b)) * (c^4 / (27*a^3*d^7 \\
& - 27*b^3*c^3*d^4 + 81*a*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^{(1/3)} * (3^{(1/2)} * 1i \\
& - 1)) / 2 - (\log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4)) / (b*d) - (3*a^2*c*(3^{(1/2)} * \\
& 1i + 1) * (c^4 / (d^4 * (a*d - b*c)^3))^{(1/3)} * (a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - \\
& a^4*b*c*d^4)) / (2*b)) * (c^4 / (27*a^3*d^7 - 27*b^3*c^3*d^4 + 81*a*b^2*c^2*d^5 - \\
& 81*a^2*b*c*d^6))^{(1/3)} * (3^{(1/2)} * 1i + 1)) / 2
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(b*x**3+a)/(d*x**3+c),x)

[Out] Timed out

$$3.110 \quad \int \frac{x^5}{(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=53

$$\frac{c \log(c + dx^3)}{3d(bc - ad)} - \frac{a \log(a + bx^3)}{3b(bc - ad)}$$

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 72}

$$\frac{c \log(c + dx^3)}{3d(bc - ad)} - \frac{a \log(a + bx^3)}{3b(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[x^5/((a + b*x^3)*(c + d*x^3)),x]

[Out] -(a*Log[a + b*x^3]/(3*b*(b*c - a*d)) + (c*Log[c + d*x^3]/(3*d*(b*c - a*d)))

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(a+bx^3)(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{(a+bx)(c+dx)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{a}{(bc-ad)(a+bx)} + \frac{c}{(bc-ad)(c+dx)} \right) dx, x, x^3 \right) \\ &= -\frac{a \log(a+bx^3)}{3b(bc-ad)} + \frac{c \log(c+dx^3)}{3d(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.06, size = 43, normalized size = 0.81

$$-\frac{ad \log(a + bx^3) - bc \log(c + dx^3)}{3b^2cd - 3abd^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^3)*(c + d*x^3)),x]

[Out] -((a*d*Log[a + b*x^3] - b*c*Log[c + d*x^3])/(3*b^2*c*d - 3*a*b*d^2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + bx^3)(c + dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/((a + b*x^3)*(c + d*x^3)),x]

[Out] IntegrateAlgebraic[x^5/((a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 0.99, size = 42, normalized size = 0.79

$$-\frac{ad \log(bx^3 + a) - bc \log(dx^3 + c)}{3(b^2cd - abd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")

[Out] -1/3*(a*d*log(b*x^3 + a) - b*c*log(d*x^3 + c))/(b^2*c*d - a*b*d^2)

giac [A] time = 0.19, size = 51, normalized size = 0.96

$$-\frac{a \log(|bx^3 + a|)}{3(b^2c - abd)} + \frac{c \log(|dx^3 + c|)}{3(bcd - ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")

[Out] -1/3*a*log(abs(b*x^3 + a))/(b^2*c - a*b*d) + 1/3*c*log(abs(d*x^3 + c))/(b*c*d - a*d^2)

maple [A] time = 0.05, size = 50, normalized size = 0.94

$$\frac{a \ln(bx^3 + a)}{3(ad - bc)b} - \frac{c \ln(dx^3 + c)}{3(ad - bc)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^3+a)/(d*x^3+c),x)

[Out] 1/3*a/(a*d-b*c)/b*ln(b*x^3+a)-1/3*c/(a*d-b*c)/d*ln(d*x^3+c)

maxima [A] time = 0.46, size = 49, normalized size = 0.92

$$-\frac{a \log(bx^3 + a)}{3(b^2c - abd)} + \frac{c \log(dx^3 + c)}{3(bcd - ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")

[Out] -1/3*a*log(b*x^3 + a)/(b^2*c - a*b*d) + 1/3*c*log(d*x^3 + c)/(b*c*d - a*d^2)

mupad [B] time = 0.31, size = 51, normalized size = 0.96

$$-\frac{a \ln(bx^3 + a)}{3b^2c - 3abd} - \frac{c \ln(dx^3 + c)}{3ad^2 - 3bcd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((a + b*x^3)*(c + d*x^3)),x)

[Out] - (a*log(a + b*x^3))/(3*b^2*c - 3*a*b*d) - (c*log(c + d*x^3))/(3*a*d^2 - 3*b*c*d)

sympy [B] time = 6.75, size = 144, normalized size = 2.72

$$\frac{a \log\left(x^3 + \frac{\frac{a^3d^2}{b(ad-bc)} - \frac{2a^2cd}{ad-bc} + \frac{abc^2}{ad-bc} + 2ac}{ad+bc}\right)}{3b(ad-bc)} - \frac{c \log\left(x^3 + \frac{-\frac{a^2cd}{ad-bc} + \frac{2abc^2}{ad-bc} + 2ac - \frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{3d(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x**3+a)/(d*x**3+c),x)

[Out] a*log(x**3 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**2/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(3*b*(a*d - b*c)) - c*log(x**3 + (-a**2*c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b*c)))/(a*d + b*c))/(3*d*(a*d - b*c))

$$3.111 \quad \int \frac{x^4}{(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=288

$$-\frac{a^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{2/3}(bc - ad)} + \frac{a^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{2/3}(bc - ad)} + \frac{a^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc - ad)} + \frac{c^{2/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6d^{2/3}(bc - ad)}$$

Rubi [A] time = 0.15, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {481, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{2/3}(bc - ad)} + \frac{a^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{2/3}(bc - ad)} + \frac{a^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc - ad)} + \frac{c^{2/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6d^{2/3}(bc - ad)} - \frac{c^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3d^{2/3}(bc - ad)} - \frac{c^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{2/3}(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[x^4/((a + b*x^3)*(c + d*x^3)),x]

[Out] (a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^(2/3)*(b*c - a*d)) - (c^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]/(Sqrt[3]*d^(2/3)*(b*c - a*d)) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(2/3)*(b*c - a*d)) - (c^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(2/3)*(b*c - a*d)) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(b*c - a*d)) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d^(2/3)*(b*c - a*d))

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^-1, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.))),
  x_Symbol] := -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
  x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(a+bx^3)(c+dx^3)} dx &= -\frac{a \int \frac{x}{a+bx^3} dx}{bc-ad} + \frac{c \int \frac{x}{c+dx^3} dx}{bc-ad} \\
&= \frac{a^{2/3} \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3\sqrt[3]{b}(bc-ad)} - \frac{a^{2/3} \int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2} dx}{3\sqrt[3]{b}(bc-ad)} - \frac{c^{2/3} \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{3\sqrt[3]{d}(bc-ad)} + \frac{c^{2/3} \int \frac{\sqrt[3]{c} + \sqrt[3]{d}x}{c^{2/3} - \sqrt[3]{c} \sqrt[3]{d}x + d^{2/3}x^2} dx}{3\sqrt[3]{d}(bc-ad)} \\
&= \frac{a^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3d^{2/3}(bc-ad)} - \frac{a^{2/3} \int \frac{-\sqrt[3]{a} \sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2} dx}{6b^{2/3}(bc-ad)} - \frac{a \int \frac{1}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2} dx}{2\sqrt[3]{b}(bc-ad)} \\
&= \frac{a^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3d^{2/3}(bc-ad)} - \frac{a^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{6b^{2/3}(bc-ad)} + \\
&= \frac{a^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc-ad)} - \frac{c^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{2/3}(bc-ad)} + \frac{a^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3d^{2/3}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 224, normalized size = 0.78

$$\frac{-\frac{a^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{b^{2/3}} + \frac{2a^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{b^{2/3}} + \frac{2\sqrt{3}a^{2/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} + \frac{c^{2/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d}x + d^{2/3}x^2)}{d^{2/3}} - \frac{2c^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{d^{2/3}} - \frac{2\sqrt{3}c^{2/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{d^{2/3}}}{6bc - 6ad}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((a + b*x^3)*(c + d*x^3)),x]

[Out] ((2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) - (2*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(2/3) + (2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (2*c^(2/3)*Log[c^(1/3) + d^(1/3)*x])/d^(2/3) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(2/3) / (6*b*c - 6*a*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/((a + b*x^3)*(c + d*x^3)), x]

[Out] IntegrateAlgebraic[x^4/((a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 1.05, size = 244, normalized size = 0.85

$$\frac{2\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}a}{3a}\right)-2\sqrt{3}\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}c}{3c}\right)-\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\log\left(cx^2-dx\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}+c\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\right)+2\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+2\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\log\left(cx+d\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\right)}{6(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a)/(d*x^3+c), x, algorithm="fricas")

[Out] $-1/6*(2*\sqrt{3})*(-a^2/b^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3})*b*x*(-a^2/b^2)^{(1/3)} + \sqrt{3}*a)/a - 2*\sqrt{3}*(c^2/d^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3})*d*x*(c^2/d^2)^{(1/3)} - \sqrt{3}*c)/c - (-a^2/b^2)^{(1/3)}*\log(a*x^2 - b*x*(-a^2/b^2)^{(2/3)} - a*(-a^2/b^2)^{(1/3)}) - (c^2/d^2)^{(1/3)}*\log(c*x^2 - d*x*(c^2/d^2)^{(2/3)} + c*(c^2/d^2)^{(1/3)}) + 2*(-a^2/b^2)^{(1/3)}*\log(a*x + b*(-a^2/b^2)^{(2/3)}) + 2*(c^2/d^2)^{(1/3)}*\log(c*x + d*(c^2/d^2)^{(2/3)})/(b*c - a*d)$

giac [A] time = 0.25, size = 286, normalized size = 0.99

$$\frac{a\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\log\left(x-\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)}{3(abc-a^2d)} - \frac{c\left(\frac{c^2}{d^2}\right)^{\frac{2}{3}}\log\left(x-\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\right)}{3(bc^2-acd)} + \frac{(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c-\sqrt{3}ab^2d} - \frac{(-cd^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3} - \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)}{6(b^3c-ab^2d)} + \frac{(-cd^2)^{\frac{2}{3}}\log\left(x^2+x\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}+\left(\frac{c^2}{d^2}\right)^{\frac{2}{3}}\right)}{6(bcd^2-ad^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a)/(d*x^3+c), x, algorithm="giac")

[Out] $1/3*a*(-a/b)^{(2/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a*b*c - a^2*d) - 1/3*c*(-c/d)^{(2/3)}*\log(\text{abs}(x - (-c/d)^{(1/3)}))/(b*c^2 - a*c*d) + (-a*b^2)^{(2/3)}*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(\sqrt{3}*b^3*c - \sqrt{3}*a*b^2*d) - (-c*d^2)^{(2/3)}*\arctan(1/3*\sqrt{3}*(2*x + (-c/d)^{(1/3)})/(-c/d)^{(1/3)})/(\sqrt{3}*b*c*d^2 - \sqrt{3}*a*d^3) - 1/6*(-a*b^2)^{(2/3)}*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(b^3*c - a*b^2*d) + 1/6*(-c*d^2)^{(2/3)}*\log(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)})/(b*c*d^2 - a*d^3)$

maple [A] time = 0.05, size = 246, normalized size = 0.85

$$\frac{\sqrt{3} a \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b} - \frac{a \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b} + \frac{a \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b} - \frac{\sqrt{3} c \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}d} + \frac{c \ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}d} - \frac{c \ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^4/(b*x^3+a)/(d*x^3+c), x)$

[Out] $-1/3*a/(a*d-b*c)/b/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)})+1/6*a/(a*d-b*c)/b/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+1/3*a/(a*d-b*c)*3^{(1/2)}/b/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))+1/3*c/(a*d-b*c)/d/(c/d)^{(1/3)}*\ln(x+(c/d)^{(1/3)})-1/6*c/(a*d-b*c)/d/(c/d)^{(1/3)}*\ln(x^2-(c/d)^{(1/3)}*x+(c/d)^{(2/3)})-1/3*c/(a*d-b*c)*3^{(1/2)}/d/(c/d)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))$

maxima [A] time = 1.27, size = 289, normalized size = 1.00

$$-\frac{\sqrt{3} a \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c-abd\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3} c \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd-ad^2\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{a \log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{c \log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{a \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{c \log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^4/(b*x^3+a)/(d*x^3+c), x, \text{algorithm}="maxima")$

[Out] $-1/3*\text{sqrt}(3)*a*\arctan(1/3*\text{sqrt}(3)*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/((b^2*c - a*b*d)*(a/b)^{(1/3)}) + 1/3*\text{sqrt}(3)*c*\arctan(1/3*\text{sqrt}(3)*(2*x - (c/d)^{(1/3)})/(c/d)^{(1/3)})/((b*c*d - a*d^2)*(c/d)^{(1/3)}) - 1/6*a*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^2*c*(a/b)^{(1/3)} - a*b*d*(a/b)^{(1/3)}) + 1/6*c*\log(x^2 - x*(c/d)^{(1/3)} + (c/d)^{(2/3)})/(b*c*d*(c/d)^{(1/3)} - a*d^2*(c/d)^{(1/3)}) + 1/3*a*\log(x + (a/b)^{(1/3)})/(b^2*c*(a/b)^{(1/3)} - a*b*d*(a/b)^{(1/3)}) - 1/3*c*\log(x + (c/d)^{(1/3)})/(b*c*d*(c/d)^{(1/3)} - a*d^2*(c/d)^{(1/3)})$

mupad [B] time = 9.05, size = 1364, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^4/((a + b*x^3)*(c + d*x^3)), x)$

[Out] $\log(a*x + b^3*c^2*(-a^2/(b^2*(a*d - b*c)^3))^{(2/3)} + a^2*b*d^2*(-a^2/(b^2*(a*d - b*c)^3))^{(2/3)} - 2*a*b^2*c*d*(-a^2/(b^2*(a*d - b*c)^3))^{(2/3)})*(a^2/(27*b^5*c^3 - 27*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^{(1/3)} + \log(c*x + a^2*d^3*(c^2/(d^2*(a*d - b*c)^3))^{(2/3)} + b^2*c^2*d*(c^2/(d^2*(a*d - b*c)^3))^{(2/3)} - 2*a*b*c*d^2*(c^2/(d^2*(a*d - b*c)^3))^{(2/3)})*(c^2/(27*a^3*d^5 - 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^{(1/3)} + (\log(((3^{(1/2)}*1i - 1)^2*(-a^2/(b^2*(a*d - b*c)^3))^{(2/3)})*((3^{(1/2)}*1i - 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^{(1/2)}*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-a^2/(b^2*(a*d - b*c)^3))^{(2/3)}))/4)*(-a^2/(b^2*(a*d - b*c)^3))^{(1/3)})/6 - 9*a^2*b^4*c^4*d^2 - 9*a^4*b^2*c^2*d^4 + 9*a*b^5*c^5*d + 9*a^5*b*c*d^5)/36 + a^2*b*c^2*d*x*(a*d + b*c))*(a^2/(27*b^5*c^3 - 27*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^{(1/3)}*(3^{(1/2)}*1i - 1))/2 - (\log(((3^{(1/2)}*1i + 1)^2*(-a^2/(b^2*(a*d - b*c)^3))^{(2/3)})*((3^{(1/2)}*1i + 1)$

$$\begin{aligned}
&) * (54 * a^2 * b^3 * c^2 * d^3 * x * (a * d - b * c)^2 + (27 * a * b^3 * c * d^3 * (3^{(1/2)} * i + 1)^2 * \\
& (a * d + b * c) * (a * d - b * c)^4 * (-a^2 / (b^2 * (a * d - b * c)^3))^{(2/3)}) / 4 * (-a^2 / (b^2 * (\\
& a * d - b * c)^3))^{(1/3)} / 6 + 9 * a^2 * b^4 * c^4 * d^2 + 9 * a^4 * b^2 * c^2 * d^4 - 9 * a * b^5 * c \\
& ^5 * d - 9 * a^5 * b * c * d^5) / 36 - a^2 * b * c^2 * d * x * (a * d + b * c) * (a^2 / (27 * b^5 * c^3 - 2 \\
& 7 * a^3 * b^2 * d^3 + 81 * a^2 * b^3 * c * d^2 - 81 * a * b^4 * c^2 * d))^{(1/3)} * (3^{(1/2)} * i + 1) \\
& / 2 + (\log(((3^{(1/2)} * i - 1)^2 * (c^2 / (d^2 * (a * d - b * c)^3))^{(2/3)} * (((3^{(1/2)} * i \\
& - 1) * (54 * a^2 * b^3 * c^2 * d^3 * x * (a * d - b * c)^2 + (27 * a * b^3 * c * d^3 * (3^{(1/2)} * i - 1) \\
&)^2 * (a * d + b * c) * (a * d - b * c)^4 * (c^2 / (d^2 * (a * d - b * c)^3))^{(2/3)}) / 4 * (c^2 / (d^2 \\
& * (a * d - b * c)^3))^{(1/3)} / 6 - 9 * a^2 * b^4 * c^4 * d^2 - 9 * a^4 * b^2 * c^2 * d^4 + 9 * a * b^5 * \\
& c^5 * d + 9 * a^5 * b * c * d^5) / 36 + a^2 * b * c^2 * d * x * (a * d + b * c) * (c^2 / (27 * a^3 * d^5 - \\
& 27 * b^3 * c^3 * d^2 + 81 * a * b^2 * c^2 * d^3 - 81 * a^2 * b * c * d^4))^{(1/3)} * (3^{(1/2)} * i - 1) \\
&)) / 2 - (\log(((3^{(1/2)} * i + 1)^2 * (c^2 / (d^2 * (a * d - b * c)^3))^{(2/3)} * (((3^{(1/2)} * i \\
& + 1) * (54 * a^2 * b^3 * c^2 * d^3 * x * (a * d - b * c)^2 + (27 * a * b^3 * c * d^3 * (3^{(1/2)} * i + 1) \\
&)^2 * (a * d + b * c) * (a * d - b * c)^4 * (c^2 / (d^2 * (a * d - b * c)^3))^{(2/3)}) / 4 * (c^2 / (d \\
& ^2 * (a * d - b * c)^3))^{(1/3)} / 6 + 9 * a^2 * b^4 * c^4 * d^2 + 9 * a^4 * b^2 * c^2 * d^4 - 9 * a * b \\
& ^5 * c^5 * d - 9 * a^5 * b * c * d^5) / 36 - a^2 * b * c^2 * d * x * (a * d + b * c) * (c^2 / (27 * a^3 * d^5 \\
& - 27 * b^3 * c^3 * d^2 + 81 * a * b^2 * c^2 * d^3 - 81 * a^2 * b * c * d^4))^{(1/3)} * (3^{(1/2)} * i + \\
& 1)) / 2
\end{aligned}$$

sympy [B] time = 123.98, size = 573, normalized size = 1.99

RootSum(1/27*a^2*b^3*c^2*d^3*x*(a*d-b*c)^2+(27*a*b^3*c*d^3*(3^(1/2)*i+1)^2*(a*d+b*c)*(a*d-b*c)^4*(-a^2/(b^2*(a*d-b*c)^3))^(2/3))/4*(-a^2/(b^2*(a*d-b*c)^3))^(1/3)/6+9*a^2*b^4*c^4*d^2+9*a^4*b^2*c^2*d^4-9*a*b^5*c^5*d-9*a^5*b*c*d^5)/36-a^2*b*c^2*d*x*(a*d+b*c)*(a^2/(27*b^5*c^3-27*a^3*b^2*d^3+81*a^2*b^3*c*d^2-81*a*b^4*c^2*d))^(1/3)*(3^(1/2)*i+1)/2+(log(((3^(1/2)*i-1)^2*(c^2/(d^2*(a*d-b*c)^3))^(2/3)*(((3^(1/2)*i-1)*(54*a^2*b^3*c^2*d^3*x*(a*d-b*c)^2+(27*a*b^3*c*d^3*(3^(1/2)*i-1)^2*(a*d+b*c)*(a*d-b*c)^4*(c^2/(d^2*(a*d-b*c)^3))^(2/3))/4*(c^2/(d^2*(a*d-b*c)^3))^(1/3)/6-9*a^2*b^4*c^4*d^2-9*a^4*b^2*c^2*d^4+9*a*b^5*c^5*d+9*a^5*b*c*d^5)/36+a^2*b*c^2*d*x*(a*d+b*c)*(c^2/(27*a^3*d^5-27*b^3*c^3*d^2+81*a*b^2*c^2*d^3-81*a^2*b*c*d^4))^(1/3)*(3^(1/2)*i-1)))/2-(log(((3^(1/2)*i+1)^2*(c^2/(d^2*(a*d-b*c)^3))^(2/3)*(((3^(1/2)*i+1)*(54*a^2*b^3*c^2*d^3*x*(a*d-b*c)^2+(27*a*b^3*c*d^3*(3^(1/2)*i+1)^2*(a*d+b*c)*(a*d-b*c)^4*(c^2/(d^2*(a*d-b*c)^3))^(2/3))/4*(c^2/(d^2*(a*d-b*c)^3))^(1/3)/6+9*a^2*b^4*c^4*d^2+9*a^4*b^2*c^2*d^4-9*a*b^5*c^5*d-9*a^5*b*c*d^5)/36-a^2*b*c^2*d*x*(a*d+b*c)*(c^2/(27*a^3*d^5-27*b^3*c^3*d^2+81*a*b^2*c^2*d^3-81*a^2*b*c*d^4))^(1/3)*(3^(1/2)*i+1)))/2)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**3+a)/(d*x**3+c), x)

[Out] RootSum(_t**3*(27*a**3*d**5 - 81*a**2*b*c*d**4 + 81*a*b**2*c**2*d**3 - 27*b**3*c**3*d**2) - c**2, Lambda(_t, _t*log(x + (243*_t**5*a**6*b**2*d**8 - 1458*_t**5*a**5*b**3*c*d**7 + 3645*_t**5*a**4*b**4*c**2*d**6 - 4860*_t**5*a**3*b**5*c**3*d**5 + 3645*_t**5*a**2*b**6*c**4*d**4 - 1458*_t**5*a*b**7*c**5*d**3 + 243*_t**5*b**8*c**6*d**2 + 9*_t**2*a**5*d**5 - 18*_t**2*a**4*b*c*d**4 + 9*_t**2*a**3*b**2*c**2*d**3 + 9*_t**2*a**2*b**3*c**3*d**2 - 18*_t**2*a*b**4*c**4*d + 9*_t**2*b**5*c**5)/(a**3*c*d**2 + a*b**2*c**3)))) + RootSum(_t**3*(27*a**3*b**2*d**3 - 81*a**2*b**3*c*d**2 + 81*a*b**4*c**2*d - 27*b**5*c**3) + a**2, Lambda(_t, _t*log(x + (243*_t**5*a**6*b**2*d**8 - 1458*_t**5*a**5*b**3*c*d**7 + 3645*_t**5*a**4*b**4*c**2*d**6 - 4860*_t**5*a**3*b**5*c**3*d**5 + 3645*_t**5*a**2*b**6*c**4*d**4 - 1458*_t**5*a*b**7*c**5*d**3 + 243*_t**5*b**8*c**6*d**2 + 9*_t**2*a**5*d**5 - 18*_t**2*a**4*b*c*d**4 + 9*_t**2*a**3*b**2*c**2*d**3 + 9*_t**2*a**2*b**3*c**3*d**2 - 18*_t**2*a*b**4*c**4*d + 9*_t**2*b**5*c**5)/(a**3*c*d**2 + a*b**2*c**3))))

$$3.112 \quad \int \frac{x^3}{(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=288

$$\frac{\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6\sqrt[3]{d}(bc-ad)} - \frac{\sqrt[3]{a} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c} \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3\sqrt[3]{d}(bc-ad)}$$

Rubi [A] time = 0.15, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {481, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6\sqrt[3]{d}(bc-ad)} - \frac{\sqrt[3]{a} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c} \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3\sqrt[3]{d}(bc-ad)} + \frac{\sqrt[3]{a} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c} \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^3/((a + b*x^3)*(c + d*x^3)),x]

[Out] (a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^(1/3)*(b*c - a*d)) - (c^(1/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]/(Sqrt[3]*d^(1/3)*(b*c - a*d)) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*x]/(3*b^(1/3)*(b*c - a*d)) + (c^(1/3)*Log[c^(1/3) + d^(1/3)*x]/(3*d^(1/3)*(b*c - a*d)) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(1/3)*(b*c - a*d)) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(6*d^(1/3)*(b*c - a*d)))

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^-1, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.))),
  x_Symbol] := -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
  x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx^3)(c+dx^3)} dx &= -\frac{a \int \frac{1}{a+bx^3} dx}{bc-ad} + \frac{c \int \frac{1}{c+dx^3} dx}{bc-ad} \\
&= -\frac{\sqrt[3]{a} \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x} dx}{3(bc-ad)} - \frac{\sqrt[3]{a} \int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx}{3(bc-ad)} + \frac{\sqrt[3]{c} \int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x} dx}{3(bc-ad)} + \frac{\sqrt[3]{c} \int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2} dx}{3(bc-ad)} \\
&= -\frac{\sqrt[3]{a} \log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c} \log(\sqrt[3]{c}+\sqrt[3]{d}x)}{3\sqrt[3]{d}(bc-ad)} - \frac{a^{2/3} \int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx}{2(bc-ad)} + \frac{\sqrt[3]{a} \int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx}{6\sqrt[3]{b}(bc-ad)} \\
&= -\frac{\sqrt[3]{a} \log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c} \log(\sqrt[3]{c}+\sqrt[3]{d}x)}{3\sqrt[3]{d}(bc-ad)} + \frac{\sqrt[3]{a} \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{6\sqrt[3]{b}(bc-ad)} \\
&= \frac{\sqrt[3]{a} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c} \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)} - \frac{\sqrt[3]{a} \log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c} \log(\sqrt[3]{c}+\sqrt[3]{d}x)}{3\sqrt[3]{d}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 224, normalized size = 0.78

$$\frac{\frac{\sqrt[3]{a} \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{\sqrt[3]{b}} - \frac{2\sqrt[3]{a} \log(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{b}} + \frac{2\sqrt{3}\sqrt[3]{a} \tan^{-1}\left(\frac{1-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{c} \log(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{c} \log(\sqrt[3]{c}+\sqrt[3]{d}x)}{\sqrt[3]{d}} - \frac{2\sqrt{3}\sqrt[3]{c} \tan^{-1}\left(\frac{1-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{6bc-6ad}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((a + b*x^3)*(c + d*x^3)),x]

[Out] ((2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - (2*Sqrt[3]*c^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3) - (2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (2*c^(1/3)*Log[c^(1/3) + d^(1/3)*x])/d^(1/3) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(1/3) / (6*b*c - 6*a*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/((a + b*x^3)*(c + d*x^3)), x]

[Out] IntegrateAlgebraic[x^3/((a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 0.80, size = 199, normalized size = 0.69

$$\frac{2\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\ln\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+2\sqrt{3}\left(-\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\ln\left(-\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)+2\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{6(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a)/(d*x^3+c), x, algorithm="fricas")

[Out] $-1/6*(2*\sqrt{3}*(a/b)^{(1/3)}*\arctan(1/3*(2*\sqrt{3})*b*x*(a/b)^{(2/3)} - \sqrt{3}*(a/b)^{(1/3)})/a + 2*\sqrt{3}*(-c/d)^{(1/3)}*\arctan(1/3*(2*\sqrt{3})*d*x*(-c/d)^{(2/3)} - \sqrt{3}*(-c/d)^{(1/3)})/c - (a/b)^{(1/3)}*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)}) - (-c/d)^{(1/3)}*\log(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)}) + 2*(a/b)^{(1/3)}*\log(x + (a/b)^{(1/3)}) + 2*(-c/d)^{(1/3)}*\log(x - (-c/d)^{(1/3)})/(b*c - a*d)$

giac [A] time = 0.20, size = 278, normalized size = 0.97

$$\frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(abc-a^2d)} - \frac{c\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(bc^2-acd)} - \frac{\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c-\sqrt{3}abd} + \frac{\left(-cd^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd-\sqrt{3}ad^2} - \frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(b^2c-abd)} + \frac{\left(-cd^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bcd-ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a)/(d*x^3+c), x, algorithm="giac")

[Out] $1/3*a*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a*b*c - a^2*d) - 1/3*c*(-c/d)^{(1/3)}*\log(\text{abs}(x - (-c/d)^{(1/3)}))/(b*c^2 - a*c*d) - (-a*b^2)^{(1/3)}*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(\sqrt{3}*b^2*c - \sqrt{3}*a*b*d) + (-c*d^2)^{(1/3)}*\arctan(1/3*\sqrt{3}*(2*x + (-c/d)^{(1/3)})/(-c/d)^{(1/3)})/(\sqrt{3}*b*c*d - \sqrt{3}*a*d^2) - 1/6*(-a*b^2)^{(1/3)}*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(b^2*c - a*b*d) + 1/6*(-c*d^2)^{(1/3)}*\log(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)})/(b*c*d - a*d^2)$

maple [A] time = 0.05, size = 246, normalized size = 0.85

$$\frac{\sqrt{3} a \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{b}-1\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b} + \frac{a \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b} - \frac{a \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b} - \frac{\sqrt{3} c \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{d}-1\right)}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}d} - \frac{c \ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}d} + \frac{c \ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^3+a)/(d*x^3+c), x)

[Out] $\frac{1}{3}a/(a*d-b*c)/b/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})-1/6*a/(a*d-b*c)/b/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+1/3*a/(a*d-b*c)/b/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))-1/3*c/(a*d-b*c)/d/(c/d)^{(2/3)}*\ln(x+(c/d)^{(1/3)})+1/6*c/(a*d-b*c)/d/(c/d)^{(2/3)}*\ln(x^2-(c/d)^{(1/3)}*x+(c/d)^{(2/3)})-1/3*c/(a*d-b*c)/d/(c/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))$

maxima [A] time = 1.21, size = 317, normalized size = 1.10

$$\frac{\sqrt{3}a \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}c \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{a \log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{c \log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} - \frac{a \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{c \log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")

[Out] $-\frac{1}{3}\sqrt{3}a*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/((b^2*c*(a/b)^{(1/3)} - a*b*d*(a/b)^{(1/3)}*(a/b)^{(1/3)}) + 1/3*\sqrt{3}c*\arctan(1/3*\sqrt{3}*(2*x - (c/d)^{(1/3)})/(c/d)^{(1/3)})/((b*c*d*(c/d)^{(1/3)} - a*d^2*(c/d)^{(1/3)}*(c/d)^{(1/3)}) + 1/6*a*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^2*c*(a/b)^{(2/3)} - a*b*d*(a/b)^{(2/3)}) - 1/6*c*\log(x^2 - x*(c/d)^{(1/3)} + (c/d)^{(2/3)})/(b*c*d*(c/d)^{(2/3)} - a*d^2*(c/d)^{(2/3)}) - 1/3*a*\log(x + (a/b)^{(1/3)})/(b^2*c*(a/b)^{(2/3)} - a*b*d*(a/b)^{(2/3)}) + 1/3*c*\log(x + (c/d)^{(1/3)})/(b*c*d*(c/d)^{(2/3)} - a*d^2*(c/d)^{(2/3)})$

mupad [B] time = 8.12, size = 1265, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + b*x^3)*(c + d*x^3)),x)

[Out] $\log(x + a*d*(a/(b*(a*d - b*c)^3))^{(1/3)} - b*c*(a/(b*(a*d - b*c)^3))^{(1/3)})*(-a/(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^{(1/3)} + \log(x - a*d*(-c/(d*(a*d - b*c)^3))^{(1/3)} + b*c*(-c/(d*(a*d - b*c)^3))^{(1/3)})*(-c/(27*a^3*d^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^{(1/3)} + (\log(((3^{(1/2)}*1i - 1)*(a/(b*(a*d - b*c)^3))^{(1/3)}*((3^{(1/2)}*1i - 1)^2*(81*a*b^3*c*d^3*x*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^{(1/2)}*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(a/(b*(a*d - b*c)^3))^{(1/3)})/2)*(a/(b*(a*d - b*c)^3))^{(2/3)})/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^2*d^4)/6 - 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*((3^{(1/2)}*1i - 1)*(a/(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^{(1/3)})/2 - (\log(((3^{(1/2)}*1i + 1)*(a/(b*(a*d - b*c)^3))^{(1/3)}*((3^{(1/2)}*1i + 1)^2*(81*a*b^3*c*d^3*x*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^{(1/2)}*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(a/(b*(a*d - b*c)^3))^{(1/3)})/2)*(a/(b*(a*d - b*c)^3))^{(2/3)})$

$$\begin{aligned} &)/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^2*d^4)/6 + 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^{(1/2)*1i} + 1)*(-a/(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^{(1/3)}/2 + (\log(((3^{(1/2)*1i} - 1)*(-c/(d*(a*d - b*c)^3))^{(1/3)}*((3^{(1/2)*1i} - 1)^2*(81*a*b^3*c*d^3*x*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^{(1/2)*1i} - 1)*(a*d + b*c)*(a*d - b*c)^4*(-c/(d*(a*d - b*c)^3))^{(1/3)}/2)*(-c/(d*(a*d - b*c)^3))^{(2/3)}/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^2*d^4))/6 - 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^{(1/2)*1i} - 1)*(-c/(27*a^3*d^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^{(1/3)}/2 - (\log(((3^{(1/2)*1i} + 1)*(-c/(d*(a*d - b*c)^3))^{(1/3)}*((3^{(1/2)*1i} + 1)^2*(81*a*b^3*c*d^3*x*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^{(1/2)*1i} + 1)*(a*d + b*c)*(a*d - b*c)^4*(-c/(d*(a*d - b*c)^3))^{(1/3)}/2)*(-c/(d*(a*d - b*c)^3))^{(2/3)}/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^2*d^4))/6 + 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^{(1/2)*1i} + 1)*(-c/(27*a^3*d^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^{(1/3)}/2 \end{aligned}$$

sympy [A] time = 20.20, size = 342, normalized size = 1.19

$$\text{RootSum}\left(x^3(27a^3d^4 - 81a^2b^2cd^3 + 81ab^3c^2d^2 - 27b^4c^3) + c, \log\left(x + \frac{162a^4b^5d^5 - 648a^3b^4c^2d^4 + 972a^2b^3c^3d^3 - 648a^2b^4c^3d^2 + 162a^3b^5c^4d - 3a^2d^5 + 6abcd - 3b^2c^2}{ad + bc}\right)\right) + \text{RootSum}\left(x^3(27a^3b^3d^3 - 81a^2b^2c^2d^2 + 81ab^3c^2d - 27b^4c^3) - a, \log\left(x + \frac{162a^4b^5d^5 - 648a^3b^4c^2d^4 + 972a^2b^3c^3d^3 - 648a^2b^4c^3d^2 + 162a^3b^5c^4d - 3a^2d^5 + 6abcd - 3b^2c^2}{ad + bc}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x**3+a)/(d*x**3+c), x)

[Out] RootSum(_t**3*(27*a**3*d**4 - 81*a**2*b*c*d**3 + 81*a*b**2*c**2*d**2 - 27*b**3*c**3*d) + c, Lambda(_t, _t*log(x + (162*_t**4*a**4*b*d**5 - 648*_t**4*a**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 - 648*_t**4*a*b**4*c**3*d**2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t*a*b*c*d - 3*_t*b**2*c**2)/(a*d + b*c)))) + RootSum(_t**3*(27*a**3*b*d**3 - 81*a**2*b**2*c*d**2 + 81*a*b**3*c**2*d - 27*b**4*c**3) - a, Lambda(_t, _t*log(x + (162*_t**4*a**4*b*d**5 - 648*_t**4*a**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 - 648*_t**4*a*b**4*c**3*d**2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t*a*b*c*d - 3*_t*b**2*c**2)/(a*d + b*c))))

$$3.113 \quad \int \frac{x^2}{(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=45

$$\frac{\log(a+bx^3)}{3(bc-ad)} - \frac{\log(c+dx^3)}{3(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {444, 36, 31}

$$\frac{\log(a+bx^3)}{3(bc-ad)} - \frac{\log(c+dx^3)}{3(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x^3)*(c + d*x^3)),x]

[Out] Log[a + b*x^3]/(3*(b*c - a*d)) - Log[c + d*x^3]/(3*(b*c - a*d))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+bx^3)(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(a+bx)(c+dx)} dx, x, x^3 \right) \\
&= \frac{b \text{Subst} \left(\int \frac{1}{a+bx} dx, x, x^3 \right) - d \text{Subst} \left(\int \frac{1}{c+dx} dx, x, x^3 \right)}{3(bc-ad)} \\
&= \frac{\log(a+bx^3)}{3(bc-ad)} - \frac{\log(c+dx^3)}{3(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.69

$$\frac{\log(a+bx^3) - \log(c+dx^3)}{3bc - 3ad}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + b*x^3)*(c + d*x^3)),x]

[Out] (Log[a + b*x^3] - Log[c + d*x^3])/(3*b*c - 3*a*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/((a + b*x^3)*(c + d*x^3)),x]

[Out] IntegrateAlgebraic[x^2/((a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 0.84, size = 31, normalized size = 0.69

$$\frac{\log(bx^3 + a) - \log(dx^3 + c)}{3(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")

[Out] 1/3*(log(b*x^3 + a) - log(d*x^3 + c))/(b*c - a*d)

giac [A] time = 0.18, size = 51, normalized size = 1.13

$$\frac{b \log(|bx^3 + a|)}{3(b^2c - abd)} - \frac{d \log(|dx^3 + c|)}{3(bcd - ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")

[Out] 1/3*b*log(abs(b*x^3 + a))/(b^2*c - a*b*d) - 1/3*d*log(abs(d*x^3 + c))/(b*c*d - a*d^2)

maple [A] time = 0.05, size = 42, normalized size = 0.93

$$-\frac{\ln(bx^3 + a)}{3(ad - bc)} + \frac{\ln(dx^3 + c)}{3ad - 3bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^3+a)/(d*x^3+c),x)

[Out] -1/3/(a*d-b*c)*ln(b*x^3+a)+1/3/(a*d-b*c)*ln(d*x^3+c)

maxima [A] time = 0.48, size = 41, normalized size = 0.91

$$\frac{\log(bx^3 + a)}{3(bc - ad)} - \frac{\log(dx^3 + c)}{3(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")

[Out] 1/3*log(b*x^3 + a)/(b*c - a*d) - 1/3*log(d*x^3 + c)/(b*c - a*d)

mupad [B] time = 0.26, size = 602, normalized size = 13.38

$$\operatorname{atan} \left(\frac{\left(\frac{x^3(36cb^4d^3+36ab^3d^4)+x^3(54a^2b^3d^5+108ab^4cd^4+54b^5c^2d^3)+108ab^4c^2d^3+108a^2b^3cd^4+36ab^3cd^3}{3ad-3bc} + 6b^3d^3x^3 \right)^{11} - \left(\frac{x^3(36cb^4d^3+36ab^3d^4)+x^3(54a^2b^3d^5+108ab^4cd^4+54b^5c^2d^3)+108ab^4c^2d^3+108a^2b^3cd^4+36ab^3cd^3}{3ad-3bc} - 6b^3d^3x^3 \right)^{11}}{\frac{x^3(36cb^4d^3+36ab^3d^4)+x^3(54a^2b^3d^5+108ab^4cd^4+54b^5c^2d^3)+108ab^4c^2d^3+108a^2b^3cd^4+36ab^3cd^3}{3ad-3bc} + 6b^3d^3x^3} + \frac{\left(\frac{x^3(36cb^4d^3+36ab^3d^4)+x^3(54a^2b^3d^5+108ab^4cd^4+54b^5c^2d^3)+108ab^4c^2d^3+108a^2b^3cd^4+36ab^3cd^3}{3ad-3bc} - 6b^3d^3x^3 \right)^{11}}{\frac{x^3(36cb^4d^3+36ab^3d^4)+x^3(54a^2b^3d^5+108ab^4cd^4+54b^5c^2d^3)+108ab^4c^2d^3+108a^2b^3cd^4+36ab^3cd^3}{3ad-3bc} - 6b^3d^3x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x^3)*(c + d*x^3)),x)

[Out] -(atan((((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) + (x^3*(54*a^2*b^3*d^5 + 54*b^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a*

$$d - 3bc) + 36ab^3cd^3)/(3ad - 3bc) + 6b^3d^3x^3)i)/(3ad - 3bc) - (((x^3(36ab^3d^4 + 36b^4cd^3) - (x^3(54a^2b^3d^5 + 54b^5c^2d^3 + 108ab^4cd^4) + 108ab^4c^2d^3 + 108a^2b^3cd^4)/(3ad - 3bc) + 36ab^3cd^3)/(3ad - 3bc) - 6b^3d^3x^3)i)/(3ad - 3bc)))/(((x^3(36ab^3d^4 + 36b^4cd^3) + (x^3(54a^2b^3d^5 + 54b^5c^2d^3 + 108ab^4cd^4) + 108ab^4c^2d^3 + 108a^2b^3cd^4)/(3ad - 3bc) + 36ab^3cd^3)/(3ad - 3bc) + 6b^3d^3x^3)/(3ad - 3bc) + ((x^3(36ab^3d^4 + 36b^4cd^3) - (x^3(54a^2b^3d^5 + 54b^5c^2d^3 + 108ab^4cd^4) + 108ab^4c^2d^3 + 108a^2b^3cd^4)/(3ad - 3bc) + 36ab^3cd^3)/(3ad - 3bc) - 6b^3d^3x^3)/(3ad - 3bc)))*2i)/(3ad - 3bc)$$

sympy [B] time = 2.47, size = 138, normalized size = 3.07

$$\frac{\log\left(x^3 + \frac{-\frac{a^2d^2}{ad-bc} + \frac{2abcd}{ad-bc} + ad - \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{3(ad-bc)} - \frac{\log\left(x^3 + \frac{\frac{a^2d^2}{ad-bc} - \frac{2abcd}{ad-bc} + ad + \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{3(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**3+a)/(d*x**3+c),x)

[Out] log(x**3 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**2/(a*d - b*c) + b*c)/(2*b*d))/(3*(a*d - b*c)) - log(x**3 + (a**2*d**2/(a*d - b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d))/(3*(a*d - b*c))

$$3.114 \quad \int \frac{x}{(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=288

$$\frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6\sqrt[3]{a}(bc-ad)} - \frac{\sqrt[3]{d} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6\sqrt[3]{c}(bc-ad)} - \frac{\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d} \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3\sqrt[3]{c}(bc-ad)}$$

Rubi [A] time = 0.14, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {482, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6\sqrt[3]{a}(bc-ad)} - \frac{\sqrt[3]{d} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6\sqrt[3]{c}(bc-ad)} - \frac{\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d} \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3\sqrt[3]{c}(bc-ad)} - \frac{\sqrt[3]{b} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d} \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x/((a + b*x^3)*(c + d*x^3)),x]

[Out] $-\left(\frac{b^{1/3} \operatorname{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3}x}{\sqrt{3}a^{1/3}}\right]}{\sqrt{3}a^{1/3}(bc-ad)}\right) + \left(\frac{d^{1/3} \operatorname{ArcTan}\left[\frac{c^{1/3} - 2d^{1/3}x}{\sqrt{3}c^{1/3}}\right]}{\sqrt{3}c^{1/3}(bc-ad)}\right) - \left(\frac{b^{1/3} \operatorname{Log}\left[\frac{a^{1/3} + b^{1/3}x}{3a^{1/3}(bc-ad)}\right]}{3a^{1/3}(bc-ad)}\right) + \left(\frac{d^{1/3} \operatorname{Log}\left[\frac{c^{1/3} + d^{1/3}x}{3c^{1/3}(bc-ad)}\right]}{3c^{1/3}(bc-ad)}\right) + \left(\frac{b^{1/3} \operatorname{Log}\left[\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{6a^{1/3}(bc-ad)}\right]}{6a^{1/3}(bc-ad)}\right) - \left(\frac{d^{1/3} \operatorname{Log}\left[\frac{c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2}{6c^{1/3}(bc-ad)}\right]}{6c^{1/3}(bc-ad)}\right)$

Rule 31

Int[((a_) + (b_.)*(x_))⁻¹, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)⁻¹, x_Symbol] := -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])⁻¹, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 482

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.))),
  x_Symbol] := Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
&& NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a+bx^3)(c+dx^3)} dx &= \frac{b \int \frac{x}{a+bx^3} dx}{bc-ad} - \frac{d \int \frac{x}{c+dx^3} dx}{bc-ad} \\
&= -\frac{b^{2/3} \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3\sqrt[3]{a}(bc-ad)} + \frac{b^{2/3} \int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3\sqrt[3]{a}(bc-ad)} + \frac{d^{2/3} \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{3\sqrt[3]{c}(bc-ad)} - \frac{d^{2/3} \int \frac{\sqrt[3]{c} + \sqrt[3]{d}x}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2} dx}{3\sqrt[3]{c}(bc-ad)} \\
&= -\frac{\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3\sqrt[3]{c}(bc-ad)} + \frac{\sqrt[3]{b} \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6\sqrt[3]{a}(bc-ad)} + \frac{b^{2/3} \int \frac{\sqrt[3]{c}\sqrt[3]{d} - 2d^{2/3}x}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2} dx}{6\sqrt[3]{c}(bc-ad)} \\
&= -\frac{\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3\sqrt[3]{c}(bc-ad)} + \frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6\sqrt[3]{a}(bc-ad)} - \frac{\sqrt[3]{d} \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2)}{6\sqrt[3]{c}(bc-ad)} \\
&= -\frac{\sqrt[3]{b} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)} - \frac{\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3\sqrt[3]{c}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 224, normalized size = 0.78

$$\frac{-\frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{\sqrt[3]{a}} + \frac{2\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{\sqrt[3]{a}} + \frac{2\sqrt{3}\sqrt[3]{b} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}} + \frac{\sqrt[3]{d} \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2)}{\sqrt[3]{c}} - \frac{2\sqrt[3]{d} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{\sqrt[3]{c}} - \frac{2\sqrt{3}\sqrt[3]{d} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt[3]{c}}}{6ad - 6bc}$$

Antiderivative was successfully verified.

[In] Integrate[x/((a + b*x^3)*(c + d*x^3)), x]

[Out] ((2*sqrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/a^(1/3) - (2*sqrt[3]*d^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/sqrt[3]])/c^(1/3) + (2*b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) - (2*d^(1/3)*Log[c^(1/3) + d^(1/3)*x])/c^(1/3) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3) + (d^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(1/3) /(-6*b*c + 6*a*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^3)(c+dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/((a + b*x^3)*(c + d*x^3)),x]

[Out] IntegrateAlgebraic[x/((a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 0.84, size = 201, normalized size = 0.70

$$\frac{2\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-2\sqrt{3}\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)+\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx^2-cx\left(-\frac{d}{c}\right)^{\frac{2}{3}}-c\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)-2\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)-2\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx+c\left(-\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")

[Out] $\frac{1}{6}*(2*\sqrt{3}*(b/a)^{(1/3)}*\arctan(2/3*\sqrt{3}*x*(b/a)^{(1/3)} - 1/3*\sqrt{3}) - 2*\sqrt{3}*(-d/c)^{(1/3)}*\arctan(2/3*\sqrt{3}*x*(-d/c)^{(1/3)} + 1/3*\sqrt{3}) + (b/a)^{(1/3)}*\log(b*x^2 - a*x*(b/a)^{(2/3)} + a*(b/a)^{(1/3)}) + (-d/c)^{(1/3)}*\log(d*x^2 - c*x*(-d/c)^{(2/3)} - c*(-d/c)^{(1/3)}) - 2*(b/a)^{(1/3)}*\log(b*x + a*(b/a)^{(2/3)}) - 2*(-d/c)^{(1/3)}*\log(d*x + c*(-d/c)^{(2/3)})/(b*c - a*d)$

giac [A] time = 0.25, size = 290, normalized size = 1.01

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(abc-a^2d)} + \frac{d\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(bc^2-acd)} - \frac{(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c-\sqrt{3}a^2bd} + \frac{(-cd^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2d-\sqrt{3}acd^2} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ab^2c-a^2bd)} - \frac{(-cd^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bc^2d-acd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")

[Out] $-1/3*b*(-a/b)^{(2/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a*b*c - a^2*d) + 1/3*d*(-c/d)^{(2/3)}*\log(\text{abs}(x - (-c/d)^{(1/3)}))/(b*c^2 - a*c*d) - (-a*b^2)^{(2/3)}*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(sqrt(3)*a*b^2*c - sqrt(3)*a^2*b*d) + (-c*d^2)^{(2/3)}*\arctan(1/3*\sqrt{3}*(2*x + (-c/d)^{(1/3)})/(-c/d)^{(1/3)})/(sqrt(3)*b*c^2*d - sqrt(3)*a*c*d^2) + 1/6*(-a*b^2)^{(2/3)}*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a*b^2*c - a^2*b*d) - 1/6*(-c*d^2)^{(2/3)}*\log(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)})/(b*c^2*d - a*c*d^2)$

maple [A] time = 0.05, size = 222, normalized size = 0.77

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^3+a)/(d*x^3+c),x)

[Out] $1/3/(a*d-b*c)/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)})-1/6/(a*d-b*c)/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})-1/3/(a*d-b*c)*3^{(1/2)}/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))-1/3/(a*d-b*c)/(c/d)^{(1/3)}*\ln(x+(c/d)^{(1/3)})+1/6/(a*d-b*c)/(c/d)^{(1/3)}*\ln(x^2-(c/d)^{(1/3)}*x+(c/d)^{(2/3)})+1/3/(a*d-b*c)*3^{(1/2)}/(c/d)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))$

maxima [A] time = 1.29, size = 265, normalized size = 0.92

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(bc-ad)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3(bc-ad)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")`

[Out] $1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/((b*c - a*d)*(a/b)^{(1/3)}) - 1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - (c/d)^{(1/3)})/(c/d)^{(1/3)})/((b*c - a*d)*(c/d)^{(1/3)}) + 1/6*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b*c*(a/b)^{(1/3)} - a*d*(a/b)^{(1/3)}) - 1/6*\log(x^2 - x*(c/d)^{(1/3)} + (c/d)^{(2/3)})/(b*c*(c/d)^{(1/3)} - a*d*(c/d)^{(1/3)}) - 1/3*\log(x + (a/b)^{(1/3)})/(b*c*(a/b)^{(1/3)} - a*d*(a/b)^{(1/3)}) + 1/3*\log(x + (c/d)^{(1/3)})/(b*c*(c/d)^{(1/3)} - a*d*(c/d)^{(1/3)})$

mupad [B] time = 5.42, size = 982, normalized size = 3.41



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((a + b*x^3)*(c + d*x^3)),x)`

[Out] $\log(b*x + a^3*d^2*(b/(a*(a*d - b*c)^3))^{(2/3)} + a*b^2*c^2*(b/(a*(a*d - b*c)^3))^{(2/3)} - 2*a^2*b*c*d*(b/(a*(a*d - b*c)^3))^{(2/3)})*(b/(27*a^4*d^3 - 27*a*b^3*c^3 + 81*a^2*b^2*c^2*d - 81*a^3*b*c*d^2))^{(1/3)} + \log(d*x + b^2*c^3*(-d/(c*(a*d - b*c)^3))^{(2/3)} + a^2*c*d^2*(-d/(c*(a*d - b*c)^3))^{(2/3)} - 2*a*b*c^2*d*(-d/(c*(a*d - b*c)^3))^{(2/3)})*(d/(27*b^3*c^4 - 27*a^3*c*d^3 + 81*a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^{(1/3)} + (\log(b^4*d^4*x - (b*(3^{(1/2)}*1i - 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^{(1/2)}*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b/(a*(a*d - b*c)^3))^{(2/3)}/4)))/(216*a*(a*d - b*c)^3))*3^{(1/2)}*1i - 1)*(b/(27*a^4*d^3 - 27*a*b^3*c^3 + 81*a^2*b^2*c^2*d - 81*a^3*b*c*d^2))^{(1/3)})/2 - (\log(b^4*d^4*x + (b*(3^{(1/2)}*1i + 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^{(1/2)}*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b/(a*(a*d - b*c)^3))^{(2/3)}/4)))/(216*a*(a*d - b*c)^3))*3^{(1/2)}*1i + 1)*(b/(27*a^4*d^3 - 27*a*b^3*c^3 + 81*a^2*b^2*c^2*d - 81*a^3*b*c*d^2))^{(1/3)})/2 + (\log(b^4*d^4*x + (d*(3^{(1/2)}*1i -$

$$\begin{aligned} & 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^{(1/2)*1i} - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d/(c*(a*d - b*c)^3))^{(2/3)}/4)) \\ & / (216*c*(a*d - b*c)^3)*(3^{(1/2)*1i} - 1)*(d/(27*b^3*c^4 - 27*a^3*c*d^3 + 81 \\ & *a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^{(1/3)}/2 - (\log(b^4*d^4*x - (d*(3^{(1/2)*1 \\ & i} + 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3 \\ & (3^{(1/2)*1i} + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d/(c*(a*d - b*c)^3))^{(2/3)}/ \\ & 4)))/(216*c*(a*d - b*c)^3)*(3^{(1/2)*1i} + 1)*(d/(27*b^3*c^4 - 27*a^3*c*d^3 + \\ & 81*a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^{(1/3)}/2 \end{aligned}$$

sympy [A] time = 14.51, size = 515, normalized size = 1.79

RootSum($\sqrt[3]{(27*a^4*d^3 - 81*a^3*b*c*d^2 + 81*a^2*b^2*c^2*d - 27*a*b^3*c^3) - b}$, Lambda($_t, _t*\log(x + (243*_t^5*a^7*c*d^6 - 1458*_t^5*a^6*b*c^2*d^5 + 3645*_t^5*a^5*b^2*c^3*d^4 - 4860*_t^5*a^4*b^3*c^4*d^3 + 3645*_t^5*a^3*b^4*c^5*d^2 - 1458*_t^5*a^2*b^5*c^6*d + 243*_t^5*a*b^6*c^7 + 9*_t^2*a^4*d^4 - 18*_t^2*a^3*b*c*d^3 + 18*_t^2*a^2*b^2*c^2*d^2 - 18*_t^2*a*b^3*c^3*d + 9*_t^2*b^4*c^4)/(a*b*d^2 + b^2*c*d))$)) + RootSum($\sqrt[3]{(27*a^3*c*d^3 - 81*a^2*b*c^2*d^2 + 81*a*b^2*c^3*d - 27*b^3*c^4) + d}$, Lambda($_t, _t*\log(x + (243*_t^5*a^7*c*d^6 - 1458*_t^5*a^6*b*c^2*d^5 + 3645*_t^5*a^5*b^2*c^3*d^4 - 4860*_t^5*a^4*b^3*c^4*d^3 + 3645*_t^5*a^3*b^4*c^5*d^2 - 1458*_t^5*a^2*b^5*c^6*d + 243*_t^5*a*b^6*c^7 + 9*_t^2*a^4*d^4 - 18*_t^2*a^3*b*c*d^3 + 18*_t^2*a^2*b^2*c^2*d^2 - 18*_t^2*a*b^3*c^3*d + 9*_t^2*b^4*c^4)/(a*b*d^2 + b^2*c*d))$))

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**3+a)/(d*x**3+c),x)

[Out] RootSum($_t^3*(27*a^4*d^3 - 81*a^3*b*c*d^2 + 81*a^2*b^2*c^2*d - 27*a*b^3*c^3) - b$, Lambda($_t, _t*\log(x + (243*_t^5*a^7*c*d^6 - 1458*_t^5*a^6*b*c^2*d^5 + 3645*_t^5*a^5*b^2*c^3*d^4 - 4860*_t^5*a^4*b^3*c^4*d^3 + 3645*_t^5*a^3*b^4*c^5*d^2 - 1458*_t^5*a^2*b^5*c^6*d + 243*_t^5*a*b^6*c^7 + 9*_t^2*a^4*d^4 - 18*_t^2*a^3*b*c*d^3 + 18*_t^2*a^2*b^2*c^2*d^2 - 18*_t^2*a*b^3*c^3*d + 9*_t^2*b^4*c^4)/(a*b*d^2 + b^2*c*d))$)) + RootSum($_t^3*(27*a^3*c*d^3 - 81*a^2*b*c^2*d^2 + 81*a*b^2*c^3*d - 27*b^3*c^4) + d$, Lambda($_t, _t*\log(x + (243*_t^5*a^7*c*d^6 - 1458*_t^5*a^6*b*c^2*d^5 + 3645*_t^5*a^5*b^2*c^3*d^4 - 4860*_t^5*a^4*b^3*c^4*d^3 + 3645*_t^5*a^3*b^4*c^5*d^2 - 1458*_t^5*a^2*b^5*c^6*d + 243*_t^5*a*b^6*c^7 + 9*_t^2*a^4*d^4 - 18*_t^2*a^3*b*c*d^3 + 18*_t^2*a^2*b^2*c^2*d^2 - 18*_t^2*a*b^3*c^3*d + 9*_t^2*b^4*c^4)/(a*b*d^2 + b^2*c*d))$))

$$3.115 \quad \int \frac{1}{(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=288

$$-\frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3}(bc - ad)} + \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3}(bc - ad)} - \frac{b^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3}(bc - ad)} + \frac{d^{2/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6c^{2/3}(bc - ad)}$$

Rubi [A] time = 0.14, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {391, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3}(bc - ad)} + \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3}(bc - ad)} - \frac{b^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3}(bc - ad)} + \frac{d^{2/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6c^{2/3}(bc - ad)} - \frac{d^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3c^{2/3}(bc - ad)} + \frac{d^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3} c^{2/3}(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^3)*(c + d*x^3)),x]

[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)*Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log[c^(1/3) + d^(1/3)*x]/(3*c^(2/3)*(b*c - a*d)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(2/3)*(b*c - a*d)) + (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(6*c^(2/3)*(b*c - a*d)))

Rule 31

Int[((a_) + (b_)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 391

```
Int[1/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist
[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^3)(c+dx^3)} dx &= \frac{b \int \frac{1}{a+bx^3} dx}{bc-ad} - \frac{d \int \frac{1}{c+dx^3} dx}{bc-ad} \\
&= \frac{b \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{2/3}(bc-ad)} + \frac{b \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{2/3}(bc-ad)} - \frac{d \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{3c^{2/3}(bc-ad)} - \frac{d \int \frac{2\sqrt[3]{c} - \sqrt[3]{d}x}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2} dx}{3c^{2/3}(bc-ad)} \\
&= \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3} \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{2/3}(bc-ad)} + \frac{b \int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{2a^{2/3}(bc-ad)} \\
&= \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{2/3}(bc-ad)} + \\
&= -\frac{b^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)} + \frac{d^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)} + \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{2/3}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 224, normalized size = 0.78

$$\frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{a^{2/3}} - \frac{2b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{a^{2/3}} + \frac{2\sqrt{3}b^{2/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}} - \frac{d^{2/3} \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2)}{c^{2/3}} + \frac{2d^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{c^{2/3}} - \frac{2\sqrt{3}d^{2/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{c^{2/3}}$$

$6ad - 6bc$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x^3)*(c + d*x^3)),x]

[Out] ((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) - (2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) - (2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d^(1/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(2/3) /(-6*b*c + 6*a*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^3)(c+dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((a + b*x^3)*(c + d*x^3)),x]

[Out] IntegrateAlgebraic[1/((a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 0.98, size = 254, normalized size = 0.88

$$\frac{2\sqrt{3}\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+2\sqrt{3}\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(d^2x^2-cdx\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}+c^2\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}\right)+2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)+2\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\right)}{6(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")

[Out] $-1/6*(2*\sqrt{3}*(-b^2/a^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3})*a*x*(-b^2/a^2)^{(2/3)} - \sqrt{3}*b)/b + 2*\sqrt{3}*(d^2/c^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3})*c*x*(d^2/c^2)^{(2/3)} - \sqrt{3}*d)/d - (-b^2/a^2)^{(1/3)}*\log(b^2*x^2 + a*b*x*(-b^2/a^2)^{(1/3)} + a^2*(-b^2/a^2)^{(2/3)}) - (d^2/c^2)^{(1/3)}*\log(d^2*x^2 - c*d*x*(d^2/c^2)^{(1/3)} + c^2*(d^2/c^2)^{(2/3)}) + 2*(-b^2/a^2)^{(1/3)}*\log(b*x - a*(-b^2/a^2)^{(1/3)}) + 2*(d^2/c^2)^{(1/3)}*\log(d*x + c*(d^2/c^2)^{(1/3)})/(b*c - a*d)$

giac [A] time = 0.21, size = 278, normalized size = 0.97

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(abc-a^2d)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(bc^2-acd)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}abc-\sqrt{3}a^2d}-\frac{\left(-cd^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2-\sqrt{3}acd}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(abc-a^2d)}-\frac{\left(-cd^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bc^2-acd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")

[Out] $-1/3*b*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a*b*c - a^2*d) + 1/3*d*(-c/d)^{(1/3)}*\log(\text{abs}(x - (-c/d)^{(1/3)}))/(b*c^2 - a*c*d) + (-a*b^2)^{(1/3)}*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(\sqrt{3}*a*b*c - \sqrt{3}*a^2*d) - (-c*d^2)^{(1/3)}*\arctan(1/3*\sqrt{3}*(2*x + (-c/d)^{(1/3)})/(-c/d)^{(1/3)})/(\sqrt{3}*b*c^2 - \sqrt{3}*a*c*d) + 1/6*(-a*b^2)^{(1/3)}*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a*b*c - a^2*d) - 1/6*(-c*d^2)^{(1/3)}*\log(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)})/(b*c^2 - a*c*d)$

maple [A] time = 0.05, size = 222, normalized size = 0.77

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\frac{a}{b}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\frac{c}{d}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^3+a)/(d*x^3+c),x)

[Out] $-1/3/(a*d-b*c)/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})+1/6/(a*d-b*c)/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})-1/3/(a*d-b*c)/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))+1/3/(a*d-b*c)/(c/d)^{(2/3)}*\ln(x+(c/d)^{(1/3)})-1/6/(a*d-b*c)/(c/d)^{(2/3)}*\ln(x^2-(c/d)^{(1/3)}*x+(c/d)^{(2/3)})+1/3/(a*d-b*c)/(c/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))$

maxima [A] time = 1.34, size = 293, normalized size = 1.02

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")

[Out] $1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/((b*c*(a/b)^{(1/3)} - a*d*(a/b)^{(1/3)})*(a/b)^{(1/3)}) - 1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - (c/d)^{(1/3)})/(c/d)^{(1/3)})/((b*c*(c/d)^{(1/3)} - a*d*(c/d)^{(1/3)})*(c/d)^{(1/3)}) - 1/6*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b*c*(a/b)^{(2/3)} - a*d*(a/b)^{(2/3)}) + 1/6*\log(x^2 - x*(c/d)^{(1/3)} + (c/d)^{(2/3)})/(b*c*(c/d)^{(2/3)} - a*d*(c/d)^{(2/3)}) + 1/3*\log(x + (a/b)^{(1/3)})/(b*c*(a/b)^{(2/3)} - a*d*(a/b)^{(2/3)}) - 1/3*\log(x + (c/d)^{(1/3)})/(b*c*(c/d)^{(2/3)} - a*d*(c/d)^{(2/3)})$

mupad [B] time = 9.01, size = 1364, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x^3)*(c + d*x^3)),x)

[Out] $\log\left(\left(\left(-b^2/(a^2*(a*d - b*c)^3)\right)^{(1/3)}*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(-b^2/(a^2*(a*d - b*c)^3))^{(1/3)})*(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^{(2/3)})\right)/3 - 6*b^5*d^5*x*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^{(1/3)} + \log\left(\left(\left(d^2/(c^2*(a*d - b*c)^3)\right)^{(1/3)}*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(d^2/(c^2*(a*d - b*c)^3))^{(1/3)})*(a*d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^{(2/3)})\right)/3 - 6*b^5*d^5*x*(-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^{(1/3)} + \log(6*b^5*d^5*x + ((3^{(1/2)}*1i - 1)*(-b^2/(a^2*(a*d - b*c)^3))^{(1/3)}*((3^{(1/2)}*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^{(1/2)}*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^{(1/3)}))/2)*(-b^2/(a^2*(a*d - b*c)^3))^{(2/3)})/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5)/6*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81$

$$\begin{aligned} & *a^4*b*c*d^2)^{(1/3)}*(3^{(1/2)*1i} - 1))/2 - (\log(6*b^5*d^5*x - ((3^{(1/2)*1i} \\ & + 1)*(-b^2/(a^2*(a*d - b*c)^3))^{(1/3)}*(((3^{(1/2)*1i} + 1)^2*(81*b^3*d^3*x*(a \\ & *d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^{(1/2)*1i} + 1)*(a*d + b*c)*(a*d \\ & - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^{(1/3)}))/2)*(-b^2/(a^2*(a*d - b*c)^3))^{(\\ & 2/3)))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-b^2/(27*a^ \\ & 5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^{(1/3)}*(3^{(1/2) \\ & *1i} + 1))/2 + (\log(6*b^5*d^5*x + ((3^{(1/2)*1i} - 1)*(d^2/(c^2*(a*d - b*c)^3) \\ &)^{(1/3)}*(((3^{(1/2)*1i} - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81* \\ & a*b^3*c*d^3*(3^{(1/2)*1i} - 1)*(a*d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c \\ &)^3))^{(1/3)}))/2)*(d^2/(c^2*(a*d - b*c)^3))^{(2/3)))/36 - 9*a^2*b^4*d^6 - 9*b^6 \\ & *c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2* \\ & b*c^3*d^2 - 81*a*b^2*c^4*d))^{(1/3)}*(3^{(1/2)*1i} - 1))/2 - (\log(6*b^5*d^5*x - \\ & ((3^{(1/2)*1i} + 1)*(d^2/(c^2*(a*d - b*c)^3))^{(1/3)}*(((3^{(1/2)*1i} + 1)^2*(81 \\ & *b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^{(1/2)*1i} + 1)*(a* \\ & d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^{(1/3)}))/2)*(d^2/(c^2*(a*d - \\ & b*c)^3))^{(2/3)))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)* \\ & (-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^{(1/ \\ & 3)}*(3^{(1/2)*1i} + 1))/2 \end{aligned}$$

sympy [A] time = 133.33, size = 447, normalized size = 1.55

RootSum($\sqrt[3]{(27*d^6 - 81*a*b*c*d^5 + 81*a^2*b^3*c^3 - 27*b^6*c^5)}$) + $\sqrt[3]{(1 + 13*\sqrt[3]{(81*d^2*d^2 - 243*a*b*c*d^2 + 162*a^2*b^3*c^3 - 243*d^2*d^2 + 81*d^2*d^2 - 36*a^2*d^2 + 36*a^2*d^2 - 36*a^2*d^2)})}$) + RootSum($\sqrt[3]{(27*d^6 - 81*a*b*c*d^5 + 81*a^2*b^3*c^3 - 27*b^6*c^5)}$) - $\sqrt[3]{(1 + 13*\sqrt[3]{(81*d^2*d^2 - 243*a*b*c*d^2 + 162*a^2*b^3*c^3 - 243*d^2*d^2 + 81*d^2*d^2 - 36*a^2*d^2 + 36*a^2*d^2 - 36*a^2*d^2)})}$)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**3+a)/(d*x**3+c),x)

[Out] RootSum(_t**3*(27*a**5*d**3 - 81*a**4*b*c*d**2 + 81*a**3*b**2*c**2*d - 27*a**2*b**3*c**3) + b**2, Lambda(_t, _t*log(x + (81*_t**4*a**7*c**2*d**5 - 243*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c**4*d**3 + 162*_t**4*a**4*b**3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*_t**4*a**2*b**5*c**7 - 3*_t**4*a**4*d**4 + 3*_t**4*a**3*b*c*d**3 + 3*_t**4*a*b**3*c**3*d - 3*_t**4*b**4*c**4)/(a**2*b*d**3 + b**3*c**2*d))) + RootSum(_t**3*(27*a**3*c**2*d**3 - 81*a**2*b*c**3*d**2 + 81*a*b**2*c**4*d - 27*b**3*c**5) - d**2, Lambda(_t, _t*log(x + (81*_t**4*a**7*c**2*d**5 - 243*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c**4*d**3 + 162*_t**4*a**4*b**3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*_t**4*a**2*b**5*c**7 - 3*_t**4*a**4*d**4 + 3*_t**4*a**3*b*c*d**3 + 3*_t**4*a*b**3*c**3*d - 3*_t**4*b**4*c**4)/(a**2*b*d**3 + b**3*c**2*d)))

$$3.116 \quad \int \frac{1}{x(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=62

$$-\frac{b \log(a+bx^3)}{3a(bc-ad)} + \frac{d \log(c+dx^3)}{3c(bc-ad)} + \frac{\log(x)}{ac}$$

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 72}

$$-\frac{b \log(a+bx^3)}{3a(bc-ad)} + \frac{d \log(c+dx^3)}{3c(bc-ad)} + \frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^3)*(c + d*x^3)),x]

[Out] Log[x]/(a*c) - (b*Log[a + b*x^3])/(3*a*(b*c - a*d)) + (d*Log[c + d*x^3])/(3*c*(b*c - a*d))

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a+bx^3)(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(a+bx)(c+dx)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{acx} + \frac{b^2}{a(-bc+ad)(a+bx)} + \frac{d^2}{c(bc-ad)(c+dx)} \right) dx, x, x^3 \right) \\ &= \frac{\log(x)}{ac} - \frac{b \log(a+bx^3)}{3a(bc-ad)} + \frac{d \log(c+dx^3)}{3c(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.87

$$\frac{-bc \log(a + bx^3) + ad \log(c + dx^3) - 3ad \log(x) + 3bc \log(x)}{3abc^2 - 3a^2cd}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^3)*(c + d*x^3)),x]

[Out] (3*b*c*Log[x] - 3*a*d*Log[x] - b*c*Log[a + b*x^3] + a*d*Log[c + d*x^3])/(3*a*b*c^2 - 3*a^2*c*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + bx^3)(c + dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(a + b*x^3)*(c + d*x^3)),x]

[Out] IntegrateAlgebraic[1/(x*(a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 1.56, size = 54, normalized size = 0.87

$$\frac{bc \log(bx^3 + a) - ad \log(dx^3 + c) - 3(bc - ad) \log(x)}{3(abc^2 - a^2cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")

[Out] -1/3*(b*c*log(b*x^3 + a) - a*d*log(d*x^3 + c) - 3*(b*c - a*d)*log(x))/(a*b*c^2 - a^2*c*d)

giac [A] time = 0.18, size = 71, normalized size = 1.15

$$-\frac{b^2 \log(|bx^3 + a|)}{3(ab^2c - a^2bd)} + \frac{d^2 \log(|dx^3 + c|)}{3(bc^2d - acd^2)} + \frac{\log(|x|)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")

[Out] -1/3*b^2*log(abs(b*x^3 + a))/(a*b^2*c - a^2*b*d) + 1/3*d^2*log(abs(d*x^3 + c))/(b*c^2*d - a*c*d^2) + log(abs(x))/(a*c)

maple [A] time = 0.05, size = 59, normalized size = 0.95

$$\frac{b \ln(bx^3 + a)}{3(ad - bc)a} - \frac{d \ln(dx^3 + c)}{3(ad - bc)c} + \frac{\ln(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^3+a)/(d*x^3+c),x)

[Out] 1/3*b/a/(a*d-b*c)*ln(b*x^3+a)-1/3*d/c/(a*d-b*c)*ln(d*x^3+c)+ln(x)/a/c

maxima [A] time = 0.54, size = 61, normalized size = 0.98

$$-\frac{b \log(bx^3 + a)}{3(abc - a^2d)} + \frac{d \log(dx^3 + c)}{3(bc^2 - acd)} + \frac{\log(x^3)}{3ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")

[Out] -1/3*b*log(b*x^3 + a)/(a*b*c - a^2*d) + 1/3*d*log(d*x^3 + c)/(b*c^2 - a*c*d) + 1/3*log(x^3)/(a*c)

mupad [B] time = 2.84, size = 58, normalized size = 0.94

$$\frac{b \ln(bx^3 + a)}{3a^2d - 3abc} + \frac{d \ln(dx^3 + c)}{3bc^2 - 3acd} + \frac{\ln(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^3)*(c + d*x^3)),x)

[Out] (b*log(a + b*x^3))/(3*a^2*d - 3*a*b*c) + (d*log(c + d*x^3))/(3*b*c^2 - 3*a*c*d) + log(x)/(a*c)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**3+a)/(d*x**3+c),x)

[Out] Timed out

$$3.117 \quad \int \frac{1}{x^2(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=299

$$\frac{b^{4/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{4/3}(bc - ad)} + \frac{b^{4/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{4/3}(bc - ad)} + \frac{b^{4/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}(bc - ad)} + \frac{d^{4/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6c^{4/3}(bc - ad)}$$

Rubi [A] time = 0.27, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {480, 584, 292, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{4/3}(bc - ad)} + \frac{b^{4/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{4/3}(bc - ad)} + \frac{b^{4/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}(bc - ad)} + \frac{d^{4/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6c^{4/3}(bc - ad)} - \frac{d^{4/3} \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3c^{4/3}(bc - ad)} - \frac{d^{4/3} \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{4/3}(bc - ad)} - \frac{1}{acx}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*x^3)*(c + d*x^3)), x]

[Out] $-(1/(a*c*x)) + (b^{(4/3)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)*x})/(Sqrt[3]*a^{(1/3)})]) / (Sqrt[3]*a^{(4/3)*(b*c - a*d)} - (d^{(4/3)*ArcTan[(c^{(1/3)} - 2*d^{(1/3)*x})/(Sqrt[3]*c^{(1/3)})]) / (Sqrt[3]*c^{(4/3)*(b*c - a*d)} + (b^{(4/3)*Log[a^{(1/3)} + b^{(1/3)*x}) / (3*a^{(4/3)*(b*c - a*d)} - (d^{(4/3)*Log[c^{(1/3)} + d^{(1/3)*x}) / (3*c^{(4/3)*(b*c - a*d)} - (b^{(4/3)*Log[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}] / (6*a^{(4/3)*(b*c - a*d)} + (d^{(4/3)*Log[c^{(2/3)} - c^{(1/3)*d^{(1/3)*x} + d^{(2/3)*x^2}] / (6*c^{(4/3)*(b*c - a*d)}))$

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^-1, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e^(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((g*x)^m*(a + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a + bx^3)(c + dx^3)} dx &= -\frac{1}{acx} + \frac{\int \frac{x(-bc-ad-bdx^3)}{(a+bx^3)(c+dx^3)} dx}{ac} \\
&= -\frac{1}{acx} + \frac{\int \left(-\frac{b^2cx}{(bc-ad)(a+bx^3)} - \frac{ad^2x}{(-bc+ad)(c+dx^3)} \right) dx}{ac} \\
&= -\frac{1}{acx} - \frac{b^2 \int \frac{x}{a+bx^3} dx}{a(bc-ad)} + \frac{d^2 \int \frac{x}{c+dx^3} dx}{c(bc-ad)} \\
&= -\frac{1}{acx} + \frac{b^{5/3} \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{4/3}(bc-ad)} - \frac{b^{5/3} \int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{4/3}(bc-ad)} - \frac{d^{5/3} \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{3c^{4/3}(bc-ad)} + \frac{d^{5/3} \int \frac{1}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{d}x + d^{2/3}x^2} dx}{3c^{4/3}(bc-ad)} \\
&= -\frac{1}{acx} + \frac{b^{4/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{4/3}(bc-ad)} - \frac{d^{4/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{4/3}(bc-ad)} - \frac{b^{4/3} \int \frac{-\sqrt[3]{a} \sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{4/3}(bc-ad)} \\
&= -\frac{1}{acx} + \frac{b^{4/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{4/3}(bc-ad)} - \frac{d^{4/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{4/3}(bc-ad)} - \frac{b^{4/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x)}{6a^{4/3}(bc-ad)} \\
&= -\frac{1}{acx} + \frac{b^{4/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{4/3}(bc-ad)} - \frac{d^{4/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3} \sqrt[3]{c}}\right)}{\sqrt{3} c^{4/3}(bc-ad)} + \frac{b^{4/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{4/3}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 244, normalized size = 0.82

$$\frac{\frac{b^{4/3}x \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{a^{4/3}} - \frac{2b^{4/3}x \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{a^{4/3}} - \frac{2\sqrt{3}b^{4/3}x \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}}\right)}{a^{4/3}} + \frac{6b}{a} - \frac{d^{4/3}x \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d}x + d^{2/3}x^2)}{c^{4/3}} + \frac{2d^{4/3}x \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{c^{4/3}} + \frac{2\sqrt{3}d^{4/3}x \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d}x}{\sqrt{3}}\right)}{c^{4/3}} - \frac{6d}{c}}{6adx - 6bcx}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + b*x^3)*(c + d*x^3)), x]

[Out] ((6*b)/a - (6*d)/c - (2*sqrt[3]*b^(4/3)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/a^(4/3) + (2*sqrt[3]*d^(4/3)*x*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/sqrt[3]])/c^(4/3) - (2*b^(4/3)*x*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + (2*d^(4/3)*x*Log[c^(1/3) + d^(1/3)*x])/c^(4/3) + (b^(4/3)*x*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3) - (d^(4/3)*x*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(4/3))/(-6*b*c*x + 6*a*d*x)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + bx^3)(c + dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(a + b*x^3)*(c + d*x^3)), x]

[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 0.89, size = 238, normalized size = 0.80

$$\frac{2\sqrt{3}bcx\left(-\frac{d}{a}\right)^{\frac{1}{3}} \arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{d}{a}\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) - 2\sqrt{3}adx\left(\frac{d}{c}\right)^{\frac{1}{3}} \arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{d}{c}\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) - bcx\left(-\frac{d}{a}\right)^{\frac{1}{3}} \log\left(bx^2 - ax\left(-\frac{d}{a}\right)^{\frac{2}{3}} - a\left(-\frac{d}{a}\right)^{\frac{1}{3}}\right) - adx\left(\frac{d}{c}\right)^{\frac{1}{3}} \log\left(dx^2 - cx\left(\frac{d}{c}\right)^{\frac{2}{3}} + c\left(\frac{d}{c}\right)^{\frac{1}{3}}\right) + 2bcx\left(-\frac{d}{a}\right)^{\frac{1}{3}} \log\left(bx + a\left(-\frac{d}{a}\right)^{\frac{2}{3}}\right) + 2adx\left(\frac{d}{c}\right)^{\frac{1}{3}} \log\left(dx + c\left(\frac{d}{c}\right)^{\frac{2}{3}}\right) + 6bc - 6ad}{6(abc^2 - a^2cd)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c), x, algorithm="fricas")

[Out] $-1/6*(2*\sqrt{3}*b*c*x*(-b/a)^{(1/3)}*\arctan(2/3*\sqrt{3}*x*(-b/a)^{(1/3)} + 1/3*\sqrt{3}) - 2*\sqrt{3}*a*d*x*(d/c)^{(1/3)}*\arctan(2/3*\sqrt{3}*x*(d/c)^{(1/3)} - 1/3*\sqrt{3}) - b*c*x*(-b/a)^{(1/3)}*\log(b*x^2 - a*x*(-b/a)^{(2/3)} - a*(-b/a)^{(1/3)}) - a*d*x*(d/c)^{(1/3)}*\log(d*x^2 - c*x*(d/c)^{(2/3)} + c*(d/c)^{(1/3)}) + 2*b*c*x*(-b/a)^{(1/3)}*\log(b*x + a*(-b/a)^{(2/3)}) + 2*a*d*x*(d/c)^{(1/3)}*\log(d*x + c*(d/c)^{(2/3)}) + 6*b*c - 6*a*d)/((a*b*c^2 - a^2*c*d)*x)$

giac [A] time = 0.25, size = 305, normalized size = 1.02

$$\frac{b^2\left(-\frac{d}{a}\right)^{\frac{2}{3}} \log\left(x - \left(-\frac{d}{a}\right)^{\frac{1}{3}}\right)}{3(a^2bc - a^3d)} - \frac{d^2\left(-\frac{c}{d}\right)^{\frac{2}{3}} \log\left(x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(bc^3 - ac^2d)} + \frac{(-ab^2)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{d}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{a}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^2bc - \sqrt{3}a^3d} - \frac{(-cd^2)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^3 - \sqrt{3}ac^2d} - \frac{(-ab^2)^{\frac{2}{3}} \log\left(x^2 + x\left(-\frac{d}{a}\right)^{\frac{1}{3}} + \left(-\frac{d}{a}\right)^{\frac{2}{3}}\right)}{6(a^2bc - a^3d)} + \frac{(-cd^2)^{\frac{2}{3}} \log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bc^3 - ac^2d)} - \frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c), x, algorithm="giac")

[Out] $1/3*b^2*(-a/b)^{(2/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a^2*b*c - a^3*d) - 1/3*d^2*(-c/d)^{(2/3)}*\log(\text{abs}(x - (-c/d)^{(1/3)}))/(b*c^3 - a*c^2*d) + (-a*b^2)^{(2/3)}*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(sqrt(3)*a^2*b*c - sqrt(3)*a^3*d) - (-c*d^2)^{(2/3)}*\arctan(1/3*\sqrt{3}*(2*x + (-c/d)^{(1/3)})/(-c/d)^{(1/3)})/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^{(2/3)}*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)^{(2/3)}*\log(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)})/(b*c^3 - a*c^2*d) - 1/(a*c*x)$

maple [A] time = 0.06, size = 257, normalized size = 0.86

$$\frac{\sqrt{3} b \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}a} - \frac{b \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}a} + \frac{b \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}a} - \frac{\sqrt{3} d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}c} + \frac{d \ln\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}c} - \frac{d \ln\left(x^2 - \left(\frac{c}{d}\right)^{\frac{1}{3}}x + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}c} - \frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^3+a)/(d*x^3+c), x)

[Out] $-1/3*b/a/(a*d-b*c)/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)})+1/6*b/a/(a*d-b*c)/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+1/3*b/a/(a*d-b*c)*3^{(1/2)}/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))+1/3*d/c/(a*d-b*c)/(c/d)^{(1/3)}*\ln(x+(c/d)^{(1/3)})-1/6*d/c/(a*d-b*c)/(c/d)^{(1/3)}*\ln(x^2-(c/d)^{(1/3)}*x+(c/d)^{(2/3)})-1/3*d/c/(a*d-b*c)*3^{(1/2)}/(c/d)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))-1/a/c/x$

maxima [A] time = 1.20, size = 300, normalized size = 1.00

$$\frac{\sqrt{3} b \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(abc-a^2d)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3} d \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3(bc^2-acd)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{b \log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{d \log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{b \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{d \log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} - \frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c), x, algorithm="maxima")

[Out] $-1/3*\sqrt{3}*b*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/((a*b*c - a^2*d)*(a/b)^{(1/3)}) + 1/3*\sqrt{3}*d*\arctan(1/3*\sqrt{3}*(2*x - (c/d)^{(1/3)})/(c/d)^{(1/3)})/((b*c^2 - a*c*d)*(c/d)^{(1/3)}) - 1/6*b*log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a*b*c*(a/b)^{(1/3)} - a^2*d*(a/b)^{(1/3)}) + 1/6*d*log(x^2 - x*(c/d)^{(1/3)} + (c/d)^{(2/3)})/(b*c^2*(c/d)^{(1/3)} - a*c*d*(c/d)^{(1/3)}) + 1/3*b*log(x + (a/b)^{(1/3)})/(a*b*c*(a/b)^{(1/3)} - a^2*d*(a/b)^{(1/3)}) - 1/3*d*log(x + (c/d)^{(1/3)})/(b*c^2*(c/d)^{(1/3)} - a*c*d*(c/d)^{(1/3)}) - 1/(a*c*x)$

mupad [B] time = 3.85, size = 716, normalized size = 2.39

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*x^3)*(c + d*x^3)), x)

[Out] $\log(b - a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^{(1/3)} + a*b*c*x*(-b^4/(a^4*(a*d - b*c)^3))^{(1/3)})*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d - 8$

$$\begin{aligned}
& 1*a^6*b*c*d^2)^{(1/3)} + \log(d - b*c^2*x*(d^4/(c^4*(a*d - b*c)^3))^{(1/3)} + a \\
& *c*d*x*(d^4/(c^4*(a*d - b*c)^3))^{(1/3)})*(-d^4/(27*b^3*c^7 - 27*a^3*c^4*d^3 \\
& + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^{(1/3)} - 1/(a*c*x) - (\log(b - 3^{(1/2)}* \\
& b*1i + 2*a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^{(1/3)} - 2*a*b*c*x*(-b^4/(a^4*(a \\
& *d - b*c)^3))^{(1/3)})*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d \\
& - 81*a^6*b*c*d^2))^{(1/3)}*(3^{(1/2)}*1i + 1))/2 + (\log(b + 3^{(1/2)}*b*1i + 2*a^ \\
& 2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^{(1/3)} - 2*a*b*c*x*(-b^4/(a^4*(a*d - b*c)^3 \\
&))^{(1/3)})*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d - 81*a^6*b* \\
& c*d^2))^{(1/3)}*(3^{(1/2)}*1i - 1))/2 - (\log(d - 3^{(1/2)}*d*1i + 2*b*c^2*x*(d^4/ \\
& (c^4*(a*d - b*c)^3))^{(1/3)} - 2*a*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^{(1/3)})*(-d \\
& ^4/(27*b^3*c^7 - 27*a^3*c^4*d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^{(1/3)} \\
& *(3^{(1/2)}*1i + 1))/2 + (\log(d + 3^{(1/2)}*d*1i + 2*b*c^2*x*(d^4/(c^4*(a*d - b \\
& *c)^3))^{(1/3)} - 2*a*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^{(1/3)})*(-d^4/(27*b^3*c^ \\
& 7 - 27*a^3*c^4*d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^{(1/3)}*(3^{(1/2)}*1i \\
& - 1))/2
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**3+a)/(d*x**3+c),x)

[Out] Timed out

$$3.118 \quad \int \frac{1}{x^3(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=301

$$\frac{b^{5/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{5/3}(bc - ad)} - \frac{b^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{5/3}(bc - ad)} + \frac{b^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{5/3}(bc - ad)} - \frac{d^{5/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6c^{5/3}(bc - ad)}$$

Rubi [A] time = 0.25, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {480, 522, 200, 31, 634, 617, 204, 628}

$$\frac{b^{5/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{5/3}(bc - ad)} - \frac{b^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{5/3}(bc - ad)} + \frac{b^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{5/3}(bc - ad)} - \frac{d^{5/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6c^{5/3}(bc - ad)} + \frac{d^{5/3} \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3c^{5/3}(bc - ad)} - \frac{d^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d} x}{\sqrt{3} \sqrt[3]{c}}\right)}{\sqrt{3} c^{5/3}(bc - ad)} - \frac{1}{2acx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a + b*x^3)*(c + d*x^3)), x]

[Out] $-1/(2*a*c*x^2) + (b^{(5/3)}*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(5/3)}*(b*c - a*d)) - (d^{(5/3)}*ArcTan[(c^{(1/3)} - 2*d^{(1/3)}*x)/(Sqrt[3]*c^{(1/3)})])/(Sqrt[3]*c^{(5/3)}*(b*c - a*d)) - (b^{(5/3)}*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(5/3)}*(b*c - a*d)) + (d^{(5/3)}*Log[c^{(1/3)} + d^{(1/3)}*x])/(3*c^{(5/3)}*(b*c - a*d)) + (b^{(5/3)}*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(5/3)}*(b*c - a*d)) - (d^{(5/3)}*Log[c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x + d^{(2/3)}*x^2])/(6*c^{(5/3)}*(b*c - a*d))$

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^-1, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 480

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(a+bx^3)(c+dx^3)} dx &= -\frac{1}{2acx^2} + \frac{\int \frac{-2(bc+ad)-2bdx^3}{(a+bx^3)(c+dx^3)} dx}{2ac} \\
&= -\frac{1}{2acx^2} - \frac{b^2 \int \frac{1}{a+bx^3} dx}{a(bc-ad)} + \frac{d^2 \int \frac{1}{c+dx^3} dx}{c(bc-ad)} \\
&= -\frac{1}{2acx^2} - \frac{b^2 \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{5/3}(bc-ad)} - \frac{b^2 \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{5/3}(bc-ad)} + \frac{d^2 \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{3c^{5/3}(bc-ad)} + \frac{d^2 \int \frac{1}{\sqrt[3]{c} - \sqrt[3]{d}x} dx}{3c^{5/3}(bc-ad)} \\
&= -\frac{1}{2acx^2} - \frac{b^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{5/3}(bc-ad)} + \frac{d^{5/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{5/3}(bc-ad)} + \frac{b^{5/3} \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{5/3}(bc-ad)} \\
&= -\frac{1}{2acx^2} - \frac{b^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{5/3}(bc-ad)} + \frac{d^{5/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{5/3}(bc-ad)} + \frac{b^{5/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x)}{6a^{5/3}(bc-ad)} \\
&= -\frac{1}{2acx^2} + \frac{b^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}(bc-ad)} - \frac{d^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{5/3}(bc-ad)} - \frac{b^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{5/3}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 259, normalized size = 0.86

$$\frac{\frac{2b^{5/3}x^2 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{a^{5/3}} - \frac{b^{5/3}x^2 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{a^{5/3}} - \frac{2\sqrt{3}b^{5/3}x^2 \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{3b}{a} - \frac{2d^{5/3}x^2 \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{c^{5/3}} + \frac{d^{5/3}x^2 \log(a^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2)}{c^{5/3}} + \frac{2\sqrt{3}d^{5/3}x^2 \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}} - \frac{3d}{c}}{6x^2(ad-bc)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a + b*x^3)*(c + d*x^3)), x]

[Out] ((3*b)/a - (3*d)/c - (2*sqrt[3]*b^(5/3)*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/a^(5/3) + (2*sqrt[3]*d^(5/3)*x^2*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/sqrt[3]])/c^(5/3) + (2*b^(5/3)*x^2*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (2*d^(5/3)*x^2*Log[c^(1/3) + d^(1/3)*x])/c^(5/3) - (b^(5/3)*x^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + (d^(5/3)*x^2*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(5/3))/(6*(-(b*c) + a*d)*x^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a+bx^3)(c+dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(a + b*x^3)*(c + d*x^3)), x]

[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 2.62, size = 301, normalized size = 1.00

$$\frac{2\sqrt{3}bcx^2\left(\frac{d}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}x\left(\frac{d}{a}\right)^{\frac{1}{3}}-\sqrt{3}d}{3d}\right)+2\sqrt{3}adx^2\left(\frac{d}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}x\left(\frac{d}{a}\right)^{\frac{1}{3}}-\sqrt{3}d}{3d}\right)-bcx^2\left(\frac{d}{a}\right)^{\frac{1}{3}}\log\left(d^2x^2-abx\left(\frac{d}{a}\right)^{\frac{1}{3}}+d^2\left(\frac{d}{a}\right)^{\frac{1}{3}}\right)-adx^2\left(\frac{d}{a}\right)^{\frac{1}{3}}\log\left(d^2x^2+cdx\left(\frac{d}{a}\right)^{\frac{1}{3}}+d^2\left(\frac{d}{a}\right)^{\frac{1}{3}}\right)+2bcx^2\left(\frac{d}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{d}{a}\right)^{\frac{1}{3}}\right)+2adx^2\left(\frac{d}{a}\right)^{\frac{1}{3}}\log\left(dx-c\left(\frac{d}{a}\right)^{\frac{1}{3}}\right)+3bc-3ad}{6(abc^2-a^2cd)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")

[Out] $-1/6*(2*\sqrt{3}*b*c*x^2*(b^2/a^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3})*a*x*(b^2/a^2)^{(2/3)} - \sqrt{3}*b)/b) + 2*\sqrt{3}*a*d*x^2*(-d^2/c^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3})*c*x*(-d^2/c^2)^{(2/3)} - \sqrt{3}*d)/d) - b*c*x^2*(b^2/a^2)^{(1/3)}*\log(b^2*x^2 - a*b*x*(b^2/a^2)^{(1/3)} + a^2*(b^2/a^2)^{(2/3)}) - a*d*x^2*(-d^2/c^2)^{(1/3)}*\log(d^2*x^2 + c*d*x*(-d^2/c^2)^{(1/3)} + c^2*(-d^2/c^2)^{(2/3)}) + 2*b*c*x^2*(b^2/a^2)^{(1/3)}*\log(b*x + a*(b^2/a^2)^{(1/3)}) + 2*a*d*x^2*(-d^2/c^2)^{(1/3)}*\log(d*x - c*(-d^2/c^2)^{(1/3)}) + 3*b*c - 3*a*d)/((a*b*c^2 - a^2*c*d)*x^2)$

giac [A] time = 0.21, size = 309, normalized size = 1.03

$$\frac{b^2\left(-\frac{d}{a}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(a^2bc-a^3d)} - \frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3(bc^3-ac^2d)} - \frac{(-ab^2)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^2bc-\sqrt{3}a^3d} + \frac{(-cd^2)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^3-\sqrt{3}ac^2d} - \frac{(-ab^2)^{\frac{1}{3}}b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(a^2bc-a^3d)} + \frac{(-cd^2)^{\frac{1}{3}}d\log\left(x^2+x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\left(-\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6(bc^3-ac^2d)} - \frac{1}{2acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")

[Out] $1/3*b^2*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a^2*b*c - a^3*d) - 1/3*d^2*(-c/d)^{(1/3)}*\log(\text{abs}(x - (-c/d)^{(1/3)}))/(b*c^3 - a*c^2*d) - (-a*b^2)^{(1/3)}*b*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(\sqrt{3}*a^2*b*c - \sqrt{3}*a^3*d) + (-c*d^2)^{(1/3)}*d*\arctan(1/3*\sqrt{3}*(2*x + (-c/d)^{(1/3)})/(-c/d)^{(1/3)})/(\sqrt{3}*b*c^3 - \sqrt{3}*a*c^2*d) - 1/6*(-a*b^2)^{(1/3)}*b*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)^{(1/3)}*d*\log(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)})/(b*c^3 - a*c^2*d) - 1/2/(a*c*x^2)$

maple [A] time = 0.05, size = 257, normalized size = 0.85

$$\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}a} + \frac{b\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}a} - \frac{b\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}a} - \frac{\sqrt{3}d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}c} - \frac{d\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}c} + \frac{d\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}c} - \frac{1}{2acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/x^3/(b*x^3+a)/(d*x^3+c), x)$

[Out] $\frac{1}{3} \frac{a*b}{a*d-b*c} \frac{1}{(a/b)^{2/3}} \ln(x+(a/b)^{1/3}) - \frac{1}{6} \frac{a*b}{a*d-b*c} \frac{1}{(a/b)^{2/3}} \ln(x^2-(a/b)^{1/3}*x+(a/b)^{2/3}) + \frac{1}{3} \frac{a*b}{a*d-b*c} \frac{1}{(a/b)^{2/3}} * 3^{1/2} * \arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1)) - \frac{1}{2} \frac{a*c}{a*d-b*c} \frac{1}{(c/d)^{2/3}} \ln(x+(c/d)^{1/3}) + \frac{1}{6} \frac{c*d}{a*d-b*c} \frac{1}{(c/d)^{2/3}} \ln(x^2-(c/d)^{1/3}*x+(c/d)^{2/3}) - \frac{1}{3} \frac{c*d}{a*d-b*c} \frac{1}{(c/d)^{2/3}} * 3^{1/2} * \arctan(1/3*3^{1/2}*(2/(c/d)^{1/3}*x-1))$

maxima [A] time = 1.14, size = 328, normalized size = 1.09

$$-\frac{\sqrt{3} b \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3} d \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3\left(bc^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{b \log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(abc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{d \log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-acd\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} - \frac{b \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{d \log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-acd\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} - \frac{1}{2acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/x^3/(b*x^3+a)/(d*x^3+c), x, \text{algorithm}="maxima")$

[Out] $-1/3*\text{sqrt}(3)*b*\arctan(1/3*\text{sqrt}(3)*(2*x - (a/b)^{1/3}))/((a/b)^{1/3})/((a*b*c*(a/b)^{1/3} - a^2*d*(a/b)^{1/3})*(a/b)^{1/3}) + 1/3*\text{sqrt}(3)*d*\arctan(1/3*\text{sqrt}(3)*(2*x - (c/d)^{1/3}))/((c/d)^{1/3})/((b*c^2*(c/d)^{1/3} - a*c*d*(c/d)^{1/3})*(c/d)^{1/3}) + 1/6*b*\log(x^2 - x*(a/b)^{1/3} + (a/b)^{2/3})/(a*b*c*(a/b)^{2/3} - a^2*d*(a/b)^{2/3}) - 1/6*d*\log(x^2 - x*(c/d)^{1/3} + (c/d)^{2/3})/(b*c^2*(c/d)^{2/3} - a*c*d*(c/d)^{2/3}) - 1/3*b*\log(x + (a/b)^{1/3})/(a*b*c*(a/b)^{2/3} - a^2*d*(a/b)^{2/3}) + 1/3*d*\log(x + (c/d)^{1/3})/(b*c^2*(c/d)^{2/3} - a*c*d*(c/d)^{2/3}) - 1/2/(a*c*x^2)$

mupad [B] time = 11.83, size = 1829, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^3*(a + b*x^3)*(c + d*x^3)), x)$

[Out] $\log\left(\left(\left(b^5/(a^5*(a*d - b*c)^3)\right)^{1/3}\right)\left(\left(\left(81*a^{10}*b^3*c^{10}*d^3*(a*d + b*c)*(a*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3)\right)^{1/3} - 81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d)\right)*(b^5/(a^5*(a*d - b*c)^3)\right)^{2/3}\right)/9 + 9*a^6*b^9*c^{11}*d^4 - 9*a^7*b^8*c^{10}*d^5 - 9*a^{10}*b^5*c^7*d^8 + 9*a^{11}*b^4*c^6*d^9)/3 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2)*(b^5/(27*a^8*d^3 - 27*a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2))^{1/3} + \log\left(\left(-d^5/(c^5*(a*d - b*c)^3)\right)^{1/3}\right)\left(\left(\left(81*a^{10}*b^3*c^{10}*d^3*(a*d + b*c)*(a*d - b*c)^4*(-d^5/(c^5*(a*d - b*c)^3)\right)^{1/3} - 81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d)\right)*(-d^5/(c^5*(a*d - b*c)^3)\right)^{2/3}\right)/9 + 9*a^6*b^9*c^{11}*d^4 - 9*a^7*b^8*c^{10}*d^5 - 9*a^{10}*b^5*c^7*d^8 + 9*a^{11}*b^4*c^6*d^9)/3 + 3*$

$$\begin{aligned}
& a^6 b^6 c^6 d^6 x (a^2 d^2 + b^2 c^2) (d^5 / (27 b^3 c^8 - 27 a^3 c^5 d^3 + 81 a^2 b c^6 d^2 - 81 a b^2 c^7 d))^{1/3} + (\log(((3^{1/2}) i - 1) (b^5 / (a^5 (a d - b c)^3)))^{1/3} * (((3^{1/2}) i - 1)^2 * (81 a^8 b^3 c^8 d^3 x (a d - b c)^4 * (a^2 d^2 + b^2 c^2 + a b c d) - (81 a^{10} b^3 c^{10} d^3 * (3^{1/2}) i - 1) * (a d + b c) * (a d - b c)^4 * (b^5 / (a^5 (a d - b c)^3)))^{1/3}) / 2) * (b^5 / (a^5 (a d - b c)^3))^{2/3} / 36 - 9 a^6 b^9 c^{11} d^4 + 9 a^7 b^8 c^{10} d^5 + 9 a^{10} b^5 c^7 d^8 - 9 a^{11} b^4 c^6 d^9) / 6 - 3 a^6 b^6 c^6 d^6 x (a^2 d^2 + b^2 c^2) * (b^5 / (27 a^8 d^3 - 27 a^5 b^3 c^3 + 81 a^6 b^2 c^2 d - 81 a^7 b c d^2))^{1/3} * (3^{1/2} i - 1) / 2 - (\log(((3^{1/2}) i + 1) (b^5 / (a^5 (a d - b c)^3)))^{1/3} * (((3^{1/2}) i + 1)^2 * (81 a^8 b^3 c^8 d^3 x (a d - b c)^4 * (a^2 d^2 + b^2 c^2 + a b c d) + (81 a^{10} b^3 c^{10} d^3 * (3^{1/2}) i + 1) * (a d + b c) * (a d - b c)^4 * (b^5 / (a^5 (a d - b c)^3)))^{1/3}) / 2) * (b^5 / (a^5 (a d - b c)^3))^{2/3} / 36 - 9 a^6 b^9 c^{11} d^4 + 9 a^7 b^8 c^{10} d^5 + 9 a^{10} b^5 c^7 d^8 - 9 a^{11} b^4 c^6 d^9) / 6 + 3 a^6 b^6 c^6 d^6 x (a^2 d^2 + b^2 c^2) * (b^5 / (27 a^8 d^3 - 27 a^5 b^3 c^3 + 81 a^6 b^2 c^2 d - 81 a^7 b c d^2))^{1/3} * (3^{1/2} i + 1) / 2 + (\log(((3^{1/2}) i - 1) (-d^5 / (c^5 (a d - b c)^3)))^{1/3} * (((3^{1/2}) i - 1)^2 * (81 a^8 b^3 c^8 d^3 x (a d - b c)^4 * (a^2 d^2 + b^2 c^2 + a b c d) - (81 a^{10} b^3 c^{10} d^3 * (3^{1/2}) i - 1) * (a d + b c) * (a d - b c)^4 * (-d^5 / (c^5 (a d - b c)^3)))^{1/3}) / 2) * (-d^5 / (c^5 (a d - b c)^3))^{2/3} / 36 - 9 a^6 b^9 c^{11} d^4 + 9 a^7 b^8 c^{10} d^5 + 9 a^{10} b^5 c^7 d^8 - 9 a^{11} b^4 c^6 d^9) / 6 - 3 a^6 b^6 c^6 d^6 x (a^2 d^2 + b^2 c^2) * (d^5 / (27 b^3 c^8 - 27 a^3 c^5 d^3 + 81 a^2 b c^6 d^2 - 81 a b^2 c^7 d))^{1/3} * (3^{1/2} i - 1) / 2 - (\log(((3^{1/2}) i + 1) (-d^5 / (c^5 (a d - b c)^3)))^{1/3} * (((3^{1/2}) i + 1)^2 * (81 a^8 b^3 c^8 d^3 x (a d - b c)^4 * (a^2 d^2 + b^2 c^2 + a b c d) + (81 a^{10} b^3 c^{10} d^3 * (3^{1/2}) i + 1) * (a d + b c) * (a d - b c)^4 * (-d^5 / (c^5 (a d - b c)^3)))^{1/3}) / 2) * (-d^5 / (c^5 (a d - b c)^3))^{2/3} / 36 - 9 a^6 b^9 c^{11} d^4 + 9 a^7 b^8 c^{10} d^5 + 9 a^{10} b^5 c^7 d^8 - 9 a^{11} b^4 c^6 d^9) / 6 + 3 a^6 b^6 c^6 d^6 x (a^2 d^2 + b^2 c^2) * (d^5 / (27 b^3 c^8 - 27 a^3 c^5 d^3 + 81 a^2 b c^6 d^2 - 81 a b^2 c^7 d))^{1/3} * (3^{1/2} i + 1) / 2 - 1 / (2 a c x^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x**3+a)/(d*x**3+c),x)

[Out] Timed out

$$3.119 \quad \int \frac{1}{x^4(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=87

$$\frac{b^2 \log(a+bx^3)}{3a^2(bc-ad)} - \frac{\log(x)(ad+bc)}{a^2c^2} - \frac{d^2 \log(c+dx^3)}{3c^2(bc-ad)} - \frac{1}{3acx^3}$$

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 72}

$$\frac{b^2 \log(a+bx^3)}{3a^2(bc-ad)} - \frac{\log(x)(ad+bc)}{a^2c^2} - \frac{d^2 \log(c+dx^3)}{3c^2(bc-ad)} - \frac{1}{3acx^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^3)*(c + d*x^3)), x]

[Out] -1/(3*a*c*x^3) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^3])/(3*a^2*(b*c - a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^4(a+bx^3)(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2(a+bx)(c+dx)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{acx^2} + \frac{-bc-ad}{a^2c^2x} - \frac{b^3}{a^2(-bc+ad)(a+bx)} - \frac{d^3}{c^2(bc-ad)(c+dx)} \right) dx, x, x^3 \right) \\ &= -\frac{1}{3acx^3} - \frac{(bc+ad)\log(x)}{a^2c^2} + \frac{b^2 \log(a+bx^3)}{3a^2(bc-ad)} - \frac{d^2 \log(c+dx^3)}{3c^2(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 1.01

$$-\frac{b^2 \log(a + bx^3)}{3a^2(ad - bc)} + \frac{\log(x)(-ad - bc)}{a^2c^2} - \frac{d^2 \log(c + dx^3)}{3c^2(bc - ad)} - \frac{1}{3acx^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]

[Out] -1/3*1/(a*c*x^3) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^3])/(3*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + bx^3)(c + dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]

[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 4.79, size = 99, normalized size = 1.14

$$\frac{b^2c^2x^3 \log(bx^3 + a) - a^2d^2x^3 \log(dx^3 + c) - 3(b^2c^2 - a^2d^2)x^3 \log(x) - abc^2 + a^2cd}{3(a^2bc^3 - a^3c^2d)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")

[Out] 1/3*(b^2*c^2*x^3*log(b*x^3 + a) - a^2*d^2*x^3*log(d*x^3 + c) - 3*(b^2*c^2 - a^2*d^2)*x^3*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^3)

giac [A] time = 0.19, size = 111, normalized size = 1.28

$$\frac{b^3 \log(|bx^3 + a|)}{3(a^2b^2c - a^3bd)} - \frac{d^3 \log(|dx^3 + c|)}{3(bc^3d - ac^2d^2)} - \frac{(bc + ad) \log(|x|)}{a^2c^2} + \frac{bcx^3 + adx^3 - ac}{3a^2c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")

[Out] 1/3*b^3*log(abs(b*x^3 + a))/(a^2*b^2*c - a^3*b*d) - 1/3*d^3*log(abs(d*x^3 + c))/(b*c^3*d - a*c^2*d^2) - (b*c + a*d)*log(abs(x))/(a^2*c^2) + 1/3*(b*c*x^3 + a*d*x^3 - a*c)/(a^2*c^2*x^3)

maple [A] time = 0.06, size = 87, normalized size = 1.00

$$-\frac{b^2 \ln(bx^3 + a)}{3(ad - bc)a^2} + \frac{d^2 \ln(dx^3 + c)}{3(ad - bc)c^2} - \frac{d \ln(x)}{ac^2} - \frac{b \ln(x)}{a^2c} - \frac{1}{3acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x^3+a)/(d*x^3+c), x)

[Out] -1/3*b^2/a^2/(a*d-b*c)*ln(b*x^3+a)+1/3*d^2/c^2/(a*d-b*c)*ln(d*x^3+c)-1/3/a/c/x^3-1/a/c^2*ln(x)*d-1/a^2/c*ln(x)*b

maxima [A] time = 0.49, size = 87, normalized size = 1.00

$$\frac{b^2 \log(bx^3 + a)}{3(a^2bc - a^3d)} - \frac{d^2 \log(dx^3 + c)}{3(bc^3 - ac^2d)} - \frac{(bc + ad) \log(x^3)}{3a^2c^2} - \frac{1}{3acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c), x, algorithm="maxima")

[Out] 1/3*b^2*log(b*x^3 + a)/(a^2*b*c - a^3*d) - 1/3*d^2*log(d*x^3 + c)/(b*c^3 - a*c^2*d) - 1/3*(b*c + a*d)*log(x^3)/(a^2*c^2) - 1/3/(a*c*x^3)

mupad [B] time = 3.22, size = 87, normalized size = 1.00

$$-\frac{b^2 \ln(bx^3 + a)}{3(a^3d - a^2bc)} - \frac{d^2 \ln(dx^3 + c)}{3(bc^3 - ac^2d)} - \frac{1}{3acx^3} - \frac{\ln(x)(ad + bc)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a + b*x^3)*(c + d*x^3)), x)

[Out] - (b^2*log(a + b*x^3))/(3*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^3))/(3*(b*c^3 - a*c^2*d)) - 1/(3*a*c*x^3) - (log(x)*(a*d + b*c))/(a^2*c^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c), x)

[Out] Timed out

$$3.120 \quad \int \frac{1}{x^5(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=318

$$\frac{b^{7/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{7/3}(bc - ad)} - \frac{b^{7/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{7/3}(bc - ad)} - \frac{b^{7/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{7/3}(bc - ad)} + \frac{ad + bc}{a^2 c^2 x} - \frac{d^{7/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x)}{6c^{7/3}(bc - ad)}$$

Rubi [A] time = 0.38, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {480, 583, 584, 292, 31, 634, 617, 204, 628}

$$\frac{b^{7/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{7/3}(bc - ad)} - \frac{b^{7/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{7/3}(bc - ad)} - \frac{b^{7/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{7/3}(bc - ad)} + \frac{ad + bc}{a^2 c^2 x} - \frac{d^{7/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6c^{7/3}(bc - ad)} + \frac{d^{7/3} \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3c^{7/3}(bc - ad)} + \frac{d^{7/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d} x}{\sqrt{3} \sqrt[3]{c}}\right)}{\sqrt{3} c^{7/3}(bc - ad)} - \frac{1}{4acx^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]

[Out] -1/(4*a*c*x^4) + (b*c + a*d)/(a^2*c^2*x) - (b^(7/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(7/3)*(b*c - a*d)) + (d^(7/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]/(Sqrt[3]*c^(7/3)*(b*c - a*d)) - (b^(7/3)*Log[a^(1/3) + b^(1/3)*x]/(3*a^(7/3)*(b*c - a*d)) + (d^(7/3)*Log[c^(1/3) + d^(1/3)*x]/(3*c^(7/3)*(b*c - a*d)) + (b^(7/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(7/3)*(b*c - a*d)) - (d^(7/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(6*c^(7/3)*(b*c - a*d))

Rule 31

Int[((a_) + (b_.)*(x_))^-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^-1, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[(g*x)^m*(a + b*x^n)^p*(e + f*x^n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^5 (a + bx^3)(c + dx^3)} dx &= -\frac{1}{4acx^4} + \frac{\int \frac{-4(bc+ad)-4bdx^3}{x^2(a+bx^3)(c+dx^3)} dx}{4ac} \\
 &= -\frac{1}{4acx^4} + \frac{bc+ad}{a^2c^2x} - \frac{\int \frac{x(-4(b^2c^2+abcd+a^2d^2)-4bd(bc+ad)x^3)}{(a+bx^3)(c+dx^3)} dx}{4a^2c^2} \\
 &= -\frac{1}{4acx^4} + \frac{bc+ad}{a^2c^2x} - \frac{\int \left(-\frac{4b^3c^2x}{(bc-ad)(a+bx^3)} - \frac{4a^2d^3x}{(-bc+ad)(c+dx^3)} \right) dx}{4a^2c^2} \\
 &= -\frac{1}{4acx^4} + \frac{bc+ad}{a^2c^2x} + \frac{b^3 \int \frac{x}{a+bx^3} dx}{a^2(bc-ad)} - \frac{d^3 \int \frac{x}{c+dx^3} dx}{c^2(bc-ad)} \\
 &= -\frac{1}{4acx^4} + \frac{bc+ad}{a^2c^2x} - \frac{b^{8/3} \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{7/3}(bc-ad)} + \frac{b^{8/3} \int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{7/3}(bc-ad)} + \frac{d^{8/3} \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{3c^{7/3}(bc-ad)} \\
 &= -\frac{1}{4acx^4} + \frac{bc+ad}{a^2c^2x} - \frac{b^{7/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{7/3}(bc-ad)} + \frac{d^{7/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{7/3}(bc-ad)} + \frac{b^{7/3} \int \frac{-\sqrt[3]{a}}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{7/3}(bc-ad)} \\
 &= -\frac{1}{4acx^4} + \frac{bc+ad}{a^2c^2x} - \frac{b^{7/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{7/3}(bc-ad)} + \frac{d^{7/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{7/3}(bc-ad)} + \frac{b^{7/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{6a^{7/3}(bc-ad)} \\
 &= -\frac{1}{4acx^4} + \frac{bc+ad}{a^2c^2x} - \frac{b^{7/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}(bc-ad)} + \frac{d^{7/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{7/3}(bc-ad)} - \frac{b^{7/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{3a^{7/3}(bc-ad)}
 \end{aligned}$$

Mathematica [A] time = 0.26, size = 282, normalized size = 0.89

$$\frac{4b^{7/3}x^4 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{a^{7/3}} + \frac{4\sqrt{3}b^{7/3}x^4 \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{7/3}} - \frac{2b^{7/3}x^4 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{a^{7/3}} - \frac{12b^2x^3}{a^2} + \frac{3b}{a} - \frac{4d^{7/3}x^4 \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{c^{7/3}} - \frac{4\sqrt{3}d^{7/3}x^4 \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{c^{7/3}} + \frac{2d^{7/3}x^4 \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2)}{c^{7/3}} + \frac{12d^2x^3}{c^2} - \frac{3d}{c}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(a + b*x^3)*(c + d*x^3)), x]

[Out] ((3*b)/a - (3*d)/c - (12*b^2*x^3)/a^2 + (12*d^2*x^3)/c^2 + (4*sqrt[3]*b^(7/3)*x^4*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/a^(7/3) - (4*sqrt[3]*d^(7/3)*x^4*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/sqrt[3]])/c^(7/3) - (2*b^(7/3)*x^4*log(a^(2/3) - sqrt[3]*b^(1/3)*x + b^(2/3)*x^2))/a^(7/3) - (2*d^(7/3)*x^4*log(c^(2/3) - sqrt[3]*d^(1/3)*x + d^(2/3)*x^2))/c^(7/3) - 12*b^2*x^3/a^2 + 3*b/a - 12*d^2*x^3/c^2 + 3*d/c

$$\begin{aligned} & (7/3)*x^4*\text{ArcTan}[(1 - (2*d^{(1/3)}*x)/c^{(1/3)})/\text{Sqrt}[3]]/c^{(7/3)} + (4*b^{(7/3)} \\ & *x^4*\text{Log}[a^{(1/3)} + b^{(1/3)}*x])/a^{(7/3)} - (4*d^{(7/3)}*x^4*\text{Log}[c^{(1/3)} + d^{(1/3)} \\ & *x])/c^{(7/3)} - (2*b^{(7/3)}*x^4*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x \\ & ^2])/a^{(7/3)} + (2*d^{(7/3)}*x^4*\text{Log}[c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x + d^{(2/3)}*x^2 \\ &])/c^{(7/3)})/(12*(-(b*c) + a*d)*x^4) \end{aligned}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 (a + bx^3)(c + dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(a + b*x^3)*(c + d*x^3)), x]

[Out] IntegrateAlgebraic[1/(x^5*(a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 3.41, size = 305, normalized size = 0.96

$$\frac{4\sqrt{3}b^{2/3}x^{1/3}\left(\frac{2}{3}\right)^{1/3}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{2}{3}\right)^{1/3}-\frac{1}{3}\sqrt{3}\right)-4\sqrt{3}a^{2/3}d^{1/3}x^{1/3}\left(\frac{2}{3}\right)^{1/3}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{2}{3}\right)^{1/3}+\frac{1}{3}\sqrt{3}\right)+2b^{2/3}c^{1/3}x^{1/3}\left(\frac{2}{3}\right)^{1/3}\log\left(bx^2-ax\left(\frac{2}{3}\right)^{1/3}+a\left(\frac{2}{3}\right)^{1/3}\right)+2a^{2/3}d^{1/3}x^{1/3}\left(\frac{2}{3}\right)^{1/3}\log\left(dx^2-cx\left(\frac{2}{3}\right)^{1/3}-c\left(\frac{2}{3}\right)^{1/3}\right)-4b^{2/3}d^{1/3}x^{1/3}\left(\frac{2}{3}\right)^{1/3}\log\left(bx+d\left(\frac{2}{3}\right)^{1/3}\right)-4a^{2/3}d^{1/3}x^{1/3}\left(\frac{2}{3}\right)^{1/3}\log\left(dx+c\left(\frac{2}{3}\right)^{1/3}\right)-3ab^{2/3}c^{1/3}+3a^{2/3}cd+12(b^{2/3}c^{1/3}-a^{2/3}d^{1/3})x^3}{12(a^{2/3}bc^{1/3}-a^{2/3}d^{1/3})x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c), x, algorithm="fricas")

$$\begin{aligned} & 1/12*(4*\text{sqrt}(3)*b^2*c^2*x^4*(b/a)^{(1/3)}*\arctan(2/3*\text{sqrt}(3)*x*(b/a)^{(1/3)} - \\ & 1/3*\text{sqrt}(3)) - 4*\text{sqrt}(3)*a^2*d^2*x^4*(-d/c)^{(1/3)}*\arctan(2/3*\text{sqrt}(3)*x*(-d/ \\ & c)^{(1/3)} + 1/3*\text{sqrt}(3)) + 2*b^2*c^2*x^4*(b/a)^{(1/3)}*\log(b*x^2 - a*x*(b/a)^{(1/3)} \\ & + a*(b/a)^{(1/3)}) + 2*a^2*d^2*x^4*(-d/c)^{(1/3)}*\log(d*x^2 - c*x*(-d/c)^{(1/3)} \\ & - c*(-d/c)^{(1/3)}) - 4*b^2*c^2*x^4*(b/a)^{(1/3)}*\log(b*x + a*(b/a)^{(2/3)}) \\ & - 4*a^2*d^2*x^4*(-d/c)^{(1/3)}*\log(d*x + c*(-d/c)^{(2/3)}) - 3*a*b*c^2 + 3*a^2 \\ & *c*d + 12*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*d)*x^4) \end{aligned}$$

giac [A] time = 0.24, size = 328, normalized size = 1.03

$$\frac{b^3\left(\frac{a}{b}\right)^{2/3}\log\left(x-\left(\frac{a}{b}\right)^{1/3}\right)+a^3\left(\frac{c}{d}\right)^{2/3}\log\left(x-\left(\frac{c}{d}\right)^{1/3}\right)-\frac{(-ab^2)^{2/3}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{1/3}\right)}{3\left(\frac{a}{b}\right)^{1/3}}\right)}{\sqrt{3}a^3bc-\sqrt{3}a^4d}}{\frac{b^3\left(\frac{a}{b}\right)^{2/3}\log\left(x-\left(\frac{a}{b}\right)^{1/3}\right)+a^3\left(\frac{c}{d}\right)^{2/3}\log\left(x-\left(\frac{c}{d}\right)^{1/3}\right)-\frac{(-ab^2)^{2/3}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{1/3}\right)}{3\left(\frac{a}{b}\right)^{1/3}}\right)}{\sqrt{3}a^3bc-\sqrt{3}a^4d}}{\sqrt{3}bc^4-\sqrt{3}ac^3d}}+\frac{(-cd^2)^{2/3}d\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{c}{d}\right)^{1/3}\right)}{3\left(\frac{c}{d}\right)^{1/3}}\right)}{\sqrt{3}bc^4-\sqrt{3}ac^3d}}+\frac{(-ab^2)^{2/3}b\log\left(x^2+x\left(\frac{a}{b}\right)^{1/3}+\left(\frac{a}{b}\right)^{2/3}\right)-(-cd^2)^{2/3}d\log\left(x^2+x\left(\frac{c}{d}\right)^{1/3}+\left(\frac{c}{d}\right)^{2/3}\right)+\frac{4bcx^3+4adx^3-ac}{4a^2c^2x^4}}{6(a^3bc-a^4d)}-\frac{(-cd^2)^{2/3}d\log\left(x^2+x\left(\frac{c}{d}\right)^{1/3}+\left(\frac{c}{d}\right)^{2/3}\right)+\frac{4bcx^3+4adx^3-ac}{4a^2c^2x^4}}{6(bc^4-ac^3d)}+\frac{4bcx^3+4adx^3-ac}{4a^2c^2x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c), x, algorithm="giac")

$$\begin{aligned} & -1/3*b^3*(-a/b)^{(2/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/ (a^3*b*c - a^4*d) + 1/3*d^3 \\ & *(-c/d)^{(2/3)}*\log(\text{abs}(x - (-c/d)^{(1/3)}))/ (b*c^4 - a*c^3*d) - (-a*b^2)^{(2/3)} \\ & *b*\arctan(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(\text{sqrt}(3)*a^3*b*c \\ & - \text{sqrt}(3)*a^4*d) + (-c*d^2)^{(2/3)}*d*\arctan(1/3*\text{sqrt}(3)*(2*x + (-c/d)^{(1/3)})) \end{aligned}$$

$$\frac{1}{(-c/d)^{1/3}} \frac{1}{(\sqrt{3} b^2 c^4 - \sqrt{3} a^3 c^3 d)} + \frac{1}{6} (-a b^2)^{2/3} b \log(x^2 + x(-a/b)^{1/3} + (-a/b)^{2/3}) / (a^3 b^2 c - a^4 d) - \frac{1}{6} (-c d^2)^{2/3} d \log(x^2 + x(-c/d)^{1/3} + (-c/d)^{2/3}) / (b^2 c^4 - a^3 c^3 d) + \frac{1}{4} (4 b^2 c^3 + 4 a^2 d^2 x^3 - a^3 c) / (a^2 c^2 x^4)$$

maple [A] time = 0.05, size = 291, normalized size = 0.92

$$\frac{\sqrt{3} b^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{b^3}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}} a^2} + \frac{b^2 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - b^2 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}} a^2} - \frac{\sqrt{3} d^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{d^3}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}} c^2} - \frac{d^2 \ln\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right) - d^2 \ln\left(x^2 - \left(\frac{c}{d}\right)^{\frac{1}{3}} x + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}} c^2} + \frac{d}{a^2 c^2 x} + \frac{b}{a^2 c x} - \frac{1}{4 a c x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(b*x^3+a)/(d*x^3+c), x)

[Out] $\frac{1}{3} b^2 a^2 / (a d - b^2 c) / (a/b)^{1/3} * \ln(x + (a/b)^{1/3}) - \frac{1}{6} b^2 a^2 / (a d - b^2 c) / (a/b)^{1/3} * \ln(x^2 - (a/b)^{1/3} x + (a/b)^{2/3}) - \frac{1}{3} b^2 a^2 / (a d - b^2 c) * 3^{1/2} / (a/b)^{1/3} * \arctan(1/3 * 3^{1/2} * (2/(a/b)^{1/3} * x - 1)) - \frac{1}{3} d^2 c^2 / (a d - b^2 c) / (c/d)^{1/3} * \ln(x + (c/d)^{1/3}) + \frac{1}{6} d^2 c^2 / (a d - b^2 c) / (c/d)^{1/3} * \ln(x^2 - (c/d)^{1/3} x + (c/d)^{2/3}) + \frac{1}{3} d^2 c^2 / (a d - b^2 c) * 3^{1/2} / (c/d)^{1/3} * \arctan(1/3 * 3^{1/2} * (2/(c/d)^{1/3} * x - 1)) - \frac{1}{4} a/c/x^4 + \frac{1}{a/c^2/x} + \frac{1}{a^2/c/x^2} + \frac{1}{4 a c x^4}$

maxima [A] time = 1.13, size = 341, normalized size = 1.07

$$\frac{\sqrt{3} b^2 \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(a^2 b c - a^3 d)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3} d^2 \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3(bc^3 - ac^2 d)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{b^2 \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - d^2 \log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(a^2 b c\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^3 d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{b^2 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - d^2 \log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(a^2 b c\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^3 d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{4(bc+ad)x^3 - ac}{4a^2 c^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c), x, algorithm="maxima")

[Out] $\frac{1}{3} \sqrt{3} b^2 \arctan(1/3 \sqrt{3} (2x - (a/b)^{1/3}) / (a/b)^{1/3}) / ((a^2 b^2 c - a^3 d) (a/b)^{1/3}) - \frac{1}{3} \sqrt{3} d^2 \arctan(1/3 \sqrt{3} (2x - (c/d)^{1/3}) / (c/d)^{1/3}) / ((b^2 c^3 - a^3 d) (c/d)^{1/3}) + \frac{1}{6} b^2 \log(x^2 - x(a/b)^{1/3} + (a/b)^{2/3}) / (a^2 b^2 c (a/b)^{1/3} - a^3 d (a/b)^{1/3}) - \frac{1}{6} d^2 \log(x^2 - x(c/d)^{1/3} + (c/d)^{2/3}) / (b^2 c^3 (c/d)^{1/3} - a^3 d (c/d)^{1/3}) - \frac{1}{3} b^2 \log(x + (a/b)^{1/3}) / (a^2 b^2 c (a/b)^{1/3} - a^3 d (a/b)^{1/3}) + \frac{1}{3} d^2 \log(x + (c/d)^{1/3}) / (b^2 c^3 (c/d)^{1/3} - a^3 d (c/d)^{1/3}) + \frac{1}{4} (4(b^2 c + a^2 d) x^3 - a^3 c) / (a^2 c^2 x^4)$

mupad [B] time = 11.37, size = 1734, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^5*(a + b*x^3)*(c + d*x^3)),x)$

[Out] $\log\left(\left(\frac{b^7}{a^7*(a*d - b*c)^3}\right)^{2/3} * \left(\left(27*a^{14}*b^3*c^{14}*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + 27*a^{19}*b^3*c^{19}*d^3*(a*d + b*c)*(a*d - b*c)^4*(b^7/(a^7*(a*d - b*c)^3))^{2/3}\right)^{1/3}\right) / 3 + 9*a^{13}*b^{11}*c^{20}*d^4 - 9*a^{14}*b^{10}*c^{19}*d^5 - 9*a^{19}*b^5*c^{14}*d^{10} + 9*a^{20}*b^4*c^{13}*d^{11}) / 9 + a^{13}*b^9*c^{13}*d^9*x*(b^7/(27*a^{10}*d^3 - 27*a^7*b^3*c^3 + 81*a^8*b^2*c^2*d - 81*a^9*b*c*d^2))^{1/3} + \log\left(\left(-d^7/(c^7*(a*d - b*c)^3)\right)^{2/3} * \left(\left(27*a^{14}*b^3*c^{14}*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + 27*a^{19}*b^3*c^{19}*d^3*(a*d + b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^{2/3}\right)^{1/3}\right) / 3 + 9*a^{13}*b^{11}*c^{20}*d^4 - 9*a^{14}*b^{10}*c^{19}*d^5 - 9*a^{19}*b^5*c^{14}*d^{10} + 9*a^{20}*b^4*c^{13}*d^{11}) / 9 + a^{13}*b^9*c^{13}*d^9*x*(d^7/(27*b^3*c^{10} - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*a*b^2*c^9*d))^{1/3} - (1/(4*a*c) - (x^3*(a*d + b*c))/(a^2*c^2))/x^4 + (\log\left(\left(3^{1/2}*1i - 1\right)^2 * \left(\frac{b^7}{a^7*(a*d - b*c)^3}\right)^{2/3} * \left(\left(3^{1/2}*1i - 1\right)*(27*a^{14}*b^3*c^{14}*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^{19}*b^3*c^{19}*d^3*(3^{1/2}*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b^7/(a^7*(a*d - b*c)^3))^{2/3}\right)^{1/3}\right) / 4 * \left(\frac{b^7}{a^7*(a*d - b*c)^3}\right)^{1/3}) / 6 + 9*a^{13}*b^{11}*c^{20}*d^4 - 9*a^{14}*b^{10}*c^{19}*d^5 - 9*a^{19}*b^5*c^{14}*d^{10} + 9*a^{20}*b^4*c^{13}*d^{11}) / 36 + a^{13}*b^9*c^{13}*d^9*x*(b^7/(27*a^{10}*d^3 - 27*a^7*b^3*c^3 + 81*a^8*b^2*c^2*d - 81*a^9*b*c*d^2))^{1/3} * (3^{1/2}*1i - 1) / 2 - (\log\left(\left(3^{1/2}*1i + 1\right)^2 * \left(\frac{b^7}{a^7*(a*d - b*c)^3}\right)^{2/3} * \left(\left(3^{1/2}*1i + 1\right)*(27*a^{14}*b^3*c^{14}*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^{19}*b^3*c^{19}*d^3*(3^{1/2}*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b^7/(a^7*(a*d - b*c)^3))^{2/3}\right)^{1/3}\right) / 4 * \left(\frac{b^7}{a^7*(a*d - b*c)^3}\right)^{1/3}) / 6 - 9*a^{13}*b^{11}*c^{20}*d^4 + 9*a^{14}*b^{10}*c^{19}*d^5 + 9*a^{19}*b^5*c^{14}*d^{10} - 9*a^{20}*b^4*c^{13}*d^{11}) / 36 - a^{13}*b^9*c^{13}*d^9*x*(b^7/(27*a^{10}*d^3 - 27*a^7*b^3*c^3 + 81*a^8*b^2*c^2*d - 81*a^9*b*c*d^2))^{1/3} * (3^{1/2}*1i + 1) / 2 + (\log\left(\left(3^{1/2}*1i - 1\right)^2 * \left(-d^7/(c^7*(a*d - b*c)^3)\right)^{2/3} * \left(\left(3^{1/2}*1i - 1\right)*(27*a^{14}*b^3*c^{14}*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^{19}*b^3*c^{19}*d^3*(3^{1/2}*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^{2/3}\right)^{1/3}\right) / 4 * \left(-d^7/(c^7*(a*d - b*c)^3)\right)^{1/3}) / 6 + 9*a^{13}*b^{11}*c^{20}*d^4 - 9*a^{14}*b^{10}*c^{19}*d^5 - 9*a^{19}*b^5*c^{14}*d^{10} + 9*a^{20}*b^4*c^{13}*d^{11}) / 36 + a^{13}*b^9*c^{13}*d^9*x*(d^7/(27*b^3*c^{10} - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*a*b^2*c^9*d))^{1/3} * (3^{1/2}*1i - 1) / 2 - (\log\left(\left(3^{1/2}*1i + 1\right)^2 * \left(-d^7/(c^7*(a*d - b*c)^3)\right)^{2/3} * \left(\left(3^{1/2}*1i + 1\right)*(27*a^{14}*b^3*c^{14}*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^{19}*b^3*c^{19}*d^3*(3^{1/2}*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^{2/3}\right)^{1/3}\right) / 4 * \left(-d^7/(c^7*(a*d - b*c)^3)\right)^{1/3}) / 6 - 9*a^{13}*b^{11}*c^{20}*d^4 + 9*a^{14}*b^{10}*c^{19}*d^5 + 9*a^{19}*b^5*c^{14}*d^{10} - 9*a^{20}*b^4*c^{13}*d^{11}) / 36 - a^{13}*b^9*c^{13}*d^9*x*(d^7/(27*b^3*c^{10} - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*a*b^2*c^9*d))^{1/3} * (3^{1/2}*1i + 1) / 2$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

$$3.121 \quad \int \frac{1}{x^6(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=321

$$-\frac{b^{8/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{8/3}(bc - ad)} + \frac{b^{8/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{8/3}(bc - ad)} - \frac{b^{8/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc - ad)} + \frac{ad + bc}{2a^2c^2x^2} + \frac{d^{8/3} \log(c^{2/3} - \sqrt[3]{c} x)}{6c^{8/3}(bc - ad)}$$

Rubi [A] time = 0.46, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {480, 583, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{8/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{8/3}(bc - ad)} + \frac{b^{8/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{8/3}(bc - ad)} - \frac{b^{8/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc - ad)} + \frac{ad + bc}{2a^2c^2x^2} + \frac{d^{8/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6c^{8/3}(bc - ad)} - \frac{d^{8/3} \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3c^{8/3}(bc - ad)} + \frac{d^{8/3} \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{8/3}(bc - ad)} - \frac{1}{5acx^5}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]

[Out] $-1/(5*a*c*x^5) + (b*c + a*d)/(2*a^2*c^2*x^2) - (b^{(8/3)}*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(8/3)}*(b*c - a*d)) + (d^{(8/3)}*ArcTan[(c^{(1/3)} - 2*d^{(1/3)}*x)/(Sqrt[3]*c^{(1/3)})])/(Sqrt[3]*c^{(8/3)}*(b*c - a*d)) + (b^{(8/3)}*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(8/3)}*(b*c - a*d)) - (d^{(8/3)}*Log[c^{(1/3)} + d^{(1/3)}*x])/(3*c^{(8/3)}*(b*c - a*d)) - (b^{(8/3)}*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(8/3)}*(b*c - a*d)) + (d^{(8/3)}*Log[c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x + d^{(2/3)}*x^2])/(6*c^{(8/3)}*(b*c - a*d))$

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^-1, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 480

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e^(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^6 (a + bx^3)(c + dx^3)} dx &= -\frac{1}{5acx^5} + \frac{\int \frac{-5(bc+ad)-5bdx^3}{x^3(a+bx^3)(c+dx^3)} dx}{5ac} \\
 &= -\frac{1}{5acx^5} + \frac{bc+ad}{2a^2c^2x^2} - \frac{\int \frac{-10(b^2c^2+abcd+a^2d^2)-10bd(bc+ad)x^3}{(a+bx^3)(c+dx^3)} dx}{10a^2c^2} \\
 &= -\frac{1}{5acx^5} + \frac{bc+ad}{2a^2c^2x^2} + \frac{b^3 \int \frac{1}{a+bx^3} dx}{a^2(bc-ad)} - \frac{d^3 \int \frac{1}{c+dx^3} dx}{c^2(bc-ad)} \\
 &= -\frac{1}{5acx^5} + \frac{bc+ad}{2a^2c^2x^2} + \frac{b^3 \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{8/3}(bc-ad)} + \frac{b^3 \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{8/3}(bc-ad)} - \frac{d^3 \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{3c^{8/3}(bc-ad)} \\
 &= -\frac{1}{5acx^5} + \frac{bc+ad}{2a^2c^2x^2} + \frac{b^{8/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{8/3}(bc-ad)} - \frac{d^{8/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{8/3}(bc-ad)} - \frac{b^{8/3} \int \frac{-1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{8/3}(bc-ad)} \\
 &= -\frac{1}{5acx^5} + \frac{bc+ad}{2a^2c^2x^2} + \frac{b^{8/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{8/3}(bc-ad)} - \frac{d^{8/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{8/3}(bc-ad)} - \frac{b^{8/3} \log(a - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{8/3}(bc-ad)} \\
 &= -\frac{1}{5acx^5} + \frac{bc+ad}{2a^2c^2x^2} - \frac{b^{8/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc-ad)} + \frac{d^{8/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{8/3}(bc-ad)} + \frac{b^{8/3} \log(a - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{3a^{8/3}(bc-ad)}
 \end{aligned}$$

Mathematica [A] time = 0.22, size = 282, normalized size = 0.88

$$\frac{-\frac{10b^{8/3}x^5 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{a^{8/3}} + \frac{10\sqrt{3}b^{8/3}x^5 \tan^{-1}\left(\frac{1-2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{8/3}} + \frac{5b^{8/3}x^5 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{a^{8/3}} - \frac{15b^2x^3}{a^2} + \frac{6b}{a} + \frac{10a^{8/3}x^5 \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{c^{8/3}} - \frac{10\sqrt{3}d^{8/3}x^5 \tan^{-1}\left(\frac{1-2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{8/3}} - \frac{5d^{8/3}x^5 \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2)}{c^{8/3}} + \frac{15d^2x^3}{c^2} - \frac{6d}{c}}{30x^5(ad-bc)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6*(a + b*x^3)*(c + d*x^3)), x]

[Out] ((6*b)/a - (6*d)/c - (15*b^2*x^3)/a^2 + (15*d^2*x^3)/c^2 + (10*sqrt[3]*b^(8/3)*x^5*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/a^(8/3) - (10*sqrt[3]*d^(8/3)*x^5*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/sqrt[3]])/c^(8/3) - (10*b^(8/3)*x^5*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) + (10*d^(8/3)*x^5*Log[c^(1/3) + d

$$\frac{(a^{1/3}x)^{1/3}}{c^{8/3}} + (5b^{8/3}x^5 \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2]) / a^{8/3} - (5d^{8/3}x^5 \operatorname{Log}[c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2]) / c^{8/3} / (30(-b^*c) + a*d)x^5$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 (a + bx^3)(c + dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^6*(a + b*x^3)*(c + d*x^3)), x]

[Out] IntegrateAlgebraic[1/(x^6*(a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 1.04, size = 356, normalized size = 1.11

$$\frac{10\sqrt{3}b^2c^2x^5\left(-\frac{a}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}a\left(\frac{a}{c}\right)^{\frac{1}{3}}-\sqrt{3}a}{3}\right)+10\sqrt{3}a^2d^2x^5\left(\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}c\left(\frac{c}{d}\right)^{\frac{1}{3}}-\sqrt{3}c}{3}\right)-5b^2c^2x^5\left(-\frac{a}{c}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{a}{c}\right)^{\frac{1}{3}}+a^2\left(-\frac{a}{c}\right)^{\frac{2}{3}}\right)-5a^2d^2x^5\left(\frac{c}{d}\right)^{\frac{1}{3}}\log\left(d^2x^2-cdx\left(\frac{c}{d}\right)^{\frac{1}{3}}+c^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)+10b^2c^2x^5\left(-\frac{a}{c}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)+10a^2d^2x^5\left(\frac{c}{d}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)+6ab^2c-6a^2cd-15(b^2c^2-a^2d^2)x^3}{30(a^2bc^3-a^2c^2d)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c), x, algorithm="fricas")

[Out] $-1/30*(10*\sqrt{3}*b^2*c^2*x^5*(-b^2/a^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3})*a*x*(-b^2/a^2)^{(2/3)} - \sqrt{3}*b)/b) + 10*\sqrt{3}*a^2*d^2*x^5*(d^2/c^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3})*c*x*(d^2/c^2)^{(2/3)} - \sqrt{3}*d)/d) - 5*b^2*c^2*x^5*(-b^2/a^2)^{(1/3)}*\log(b^2*x^2 + a*b*x*(-b^2/a^2)^{(1/3)} + a^2*(-b^2/a^2)^{(2/3)}) - 5*a^2*d^2*x^5*(d^2/c^2)^{(1/3)}*\log(d^2*x^2 - c*d*x*(d^2/c^2)^{(1/3)} + c^2*(d^2/c^2)^{(2/3)}) + 10*b^2*c^2*x^5*(-b^2/a^2)^{(1/3)}*\log(b*x - a*(-b^2/a^2)^{(1/3)}) + 10*a^2*d^2*x^5*(d^2/c^2)^{(1/3)}*\log(d*x + c*(d^2/c^2)^{(1/3)}) + 6*a*b*c^2 - 6*a^2*c*d - 15*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*d)*x^5)$

giac [A] time = 0.21, size = 336, normalized size = 1.05

$$\frac{b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(a^3bc-a^4d)} + \frac{d^3\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(bc^4-ac^3d)} + \frac{(-ab^2)^{\frac{1}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^3bc-\sqrt{3}a^4d} - \frac{(-cd^2)^{\frac{1}{3}}d^2\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^4-\sqrt{3}ac^3d} + \frac{(-ab^2)^{\frac{1}{3}}b^2\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(a^3bc-a^4d)} - \frac{(-cd^2)^{\frac{1}{3}}d^2\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bc^4-ac^3d)} + \frac{5bcx^3+5adx^3-2ac}{10a^2c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c), x, algorithm="giac")

[Out] $-1/3*b^3*(-a/b)^{(1/3)}*\log(\operatorname{abs}(x - (-a/b)^{(1/3)}))/ (a^3*b*c - a^4*d) + 1/3*d^3*(-c/d)^{(1/3)}*\log(\operatorname{abs}(x - (-c/d)^{(1/3)}))/ (b*c^4 - a*c^3*d) + (-a*b^2)^{(1/3)}*b^2*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)}))/ (-a/b)^{(1/3)} / (\sqrt{3}*a^3*b*c - \sqrt{3}*a^4*d) - (-c*d^2)^{(1/3)}*d^2*\arctan(1/3*\sqrt{3}*(2*x + (-c/d)^{(1/3)}))/ (-c/d)^{(1/3)} / (\sqrt{3}*b*c^4 - \sqrt{3}*a*c^3*d) + 1/6*(-a*b^2)^{(1/3)}*b$

$$\sqrt{3} \log(x^2 + x(-a/b)^{1/3} + (-a/b)^{2/3}) / (a^3 b c - a^4 d) - 1/6 (-c d^2)^{1/3} d^2 \log(x^2 + x(-c/d)^{1/3} + (-c/d)^{2/3}) / (b c^4 - a c^3 d) + 1/10 (5 b c x^3 + 5 a d x^3 - 2 a c) / (a^2 c^2 x^5)$$

maple [A] time = 0.05, size = 293, normalized size = 0.91

$$\frac{\sqrt{3} b^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{b}-1\right)}{\left(\frac{b}{b}\right)^{\frac{2}{3}}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}} a^2} - \frac{b^2 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}} a^2} + \frac{b^2 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}} a^2} + \frac{\sqrt{3} d^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{d}-1\right)}{\left(\frac{d}{d}\right)^{\frac{2}{3}}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}} c^2} + \frac{d^2 \ln\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}} c^2} - \frac{d^2 \ln\left(x^2 - \left(\frac{c}{d}\right)^{\frac{1}{3}} x + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}} c^2} + \frac{d}{2a c^2 x^2} + \frac{b}{2a^2 c x^2} - \frac{1}{5ac x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(b*x^3+a)/(d*x^3+c), x)

[Out] $-1/3/a^2 b^2/(a d - b^3 c)/(a/b)^{2/3} \ln(x + (a/b)^{1/3}) + 1/6/a^2 b^2/(a d - b^3 c)/(a/b)^{2/3} \ln(x^2 - (a/b)^{1/3} x + (a/b)^{2/3}) - 1/3/a^2 b^2/(a d - b^3 c)/(a/b)^{2/3} \ln(x^2 - (a/b)^{1/3} x + (a/b)^{2/3}) - 1/3/a^2 b^2/(a d - b^3 c)/(a/b)^{2/3} \ln(x^2 - (a/b)^{1/3} x + (a/b)^{2/3}) + 1/3/c^2 d^2/(a d - b^3 c)/(c/d)^{2/3} \ln(x + (c/d)^{1/3}) - 1/6/c^2 d^2/(a d - b^3 c)/(c/d)^{2/3} \ln(x^2 - (c/d)^{1/3} x + (c/d)^{2/3}) + 1/3/c^2 d^2/(a d - b^3 c)/(c/d)^{2/3} \ln(x^2 - (c/d)^{1/3} x + (c/d)^{2/3}) + 1/3/c^2 d^2/(a d - b^3 c)/(c/d)^{2/3} \ln(x^2 - (c/d)^{1/3} x + (c/d)^{2/3}) + 1/3/c^2 d^2/(a d - b^3 c)/(c/d)^{2/3} \ln(x^2 - (c/d)^{1/3} x + (c/d)^{2/3}) - 1/5/a/c/x^5 + 1/2/a/c^2/x^2 d + 1/2/a^2/c/x^2 b$

maxima [A] time = 1.23, size = 369, normalized size = 1.15

$$\frac{\sqrt{3} b^2 \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3\left(a^2 b c \left(\frac{a}{b}\right)^{\frac{1}{3}} - a^3 d \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) \left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt{3} d^2 \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3\left(b c^3 \left(\frac{c}{d}\right)^{\frac{1}{3}} - a c^2 d \left(\frac{c}{d}\right)^{\frac{1}{3}}\right) \left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{b^2 \log\left(x^2 - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2 b c \left(\frac{a}{b}\right)^{\frac{1}{3}} - a^3 d \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{d^2 \log\left(x^2 - x \left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(b c^3 \left(\frac{c}{d}\right)^{\frac{1}{3}} - a c^2 d \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{b^2 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^2 b c \left(\frac{a}{b}\right)^{\frac{1}{3}} - a^3 d \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{d^2 \log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(b c^3 \left(\frac{c}{d}\right)^{\frac{1}{3}} - a c^2 d \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{5(bc+ad)x^3 - 2ac}{10 a^2 c^2 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c), x, algorithm="maxima")

[Out] $1/3 \sqrt{3} b^2 \arctan(1/3 \sqrt{3} (2x - (a/b)^{1/3}) / (a/b)^{1/3}) / ((a^2 b c (a/b)^{1/3} - a^3 d (a/b)^{1/3}) (a/b)^{1/3}) - 1/3 \sqrt{3} d^2 \arctan(1/3 \sqrt{3} (2x - (c/d)^{1/3}) / (c/d)^{1/3}) / ((b c^3 (c/d)^{1/3} - a c^2 d (c/d)^{1/3}) (c/d)^{1/3}) - 1/6 b^2 \log(x^2 - x (a/b)^{1/3} + (a/b)^{2/3}) / (a^2 b c (a/b)^{1/3} - a^3 d (a/b)^{1/3}) + 1/6 d^2 \log(x^2 - x (c/d)^{1/3} + (c/d)^{2/3}) / (b c^3 (c/d)^{1/3} - a c^2 d (c/d)^{1/3}) + 1/3 b^2 \log(x + (a/b)^{1/3}) / (a^2 b c (a/b)^{1/3} - a^3 d (a/b)^{1/3}) - 1/3 d^2 \log(x + (c/d)^{1/3}) / (b c^3 (c/d)^{1/3} - a c^2 d (c/d)^{1/3}) + 1/10 (5 (b c + a d) x^3 - 2 a c) / (a^2 c^2 x^5)$

mupad [B] time = 11.57, size = 1860, normalized size = 5.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6*(a + b*x^3)*(c + d*x^3)),x)

[Out] $\log\left(\left(\frac{-b^8}{a^8(a^3d - b^3c)^3}\right)^{1/3} \cdot (9a^{13}b^{11}c^{19}d^5 - 9a^{12}b^{12}c^{20}d^4 + 9a^{19}b^5c^{13}d^{11} - 9a^{20}b^4c^{12}d^{12} + 9a^{16}b^3c^{16}d^3 \cdot (ad + bc)(ad - bc)^4(a^3c^3(-b^8/(a^8(ad - b^3c)^3))^{1/3} + a^2d^2x + b^2c^2x)(-b^8/(a^8(ad - b^3c)^3))^{2/3})\right)^{1/3} + 3a^{12}b^7c^{12}d^7x \cdot (a^4d^4 + b^4c^4) \cdot (-b^8/(27a^{11}d^3 - 27a^8b^3c^3 + 81a^9b^2c^2d - 81a^{10}b^2c^2d^2))^{1/3} - (1/(5ac) - (x^3(ad + bc))/(2a^2c^2)) / x^5 + \log\left(\left(\frac{d^8}{c^8(ad - b^3c)^3}\right)^{1/3} \cdot (9a^{13}b^{11}c^{19}d^5 - 9a^{12}b^{12}c^{20}d^4 + 9a^{19}b^5c^{13}d^{11} - 9a^{20}b^4c^{12}d^{12} + 9a^{16}b^3c^{16}d^3 \cdot (ad + bc)(ad - bc)^4(a^3c^3(d^8/(c^8(ad - b^3c)^3))^{1/3} + a^2d^2x + b^2c^2x)(d^8/(c^8(ad - b^3c)^3))^{2/3})\right)^{1/3} + 3a^{12}b^7c^{12}d^7x \cdot (a^4d^4 + b^4c^4) \cdot (-d^8/(27b^3c^{11} - 27a^3c^8d^3 + 81a^2b^2c^9d^2 - 81ab^2c^{10}d))^{1/3} + (\log\left(\left(3^{1/2}\right)^{1i} - 1\right) \cdot (-b^8/(a^8(ad - b^3c)^3))^{1/3} \cdot \left(\left(3^{1/2}\right)^{1i} - 1\right)^2 \cdot (81a^{16}b^3c^{16}d^3x \cdot (ad - bc)^4 \cdot (a^3d^3 + b^3c^3 + ab^2c^2d + a^2b^2c^2d^2) + (81a^{19}b^3c^{19}d^3 \cdot (3^{1/2})^{1i} - 1) \cdot (ad + bc) \cdot (ad - bc)^4 \cdot (-b^8/(a^8(ad - b^3c)^3))^{1/3}) / 2) \cdot (-b^8/(a^8(ad - b^3c)^3))^{2/3}) / 36 - 9a^{12}b^{12}c^{20}d^4 + 9a^{13}b^{11}c^{19}d^5 + 9a^{19}b^5c^{13}d^{11} - 9a^{20}b^4c^{12}d^{12}) / 6 + 3a^{12}b^7c^{12}d^7x \cdot (a^4d^4 + b^4c^4) \cdot (-b^8/(27a^{11}d^3 - 27a^8b^3c^3 + 81a^9b^2c^2d - 81a^{10}b^2c^2d^2))^{1/3} \cdot (3^{1/2})^{1i} - 1) / 2 - (\log\left(\left(3^{1/2}\right)^{1i} + 1\right) \cdot (-b^8/(a^8(ad - b^3c)^3))^{1/3} \cdot \left(\left(3^{1/2}\right)^{1i} + 1\right)^2 \cdot (81a^{16}b^3c^{16}d^3x \cdot (ad - bc)^4 \cdot (a^3d^3 + b^3c^3 + ab^2c^2d + a^2b^2c^2d^2) - (81a^{19}b^3c^{19}d^3 \cdot (3^{1/2})^{1i} + 1) \cdot (ad + bc) \cdot (ad - bc)^4 \cdot (-b^8/(a^8(ad - b^3c)^3))^{1/3}) / 2) \cdot (-b^8/(a^8(ad - b^3c)^3))^{2/3}) / 36 - 9a^{12}b^{12}c^{20}d^4 + 9a^{13}b^{11}c^{19}d^5 + 9a^{19}b^5c^{13}d^{11} - 9a^{20}b^4c^{12}d^{12}) / 6 - 3a^{12}b^7c^{12}d^7x \cdot (a^4d^4 + b^4c^4) \cdot (-b^8/(27a^{11}d^3 - 27a^8b^3c^3 + 81a^9b^2c^2d - 81a^{10}b^2c^2d^2))^{1/3} \cdot (3^{1/2})^{1i} + 1) / 2 + (\log\left(\left(3^{1/2}\right)^{1i} - 1\right) \cdot (d^8/(c^8(ad - b^3c)^3))^{1/3} \cdot \left(\left(3^{1/2}\right)^{1i} - 1\right)^2 \cdot (81a^{16}b^3c^{16}d^3x \cdot (ad - bc)^4 \cdot (a^3d^3 + b^3c^3 + ab^2c^2d + a^2b^2c^2d^2) + (81a^{19}b^3c^{19}d^3 \cdot (3^{1/2})^{1i} - 1) \cdot (ad + bc) \cdot (ad - bc)^4 \cdot (d^8/(c^8(ad - b^3c)^3))^{1/3}) / 2) \cdot (d^8/(c^8(ad - b^3c)^3))^{2/3}) / 36 - 9a^{12}b^{12}c^{20}d^4 + 9a^{13}b^{11}c^{19}d^5 + 9a^{19}b^5c^{13}d^{11} - 9a^{20}b^4c^{12}d^{12}) / 6 + 3a^{12}b^7c^{12}d^7x \cdot (a^4d^4 + b^4c^4) \cdot (-d^8/(27b^3c^{11} - 27a^3c^8d^3 + 81a^2b^2c^9d^2 - 81ab^2c^{10}d))^{1/3} \cdot (3^{1/2})^{1i} - 1) / 2 - (\log\left(\left(3^{1/2}\right)^{1i} + 1\right) \cdot (d^8/(c^8(ad - b^3c)^3))^{1/3} \cdot \left(\left(3^{1/2}\right)^{1i} + 1\right)^2 \cdot (81a^{16}b^3c^{16}d^3x \cdot (ad - bc)^4 \cdot (a^3d^3 + b^3c^3 + ab^2c^2d + a^2b^2c^2d^2) - (81a^{19}b^3c^{19}d^3 \cdot (3^{1/2})^{1i} + 1) \cdot (ad + bc) \cdot (ad - bc)^4 \cdot (d^8/(c^8(ad - b^3c)^3))^{1/3}) / 2) \cdot (d^8/(c^8(ad - b^3c)^3))^{2/3}) / 36 - 9a^{12}b^{12}c^{20}d^4 + 9a^{13}b^{11}c^{19}d^5 + 9a^{19}b^5c^{13}d^{11} - 9a^{20}b^4c^{12}d^{12}) / 6 - 3a^{12}b^7c^{12}d^7x \cdot (a^4d^4 + b^4c^4) \cdot (-d^8/(27b^3c^{11} - 27a^3c^8d^3 + 81a^2b^2c^9d^2 - 81ab^2c^{10}d))^{1/3} \cdot (3^{1/2})^{1i} + 1) / 2$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**6/(b*x**3+a)/(d*x**3+c),x)

[Out] Timed out

$$3.122 \quad \int \frac{1}{x^7(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=119

$$-\frac{b^3 \log(a+bx^3)}{3a^3(bc-ad)} + \frac{ad+bc}{3a^2c^2x^3} + \frac{\log(x)(a^2d^2+abcd+b^2c^2)}{a^3c^3} + \frac{d^3 \log(c+dx^3)}{3c^3(bc-ad)} - \frac{1}{6acx^6}$$

Rubi [A] time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 72}

$$\frac{\log(x)(a^2d^2+abcd+b^2c^2)}{a^3c^3} - \frac{b^3 \log(a+bx^3)}{3a^3(bc-ad)} + \frac{ad+bc}{3a^2c^2x^3} + \frac{d^3 \log(c+dx^3)}{3c^3(bc-ad)} - \frac{1}{6acx^6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(a + b*x^3)*(c + d*x^3)),x]

[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(3*a^2*c^2*x^3) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*Log[x])/(a^3*c^3) - (b^3*Log[a + b*x^3])/(3*a^3*(b*c - a*d)) + (d^3*Log[c + d*x^3])/(3*c^3*(b*c - a*d))

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^7(a+bx^3)(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^3(a+bx)(c+dx)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{acx^3} + \frac{-bc-ad}{a^2c^2x^2} + \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} + \frac{b^4}{a^3(-bc+ad)(a+bx)} + \frac{d^4}{a^3(-bc+ad)(c+dx)} \right) dx, x, x^3 \right) \\ &= -\frac{1}{6acx^6} + \frac{bc+ad}{3a^2c^2x^3} + \frac{(b^2c^2+abcd+a^2d^2)\log(x)}{a^3c^3} - \frac{b^3\log(a+bx^3)}{3a^3(bc-ad)} + \frac{d^3\log(c+dx^3)}{3c^3(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.06, size = 119, normalized size = 1.00

$$\frac{b^3 \log(a+bx^3)}{3a^3(ad-bc)} + \frac{ad+bc}{3a^2c^2x^3} + \frac{\log(x)(a^2d^2+abcd+b^2c^2)}{a^3c^3} + \frac{d^3 \log(c+dx^3)}{3c^3(bc-ad)} - \frac{1}{6acx^6}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(a + b*x^3)*(c + d*x^3)), x]

[Out] -1/6*1/(a*c*x^6) + (b*c + a*d)/(3*a^2*c^2*x^3) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*Log[x])/(a^3*c^3) + (b^3*Log[a + b*x^3])/(3*a^3*(-(b*c) + a*d)) + (d^3*Log[c + d*x^3])/(3*c^3*(b*c - a*d))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7(a+bx^3)(c+dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^7*(a + b*x^3)*(c + d*x^3)), x]

[Out] IntegrateAlgebraic[1/(x^7*(a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 11.77, size = 127, normalized size = 1.07

$$\frac{2b^3c^3x^6 \log(bx^3+a) - 2a^3d^3x^6 \log(dx^3+c) - 6(b^3c^3 - a^3d^3)x^6 \log(x) + a^2bc^3 - a^3c^2d - 2(ab^2c^3 - a^3cd^2)x^3}{6(a^3bc^4 - a^4c^3d)x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c), x, algorithm="fricas")

[Out] -1/6*(2*b^3*c^3*x^6*log(b*x^3 + a) - 2*a^3*d^3*x^6*log(d*x^3 + c) - 6*(b^3*c^3 - a^3*d^3)*x^6*log(x) + a^2*b*c^3 - a^3*c^2*d - 2*(a*b^2*c^3 - a^3*c*d^2)*x^3)/((a^3*b*c^4 - a^4*c^3*d)*x^6)

giac [A] time = 0.26, size = 165, normalized size = 1.39

$$-\frac{b^4 \log(|bx^3 + a|)}{3(a^3b^2c - a^4bd)} + \frac{d^4 \log(|dx^3 + c|)}{3(bc^4d - ac^3d^2)} + \frac{(b^2c^2 + abcd + a^2d^2) \log(|x|)}{a^3c^3} - \frac{3b^2c^2x^6 + 3abcdx^6 + 3a^2d^2x^6 - 2abc^2x^3 - 2a^2cdx^3 + a^2c^2}{6a^3c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")

[Out] $-\frac{1}{3}b^4 \log(\text{abs}(bx^3 + a)) / (a^3b^2c - a^4bd) + \frac{1}{3}d^4 \log(\text{abs}(dx^3 + c)) / (bc^4d - ac^3d^2) + (b^2c^2 + abc^2d + a^2d^2) \log(\text{abs}(x)) / (a^3c^3) - \frac{1}{6} * (3b^2c^2x^6 + 3a^2b^2cdx^6 + 3a^2d^2cdx^6 - 2abc^2x^3 - 2a^2cdx^3 + a^2c^2) / (a^3c^3x^6)$

maple [A] time = 0.06, size = 124, normalized size = 1.04

$$\frac{b^3 \ln(bx^3 + a)}{3(ad - bc)a^3} - \frac{d^3 \ln(dx^3 + c)}{3(ad - bc)c^3} + \frac{d^2 \ln(x)}{a^3c} + \frac{bd \ln(x)}{a^2c^2} + \frac{b^2 \ln(x)}{a^3c} + \frac{d}{3ac^2x^3} + \frac{b}{3a^2cx^3} - \frac{1}{6acx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^7/(b*x^3+a)/(d*x^3+c),x)

[Out] $\frac{1}{3}b^3/a^3/(a*d-b*c)*\ln(b*x^3+a) - \frac{1}{3}d^3/c^3/(a*d-b*c)*\ln(d*x^3+c) - \frac{1}{6}a/c/x^6 + \frac{1}{3}a/c^2/x^3*d + \frac{1}{3}a^2/c/x^3*b + \frac{1}{a/c^3*\ln(x)*d^2 + 1/a^2/c^2*\ln(x)*b*d + 1/a^3/c*\ln(x)*b^2}$

maxima [A] time = 0.52, size = 117, normalized size = 0.98

$$-\frac{b^3 \log(bx^3 + a)}{3(a^3bc - a^4d)} + \frac{d^3 \log(dx^3 + c)}{3(bc^4 - ac^3d)} + \frac{(b^2c^2 + abcd + a^2d^2) \log(x^3)}{3a^3c^3} + \frac{2(bc + ad)x^3 - ac}{6a^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")

[Out] $-\frac{1}{3}b^3 \log(bx^3 + a) / (a^3bc - a^4d) + \frac{1}{3}d^3 \log(dx^3 + c) / (bc^4 - ac^3d) + \frac{1}{3} * (b^2c^2 + abc^2d + a^2d^2) \log(x^3) / (a^3c^3) + \frac{1}{6} * (2 * (bc + ad) * x^3 - ac) / (a^2c^2x^6)$

mupad [B] time = 3.21, size = 118, normalized size = 0.99

$$\frac{b^3 \ln(bx^3 + a)}{3a^4d - 3a^3bc} - \frac{\frac{1}{6ac} - \frac{x^3(ad+bc)}{3a^2c^2}}{x^6} + \frac{d^3 \ln(dx^3 + c)}{3bc^4 - 3ac^3d} + \frac{\ln(x) (a^2d^2 + abc^2d + b^2c^2)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] (b^3*log(a + b*x^3))/(3*a^4*d - 3*a^3*b*c) - (1/(6*a*c) - (x^3*(a*d + b*c))
/(3*a^2*c^2))/x^6 + (d^3*log(c + d*x^3))/(3*b*c^4 - 3*a*c^3*d) + (log(x)*(a
^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

$$3.123 \quad \int \frac{1}{x^8(a+bx^3)(c+dx^3)} dx$$

Optimal. Leaf size=352

$$-\frac{b^{10/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{10/3}(bc-ad)} + \frac{b^{10/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{10/3}(bc-ad)} + \frac{b^{10/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)} + \frac{ad+bc}{4a^2c^2x^4} - \frac{a^2d^2+abcd+b^2}{a^3c^3x}$$

Rubi [A] time = 0.50, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {480, 583, 584, 292, 31, 634, 617, 204, 628}

$$-\frac{a^2d^2+abcd+b^2c^2}{a^3c^3x} - \frac{b^{10/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{10/3}(bc-ad)} + \frac{b^{10/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{10/3}(bc-ad)} + \frac{b^{10/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)} + \frac{ad+bc}{4a^2c^2x^4} + \frac{d^{10/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6c^{10/3}(bc-ad)} - \frac{d^{10/3} \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3c^{10/3}(bc-ad)} - \frac{d^{10/3} \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{10/3}(bc-ad)} - \frac{1}{7acx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]

[Out] $-1/(7*a*c*x^7) + (b*c + a*d)/(4*a^2*c^2*x^4) - (b^2*c^2 + a*b*c*d + a^2*d^2)/(a^3*c^3*x) + (b^{(10/3)}*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(10/3)}*(b*c - a*d)) - (d^{(10/3)}*ArcTan[(c^{(1/3)} - 2*d^{(1/3)}*x)/(Sqrt[3]*c^{(1/3)})])/(Sqrt[3]*c^{(10/3)}*(b*c - a*d)) + (b^{(10/3)}*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(10/3)}*(b*c - a*d)) - (d^{(10/3)}*Log[c^{(1/3)} + d^{(1/3)}*x])/(3*c^{(10/3)}*(b*c - a*d)) - (b^{(10/3)}*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(10/3)}*(b*c - a*d)) + (d^{(10/3)}*Log[c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x + d^{(2/3)}*x^2])/(6*c^{(10/3)}*(b*c - a*d))$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])⁽⁻¹⁾, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x

$\wedge 2$), x], x] /; FreeQ[{a, b}, x]

Rule 480

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 583

Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 584

Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((g*x)^m*(a + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && IGtQ[n, 0]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8 (a + bx^3)(c + dx^3)} dx &= -\frac{1}{7acx^7} + \frac{\int \frac{-7(bc+ad)-7bdx^3}{x^5(a+bx^3)(c+dx^3)} dx}{7ac} \\
&= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{\int \frac{-28(b^2c^2+abcd+a^2d^2)-28bd(bc+ad)x^3}{x^2(a+bx^3)(c+dx^3)} dx}{28a^2c^2} \\
&= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} + \frac{\int \frac{x(-28(bc+ad)(b^2c^2+a^2d^2)-28bd(b^2c^2+abcd+a^2d^2))}{(a+bx^3)(c+dx^3)} dx}{28a^3c^3} \\
&= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} + \frac{\int \left(-\frac{28b^4c^3x}{(bc-ad)(a+bx^3)} - \frac{28a^3d^4x}{(-bc+ad)(c+dx^3)} \right) dx}{28a^3c^3} \\
&= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} - \frac{b^4}{a^3(bc-ad)} \int \frac{x}{a+bx^3} dx + \frac{d^4}{c^3(bc-ad)} \int \frac{x}{c+dx^3} dx \\
&= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} + \frac{b^{11/3}}{3a^{10/3}(bc-ad)} \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx - \frac{b^{11/3}}{3a^{10/3}(bc-ad)} \int \frac{\sqrt[3]{a} + \sqrt[3]{b}}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}} dx \\
&= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} + \frac{b^{10/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{10/3}(bc-ad)} - \frac{d^{10/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{10/3}(bc-ad)} \\
&= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} + \frac{b^{10/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{10/3}(bc-ad)} - \frac{d^{10/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{10/3}(bc-ad)} \\
&= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} + \frac{b^{10/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)} - \frac{d^{10/3} \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{10/3}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 304, normalized size = 0.86

$$\frac{28b^{10/3}x^7 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{a^{10/3}} - \frac{28\sqrt{3}b^{10/3}x^7 \tan^{-1}\left(\frac{1-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{10/3}} + \frac{14bd^{10/3}x^7 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + d^{2/3}x^2)}{a^{10/3}} + \frac{84b^3x^6}{a^3} - \frac{21b^2x^5}{a^2} + \frac{12b}{a} + \frac{28d^{10/3}x^7 \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{c^{10/3}} + \frac{28\sqrt{3}d^{10/3}x^7 \tan^{-1}\left(\frac{1-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{c^{10/3}} - \frac{14d^{10/3}x^7 \log(a^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2)}{c^{10/3}} - \frac{84d^3x^6}{c^3} + \frac{21d^2x^5}{c^2} - \frac{12d}{c}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^8*(a + b*x^3)*(c + d*x^3)), x]

[Out]
$$\begin{aligned} & \left(\frac{(12*b)}{a} - \frac{(12*d)}{c} - \frac{(21*b^2*x^3)}{a^2} + \frac{(21*d^2*x^3)}{c^2} + \frac{(84*b^3*x^6)}{a^3} - \frac{(84*d^3*x^6)}{c^3} - \frac{(28*\sqrt{3}*b^{(10/3)}*x^7*\text{ArcTan}[(1 - (2*b^{(1/3)}*x)/a^{(1/3)})/\sqrt{3}])}{a^{(10/3)}} + \frac{(28*\sqrt{3}*d^{(10/3)}*x^7*\text{ArcTan}[(1 - (2*d^{(1/3)}*x)/c^{(1/3)})/\sqrt{3}])}{c^{(10/3)}} - \frac{(28*b^{(10/3)}*x^7*\text{Log}[a^{(1/3)} + b^{(1/3)}*x])}{a^{(10/3)}} + \frac{(28*d^{(10/3)}*x^7*\text{Log}[c^{(1/3)} + d^{(1/3)}*x])}{c^{(10/3)}} + \frac{(14*b^{(10/3)}*x^7*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])}{a^{(10/3)}} - \frac{(14*d^{(10/3)}*x^7*\text{Log}[c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x + d^{(2/3)}*x^2])}{c^{(10/3)}} \right) / (84*(b*c + a*d)*x^7) \end{aligned}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8(a + bx^3)(c + dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^8*(a + b*x^3)*(c + d*x^3)), x]

[Out] IntegrateAlgebraic[1/(x^8*(a + b*x^3)*(c + d*x^3)), x]

fricas [A] time = 1.20, size = 332, normalized size = 0.94

$$\frac{28\sqrt{3}b^{10/3}c^{2/3}\left(-\frac{1}{3}\right)^{1/3}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{1}{3}\right)^{1/3} + \frac{1}{3}\sqrt{3}\right) - 28\sqrt{3}d^{10/3}c^{2/3}\left(-\frac{1}{3}\right)^{1/3}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{1}{3}\right)^{1/3} - \frac{1}{3}\sqrt{3}\right) - 14b^{10/3}c^{2/3}\left(-\frac{1}{3}\right)^{1/3}\log\left(bx^2 - ax\left(-\frac{1}{3}\right)^{1/3} - a\left(-\frac{1}{3}\right)^{1/3}\right) - 14d^{10/3}c^{2/3}\left(-\frac{1}{3}\right)^{1/3}\log\left(dx^2 - cx\left(-\frac{1}{3}\right)^{1/3} + c\left(-\frac{1}{3}\right)^{1/3}\right) + 28b^{10/3}c^{2/3}\left(-\frac{1}{3}\right)^{1/3}\log\left(bx + a\left(-\frac{1}{3}\right)^{1/3}\right) + 28d^{10/3}c^{2/3}\left(-\frac{1}{3}\right)^{1/3}\log\left(dx + c\left(-\frac{1}{3}\right)^{1/3}\right) + 84(b^{10/3} - a^{10/3})c^2 + 12a^2b^2c^3 - 12a^3c^2d - 21(a^{10/3} - a^2cd^2)x^6}{84(a^3bc^4 - a^4c^3d)x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c), x, algorithm="fricas")

[Out]
$$\begin{aligned} & -\frac{1}{84} * (28 * \sqrt{3} * b^3 * c^3 * x^7 * (-b/a)^{(1/3)} * \arctan(2/3 * \sqrt{3} * x * (-b/a)^{(1/3)} + 1/3 * \sqrt{3}) - 28 * \sqrt{3} * a^3 * d^3 * x^7 * (d/c)^{(1/3)} * \arctan(2/3 * \sqrt{3} * x * (d/c)^{(1/3)} - 1/3 * \sqrt{3}) - 14 * b^3 * c^3 * x^7 * (-b/a)^{(1/3)} * \log(b * x^2 - a * x * (-b/a)^{(2/3)} - a * (-b/a)^{(1/3)}) - 14 * a^3 * d^3 * x^7 * (d/c)^{(1/3)} * \log(d * x^2 - c * x * (d/c)^{(2/3)} + c * (d/c)^{(1/3)}) + 28 * b^3 * c^3 * x^7 * (-b/a)^{(1/3)} * \log(b * x + a * (-b/a)^{(2/3)}) + 28 * a^3 * d^3 * x^7 * (d/c)^{(1/3)} * \log(d * x + c * (d/c)^{(2/3)}) + 84 * (b^3 * c^3 - a^3 * d^3) * x^6 + 12 * a^2 * b * c^3 - 12 * a^3 * c^2 * d - 21 * (a * b^2 * c^3 - a^3 * c * d^2) * x^3) / ((a^3 * b * c^4 - a^4 * c^3 * d) * x^7) \end{aligned}$$

giac [A] time = 0.22, size = 377, normalized size = 1.07

$$\frac{b^4 \left(-\frac{1}{3}\right)^{1/3} \log\left(x - \left(-\frac{1}{3}\right)^{1/3}\right) - a^4 \left(-\frac{1}{3}\right)^{1/3} \log\left(x - \left(-\frac{1}{3}\right)^{1/3}\right) + \frac{(-ab^2)^{2/3} b^2 \arctan\left(\frac{\sqrt{3}(2x + (-\frac{1}{3})^{1/3})}{3(-\frac{1}{3})^{1/3}}\right)}{\sqrt{3}abc - \sqrt{3}ad} - \frac{(-cd)^{2/3} d^2 \arctan\left(\frac{\sqrt{3}(2x + (-\frac{1}{3})^{1/3})}{3(-\frac{1}{3})^{1/3}}\right)}{\sqrt{3}bc^3 - \sqrt{3}cd} - \frac{(-ab^2)^{2/3} b^2 \log\left(x^2 + x\left(-\frac{1}{3}\right)^{1/3} + \left(-\frac{1}{3}\right)^{2/3}\right)}{6(a^3bc - a^2d)} + \frac{(-cd)^{2/3} d^2 \log\left(x^2 + x\left(-\frac{1}{3}\right)^{1/3} + \left(-\frac{1}{3}\right)^{2/3}\right)}{6(bc^3 - ac^4d)} - \frac{28b^2c^2a^6 + 28abcd^6 + 28a^2d^2x^6 - 7abc^2x^3 - 7a^2cdx^3 + 4a^2c^2}{28a^3c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")

[Out] $\frac{1}{3}b^4(-a/b)^{2/3}\log(\text{abs}(x - (-a/b)^{1/3}))/ (a^4bc - a^5d) - \frac{1}{3}d^4(-c/d)^{2/3}\log(\text{abs}(x - (-c/d)^{1/3}))/ (b^4c^5 - a^4d) + (-ab^2)^{2/3}b^2\arctan(1/3\sqrt{3}(2x + (-a/b)^{1/3}))/ (-a/b)^{1/3} / (\sqrt{3}a^4bc - \sqrt{3}a^5d) - (-cd^2)^{2/3}d^2\arctan(1/3\sqrt{3}(2x + (-c/d)^{1/3}))/ (-c/d)^{1/3} / (\sqrt{3}b^4c^5 - \sqrt{3}a^4d) - \frac{1}{6}(-ab^2)^{2/3}b^2\log(x^2 + x(-a/b)^{1/3} + (-a/b)^{2/3}) / (a^4bc - a^5d) + \frac{1}{6}(-cd^2)^{2/3}d^2\log(x^2 + x(-c/d)^{1/3} + (-c/d)^{2/3}) / (b^4c^5 - a^4d) - \frac{1}{8}(28b^2c^2x^6 + 28ab^2cdx^6 + 28a^2d^2x^6 - 7ab^2c^2x^3 - 7a^2cd^2x^3 + 4a^2c^2) / (a^3c^3x^7)$

maple [A] time = 0.05, size = 334, normalized size = 0.95

$$\frac{\sqrt{3} b^3 \arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3} - \frac{b^3 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + b^3 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3} + \frac{\sqrt{3} d^3 \arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{3}\right)}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}c^3} + \frac{d^3 \ln\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right) + d^3 \ln\left(x^2 - \left(\frac{c}{d}\right)^{\frac{1}{3}}x + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}c^3} - \frac{d^2}{a^2c^2x} - \frac{bd}{a^2c^2x} - \frac{b^2}{a^2cx} + \frac{d}{4a^2c^2x^4} + \frac{b}{4a^2cx^4} - \frac{1}{7acx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^8/(b*x^3+a)/(d*x^3+c),x)

[Out] $-\frac{1}{3}b^3/a^3/(ad-bc)/(a/b)^{1/3}\ln(x+(a/b)^{1/3}) + \frac{1}{6}b^3/a^3/(ad-bc)/(a/b)^{1/3}\ln(x^2-(a/b)^{1/3}x+(a/b)^{2/3}) + \frac{1}{3}b^3/a^3/(ad-bc)*3^{1/2}/(a/b)^{1/3}\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}x-1)) + \frac{1}{3}d^3/c^3/(ad-bc)/(c/d)^{1/3}\ln(x+(c/d)^{1/3}) - \frac{1}{6}d^3/c^3/(ad-bc)/(c/d)^{1/3}\ln(x^2-(c/d)^{1/3}x+(c/d)^{2/3}) - \frac{1}{3}d^3/c^3/(ad-bc)*3^{1/2}/(c/d)^{1/3}\arctan(1/3*3^{1/2}*(2/(c/d)^{1/3}x-1)) - \frac{1}{7}a/c/x^7 + \frac{1}{4}a/c^2/x^4d + \frac{1}{4}a^2/c/x^4b - \frac{1}{a/c^3/x*d^2} - \frac{1}{a^2/c^2/x*b*d} - \frac{1}{a^3/c/x*b^2}$

maxima [A] time = 1.16, size = 376, normalized size = 1.07

$$\frac{\sqrt{3} b^3 \arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(a^3bc - a^4d)\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + \frac{\sqrt{3} d^3 \arctan\left(\frac{\sqrt{3}\left(\frac{2x-1}{3}\right)}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3(bc^4 - ac^3d)\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right) - \frac{b^3 \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + d^3 \log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(a^3bc\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^4d\left(\frac{a}{b}\right)^{\frac{1}{3}})} + \frac{b^3 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + d^3 \log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(a^3bc\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^4d\left(\frac{a}{b}\right)^{\frac{1}{3}})} - \frac{d^3 \log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(bc^4\left(\frac{c}{d}\right)^{\frac{1}{3}} - ac^3d\left(\frac{c}{d}\right)^{\frac{1}{3}})} - \frac{28(b^2c^2 + abcd + a^2d^2)x^6 + 4a^2c^2 - 7(abc^2 + a^2cd)x^3}{28a^3c^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")

[Out] $-\frac{1}{3}\sqrt{3}b^3\arctan(1/3\sqrt{3}(2x - (a/b)^{1/3}))/ (a/b)^{1/3} / ((a^3bc - a^4d)*(a/b)^{1/3}) + \frac{1}{3}\sqrt{3}d^3\arctan(1/3\sqrt{3}(2x - (c/d)^{1/3}))/ (c/d)^{1/3} / ((b^4c^4 - a^4c^3d)*(c/d)^{1/3}) - \frac{1}{6}b^3\log(x^2 - x(a/b)^{1/3} + (a/b)^{2/3}) / (a^3bc*(a/b)^{1/3} - a^4d*(a/b)^{1/3}) + \frac{1}{6}d^3\log(x^2 - x(c/d)^{1/3} + (c/d)^{2/3}) / (b^4c^4*(c/d)^{1/3} - a^4c^3d*(c/d)^{1/3}) + \frac{1}{3}b^3\log(x + (a/b)^{1/3}) / (a^3bc*(a/b)^{1/3} - a^4d*(a/b)^{1/3}) - \frac{1}{3}d^3\log(x + (c/d)^{1/3}) / (b^4c^4*(c/d)^{1/3} - a^4c^3d*(c/d)^{1/3})$

$$\frac{1}{3}) - \frac{1}{28}(28*(b^2*c^2 + a*b*c*d + a^2*d^2)*x^6 + 4*a^2*c^2 - 7*(a*b*c^2 + a^2*c*d)*x^3)/(a^3*c^3*x^7)$$

mupad [B] time = 11.91, size = 1814, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8*(a + b*x^3)*(c + d*x^3)),x)

[Out] $\log\left(\left(-b^{10}/(a^{10}(a*d - b*c)^3)\right)^{2/3} * \left(\left(27*a^{21}*b^3*c^{21}*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + 27*a^{28}*b^3*c^{28}*d^3*(a*d + b*c)*(a*d - b*c)^4 * (-b^{10}/(a^{10}(a*d - b*c)^3)\right)^{2/3} * (-b^{10}/(a^{10}(a*d - b*c)^3)\right)^{1/3}\right) / 3 - 9*a^{19}*b^{14}*c^{29}*d^4 + 9*a^{20}*b^{13}*c^{28}*d^5 + 9*a^{28}*b^5*c^{20}*d^{13} - 9*a^{29}*b^4*c^{19}*d^{14}) / 9 - a^{19}*b^{11}*c^{19}*d^{11}*x*(a*d + b*c)) * (-b^{10}/(27*a^{13}*d^3 - 27*a^{10}*b^3*c^3 + 81*a^{11}*b^2*c^2*d - 81*a^{12}*b*c*d^2))^{1/3} + \log\left(\left(d^{10}/(c^{10}(a*d - b*c)^3)\right)^{2/3} * \left(\left(27*a^{21}*b^3*c^{21}*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + 27*a^{28}*b^3*c^{28}*d^3*(a*d + b*c)*(a*d - b*c)^4 * (d^{10}/(c^{10}(a*d - b*c)^3)\right)^{2/3} * (d^{10}/(c^{10}(a*d - b*c)^3)\right)^{1/3}\right) / 3 - 9*a^{19}*b^{14}*c^{29}*d^4 + 9*a^{20}*b^{13}*c^{28}*d^5 + 9*a^{28}*b^5*c^{20}*d^{13} - 9*a^{29}*b^4*c^{19}*d^{14}) / 9 - a^{19}*b^{11}*c^{19}*d^{11}*x*(a*d + b*c)) * (-d^{10}/(27*b^3*c^{13} - 27*a^3*c^{10}*d^3 + 81*a^2*b*c^{11}*d^2 - 81*a*b^2*c^{12}*d))^{1/3} - (1/(7*a*c) - (x^3*(a*d + b*c))/(4*a^2*c^2) + (x^6*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3))/x^7 - (\log\left(\left(3^{1/2}*1i + 1\right)^2 * (-b^{10}/(a^{10}(a*d - b*c)^3)\right)^{2/3} * \left(\left(3^{1/2}*1i + 1\right) * (27*a^{21}*b^3*c^{21}*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^{28}*b^3*c^{28}*d^3 * (3^{1/2}*1i + 1)^2 * (a*d + b*c)*(a*d - b*c)^4 * (-b^{10}/(a^{10}(a*d - b*c)^3)\right)^{2/3}\right) / 4 * (-b^{10}/(a^{10}(a*d - b*c)^3)\right)^{1/3}\right) / 6 + 9*a^{19}*b^{14}*c^{29}*d^4 - 9*a^{20}*b^{13}*c^{28}*d^5 - 9*a^{28}*b^5*c^{20}*d^{13} + 9*a^{29}*b^4*c^{19}*d^{14}) / 36 + a^{19}*b^{11}*c^{19}*d^{11}*x*(a*d + b*c)) * (-b^{10}/(27*a^{13}*d^3 - 27*a^{10}*b^3*c^3 + 81*a^{11}*b^2*c^2*d - 81*a^{12}*b*c*d^2))^{1/3} * (3^{1/2}*1i + 1) / 2 + (\log\left(\left(3^{1/2}*1i - 1\right)^2 * (-b^{10}/(a^{10}(a*d - b*c)^3)\right)^{2/3} * \left(\left(3^{1/2}*1i - 1\right) * (27*a^{21}*b^3*c^{21}*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^{28}*b^3*c^{28}*d^3 * (3^{1/2}*1i - 1)^2 * (a*d + b*c)*(a*d - b*c)^4 * (-b^{10}/(a^{10}(a*d - b*c)^3)\right)^{2/3}\right) / 4 * (-b^{10}/(a^{10}(a*d - b*c)^3)\right)^{1/3}\right) / 6 - 9*a^{19}*b^{14}*c^{29}*d^4 + 9*a^{20}*b^{13}*c^{28}*d^5 + 9*a^{28}*b^5*c^{20}*d^{13} - 9*a^{29}*b^4*c^{19}*d^{14}) / 36 - a^{19}*b^{11}*c^{19}*d^{11}*x*(a*d + b*c)) * (-b^{10}/(27*a^{13}*d^3 - 27*a^{10}*b^3*c^3 + 81*a^{11}*b^2*c^2*d - 81*a^{12}*b*c*d^2))^{1/3} * (3^{1/2}*1i - 1) / 2 - (\log\left(\left(3^{1/2}*1i + 1\right)^2 * (d^{10}/(c^{10}(a*d - b*c)^3)\right)^{2/3} * \left(\left(3^{1/2}*1i + 1\right) * (27*a^{21}*b^3*c^{21}*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^{28}*b^3*c^{28}*d^3 * (3^{1/2}*1i + 1)^2 * (a*d + b*c)*(a*d - b*c)^4 * (d^{10}/(c^{10}(a*d - b*c)^3)\right)^{2/3}\right) / 4 * (d^{10}/(c^{10}(a*d - b*c)^3)\right)^{1/3}\right) / 6 + 9*a^{19}*b^{14}*c^{29}*d^4 - 9*a^{20}*b^{13}*c^{28}*d^5 - 9*a^{28}*b^5*c^{20}*d^{13} + 9*a^{29}*b^4*c^{19}*d^{14}) / 36 + a^{19}*b^{11}*c^{19}*d^{11}*x*(a*d + b*c)) * (-d^{10}/(27*b^3*c^{13} - 27*a^3*c^{10}*d^3 + 81*a^2*b*c^{11}*d^2 - 81*a*b^2*c^{12}*d))^{1/3} * (3^{1/2}*1i + 1) / 2 + (\log\left(\left(3^{1/2}*1i - 1\right)^2 * (d^{10}/(c^{10}(a*d - b*c)^3)\right)^{2/3} * \left(\left(3^{1/2}*1i - 1\right) * (27*a^{21}*b^3*c^{21}*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^{28}*b^3*c^{28}*d^3 * (3^{1/2}*1i - 1)^2 * (a*d + b*c)*(a*d - b*c)^4 * (-b^{10}/(a^{10}(a*d - b*c)^3)\right)^{2/3}\right) / 4 * (-b^{10}/(a^{10}(a*d - b*c)^3)\right)^{1/3}\right) / 6 - 9*a^{19}*b^{14}*c^{29}*d^4 + 9*a^{20}*b^{13}*c^{28}*d^5 + 9*a^{28}*b^5*c^{20}*d^{13} - 9*a^{29}*b^4*c^{19}*d^{14}) / 36 - a^{19}*b^{11}*c^{19}*d^{11}*x*(a*d + b*c)) * (-d^{10}/(27*b^3*c^{13} - 27*a^3*c^{10}*d^3 + 81*a^2*b*c^{11}*d^2 - 81*a*b^2*c^{12}*d))^{1/3} * (3^{1/2}*1i - 1) / 2$

$$7*a^{21}*b^3*c^{21}*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^{28}*b^3*c^{28} *d^3*(3^{(1/2)}*i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(d^{10}/(c^{10}*(a*d - b*c)^3))^{(2/3)}/4*(d^{10}/(c^{10}*(a*d - b*c)^3))^{(1/3)}/6 - 9*a^{19}*b^{14}*c^{29}*d^4 + 9*a^{20}*b^{13}*c^{28}*d^5 + 9*a^{28}*b^5*c^{20}*d^{13} - 9*a^{29}*b^4*c^{19}*d^{14}))/36 - a^{19}*b^{11}*c^{19}*d^{11}*x*(a*d + b*c))*(-d^{10}/(27*b^3*c^{13} - 27*a^3*c^{10}*d^3 + 81*a^2*b*c^{11}*d^2 - 81*a*b^2*c^{12}*d))^{(1/3)}*(3^{(1/2)}*i - 1))/2$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**8/(b*x**3+a)/(d*x**3+c),x)

[Out] Timed out

$$3.124 \quad \int x^m (a + bx^3)^5 (A + Bx^3) dx$$

Optimal. Leaf size=148

$$\frac{a^5 Ax^{m+1}}{m+1} + \frac{a^4 x^{m+4}(aB + 5Ab)}{m+4} + \frac{5a^3 bx^{m+7}(aB + 2Ab)}{m+7} + \frac{10a^2 b^2 x^{m+10}(aB + Ab)}{m+10} + \frac{b^4 x^{m+16}(5aB + Ab)}{m+16} + \frac{5ab^3 x^{m+19}}{m+19}$$

Rubi [A] time = 0.10, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{10a^2 b^2 x^{m+10}(aB + Ab)}{m+10} + \frac{a^4 x^{m+4}(aB + 5Ab)}{m+4} + \frac{5a^3 bx^{m+7}(aB + 2Ab)}{m+7} + \frac{a^5 Ax^{m+1}}{m+1} + \frac{5ab^3 x^{m+13}(2aB + Ab)}{m+13} + \frac{b^4 x^{m+16}(5aB + Ab)}{m+16} + \frac{b^5 Bx^{m+19}}{m+19}$$

Antiderivative was successfully verified.

[In] Int[x^m*(a + b*x^3)^5*(A + B*x^3), x]

[Out] (a^5*A*x^(1 + m))/(1 + m) + (a^4*(5*A*b + a*B)*x^(4 + m))/(4 + m) + (5*a^3*b*(2*A*b + a*B)*x^(7 + m))/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^(10 + m))/(10 + m) + (5*a*b^3*(A*b + 2*a*B)*x^(13 + m))/(13 + m) + (b^4*(A*b + 5*a*B)*x^(16 + m))/(16 + m) + (b^5*B*x^(19 + m))/(19 + m)

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^m (a + bx^3)^5 (A + Bx^3) dx &= \int (a^5 Ax^m + a^4(5Ab + aB)x^{3+m} + 5a^3b(2Ab + aB)x^{6+m} + 10a^2b^2(Ab + aB)x^9 + 5ab^3(2aB + Ab)x^{12+m} + b^4x^{15}(5aB + Ab) + b^5Bx^{18}) x^m dx \\ &= \frac{a^5 Ax^{1+m}}{1+m} + \frac{a^4(5Ab + aB)x^{4+m}}{4+m} + \frac{5a^3b(2Ab + aB)x^{7+m}}{7+m} + \frac{10a^2b^2(Ab + aB)x^{10+m}}{10+m} + \frac{5ab^3(2aB + Ab)x^{13+m}}{13+m} + \frac{b^4x^{16}(5aB + Ab)}{16+m} + \frac{b^5Bx^{19+m}}{19+m} \end{aligned}$$

Mathematica [A] time = 0.26, size = 137, normalized size = 0.93

$$x^{m+1} \left(\frac{a^5 A}{m+1} + \frac{a^4 x^3 (aB + 5Ab)}{m+4} + \frac{5a^3 bx^6 (aB + 2Ab)}{m+7} + \frac{10a^2 b^2 x^9 (aB + Ab)}{m+10} + \frac{b^4 x^{15} (5aB + Ab)}{m+16} + \frac{5ab^3 x^{12} (2aB + Ab)}{m+13} + \frac{b^5 Bx^{18}}{m+19} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m*(a + b*x^3)^5*(A + B*x^3),x]

[Out] $x^{(1+m)} \left(\frac{a^5 A}{(1+m)} + \frac{a^4 (5A*b + a*B) x^3}{(4+m)} + \frac{5a^3 b (2A*b + a*B) x^6}{(7+m)} + \frac{10a^2 b^2 (A*b + a*B) x^9}{(10+m)} + \frac{5a b^3 (A*b + 2a*B) x^{12}}{(13+m)} + \frac{b^4 (A*b + 5a*B) x^{15}}{(16+m)} + \frac{b^5 B x^{18}}{(19+m)} \right)$

IntegrateAlgebraic [F] time = 0.35, size = 0, normalized size = 0.00

$$\int x^m (a + bx^3)^5 (A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^m*(a + b*x^3)^5*(A + B*x^3),x]

[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^3)^5*(A + B*x^3), x]

fricas [B] time = 0.91, size = 851, normalized size = 5.75

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")

[Out] $((B*b^5*m^6 + 51*B*b^5*m^5 + 1005*B*b^5*m^4 + 9605*B*b^5*m^3 + 45474*B*b^5*m^2 + 95064*B*b^5*m + 58240*B*b^5)*x^{19} + ((5*B*a*b^4 + A*b^5)*m^6 + 345800*B*a*b^4 + 69160*A*b^5 + 54*(5*B*a*b^4 + A*b^5)*m^5 + 1110*(5*B*a*b^4 + A*b^5)*m^4 + 10940*(5*B*a*b^4 + A*b^5)*m^3 + 52929*(5*B*a*b^4 + A*b^5)*m^2 + 12206*(5*B*a*b^4 + A*b^5)*m)*x^{16} + 5*((2*B*a^2*b^3 + A*a*b^4)*m^6 + 170240*B*a^2*b^3 + 85120*A*a*b^4 + 57*(2*B*a^2*b^3 + A*a*b^4)*m^5 + 1233*(2*B*a^2*b^3 + A*a*b^4)*m^4 + 12671*(2*B*a^2*b^3 + A*a*b^4)*m^3 + 63246*(2*B*a^2*b^3 + A*a*b^4)*m^2 + 136872*(2*B*a^2*b^3 + A*a*b^4)*m)*x^{13} + 10*((B*a^3*b^2 + A*a^2*b^3)*m^6 + 110656*B*a^3*b^2 + 110656*A*a^2*b^3 + 60*(B*a^3*b^2 + A*a^2*b^3)*m^5 + 1374*(B*a^3*b^2 + A*a^2*b^3)*m^4 + 14960*(B*a^3*b^2 + A*a^2*b^3)*m^3 + 78369*(B*a^3*b^2 + A*a^2*b^3)*m^2 + 175380*(B*a^3*b^2 + A*a^2*b^3)*m)*x^{10} + 5*((B*a^4*b + 2*A*a^3*b^2)*m^6 + 158080*B*a^4*b + 316160*A*a^3*b^2 + 63*(B*a^4*b + 2*A*a^3*b^2)*m^5 + 1533*(B*a^4*b + 2*A*a^3*b^2)*m^4 + 17969*(B*a^4*b + 2*A*a^3*b^2)*m^3 + 102186*(B*a^4*b + 2*A*a^3*b^2)*m^2 + 243768*(B*a^4*b + 2*A*a^3*b^2)*m)*x^7 + ((B*a^5 + 5*A*a^4*b)*m^6 + 276640*B*a^5 + 1383200*A*a^4*b + 66*(B*a^5 + 5*A*a^4*b)*m^5 + 1710*(B*a^5 + 5*A*a^4*b)*m^4 + 21860*(B*a^5 + 5*A*a^4*b)*m^3 + 140529*(B*a^5 + 5*A*a^4*b)*m^2 + 396954*(B*a^5 + 5*A*a^4*b)*m)*x^4 + (A*a^5*m^6 + 69*A*a^5*m^5 + 1905*A*a^5*m^4 + 26795*A*a^5*m^3 + 201174*A*a^5*m^2 + 757896*A*a^5*m + 1106560*A*a^5)*x)*x^m/(m^7 + 70*m^6 + 1974*m^5 + 28700*m^4 + 227969*m^3 + 959070*m^2 + 1864456*m + 1106560)$

giac [B] time = 0.29, size = 1331, normalized size = 8.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x^m(bx^3+a)^5(Bx^3+A)$, x, algorithm="giac")

[Out] $(Bb^5m^6x^{19x^m} + 51Bb^5m^5x^{19x^m} + 1005Bb^5m^4x^{19x^m} + 5Ba^b^4m^6x^{16x^m} + A^b^5m^6x^{16x^m} + 9605Bb^5m^3x^{19x^m} + 270Ba^b^4m^5x^{16x^m} + 54A^b^5m^5x^{16x^m} + 45474Bb^5m^2x^{19x^m} + 5550Ba^b^4m^4x^{16x^m} + 1110A^b^5m^4x^{16x^m} + 95064Bb^5m^1x^{19x^m} + 10Ba^2b^3m^6x^{13x^m} + 5A^2b^3m^6x^{13x^m} + 54700Ba^2b^3m^5x^{16x^m} + 10940A^2b^3m^5x^{16x^m} + 58240Bb^5m^4x^{19x^m} + 570Ba^2b^3m^5x^{13x^m} + 285A^2b^3m^5x^{13x^m} + 264645Ba^2b^3m^4x^{16x^m} + 52929A^2b^3m^4x^{16x^m} + 12330Ba^2b^3m^4x^{13x^m} + 6165A^2b^3m^4x^{13x^m} + 561030Ba^2b^3m^3x^{16x^m} + 112206A^2b^3m^3x^{16x^m} + 10Ba^3b^2m^6x^{10x^m} + 10A^3b^2m^6x^{10x^m} + 126710Ba^3b^2m^3x^{13x^m} + 63355A^3b^2m^3x^{13x^m} + 345800Ba^3b^2m^2x^{16x^m} + 69160A^3b^2m^2x^{16x^m} + 600Ba^3b^2m^2x^{10x^m} + 600A^3b^2m^2x^{10x^m} + 632460Ba^3b^2m^2x^{13x^m} + 316230A^3b^2m^2x^{13x^m} + 13740Ba^3b^2m^4x^{10x^m} + 13740A^3b^2m^4x^{10x^m} + 1368720Ba^3b^2m^3x^{13x^m} + 684360A^3b^2m^3x^{13x^m} + 5Ba^4b^1m^6x^7x^m + 10A^4b^1m^6x^7x^m + 149600Ba^4b^1m^3x^{10x^m} + 149600A^4b^1m^3x^{10x^m} + 851200Ba^4b^1m^2x^{13x^m} + 425600A^4b^1m^2x^{13x^m} + 315Ba^4b^1m^5x^7x^m + 630A^4b^1m^5x^7x^m + 783690Ba^4b^1m^2x^{10x^m} + 783690A^4b^1m^2x^{10x^m} + 7665Ba^4b^1m^4x^7x^m + 15330A^4b^1m^4x^7x^m + 1753800Ba^4b^1m^3x^{10x^m} + 1753800A^4b^1m^3x^{10x^m} + Ba^5m^6x^4x^m + 5A^5m^6x^4x^m + 89845Ba^4b^1m^3x^7x^m + 179690A^4b^1m^3x^7x^m + 1106560Ba^4b^1m^2x^{10x^m} + 1106560A^4b^1m^2x^{10x^m} + 66Ba^5m^5x^4x^m + 330A^5m^5x^4x^m + 510930Ba^4b^1m^2x^7x^m + 1021860A^4b^1m^2x^7x^m + 1710Ba^5m^4x^4x^m + 8550A^5m^4x^4x^m + 1218840Ba^4b^1m^1x^7x^m + 2437680A^4b^1m^1x^7x^m + A^5m^6x^4x^m + 21860Ba^5m^3x^4x^m + 109300A^4b^1m^3x^4x^m + 790400Ba^4b^1m^1x^7x^m + 1580800A^4b^1m^3x^7x^m + 69A^5m^5x^4x^m + 140529Ba^5m^2x^4x^m + 702645A^4b^1m^2x^4x^m + 1905A^5m^4x^4x^m + 396954Ba^5m^1x^4x^m + 1984770A^4b^1m^1x^4x^m + 26795A^5m^3x^4x^m + 276640Ba^5m^1x^4x^m + 1383200A^4b^1m^1x^4x^m + 201174A^5m^2x^4x^m + 757896A^5m^1x^4x^m + 1106560A^4b^1m^1x^4x^m)/(m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560)$

maple [B] time = 0.05, size = 1078, normalized size = 7.28

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^m(bx^3+a)^5(Bx^3+A), x)$

[Out] $x^{(m+1)}(B^5b^5m^6x^{18}+51B^5b^5m^5x^{18}+1005B^5b^5m^4x^{18}+A^5b^5m^6x^{15}+5B^5a^5b^4m^6x^{15}+9605B^5b^5m^3x^{18}+54A^5b^5m^5x^{15}+270B^5a^5b^4m^5x^{15}+45474B^5b^5m^2x^{18}+1110A^5b^5m^4x^{15}+5550B^5a^5b^4m^4x^{15}+95064B^5b^5m^2x^{18}+5A^5a^5b^4m^6x^{12}+10940A^5b^5m^3x^{15}+10B^5a^5b^3m^6x^{12}+54700B^5a^5b^4m^3x^{15}+58240B^5b^5m^5x^{18}+285A^5a^5b^4m^5x^{12}+52929A^5b^5m^2x^{15}+570B^5a^5b^3m^5x^{12}+264645B^5a^5b^4m^2x^{15}+6165A^5a^5b^4m^4x^{12}+112206A^5b^5m^2x^{15}+12330B^5a^5b^3m^4x^{12}+561030B^5a^5b^4m^2x^{15}+10A^5a^5b^3m^6x^9+63355A^5a^5b^4m^3x^{12}+69160A^5b^5m^5x^{15}+10B^5a^5b^3m^2x^9+126710B^5a^5b^3m^3x^{12}+345800B^5a^5b^4m^2x^{15}+600A^5a^5b^3m^5x^9+316230A^5a^5b^4m^2x^{12}+600B^5a^5b^3m^2x^9+632460B^5a^5b^3m^2x^{12}+13740A^5a^5b^3m^4x^9+684360A^5a^5b^4m^2x^{12}+13740B^5a^5b^3m^4x^9+1368720B^5a^5b^3m^3x^{12}+10A^5a^5b^3m^6x^6+149600A^5a^5b^3m^3x^9+425600A^5a^5b^4x^{12}+5B^5a^5b^4m^6x^6+149600B^5a^5b^3m^3x^9+851200B^5a^5b^3x^{12}+630A^5a^5b^3m^2x^6+783690A^5a^5b^3m^2x^9+315B^5a^5b^4m^5x^6+783690B^5a^5b^3m^2x^9+15330A^5a^5b^3m^4x^6+1753800A^5a^5b^3m^3x^9+7665B^5a^5b^4m^4x^6+1753800B^5a^5b^3m^2x^9+5A^5a^5b^4m^6x^3+179690A^5a^5b^3m^3x^6+1106560A^5a^5b^3m^3x^9+B^5a^5m^6x^3+89845B^5a^5b^4m^3x^6+1106560B^5a^5b^3m^2x^9+330A^5a^5b^4m^5x^3+1021860A^5a^5b^3m^2x^6+66B^5a^5m^5x^3+510930B^5a^5b^4m^2x^6+8550A^5a^5b^4m^4x^3+2437680A^5a^5b^3m^2x^6+1710B^5a^5m^4x^3+1218840B^5a^5b^4m^3x^6+A^5a^5m^6+109300A^5a^5b^4m^3x^3+1580800A^5a^5b^3m^2x^6+21860B^5a^5m^3x^3+790400B^5a^5b^4m^2x^6+69A^5a^5m^5+702645A^5a^5b^4m^2x^3+140529B^5a^5m^2x^3+1905A^5a^5m^4+1984770A^5a^5b^4m^2x^3+396954B^5a^5m^2x^3+26795A^5a^5m^3+1383200A^5a^5b^4m^2x^3+276640B^5a^5m^2x^3+201174A^5a^5m^2+757896A^5a^5m+1106560A^5a^5)/((m+1)((m+4)((m+7)((m+10)((m+13)((m+16)((m+19))$

maxima [A] time = 0.52, size = 205, normalized size = 1.39

$$\frac{Bb^5x^{m+19}}{m+19} + \frac{5Bab^4x^{m+16}}{m+16} + \frac{Ab^5x^{m+16}}{m+16} + \frac{10Ba^2b^3x^{m+13}}{m+13} + \frac{5Aab^4x^{m+13}}{m+13} + \frac{10Ba^3b^2x^{m+10}}{m+10} + \frac{10Aa^2b^3x^{m+10}}{m+10} + \frac{5Ba^4bx^{m+7}}{m+7} + \frac{10Aa^3b^2x^{m+7}}{m+7} + \frac{Ba^5x^{m+4}}{m+4} + \frac{5Aa^4bx^{m+4}}{m+4} + \frac{Aa^5x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^m(bx^3+a)^5(Bx^3+A), x, \text{algorithm}="maxima")$

[Out] $B^5b^5x^{(m+19)}/(m+19) + 5B^5a^5b^4x^{(m+16)}/(m+16) + A^5b^5x^{(m+16)}/(m+16) + 10B^5a^5b^3x^{(m+13)}/(m+13) + 5A^5a^5b^4x^{(m+13)}/(m+13) + 10B^5a^5b^3m^2x^{(m+10)}/(m+10) + 10A^5a^5b^3x^{(m+10)}/(m+10) + 5B^5a^5b^4m^2x^{(m+7)}/(m+7) + 10A^5a^5b^3m^2x^{(m+7)}/(m+7) + B^5a^5m^2x^{(m+4)}/(m+4) + 5A^5a^5b^4m^2x^{(m+4)}/(m+4) + A^5a^5m^2x^{(m+1)}/(m+1)$

mupad [B] time = 3.21, size = 559, normalized size = 3.78

$$\frac{Bb^5x^{m+19}}{m+19} + \frac{5Bab^4x^{m+16}}{m+16} + \frac{Ab^5x^{m+16}}{m+16} + \frac{10Ba^2b^3x^{m+13}}{m+13} + \frac{5Aab^4x^{m+13}}{m+13} + \frac{10Ba^3b^2x^{m+10}}{m+10} + \frac{10Aa^2b^3x^{m+10}}{m+10} + \frac{5Ba^4bx^{m+7}}{m+7} + \frac{10Aa^3b^2x^{m+7}}{m+7} + \frac{Ba^5x^{m+4}}{m+4} + \frac{5Aa^4bx^{m+4}}{m+4} + \frac{Aa^5x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*(A + B*x^3)*(a + b*x^3)^5,x)`

[Out]
$$\frac{(B*b^5*x^m*x^{19}*(95064*m + 45474*m^2 + 9605*m^3 + 1005*m^4 + 51*m^5 + m^6 + 58240))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (a^4*x^m*x^4*(5*A*b + B*a)*(396954*m + 140529*m^2 + 21860*m^3 + 1710*m^4 + 66*m^5 + m^6 + 276640))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (b^4*x^m*x^{16}*(A*b + 5*B*a)*(112206*m + 52929*m^2 + 10940*m^3 + 1110*m^4 + 54*m^5 + m^6 + 69160))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (A*a^5*x*x^m*(757896*m + 201174*m^2 + 26795*m^3 + 1905*m^4 + 69*m^5 + m^6 + 1106560))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (10*a^2*b^2*x^m*x^{10}*(A*b + B*a)*(175380*m + 78369*m^2 + 14960*m^3 + 1374*m^4 + 60*m^5 + m^6 + 1106560))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (5*a*b^3*x^m*x^{13}*(A*b + 2*B*a)*(136872*m + 63246*m^2 + 12671*m^3 + 1233*m^4 + 57*m^5 + m^6 + 85120))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (5*a^3*b*x^m*x^7*(2*A*b + B*a)*(243768*m + 102186*m^2 + 17969*m^3 + 1533*m^4 + 63*m^5 + m^6 + 158080))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560)$$

sympy [A] time = 23.94, size = 5418, normalized size = 36.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*(b*x**3+a)**5*(B*x**3+A),x)`

[Out]
$$\text{Piecewise}\left(\left(-A*a**5/(18*x**18) - A*a**4*b/(3*x**15) - 5*A*a**3*b**2/(6*x**12) - 10*A*a**2*b**3/(9*x**9) - 5*A*a*b**4/(6*x**6) - A*b**5/(3*x**3) - B*a**5/(15*x**15) - 5*B*a**4*b/(12*x**12) - 10*B*a**3*b**2/(9*x**9) - 5*B*a**2*b**3/(3*x**6) - 5*B*a*b**4/(3*x**3) + B*b**5*\log(x), \text{Eq}(m, -19)\right), \left(-A*a**5/(15*x**15) - 5*A*a**4*b/(12*x**12) - 10*A*a**3*b**2/(9*x**9) - 5*A*a**2*b**3/(3*x**6) - 5*A*a*b**4/(3*x**3) + A*b**5*\log(x) - B*a**5/(12*x**12) - 5*B*a**4*b/(9*x**9) - 5*B*a**3*b**2/(3*x**6) - 10*B*a**2*b**3/(3*x**3) + 5*B*a*b**4*\log(x) + B*b**5*x**3/3, \text{Eq}(m, -16)\right), \left(-A*a**5/(12*x**12) - 5*A*a**4*b/(9*x**9) - 5*A*a**3*b**2/(3*x**6) - 10*A*a**2*b**3/(3*x**3) + 5*A*a*b**4*\log(x) + A*b**5*x**3/3 - B*a**5/(9*x**9) - 5*B*a**4*b/(6*x**6) - 10*B*a**3*b**2/(3*x**3) + 10*B*a**2*b**3*\log(x) + 5*B*a*b**4*x**3/3 + B*b**5*x**6/6, \text{Eq}(m, -13)\right), \left(-A*a**5/(9*x**9) - 5*A*a**4*b/(6*x**6) - 10*A*a**3*b**2/(3*x**3) + 10*A*a**2*b**3*\log(x) + 5*A*a*b**4*x**3/3 + A*b**5*x**6/6 - B*a**5/(6*x**6) - 5*B*a**4*b/(3*x**3) + 10*B*a**3*b**2*\log(x) + 10*B*a**2*b**3*x**3/3 + 5*B*a*b**4*x**6/6 + B*b**5*x**9/9, \text{Eq}(m, -10)\right), \left(-A*a**5/(6*x**6) - 5*A*a**4*b/(3*x**3) + 10*A*a**3*b**2*\log(x) + 10*A*a**2*b**3*x**3/3 + 5*A*a*b**4*x**6/6 + A*b**5*x**9/9 - B*a**5/(3*x**3) + 5*B*a**4*b*\log(x) + 10*B*a**3*b**2*x**3/3 + 5*B*a**2*b**3*x**6/3 + 5*B*a*b**4*x**9/9 + B*b**5*x**12/12, \text{Eq}(m, -7)\right)$$

$m, -7)), (-A^{*5}/(3x^{*3}) + 5A^{*4}b \log(x) + 10A^{*3}b^2x^{*3}/3 + 5A^{*2}b^3x^{*6}/3 + 5A^*ab^4x^{*9}/9 + Ab^5x^{*12}/12 + B^{*5} \log(x) + 5B^{*4}bx^{*3}/3 + 5B^{*3}b^2x^{*6}/3 + 10B^{*2}b^3x^{*9}/9 + 5B^*ab^4x^{*12}/12 + Bb^5x^{*15}/15, \text{Eq}(m, -4)), (A^{*5} \log(x) + 5A^{*4}bx^{*3}/3 + 5A^{*3}b^2x^{*6}/3 + 10A^{*2}b^3x^{*9}/9 + 5A^*ab^4x^{*12}/12 + Ab^5x^{*15}/15 + B^{*5}x^{*3}/3 + 5B^{*4}bx^{*6}/6 + 10B^{*3}b^2x^{*9}/9 + 5B^{*2}b^3x^{*12}/6 + B^*ab^4x^{*15}/3 + Bb^5x^{*18}/18, \text{Eq}(m, -1)), (A^{*5}m^{*6}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 69A^{*5}m^{*5}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 1905A^{*5}m^{*4}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 26795A^{*5}m^{*3}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 201174A^{*5}m^{*2}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 757896A^{*5}m^{*1}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 1106560A^{*5}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 5A^{*4}b^{*6}x^{*4}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 330A^{*4}b^{*5}x^{*4}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 8550A^{*4}b^{*4}x^{*4}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 109300A^{*4}b^{*3}x^{*4}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 702645A^{*4}b^{*2}x^{*4}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 1984770A^{*4}b^{*1}x^{*4}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 1383200A^{*4}b^{*0}x^{*4}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 10A^{*3}b^{*2}m^{*6}x^{*7}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 630A^{*3}b^{*2}m^{*5}x^{*7}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 15330A^{*3}b^{*2}m^{*4}x^{*7}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 179690A^{*3}b^{*2}m^{*3}x^{*7}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 1021860A^{*3}b^{*2}m^{*2}x^{*7}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 2437680A^{*3}b^{*2}m^{*1}x^{*7}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 1580800A^{*3}b^{*2}x^{*7}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 10A^{*2}b^{*3}m^{*6}x^{*10}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 600A^{*2}b^{*3}m^{*5}x^{*10}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560) + 13740A^{*2}b^{*3}m^{*4}x^{*10}x^{*x}m/(m^{*7} + 70m^{*6} + 1974m^{*5} + 28700m^{*4} + 227969m^{*3} + 959070m^{*2} + 1864456m + 1106560)$

$$\begin{aligned}
& 7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 149600Aa^2b^3m^3x^{10}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 783690 \\
& *Aa^2b^3m^2x^{10}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 1753800Aa^2b^3mx^{10} \\
& *x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 1106560Aa^2b^3x^{10}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + \\
& 5Aab^4m^6x^{13}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 285Aab^4m^5x^{13}x^m / \\
& (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 6165Aab^4m^4x^{13}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 63355A \\
& *ab^4m^3x^{13}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 316230Aab^4m^2x^{13}x^m / \\
& (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 684360Aab^4mx^{13}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 425600A \\
& *ab^4x^{13}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + Ab^5m^6x^{16}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + \\
& 54Ab^5m^5x^{16}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 1110Ab^5m^4x^{16}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + \\
& 10940Ab^5m^3x^{16}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 52929 \\
& *Ab^5m^2x^{16}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 112206Ab^5mx^{16}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + \\
& 69160Ab^5x^{16}x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + B^5m^6x^4x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + \\
& 66B^5m^5x^4x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 1710B^5 \\
& *m^4x^4x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 21860B^5m^3x^4x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + \\
& 140529B^5m^2x^4x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 396954B^5mx^4x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + \\
& 276640B^5x^4x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 5B^5 \\
& *ab^6m^7x^7x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 315B^5ab^6m^5x^7x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 315B^5ab^6m^5x^7x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560) + 315B^5ab^6m^5x^7x^m / (m^7 + 70m^6 + 1974m^5 + 28700m^4 + 227969m^3 + 959070m^2 + 1864456m + 1106560)
\end{aligned}$$

+ 1106560) + 7665*B*a**4*b*m**4*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 89845*B*a**4*b*m**3*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 510930*B*a**4*b*m**2*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1218840*B*a**4*b*m*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 790400*B*a**4*b*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 10*B*a**3*b**2*m**6*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 600*B*a**3*b**2*m**5*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 13740*B*a**3*b**2*m**4*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 149600*B*a**3*b**2*m**3*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 783690*B*a**3*b**2*m**2*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1753800*B*a**3*b**2*m*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1106560*B*a**3*b**2*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 10*B*a**2*b**3*m**6*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 570*B*a**2*b**3*m**5*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 12330*B*a**2*b**3*m**4*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 126710*B*a**2*b**3*m**3*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 632460*B*a**2*b**3*m**2*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1368720*B*a**2*b**3*m*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 851200*B*a**2*b**3*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 5*B*a*b**4*m**6*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 270*B*a*b**4*m**5*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 5550*B*a*b**4*m**4*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 54700*B*a*b**4*m**3*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 264645*B*a*b**4*m**2*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 561030*B*a*b**4*m*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 345800*B*a*b**4*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + B*b**5*m**6*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1

```

106560) + 51*B*b**5*m**5*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**
4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1005*B*b**5*m**4*x**
19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**
2 + 1864456*m + 1106560) + 9605*B*b**5*m**3*x**19*x**m/(m**7 + 70*m**6 + 19
74*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 4
5474*B*b**5*m**2*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 2279
69*m**3 + 959070*m**2 + 1864456*m + 1106560) + 95064*B*b**5*m*x**19*x**m/(m
**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 186445
6*m + 1106560) + 58240*B*b**5*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 2870
0*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560), True))

```

$$3.125 \quad \int x^m (a + bx^3)^2 (A + Bx^3) dx$$

Optimal. Leaf size=71

$$\frac{a^2 Ax^{m+1}}{m+1} + \frac{ax^{m+4}(aB + 2Ab)}{m+4} + \frac{bx^{m+7}(2aB + Ab)}{m+7} + \frac{b^2 Bx^{m+10}}{m+10}$$

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{a^2 Ax^{m+1}}{m+1} + \frac{ax^{m+4}(aB + 2Ab)}{m+4} + \frac{bx^{m+7}(2aB + Ab)}{m+7} + \frac{b^2 Bx^{m+10}}{m+10}$$

Antiderivative was successfully verified.

[In] Int[x^m*(a + b*x^3)^2*(A + B*x^3), x]

[Out] (a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(4 + m))/(4 + m) + (b*(A*b + 2*a*B)*x^(7 + m))/(7 + m) + (b^2*B*x^(10 + m))/(10 + m)

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^m (a + bx^3)^2 (A + Bx^3) dx &= \int (a^2 Ax^m + a(2Ab + aB)x^{3+m} + b(Ab + 2aB)x^{6+m} + b^2 Bx^{9+m}) dx \\ &= \frac{a^2 Ax^{1+m}}{1+m} + \frac{a(2Ab + aB)x^{4+m}}{4+m} + \frac{b(Ab + 2aB)x^{7+m}}{7+m} + \frac{b^2 Bx^{10+m}}{10+m} \end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.93

$$x^{m+1} \left(\frac{a^2 A}{m+1} + \frac{bx^6(2aB + Ab)}{m+7} + \frac{ax^3(aB + 2Ab)}{m+4} + \frac{b^2 Bx^9}{m+10} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m*(a + b*x^3)^2*(A + B*x^3), x]

[Out] $x^{(1+m)} \cdot ((a^2 \cdot A) / (1+m) + (a \cdot (2 \cdot A \cdot b + a \cdot B) \cdot x^3) / (4+m) + (b \cdot (A \cdot b + 2 \cdot a \cdot B) \cdot x^6) / (7+m) + (b^2 \cdot B \cdot x^9) / (10+m))$

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^m (a + bx^3)^2 (A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^m*(a + b*x^3)^2*(A + B*x^3), x]

[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^3)^2*(A + B*x^3), x]

fricas [B] time = 0.89, size = 215, normalized size = 3.03

$$\frac{((B^2 m^3 + 12 B b^2 m^2 + 39 B^2 m + 28 B^2) x^{10} + ((2 B a b + A b^2) m^3 + 80 B a b + 40 A b^2 + 15 (2 B a b + A b^2) m^2 + 54 (2 B a b + A b^2) m) x^7 + ((B a^2 + 2 A a b) m^3 + 70 B a^2 + 140 A a b + 18 (B a^2 + 2 A a b) m^2 + 87 (B a^2 + 2 A a b) m) x^4 + (A a^2 m^3 + 21 A a^2 m^2 + 138 A a^2 m + 280 A a^2) x)}{m^4 + 22 m^3 + 159 m^2 + 418 m + 280} x^m$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A), x, algorithm="fricas")

[Out] $((B \cdot b^2 \cdot m^3 + 12 \cdot B \cdot b^2 \cdot m^2 + 39 \cdot B \cdot b^2 \cdot m + 28 \cdot B \cdot b^2) \cdot x^{10} + ((2 \cdot B \cdot a \cdot b + A \cdot b^2) \cdot m^3 + 80 \cdot B \cdot a \cdot b + 40 \cdot A \cdot b^2 + 15 \cdot (2 \cdot B \cdot a \cdot b + A \cdot b^2) \cdot m^2 + 54 \cdot (2 \cdot B \cdot a \cdot b + A \cdot b^2) \cdot m) \cdot x^7 + ((B \cdot a^2 + 2 \cdot A \cdot a \cdot b) \cdot m^3 + 70 \cdot B \cdot a^2 + 140 \cdot A \cdot a \cdot b + 18 \cdot (B \cdot a^2 + 2 \cdot A \cdot a \cdot b) \cdot m^2 + 87 \cdot (B \cdot a^2 + 2 \cdot A \cdot a \cdot b) \cdot m) \cdot x^4 + (A \cdot a^2 \cdot m^3 + 21 \cdot A \cdot a^2 \cdot m^2 + 138 \cdot A \cdot a^2 \cdot m + 280 \cdot A \cdot a^2) \cdot x) \cdot x^m / (m^4 + 22 \cdot m^3 + 159 \cdot m^2 + 418 \cdot m + 280)$

giac [B] time = 0.20, size = 332, normalized size = 4.68

$$\frac{(B^2 m^3 x^{10} + 12 B b^2 m^2 x^{10} + 39 B^2 m x^{10} + 28 B^2 x^{10} + A b^2 m^3 x^7 + 2 B a b m^2 x^7 + 28 B a b m x^7 + 28 B a b x^7 + 15 A b^2 m^2 x^7 + 54 A b^2 m x^7 + 108 B a b m^2 x^4 + 80 B a b m x^4 + 40 A b^2 m^3 x^4 + 40 A b^2 m^2 x^4 + 18 B a^2 m^3 x^4 + 140 A a b m^2 x^4 + 140 A a b m x^4 + 18 A a^2 m^3 x^4 + 21 A a^2 m^2 x^4 + 138 A a^2 m x^4 + 280 A a^2 x^4) x^m}{m^4 + 22 m^3 + 159 m^2 + 418 m + 280}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A), x, algorithm="giac")

[Out] $(B \cdot b^2 \cdot m^3 \cdot x^{10} \cdot x^m + 12 \cdot B \cdot b^2 \cdot m^2 \cdot x^{10} \cdot x^m + 39 \cdot B \cdot b^2 \cdot m \cdot x^{10} \cdot x^m + 2 \cdot B \cdot a \cdot b \cdot m^3 \cdot x^7 \cdot x^m + A \cdot b^2 \cdot m^3 \cdot x^7 \cdot x^m + 28 \cdot B \cdot b^2 \cdot m^2 \cdot x^{10} \cdot x^m + 30 \cdot B \cdot a \cdot b \cdot m^2 \cdot x^7 \cdot x^m + 15 \cdot A \cdot b^2 \cdot m^2 \cdot x^7 \cdot x^m + 108 \cdot B \cdot a \cdot b \cdot m \cdot x^7 \cdot x^m + 54 \cdot A \cdot b^2 \cdot m \cdot x^7 \cdot x^m + B \cdot a^2 \cdot m^3 \cdot x^4 \cdot x^m + 2 \cdot A \cdot a \cdot b \cdot m^3 \cdot x^4 \cdot x^m + 80 \cdot B \cdot a \cdot b \cdot m^2 \cdot x^4 \cdot x^m + 40 \cdot A \cdot b^2 \cdot m^2 \cdot x^4 \cdot x^m + 18 \cdot B \cdot a^2 \cdot m^2 \cdot x^4 \cdot x^m + 36 \cdot A \cdot a \cdot b \cdot m^2 \cdot x^4 \cdot x^m + 87 \cdot B \cdot a^2 \cdot m \cdot x^4 \cdot x^m + 174 \cdot A \cdot a \cdot b \cdot m \cdot x^4 \cdot x^m + A \cdot a^2 \cdot m^3 \cdot x^4 \cdot x^m + 70 \cdot B \cdot a^2 \cdot m^2 \cdot x^4 \cdot x^m + 140 \cdot A \cdot a \cdot b \cdot m^2 \cdot x^4 \cdot x^m + 21 \cdot A \cdot a^2 \cdot m^2 \cdot x^4 \cdot x^m + 138 \cdot A \cdot a^2 \cdot m \cdot x^4 \cdot x^m + 280 \cdot A \cdot a^2 \cdot x^4 \cdot x^m) / (m^4 + 22 \cdot m^3 + 159 \cdot m^2 + 418 \cdot m + 280)$

maple [B] time = 0.05, size = 262, normalized size = 3.69

$$\frac{(B^2 m^3 x^9 + 12 B b^2 m^2 x^9 + 39 B^2 m x^9 + A b^2 m^3 x^6 + 2 B a b m^2 x^6 + 28 B^2 B x^6 + 15 A b^2 m^2 x^6 + 30 B a b m^2 x^6 + 54 A b^2 m x^6 + 108 B a b m x^6 + 2 A a b m^3 x^6 + 40 A b^2 x^6 + B a^2 m^3 x^6 + 80 B a b x^6 + 36 A a b m^2 x^6 + 18 B a^2 m^2 x^6 + 174 A a b m x^6 + 87 B a^2 m x^6 + A a^2 m^3 + 140 A a b x^6 + 70 B a^2 x^6 + 21 A a^2 m^2 + 138 A a^2 m + 280 B^2 A) x^{m+1}}{(m+10)(m+7)(m+4)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^m \cdot (b \cdot x^3 + a)^2 \cdot (B \cdot x^3 + A), x)$

[Out] $x^{(m+1)} \cdot (B \cdot b^2 \cdot m^3 \cdot x^9 + 12 \cdot B \cdot b^2 \cdot m^2 \cdot x^9 + 39 \cdot B \cdot b^2 \cdot m \cdot x^9 + A \cdot b^2 \cdot m^3 \cdot x^6 + 2 \cdot B \cdot a \cdot b \cdot m^3 \cdot x^6 + 28 \cdot B \cdot b^2 \cdot x^9 + 15 \cdot A \cdot b^2 \cdot m^2 \cdot x^6 + 30 \cdot B \cdot a \cdot b \cdot m^2 \cdot x^6 + 54 \cdot A \cdot b^2 \cdot m \cdot x^6 + 108 \cdot B \cdot a \cdot b \cdot m \cdot x^6 + 2 \cdot A \cdot a \cdot b \cdot m^3 \cdot x^3 + 40 \cdot A \cdot b^2 \cdot x^6 + B \cdot a^2 \cdot m^3 \cdot x^3 + 80 \cdot B \cdot a \cdot b \cdot x^6 + 36 \cdot A \cdot a \cdot b \cdot m^2 \cdot x^3 + 18 \cdot B \cdot a^2 \cdot m^2 \cdot x^3 + 174 \cdot A \cdot a \cdot b \cdot m \cdot x^3 + 87 \cdot B \cdot a^2 \cdot m \cdot x^3 + A \cdot a^2 \cdot m^3 + 140 \cdot A \cdot a \cdot b \cdot x^3 + 70 \cdot B \cdot a^2 \cdot x^3 + 21 \cdot A \cdot a^2 \cdot m^2 + 138 \cdot A \cdot a^2 \cdot m + 280 \cdot A \cdot a^2) / (m+10) / (m+7) / (m+4) / (m+1)$

maxima [A] time = 0.49, size = 91, normalized size = 1.28

$$\frac{Bb^2x^{m+10}}{m+10} + \frac{2Babx^{m+7}}{m+7} + \frac{Ab^2x^{m+7}}{m+7} + \frac{Ba^2x^{m+4}}{m+4} + \frac{2Aabx^{m+4}}{m+4} + \frac{Aa^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^m \cdot (b \cdot x^3 + a)^2 \cdot (B \cdot x^3 + A), x, \text{algorithm}="maxima")$

[Out] $B \cdot b^2 \cdot x^{(m+10)} / (m+10) + 2 \cdot B \cdot a \cdot b \cdot x^{(m+7)} / (m+7) + A \cdot b^2 \cdot x^{(m+7)} / (m+7) + B \cdot a^2 \cdot x^{(m+4)} / (m+4) + 2 \cdot A \cdot a \cdot b \cdot x^{(m+4)} / (m+4) + A \cdot a^2 \cdot x^{(m+1)} / (m+1)$

mupad [B] time = 2.72, size = 177, normalized size = 2.49

$$x^m \left(\frac{B b^2 x^{10} (m^3 + 12 m^2 + 39 m + 28)}{m^4 + 22 m^3 + 159 m^2 + 418 m + 280} + \frac{A a^2 x (m^3 + 21 m^2 + 138 m + 280)}{m^4 + 22 m^3 + 159 m^2 + 418 m + 280} + \frac{a x^4 (2 A b + B a) (m^3 + 18 m^2 + 87 m + 70)}{m^4 + 22 m^3 + 159 m^2 + 418 m + 280} + \frac{b x^7 (A b + 2 B a) (m^3 + 15 m^2 + 54 m + 40)}{m^4 + 22 m^3 + 159 m^2 + 418 m + 280} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^m \cdot (A + B \cdot x^3) \cdot (a + b \cdot x^3)^2, x)$

[Out] $x^m \cdot ((B \cdot b^2 \cdot x^{10} \cdot (39 \cdot m + 12 \cdot m^2 + m^3 + 28)) / (418 \cdot m + 159 \cdot m^2 + 22 \cdot m^3 + m^4 + 280) + (A \cdot a^2 \cdot x \cdot (138 \cdot m + 21 \cdot m^2 + m^3 + 280)) / (418 \cdot m + 159 \cdot m^2 + 22 \cdot m^3 + m^4 + 280) + (a \cdot x^4 \cdot (2 \cdot A \cdot b + B \cdot a) \cdot (87 \cdot m + 18 \cdot m^2 + m^3 + 70)) / (418 \cdot m + 159 \cdot m^2 + 22 \cdot m^3 + m^4 + 280) + (b \cdot x^7 \cdot (A \cdot b + 2 \cdot B \cdot a) \cdot (54 \cdot m + 15 \cdot m^2 + m^3 + 40)) / (418 \cdot m + 159 \cdot m^2 + 22 \cdot m^3 + m^4 + 280))$

sympy [A] time = 6.31, size = 1057, normalized size = 14.89



Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**m \cdot (b*x**3+a)**2 \cdot (B*x**3+A), x)$

[Out] $\text{Piecewise}((-A*a**2/(9*x**9) - A*a*b/(3*x**6) - A*b**2/(3*x**3) - B*a**2/(6*x**6) - 2*B*a*b/(3*x**3) + B*b**2*\log(x), \text{Eq}(m, -10)), (-A*a**2/(6*x**6) -$


```

2*A*a*b/(3*x**3) + A*b**2*log(x) - B*a**2/(3*x**3) + 2*B*a*b*log(x) + B*b**
2*x**3/3, Eq(m, -7)), (-A*a**2/(3*x**3) + 2*A*a*b*log(x) + A*b**2*x**3/3 +
B*a**2*log(x) + 2*B*a*b*x**3/3 + B*b**2*x**6/6, Eq(m, -4)), (A*a**2*log(x)
+ 2*A*a*b*x**3/3 + A*b**2*x**6/6 + B*a**2*x**3/3 + B*a*b*x**6/3 + B*b**2*x*
*9/9, Eq(m, -1)), (A*a**2*m**3*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m +
280) + 21*A*a**2*m**2*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 13
8*A*a**2*m*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 280*A*a**2*x*
x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 2*A*a*b*m**3*x**4*x**m/(m**
4 + 22*m**3 + 159*m**2 + 418*m + 280) + 36*A*a*b*m**2*x**4*x**m/(m**4 + 22
*m**3 + 159*m**2 + 418*m + 280) + 174*A*a*b*m*x**4*x**m/(m**4 + 22*m**3 + 1
59*m**2 + 418*m + 280) + 140*A*a*b*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 4
18*m + 280) + A*b**2*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 28
0) + 15*A*b**2*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 5
4*A*b**2*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 40*A*b**2*
x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + B*a**2*m**3*x**4*x**m
/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 18*B*a**2*m**2*x**4*x**m/(m**4
+ 22*m**3 + 159*m**2 + 418*m + 280) + 87*B*a**2*m*x**4*x**m/(m**4 + 22*m**
3 + 159*m**2 + 418*m + 280) + 70*B*a**2*x**4*x**m/(m**4 + 22*m**3 + 159*m**
2 + 418*m + 280) + 2*B*a*b*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*
m + 280) + 30*B*a*b*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280
) + 108*B*a*b*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 80*B*
a*b*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + B*b**2*m**3*x**10
*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 12*B*b**2*m**2*x**10*x**m
/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 39*B*b**2*m*x**10*x**m/(m**4 +
22*m**3 + 159*m**2 + 418*m + 280) + 28*B*b**2*x**10*x**m/(m**4 + 22*m**3 +
159*m**2 + 418*m + 280), True))

```

$$3.126 \quad \int x^m (a + bx^3) (A + Bx^3) dx$$

Optimal. Leaf size=45

$$\frac{x^{m+4}(aB + Ab)}{m + 4} + \frac{aAx^{m+1}}{m + 1} + \frac{bBx^{m+7}}{m + 7}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {448}

$$\frac{x^{m+4}(aB + Ab)}{m + 4} + \frac{aAx^{m+1}}{m + 1} + \frac{bBx^{m+7}}{m + 7}$$

Antiderivative was successfully verified.

[In] Int[x^m*(a + b*x^3)*(A + B*x^3), x]

[Out] (a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(4 + m))/(4 + m) + (b*B*x^(7 + m))/(7 + m)

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^m (a + bx^3) (A + Bx^3) dx &= \int (aAx^m + (Ab + aB)x^{3+m} + bBx^{6+m}) dx \\ &= \frac{aAx^{1+m}}{1 + m} + \frac{(Ab + aB)x^{4+m}}{4 + m} + \frac{bBx^{7+m}}{7 + m} \end{aligned}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 0.93

$$x^{m+1} \left(\frac{x^3(aB + Ab)}{m + 4} + \frac{aA}{m + 1} + \frac{bBx^6}{m + 7} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m*(a + b*x^3)*(A + B*x^3), x]

[Out] $x^{(1+m)} \cdot ((aA)/(1+m) + ((A*b + a*B)*x^3)/(4+m) + (b*B*x^6)/(7+m))$

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^m (a + bx^3) (A + Bx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^m*(a + b*x^3)*(A + B*x^3), x]

[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^3)*(A + B*x^3), x]

fricas [B] time = 0.85, size = 92, normalized size = 2.04

$$\frac{((Bbm^2 + 5Bbm + 4Bb)x^7 + ((Ba + Ab)m^2 + 7Ba + 7Ab + 8(Ba + Ab)m)x^4 + (Aam^2 + 11Aam + 28Aa)x)x^m}{m^3 + 12m^2 + 39m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b*x^3+a)*(B*x^3+A), x, algorithm="fricas")

[Out] $((B*b*m^2 + 5*B*b*m + 4*B*b)*x^7 + ((B*a + A*b)*m^2 + 7*B*a + 7*A*b + 8*(B*a + A*b)*m)*x^4 + (A*a*m^2 + 11*A*a*m + 28*A*a)*x)*x^m/(m^3 + 12*m^2 + 39*m + 28)$

giac [B] time = 0.17, size = 143, normalized size = 3.18

$$\frac{Bbm^2x^7x^m + 5Bbm^2x^7x^m + 4Bbx^7x^m + Bam^2x^4x^m + Abm^2x^4x^m + 8Bamx^4x^m + 8Abmx^4x^m + 7Bax^4x^m + 7Abx^4x^m + Aam^2xx^m + 11Aamxx^m + 28Aaxx^m}{m^3 + 12m^2 + 39m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b*x^3+a)*(B*x^3+A), x, algorithm="giac")

[Out] $(B*b*m^2*x^7*x^m + 5*B*b*m*x^7*x^m + 4*B*b*x^7*x^m + B*a*m^2*x^4*x^m + A*b*m^2*x^4*x^m + 8*B*a*m*x^4*x^m + 8*A*b*m*x^4*x^m + 7*B*a*x^4*x^m + 7*A*b*x^4*x^m + A*a*m^2*x*x^m + 11*A*a*m*x*x^m + 28*A*a*x*x^m)/(m^3 + 12*m^2 + 39*m + 28)$

maple [B] time = 0.04, size = 110, normalized size = 2.44

$$\frac{(Bb m^2 x^6 + 5Bbm x^6 + 4Bb x^6 + Ab m^2 x^3 + Ba m^2 x^3 + 8Abm x^3 + 8Bam x^3 + 7Ab x^3 + 7Ba x^3 + Aa m^2 + 11Aam + 28Aa) x^{m+1}}{(m+7)(m+4)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(b*x^3+a)*(B*x^3+A), x)

[Out] $x^{(m+1)} * (B*b*m^2*x^6 + 5*B*b*m*x^6 + 4*B*b*x^6 + A*b*m^2*x^3 + B*a*m^2*x^3 + 8*A*b*m*x^3 + 8*B*a*m*x^3 + 7*A*b*x^3 + 7*B*a*x^3 + A*a*m^2 + 11*A*a*m + 28*A*a) / (m+7) / (m+4) / (m+1)$

maxima [A] time = 0.61, size = 53, normalized size = 1.18

$$\frac{Bbx^{m+7}}{m+7} + \frac{Bax^{m+4}}{m+4} + \frac{Abx^{m+4}}{m+4} + \frac{Aax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x^m*(b*x^3+a)*(B*x^3+A)$, x, algorithm="maxima")

[Out] $B*b*x^{(m+7)} / (m+7) + B*a*x^{(m+4)} / (m+4) + A*b*x^{(m+4)} / (m+4) + A*a*x^{(m+1)} / (m+1)$

mupad [B] time = 2.65, size = 95, normalized size = 2.11

$$x^m \left(\frac{x^4 (Ab + Ba) (m^2 + 8m + 7)}{m^3 + 12m^2 + 39m + 28} + \frac{Bbx^7 (m^2 + 5m + 4)}{m^3 + 12m^2 + 39m + 28} + \frac{Aax (m^2 + 11m + 28)}{m^3 + 12m^2 + 39m + 28} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int($x^m*(A + B*x^3)*(a + b*x^3)$, x)

[Out] $x^m * ((x^4 * (A*b + B*a) * (8*m + m^2 + 7)) / (39*m + 12*m^2 + m^3 + 28) + (B*b*x^7 * (5*m + m^2 + 4)) / (39*m + 12*m^2 + m^3 + 28) + (A*a*x * (11*m + m^2 + 28)) / (39*m + 12*m^2 + m^3 + 28))$

sympy [A] time = 2.89, size = 410, normalized size = 9.11

$$\begin{cases} -\frac{Aa}{6a^m} - \frac{Ab}{3a^3} - \frac{Ba}{3a^3} + Bb \log(x) & \text{for } m = -7 \\ -\frac{Aa}{3a^2} + Ab \log(x) + Ba \log(x) + \frac{Bbx^3}{3} & \text{for } m = -4 \\ Aa \log(x) + \frac{Abx^3}{3} + \frac{Bax^3}{3} + \frac{Bbx^6}{6} & \text{for } m = -1 \\ \frac{Aam^2x^m}{m^3+12m^2+39m+28} + \frac{11Aamx^m}{m^3+12m^2+39m+28} + \frac{28Aax^m}{m^3+12m^2+39m+28} + \frac{Abm^2x^m}{m^3+12m^2+39m+28} + \frac{8Abmx^m}{m^3+12m^2+39m+28} + \frac{7Abx^4x^m}{m^3+12m^2+39m+28} + \frac{Bam^2x^4x^m}{m^3+12m^2+39m+28} + \frac{8Bamx^4x^m}{m^3+12m^2+39m+28} + \frac{7Bax^4x^m}{m^3+12m^2+39m+28} + \frac{Bbm^2x^7x^m}{m^3+12m^2+39m+28} + \frac{5Bbm x^7x^m}{m^3+12m^2+39m+28} + \frac{4Bbx^7x^m}{m^3+12m^2+39m+28} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x**m*(b*x**3+a)*(B*x**3+A)$, x)

[Out] Piecewise((-A*a/(6*x**6) - A*b/(3*x**3) - B*a/(3*x**3) + B*b*log(x), Eq(m, -7)), (-A*a/(3*x**3) + A*b*log(x) + B*a*log(x) + B*b*x**3/3, Eq(m, -4)), (A*a*log(x) + A*b*x**3/3 + B*a*x**3/3 + B*b*x**6/6, Eq(m, -1)), (A*a*m**2*x**m/(m**3 + 12*m**2 + 39*m + 28) + 11*A*a*m*x**m/(m**3 + 12*m**2 + 39*m + 28) + 28*A*a*x**m/(m**3 + 12*m**2 + 39*m + 28) + A*b*m**2*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 8*A*b*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 7*A*b*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*a*m**2*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 8*B*a*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 7*B

```
*a*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*b*m**2*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 5*B*b*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 4*B*b*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28), True))
```

$$3.127 \quad \int x^{7/2} (a + bx^3) (A + Bx^3) dx$$

Optimal. Leaf size=39

$$\frac{2}{15}x^{15/2}(aB + Ab) + \frac{2}{9}aAx^{9/2} + \frac{2}{21}bBx^{21/2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{2}{15}x^{15/2}(aB + Ab) + \frac{2}{9}aAx^{9/2} + \frac{2}{21}bBx^{21/2}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)*(a + b*x^3)*(A + B*x^3), x]

[Out] (2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(15/2))/15 + (2*b*B*x^(21/2))/21

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^{7/2} (a + bx^3) (A + Bx^3) dx &= \int (aAx^{7/2} + (Ab + aB)x^{13/2} + bBx^{19/2}) dx \\ &= \frac{2}{9}aAx^{9/2} + \frac{2}{15}(Ab + aB)x^{15/2} + \frac{2}{21}bBx^{21/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2}{315}x^{9/2} (21x^3(aB + Ab) + 35aA + 15bBx^6)$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)*(a + b*x^3)*(A + B*x^3), x]

[Out] (2*x^(9/2)*(35*a*A + 21*(A*b + a*B)*x^3 + 15*b*B*x^6))/315

IntegrateAlgebraic [A] time = 0.02, size = 35, normalized size = 0.90

$$\frac{2}{315}x^{9/2}(35aA + 21aBx^3 + 21Abx^3 + 15bBx^6)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^3)*(A + B*x^3), x]

[Out] (2*x^(9/2)*(35*a*A + 21*A*b*x^3 + 21*a*B*x^3 + 15*b*B*x^6))/315

fricas [A] time = 0.88, size = 32, normalized size = 0.82

$$\frac{2}{315}(15Bbx^{10} + 21(Ba + Ab)x^7 + 35Aax^4)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A), x, algorithm="fricas")

[Out] 2/315*(15*B*b*x^10 + 21*(B*a + A*b)*x^7 + 35*A*a*x^4)*sqrt(x)

giac [A] time = 0.17, size = 29, normalized size = 0.74

$$\frac{2}{21}Bbx^{\frac{21}{2}} + \frac{2}{15}Bax^{\frac{15}{2}} + \frac{2}{15}Abx^{\frac{15}{2}} + \frac{2}{9}Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A), x, algorithm="giac")

[Out] 2/21*B*b*x^(21/2) + 2/15*B*a*x^(15/2) + 2/15*A*b*x^(15/2) + 2/9*A*a*x^(9/2)

maple [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2(15Bbx^6 + 21Abx^3 + 21Bax^3 + 35Aa)x^{\frac{9}{2}}}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)*(b*x^3+a)*(B*x^3+A), x)

[Out] 2/315*x^(9/2)*(15*B*b*x^6+21*A*b*x^3+21*B*a*x^3+35*A*a)

maxima [A] time = 0.45, size = 27, normalized size = 0.69

$$\frac{2}{21}Bbx^{\frac{21}{2}} + \frac{2}{15}(Ba + Ab)x^{\frac{15}{2}} + \frac{2}{9}Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")`

[Out] $2/21*B*b*x^{(21/2)} + 2/15*(B*a + A*b)*x^{(15/2)} + 2/9*A*a*x^{(9/2)}$

mupad [B] time = 0.05, size = 31, normalized size = 0.79

$$\frac{2x^{9/2} (35Aa + 21Abx^3 + 21Bax^3 + 15Bbx^6)}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(7/2)*(A + B*x^3)*(a + b*x^3),x)`

[Out] $(2*x^{(9/2)}*(35*A*a + 21*A*b*x^3 + 21*B*a*x^3 + 15*B*b*x^6))/315$

sympy [A] time = 20.86, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{9}{2}}}{9} + \frac{2Abx^{\frac{15}{2}}}{15} + \frac{2Bax^{\frac{15}{2}}}{15} + \frac{2Bbx^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(7/2)*(b*x**3+a)*(B*x**3+A),x)`

[Out] $2*A*a*x^{(9/2)}/9 + 2*A*b*x^{(15/2)}/15 + 2*B*a*x^{(15/2)}/15 + 2*B*b*x^{(21/2)}/21$

$$3.128 \quad \int x^{5/2} (a + bx^3) (A + Bx^3) dx$$

Optimal. Leaf size=39

$$\frac{2}{13}x^{13/2}(aB + Ab) + \frac{2}{7}aAx^{7/2} + \frac{2}{19}bBx^{19/2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{2}{13}x^{13/2}(aB + Ab) + \frac{2}{7}aAx^{7/2} + \frac{2}{19}bBx^{19/2}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)*(a + b*x^3)*(A + B*x^3), x]

[Out] (2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(19/2))/19

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^{5/2} (a + bx^3) (A + Bx^3) dx &= \int (aAx^{5/2} + (Ab + aB)x^{11/2} + bBx^{17/2}) dx \\ &= \frac{2}{7}aAx^{7/2} + \frac{2}{13}(Ab + aB)x^{13/2} + \frac{2}{19}bBx^{19/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2x^{7/2} (133x^3(aB + Ab) + 247aA + 91bBx^6)}{1729}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)*(a + b*x^3)*(A + B*x^3), x]

[Out] (2*x^(7/2)*(247*a*A + 133*(A*b + a*B)*x^3 + 91*b*B*x^6))/1729

IntegrateAlgebraic [A] time = 0.02, size = 41, normalized size = 1.05

$$\frac{2(247aAx^{7/2} + 133aBx^{13/2} + 133Abx^{13/2} + 91bBx^{19/2})}{1729}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^3)*(A + B*x^3), x]

[Out] (2*(247*a*A*x^(7/2) + 133*A*b*x^(13/2) + 133*a*B*x^(13/2) + 91*b*B*x^(19/2)))/1729

fricas [A] time = 1.32, size = 32, normalized size = 0.82

$$\frac{2}{1729} (91 Bbx^9 + 133 (Ba + Ab)x^6 + 247 Aax^3) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A), x, algorithm="fricas")

[Out] 2/1729*(91*B*b*x^9 + 133*(B*a + A*b)*x^6 + 247*A*a*x^3)*sqrt(x)

giac [A] time = 0.15, size = 29, normalized size = 0.74

$$\frac{2}{19} Bbx^{\frac{19}{2}} + \frac{2}{13} Bax^{\frac{13}{2}} + \frac{2}{13} Abx^{\frac{13}{2}} + \frac{2}{7} Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A), x, algorithm="giac")

[Out] 2/19*B*b*x^(19/2) + 2/13*B*a*x^(13/2) + 2/13*A*b*x^(13/2) + 2/7*A*a*x^(7/2)

maple [A] time = 0.04, size = 32, normalized size = 0.82

$$\frac{2(91Bbx^6 + 133Abx^3 + 133Bax^3 + 247Aa)x^{\frac{7}{2}}}{1729}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(b*x^3+a)*(B*x^3+A), x)

[Out] 2/1729*x^(7/2)*(91*B*b*x^6+133*A*b*x^3+133*B*a*x^3+247*A*a)

maxima [A] time = 0.47, size = 27, normalized size = 0.69

$$\frac{2}{19} Bbx^{\frac{19}{2}} + \frac{2}{13} (Ba + Ab)x^{\frac{13}{2}} + \frac{2}{7} Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")`

[Out] $2/19*B*b*x^{(19/2)} + 2/13*(B*a + A*b)*x^{(13/2)} + 2/7*A*a*x^{(7/2)}$

mupad [B] time = 2.56, size = 31, normalized size = 0.79

$$\frac{2x^{7/2} (247Aa + 133Abx^3 + 133Bax^3 + 91Bbx^6)}{1729}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(5/2)*(A + B*x^3)*(a + b*x^3),x)`

[Out] $(2*x^{(7/2)}*(247*A*a + 133*A*b*x^3 + 133*B*a*x^3 + 91*B*b*x^6))/1729$

sympy [A] time = 12.71, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{7}{2}}}{7} + \frac{2Abx^{\frac{13}{2}}}{13} + \frac{2Bax^{\frac{13}{2}}}{13} + \frac{2Bbx^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(5/2)*(b*x**3+a)*(B*x**3+A),x)`

[Out] $2*A*a*x^{(7/2)}/7 + 2*A*b*x^{(13/2)}/13 + 2*B*a*x^{(13/2)}/13 + 2*B*b*x^{(19/2)}/19$

$$3.129 \quad \int x^{3/2} (a + bx^3) (A + Bx^3) dx$$

Optimal. Leaf size=39

$$\frac{2}{11}x^{11/2}(aB + Ab) + \frac{2}{5}aAx^{5/2} + \frac{2}{17}bBx^{17/2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{2}{11}x^{11/2}(aB + Ab) + \frac{2}{5}aAx^{5/2} + \frac{2}{17}bBx^{17/2}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)*(a + b*x^3)*(A + B*x^3), x]

[Out] (2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(17/2))/17

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^{3/2} (a + bx^3) (A + Bx^3) dx &= \int (aAx^{3/2} + (Ab + aB)x^{9/2} + bBx^{15/2}) dx \\ &= \frac{2}{5}aAx^{5/2} + \frac{2}{11}(Ab + aB)x^{11/2} + \frac{2}{17}bBx^{17/2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.85

$$\frac{2}{935}x^{5/2} (85x^3(aB + Ab) + 187aA + 55bBx^6)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)*(a + b*x^3)*(A + B*x^3), x]

[Out] (2*x^(5/2)*(187*a*A + 85*(A*b + a*B)*x^3 + 55*b*B*x^6))/935

IntegrateAlgebraic [A] time = 0.02, size = 35, normalized size = 0.90

$$\frac{2}{935}x^{5/2}(187aA + 85aBx^3 + 85Abx^3 + 55bBx^6)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]

[Out] (2*x^(5/2)*(187*a*A + 85*A*b*x^3 + 85*a*B*x^3 + 55*b*B*x^6))/935

fricas [A] time = 0.93, size = 32, normalized size = 0.82

$$\frac{2}{935}(55Bbx^8 + 85(Ba + Ab)x^5 + 187Aax^2)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")

[Out] 2/935*(55*B*b*x^8 + 85*(B*a + A*b)*x^5 + 187*A*a*x^2)*sqrt(x)

giac [A] time = 0.18, size = 29, normalized size = 0.74

$$\frac{2}{17}Bbx^{\frac{17}{2}} + \frac{2}{11}Bax^{\frac{11}{2}} + \frac{2}{11}Abx^{\frac{11}{2}} + \frac{2}{5}Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")

[Out] 2/17*B*b*x^(17/2) + 2/11*B*a*x^(11/2) + 2/11*A*b*x^(11/2) + 2/5*A*a*x^(5/2)

maple [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2(55Bbx^6 + 85Abx^3 + 85Bax^3 + 187Aa)x^{\frac{5}{2}}}{935}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(b*x^3+a)*(B*x^3+A),x)

[Out] 2/935*x^(5/2)*(55*B*b*x^6+85*A*b*x^3+85*B*a*x^3+187*A*a)

maxima [A] time = 0.45, size = 27, normalized size = 0.69

$$\frac{2}{17}Bbx^{\frac{17}{2}} + \frac{2}{11}(Ba + Ab)x^{\frac{11}{2}} + \frac{2}{5}Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")`

[Out] $2/17*B*b*x^{17/2} + 2/11*(B*a + A*b)*x^{11/2} + 2/5*A*a*x^{5/2}$

mupad [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2x^{5/2}(187Aa + 85Abx^3 + 85Bax^3 + 55Bbx^6)}{935}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)*(A + B*x^3)*(a + b*x^3),x)`

[Out] $(2*x^{5/2}*(187*A*a + 85*A*b*x^3 + 85*B*a*x^3 + 55*B*b*x^6))/935$

sympy [A] time = 6.74, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{5}{2}}}{5} + \frac{2Abx^{\frac{11}{2}}}{11} + \frac{2Bax^{\frac{11}{2}}}{11} + \frac{2Bbx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2)*(b*x**3+a)*(B*x**3+A),x)`

[Out] $2*A*a*x^{5/2}/5 + 2*A*b*x^{11/2}/11 + 2*B*a*x^{11/2}/11 + 2*B*b*x^{17/2}/17$

$$3.130 \quad \int \sqrt{x} (a + bx^3) (A + Bx^3) dx$$

Optimal. Leaf size=39

$$\frac{2}{9}x^{9/2}(aB + Ab) + \frac{2}{3}aAx^{3/2} + \frac{2}{15}bBx^{15/2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{2}{9}x^{9/2}(aB + Ab) + \frac{2}{3}aAx^{3/2} + \frac{2}{15}bBx^{15/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]*(a + b*x^3)*(A + B*x^3), x]

[Out] (2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(15/2))/15

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{x} (a + bx^3) (A + Bx^3) dx &= \int (aA\sqrt{x} + (Ab + aB)x^{7/2} + bBx^{13/2}) dx \\ &= \frac{2}{3}aAx^{3/2} + \frac{2}{9}(Ab + aB)x^{9/2} + \frac{2}{15}bBx^{15/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2}{45}x^{3/2} (5x^3(aB + Ab) + 15aA + 3bBx^6)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]*(a + b*x^3)*(A + B*x^3), x]

[Out] (2*x^(3/2)*(15*a*A + 5*(A*b + a*B)*x^3 + 3*b*B*x^6))/45

IntegrateAlgebraic [A] time = 0.02, size = 41, normalized size = 1.05

$$\frac{2}{45} (15aAx^{3/2} + 5aBx^{9/2} + 5Abx^{9/2} + 3bBx^{15/2})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^3)*(A + B*x^3), x]

[Out] (2*(15*a*A*x^(3/2) + 5*A*b*x^(9/2) + 5*a*B*x^(9/2) + 3*b*B*x^(15/2)))/45

fricas [A] time = 0.89, size = 30, normalized size = 0.77

$$\frac{2}{45} (3Bbx^7 + 5(Ba + Ab)x^4 + 15Aax)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2), x, algorithm="fricas")

[Out] 2/45*(3*B*b*x^7 + 5*(B*a + A*b)*x^4 + 15*A*a*x)*sqrt(x)

giac [A] time = 0.15, size = 29, normalized size = 0.74

$$\frac{2}{15} Bbx^{\frac{15}{2}} + \frac{2}{9} Bax^{\frac{9}{2}} + \frac{2}{9} Abx^{\frac{9}{2}} + \frac{2}{3} Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2), x, algorithm="giac")

[Out] 2/15*B*b*x^(15/2) + 2/9*B*a*x^(9/2) + 2/9*A*b*x^(9/2) + 2/3*A*a*x^(3/2)

maple [A] time = 0.04, size = 32, normalized size = 0.82

$$\frac{2(3Bbx^6 + 5Abx^3 + 5Bax^3 + 15Aa)x^{\frac{3}{2}}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(B*x^3+A)*x^(1/2), x)

[Out] 2/45*x^(3/2)*(3*B*b*x^6+5*A*b*x^3+5*B*a*x^3+15*A*a)

maxima [A] time = 0.47, size = 27, normalized size = 0.69

$$\frac{2}{15} Bbx^{\frac{15}{2}} + \frac{2}{9} (Ba + Ab)x^{\frac{9}{2}} + \frac{2}{3} Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="maxima")`

[Out] $2/15*B*b*x^{(15/2)} + 2/9*(B*a + A*b)*x^{(9/2)} + 2/3*A*a*x^{(3/2)}$

mupad [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2x^{3/2} (15Aa + 5Abx^3 + 5Bax^3 + 3Bbx^6)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(A + B*x^3)*(a + b*x^3),x)`

[Out] $(2*x^{(3/2)}*(15*A*a + 5*A*b*x^3 + 5*B*a*x^3 + 3*B*b*x^6))/45$

sympy [A] time = 3.37, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{3}{2}}}{3} + \frac{2Abx^{\frac{9}{2}}}{9} + \frac{2Bax^{\frac{9}{2}}}{9} + \frac{2Bbx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)*(B*x**3+A)*x**(1/2),x)`

[Out] $2*A*a*x^{(3/2)}/3 + 2*A*b*x^{(9/2)}/9 + 2*B*a*x^{(9/2)}/9 + 2*B*b*x^{(15/2)}/15$

$$3.131 \quad \int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}} dx$$

Optimal. Leaf size=37

$$\frac{2}{7}x^{7/2}(aB + Ab) + 2aA\sqrt{x} + \frac{2}{13}bBx^{13/2}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{2}{7}x^{7/2}(aB + Ab) + 2aA\sqrt{x} + \frac{2}{13}bBx^{13/2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*(A + B*x^3))/Sqrt[x], x]

[Out] 2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(13/2))/13

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)(A + Bx^3)}{\sqrt{x}} dx &= \int \left(\frac{aA}{\sqrt{x}} + (Ab + aB)x^{5/2} + bBx^{11/2} \right) dx \\ &= 2aA\sqrt{x} + \frac{2}{7}(Ab + aB)x^{7/2} + \frac{2}{13}bBx^{13/2} \end{aligned}$$

Mathematica [A] time = 0.07, size = 33, normalized size = 0.89

$$\frac{2}{91}\sqrt{x} (13x^3(aB + Ab) + 91aA + 7bBx^6)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(A + B*x^3))/Sqrt[x], x]

[Out] $(2*\text{Sqrt}[x]*(91*a*A + 13*(A*b + a*B)*x^3 + 7*b*B*x^6))/91$

IntegrateAlgebraic [A] time = 0.02, size = 41, normalized size = 1.11

$$\frac{2}{91} (91aA\sqrt{x} + 13aBx^{7/2} + 13Abx^{7/2} + 7bBx^{13/2})$$

Antiderivative was successfully verified.

[In] `IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/Sqrt[x], x]`

[Out] $(2*(91*a*A*\text{Sqrt}[x] + 13*A*b*x^{(7/2)} + 13*a*B*x^{(7/2)} + 7*b*B*x^{(13/2)}))/91$

fricas [A] time = 0.75, size = 29, normalized size = 0.78

$$\frac{2}{91} (7Bbx^6 + 13(Ba + Ab)x^3 + 91Aa)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)*(B*x^3+A)/x^(1/2), x, algorithm="fricas")`

[Out] $2/91*(7*B*b*x^6 + 13*(B*a + A*b)*x^3 + 91*A*a)*\text{sqrt}(x)$

giac [A] time = 0.15, size = 29, normalized size = 0.78

$$\frac{2}{13} Bbx^{\frac{13}{2}} + \frac{2}{7} Bax^{\frac{7}{2}} + \frac{2}{7} Abx^{\frac{7}{2}} + 2Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)*(B*x^3+A)/x^(1/2), x, algorithm="giac")`

[Out] $2/13*B*b*x^{(13/2)} + 2/7*B*a*x^{(7/2)} + 2/7*A*b*x^{(7/2)} + 2*A*a*\text{sqrt}(x)$

maple [A] time = 0.04, size = 32, normalized size = 0.86

$$\frac{2(7Bbx^6 + 13Abx^3 + 13Bax^3 + 91Aa)\sqrt{x}}{91}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)*(B*x^3+A)/x^(1/2), x)`

[Out] $2/91*x^{(1/2)}*(7*B*b*x^6+13*A*b*x^3+13*B*a*x^3+91*A*a)$

maxima [A] time = 0.50, size = 27, normalized size = 0.73

$$\frac{2}{13} Bbx^{\frac{13}{2}} + \frac{2}{7} (Ba + Ab)x^{\frac{7}{2}} + 2Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="maxima")

[Out] 2/13*B*b*x^(13/2) + 2/7*(B*a + A*b)*x^(7/2) + 2*A*a*sqrt(x)

mupad [B] time = 2.59, size = 31, normalized size = 0.84

$$\frac{2\sqrt{x} (91 A a + 13 A b x^3 + 13 B a x^3 + 7 B b x^6)}{91}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3))/x^(1/2),x)

[Out] (2*x^(1/2)*(91*A*a + 13*A*b*x^3 + 13*B*a*x^3 + 7*B*b*x^6))/91

sympy [A] time = 2.30, size = 44, normalized size = 1.19

$$2Aa\sqrt{x} + \frac{2Abx^{\frac{7}{2}}}{7} + \frac{2Bax^{\frac{7}{2}}}{7} + \frac{2Bbx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(B*x**3+A)/x**(1/2),x)

[Out] 2*A*a*sqrt(x) + 2*A*b*x**(7/2)/7 + 2*B*a*x**(7/2)/7 + 2*B*b*x**(13/2)/13

$$3.132 \quad \int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}} dx$$

Optimal. Leaf size=37

$$\frac{2}{5}x^{5/2}(aB + Ab) - \frac{2aA}{\sqrt{x}} + \frac{2}{11}bBx^{11/2}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{2}{5}x^{5/2}(aB + Ab) - \frac{2aA}{\sqrt{x}} + \frac{2}{11}bBx^{11/2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*(A + B*x^3))/x^(3/2), x]

[Out] (-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(11/2))/11

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)(A + Bx^3)}{x^{3/2}} dx &= \int \left(\frac{aA}{x^{3/2}} + (Ab + aB)x^{3/2} + bBx^{9/2} \right) dx \\ &= -\frac{2aA}{\sqrt{x}} + \frac{2}{5}(Ab + aB)x^{5/2} + \frac{2}{11}bBx^{11/2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.95

$$\frac{2(-55aA + 11aBx^3 + 11Abx^3 + 5bBx^6)}{55\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(3/2), x]

[Out] $(2*(-55*a*A + 11*A*b*x^3 + 11*a*B*x^3 + 5*b*B*x^6))/(55*\text{Sqrt}[x])$

IntegrateAlgebraic [A] time = 0.02, size = 35, normalized size = 0.95

$$\frac{2(-55aA + 11aBx^3 + 11Abx^3 + 5bBx^6)}{55\sqrt{x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^(3/2), x]

[Out] $(2*(-55*a*A + 11*A*b*x^3 + 11*a*B*x^3 + 5*b*B*x^6))/(55*\text{Sqrt}[x])$

fricas [A] time = 0.78, size = 29, normalized size = 0.78

$$\frac{2(5Bbx^6 + 11(Ba + Ab)x^3 - 55Aa)}{55\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2), x, algorithm="fricas")

[Out] $2/55*(5*B*b*x^6 + 11*(B*a + A*b)*x^3 - 55*A*a)/\text{sqrt}(x)$

giac [A] time = 0.15, size = 29, normalized size = 0.78

$$\frac{2}{11} Bbx^{\frac{11}{2}} + \frac{2}{5} Bax^{\frac{5}{2}} + \frac{2}{5} Abx^{\frac{5}{2}} - \frac{2Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2), x, algorithm="giac")

[Out] $2/11*B*b*x^{(11/2)} + 2/5*B*a*x^{(5/2)} + 2/5*A*b*x^{(5/2)} - 2*A*a/\text{sqrt}(x)$

maple [A] time = 0.04, size = 32, normalized size = 0.86

$$\frac{2(-5Bbx^6 - 11Abx^3 - 11Bax^3 + 55Aa)}{55\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(B*x^3+A)/x^(3/2), x)

[Out] $-2/55*(-5*B*b*x^6 - 11*A*b*x^3 - 11*B*a*x^3 + 55*A*a)/x^{(1/2)}$

maxima [A] time = 0.46, size = 27, normalized size = 0.73

$$\frac{2}{11} B b x^{\frac{11}{2}} + \frac{2}{5} (B a + A b) x^{\frac{5}{2}} - \frac{2 A a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="maxima")

[Out] 2/11*B*b*x^(11/2) + 2/5*(B*a + A*b)*x^(5/2) - 2*A*a/sqrt(x)

mupad [B] time = 2.60, size = 31, normalized size = 0.84

$$\frac{22 A b x^3 - 110 A a + 22 B a x^3 + 10 B b x^6}{55 \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3))/x^(3/2),x)

[Out] (22*A*b*x^3 - 110*A*a + 22*B*a*x^3 + 10*B*b*x^6)/(55*x^(1/2))

sympy [A] time = 2.77, size = 44, normalized size = 1.19

$$-\frac{2 A a}{\sqrt{x}} + \frac{2 A b x^{\frac{5}{2}}}{5} + \frac{2 B a x^{\frac{5}{2}}}{5} + \frac{2 B b x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(B*x**3+A)/x**(3/2),x)

[Out] -2*A*a/sqrt(x) + 2*A*b*x**(5/2)/5 + 2*B*a*x**(5/2)/5 + 2*B*b*x**(11/2)/11

$$3.133 \quad \int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}} dx$$

Optimal. Leaf size=39

$$\frac{2}{3}x^{3/2}(aB + Ab) - \frac{2aA}{3x^{3/2}} + \frac{2}{9}bBx^{9/2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{2}{3}x^{3/2}(aB + Ab) - \frac{2aA}{3x^{3/2}} + \frac{2}{9}bBx^{9/2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*(A + B*x^3))/x^(5/2), x]

[Out] (-2*a*A)/(3*x^(3/2)) + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(9/2))/9

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)(A + Bx^3)}{x^{5/2}} dx &= \int \left(\frac{aA}{x^{5/2}} + (Ab + aB)\sqrt{x} + bBx^{7/2} \right) dx \\ &= -\frac{2aA}{3x^{3/2}} + \frac{2}{3}(Ab + aB)x^{3/2} + \frac{2}{9}bBx^{9/2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.87

$$\frac{2(-3aA + 3aBx^3 + 3Abx^3 + bBx^6)}{9x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(5/2), x]

[Out] $(2*(-3*a*A + 3*A*b*x^3 + 3*a*B*x^3 + b*B*x^6))/(9*x^(3/2))$

IntegrateAlgebraic [A] time = 0.02, size = 34, normalized size = 0.87

$$\frac{2(-3aA + 3aBx^3 + 3Abx^3 + bBx^6)}{9x^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^(5/2), x]

[Out] $(2*(-3*a*A + 3*A*b*x^3 + 3*a*B*x^3 + b*B*x^6))/(9*x^(3/2))$

fricas [A] time = 0.89, size = 28, normalized size = 0.72

$$\frac{2(Bbx^6 + 3(Ba + Ab)x^3 - 3Aa)}{9x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2), x, algorithm="fricas")

[Out] $2/9*(B*b*x^6 + 3*(B*a + A*b)*x^3 - 3*A*a)/x^(3/2)$

giac [A] time = 0.16, size = 29, normalized size = 0.74

$$\frac{2}{9}Bbx^{9/2} + \frac{2}{3}Bax^{3/2} + \frac{2}{3}Abx^{3/2} - \frac{2Aa}{3x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2), x, algorithm="giac")

[Out] $2/9*B*b*x^(9/2) + 2/3*B*a*x^(3/2) + 2/3*A*b*x^(3/2) - 2/3*A*a/x^(3/2)$

maple [A] time = 0.04, size = 32, normalized size = 0.82

$$\frac{2(-Bbx^6 - 3Abx^3 - 3Bax^3 + 3Aa)}{9x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(B*x^3+A)/x^(5/2), x)

[Out] $-2/9*(-B*b*x^6 - 3*A*b*x^3 - 3*B*a*x^3 + 3*A*a)/x^(3/2)$

maxima [A] time = 0.48, size = 27, normalized size = 0.69

$$\frac{2}{9} Bbx^{\frac{9}{2}} + \frac{2}{3} (Ba + Ab)x^{\frac{3}{2}} - \frac{2Aa}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="maxima")

[Out] 2/9*B*b*x^(9/2) + 2/3*(B*a + A*b)*x^(3/2) - 2/3*A*a/x^(3/2)

mupad [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{6Abx^3 - 6Aa + 6Bax^3 + 2Bbx^6}{9x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3))/x^(5/2),x)

[Out] (6*A*b*x^3 - 6*A*a + 6*B*a*x^3 + 2*B*b*x^6)/(9*x^(3/2))

sympy [A] time = 3.74, size = 46, normalized size = 1.18

$$-\frac{2Aa}{3x^{\frac{3}{2}}} + \frac{2Abx^{\frac{3}{2}}}{3} + \frac{2Bax^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(B*x**3+A)/x**(5/2),x)

[Out] -2*A*a/(3*x**(3/2)) + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + 2*B*b*x**(9/2)/9

$$3.134 \quad \int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}} dx$$

Optimal. Leaf size=37

$$2\sqrt{x}(aB + Ab) - \frac{2aA}{5x^{5/2}} + \frac{2}{7}bBx^{7/2}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$2\sqrt{x}(aB + Ab) - \frac{2aA}{5x^{5/2}} + \frac{2}{7}bBx^{7/2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*(A + B*x^3))/x^(7/2), x]

[Out] (-2*a*A)/(5*x^(5/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(7/2))/7

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)(A + Bx^3)}{x^{7/2}} dx &= \int \left(\frac{aA}{x^{7/2}} + \frac{Ab + aB}{\sqrt{x}} + bBx^{5/2} \right) dx \\ &= -\frac{2aA}{5x^{5/2}} + 2(Ab + aB)\sqrt{x} + \frac{2}{7}bBx^{7/2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.97

$$\frac{2(5bx^3(7A + Bx^3) - 7a(A - 5Bx^3))}{35x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(7/2), x]

[Out] $(2*(-7*a*(A - 5*B*x^3) + 5*b*x^3*(7*A + B*x^3)))/(35*x^(5/2))$

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.95

$$\frac{2(-7aA + 35aBx^3 + 35Abx^3 + 5bBx^6)}{35x^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic(((a + b*x^3)*(A + B*x^3))/x^(7/2), x)

[Out] $(2*(-7*a*A + 35*A*b*x^3 + 35*a*B*x^3 + 5*b*B*x^6))/(35*x^(5/2))$

fricas [A] time = 0.75, size = 29, normalized size = 0.78

$$\frac{2(5Bbx^6 + 35(Ba + Ab)x^3 - 7Aa)}{35x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2), x, algorithm="fricas")

[Out] $2/35*(5*B*b*x^6 + 35*(B*a + A*b)*x^3 - 7*A*a)/x^(5/2)$

giac [A] time = 0.26, size = 29, normalized size = 0.78

$$\frac{2}{7}Bbx^{7/2} + 2Ba\sqrt{x} + 2Ab\sqrt{x} - \frac{2Aa}{5x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2), x, algorithm="giac")

[Out] $2/7*B*b*x^(7/2) + 2*B*a*sqrt(x) + 2*A*b*sqrt(x) - 2/5*A*a/x^(5/2)$

maple [A] time = 0.04, size = 32, normalized size = 0.86

$$\frac{2(-5Bbx^6 - 35Abx^3 - 35Bax^3 + 7Aa)}{35x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(B*x^3+A)/x^(7/2), x)

[Out] $-2/35*(-5*B*b*x^6 - 35*A*b*x^3 - 35*B*a*x^3 + 7*A*a)/x^(5/2)$

maxima [A] time = 0.45, size = 27, normalized size = 0.73

$$\frac{2}{7} B b x^{\frac{7}{2}} + 2 (B a + A b) \sqrt{x} - \frac{2 A a}{5 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="maxima")

[Out] 2/7*B*b*x^(7/2) + 2*(B*a + A*b)*sqrt(x) - 2/5*A*a/x^(5/2)

mupad [B] time = 0.04, size = 30, normalized size = 0.81

$$\frac{2 A b x^3 - \frac{2 A a}{5} + 2 B a x^3 + \frac{2 B b x^6}{7}}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3))/x^(7/2),x)

[Out] (2*A*b*x^3 - (2*A*a)/5 + 2*B*a*x^3 + (2*B*b*x^6)/7)/x^(5/2)

sympy [A] time = 4.21, size = 42, normalized size = 1.14

$$-\frac{2 A a}{5 x^{\frac{5}{2}}} + 2 A b \sqrt{x} + 2 B a \sqrt{x} + \frac{2 B b x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(B*x**3+A)/x**(7/2),x)

[Out] -2*A*a/(5*x**(5/2)) + 2*A*b*sqrt(x) + 2*B*a*sqrt(x) + 2*B*b*x**(7/2)/7

$$3.135 \quad \int x^{7/2} (a + bx^3)^2 (A + Bx^3) dx$$

Optimal. Leaf size=63

$$\frac{2}{9}a^2Ax^{9/2} + \frac{2}{21}bx^{21/2}(2aB + Ab) + \frac{2}{15}ax^{15/2}(aB + 2Ab) + \frac{2}{27}b^2Bx^{27/2}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$\frac{2}{9}a^2Ax^{9/2} + \frac{2}{21}bx^{21/2}(2aB + Ab) + \frac{2}{15}ax^{15/2}(aB + 2Ab) + \frac{2}{27}b^2Bx^{27/2}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]

[Out] (2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(15/2))/15 + (2*b*(A*b + 2*a*B)*x^(21/2))/21 + (2*b^2*B*x^(27/2))/27

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^{7/2} (a + bx^3)^2 (A + Bx^3) dx &= \int (a^2Ax^{7/2} + a(2Ab + aB)x^{13/2} + b(Ab + 2aB)x^{19/2} + b^2Bx^{25/2}) dx \\ &= \frac{2}{9}a^2Ax^{9/2} + \frac{2}{15}a(2Ab + aB)x^{15/2} + \frac{2}{21}b(Ab + 2aB)x^{21/2} + \frac{2}{27}b^2Bx^{27/2} \end{aligned}$$

Mathematica [A] time = 0.06, size = 53, normalized size = 0.84

$$\frac{2}{945}x^{9/2} (105a^2A + 45bx^6(2aB + Ab) + 63ax^3(aB + 2Ab) + 35b^2Bx^9)$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]

[Out] $(2*x^{9/2}*(105*a^2*A + 63*a*(2*A*b + a*B)*x^3 + 45*b*(A*b + 2*a*B)*x^6 + 35*b^2*B*x^9))/945$

IntegrateAlgebraic [A] time = 0.03, size = 59, normalized size = 0.94

$$\frac{2}{945}x^{9/2} (105a^2A + 63a^2Bx^3 + 126aAbx^3 + 90abBx^6 + 45Ab^2x^6 + 35b^2Bx^9)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^3)^2*(A + B*x^3), x]

[Out] $(2*x^{9/2}*(105*a^2*A + 126*a*A*b*x^3 + 63*a^2*B*x^3 + 45*A*b^2*x^6 + 90*a*b*B*x^6 + 35*b^2*B*x^9))/945$

fricas [A] time = 0.76, size = 56, normalized size = 0.89

$$\frac{2}{945} (35Bb^2x^{13} + 45(2Bab + Ab^2)x^{10} + 63(Ba^2 + 2Aab)x^7 + 105Aa^2x^4)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A), x, algorithm="fricas")

[Out] $2/945*(35*B*b^2*x^{13} + 45*(2*B*a*b + A*b^2)*x^{10} + 63*(B*a^2 + 2*A*a*b)*x^7 + 105*A*a^2*x^4)*\text{sqrt}(x)$

giac [A] time = 0.15, size = 53, normalized size = 0.84

$$\frac{2}{27}Bb^2x^{\frac{27}{2}} + \frac{4}{21}Babx^{\frac{21}{2}} + \frac{2}{21}Ab^2x^{\frac{21}{2}} + \frac{2}{15}Ba^2x^{\frac{15}{2}} + \frac{4}{15}Aabx^{\frac{15}{2}} + \frac{2}{9}Aa^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A), x, algorithm="giac")

[Out] $2/27*B*b^2*x^{(27/2)} + 4/21*B*a*b*x^{(21/2)} + 2/21*A*b^2*x^{(21/2)} + 2/15*B*a^2*x^{(15/2)} + 4/15*A*a*b*x^{(15/2)} + 2/9*A*a^2*x^{(9/2)}$

maple [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2(35b^2Bx^9 + 45Ab^2x^6 + 90Babx^6 + 126Aabx^3 + 63Ba^2x^3 + 105a^2A)x^{\frac{9}{2}}}{945}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)*(b*x^3+a)^2*(B*x^3+A), x)

[Out] $2/945*x^{(9/2)}*(35*B*b^2*x^9+45*A*b^2*x^6+90*B*a*b*x^6+126*A*a*b*x^3+63*B*a^2*x^3+105*A*a^2)$

maxima [A] time = 0.56, size = 51, normalized size = 0.81

$$\frac{2}{27} B b^2 x^{\frac{27}{2}} + \frac{2}{21} (2 B a b + A b^2) x^{\frac{21}{2}} + \frac{2}{15} (B a^2 + 2 A a b) x^{\frac{15}{2}} + \frac{2}{9} A a^2 x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")`

[Out] $2/27*B*b^2*x^{(27/2)} + 2/21*(2*B*a*b + A*b^2)*x^{(21/2)} + 2/15*(B*a^2 + 2*A*a*b)*x^{(15/2)} + 2/9*A*a^2*x^{(9/2)}$

mupad [B] time = 2.57, size = 51, normalized size = 0.81

$$x^{15/2} \left(\frac{2 B a^2}{15} + \frac{4 A b a}{15} \right) + x^{21/2} \left(\frac{2 A b^2}{21} + \frac{4 B a b}{21} \right) + \frac{2 A a^2 x^{9/2}}{9} + \frac{2 B b^2 x^{27/2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(7/2)*(A + B*x^3)*(a + b*x^3)^2,x)`

[Out] $x^{(15/2)}*((2*B*a^2)/15 + (4*A*a*b)/15) + x^{(21/2)}*((2*A*b^2)/21 + (4*B*a*b)/21) + (2*A*a^2*x^{(9/2)})/9 + (2*B*b^2*x^{(27/2)})/27$

sympy [A] time = 47.81, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{9}{2}}}{9} + \frac{4Aabx^{\frac{15}{2}}}{15} + \frac{2Ab^2x^{\frac{21}{2}}}{21} + \frac{2Ba^2x^{\frac{15}{2}}}{15} + \frac{4Babx^{\frac{21}{2}}}{21} + \frac{2Bb^2x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(7/2)*(b*x**3+a)**2*(B*x**3+A),x)`

[Out] $2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(15/2)/15 + 2*A*b**2*x**(21/2)/21 + 2*B*a**2*x**(15/2)/15 + 4*B*a*b*x**(21/2)/21 + 2*B*b**2*x**(27/2)/27$

$$3.136 \quad \int x^{5/2} (a + bx^3)^2 (A + Bx^3) dx$$

Optimal. Leaf size=63

$$\frac{2}{7}a^2Ax^{7/2} + \frac{2}{19}bx^{19/2}(2aB + Ab) + \frac{2}{13}ax^{13/2}(aB + 2Ab) + \frac{2}{25}b^2Bx^{25/2}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$\frac{2}{7}a^2Ax^{7/2} + \frac{2}{19}bx^{19/2}(2aB + Ab) + \frac{2}{13}ax^{13/2}(aB + 2Ab) + \frac{2}{25}b^2Bx^{25/2}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)*(a + b*x^3)^2*(A + B*x^3), x]

[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)*x^(19/2))/19 + (2*b^2*B*x^(25/2))/25

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^{5/2} (a + bx^3)^2 (A + Bx^3) dx &= \int (a^2Ax^{5/2} + a(2Ab + aB)x^{11/2} + b(Ab + 2aB)x^{17/2} + b^2Bx^{23/2}) dx \\ &= \frac{2}{7}a^2Ax^{7/2} + \frac{2}{13}a(2Ab + aB)x^{13/2} + \frac{2}{19}b(Ab + 2aB)x^{19/2} + \frac{2}{25}b^2Bx^{25/2} \end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 1.00

$$\frac{2}{7}a^2Ax^{7/2} + \frac{2}{19}bx^{19/2}(2aB + Ab) + \frac{2}{13}ax^{13/2}(aB + 2Ab) + \frac{2}{25}b^2Bx^{25/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)*(a + b*x^3)^2*(A + B*x^3), x]

[Out] $(2*a^2*A*x^{(7/2)})/7 + (2*a*(2*A*b + a*B)*x^{(13/2)})/13 + (2*b*(A*b + 2*a*B)*x^{(19/2)})/19 + (2*b^2*B*x^{(25/2)})/25$

IntegrateAlgebraic [A] time = 0.03, size = 69, normalized size = 1.10

$$\frac{2 \left(6175a^2Ax^{7/2} + 3325a^2Bx^{13/2} + 6650aAbx^{13/2} + 4550abBx^{19/2} + 2275Ab^2x^{19/2} + 1729b^2Bx^{25/2} \right)}{43225}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^3)^2*(A + B*x^3), x]

[Out] $(2*(6175*a^2*A*x^{(7/2)} + 6650*a*A*b*x^{(13/2)} + 3325*a^2*B*x^{(13/2)} + 2275*A*b^2*x^{(19/2)} + 4550*a*b*B*x^{(19/2)} + 1729*b^2*B*x^{(25/2)}))/43225$

fricas [A] time = 0.76, size = 56, normalized size = 0.89

$$\frac{2}{43225} \left(1729 Bb^2x^{12} + 2275 (2 Bab + Ab^2)x^9 + 3325 (Ba^2 + 2 Aab)x^6 + 6175 Aa^2x^3 \right) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A), x, algorithm="fricas")

[Out] $2/43225*(1729*B*b^2*x^{12} + 2275*(2*B*a*b + A*b^2)*x^9 + 3325*(B*a^2 + 2*A*a*b)*x^6 + 6175*A*a^2*x^3)*\text{sqrt}(x)$

giac [A] time = 0.18, size = 53, normalized size = 0.84

$$\frac{2}{25} Bb^2x^{25} + \frac{4}{19} Babx^{19} + \frac{2}{19} Ab^2x^{19} + \frac{2}{13} Ba^2x^{13} + \frac{4}{13} Aabx^{13} + \frac{2}{7} Aa^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A), x, algorithm="giac")

[Out] $2/25*B*b^2*x^{(25/2)} + 4/19*B*a*b*x^{(19/2)} + 2/19*A*b^2*x^{(19/2)} + 2/13*B*a^2*x^{(13/2)} + 4/13*A*a*b*x^{(13/2)} + 2/7*A*a^2*x^{(7/2)}$

maple [A] time = 0.04, size = 56, normalized size = 0.89

$$\frac{2 \left(1729b^2Bx^9 + 2275Ab^2x^6 + 4550Babx^6 + 6650Aabx^3 + 3325Ba^2x^3 + 6175a^2A \right) x^{7/2}}{43225}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(b*x^3+a)^2*(B*x^3+A), x)

[Out] $2/43225*x^{(7/2)}*(1729*B*b^2*x^9+2275*A*b^2*x^6+4550*B*a*b*x^6+6650*A*a*b*x^3+3325*B*a^2*x^3+6175*A*a^2)$

maxima [A] time = 0.54, size = 51, normalized size = 0.81

$$\frac{2}{25} B b^2 x^{\frac{25}{2}} + \frac{2}{19} (2 B a b + A b^2) x^{\frac{19}{2}} + \frac{2}{13} (B a^2 + 2 A a b) x^{\frac{13}{2}} + \frac{2}{7} A a^2 x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")`

[Out] $2/25*B*b^2*x^{(25/2)} + 2/19*(2*B*a*b + A*b^2)*x^{(19/2)} + 2/13*(B*a^2 + 2*A*a*b)*x^{(13/2)} + 2/7*A*a^2*x^{(7/2)}$

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{13/2} \left(\frac{2 B a^2}{13} + \frac{4 A b a}{13} \right) + x^{19/2} \left(\frac{2 A b^2}{19} + \frac{4 B a b}{19} \right) + \frac{2 A a^2 x^{7/2}}{7} + \frac{2 B b^2 x^{25/2}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(5/2)*(A + B*x^3)*(a + b*x^3)^2,x)`

[Out] $x^{(13/2)}*((2*B*a^2)/13 + (4*A*a*b)/13) + x^{(19/2)}*((2*A*b^2)/19 + (4*B*a*b)/19) + (2*A*a^2*x^{(7/2)})/7 + (2*B*b^2*x^{(25/2)})/25$

sympy [A] time = 29.39, size = 80, normalized size = 1.27

$$\frac{2 A a^2 x^{\frac{7}{2}}}{7} + \frac{4 A a b x^{\frac{13}{2}}}{13} + \frac{2 A b^2 x^{\frac{19}{2}}}{19} + \frac{2 B a^2 x^{\frac{13}{2}}}{13} + \frac{4 B a b x^{\frac{19}{2}}}{19} + \frac{2 B b^2 x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(5/2)*(b*x**3+a)**2*(B*x**3+A),x)`

[Out] $2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(13/2)/13 + 2*A*b**2*x**(19/2)/19 + 2*B*a**2*x**(13/2)/13 + 4*B*a*b*x**(19/2)/19 + 2*B*b**2*x**(25/2)/25$

$$3.137 \quad \int x^{3/2} (a + bx^3)^2 (A + Bx^3) dx$$

Optimal. Leaf size=63

$$\frac{2}{5}a^2Ax^{5/2} + \frac{2}{17}bx^{17/2}(2aB + Ab) + \frac{2}{11}ax^{11/2}(aB + 2Ab) + \frac{2}{23}b^2Bx^{23/2}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$\frac{2}{5}a^2Ax^{5/2} + \frac{2}{17}bx^{17/2}(2aB + Ab) + \frac{2}{11}ax^{11/2}(aB + 2Ab) + \frac{2}{23}b^2Bx^{23/2}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)*(a + b*x^3)^2*(A + B*x^3), x]

[Out] (2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)*x^(17/2))/17 + (2*b^2*B*x^(23/2))/23

Rule 448

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^{3/2} (a + bx^3)^2 (A + Bx^3) dx &= \int (a^2Ax^{3/2} + a(2Ab + aB)x^{9/2} + b(Ab + 2aB)x^{15/2} + b^2Bx^{21/2}) dx \\ &= \frac{2}{5}a^2Ax^{5/2} + \frac{2}{11}a(2Ab + aB)x^{11/2} + \frac{2}{17}b(Ab + 2aB)x^{17/2} + \frac{2}{23}b^2Bx^{23/2} \end{aligned}$$

Mathematica [A] time = 0.06, size = 53, normalized size = 0.84

$$\frac{2x^{5/2} (4301a^2A + 1265bx^6(2aB + Ab) + 1955ax^3(aB + 2Ab) + 935b^2Bx^9)}{21505}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)*(a + b*x^3)^2*(A + B*x^3), x]

[Out] $(2*x^{5/2}*(4301*a^2*A + 1955*a*(2*A*b + a*B)*x^3 + 1265*b*(A*b + 2*a*B)*x^6 + 935*b^2*B*x^9))/21505$

IntegrateAlgebraic [A] time = 0.03, size = 69, normalized size = 1.10

$$\frac{2(4301a^2Ax^{5/2} + 1955a^2Bx^{11/2} + 3910aAbx^{11/2} + 2530abBx^{17/2} + 1265Ab^2x^{17/2} + 935b^2Bx^{23/2})}{21505}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^3)^2*(A + B*x^3), x]

[Out] $(2*(4301*a^2*A*x^{5/2} + 3910*a*A*b*x^{11/2} + 1955*a^2*B*x^{11/2} + 1265*A*b^2*x^{17/2} + 2530*a*b*B*x^{17/2} + 935*b^2*B*x^{23/2}))/21505$

fricas [A] time = 0.78, size = 56, normalized size = 0.89

$$\frac{2}{21505} (935 Bb^2x^{11} + 1265 (2 Bab + Ab^2)x^8 + 1955 (Ba^2 + 2 Aab)x^5 + 4301 Aa^2x^2)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A), x, algorithm="fricas")

[Out] $2/21505*(935*B*b^2*x^{11} + 1265*(2*B*a*b + A*b^2)*x^8 + 1955*(B*a^2 + 2*A*a*b)*x^5 + 4301*A*a^2*x^2)*\text{sqrt}(x)$

giac [A] time = 0.16, size = 53, normalized size = 0.84

$$\frac{2}{23} Bb^2x^{\frac{23}{2}} + \frac{4}{17} Babx^{\frac{17}{2}} + \frac{2}{17} Ab^2x^{\frac{17}{2}} + \frac{2}{11} Ba^2x^{\frac{11}{2}} + \frac{4}{11} Aabx^{\frac{11}{2}} + \frac{2}{5} Aa^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A), x, algorithm="giac")

[Out] $2/23*B*b^2*x^{23/2} + 4/17*B*a*b*x^{17/2} + 2/17*A*b^2*x^{17/2} + 2/11*B*a^2*x^{11/2} + 4/11*A*a*b*x^{11/2} + 2/5*A*a^2*x^{5/2}$

maple [A] time = 0.04, size = 56, normalized size = 0.89

$$\frac{2(935b^2Bx^9 + 1265A b^2x^6 + 2530Babx^6 + 3910Aabx^3 + 1955B a^2x^3 + 4301a^2A)x^{\frac{5}{2}}}{21505}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(b*x^3+a)^2*(B*x^3+A), x)

[Out] $2/21505*x^{(5/2)}*(935*B*b^2*x^9+1265*A*b^2*x^6+2530*B*a*b*x^6+3910*A*a*b*x^3+1955*B*a^2*x^3+4301*A*a^2)$

maxima [A] time = 0.50, size = 51, normalized size = 0.81

$$\frac{2}{23} B b^2 x^{\frac{23}{2}} + \frac{2}{17} (2 B a b + A b^2) x^{\frac{17}{2}} + \frac{2}{11} (B a^2 + 2 A a b) x^{\frac{11}{2}} + \frac{2}{5} A a^2 x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")`

[Out] $2/23*B*b^2*x^{(23/2)} + 2/17*(2*B*a*b + A*b^2)*x^{(17/2)} + 2/11*(B*a^2 + 2*A*a*b)*x^{(11/2)} + 2/5*A*a^2*x^{(5/2)}$

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{11/2} \left(\frac{2 B a^2}{11} + \frac{4 A b a}{11} \right) + x^{17/2} \left(\frac{2 A b^2}{17} + \frac{4 B a b}{17} \right) + \frac{2 A a^2 x^{5/2}}{5} + \frac{2 B b^2 x^{23/2}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)*(A + B*x^3)*(a + b*x^3)^2,x)`

[Out] $x^{(11/2)}*((2*B*a^2)/11 + (4*A*a*b)/11) + x^{(17/2)}*((2*A*b^2)/17 + (4*B*a*b)/17) + (2*A*a^2*x^{(5/2)})/5 + (2*B*b^2*x^{(23/2)})/23$

sympy [A] time = 21.96, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{5}{2}}}{5} + \frac{4Aabx^{\frac{11}{2}}}{11} + \frac{2Ab^2x^{\frac{17}{2}}}{17} + \frac{2Ba^2x^{\frac{11}{2}}}{11} + \frac{4Babx^{\frac{17}{2}}}{17} + \frac{2Bb^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2)*(b*x**3+a)**2*(B*x**3+A),x)`

[Out] $2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(17/2)/17 + 2*B*a**2*x**(11/2)/11 + 4*B*a*b*x**(17/2)/17 + 2*B*b**2*x**(23/2)/23$

$$3.138 \quad \int \sqrt{x} (a + bx^3)^2 (A + Bx^3) dx$$

Optimal. Leaf size=63

$$\frac{2}{3}a^2Ax^{3/2} + \frac{2}{15}bx^{15/2}(2aB + Ab) + \frac{2}{9}ax^{9/2}(aB + 2Ab) + \frac{2}{21}b^2Bx^{21/2}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$\frac{2}{3}a^2Ax^{3/2} + \frac{2}{15}bx^{15/2}(2aB + Ab) + \frac{2}{9}ax^{9/2}(aB + 2Ab) + \frac{2}{21}b^2Bx^{21/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3), x]

[Out] (2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x^(15/2))/15 + (2*b^2*B*x^(21/2))/21

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{x} (a + bx^3)^2 (A + Bx^3) dx &= \int (a^2A\sqrt{x} + a(2Ab + aB)x^{7/2} + b(Ab + 2aB)x^{13/2} + b^2Bx^{19/2}) dx \\ &= \frac{2}{3}a^2Ax^{3/2} + \frac{2}{9}a(2Ab + aB)x^{9/2} + \frac{2}{15}b(Ab + 2aB)x^{15/2} + \frac{2}{21}b^2Bx^{21/2} \end{aligned}$$

Mathematica [A] time = 0.11, size = 53, normalized size = 0.84

$$\frac{2}{315}x^{3/2} (105a^2A + 21bx^6(2aB + Ab) + 35ax^3(aB + 2Ab) + 15b^2Bx^9)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3), x]

[Out] $(2x^{(3/2)}*(105a^2A + 35a*(2Ab + aB)*x^3 + 21b*(Ab + 2aB)*x^6 + 15b^2B*x^9))/315$

IntegrateAlgebraic [A] time = 0.04, size = 69, normalized size = 1.10

$$\frac{2}{315} (105a^2Ax^{3/2} + 35a^2Bx^{9/2} + 70aAbx^{9/2} + 42abBx^{15/2} + 21Ab^2x^{15/2} + 15b^2Bx^{21/2})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3), x]

[Out] $(2*(105a^2A*x^{(3/2)} + 70aAb*x^{(9/2)} + 35a^2B*x^{(9/2)} + 21Ab^2*x^{(15/2)} + 42a*b*B*x^{(15/2)} + 15b^2*B*x^{(21/2)}))/315$

fricas [A] time = 0.63, size = 54, normalized size = 0.86

$$\frac{2}{315} (15Bb^2x^{10} + 21(2Bab + Ab^2)x^7 + 35(Ba^2 + 2Aab)x^4 + 105Aa^2x)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2), x, algorithm="fricas")

[Out] $2/315*(15B*b^2*x^{10} + 21*(2B*a*b + A*b^2)*x^7 + 35*(B*a^2 + 2A*a*b)*x^4 + 105*A*a^2*x)*sqrt(x)$

giac [A] time = 0.16, size = 53, normalized size = 0.84

$$\frac{2}{21} Bb^2x^{\frac{21}{2}} + \frac{4}{15} Babx^{\frac{15}{2}} + \frac{2}{15} Ab^2x^{\frac{15}{2}} + \frac{2}{9} Ba^2x^{\frac{9}{2}} + \frac{4}{9} Aabx^{\frac{9}{2}} + \frac{2}{3} Aa^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2), x, algorithm="giac")

[Out] $2/21*B*b^2*x^{(21/2)} + 4/15*B*a*b*x^{(15/2)} + 2/15*A*b^2*x^{(15/2)} + 2/9*B*a^2*x^{(9/2)} + 4/9*A*a*b*x^{(9/2)} + 2/3*A*a^2*x^{(3/2)}$

maple [A] time = 0.04, size = 56, normalized size = 0.89

$$\frac{2(15b^2Bx^9 + 21Ab^2x^6 + 42Babx^6 + 70Aabx^3 + 35Ba^2x^3 + 105a^2A)x^{\frac{3}{2}}}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(B*x^3+A)*x^(1/2), x)

[Out] $\frac{2}{315}x^{3/2}*(15*B*b^2*x^9+21*A*b^2*x^6+42*B*a*b*x^6+70*A*a*b*x^3+35*B*a^2*x^3+105*A*a^2)$

maxima [A] time = 0.54, size = 51, normalized size = 0.81

$$\frac{2}{21}Bb^2x^{\frac{21}{2}} + \frac{2}{15}(2Bab + Ab^2)x^{\frac{15}{2}} + \frac{2}{9}(Ba^2 + 2Aab)x^{\frac{9}{2}} + \frac{2}{3}Aa^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="maxima")`

[Out] $\frac{2}{21}B*b^2*x^{21/2} + \frac{2}{15}*(2*B*a*b + A*b^2)*x^{15/2} + \frac{2}{9}*(B*a^2 + 2*A*a*b)*x^{9/2} + \frac{2}{3}*A*a^2*x^{3/2}$

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{9/2} \left(\frac{2Ba^2}{9} + \frac{4Aba}{9} \right) + x^{15/2} \left(\frac{2Ab^2}{15} + \frac{4Bab}{15} \right) + \frac{2Aa^2x^{3/2}}{3} + \frac{2Bb^2x^{21/2}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(A + B*x^3)*(a + b*x^3)^2,x)`

[Out] $x^{9/2}*((2*B*a^2)/9 + (4*A*a*b)/9) + x^{15/2}*((2*A*b^2)/15 + (4*B*a*b)/15) + (2*A*a^2*x^{3/2})/3 + (2*B*b^2*x^{21/2})/21$

sympy [A] time = 5.31, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{3}{2}}}{3} + \frac{4Aabx^{\frac{9}{2}}}{9} + \frac{2Ab^2x^{\frac{15}{2}}}{15} + \frac{2Ba^2x^{\frac{9}{2}}}{9} + \frac{4Babx^{\frac{15}{2}}}{15} + \frac{2Bb^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*(B*x**3+A)*x**(1/2),x)`

[Out] $2*A*a**2*x**(3/2)/3 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(15/2)/15 + 2*B*a**2*x**(9/2)/9 + 4*B*a*b*x**(15/2)/15 + 2*B*b**2*x**(21/2)/21$

$$3.139 \quad \int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}} dx$$

Optimal. Leaf size=61

$$2a^2A\sqrt{x} + \frac{2}{13}bx^{13/2}(2aB + Ab) + \frac{2}{7}ax^{7/2}(aB + 2Ab) + \frac{2}{19}b^2Bx^{19/2}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$2a^2A\sqrt{x} + \frac{2}{13}bx^{13/2}(2aB + Ab) + \frac{2}{7}ax^{7/2}(aB + 2Ab) + \frac{2}{19}b^2Bx^{19/2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x], x]

[Out] 2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B))*x^(7/2))/7 + (2*b*(A*b + 2*a*B))*x^(13/2))/13 + (2*b^2*B*x^(19/2))/19

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}} dx &= \int \left(\frac{a^2A}{\sqrt{x}} + a(2Ab + aB)x^{5/2} + b(Ab + 2aB)x^{11/2} + b^2Bx^{17/2} \right) dx \\ &= 2a^2A\sqrt{x} + \frac{2}{7}a(2Ab + aB)x^{7/2} + \frac{2}{13}b(Ab + 2aB)x^{13/2} + \frac{2}{19}b^2Bx^{19/2} \end{aligned}$$

Mathematica [A] time = 0.07, size = 53, normalized size = 0.87

$$\frac{2\sqrt{x} (1729a^2A + 133bx^6(2aB + Ab) + 247ax^3(aB + 2Ab) + 91b^2Bx^9)}{1729}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x], x]

[Out] (2*Sqrt[x]*(1729*a^2*A + 247*a*(2*A*b + a*B)*x^3 + 133*b*(A*b + 2*a*B)*x^6 + 91*b^2*B*x^9))/1729

IntegrateAlgebraic [A] time = 0.03, size = 69, normalized size = 1.13

$$\frac{2(1729a^2A\sqrt{x} + 247a^2Bx^{7/2} + 494aAbx^{7/2} + 266abBx^{13/2} + 133Ab^2x^{13/2} + 91b^2Bx^{19/2})}{1729}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x], x]

[Out] (2*(1729*a^2*A*Sqrt[x] + 494*a*A*b*x^(7/2) + 247*a^2*B*x^(7/2) + 133*A*b^2*x^(13/2) + 266*a*b*B*x^(13/2) + 91*b^2*B*x^(19/2)))/1729

fricas [A] time = 0.82, size = 53, normalized size = 0.87

$$\frac{2}{1729} (91 B b^2 x^9 + 133 (2 B a b + A b^2) x^6 + 247 (B a^2 + 2 A a b) x^3 + 1729 A a^2) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2), x, algorithm="fricas")

[Out] 2/1729*(91*B*b^2*x^9 + 133*(2*B*a*b + A*b^2)*x^6 + 247*(B*a^2 + 2*A*a*b)*x^3 + 1729*A*a^2)*sqrt(x)

giac [A] time = 0.23, size = 53, normalized size = 0.87

$$\frac{2}{19} B b^2 x^{\frac{19}{2}} + \frac{4}{13} B a b x^{\frac{13}{2}} + \frac{2}{13} A b^2 x^{\frac{13}{2}} + \frac{2}{7} B a^2 x^{\frac{7}{2}} + \frac{4}{7} A a b x^{\frac{7}{2}} + 2 A a^2 \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2), x, algorithm="giac")

[Out] 2/19*B*b^2*x^(19/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) + 2/7*B*a^2*x^(7/2) + 4/7*A*a*b*x^(7/2) + 2*A*a^2*sqrt(x)

maple [A] time = 0.05, size = 56, normalized size = 0.92

$$\frac{2(91b^2Bx^9 + 133Ab^2x^6 + 266Babx^6 + 494Aabx^3 + 247Ba^2x^3 + 1729a^2A)\sqrt{x}}{1729}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x)`

[Out] $2/1729*x^{(1/2)}*(91*B*b^2*x^9+133*A*b^2*x^6+266*B*a*b*x^6+494*A*a*b*x^3+247*B*a^2*x^3+1729*A*a^2)$

maxima [A] time = 0.51, size = 51, normalized size = 0.84

$$\frac{2}{19} B b^2 x^{\frac{19}{2}} + \frac{2}{13} (2 B a b + A b^2) x^{\frac{13}{2}} + \frac{2}{7} (B a^2 + 2 A a b) x^{\frac{7}{2}} + 2 A a^2 \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="maxima")`

[Out] $2/19*B*b^2*x^{(19/2)} + 2/13*(2*B*a*b + A*b^2)*x^{(13/2)} + 2/7*(B*a^2 + 2*A*a*b)*x^{(7/2)} + 2*A*a^2*\text{sqrt}(x)$

mupad [B] time = 0.05, size = 51, normalized size = 0.84

$$x^{7/2} \left(\frac{2 B a^2}{7} + \frac{4 A b a}{7} \right) + x^{13/2} \left(\frac{2 A b^2}{13} + \frac{4 B a b}{13} \right) + 2 A a^2 \sqrt{x} + \frac{2 B b^2 x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^2)/x^(1/2),x)`

[Out] $x^{(7/2)}*((2*B*a^2)/7 + (4*A*a*b)/7) + x^{(13/2)}*((2*A*b^2)/13 + (4*B*a*b)/13) + 2*A*a^2*x^{(1/2)} + (2*B*b^2*x^{(19/2)})/19$

sympy [A] time = 9.22, size = 78, normalized size = 1.28

$$2 A a^2 \sqrt{x} + \frac{4 A a b x^{\frac{7}{2}}}{7} + \frac{2 A b^2 x^{\frac{13}{2}}}{13} + \frac{2 B a^2 x^{\frac{7}{2}}}{7} + \frac{4 B a b x^{\frac{13}{2}}}{13} + \frac{2 B b^2 x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*(B*x**3+A)/x**(1/2),x)`

[Out] $2*A*a**2*\text{sqrt}(x) + 4*A*a*b*x**(7/2)/7 + 2*A*b**2*x**(13/2)/13 + 2*B*a**2*x***(7/2)/7 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(19/2)/19$

$$3.140 \quad \int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}} dx$$

Optimal. Leaf size=61

$$-\frac{2a^2A}{\sqrt{x}} + \frac{2}{11}bx^{11/2}(2aB + Ab) + \frac{2}{5}ax^{5/2}(aB + 2Ab) + \frac{2}{17}b^2Bx^{17/2}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$-\frac{2a^2A}{\sqrt{x}} + \frac{2}{11}bx^{11/2}(2aB + Ab) + \frac{2}{5}ax^{5/2}(aB + 2Ab) + \frac{2}{17}b^2Bx^{17/2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(3/2), x]

[Out] (-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x^(11/2))/11 + (2*b^2*B*x^(17/2))/17

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}} dx &= \int \left(\frac{a^2A}{x^{3/2}} + a(2Ab + aB)x^{3/2} + b(Ab + 2aB)x^{9/2} + b^2Bx^{15/2} \right) dx \\ &= -\frac{2a^2A}{\sqrt{x}} + \frac{2}{5}a(2Ab + aB)x^{5/2} + \frac{2}{11}b(Ab + 2aB)x^{11/2} + \frac{2}{17}b^2Bx^{17/2} \end{aligned}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 0.98

$$\frac{-374a^2(5A - Bx^3) + 68abx^3(11A + 5Bx^3) + 10b^2x^6(17A + 11Bx^3)}{935\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(3/2), x]

[Out] (-374*a^2*(5*A - B*x^3) + 68*a*b*x^3*(11*A + 5*B*x^3) + 10*b^2*x^6*(17*A + 11*B*x^3))/(935*sqrt[x])

IntegrateAlgebraic [A] time = 0.04, size = 59, normalized size = 0.97

$$\frac{2(-935a^2A + 187a^2Bx^3 + 374aAbx^3 + 170abBx^6 + 85Ab^2x^6 + 55b^2Bx^9)}{935\sqrt{x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^(3/2), x]

[Out] (2*(-935*a^2*A + 374*a*A*b*x^3 + 187*a^2*B*x^3 + 85*A*b^2*x^6 + 170*a*b*B*x^6 + 55*b^2*B*x^9))/(935*sqrt[x])

fricas [A] time = 0.71, size = 53, normalized size = 0.87

$$\frac{2(55Bb^2x^9 + 85(2Bab + Ab^2)x^6 + 187(Ba^2 + 2Aab)x^3 - 935Aa^2)}{935\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2), x, algorithm="fricas")

[Out] 2/935*(55*B*b^2*x^9 + 85*(2*B*a*b + A*b^2)*x^6 + 187*(B*a^2 + 2*A*a*b)*x^3 - 935*A*a^2)/sqrt(x)

giac [A] time = 0.15, size = 53, normalized size = 0.87

$$\frac{2}{17}Bb^2x^{\frac{17}{2}} + \frac{4}{11}Babx^{\frac{11}{2}} + \frac{2}{11}Ab^2x^{\frac{11}{2}} + \frac{2}{5}Ba^2x^{\frac{5}{2}} + \frac{4}{5}Aabx^{\frac{5}{2}} - \frac{2Aa^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2), x, algorithm="giac")

[Out] 2/17*B*b^2*x^(17/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) + 2/5*B*a^2*x^(5/2) + 4/5*A*a*b*x^(5/2) - 2*A*a^2/sqrt(x)

maple [A] time = 0.05, size = 56, normalized size = 0.92

$$\frac{2(-55b^2Bx^9 - 85Ab^2x^6 - 170Babx^6 - 374Aabx^3 - 187Ba^2x^3 + 935a^2A)}{935\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x)`

[Out] $-2/935*(-55*B*b^2*x^9-85*A*b^2*x^6-170*B*a*b*x^6-374*A*a*b*x^3-187*B*a^2*x^3+935*A*a^2)/x^{(1/2)}$

maxima [A] time = 0.54, size = 51, normalized size = 0.84

$$\frac{2}{17} B b^2 x^{\frac{17}{2}} + \frac{2}{11} (2 B a b + A b^2) x^{\frac{11}{2}} + \frac{2}{5} (B a^2 + 2 A a b) x^{\frac{5}{2}} - \frac{2 A a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="maxima")`

[Out] $2/17*B*b^2*x^{(17/2)} + 2/11*(2*B*a*b + A*b^2)*x^{(11/2)} + 2/5*(B*a^2 + 2*A*a*b)*x^{(5/2)} - 2*A*a^2/\text{sqrt}(x)$

mupad [B] time = 0.05, size = 51, normalized size = 0.84

$$x^{5/2} \left(\frac{2 B a^2}{5} + \frac{4 A b a}{5} \right) + x^{11/2} \left(\frac{2 A b^2}{11} + \frac{4 B a b}{11} \right) - \frac{2 A a^2}{\sqrt{x}} + \frac{2 B b^2 x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^2)/x^(3/2),x)`

[Out] $x^{(5/2)}*((2*B*a^2)/5 + (4*A*a*b)/5) + x^{(11/2)}*((2*A*b^2)/11 + (4*B*a*b)/11) - (2*A*a^2)/x^{(1/2)} + (2*B*b^2*x^{(17/2)})/17$

sympy [A] time = 7.80, size = 78, normalized size = 1.28

$$-\frac{2 A a^2}{\sqrt{x}} + \frac{4 A a b x^{\frac{5}{2}}}{5} + \frac{2 A b^2 x^{\frac{11}{2}}}{11} + \frac{2 B a^2 x^{\frac{5}{2}}}{5} + \frac{4 B a b x^{\frac{11}{2}}}{11} + \frac{2 B b^2 x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*(B*x**3+A)/x**(3/2),x)`

[Out] $-2*A*a**2/\text{sqrt}(x) + 4*A*a*b*x**(5/2)/5 + 2*A*b**2*x**(11/2)/11 + 2*B*a**2*x**(5/2)/5 + 4*B*a*b*x**(11/2)/11 + 2*B*b**2*x**(17/2)/17$

$$3.141 \quad \int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}} dx$$

Optimal. Leaf size=63

$$-\frac{2a^2A}{3x^{3/2}} + \frac{2}{9}bx^{9/2}(2aB + Ab) + \frac{2}{3}ax^{3/2}(aB + 2Ab) + \frac{2}{15}b^2Bx^{15/2}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$-\frac{2a^2A}{3x^{3/2}} + \frac{2}{9}bx^{9/2}(2aB + Ab) + \frac{2}{3}ax^{3/2}(aB + 2Ab) + \frac{2}{15}b^2Bx^{15/2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(5/2), x]

[Out] (-2*a^2*A)/(3*x^(3/2)) + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B)*x^(9/2))/9 + (2*b^2*B*x^(15/2))/15

Rule 448

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}} dx &= \int \left(\frac{a^2A}{x^{5/2}} + a(2Ab + aB)\sqrt{x} + b(Ab + 2aB)x^{7/2} + b^2Bx^{13/2} \right) dx \\ &= -\frac{2a^2A}{3x^{3/2}} + \frac{2}{3}a(2Ab + aB)x^{3/2} + \frac{2}{9}b(Ab + 2aB)x^{9/2} + \frac{2}{15}b^2Bx^{15/2} \end{aligned}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 0.90

$$\frac{-30a^2(A - Bx^3) + 20abx^3(3A + Bx^3) + 2b^2x^6(5A + 3Bx^3)}{45x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]

[Out] $(-30*a^2*(A - B*x^3) + 20*a*b*x^3*(3*A + B*x^3) + 2*b^2*x^6*(5*A + 3*B*x^3))/(45*x^(3/2))$

IntegrateAlgebraic [A] time = 0.04, size = 59, normalized size = 0.94

$$\frac{2(-15a^2A + 15a^2Bx^3 + 30aAbx^3 + 10abBx^6 + 5Ab^2x^6 + 3b^2Bx^9)}{45x^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]

[Out] $(2*(-15*a^2*A + 30*a*A*b*x^3 + 15*a^2*B*x^3 + 5*A*b^2*x^6 + 10*a*b*B*x^6 + 3*b^2*B*x^9))/(45*x^(3/2))$

fricas [A] time = 0.77, size = 53, normalized size = 0.84

$$\frac{2(3Bb^2x^9 + 5(2Bab + Ab^2)x^6 + 15(Ba^2 + 2Aab)x^3 - 15Aa^2)}{45x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="fricas")

[Out] $2/45*(3*B*b^2*x^9 + 5*(2*B*a*b + A*b^2)*x^6 + 15*(B*a^2 + 2*A*a*b)*x^3 - 15*A*a^2)/x^(3/2)$

giac [A] time = 0.16, size = 53, normalized size = 0.84

$$\frac{2}{15}Bb^2x^{\frac{15}{2}} + \frac{4}{9}Babx^{\frac{9}{2}} + \frac{2}{9}Ab^2x^{\frac{9}{2}} + \frac{2}{3}Ba^2x^{\frac{3}{2}} + \frac{4}{3}Aabx^{\frac{3}{2}} - \frac{2Aa^2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="giac")

[Out] $2/15*B*b^2*x^(15/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/3*B*a^2*x^(3/2) + 4/3*A*a*b*x^(3/2) - 2/3*A*a^2/x^(3/2)$

maple [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2(-3b^2Bx^9 - 5Ab^2x^6 - 10Babx^6 - 30Aabx^3 - 15Ba^2x^3 + 15a^2A)}{45x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x)`

[Out] $-2/45*(-3*B*b^2*x^9-5*A*b^2*x^6-10*B*a*b*x^6-30*A*a*b*x^3-15*B*a^2*x^3+15*A*a^2)/x^(3/2)$

maxima [A] time = 0.51, size = 51, normalized size = 0.81

$$\frac{2}{15} B b^2 x^{\frac{15}{2}} + \frac{2}{9} (2 B a b + A b^2) x^{\frac{9}{2}} + \frac{2}{3} (B a^2 + 2 A a b) x^{\frac{3}{2}} - \frac{2 A a^2}{3 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="maxima")`

[Out] $2/15*B*b^2*x^(15/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) + 2/3*(B*a^2 + 2*A*a*b)*x^(3/2) - 2/3*A*a^2/x^(3/2)$

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{3/2} \left(\frac{2 B a^2}{3} + \frac{4 A b a}{3} \right) + x^{9/2} \left(\frac{2 A b^2}{9} + \frac{4 B a b}{9} \right) - \frac{2 A a^2}{3 x^{3/2}} + \frac{2 B b^2 x^{15/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^2)/x^(5/2),x)`

[Out] $x^(3/2)*((2*B*a^2)/3 + (4*A*a*b)/3) + x^(9/2)*((2*A*b^2)/9 + (4*B*a*b)/9) - (2*A*a^2)/(3*x^(3/2)) + (2*B*b^2*x^(15/2))/15$

sympy [A] time = 11.74, size = 80, normalized size = 1.27

$$-\frac{2 A a^2}{3 x^{\frac{3}{2}}} + \frac{4 A a b x^{\frac{3}{2}}}{3} + \frac{2 A b^2 x^{\frac{9}{2}}}{9} + \frac{2 B a^2 x^{\frac{3}{2}}}{3} + \frac{4 B a b x^{\frac{9}{2}}}{9} + \frac{2 B b^2 x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*(B*x**3+A)/x**(5/2),x)`

[Out] $-2*A*a**2/(3*x**(3/2)) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(9/2)/9 + 2*B*a**2*x**(3/2)/3 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(15/2)/15$

$$3.142 \quad \int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}} dx$$

Optimal. Leaf size=61

$$-\frac{2a^2A}{5x^{5/2}} + \frac{2}{7}bx^{7/2}(2aB + Ab) + 2a\sqrt{x}(aB + 2Ab) + \frac{2}{13}b^2Bx^{13/2}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$-\frac{2a^2A}{5x^{5/2}} + \frac{2}{7}bx^{7/2}(2aB + Ab) + 2a\sqrt{x}(aB + 2Ab) + \frac{2}{13}b^2Bx^{13/2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(7/2), x]

[Out] (-2*a^2*A)/(5*x^(5/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^(7/2))/7 + (2*b^2*B*x^(13/2))/13

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}} dx &= \int \left(\frac{a^2A}{x^{7/2}} + \frac{a(2Ab+aB)}{\sqrt{x}} + b(Ab+2aB)x^{5/2} + b^2Bx^{11/2} \right) dx \\ &= -\frac{2a^2A}{5x^{5/2}} + 2a(2Ab+aB)\sqrt{x} + \frac{2}{7}b(Ab+2aB)x^{7/2} + \frac{2}{13}b^2Bx^{13/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.93

$$\frac{2(-91a^2(A-5Bx^3) + 130abx^3(7A+Bx^3) + 5b^2x^6(13A+7Bx^3))}{455x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(7/2), x]

[Out] (2*(-91*a^2*(A - 5*B*x^3) + 130*a*b*x^3*(7*A + B*x^3) + 5*b^2*x^6*(13*A + 7*B*x^3)))/(455*x^(5/2))

IntegrateAlgebraic [A] time = 0.04, size = 59, normalized size = 0.97

$$\frac{2(-91a^2A + 455a^2Bx^3 + 910aAbx^3 + 130abBx^6 + 65Ab^2x^6 + 35b^2Bx^9)}{455x^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^(7/2), x]

[Out] (2*(-91*a^2*A + 910*a*A*b*x^3 + 455*a^2*B*x^3 + 65*A*b^2*x^6 + 130*a*b*B*x^6 + 35*b^2*B*x^9))/(455*x^(5/2))

fricas [A] time = 0.88, size = 53, normalized size = 0.87

$$\frac{2(35Bb^2x^9 + 65(2Bab + Ab^2)x^6 + 455(Ba^2 + 2Aab)x^3 - 91Aa^2)}{455x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2), x, algorithm="fricas")

[Out] 2/455*(35*B*b^2*x^9 + 65*(2*B*a*b + A*b^2)*x^6 + 455*(B*a^2 + 2*A*a*b)*x^3 - 91*A*a^2)/x^(5/2)

giac [A] time = 0.15, size = 53, normalized size = 0.87

$$\frac{2}{13}Bb^2x^{13/2} + \frac{4}{7}Babx^{7/2} + \frac{2}{7}Ab^2x^{7/2} + 2Ba^2\sqrt{x} + 4Aab\sqrt{x} - \frac{2Aa^2}{5x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2), x, algorithm="giac")

[Out] 2/13*B*b^2*x^(13/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2*B*a^2*sqrt(x) + 4*A*a*b*sqrt(x) - 2/5*A*a^2/x^(5/2)

maple [A] time = 0.05, size = 56, normalized size = 0.92

$$\frac{2(-35b^2Bx^9 - 65Ab^2x^6 - 130Babx^6 - 910Aabx^3 - 455Ba^2x^3 + 91a^2A)}{455x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x)`

[Out] $-2/455*(-35*B*b^2*x^9-65*A*b^2*x^6-130*B*a*b*x^6-910*A*a*b*x^3-455*B*a^2*x^3+91*A*a^2)/x^{5/2}$

maxima [A] time = 0.46, size = 51, normalized size = 0.84

$$\frac{2}{13} B b^2 x^{\frac{13}{2}} + \frac{2}{7} (2 B a b + A b^2) x^{\frac{7}{2}} + 2 (B a^2 + 2 A a b) \sqrt{x} - \frac{2 A a^2}{5 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="maxima")`

[Out] $2/13*B*b^2*x^{13/2} + 2/7*(2*B*a*b + A*b^2)*x^{7/2} + 2*(B*a^2 + 2*A*a*b)*\text{sqrt}(x) - 2/5*A*a^2/x^{5/2}$

mupad [B] time = 0.05, size = 51, normalized size = 0.84

$$\sqrt{x} (2 B a^2 + 4 A b a) + x^{7/2} \left(\frac{2 A b^2}{7} + \frac{4 B a b}{7} \right) - \frac{2 A a^2}{5 x^{5/2}} + \frac{2 B b^2 x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^2)/x^(7/2),x)`

[Out] $x^{1/2}*(2*B*a^2 + 4*A*a*b) + x^{7/2}*((2*A*b^2)/7 + (4*B*a*b)/7) - (2*A*a^2)/(5*x^{5/2}) + (2*B*b^2*x^{13/2})/13$

sympy [A] time = 18.41, size = 76, normalized size = 1.25

$$-\frac{2 A a^2}{5 x^{\frac{5}{2}}} + 4 A a b \sqrt{x} + \frac{2 A b^2 x^{\frac{7}{2}}}{7} + 2 B a^2 \sqrt{x} + \frac{4 B a b x^{\frac{7}{2}}}{7} + \frac{2 B b^2 x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*(B*x**3+A)/x**(7/2),x)`

[Out] $-2*A*a**2/(5*x**(5/2)) + 4*A*a*b*\text{sqrt}(x) + 2*A*b**2*x**(7/2)/7 + 2*B*a**2*\text{sqrt}(x) + 4*B*a*b*x**(7/2)/7 + 2*B*b**2*x**(13/2)/13$

$$3.143 \quad \int x^{7/2} (a + bx^3)^3 (A + Bx^3) dx$$

Optimal. Leaf size=85

$$\frac{2}{9}a^3Ax^{9/2} + \frac{2}{15}a^2x^{15/2}(aB + 3Ab) + \frac{2}{27}b^2x^{27/2}(3aB + Ab) + \frac{2}{7}abx^{21/2}(aB + Ab) + \frac{2}{33}b^3Bx^{33/2}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$\frac{2}{15}a^2x^{15/2}(aB + 3Ab) + \frac{2}{9}a^3Ax^{9/2} + \frac{2}{27}b^2x^{27/2}(3aB + Ab) + \frac{2}{7}abx^{21/2}(aB + Ab) + \frac{2}{33}b^3Bx^{33/2}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)*(a + b*x^3)^3*(A + B*x^3), x]

[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(15/2))/15 + (2*a*b*(A*b + a*B)*x^(21/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(27/2))/27 + (2*b^3*B*x^(33/2))/33

Rule 448

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^{7/2} (a + bx^3)^3 (A + Bx^3) dx &= \int (a^3Ax^{7/2} + a^2(3Ab + aB)x^{13/2} + 3ab(Ab + aB)x^{19/2} + b^2(Ab + 3aB)x^{25/2} + Bb^3x^{31/2}) dx \\ &= \frac{2}{9}a^3Ax^{9/2} + \frac{2}{15}a^2(3Ab + aB)x^{15/2} + \frac{2}{7}ab(Ab + aB)x^{21/2} + \frac{2}{27}b^2(Ab + 3aB)x^{27/2} + \frac{2}{33}Bb^3x^{33/2} \end{aligned}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.84

$$\frac{2x^{9/2} (1155a^3A + 693a^2x^3(aB + 3Ab) + 385b^2x^9(3aB + Ab) + 1485abx^6(aB + Ab) + 315b^3Bx^{12})}{10395}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)*(a + b*x^3)^3*(A + B*x^3), x]

[Out] $(2*x^{(9/2)}*(1155*a^3*A + 693*a^2*(3*A*b + a*B)*x^3 + 1485*a*b*(A*b + a*B)*x^6 + 385*b^2*(A*b + 3*a*B)*x^9 + 315*b^3*B*x^{12}))/10395$

IntegrateAlgebraic [A] time = 0.04, size = 97, normalized size = 1.14

$$\frac{2(1155a^3Ax^{9/2} + 693a^3Bx^{15/2} + 2079a^2Abx^{15/2} + 1485a^2bBx^{21/2} + 1485aAb^2x^{21/2} + 1155ab^2Bx^{27/2} + 385Ab^3x^{27/2} + 315b^3Bx^{33/2})}{10395}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^3)^3*(A + B*x^3), x]

[Out] $(2*(1155*a^3*A*x^{(9/2)} + 2079*a^2*A*b*x^{(15/2)} + 693*a^3*B*x^{(15/2)} + 1485*a*A*b^2*x^{(21/2)} + 1485*a^2*b*B*x^{(21/2)} + 385*A*b^3*x^{(27/2)} + 1155*a*b^2*B*x^{(27/2)} + 315*b^3*B*x^{(33/2)}))/10395$

fricas [A] time = 0.72, size = 78, normalized size = 0.92

$$\frac{2}{10395} (315 Bb^3x^{16} + 385 (3 Bab^2 + Ab^3)x^{13} + 1485 (Ba^2b + Aab^2)x^{10} + 1155 Aa^3x^4 + 693 (Ba^3 + 3 Aa^2b)x^7)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A), x, algorithm="fricas")

[Out] $2/10395*(315*B*b^3*x^{16} + 385*(3*B*a*b^2 + A*b^3)*x^{13} + 1485*(B*a^2*b + A*a*b^2)*x^{10} + 1155*A*a^3*x^4 + 693*(B*a^3 + 3*A*a^2*b)*x^7)*\text{sqrt}(x)$

giac [A] time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{33} Bb^3x^{\frac{33}{2}} + \frac{2}{9} Bab^2x^{\frac{27}{2}} + \frac{2}{27} Ab^3x^{\frac{27}{2}} + \frac{2}{7} Ba^2bx^{\frac{21}{2}} + \frac{2}{7} Aab^2x^{\frac{21}{2}} + \frac{2}{15} Ba^3x^{\frac{15}{2}} + \frac{2}{5} Aa^2bx^{\frac{15}{2}} + \frac{2}{9} Aa^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A), x, algorithm="giac")

[Out] $2/33*B*b^3*x^{(33/2)} + 2/9*B*a*b^2*x^{(27/2)} + 2/27*A*b^3*x^{(27/2)} + 2/7*B*a^2*b*x^{(21/2)} + 2/7*A*a*b^2*x^{(21/2)} + 2/15*B*a^3*x^{(15/2)} + 2/5*A*a^2*b*x^{(15/2)} + 2/9*A*a^3*x^{(9/2)}$

maple [A] time = 0.04, size = 80, normalized size = 0.94

$$\frac{2(315Bb^3x^{12} + 385x^9Ab^3 + 1155x^9Bab^2 + 1485x^6Aab^2 + 1485x^6B a^2b + 2079x^3A a^2b + 693B a^3x^3 + 1155A a^3)x^{\frac{9}{2}}}{10395}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)*(b*x^3+a)^3*(B*x^3+A), x)

[Out] $2/10395*x^{(9/2)}*(315*B*b^3*x^{12}+385*A*b^3*x^9+1155*B*a*b^2*x^9+1485*A*a*b^2*x^6+1485*B*a^2*b*x^6+2079*A*a^2*b*x^3+693*B*a^3*x^3+1155*A*a^3)$

maxima [A] time = 0.57, size = 73, normalized size = 0.86

$$\frac{2}{33} B b^3 x^{\frac{33}{2}} + \frac{2}{27} (3 B a b^2 + A b^3) x^{\frac{27}{2}} + \frac{2}{7} (B a^2 b + A a b^2) x^{\frac{21}{2}} + \frac{2}{9} A a^3 x^{\frac{9}{2}} + \frac{2}{15} (B a^3 + 3 A a^2 b) x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")`

[Out] $2/33*B*b^3*x^{(33/2)} + 2/27*(3*B*a*b^2 + A*b^3)*x^{(27/2)} + 2/7*(B*a^2*b + A*a*b^2)*x^{(21/2)} + 2/9*A*a^3*x^{(9/2)} + 2/15*(B*a^3 + 3*A*a^2*b)*x^{(15/2)}$

mupad [B] time = 2.52, size = 69, normalized size = 0.81

$$x^{15/2} \left(\frac{2 B a^3}{15} + \frac{2 A b a^2}{5} \right) + x^{27/2} \left(\frac{2 A b^3}{27} + \frac{2 B a b^2}{9} \right) + \frac{2 A a^3 x^{9/2}}{9} + \frac{2 B b^3 x^{33/2}}{33} + \frac{2 a b x^{21/2} (A b + B a)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(7/2)*(A + B*x^3)*(a + b*x^3)^3,x)`

[Out] $x^{(15/2)}*((2*B*a^3)/15 + (2*A*a^2*b)/5) + x^{(27/2)}*((2*A*b^3)/27 + (2*B*a*b^2)/9) + (2*A*a^3*x^{(9/2)})/9 + (2*B*b^3*x^{(33/2)})/33 + (2*a*b*x^{(21/2)}*(A*b + B*a))/7$

sympy [A] time = 92.99, size = 114, normalized size = 1.34

$$\frac{2 A a^3 x^{\frac{9}{2}}}{9} + \frac{2 A a^2 b x^{\frac{15}{2}}}{5} + \frac{2 A a b^2 x^{\frac{21}{2}}}{7} + \frac{2 A b^3 x^{\frac{27}{2}}}{27} + \frac{2 B a^3 x^{\frac{15}{2}}}{15} + \frac{2 B a^2 b x^{\frac{21}{2}}}{7} + \frac{2 B a b^2 x^{\frac{27}{2}}}{9} + \frac{2 B b^3 x^{\frac{33}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(7/2)*(b*x**3+a)**3*(B*x**3+A),x)`

[Out] $2*A*a**3*x**(9/2)/9 + 2*A*a**2*b*x**(15/2)/5 + 2*A*a*b**2*x**(21/2)/7 + 2*A*b**3*x**(27/2)/27 + 2*B*a**3*x**(15/2)/15 + 2*B*a**2*b*x**(21/2)/7 + 2*B*a*b**2*x**(27/2)/9 + 2*B*b**3*x**(33/2)/33$

$$3.144 \quad \int x^{5/2} (a + bx^3)^3 (A + Bx^3) dx$$

Optimal. Leaf size=85

$$\frac{2}{7}a^3Ax^{7/2} + \frac{2}{13}a^2x^{13/2}(aB + 3Ab) + \frac{2}{25}b^2x^{25/2}(3aB + Ab) + \frac{6}{19}abx^{19/2}(aB + Ab) + \frac{2}{31}b^3Bx^{31/2}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$\frac{2}{13}a^2x^{13/2}(aB + 3Ab) + \frac{2}{7}a^3Ax^{7/2} + \frac{2}{25}b^2x^{25/2}(3aB + Ab) + \frac{6}{19}abx^{19/2}(aB + Ab) + \frac{2}{31}b^3Bx^{31/2}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)*(a + b*x^3)^3*(A + B*x^3), x]

[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B)*x^(19/2))/19 + (2*b^2*(A*b + 3*a*B)*x^(25/2))/25 + (2*b^3*B*x^(31/2))/31

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^{5/2} (a + bx^3)^3 (A + Bx^3) dx &= \int (a^3Ax^{5/2} + a^2(3Ab + aB)x^{11/2} + 3ab(Ab + aB)x^{17/2} + b^2(Ab + 3aB)x^{23/2} + \\ &= \frac{2}{7}a^3Ax^{7/2} + \frac{2}{13}a^2(3Ab + aB)x^{13/2} + \frac{6}{19}ab(Ab + aB)x^{19/2} + \frac{2}{25}b^2(Ab + 3aB)x^{25/2} + \frac{2}{31}b^3Bx^{31/2} \end{aligned}$$

Mathematica [A] time = 0.10, size = 85, normalized size = 1.00

$$\frac{2}{7}a^3Ax^{7/2} + \frac{2}{13}a^2x^{13/2}(aB + 3Ab) + \frac{2}{25}b^2x^{25/2}(3aB + Ab) + \frac{6}{19}abx^{19/2}(aB + Ab) + \frac{2}{31}b^3Bx^{31/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)*(a + b*x^3)^3*(A + B*x^3), x]

[Out] $(2*a^3*A*x^{(7/2)})/7 + (2*a^2*(3*A*b + a*B)*x^{(13/2)})/13 + (6*a*b*(A*b + a*B)*x^{(19/2)})/19 + (2*b^2*(A*b + 3*a*B)*x^{(25/2)})/25 + (2*b^3*B*x^{(31/2)})/31$

IntegrateAlgebraic [A] time = 0.04, size = 97, normalized size = 1.14

$$\frac{2(191425a^3Ax^{7/2} + 103075a^3Bx^{13/2} + 309225a^2Abx^{13/2} + 211575a^2bBx^{19/2} + 211575aAb^2x^{19/2} + 160797ab^2Bx^{25/2} + 53599Ab^3x^{25/2} + 43225b^3Bx^{31/2})}{1339975}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^3)^3*(A + B*x^3), x]

[Out] $(2*(191425*a^3*A*x^{(7/2)} + 309225*a^2*A*b*x^{(13/2)} + 103075*a^3*B*x^{(13/2)} + 211575*a*A*b^2*x^{(19/2)} + 211575*a^2*b*B*x^{(19/2)} + 53599*A*b^3*x^{(25/2)} + 160797*a*b^2*B*x^{(25/2)} + 43225*b^3*B*x^{(31/2)}))/1339975$

fricas [A] time = 0.82, size = 78, normalized size = 0.92

$$\frac{2}{1339975} (43225 B b^3 x^{15} + 53599 (3 B a b^2 + A b^3) x^{12} + 211575 (B a^2 b + A a b^2) x^9 + 191425 A a^3 x^3 + 103075 (B a^3 + 3 A a^2 b) x^6) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A), x, algorithm="fricas")

[Out] $2/1339975*(43225*B*b^3*x^{15} + 53599*(3*B*a*b^2 + A*b^3)*x^{12} + 211575*(B*a^2*b + A*a*b^2)*x^9 + 191425*A*a^3*x^3 + 103075*(B*a^3 + 3*A*a^2*b)*x^6)*\sqrt{x}$

giac [A] time = 0.15, size = 77, normalized size = 0.91

$$\frac{2}{31} B b^3 x^{31/2} + \frac{6}{25} B a b^2 x^{25/2} + \frac{2}{25} A b^3 x^{25/2} + \frac{6}{19} B a^2 b x^{19/2} + \frac{6}{19} A a b^2 x^{19/2} + \frac{2}{13} B a^3 x^{13/2} + \frac{6}{13} A a^2 b x^{13/2} + \frac{2}{7} A a^3 x^{7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A), x, algorithm="giac")

[Out] $2/31*B*b^3*x^{(31/2)} + 6/25*B*a*b^2*x^{(25/2)} + 2/25*A*b^3*x^{(25/2)} + 6/19*B*a^2*b*x^{(19/2)} + 6/19*A*a*b^2*x^{(19/2)} + 2/13*B*a^3*x^{(13/2)} + 6/13*A*a^2*b*x^{(13/2)} + 2/7*A*a^3*x^{(7/2)}$

maple [A] time = 0.04, size = 80, normalized size = 0.94

$$\frac{2(43225Bb^3x^{12} + 53599x^9Ab^3 + 160797x^9Ba^2b + 211575x^6Aa^2b + 211575x^6Ba^2b + 309225x^3Aa^2b + 103075Ba^3x^3 + 191425Aa^3)x^2}{1339975}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(b*x^3+a)^3*(B*x^3+A), x)

[Out] $2/1339975*x^{(7/2)}*(43225*B*b^3*x^{12}+53599*A*b^3*x^9+160797*B*a*b^2*x^9+211575*A*a*b^2*x^6+211575*B*a^2*b*x^6+309225*A*a^2*b*x^3+103075*B*a^3*x^3+191425*A*a^3)$

maxima [A] time = 0.50, size = 73, normalized size = 0.86

$$\frac{2}{31} B b^3 x^{\frac{31}{2}} + \frac{2}{25} (3 B a b^2 + A b^3) x^{\frac{25}{2}} + \frac{6}{19} (B a^2 b + A a b^2) x^{\frac{19}{2}} + \frac{2}{7} A a^3 x^{\frac{7}{2}} + \frac{2}{13} (B a^3 + 3 A a^2 b) x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")`

[Out] $2/31*B*b^3*x^{(31/2)} + 2/25*(3*B*a*b^2 + A*b^3)*x^{(25/2)} + 6/19*(B*a^2*b + A*a*b^2)*x^{(19/2)} + 2/7*A*a^3*x^{(7/2)} + 2/13*(B*a^3 + 3*A*a^2*b)*x^{(13/2)}$

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{13/2} \left(\frac{2 B a^3}{13} + \frac{6 A b a^2}{13} \right) + x^{25/2} \left(\frac{2 A b^3}{25} + \frac{6 B a b^2}{25} \right) + \frac{2 A a^3 x^{7/2}}{7} + \frac{2 B b^3 x^{31/2}}{31} + \frac{6 a b x^{19/2} (A b + B a)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(5/2)*(A + B*x^3)*(a + b*x^3)^3,x)`

[Out] $x^{(13/2)}*((2*B*a^3)/13 + (6*A*a^2*b)/13) + x^{(25/2)}*((2*A*b^3)/25 + (6*B*a*b^2)/25) + (2*A*a^3*x^{(7/2)})/7 + (2*B*b^3*x^{(31/2)})/31 + (6*a*b*x^{(19/2)}*(A*b + B*a))/19$

sympy [A] time = 58.70, size = 114, normalized size = 1.34

$$\frac{2 A a^3 x^{\frac{7}{2}}}{7} + \frac{6 A a^2 b x^{\frac{13}{2}}}{13} + \frac{6 A a b^2 x^{\frac{19}{2}}}{19} + \frac{2 A b^3 x^{\frac{25}{2}}}{25} + \frac{2 B a^3 x^{\frac{13}{2}}}{13} + \frac{6 B a^2 b x^{\frac{19}{2}}}{19} + \frac{6 B a b^2 x^{\frac{25}{2}}}{25} + \frac{2 B b^3 x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(5/2)*(b*x**3+a)**3*(B*x**3+A),x)`

[Out] $2*A*a**3*x**(7/2)/7 + 6*A*a**2*b*x**(13/2)/13 + 6*A*a*b**2*x**(19/2)/19 + 2*A*b**3*x**(25/2)/25 + 2*B*a**3*x**(13/2)/13 + 6*B*a**2*b*x**(19/2)/19 + 6*B*a*b**2*x**(25/2)/25 + 2*B*b**3*x**(31/2)/31$

$$3.145 \quad \int x^{3/2} (a + bx^3)^3 (A + Bx^3) dx$$

Optimal. Leaf size=85

$$\frac{2}{5}a^3Ax^{5/2} + \frac{2}{11}a^2x^{11/2}(aB + 3Ab) + \frac{2}{23}b^2x^{23/2}(3aB + Ab) + \frac{6}{17}abx^{17/2}(aB + Ab) + \frac{2}{29}b^3Bx^{29/2}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$\frac{2}{11}a^2x^{11/2}(aB + 3Ab) + \frac{2}{5}a^3Ax^{5/2} + \frac{2}{23}b^2x^{23/2}(3aB + Ab) + \frac{6}{17}abx^{17/2}(aB + Ab) + \frac{2}{29}b^3Bx^{29/2}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)*(a + b*x^3)^3*(A + B*x^3), x]

[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (6*a*b*(A*b + a*B)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(23/2))/23 + (2*b^3*B*x^(29/2))/29

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^{3/2} (a + bx^3)^3 (A + Bx^3) dx &= \int (a^3Ax^{3/2} + a^2(3Ab + aB)x^{9/2} + 3ab(Ab + aB)x^{15/2} + b^2(Ab + 3aB)x^{21/2} + b^3Bx^{27/2}) dx \\ &= \frac{2}{5}a^3Ax^{5/2} + \frac{2}{11}a^2(3Ab + aB)x^{11/2} + \frac{6}{17}ab(Ab + aB)x^{17/2} + \frac{2}{23}b^2(Ab + 3aB)x^{23/2} + \frac{2}{29}b^3Bx^{29/2} \end{aligned}$$

Mathematica [A] time = 0.08, size = 85, normalized size = 1.00

$$\frac{2}{5}a^3Ax^{5/2} + \frac{2}{11}a^2x^{11/2}(aB + 3Ab) + \frac{2}{23}b^2x^{23/2}(3aB + Ab) + \frac{6}{17}abx^{17/2}(aB + Ab) + \frac{2}{29}b^3Bx^{29/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)*(a + b*x^3)^3*(A + B*x^3), x]

[Out] $(2*a^3*A*x^{(5/2)})/5 + (2*a^2*(3*A*b + a*B)*x^{(11/2)})/11 + (6*a*b*(A*b + a*B)*x^{(17/2)})/17 + (2*b^2*(A*b + 3*a*B)*x^{(23/2)})/23 + (2*b^3*B*x^{(29/2)})/29$

IntegrateAlgebraic [A] time = 0.04, size = 97, normalized size = 1.14

$$\frac{2(124729a^3Ax^{5/2} + 56695a^3Bx^{11/2} + 170085a^2Abx^{17/2} + 110055a^2bBx^{17/2} + 110055aAb^2x^{23/2} + 81345ab^2Bx^{23/2} + 27115Ab^3x^{29/2} + 21505b^3Bx^{29/2})}{623645}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^3)^3*(A + B*x^3), x]

[Out] $(2*(124729*a^3*A*x^{(5/2)} + 170085*a^2*A*b*x^{(11/2)} + 56695*a^3*B*x^{(11/2)} + 110055*a*A*b^2*x^{(17/2)} + 110055*a^2*b*B*x^{(17/2)} + 27115*A*b^3*x^{(23/2)} + 81345*a*b^2*B*x^{(23/2)} + 21505*b^3*B*x^{(29/2)}))/623645$

fricas [A] time = 0.83, size = 78, normalized size = 0.92

$$\frac{2}{623645} (21505 Bb^3x^{14} + 27115 (3 Bab^2 + Ab^3)x^{11} + 110055 (Ba^2b + Aab^2)x^8 + 124729 Aa^3x^2 + 56695 (Ba^3 + 3 Aa^2b)x^5)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A), x, algorithm="fricas")

[Out] $2/623645*(21505*B*b^3*x^{14} + 27115*(3*B*a*b^2 + A*b^3)*x^{11} + 110055*(B*a^2*b + A*a*b^2)*x^8 + 124729*A*a^3*x^2 + 56695*(B*a^3 + 3*A*a^2*b)*x^5)*\sqrt{x}$

giac [A] time = 0.17, size = 77, normalized size = 0.91

$$\frac{2}{29} Bb^3x^{29} + \frac{6}{23} Bab^2x^{23} + \frac{2}{23} Ab^3x^{23} + \frac{6}{17} Ba^2bx^{17} + \frac{6}{17} Aab^2x^{17} + \frac{2}{11} Ba^3x^{11} + \frac{6}{11} Aa^2bx^{11} + \frac{2}{5} Aa^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A), x, algorithm="giac")

[Out] $2/29*B*b^3*x^{(29/2)} + 6/23*B*a*b^2*x^{(23/2)} + 2/23*A*b^3*x^{(23/2)} + 6/17*B*a^2*b*x^{(17/2)} + 6/17*A*a*b^2*x^{(17/2)} + 2/11*B*a^3*x^{(11/2)} + 6/11*A*a^2*b*x^{(11/2)} + 2/5*A*a^3*x^{(5/2)}$

maple [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2(21505Bb^3x^{12} + 27115x^9Ab^3 + 81345x^9Ba^2b^2 + 110055x^6Aa^2b^2 + 110055x^6Ba^2b + 170085x^3Aa^2b + 56695Ba^3x^3 + 124729Aa^3)x^2}{623645}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(b*x^3+a)^3*(B*x^3+A), x)

[Out] $2/623645*x^{(5/2)}*(21505*B*b^3*x^{12}+27115*A*b^3*x^9+81345*B*a*b^2*x^9+110055*A*a*b^2*x^6+110055*B*a^2*b*x^6+170085*A*a^2*b*x^3+56695*B*a^3*x^3+124729*A*a^3)$

maxima [A] time = 0.48, size = 73, normalized size = 0.86

$$\frac{2}{29} B b^3 x^{\frac{29}{2}} + \frac{2}{23} (3 B a b^2 + A b^3) x^{\frac{23}{2}} + \frac{6}{17} (B a^2 b + A a b^2) x^{\frac{17}{2}} + \frac{2}{5} A a^3 x^{\frac{5}{2}} + \frac{2}{11} (B a^3 + 3 A a^2 b) x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")`

[Out] $2/29*B*b^3*x^{(29/2)} + 2/23*(3*B*a*b^2 + A*b^3)*x^{(23/2)} + 6/17*(B*a^2*b + A*a*b^2)*x^{(17/2)} + 2/5*A*a^3*x^{(5/2)} + 2/11*(B*a^3 + 3*A*a^2*b)*x^{(11/2)}$

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{11/2} \left(\frac{2 B a^3}{11} + \frac{6 A b a^2}{11} \right) + x^{23/2} \left(\frac{2 A b^3}{23} + \frac{6 B a b^2}{23} \right) + \frac{2 A a^3 x^{5/2}}{5} + \frac{2 B b^3 x^{29/2}}{29} + \frac{6 a b x^{17/2} (A b + B a)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)*(A + B*x^3)*(a + b*x^3)^3,x)`

[Out] $x^{(11/2)}*((2*B*a^3)/11 + (6*A*a^2*b)/11) + x^{(23/2)}*((2*A*b^3)/23 + (6*B*a*b^2)/23) + (2*A*a^3*x^{(5/2)})/5 + (2*B*b^3*x^{(29/2)})/29 + (6*a*b*x^{(17/2)}*(A*b + B*a))/17$

sympy [A] time = 39.60, size = 114, normalized size = 1.34

$$\frac{2 A a^3 x^{\frac{5}{2}}}{5} + \frac{6 A a^2 b x^{\frac{11}{2}}}{11} + \frac{6 A a b^2 x^{\frac{17}{2}}}{17} + \frac{2 A b^3 x^{\frac{23}{2}}}{23} + \frac{2 B a^3 x^{\frac{11}{2}}}{11} + \frac{6 B a^2 b x^{\frac{17}{2}}}{17} + \frac{6 B a b^2 x^{\frac{23}{2}}}{23} + \frac{2 B b^3 x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2)*(b*x**3+a)**3*(B*x**3+A),x)`

[Out] $2*A*a**3*x**(5/2)/5 + 6*A*a**2*b*x**(11/2)/11 + 6*A*a*b**2*x**(17/2)/17 + 2*A*b**3*x**(23/2)/23 + 2*B*a**3*x**(11/2)/11 + 6*B*a**2*b*x**(17/2)/17 + 6*B*a*b**2*x**(23/2)/23 + 2*B*b**3*x**(29/2)/29$

$$3.146 \quad \int \sqrt{x} (a + bx^3)^3 (A + Bx^3) dx$$

Optimal. Leaf size=85

$$\frac{2}{3}a^3Ax^{3/2} + \frac{2}{9}a^2x^{9/2}(aB + 3Ab) + \frac{2}{21}b^2x^{21/2}(3aB + Ab) + \frac{2}{5}abx^{15/2}(aB + Ab) + \frac{2}{27}b^3Bx^{27/2}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$\frac{2}{9}a^2x^{9/2}(aB + 3Ab) + \frac{2}{3}a^3Ax^{3/2} + \frac{2}{21}b^2x^{21/2}(3aB + Ab) + \frac{2}{5}abx^{15/2}(aB + Ab) + \frac{2}{27}b^3Bx^{27/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3), x]

[Out] (2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (2*a*b*(A*b + a*B)*x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(27/2))/27

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{x} (a + bx^3)^3 (A + Bx^3) dx &= \int (a^3A\sqrt{x} + a^2(3Ab + aB)x^{7/2} + 3ab(Ab + aB)x^{13/2} + b^2(Ab + 3aB)x^{19/2} + \\ &= \frac{2}{3}a^3Ax^{3/2} + \frac{2}{9}a^2(3Ab + aB)x^{9/2} + \frac{2}{5}ab(Ab + aB)x^{15/2} + \frac{2}{21}b^2(Ab + 3aB)x^{21/2} \end{aligned}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 0.84

$$\frac{2}{945}x^{3/2} (315a^3A + 105a^2x^3(aB + 3Ab) + 45b^2x^9(3aB + Ab) + 189abx^6(aB + Ab) + 35b^3Bx^{12})$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3), x]

[Out] $(2*x^{(3/2)}*(315*a^3*A + 105*a^2*(3*A*b + a*B)*x^3 + 189*a*b*(A*b + a*B)*x^6 + 45*b^2*(A*b + 3*a*B)*x^9 + 35*b^3*B*x^{12}))/945$

IntegrateAlgebraic [A] time = 0.05, size = 97, normalized size = 1.14

$$\frac{2}{945} (315a^3Ax^{3/2} + 105a^3Bx^{9/2} + 315a^2Abx^{9/2} + 189a^2bBx^{15/2} + 189aAb^2x^{15/2} + 135ab^2Bx^{21/2} + 45Ab^3x^{21/2} + 35b^3Bx^{27/2})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3), x]

[Out] $(2*(315*a^3*A*x^{(3/2)} + 315*a^2*A*b*x^{(9/2)} + 105*a^3*B*x^{(9/2)} + 189*a*A*b^2*x^{(15/2)} + 189*a^2*b*B*x^{(15/2)} + 45*A*b^3*x^{(21/2)} + 135*a*b^2*B*x^{(21/2)} + 35*b^3*B*x^{(27/2)}))/945$

fricas [A] time = 0.80, size = 76, normalized size = 0.89

$$\frac{2}{945} (35Bb^3x^{13} + 45(3Bab^2 + Ab^3)x^{10} + 189(Ba^2b + Aab^2)x^7 + 315Aa^3x + 105(Ba^3 + 3Aa^2b)x^4)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2), x, algorithm="fricas")

[Out] $2/945*(35*B*b^3*x^{13} + 45*(3*B*a*b^2 + A*b^3)*x^{10} + 189*(B*a^2*b + A*a*b^2)*x^7 + 315*A*a^3*x + 105*(B*a^3 + 3*A*a^2*b)*x^4)*\text{sqrt}(x)$

giac [A] time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{27}Bb^3x^{\frac{27}{2}} + \frac{2}{7}Bab^2x^{\frac{21}{2}} + \frac{2}{21}Ab^3x^{\frac{21}{2}} + \frac{2}{5}Ba^2bx^{\frac{15}{2}} + \frac{2}{5}Aab^2x^{\frac{15}{2}} + \frac{2}{9}Ba^3x^{\frac{9}{2}} + \frac{2}{3}Aa^2bx^{\frac{9}{2}} + \frac{2}{3}Aa^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2), x, algorithm="giac")

[Out] $2/27*B*b^3*x^{(27/2)} + 2/7*B*a*b^2*x^{(21/2)} + 2/21*A*b^3*x^{(21/2)} + 2/5*B*a^2*b*x^{(15/2)} + 2/5*A*a*b^2*x^{(15/2)} + 2/9*B*a^3*x^{(9/2)} + 2/3*A*a^2*b*x^{(9/2)} + 2/3*A*a^3*x^{(3/2)}$

maple [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2(35Bb^3x^{12} + 45x^9Ab^3 + 135x^9Bab^2 + 189x^6Aab^2 + 189x^6Ba^2b + 315x^3Aa^2b + 105Ba^3x^3 + 315Aa^3)x^{\frac{3}{2}}}{945}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^3*(B*x^3+A)*x^(1/2), x)

[Out] $2/945*x^{(3/2)}*(35*B*b^3*x^{12}+45*A*b^3*x^9+135*B*a*b^2*x^9+189*A*a*b^2*x^6+189*B*a^2*b*x^6+315*A*a^2*b*x^3+105*B*a^3*x^3+315*A*a^3)$

maxima [A] time = 0.67, size = 73, normalized size = 0.86

$$\frac{2}{27} B b^3 x^{\frac{27}{2}} + \frac{2}{21} (3 B a b^2 + A b^3) x^{\frac{21}{2}} + \frac{2}{5} (B a^2 b + A a b^2) x^{\frac{15}{2}} + \frac{2}{3} A a^3 x^{\frac{9}{2}} + \frac{2}{9} (B a^3 + 3 A a^2 b) x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="maxima")`

[Out] $2/27*B*b^3*x^{(27/2)} + 2/21*(3*B*a*b^2 + A*b^3)*x^{(21/2)} + 2/5*(B*a^2*b + A*a*b^2)*x^{(15/2)} + 2/3*A*a^3*x^{(3/2)} + 2/9*(B*a^3 + 3*A*a^2*b)*x^{(9/2)}$

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{9/2} \left(\frac{2 B a^3}{9} + \frac{2 A b a^2}{3} \right) + x^{21/2} \left(\frac{2 A b^3}{21} + \frac{2 B a b^2}{7} \right) + \frac{2 A a^3 x^{3/2}}{3} + \frac{2 B b^3 x^{27/2}}{27} + \frac{2 a b x^{15/2} (A b + B a)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(A + B*x^3)*(a + b*x^3)^3,x)`

[Out] $x^{(9/2)}*((2*B*a^3)/9 + (2*A*a^2*b)/3) + x^{(21/2)}*((2*A*b^3)/21 + (2*B*a*b^2)/7) + (2*A*a^3*x^{(3/2)})/3 + (2*B*b^3*x^{(27/2)})/27 + (2*a*b*x^{(15/2)}*(A*b + B*a))/5$

sympy [A] time = 6.34, size = 114, normalized size = 1.34

$$\frac{2 A a^3 x^{\frac{3}{2}}}{3} + \frac{2 A a^2 b x^{\frac{9}{2}}}{3} + \frac{2 A a b^2 x^{\frac{15}{2}}}{5} + \frac{2 A b^3 x^{\frac{21}{2}}}{21} + \frac{2 B a^3 x^{\frac{9}{2}}}{9} + \frac{2 B a^2 b x^{\frac{15}{2}}}{5} + \frac{2 B a b^2 x^{\frac{21}{2}}}{7} + \frac{2 B b^3 x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**3*(B*x**3+A)*x**(1/2),x)`

[Out] $2*A*a**3*x**(3/2)/3 + 2*A*a**2*b*x**(9/2)/3 + 2*A*a*b**2*x**(15/2)/5 + 2*A*b**3*x**(21/2)/21 + 2*B*a**3*x**(9/2)/9 + 2*B*a**2*b*x**(15/2)/5 + 2*B*a*b**2*x**(21/2)/7 + 2*B*b**3*x**(27/2)/27$

$$3.147 \quad \int \frac{(a+bx^3)^3 (A+Bx^3)}{\sqrt{x}} dx$$

Optimal. Leaf size=83

$$2a^3 A\sqrt{x} + \frac{2}{7}a^2 x^{7/2}(aB + 3Ab) + \frac{2}{19}b^2 x^{19/2}(3aB + Ab) + \frac{6}{13}abx^{13/2}(aB + Ab) + \frac{2}{25}b^3 Bx^{25/2}$$

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$\frac{2}{7}a^2 x^{7/2}(aB + 3Ab) + 2a^3 A\sqrt{x} + \frac{2}{19}b^2 x^{19/2}(3aB + Ab) + \frac{6}{13}abx^{13/2}(aB + Ab) + \frac{2}{25}b^3 Bx^{25/2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x], x]

[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*x^(13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(25/2))/25

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^(m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^3 (A+Bx^3)}{\sqrt{x}} dx &= \int \left(\frac{a^3 A}{\sqrt{x}} + a^2(3Ab + aB)x^{5/2} + 3ab(Ab + aB)x^{11/2} + b^2(Ab + 3aB)x^{17/2} + b^3 Bx^{23/2} \right) dx \\ &= 2a^3 A\sqrt{x} + \frac{2}{7}a^2(3Ab + aB)x^{7/2} + \frac{6}{13}ab(Ab + aB)x^{13/2} + \frac{2}{19}b^2(Ab + 3aB)x^{19/2} + \frac{2}{25}b^3 Bx^{25/2} \end{aligned}$$

Mathematica [A] time = 0.06, size = 83, normalized size = 1.00

$$2a^3 A\sqrt{x} + \frac{2}{7}a^2 x^{7/2}(aB + 3Ab) + \frac{2}{19}b^2 x^{19/2}(3aB + Ab) + \frac{6}{13}abx^{13/2}(aB + Ab) + \frac{2}{25}b^3 Bx^{25/2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x], x]

[Out] $2*a^3*A*\text{Sqrt}[x] + (2*a^2*(3*A*b + a*B))*x^{(7/2)}/7 + (6*a*b*(A*b + a*B))*x^{(13/2)}/13 + (2*b^2*(A*b + 3*a*B))*x^{(19/2)}/19 + (2*b^3*B*x^{(25/2)})/25$

IntegrateAlgebraic [A] time = 0.04, size = 97, normalized size = 1.17

$$\frac{2(43225a^3A\sqrt{x} + 6175a^3Bx^{7/2} + 18525a^2Abx^{7/2} + 9975a^2bBx^{13/2} + 9975aAb^2x^{13/2} + 6825ab^2Bx^{19/2} + 2275Ab^3x^{19/2} + 1729b^3Bx^{25/2})}{43225}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x], x]

[Out] $(2*(43225*a^3*A*\text{Sqrt}[x] + 18525*a^2*A*b*x^{(7/2)} + 6175*a^3*B*x^{(7/2)} + 9975*a*A*b^2*x^{(13/2)} + 9975*a^2*b*B*x^{(13/2)} + 2275*A*b^3*x^{(19/2)} + 6825*a*b^2*B*x^{(19/2)} + 1729*b^3*B*x^{(25/2)}))/43225$

fricas [A] time = 0.80, size = 75, normalized size = 0.90

$$\frac{2}{43225} (1729 B b^3 x^{12} + 2275 (3 B a b^2 + A b^3) x^9 + 9975 (B a^2 b + A a b^2) x^6 + 43225 A a^3 + 6175 (B a^3 + 3 A a^2 b) x^3) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2), x, algorithm="fricas")

[Out] $2/43225*(1729*B*b^3*x^{12} + 2275*(3*B*a*b^2 + A*b^3))*x^9 + 9975*(B*a^2*b + A*a*b^2)*x^6 + 43225*A*a^3 + 6175*(B*a^3 + 3*A*a^2*b)*x^3)*\text{sqrt}(x)$

giac [A] time = 0.16, size = 77, normalized size = 0.93

$$\frac{2}{25} B b^3 x^{25/2} + \frac{6}{19} B a b^2 x^{19/2} + \frac{2}{19} A b^3 x^{19/2} + \frac{6}{13} B a^2 b x^{13/2} + \frac{6}{13} A a b^2 x^{13/2} + \frac{2}{7} B a^3 x^{7/2} + \frac{6}{7} A a^2 b x^{7/2} + 2 A a^3 \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2), x, algorithm="giac")

[Out] $2/25*B*b^3*x^{(25/2)} + 6/19*B*a*b^2*x^{(19/2)} + 2/19*A*b^3*x^{(19/2)} + 6/13*B*a^2*b*x^{(13/2)} + 6/13*A*a*b^2*x^{(13/2)} + 2/7*B*a^3*x^{(7/2)} + 6/7*A*a^2*b*x^{(7/2)} + 2*A*a^3*\text{sqrt}(x)$

maple [A] time = 0.04, size = 80, normalized size = 0.96

$$\frac{2(1729B b^3 x^{12} + 2275x^9 A b^3 + 6825x^9 B a b^2 + 9975x^6 A a b^2 + 9975x^6 B a^2 b + 18525x^3 A a^2 b + 6175B a^3 x^3 + 43225A a^3) \sqrt{x}}{43225}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x)`

[Out] $2/43225*x^{1/2}*(1729*B*b^3*x^{12}+2275*A*b^3*x^9+6825*B*a*b^2*x^9+9975*A*a*b^2*x^6+9975*B*a^2*b*x^6+18525*A*a^2*b*x^3+6175*B*a^3*x^3+43225*A*a^3)$

maxima [A] time = 0.56, size = 73, normalized size = 0.88

$$\frac{2}{25} B b^3 x^{\frac{25}{2}} + \frac{2}{19} (3 B a b^2 + A b^3) x^{\frac{19}{2}} + \frac{6}{13} (B a^2 b + A a b^2) x^{\frac{13}{2}} + 2 A a^3 \sqrt{x} + \frac{2}{7} (B a^3 + 3 A a^2 b) x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="maxima")`

[Out] $2/25*B*b^3*x^{25/2} + 2/19*(3*B*a*b^2 + A*b^3)*x^{19/2} + 6/13*(B*a^2*b + A*a*b^2)*x^{13/2} + 2*A*a^3*\text{sqrt}(x) + 2/7*(B*a^3 + 3*A*a^2*b)*x^{7/2}$

mupad [B] time = 0.03, size = 69, normalized size = 0.83

$$x^{7/2} \left(\frac{2 B a^3}{7} + \frac{6 A b a^2}{7} \right) + x^{19/2} \left(\frac{2 A b^3}{19} + \frac{6 B a b^2}{19} \right) + 2 A a^3 \sqrt{x} + \frac{2 B b^3 x^{25/2}}{25} + \frac{6 a b x^{13/2} (A b + B a)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^3)/x^(1/2),x)`

[Out] $x^{7/2}*((2*B*a^3)/7 + (6*A*a^2*b)/7) + x^{19/2}*((2*A*b^3)/19 + (6*B*a*b^2)/19) + 2*A*a^3*x^{1/2} + (2*B*b^3*x^{25/2})/25 + (6*a*b*x^{13/2}*(A*b + B*a))/13$

sympy [A] time = 23.64, size = 112, normalized size = 1.35

$$2 A a^3 \sqrt{x} + \frac{6 A a^2 b x^{\frac{7}{2}}}{7} + \frac{6 A a b^2 x^{\frac{13}{2}}}{13} + \frac{2 A b^3 x^{\frac{19}{2}}}{19} + \frac{2 B a^3 x^{\frac{7}{2}}}{7} + \frac{6 B a^2 b x^{\frac{13}{2}}}{13} + \frac{6 B a b^2 x^{\frac{19}{2}}}{19} + \frac{2 B b^3 x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**3*(B*x**3+A)/x**(1/2),x)`

[Out] $2*A*a**3*\text{sqrt}(x) + 6*A*a**2*b*x**(7/2)/7 + 6*A*a*b**2*x**(13/2)/13 + 2*A*b**3*x**(19/2)/19 + 2*B*a**3*x**(7/2)/7 + 6*B*a**2*b*x**(13/2)/13 + 6*B*a*b**2*x**(19/2)/19 + 2*B*b**3*x**(25/2)/25$

$$3.148 \quad \int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}} dx$$

Optimal. Leaf size=83

$$-\frac{2a^3A}{\sqrt{x}} + \frac{2}{5}a^2x^{5/2}(aB + 3Ab) + \frac{2}{17}b^2x^{17/2}(3aB + Ab) + \frac{6}{11}abx^{11/2}(aB + Ab) + \frac{2}{23}b^3Bx^{23/2}$$

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$\frac{2}{5}a^2x^{5/2}(aB + 3Ab) - \frac{2a^3A}{\sqrt{x}} + \frac{2}{17}b^2x^{17/2}(3aB + Ab) + \frac{6}{11}abx^{11/2}(aB + Ab) + \frac{2}{23}b^3Bx^{23/2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(3/2), x]

[Out] (-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (6*a*b*(A*b + a*B)*x^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(23/2))/23

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}} dx &= \int \left(\frac{a^3A}{x^{3/2}} + a^2(3Ab + aB)x^{3/2} + 3ab(Ab + aB)x^{9/2} + b^2(Ab + 3aB)x^{15/2} + b^3Bx^{21/2} \right) dx \\ &= -\frac{2a^3A}{\sqrt{x}} + \frac{2}{5}a^2(3Ab + aB)x^{5/2} + \frac{6}{11}ab(Ab + aB)x^{11/2} + \frac{2}{17}b^2(Ab + 3aB)x^{17/2} + \frac{2}{23}b^3Bx^{23/2} \end{aligned}$$

Mathematica [A] time = 0.04, size = 81, normalized size = 0.98

$$\frac{-8602a^3(5A - Bx^3) + 2346a^2bx^3(11A + 5Bx^3) + 690ab^2x^6(17A + 11Bx^3) + 110b^3x^9(23A + 17Bx^3)}{21505\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(3/2), x]

[Out] (-8602*a^3*(5*A - B*x^3) + 2346*a^2*b*x^3*(11*A + 5*B*x^3) + 690*a*b^2*x^6*(17*A + 11*B*x^3) + 110*b^3*x^9*(23*A + 17*B*x^3))/(21505*Sqrt[x])

IntegrateAlgebraic [A] time = 0.05, size = 83, normalized size = 1.00

$$\frac{2(-21505a^3A + 4301a^3Bx^3 + 12903a^2Abx^3 + 5865a^2bBx^6 + 5865aAb^2x^6 + 3795ab^2Bx^9 + 1265Ab^3x^9 + 935b^3Bx^{12})}{21505\sqrt{x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)^3*(A + B*x^3))/x^(3/2), x]

[Out] (2*(-21505*a^3*A + 12903*a^2*A*b*x^3 + 4301*a^3*B*x^3 + 5865*a*A*b^2*x^6 + 5865*a^2*b*B*x^6 + 1265*A*b^3*x^9 + 3795*a*b^2*B*x^9 + 935*b^3*B*x^12))/(21505*Sqrt[x])

fricas [A] time = 1.00, size = 75, normalized size = 0.90

$$\frac{2(935Bb^3x^{12} + 1265(3Bab^2 + Ab^3)x^9 + 5865(Ba^2b + Aab^2)x^6 - 21505Aa^3 + 4301(Ba^3 + 3Aa^2b)x^3)}{21505\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2), x, algorithm="fricas")

[Out] 2/21505*(935*B*b^3*x^12 + 1265*(3*B*a*b^2 + A*b^3)*x^9 + 5865*(B*a^2*b + A*a*b^2)*x^6 - 21505*A*a^3 + 4301*(B*a^3 + 3*A*a^2*b)*x^3)/sqrt(x)

giac [A] time = 0.16, size = 77, normalized size = 0.93

$$\frac{2}{23}Bb^3x^{\frac{23}{2}} + \frac{6}{17}Bab^2x^{\frac{17}{2}} + \frac{2}{17}Ab^3x^{\frac{17}{2}} + \frac{6}{11}Ba^2bx^{\frac{11}{2}} + \frac{6}{11}Aab^2x^{\frac{11}{2}} + \frac{2}{5}Ba^3x^{\frac{5}{2}} + \frac{6}{5}Aa^2bx^{\frac{5}{2}} - \frac{2Aa^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2), x, algorithm="giac")

[Out] 2/23*B*b^3*x^(23/2) + 6/17*B*a*b^2*x^(17/2) + 2/17*A*b^3*x^(17/2) + 6/11*B*a^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2) + 2/5*B*a^3*x^(5/2) + 6/5*A*a^2*b*x^(5/2) - 2*A*a^3/sqrt(x)

maple [A] time = 0.04, size = 80, normalized size = 0.96

$$\frac{2(-935Bb^3x^{12} - 1265x^9Ab^3 - 3795x^9Ba^2b^2 - 5865x^6Aa^2b^2 - 5865x^6Ba^2b - 12903x^3Aa^2b - 4301Ba^3x^3 + 21505Aa^3)}{21505\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x)`

[Out] $-2/21505*(-935*B*b^3*x^{12}-1265*A*b^3*x^9-3795*B*a*b^2*x^9-5865*A*a*b^2*x^6-5865*B*a^2*b*x^6-12903*A*a^2*b*x^3-4301*B*a^3*x^3+21505*A*a^3)/x^{1/2}$

maxima [A] time = 0.56, size = 73, normalized size = 0.88

$$\frac{2}{23} B b^3 x^{\frac{23}{2}} + \frac{2}{17} (3 B a b^2 + A b^3) x^{\frac{17}{2}} + \frac{6}{11} (B a^2 b + A a b^2) x^{\frac{11}{2}} - \frac{2 A a^3}{\sqrt{x}} + \frac{2}{5} (B a^3 + 3 A a^2 b) x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="maxima")`

[Out] $2/23*B*b^3*x^{23/2} + 2/17*(3*B*a*b^2 + A*b^3)*x^{17/2} + 6/11*(B*a^2*b + A*a*b^2)*x^{11/2} - 2*A*a^3/\text{sqrt}(x) + 2/5*(B*a^3 + 3*A*a^2*b)*x^{5/2}$

mupad [B] time = 0.03, size = 69, normalized size = 0.83

$$x^{5/2} \left(\frac{2 B a^3}{5} + \frac{6 A b a^2}{5} \right) + x^{17/2} \left(\frac{2 A b^3}{17} + \frac{6 B a b^2}{17} \right) - \frac{2 A a^3}{\sqrt{x}} + \frac{2 B b^3 x^{23/2}}{23} + \frac{6 a b x^{11/2} (A b + B a)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^3)/x^(3/2),x)`

[Out] $x^{5/2}*((2*B*a^3)/5 + (6*A*a^2*b)/5) + x^{17/2}*((2*A*b^3)/17 + (6*B*a*b^2)/17) - (2*A*a^3)/x^{1/2} + (2*B*b^3*x^{23/2})/23 + (6*a*b*x^{11/2}*(A*b + B*a))/11$

sympy [A] time = 19.49, size = 112, normalized size = 1.35

$$-\frac{2 A a^3}{\sqrt{x}} + \frac{6 A a^2 b x^{\frac{5}{2}}}{5} + \frac{6 A a b^2 x^{\frac{11}{2}}}{11} + \frac{2 A b^3 x^{\frac{17}{2}}}{17} + \frac{2 B a^3 x^{\frac{5}{2}}}{5} + \frac{6 B a^2 b x^{\frac{11}{2}}}{11} + \frac{6 B a b^2 x^{\frac{17}{2}}}{17} + \frac{2 B b^3 x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**3*(B*x**3+A)/x**(3/2),x)`

[Out] $-2*A*a**3/\text{sqrt}(x) + 6*A*a**2*b*x**(5/2)/5 + 6*A*a*b**2*x**(11/2)/11 + 2*A*b**3*x**(17/2)/17 + 2*B*a**3*x**(5/2)/5 + 6*B*a**2*b*x**(11/2)/11 + 6*B*a*b**2*x**(17/2)/17 + 2*B*b**3*x**(23/2)/23$

$$3.149 \quad \int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}} dx$$

Optimal. Leaf size=85

$$-\frac{2a^3A}{3x^{3/2}} + \frac{2}{3}a^2x^{3/2}(aB + 3Ab) + \frac{2}{15}b^2x^{15/2}(3aB + Ab) + \frac{2}{3}abx^{9/2}(aB + Ab) + \frac{2}{21}b^3Bx^{21/2}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$\frac{2}{3}a^2x^{3/2}(aB + 3Ab) - \frac{2a^3A}{3x^{3/2}} + \frac{2}{15}b^2x^{15/2}(3aB + Ab) + \frac{2}{3}abx^{9/2}(aB + Ab) + \frac{2}{21}b^3Bx^{21/2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(5/2), x]

[Out] (-2*a^3*A)/(3*x^(3/2)) + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (2*a*b*(A*b + a*B)*x^(9/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(21/2))/21

Rule 448

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}} dx &= \int \left(\frac{a^3A}{x^{5/2}} + a^2(3Ab + aB)\sqrt{x} + 3ab(Ab + aB)x^{7/2} + b^2(Ab + 3aB)x^{13/2} + b^3Bx^{19/2} \right) dx \\ &= -\frac{2a^3A}{3x^{3/2}} + \frac{2}{3}a^2(3Ab + aB)x^{3/2} + \frac{2}{3}ab(Ab + aB)x^{9/2} + \frac{2}{15}b^2(Ab + 3aB)x^{15/2} + \frac{2}{21}b^3Bx^{21/2} \end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.91

$$\frac{2(-35a^3(A - Bx^3) + 35a^2bx^3(3A + Bx^3) + 7ab^2x^6(5A + 3Bx^3) + b^3x^9(7A + 5Bx^3))}{105x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(5/2), x]

[Out] (2*(-35*a^3*(A - B*x^3) + 35*a^2*b*x^3*(3*A + B*x^3) + 7*a*b^2*x^6*(5*A + 3*B*x^3) + b^3*x^9*(7*A + 5*B*x^3)))/(105*x^(3/2))

IntegrateAlgebraic [A] time = 0.04, size = 83, normalized size = 0.98

$$\frac{2(-35a^3A + 35a^3Bx^3 + 105a^2Abx^3 + 35a^2bBx^6 + 35aAb^2x^6 + 21ab^2Bx^9 + 7Ab^3x^9 + 5b^3Bx^{12})}{105x^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)^3*(A + B*x^3))/x^(5/2), x]

[Out] (2*(-35*a^3*A + 105*a^2*A*b*x^3 + 35*a^3*B*x^3 + 35*a*A*b^2*x^6 + 35*a^2*b*B*x^6 + 7*A*b^3*x^9 + 21*a*b^2*B*x^9 + 5*b^3*B*x^12))/(105*x^(3/2))

fricas [A] time = 0.79, size = 75, normalized size = 0.88

$$\frac{2(5Bb^3x^{12} + 7(3Bab^2 + Ab^3)x^9 + 35(Ba^2b + Aab^2)x^6 - 35Aa^3 + 35(Ba^3 + 3Aa^2b)x^3)}{105x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2), x, algorithm="fricas")

[Out] 2/105*(5*B*b^3*x^12 + 7*(3*B*a*b^2 + A*b^3)*x^9 + 35*(B*a^2*b + A*a*b^2)*x^6 - 35*A*a^3 + 35*(B*a^3 + 3*A*a^2*b)*x^3)/x^(3/2)

giac [A] time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{21}Bb^3x^{21/2} + \frac{2}{5}Bab^2x^{15/2} + \frac{2}{15}Ab^3x^{15/2} + \frac{2}{3}Ba^2bx^{9/2} + \frac{2}{3}Aab^2x^{9/2} + \frac{2}{3}Ba^3x^{3/2} + 2Aa^2bx^{3/2} - \frac{2Aa^3}{3x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2), x, algorithm="giac")

[Out] 2/21*B*b^3*x^(21/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2) + 2/3*B*a^2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) + 2/3*B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2) - 2/3*A*a^3/x^(3/2)

maple [A] time = 0.04, size = 80, normalized size = 0.94

$$\frac{2(-5Bb^3x^{12} - 7x^9Ab^3 - 21x^9Ba^2b^2 - 35x^6Aa^2b^2 - 35x^6Ba^2b - 105x^3Aa^2b - 35Ba^3x^3 + 35Aa^3)}{105x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x)`

[Out] $-2/105*(-5*B*b^3*x^{12}-7*A*b^3*x^9-21*B*a*b^2*x^9-35*A*a*b^2*x^6-35*B*a^2*b*x^6-105*A*a^2*b*x^3-35*B*a^3*x^3+35*A*a^3)/x^{3/2}$

maxima [A] time = 0.64, size = 73, normalized size = 0.86

$$\frac{2}{21} B b^3 x^{\frac{21}{2}} + \frac{2}{15} (3 B a b^2 + A b^3) x^{\frac{15}{2}} + \frac{2}{3} (B a^2 b + A a b^2) x^{\frac{9}{2}} - \frac{2 A a^3}{3 x^{\frac{3}{2}}} + \frac{2}{3} (B a^3 + 3 A a^2 b) x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="maxima")`

[Out] $2/21*B*b^3*x^{21/2} + 2/15*(3*B*a*b^2 + A*b^3)*x^{15/2} + 2/3*(B*a^2*b + A*a*b^2)*x^{9/2} - 2/3*A*a^3/x^{3/2} + 2/3*(B*a^3 + 3*A*a^2*b)*x^{3/2}$

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{3/2} \left(\frac{2 B a^3}{3} + 2 A b a^2 \right) + x^{15/2} \left(\frac{2 A b^3}{15} + \frac{2 B a b^2}{5} \right) - \frac{2 A a^3}{3 x^{3/2}} + \frac{2 B b^3 x^{21/2}}{21} + \frac{2 a b x^{9/2} (A b + B a)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^3)/x^(5/2),x)`

[Out] $x^{3/2}*((2*B*a^3)/3 + 2*A*a^2*b) + x^{15/2}*((2*A*b^3)/15 + (2*B*a*b^2)/5) - (2*A*a^3)/(3*x^{3/2}) + (2*B*b^3*x^{21/2})/21 + (2*a*b*x^{9/2}*(A*b + B*a))/3$

sympy [A] time = 28.37, size = 112, normalized size = 1.32

$$-\frac{2 A a^3}{3 x^{\frac{3}{2}}} + 2 A a^2 b x^{\frac{3}{2}} + \frac{2 A a b^2 x^{\frac{9}{2}}}{3} + \frac{2 A b^3 x^{\frac{15}{2}}}{15} + \frac{2 B a^3 x^{\frac{3}{2}}}{3} + \frac{2 B a^2 b x^{\frac{9}{2}}}{3} + \frac{2 B a b^2 x^{\frac{15}{2}}}{5} + \frac{2 B b^3 x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**3*(B*x**3+A)/x**(5/2),x)`

[Out] $-2*A*a**3/(3*x**(3/2)) + 2*A*a**2*b*x**(3/2) + 2*A*a*b**2*x**(9/2)/3 + 2*A*b**3*x**(15/2)/15 + 2*B*a**3*x**(3/2)/3 + 2*B*a**2*b*x**(9/2)/3 + 2*B*a*b**2*x**(15/2)/5 + 2*B*b**3*x**(21/2)/21$

$$3.150 \quad \int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}} dx$$

Optimal. Leaf size=83

$$-\frac{2a^3A}{5x^{5/2}} + 2a^2\sqrt{x}(aB + 3Ab) + \frac{2}{13}b^2x^{13/2}(3aB + Ab) + \frac{6}{7}abx^{7/2}(aB + Ab) + \frac{2}{19}b^3Bx^{19/2}$$

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$2a^2\sqrt{x}(aB + 3Ab) - \frac{2a^3A}{5x^{5/2}} + \frac{2}{13}b^2x^{13/2}(3aB + Ab) + \frac{6}{7}abx^{7/2}(aB + Ab) + \frac{2}{19}b^3Bx^{19/2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(7/2), x]

[Out] (-2*a^3*A)/(5*x^(5/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*b + a*B)*x^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(19/2))/19

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}} dx &= \int \left(\frac{a^3A}{x^{7/2}} + \frac{a^2(3Ab+aB)}{\sqrt{x}} + 3ab(Ab+aB)x^{5/2} + b^2(Ab+3aB)x^{11/2} + b^3Bx^{17/2} \right) dx \\ &= -\frac{2a^3A}{5x^{5/2}} + 2a^2(3Ab+aB)\sqrt{x} + \frac{6}{7}ab(Ab+aB)x^{7/2} + \frac{2}{13}b^2(Ab+3aB)x^{13/2} + \frac{2}{19}b^3Bx^{19/2} \end{aligned}$$

Mathematica [A] time = 0.09, size = 78, normalized size = 0.94

$$\frac{-3458a^3(A - 5Bx^3) + 7410a^2bx^3(7A + Bx^3) + 570ab^2x^6(13A + 7Bx^3) + 70b^3x^9(19A + 13Bx^3)}{8645x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(7/2), x]

[Out] (-3458*a^3*(A - 5*B*x^3) + 7410*a^2*b*x^3*(7*A + B*x^3) + 570*a*b^2*x^6*(13*A + 7*B*x^3) + 70*b^3*x^9*(19*A + 13*B*x^3))/(8645*x^(5/2))

IntegrateAlgebraic [A] time = 0.05, size = 83, normalized size = 1.00

$$\frac{2(-1729a^3A + 8645a^3Bx^3 + 25935a^2Abx^3 + 3705a^2bBx^6 + 3705aAb^2x^6 + 1995ab^2Bx^9 + 665Ab^3x^9 + 455b^3Bx^{12})}{8645x^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)^3*(A + B*x^3))/x^(7/2), x]

[Out] (2*(-1729*a^3*A + 25935*a^2*A*b*x^3 + 8645*a^3*B*x^3 + 3705*a*A*b^2*x^6 + 3705*a^2*b*B*x^6 + 665*A*b^3*x^9 + 1995*a*b^2*B*x^9 + 455*b^3*B*x^12))/(8645*x^(5/2))

fricas [A] time = 0.68, size = 75, normalized size = 0.90

$$\frac{2(455Bb^3x^{12} + 665(3Bab^2 + Ab^3)x^9 + 3705(Ba^2b + Aab^2)x^6 - 1729Aa^3 + 8645(Ba^3 + 3Aa^2b)x^3)}{8645x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2), x, algorithm="fricas")

[Out] 2/8645*(455*B*b^3*x^12 + 665*(3*B*a*b^2 + A*b^3)*x^9 + 3705*(B*a^2*b + A*a*b^2)*x^6 - 1729*A*a^3 + 8645*(B*a^3 + 3*A*a^2*b)*x^3)/x^(5/2)

giac [A] time = 0.16, size = 77, normalized size = 0.93

$$\frac{2}{19}Bb^3x^{\frac{19}{2}} + \frac{6}{13}Bab^2x^{\frac{13}{2}} + \frac{2}{13}Ab^3x^{\frac{13}{2}} + \frac{6}{7}Ba^2bx^{\frac{7}{2}} + \frac{6}{7}Aab^2x^{\frac{7}{2}} + 2Ba^3\sqrt{x} + 6Aa^2b\sqrt{x} - \frac{2Aa^3}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2), x, algorithm="giac")

[Out] 2/19*B*b^3*x^(19/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2) + 6/7*B*a^2*b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 2*B*a^3*sqrt(x) + 6*A*a^2*b*sqrt(x) - 2/5*A*a^3/x^(5/2)

maple [A] time = 0.04, size = 80, normalized size = 0.96

$$\frac{2(-455Bb^3x^{12} - 665x^9Ab^3 - 1995x^9Bab^2 - 3705x^6Aa^2b^2 - 3705x^6Ba^2b - 25935x^3Aa^2b - 8645Ba^3x^3 + 1729Aa^3)}{8645x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x)`

[Out] $-2/8645*(-455*B*b^3*x^{12}-665*A*b^3*x^9-1995*B*a*b^2*x^9-3705*A*a*b^2*x^6-3705*B*a^2*b*x^6-25935*A*a^2*b*x^3-8645*B*a^3*x^3+1729*A*a^3)/x^{5/2}$

maxima [A] time = 0.55, size = 73, normalized size = 0.88

$$\frac{2}{19} B b^3 x^{\frac{19}{2}} + \frac{2}{13} (3 B a b^2 + A b^3) x^{\frac{13}{2}} + \frac{6}{7} (B a^2 b + A a b^2) x^{\frac{7}{2}} - \frac{2 A a^3}{5 x^{\frac{5}{2}}} + 2 (B a^3 + 3 A a^2 b) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="maxima")`

[Out] $2/19*B*b^3*x^{19/2} + 2/13*(3*B*a*b^2 + A*b^3)*x^{13/2} + 6/7*(B*a^2*b + A*a*b^2)*x^{7/2} - 2/5*A*a^3/x^{5/2} + 2*(B*a^3 + 3*A*a^2*b)*sqrt(x)$

mupad [B] time = 0.03, size = 69, normalized size = 0.83

$$\sqrt{x} (2 B a^3 + 6 A b a^2) + x^{13/2} \left(\frac{2 A b^3}{13} + \frac{6 B a b^2}{13} \right) - \frac{2 A a^3}{5 x^{5/2}} + \frac{2 B b^3 x^{19/2}}{19} + \frac{6 a b x^{7/2} (A b + B a)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^3)/x^(7/2),x)`

[Out] $x^{1/2}*(2*B*a^3 + 6*A*a^2*b) + x^{13/2}*((2*A*b^3)/13 + (6*B*a*b^2)/13) - (2*A*a^3)/(5*x^{5/2}) + (2*B*b^3*x^{19/2})/19 + (6*a*b*x^{7/2}*(A*b + B*a))/7$

sympy [A] time = 29.28, size = 110, normalized size = 1.33

$$-\frac{2 A a^3}{5 x^{\frac{5}{2}}} + 6 A a^2 b \sqrt{x} + \frac{6 A a b^2 x^{\frac{7}{2}}}{7} + \frac{2 A b^3 x^{\frac{13}{2}}}{13} + 2 B a^3 \sqrt{x} + \frac{6 B a^2 b x^{\frac{7}{2}}}{7} + \frac{6 B a b^2 x^{\frac{13}{2}}}{13} + \frac{2 B b^3 x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**3*(B*x**3+A)/x**(7/2),x)`

[Out] $-2*A*a**3/(5*x**(5/2)) + 6*A*a**2*b*sqrt(x) + 6*A*a*b**2*x**(7/2)/7 + 2*A*b**3*x**(13/2)/13 + 2*B*a**3*sqrt(x) + 6*B*a**2*b*x**(7/2)/7 + 6*B*a*b**2*x**(13/2)/13 + 2*B*b**3*x**(19/2)/19$

$$3.151 \quad \int \frac{x^{7/2}(A+Bx^3)}{a+bx^3} dx$$

Optimal. Leaf size=73

$$-\frac{2\sqrt{a}(Ab - aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}} + \frac{2x^{3/2}(Ab - aB)}{3b^2} + \frac{2Bx^{9/2}}{9b}$$

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {459, 321, 329, 275, 205}

$$\frac{2x^{3/2}(Ab - aB)}{3b^2} - \frac{2\sqrt{a}(Ab - aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}} + \frac{2Bx^{9/2}}{9b}$$

Antiderivative was successfully verified.

[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3), x]

[Out] (2*(A*b - a*B)*x^(3/2))/(3*b^2) + (2*B*x^(9/2))/(9*b) - (2*sqrt[a]*(A*b - a*B)*ArcTan[(sqrt[b]*x^(3/2))/sqrt[a]])/(3*b^(5/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^

$n)^p, x], x, (c*x)^{(1/k)], x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{Fractio} \\ \text{nQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 459

$\text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^p \cdot (c + d \cdot x^n), x_Symbol] :> \text{Simp}[(d \cdot (e \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1}) / (b \cdot e \cdot (m + n \cdot (p + 1) + 1)), x] - \text{Dist}[(a \cdot d \cdot (m + 1) - b \cdot c \cdot (m + n \cdot (p + 1) + 1)) / (b \cdot (m + n \cdot (p + 1) + 1)), \text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, n, p\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{NeQ}[m + n \cdot (p + 1) + 1, 0]$

Rubi steps

$$\begin{aligned} \int \frac{x^{7/2} (A + Bx^3)}{a + bx^3} dx &= \frac{2Bx^{9/2}}{9b} - \frac{\left(2 \left(-\frac{9Ab}{2} + \frac{9aB}{2}\right)\right) \int \frac{x^{7/2}}{a+bx^3} dx}{9b} \\ &= \frac{2(Ab - aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(a(Ab - aB)) \int \frac{\sqrt{x}}{a+bx^3} dx}{b^2} \\ &= \frac{2(Ab - aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(2a(Ab - aB)) \text{Subst}\left(\int \frac{x^2}{a+bx^6} dx, x, \sqrt{x}\right)}{b^2} \\ &= \frac{2(Ab - aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(2a(Ab - aB)) \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, x^{3/2}\right)}{3b^2} \\ &= \frac{2(Ab - aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{2\sqrt{a} (Ab - aB) \tan^{-1}\left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.12, size = 67, normalized size = 0.92

$$\frac{2\sqrt{a} (aB - Ab) \tan^{-1}\left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}} + \frac{2x^{3/2} (-3aB + 3Ab + bBx^3)}{9b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3), x]

[Out] (2*x^(3/2)*(3*A*b - 3*a*B + b*B*x^3))/(9*b^2) + (2*sqrt[a]*(-(A*b) + a*B)*ArcTan[(sqrt[b]*x^(3/2))/sqrt[a]])/(3*b^(5/2))

IntegrateAlgebraic [A] time = 0.07, size = 71, normalized size = 0.97

$$\frac{2(a^{3/2}B - \sqrt{a}Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}} + \frac{2x^{3/2}(-3aB + 3Ab + bBx^3)}{9b^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(7/2)*(A + B*x^3))/(a + b*x^3), x]

[Out] (2*x^(3/2)*(3*A*b - 3*a*B + b*B*x^3))/(9*b^2) + (2*(-(Sqrt[a]*A*b) + a^(3/2)*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))

fricas [A] time = 0.68, size = 143, normalized size = 1.96

$$\left[\frac{3(Ba - Ab)\sqrt{-\frac{a}{b}} \log\left(\frac{bx^3 - 2bx^2\sqrt{-\frac{a}{b}} - a}{bx^3 + a}\right) - 2(Bbx^4 - 3(Ba - Ab)x)\sqrt{x}}{9b^2}, \frac{2\left(3(Ba - Ab)\sqrt{\frac{a}{b}} \arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right) + (Bbx^4 - 3(Ba - Ab)x)\sqrt{x}\right)}{9b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a), x, algorithm="fricas")

[Out] [-1/9*(3*(B*a - A*b)*sqrt(-a/b)*log((b*x^3 - 2*b*x^(3/2)*sqrt(-a/b) - a)/(b*x^3 + a)) - 2*(B*b*x^4 - 3*(B*a - A*b)*x)*sqrt(x))/b^2, 2/9*(3*(B*a - A*b)*sqrt(a/b)*arctan(b*x^(3/2)*sqrt(a/b)/a) + (B*b*x^4 - 3*(B*a - A*b)*x)*sqrt(x))/b^2]

giac [A] time = 0.17, size = 64, normalized size = 0.88

$$\frac{2(Ba^2 - Aab)\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{3\sqrt{ab}b^2} + \frac{2(Bb^2x^9 - 3Babx^3 + 3Ab^2x^3)}{9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a), x, algorithm="giac")

[Out] 2/3*(B*a^2 - A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/9*(B*b^2*x^(9/2) - 3*B*a*b*x^(3/2) + 3*A*b^2*x^(3/2))/b^3

maple [A] time = 0.05, size = 78, normalized size = 1.07

$$\frac{2Bx^9}{9b} - \frac{2Aa\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{3\sqrt{ab}b} + \frac{2Ba^2\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{3\sqrt{ab}b^2} + \frac{2Ax^3}{3b} - \frac{2Ba^2x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(7/2)*(B*x^3+A)/(b*x^3+a),x)`

[Out] $2/9*B*x^{(9/2)}/b+2/3*b*A*x^{(3/2)}-2/3/b^2*B*a*x^{(3/2)}-2/3*a/b/(a*b)^{(1/2)}*\arctan(x^{(3/2)*b/(a*b)^{(1/2)})*A+2/3*a^2/b^2/(a*b)^{(1/2)}*\arctan(x^{(3/2)*b/(a*b)^{(1/2)})*B$

maxima [A] time = 1.32, size = 58, normalized size = 0.79

$$\frac{2(Ba^2 - Aab) \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}b^2} + \frac{2(Bbx^{\frac{9}{2}} - 3(Ba - Ab)x^{\frac{3}{2}})}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")`

[Out] $2/3*(B*a^2 - A*a*b)*\arctan(b*x^{(3/2)}/\sqrt{a*b})/(\sqrt{a*b}*b^2) + 2/9*(B*b*x^{(9/2)} - 3*(B*a - A*b)*x^{(3/2)})/b^2$

mupad [B] time = 2.61, size = 111, normalized size = 1.52

$$x^{3/2} \left(\frac{2A}{3b} - \frac{2Ba}{3b^2} \right) + \frac{2Bx^{9/2}}{9b} - \frac{2\sqrt{a} \operatorname{atan}\left(\frac{72b^{3/2}x^{3/2}(A^2a^2b^2 - 2ABa^3b + B^2a^4)}{\sqrt{a}(72Aa^2b^2 - 72Ba^3b)(Ab - Ba)}\right)}{3b^{5/2}} (Ab - Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(7/2)*(A + B*x^3))/(a + b*x^3),x)`

[Out] $x^{(3/2)*((2*A)/(3*b) - (2*B*a)/(3*b^2)) + (2*B*x^{(9/2)})/(9*b) - (2*a^{(1/2)}*\operatorname{atan}((72*b^{(3/2)}*x^{(3/2)}*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/(a^{(1/2)}*(72*A*a^2*b^2 - 72*B*a^3*b)*(A*b - B*a)))* (A*b - B*a))/(3*b^{(5/2)})$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a),x)`

[Out] Timed out

$$3.152 \quad \int \frac{x^{5/2}(A+Bx^3)}{a+bx^3} dx$$

Optimal. Leaf size=288

$$\frac{\sqrt[6]{a}(Ab - aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} b^{13/6}} - \frac{\sqrt[6]{a}(Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} b^{13/6}} + \frac{\sqrt[6]{a}(Ab - aB)}{7b}$$

Rubi [A] time = 0.52, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {459, 321, 329, 209, 634, 618, 204, 628, 205}

$$\frac{2\sqrt{x}(Ab - aB)}{b^2} + \frac{\sqrt[6]{a}(Ab - aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} b^{13/6}} - \frac{\sqrt[6]{a}(Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} b^{13/6}} + \frac{\sqrt[6]{a}(Ab - aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{a}\sqrt{x}}{\sqrt[6]{b}}\right)}{3b^{13/6}} - \frac{\sqrt[6]{a}(Ab - aB) \tan^{-1}\left(\frac{2\sqrt[6]{a}\sqrt{x}}{\sqrt[6]{b}} + \sqrt{3}\right)}{3b^{13/6}} - \frac{2\sqrt[6]{a}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{a}\sqrt{x}}{\sqrt[6]{b}}\right)}{3b^{13/6}} + \frac{2Bx^{7/2}}{7b}$$

Antiderivative was successfully verified.

[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3), x]

[Out] (2*(A*b - a*B)*Sqrt[x])/b^2 + (2*B*x^(7/2))/(7*b) + (a^(1/6)*(A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) - (a^(1/6)*(A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) - (2*a^(1/6)*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) + (a^(1/6)*(A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*b^(13/6)) - (a^(1/6)*(A*b - a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*b^(13/6))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 209

Int[((a_) + (b_.)*(x_)^(n_))(-1), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*k - 1)*Pi)/n]*x])/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[((2*k - 1)*Pi)/n]*x])/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u

, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a/b]

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2} (A + Bx^3)}{a + bx^3} dx &= \frac{2Bx^{7/2}}{7b} - \frac{\left(2\left(-\frac{7Ab}{2} + \frac{7aB}{2}\right)\right) \int \frac{x^{5/2}}{a+bx^3} dx}{7b} \\
&= \frac{2(Ab - aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(a(Ab - aB)) \int \frac{1}{\sqrt{x}(a+bx^3)} dx}{b^2} \\
&= \frac{2(Ab - aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(2a(Ab - aB)) \text{Subst}\left(\int \frac{1}{a+bx^6} dx, x, \sqrt{x}\right)}{b^2} \\
&= \frac{2(Ab - aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(2\sqrt[6]{a}(Ab - aB)) \text{Subst}\left(\int \frac{\sqrt[6]{a} - \frac{1}{2}\sqrt{3} \sqrt[6]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{3b^2} \\
&= \frac{2(Ab - aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{2\sqrt[6]{a}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{(\sqrt[6]{a}(Ab - aB)) \text{Subst}\left(\int \frac{1}{\sqrt[3]{a}} dx, x, \sqrt{x}\right)}{2\sqrt{3}} \\
&= \frac{2(Ab - aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{2\sqrt[6]{a}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{\sqrt[6]{a}(Ab - aB) \log\left(\sqrt[3]{a} - \sqrt{3}\right)}{2\sqrt{3} b^{13/6}} \\
&= \frac{2(Ab - aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} + \frac{\sqrt[6]{a}(Ab - aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} - \frac{\sqrt[6]{a}(Ab - aB) \tan^{-1}\left(\sqrt[6]{b} \sqrt{x}\right)}{3b^{13/6}}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 54, normalized size = 0.19

$$\frac{2\sqrt{x} \left((7aB - 7Ab) {}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; -\frac{bx^3}{a}\right) - 7aB + 7Ab + bBx^3 \right)}{7b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3), x]

[Out] (2*Sqrt[x]*(7*A*b - 7*a*B + b*B*x^3 + (-7*A*b + 7*a*B)*Hypergeometric2F1[1/6, 1, 7/6, -(b*x^3)/a]))/(7*b^2)

IntegrateAlgebraic [A] time = 0.21, size = 193, normalized size = 0.67

$$\frac{2(a^{7/6}B - \sqrt[6]{a}Ab) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{(\sqrt[6]{a}Ab - a^{7/6}B) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b}x}{\sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}\right)}{3b^{13/6}} + \frac{(a^{7/6}B - \sqrt[6]{a}Ab) \tanh^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{b}x}\right)}{\sqrt{3} b^{13/6}} + \frac{2\sqrt{x} (-7aB + 7Ab + bBx^3)}{7b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*sqrt[x]*(7*A*b - 7*a*B + b*B*x^3))/(7*b^2) + (2*(-(a^(1/6)*A*b) + a^(7/6)
)*B)*ArcTan[(b^(1/6)*sqrt[x])/a^(1/6)]/(3*b^(13/6)) + ((a^(1/6)*A*b - a^(7/6)
)*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*sqrt[x])]/(3*b^(13/6)
) + ((-(a^(1/6)*A*b) + a^(7/6)*B)*ArcTanh[(sqrt[3]*a^(1/6)*b^(1/6)*sqrt[x])
]/(a^(1/3) + b^(1/3)*x)]/(sqrt[3]*b^(13/6))
```

fricas [B] time = 1.07, size = 2433, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/42*(28*sqrt(3)*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*
A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(
1/6)*arctan(1/3*(2*sqrt(3)*sqrt(b^4*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4
*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*
a*b^6)/b^13)^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a*b^2 - A*b^3)*
sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^
3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6))*b^11*(-(
B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*
B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/6) + 2*sqrt(3)*(B*a*b^1
1 - A*b^12)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/
6) - sqrt(3)*(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4
*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6))/(B^6*a^7 - 6*A*B^
5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*
A^5*B*a^2*b^5 + A^6*a*b^6)) + 28*sqrt(3)*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 1
5*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b
^5 + A^6*a*b^6)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(b^4*(-(B^6*a^7 - 6*A
*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 -
6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)
*x - (B*a*b^2 - A*b^3)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*
b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6
)/b^13)^(1/6))*b^11*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/
6) + 2*sqrt(3)*(B*a*b^11 - A*b^12)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*
A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5
+ A^6*a*b^6)/b^13)^(5/6) + sqrt(3)*(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a
^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*
b^6))/(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 +
15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)) - 7*b^2*(-(B^6*a^7 - 6*A
```

$$\begin{aligned}
& *B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - \\
& 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^{13})^{(1/6)}*\log(4*b^4*(-(B^6*a^7 - 6*A*B^5*a^6*b \\
& + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + \\
& A^6*a*b^6)/b^{13})^{(1/3)} + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + \\
& 4*(B*a*b^2 - A*b^3)*\sqrt{x}*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 \\
& + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^{13})^{(1/6)})) + 7*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 \\
& + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^{13})^{(1/6)})*\log(4*b^4*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 \\
& + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^{13})^{(1/3)} + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - 4*(B*a*b^2 - A*b^3)*\sqrt{x}*(-(B^6*a^7 \\
& - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^{13})^{(1/6)})) + 14*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b \\
& + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^{13})^{(1/6)})*\log(b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b \\
& + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^{13})^{(1/6)} - (B*a - A*b)*\sqrt{x}) - 14*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b \\
& + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^{13})^{(1/6)})*\log(-b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 \\
& + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^{13})^{(1/6)} - (B*a - A*b)*\sqrt{x}) - 12*(B*b*x^3 - 7*B*a + 7*A*b)*\sqrt{x})/b^2
\end{aligned}$$

giac [A] time = 0.21, size = 289, normalized size = 1.00

$$\frac{\sqrt{3} \left((ab^5)^2 Ba - (ab^5)^2 Ab \right) \log \left(\sqrt{3} \sqrt{\left(\frac{a}{b}\right)^2 + x + \left(\frac{a}{b}\right)^2} \right)}{6b^5} - \frac{\sqrt{3} \left((ab^5)^2 Ba - (ab^5)^2 Ab \right) \log \left(-\sqrt{3} \sqrt{\left(\frac{a}{b}\right)^2 + x + \left(\frac{a}{b}\right)^2} \right)}{6b^5} + \frac{\left((ab^5)^2 Ba - (ab^5)^2 Ab \right) \arctan \left(\frac{\sqrt{3} \left(\frac{a}{b}\right)^2 \sqrt{x}}{\left(\frac{a}{b}\right)^2} \right)}{3b^5} + \frac{\left((ab^5)^2 Ba - (ab^5)^2 Ab \right) \arctan \left(-\frac{\sqrt{3} \left(\frac{a}{b}\right)^2 \sqrt{x}}{\left(\frac{a}{b}\right)^2} \right)}{3b^5} + \frac{2 \left((ab^5)^2 Ba - (ab^5)^2 Ab \right) \arctan \left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^2} \right)}{3b^5} + \frac{2 \left(8b^6 x^2 - 7Bab^5 \sqrt{x} + 7Ab^6 \sqrt{x} \right)}{7b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")

[Out] 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/b^3 - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/b^3 + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/b^3 + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/b^3 + 2/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/b^3 + 2/7*(B*b^6*x^(7/2) - 7*B*a*b^5*sqrt(x) + 7*A*b^6*sqrt(x))/b^7

maple [A] time = 0.18, size = 377, normalized size = 1.31

$$\frac{2b^5 x^2}{7b^7} + \frac{2 \left(\frac{a}{b}\right)^2 \operatorname{Arctan} \left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^2} \right)}{3b} + \frac{\left(\frac{a}{b}\right)^2 \operatorname{Arctan} \left(\frac{2\sqrt{x} - \sqrt{3}}{\left(\frac{a}{b}\right)^2} \right)}{3b} + \frac{\left(\frac{a}{b}\right)^2 \operatorname{Arctan} \left(\frac{2\sqrt{x} + \sqrt{3}}{\left(\frac{a}{b}\right)^2} \right)}{3b} + \frac{\sqrt{3} \left(\frac{a}{b}\right)^2 \operatorname{Atan} \left(\frac{x + \sqrt{3} \left(\frac{a}{b}\right)^2 \sqrt{x} + \left(\frac{a}{b}\right)^2}{\left(\frac{a}{b}\right)^2} \right)}{6b} + \frac{\sqrt{3} \left(\frac{a}{b}\right)^2 \operatorname{Atan} \left(\frac{-x + \sqrt{3} \left(\frac{a}{b}\right)^2 \sqrt{x} - \left(\frac{a}{b}\right)^2}{\left(\frac{a}{b}\right)^2} \right)}{6b} + \frac{2 \left(\frac{a}{b}\right)^2 \operatorname{Re} \operatorname{arctan} \left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^2} \right)}{3b} + \frac{\left(\frac{a}{b}\right)^2 \operatorname{Re} \operatorname{arctan} \left(\frac{2\sqrt{x} - \sqrt{3}}{\left(\frac{a}{b}\right)^2} \right)}{3b} + \frac{\left(\frac{a}{b}\right)^2 \operatorname{Re} \operatorname{arctan} \left(\frac{2\sqrt{x} + \sqrt{3}}{\left(\frac{a}{b}\right)^2} \right)}{3b} + \frac{\sqrt{3} \left(\frac{a}{b}\right)^2 \operatorname{Re} \operatorname{Im} \left(\frac{x + \sqrt{3} \left(\frac{a}{b}\right)^2 \sqrt{x} + \left(\frac{a}{b}\right)^2}{\left(\frac{a}{b}\right)^2} \right)}{6b} + \frac{\sqrt{3} \left(\frac{a}{b}\right)^2 \operatorname{Re} \operatorname{Im} \left(\frac{-x + \sqrt{3} \left(\frac{a}{b}\right)^2 \sqrt{x} - \left(\frac{a}{b}\right)^2}{\left(\frac{a}{b}\right)^2} \right)}{6b} + \frac{2A\sqrt{x}}{b} + \frac{2Bab\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{(5/2)}*(B*x^3+A)/(b*x^3+a), x)$

[Out] $2/7*B*x^{(7/2)}/b+2/b*A*x^{(1/2)}-2/b^2*B*a*x^{(1/2)}-2/3/b*(a/b)^{(1/6)}*\arctan(x^{(1/2)}/(a/b)^{(1/6)})*A+2/3*a/b^2*(a/b)^{(1/6)}*\arctan(x^{(1/2)}/(a/b)^{(1/6)})*B+1/6/b*3^{(1/2)}*(a/b)^{(1/6)}*\ln(3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)}-x-(a/b)^{(1/3)})*A-1/6*a/b^2*3^{(1/2)}*(a/b)^{(1/6)}*\ln(3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)}-x-(a/b)^{(1/3)})*B-1/3/b*(a/b)^{(1/6)}*\arctan(-3^{(1/2)}+2*x^{(1/2)}/(a/b)^{(1/6)})*A+1/3*a/b^2*(a/b)^{(1/6)}*\arctan(-3^{(1/2)}+2*x^{(1/2)}/(a/b)^{(1/6)})*B-1/6/b*3^{(1/2)}*(a/b)^{(1/6)}*\ln(x+3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)}+(a/b)^{(1/3)})*A+1/6*a/b^2*3^{(1/2)}*(a/b)^{(1/6)}*\ln(x+3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)}+(a/b)^{(1/3)})*B-1/3/b*(a/b)^{(1/6)}*\arctan(2*x^{(1/2)}/(a/b)^{(1/6)}+3^{(1/2)})*A+1/3*a/b^2*(a/b)^{(1/6)}*\arctan(2*x^{(1/2)}/(a/b)^{(1/6)}+3^{(1/2)})*B$

maxima [A] time = 1.17, size = 295, normalized size = 1.02

$$\frac{\left(\frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{c+b^{\frac{1}{3}}x+a^{\frac{1}{3}}}\right)}{a^{\frac{1}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{c+b^{\frac{1}{3}}x+a^{\frac{1}{3}}}\right)}{a^{\frac{1}{6}}b^{\frac{1}{6}}} + \frac{4\left(Ba^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{c}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}} + \frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{c}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}} + \frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{c}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} \right)}{6b^2} + \frac{2\left(Bbx^{\frac{7}{2}}-7(Ba-Ab)\sqrt{x}\right)}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{(5/2)}*(B*x^3+A)/(b*x^3+a), x, \text{algorithm}="maxima")$

[Out] $1/6*(\text{sqrt}(3)*(B*a - A*b)*\log(\text{sqrt}(3)*a^{(1/6)}*b^{(1/6)}*\text{sqrt}(x) + b^{(1/3)}*x + a^{(1/3)})/(a^{(5/6)}*b^{(1/6)}) - \text{sqrt}(3)*(B*a - A*b)*\log(-\text{sqrt}(3)*a^{(1/6)}*b^{(1/6)}*\text{sqrt}(x) + b^{(1/3)}*x + a^{(1/3)})/(a^{(5/6)}*b^{(1/6)}) + 4*(B*a*b^{(1/3)} - A*b^{(4/3)})*\arctan(b^{(1/3)}*\text{sqrt}(x)/\text{sqrt}(a^{(1/3)}*b^{(1/3)}))/ (a^{(2/3)}*b^{(1/3)}*\text{sqrt}(a^{(1/3)}*b^{(1/3)})) + 2*(B*a^{(4/3)}*b^{(1/3)} - A*a^{(1/3)}*b^{(4/3)})*\arctan((\text{sqrt}(3)*a^{(1/6)}*b^{(1/6)} + 2*b^{(1/3)}*\text{sqrt}(x))/\text{sqrt}(a^{(1/3)}*b^{(1/3)}))/ (a*b^{(1/3)}*\text{sqrt}(a^{(1/3)}*b^{(1/3)})) + 2*(B*a^{(4/3)}*b^{(1/3)} - A*a^{(1/3)}*b^{(4/3)})*\arctan(-(\text{sqrt}(3)*a^{(1/6)}*b^{(1/6)} - 2*b^{(1/3)}*\text{sqrt}(x))/\text{sqrt}(a^{(1/3)}*b^{(1/3)}))/ (a*b^{(1/3)}*\text{sqrt}(a^{(1/3)}*b^{(1/3)})))*a/b^2 + 2/7*(B*b*x^{(7/2)} - 7*(B*a - A*b)*\text{sqrt}(x))/b^2$

mupad [B] time = 2.89, size = 1933, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^{(5/2)}*(A + B*x^3))/(a + b*x^3), x)$

[Out] $x^{(1/2)}*((2*A)/b - (2*B*a)/b^2) + (2*B*x^{(7/2)})/(7*b) + ((-a)^{(1/6)}*\text{atan}(((-a)^{(1/6)}*(A*b - B*a)*((96*x^{(1/2)}*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^{(1/6)}*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^{(19/6)})*1i)/(3*b^{(13/6)}) + ((-a)^{(1/6)}*(A*b - B*a)*((96*x^{(1/2)}*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^{(1/6)}*(A$

$$\begin{aligned}
& *b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^{(19/6)} \\
& (6)*1i)/(3*b^{(13/6)})))/(((-a)^{(1/6)}*(A*b - B*a)*((96*x^{(1/2)}*(B^4*a^8 + A^4*a^4*b^4 \\
& + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^{(1/6)}*(A*b - B*a)* \\
& (B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^{(19/6)})))/(3*b^{(13/6)}) - ((-a)^{(1/6)}*(A*b - B*a)*((96*x^{(1/2)}*(B^4*a^8 + A^4*a^4*b^4 \\
& + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^{(1/6)}*(A*b - B*a)* \\
& (B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^{(19/6)})))/(3*b^{(13/6)})))* \\
& (A*b - B*a)*2i)/(3*b^{(13/6)}) + ((-a)^{(1/6)}*atan((((-a)^{(1/6)}*((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a)* \\
& ((96*x^{(1/2)}*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - \\
& (96*(-a)^{(1/6)}*((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + \\
& 3*A^2*B*a^5*b^2))/b^{(19/6)})*1i)/(3*b^{(13/6)}) + ((-a)^{(1/6)}*((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a)* \\
& ((96*x^{(1/2)}*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - \\
& (96*(-a)^{(1/6)}*((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + \\
& 3*A^2*B*a^5*b^2))/b^{(19/6)})*1i)/(3*b^{(13/6)})))/(((-a)^{(1/6)}*((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a)* \\
& ((96*x^{(1/2)}*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - \\
& (96*(-a)^{(1/6)}*((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + \\
& 3*A^2*B*a^5*b^2))/b^{(19/6)})))/(3*b^{(13/6)})) - ((-a)^{(1/6)}*((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a)* \\
& ((96*x^{(1/2)}*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + \\
& (96*(-a)^{(1/6)}*((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + \\
& 3*A^2*B*a^5*b^2))/b^{(19/6)})))/(3*b^{(13/6)})))*((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*b^{(13/6)}) + ((-a)^{(1/6)}*atan((((-a)^{(1/6)}*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)* \\
& ((96*x^{(1/2)}*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - \\
& (96*(-a)^{(1/6)}*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + \\
& 3*A^2*B*a^5*b^2))/b^{(19/6)})*1i)/(3*b^{(13/6)}) + ((-a)^{(1/6)}*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)* \\
& ((96*x^{(1/2)}*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + \\
& (96*(-a)^{(1/6)}*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + \\
& 3*A^2*B*a^5*b^2))/b^{(19/6)})*1i)/(3*b^{(13/6)})))/(((-a)^{(1/6)}*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)* \\
& ((96*x^{(1/2)}*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - \\
& (96*(-a)^{(1/6)}*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + \\
& 3*A^2*B*a^5*b^2))/b^{(19/6)})))/(3*b^{(13/6)})) - ((-a)^{(1/6)}*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)* \\
& ((96*x^{(1/2)}*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + \\
& (96*(-a)^{(1/6)}*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + \\
& 3*A^2*B*a^5*b^2))/b^{(19/6)})))/(3*b^{(13/6)})))*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*b^{(13/6)})
\end{aligned}$$

sympy [A] time = 177.80, size = 881, normalized size = 3.06

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a),x)`

[Out] `Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(7/2)/7), Eq(a, 0) & Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(7/2)/7)/b, Eq(a, 0)), ((2*A*x**(7/2)/7 + 2*B*x**(13/2)/13)/a, Eq(b, 0)), ((-1)**(1/6)*A*a**(1/6)*(1/b)**(1/6)*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b) - (-1)**(1/6)*A*a**(1/6)*(1/b)**(1/6)*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b) + (-1)**(1/6)*A*a**(1/6)*(1/b)**(1/6)*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b) - (-1)**(1/6)*A*a**(1/6)*(1/b)**(1/6)*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b) - (-1)**(1/6)*sqrt(3)*A*a**(1/6)*(1/b)**(1/6)*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b) + (-1)**(1/6)*sqrt(3)*A*a**(1/6)*(1/b)**(1/6)*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b) + 2*A*sqrt(x)/b - (-1)**(1/6)*B*a**(7/6)*(1/b)**(1/6)*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**2) + (-1)**(1/6)*B*a**(7/6)*(1/b)**(1/6)*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**2) - (-1)**(1/6)*B*a**(7/6)*(1/b)**(1/6)*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b**2) + (-1)**(1/6)*B*a**(7/6)*(1/b)**(1/6)*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b**2) + (-1)**(1/6)*sqrt(3)*B*a**(7/6)*(1/b)**(1/6)*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b**2) - (-1)**(1/6)*sqrt(3)*B*a**(7/6)*(1/b)**(1/6)*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b**2) - 2*B*a*sqrt(x)/b**2 + 2*B*x**(7/2)/(7*b), True))`

$$3.153 \quad \int \frac{x^{3/2}(A+Bx^3)}{a+bx^3} dx$$

Optimal. Leaf size=270

$$\frac{(Ab - aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} \sqrt[6]{a} b^{11/6}} - \frac{(Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} \sqrt[6]{a} b^{11/6}} - \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x}{\sqrt[6]{a} b^{11/6}}\right)}{3\sqrt[6]{a} b^{11/6}}$$

Rubi [A] time = 0.55, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {459, 329, 295, 634, 618, 204, 628, 205}

$$\frac{(Ab - aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} \sqrt[6]{a} b^{11/6}} - \frac{(Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} \sqrt[6]{a} b^{11/6}} - \frac{(Ab - aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a} b^{11/6}} + \frac{(Ab - aB) \tan^{-1}\left(\frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}} + \sqrt{3}\right)}{3\sqrt[6]{a} b^{11/6}} + \frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a} b^{11/6}} + \frac{2Bx^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3), x]

[Out] (2*B*x^(5/2))/(5*b) - ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(1/6)*b^(11/6)) + ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(1/6)*b^(11/6)) + (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(1/6)*b^(11/6)) + ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(1/6)*b^(11/6)) - ((A*b - a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(1/6)*b^(11/6))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 295

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*cos[((2*k - 1)*m*Pi)/n] - s*cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*cos[((2*k - 1)*m*Pi)/n] + s*cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]

; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x, x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^(n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2} (A + Bx^3)}{a + bx^3} dx &= \frac{2Bx^{5/2}}{5b} - \frac{\left(2\left(-\frac{5Ab}{2} + \frac{5aB}{2}\right)\right) \int \frac{x^{3/2}}{a+bx^3} dx}{5b} \\
&= \frac{2Bx^{5/2}}{5b} - \frac{\left(4\left(-\frac{5Ab}{2} + \frac{5aB}{2}\right)\right) \text{Subst}\left(\int \frac{x^4}{a+bx^6} dx, x, \sqrt{x}\right)}{5b} \\
&= \frac{2Bx^{5/2}}{5b} + \frac{(2(Ab - aB)) \text{Subst}\left(\int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x^2} dx, x, \sqrt{x}\right)}{3b^{5/3}} + \frac{(2(Ab - aB)) \text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2} + \frac{1}{2}}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a}} dx, x, \sqrt{x}\right)}{3\sqrt[6]{a} b^{5/3}} \\
&= \frac{2Bx^{5/2}}{5b} + \frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a} b^{11/6}} + \frac{(Ab - aB) \text{Subst}\left(\int \frac{-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} + 2\sqrt[3]{b}x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b}x + \sqrt[3]{b}x^2} dx, x, \sqrt{x}\right)}{2\sqrt{3} \sqrt[6]{a} b^{11/6}} \\
&= \frac{2Bx^{5/2}}{5b} + \frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a} b^{11/6}} + \frac{(Ab - aB) \log\left(\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{b}x\right)}{2\sqrt{3} \sqrt[6]{a} b^{11/6}} - \frac{(Ab - aB) \log\left(\sqrt[3]{a} + \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{b}x\right)}{2\sqrt{3} \sqrt[6]{a} b^{11/6}} \\
&= \frac{2Bx^{5/2}}{5b} - \frac{(Ab - aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a} b^{11/6}} + \frac{(Ab - aB) \tan^{-1}\left(\sqrt{3} + \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a} b^{11/6}} + \frac{2(Ab - aB)}{3\sqrt[6]{a} b^{11/6}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 45, normalized size = 0.17

$$\frac{2x^{5/2} \left((Ab - aB) {}_2F_1\left(\frac{5}{6}, 1; \frac{11}{6}; -\frac{bx^3}{a}\right) + aB \right)}{5ab}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3), x]

[Out] (2*x^(5/2)*(a*B + (A*b - a*B)*Hypergeometric2F1[5/6, 1, 11/6, -(b*x^3)/a]))/(5*a*b)

IntegrateAlgebraic [A] time = 0.23, size = 167, normalized size = 0.62

$$-\frac{2(aB - Ab) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a} b^{11/6}} + \frac{(aB - Ab) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b}x}{\sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}\right)}{3\sqrt[6]{a} b^{11/6}} + \frac{(aB - Ab) \tanh^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{b}x}\right)}{\sqrt{3} \sqrt[6]{a} b^{11/6}} + \frac{2Bx^{5/2}}{5b}$$

Antiderivative was successfully verified.

$$\begin{aligned}
& ^{13}) * (- (B^6 * a^6 - 6 * A * B^5 * a^5 * b + 15 * A^2 * B^4 * a^4 * b^2 - 20 * A^3 * B^3 * a^3 * b^3 + \\
& 15 * A^4 * B^2 * a^2 * b^4 - 6 * A^5 * B * a * b^5 + A^6 * b^6) / (a * b^{11}))^{(2/3)} * b^2 * (- (B^6 * \\
& a^6 - 6 * A * B^5 * a^5 * b + 15 * A^2 * B^4 * a^4 * b^2 - 20 * A^3 * B^3 * a^3 * b^3 + 15 * A^4 * B^2 * \\
& a^2 * b^4 - 6 * A^5 * B * a * b^5 + A^6 * b^6) / (a * b^{11}))^{(1/6)} + 2 * \sqrt{3} * (B^5 * a^5 * b^2 \\
& - 5 * A * B^4 * a^4 * b^3 + 10 * A^2 * B^3 * a^3 * b^4 - 10 * A^3 * B^2 * a^2 * b^5 + 5 * A^4 * B * a * b^6 \\
& - A^5 * b^7) * \sqrt{x} * (- (B^6 * a^6 - 6 * A * B^5 * a^5 * b + 15 * A^2 * B^4 * a^4 * b^2 - 20 * A \\
& ^3 * B^3 * a^3 * b^3 + 15 * A^4 * B^2 * a^2 * b^4 - 6 * A^5 * B * a * b^5 + A^6 * b^6) / (a * b^{11}))^{(1 \\
& /6)} + \sqrt{3} * (B^6 * a^6 - 6 * A * B^5 * a^5 * b + 15 * A^2 * B^4 * a^4 * b^2 - 20 * A^3 * B^3 * a^ \\
& 3 * b^3 + 15 * A^4 * B^2 * a^2 * b^4 - 6 * A^5 * B * a * b^5 + A^6 * b^6) / (B^6 * a^6 - 6 * A * B^5 * a \\
& ^5 * b + 15 * A^2 * B^4 * a^4 * b^2 - 20 * A^3 * B^3 * a^3 * b^3 + 15 * A^4 * B^2 * a^2 * b^4 - 6 * A^5 \\
& * B * a * b^5 + A^6 * b^6) + 10 * b * (- (B^6 * a^6 - 6 * A * B^5 * a^5 * b + 15 * A^2 * B^4 * a^4 * b^2 \\
& - 20 * A^3 * B^3 * a^3 * b^3 + 15 * A^4 * B^2 * a^2 * b^4 - 6 * A^5 * B * a * b^5 + A^6 * b^6) / (a * b^ \\
& 11))^{(1/6)} * \log(a * b^9 * (- (B^6 * a^6 - 6 * A * B^5 * a^5 * b + 15 * A^2 * B^4 * a^4 * b^2 - 20 * A \\
& ^3 * B^3 * a^3 * b^3 + 15 * A^4 * B^2 * a^2 * b^4 - 6 * A^5 * B * a * b^5 + A^6 * b^6) / (a * b^{11}))^{(5 \\
& /6)} - (B^5 * a^5 - 5 * A * B^4 * a^4 * b + 10 * A^2 * B^3 * a^3 * b^2 - 10 * A^3 * B^2 * a^2 * b^3 + \\
& 5 * A^4 * B * a * b^4 - A^5 * b^5) * \sqrt{x}) - 10 * b * (- (B^6 * a^6 - 6 * A * B^5 * a^5 * b + 15 * A^ \\
& 2 * B^4 * a^4 * b^2 - 20 * A^3 * B^3 * a^3 * b^3 + 15 * A^4 * B^2 * a^2 * b^4 - 6 * A^5 * B * a * b^5 + A \\
& ^6 * b^6) / (a * b^{11}))^{(1/6)} * \log(- a * b^9 * (- (B^6 * a^6 - 6 * A * B^5 * a^5 * b + 15 * A^2 * B^4 * \\
& a^4 * b^2 - 20 * A^3 * B^3 * a^3 * b^3 + 15 * A^4 * B^2 * a^2 * b^4 - 6 * A^5 * B * a * b^5 + A^6 * b^6 \\
&) / (a * b^{11}))^{(5/6)} - (B^5 * a^5 - 5 * A * B^4 * a^4 * b + 10 * A^2 * B^3 * a^3 * b^2 - 10 * A^3 * \\
& B^2 * a^2 * b^3 + 5 * A^4 * B * a * b^4 - A^5 * b^5) * \sqrt{x}) - 5 * b * (- (B^6 * a^6 - 6 * A * B^5 * \\
& a^5 * b + 15 * A^2 * B^4 * a^4 * b^2 - 20 * A^3 * B^3 * a^3 * b^3 + 15 * A^4 * B^2 * a^2 * b^4 - 6 * A^ \\
& 5 * B * a * b^5 + A^6 * b^6) / (a * b^{11}))^{(1/6)} * \log(4 * (B^5 * a^6 * b^9 - 5 * A * B^4 * a^5 * b^{10} \\
& + 10 * A^2 * B^3 * a^4 * b^{11} - 10 * A^3 * B^2 * a^3 * b^{12} + 5 * A^4 * B * a^2 * b^{13} - A^5 * a * b^{14} \\
&) * \sqrt{x} * (- (B^6 * a^6 - 6 * A * B^5 * a^5 * b + 15 * A^2 * B^4 * a^4 * b^2 - 20 * A^3 * B^3 * a^3 * \\
& b^3 + 15 * A^4 * B^2 * a^2 * b^4 - 6 * A^5 * B * a * b^5 + A^6 * b^6) / (a * b^{11}))^{(5/6)} + 4 * (B^ \\
& 10 * a^{10} - 10 * A * B^9 * a^9 * b + 45 * A^2 * B^8 * a^8 * b^2 - 120 * A^3 * B^7 * a^7 * b^3 + 210 * A \\
& ^4 * B^6 * a^6 * b^4 - 252 * A^5 * B^5 * a^5 * b^5 + 210 * A^6 * B^4 * a^4 * b^6 - 120 * A^7 * B^3 * a^ \\
& 3 * b^7 + 45 * A^8 * B^2 * a^2 * b^8 - 10 * A^9 * B * a * b^9 + A^{10} * b^{10}) * x - 4 * (B^6 * a^7 * b^7 \\
& - 6 * A * B^5 * a^6 * b^8 + 15 * A^2 * B^4 * a^5 * b^9 - 20 * A^3 * B^3 * a^4 * b^{10} + 15 * A^4 * B^2 * \\
& a^3 * b^{11} - 6 * A^5 * B * a^2 * b^{12} + A^6 * a * b^{13}) * (- (B^6 * a^6 - 6 * A * B^5 * a^5 * b + 15 * A \\
& ^2 * B^4 * a^4 * b^2 - 20 * A^3 * B^3 * a^3 * b^3 + 15 * A^4 * B^2 * a^2 * b^4 - 6 * A^5 * B * a * b^5 + \\
& A^6 * b^6) / (a * b^{11}))^{(2/3)} + 5 * b * (- (B^6 * a^6 - 6 * A * B^5 * a^5 * b + 15 * A^2 * B^4 * a^4 \\
& * b^2 - 20 * A^3 * B^3 * a^3 * b^3 + 15 * A^4 * B^2 * a^2 * b^4 - 6 * A^5 * B * a * b^5 + A^6 * b^6) / (\\
& a * b^{11}))^{(1/6)} * \log(- 4 * (B^5 * a^6 * b^9 - 5 * A * B^4 * a^5 * b^{10} + 10 * A^2 * B^3 * a^4 * b^{11} \\
& - 10 * A^3 * B^2 * a^3 * b^{12} + 5 * A^4 * B * a^2 * b^{13} - A^5 * a * b^{14}) * \sqrt{x} * (- (B^6 * a^6 \\
& - 6 * A * B^5 * a^5 * b + 15 * A^2 * B^4 * a^4 * b^2 - 20 * A^3 * B^3 * a^3 * b^3 + 15 * A^4 * B^2 * a^2 * \\
& b^4 - 6 * A^5 * B * a * b^5 + A^6 * b^6) / (a * b^{11}))^{(5/6)} + 4 * (B^{10} * a^{10} - 10 * A * B^9 * a^ \\
& 9 * b + 45 * A^2 * B^8 * a^8 * b^2 - 120 * A^3 * B^7 * a^7 * b^3 + 210 * A^4 * B^6 * a^6 * b^4 - 252 * \\
& A^5 * B^5 * a^5 * b^5 + 210 * A^6 * B^4 * a^4 * b^6 - 120 * A^7 * B^3 * a^3 * b^7 + 45 * A^8 * B^2 * a^ \\
& 2 * b^8 - 10 * A^9 * B * a * b^9 + A^{10} * b^{10}) * x - 4 * (B^6 * a^7 * b^7 - 6 * A * B^5 * a^6 * b^8 + \\
& 15 * A^2 * B^4 * a^5 * b^9 - 20 * A^3 * B^3 * a^4 * b^{10} + 15 * A^4 * B^2 * a^3 * b^{11} - 6 * A^5 * B * a^ \\
& 2 * b^{12} + A^6 * a * b^{13}) * (- (B^6 * a^6 - 6 * A * B^5 * a^5 * b + 15 * A^2 * B^4 * a^4 * b^2 - 20 * A \\
& ^3 * B^3 * a^3 * b^3 + 15 * A^4 * B^2 * a^2 * b^4 - 6 * A^5 * B * a * b^5 + A^6 * b^6) / (a * b^{11}))^{(2 \\
& /3)) / b
\end{aligned}$$

giac [A] time = 1.35, size = 280, normalized size = 1.04

$$\frac{2Bx^{\frac{5}{2}}}{5b} - \frac{2\left(Ba\left(\frac{x}{b}\right)^{\frac{5}{2}} - Ab\left(\frac{x}{b}\right)^{\frac{5}{2}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{2}}}\right)}{3ab} + \frac{\sqrt{5}\left((ab^5)^{\frac{5}{2}}Ba - (ab^5)^{\frac{5}{2}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{x}{b}\right)^{\frac{1}{2}} + x + \left(\frac{x}{b}\right)^{\frac{1}{2}}\right)}{6ab^6} - \frac{\sqrt{3}\left((ab^5)^{\frac{5}{2}}Ba - (ab^5)^{\frac{5}{2}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{x}{b}\right)^{\frac{1}{2}} + x + \left(\frac{x}{b}\right)^{\frac{1}{2}}\right)}{6ab^6} - \frac{\left((ab^5)^{\frac{5}{2}}Ba - (ab^5)^{\frac{5}{2}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{2}} + 2\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{2}}}\right)}{3ab^6} - \frac{\left((ab^5)^{\frac{5}{2}}Ba - (ab^5)^{\frac{5}{2}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{2}} - 2\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{2}}}\right)}{3ab^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{2}{5}Bx^{\frac{5}{2}}/b - \frac{2}{3}*(B*a*(a/b)^{\frac{5}{6}} - A*b*(a/b)^{\frac{5}{6}})*\arctan(\sqrt{x}/(a/b)^{\frac{1}{6}})/(a*b) + \frac{1}{6}*\sqrt{3}*((a*b^5)^{\frac{5}{6}}*B*a - (a*b^5)^{\frac{5}{6}}*A*b)*\log(\sqrt{3}*\sqrt{x}*(a/b)^{\frac{1}{6}} + x + (a/b)^{\frac{1}{3}})/(a*b^6) - \frac{1}{6}*\sqrt{3}*((a*b^5)^{\frac{5}{6}}*B*a - (a*b^5)^{\frac{5}{6}}*A*b)*\log(-\sqrt{3}*\sqrt{x}*(a/b)^{\frac{1}{6}} + x + (a/b)^{\frac{1}{3}})/(a*b^6) - \frac{1}{3}*((a*b^5)^{\frac{5}{6}}*B*a - (a*b^5)^{\frac{5}{6}}*A*b)*\arctan((\sqrt{3}*(a/b)^{\frac{1}{6}} + 2*\sqrt{x})/(a/b)^{\frac{1}{6}})/(a*b^6) - \frac{1}{3}*((a*b^5)^{\frac{5}{6}}*B*a - (a*b^5)^{\frac{5}{6}}*A*b)*\arctan(-(\sqrt{3}*(a/b)^{\frac{1}{6}} - 2*\sqrt{x})/(a/b)^{\frac{1}{6}})/(a*b^6)$

maple [A] time = 0.16, size = 356, normalized size = 1.32

$$\frac{2Bx^{\frac{5}{2}}}{5b} - \frac{\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{2}}A\ln\left(x + \sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{2}}\sqrt{x} + \left(\frac{x}{b}\right)^{\frac{1}{2}}\right)}{6a} - \frac{\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{2}}A\ln\left(-x + \sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{2}}\sqrt{x} - \left(\frac{x}{b}\right)^{\frac{1}{2}}\right)}{6a} - \frac{2A\arctan\left(\frac{\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{2}}}\right)}{3\left(\frac{x}{b}\right)^{\frac{1}{2}}b} - \frac{A\arctan\left(\frac{2\sqrt{x} - \sqrt{3}}{\left(\frac{x}{b}\right)^{\frac{1}{2}}}\right)}{3\left(\frac{x}{b}\right)^{\frac{1}{2}}b} - \frac{A\arctan\left(\frac{2\sqrt{x} + \sqrt{3}}{\left(\frac{x}{b}\right)^{\frac{1}{2}}}\right)}{3\left(\frac{x}{b}\right)^{\frac{1}{2}}b} - \frac{2Ba\arctan\left(\frac{\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{2}}}\right)}{3\left(\frac{x}{b}\right)^{\frac{1}{2}}b^2} - \frac{Ba\arctan\left(\frac{2\sqrt{x} - \sqrt{3}}{\left(\frac{x}{b}\right)^{\frac{1}{2}}}\right)}{3\left(\frac{x}{b}\right)^{\frac{1}{2}}b^2} - \frac{Ba\arctan\left(\frac{2\sqrt{x} + \sqrt{3}}{\left(\frac{x}{b}\right)^{\frac{1}{2}}}\right)}{3\left(\frac{x}{b}\right)^{\frac{1}{2}}b^2} + \frac{\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{2}}B\ln\left(x + \sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{2}}\sqrt{x} + \left(\frac{x}{b}\right)^{\frac{1}{2}}\right)}{6b} - \frac{\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{2}}B\ln\left(-x + \sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{2}}\sqrt{x} - \left(\frac{x}{b}\right)^{\frac{1}{2}}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a),x)

[Out] $\frac{2}{5}Bx^{\frac{5}{2}}/b + \frac{2}{3}b/(a/b)^{\frac{1}{6}}*\arctan(1/(a/b)^{\frac{1}{6}}*x^{\frac{1}{2}})*A - \frac{2}{3}b^2/(a/b)^{\frac{1}{6}}*\arctan(1/(a/b)^{\frac{1}{6}}*x^{\frac{1}{2}})*B*a + \frac{1}{6}a*3^{\frac{1}{2}}*(a/b)^{\frac{5}{6}}*\ln(-x+3^{\frac{1}{2}}*(a/b)^{\frac{1}{6}}*x^{\frac{1}{2}}-(a/b)^{\frac{1}{3}})*A - \frac{1}{6}b*3^{\frac{1}{2}}*(a/b)^{\frac{5}{6}}*\ln(-x+3^{\frac{1}{2}}*(a/b)^{\frac{1}{6}}*x^{\frac{1}{2}}-(a/b)^{\frac{1}{3}})*B + \frac{1}{3}b/(a/b)^{\frac{1}{6}}*\arctan(2/(a/b)^{\frac{1}{6}}*x^{\frac{1}{2}}-3^{\frac{1}{2}}*(a/b)^{\frac{1}{6}})*A - \frac{1}{3}b^2/(a/b)^{\frac{1}{6}}*\arctan(2/(a/b)^{\frac{1}{6}}*x^{\frac{1}{2}}-3^{\frac{1}{2}}*(a/b)^{\frac{1}{6}})*B*a - \frac{1}{6}a*3^{\frac{1}{2}}*(a/b)^{\frac{5}{6}}*\ln(x+3^{\frac{1}{2}}*(a/b)^{\frac{1}{6}}*x^{\frac{1}{2}}+(a/b)^{\frac{1}{3}})*A + \frac{1}{6}b*3^{\frac{1}{2}}*(a/b)^{\frac{5}{6}}*\ln(x+3^{\frac{1}{2}}*(a/b)^{\frac{1}{6}}*x^{\frac{1}{2}}+(a/b)^{\frac{1}{3}})*B + \frac{1}{3}b/(a/b)^{\frac{1}{6}}*\arctan(2/(a/b)^{\frac{1}{6}}*x^{\frac{1}{2}}+3^{\frac{1}{2}}*(a/b)^{\frac{1}{6}})*A - \frac{1}{3}b^2/(a/b)^{\frac{1}{6}}*\arctan(2/(a/b)^{\frac{1}{6}}*x^{\frac{1}{2}}+3^{\frac{1}{2}}*(a/b)^{\frac{1}{6}})*B*a$

maxima [A] time = 1.14, size = 212, normalized size = 0.79

$$\frac{2Bx^{\frac{5}{2}}}{5b} + \frac{(Ba - Ab) \left(\frac{\sqrt{3} \log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x} + b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right)}{\frac{1}{a^{\frac{1}{6}}b^{\frac{1}{6}}}} - \frac{\sqrt{3} \log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x} + b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right)}{\frac{1}{a^{\frac{1}{6}}b^{\frac{1}{6}}}} - \frac{2 \arctan\left(\frac{\frac{1}{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}} + 2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}\right)}{\frac{2}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}} - \frac{2 \arctan\left(-\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}} - 2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{\frac{2}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}} - \frac{4 \arctan\left(\frac{\frac{1}{b^{\frac{1}{3}}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{\frac{2}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} \right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")

[Out] $\frac{2}{5}Bx^{5/2}/b + \frac{1}{6}(Ba - Ab)(\sqrt{3})\log(\sqrt{3})a^{1/6}b^{1/6}\sqrt{x + b^{1/3}x + a^{1/3}}/(a^{1/6}b^{5/6}) - \sqrt{3}\log(-\sqrt{3})a^{1/6}b^{1/6}\sqrt{x + b^{1/3}x + a^{1/3}}/(a^{1/6}b^{5/6}) - 2\arctan((\sqrt{3})a^{1/6}b^{1/6} + 2b^{1/3}\sqrt{x})/\sqrt{a^{1/3}b^{1/3}}/(b^{2/3}\sqrt{a^{1/3}b^{1/3}}) - 2\arctan(-(\sqrt{3})a^{1/6}b^{1/6} - 2b^{1/3}\sqrt{x})/\sqrt{a^{1/3}b^{1/3}}/(b^{2/3}\sqrt{a^{1/3}b^{1/3}}) - 4\arctan(b^{1/3}\sqrt{x})/\sqrt{a^{1/3}b^{1/3}}/(b^{2/3}\sqrt{a^{1/3}b^{1/3}})/b$

mupad [B] time = 2.85, size = 1640, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^{3/2}(A + Bx^3))/(a + bx^3), x)$

[Out] $(2Bx^{5/2})/(5b) + (\text{atan}(\frac{(Ab - Ba)^2(32A^3a^3b^3 - 32B^3a^6 + 96A^2B^2a^5b - 96A^2B^2a^4b^2 + (x^{1/2}(Ab - Ba)(864A^2a^3b^4 + 864B^2a^5b^2 - 1728ABa^4b^3))}{27(-a)^{1/6}b^{11/6}})) * 1i) / ((-a)^{1/3}b^{11/3}) + ((Ab - Ba)^2(32B^3a^6 - 32A^3a^3b^3 - 96A^2B^2a^5b + 96A^2B^2a^4b^2 + (x^{1/2}(Ab - Ba)(864A^2a^3b^4 + 864B^2a^5b^2 - 1728ABa^4b^3))}{27(-a)^{1/6}b^{11/6}})) * 1i) / ((-a)^{1/3}b^{11/3})) / (((Ab - Ba)^2(32A^3a^3b^3 - 32B^3a^6 + 96A^2B^2a^5b - 96A^2B^2a^4b^2 + (x^{1/2}(Ab - Ba)(864A^2a^3b^4 + 864B^2a^5b^2 - 1728ABa^4b^3))}{27(-a)^{1/6}b^{11/6}})) / ((-a)^{1/3}b^{11/3}) - ((Ab - Ba)^2(32B^3a^6 - 32A^3a^3b^3 - 96A^2B^2a^5b + 96A^2B^2a^4b^2 + (x^{1/2}(Ab - Ba)(864A^2a^3b^4 + 864B^2a^5b^2 - 1728ABa^4b^3))}{27(-a)^{1/6}b^{11/6}})) / ((-a)^{1/3}b^{11/3})) * (Ab - Ba) * 2i) / (3(-a)^{1/6}b^{11/6}) + (\text{atan}(\frac{(3^{1/2} * 1i) / 2 - 1/2)^2(Ab - Ba)^2(32A^3a^3b^3 - 32B^3a^6 + 96A^2B^2a^5b - 96A^2B^2a^4b^2 + (x^{1/2} * ((3^{1/2} * 1i) / 2 - 1/2) * (Ab - Ba) * (864A^2a^3b^4 + 864B^2a^5b^2 - 1728ABa^4b^3))}{27(-a)^{1/6}b^{11/6}})) * 1i) / ((-a)^{1/3}b^{11/3}) + (((3^{1/2} * 1i) / 2 - 1/2)^2(Ab - Ba)^2(32B^3a^6 - 32A^3a^3b^3 - 96A^2B^2a^5b + 96A^2B^2a^4b^2 + (x^{1/2} * ((3^{1/2} * 1i) / 2 - 1/2) * (Ab - Ba) * (864A^2a^3b^4 + 864B^2a^5b^2 - 1728ABa^4b^3))}{27(-a)^{1/6}b^{11/6}})) / ((-a)^{1/3}b^{11/3}) - (((3^{1/2} * 1i) / 2 - 1/2)^2(Ab - Ba)^2(32B^3a^6 - 32A^3a^3b^3 - 96A^2B^2a^5b + 96A^2B^2a^4b^2 + (x^{1/2} * ((3^{1/2} * 1i) / 2 - 1/2) * (Ab - Ba) * (864A^2a^3b^4 + 864B^2a^5b^2 - 1728ABa^4b^3))}{27(-a)^{1/6}b^{11/6}})) / ((-a)^{1/3}b^{11/3})) * ((3^{1/2} * 1i) / 2 - 1/2) * (Ab - Ba) * 2i) / (3(-a)^{1/6}b^{11/6}) + (\text{atan}(\frac{(3^{1/2} * 1i) / 2 + 1/2)^2(Ab - Ba)^2(32A^3a^3b^3 - 32B^3a^6 + 96A^2B^2a^5b - 96A^2B^2a^4b^2 + (x^{1/2} * ((3^{1/2} * 1i) / 2 + 1/2) * (Ab - Ba) * (864A^2a^3b^4 + 864B^2a^5b^2 - 1728ABa^4b^3))}{27(-a)^{1/6}b^{11/6}})) * 1i) / ((-a)^{1/3}b^{11/3}) + (((3^{1/2} * 1i) / 2 + 1/2)^2(Ab - Ba)^2(32B^3a^6 - 32A^3a^3b^3 - 96A^2B^2a^5b + 96A^2B^2a^4b^2 + (x^{1/2} * ((3^{1/2} * 1i) / 2 + 1/2) * (Ab - Ba) * (864A^2a^3b^4 + 864B^2a^5b^2 - 1728ABa^4b^3))}{27(-a)^{1/6}b^{11/6}})) / ((-a)^{1/3}b^{11/3})) * ((3^{1/2} * 1i) / 2 + 1/2) * (Ab - Ba) * 2i) / (3(-a)^{1/6}b^{11/6})$

$$\begin{aligned} & (1/6)*b^{(11/6)}) * i) / ((-a)^{(1/3)} * b^{(11/3)}) + (((3^{(1/2)} * i) / 2 + 1/2)^2 * (A * b \\ & - B * a)^2 * (32 * B^3 * a^6 - 32 * A^3 * a^3 * b^3 - 96 * A * B^2 * a^5 * b + 96 * A^2 * B * a^4 * b^2 \\ & + (x^{(1/2)} * ((3^{(1/2)} * i) / 2 + 1/2) * (A * b - B * a) * (864 * A^2 * a^3 * b^4 + 864 * B^2 * a^5 * b^2 \\ & - 1728 * A * B * a^4 * b^3))) / (27 * (-a)^{(1/6)} * b^{(11/6)}) * i) / ((-a)^{(1/3)} * b^{(11/3)}) \\ & / (((3^{(1/2)} * i) / 2 + 1/2)^2 * (A * b - B * a)^2 * (32 * A^3 * a^3 * b^3 - 32 * B^3 * a^6 \\ & + 96 * A * B^2 * a^5 * b - 96 * A^2 * B * a^4 * b^2 + (x^{(1/2)} * ((3^{(1/2)} * i) / 2 + 1/2) * (A * b \\ & - B * a) * (864 * A^2 * a^3 * b^4 + 864 * B^2 * a^5 * b^2 - 1728 * A * B * a^4 * b^3))) / (27 * (-a)^{(1/6)} * b^{(11/6)}) \\ & / ((-a)^{(1/3)} * b^{(11/3)}) - (((3^{(1/2)} * i) / 2 + 1/2)^2 * (A * b - B * a)^2 * (32 * B^3 * a^6 - 32 * A^3 * a^3 * b^3 \\ & - 96 * A * B^2 * a^5 * b + 96 * A^2 * B * a^4 * b^2 + (x^{(1/2)} * ((3^{(1/2)} * i) / 2 + 1/2) * (A * b - B * a) * (864 * A^2 * a^3 * b^4 \\ & + 864 * B^2 * a^5 * b^2 - 1728 * A * B * a^4 * b^3))) / (27 * (-a)^{(1/6)} * b^{(11/6)}) / ((-a)^{(1/3)} * b^{(11/3)}) * ((3^{(1/2)} * i) / 2 + 1/2) * (A * b - B * a) * 2i) / (3 * (-a)^{(1/6)} * b^{(11/6)}) \end{aligned}$$

sympy [A] time = 64.30, size = 857, normalized size = 3.17

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a), x)

[Out] Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*x**(5/2)/5)/b, Eq(a, 0)), ((2*A*x**(5/2)/5 + 2*B*x**(11/2)/11)/a, Eq(b, 0)), (-(-1)**(5/6)*A*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*A*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(1/6)*b*(1/b)**(1/6)) - (-1)**(5/6)*A*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*A*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(1/6)*b*(1/b)**(1/6)) - (-1)**(5/6)*sqrt(3)*A*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*sqrt(3)*A*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*B*a**(5/6)*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**2*(1/b)**(1/6)) - (-1)**(5/6)*B*a**(5/6)*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**2*(1/b)**(1/6)) + (-1)**(5/6)*B*a**(5/6)*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b**2*(1/b)**(1/6)) - (-1)**(5/6)*B*a**(5/6)*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b**2*(1/b)**(1/6)) + (-1)**(5/6)*sqrt(3)*B*a**(5/6)*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b**2*(1/b)**(1/6)) - (-1)**(5/6)*sqrt(3)*B*a**(5/6)*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b**2*(1/b)**(1/6)) + 2*B*x**(5/2)/(5*b), True))

$$3.154 \quad \int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3} dx$$

Optimal. Leaf size=53

$$\frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}} + \frac{2Bx^{3/2}}{3b}$$

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {459, 329, 275, 205}

$$\frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}} + \frac{2Bx^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]

[Out] (2*B*x^(3/2))/(3*b) + (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(3/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 329

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), x]

+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{x} (A + Bx^3)}{a + bx^3} dx &= \frac{2Bx^{3/2}}{3b} - \frac{\left(2\left(-\frac{3Ab}{2} + \frac{3aB}{2}\right)\right) \int \frac{\sqrt{x}}{a+bx^3} dx}{3b} \\
 &= \frac{2Bx^{3/2}}{3b} - \frac{\left(4\left(-\frac{3Ab}{2} + \frac{3aB}{2}\right)\right) \text{Subst}\left(\int \frac{x^2}{a+bx^6} dx, x, \sqrt{x}\right)}{3b} \\
 &= \frac{2Bx^{3/2}}{3b} + \frac{(2(Ab - aB)) \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, x^{3/2}\right)}{3b} \\
 &= \frac{2Bx^{3/2}}{3b} + \frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.08, size = 52, normalized size = 0.98

$$\frac{2}{3} \left(\frac{Bx^{3/2}}{b} - \frac{(aB - Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3), x]

[Out] (2*((B*x^(3/2))/b - ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(Sqrt[a]*b^(3/2)))/3

IntegrateAlgebraic [A] time = 0.05, size = 53, normalized size = 1.00

$$\frac{2Bx^{3/2}}{3b} - \frac{2(aB - Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[x]*(A + B*x^3))/(a + b*x^3), x]

[Out] (2*B*x^(3/2))/(3*b) - (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(3/2))

fricas [A] time = 0.81, size = 108, normalized size = 2.04

$$\left[\frac{2 Babx^{\frac{3}{2}} + (Ba - Ab)\sqrt{-ab} \log\left(\frac{bx^3 - 2\sqrt{-ab}x^{\frac{3}{2}} - a}{bx^3 + a}\right)}{3 ab^2}, \frac{2\left(Babx^{\frac{3}{2}} - (Ba - Ab)\sqrt{ab} \arctan\left(\frac{\sqrt{ab}x^{\frac{3}{2}}}{a}\right)\right)}{3 ab^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="fricas")

[Out] [1/3*(2*B*a*b*x^(3/2) + (B*a - A*b)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)))/(a*b^2), 2/3*(B*a*b*x^(3/2) - (B*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a))/(a*b^2)]

giac [A] time = 0.17, size = 39, normalized size = 0.74

$$\frac{2 Bx^{\frac{3}{2}}}{3b} - \frac{2(Ba - Ab) \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="giac")

[Out] 2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b)

maple [A] time = 0.05, size = 53, normalized size = 1.00

$$\frac{2A \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}} - \frac{2Ba \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}b} + \frac{2Bx^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a),x)

[Out] 2/3*B*x^(3/2)/b+2/3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*A-2/3/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*B*a

maxima [A] time = 1.21, size = 39, normalized size = 0.74

$$\frac{2 Bx^{\frac{3}{2}}}{3b} - \frac{2(Ba - Ab) \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a), x, algorithm="maxima")
```

```
[Out] 2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [B] time = 2.60, size = 93, normalized size = 1.75

$$\frac{2 B x^{3/2}}{3 b} - \frac{2 \operatorname{atan}\left(\frac{3 \sqrt{a} b^{3/2} x^{3/2} (24 A^2 b^3 - 48 A B a b^2 + 24 B^2 a^2 b)}{(72 B a^2 b^2 - 72 A a b^3)(A b - B a)}\right) (A b - B a)}{3 \sqrt{a} b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^3))/(a + b*x^3), x)
```

```
[Out] (2*B*x^(3/2))/(3*b) - (2*atan((3*a^(1/2)*b^(3/2)*x^(3/2)*(24*A^2*b^3 + 24*B^2*a^2*b - 48*A*B*a*b^2))/(72*B*a^2*b^2 - 72*A*a*b^3)*(A*b - B*a)))/(3*a^(1/2)*b^(3/2))
```

sympy [A] time = 22.39, size = 537, normalized size = 10.13

$$\begin{cases} \frac{24}{3b^2} + \frac{20x^{3/2}}{3} & \text{for } a = 0 \wedge b = 0 \\ \frac{24}{3b^2} + \frac{20x^{3/2}}{3} & \text{for } a = 0 \\ \frac{24}{3b^2} + \frac{20x^{3/2}}{3} & \text{for } b = 0 \\ \frac{24}{3\sqrt{a}\sqrt{b^2}} + \frac{I A \log(\sqrt{7}\sqrt{a}\sqrt{b^2} + \sqrt{a})}{3\sqrt{a}\sqrt{b^2}} + \frac{I A \log(-4\sqrt{7}\sqrt{a}\sqrt{b^2} + 4\sqrt{7}\sqrt{a}\sqrt{b^2} + 4)}{3\sqrt{a}\sqrt{b^2}} - \frac{I A \log(4\sqrt{7}\sqrt{a}\sqrt{b^2} + 4\sqrt{7}\sqrt{a}\sqrt{b^2} + 4)}{3\sqrt{a}\sqrt{b^2}} + \frac{B \sqrt{a} \log(-\sqrt{7}\sqrt{a}\sqrt{b^2} + \sqrt{a})}{B \sqrt{a}\sqrt{b^2}} - \frac{B \sqrt{a} \log(\sqrt{7}\sqrt{a}\sqrt{b^2} + \sqrt{a})}{B \sqrt{a}\sqrt{b^2}} - \frac{B \sqrt{a} \log(-4\sqrt{7}\sqrt{a}\sqrt{b^2} + 4\sqrt{7}\sqrt{a}\sqrt{b^2} + 4)}{B \sqrt{a}\sqrt{b^2}} + \frac{B \sqrt{a} \log(4\sqrt{7}\sqrt{a}\sqrt{b^2} + 4\sqrt{7}\sqrt{a}\sqrt{b^2} + 4)}{B \sqrt{a}\sqrt{b^2}} + \frac{20x^{3/2}}{30} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a), x)
```

```
[Out] Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*x**(3/2)/3), Eq(a, 0) & Eq(b, 0)), ((-2*A/(3*x**(3/2)) + 2*B*x**(3/2)/3)/b, Eq(a, 0)), ((2*A*x**(3/2)/3 + 2*B*x**(9/2)/9)/a, Eq(b, 0)), (-I*A*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*sqrt(a)*b*sqrt(1/b)) + I*A*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*sqrt(a)*b*sqrt(1/b)) + I*A*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*sqrt(a)*b*sqrt(1/b)) - I*A*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*sqrt(a)*b*sqrt(1/b)) + I*B*sqrt(a)*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**2*sqrt(1/b)) - I*B*sqrt(a)*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**2*sqrt(1/b)) - I*B*sqrt(a)*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*b**2*sqrt(1/b)) + I*B*sqrt(a)*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*b**2*sqrt(1/b)) + 2*B*x**(3/2)/(3*b), True))
```

$$3.155 \quad \int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)} dx$$

Optimal. Leaf size=268

$$\frac{(Ab - aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{5/6} b^{7/6}} + \frac{(Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{5/6} b^{7/6}} - \frac{(Ab - aB) \tan^{-1}}{3a^{5/6}}$$

Rubi [A] time = 0.47, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {459, 329, 209, 634, 618, 204, 628, 205}

$$\frac{(Ab - aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{5/6} b^{7/6}} + \frac{(Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{5/6} b^{7/6}} - \frac{(Ab - aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6} b^{7/6}} + \frac{(Ab - aB) \tan^{-1}\left(\frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}} + \sqrt{3}\right)}{3a^{5/6} b^{7/6}} + \frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6} b^{7/6}} + \frac{2B\sqrt{x}}{b}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)), x]

[Out] (2*B*Sqrt[x])/b - ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/((3*a^(5/6)*b^(7/6)) + ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/((3*a^(5/6)*b^(7/6)) + (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]))/((3*a^(5/6)*b^(7/6)) - ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(5/6)*b^(7/6)) + ((A*b - a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(5/6)*b^(7/6))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 209

Int[((a_) + (b_.)*(x_)^(n_))^(m_), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u

, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a/b]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{\sqrt{x}(a + bx^3)} dx &= \frac{2B\sqrt{x}}{b} - \frac{\left(2\left(-\frac{Ab}{2} + \frac{aB}{2}\right)\right) \int \frac{1}{\sqrt{x}(a+bx^3)} dx}{b} \\
&= \frac{2B\sqrt{x}}{b} - \frac{\left(4\left(-\frac{Ab}{2} + \frac{aB}{2}\right)\right) \text{Subst}\left(\int \frac{1}{a+bx^6} dx, x, \sqrt{x}\right)}{b} \\
&= \frac{2B\sqrt{x}}{b} + \frac{(2(Ab - aB)) \text{Subst}\left(\int \frac{\sqrt[6]{a} - \frac{1}{2}\sqrt{3} \sqrt[6]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{3a^{5/6}b} + \frac{(2(Ab - aB)) \text{Subst}\left(\int \frac{1}{a+bx^6} dx, x, \sqrt{x}\right)}{b} \\
&= \frac{2B\sqrt{x}}{b} + \frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} - \frac{(Ab - aB) \text{Subst}\left(\int \frac{-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} + 2\sqrt[3]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{2\sqrt{3} a^{5/6}b^{7/6}} \\
&= \frac{2B\sqrt{x}}{b} + \frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} - \frac{(Ab - aB) \log\left(\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{5/6}b^{7/6}} + \frac{(Ab - aB) \text{Subst}\left(\int \frac{1}{a+bx^6} dx, x, \sqrt{x}\right)}{b} \\
&= \frac{2B\sqrt{x}}{b} - \frac{(Ab - aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} + \frac{(Ab - aB) \tan^{-1}\left(\sqrt{3} + \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} + \frac{2(Ab - aB) \text{Subst}\left(\int \frac{1}{a+bx^6} dx, x, \sqrt{x}\right)}{b}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 43, normalized size = 0.16

$$\frac{2\sqrt{x} \left((Ab - aB) {}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; -\frac{bx^3}{a}\right) + aB \right)}{ab}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)), x]

[Out] (2*Sqrt[x]*(a*B + (A*b - a*B)*Hypergeometric2F1[1/6, 1, 7/6, -(b*x^3)/a]))/(a*b)

IntegrateAlgebraic [A] time = 0.25, size = 166, normalized size = 0.62

$$-\frac{2(aB - Ab) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} + \frac{(aB - Ab) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b} x}{\sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}\right)}{3a^{5/6}b^{7/6}} - \frac{(aB - Ab) \tanh^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{b} x}\right)}{\sqrt{3} a^{5/6}b^{7/6}} + \frac{2B\sqrt{x}}{b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)), x]


```
[Out] (2*B*Sqrt[x])/b - (2*(-(A*b) + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(3*a
^(5/6)*b^(7/6)) + ((-(A*b) + a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(
1/6)*Sqrt[x]])/(3*a^(5/6)*b^(7/6)) - ((-(A*b) + a*B)*ArcTanh[(Sqrt[3]*a^(1
/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)]/(Sqrt[3]*a^(5/6)*b^(7/6))
```

fricas [B] time = 1.00, size = 2424, normalized size = 9.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B
^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)
*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4
*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^
6)/(a^5*b^7))^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a^2*b - A*a*b^
2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3
*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6))*a^4*
b^6*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(5/6) + 2*sqrt(3)*
(B*a^5*b^6 - A*a^4*b^7)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4
*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(
a^5*b^7))^(5/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 2
0*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(B^6*a^6
- 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2
*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) + 4*sqrt(3)*b*(-(B^6*a^6 - 6*A*B^5*a^5*b +
15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b
^5 + A^6*b^6)/(a^5*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^2*(-(B^6*a^
6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^
2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/3) + (B^2*a^2 - 2*A*B*a*b +
A^2*b^2)*x - (B*a^2*b - A*a*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^
2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A
^6*b^6)/(a^5*b^7))^(1/6))*a^4*b^6*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a
^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)
/(a^5*b^7))^(5/6) + 2*sqrt(3)*(B*a^5*b^6 - A*a^4*b^7)*sqrt(x)*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(5/6) + sqrt(3)*(B^6*a^6 - 6*A*B^5*
a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^
5*B*a*b^5 + A^6*b^6)/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^
3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) - b*(-(B^6*a
^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a
^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(4*a^2*b^2*(-(B^6*a^6
- 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2
*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b +
A^2*b^2)*x + 4*(B*a^2*b - A*a*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15
```

$$\begin{aligned}
& *A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 \\
& + A^6*b^6)/(a^5*b^7))^{(1/6)} + b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4 \\
& *b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/ \\
& (a^5*b^7))^{(1/6)}*\log(4*a^2*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4* \\
& b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a \\
& ^5*b^7))^{(1/3)} + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - 4*(B*a^2*b - A*a*b^2 \\
&)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3* \\
& b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^{(1/6)} + 2*b \\
& *(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15* \\
& A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^{(1/6)}*\log(a*b*(-(B^6* \\
& a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2* \\
& a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^{(1/6)} - (B*a - A*b)*sqrt(x)) \\
& - 2*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 \\
& + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^{(1/6)}*\log(-a*b*(\\
& -(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^ \\
& 4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^{(1/6)} - (B*a - A*b)*sq \\
& rt(x)) + 12*B*sqrt(x))/b
\end{aligned}$$

giac [A] time = 0.21, size = 280, normalized size = 1.04

$$\frac{2B\sqrt{x}}{b} - \frac{\sqrt{3}\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{x}{b}\right)^{\frac{1}{6}} + x + \left(\frac{x}{b}\right)^{\frac{1}{3}}\right)}{6ab^2} + \frac{\sqrt{3}\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{x}{b}\right)^{\frac{1}{6}} + x + \left(\frac{x}{b}\right)^{\frac{1}{3}}\right)}{6ab^2} - \frac{\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{6}} + 2\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} - \frac{\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{-\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{6}} - 2\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} - \frac{2\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="giac")

[Out] 2*B*sqrt(x)/b - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^2) + 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^2) - 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^2) - 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a*b^2) - 2/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a*b^2)

maple [A] time = 0.15, size = 353, normalized size = 1.32

$$\frac{2\left(\frac{x}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{3}}}\right)}{3a} + \frac{\left(\frac{x}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{3}} - \sqrt{3}}\right)}{3a} + \frac{\left(\frac{x}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{3}} + \sqrt{3}}\right)}{3a} + \frac{\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{6}}A\ln\left(x + \sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{3}}\sqrt{x} + \left(\frac{x}{b}\right)^{\frac{1}{3}}\right)}{6a} + \frac{\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{6}}A\ln\left(-x + \sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{3}}\sqrt{x} - \left(\frac{x}{b}\right)^{\frac{1}{3}}\right)}{6a} - \frac{2\left(\frac{x}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{3}}}\right)}{3b} + \frac{\left(\frac{x}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{3}} - \sqrt{3}}\right)}{3b} + \frac{\left(\frac{x}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{3}} + \sqrt{3}}\right)}{3b} + \frac{\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{6}}B\ln\left(x + \sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{3}}\sqrt{x} + \left(\frac{x}{b}\right)^{\frac{1}{3}}\right)}{6b} + \frac{\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{6}}B\ln\left(-x + \sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{3}}\sqrt{x} - \left(\frac{x}{b}\right)^{\frac{1}{3}}\right)}{6b} + \frac{2B\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/(b*x^3+a)/x^(1/2),x)

[Out] 2*B*x^(1/2)/b+2/3/a*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A-2/3/b*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B-1/6/a*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2))*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*A+1/6/b*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1

$$\frac{1}{2} \cdot \left(\frac{a}{b}\right)^{1/6} \cdot x^{1/2} - \left(\frac{a}{b}\right)^{1/3} \Big) \cdot B + \frac{1}{3} \cdot a \cdot \left(\frac{a}{b}\right)^{1/6} \cdot \arctan\left(\frac{2}{\left(\frac{a}{b}\right)^{1/6} \cdot x^{1/2} - 3^{1/2}}\right) \cdot A - \frac{1}{3} \cdot b \cdot \left(\frac{a}{b}\right)^{1/6} \cdot \arctan\left(\frac{2}{\left(\frac{a}{b}\right)^{1/6} \cdot x^{1/2} - 3^{1/2}}\right) \cdot B + \frac{1}{6} \cdot a \cdot 3^{1/2} \cdot \left(\frac{a}{b}\right)^{1/6} \cdot \ln\left(x + 3^{1/2}\right) \cdot \left(\frac{a}{b}\right)^{1/6} \cdot x^{1/2} + \left(\frac{a}{b}\right)^{1/3} \Big) \cdot A - \frac{1}{6} \cdot b \cdot 3^{1/2} \cdot \left(\frac{a}{b}\right)^{1/6} \cdot \ln\left(x + 3^{1/2}\right) \cdot \left(\frac{a}{b}\right)^{1/6} \cdot x^{1/2} + \left(\frac{a}{b}\right)^{1/3} \Big) \cdot B + \frac{1}{3} \cdot a \cdot \left(\frac{a}{b}\right)^{1/6} \cdot \arctan\left(\frac{2}{\left(\frac{a}{b}\right)^{1/6} \cdot x^{1/2} + 3^{1/2}}\right) \cdot A - \frac{1}{3} \cdot b \cdot \left(\frac{a}{b}\right)^{1/6} \cdot \arctan\left(\frac{2}{\left(\frac{a}{b}\right)^{1/6} \cdot x^{1/2} + 3^{1/2}}\right) \cdot B$$

maxima [A] time = 1.42, size = 278, normalized size = 1.04

$$\frac{2B\sqrt{x}}{b} - \frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{\frac{1}{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+\frac{1}{3}x+a^{\frac{1}{3}}}\right)}{\frac{5}{a^{\frac{5}{6}}b^{\frac{5}{6}}}} - \frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{\frac{1}{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+\frac{1}{3}x+a^{\frac{1}{3}}}\right)}{\frac{5}{a^{\frac{5}{6}}b^{\frac{5}{6}}}} + \frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{\frac{1}{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}}\right)}{\frac{2}{a^{\frac{2}{3}}b^{\frac{2}{3}}}\sqrt{\frac{1}{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}} + \frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{\frac{1}{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}}{\sqrt{\frac{1}{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}}\right)}{ab^{\frac{1}{3}}\sqrt{\frac{1}{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}} + \frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{\frac{1}{3}a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}}{\sqrt{\frac{1}{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}}\right)}{ab^{\frac{1}{3}}\sqrt{\frac{1}{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="maxima")

[Out]
$$2B\sqrt{x}/b - 1/6 \cdot (\sqrt{3} \cdot (B \cdot a - A \cdot b) \cdot \log(\sqrt{3} \cdot a^{1/6} \cdot b^{1/6} \cdot \sqrt{x} + b^{1/3} \cdot x + a^{1/3})) / (a^{5/6} \cdot b^{1/6}) - \sqrt{3} \cdot (B \cdot a - A \cdot b) \cdot \log(-\sqrt{3} \cdot a^{1/6} \cdot b^{1/6} \cdot \sqrt{x} + b^{1/3} \cdot x + a^{1/3}) / (a^{5/6} \cdot b^{1/6}) + 4 \cdot (B \cdot a \cdot b^{1/3} - A \cdot b^{4/3}) \cdot \arctan(b^{1/3} \cdot \sqrt{x} / \sqrt{a^{1/3} \cdot b^{1/3}}) / (a^{2/3} \cdot b^{1/3} \cdot \sqrt{a^{1/3} \cdot b^{1/3}}) + 2 \cdot (B \cdot a^{4/3} \cdot b^{1/3} - A \cdot a^{1/3} \cdot b^{4/3}) \cdot \arctan((\sqrt{3} \cdot a^{1/6} \cdot b^{1/6} + 2 \cdot b^{1/3} \cdot \sqrt{x}) / \sqrt{a^{1/3} \cdot b^{1/3}}) / (a \cdot b^{1/3} \cdot \sqrt{a^{1/3} \cdot b^{1/3}}) + 2 \cdot (B \cdot a^{4/3} \cdot b^{1/3} - A \cdot a^{1/3} \cdot b^{4/3}) \cdot \arctan(-(\sqrt{3} \cdot a^{1/6} \cdot b^{1/6} - 2 \cdot b^{1/3} \cdot \sqrt{x}) / \sqrt{a^{1/3} \cdot b^{1/3}}) / (a \cdot b^{1/3} \cdot \sqrt{a^{1/3} \cdot b^{1/3}}) / b$$

mupad [B] time = 2.88, size = 1915, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^(1/2)*(a + b*x^3)),x)

[Out]
$$(2Bx^{1/2})/b + (\operatorname{atan}(\frac{(x^{1/2} \cdot (96A^4b^5 + 96B^4a^4b + 576A^2B^2a^2b^3 - 384A^3Bab^4 - 384AB^3a^3b^2) - ((Ab - Ba) \cdot (288A^3ab^5 - 288B^3a^4b^2 + 864AB^2a^3b^3 - 864A^2B^2ab^4))}{3(-a)^{5/6}b^{7/6}})) \cdot (Ab - Ba) \cdot i)}{3(-a)^{5/6}b^{7/6}} + ((x^{1/2} \cdot (96A^4b^5 + 96B^4a^4b + 576A^2B^2a^2b^3 - 384A^3Bab^4 - 384AB^3a^3b^2) + ((Ab - Ba) \cdot (288A^3ab^5 - 288B^3a^4b^2 + 864AB^2a^3b^3 - 864A^2B^2ab^4))}{3(-a)^{5/6}b^{7/6}})) \cdot (Ab - Ba) \cdot i)}{3(-a)^{5/6}b^{7/6}}) / (((x^{1/2} \cdot (96A^4b^5 + 96B^4a^4b + 576A^2B^2a^2b^3 - 384A^3Bab^4 - 384AB^3a^3b^2) - ((Ab - Ba) \cdot (288A^3ab^5 - 288B^3a^4b^2 + 864AB^2a^3b^3 - 864A^2B^2ab^4))}{3(-a)^{5/6}b^{7/6}})) \cdot (Ab - Ba)) / (3(-a)^{5/6}b^{7/6}) - ((x^{1/2} \cdot (96A^4b^5 + 96B^4a^4b + 576A^2B^2a^2b^3 - 384A^3Bab^4 - 384AB^3a^3b^2) + ((Ab - Ba) \cdot (288A^3ab^5 - 288B^3a^4b^2 + 864AB^2a^3b^3 - 864A^2B^2ab^4))}{3(-a)^{5/6}b^{7/6}})) \cdot (Ab - Ba)) / (3(-a)^{5/6}b^{7/6}) - ((x^{1/2} \cdot (96A^4b^5 + 96B^4a^4b + 576A^2B^2a^2b^3 - 384A^3Bab^4 - 384AB^3a^3b^2) + ((Ab - Ba) \cdot (288A^3ab^5 - 288B^3a^4b^2 + 864AB^2a^3b^3 - 864A^2B^2ab^4))}{3(-a)^{5/6}b^{7/6}})) \cdot (Ab - Ba)) / (3(-a)^{5/6}b^{7/6})$$

$$\begin{aligned} &)^{(5/6)*b^{(7/6)}}*(A*b - B*a)/(3*(-a)^{(5/6)*b^{(7/6)}}))*(A*b - B*a)*2i)/(3* \\ & (-a)^{(5/6)*b^{(7/6)}} + (\operatorname{atan}(\frac{((3^{(1/2)*1i})/2 - 1/2)*(A*b - B*a)*(x^{(1/2)}*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((3^{(1/2)*1i})/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))}{(3*(-a)^{(5/6)*b^{(7/6)}})*1i})/(3*(-a)^{(5/6)*b^{(7/6)}}) + ((3^{(1/2)*1i})/2 - 1/2)*(A*b - B*a)*(x^{(1/2)}*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + ((3^{(1/2)*1i})/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))}{(3*(-a)^{(5/6)*b^{(7/6)}})*1i})/(3*(-a)^{(5/6)*b^{(7/6)}})))/((3^{(1/2)*1i})/2 - 1/2)*(A*b - B*a)*(x^{(1/2)}*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((3^{(1/2)*1i})/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))}{(3*(-a)^{(5/6)*b^{(7/6)}})))/((3^{(1/2)*1i})/2 - 1/2)*(A*b - B*a)*(x^{(1/2)}*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + ((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))}{(3*(-a)^{(5/6)*b^{(7/6)}})*1i})/(3*(-a)^{(5/6)*b^{(7/6)}}) + ((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(x^{(1/2)}*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))}{(3*(-a)^{(5/6)*b^{(7/6)}})*1i})/(3*(-a)^{(5/6)*b^{(7/6)}}) + ((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(x^{(1/2)}*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))}{(3*(-a)^{(5/6)*b^{(7/6)}})))/((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(x^{(1/2)}*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))}{(3*(-a)^{(5/6)*b^{(7/6)}})))/((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(x^{(1/2)}*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + ((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))}{(3*(-a)^{(5/6)*b^{(7/6)}})))/((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^{(5/6)*b^{(7/6)}}) + (\operatorname{atan}(\frac{((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(x^{(1/2)}*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))}{(3*(-a)^{(5/6)*b^{(7/6)}})*1i})/(3*(-a)^{(5/6)*b^{(7/6)}}) + ((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(x^{(1/2)}*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + ((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))}{(3*(-a)^{(5/6)*b^{(7/6)}})*1i})/(3*(-a)^{(5/6)*b^{(7/6)}})))/((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(x^{(1/2)}*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))}{(3*(-a)^{(5/6)*b^{(7/6)}})))/((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(x^{(1/2)}*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + ((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))}{(3*(-a)^{(5/6)*b^{(7/6)}})))/((3^{(1/2)*1i})/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^{(5/6)*b^{(7/6)}}) \end{aligned}$$

sympy [A] time = 23.16, size = 833, normalized size = 3.11

$$\frac{\left(\frac{23}{12} + 23\sqrt{3}\right) \sqrt{a}}{\sqrt{a^2 + 2ab + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/(b*x**3+a)/x**(1/2), x)

```
[Out] Piecewise((zoo*(-2*A/(5*x**(5/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-
2*A/(5*x**(5/2)) + 2*B*sqrt(x))/b, Eq(a, 0)), ((2*A*sqrt(x) + 2*B*x**(7/2)/
7)/a, Eq(b, 0)), (-(-1)**(1/6)*A*(1/b)**(1/6)*log(-(-1)**(1/6)*a**(1/6)*(1/
b)**(1/6) + sqrt(x))/(3*a**(5/6)) + (-1)**(1/6)*A*(1/b)**(1/6)*log((-1)**(1
/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(5/6)) - (-1)**(1/6)*A*(1/b)**(1
/6)*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/
3)*(1/b)**(1/3) + 4*x)/(6*a**(5/6)) + (-1)**(1/6)*A*(1/b)**(1/6)*log(4*(-1)
** (1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3)
+ 4*x)/(6*a**(5/6)) + (-1)**(1/6)*sqrt(3)*A*(1/b)**(1/6)*atan(sqrt(3)/3 -
2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(5/6)) - (-1
)**(1/6)*sqrt(3)*A*(1/b)**(1/6)*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt
(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(5/6)) + (-1)**(1/6)*B*a**(1/6)*(1/b)*
*(1/6)*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b) - (-1)**(1/6
)*B*a**(1/6)*(1/b)**(1/6)*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/
(3*b) + (-1)**(1/6)*B*a**(1/6)*(1/b)**(1/6)*log(-4*(-1)**(1/6)*a**(1/6)*sqr
t(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b) - (-1)
** (1/6)*B*a**(1/6)*(1/b)**(1/6)*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(
1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b) - (-1)**(1/6)*sqrt(
3)*B*a**(1/6)*(1/b)**(1/6)*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(
3*a**(1/6)*(1/b)**(1/6)))/(3*b) + (-1)**(1/6)*sqrt(3)*B*a**(1/6)*(1/b)**(1/
6)*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)
))/(3*b) + 2*B*sqrt(x)/b, True))
```

$$3.156 \quad \int \frac{A+Bx^3}{x^{3/2}(a+bx^3)} dx$$

Optimal. Leaf size=268

$$\frac{(Ab - aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{7/6} b^{5/6}} + \frac{(Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{7/6} b^{5/6}} + \frac{(Ab - aB) \tan^{-1}}{3a^{7/6}}$$

Rubi [A] time = 0.55, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {453, 329, 295, 634, 618, 204, 628, 205}

$$\frac{(Ab - aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{7/6} b^{5/6}} + \frac{(Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{7/6} b^{5/6}} + \frac{(Ab - aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6} b^{5/6}} - \frac{(Ab - aB) \tan^{-1}\left(\frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}} + \sqrt{3}\right)}{3a^{7/6} b^{5/6}} - \frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6} b^{5/6}} - \frac{2A}{a\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)), x]

[Out] $(-2*A)/(a*\text{Sqrt}[x]) + ((A*b - a*B)*\text{ArcTan}[\text{Sqrt}[3] - (2*b^{(1/6)}*\text{Sqrt}[x])/a^{(1/6)}])/(3*a^{(7/6)}*b^{(5/6)}) - ((A*b - a*B)*\text{ArcTan}[\text{Sqrt}[3] + (2*b^{(1/6)}*\text{Sqrt}[x])/a^{(1/6)}])/(3*a^{(7/6)}*b^{(5/6)}) - (2*(A*b - a*B)*\text{ArcTan}[(b^{(1/6)}*\text{Sqrt}[x])/a^{(1/6)}])/(3*a^{(7/6)}*b^{(5/6)}) - ((A*b - a*B)*\text{Log}[a^{(1/3)} - \text{Sqrt}[3]*a^{(1/6)}*b^{(1/6)}*\text{Sqrt}[x] + b^{(1/3)}*x])/(2*\text{Sqrt}[3]*a^{(7/6)}*b^{(5/6)}) + ((A*b - a*B)*\text{Log}[a^{(1/3)} + \text{Sqrt}[3]*a^{(1/6)}*b^{(1/6)}*\text{Sqrt}[x] + b^{(1/3)}*x])/(2*\text{Sqrt}[3]*a^{(7/6)}*b^{(5/6)})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 295

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]

```
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x, x] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^{3/2}(a + bx^3)} dx &= -\frac{2A}{a\sqrt{x}} - \frac{\left(2\left(\frac{Ab}{2} - \frac{aB}{2}\right)\right) \int \frac{x^{3/2}}{a+bx^3} dx}{a} \\
&= -\frac{2A}{a\sqrt{x}} - \frac{\left(4\left(\frac{Ab}{2} - \frac{aB}{2}\right)\right) \text{Subst}\left(\int \frac{x^4}{a+bx^6} dx, x, \sqrt{x}\right)}{a} \\
&= -\frac{2A}{a\sqrt{x}} - \frac{(2(Ab - aB)) \text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2} + \frac{1}{2}\sqrt{3} \sqrt[6]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{3a^{7/6}b^{2/3}} - \frac{(2(Ab - aB)) \text{Subst}\left(\int \frac{x^4}{a+bx^6} dx, x, \sqrt{x}\right)}{3a^{7/6}b^{2/3}} \\
&= -\frac{2A}{a\sqrt{x}} - \frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab - aB) \text{Subst}\left(\int \frac{-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} + 2\sqrt[3]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{2\sqrt{3} a^{7/6}b^{5/6}} \\
&= -\frac{2A}{a\sqrt{x}} - \frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab - aB) \log\left(\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{7/6}b^{5/6}} + \frac{(Ab - aB) \log\left(\sqrt[3]{a} + \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{7/6}b^{5/6}} \\
&= -\frac{2A}{a\sqrt{x}} + \frac{(Ab - aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab - aB) \tan^{-1}\left(\sqrt{3} + \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{2(Ab - aB) \log\left(\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{7/6}b^{5/6}} + \frac{2(Ab - aB) \log\left(\sqrt[3]{a} + \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{7/6}b^{5/6}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.17

$$\frac{2\left(x^3(aB - Ab) {}_2F_1\left(\frac{5}{6}, 1; \frac{11}{6}; -\frac{bx^3}{a}\right) - 5aA\right)}{5a^2\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)), x]

[Out] (2*(-5*a*A + (-A*b) + a*B)*x^3*Hypergeometric2F1[5/6, 1, 11/6, -(b*x^3)/a])/(5*a^2*Sqrt[x])

IntegrateAlgebraic [A] time = 0.24, size = 166, normalized size = 0.62

$$\frac{2(aB - Ab) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} + \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b} x}{\sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}\right)}{3a^{7/6}b^{5/6}} - \frac{(aB - Ab) \tanh^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{b} x}\right)}{\sqrt{3} a^{7/6}b^{5/6}} - \frac{2A}{a\sqrt{x}}$$

Antiderivative was successfully verified.

$$\begin{aligned}
& *B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^{(2/3)})*a*b*(-(B^6*a^6 - \\
& 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 \\
& 4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^{(1/6)} + 2*\sqrt{3}*(B^5*a^6*b - 5*A \\
& B^4*a^5*b^2 + 10*A^2*B^3*a^4*b^3 - 10*A^3*B^2*a^3*b^4 + 5*A^4*B*a^2*b^5 - A \\
& ^5*a*b^6)*\sqrt{x}*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3* \\
& B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^{(1/6} \\
&) + \sqrt{3}*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3* \\
& b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5 \\
& *b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B \\
& *a*b^5 + A^6*b^6)) - 2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 \\
& - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b \\
& ^5))^{(1/6)}*\log(a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20 \\
& *A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5)) \\
& ^{(5/6)} - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 \\
& + 5*A^4*B*a*b^4 - A^5*b^5)*\sqrt{x})) + 2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1 \\
& 5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 \\
& + A^6*b^6)/(a^7*b^5))^{(1/6)}*\log(-a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A \\
& ^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + \\
& A^6*b^6)/(a^7*b^5))^{(5/6)} - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - \\
& 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*\sqrt{x})) + a*x*(-(B^6*a^6 - \\
& 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^ \\
& 4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^{(1/6)}*\log(4*(B^5*a^11*b^4 - 5*A*B^4 \\
& *a^10*b^5 + 10*A^2*B^3*a^9*b^6 - 10*A^3*B^2*a^8*b^7 + 5*A^4*B*a^7*b^8 - A^5 \\
& *a^6*b^9)*\sqrt{x}*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3* \\
& B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^{(5/6} \\
&) + 4*(B^10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^ \\
& 3 + 210*A^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A \\
& ^7*B^3*a^3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^ \\
& 6*a^11*b^3 - 6*A*B^5*a^10*b^4 + 15*A^2*B^4*a^9*b^5 - 20*A^3*B^3*a^8*b^6 + 1 \\
& 5*A^4*B^2*a^7*b^7 - 6*A^5*B*a^6*b^8 + A^6*a^5*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5 \\
& *b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B \\
& *a*b^5 + A^6*b^6)/(a^7*b^5))^{(2/3)} - a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A \\
& ^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + \\
& A^6*b^6)/(a^7*b^5))^{(1/6)}*\log(-4*(B^5*a^11*b^4 - 5*A*B^4*a^10*b^5 + 10*A^2* \\
& B^3*a^9*b^6 - 10*A^3*B^2*a^8*b^7 + 5*A^4*B*a^7*b^8 - A^5*a^6*b^9)*\sqrt{x}*(\\
& -(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^ \\
& 4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^{(5/6)} + 4*(B^10*a^10 - \\
& 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6 \\
& *b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45 \\
& *A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^6*a^11*b^3 - 6*A*B^ \\
& 5*a^10*b^4 + 15*A^2*B^4*a^9*b^5 - 20*A^3*B^3*a^8*b^6 + 15*A^4*B^2*a^7*b^7 - \\
& 6*A^5*B*a^6*b^8 + A^6*a^5*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4 \\
& *b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(\\
& a^7*b^5))^{(2/3)} - 12*A*\sqrt{x})/(a*x)
\end{aligned}$$

giac [A] time = 0.41, size = 280, normalized size = 1.04

$$\frac{2A}{a\sqrt{x}} - \frac{\sqrt{3}\left((ab^5)^{\frac{5}{6}}Ba - (ab^5)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{x}{a}\right)^{\frac{1}{6}} + x + \left(\frac{x}{a}\right)^{\frac{1}{3}}\right)}{6a^2b^5} + \frac{\sqrt{3}\left((ab^5)^{\frac{5}{6}}Ba - (ab^5)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{x}{a}\right)^{\frac{1}{6}} + x + \left(\frac{x}{a}\right)^{\frac{1}{3}}\right)}{6a^2b^5} + \frac{\left((ab^5)^{\frac{5}{6}}Ba - (ab^5)^{\frac{5}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{x}{a}\right)^{\frac{1}{6}} + 2\sqrt{x}}{\left(\frac{x}{a}\right)^{\frac{1}{3}}}\right)}{3a^2b^5} + \frac{\left((ab^5)^{\frac{5}{6}}Ba - (ab^5)^{\frac{5}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{x}{a}\right)^{\frac{1}{6}} + 2\sqrt{x}}{\left(\frac{x}{a}\right)^{\frac{1}{3}}}\right)}{3a^2b^5} + \frac{2\left((ab^5)^{\frac{5}{6}}Ba - (ab^5)^{\frac{5}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{x}{a}\right)^{\frac{1}{3}}}\right)}{3a^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="giac")

[Out] $-2A/(a*\sqrt{x}) - 1/6*\sqrt{3}*((a*b^5)^{(5/6)}*B*a - (a*b^5)^{(5/6)}*A*b)*\log(\sqrt{3}*\sqrt{x}*(a/b)^{(1/6)} + x + (a/b)^{(1/3)})/(a^2*b^5) + 1/6*\sqrt{3}*((a*b^5)^{(5/6)}*B*a - (a*b^5)^{(5/6)}*A*b)*\log(-\sqrt{3}*\sqrt{x}*(a/b)^{(1/6)} + x + (a/b)^{(1/3)})/(a^2*b^5) + 1/3*((a*b^5)^{(5/6)}*B*a - (a*b^5)^{(5/6)}*A*b)*\arctan((\sqrt{3}*(a/b)^{(1/6)} + 2*\sqrt{x})/(a/b)^{(1/6)})/(a^2*b^5) + 1/3*((a*b^5)^{(5/6)}*B*a - (a*b^5)^{(5/6)}*A*b)*\arctan(-(\sqrt{3}*(a/b)^{(1/6)} - 2*\sqrt{x})/(a/b)^{(1/6)})/(a^2*b^5) + 2/3*((a*b^5)^{(5/6)}*B*a - (a*b^5)^{(5/6)}*A*b)*\arctan(\sqrt{x}/(a/b)^{(1/6)})/(a^2*b^5)$

maple [A] time = 0.16, size = 355, normalized size = 1.32

$$\frac{2A\arctan\left(\frac{\sqrt{x}}{\left(\frac{x}{a}\right)^{\frac{1}{3}}}\right)}{3\left(\frac{x}{a}\right)^{\frac{1}{3}}a} - \frac{A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{x}{a}\right)^{\frac{1}{3}} - \sqrt{3}}\right)}{3\left(\frac{x}{a}\right)^{\frac{1}{3}}a} - \frac{A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{x}{a}\right)^{\frac{1}{3}} + \sqrt{3}}\right)}{3\left(\frac{x}{a}\right)^{\frac{1}{3}}a} + \frac{\sqrt{3}\left(\frac{x}{a}\right)^{\frac{1}{6}}Ab\ln\left(x + \sqrt{3}\left(\frac{x}{a}\right)^{\frac{1}{6}}\sqrt{x} + \left(\frac{x}{a}\right)^{\frac{1}{3}}\right)}{6a^2} - \frac{\sqrt{3}\left(\frac{x}{a}\right)^{\frac{1}{6}}Ab\ln\left(-x + \sqrt{3}\left(\frac{x}{a}\right)^{\frac{1}{6}}\sqrt{x} - \left(\frac{x}{a}\right)^{\frac{1}{3}}\right)}{6a^2} - \frac{\sqrt{3}\left(\frac{x}{a}\right)^{\frac{1}{6}}B\ln\left(x + \sqrt{3}\left(\frac{x}{a}\right)^{\frac{1}{6}}\sqrt{x} + \left(\frac{x}{a}\right)^{\frac{1}{3}}\right)}{6a} - \frac{\sqrt{3}\left(\frac{x}{a}\right)^{\frac{1}{6}}B\ln\left(-x + \sqrt{3}\left(\frac{x}{a}\right)^{\frac{1}{6}}\sqrt{x} - \left(\frac{x}{a}\right)^{\frac{1}{3}}\right)}{6a} + \frac{2B\arctan\left(\frac{\sqrt{x}}{\left(\frac{x}{a}\right)^{\frac{1}{3}}}\right)}{3\left(\frac{x}{a}\right)^{\frac{1}{3}}b} + \frac{B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{x}{a}\right)^{\frac{1}{3}} - \sqrt{3}}\right)}{3\left(\frac{x}{a}\right)^{\frac{1}{3}}b} + \frac{B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{x}{a}\right)^{\frac{1}{3}} + \sqrt{3}}\right)}{3\left(\frac{x}{a}\right)^{\frac{1}{3}}b} - \frac{2A}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a),x)

[Out] $-2/3/a/(a/b)^{(1/6)}*\arctan(1/(a/b)^{(1/6)}*x^{(1/2)})*A + 2/3/b/(a/b)^{(1/6)}*\arctan(1/(a/b)^{(1/6)}*x^{(1/2)})*B - 1/6/a^2*3^{(1/2)}*(a/b)^{(5/6)}*\ln(-x + 3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)} - (a/b)^{(1/3)})*A*b + 1/6/a*3^{(1/2)}*(a/b)^{(5/6)}*\ln(-x + 3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)} - (a/b)^{(1/3)})*B - 1/3/a/(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)} - 3^{(1/2)})*A + 1/3/b/(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)} - 3^{(1/2)})*B + 1/6/a^2*3^{(1/2)}*(a/b)^{(5/6)}*\ln(x + 3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)} + (a/b)^{(1/3)})*A*b - 1/6/a*3^{(1/2)}*(a/b)^{(5/6)}*\ln(x + 3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)} + (a/b)^{(1/3)})*B - 1/3/a/(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)} + 3^{(1/2)})*A + 1/3/b/(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)} + 3^{(1/2)})*B - 2*A/a/x^{(1/2)}$

maxima [A] time = 1.34, size = 212, normalized size = 0.79

$$\frac{(Ba - Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x} + b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right)}{\frac{1}{a^6b^6}} - \frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x} + b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right)}{\frac{1}{a^6b^6}} - \frac{2\arctan\left(\frac{\frac{1}{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}} + 2b^{\frac{1}{3}}\sqrt{x}}{\frac{1}{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{\frac{2}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}\right)}{6a} - \frac{2\arctan\left(\frac{-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}} - 2b^{\frac{1}{3}}\sqrt{x}}{\frac{1}{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{\frac{2}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}\right)}{6a} - \frac{4\arctan\left(\frac{\frac{1}{b^{\frac{1}{3}}}\sqrt{x}}{\frac{1}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}\right)}{\frac{2}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}\right)}{6a} - \frac{2A}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="maxima")

$$\begin{aligned}
& + 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5) \\
&)/(27*(-a)^{(7/6)}*b^{(5/6)})) * 1i) / (((-a)^{(7/3)}*b^{(5/3)}) + (((3^{(1/2)}*1i)/2 + \\
& 1/2)^2*(A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 \\
& - 96*A^2*B*a^10*b^5 + (x^{(1/2)}*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2* \\
& a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5)) / (27*(-a)^{(7/6)}*b^{(5/6)})) * \\
& 1i) / (((-a)^{(7/3)}*b^{(5/3)})) / (((3^{(1/2)}*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*B^3* \\
& a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^{(1/2)} \\
&)*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - \\
& 1728*A*B*a^11*b^5)) / (27*(-a)^{(7/6)}*b^{(5/6)})) / ((-a)^{(7/3)}*b^{(5/3)}) - (((3^{(1/2)} \\
& (1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A \\
& *B^2*a^11*b^4 - 96*A^2*B*a^10*b^5 + (x^{(1/2)}*((3^{(1/2)}*1i)/2 + 1/2)*(A*b - \\
& B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5)) / (27*(-a)^{(7/6)} \\
& *b^{(5/6)})) / ((-a)^{(7/3)}*b^{(5/3)})) * ((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)*2i \\
&) / (3*(-a)^{(7/6)}*b^{(5/6)})
\end{aligned}$$

sympy [A] time = 59.23, size = 836, normalized size = 3.12

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a), x)

[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-2*A/(7*x**(7/2)) - 2*B/sqrt(x))/b, Eq(a, 0)), ((-2*A/sqrt(x) + 2*B*x**(5/2)/5)/a, Eq(b, 0)), (-2*A/(a*sqrt(x)) + (-1)**(5/6)*A*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(7/6)*(1/b)**(1/6)) - (-1)**(5/6)*A*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(7/6)*(1/b)**(1/6)) + (-1)**(5/6)*A*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(7/6)*(1/b)**(1/6)) - (-1)**(5/6)*A*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(7/6)*(1/b)**(1/6)) + (-1)**(5/6)*sqrt(3)*A*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(7/6)*(1/b)**(1/6)) - (-1)**(5/6)*sqrt(3)*A*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(7/6)*(1/b)**(1/6)) - (-1)**(5/6)*B*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*B*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(1/6)*b*(1/b)**(1/6)) - (-1)**(5/6)*B*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*B*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(1/6)*b*(1/b)**(1/6)) - (-1)**(5/6)*sqrt(3)*B*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*sqrt(3)*B

```
atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(  
3*a**(1/6)*b*(1/b)**(1/6)), True))
```

$$3.157 \quad \int \frac{A+Bx^3}{x^{5/2}(a+bx^3)} dx$$

Optimal. Leaf size=53

$$-\frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}} - \frac{2A}{3ax^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {453, 329, 275, 205}

$$-\frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}} - \frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)), x]

[Out] (-2*A)/(3*a*x^(3/2)) - (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(3/2)*Sqrt[b])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 329

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^(p), x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p*((c_) + (d_.)*(x_)^(n_.)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),

x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^3}{x^{5/2}(a + bx^3)} dx &= -\frac{2A}{3ax^{3/2}} - \frac{\left(2\left(\frac{3Ab}{2} - \frac{3aB}{2}\right)\right) \int \frac{\sqrt{x}}{a+bx^3} dx}{3a} \\ &= -\frac{2A}{3ax^{3/2}} - \frac{\left(4\left(\frac{3Ab}{2} - \frac{3aB}{2}\right)\right) \text{Subst}\left(\int \frac{x^2}{a+bx^6} dx, x, \sqrt{x}\right)}{3a} \\ &= -\frac{2A}{3ax^{3/2}} - \frac{(2(Ab - aB)) \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, x^{3/2}\right)}{3a} \\ &= -\frac{2A}{3ax^{3/2}} - \frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 1.00

$$\frac{2(aB - Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}} - \frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)), x]

[Out] (-2*A)/(3*a*x^(3/2)) + (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(3/2)*Sqrt[b])

IntegrateAlgebraic [A] time = 0.05, size = 53, normalized size = 1.00

$$\frac{2(aB - Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}} - \frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^(5/2)*(a + b*x^3)), x]

[Out] $(-2A)/(3ax^{3/2}) + (2(-Ab) + aB) \operatorname{ArcTan}[(\operatorname{Sqrt}[b] * x^{3/2})/\operatorname{Sqrt}[a]] / (3a^{3/2} \operatorname{Sqrt}[b])$

fricas [A] time = 0.78, size = 120, normalized size = 2.26

$$\left[\frac{(Ba - Ab)\sqrt{-ab} x^2 \log\left(\frac{bx^3 + 2\sqrt{-ab} x^2 - a}{bx^3 + a}\right) - 2Aab\sqrt{x}}{3a^2bx^2}, \frac{2\left((Ba - Ab)\sqrt{ab} x^2 \arctan\left(\frac{\sqrt{ab} x^2}{a}\right) - Aab\sqrt{x}\right)}{3a^2bx^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="fricas")`

[Out] $[1/3((B*a - A*b)*\operatorname{sqrt}(-a*b)*x^2*\log((b*x^3 + 2*\operatorname{sqrt}(-a*b)*x^{3/2} - a)/(b*x^3 + a)) - 2*A*a*b*\operatorname{sqrt}(x))/(a^2*b*x^2), 2/3((B*a - A*b)*\operatorname{sqrt}(a*b)*x^2*\operatorname{arctan}(\operatorname{sqrt}(a*b)*x^{3/2}/a) - A*a*b*\operatorname{sqrt}(x))/(a^2*b*x^2)]$

giac [A] time = 0.17, size = 39, normalized size = 0.74

$$\frac{2(Ba - Ab) \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}a} - \frac{2A}{3ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="giac")`

[Out] $2/3*(B*a - A*b)*\operatorname{arctan}(b*x^{3/2}/\operatorname{sqrt}(a*b))/(\operatorname{sqrt}(a*b)*a) - 2/3*A/(a*x^{3/2})$

maple [A] time = 0.06, size = 53, normalized size = 1.00

$$-\frac{2Ab \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}a} + \frac{2B \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}} - \frac{2A}{3ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^3+A)/x^(5/2)/(b*x^3+a),x)`

[Out] $-2/3*A/a/x^{3/2} - 2/3/a/(a*b)^{1/2}*\operatorname{arctan}(1/(a*b)^{1/2}*b*x^{3/2})*A*b + 2/3/(a*b)^{1/2}*\operatorname{arctan}(1/(a*b)^{1/2}*b*x^{3/2})*B$

maxima [A] time = 1.40, size = 39, normalized size = 0.74

$$\frac{2(Ba - Ab) \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}a} - \frac{2A}{3ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="maxima")

[Out] 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a) - 2/3*A/(a*x^(3/2))

mupad [B] time = 0.10, size = 102, normalized size = 1.92

$$\frac{2A}{3ax^{3/2}} - \frac{2 \operatorname{atan}\left(\frac{3a^{3/2}\sqrt{b}x^{3/2}(24A^2a^3b^5 - 48ABa^4b^4 + 24B^2a^5b^3)}{(Ab - Ba)(72Aa^5b^4 - 72Ba^6b^3)}\right)(Ab - Ba)}{3a^{3/2}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^(5/2)*(a + b*x^3)),x)

[Out] - (2*A)/(3*a*x^(3/2)) - (2*atan((3*a^(3/2)*b^(1/2)*x^(3/2)*(24*A^2*a^3*b^5 + 24*B^2*a^5*b^3 - 48*A*B*a^4*b^4))/(A*b - B*a)*(72*A*a^5*b^4 - 72*B*a^6*b^3)))/(3*a^(3/2)*b^(1/2))

sympy [A] time = 156.58, size = 527, normalized size = 9.94

$$\begin{cases} \frac{2A}{3ax^{\frac{3}{2}}} - \frac{2B}{3\sqrt{ab}a} & \text{for } a = 0 \wedge b = 0 \\ \frac{2A}{3ax^{\frac{3}{2}}} - \frac{2B}{3\sqrt{ab}a} & \text{for } a = 0 \\ \frac{2A}{3ax^{\frac{3}{2}}} - \frac{2B}{3\sqrt{ab}a} & \text{for } b = 0 \\ \frac{2A}{3ax^{\frac{3}{2}}} + \frac{iA \log\left(\frac{\sqrt{-1}\sqrt{b}\sqrt{x^2+\sqrt{a}}}{\sqrt{a}}\right) - iA \log\left(\frac{\sqrt{-1}\sqrt{b}\sqrt{x^2+\sqrt{a}}}{\sqrt{a}}\right) - iA \log\left(\frac{-1\sqrt{-1}\sqrt{b}\sqrt{x^2+\sqrt{a}} + \sqrt{-1}\sqrt{b}\sqrt{x^2+\sqrt{a}}}{\sqrt{a}}\right) + iA \log\left(\frac{1\sqrt{-1}\sqrt{b}\sqrt{x^2+\sqrt{a}} + \sqrt{-1}\sqrt{b}\sqrt{x^2+\sqrt{a}}}{\sqrt{a}}\right) - iA \log\left(\frac{\sqrt{-1}\sqrt{b}\sqrt{x^2+\sqrt{a}}}{\sqrt{a}}\right) + iA \log\left(\frac{\sqrt{-1}\sqrt{b}\sqrt{x^2+\sqrt{a}}}{\sqrt{a}}\right) - iA \log\left(\frac{-1\sqrt{-1}\sqrt{b}\sqrt{x^2+\sqrt{a}} + \sqrt{-1}\sqrt{b}\sqrt{x^2+\sqrt{a}}}{\sqrt{a}}\right) - iA \log\left(\frac{1\sqrt{-1}\sqrt{b}\sqrt{x^2+\sqrt{a}} + \sqrt{-1}\sqrt{b}\sqrt{x^2+\sqrt{a}}}{\sqrt{a}}\right)}{3\sqrt{ab}\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a),x)

[Out] Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0)), ((-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2)))/b, Eq(a, 0)), ((-2*A/(3*x**(3/2)) + 2*B*x**(3/2)/3)/a, Eq(b, 0)), (-2*A/(3*a*x**(3/2)) + I*A*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(3/2)*sqrt(1/b)) - I*A*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(3/2)*sqrt(1/b)) - I*A*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*a**(3/2)*sqrt(1/b)) + I*A*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*

```

(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*a**(3/2)*sqrt(
1/b)) - I*B*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*sqrt(a)*b*
sqrt(1/b)) + I*B*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*sqrt(a)
)*b*sqrt(1/b)) + I*B*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(
-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*sqrt(a)*b*sqrt(1/b)) - I*B*log(4
*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**
(1/3) + 4*x)/(3*sqrt(a)*b*sqrt(1/b)), True))

```

$$3.158 \quad \int \frac{A+Bx^3}{x^{7/2}(a+bx^3)} dx$$

Optimal. Leaf size=270

$$\frac{(Ab - aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b}x\right)}{2\sqrt{3} a^{11/6} \sqrt[6]{b}} - \frac{(Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b}x\right)}{2\sqrt{3} a^{11/6} \sqrt[6]{b}} + \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b}x}\right)}{3a^{11/6} \sqrt[6]{b}}$$

Rubi [A] time = 0.48, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {453, 329, 209, 634, 618, 204, 628, 205}

$$\frac{(Ab - aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b}x\right)}{2\sqrt{3} a^{11/6} \sqrt[6]{b}} - \frac{(Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b}x\right)}{2\sqrt{3} a^{11/6} \sqrt[6]{b}} + \frac{(Ab - aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6} \sqrt[6]{b}} - \frac{(Ab - aB) \tan^{-1}\left(\frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}} + \sqrt{3}\right)}{3a^{11/6} \sqrt[6]{b}} - \frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6} \sqrt[6]{b}} - \frac{2A}{5ax^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)), x]

[Out] $\frac{(-2*A)/(5*a*x^{5/2}) + ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^{1/6})*Sqrt[x])/a^{1/6}]}{(3*a^{11/6}*b^{1/6})} - \frac{((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^{1/6})*Sqrt[x])/a^{1/6}]}{(3*a^{11/6}*b^{1/6})} - \frac{(2*(A*b - a*B)*ArcTan[(b^{1/6})*Sqrt[x])/a^{1/6}]}{(3*a^{11/6}*b^{1/6})} + \frac{((A*b - a*B)*Log[a^{1/3} - Sqrt[3]*a^{1/6}*b^{1/6}*Sqrt[x] + b^{1/3}*x])}{(2*Sqrt[3]*a^{11/6}*b^{1/6})} - \frac{((A*b - a*B)*Log[a^{1/3} + Sqrt[3]*a^{1/6}*b^{1/6}*Sqrt[x] + b^{1/3}*x])}{(2*Sqrt[3]*a^{11/6}*b^{1/6})}$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 209

Int[((a_) + (b_.)*(x_)^(n_))^(m_), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u

, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a/b]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^{7/2}(a + bx^3)} dx &= -\frac{2A}{5ax^{5/2}} - \frac{\left(2\left(\frac{5Ab}{2} - \frac{5aB}{2}\right)\right) \int \frac{1}{\sqrt{x}(a+bx^3)} dx}{5a} \\
&= -\frac{2A}{5ax^{5/2}} - \frac{\left(4\left(\frac{5Ab}{2} - \frac{5aB}{2}\right)\right) \text{Subst}\left(\int \frac{1}{a+bx^6} dx, x, \sqrt{x}\right)}{5a} \\
&= -\frac{2A}{5ax^{5/2}} - \frac{(2(Ab - aB)) \text{Subst}\left(\int \frac{\sqrt[6]{a} - \frac{1}{2}\sqrt{3} \sqrt[6]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{3a^{11/6}} - \frac{(2(Ab - aB)) \text{Subst}\left(\int \frac{1}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{6a^{5/3}} \\
&= -\frac{2A}{5ax^{5/2}} - \frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6} \sqrt[6]{b}} - \frac{(Ab - aB) \text{Subst}\left(\int \frac{1}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{6a^{5/3}} \\
&= -\frac{2A}{5ax^{5/2}} - \frac{2(Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6} \sqrt[6]{b}} + \frac{(Ab - aB) \log\left(\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{11/6} \sqrt[6]{b}} - \frac{(Ab - aB) \log\left(\sqrt[3]{a} + \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{11/6} \sqrt[6]{b}} \\
&= -\frac{2A}{5ax^{5/2}} + \frac{(Ab - aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6} \sqrt[6]{b}} - \frac{(Ab - aB) \tan^{-1}\left(\sqrt{3} + \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6} \sqrt[6]{b}} - \frac{2(Ab - aB) \log\left(\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{11/6} \sqrt[6]{b}} + \frac{2(Ab - aB) \log\left(\sqrt[3]{a} + \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{b} x\right)}{2\sqrt{3} a^{11/6} \sqrt[6]{b}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.17

$$\frac{10x^3(aB - Ab) {}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; -\frac{bx^3}{a}\right) - 2aA}{5a^2x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)), x]

[Out] (-2*a*A + 10*(-(A*b) + a*B)*x^3*Hypergeometric2F1[1/6, 1, 7/6, -(b*x^3)/a])/ (5*a^2*x^(5/2))

IntegrateAlgebraic [A] time = 0.19, size = 167, normalized size = 0.62

$$\frac{2(aB - Ab) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6} \sqrt[6]{b}} + \frac{(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b} x}{\sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}\right)}{3a^{11/6} \sqrt[6]{b}} + \frac{(aB - Ab) \tanh^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{b} x}\right)}{\sqrt{3} a^{11/6} \sqrt[6]{b}} - \frac{2A}{5ax^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]

[Out] $(-2A)/(5a*x^{5/2}) + (2*(-(A*b) + a*B)*\text{ArcTan}[(b^{1/6}*\text{Sqrt}[x])/a^{1/6}]) / (3a^{11/6}*b^{1/6}) + ((A*b - a*B)*\text{ArcTan}[(a^{1/3} - b^{1/3}*x)/(a^{1/6}*b^{1/6}*\text{Sqrt}[x])]) / (3a^{11/6}*b^{1/6}) + ((-(A*b) + a*B)*\text{ArcTanh}[(\text{Sqrt}[3]*a^{1/6}*b^{1/6}*\text{Sqrt}[x])/(a^{1/3} + b^{1/3}*x)]) / (\text{Sqrt}[3]*a^{11/6}*b^{1/6})$

fricas [B] time = 0.95, size = 2424, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="fricas")

[Out] $-1/30*(20*\text{sqrt}(3)*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^{11*b}))^{1/6}*\arctan(1/3*(2*\text{sqrt}(3)*\text{sqrt}(a^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^{11*b}))^{1/3} + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a^3 - A*a^2*b)*\text{sqrt}(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^{11*b}))^{1/6})*a^9*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^{11*b}))^{5/6} + 2*\text{sqrt}(3)*(B*a^{10*b} - A*a^9*b^2)*\text{sqrt}(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^{11*b}))^{5/6} - \text{sqrt}(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) + 20*\text{sqrt}(3)*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^{11*b}))^{1/6}*\arctan(1/3*(2*\text{sqrt}(3)*\text{sqrt}(a^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^{11*b}))^{1/3} + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - (B*a^3 - A*a^2*b)*\text{sqrt}(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^{11*b}))^{1/6})*a^9*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^{11*b}))^{5/6} + 2*\text{sqrt}(3)*(B*a^{10*b} - A*a^9*b^2)*\text{sqrt}(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^{11*b}))^{5/6} + \text{sqrt}(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) - 5*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^{11*b}))^{1/6})*\log(4*a^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B$

$$\begin{aligned}
& a^5b^5 + A^6b^6)/(a^{11}b))^{1/3} + 4*(B^2a^2 - 2*AB*a*b + A^2b^2)*x + 4* \\
& (B^3a^3 - A^3a^2*b)*\sqrt{x}*(-(B^6a^6 - 6*AB^5a^5*b + 15*A^2B^4a^4*b^2 - \\
& 20*A^3B^3a^3*b^3 + 15*A^4B^2a^2*b^4 - 6*A^5B*a*b^5 + A^6b^6)/(a^{11}b \\
&))^{1/6}) + 5*a*x^3*(-(B^6a^6 - 6*AB^5a^5*b + 15*A^2B^4a^4*b^2 - 20*A^3 \\
& B^3a^3*b^3 + 15*A^4B^2a^2*b^4 - 6*A^5B*a*b^5 + A^6b^6)/(a^{11}b))^{1/6} \\
& * \log(4*a^4*(-(B^6a^6 - 6*AB^5a^5*b + 15*A^2B^4a^4*b^2 - 20*A^3B^3a^3 \\
& b^3 + 15*A^4B^2a^2*b^4 - 6*A^5B*a*b^5 + A^6b^6)/(a^{11}b))^{1/3} + 4* \\
& (B^2a^2 - 2*AB*a*b + A^2b^2)*x - 4*(B^3a^3 - A^3a^2*b)*\sqrt{x}*(-(B^6a^6 \\
& - 6*AB^5a^5*b + 15*A^2B^4a^4*b^2 - 20*A^3B^3a^3*b^3 + 15*A^4B^2a^2* \\
& b^4 - 6*A^5B*a*b^5 + A^6b^6)/(a^{11}b))^{1/6}) + 10*a*x^3*(-(B^6a^6 - 6* \\
& AB^5a^5*b + 15*A^2B^4a^4*b^2 - 20*A^3B^3a^3*b^3 + 15*A^4B^2a^2*b^4 - \\
& 6*A^5B*a*b^5 + A^6b^6)/(a^{11}b))^{1/6} * \log(a^2*(-(B^6a^6 - 6*AB^5a^5* \\
& b + 15*A^2B^4a^4*b^2 - 20*A^3B^3a^3*b^3 + 15*A^4B^2a^2*b^4 - 6*A^5B* \\
& a*b^5 + A^6b^6)/(a^{11}b))^{1/6} - (B*a - A*b)*\sqrt{x}) - 10*a*x^3*(-(B^6a^6 \\
& - 6*AB^5a^5*b + 15*A^2B^4a^4*b^2 - 20*A^3B^3a^3*b^3 + 15*A^4B^2a^2* \\
& b^4 - 6*A^5B*a*b^5 + A^6b^6)/(a^{11}b))^{1/6} * \log(-a^2*(-(B^6a^6 - 6* \\
& AB^5a^5*b + 15*A^2B^4a^4*b^2 - 20*A^3B^3a^3*b^3 + 15*A^4B^2a^2*b^4 - \\
& 6*A^5B*a*b^5 + A^6b^6)/(a^{11}b))^{1/6} - (B*a - A*b)*\sqrt{x}) + 12*A*\sqrt{x} \\
& t(x))/(a*x^3)
\end{aligned}$$

giac [A] time = 0.21, size = 280, normalized size = 1.04

$$\frac{\sqrt{3} \left((ab^5)^{\frac{1}{2}} Ba - (ab^5)^{\frac{1}{2}} Ab \right) \log \left(\sqrt{3} \sqrt{\frac{a}{b}} \left(\frac{a}{b} \right)^{\frac{1}{2}} + x + \left(\frac{a}{b} \right)^{\frac{1}{2}} \right) - \sqrt{3} \left((ab^5)^{\frac{1}{2}} Ba - (ab^5)^{\frac{1}{2}} Ab \right) \log \left(-\sqrt{3} \sqrt{\frac{a}{b}} \left(\frac{a}{b} \right)^{\frac{1}{2}} + x + \left(\frac{a}{b} \right)^{\frac{1}{2}} \right)}{6 a^2 b} + \frac{\left((ab^5)^{\frac{1}{2}} Ba - (ab^5)^{\frac{1}{2}} Ab \right) \arctan \left(\frac{\sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} + 2 \sqrt{a}}{\left(\frac{a}{b} \right)^{\frac{1}{2}}} \right)}{3 a^2 b} + \frac{\left((ab^5)^{\frac{1}{2}} Ba - (ab^5)^{\frac{1}{2}} Ab \right) \arctan \left(-\frac{\sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} - 2 \sqrt{a}}{\left(\frac{a}{b} \right)^{\frac{1}{2}}} \right)}{3 a^2 b} + \frac{2 \left((ab^5)^{\frac{1}{2}} Ba - (ab^5)^{\frac{1}{2}} Ab \right) \arctan \left(\frac{\sqrt{a}}{\left(\frac{a}{b} \right)^{\frac{1}{2}}} \right)}{3 a^2 b} - \frac{2 A}{5 a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="giac")

[Out] 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b) - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b) + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) + 2/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b) - 2/5*A/(a*x^(5/2))

maple [A] time = 0.18, size = 358, normalized size = 1.33

$$\frac{2 \left(\frac{a}{b} \right)^{\frac{1}{2}} Ab \arctan \left(\frac{\sqrt{a}}{\left(\frac{a}{b} \right)^{\frac{1}{2}}} \right) - \left(\frac{a}{b} \right)^{\frac{1}{2}} Ab \arctan \left(\frac{2 \sqrt{a} - \sqrt{3}}{\left(\frac{a}{b} \right)^{\frac{1}{2}}} \right) - \left(\frac{a}{b} \right)^{\frac{1}{2}} Ab \arctan \left(\frac{2 \sqrt{a} + \sqrt{3}}{\left(\frac{a}{b} \right)^{\frac{1}{2}}} \right)}{3 a^2} + \frac{\sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} Ab \ln \left(x + \sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} \sqrt{a} + \left(\frac{a}{b} \right)^{\frac{1}{2}} \right) - \sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} Ab \ln \left(-x + \sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} \sqrt{a} - \left(\frac{a}{b} \right)^{\frac{1}{2}} \right)}{6 a^2} + \frac{2 \left(\frac{a}{b} \right)^{\frac{1}{2}} B \arctan \left(\frac{\sqrt{a}}{\left(\frac{a}{b} \right)^{\frac{1}{2}}} \right) - \left(\frac{a}{b} \right)^{\frac{1}{2}} B \arctan \left(\frac{2 \sqrt{a} - \sqrt{3}}{\left(\frac{a}{b} \right)^{\frac{1}{2}}} \right) - \left(\frac{a}{b} \right)^{\frac{1}{2}} B \arctan \left(\frac{2 \sqrt{a} + \sqrt{3}}{\left(\frac{a}{b} \right)^{\frac{1}{2}}} \right)}{3 a} + \frac{\sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} B \ln \left(x + \sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} \sqrt{a} + \left(\frac{a}{b} \right)^{\frac{1}{2}} \right) - \sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} B \ln \left(-x + \sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} \sqrt{a} - \left(\frac{a}{b} \right)^{\frac{1}{2}} \right)}{6 a} - \frac{2 A}{5 a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a),x)

[Out] $-2/5*A/a/x^{(5/2)}-2/3/a^2*(a/b)^{(1/6)}*\arctan(1/(a/b)^{(1/6)}*x^{(1/2)})*A*b+2/3/a*(a/b)^{(1/6)}*\arctan(1/(a/b)^{(1/6)}*x^{(1/2)})*B+1/6/a^2*3^{(1/2)}*(a/b)^{(1/6)}*\ln(-x+3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)}-(a/b)^{(1/3)})*A*b-1/6/a*3^{(1/2)}*(a/b)^{(1/6)}*\ln(-x+3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)}-(a/b)^{(1/3)})*B-1/3/a^2*(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)}-3^{(1/2)})*A*b+1/3/a*(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)}-3^{(1/2)})*B-1/6/a^2*3^{(1/2)}*(a/b)^{(1/6)}*\ln(x+3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)}+(a/b)^{(1/3)})*A*b+1/6/a*3^{(1/2)}*(a/b)^{(1/6)}*\ln(x+3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)}+(a/b)^{(1/3)})*B-1/3/a^2*(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)}+3^{(1/2)})*A*b+1/3/a*(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)}+3^{(1/2)})*B$

maxima [A] time = 1.42, size = 278, normalized size = 1.03

$$\frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{\frac{1}{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x+b^{\frac{1}{3}}x+a^{\frac{1}{3}}}}\right)}{\frac{5}{a^{\frac{5}{6}}b^{\frac{1}{6}}}} - \frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{\frac{1}{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x+b^{\frac{1}{3}}x+a^{\frac{1}{3}}}}\right)}{\frac{5}{a^{\frac{5}{6}}b^{\frac{1}{6}}}} + \frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{\frac{1}{b^{\frac{1}{3}}}\sqrt{x}}{\sqrt{\frac{1}{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}\right)}{\frac{2}{a^{\frac{2}{3}}b^{\frac{1}{3}}}\sqrt{\frac{1}{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}} + \frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{\frac{1}{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}}\sqrt{x}}{\sqrt{\frac{1}{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}\right)}{ab^{\frac{1}{3}}\sqrt{\frac{1}{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}} + \frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{\frac{1}{3}a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}}\sqrt{x}}{\sqrt{\frac{1}{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}\right)}{ab^{\frac{1}{3}}\sqrt{\frac{1}{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}} - \frac{2A}{5ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="maxima")

[Out] $1/6*(\sqrt{3}*(B*a - A*b)*\log(\sqrt{3}*a^{(1/6)}*b^{(1/6)}*\sqrt{x} + b^{(1/3)}*x + a^{(1/3)})/(a^{(5/6)}*b^{(1/6)}) - \sqrt{3}*(B*a - A*b)*\log(-\sqrt{3}*a^{(1/6)}*b^{(1/6)}*\sqrt{x} + b^{(1/3)}*x + a^{(1/3)})/(a^{(5/6)}*b^{(1/6)}) + 4*(B*a*b^{(1/3)} - A*b^{(4/3)})*\arctan(b^{(1/3)}*\sqrt{x}/\sqrt{a^{(1/3)}*b^{(1/3)}})/(a^{(2/3)}*b^{(1/3)}*\sqrt{a^{(1/3)}*b^{(1/3)}}) + 2*(B*a^{(4/3)}*b^{(1/3)} - A*a^{(1/3)}*b^{(4/3)})*\arctan((\sqrt{3}*a^{(1/6)}*b^{(1/6)} + 2*b^{(1/3)}*\sqrt{x})/\sqrt{a^{(1/3)}*b^{(1/3)}})/(a*b^{(1/3)}*\sqrt{a^{(1/3)}*b^{(1/3)}}) + 2*(B*a^{(4/3)}*b^{(1/3)} - A*a^{(1/3)}*b^{(4/3)})*\arctan(-(\sqrt{3}*a^{(1/6)}*b^{(1/6)} - 2*b^{(1/3)}*\sqrt{x})/\sqrt{a^{(1/3)}*b^{(1/3)}})/(a*b^{(1/3)}*\sqrt{a^{(1/3)}*b^{(1/3)}}))/a - 2/5*A/(a*x^{(5/2)})$

mupad [B] time = 2.91, size = 2023, normalized size = 7.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^(7/2)*(a + b*x^3)),x)

[Out] $-(2*A)/(5*a*x^{(5/2)}) - (\operatorname{atan}(\frac{(x^{(1/2)}*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^{10}*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))}{3*(-a)^{(11/6)}*b^{(1/6)}}))*(A*b - B*a)*i)/(3*(-a)^{(11/6)}*b^{(1/6)}) + ((x^{(1/2)}*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^{10}*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7)))/3*(-a)^{(11/6)}*b^{(1/6)}))*(A*b - B*a)*i)/(3*(-a)^{(11/6)}*b^{(1/6)})/(((x^{(1/2)}*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^{10}*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7)) - ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^{10}*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7)))/3*(-a)^{(11/6)}*b^{(1/6)}))$

$$\begin{aligned}
& B*a^8*b^7)/(3*(-a)^{(11/6)}*b^{(1/6)}))*(A*b - B*a))/(3*(-a)^{(11/6)}*b^{(1/6)}) - \\
& ((x^{(1/2)}*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7)))/(3*(-a)^{(11/6)}*b^{(1/6)})))* \\
& (A*b - B*a))/(3*(-a)^{(11/6)}*b^{(1/6)})))*((A*b - B*a)*2i)/(3*(-a)^{(11/6)}*b^{(1/6)}) - (\operatorname{atan}((((3^{(1/2)}*1i)/2 - 1/2)*(x^{(1/2)}*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7)))/(3*(-a)^{(11/6)}*b^{(1/6)})))*((A*b - B*a)*1i)/(3*(-a)^{(11/6)}*b^{(1/6)}) + (((3^{(1/2)}*1i)/2 - 1/2)*(x^{(1/2)}*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + (((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7)))/(3*(-a)^{(11/6)}*b^{(1/6)})))*((A*b - B*a)*1i)/(3*(-a)^{(11/6)}*b^{(1/6)})))/((((3^{(1/2)}*1i)/2 - 1/2)*(x^{(1/2)}*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7)))/(3*(-a)^{(11/6)}*b^{(1/6)})))*((A*b - B*a)*1i)/(3*(-a)^{(11/6)}*b^{(1/6)}) - (((3^{(1/2)}*1i)/2 - 1/2)*(x^{(1/2)}*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + (((3^{(1/2)}*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7)))/(3*(-a)^{(11/6)}*b^{(1/6)})))*((A*b - B*a)*2i)/(3*(-a)^{(11/6)}*b^{(1/6)}) - (\operatorname{atan}((((3^{(1/2)}*1i)/2 + 1/2)*(x^{(1/2)}*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7)))/(3*(-a)^{(11/6)}*b^{(1/6)})))*((A*b - B*a)*1i)/(3*(-a)^{(11/6)}*b^{(1/6)}) + (((3^{(1/2)}*1i)/2 + 1/2)*(x^{(1/2)}*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + (((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7)))/(3*(-a)^{(11/6)}*b^{(1/6)})))*((A*b - B*a)*1i)/(3*(-a)^{(11/6)}*b^{(1/6)})))/((((3^{(1/2)}*1i)/2 + 1/2)*(x^{(1/2)}*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7)))/(3*(-a)^{(11/6)}*b^{(1/6)})))*((A*b - B*a)*1i)/(3*(-a)^{(11/6)}*b^{(1/6)}) - (((3^{(1/2)}*1i)/2 + 1/2)*(x^{(1/2)}*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + (((3^{(1/2)}*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7)))/(3*(-a)^{(11/6)}*b^{(1/6)})))*((A*b - B*a)*2i)/(3*(-a)^{(11/6)}*b^{(1/6)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a),x)
```

```
[Out] Timed out
```

$$3.159 \quad \int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=95

$$\frac{(Ab - 3aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}} - \frac{x^{3/2}(Ab - 3aB)}{3ab^2} + \frac{x^{9/2}(Ab - aB)}{3ab(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 321, 329, 275, 205}

$$-\frac{x^{3/2}(Ab - 3aB)}{3ab^2} + \frac{(Ab - 3aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}} + \frac{x^{9/2}(Ab - aB)}{3ab(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] -((A*b - 3*a*B)*x^(3/2))/(3*a*b^2) + ((A*b - a*B)*x^(9/2))/(3*a*b*(a + b*x^3)) + ((A*b - 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(5/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n)^(p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^(m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^{7/2} (A + Bx^3)}{(a + bx^3)^2} dx &= \frac{(Ab - aB)x^{9/2}}{3ab(a + bx^3)} + \frac{\left(-\frac{3Ab}{2} + \frac{9aB}{2}\right) \int \frac{x^{7/2}}{a + bx^3} dx}{3ab} \\
 &= -\frac{(Ab - 3aB)x^{3/2}}{3ab^2} + \frac{(Ab - aB)x^{9/2}}{3ab(a + bx^3)} + \frac{(Ab - 3aB) \int \frac{\sqrt{x}}{a + bx^3} dx}{2b^2} \\
 &= -\frac{(Ab - 3aB)x^{3/2}}{3ab^2} + \frac{(Ab - aB)x^{9/2}}{3ab(a + bx^3)} + \frac{(Ab - 3aB) \operatorname{Subst}\left(\int \frac{x^2}{a + bx^6} dx, x, \sqrt{x}\right)}{b^2} \\
 &= -\frac{(Ab - 3aB)x^{3/2}}{3ab^2} + \frac{(Ab - aB)x^{9/2}}{3ab(a + bx^3)} + \frac{(Ab - 3aB) \operatorname{Subst}\left(\int \frac{1}{a + bx^2} dx, x, x^{3/2}\right)}{3b^2} \\
 &= -\frac{(Ab - 3aB)x^{3/2}}{3ab^2} + \frac{(Ab - aB)x^{9/2}}{3ab(a + bx^3)} + \frac{(Ab - 3aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 0.18, size = 77, normalized size = 0.81

$$\frac{\frac{(Ab - 3aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{b}x^{3/2}(3aB - Ab + 2bBx^3)}{a + bx^3}}{3b^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] ((Sqrt[b]*x^(3/2)*(-A*b) + 3*a*B + 2*b*B*x^3))/(a + b*x^3) + ((A*b - 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/Sqrt[a]/(3*b^(5/2))

IntegrateAlgebraic [A] time = 0.11, size = 77, normalized size = 0.81

$$\frac{(Ab - 3aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}} + \frac{x^{3/2}(3aB - Ab + 2bBx^3)}{3b^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] (x^(3/2)*(-A*b) + 3*a*B + 2*b*B*x^3)/(3*b^2*(a + b*x^3)) + ((A*b - 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(5/2))

fricas [A] time = 0.72, size = 222, normalized size = 2.34

$$\left[\frac{\left((3Bab - Ab^2)x^3 + 3Ba^2 - Aab \right) \sqrt{-ab} \log\left(\frac{bx^3 - 2\sqrt{-ab}x^2 - a}{bx^3 + a} \right) + 2(2Babx^4 + (3Ba^2b - Aab^2)x)\sqrt{x} - \left((3Bab - Ab^2)x^3 + 3Ba^2 - Aab \right) \sqrt{ab} \arctan\left(\frac{\sqrt{ab}x^2}{a} \right) - (2Babx^4 + (3Ba^2b - Aab^2)x)\sqrt{x}}{6(ab^4x^3 + a^2b^3)}, \frac{\left((3Bab - Ab^2)x^3 + 3Ba^2 - Aab \right) \sqrt{ab} \arctan\left(\frac{\sqrt{ab}x^2}{a} \right) - (2Babx^4 + (3Ba^2b - Aab^2)x)\sqrt{x}}{3(ab^4x^3 + a^2b^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [1/6*(((3*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) + 2*(2*B*a*b^2*x^4 + (3*B*a^2*b - A*a*b^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3), -1/3*(((3*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*B*a*b^2*x^4 + (3*B*a^2*b - A*a*b^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3)]

giac [A] time = 0.18, size = 68, normalized size = 0.72

$$\frac{2Bx^{\frac{3}{2}}}{3b^2} - \frac{(3Ba - Ab) \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}b^2} + \frac{Bax^{\frac{3}{2}} - Abx^{\frac{3}{2}}}{3(bx^3 + a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")

[Out] 2/3*B*x^(3/2)/b^2 - 1/3*(3*B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^3 + a)*b^2)

maple [A] time = 0.06, size = 93, normalized size = 0.98

$$-\frac{Ax^{\frac{3}{2}}}{3(bx^3+a)b} + \frac{Bax^{\frac{3}{2}}}{3(bx^3+a)b^2} + \frac{A \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}b} - \frac{Ba \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2Bx^{\frac{3}{2}}}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x)`

[Out] $2/3*B*x^{(3/2)}/b^2-1/3/b*x^{(3/2)}/(b*x^3+a)*A+1/3/b^2*x^{(3/2)}/(b*x^3+a)*B*a+1/3/b/(a*b)^{(1/2)}*\arctan(1/(a*b)^{(1/2)}*b*x^{(3/2)})*A-1/b^2/(a*b)^{(1/2)}*\arctan(1/(a*b)^{(1/2)}*b*x^{(3/2)})*B*a$

maxima [A] time = 1.24, size = 68, normalized size = 0.72

$$\frac{(Ba - Ab)x^{\frac{3}{2}}}{3(b^3x^3 + ab^2)} + \frac{2Bx^{\frac{3}{2}}}{3b^2} - \frac{(3Ba - Ab) \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] $1/3*(B*a - A*b)*x^{(3/2)}/(b^3*x^3 + a*b^2) + 2/3*B*x^{(3/2)}/b^2 - 1/3*(3*B*a - A*b)*\arctan(b*x^{(3/2)}/\sqrt{a*b})/(\sqrt{a*b}*b^2)$

mupad [B] time = 2.65, size = 116, normalized size = 1.22

$$\frac{2Bx^{3/2}}{3b^2} - \frac{x^{3/2}\left(\frac{Ab}{3} - \frac{Ba}{3}\right)}{b^3x^3 + ab^2} + \frac{\operatorname{atan}\left(\frac{36\sqrt{a}b^{3/2}x^{3/2}(A^2b^2 - 6ABab + 9B^2a^2)}{(Ab - 3Ba)(36Aab^2 - 108Ba^2b)}\right)(Ab - 3Ba)}{3\sqrt{a}b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x)`

[Out] $(2*B*x^{(3/2)})/(3*b^2) - (x^{(3/2)}*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) + (\operatorname{atan}((36*a^{(1/2)}*b^{(3/2)}*x^{(3/2)}*(A^2*b^2 + 9*B^2*a^2 - 6*A*B*a*b))/((A*b - 3*B*a)*(36*A*a*b^2 - 108*B*a^2*b)))*(A*b - 3*B*a))/(3*a^{(1/2)}*b^{(5/2)})$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```


$$3.160 \quad \int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=312

$$\frac{(Ab - 7aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{5/6} b^{13/6}} + \frac{(Ab - 7aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{5/6} b^{13/6}} - \frac{(Ab - 7aB) \tan^{-1}\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{18a^{5/6} b^{13/6}} + \frac{(Ab - 7aB) \tan^{-1}\left(\frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}} + \sqrt{3}\right)}{18a^{5/6} b^{13/6}} + \frac{(Ab - 7aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6} b^{13/6}} - \frac{\sqrt{x}(Ab - 7aB)}{3ab^2} + \frac{x^{7/2}(Ab - aB)}{3ab(a + bx^3)}$$

Rubi [A] time = 0.50, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, number of rules / integrand size = 0.409, Rules used = {457, 321, 329, 209, 634, 618, 204, 628, 205}

$$\frac{(Ab - 7aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{5/6} b^{13/6}} + \frac{(Ab - 7aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{5/6} b^{13/6}} - \frac{(Ab - 7aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6} b^{13/6}} + \frac{(Ab - 7aB) \tan^{-1}\left(\frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}} + \sqrt{3}\right)}{18a^{5/6} b^{13/6}} + \frac{(Ab - 7aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6} b^{13/6}} - \frac{\sqrt{x}(Ab - 7aB)}{3ab^2} + \frac{x^{7/2}(Ab - aB)}{3ab(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2, x]

[Out] -((A*b - 7*a*B)*Sqrt[x])/(3*a*b^2) + ((A*b - a*B)*x^(7/2))/(3*a*b*(a + b*x^3)) - ((A*b - 7*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(5/6)*b^(13/6)) - ((A*b - 7*a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(5/6)*b^(13/6))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 209

Int[((a_) + (b_.)*(x_)^(n_))(-1), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*k - 1)*Pi)/n]*x])/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[

$(r + s \cos((2k - 1)\pi/n)x) / (r^2 + 2rs \cos((2k - 1)\pi/n)x + s^2 x^2)$, x]; $(2r^2 \int 1/(r^2 + s^2 x^2), x) / (a^n) + \text{Dist}[(2r)/(a^n), \text{Sum}[u, \{k, 1, (n - 2)/4\}], x, x] /;$ $\text{FreeQ}\{a, b\}, x$ && $\text{IGtQ}[(n - 2)/4, 0]$ && $\text{PosQ}[a/b]$

Rule 321

$\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^p, x_Symbol] :> \text{Simp}[(c^{n-1} \cdot (c \cdot x)^{m-n+1} \cdot (a + b \cdot x^n)^{p+1}) / (b \cdot (m + n \cdot p + 1)), x] - \text{Dist}[(a \cdot c^{n-1} \cdot (c \cdot x)^{m-n+1}) / (b \cdot (m + n \cdot p + 1)), \text{Int}[(c \cdot x)^{m-n} \cdot (a + b \cdot x^n)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, p\}, x$ && $\text{IGtQ}[n, 0]$ && $\text{GtQ}[m, n - 1]$ && $\text{NeQ}[m + n \cdot p + 1, 0]$ && $\text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 329

$\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^p, x_Symbol] :> \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{k \cdot (m+1) - 1} \cdot (a + b \cdot x^{k \cdot n}) / c^n]^p, x], x, (c \cdot x)^{1/k}], x] /;$ $\text{FreeQ}\{a, b, c, p\}, x$ && $\text{IGtQ}[n, 0]$ && $\text{FractionQ}[m]$ && $\text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 457

$\text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^p \cdot (c + d \cdot x^n), x_Symbol] :> -\text{Simp}[(b \cdot c - a \cdot d) \cdot (e \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} / (a \cdot b \cdot e \cdot n \cdot (p + 1)), x] - \text{Dist}[(a \cdot d \cdot (m + 1) - b \cdot c \cdot (m + n \cdot (p + 1) + 1)) / (a \cdot b \cdot n \cdot (p + 1)), \text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^{p+1}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, m, n\}, x$ && $\text{NeQ}[b \cdot c - a \cdot d, 0]$ && $\text{LtQ}[p, -1]$ && $(\text{!IntegerQ}[p + 1/2] \&\& \text{NeQ}[p, -5/4]) \text{ || } \text{!RationalQ}[m] \text{ || } (\text{IGtQ}[n, 0] \&\& \text{ILtQ}[p + 1/2, 0] \&\& \text{LeQ}[-1, m, -(n \cdot (p + 1))])$

Rule 618

$\text{Int}[(a + b \cdot x + c \cdot x^2)^{-1}, x_Symbol] :> \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4 \cdot a \cdot c - x^2, x], x], x, b + 2 \cdot c \cdot x], x] /;$ $\text{FreeQ}\{a, b, c\}, x$ && $\text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 628

$\text{Int}[(d + e \cdot x) / (a + b \cdot x + c \cdot x^2), x_Symbol] :> \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] /;$ $\text{FreeQ}\{a, b, c, d, e\}, x$ && $\text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2} (A + Bx^3)}{(a + bx^3)^2} dx &= \frac{(Ab - aB)x^{7/2}}{3ab(a + bx^3)} + \frac{\left(-\frac{Ab}{2} + \frac{7aB}{2}\right) \int \frac{x^{5/2}}{a+bx^3} dx}{3ab} \\
&= -\frac{(Ab - 7aB)\sqrt{x}}{3ab^2} + \frac{(Ab - aB)x^{7/2}}{3ab(a + bx^3)} + \frac{(Ab - 7aB) \int \frac{1}{\sqrt{x}(a+bx^3)} dx}{6b^2} \\
&= -\frac{(Ab - 7aB)\sqrt{x}}{3ab^2} + \frac{(Ab - aB)x^{7/2}}{3ab(a + bx^3)} + \frac{(Ab - 7aB) \operatorname{Subst}\left(\int \frac{1}{a+bx^6} dx, x, \sqrt{x}\right)}{3b^2} \\
&= -\frac{(Ab - 7aB)\sqrt{x}}{3ab^2} + \frac{(Ab - aB)x^{7/2}}{3ab(a + bx^3)} + \frac{(Ab - 7aB) \operatorname{Subst}\left(\int \frac{\sqrt[6]{a} - \frac{1}{2}\sqrt{3} \sqrt[6]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{9a^{5/6}b^2} \\
&= -\frac{(Ab - 7aB)\sqrt{x}}{3ab^2} + \frac{(Ab - aB)x^{7/2}}{3ab(a + bx^3)} + \frac{(Ab - 7aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} - \frac{(Ab - 7aB) \operatorname{Subst}\left(\int \frac{1}{a+bx^6} dx, x, \sqrt{x}\right)}{12\sqrt{3}} \\
&= -\frac{(Ab - 7aB)\sqrt{x}}{3ab^2} + \frac{(Ab - aB)x^{7/2}}{3ab(a + bx^3)} + \frac{(Ab - 7aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} - \frac{(Ab - 7aB) \log\left(\sqrt[3]{a} - \sqrt[6]{b} \sqrt{x}\right)}{12\sqrt{3}} \\
&= -\frac{(Ab - 7aB)\sqrt{x}}{3ab^2} + \frac{(Ab - aB)x^{7/2}}{3ab(a + bx^3)} - \frac{(Ab - 7aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}} + \frac{(Ab - 7aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 76, normalized size = 0.24

$$\frac{\sqrt{x} \left((a + bx^3) (Ab - 7aB) {}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; -\frac{bx^3}{a}\right) + a(7aB - Ab + 6bBx^3) \right)}{3ab^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] (Sqrt[x]*(a*(-A*b) + 7*a*B + 6*b*B*x^3) + (A*b - 7*a*B)*(a + b*x^3)*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)])/(3*a*b^2*(a + b*x^3))

IntegrateAlgebraic [A] time = 0.65, size = 196, normalized size = 0.63

$$\frac{(Ab - 7aB) \tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} + \frac{(7aB - Ab) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{18a^{5/6}b^{13/6}} - \frac{(7aB - Ab) \tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{b}x}\right)}{6\sqrt{3}a^{5/6}b^{13/6}} + \frac{\sqrt{x}(7aB - Ab + 6bBx^3)}{3b^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] (Sqrt[x]*(-(A*b) + 7*a*B + 6*b*B*x^3))/(3*b^2*(a + b*x^3)) + ((A*b - 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(5/6)*b^(13/6)) + (((-A*b) + 7*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x]])/(18*a^(5/6)*b^(13/6)) - (((-A*b) + 7*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)]/(6*Sqrt[3]*a^(5/6)*b^(13/6)))

fricas [B] time = 0.84, size = 2566, normalized size = 8.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/36*(4*sqrt(3)*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^4*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/3) + (49*B^2*a^2 - 14*A*B*a*b + A^2*b^2)*x + (7*B*a^2*b^2 - A*a*b^3)*sqrt(x))*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6))*a^4*b^11*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(5/6) + 2*sqrt(3)*(7*B*a^5*b^11 - A*a^4*b^12)*sqrt(x))*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(5/6) - sqrt(3)*(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)) + 4*sqrt(3)*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))

$$\begin{aligned}
& 3))^{(1/6)} * \arctan(1/3 * (2 * \sqrt{3}) * \sqrt{a^2 * b^4 * (- (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (a^5 * b^{13}))^{(1/3)} + (49 * B^2 * a^2 - 14 * A * B * a * b + A^2 * b^2) * x - (7 * B * a^2 * b^2 - A * a * b^3) * \sqrt{x} * (- (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (a^5 * b^{13}))^{(1/6)}) * a^4 * b^{11} * (- (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (a^5 * b^{13}))^{(5/6)} + 2 * \sqrt{3} * (3) * (7 * B * a^5 * b^{11} - A * a^4 * b^{12}) * \sqrt{x} * (- (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (a^5 * b^{13}))^{(5/6)} + \sqrt{3} * (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6)) - (b^3 * x^3 + a * b^2) * (- (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (a^5 * b^{13}))^{(1/6)} * \log(a^2 * b^4 * (- (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (a^5 * b^{13}))^{(1/3)} + (49 * B^2 * a^2 - 14 * A * B * a * b + A^2 * b^2) * x + (7 * B * a^2 * b^2 - A * a * b^3) * \sqrt{x} * (- (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (a^5 * b^{13}))^{(1/6)}) + (b^3 * x^3 + a * b^2) * (- (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (a^5 * b^{13}))^{(1/6)} * \log(a^2 * b^4 * (- (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (a^5 * b^{13}))^{(1/3)} + (49 * B^2 * a^2 - 14 * A * B * a * b + A^2 * b^2) * x - (7 * B * a^2 * b^2 - A * a * b^3) * \sqrt{x} * (- (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (a^5 * b^{13}))^{(1/6)}) + 2 * (b^3 * x^3 + a * b^2) * (- (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (a^5 * b^{13}))^{(1/6)} * \log(a * b^2 * (- (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (a^5 * b^{13}))^{(1/6)} - (7 * B * a - A * b) * \sqrt{x}) - 2 * (b^3 * x^3 + a * b^2) * (- (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (a^5 * b^{13}))^{(1/6)} * \log(- a * b^2 * (- (117649 * B^6 * a^6 - 100842 * A * B^5 * a^5 * b + 36015 * A^2 * B^4 * a^4 * b^2 - 6860 * A^3 * B^3 * a^3 * b^3 + 735 * A^4 * B^2 * a^2 * b^4 - 42 * A^5 * B * a * b^5 + A^6 * b^6) / (a^5 * b^{13}))^{(1/6)} - (7 * B * a - A * b) * \sqrt{x}) + 12 * (6 * B * b * x^3 + 7 * B * a - A * b) * \sqrt{x} / (b^3 * x^3 + a * b^2)
\end{aligned}$$

giac [A] time = 0.33, size = 313, normalized size = 1.00

$$\frac{2B\sqrt{c}}{b^2} - \frac{\sqrt{3} \left(7 (ab)^{\frac{1}{2}} Ba - (ab)^{\frac{1}{2}} Ab \right) \log \left(\sqrt{3} \sqrt{\left(\frac{c}{b}\right)^{\frac{1}{2}} + x + \left(\frac{c}{b}\right)^{\frac{1}{2}}} \right)}{36ab^3} + \frac{\sqrt{3} \left(7 (ab)^{\frac{1}{2}} Ba - (ab)^{\frac{1}{2}} Ab \right) \log \left(-\sqrt{3} \sqrt{\left(\frac{c}{b}\right)^{\frac{1}{2}} + x + \left(\frac{c}{b}\right)^{\frac{1}{2}}} \right)}{36ab^3} + \frac{Ba\sqrt{c} - Ab\sqrt{c}}{3(b^3 + a)^2} - \frac{\left(7 (ab)^{\frac{1}{2}} Ba - (ab)^{\frac{1}{2}} Ab \right) \arctan \left(\frac{\sqrt{3} \left(\frac{c}{b}\right)^{\frac{1}{2}} + 2\sqrt{c}}{\left(\frac{c}{b}\right)^{\frac{1}{2}}} \right)}{18ab^3} - \frac{\left(7 (ab)^{\frac{1}{2}} Ba - (ab)^{\frac{1}{2}} Ab \right) \arctan \left(-\frac{\sqrt{3} \left(\frac{c}{b}\right)^{\frac{1}{2}} + 2\sqrt{c}}{\left(\frac{c}{b}\right)^{\frac{1}{2}}} \right)}{18ab^3} - \frac{\left(7 (ab)^{\frac{1}{2}} Ba - (ab)^{\frac{1}{2}} Ab \right) \arctan \left(\frac{\sqrt{c}}{\left(\frac{c}{b}\right)^{\frac{1}{2}}} \right)}{9ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")

[Out] $2*B*\sqrt{x}/b^2 - 1/36*\sqrt{3}*(7*(a*b^5)^{(1/6)}*B*a - (a*b^5)^{(1/6)}*A*b)*\log(\sqrt{3}*\sqrt{x}*(a/b)^{(1/6)} + x + (a/b)^{(1/3)})/(a*b^3) + 1/36*\sqrt{3}*(7*(a*b^5)^{(1/6)}*B*a - (a*b^5)^{(1/6)}*A*b)*\log(-\sqrt{3}*\sqrt{x}*(a/b)^{(1/6)} + x + (a/b)^{(1/3)})/(a*b^3) + 1/3*(B*a*\sqrt{x} - A*b*\sqrt{x})/((b*x^3 + a)*b^2) - 1/18*(7*(a*b^5)^{(1/6)}*B*a - (a*b^5)^{(1/6)}*A*b)*\arctan((\sqrt{3}*(a/b)^{(1/6)} + 2*\sqrt{x})/(a/b)^{(1/6)})/(a*b^3) - 1/18*(7*(a*b^5)^{(1/6)}*B*a - (a*b^5)^{(1/6)}*A*b)*\arctan(-(\sqrt{3}*(a/b)^{(1/6)} - 2*\sqrt{x})/(a/b)^{(1/6)})/(a*b^3) - 1/9*(7*(a*b^5)^{(1/6)}*B*a - (a*b^5)^{(1/6)}*A*b)*\arctan(\sqrt{x}/(a/b)^{(1/6)})/(a*b^3)$

maple [A] time = 0.17, size = 405, normalized size = 1.30

$$\frac{A\sqrt{x}}{3(b^2x^3+a)^2} + \frac{B\sqrt{x}}{3(b^2x^3+a)^2} - \frac{(b^2x^3+a)\arctan\left(\frac{\sqrt{x}}{(b^2x^3+a)^{1/6}}\right)}{9ab} + \frac{(b^2x^3+a)\arctan\left(\frac{2\sqrt{x}}{(b^2x^3+a)^{1/6}} - \sqrt{3}\right)}{18ab} + \frac{(b^2x^3+a)\arctan\left(\frac{2\sqrt{x}}{(b^2x^3+a)^{1/6}} + \sqrt{3}\right)}{18ab} + \frac{\sqrt{3}(b^2x^3+a)\ln\left(x + \sqrt{3}(b^2x^3+a)^{1/6}\sqrt{x} + (b^2x^3+a)^{1/3}\right)}{36ab} + \frac{\sqrt{3}(b^2x^3+a)\ln\left(-x + \sqrt{3}(b^2x^3+a)^{1/6}\sqrt{x} - (b^2x^3+a)^{1/3}\right)}{36ab} + \frac{7(b^2x^3+a)\arctan\left(\frac{\sqrt{x}}{(b^2x^3+a)^{1/6}}\right)}{9ab} + \frac{7(b^2x^3+a)\arctan\left(\frac{2\sqrt{x}}{(b^2x^3+a)^{1/6}} - \sqrt{3}\right)}{18ab} + \frac{7(b^2x^3+a)\arctan\left(\frac{2\sqrt{x}}{(b^2x^3+a)^{1/6}} + \sqrt{3}\right)}{18ab} + \frac{7\sqrt{3}(b^2x^3+a)\ln\left(x + \sqrt{3}(b^2x^3+a)^{1/6}\sqrt{x} + (b^2x^3+a)^{1/3}\right)}{36ab} + \frac{7\sqrt{3}(b^2x^3+a)\ln\left(-x + \sqrt{3}(b^2x^3+a)^{1/6}\sqrt{x} - (b^2x^3+a)^{1/3}\right)}{36ab} + \frac{2B\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x)

[Out] $2*B/b^2*x^{(1/2)} - 1/3/b*x^{(1/2)}/(b*x^3+a)*A + 1/3/b^2*x^{(1/2)}/(b*x^3+a)*B*a - 7/9/b^2*B*(a/b)^{(1/6)}*\arctan(1/(a/b)^{(1/6)}*x^{(1/2)}) + 7/36/b^2*B*3^{(1/2)}*(a/b)^{(1/6)}*\ln(-x+3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)} - (a/b)^{(1/3)}) - 7/18/b^2*B*(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)} - 3^{(1/2)}) - 7/36/b^2*B*3^{(1/2)}*(a/b)^{(1/6)}*\ln(x+3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)} + (a/b)^{(1/3)}) - 7/18/b^2*B*(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)} + 3^{(1/2)}) + 1/9/b*A/a*(a/b)^{(1/6)}*\arctan(1/(a/b)^{(1/6)}*x^{(1/2)}) - 1/36/b*A/a*3^{(1/2)}*(a/b)^{(1/6)}*\ln(-x+3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)} - (a/b)^{(1/3)}) + 1/18/b*A/a*(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)} - 3^{(1/2)}) + 1/36/b*A/a*3^{(1/2)}*(a/b)^{(1/6)}*\ln(x+3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)} + (a/b)^{(1/3)}) + 1/18/b*A/a*(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)} + 3^{(1/2)})$

maxima [A] time = 1.35, size = 311, normalized size = 1.00

$$\frac{(Ba - Ab)\sqrt{x}}{3(b^2x^3 + ab^2)} + \frac{2B\sqrt{x}}{b^2} - \frac{\sqrt{3}(7Ba - Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x} + b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(7Ba - Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x} + b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(7Ba^{\frac{1}{3}} - Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{6}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{36b^2} + \frac{2\left(7Ba^{\frac{4}{3}}b^{\frac{1}{3}} - Ab^{\frac{4}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}} + 2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{5}{6}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{2\left(7Ba^{\frac{4}{3}}b^{\frac{1}{3}} - Ab^{\frac{4}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}} - 2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{5}{6}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] $1/3*(B*a - A*b)*\sqrt{x}/(b^3*x^3 + a*b^2) + 2*B*\sqrt{x}/b^2 - 1/36*(\sqrt{3}*(7*B*a - A*b)*\log(\sqrt{3}*a^{(1/6)}*b^{(1/6)}*\sqrt{x} + b^{(1/3)}*x + a^{(1/3)})/(a^{(5/6)}*b^{(1/6)}) - \sqrt{3}*(7*B*a - A*b)*\log(-\sqrt{3}*a^{(1/6)}*b^{(1/6)}*\sqrt{x} + b^{(1/3)}*x + a^{(1/3)})/(a^{(5/6)}*b^{(1/6)}) + 4*(7*B*a*b^{(1/3)} - A*b^{(4/3)})$

$$\begin{aligned} &) * b^{(19/6)}) * (A * b - 7 * B * a)) / (18 * (-a)^{(5/6)} * b^{(13/6)}) * ((3^{(1/2)} * i) / 2 - 1 / \\ & 2) * (A * b - 7 * B * a) * i) / (9 * (-a)^{(5/6)} * b^{(13/6)}) - (\operatorname{atan}(\frac{((3^{(1/2)} * i) / 2 + 1 / \\ & 2) * ((2 * x^{(1/2)} * (A^4 * b^4 + 2401 * B^4 * a^4 + 294 * A^2 * B^2 * a^2 * b^2 - 1372 * A * B^3 * a^3 * b - \\ & 28 * A^3 * B * a * b^3)) / (27 * b^3) - (2 * ((3^{(1/2)} * i) / 2 + 1 / 2) * (A * b - 7 * B * a) * \\ & (343 * B^3 * a^4 - A^3 * a * b^3 - 147 * A * B^2 * a^3 * b + 21 * A^2 * B * a^2 * b^2)) / (27 * (-a)^{(5/6)} * b^{(19/6)}) * \\ & (A * b - 7 * B * a) * i) / (18 * (-a)^{(5/6)} * b^{(13/6)}) + (((3^{(1/2)} * i) / 2 + 1 / 2) * ((2 * x^{(1/2)} * (A^4 * b^4 + \\ & 2401 * B^4 * a^4 + 294 * A^2 * B^2 * a^2 * b^2 - 1372 * A * B^3 * a^3 * b - 28 * A^3 * B * a * b^3)) / (27 * b^3) + (2 * ((3^{(1/2)} * i) / 2 + \\ & 1 / 2) * (A * b - 7 * B * a) * (343 * B^3 * a^4 - A^3 * a * b^3 - 147 * A * B^2 * a^3 * b + 21 * A^2 * B * a^2 * b^2)) / (27 * (-a)^{(5/6)} * b^{(19/6)}) * \\ & (A * b - 7 * B * a) * i) / (18 * (-a)^{(5/6)} * b^{(13/6)})) / (((3^{(1/2)} * i) / 2 + 1 / 2) * ((2 * x^{(1/2)} * (A^4 * b^4 + \\ & 2401 * B^4 * a^4 + 294 * A^2 * B^2 * a^2 * b^2 - 1372 * A * B^3 * a^3 * b - 28 * A^3 * B * a * b^3)) / (27 * b^3) - (2 * ((3^{(1/2)} * i) / 2 + \\ & 1 / 2) * (A * b - 7 * B * a) * (343 * B^3 * a^4 - A^3 * a * b^3 - 147 * A * B^2 * a^3 * b + 21 * A^2 * B * a^2 * b^2)) / (27 * (-a)^{(5/6)} * b^{(19/6)}) * \\ & (A * b - 7 * B * a)) / (18 * (-a)^{(5/6)} * b^{(13/6)}) - (((3^{(1/2)} * i) / 2 + 1 / 2) * ((2 * x^{(1/2)} * (A^4 * b^4 + \\ & 2401 * B^4 * a^4 + 294 * A^2 * B^2 * a^2 * b^2 - 1372 * A * B^3 * a^3 * b - 28 * A^3 * B * a * b^3)) / (27 * b^3) + (2 * ((3^{(1/2)} * i) / 2 + \\ & 1 / 2) * (A * b - 7 * B * a) * (343 * B^3 * a^4 - A^3 * a * b^3 - 147 * A * B^2 * a^3 * b + 21 * A^2 * B * a^2 * b^2)) / (27 * (-a)^{(5/6)} * b^{(19/6)}) * \\ & (A * b - 7 * B * a)) / (18 * (-a)^{(5/6)} * b^{(13/6)})) * ((3^{(1/2)} * i) / 2 + 1 / 2) * (A * b - 7 * B * a) * i) / (9 * (-a)^{(5/6)} * b^{(13/6)}) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**2,x)

[Out] Timed out

$$3.161 \quad \int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=289

$$\frac{(5aB + Ab) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{7/6} b^{11/6}} - \frac{(5aB + Ab) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{7/6} b^{11/6}} - \frac{(5aB + Ab) \tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}{\sqrt[3]{a}} + \sqrt{3}\right)}{18a^{7/6} b^{11/6}} + \frac{(5aB + Ab) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[3]{a}} + \sqrt{3}\right)}{18a^{7/6} b^{11/6}} + \frac{(5aB + Ab) \tan^{-1}\left(\frac{\sqrt[6]{a} \sqrt{x}}{\sqrt[3]{b}} + \sqrt{3}\right)}{9a^{7/6} b^{11/6}} + \frac{x^{5/2}(Ab - aB)}{3ab(a + bx^3)}$$

Rubi [A] time = 0.55, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, number of rules / integrand size = 0.364, Rules used = {457, 329, 295, 634, 618, 204, 628, 205}

$$\frac{(5aB + Ab) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{7/6} b^{11/6}} - \frac{(5aB + Ab) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{7/6} b^{11/6}} - \frac{(5aB + Ab) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[3]{a}}\right)}{18a^{7/6} b^{11/6}} + \frac{(5aB + Ab) \tan^{-1}\left(\frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[3]{a}} + \sqrt{3}\right)}{18a^{7/6} b^{11/6}} + \frac{(5aB + Ab) \tan^{-1}\left(\frac{\sqrt[6]{a} \sqrt{x}}{\sqrt[3]{b}} + \sqrt{3}\right)}{9a^{7/6} b^{11/6}} + \frac{x^{5/2}(Ab - aB)}{3ab(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2, x]

[Out] ((A*b - a*B)*x^(5/2))/(3*a*b*(a + b*x^3)) - ((A*b + 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)]/(18*a^(7/6)*b^(11/6)) + ((A*b + 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)]/(18*a^(7/6)*b^(11/6)) + ((A*b + 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(9*a^(7/6)*b^(11/6)) + ((A*b + 5*a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(7/6)*b^(11/6)) - ((A*b + 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(7/6)*b^(11/6))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 295

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*cos[((2*k - 1)*m*Pi)/n] - s*cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*cos[((2*k - 1)*m*Pi)/n] + s*cos[((2*k - 1)*Pi)/n])/(r^2 - 2*r*s*cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]

```
- 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^(
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2} (A + Bx^3)}{(a + bx^3)^2} dx &= \frac{(Ab - aB)x^{5/2}}{3ab(a + bx^3)} + \frac{\left(\frac{Ab}{2} + \frac{5aB}{2}\right) \int \frac{x^{3/2}}{a+bx^3} dx}{3ab} \\
&= \frac{(Ab - aB)x^{5/2}}{3ab(a + bx^3)} + \frac{\left(2\left(\frac{Ab}{2} + \frac{5aB}{2}\right)\right) \text{Subst}\left(\int \frac{x^4}{a+bx^6} dx, x, \sqrt{x}\right)}{3ab} \\
&= \frac{(Ab - aB)x^{5/2}}{3ab(a + bx^3)} + \frac{(Ab + 5aB) \text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2} + \frac{1}{2}\sqrt{3} \sqrt[6]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{9a^{7/6}b^{5/3}} + \frac{(Ab + 5aB) \text{Subst}\left(\int \frac{x^4}{a+bx^6} dx, x, \sqrt{x}\right)}{3ab} \\
&= \frac{(Ab - aB)x^{5/2}}{3ab(a + bx^3)} + \frac{(Ab + 5aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}} + \frac{(Ab + 5aB) \text{Subst}\left(\int \frac{-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} + 2\sqrt[3]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{12\sqrt{3} a^{7/6}b^{11/6}} \\
&= \frac{(Ab - aB)x^{5/2}}{3ab(a + bx^3)} + \frac{(Ab + 5aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}} + \frac{(Ab + 5aB) \log\left(\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{7/6}b^{11/6}} \\
&= \frac{(Ab - aB)x^{5/2}}{3ab(a + bx^3)} - \frac{(Ab + 5aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}} + \frac{(Ab + 5aB) \tan^{-1}\left(\sqrt{3} + \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}}
\end{aligned}$$

Mathematica [C] time = 0.10, size = 62, normalized size = 0.21

$$\frac{2x^{5/2} \left((Ab - aB) {}_2F_1\left(\frac{5}{6}, 2; \frac{11}{6}; -\frac{bx^3}{a}\right) + aB {}_2F_1\left(\frac{5}{6}, 1; \frac{11}{6}; -\frac{bx^3}{a}\right) \right)}{5a^2b}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] (2*x^(5/2)*(a*B*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a)] + (A*b - a*B)*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a)]))/(5*a^2*b)

IntegrateAlgebraic [A] time = 0.57, size = 190, normalized size = 0.66

$$\frac{(5aB + Ab) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}} + \frac{(-5aB - Ab) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b} x}{\sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}\right)}{18a^{7/6}b^{11/6}} - \frac{(5aB + Ab) \tanh^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{b} x}\right)}{6\sqrt{3} a^{7/6}b^{11/6}} - \frac{x^{5/2}(aB - Ab)}{3ab(a + bx^3)}$$

$$\begin{aligned}
& *b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30 \\
& *A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^{(5/6)} + (9765625*B^10*a^10 + 19531250*A \\
& *B^9*a^9*b + 17578125*A^2*B^8*a^8*b^2 + 9375000*A^3*B^7*a^7*b^3 + 3281250*A \\
& ^4*B^6*a^6*b^4 + 787500*A^5*B^5*a^5*b^5 + 131250*A^6*B^4*a^4*b^6 + 15000*A^ \\
& 7*B^3*a^3*b^7 + 1125*A^8*B^2*a^2*b^8 + 50*A^9*B*a*b^9 + A^10*b^10)*x - (156 \\
& 25*B^6*a^11*b^7 + 18750*A*B^5*a^10*b^8 + 9375*A^2*B^4*a^9*b^9 + 2500*A^3*B^ \\
& 3*a^8*b^10 + 375*A^4*B^2*a^7*b^11 + 30*A^5*B*a^6*b^12 + A^6*a^5*b^13)*(- (15 \\
& 625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b \\
& ^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^{(2/3))*a*b \\
& ^2*(- (15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B \\
& ^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^{(1 \\
& /6)} - 2*sqrt(3)*(3125*B^5*a^6*b^2 + 3125*A*B^4*a^5*b^3 + 1250*A^2*B^3*a^4*b \\
& ^4 + 250*A^3*B^2*a^3*b^5 + 25*A^4*B*a^2*b^6 + A^5*a*b^7)*sqrt(x)*(- (15625*B \\
& ^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + \\
& 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^{(1/6)} - sqrt(3) \\
& *(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a \\
& ^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6))/(15625*B^6*a^6 + \\
& 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B \\
& ^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)) - 2*(a*b^2*x^3 + a^2*b)*(- (15625*B \\
& ^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 3 \\
& 75*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^{(1/6)}*log(a^6*b^ \\
& 9*(- (15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^ \\
& 3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^{(5/ \\
& 6)} + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^3*B^2* \\
& a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) + 2*(a*b^2*x^3 + a^2*b)*(- (156 \\
& 25*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^ \\
& 3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^{(1/6)}*log(- \\
& a^6*b^9*(- (15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500* \\
& A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11 \\
&))^{(5/6)} + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^ \\
& 3*B^2*a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) - (a*b^2*x^3 + a^2*b)*(- \\
& (15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^ \\
& 3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^{(1/6)}*l \\
& og(((3125*B^5*a^11*b^9 + 3125*A*B^4*a^10*b^10 + 1250*A^2*B^3*a^9*b^11 + 250* \\
& A^3*B^2*a^8*b^12 + 25*A^4*B*a^7*b^13 + A^5*a^6*b^14)*sqrt(x)*(- (15625*B^6*a \\
& ^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375* \\
& A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^{(5/6)} + (9765625*B^ \\
& 10*a^10 + 19531250*A*B^9*a^9*b + 17578125*A^2*B^8*a^8*b^2 + 9375000*A^3*B^7 \\
& *a^7*b^3 + 3281250*A^4*B^6*a^6*b^4 + 787500*A^5*B^5*a^5*b^5 + 131250*A^6*B^ \\
& 4*a^4*b^6 + 15000*A^7*B^3*a^3*b^7 + 1125*A^8*B^2*a^2*b^8 + 50*A^9*B*a*b^9 + \\
& A^10*b^10)*x - (15625*B^6*a^11*b^7 + 18750*A*B^5*a^10*b^8 + 9375*A^2*B^4*a \\
& ^9*b^9 + 2500*A^3*B^3*a^8*b^10 + 375*A^4*B^2*a^7*b^11 + 30*A^5*B*a^6*b^12 + \\
& A^6*a^5*b^13)*(- (15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 \\
& + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a \\
& ^7*b^11))^{(2/3)} + (a*b^2*x^3 + a^2*b)*(- (15625*B^6*a^6 + 18750*A*B^5*a^5*b
\end{aligned}$$

$$+ 9375A^2B^4a^4b^2 + 2500A^3B^3a^3b^3 + 375A^4B^2a^2b^4 + 30A^5B^1a^1b^5 + A^6b^6)/(a^7b^{11})^{1/6} \log(- (3125B^5a^{11}b^9 + 3125A^4B^4a^{10}b^{10} + 1250A^2B^3a^9b^{11} + 250A^3B^2a^8b^{12} + 25A^4B^1a^7b^{13} + A^5a^6b^{14}) \sqrt{x} \cdot (- (15625B^6a^6 + 18750A^5a^5b + 9375A^2B^4a^4b^2 + 2500A^3B^3a^3b^3 + 375A^4B^2a^2b^4 + 30A^5B^1a^1b^5 + A^6b^6)/(a^7b^{11}))^{5/6} + (9765625B^{10}a^{10} + 19531250A^9a^9b + 17578125A^2B^8a^8b^2 + 9375000A^3B^7a^7b^3 + 3281250A^4B^6a^6b^4 + 787500A^5B^5a^5b^5 + 131250A^6B^4a^4b^6 + 15000A^7B^3a^3b^7 + 1125A^8B^2a^2b^8 + 50A^9B^1a^1b^9 + A^{10}b^{10}) \cdot x - (15625B^6a^{11}b^7 + 18750A^5B^5a^{10}b^8 + 9375A^2B^4a^9b^9 + 2500A^3B^3a^8b^{10} + 375A^4B^2a^7b^{11} + 30A^5B^1a^6b^{12} + A^6a^5b^{13}) \cdot (- (15625B^6a^6 + 18750A^5B^5a^5b + 9375A^2B^4a^4b^2 + 2500A^3B^3a^3b^3 + 375A^4B^2a^2b^4 + 30A^5B^1a^1b^5 + A^6b^6)/(a^7b^{11}))^{2/3}))/ (a^2b^2x^3 + a^2b)$$

giac [A] time = 1.38, size = 302, normalized size = 1.04

$$\frac{\left(5Ba\left(\frac{5}{6}\right) + Ab\left(\frac{5}{6}\right)\right) \arctan\left(\frac{\sqrt{x}}{\left(\frac{5}{6}\right)^{1/6}}\right)}{9a^2b} + \frac{Bax^{\frac{5}{6}} - Abx^{\frac{5}{6}}}{3(bx^3 + a)ab} \cdot \frac{\sqrt{5}\left(5(ab^5)^{\frac{5}{6}}Ba + (ab^5)^{\frac{5}{6}}Ab\right) \log\left(\sqrt{3}\sqrt{x}\left(\frac{5}{6}\right)^{\frac{1}{6}} + x + \left(\frac{5}{6}\right)^{\frac{1}{6}}\right)}{36a^2b^6} + \frac{\sqrt{5}\left(5(ab^5)^{\frac{5}{6}}Ba + (ab^5)^{\frac{5}{6}}Ab\right) \log\left(-\sqrt{3}\sqrt{x}\left(\frac{5}{6}\right)^{\frac{1}{6}} + x + \left(\frac{5}{6}\right)^{\frac{1}{6}}\right)}{36a^2b^6} + \frac{\left(5(ab^5)^{\frac{5}{6}}Ba + (ab^5)^{\frac{5}{6}}Ab\right) \arctan\left(\frac{\sqrt{3}\left(\frac{5}{6}\right)^{\frac{1}{6}} + 2\sqrt{x}}{\left(\frac{5}{6}\right)^{\frac{1}{6}}}\right)}{18a^2b^6} + \frac{\left(5(ab^5)^{\frac{5}{6}}Ba + (ab^5)^{\frac{5}{6}}Ab\right) \arctan\left(\frac{-\sqrt{3}\left(\frac{5}{6}\right)^{\frac{1}{6}} - 2\sqrt{x}}{\left(\frac{5}{6}\right)^{\frac{1}{6}}}\right)}{18a^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")

[Out] $\frac{1}{9} * (5 * B * a * (a/b)^{(5/6)} + A * b * (a/b)^{(5/6)}) * \arctan(\sqrt{x} / (a/b)^{(1/6)}) / (a^2 * b) - \frac{1}{3} * (B * a * x^{(5/2)} - A * b * x^{(5/2)}) / ((b * x^3 + a) * a * b) - \frac{1}{36} * \sqrt{3} * (5 * (a * b^5)^{(5/6)} * B * a + (a * b^5)^{(5/6)} * A * b) * \log(\sqrt{3} * \sqrt{x} * (a/b)^{(1/6)} + x + (a/b)^{(1/3)}) / (a^2 * b^6) + \frac{1}{36} * \sqrt{3} * (5 * (a * b^5)^{(5/6)} * B * a + (a * b^5)^{(5/6)} * A * b) * \log(-\sqrt{3} * \sqrt{x} * (a/b)^{(1/6)} + x + (a/b)^{(1/3)}) / (a^2 * b^6) + \frac{1}{18} * (5 * (a * b^5)^{(5/6)} * B * a + (a * b^5)^{(5/6)} * A * b) * \arctan((\sqrt{3} * (a/b)^{(1/6)} + 2 * \sqrt{x}) / (a/b)^{(1/6)}) / (a^2 * b^6) + \frac{1}{18} * (5 * (a * b^5)^{(5/6)} * B * a + (a * b^5)^{(5/6)} * A * b) * \arctan(-(\sqrt{3} * (a/b)^{(1/6)} - 2 * \sqrt{x}) / (a/b)^{(1/6)}) / (a^2 * b^6)$

maple [A] time = 0.16, size = 387, normalized size = 1.34

$$\frac{(Ab - Ba)x^{\frac{5}{2}}}{3(bx^3 + a)ab} + \frac{A \arctan\left(\frac{\sqrt{x}}{\left(\frac{5}{6}\right)^{1/6}}\right)}{9\left(\frac{5}{6}\right)^{1/6}ab} + \frac{A \arctan\left(\frac{2\sqrt{x} - \sqrt{3}}{\left(\frac{5}{6}\right)^{1/6}}\right)}{18\left(\frac{5}{6}\right)^{1/6}ab} + \frac{A \arctan\left(\frac{2\sqrt{x} + \sqrt{3}}{\left(\frac{5}{6}\right)^{1/6}}\right)}{18\left(\frac{5}{6}\right)^{1/6}ab} + \frac{\sqrt{5}\left(\frac{5}{6}\right)^{1/6}A \ln\left(x + \sqrt{3}\left(\frac{5}{6}\right)^{1/6}\sqrt{x} + \left(\frac{5}{6}\right)^{1/6}\right)}{36a^2} + \frac{\sqrt{5}\left(\frac{5}{6}\right)^{1/6}A \ln\left(-x + \sqrt{3}\left(\frac{5}{6}\right)^{1/6}\sqrt{x} - \left(\frac{5}{6}\right)^{1/6}\right)}{36a^2} + \frac{5\sqrt{3}\left(\frac{5}{6}\right)^{1/6}B \ln\left(x + \sqrt{3}\left(\frac{5}{6}\right)^{1/6}\sqrt{x} + \left(\frac{5}{6}\right)^{1/6}\right)}{36ab} + \frac{5\sqrt{3}\left(\frac{5}{6}\right)^{1/6}B \ln\left(-x + \sqrt{3}\left(\frac{5}{6}\right)^{1/6}\sqrt{x} - \left(\frac{5}{6}\right)^{1/6}\right)}{36ab} + \frac{5B \arctan\left(\frac{\sqrt{x}}{\left(\frac{5}{6}\right)^{1/6}}\right)}{9\left(\frac{5}{6}\right)^{1/6}a^2} + \frac{5B \arctan\left(\frac{2\sqrt{x} - \sqrt{3}}{\left(\frac{5}{6}\right)^{1/6}}\right)}{18\left(\frac{5}{6}\right)^{1/6}a^2} + \frac{5B \arctan\left(\frac{2\sqrt{x} + \sqrt{3}}{\left(\frac{5}{6}\right)^{1/6}}\right)}{18\left(\frac{5}{6}\right)^{1/6}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x)

[Out] $\frac{1}{3} * (A * b - B * a) * x^{(5/2)} / a / b / (b * x^3 + a) + \frac{1}{9} * b / a / (a/b)^{(1/6)} * \arctan(1 / (a/b)^{(1/6)}) * x^{(1/2)} * A + \frac{5}{9} * b^2 / (a/b)^{(1/6)} * \arctan(1 / (a/b)^{(1/6)}) * x^{(1/2)} * B + \frac{1}{36} * a^2 * 3^{(1/2)} * (a/b)^{(5/6)} * \ln(-x + 3^{(1/2)} * (a/b)^{(1/6)} * x^{(1/2)} - (a/b)^{(1/3)}) * A + \frac{5}{36} * b / a * 3^{(1/2)} * (a/b)^{(5/6)} * \ln(-x + 3^{(1/2)} * (a/b)^{(1/6)} * x^{(1/2)} - (a/b)^{(1/3)}) * B + \frac{1}{18} * b / a / (a/b)^{(1/6)} * \arctan(2 / (a/b)^{(1/6)}) * x^{(1/2)} - 3^{(1/2)} * A + \frac{5}{18} * b^2 / (a/b)^{(1/6)}$

6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*B-1/36/a^2*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A-5/36/b/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+1/18/b/a/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*A+5/18/b^2/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*B

maxima [A] time = 1.47, size = 235, normalized size = 0.81

$$\frac{(Ba - Ab)x^{\frac{5}{2}}}{3(ab^2x^3 + a^2b)} \left(\frac{\sqrt{3} \log\left(\sqrt{3a^6b^6}\sqrt{x+b^{\frac{1}{3}}x+a^{\frac{1}{3}}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{\sqrt{3} \log\left(-\sqrt{3a^6b^6}\sqrt{x+b^{\frac{1}{3}}x+a^{\frac{1}{3}}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{2 \arctan\left(\frac{\sqrt{3a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} - \frac{2 \arctan\left(-\frac{\sqrt{3a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} - \frac{4 \arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] -1/3*(B*a - A*b)*x^(5/2)/(a*b^2*x^3 + a^2*b) - 1/36*(5*B*a + A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3)))/(a*b)

mupad [B] time = 2.87, size = 1578, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x)

[Out] (x^(5/2)*(A*b - B*a))/(3*a*b*(a + b*x^3)) - (atan((((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3)))/(18*(-a)^(7/6)*b^(11/6))))*1i)/(324*(-a)^(7/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3)))/(18*(-a)^(7/6)*b^(11/6))))*1i)/(324*(-a)^(7/3)*b^(11/3)))/((((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3)))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3)))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3))

$$\begin{aligned}
& 3^{(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A* \\
& B*a^2*b^3))/(18*(-a)^{(7/6)*b^{(11/6)}})/(324*(-a)^{(7/3)*b^{(11/3)}}))*((3^{(1/2) \\
&)*1i)/2 - 1/2)*(A*b + 5*B*a)*1i)/(9*(-a)^{(7/6)*b^{(11/6)}} - (\operatorname{atan}((((3^{(1/2) \\
&)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B \\
& ^2*a^2*b + 20*A^2*B*a*b^2 - (x^{(1/2)}*((3^{(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(\\
& 24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^{(7/6)*b^{(11/6)}}) \\
&)*1i)/(324*(-a)^{(7/3)*b^{(11/3)}} - (((3^{(1/2)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2 \\
& *((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^{(\\
& 1/2)*((3^{(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + \\
& 240*A*B*a^2*b^3))/(18*(-a)^{(7/6)*b^{(11/6)}}))*1i)/(324*(-a)^{(7/3)*b^{(11/3)}}) \\
& /((((3^{(1/2)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/ \\
& 3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 - (x^{(1/2)}*((3^{(1/2)*1i)/2 + 1/2)*(A*b \\
& + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^{(7/6) \\
&)*b^{(11/6)})))/(324*(-a)^{(7/3)*b^{(11/3)}} + (((3^{(1/2)*1i)/2 + 1/2)^2*(A*b + \\
& 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^ \\
& 2 + (x^{(1/2)}*((3^{(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a \\
& ^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^{(7/6)*b^{(11/6)}})))/(324*(-a)^{(7/3)*b^{(11 \\
& /3)}}))*((3^{(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*1i)/(9*(-a)^{(7/6)*b^{(11/6)}} - (\\
& \operatorname{atan}(((A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + \\
& 20*A^2*B*a*b^2 - (x^{(1/2)}*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + \\
& 240*A*B*a^2*b^3))/(18*(-a)^{(7/6)*b^{(11/6)}}))*1i)/(324*(-a)^{(7/3)*b^{(11/3)}} - \\
& ((A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A \\
& ^2*B*a*b^2 + (x^{(1/2)}*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A \\
& *B*a^2*b^3))/(18*(-a)^{(7/6)*b^{(11/6)}}))*1i)/(324*(-a)^{(7/3)*b^{(11/3)}})/(((A* \\
& b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B* \\
& a*b^2 - (x^{(1/2)}*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^ \\
& 2*b^3))/(18*(-a)^{(7/6)*b^{(11/6)}})))/(324*(-a)^{(7/3)*b^{(11/3)}} + ((A*b + 5*B* \\
& a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + \\
& (x^{(1/2)}*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/ \\
& (18*(-a)^{(7/6)*b^{(11/6)}})))/(324*(-a)^{(7/3)*b^{(11/3)}}))*((A*b + 5*B*a)*1i)/(9 \\
& *(-a)^{(7/6)*b^{(11/6)}})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**2,x)

[Out] Timed out

$$3.162 \quad \int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=71

$$\frac{(aB + Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}} + \frac{x^{3/2}(Ab - aB)}{3ab(a + bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {457, 329, 275, 205}

$$\frac{(aB + Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}} + \frac{x^{3/2}(Ab - aB)}{3ab(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] ((A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 329

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x} (A + Bx^3)}{(a + bx^3)^2} dx &= \frac{(Ab - aB)x^{3/2}}{3ab(a + bx^3)} + \frac{(Ab + aB) \int \frac{\sqrt{x}}{a+bx^3} dx}{2ab} \\ &= \frac{(Ab - aB)x^{3/2}}{3ab(a + bx^3)} + \frac{(Ab + aB) \text{Subst}\left(\int \frac{x^2}{a+bx^6} dx, x, \sqrt{x}\right)}{ab} \\ &= \frac{(Ab - aB)x^{3/2}}{3ab(a + bx^3)} + \frac{(Ab + aB) \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, x^{3/2}\right)}{3ab} \\ &= \frac{(Ab - aB)x^{3/2}}{3ab(a + bx^3)} + \frac{(Ab + aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 1.00

$$\frac{(aB + Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}} + \frac{x^{3/2}(Ab - aB)}{3ab(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2, x]

[Out] ((A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))

IntegrateAlgebraic [A] time = 0.10, size = 71, normalized size = 1.00

$$\frac{(aB + Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}} - \frac{x^{3/2}(aB - Ab)}{3ab(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] $-1/3*((-(A*b) + a*B)*x^{(3/2)})/(a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x^{(3/2)})/Sqrt[a]])/(3*a^{(3/2)}*b^{(3/2)})$

fricas [A] time = 0.80, size = 190, normalized size = 2.68

$$\left[\frac{2(Ba^2b - Aab^2)x^{\frac{3}{2}} + ((Bab + Ab^2)x^3 + Ba^2 + Aab)\sqrt{-ab} \log\left(\frac{bx^3 - 2\sqrt{-ab}x^{\frac{3}{2}} - a}{bx^3 + a}\right)}{6(a^2b^3x^3 + a^3b^2)}, -\frac{(Ba^2b - Aab^2)x^{\frac{3}{2}} - ((Bab + Ab^2)x^3 + Ba^2 + Aab)\sqrt{ab} \arctan\left(\frac{\sqrt{ab}x^{\frac{3}{2}}}{a}\right)}{3(a^2b^3x^3 + a^3b^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] $[-1/6*(2*(B*a^2*b - A*a*b^2)*x^{(3/2)} + ((B*a*b + A*b^2)*x^3 + B*a^2 + A*a*b)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^{(3/2)} - a)/(b*x^3 + a)))/(a^2*b^3*x^3 + a^3*b^2), -1/3*((B*a^2*b - A*a*b^2)*x^{(3/2)} - ((B*a*b + A*b^2)*x^3 + B*a^2 + A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^{(3/2)}/a))/(a^2*b^3*x^3 + a^3*b^2)]$

giac [A] time = 0.28, size = 63, normalized size = 0.89

$$\frac{(Ba + Ab) \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}ab} - \frac{Bax^{\frac{3}{2}} - Abx^{\frac{3}{2}}}{3(bx^3 + a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="giac")

[Out] $1/3*(B*a + A*b)*arctan(b*x^{(3/2)}/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/3*(B*a*x^{(3/2)} - A*b*x^{(3/2)})/((b*x^3 + a)*a*b)$

maple [A] time = 0.06, size = 74, normalized size = 1.04

$$\frac{A \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}a} + \frac{B \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}b} + \frac{(Ab - Ba)x^{\frac{3}{2}}}{3(bx^3 + a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x)

[Out] $1/3*(A*b-B*a)*x^{(3/2)}/a/b/(b*x^3+a)+1/3/a/(a*b)^{(1/2)}*\arctan(1/(a*b)^{(1/2)}*b*x^{(3/2)})*A+1/3/b/(a*b)^{(1/2)}*\arctan(1/(a*b)^{(1/2)}*b*x^{(3/2)})*B$

maxima [A] time = 1.40, size = 61, normalized size = 0.86

$$-\frac{(Ba - Ab)x^{\frac{3}{2}}}{3(ab^2x^3 + a^2b)} + \frac{(Ba + Ab) \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] $-1/3*(B*a - A*b)*x^{(3/2)}/(a*b^2*x^3 + a^2*b) + 1/3*(B*a + A*b)*\arctan(b*x^{(3/2)}/\sqrt{a*b})/(\sqrt{a*b}*a*b)$

mupad [B] time = 0.14, size = 115, normalized size = 1.62

$$\frac{B a^2 \operatorname{atan}\left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}}\right) + A b^2 x^3 \operatorname{atan}\left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}}\right) + A a b \operatorname{atan}\left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}}\right) + A \sqrt{a} b^{3/2} x^{3/2} - B a^{3/2} \sqrt{b} x^{3/2} + B a b x^3 \operatorname{atan}\left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}}\right)}{3 a^{5/2} b^{3/2} + 3 a^{3/2} b^{5/2} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(1/2)*(A + B*x^3))/(a + b*x^3)^2,x)`

[Out] $(B*a^2*\operatorname{atan}((b^{(1/2)}*x^{(3/2)})/a^{(1/2)}) + A*b^2*x^3*\operatorname{atan}((b^{(1/2)}*x^{(3/2)})/a^{(1/2)}) + A*a*b*\operatorname{atan}((b^{(1/2)}*x^{(3/2)})/a^{(1/2)}) + A*a^{(1/2)}*b^{(3/2)}*x^{(3/2)} - B*a^{(3/2)}*b^{(1/2)}*x^{(3/2)} + B*a*b*x^3*\operatorname{atan}((b^{(1/2)}*x^{(3/2)})/a^{(1/2)}))/ (3*a^{(5/2)}*b^{(3/2)} + 3*a^{(3/2)}*b^{(5/2)}*x^3)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)*x**(1/2)/(b*x**3+a)**2,x)`

[Out] Timed out

$$3.163 \quad \int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)^2} dx$$

Optimal. Leaf size=289

$$\frac{(aB + 5Ab) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{11/6} b^{7/6}} + \frac{(aB + 5Ab) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{11/6} b^{7/6}} - \frac{(aB + 5Ab)}{3ab(a + bx^3)}$$

Rubi [A] time = 0.47, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {457, 329, 209, 634, 618, 204, 628, 205}

$$\frac{(aB + 5Ab) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{11/6} b^{7/6}} + \frac{(aB + 5Ab) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{11/6} b^{7/6}} - \frac{(aB + 5Ab) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{a}}{\sqrt[6]{b}}\right)}{18a^{11/6} b^{7/6}} + \frac{(aB + 5Ab) \tan^{-1}\left(\frac{2\sqrt[6]{a}}{\sqrt[6]{b}} + \sqrt{3}\right)}{18a^{11/6} b^{7/6}} + \frac{(aB + 5Ab) \tan^{-1}\left(\frac{\sqrt[6]{a}}{\sqrt[6]{b}}\right)}{9a^{11/6} b^{7/6}} + \frac{\sqrt{x}(Ab - aB)}{3ab(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2), x]

[Out] ((A*b - a*B)*Sqrt[x])/(3*a*b*(a + b*x^3)) - ((5*A*b + a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)]/(18*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)]/(18*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(9*a^(11/6)*b^(7/6)) - ((5*A*b + a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(11/6)*b^(7/6))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 209

Int[((a_) + (b_.)*(x_)^(n_))^(n_+1), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*k - 1)*Pi)/n]*x])/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]

x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}], x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a/b]

Rule 329

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 457

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ((!IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{\sqrt{x} (a + bx^3)^2} dx &= \frac{(Ab - aB)\sqrt{x}}{3ab(a + bx^3)} + \frac{\left(\frac{5Ab}{2} + \frac{aB}{2}\right) \int \frac{1}{\sqrt{x}(a+bx^3)} dx}{3ab} \\
&= \frac{(Ab - aB)\sqrt{x}}{3ab(a + bx^3)} + \frac{\left(2\left(\frac{5Ab}{2} + \frac{aB}{2}\right)\right) \text{Subst}\left(\int \frac{1}{a+bx^6} dx, x, \sqrt{x}\right)}{3ab} \\
&= \frac{(Ab - aB)\sqrt{x}}{3ab(a + bx^3)} + \frac{(5Ab + aB) \text{Subst}\left(\int \frac{\sqrt[6]{a} - \frac{1}{2}\sqrt{3} \sqrt[6]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{9a^{11/6}b} + \frac{(5Ab + aB) \text{Subst}\left(\int \frac{1}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{12\sqrt{3} a^{11/6} b^{7/6}} \\
&= \frac{(Ab - aB)\sqrt{x}}{3ab(a + bx^3)} + \frac{(5Ab + aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6} b^{7/6}} - \frac{(5Ab + aB) \text{Subst}\left(\int \frac{-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} + 2\sqrt[3]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{12\sqrt{3} a^{11/6} b^{7/6}} \\
&= \frac{(Ab - aB)\sqrt{x}}{3ab(a + bx^3)} + \frac{(5Ab + aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6} b^{7/6}} - \frac{(5Ab + aB) \log\left(\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{11/6} b^{7/6}} \\
&= \frac{(Ab - aB)\sqrt{x}}{3ab(a + bx^3)} - \frac{(5Ab + aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6} b^{7/6}} + \frac{(5Ab + aB) \tan^{-1}\left(\sqrt{3} + \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6} b^{7/6}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.24

$$\frac{\sqrt{x} \left((a + bx^3) (aB + 5Ab) {}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; -\frac{bx^3}{a}\right) + a(Ab - aB) \right)}{3a^2b(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2), x]

[Out] (Sqrt[x]*(a*(A*b - a*B) + (5*A*b + a*B)*(a + b*x^3)*Hypergeometric2F1[1/6, 1, 7/6, -(b*x^3)/a]))/(3*a^2*b*(a + b*x^3))

IntegrateAlgebraic [A] time = 0.57, size = 190, normalized size = 0.66

$$\frac{(aB + 5Ab) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}} + \frac{(-aB - 5Ab) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b} x}{\sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}\right)}{18a^{11/6}b^{7/6}} + \frac{(aB + 5Ab) \tanh^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{b} x}\right)}{6\sqrt{3} a^{11/6} b^{7/6}} - \frac{\sqrt{x} (aB - Ab)}{3ab(a + bx^3)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

```
[Out] -1/3*((-(A*b) + a*B)*Sqrt[x])/(a*b*(a + b*x^3)) + ((5*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(9*a^(11/6)*b^(7/6)) + ((-5*A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x]])/(18*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(6*Sqrt[3]*a^(11/6)*b^(7/6))
```

fricas [B] time = 1.07, size = 2555, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/36*(4*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7)))^(1/3) + (B^2*a^2 + 10*A*B*a*b + 25*A^2*b^2)*x + (B*a^3*b + 5*A*a^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7)))^(1/6))*a^9*b^6*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7)))^(5/6) - 2*sqrt(3)*(B*a^10*b^6 + 5*A*a^9*b^7)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7)))^(5/6) + sqrt(3)*(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)) + 4*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7)))^(1/3) + (B^2*a^2 + 10*A*B*a*b + 25*A^2*b^2)*x - (B*a^3*b + 5*A*a^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7)))^(1/6))*a^9*b^6*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7)))^(5/6) - 2*sqrt(3)*(B*a^10*b^6 + 5*A*a^9*b^7)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7)))
```


$$\begin{aligned} & \left(\frac{5}{6} - \sqrt{3} \right) \cdot (B^6 a^6 + 30 A B^5 a^5 b + 375 A^2 B^4 a^4 b^2 + 2500 A^3 B^3 a^3 b^3 + 9375 A^4 B^2 a^2 b^4 + 18750 A^5 B a b^5 + 15625 A^6 b^6) / \\ & (B^6 a^6 + 30 A B^5 a^5 b + 375 A^2 B^4 a^4 b^2 + 2500 A^3 B^3 a^3 b^3 + 9375 A^4 B^2 a^2 b^4 + 18750 A^5 B a b^5 + 15625 A^6 b^6) + (a b^2 x^3 + a^2 b) \cdot \\ & (- (B^6 a^6 + 30 A B^5 a^5 b + 375 A^2 B^4 a^4 b^2 + 2500 A^3 B^3 a^3 b^3 + 9375 A^4 B^2 a^2 b^4 + 18750 A^5 B a b^5 + 15625 A^6 b^6) / (a^{11} b^7))^{(1/6)} \cdot \\ & \log(a^4 b^2 \cdot (- (B^6 a^6 + 30 A B^5 a^5 b + 375 A^2 B^4 a^4 b^2 + 2500 A^3 B^3 a^3 b^3 + 9375 A^4 B^2 a^2 b^4 + 18750 A^5 B a b^5 + 15625 A^6 b^6) / \\ & (a^{11} b^7))^{(1/3)} + (B^2 a^2 + 10 A B a b + 25 A^2 b^2) \cdot x + (B a^3 b + 5 A a^2 b^2) \cdot \sqrt{x} \cdot (- (B^6 a^6 + 30 A B^5 a^5 b + 375 A^2 B^4 a^4 b^2 + 2500 A^3 B^3 a^3 b^3 + \\ & 9375 A^4 B^2 a^2 b^4 + 18750 A^5 B a b^5 + 15625 A^6 b^6) / (a^{11} b^7))^{(1/6)} - (a b^2 x^3 + a^2 b) \cdot (- (B^6 a^6 + 30 A B^5 a^5 b + 375 A^2 B^4 a^4 b^2 + 2500 A^3 B^3 a^3 b^3 + \\ & 9375 A^4 B^2 a^2 b^4 + 18750 A^5 B a b^5 + 15625 A^6 b^6) / (a^{11} b^7))^{(1/6)} \cdot \log(a^4 b^2 \cdot (- (B^6 a^6 + 30 A B^5 a^5 b + 375 A^2 B^4 a^4 b^2 + 2500 A^3 B^3 a^3 b^3 + \\ & 9375 A^4 B^2 a^2 b^4 + 18750 A^5 B a b^5 + 15625 A^6 b^6) / (a^{11} b^7))^{(1/3)} + (B^2 a^2 + 10 A B a b + 25 A^2 b^2) \cdot x - (B a^3 b + 5 A a^2 b^2) \cdot \sqrt{x} \cdot (- (B^6 a^6 + 30 A B^5 a^5 b + \\ & 375 A^2 B^4 a^4 b^2 + 2500 A^3 B^3 a^3 b^3 + 9375 A^4 B^2 a^2 b^4 + 18750 A^5 B a b^5 + 15625 A^6 b^6) / (a^{11} b^7))^{(1/6)} + 2 \cdot (a b^2 x^3 + a^2 b) \cdot \\ & (- (B^6 a^6 + 30 A B^5 a^5 b + 375 A^2 B^4 a^4 b^2 + 2500 A^3 B^3 a^3 b^3 + 9375 A^4 B^2 a^2 b^4 + 18750 A^5 B a b^5 + 15625 A^6 b^6) / (a^{11} b^7))^{(1/6)} \cdot \log(a^2 b \cdot (- (B^6 a^6 + 30 A B^5 a^5 b + \\ & 375 A^2 B^4 a^4 b^2 + 2500 A^3 B^3 a^3 b^3 + 9375 A^4 B^2 a^2 b^4 + 18750 A^5 B a b^5 + 15625 A^6 b^6) / (a^{11} b^7))^{(1/6)} + (B a + 5 A b) \cdot \sqrt{x} - 2 \cdot (a b^2 x^3 + a^2 b) \cdot (- (B^6 a^6 + \\ & 30 A B^5 a^5 b + 375 A^2 B^4 a^4 b^2 + 2500 A^3 B^3 a^3 b^3 + 9375 A^4 B^2 a^2 b^4 + 18750 A^5 B a b^5 + 15625 A^6 b^6) / (a^{11} b^7))^{(1/6)} \cdot \log(- a^2 b \cdot (- (B^6 a^6 + 30 A B^5 a^5 b + \\ & 375 A^2 B^4 a^4 b^2 + 2500 A^3 B^3 a^3 b^3 + 9375 A^4 B^2 a^2 b^4 + 18750 A^5 B a b^5 + 15625 A^6 b^6) / (a^{11} b^7))^{(1/6)} + (B a + 5 A b) \cdot \sqrt{x} - 12 \cdot (B a - A b) \cdot \sqrt{x} / (a b^2 x^3 + a^2 b) \end{aligned}$$

giac [A] time = 0.31, size = 302, normalized size = 1.04

$$\frac{\sqrt{3} \left((ab)^{\frac{1}{2}} B a + 5 (ab)^{\frac{1}{2}} A b \right) \log \left(\sqrt{3} \sqrt{\frac{a}{b}} \left(\frac{a}{b} \right)^{\frac{1}{2}} + x + \left(\frac{a}{b} \right)^{\frac{1}{2}} \right) - \sqrt{3} \left((ab)^{\frac{1}{2}} B a + 5 (ab)^{\frac{1}{2}} A b \right) \log \left(-\sqrt{3} \sqrt{\frac{a}{b}} \left(\frac{a}{b} \right)^{\frac{1}{2}} + x + \left(\frac{a}{b} \right)^{\frac{1}{2}} \right)}{36 a^2 b^2} + \frac{B a \sqrt{x} - A b \sqrt{x}}{3 (b x^3 + a) a b} + \frac{\left((ab)^{\frac{1}{2}} B a + 5 (ab)^{\frac{1}{2}} A b \right) \arctan \left(\frac{\sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} + 2 \sqrt{x}}{\left(\frac{a}{b} \right)^{\frac{1}{2}}} \right)}{18 a^2 b^2} + \frac{\left((ab)^{\frac{1}{2}} B a + 5 (ab)^{\frac{1}{2}} A b \right) \arctan \left(-\frac{\sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} + 2 \sqrt{x}}{\left(\frac{a}{b} \right)^{\frac{1}{2}}} \right)}{18 a^2 b^2} + \frac{\left((ab)^{\frac{1}{2}} B a + 5 (ab)^{\frac{1}{2}} A b \right) \arctan \left(\frac{\sqrt{x}}{\left(\frac{a}{b} \right)^{\frac{1}{2}}} \right)}{9 a^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="giac")

[Out] 1/36*sqrt(3)*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^2) - 1/36*sqrt(3)*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^2) - 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a*b) + 1/18*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^2) + 1/18*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(-sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^2) + 1/9*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b^2)

maple [A] time = 0.16, size = 387, normalized size = 1.34

$$\frac{5 \left(\frac{1}{9a^2}\right)^{\frac{1}{2}} \arctan\left(\frac{\sqrt{a}}{\sqrt{b}}\right) - 5 \left(\frac{1}{18a^2}\right)^{\frac{1}{2}} \arctan\left(\frac{2\sqrt{a} - \sqrt{b}}{\sqrt{b}}\right) + 5 \left(\frac{1}{18a^2}\right)^{\frac{1}{2}} \arctan\left(\frac{2\sqrt{a} + \sqrt{b}}{\sqrt{b}}\right) + 5\sqrt{3} \left(\frac{1}{36a^2}\right)^{\frac{1}{2}} \operatorname{Atan}\left(\frac{1}{1+\sqrt{3}} \left(\frac{1}{\sqrt{b}}\sqrt{x} + \left(\frac{1}{b}\right)^{\frac{1}{2}}\right)\right) - 5\sqrt{3} \left(\frac{1}{36a^2}\right)^{\frac{1}{2}} \operatorname{Atan}\left(\frac{1}{1+\sqrt{3}} \left(\frac{1}{\sqrt{b}}\sqrt{x} - \left(\frac{1}{b}\right)^{\frac{1}{2}}\right)\right) + \left(\frac{1}{9ab}\right)^{\frac{1}{2}} \arctan\left(\frac{\sqrt{a}}{\sqrt{b}}\right) + \left(\frac{1}{18ab}\right)^{\frac{1}{2}} \arctan\left(\frac{2\sqrt{a} - \sqrt{b}}{\sqrt{b}}\right) + \left(\frac{1}{18ab}\right)^{\frac{1}{2}} \arctan\left(\frac{2\sqrt{a} + \sqrt{b}}{\sqrt{b}}\right) + \sqrt{3} \left(\frac{1}{36ab}\right)^{\frac{1}{2}} \operatorname{Atan}\left(\frac{1}{1+\sqrt{3}} \left(\frac{1}{\sqrt{b}}\sqrt{x} + \left(\frac{1}{b}\right)^{\frac{1}{2}}\right)\right) - \sqrt{3} \left(\frac{1}{36ab}\right)^{\frac{1}{2}} \operatorname{Atan}\left(\frac{1}{1+\sqrt{3}} \left(\frac{1}{\sqrt{b}}\sqrt{x} - \left(\frac{1}{b}\right)^{\frac{1}{2}}\right)\right) + \frac{(Ab - Ba)\sqrt{a}}{3(b^2x^3 + a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x)`

[Out] $\frac{1}{3}*(A*b-B*a)*x^{(1/2)}/a/b/(b*x^3+a)+5/9/a^2*(a/b)^{(1/6)}*\arctan(1/(a/b)^{(1/6)})*x^{(1/2)})*A+1/9/b/a*(a/b)^{(1/6)}*\arctan(1/(a/b)^{(1/6)})*x^{(1/2)})*B-5/36/a^2*3^{(1/2)}*(a/b)^{(1/6)}*\ln(-x+3^{(1/2)}*(a/b)^{(1/6)})*x^{(1/2)}-(a/b)^{(1/3)})*A-1/36/b/a*3^{(1/2)}*(a/b)^{(1/6)}*\ln(-x+3^{(1/2)}*(a/b)^{(1/6)})*x^{(1/2)}-(a/b)^{(1/3)})*B+5/18/a^2*(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)})*x^{(1/2)}-3^{(1/2)})*A+1/18/b/a*(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)})*x^{(1/2)}-3^{(1/2)})*B+5/36/a^2*3^{(1/2)}*(a/b)^{(1/6)}*\ln(x+3^{(1/2)}*(a/b)^{(1/6)})*x^{(1/2)}+(a/b)^{(1/3)})*A+1/36/b/a*3^{(1/2)}*(a/b)^{(1/6)}*\ln(x+3^{(1/2)}*(a/b)^{(1/6)})*x^{(1/2)}+(a/b)^{(1/3)})*B+5/18/a^2*(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)})*x^{(1/2)}+3^{(1/2)})*A+1/18/b/a*(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)})*x^{(1/2)}+3^{(1/2)})*B$

maxima [A] time = 1.20, size = 301, normalized size = 1.04

$$\frac{(Ba - Ab)\sqrt{x}}{3(ab^2x^3 + a^2b)} + \frac{\sqrt{3}(Ba+5Ab)\log\left(\sqrt[5]{3a^6b^6}\sqrt{x} + b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right) - \sqrt{3}(Ba+5Ab)\log\left(-\sqrt[5]{3a^6b^6}\sqrt{x} + b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{5}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}+5Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt[5]{a^3b^3}}\right)}{36ab} + \frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}+5Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt[5]{3a^6b^6}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt[5]{a^3b^3}}\right)}{ab^{\frac{1}{3}}\sqrt[5]{a^3b^3}} + \frac{2\left(Ba^{\frac{1}{3}}b^{\frac{4}{3}}+5Aa^{\frac{4}{3}}b^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt[5]{3a^6b^6}-2b^{\frac{1}{3}}\sqrt{x}}{\sqrt[5]{a^3b^3}}\right)}{ab^{\frac{1}{3}}\sqrt[5]{a^3b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="maxima")`

[Out] $-1/3*(B*a - A*b)*\sqrt{x}/(a*b^2*x^3 + a^2*b) + 1/36*(\sqrt{3}*(B*a + 5*A*b)*\log(\sqrt{3}*a^{(1/6)}*b^{(1/6)}*\sqrt{x} + b^{(1/3)}*x + a^{(1/3)})/(a^{(5/6)}*b^{(1/6)}) - \sqrt{3}*(B*a + 5*A*b)*\log(-\sqrt{3}*a^{(1/6)}*b^{(1/6)}*\sqrt{x} + b^{(1/3)}*x + a^{(1/3)})/(a^{(5/6)}*b^{(1/6)}) + 4*(B*a*b^{(1/3)} + 5*A*b^{(4/3)})*\arctan(b^{(1/3)}*\sqrt{x}/\sqrt{a^{(1/3)}*b^{(1/3)}})/(a^{(2/3)}*b^{(1/3)}*\sqrt{a^{(1/3)}*b^{(1/3)}}) + 2*(B*a^{(4/3)}*b^{(1/3)} + 5*A*a^{(1/3)}*b^{(4/3)})*\arctan((\sqrt{3}*a^{(1/6)}*b^{(1/6)} + 2*b^{(1/3)}*\sqrt{x})/\sqrt{a^{(1/3)}*b^{(1/3)}})/(a*b^{(1/3)}*\sqrt{a^{(1/3)}*b^{(1/3)}}) + 2*(B*a^{(4/3)}*b^{(1/3)} + 5*A*a^{(1/3)}*b^{(4/3)})*\arctan(-(\sqrt{3}*a^{(1/6)}*b^{(1/6)} - 2*b^{(1/3)}*\sqrt{x})/\sqrt{a^{(1/3)}*b^{(1/3)}})/(a*b^{(1/3)}*\sqrt{a^{(1/3)}*b^{(1/3)}})))/(a*b)$

mupad [B] time = 2.92, size = 1922, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)/(x^(1/2)*(a + b*x^3)^2),x)`

$$\begin{aligned} & \sqrt{\frac{625A^4b^5 + B^4a^4b + 150A^2B^2a^2b^3 + 500A^3Bab^4 + 20AB^3a^3b^2}{27a^4}} + \frac{2\left(\sqrt{3}i/2 + 1/2\right)\left(5Ab + Ba\right)\left(125A^3b^5 + B^3a^3b^2 + 75A^2Bab^4 + 15AB^2a^2b^3\right)}{27(-a)^{23/6}b^{7/6}} \\ & \left. \right) \frac{2\left(\sqrt{3}i/2 + 1/2\right)\left(5Ab + Ba\right)}{18(-a)^{11/6}b^{7/6}} + \frac{2\left(\sqrt{3}i/2 + 1/2\right)\left(5Ab + Ba\right)}{9(-a)^{11/6}b^{7/6}} + \frac{x\sqrt{Ab - Ba}}{3ab(a + bx^3)} \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/(b*x**3+a)**2/x**(1/2),x)

[Out] Timed out

$$3.164 \quad \int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^2} dx$$

Optimal. Leaf size=318

$$\frac{(7Ab - aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{13/6} b^{5/6}} + \frac{(7Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{13/6} b^{5/6}} + \frac{(7Ab - aB) \tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x}{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x}\right)}{18a^{13/6} b^{5/6}}$$

Rubi [A] time = 0.69, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {457, 325, 329, 295, 634, 618, 204, 628, 205}

$$\frac{(7Ab - aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{13/6} b^{5/6}} + \frac{(7Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{13/6} b^{5/6}} + \frac{(7Ab - aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt{6}\sqrt{x}}{\sqrt{a}}\right)}{18a^{13/6} b^{5/6}} - \frac{(7Ab - aB) \tan^{-1}\left(\frac{2\sqrt{6}\sqrt{x}}{\sqrt{a}} + \sqrt{3}\right)}{18a^{13/6} b^{5/6}} - \frac{(7Ab - aB) \tan^{-1}\left(\frac{\sqrt{6}\sqrt{x}}{\sqrt{a}}\right)}{9a^{13/6} b^{5/6}} - \frac{7Ab - aB}{3a^2 b \sqrt{x}} + \frac{Ab - aB}{3ab \sqrt{x} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2), x]

[Out] $-(7Ab - aB)/(3a^2 b \sqrt{x}) + (Ab - aB)/(3a^2 b \sqrt{x} (a + b x^3)) + ((7Ab - aB) \operatorname{ArcTan}[\sqrt{3} - (2b^{1/6} \sqrt{x})/a^{1/6}]) / (18a^{13/6} b^{5/6}) - ((7Ab - aB) \operatorname{ArcTan}[\sqrt{3} + (2b^{1/6} \sqrt{x})/a^{1/6}]) / (18a^{13/6} b^{5/6}) - ((7Ab - aB) \operatorname{ArcTan}[(b^{1/6} \sqrt{x})/a^{1/6}]) / (9a^{13/6} b^{5/6}) - ((7Ab - aB) \operatorname{Log}[a^{1/3} - \sqrt{3} a^{1/6} b^{1/6} \sqrt{x} + b^{1/3} x]) / (12\sqrt{3} a^{13/6} b^{5/6}) + ((7Ab - aB) \operatorname{Log}[a^{1/3} + \sqrt{3} a^{1/6} b^{1/6} \sqrt{x} + b^{1/3} x]) / (12\sqrt{3} a^{13/6} b^{5/6})$

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 295

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*cos[((2*k - 1)*m*Pi)/n] - s*cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*cos[((2*k

$- 1) \cdot \pi) / n] \cdot x + s^2 \cdot x^2), x] + \text{Int}[(r \cdot \cos[((2 \cdot k - 1) \cdot m \cdot \pi) / n] + s \cdot \cos[((2 \cdot k - 1) \cdot (m + 1) \cdot \pi) / n] \cdot x) / (r^2 + 2 \cdot r \cdot s \cdot \cos[((2 \cdot k - 1) \cdot \pi) / n] \cdot x + s^2 \cdot x^2), x]$
 $; (2 \cdot (-1)^{(m/2)} \cdot r^{(m+2)} \cdot \text{Int}[1 / (r^2 + s^2 \cdot x^2), x]) / (a \cdot n \cdot s^m) + \text{Dist}[(2 \cdot r^{(m+1)}) / (a \cdot n \cdot s^m), \text{Sum}[u, \{k, 1, (n-2)/4\}], x], x] /;$ FreeQ[{a, b}, x] && IGtQ[(n-2)/4, 0] && IGtQ[m, 0] && LtQ[m, n-1] && PosQ[a/b]

Rule 325

$\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^p, x_Symbol] := \text{Simp}[(c \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} / (a \cdot c \cdot (m+1)), x] - \text{Dist}[(b \cdot (m+n \cdot (p+1) + 1)) / (a \cdot c^n \cdot (m+1)), \text{Int}[(c \cdot x)^{m+n} \cdot (a + b \cdot x^n)^p, x], x] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

$\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^p, x_Symbol] := \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{k \cdot (m+1)} - 1] \cdot (a + (b \cdot x^{k \cdot n})) / c^n)^p, x], x, (c \cdot x)^{1/k}], x] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 457

$\text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^p \cdot (c + d \cdot x^n), x_Symbol] := -\text{Simp}[(b \cdot c - a \cdot d) \cdot (e \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} / (a \cdot b \cdot e \cdot n \cdot (p+1)), x] - \text{Dist}[(a \cdot d \cdot (m+1) - b \cdot c \cdot (m+n \cdot (p+1) + 1)) / (a \cdot b \cdot n \cdot (p+1)), \text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^{p+1}, x], x] /;$ FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b \cdot c - a \cdot d, 0] && LtQ[p, -1] && (!IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n \cdot (p+1))])

Rule 618

$\text{Int}[(a + b \cdot x + c \cdot x^2)^{-1}, x_Symbol] := \text{Dist}[-2, \text{Subst}[\text{Int}[1 / \text{Simp}[b^2 - 4 \cdot a \cdot c - x^2, x], x], x, b + 2 \cdot c \cdot x], x] /;$ FreeQ[{a, b, c}, x] && NeQ[b^2 - 4 \cdot a \cdot c, 0]

Rule 628

$\text{Int}[(d + e \cdot x) / (a + b \cdot x + c \cdot x^2), x_Symbol] := \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[2 \cdot c \cdot d - b \cdot e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^{3/2}(a + bx^3)^2} dx &= \frac{Ab - aB}{3ab\sqrt{x}(a + bx^3)} + \frac{\left(\frac{7Ab}{2} - \frac{aB}{2}\right) \int \frac{1}{x^{3/2}(a+bx^3)} dx}{3ab} \\
&= -\frac{7Ab - aB}{3a^2b\sqrt{x}} + \frac{Ab - aB}{3ab\sqrt{x}(a + bx^3)} - \frac{(7Ab - aB) \int \frac{x^{3/2}}{a+bx^3} dx}{6a^2} \\
&= -\frac{7Ab - aB}{3a^2b\sqrt{x}} + \frac{Ab - aB}{3ab\sqrt{x}(a + bx^3)} - \frac{(7Ab - aB) \operatorname{Subst}\left(\int \frac{x^4}{a+bx^6} dx, x, \sqrt{x}\right)}{3a^2} \\
&= -\frac{7Ab - aB}{3a^2b\sqrt{x}} + \frac{Ab - aB}{3ab\sqrt{x}(a + bx^3)} - \frac{(7Ab - aB) \operatorname{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2} + \frac{1}{2}\sqrt{3} \sqrt[6]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{9a^{13/6}b^{2/3}} \\
&= -\frac{7Ab - aB}{3a^2b\sqrt{x}} + \frac{Ab - aB}{3ab\sqrt{x}(a + bx^3)} - \frac{(7Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}} - \frac{(7Ab - aB) \operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{12\sqrt{3} a^{13/6} b^{2/3}} \\
&= -\frac{7Ab - aB}{3a^2b\sqrt{x}} + \frac{Ab - aB}{3ab\sqrt{x}(a + bx^3)} - \frac{(7Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}} - \frac{(7Ab - aB) \log\left(\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2\right)}{12\sqrt{3} a^{13/6} b^{2/3}} \\
&= -\frac{7Ab - aB}{3a^2b\sqrt{x}} + \frac{Ab - aB}{3ab\sqrt{x}(a + bx^3)} + \frac{(7Ab - aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}} - \frac{(7Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}}
\end{aligned}$$

Mathematica [C] time = 0.14, size = 70, normalized size = 0.22

$$\frac{2\left(x^3(aB - Ab) {}_2F_1\left(\frac{5}{6}, 2; \frac{11}{6}; -\frac{bx^3}{a}\right) - Abx^3 {}_2F_1\left(\frac{5}{6}, 1; \frac{11}{6}; -\frac{bx^3}{a}\right) - 5aA\right)}{5a^3\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2), x]

[Out] (2*(-5*a*A - A*b*x^3*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a)] + (-A*b) + a*B)*x^3*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a)])/(5*a^3*Sqrt[x])

IntegrateAlgebraic [A] time = 0.61, size = 197, normalized size = 0.62

$$\frac{(aB - 7Ab) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}} + \frac{(7Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b}x}{\sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}\right)}{18a^{13/6}b^{5/6}} - \frac{(aB - 7Ab) \tanh^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{b}x}\right)}{6\sqrt{3} a^{13/6}b^{5/6}} + \frac{-6aA + aBx^3 - 7Abx^3}{3a^2\sqrt{x}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2), x]

[Out] (-6*a*A - 7*A*b*x^3 + a*B*x^3)/(3*a^2*Sqrt[x]*(a + b*x^3)) + ((-7*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(13/6)*b^(5/6)) + ((7*A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x]])/(18*a^(13/6)*b^(5/6)) - ((-7*A*b + a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(6*Sqrt[3]*a^(13/6)*b^(5/6))

fricas [B] time = 0.97, size = 3798, normalized size = 11.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/36*(4*sqrt(3)*(a^2*b*x^4 + a^3*x)*(-B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt((B^5*a^16*b^4 - 35*A*B^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7 + 12005*A^4*B*a^12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*A*B^9*a^9*b + 2205*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6*a^6*b^4 - 4235364*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^7*B^3*a^3*b^7 + 259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 282475249*A^10*b^10)*x - (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5 - 6860*A^3*B^3*a^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 + 117649*A^6*a^9*b^9)*(-B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(2/3)*a^2*b*(-B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^7*b - 35*A*B^4*a^6*b^2 + 490*A^2*B^3*a^5*b^3 - 3430*A^3*B^2*a^4*b^4 + 12005*A^4*B*a^3*b^5 - 16807*

$$\begin{aligned}
& A^5 a^2 b^6) \sqrt{x} * (- (B^6 a^6 - 42 A B^5 a^5 b + 735 A^2 B^4 a^4 b^2 - 68 \\
& 60 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - 100842 A^5 B a b^5 + 117649 A^6 \\
& b^6) / (a^{13} b^5))^{(1/6)} - \sqrt{3} * (B^6 a^6 - 42 A B^5 a^5 b + 735 A^2 B^4 a^4 \\
& b^2 - 6860 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - 100842 A^5 B a b^5 + 117649 A^6 b^6) / \\
& (B^6 a^6 - 42 A B^5 a^5 b + 735 A^2 B^4 a^4 b^2 - 6860 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - \\
& 100842 A^5 B a b^5 + 117649 A^6 b^6)) + 4 \sqrt{3} * (a^2 b x^4 + a^3 x) * (- (B^6 a^6 - 42 A B^5 a^5 b + 735 A^2 \\
& B^4 a^4 b^2 - 6860 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - 100842 A^5 B a \\
& b^5 + 117649 A^6 b^6) / (a^{13} b^5))^{(1/6)} * \arctan(1/3 * (2 \sqrt{3}) \sqrt{-(B^5 a \\
& ^{16} b^4 - 35 A B^4 a^{15} b^5 + 490 A^2 B^3 a^{14} b^6 - 3430 A^3 B^2 a^{13} b^7 \\
& + 12005 A^4 B a^{12} b^8 - 16807 A^5 a^{11} b^9)}) \sqrt{x} * (- (B^6 a^6 - 42 A B^5 a^5 \\
& b + 735 A^2 B^4 a^4 b^2 - 6860 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - 100842 A^5 B a b^5 + 117649 A^6 b^6) / \\
& (a^{13} b^5))^{(5/6)} + (B^{10} a^{10} - 70 A B^9 a^9 b + 2205 A^2 B^8 a^8 b^2 - 41160 A^3 B^7 a^7 b^3 + 504210 A^4 B^6 \\
& a^6 b^4 - 4235364 A^5 B^5 a^5 b^5 + 24706290 A^6 B^4 a^4 b^6 - 98825160 A^7 B^3 a^3 b^7 + 259416045 A^8 B^2 a^2 b^8 - 403536070 A^9 B a b^9 + 2824752 \\
& 49 A^{10} b^{10}) * x - (B^6 a^{15} b^3 - 42 A B^5 a^{14} b^4 + 735 A^2 B^4 a^{13} b^5 - 6860 A^3 B^3 a^{12} b^6 + 36015 A^4 B^2 a^{11} b^7 - 100842 A^5 B a^{10} b^8 + \\
& 117649 A^6 a^9 b^9) * (- (B^6 a^6 - 42 A B^5 a^5 b + 735 A^2 B^4 a^4 b^2 - 6860 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - 100842 A^5 B a b^5 + 117649 A^6 \\
& b^6) / (a^{13} b^5))^{(2/3)}) * a^2 b * (- (B^6 a^6 - 42 A B^5 a^5 b + 735 A^2 B^4 a^4 \\
& b^2 - 6860 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - 100842 A^5 B a b^5 + 117649 A^6 b^6) / (a^{13} b^5))^{(1/6)} + 2 \sqrt{3} * (B^5 a^7 b - 35 A B^4 a^6 b^2 \\
& + 490 A^2 B^3 a^5 b^3 - 3430 A^3 B^2 a^4 b^4 + 12005 A^4 B a^3 b^5 - 16807 A^5 a^2 b^6) \sqrt{x} * (- (B^6 a^6 - 42 A B^5 a^5 b + 735 A^2 B^4 a^4 b^2 - 6860 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - 100842 A^5 B a b^5 + 117649 A^6 \\
& b^6) / (a^{13} b^5))^{(1/6)} + \sqrt{3} * (B^6 a^6 - 42 A B^5 a^5 b + 735 A^2 B^4 a^4 b^2 - 6860 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - 100842 A^5 B a b^5 + 117649 A^6 b^6) / \\
& (B^6 a^6 - 42 A B^5 a^5 b + 735 A^2 B^4 a^4 b^2 - 6860 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - 100842 A^5 B a b^5 + 117649 A^6 b^6)) - 2 * (a^2 b x^4 + a^3 x) * (- (B^6 a^6 - 42 A B^5 a^5 b + 735 A^2 B^4 a^4 \\
& b^2 - 6860 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - 100842 A^5 B a b^5 + 117649 A^6 b^6) / (a^{13} b^5))^{(1/6)} * \log(a^{11} b^4 * (- (B^6 a^6 - 42 A B^5 a^5 b + 735 A^2 B^4 a^4 b^2 - 6860 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - 100 \\
& 842 A^5 B a b^5 + 117649 A^6 b^6) / (a^{13} b^5))^{(5/6)} - (B^5 a^5 - 35 A B^4 a^4 b + 490 A^2 B^3 a^3 b^2 - 3430 A^3 B^2 a^2 b^3 + 12005 A^4 B a b^4 - 168 \\
& 07 A^5 b^5) \sqrt{x}) + 2 * (a^2 b x^4 + a^3 x) * (- (B^6 a^6 - 42 A B^5 a^5 b + 735 A^2 B^4 a^4 b^2 - 6860 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - 100842 \\
& A^5 B a b^5 + 117649 A^6 b^6) / (a^{13} b^5))^{(1/6)} * \log(-a^{11} b^4 * (- (B^6 a^6 - 42 A B^5 a^5 b + 735 A^2 B^4 a^4 b^2 - 6860 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - 100842 A^5 B a b^5 + 117649 A^6 b^6) / (a^{13} b^5))^{(5/6)} - (B^5 a^5 \\
& - 35 A B^4 a^4 b + 490 A^2 B^3 a^3 b^2 - 3430 A^3 B^2 a^2 b^3 + 12005 A^4 B a b^4 - 16807 A^5 b^5) \sqrt{x}) + (a^2 b x^4 + a^3 x) * (- (B^6 a^6 - 42 A \\
& B^5 a^5 b + 735 A^2 B^4 a^4 b^2 - 6860 A^3 B^3 a^3 b^3 + 36015 A^4 B^2 a^2 b^4 - 100842 A^5 B a b^5 + 117649 A^6 b^6) / (a^{13} b^5))^{(1/6)} * \log((B^5 a^{16}
\end{aligned}$$

$$\begin{aligned}
& *b^4 - 35*A*B^4*a^{15}*b^5 + 490*A^2*B^3*a^{14}*b^6 - 3430*A^3*B^2*a^{13}*b^7 + 12005*A^4*B*a^{12}*b^8 - 16807*A^5*a^{11}*b^9) * \text{sqrt}(x) * (- (B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6) / (a^{13}*b^5))^{(5/6)} + (B^{10}*a^{10} - 70*A*B^9*a^9*b + 2205*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6*a^6*b^4 - 4235364*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^7*B^3*a^3*b^7 + 259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 282475249*A^{10}*b^{10}) * x - (B^6*a^{15}*b^3 - 42*A*B^5*a^{14}*b^4 + 735*A^2*B^4*a^{13}*b^5 - 6860*A^3*B^3*a^{12}*b^6 + 36015*A^4*B^2*a^{11}*b^7 - 100842*A^5*B*a^{10}*b^8 + 117649*A^6*a^9*b^9) * (- (B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6) / (a^{13}*b^5))^{(2/3)} - (a^2*b*x^4 + a^3*x) * (- (B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6) / (a^{13}*b^5))^{(1/6)} * \log(- (B^5*a^{16}*b^4 - 35*A*B^4*a^{15}*b^5 + 490*A^2*B^3*a^{14}*b^6 - 3430*A^3*B^2*a^{13}*b^7 + 12005*A^4*B*a^{12}*b^8 - 16807*A^5*a^{11}*b^9) * \text{sqrt}(x) * (- (B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6) / (a^{13}*b^5))^{(5/6)} + (B^{10}*a^{10} - 70*A*B^9*a^9*b + 2205*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6*a^6*b^4 - 4235364*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^7*B^3*a^3*b^7 + 259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 282475249*A^{10}*b^{10}) * x - (B^6*a^{15}*b^3 - 42*A*B^5*a^{14}*b^4 + 735*A^2*B^4*a^{13}*b^5 - 6860*A^3*B^3*a^{12}*b^6 + 36015*A^4*B^2*a^{11}*b^7 - 100842*A^5*B*a^{10}*b^8 + 117649*A^6*a^9*b^9) * (- (B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6) / (a^{13}*b^5))^{(2/3)} + 12 * ((B*a - 7*A*b) * x^3 - 6*A*a) * \text{sqrt}(x) / (a^2*b*x^4 + a^3*x)
\end{aligned}$$

giac [A] time = 0.47, size = 307, normalized size = 0.97

$$\frac{Bax^3 - 7Abx^2 - 6Aa}{3(bx^2 + a\sqrt{3})^2} - \frac{\sqrt{3}((ab^3)^{\frac{1}{3}}Ba - 7(ab^3)^{\frac{2}{3}}Ab) \log(\sqrt{3}\sqrt{\frac{a}{b}}(\frac{x}{b})^{\frac{1}{3}} + x + (\frac{x}{b})^{\frac{1}{3}})}{36a^3b^5} + \frac{\sqrt{3}((ab^3)^{\frac{1}{3}}Ba - 7(ab^3)^{\frac{2}{3}}Ab) \log(-\sqrt{3}\sqrt{\frac{a}{b}}(\frac{x}{b})^{\frac{1}{3}} + x + (\frac{x}{b})^{\frac{1}{3}})}{36a^3b^5} + \frac{((ab^3)^{\frac{1}{3}}Ba - 7(ab^3)^{\frac{2}{3}}Ab) \arctan(\frac{\sqrt{3}(\frac{x}{b})^{\frac{1}{3}} + 2\sqrt{3}}{(\frac{x}{b})^{\frac{1}{3}})}}{18a^3b^5} + \frac{((ab^3)^{\frac{1}{3}}Ba - 7(ab^3)^{\frac{2}{3}}Ab) \arctan(-\frac{\sqrt{3}(\frac{x}{b})^{\frac{1}{3}} + 2\sqrt{3}}{(\frac{x}{b})^{\frac{1}{3}})}}{18a^3b^5} + \frac{((ab^3)^{\frac{1}{3}}Ba - 7(ab^3)^{\frac{2}{3}}Ab) \arctan(\frac{\sqrt{3}}{(\frac{x}{b})^{\frac{1}{3}})}}{9a^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/3*(B*a*x^3 - 7*A*b*x^3 - 6*A*a)/((b*x^(7/2) + a*sqrt(x))*a^2) - 1/36*sqrt(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^5) + 1/36*sqrt(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^5) + 1/18*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^5) + 1/18*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^5) + 1/9*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^3*b^5)

maple [A] time = 0.16, size = 401, normalized size = 1.26

$$\frac{A b^2}{3(b^2 x^2 + a)^2} - \frac{B x^2}{3(b^2 x^2 + a)^2} - \frac{7 A \arctan\left(\frac{\sqrt{x}}{b}\right)}{9(b^2 x^2 + a)^2} - \frac{7 A \arctan\left(\frac{2\sqrt{x} - \sqrt{3}}{b}\right)}{18(b^2 x^2 + a)^2} - \frac{7 A \arctan\left(\frac{2\sqrt{x} + \sqrt{3}}{b}\right)}{18(b^2 x^2 + a)^2} - \frac{7\sqrt{3} \left(\frac{1}{3}\right)^2 A b \ln\left(x + \sqrt{3} \left(\frac{1}{3}\right)^2 \sqrt{x} + \left(\frac{1}{3}\right)^2\right)}{36 a^2} - \frac{7\sqrt{3} \left(\frac{1}{3}\right)^2 A b \ln\left(-x + \sqrt{3} \left(\frac{1}{3}\right)^2 \sqrt{x} - \left(\frac{1}{3}\right)^2\right)}{36 a^2} - \frac{B \arctan\left(\frac{\sqrt{x}}{b}\right)}{9(b^2 x^2 + a)^2} - \frac{B \arctan\left(\frac{2\sqrt{x} - \sqrt{3}}{b}\right)}{18(b^2 x^2 + a)^2} - \frac{B \arctan\left(\frac{2\sqrt{x} + \sqrt{3}}{b}\right)}{18(b^2 x^2 + a)^2} - \frac{\sqrt{3} \left(\frac{1}{3}\right)^2 B \ln\left(x + \sqrt{3} \left(\frac{1}{3}\right)^2 \sqrt{x} + \left(\frac{1}{3}\right)^2\right)}{36 a^2} - \frac{\sqrt{3} \left(\frac{1}{3}\right)^2 B \ln\left(-x + \sqrt{3} \left(\frac{1}{3}\right)^2 \sqrt{x} - \left(\frac{1}{3}\right)^2\right)}{36 a^2} - \frac{2A}{a^2 \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x)

[Out]
$$-1/3/a^2*x^{5/2}/(b*x^3+a)*A*b+1/3/a*x^{5/2}/(b*x^3+a)*B-7/9/a^2*A/(a/b)^{(1/6)}*\arctan(1/(a/b)^{(1/6)}*x^{(1/2)})-7/36/a^3*A*b*3^{(1/2)}*(a/b)^{(5/6)}*\ln(-x+3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)}-(a/b)^{(1/3)})-7/18/a^2*A/(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)}-3^{(1/2)})+7/36/a^3*A*b*3^{(1/2)}*(a/b)^{(5/6)}*\ln(x+3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)}+(a/b)^{(1/3)})-7/18/a^2*A/(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)}+3^{(1/2)})+1/9/a*B/b/(a/b)^{(1/6)}*\arctan(1/(a/b)^{(1/6)}*x^{(1/2)})+1/36/a^2*B*3^{(1/2)}*(a/b)^{(5/6)}*\ln(-x+3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)}-(a/b)^{(1/3)})+1/18/a*B/b/(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)}-3^{(1/2)})-1/36/a^2*B*3^{(1/2)}*(a/b)^{(5/6)}*\ln(x+3^{(1/2)}*(a/b)^{(1/6)}*x^{(1/2)}+(a/b)^{(1/3)})+1/18/a*B/b/(a/b)^{(1/6)}*\arctan(2/(a/b)^{(1/6)}*x^{(1/2)}+3^{(1/2)})-2*A/a^2/x^{(1/2)}$$

maxima [A] time = 1.16, size = 240, normalized size = 0.75

$$\frac{(Ba - 7Ab)x^3 - 6Aa}{3(a^2bx^2 + a^3\sqrt{x})} - \frac{\sqrt{3} \log\left(\sqrt{3} \frac{1}{a^6 b^6} \sqrt{x} + b^3 x + a^3\right)}{a^6 b^6} - \frac{\sqrt{3} \log\left(-\sqrt{3} \frac{1}{a^6 b^6} \sqrt{x} + b^3 x + a^3\right)}{a^6 b^6} - \frac{2 \arctan\left(\frac{\sqrt{3} \frac{1}{a^6 b^6} + 2b^3 \sqrt{x}}{\sqrt{a^3 b^3}}\right)}{b^3 \sqrt{a^3 b^3}} - \frac{2 \arctan\left(-\frac{\sqrt{3} \frac{1}{a^6 b^6} - 2b^3 \sqrt{x}}{\sqrt{a^3 b^3}}\right)}{b^3 \sqrt{a^3 b^3}} - \frac{4 \arctan\left(\frac{\frac{1}{b^3} \sqrt{x}}{\sqrt{a^3 b^3}}\right)}{b^3 \sqrt{a^3 b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")

[Out]
$$1/3*((B*a - 7*A*b)*x^3 - 6*A*a)/(a^2*b*x^{7/2} + a^3*\sqrt{x}) - 1/36*(B*a - 7*A*b)*(sqrt(3)*log(sqrt(3)*a^{1/6}*b^{1/6}*sqrt(x) + b^{1/3}*x + a^{1/3}))/a^{1/6}*b^{5/6} - sqrt(3)*log(-sqrt(3)*a^{1/6}*b^{1/6}*sqrt(x) + b^{1/3}*x + a^{1/3})/(a^{1/6}*b^{5/6}) - 2*\arctan((sqrt(3)*a^{1/6}*b^{1/6} + 2*b^{1/3}*sqrt(x))/sqrt(a^{1/3}*b^{1/3}))/b^{2/3}*sqrt(a^{1/3}*b^{1/3}) - 2*\arctan((-sqrt(3)*a^{1/6}*b^{1/6} - 2*b^{1/3}*sqrt(x))/sqrt(a^{1/3}*b^{1/3}))/b^{2/3}*sqrt(a^{1/3}*b^{1/3}) - 4*\arctan(b^{1/3}*sqrt(x)/sqrt(a^{1/3}*b^{1/3}))/b^{2/3}*sqrt(a^{1/3}*b^{1/3}))/a^2$$

mupad [B] time = 2.91, size = 1757, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x)

$$17*b^4 - 11907*A^2*B*a^{16}*b^5 + (x^{(1/2)}*((3^{(1/2)}*1i)/2 + 1/2)*(7*A*b - B*a)*(23147208*A^2*a^{17}*b^6 + 472392*B^2*a^{19}*b^4 - 6613488*A*B*a^{18}*b^5))/(5832*(-a)^{(13/6)}*b^{(5/6)})))/((-a)^{(13/3)}*b^{(5/3)}))*((3^{(1/2)}*1i)/2 + 1/2)*(7*A*b - B*a)*1i)/(9*(-a)^{(13/6)}*b^{(5/6)})$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**2,x)

[Out] Timed out

$$3.165 \quad \int \frac{A+Bx^3}{x^{5/2}(a+bx^3)^2} dx$$

Optimal. Leaf size=96

$$-\frac{(3Ab - aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}} + \frac{aB - 3Ab}{3a^2bx^{3/2}} + \frac{Ab - aB}{3abx^{3/2}(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 325, 329, 275, 205}

$$-\frac{3Ab - aB}{3a^2bx^{3/2}} - \frac{(3Ab - aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}} + \frac{Ab - aB}{3abx^{3/2}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2), x]

[Out] -(3*A*b - a*B)/(3*a^2*b*x^(3/2)) + (A*b - a*B)/(3*a*b*x^(3/2)*(a + b*x^3)) - ((3*A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(5/2)*Sqrt[b])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 325

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^(p, x), x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx^3}{x^{5/2} (a + bx^3)^2} dx &= \frac{Ab - aB}{3abx^{3/2} (a + bx^3)} + \frac{\left(\frac{9Ab}{2} - \frac{3aB}{2}\right) \int \frac{1}{x^{5/2}(a+bx^3)} dx}{3ab} \\
 &= -\frac{3Ab - aB}{3a^2bx^{3/2}} + \frac{Ab - aB}{3abx^{3/2} (a + bx^3)} - \frac{(3Ab - aB) \int \frac{\sqrt{x}}{a+bx^3} dx}{2a^2} \\
 &= -\frac{3Ab - aB}{3a^2bx^{3/2}} + \frac{Ab - aB}{3abx^{3/2} (a + bx^3)} - \frac{(3Ab - aB) \operatorname{Subst}\left(\int \frac{x^2}{a+bx^6} dx, x, \sqrt{x}\right)}{a^2} \\
 &= -\frac{3Ab - aB}{3a^2bx^{3/2}} + \frac{Ab - aB}{3abx^{3/2} (a + bx^3)} - \frac{(3Ab - aB) \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, x^{3/2}\right)}{3a^2} \\
 &= -\frac{3Ab - aB}{3a^2bx^{3/2}} + \frac{Ab - aB}{3abx^{3/2} (a + bx^3)} - \frac{(3Ab - aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}
 \end{aligned}$$

Mathematica [A] time = 0.13, size = 79, normalized size = 0.82

$$\frac{\frac{(aB-3Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{\sqrt{a}(-2aA+aBx^3-3Abx^3)}{x^{3/2}(a+bx^3)}}{3a^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2), x]

[Out] ((Sqrt[a]*(-2*a*A - 3*A*b*x^3 + a*B*x^3))/(x^(3/2)*(a + b*x^3)) + ((-3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/Sqrt[b])/(3*a^(5/2))

IntegrateAlgebraic [A] time = 0.11, size = 79, normalized size = 0.82

$$\frac{(aB - 3Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}} + \frac{-2aA + aBx^3 - 3Abx^3}{3a^2x^{3/2}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2), x]

[Out] (-2*a*A - 3*A*b*x^3 + a*B*x^3)/(3*a^2*x^(3/2)*(a + b*x^3)) + ((-3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(5/2)*Sqrt[b])

fricas [A] time = 0.82, size = 232, normalized size = 2.42

$$\left[\frac{((Bab - 3Ab^2)x^5 + (Ba^2 - 3Aab)x^2)\sqrt{-ab} \log\left(\frac{bx^3 + 2\sqrt{-ab}x^2 - a}{bx^3 + a}\right) - 2(2Aa^2b - (Ba^2b - 3Aab^2)x^3)\sqrt{x}}{6(a^3b^2x^5 + a^4bx^2)}, \frac{((Bab - 3Ab^2)x^5 + (Ba^2 - 3Aab)x^2)\sqrt{ab} \arctan\left(\frac{\sqrt{ab}x^{3/2}}{a}\right) - (2Aa^2b - (Ba^2b - 3Aab^2)x^3)\sqrt{x}}{3(a^3b^2x^5 + a^4bx^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [1/6*(((B*a*b - 3*A*b^2)*x^5 + (B*a^2 - 3*A*a*b)*x^2)*sqrt(-a*b)*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) - 2*(2*A*a^2*b - (B*a^2*b - 3*A*a*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2), 1/3*(((B*a*b - 3*A*b^2)*x^5 + (B*a^2 - 3*A*a*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*A*a^2*b - (B*a^2*b - 3*A*a*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2)]

giac [A] time = 0.16, size = 66, normalized size = 0.69

$$\frac{(Ba - 3Ab) \arctan\left(\frac{bx^{3/2}}{\sqrt{ab}}\right)}{3\sqrt{ab}a^2} + \frac{Bax^3 - 3Abx^3 - 2Aa}{3\left(bx^{\frac{9}{2}} + ax^{\frac{3}{2}}\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/3*(B*a - 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(B*a*x^3 - 3*A*b*x^3 - 2*A*a)/((b*x^(9/2) + a*x^(3/2))*a^2)

maple [A] time = 0.07, size = 93, normalized size = 0.97

$$-\frac{A b x^{\frac{3}{2}}}{3(b x^3 + a) a^2} + \frac{B x^{\frac{3}{2}}}{3(b x^3 + a) a} - \frac{A b \arctan\left(\frac{b x^{\frac{3}{2}}}{\sqrt{a b}}\right)}{\sqrt{a b} a^2} + \frac{B \arctan\left(\frac{b x^{\frac{3}{2}}}{\sqrt{a b}}\right)}{3 \sqrt{a b} a} - \frac{2 A}{3 a^2 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x)

[Out] $-1/3/a^2*x^{3/2}/(b*x^3+a)*A*b+1/3/a*x^{3/2}/(b*x^3+a)*B-1/a^2/(a*b)^{(1/2)*\arctan(1/(a*b)^{(1/2)*b*x^{3/2}})*A*b+1/3/a/(a*b)^{(1/2)*\arctan(1/(a*b)^{(1/2)*b*x^{3/2}})*B-2/3*A/a^2/x^{3/2)}$

maxima [A] time = 1.19, size = 67, normalized size = 0.70

$$\frac{(B a - 3 A b) x^3 - 2 A a}{3 \left(a^2 b x^{\frac{3}{2}} + a^3 x^{\frac{3}{2}} \right)} + \frac{(B a - 3 A b) \arctan\left(\frac{b x^{\frac{3}{2}}}{\sqrt{a b}}\right)}{3 \sqrt{a b} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] $1/3*((B*a - 3*A*b)*x^3 - 2*A*a)/(a^2*b*x^{9/2} + a^3*x^{3/2}) + 1/3*(B*a - 3*A*b)*\arctan(b*x^{3/2}/\sqrt{a*b})/(\sqrt{a*b}*a^2)$

mupad [B] time = 0.15, size = 139, normalized size = 1.45

$$\frac{2 A a^{3/2} \sqrt{b} - B a^2 x^{3/2} \operatorname{atan}\left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}}\right) + 3 A b^2 x^{9/2} \operatorname{atan}\left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}}\right) + 3 A \sqrt{a} b^{3/2} x^3 - B a^{3/2} \sqrt{b} x^3 + 3 A a b x^{3/2} \operatorname{atan}\left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}}\right) - B a b x^{9/2} \operatorname{atan}\left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}}\right)}{3 a^{7/2} \sqrt{b} x^{3/2} + 3 a^{5/2} b^{3/2} x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x)

[Out] $-(2*A*a^{3/2}*b^{1/2} - B*a^2*x^{3/2}*\operatorname{atan}((b^{1/2}*x^{3/2})/a^{1/2})) + 3*A*b^2*x^{9/2}*\operatorname{atan}((b^{1/2}*x^{3/2})/a^{1/2}) + 3*A*a^{1/2}*b^{3/2}*x^3 - B*a^{3/2}*b^{1/2}*x^3 + 3*A*a*b*x^{3/2}*\operatorname{atan}((b^{1/2}*x^{3/2})/a^{1/2}) - B*a*b*x^{9/2}*\operatorname{atan}((b^{1/2}*x^{3/2})/a^{1/2}))/((3*a^{7/2}*b^{1/2}*x^{3/2} + 3*a^{5/2}*b^{3/2}*x^{9/2}))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

$$3.166 \quad \int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^2} dx$$

Optimal. Leaf size=318

$$\frac{(11Ab - 5aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{17/6} \sqrt[6]{b}} - \frac{(11Ab - 5aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{17/6} \sqrt[6]{b}} + \frac{(11Ab - 5aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{17/6} \sqrt[6]{b}}$$

Rubi [A] time = 0.50, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {457, 325, 329, 209, 634, 618, 204, 628, 205}

$$\frac{11Ab - 5aB}{15a^2bx^{5/2}} + \frac{(11Ab - 5aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{17/6} \sqrt[6]{b}} - \frac{(11Ab - 5aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{12\sqrt{3} a^{17/6} \sqrt[6]{b}} + \frac{(11Ab - 5aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{a}\sqrt[6]{b}}{\sqrt{x}}\right)}{18a^{17/6} \sqrt[6]{b}} - \frac{(11Ab - 5aB) \tan^{-1}\left(\frac{2\sqrt[6]{a}\sqrt[6]{b}}{\sqrt{x}} + \sqrt{3}\right)}{18a^{17/6} \sqrt[6]{b}} - \frac{(11Ab - 5aB) \tan^{-1}\left(\frac{\sqrt[6]{a}\sqrt[6]{b}}{\sqrt{x}}\right)}{9a^{17/6} \sqrt[6]{b}} + \frac{Ab - aB}{3abx^{5/2}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2), x]

[Out] $-(11A*b - 5a*B)/(15*a^2*b*x^{5/2}) + (A*b - a*B)/(3*a*b*x^{5/2}*(a + b*x^3)) + ((11A*b - 5a*B)*\text{ArcTan}[\text{Sqrt}[3] - (2*b^{1/6}*\text{Sqrt}[x])/a^{1/6}])/(18*a^{17/6}*b^{1/6}) - ((11A*b - 5a*B)*\text{ArcTan}[\text{Sqrt}[3] + (2*b^{1/6}*\text{Sqrt}[x])/a^{1/6}])/(18*a^{17/6}*b^{1/6}) - ((11A*b - 5a*B)*\text{ArcTan}[(b^{1/6}*\text{Sqrt}[x])/a^{1/6}])/(9*a^{17/6}*b^{1/6}) + ((11A*b - 5a*B)*\text{Log}[a^{1/3} - \text{Sqrt}[3]*a^{1/6}*b^{1/6}*\text{Sqrt}[x] + b^{1/3}*x])/(12*\text{Sqrt}[3]*a^{17/6}*b^{1/6}) - ((11A*b - 5a*B)*\text{Log}[a^{1/3} + \text{Sqrt}[3]*a^{1/6}*b^{1/6}*\text{Sqrt}[x] + b^{1/3}*x])/(12*\text{Sqrt}[3]*a^{17/6}*b^{1/6})$

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 209

Int[((a_) + (b_)*(x_)^(n_))^(n_+1), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[

$(r + s \cos[\frac{(2k-1)\pi}{n}]x) / (r^2 + 2rs \cos[\frac{(2k-1)\pi}{n}]x + s^2 x^2), x]; (2r^2 \int 1/(r^2 + s^2 x^2), x) / (a^n) + \text{Dist}[(2r)/(a^n), \text{Sum}[u, \{k, 1, (n-2)/4\}], x, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{IGtQ}[(n-2)/4, 0] \&\& \text{PosQ}[a/b]$

Rule 325

$\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^p, x_Symbol] \rightarrow \text{Simp}[(c \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} / (a \cdot c \cdot (m+1)), x] - \text{Dist}[(b \cdot (m + n \cdot (p+1) + 1)) / (a \cdot c^n \cdot (m+1)), \text{Int}[(c \cdot x)^{m+n} \cdot (a + b \cdot x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[m, -1] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 329

$\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^p, x_Symbol] \rightarrow \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{k \cdot (m+1) - 1} \cdot (a + b \cdot x^{k \cdot n}) / c^n]^p, x], x, (c \cdot x)^{1/k}], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 457

$\text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^p \cdot (c + d \cdot x^n), x_Symbol] \rightarrow -\text{Simp}[(b \cdot c - a \cdot d) \cdot (e \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} / (a \cdot b \cdot e \cdot n \cdot (p+1)), x] - \text{Dist}[(a \cdot d \cdot (m+1) - b \cdot c \cdot (m + n \cdot (p+1) + 1)) / (a \cdot b \cdot n \cdot (p+1)), \text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^{p+1}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, n\}, x] \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{LtQ}[p, -1] \&\& ((\text{!IntegerQ}[p + 1/2] \&\& \text{NeQ}[p, -5/4]) \|\ \text{!RationalQ}[m] \|\ (\text{IGtQ}[n, 0] \&\& \text{ILtQ}[p + 1/2, 0] \&\& \text{LeQ}[-1, m, -(n \cdot (p+1))]))$

Rule 618

$\text{Int}[(a + b \cdot x + c \cdot x^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4 \cdot a \cdot c - x^2, x], x], x, b + 2 \cdot c \cdot x], x] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 628

$\text{Int}[(d + e \cdot x) / (a + b \cdot x + c \cdot x^2), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^{7/2}(a + bx^3)^2} dx &= \frac{Ab - aB}{3abx^{5/2}(a + bx^3)} + \frac{\left(\frac{11Ab}{2} - \frac{5aB}{2}\right) \int \frac{1}{x^{7/2}(a+bx^3)} dx}{3ab} \\
&= -\frac{11Ab - 5aB}{15a^2bx^{5/2}} + \frac{Ab - aB}{3abx^{5/2}(a + bx^3)} - \frac{(11Ab - 5aB) \int \frac{1}{\sqrt{x}(a+bx^3)} dx}{6a^2} \\
&= -\frac{11Ab - 5aB}{15a^2bx^{5/2}} + \frac{Ab - aB}{3abx^{5/2}(a + bx^3)} - \frac{(11Ab - 5aB) \text{Subst}\left(\int \frac{1}{a+bx^6} dx, x, \sqrt{x}\right)}{3a^2} \\
&= -\frac{11Ab - 5aB}{15a^2bx^{5/2}} + \frac{Ab - aB}{3abx^{5/2}(a + bx^3)} - \frac{(11Ab - 5aB) \text{Subst}\left(\int \frac{\sqrt[6]{a} - \frac{1}{2}\sqrt{3} \sqrt[6]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{9a^{17/6}} \\
&= -\frac{11Ab - 5aB}{15a^2bx^{5/2}} + \frac{Ab - aB}{3abx^{5/2}(a + bx^3)} - \frac{(11Ab - 5aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6} \sqrt[6]{b}} - \frac{(11Ab - 5aB) \text{Subst}\left(\int \frac{1}{a+bx^6} dx, x, \sqrt{x}\right)}{3a^2} \\
&= -\frac{11Ab - 5aB}{15a^2bx^{5/2}} + \frac{Ab - aB}{3abx^{5/2}(a + bx^3)} - \frac{(11Ab - 5aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6} \sqrt[6]{b}} + \frac{(11Ab - 5aB) \log\left(\frac{\sqrt[6]{a} + \sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a} - \sqrt[6]{b} \sqrt{x}}\right)}{12a^{17/6} \sqrt[6]{b}} \\
&= -\frac{11Ab - 5aB}{15a^2bx^{5/2}} + \frac{Ab - aB}{3abx^{5/2}(a + bx^3)} + \frac{(11Ab - 5aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6} \sqrt[6]{b}} - \frac{(11Ab - 5aB) \log\left(\frac{\sqrt[6]{a} + \sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a} - \sqrt[6]{b} \sqrt{x}}\right)}{12a^{17/6} \sqrt[6]{b}}
\end{aligned}$$

Mathematica [C] time = 0.11, size = 74, normalized size = 0.23

$$\frac{5x^3(5aB - 11Ab) {}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; -\frac{bx^3}{a}\right) + \frac{a(-6aA + 5aBx^3 - 11Abx^3)}{a+bx^3}}{15a^3x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2), x]

[Out] ((a*(-6*a*A - 11*A*b*x^3 + 5*a*B*x^3))/(a + b*x^3) + 5*(-11*A*b + 5*a*B)*x^3*Hypergeometric2F1[1/6, 1, 7/6, -(b*x^3)/a])/(15*a^3*x^(5/2))

IntegrateAlgebraic [A] time = 0.60, size = 200, normalized size = 0.63

$$\frac{(5aB - 11Ab) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6} \sqrt[6]{b}} + \frac{(11Ab - 5aB) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b}x}{\sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}\right)}{18a^{17/6} \sqrt[6]{b}} + \frac{(5aB - 11Ab) \tanh^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{b}x}\right)}{6\sqrt{3} a^{17/6} \sqrt[6]{b}} + \frac{-6aA + 5aBx^3 - 11Abx^3}{15a^2x^{5/2}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2), x]

[Out] (-6*a*A - 11*A*b*x^3 + 5*a*B*x^3)/(15*a^2*x^(5/2)*(a + b*x^3)) + ((-11*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(17/6)*b^(1/6)) + ((11*A*b - 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x]])/(18*a^(17/6)*b^(1/6)) + ((-11*A*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(6*Sqrt[3]*a^(17/6)*b^(1/6))

fricas [B] time = 1.09, size = 2584, normalized size = 8.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] -1/180*(20*sqrt(3)*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^6*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 121*A^2*b^2)*x + (5*B*a^4 - 11*A*a^3*b)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)))*a^14*b*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(5/6) + 2*sqrt(3)*(5*B*a^15*b - 11*A*a^14*b^2)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(5/6) - sqrt(3)*(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6))/(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)) + 20*sqrt(3)*(a^2*b*x^6 + a^3*x

$$5 + 1771561 \cdot A^6 \cdot b^6 / (a^{17} \cdot b) - (5 \cdot B \cdot a - 11 \cdot A \cdot b) \cdot \sqrt{x} - 12 \cdot ((5 \cdot B \cdot a - 11 \cdot A \cdot b) \cdot x^3 - 6 \cdot A \cdot a) \cdot \sqrt{x} / (a^2 \cdot b \cdot x^6 + a^3 \cdot x^3)$$

giac [A] time = 0.22, size = 313, normalized size = 0.98

$$\frac{\sqrt{3} \left(5 (ab)^{\frac{1}{2}} Ba - 11 (ab)^{\frac{1}{2}} Ab\right) \log\left(\sqrt{3} \sqrt{\left(\frac{a}{b}\right)^{\frac{1}{3}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - \sqrt{3} \left(5 (ab)^{\frac{1}{2}} Ba - 11 (ab)^{\frac{1}{2}} Ab\right) \log\left(-\sqrt{3} \sqrt{\left(\frac{a}{b}\right)^{\frac{1}{3}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{36 a^2 b} + \frac{B a \sqrt{x} - A b \sqrt{x}}{3 (a^2 + a)} + \frac{5 (ab)^{\frac{1}{2}} Ba - 11 (ab)^{\frac{1}{2}} Ab \arctan\left(\frac{\sqrt{3} \left(\frac{a}{b}\right)^{\frac{1}{3}} + \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{18 a^2 b} + \frac{5 (ab)^{\frac{1}{2}} Ba - 11 (ab)^{\frac{1}{2}} Ab \arctan\left(\frac{-\sqrt{3} \left(\frac{a}{b}\right)^{\frac{1}{3}} + \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{18 a^2 b} + \frac{5 (ab)^{\frac{1}{2}} Ba - 11 (ab)^{\frac{1}{2}} Ab \arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 a^2 b} - \frac{2 A}{5 a^2 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/36*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b) - 1/36*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a^2) + 1/18*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^3*b) + 1/18*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b) + 1/9*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^3*b) - 2/5*A/(a^2*x^(5/2))

maple [A] time = 0.16, size = 395, normalized size = 1.24

$$\frac{A b \sqrt{x}}{3 (a^2 + a) a^2} - \frac{B \sqrt{x}}{3 (b^2 + a)} + \frac{11 \left(\frac{a}{b}\right)^{\frac{1}{3}} A b \arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - 11 \left(\frac{a}{b}\right)^{\frac{1}{3}} A b \arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}, \sqrt{3}\right)}{9 a^2} - \frac{11 \left(\frac{a}{b}\right)^{\frac{1}{3}} A b \arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}, \sqrt{3}\right)}{18 a^2} - \frac{11 \sqrt{3} \left(\frac{a}{b}\right)^{\frac{1}{3}} A b \ln\left(1 + \sqrt{3} \left(\frac{a}{b}\right)^{\frac{1}{3}} \sqrt{x} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36 a^2} - \frac{11 \sqrt{3} \left(\frac{a}{b}\right)^{\frac{1}{3}} A b \ln\left(-1 + \sqrt{3} \left(\frac{a}{b}\right)^{\frac{1}{3}} \sqrt{x} - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36 a^2} + \frac{5 \left(\frac{a}{b}\right)^{\frac{1}{3}} B \arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 a^2} + \frac{5 \left(\frac{a}{b}\right)^{\frac{1}{3}} B \arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}, \sqrt{3}\right)}{18 a^2} - \frac{5 \left(\frac{a}{b}\right)^{\frac{1}{3}} B \arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}, \sqrt{3}\right)}{18 a^2} + \frac{5 \sqrt{3} \left(\frac{a}{b}\right)^{\frac{1}{3}} B \ln\left(1 + \sqrt{3} \left(\frac{a}{b}\right)^{\frac{1}{3}} \sqrt{x} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36 a^2} - \frac{5 \sqrt{3} \left(\frac{a}{b}\right)^{\frac{1}{3}} B \ln\left(-1 + \sqrt{3} \left(\frac{a}{b}\right)^{\frac{1}{3}} \sqrt{x} - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36 a^2} - \frac{2 A}{5 a^2 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x)

[Out] -1/3/a^2*x^(1/2)/(b*x^3+a)*A*b+1/3/a*x^(1/2)/(b*x^3+a)*B-11/9/a^3*A*b*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))+11/36/a^3*A*b*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))-11/18/a^3*A*b*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))-11/36/a^3*A*b*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-11/18/a^3*A*b*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))+5/9/a^2*B*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))-5/36/a^2*B*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))+5/18/a^2*B*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))+5/36/a^2*B*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+5/18/a^2*B*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))-2/5/a^2*A/x^(5/2)

maxima [A] time = 1.19, size = 312, normalized size = 0.98

$$\frac{(5 Ba - 11 Ab)x^3 - 6 Aa}{15 (a^2 b x^2 + a^3 x^{\frac{5}{2}})} + \frac{\sqrt{3} (5 Ba - 11 Ab) \log\left(\sqrt{3 a^{\frac{1}{3}} b^{\frac{1}{3}} \sqrt{x} + b^{\frac{1}{3}} x + a^{\frac{1}{3}}}\right) - \sqrt{3} (5 Ba - 11 Ab) \log\left(-\sqrt{3 a^{\frac{1}{3}} b^{\frac{1}{3}} \sqrt{x} + b^{\frac{1}{3}} x + a^{\frac{1}{3}}}\right)}{a^{\frac{5}{2}} b^{\frac{1}{2}}} - \frac{\sqrt{3} (5 Ba - 11 Ab) \log\left(-\sqrt{3 a^{\frac{1}{3}} b^{\frac{1}{3}} \sqrt{x} + b^{\frac{1}{3}} x + a^{\frac{1}{3}}}\right)}{a^{\frac{5}{2}} b^{\frac{1}{2}}} + \frac{4 \left(5 B a b^{\frac{1}{2}} - 11 A b^{\frac{3}{2}}\right) \arctan\left(\frac{b^{\frac{1}{3}} \sqrt{x}}{\sqrt{a^{\frac{1}{3}} b^{\frac{1}{3}}}}\right)}{a^{\frac{5}{2}} b^{\frac{1}{2}} \sqrt{a^{\frac{1}{3}} b^{\frac{1}{3}}}} + \frac{2 \left(5 B a^{\frac{4}{3}} b^{\frac{1}{3}} - 11 A a^{\frac{1}{3}} b^{\frac{4}{3}}\right) \arctan\left(\frac{\sqrt{3 a^{\frac{1}{3}} b^{\frac{1}{3}} + 2 b^{\frac{1}{3}} \sqrt{x}}}{\sqrt{a^{\frac{1}{3}} b^{\frac{1}{3}}}}\right)}{a b^{\frac{5}{2}} \sqrt{a^{\frac{1}{3}} b^{\frac{1}{3}}}} + \frac{2 \left(5 B a^{\frac{4}{3}} b^{\frac{1}{3}} - 11 A a^{\frac{1}{3}} b^{\frac{4}{3}}\right) \arctan\left(\frac{-\sqrt{3 a^{\frac{1}{3}} b^{\frac{1}{3}} - 2 b^{\frac{1}{3}} \sqrt{x}}}{\sqrt{a^{\frac{1}{3}} b^{\frac{1}{3}}}}\right)}{a b^{\frac{5}{2}} \sqrt{a^{\frac{1}{3}} b^{\frac{1}{3}}}} - \frac{2 A}{5 a^2 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& b^6 - 38811960A^3B^3a^{11}b^8) + (((3^{(1/2)}*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(\\
& 34930764*A^3*a^{13}*b^8 - 3280500*B^3*a^{16}*b^5 + 21651300*A*B^2*a^{15}*b^6 - 47 \\
& 632860*A^2*B*a^{14}*b^7))/(18*(-a)^{(17/6)}*b^{(1/6)}))*(11*A*b - 5*B*a)*1i)/(18* \\
& (-a)^{(17/6)}*b^{(1/6)})))/((((3^{(1/2)}*1i)/2 - 1/2)*(x^{(1/2)}*(21346578*A^4*a^{10}* \\
& b^9 + 911250*B^4*a^{14}*b^5 + 26462700*A^2*B^2*a^{12}*b^7 - 8019000*A*B^3*a^{13}* \\
& b^6 - 38811960*A^3*B^3*a^{11}*b^8) - (((3^{(1/2)}*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(\\
& 34930764*A^3*a^{13}*b^8 - 3280500*B^3*a^{16}*b^5 + 21651300*A*B^2*a^{15}*b^6 - 47 \\
& 632860*A^2*B*a^{14}*b^7))/(18*(-a)^{(17/6)}*b^{(1/6)}))*(11*A*b - 5*B*a))/(18*(-a) \\
&)^{(17/6)}*b^{(1/6)})) - (((3^{(1/2)}*1i)/2 - 1/2)*(x^{(1/2)}*(21346578*A^4*a^{10}*b^9 \\
& + 911250*B^4*a^{14}*b^5 + 26462700*A^2*B^2*a^{12}*b^7 - 8019000*A*B^3*a^{13}*b^6 \\
& - 38811960*A^3*B^3*a^{11}*b^8) + (((3^{(1/2)}*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(3493 \\
& 0764*A^3*a^{13}*b^8 - 3280500*B^3*a^{16}*b^5 + 21651300*A*B^2*a^{15}*b^6 - 47632 \\
& 860*A^2*B*a^{14}*b^7))/(18*(-a)^{(17/6)}*b^{(1/6)}))*(11*A*b - 5*B*a))/(18*(-a)^{(17/6)} \\
&)^{(17/6)}*b^{(1/6)})))*((3^{(1/2)}*1i)/2 - 1/2)*(11*A*b - 5*B*a)*1i)/(9*(-a)^{(17/6)} \\
&)^{(17/6)}*b^{(1/6)})) - (atan((((3^{(1/2)}*1i)/2 + 1/2)*(x^{(1/2)}*(21346578*A^4*a^{10}*b^9 \\
& + 911250*B^4*a^{14}*b^5 + 26462700*A^2*B^2*a^{12}*b^7 - 8019000*A*B^3*a^{13}*b^6 \\
& - 38811960*A^3*B^3*a^{11}*b^8) - (((3^{(1/2)}*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493 \\
& 0764*A^3*a^{13}*b^8 - 3280500*B^3*a^{16}*b^5 + 21651300*A*B^2*a^{15}*b^6 - 476328 \\
& 60*A^2*B*a^{14}*b^7))/(18*(-a)^{(17/6)}*b^{(1/6)}))*(11*A*b - 5*B*a)*1i)/(18*(-a) \\
&)^{(17/6)}*b^{(1/6)})) + (((3^{(1/2)}*1i)/2 + 1/2)*(x^{(1/2)}*(21346578*A^4*a^{10}*b^9 \\
& + 911250*B^4*a^{14}*b^5 + 26462700*A^2*B^2*a^{12}*b^7 - 8019000*A*B^3*a^{13}*b^6 \\
& - 38811960*A^3*B^3*a^{11}*b^8) + (((3^{(1/2)}*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493 \\
& 0764*A^3*a^{13}*b^8 - 3280500*B^3*a^{16}*b^5 + 21651300*A*B^2*a^{15}*b^6 - 476328 \\
& 60*A^2*B*a^{14}*b^7))/(18*(-a)^{(17/6)}*b^{(1/6)}))*(11*A*b - 5*B*a)*1i)/(18*(-a) \\
&)^{(17/6)}*b^{(1/6)})))/((((3^{(1/2)}*1i)/2 + 1/2)*(x^{(1/2)}*(21346578*A^4*a^{10}*b^9 \\
& + 911250*B^4*a^{14}*b^5 + 26462700*A^2*B^2*a^{12}*b^7 - 8019000*A*B^3*a^{13}*b^6 \\
& - 38811960*A^3*B^3*a^{11}*b^8) - (((3^{(1/2)}*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493 \\
& 0764*A^3*a^{13}*b^8 - 3280500*B^3*a^{16}*b^5 + 21651300*A*B^2*a^{15}*b^6 - 476328 \\
& 60*A^2*B*a^{14}*b^7))/(18*(-a)^{(17/6)}*b^{(1/6)}))*(11*A*b - 5*B*a))/(18*(-a)^{(17/6)} \\
&)^{(17/6)}*b^{(1/6)})) - (((3^{(1/2)}*1i)/2 + 1/2)*(x^{(1/2)}*(21346578*A^4*a^{10}*b^9 + 9 \\
& 11250*B^4*a^{14}*b^5 + 26462700*A^2*B^2*a^{12}*b^7 - 8019000*A*B^3*a^{13}*b^6 - 3 \\
& 8811960*A^3*B^3*a^{11}*b^8) + (((3^{(1/2)}*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493076 \\
& 4*A^3*a^{13}*b^8 - 3280500*B^3*a^{16}*b^5 + 21651300*A*B^2*a^{15}*b^6 - 47632860* \\
& A^2*B*a^{14}*b^7))/(18*(-a)^{(17/6)}*b^{(1/6)}))*(11*A*b - 5*B*a))/(18*(-a)^{(17/6)} \\
&)^{(17/6)}*b^{(1/6)})))*((3^{(1/2)}*1i)/2 + 1/2)*(11*A*b - 5*B*a)*1i)/(9*(-a)^{(17/6)}*b^{(1/6)}))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**2,x)

[Out] Timed out

$$3.167 \quad \int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=104

$$\frac{(3aB + Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}} - \frac{x^{3/2}(3aB + Ab)}{12ab^2(a + bx^3)} + \frac{x^{9/2}(Ab - aB)}{6ab(a + bx^3)^2}$$

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 288, 329, 275, 205}

$$\frac{(3aB + Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}} - \frac{x^{3/2}(3aB + Ab)}{12ab^2(a + bx^3)} + \frac{x^{9/2}(Ab - aB)}{6ab(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] ((A*b - a*B)*x^(9/2))/(6*a*b*(a + b*x^3)^2) - ((A*b + 3*a*B)*x^(3/2))/(12*a*b^2*(a + b*x^3)) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(3/2)*b^(5/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n
_.)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\begin{aligned} \int \frac{x^{7/2} (A + Bx^3)}{(a + bx^3)^3} dx &= \frac{(Ab - aB)x^{9/2}}{6ab(a + bx^3)^2} + \frac{\left(\frac{3Ab}{2} + \frac{9aB}{2}\right) \int \frac{x^{7/2}}{(a + bx^3)^2} dx}{6ab} \\ &= \frac{(Ab - aB)x^{9/2}}{6ab(a + bx^3)^2} - \frac{(Ab + 3aB)x^{3/2}}{12ab^2(a + bx^3)} + \frac{(Ab + 3aB) \int \frac{\sqrt{x}}{a + bx^3} dx}{8ab^2} \\ &= \frac{(Ab - aB)x^{9/2}}{6ab(a + bx^3)^2} - \frac{(Ab + 3aB)x^{3/2}}{12ab^2(a + bx^3)} + \frac{(Ab + 3aB) \operatorname{Subst}\left(\int \frac{x^2}{a + bx^6} dx, x, \sqrt{x}\right)}{4ab^2} \\ &= \frac{(Ab - aB)x^{9/2}}{6ab(a + bx^3)^2} - \frac{(Ab + 3aB)x^{3/2}}{12ab^2(a + bx^3)} + \frac{(Ab + 3aB) \operatorname{Subst}\left(\int \frac{1}{a + bx^2} dx, x, x^{3/2}\right)}{12ab^2} \\ &= \frac{(Ab - aB)x^{9/2}}{6ab(a + bx^3)^2} - \frac{(Ab + 3aB)x^{3/2}}{12ab^2(a + bx^3)} + \frac{(Ab + 3aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.19, size = 93, normalized size = 0.89

$$\frac{\sqrt{a} \sqrt{b} x^{3/2} (-3a^2 B - ab(A + 5Bx^3) + Ab^2 x^3)}{(a + bx^3)^2} + (3aB + Ab) \tan^{-1}\left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] ((Sqrt[a]*Sqrt[b]*x^(3/2)*(-3*a^2*B + A*b^2*x^3 - a*b*(A + 5*B*x^3)))/(a + b*x^3)^2 + (A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(3/2)*b^(5/2))

IntegrateAlgebraic [A] time = 0.17, size = 92, normalized size = 0.88

$$\frac{(3aB + Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}} - \frac{x^{3/2}(3a^2B + aAb + 5abBx^3 - Ab^2x^3)}{12ab^2(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] -1/12*(x^(3/2)*(a*A*b + 3*a^2*B - A*b^2*x^3 + 5*a*b*B*x^3))/(a*b^2*(a + b*x^3)^2) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(3/2)*b^(5/2))

fricas [A] time = 0.77, size = 314, normalized size = 3.02

$$\left[\frac{((3Ba^2 + Ab^3)x^6 + 3Ba^3 + Aa^2b + 2(3Ba^2b + Ab^3)x^3)\sqrt{-ab} \log\left(\frac{bx^2 - 2\sqrt{-ab}x - a}{bx^2 + a}\right) + 2((5Ba^2b^2 - Ab^3)x^4 + (3Ba^2b + Aa^2b^2)x)\sqrt{x} - ((3Ba^2 + Ab^3)x^6 + 3Ba^3 + Aa^2b + 2(3Ba^2b + Ab^3)x^3)\sqrt{ab} \arctan\left(\frac{\sqrt{ab}x}{a}\right) - ((5Ba^2b^2 - Ab^3)x^4 + (3Ba^2b + Aa^2b^2)x)\sqrt{x}}{24(a^2b^2x^6 + 2a^3b^2x^3 + a^4b^3)}, \frac{((3Ba^2 + Ab^3)x^6 + 3Ba^3 + Aa^2b + 2(3Ba^2b + Ab^3)x^3)\sqrt{ab} \arctan\left(\frac{\sqrt{ab}x}{a}\right) - ((5Ba^2b^2 - Ab^3)x^4 + (3Ba^2b + Aa^2b^2)x)\sqrt{x}}{12(a^2b^2x^6 + 2a^3b^2x^3 + a^4b^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] [-1/24*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) + 2*((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(x))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), 1/12*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - ((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(x))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]

giac [A] time = 0.21, size = 84, normalized size = 0.81

$$\frac{(3Ba + Ab) \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{12\sqrt{ab}ab^2} - \frac{5Babx^{\frac{9}{2}} - Ab^2x^{\frac{9}{2}} + 3Ba^2x^{\frac{3}{2}} + Aabx^{\frac{3}{2}}}{12(bx^3 + a)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $\frac{1}{12}*(3*B*a + A*b)*\arctan(b*x^{(3/2)}/\sqrt{a*b})/(\sqrt{a*b}*a*b^2) - \frac{1}{12}*(5*B*a*b*x^{(9/2)} - A*b^2*x^{(9/2)} + 3*B*a^2*x^{(3/2)} + A*a*b*x^{(3/2)})/((b*x^3 + a)^2*a*b^2)$

maple [A] time = 0.07, size = 96, normalized size = 0.92

$$\frac{A \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\sqrt{ab} ab} + \frac{B \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab} b^2} + \frac{\frac{(Ab-5Ba)x^{\frac{9}{2}}}{12ab} - \frac{(Ab+3Ba)x^{\frac{3}{2}}}{12b^2}}{(bx^3 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x)

[Out] $\frac{2}{3}*(\frac{1}{8}*(A*b-5*B*a)/a/b*x^{(9/2)}-1/8*(A*b+3*B*a)/b^2*x^{(3/2)})/(b*x^3+a)^2+1/12/b/a/(a*b)^{(1/2)*\arctan(1/(a*b)^{(1/2)*b*x^{(3/2)})}*A+1/4/b^2/(a*b)^{(1/2)*\arctan(1/(a*b)^{(1/2)*b*x^{(3/2)})}*B}$

maxima [A] time = 1.43, size = 96, normalized size = 0.92

$$-\frac{(5 Bab - Ab^2)x^{\frac{9}{2}} + (3 Ba^2 + Aab)x^{\frac{3}{2}}}{12(ab^4x^6 + 2a^2b^3x^3 + a^3b^2)} + \frac{(3 Ba + Ab) \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12 \sqrt{ab} ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] $-\frac{1}{12}*((5*B*a*b - A*b^2)*x^{(9/2)} + (3*B*a^2 + A*a*b)*x^{(3/2)})/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2) + \frac{1}{12}*(3*B*a + A*b)*\arctan(b*x^{(3/2)}/\sqrt{a*b})/(\sqrt{a*b}*a*b^2)$

mupad [B] time = 2.76, size = 133, normalized size = 1.28

$$\frac{\operatorname{atan}\left(\frac{9b^{3/2}x^{3/2}(A^2b^2+6ABab+9B^2a^2)}{\sqrt{a}(9Ab^2+27Bab)(Ab+3Ba)}\right)(Ab+3Ba)}{12a^{3/2}b^{5/2}} - \frac{x^{3/2}(Ab+3Ba)}{12b^2} - \frac{x^{9/2}(Ab-5Ba)}{12ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x)

[Out] $(\operatorname{atan}((9*b^{(3/2)}*x^{(3/2)}*(A^2*b^2 + 9*B^2*a^2 + 6*A*B*a*b))/(a^{(1/2)}*(9*A*b^2 + 27*B*a*b)*(A*b + 3*B*a)))*(A*b + 3*B*a))/(12*a^{(3/2)}*b^{(5/2)}) - ((x^{(3/2)}*(A + B*x^3))/(a + b*x^3)^3)$

$$\frac{1}{2}*(A*b + 3*B*a)/(12*b^2) - (x^{9/2}*(A*b - 5*B*a))/(12*a*b)/(a^2 + b^2*x^6 + 2*a*b*x^3)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**3,x)

[Out] Timed out

$$3.168 \quad \int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=327

$$-\frac{(7aB + 5Ab) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{11/6} b^{13/6}} + \frac{(7aB + 5Ab) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{11/6} b^{13/6}} - \frac{(7aB + 5Ab) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{11/6} b^{13/6}}$$

Rubi [A] time = 0.49, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, number of rules / integrand size = 0.409, Rules used = {457, 288, 329, 209, 634, 618, 204, 628, 205}

$$\frac{(7aB + 5Ab) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{11/6} b^{13/6}} + \frac{(7aB + 5Ab) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{11/6} b^{13/6}} - \frac{(7aB + 5Ab) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt{6}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6} b^{13/6}} + \frac{(7aB + 5Ab) \tan^{-1}\left(\frac{2\sqrt{6}\sqrt{x}}{\sqrt[6]{a}} + \sqrt{3}\right)}{216a^{11/6} b^{13/6}} + \frac{(7aB + 5Ab) \tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6} b^{13/6}} - \frac{\sqrt{x}(7aB + 5Ab)}{36ab^2(a + bx^3)} + \frac{x^{7/2}(Ab - aB)}{6ab(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] ((A*b - a*B)*x^(7/2))/(6*a*b*(a + b*x^3)^2) - ((5*A*b + 7*a*B)*Sqrt[x])/(36*a*b^2*(a + b*x^3)) - ((5*A*b + 7*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(11/6)*b^(13/6)) - ((5*A*b + 7*a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(11/6)*b^(13/6))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 209

Int[((a_) + (b_.)*(x_)^(n_))^(n_+1), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*k - 1)*Pi)/n]*x])/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[

$(r + s \cos(((2k - 1)\pi)/n)x)/(r^2 + 2rs \cos(((2k - 1)\pi)/n)x + s^2 x^2), x]; (2r^2 \int [1/(r^2 + s^2 x^2), x])/(a^n) + \text{Dist}[(2r)/(a^n), \text{Sum}[u, \{k, 1, (n - 2)/4\}], x, x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{IGtQ}[(n - 2)/4, 0] \&\& \text{PosQ}[a/b]$

Rule 288

$\text{Int}[(c \cdot x)^m (a + b x^n)^p, x_Symbol] \rightarrow \text{Simp}[(c^{n-1} (c x)^{m-n+1} (a + b x^n)^{p+1})/(b^n (p+1)), x] - \text{Dist}[(c^n (m-n+1))/(b^n (p+1)), \text{Int}[(c x)^{m-n} (a + b x^n)^{p+1}, x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[m+1, n] \&\& \text{!IntegerQ}[(m+n(p+1)+1)/n, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 329

$\text{Int}[(c \cdot x)^m (a + b x^n)^p, x_Symbol] \rightarrow \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{k(m+1)-1} (a + b x^{kn})/c^n]^p, x], x, (c x)^{1/k}], x] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 457

$\text{Int}[(e \cdot x)^m (a + b x^n)^p (c + d x^n), x_Symbol] \rightarrow -\text{Simp}[(b c - a d) (e x)^{m+1} (a + b x^n)^{p+1}/(a^m b^n (p+1)), x] - \text{Dist}[(a d (m+1) - b c (m + n(p+1) + 1))/(a^m b^n (p+1)), \text{Int}[(e x)^m (a + b x^n)^{p+1}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x] \&\& \text{NeQ}[b c - a d, 0] \&\& \text{LtQ}[p, -1] \&\& ((\text{!IntegerQ}[p + 1/2] \&\& \text{NeQ}[p, -5/4]) \|\ \text{!RationalQ}[m] \|\ (\text{IGtQ}[n, 0] \&\& \text{ILtQ}[p + 1/2, 0] \&\& \text{LeQ}[-1, m, -(n(p+1))]))$

Rule 618

$\text{Int}[(a + b x + c x^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4ac - x^2, x], x], x, b + 2cx], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0]$

Rule 628

$\text{Int}[(d + e x)/(a + b x + c x^2), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b x + c x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2cd - b^2, 0]$

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2} (A + Bx^3)}{(a + bx^3)^3} dx &= \frac{(Ab - aB)x^{7/2}}{6ab(a + bx^3)^2} + \frac{\left(\frac{5Ab}{2} + \frac{7aB}{2}\right) \int \frac{x^{5/2}}{(a+bx^3)^2} dx}{6ab} \\
&= \frac{(Ab - aB)x^{7/2}}{6ab(a + bx^3)^2} - \frac{(5Ab + 7aB)\sqrt{x}}{36ab^2(a + bx^3)} + \frac{(5Ab + 7aB) \int \frac{1}{\sqrt{x}(a+bx^3)} dx}{72ab^2} \\
&= \frac{(Ab - aB)x^{7/2}}{6ab(a + bx^3)^2} - \frac{(5Ab + 7aB)\sqrt{x}}{36ab^2(a + bx^3)} + \frac{(5Ab + 7aB) \operatorname{Subst}\left(\int \frac{1}{a+bx^6} dx, x, \sqrt{x}\right)}{36ab^2} \\
&= \frac{(Ab - aB)x^{7/2}}{6ab(a + bx^3)^2} - \frac{(5Ab + 7aB)\sqrt{x}}{36ab^2(a + bx^3)} + \frac{(5Ab + 7aB) \operatorname{Subst}\left(\int \frac{\sqrt[6]{a} - \frac{1}{2}\sqrt{3} \sqrt[6]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{108a^{11/6}b^2} \\
&= \frac{(Ab - aB)x^{7/2}}{6ab(a + bx^3)^2} - \frac{(5Ab + 7aB)\sqrt{x}}{36ab^2(a + bx^3)} + \frac{(5Ab + 7aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} - \frac{(5Ab + 7aB) \operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{144\sqrt{3}a^{11/6}b^{13/6}} \\
&= \frac{(Ab - aB)x^{7/2}}{6ab(a + bx^3)^2} - \frac{(5Ab + 7aB)\sqrt{x}}{36ab^2(a + bx^3)} + \frac{(5Ab + 7aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} - \frac{(5Ab + 7aB) \log\left(\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2\right)}{144\sqrt{3}a^{11/6}b^{13/6}} \\
&= \frac{(Ab - aB)x^{7/2}}{6ab(a + bx^3)^2} - \frac{(5Ab + 7aB)\sqrt{x}}{36ab^2(a + bx^3)} - \frac{(5Ab + 7aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}} + \frac{(5Ab + 7aB) \log\left(\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2\right)}{216a^{11/6}b^{13/6}}
\end{aligned}$$

Mathematica [C] time = 0.11, size = 92, normalized size = 0.28

$$\frac{\sqrt{x} \left(a \left(-7a^2B - ab \left(5A + 13Bx^3 \right) + Ab^2x^3 \right) + \left(a + bx^3 \right)^2 \left(7aB + 5Ab \right) {}_2F_1 \left(\frac{1}{6}, 1; \frac{7}{6}; -\frac{bx^3}{a} \right) \right)}{36a^2b^2 \left(a + bx^3 \right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] (Sqrt[x]*(a*(-7*a^2*B + A*b^2*x^3 - a*b*(5*A + 13*B*x^3)) + (5*A*b + 7*a*B)*(a + b*x^3)^2*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)]))/(36*a^2*b^2*(a + b*x^3)^2)

IntegrateAlgebraic [A] time = 0.78, size = 213, normalized size = 0.65

$$\frac{(7aB + 5Ab) \tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} + \frac{(-7aB - 5Ab) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}x}\right)}{216a^{11/6}b^{13/6}} + \frac{(7aB + 5Ab) \tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{b}x}\right)}{72\sqrt{3}a^{11/6}b^{13/6}} - \frac{\sqrt{x}(7a^2B + 5aAb + 13abBx^3 - Ab^2x^3)}{36ab^2(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] -1/36*(Sqrt[x]*(5*a*A*b + 7*a^2*B - A*b^2*x^3 + 13*a*b*B*x^3))/(a*b^2*(a + b*x^3)^2) + ((5*A*b + 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(11/6)*b^(13/6)) + ((-5*A*b - 7*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x]])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)]/(72*Sqrt[3]*a^(11/6)*b^(13/6))

fricas [B] time = 0.79, size = 2714, normalized size = 8.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/432*(4*sqrt(3)*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*b^4*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/3) + (49*B^2*a^2 + 70*A*B*a*b + 25*A^2*b^2)*x + (7*B*a^3*b^2 + 5*A*a^2*b^3)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)))*a^9*b^11*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(5/6) - 2*sqrt(3)*(7*B*a^10*b^11 + 5*A*a^9*b^12)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(5/6) + sqrt(3)*(117649*B^6*a^6 + 504210*A*B

$$\begin{aligned}
& ^5a^5b + 900375A^2B^4a^4b^2 + 857500A^3B^3a^3b^3 + 459375A^4B^2 \\
& *a^2b^4 + 131250A^5B*a*b^5 + 15625A^6*b^6) / ((117649B^6a^6 + 504210A* \\
& B^5a^5b + 900375A^2B^4a^4b^2 + 857500A^3B^3a^3b^3 + 459375A^4B^2 \\
& *a^2b^4 + 131250A^5B*a*b^5 + 15625A^6*b^6)) + 4*\sqrt{3}*(a*b^4*x^6 + 2 \\
& *a^2*b^3*x^3 + a^3*b^2)*(- (117649B^6a^6 + 504210A*B^5a^5b + 900375A^2 \\
& *B^4a^4b^2 + 857500A^3B^3a^3b^3 + 459375A^4B^2a^2b^4 + 131250A^5 \\
& *B*a*b^5 + 15625A^6*b^6) / (a^{11}b^{13}))^{(1/6)}*\arctan(1/3*(2*\sqrt{3})*\sqrt{a^4 \\
& *b^4*(- (117649B^6a^6 + 504210A*B^5a^5b + 900375A^2B^4a^4b^2 + 8575 \\
& 00A^3B^3a^3b^3 + 459375A^4B^2a^2b^4 + 131250A^5B*a*b^5 + 15625A^ \\
& 6*b^6) / (a^{11}b^{13}))^{(1/3)} + (49B^2a^2 + 70A*B*a*b + 25A^2b^2)*x - (7B \\
& *a^3b^2 + 5A*a^2b^3)*\sqrt{x}*(- (117649B^6a^6 + 504210A*B^5a^5b + 90 \\
& 0375A^2B^4a^4b^2 + 857500A^3B^3a^3b^3 + 459375A^4B^2a^2b^4 + 13 \\
& 1250A^5B*a*b^5 + 15625A^6*b^6) / (a^{11}b^{13}))^{(1/6)})*a^9b^{11}*(- (117649B^ \\
& 6a^6 + 504210A*B^5a^5b + 900375A^2B^4a^4b^2 + 857500A^3B^3a^3b^ \\
& 3 + 459375A^4B^2a^2b^4 + 131250A^5B*a*b^5 + 15625A^6*b^6) / (a^{11}b^{13} \\
&))^{(5/6)} - 2*\sqrt{3}*(7B*a^{10}b^{11} + 5A*a^9b^{12})*\sqrt{x}*(- (117649B^6a \\
& ^6 + 504210A*B^5a^5b + 900375A^2B^4a^4b^2 + 857500A^3B^3a^3b^3 + \\
& 459375A^4B^2a^2b^4 + 131250A^5B*a*b^5 + 15625A^6*b^6) / (a^{11}b^{13}))^{(\\
& 5/6)} - \sqrt{3}*(117649B^6a^6 + 504210A*B^5a^5b + 900375A^2B^4a^4b \\
& ^2 + 857500A^3B^3a^3b^3 + 459375A^4B^2a^2b^4 + 131250A^5B*a*b^5 + \\
& 15625A^6*b^6) / ((117649B^6a^6 + 504210A*B^5a^5b + 900375A^2B^4a^4* \\
& b^2 + 857500A^3B^3a^3b^3 + 459375A^4B^2a^2b^4 + 131250A^5B*a*b^5 \\
& + 15625A^6*b^6)) + (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(- (117649B^6a^6 \\
& + 504210A*B^5a^5b + 900375A^2B^4a^4b^2 + 857500A^3B^3a^3b^3 + 4 \\
& 59375A^4B^2a^2b^4 + 131250A^5B*a*b^5 + 15625A^6*b^6) / (a^{11}b^{13}))^{(1 \\
& /6)}*\log(a^4b^4*(- (117649B^6a^6 + 504210A*B^5a^5b + 900375A^2B^4a^4 \\
& *b^2 + 857500A^3B^3a^3b^3 + 459375A^4B^2a^2b^4 + 131250A^5B*a*b^5 \\
& + 15625A^6*b^6) / (a^{11}b^{13}))^{(1/3)} + (49B^2a^2 + 70A*B*a*b + 25A^2b^ \\
& 2)*x + (7B*a^3b^2 + 5A*a^2b^3)*\sqrt{x}*(- (117649B^6a^6 + 504210A*B^5 \\
& *a^5b + 900375A^2B^4a^4b^2 + 857500A^3B^3a^3b^3 + 459375A^4B^2a \\
& ^2b^4 + 131250A^5B*a*b^5 + 15625A^6*b^6) / (a^{11}b^{13}))^{(1/6)} - (a*b^4*x \\
& ^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(- (117649B^6a^6 + 504210A*B^5a^5b + 9003 \\
& 75A^2B^4a^4b^2 + 857500A^3B^3a^3b^3 + 459375A^4B^2a^2b^4 + 1312 \\
& 50A^5B*a*b^5 + 15625A^6*b^6) / (a^{11}b^{13}))^{(1/6)}*\log(a^4b^4*(- (117649B^ \\
& 6a^6 + 504210A*B^5a^5b + 900375A^2B^4a^4b^2 + 857500A^3B^3a^3b^ \\
& 3 + 459375A^4B^2a^2b^4 + 131250A^5B*a*b^5 + 15625A^6*b^6) / (a^{11}b^{13} \\
&))^{(1/3)} + (49B^2a^2 + 70A*B*a*b + 25A^2b^2)*x - (7B*a^3b^2 + 5A*a^ \\
& 2b^3)*\sqrt{x}*(- (117649B^6a^6 + 504210A*B^5a^5b + 900375A^2B^4a^4* \\
& b^2 + 857500A^3B^3a^3b^3 + 459375A^4B^2a^2b^4 + 131250A^5B*a*b^5 \\
& + 15625A^6*b^6) / (a^{11}b^{13}))^{(1/6)} + 2*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b \\
& ^2)*(- (117649B^6a^6 + 504210A*B^5a^5b + 900375A^2B^4a^4b^2 + 85750 \\
& 0A^3B^3a^3b^3 + 459375A^4B^2a^2b^4 + 131250A^5B*a*b^5 + 15625A^6 \\
& *b^6) / (a^{11}b^{13}))^{(1/6)}*\log(a^2b^2*(- (117649B^6a^6 + 504210A*B^5a^5b \\
& + 900375A^2B^4a^4b^2 + 857500A^3B^3a^3b^3 + 459375A^4B^2a^2b^4 \\
& + 131250A^5B*a*b^5 + 15625A^6*b^6) / (a^{11}b^{13}))^{(1/6)} + (7B*a + 5A*b)
\end{aligned}$$

sqrt(x)) - 2(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*log(-a^2*b^2*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6) + (7*B*a + 5*A*b)*sqrt(x)) - 12*((13*B*a*b - A*b^2)*x^3 + 7*B*a^2 + 5*A*a*b)*sqrt(x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)

giac [A] time = 0.23, size = 328, normalized size = 1.00

$$\frac{\sqrt{3} \left(7 (ab)^{\frac{1}{2}} Ba + 5 (ab)^{\frac{1}{2}} Ab \right) \log(\sqrt{3} \sqrt{\frac{a}{b}} \left(\frac{a}{b} \right)^{\frac{1}{3}} + x + \left(\frac{a}{b} \right)^{\frac{1}{3}})}{432 a^2 b^3} - \frac{\sqrt{3} \left(7 (ab)^{\frac{1}{2}} Ba + 5 (ab)^{\frac{1}{2}} Ab \right) \log(-\sqrt{3} \sqrt{\frac{a}{b}} \left(\frac{a}{b} \right)^{\frac{1}{3}} + x + \left(\frac{a}{b} \right)^{\frac{1}{3}})}{432 a^2 b^3} + \frac{\left(7 (ab)^{\frac{1}{2}} Ba + 5 (ab)^{\frac{1}{2}} Ab \right) \arctan\left(\frac{\sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{3}} + x}{\left(\frac{a}{b} \right)^{\frac{1}{3}}}\right)}{216 a^2 b^3} - \frac{\left(7 (ab)^{\frac{1}{2}} Ba + 5 (ab)^{\frac{1}{2}} Ab \right) \arctan\left(\frac{-\sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{3}} + x}{\left(\frac{a}{b} \right)^{\frac{1}{3}}}\right)}{216 a^2 b^3} + \frac{\left(7 (ab)^{\frac{1}{2}} Ba + 5 (ab)^{\frac{1}{2}} Ab \right) \arctan\left(\frac{\sqrt{3}}{\left(\frac{a}{b} \right)^{\frac{1}{3}}}\right)}{108 a^2 b^3} - \frac{13 Bab^{\frac{1}{2}} - Ab^{\frac{3}{2}} + 7 Ba^{\frac{1}{2}} \sqrt{a} + 5 Aab \sqrt{a}}{36 (b^3 + a)^{\frac{1}{2}} ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/432*sqrt(3)*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^3) - 1/432*sqrt(3)*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^3) + 1/216*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^3) + 1/216*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(-sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^3) + 1/108*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b^3) - 1/36*(13*B*a*b*x^(7/2) - A*b^2*x^(7/2) + 7*B*a^2*sqrt(x) + 5*A*a*b*sqrt(x))/((b*x^3 + a)^2*a*b^2)

maple [A] time = 0.16, size = 416, normalized size = 1.27

$$\frac{5 \left(\frac{a}{b} \right)^{\frac{1}{2}} A \arctan\left(\frac{\sqrt{3}}{\left(\frac{a}{b} \right)^{\frac{1}{3}}}\right)}{108 a^2 b^3} - \frac{5 \left(\frac{a}{b} \right)^{\frac{1}{2}} A \arctan\left(\frac{2\sqrt{3}}{\left(\frac{a}{b} \right)^{\frac{1}{3}} + \sqrt{3}}\right)}{216 a^2 b^3} + \frac{5 \left(\frac{a}{b} \right)^{\frac{1}{2}} A \arctan\left(\frac{2\sqrt{3}}{\left(\frac{a}{b} \right)^{\frac{1}{3}} - \sqrt{3}}\right)}{216 a^2 b^3} + \frac{5 \sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} A \ln\left(x - \sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{3}} \sqrt{x} + \left(\frac{a}{b} \right)^{\frac{1}{3}}\right)}{432 a^2 b^3} + \frac{5 \sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} A \ln\left(x + \sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{3}} \sqrt{x} + \left(\frac{a}{b} \right)^{\frac{1}{3}}\right)}{432 a^2 b^3} + \frac{7 \left(\frac{a}{b} \right)^{\frac{1}{2}} B \arctan\left(\frac{\sqrt{3}}{\left(\frac{a}{b} \right)^{\frac{1}{3}}}\right)}{108 a^2 b^3} + \frac{7 \left(\frac{a}{b} \right)^{\frac{1}{2}} B \arctan\left(\frac{2\sqrt{3}}{\left(\frac{a}{b} \right)^{\frac{1}{3}} + \sqrt{3}}\right)}{216 a^2 b^3} + \frac{7 \left(\frac{a}{b} \right)^{\frac{1}{2}} B \arctan\left(\frac{2\sqrt{3}}{\left(\frac{a}{b} \right)^{\frac{1}{3}} - \sqrt{3}}\right)}{216 a^2 b^3} + \frac{7 \sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} B \ln\left(x - \sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{3}} \sqrt{x} + \left(\frac{a}{b} \right)^{\frac{1}{3}}\right)}{432 a^2 b^3} + \frac{7 \sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{2}} B \ln\left(x + \sqrt{3} \left(\frac{a}{b} \right)^{\frac{1}{3}} \sqrt{x} + \left(\frac{a}{b} \right)^{\frac{1}{3}}\right)}{432 a^2 b^3} + \frac{(a^2 - 13ab + 3a^2) \sqrt{a}}{36 (b^3 + a)^{\frac{1}{2}} ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x)

[Out] 2*(1/72*(A*b-13*B*a)/a/b*x^(7/2)-1/72*(5*A*b+7*B*a)/b^2*x^(1/2))/(b*x^3+a)^2+5/108/b/a^2*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A+7/108/b^2/a*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B-5/432/b/a^2*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A-7/432/b^2/a*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+5/216/b/a^2*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*A+7/216/b^2/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*B+5/432/b/a^2*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+7/432/b^2/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+5/216/b/a^2*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*A+7/216/b^2/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*B

maxima [A] time = 1.40, size = 341, normalized size = 1.04

$$\frac{(13 Bab - Ab^2)x^2 + (7 Ba^2 + 5 Aab)\sqrt{x}}{36(ab^4x^6 + 2a^2b^3x^3 + a^3b^2)} + \frac{\sqrt{3}(7 Ba + 5 Ab)\log\left(\frac{\sqrt{3}a^{\frac{1}{2}}b^{\frac{1}{2}}\sqrt{c+bx+a^3}}{a^{\frac{1}{2}}b^{\frac{1}{2}}}\right) - \sqrt{3}(7 Ba + 5 Ab)\log\left(-\sqrt{3}a^{\frac{1}{2}}b^{\frac{1}{2}}\sqrt{c+bx+a^3}\right)}{a^{\frac{1}{2}}b^{\frac{1}{2}}} + \frac{4(7 Bab^{\frac{1}{2}} + 5 Ab^{\frac{3}{2}})\arctan\left(\frac{b^{\frac{1}{2}}\sqrt{c}}{\sqrt{a^3b^3}}\right)}{432ab^2} + \frac{2(7 Ba^{\frac{1}{2}}b^{\frac{1}{2}} + 5 Aa^{\frac{1}{2}}b^{\frac{3}{2}})\arctan\left(\frac{\sqrt{3}a^{\frac{1}{2}}b^{\frac{1}{2}} - 2b^{\frac{1}{2}}\sqrt{c}}{\sqrt{a^3b^3}}\right)}{ab^{\frac{1}{2}}\sqrt{a^3b^3}} + \frac{2(7 Ba^{\frac{1}{2}}b^{\frac{1}{2}} + 5 Aa^{\frac{1}{2}}b^{\frac{3}{2}})\arctan\left(-\frac{\sqrt{3}a^{\frac{1}{2}}b^{\frac{1}{2}} - 2b^{\frac{1}{2}}\sqrt{c}}{\sqrt{a^3b^3}}\right)}{ab^{\frac{1}{2}}\sqrt{a^3b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")

[Out]
$$-1/36*((13*B*a*b - A*b^2)*x^{7/2} + (7*B*a^2 + 5*A*a*b)*\sqrt{x})/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2) + 1/432*(\sqrt{3}*(7*B*a + 5*A*b)*\log(\sqrt{3}*a^{1/6}*b^{1/6}*\sqrt{x} + b^{1/3}*x + a^{1/3}))/a^{5/6}*b^{1/6} - \sqrt{3}*(7*B*a + 5*A*b)*\log(-\sqrt{3}*a^{1/6}*b^{1/6}*\sqrt{x} + b^{1/3}*x + a^{1/3}))/a^{5/6}*b^{1/6} + 4*(7*B*a*b^{1/3} + 5*A*b^{4/3})*\arctan(b^{1/3}*\sqrt{x}/\sqrt{a^{1/3}*b^{1/3}})/a^{2/3}*b^{1/3}*\sqrt{a^{1/3}*b^{1/3}} + 2*(7*B*a^{4/3}*b^{1/3} + 5*A*a^{1/3}*b^{4/3})*\arctan((\sqrt{3}*a^{1/6}*b^{1/6} + 2*b^{1/3}*\sqrt{x})/\sqrt{a^{1/3}*b^{1/3}})/a*b^{1/3}*\sqrt{a^{1/3}*b^{1/3}} + 2*(7*B*a^{4/3}*b^{1/3} + 5*A*a^{1/3}*b^{4/3})*\arctan(-(\sqrt{3}*a^{1/6}*b^{1/6} - 2*b^{1/3}*\sqrt{x})/\sqrt{a^{1/3}*b^{1/3}})/a*b^{1/3}*\sqrt{a^{1/3}*b^{1/3}})))/(a*b^2)$$

mupad [B] time = 2.98, size = 1944, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x)

[Out]
$$\left(\operatorname{atan}\left(\frac{(((((5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2)))/(279936*(-a)^{(23/6)}*b^{(19/6)}) - (x^{(1/2)}*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3)))/(279936*a^4*b^3)))*(5*A*b + 7*B*a)*i\right)/(216*(-a)^{(11/6)}*b^{(13/6)}) - \left(\frac{((5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))}{279936*(-a)^{(23/6)}*b^{(19/6)}} + (x^{(1/2)}*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3)\right)*(5*A*b + 7*B*a)*i\right)/(216*(-a)^{(11/6)}*b^{(13/6)})\right)/\left(\frac{(((((5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2)))/(279936*(-a)^{(23/6)}*b^{(19/6)}) - (x^{(1/2)}*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3)))/(279936*a^4*b^3)))*(5*A*b + 7*B*a)}{216*(-a)^{(11/6)}*b^{(13/6)}} + \left(\frac{((5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))}{279936*(-a)^{(23/6)}*b^{(19/6)}} + (x^{(1/2)}*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3)\right)*(5*A*b + 7*B*a)}{216*(-a)^{(11/6)}*b^{(13/6)}}\right)\right)*i\right)/(108*(-a)^{(11/6)}*b^{(13/6)}) - \left(\frac{(x^{(1/2)}*(5*A*b + 7*B*a))}{(36*b^2)} - \frac{(x^{(7/2)}*(A*b - 13*B*a))}{(36*a*b)}\right)/(a^2 + b^2*x^6 + 2*a*b*x^3) + \operatorname{atan}\left(\frac{((3^{(1/2)}*i)/2 - 1/2)*((x^{(1/2)}*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3)))/(279936*a^4*b^3)}{216*(-a)^{(11/6)}*b^{(13/6)}}\right)$$

$$\begin{aligned}
& a^4 + 7350A^2B^2a^2b^2 + 6860A^3B^3a^3b + 3500A^3B^3a^3b^3) / (279936 * \\
& a^4b^3) - (((3^{(1/2)} * i) / 2 - 1/2) * (5A * b + 7B * a) * (125A^3b^3 + 343B^3a^3 \\
& + 735A^2B^2a^2b + 525A^2B^2a^2b^2)) / (279936 * (-a)^{(23/6)} * b^{(19/6)}) * (5A * b + 7B * a) * i) / (216 * (-a)^{(11/6)} * b^{(13/6)}) + (((3^{(1/2)} * i) / 2 - 1/2) * ((x^{(1/2)} * (625A^4b^4 + 2401B^4a^4 + 7350A^2B^2a^2b^2 + 6860A^3B^3a^3b + 3500A^3B^3a^3b^3)) / (279936 * a^4 * b^3) + (((3^{(1/2)} * i) / 2 - 1/2) * (5A * b + 7B * a) * (125A^3b^3 + 343B^3a^3 + 735A^2B^2a^2b + 525A^2B^2a^2b^2)) / (279936 * (-a)^{(23/6)} * b^{(19/6)}) * (5A * b + 7B * a) * i) / (216 * (-a)^{(11/6)} * b^{(13/6)})) / (((3^{(1/2)} * i) / 2 - 1/2) * ((x^{(1/2)} * (625A^4b^4 + 2401B^4a^4 + 7350A^2B^2a^2b^2 + 6860A^3B^3a^3b + 3500A^3B^3a^3b^3)) / (279936 * a^4 * b^3) - (((3^{(1/2)} * i) / 2 - 1/2) * (5A * b + 7B * a) * (125A^3b^3 + 343B^3a^3 + 735A^2B^2a^2b + 525A^2B^2a^2b^2)) / (279936 * (-a)^{(23/6)} * b^{(19/6)}) * (5A * b + 7B * a)) / (216 * (-a)^{(11/6)} * b^{(13/6)}) - (((3^{(1/2)} * i) / 2 - 1/2) * ((x^{(1/2)} * (625A^4b^4 + 2401B^4a^4 + 7350A^2B^2a^2b^2 + 6860A^3B^3a^3b + 3500A^3B^3a^3b^3)) / (279936 * a^4 * b^3) + (((3^{(1/2)} * i) / 2 - 1/2) * (5A * b + 7B * a) * (125A^3b^3 + 343B^3a^3 + 735A^2B^2a^2b + 525A^2B^2a^2b^2)) / (279936 * (-a)^{(23/6)} * b^{(19/6)}) * (5A * b + 7B * a)) / (216 * (-a)^{(11/6)} * b^{(13/6)})) * ((3^{(1/2)} * i) / 2 - 1/2) * (5A * b + 7B * a) * i) / (108 * (-a)^{(11/6)} * b^{(13/6)}) + (atan((((3^{(1/2)} * i) / 2 + 1/2) * ((x^{(1/2)} * (625A^4b^4 + 2401B^4a^4 + 7350A^2B^2a^2b^2 + 6860A^3B^3a^3b + 3500A^3B^3a^3b^3)) / (279936 * a^4 * b^3) - (((3^{(1/2)} * i) / 2 + 1/2) * (5A * b + 7B * a) * (125A^3b^3 + 343B^3a^3 + 735A^2B^2a^2b + 525A^2B^2a^2b^2)) / (279936 * (-a)^{(23/6)} * b^{(19/6)}) * (5A * b + 7B * a) * i) / (216 * (-a)^{(11/6)} * b^{(13/6)}) + (((3^{(1/2)} * i) / 2 + 1/2) * ((x^{(1/2)} * (625A^4b^4 + 2401B^4a^4 + 7350A^2B^2a^2b^2 + 6860A^3B^3a^3b + 3500A^3B^3a^3b^3)) / (279936 * a^4 * b^3) + (((3^{(1/2)} * i) / 2 + 1/2) * (5A * b + 7B * a) * (125A^3b^3 + 343B^3a^3 + 735A^2B^2a^2b + 525A^2B^2a^2b^2)) / (279936 * (-a)^{(23/6)} * b^{(19/6)}) * (5A * b + 7B * a) * i) / (216 * (-a)^{(11/6)} * b^{(13/6)})) / (((3^{(1/2)} * i) / 2 + 1/2) * ((x^{(1/2)} * (625A^4b^4 + 2401B^4a^4 + 7350A^2B^2a^2b^2 + 6860A^3B^3a^3b + 3500A^3B^3a^3b^3)) / (279936 * a^4 * b^3) - (((3^{(1/2)} * i) / 2 + 1/2) * (5A * b + 7B * a) * (125A^3b^3 + 343B^3a^3 + 735A^2B^2a^2b + 525A^2B^2a^2b^2)) / (279936 * (-a)^{(23/6)} * b^{(19/6)}) * (5A * b + 7B * a)) / (216 * (-a)^{(11/6)} * b^{(13/6)}) - (((3^{(1/2)} * i) / 2 + 1/2) * ((x^{(1/2)} * (625A^4b^4 + 2401B^4a^4 + 7350A^2B^2a^2b^2 + 6860A^3B^3a^3b + 3500A^3B^3a^3b^3)) / (279936 * a^4 * b^3) + (((3^{(1/2)} * i) / 2 + 1/2) * (5A * b + 7B * a) * (125A^3b^3 + 343B^3a^3 + 735A^2B^2a^2b + 525A^2B^2a^2b^2)) / (279936 * (-a)^{(23/6)} * b^{(19/6)}) * (5A * b + 7B * a)) / (216 * (-a)^{(11/6)} * b^{(13/6)})) * ((3^{(1/2)} * i) / 2 + 1/2) * (5A * b + 7B * a) * i) / (108 * (-a)^{(11/6)} * b^{(13/6)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**3,x)

[Out] Timed out

$$3.169 \quad \int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=327

$$\frac{(5aB + 7Ab) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{13/6} b^{11/6}} - \frac{(5aB + 7Ab) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{13/6} b^{11/6}} - \frac{(5aB + 7Ab) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{13/6} b^{11/6}}$$

Rubi [A] time = 0.60, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, number of rules / integrand size = 0.409, Rules used = {457, 290, 329, 295, 634, 618, 204, 628, 205}

$$\frac{(5aB + 7Ab) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{13/6} b^{11/6}} - \frac{(5aB + 7Ab) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{13/6} b^{11/6}} - \frac{(5aB + 7Ab) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt{6}\sqrt{x}}{\sqrt{a}}\right)}{216a^{13/6} b^{11/6}} + \frac{(5aB + 7Ab) \tan^{-1}\left(\frac{2\sqrt{6}\sqrt{x}}{\sqrt{a}} + \sqrt{3}\right)}{216a^{13/6} b^{11/6}} + \frac{(5aB + 7Ab) \tan^{-1}\left(\frac{\sqrt{6}\sqrt{x}}{\sqrt{a}}\right)}{108a^{13/6} b^{11/6}} + \frac{x^{5/2}(5aB + 7Ab)}{36a^2 b (a + bx^3)} + \frac{x^{5/2}(Ab - aB)}{6ab (a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3, x]

[Out] ((A*b - a*B)*x^(5/2))/(6*a*b*(a + b*x^3)^2) + ((7*A*b + 5*a*B)*x^(5/2))/(36*a^2*b*(a + b*x^3)) - ((7*A*b + 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x]/(144*Sqrt[3]*a^(13/6)*b^(11/6)) - ((7*A*b + 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x]/(144*Sqrt[3]*a^(13/6)*b^(11/6))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}

, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 295

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*cos[((2*k
- 1)*m*Pi)/n] - s*cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*cos[((2*k - 1)*m*Pi)/n] + s*cos[((2*k
- 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x]]/(a*n*s^m) + Dist[(2*r^(
m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 329

```
Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^(
n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2} (A + Bx^3)}{(a + bx^3)^3} dx &= \frac{(Ab - aB)x^{5/2}}{6ab(a + bx^3)^2} + \frac{\left(\frac{7Ab}{2} + \frac{5aB}{2}\right) \int \frac{x^{3/2}}{(a+bx^3)^2} dx}{6ab} \\
&= \frac{(Ab - aB)x^{5/2}}{6ab(a + bx^3)^2} + \frac{(7Ab + 5aB)x^{5/2}}{36a^2b(a + bx^3)} + \frac{(7Ab + 5aB) \int \frac{x^{3/2}}{a+bx^3} dx}{72a^2b} \\
&= \frac{(Ab - aB)x^{5/2}}{6ab(a + bx^3)^2} + \frac{(7Ab + 5aB)x^{5/2}}{36a^2b(a + bx^3)} + \frac{(7Ab + 5aB) \text{Subst}\left(\int \frac{x^4}{a+bx^6} dx, x, \sqrt{x}\right)}{36a^2b} \\
&= \frac{(Ab - aB)x^{5/2}}{6ab(a + bx^3)^2} + \frac{(7Ab + 5aB)x^{5/2}}{36a^2b(a + bx^3)} + \frac{(7Ab + 5aB) \text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2} + \frac{1}{2}\sqrt{3} \sqrt[6]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{108a^{13/6}b^{5/3}} \\
&= \frac{(Ab - aB)x^{5/2}}{6ab(a + bx^3)^2} + \frac{(7Ab + 5aB)x^{5/2}}{36a^2b(a + bx^3)} + \frac{(7Ab + 5aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}} + \frac{(7Ab + 5aB) \text{Subst}\left(\int \frac{1}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x, \sqrt{x}\right)}{144a^{13/6}b^{11/6}} \\
&= \frac{(Ab - aB)x^{5/2}}{6ab(a + bx^3)^2} + \frac{(7Ab + 5aB)x^{5/2}}{36a^2b(a + bx^3)} + \frac{(7Ab + 5aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}} + \frac{(7Ab + 5aB) \log\left(\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2\right)}{144a^{13/6}b^{11/6}} \\
&= \frac{(Ab - aB)x^{5/2}}{6ab(a + bx^3)^2} + \frac{(7Ab + 5aB)x^{5/2}}{36a^2b(a + bx^3)} - \frac{(7Ab + 5aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}} + \frac{(7Ab + 5aB) \log\left(\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2\right)}{144a^{13/6}b^{11/6}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 62, normalized size = 0.19

$$\frac{2x^{5/2} \left((Ab - aB) {}_2F_1\left(\frac{5}{6}, 3; \frac{11}{6}; -\frac{bx^3}{a}\right) + aB {}_2F_1\left(\frac{5}{6}, 2; \frac{11}{6}; -\frac{bx^3}{a}\right) \right)}{5a^3b}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] (2*x^(5/2)*(a*B*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a)] + (A*b - a*B)*Hypergeometric2F1[5/6, 3, 11/6, -((b*x^3)/a)]))/(5*a^3*b)

IntegrateAlgebraic [A] time = 0.74, size = 212, normalized size = 0.65

$$\frac{(5aB + 7Ab) \tan^{-1}\left(\frac{\sqrt[5]{b}\sqrt{x}}{\sqrt{a}}\right)}{108a^{13/6}b^{11/6}} + \frac{(-5aB - 7Ab) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[5]{a}\sqrt[5]{b}\sqrt{x}}\right)}{216a^{13/6}b^{11/6}} - \frac{(5aB + 7Ab) \tanh^{-1}\left(\frac{\sqrt{3}\sqrt[5]{a}\sqrt[5]{b}\sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{72\sqrt{3}a^{13/6}b^{11/6}} - \frac{x^{5/2}(a^2B - 13aAb - 5abBx^3 - 7Ab^2x^3)}{36a^2b(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] -1/36*(x^(5/2)*(-13*a*A*b + a^2*B - 7*A*b^2*x^3 - 5*a*b*B*x^3))/(a^2*b*(a + b*x^3)^2) + ((7*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(13/6)*b^(11/6)) + ((-7*A*b - 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x]])/(216*a^(13/6)*b^(11/6)) - ((7*A*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)]/(72*Sqrt[3]*a^(13/6)*b^(11/6))

fricas [B] time = 0.81, size = 3951, normalized size = 12.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] -1/432*(4*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt((3125*B^5*a^16*b^9 + 21875*A*B^4*a^15*b^10 + 1250*A^2*B^3*a^14*b^11 + 85750*A^3*B^2*a^13*b^12 + 60025*A^4*B*a^12*b^13 + 16807*A^5*a^11*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (9765625*B^10*a^10 + 136718750*A*B^9*a^9*b + 861328125*A^2*B^8*a^8*b^2 + 3215625000*A^3*B^7*a^7*b^3 + 7878281250*A^4*B^6*a^6*b^4 + 13235512500*A^5*B^5*a^5*b^5 + 15441431250*A^6*B^4*a^4*b^6 + 12353145000*A^7*B^3*a^3*b^7 + 6485401125*A^8*B^2*a^2*b^8 + 2017680350*A^9*B*a*b^9 + 282475249*A^10*b^10)*x - (15625*B^6*a^15*b^7 + 131250*A*B^5*a^14*b^8 + 459375*A^2*B^4*a^13*b^9 + 857500*A^3*B^3*a^12*b^10 + 900375*A^4*B^2*a^11*b^11 + 504210*A^5*B*a^10*b^12 + 117649*A^6*a^9*b^13)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(2/3))*a^2*b^2*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210

$$\begin{aligned}
& *A^5*B*a*b^5 + 117649*A^6*b^6)/(a^{13}*b^{11})^{(1/6)} - 2*\sqrt{3}*(3125*B^5*a^7 \\
& *b^2 + 21875*A*B^4*a^6*b^3 + 61250*A^2*B^3*a^5*b^4 + 85750*A^3*B^2*a^4*b^5 \\
& + 60025*A^4*B*a^3*b^6 + 16807*A^5*a^2*b^7)*\sqrt{x}*(-(15625*B^6*a^6 + 13125 \\
& 0*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^ \\
& 4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^{13}*b^{11})^{(1/6)} + s \\
& \text{qrt}(3)*(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 85750 \\
& 0*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^ \\
& 6*b^6))/(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 8575 \\
& 00*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A \\
& ^6*b^6)) + 4*\sqrt{3}*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 \\
& + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 9 \\
& 00375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^{13}*b^{11})^{(\\
& 1/6)}*\arctan(1/3*(2*\sqrt{3})*\sqrt{-(3125*B^5*a^{16}*b^9 + 21875*A*B^4*a^{15}*b^{10} \\
& + 61250*A^2*B^3*a^{14}*b^{11} + 85750*A^3*B^2*a^{13}*b^{12} + 60025*A^4*B*a^{12}*b^{1 \\
& 3} + 16807*A^5*a^{11}*b^{14})*\sqrt{x}*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 45 \\
& 9375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 50 \\
& 4210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^{13}*b^{11})^{(5/6)} + (9765625*B^{10}*a^{10} \\
& + 136718750*A*B^9*a^9*b + 861328125*A^2*B^8*a^8*b^2 + 3215625000*A^3*B^7*a^ \\
& 7*b^3 + 7878281250*A^4*B^6*a^6*b^4 + 13235512500*A^5*B^5*a^5*b^5 + 15441431 \\
& 250*A^6*B^4*a^4*b^6 + 12353145000*A^7*B^3*a^3*b^7 + 6485401125*A^8*B^2*a^2* \\
& b^8 + 2017680350*A^9*B*a*b^9 + 282475249*A^{10}*b^{10})*x - (15625*B^6*a^{15}*b^7 \\
& + 131250*A*B^5*a^{14}*b^8 + 459375*A^2*B^4*a^{13}*b^9 + 857500*A^3*B^3*a^{12}*b^ \\
& 10 + 900375*A^4*B^2*a^{11}*b^{11} + 504210*A^5*B*a^{10}*b^{12} + 117649*A^6*a^9*b^ \\
& 13)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500* \\
& A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6* \\
& b^6)/(a^{13}*b^{11})^{(2/3)})*a^2*b^2*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 45 \\
& 9375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 50 \\
& 4210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^{13}*b^{11})^{(1/6)} - 2*\sqrt{3}*(3125*B^5 \\
& *a^7*b^2 + 21875*A*B^4*a^6*b^3 + 61250*A^2*B^3*a^5*b^4 + 85750*A^3*B^2*a^4* \\
& b^5 + 60025*A^4*B*a^3*b^6 + 16807*A^5*a^2*b^7)*\sqrt{x}*(-(15625*B^6*a^6 + 1 \\
& 31250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90037 \\
& 5*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^{13}*b^{11})^{(1/6)} \\
& - \sqrt{3}*(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 8 \\
& 57500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 11764 \\
& 9*A^6*b^6))/(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + \\
& 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 1176 \\
& 49*A^6*b^6)) - 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 + 1 \\
& 31250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90037 \\
& 5*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^{13}*b^{11})^{(1/6)} \\
& *log(a^{11}*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^ \\
& 2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + \\
& 117649*A^6*b^6)/(a^{13}*b^{11})^{(5/6)} + (3125*B^5*a^5 + 21875*A*B^4*a^4*b + 61 \\
& 250*A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*A^5 \\
& *b^5)*\sqrt{x})) + 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 + \\
& 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900
\end{aligned}$$

[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $\frac{1}{36} \cdot (5B*ab*x^{11/2} + 7A*b^2*x^{11/2} - B*a^2*x^{5/2} + 13A*a*b*x^{5/2}) / ((b*x^3 + a)^2*a^2*b) - \frac{1}{432} \cdot \sqrt{3} \cdot (5*(a*b^5)^{5/6} * B*a + 7*(a*b^5)^{5/6} * A*b) \cdot \log(\sqrt{3} \cdot \sqrt{x} \cdot (a/b)^{1/6} + x + (a/b)^{1/3}) / (a^3*b^6) + \frac{1}{432} \cdot \sqrt{3} \cdot (5*(a*b^5)^{5/6} * B*a + 7*(a*b^5)^{5/6} * A*b) \cdot \log(-\sqrt{3} \cdot \sqrt{x} \cdot (a/b)^{1/6} + x + (a/b)^{1/3}) / (a^3*b^6) + \frac{1}{216} \cdot (5*(a*b^5)^{5/6} * B*a + 7*(a*b^5)^{5/6} * A*b) \cdot \arctan(\sqrt{3} \cdot (a/b)^{1/6} + 2 \cdot \sqrt{x}) / (a/b)^{1/6} / (a^3*b^6) + \frac{1}{216} \cdot (5*(a*b^5)^{5/6} * B*a + 7*(a*b^5)^{5/6} * A*b) \cdot \arctan(-\sqrt{3} \cdot (a/b)^{1/6} - 2 \cdot \sqrt{x}) / (a/b)^{1/6} / (a^3*b^6) + \frac{1}{108} \cdot (5*(a*b^5)^{5/6} * B*a + 7*(a*b^5)^{5/6} * A*b) \cdot \arctan(\sqrt{x} / (a/b)^{1/6}) / (a^3*b^6)$

maple [A] time = 0.16, size = 411, normalized size = 1.26

$$\frac{7A \arctan\left(\frac{\sqrt{x}}{b}\right)}{108 \binom{5}{2} a^2 b} + \frac{7A \arctan\left(\frac{2\sqrt{x} - \sqrt{3}}{b}\right)}{216 \binom{5}{2} a^2 b} + \frac{7A \arctan\left(\frac{2\sqrt{x} + \sqrt{3}}{b}\right)}{216 \binom{5}{2} a^2 b} + \frac{7 \binom{5}{2} \sqrt{3} A \ln\left(\frac{x - \sqrt{3} \binom{5}{2} \sqrt{x} + \binom{5}{2}}{\binom{5}{2}}\right)}{432 a^3} + \frac{7 \sqrt{3} \binom{5}{2} A \ln\left(\frac{x + \sqrt{3} \binom{5}{2} \sqrt{x} + \binom{5}{2}}{\binom{5}{2}}\right)}{432 a^3} + \frac{5B \arctan\left(\frac{\sqrt{x}}{b}\right)}{108 \binom{5}{2} a^2 b} + \frac{5B \arctan\left(\frac{2\sqrt{x} - \sqrt{3}}{b}\right)}{216 \binom{5}{2} a^2 b} + \frac{5B \arctan\left(\frac{2\sqrt{x} + \sqrt{3}}{b}\right)}{216 \binom{5}{2} a^2 b} + \frac{5 \binom{5}{2} \sqrt{3} B \ln\left(\frac{x - \sqrt{3} \binom{5}{2} \sqrt{x} + \binom{5}{2}}{\binom{5}{2}}\right)}{432 a^3 b} + \frac{5 \sqrt{3} \binom{5}{2} B \ln\left(\frac{x + \sqrt{3} \binom{5}{2} \sqrt{x} + \binom{5}{2}}{\binom{5}{2}}\right)}{432 a^3 b} + \frac{7A \sqrt{3} \sqrt{x}}{36 a^2 b} + \frac{7A \sqrt{3} \sqrt{x}}{36 a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x)

[Out] $2 \cdot (1/72 \cdot (7A*ab+5B*a)/a^2*x^{11/2} + 1/72 \cdot (13A*b-B*a)/a*b*x^{5/2}) / (b*x^3+a)^2 + 7/108 \cdot a^2/b \cdot (a/b)^{1/6} \cdot \arctan(1/(a/b)^{1/6} * x^{1/2}) * A + 5/108 \cdot a/b^2 \cdot (a/b)^{1/6} \cdot \arctan(1/(a/b)^{1/6} * x^{1/2}) * B + 7/432 \cdot a^3 \cdot (a/b)^{5/6} * 3^{1/2} * \ln(x-3^{1/2} * (a/b)^{1/6} * x^{1/2} + (a/b)^{1/3}) * A + 5/432 \cdot a^2/b \cdot (a/b)^{5/6} * 3^{1/2} * \ln(x-3^{1/2} * (a/b)^{1/6} * x^{1/2} + (a/b)^{1/3}) * B + 7/216 \cdot a^2/b \cdot (a/b)^{1/6} \cdot \arctan(2/(a/b)^{1/6} * x^{1/2} - 3^{1/2}) * A + 5/216 \cdot a/b^2 \cdot (a/b)^{1/6} \cdot \arctan(2/(a/b)^{1/6} * x^{1/2} - 3^{1/2}) * B - 7/432 \cdot a^3 \cdot 3^{1/2} * (a/b)^{5/6} * \ln(x+3^{1/2} * (a/b)^{1/6} * x^{1/2} + (a/b)^{1/3}) * A - 5/432 \cdot a^2/b \cdot 3^{1/2} * (a/b)^{5/6} * \ln(x+3^{1/2} * (a/b)^{1/6} * x^{1/2} + (a/b)^{1/3}) * B + 7/216 \cdot a^2/b \cdot (a/b)^{1/6} \cdot \arctan(2/(a/b)^{1/6} * x^{1/2} + 3^{1/2}) * A + 5/216 \cdot a/b^2 \cdot (a/b)^{1/6} \cdot \arctan(2/(a/b)^{1/6} * x^{1/2} + 3^{1/2}) * B$

maxima [A] time = 1.23, size = 271, normalized size = 0.83

$$\frac{(5Ba + 7Ab) \left(\frac{\sqrt{3} \log\left(\sqrt{3} a^{\frac{1}{6}} b^{\frac{1}{6}} \sqrt{x} + b^{\frac{1}{3}} x + a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}} b^{\frac{5}{6}}} - \frac{\sqrt{3} \log\left(-\sqrt{3} a^{\frac{1}{6}} b^{\frac{1}{6}} \sqrt{x} + b^{\frac{1}{3}} x + a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}} b^{\frac{5}{6}}} - \frac{2 \arctan\left(\frac{\sqrt{3} a^{\frac{1}{6}} b^{\frac{1}{6}} + 2 b^{\frac{1}{3}} \sqrt{x}}{\sqrt{3} b^{\frac{1}{3}}}\right)}{2 b^{\frac{2}{3}} \sqrt{a^{\frac{1}{3}} b^{\frac{1}{3}}}} - \frac{2 \arctan\left(\frac{-\sqrt{3} a^{\frac{1}{6}} b^{\frac{1}{6}} - 2 b^{\frac{1}{3}} \sqrt{x}}{\sqrt{3} b^{\frac{1}{3}}}\right)}{2 b^{\frac{2}{3}} \sqrt{a^{\frac{1}{3}} b^{\frac{1}{3}}}} - \frac{4 \arctan\left(\frac{b^{\frac{1}{3}} \sqrt{x}}{\sqrt{a^{\frac{1}{3}} b^{\frac{1}{3}}}}\right)}{2 b^{\frac{2}{3}} \sqrt{a^{\frac{1}{3}} b^{\frac{1}{3}}}} \right)}{36 (a^2 b^3 x^6 + 2 a^3 b^2 x^3 + a^4 b)} - \frac{432 a^2 b}{36 (a^2 b^3 x^6 + 2 a^3 b^2 x^3 + a^4 b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] $\frac{1}{36} \cdot ((5B*ab + 7A*b^2)*x^{11/2} - (B*a^2 - 13A*a*b)*x^{5/2}) / (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) - \frac{1}{432} \cdot (5B*a + 7A*b) \cdot (\sqrt{3} \cdot \log(\sqrt{3} \cdot a^{1/6} \cdot b^{1/6} \cdot \sqrt{x} + b^{1/3} \cdot x + a^{1/3}) / (a^{1/6} \cdot b^{5/6}) - \sqrt{3} \cdot \log(-\sqrt{3} \cdot a^{1/6} \cdot b^{1/6} \cdot \sqrt{x} + b^{1/3} \cdot x + a^{1/3}) / (a^{1/6} \cdot b^{5/6}))$

$$\begin{aligned}
&) - (x^{(1/2)}*((3^{(1/2)}*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^{(19/6)}*b^{(11/6)})) * 1i / (46656*(-a)^{(13/3)} * b^{(11/3)}) - (((3^{(1/2)}*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^{(1/2)}*((3^{(1/2)}*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^{(19/6)}*b^{(11/6)})) * 1i / (46656*(-a)^{(13/3)} * b^{(11/3)})) / (((3^{(1/2)}*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^{(1/2)}*((3^{(1/2)}*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^{(19/6)} * b^{(11/6)})) / (46656*(-a)^{(13/3)} * b^{(11/3)}) + (((3^{(1/2)}*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^{(1/2)}*((3^{(1/2)}*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^{(19/6)}*b^{(11/6)})) / (46656*(-a)^{(13/3)} * b^{(11/3)})) * ((3^{(1/2)}*1i)/2 + 1/2)*(7*A*b + 5*B*a)*1i / (108*(-a)^{(13/6)} * b^{(11/6)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**3,x)

[Out] Timed out

$$3.170 \quad \int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=104

$$\frac{(aB + 3Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}} + \frac{x^{3/2}(aB + 3Ab)}{12a^2b(a + bx^3)} + \frac{x^{3/2}(Ab - aB)}{6ab(a + bx^3)^2}$$

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 290, 329, 275, 205}

$$\frac{(aB + 3Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}} + \frac{x^{3/2}(aB + 3Ab)}{12a^2b(a + bx^3)} + \frac{x^{3/2}(Ab - aB)}{6ab(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] ((A*b - a*B)*x^(3/2))/(6*a*b*(a + b*x^3)^2) + ((3*A*b + a*B)*x^(3/2))/(12*a^2*b*(a + b*x^3)) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(5/2)*b^(3/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 457

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ((!IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{x} (A + Bx^3)}{(a + bx^3)^3} dx &= \frac{(Ab - aB)x^{3/2}}{6ab(a + bx^3)^2} + \frac{\left(\frac{9Ab}{2} + \frac{3aB}{2}\right) \int \frac{\sqrt{x}}{(a + bx^3)^2} dx}{6ab} \\
 &= \frac{(Ab - aB)x^{3/2}}{6ab(a + bx^3)^2} + \frac{(3Ab + aB)x^{3/2}}{12a^2b(a + bx^3)} + \frac{(3Ab + aB) \int \frac{\sqrt{x}}{a + bx^3} dx}{8a^2b} \\
 &= \frac{(Ab - aB)x^{3/2}}{6ab(a + bx^3)^2} + \frac{(3Ab + aB)x^{3/2}}{12a^2b(a + bx^3)} + \frac{(3Ab + aB) \text{Subst}\left(\int \frac{x^2}{a + bx^6} dx, x, \sqrt{x}\right)}{4a^2b} \\
 &= \frac{(Ab - aB)x^{3/2}}{6ab(a + bx^3)^2} + \frac{(3Ab + aB)x^{3/2}}{12a^2b(a + bx^3)} + \frac{(3Ab + aB) \text{Subst}\left(\int \frac{1}{a + bx^2} dx, x, x^{3/2}\right)}{12a^2b} \\
 &= \frac{(Ab - aB)x^{3/2}}{6ab(a + bx^3)^2} + \frac{(3Ab + aB)x^{3/2}}{12a^2b(a + bx^3)} + \frac{(3Ab + aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.11, size = 94, normalized size = 0.90

$$\frac{\sqrt{a} \sqrt{b} x^{3/2} (-a^2 B + ab(5A + Bx^3) + 3Ab^2 x^3)}{(a + bx^3)^2} + (aB + 3Ab) \tan^{-1}\left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}}\right)$$

$$12a^{5/2}b^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] ((Sqrt[a]*Sqrt[b]*x^(3/2)*(-(a^2*B) + 3*A*b^2*x^3 + a*b*(5*A + B*x^3)))/(a + b*x^3)^2 + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(5/2)*b^(3/2))

IntegrateAlgebraic [A] time = 0.16, size = 92, normalized size = 0.88

$$\frac{(aB + 3Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}} - \frac{x^{3/2}(a^2B - 5aAb - abBx^3 - 3Ab^2x^3)}{12a^2b(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] -1/12*(x^(3/2)*(-5*a*A*b + a^2*B - 3*A*b^2*x^3 - a*b*B*x^3))/(a^2*b*(a + b*x^3)^2) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(5/2)*b^(3/2))

fricas [A] time = 0.57, size = 313, normalized size = 3.01

$$\left[\frac{\left((Ba^2 + 3Ab^2)x^6 + Ba^3 + 3Aa^2b + 2(Ba^2b + 3Aab^2)x^3 \right) \sqrt{-ab} \log\left(\frac{x^3 - 2\sqrt{-ab}x^{3/2} - a}{bx^3 + a} \right) - 2\left((Ba^2b^2 + 3Aab^2)x^4 - (Ba^2b - 5Aa^2b^2)x \right) \sqrt{x} \left((Ba^2 + 3Ab^2)x^6 + Ba^3 + 3Aa^2b + 2(Ba^2b + 3Aab^2)x^3 \right) \sqrt{ab} \arctan\left(\frac{\sqrt{ab}x^{3/2}}{a} \right) + \left((Ba^2b^2 + 3Aab^2)x^4 - (Ba^2b - 5Aa^2b^2)x \right) \sqrt{x}}{24(a^2b^4x^6 + 2a^4b^3x^3 + a^5b^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] [-1/24*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) - 2*((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2), 1/12*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) + ((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]

giac [A] time = 0.18, size = 84, normalized size = 0.81

$$\frac{(Ba + 3Ab) \arctan\left(\frac{bx^{3/2}}{\sqrt{ab}}\right)}{12\sqrt{ab}a^2b} + \frac{Babx^9 + 3Ab^2x^9 - Ba^2x^3 + 5Aabx^3}{12(bx^3 + a)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $\frac{1}{12}*(B*a + 3*A*b)*\arctan(b*x^{3/2}/\sqrt{a*b})/(\sqrt{a*b}*a^2*b) + \frac{1}{12}*(B*a*b*x^{9/2} + 3*A*b^2*x^{9/2} - B*a^2*x^{3/2} + 5*A*a*b*x^{3/2})/((b*x^3 + a)^2*a^2*b)$

maple [A] time = 0.07, size = 97, normalized size = 0.93

$$\frac{A \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab} a^2} + \frac{B \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\sqrt{ab} ab} + \frac{\frac{(3Ab+Ba)x^{\frac{9}{2}}}{12a^2} + \frac{(5Ab-Ba)x^{\frac{3}{2}}}{12ab}}{(bx^3 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x)

[Out] $\frac{2}{3}*(\frac{1}{8}*(3*A*b+B*a)/a^2*x^{9/2}+1/8*(5*A*b-B*a)/a/b*x^{3/2})/(b*x^3+a)^2+1/4/a^2/(a*b)^{(1/2)*\arctan(1/(a*b)^{(1/2)*b*x^{3/2}})*A+1/12/a/b/(a*b)^{(1/2)*\arctan(1/(a*b)^{(1/2)*b*x^{3/2}})*B}$

maxima [A] time = 1.27, size = 96, normalized size = 0.92

$$\frac{(Bab + 3 Ab^2)x^{\frac{9}{2}} - (Ba^2 - 5 Aab)x^{\frac{3}{2}}}{12(a^2b^3x^6 + 2a^3b^2x^3 + a^4b)} + \frac{(Ba + 3 Ab) \arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\sqrt{ab} a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] $\frac{1}{12}*((B*a*b + 3*A*b^2)*x^{9/2} - (B*a^2 - 5*A*a*b)*x^{3/2})/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) + \frac{1}{12}*(B*a + 3*A*b)*\arctan(b*x^{3/2}/\sqrt{a*b})/(\sqrt{a*b}*a^2*b)$

mupad [B] time = 2.71, size = 136, normalized size = 1.31

$$\frac{\frac{x^{9/2}(3Ab+Ba)}{12a^2} + \frac{x^{3/2}(5Ab-Ba)}{12ab}}{a^2 + 2abx^3 + b^2x^6} + \frac{\operatorname{atan}\left(\frac{b^{3/2}x^{3/2}(9A^2b^3+6ABab^2+B^2a^2b)}{\sqrt{a}(3Ab+Ba)(3Ab^3+Ba^2b)}\right)}{12a^{5/2}b^{3/2}}(3Ab+Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/2)*(A + B*x^3))/(a + b*x^3)^3,x)

[Out] $((x^{9/2}*(3*A*b + B*a))/(12*a^2) + (x^{3/2}*(5*A*b - B*a))/(12*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3) + (\operatorname{atan}((b^{3/2}*x^{3/2}*(9*A^2*b^3 + B^2*a^2*b + 6*$

$$\frac{A*B*a*b^2)}{(a^{(1/2)}*(3*A*b + B*a)*(3*A*b^3 + B*a*b^2)))*(3*A*b + B*a))/(12*a^{(5/2)}*b^{(3/2)})$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a)**3,x)

[Out] Timed out

$$3.171 \quad \int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)^3} dx$$

Optimal. Leaf size=321

$$\frac{5(aB + 11Ab) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{17/6} b^{7/6}} + \frac{5(aB + 11Ab) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{17/6} b^{7/6}} - \frac{5(aB + 11Ab)}{36a^2 b (a + bx^3)^2} + \frac{\sqrt{x}(aB - aB)}{6ab(a + bx^3)^2}$$

Rubi [A] time = 0.52, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, number of rules / integrand size = 0.409, Rules used = {457, 290, 329, 209, 634, 618, 204, 628, 205}

$$\frac{5(aB + 11Ab) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{17/6} b^{7/6}} + \frac{5(aB + 11Ab) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{17/6} b^{7/6}} - \frac{5(aB + 11Ab) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6} b^{7/6}} + \frac{5(aB + 11Ab) \tan^{-1}\left(\frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}} + \sqrt{3}\right)}{216a^{17/6} b^{7/6}} + \frac{5(aB + 11Ab) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6} b^{7/6}} + \frac{\sqrt{x}(aB + 11Ab)}{36a^2 b (a + bx^3)^2} + \frac{\sqrt{x}(aB - aB)}{6ab(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3), x]

[Out] ((A*b - a*B)*Sqrt[x])/(6*a*b*(a + b*x^3)^2) + ((11*A*b + a*B)*Sqrt[x])/(36*a^2*b*(a + b*x^3)) - (5*(11*A*b + a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(17/6)*b^(7/6)) - (5*(11*A*b + a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(17/6)*b^(7/6))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 209

Int[((a_) + (b_.)*(x_)^(n_))(-1), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*k - 1)*Pi)/n]*x])/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[

$(r + s \cos[(2k - 1)\pi/n]x) / (r^2 + 2rs \cos[(2k - 1)\pi/n]x + s^2x^2)$, x]; $(2r^2 \int [1/(r^2 + s^2x^2), x]) / (a^n) + \text{Dist}[(2r)/(a^n), \text{Sum}[u, \{k, 1, (n - 2)/4\}], x, x] /;$ $\text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[(n - 2)/4, 0] \ \&\& \ \text{PosQ}[a/b]$

Rule 290

$\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^p, x_Symbol] \rightarrow -\text{Simp}[(c \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} / (a \cdot c \cdot n \cdot (p+1)), x] + \text{Dist}[(m + n \cdot (p+1) + 1) / (a \cdot n \cdot (p+1)), \text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^{p+1}, x], x] /;$ $\text{FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 329

$\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^p, x_Symbol] \rightarrow \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{k \cdot (m+1) - 1} \cdot (a + b \cdot x^{k \cdot n}) / c^n]^p, x], x, (c \cdot x)^{1/k}], x] /;$ $\text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 457

$\text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^p \cdot (c + d \cdot x^n), x_Symbol] \rightarrow -\text{Simp}[(b \cdot c - a \cdot d) \cdot (e \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} / (a \cdot b \cdot e \cdot n \cdot (p+1)), x] - \text{Dist}[(a \cdot d \cdot (m+1) - b \cdot c \cdot (m + n \cdot (p+1) + 1)) / (a \cdot b \cdot n \cdot (p+1)), \text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^{p+1}, x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e, m, n\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ ((\text{!IntegerQ}[p + 1/2] \ \&\& \ \text{NeQ}[p, -5/4]) \ || \ \text{!RationalQ}[m] \ || \ (\text{IGtQ}[n, 0] \ \&\& \ \text{ILtQ}[p + 1/2, 0] \ \&\& \ \text{LeQ}[-1, m, -(n \cdot (p+1))]))$

Rule 618

$\text{Int}[(a + b \cdot x + c \cdot x^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4 \cdot a \cdot c - x^2, x], x], x, b + 2 \cdot c \cdot x], x] /;$ $\text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 628

$\text{Int}[(d + e \cdot x) / (a + b \cdot x + c \cdot x^2), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] /;$ $\text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rule 634


```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{\sqrt{x} (a + bx^3)^3} dx &= \frac{(Ab - aB)\sqrt{x}}{6ab(a + bx^3)^2} + \frac{(11Ab + aB) \int \frac{1}{\sqrt{x}(a+bx^3)^2} dx}{12ab} \\
&= \frac{(Ab - aB)\sqrt{x}}{6ab(a + bx^3)^2} + \frac{(11Ab + aB)\sqrt{x}}{36a^2b(a + bx^3)} + \frac{(5(11Ab + aB)) \int \frac{1}{\sqrt{x}(a+bx^3)} dx}{72a^2b} \\
&= \frac{(Ab - aB)\sqrt{x}}{6ab(a + bx^3)^2} + \frac{(11Ab + aB)\sqrt{x}}{36a^2b(a + bx^3)} + \frac{(5(11Ab + aB)) \text{Subst}\left(\int \frac{1}{a+bx^6} dx, x, \sqrt{x}\right)}{36a^2b} \\
&= \frac{(Ab - aB)\sqrt{x}}{6ab(a + bx^3)^2} + \frac{(11Ab + aB)\sqrt{x}}{36a^2b(a + bx^3)} + \frac{(5(11Ab + aB)) \text{Subst}\left(\int \frac{\sqrt[6]{a} - \frac{1}{2}\sqrt{3} \sqrt[6]{b} x}{\sqrt[3]{a} - \sqrt{3} \sqrt[6]{a} \sqrt[6]{b} x + \sqrt[3]{b} x^2} dx, x\right)}{108a^{17/6}b} \\
&= \frac{(Ab - aB)\sqrt{x}}{6ab(a + bx^3)^2} + \frac{(11Ab + aB)\sqrt{x}}{36a^2b(a + bx^3)} + \frac{5(11Ab + aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} - \frac{(5(11Ab + aB)) \text{Subst}\left(\int \frac{1}{\sqrt[6]{a} + \sqrt[6]{b} x} dx, x, \sqrt{x}\right)}{108a^{17/6}b^{7/6}} \\
&= \frac{(Ab - aB)\sqrt{x}}{6ab(a + bx^3)^2} + \frac{(11Ab + aB)\sqrt{x}}{36a^2b(a + bx^3)} + \frac{5(11Ab + aB) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} - \frac{5(11Ab + aB) \log\left(\frac{\sqrt[6]{a} + \sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a} - \sqrt[6]{b} \sqrt{x}}\right)}{108a^{17/6}b^{7/6}} \\
&= \frac{(Ab - aB)\sqrt{x}}{6ab(a + bx^3)^2} + \frac{(11Ab + aB)\sqrt{x}}{36a^2b(a + bx^3)} - \frac{5(11Ab + aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}} + \frac{5(11Ab + aB) \log\left(\frac{\sqrt[6]{a} + \sqrt[6]{b} \sqrt{x}}{\sqrt[6]{a} - \sqrt[6]{b} \sqrt{x}}\right)}{216a^{17/6}b^{7/6}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 91, normalized size = 0.28

$$\frac{\sqrt{x} \left(a \left(-5a^2B + ab(17A + Bx^3) + 11Ab^2x^3 \right) + 5(a + bx^3)^2 (aB + 11Ab) {}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; -\frac{bx^3}{a}\right) \right)}{36a^3b(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3), x]

[Out] (Sqrt[x]*(a*(-5*a^2*B + 11*A*b^2*x^3 + a*b*(17*A + B*x^3)) + 5*(11*A*b + a*B)*(a + b*x^3)^2*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)]))/(36*a^3*b*(a + b*x^3)^2)

IntegrateAlgebraic [A] time = 0.79, size = 210, normalized size = 0.65

$$\frac{5(aB + 11Ab) \tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} - \frac{5(aB + 11Ab) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{216a^{17/6}b^{7/6}} + \frac{5(aB + 11Ab) \tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{72\sqrt{3}a^{17/6}b^{7/6}} - \frac{\sqrt{x}(5a^2B - 17aAb - abBx^3 - 11Ab^2x^3)}{36a^2b(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3), x]

[Out] -1/36*(Sqrt[x]*(-17*a*A*b + 5*a^2*B - 11*A*b^2*x^3 - a*b*B*x^3))/(a^2*b*(a + b*x^3)^2) + (5*(11*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(17/6)*b^(7/6)) - (5*(11*A*b + a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x]])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x]/(a^(1/3) + b^(1/3)*x))]/(72*Sqrt[3]*a^(17/6)*b^(7/6))

fricas [B] time = 1.00, size = 2674, normalized size = 8.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2), x, algorithm="fricas")

[Out] 1/432*(20*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/3) + (B^2*a^2 + 22*A*B*a*b + 121*A^2*b^2)*x + (B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)))*a^14*b^6*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(5/6) - 2*sqrt(3)*(B*a^15*b^6 + 11*A*a^14*b^7)*sqrt(x)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(5/6) + sqrt(3)*(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6))/(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)) + 20*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b

$$\begin{aligned}
& 4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 \\
& *b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^{17} \\
& *b^7))^{(1/6)}*\arctan(1/3*(2*\sqrt{3})*\sqrt{a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b \\
& + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + \\
& 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^{17}*b^7))^{(1/3)} + (B^2*a^2 + 22*A*B \\
& *a*b + 121*A^2*b^2)*x - (B*a^4*b + 11*A*a^3*b^2)*\sqrt{x})*(-(B^6*a^6 + 66*A* \\
& B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a \\
& ^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^{17}*b^7))^{(1/6)})*a^{14}*b^6 \\
& (-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 \\
& + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^{17}*b^7) \\
&)^{(5/6)} - 2*\sqrt{3}*(B*a^{15}*b^6 + 11*A*a^{14}*b^7)*\sqrt{x})*(-(B^6*a^6 + 66*A* \\
& B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a \\
& ^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^{17}*b^7))^{(5/6)} - \sqrt{3}* \\
& (B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + \\
& 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6))/(B^6*a^6 + \\
& 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4* \\
& B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)) + 5*(a^2*b^3*x^6 + 2*a \\
& ^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26 \\
& 620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561 \\
& *A^6*b^6)/(a^{17}*b^7))^{(1/6)}*\log(25*a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b + 18 \\
& 15*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 96630 \\
& 6*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^{17}*b^7))^{(1/3)} + 25*(B^2*a^2 + 22*A*B*a \\
& *b + 121*A^2*b^2)*x + 25*(B*a^4*b + 11*A*a^3*b^2)*\sqrt{x})*(-(B^6*a^6 + 66*A \\
& *B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2* \\
& a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^{17}*b^7))^{(1/6)} - 5*(a^2 \\
& *b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^ \\
& 4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a \\
& *b^5 + 1771561*A^6*b^6)/(a^{17}*b^7))^{(1/6)}*\log(25*a^6*b^2*(-(B^6*a^6 + 66*A* \\
& B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a \\
& ^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^{17}*b^7))^{(1/3)} + 25*(B^2* \\
& a^2 + 22*A*B*a*b + 121*A^2*b^2)*x - 25*(B*a^4*b + 11*A*a^3*b^2)*\sqrt{x})*(-(\\
& B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 2 \\
& 19615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^{17}*b^7))^{(\\
& 1/6)} + 10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5* \\
& b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + \\
& 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^{17}*b^7))^{(1/6)}*\log(5*a^3*b*(-(B^6 \\
& *a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 2196 \\
& 15*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^{17}*b^7))^{(1/6} \\
&) + 5*(B*a + 11*A*b)*\sqrt{x}) - 10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(- \\
& (B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + \\
& 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^{17}*b^7))^{(\\
& 1/6)}*\log(-5*a^3*b*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 266 \\
& 20*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561* \\
& A^6*b^6)/(a^{17}*b^7))^{(1/6)} + 5*(B*a + 11*A*b)*\sqrt{x}) + 12*((B*a*b + 11*A* \\
& b^2)*x^3 - 5*B*a^2 + 17*A*a*b)*\sqrt{x})/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*
\end{aligned}$$

b)

giac [A] time = 0.24, size = 322, normalized size = 1.00

$$\frac{5\sqrt{3}\left((ab)^{\frac{1}{2}}Ba+11(ab)^{\frac{1}{2}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{x}{b}\right)^{\frac{1}{2}}+x+\left(\frac{x}{b}\right)^{\frac{1}{2}}\right)}{432a^3b^2} - \frac{5\sqrt{3}\left((ab)^{\frac{1}{2}}Ba+11(ab)^{\frac{1}{2}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{x}{b}\right)^{\frac{1}{2}}+x+\left(\frac{x}{b}\right)^{\frac{1}{2}}\right)}{432a^3b^2} + \frac{5\left((ab)^{\frac{1}{2}}Ba+11(ab)^{\frac{1}{2}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{2}}+2\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{2}}}\right)}{216a^3b^2} + \frac{5\left((ab)^{\frac{1}{2}}Ba+11(ab)^{\frac{1}{2}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{2}}-2\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{2}}}\right)}{216a^3b^2} + \frac{5\left((ab)^{\frac{1}{2}}Ba+11(ab)^{\frac{1}{2}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{2}}}\right)}{108a^3b^2} + \frac{Bab^2+11Aab^2x^2-5Ba^2\sqrt{x}+17Aab\sqrt{x}}{36(bx^3+a)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="giac")

[Out] $\frac{5}{432}\sqrt{3}\left((a*b^5)^{\frac{1}{6}}*B*a + 11*(a*b^5)^{\frac{1}{6}}*A*b\right)*\log(\sqrt{3}*\sqrt{x}*(a/b)^{\frac{1}{6}} + x + (a/b)^{\frac{1}{3}})/(a^3*b^2) - \frac{5}{432}\sqrt{3}\left((a*b^5)^{\frac{1}{6}}*B*a + 11*(a*b^5)^{\frac{1}{6}}*A*b\right)*\log(-\sqrt{3}*\sqrt{x}*(a/b)^{\frac{1}{6}} + x + (a/b)^{\frac{1}{3}})/(a^3*b^2) + \frac{5}{216}\left((a*b^5)^{\frac{1}{6}}*B*a + 11*(a*b^5)^{\frac{1}{6}}*A*b\right)*\arctan(\sqrt{3}*(a/b)^{\frac{1}{6}} + 2*\sqrt{x})/(a^3*b^2) + \frac{5}{216}\left((a*b^5)^{\frac{1}{6}}*B*a + 11*(a*b^5)^{\frac{1}{6}}*A*b\right)*\arctan(-\sqrt{3}*(a/b)^{\frac{1}{6}} - 2*\sqrt{x})/(a^3*b^2) + \frac{5}{108}\left((a*b^5)^{\frac{1}{6}}*B*a + 11*(a*b^5)^{\frac{1}{6}}*A*b\right)*\arctan(\sqrt{x}/(a/b)^{\frac{1}{6}})/(a^3*b^2) + \frac{1}{36}\left(B*a*b*x^{\frac{7}{2}} + 11*A*b^2*x^{\frac{7}{2}} - 5*B*a^2*\sqrt{x} + 17*A*a*b*\sqrt{x}\right)/((b*x^3 + a)^2*a^2*b)$

maple [A] time = 0.17, size = 407, normalized size = 1.27

$$\frac{5\left(\frac{1}{108}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{6}}}\right) + \frac{5}{216}A\arctan\left(\frac{2\sqrt{x}-\sqrt{3}}{\left(\frac{x}{b}\right)^{\frac{1}{6}}}\right) + \frac{5}{216}A\arctan\left(\frac{2\sqrt{x}+\sqrt{3}}{\left(\frac{x}{b}\right)^{\frac{1}{6}}}\right) + \frac{55\sqrt{3}}{432}A\ln\left(1+\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{x}{b}\right)^{\frac{1}{3}}\right) - \frac{55\sqrt{3}}{432}A\ln\left(1-\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{x}{b}\right)^{\frac{1}{3}}\right) + \frac{5\left(\frac{1}{216}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{x}{b}\right)^{\frac{1}{6}}}\right) + \frac{5}{216}B\arctan\left(\frac{2\sqrt{x}-\sqrt{3}}{\left(\frac{x}{b}\right)^{\frac{1}{6}}}\right) + \frac{5}{216}B\arctan\left(\frac{2\sqrt{x}+\sqrt{3}}{\left(\frac{x}{b}\right)^{\frac{1}{6}}}\right) + 5\sqrt{3}\left(\frac{1}{216}B\ln\left(1+\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{x}{b}\right)^{\frac{1}{3}}\right) - 5\sqrt{3}\left(\frac{1}{216}B\ln\left(1-\sqrt{3}\left(\frac{x}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{x}{b}\right)^{\frac{1}{3}}\right) + \frac{0.17498041}{36a^2b} - \frac{0.7249-0.991\sqrt{3}}{36a^2b}\right)}{(b*x^3+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x)

[Out] $\frac{2*(1/72*(11*A*b+B*a)/a^2*x^{7/2}+1/72*(17*A*b-5*B*a)/a/b*x^{1/2})/(b*x^3+a)^2+55/108/a^3*(a/b)^{1/6}*\arctan(1/(a/b)^{1/6}*x^{1/2})*A+5/108/a^2/b*(a/b)^{1/6}*\arctan(1/(a/b)^{1/6}*x^{1/2})*B-55/432/a^3*3^{1/2}*(a/b)^{1/6}*\ln(-x+3^{1/2}*(a/b)^{1/6}*x^{1/2}-(a/b)^{1/3})*A-5/432/a^2/b*3^{1/2}*(a/b)^{1/6}*\ln(-x+3^{1/2}*(a/b)^{1/6}*x^{1/2}-(a/b)^{1/3})*B+55/216/a^3*(a/b)^{1/6}*\arctan(2/(a/b)^{1/6}*x^{1/2}-3^{1/2}*(a/b)^{1/6})*A+5/216/a^2/b*(a/b)^{1/6}*\arctan(2/(a/b)^{1/6}*x^{1/2}-3^{1/2}*(a/b)^{1/6})*B+55/432/a^3*3^{1/2}*(a/b)^{1/6}*\ln(x+3^{1/2}*(a/b)^{1/6}*x^{1/2}+(a/b)^{1/3})*A+5/432/a^2/b*3^{1/2}*(a/b)^{1/6}*\ln(x+3^{1/2}*(a/b)^{1/6}*x^{1/2}+(a/b)^{1/3})*B+55/216/a^3*(a/b)^{1/6}*\arctan(2/(a/b)^{1/6}*x^{1/2}+3^{1/2}*(a/b)^{1/6})*A+5/216/a^2/b*(a/b)^{1/6}*\arctan(2/(a/b)^{1/6}*x^{1/2}+3^{1/2}*(a/b)^{1/6})*B$

maxima [A] time = 1.21, size = 336, normalized size = 1.05

$$\frac{(Bab+11Ab^2)x^2-(5Ba^2-17Aab)\sqrt{x}}{36(a^2b^3x^6+2a^3b^2x^3+a^4b)} + \frac{5\left(\frac{\sqrt{3}(Bab+11Ab^2)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}(Bab+11Ab^2)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{5}{6}}}\right)}{432a^2b} + \frac{4\left(Bab^{\frac{1}{2}}+11Ab^{\frac{3}{2}}\right)\arctan\left(\frac{2\sqrt{x}}{\sqrt{a^{\frac{1}{2}}b^{\frac{1}{2}}}}\right)}{a^{\frac{2}{3}}b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}+11Aa^{\frac{5}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}} + \frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}+11Aa^{\frac{5}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="maxima")
[Out] 1/36*((B*a*b + 11*A*b^2)*x^(7/2) - (5*B*a^2 - 17*A*a*b)*sqrt(x))/(a^2*b^3*x
^6 + 2*a^3*b^2*x^3 + a^4*b) + 5/432*(sqrt(3)*(B*a + 11*A*b)*log(sqrt(3)*a^(
1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*
a + 11*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(
5/6)*b^(1/6)) + 4*(B*a*b^(1/3) + 11*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt
(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b
^(1/3) + 11*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*
sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(
4/3)*b^(1/3) + 11*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*
b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3)))/
(a^2*b)
```

mupad [B] time = 2.95, size = 1952, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(1/2)*(a + b*x^3)^3),x)
[Out] ((x^(7/2)*(11*A*b + B*a))/(36*a^2) + (x^(1/2)*(17*A*b - 5*B*a))/(36*a*b))/
(a^2 + b^2*x^6 + 2*a*b*x^3) - (atan((((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4
*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^
8) - (625*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33
*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a)*5i)/(216*(-a)
^(17/6)*b^(7/6)) + (((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*
a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*(11*A*b
+ B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/
(279936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a)*5i)/(216*(-a)^(17/6)*b^(7/6))
)/((5*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*
A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*(11*A*b + B*a)*(1331*A
^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(4
7/6)*b^(7/6)))*(11*A*b + B*a))/(216*(-a)^(17/6)*b^(7/6)) - (5*((625*x^(1/2)
*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A
*B^3*a^3*b^2))/(279936*a^8) + (625*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b
^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*(11
*A*b + B*a))/(216*(-a)^(17/6)*b^(7/6)))*((11*A*b + B*a)*5i)/(108*(-a)^(17/6
)*b^(7/6)) - (atan((((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(1
4641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^
3*a^3*b^2))/(279936*a^8) - (625*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*(1331
*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(
47/6)*b^(7/6)))*5i)/(216*(-a)^(17/6)*b^(7/6)) + (((3^(1/2)*1i)/2 - 1/2)*(1
1*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3
+ 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*((3^(1/2)*1i)/
2 - 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*
```

$$\begin{aligned} & A*B^2*a^2*b^3)/(279936*(-a)^{(47/6)}*b^{(7/6)})) * 5i)/(216*(-a)^{(17/6)}*b^{(7/6)}) \\ &)/((5*((3^{(1/2)}*1i)/2 - 1/2)*(11*A*b + B*a)*((625*x^{(1/2)}*(14641*A^4*b^5 + \\ & B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(27 \\ & 9936*a^8) - (625*((3^{(1/2)}*1i)/2 - 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3* \\ & a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^{(47/6)}*b^{(7/6)}) \\ &))/(216*(-a)^{(17/6)}*b^{(7/6)}) - (5*((3^{(1/2)}*1i)/2 - 1/2)*(11*A*b + B*a)*((6 \\ & 25*x^{(1/2)}*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a* \\ & b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*((3^{(1/2)}*1i)/2 - 1/2)*(11*A*b \\ & + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/ \\ & (279936*(-a)^{(47/6)}*b^{(7/6)})))/(216*(-a)^{(17/6)}*b^{(7/6)})) * ((3^{(1/2)}*1i)/2 \\ & - 1/2)*(11*A*b + B*a)*5i)/(108*(-a)^{(17/6)}*b^{(7/6)}) - (atan((((3^{(1/2)}*1i) \\ & /2 + 1/2)*(11*A*b + B*a)*((625*x^{(1/2)}*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2 \\ & *B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*((\\ & 3^{(1/2)}*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B \\ & *a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^{(47/6)}*b^{(7/6)})) * 5i)/(216*(-a)^{(17 \\ & /6)}*b^{(7/6)}) + (((3^{(1/2)}*1i)/2 + 1/2)*(11*A*b + B*a)*((625*x^{(1/2)}*(14641* \\ & A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3 \\ & *b^2))/(279936*a^8) + (625*((3^{(1/2)}*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3* \\ & b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^{(47/6 \\ &)}*b^{(7/6)})) * 5i)/(216*(-a)^{(17/6)}*b^{(7/6)})))/(5*((3^{(1/2)}*1i)/2 + 1/2)*(11*A \\ & *b + B*a)*((625*x^{(1/2)}*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + \\ & 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*((3^{(1/2)}*1i)/2 + \\ & 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B \\ & ^2*a^2*b^3))/(279936*(-a)^{(47/6)}*b^{(7/6)})))/(216*(-a)^{(17/6)}*b^{(7/6)}) - (5* \\ & ((3^{(1/2)}*1i)/2 + 1/2)*(11*A*b + B*a)*((625*x^{(1/2)}*(14641*A^4*b^5 + B^4*a^ \\ & 4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a \\ & ^8) + (625*((3^{(1/2)}*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^ \\ & 2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^{(47/6)}*b^{(7/6)})))/(21 \\ & 6*(-a)^{(17/6)}*b^{(7/6)})) * ((3^{(1/2)}*1i)/2 + 1/2)*(11*A*b + B*a)*5i)/(108*(-a \\ &)^{\{17/6\}}*b^{\{7/6\}}) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/(b*x**3+a)**3/x**(1/2),x)

[Out] Timed out

$$3.172 \quad \int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^3} dx$$

Optimal. Leaf size=351

$$\frac{7(13Ab - aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{19/6} b^{5/6}} + \frac{7(13Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{19/6} b^{5/6}} + \frac{7(13Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{19/6} b^{5/6}}$$

Rubi [A] time = 0.62, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 22, number of rules / integrand size = 0.454, Rules used = {457, 290, 325, 329, 295, 634, 618, 204, 628, 205}

$$\frac{7(13Ab - aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{19/6} b^{5/6}} + \frac{7(13Ab - aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{19/6} b^{5/6}} + \frac{7(13Ab - aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt{6}\sqrt{x}}{\sqrt{a}}\right)}{216a^{19/6} b^{5/6}} - \frac{7(13Ab - aB) \tan^{-1}\left(\frac{2\sqrt{6}\sqrt{x}}{\sqrt{a}} + \sqrt{3}\right)}{216a^{19/6} b^{5/6}} - \frac{7(13Ab - aB) \tan^{-1}\left(\frac{\sqrt{6}\sqrt{x}}{\sqrt{a}}\right)}{108a^{19/6} b^{5/6}} + \frac{13Ab - aB}{36a^2 b \sqrt{x} (a + bx^3)} - \frac{7(13Ab - aB)}{36a^2 b \sqrt{x}} + \frac{Ab - aB}{6ab \sqrt{x} (a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3), x]

[Out] (-7*(13*A*b - a*B))/(36*a^3*b*Sqrt[x]) + (A*b - a*B)/(6*a*b*Sqrt[x]*(a + b*x^3)^2) + (13*A*b - a*B)/(36*a^2*b*Sqrt[x]*(a + b*x^3)) + (7*(13*A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(19/6)*b^(5/6)) - (7*(13*A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(19/6)*b^(5/6)) - (7*(13*A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(19/6)*b^(5/6)) - (7*(13*A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(19/6)*b^(5/6)) + (7*(13*A*b - a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(19/6)*b^(5/6))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(c*x)^(m+1)*(a + b*x^n)^(p+1)/(a*c*n*(p+1)), x] + Dist[(m + n*(p + 1))

```

+ 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]

```

Rule 295

```

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
- 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
- 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x)]/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]

```

Rule 325

```

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]

```

Rule 329

```

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

```

Rule 457

```

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))

```

Rule 618

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

```


$x]$ && NeQ[$b^2 - 4ac$, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[$b^2 - 4ac$, 0] && !NiceSqrtQ[$b^2 - 4ac$]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^{3/2} (a + bx^3)^3} dx &= \frac{Ab - aB}{6ab\sqrt{x} (a + bx^3)^2} + \frac{\left(\frac{13Ab}{2} - \frac{aB}{2}\right) \int \frac{1}{x^{3/2}(a+bx^3)^2} dx}{6ab} \\
&= \frac{Ab - aB}{6ab\sqrt{x} (a + bx^3)^2} + \frac{13Ab - aB}{36a^2b\sqrt{x} (a + bx^3)} + \frac{(7(13Ab - aB)) \int \frac{1}{x^{3/2}(a+bx^3)} dx}{72a^2b} \\
&= -\frac{7(13Ab - aB)}{36a^3b\sqrt{x}} + \frac{Ab - aB}{6ab\sqrt{x} (a + bx^3)^2} + \frac{13Ab - aB}{36a^2b\sqrt{x} (a + bx^3)} - \frac{(7(13Ab - aB)) \int \frac{x^{3/2}}{a+bx^3} dx}{72a^3} \\
&= -\frac{7(13Ab - aB)}{36a^3b\sqrt{x}} + \frac{Ab - aB}{6ab\sqrt{x} (a + bx^3)^2} + \frac{13Ab - aB}{36a^2b\sqrt{x} (a + bx^3)} - \frac{(7(13Ab - aB)) \text{Subst}\left(\int \frac{1}{a+bx^3} dx\right)}{36a^3} \\
&= -\frac{7(13Ab - aB)}{36a^3b\sqrt{x}} + \frac{Ab - aB}{6ab\sqrt{x} (a + bx^3)^2} + \frac{13Ab - aB}{36a^2b\sqrt{x} (a + bx^3)} - \frac{(7(13Ab - aB)) \text{Subst}\left(\int \frac{1}{a+bx^3} dx\right)}{108a} \\
&= -\frac{7(13Ab - aB)}{36a^3b\sqrt{x}} + \frac{Ab - aB}{6ab\sqrt{x} (a + bx^3)^2} + \frac{13Ab - aB}{36a^2b\sqrt{x} (a + bx^3)} - \frac{7(13Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{19/6}b^{5/6}} \\
&= -\frac{7(13Ab - aB)}{36a^3b\sqrt{x}} + \frac{Ab - aB}{6ab\sqrt{x} (a + bx^3)^2} + \frac{13Ab - aB}{36a^2b\sqrt{x} (a + bx^3)} - \frac{7(13Ab - aB) \tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{19/6}b^{5/6}} \\
&= -\frac{7(13Ab - aB)}{36a^3b\sqrt{x}} + \frac{Ab - aB}{6ab\sqrt{x} (a + bx^3)^2} + \frac{13Ab - aB}{36a^2b\sqrt{x} (a + bx^3)} + \frac{7(13Ab - aB) \tan^{-1}\left(\sqrt{3} - \frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{19/6}b^{5/6}}
\end{aligned}$$

Mathematica [C] time = 0.13, size = 113, normalized size = 0.32

$$2 \left(-\frac{x^{5/2}(Ab - aB) {}_2F_1\left(\frac{5}{6}, 3; \frac{11}{6}; -\frac{bx^3}{a}\right)}{5a^4} - \frac{Abx^{5/2} {}_2F_1\left(\frac{5}{6}, 1; \frac{11}{6}; -\frac{bx^3}{a}\right)}{5a^4} - \frac{Abx^{5/2} {}_2F_1\left(\frac{5}{6}, 2; \frac{11}{6}; -\frac{bx^3}{a}\right)}{5a^4} - \frac{A}{a^3\sqrt{x}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3), x]

[Out] $2*(-(A/(a^3\sqrt{x}))) - (A*b*x^{(5/2)}*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a)])/(5*a^4) - (A*b*x^{(5/2)}*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a)])/(5*a^4) - ((A*b - a*B)*x^{(5/2)}*Hypergeometric2F1[5/6, 3, 11/6, -((b*x^3)/a)])/(5*a^4)$

IntegrateAlgebraic [A] time = 0.63, size = 219, normalized size = 0.62

$$\frac{7(aB - 13Ab) \tan^{-1}\left(\frac{\sqrt[5]{b}\sqrt{x}}{\sqrt[5]{a}}\right)}{108a^{19/6}b^{5/6}} - \frac{7(aB - 13Ab) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b}x}{\sqrt[5]{a}\sqrt[5]{b}\sqrt{x}}\right)}{216a^{19/6}b^{5/6}} - \frac{7(aB - 13Ab) \tanh^{-1}\left(\frac{\sqrt{3}\sqrt[5]{a}\sqrt[5]{b}\sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{b}x}\right)}{72\sqrt{3}a^{19/6}b^{5/6}} + \frac{-72a^2A + 13a^2Bx^3 - 169aAbx^3 + 7abBx^6 - 91Ab^2x^6}{36a^3\sqrt{x}(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3), x]

[Out] $(-72*a^2*A - 169*a*A*b*x^3 + 13*a^2*B*x^3 - 91*A*b^2*x^6 + 7*a*b*B*x^6)/(36*a^3*\sqrt{x}*(a + b*x^3)^2) + (7*(-13*A*b + a*B)*ArcTan[(b^(1/6)*\sqrt{x})/a^(1/6)])/(108*a^(19/6)*b^(5/6)) - (7*(-13*A*b + a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*\sqrt{x}])/(216*a^(19/6)*b^(5/6)) - (7*(-13*A*b + a*B)*ArcTanh[(\sqrt{3}*a^(1/6)*b^(1/6)*\sqrt{x})/(a^(1/3) + b^(1/3)*x)]/(72*\sqrt{3}*a^(19/6)*b^(5/6))$

fricas [B] time = 0.97, size = 3904, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $1/432*(28*\sqrt{3}*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^{19}*b^5))^{(1/6)}*\arctan(1/3*(2*\sqrt{3}*\sqrt{(B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1690*A^2*B^3*a^19*b^6 - 21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371293*A^5*a^16*b^9)*\sqrt{x})*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^{19}*b^5))^{(5/6)} + (B^{10}*a^{10} - 130*A*B^9*a^9*b + 7605*A^2*B^8*a^8*b^2 - 263640*A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4 - 93565836*A^5*B^5*a^5*b^5 + 1013629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^3*a^3*b^7 + 36707882445*A^8*B^2*a^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858491849*A^{10}*b^{10})*x - (B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a^14*b^8 + 4826809*A^6*a^13*b^9)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^{19}*b^5))^{(2/3)})*a^3*b*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^{19}*b^5))^{(1/6)} + 2*\sqrt{3}*(B^5*a^8*b - 65*A*B^4*a^7*b^2 + 1690*A^2*$

$$\begin{aligned}
& B^3 a^6 b^3 - 21970 A^3 B^2 a^5 b^4 + 142805 A^4 B a^4 b^5 - 371293 A^5 a^3 b^6) \sqrt{x} \cdot (- (B^6 a^6 - 78 A B^5 a^5 b + 2535 A^2 B^4 a^4 b^2 - 43940 A^3 B^3 a^3 b^3 + 428415 A^4 B^2 a^2 b^4 - 2227758 A^5 B a b^5 + 4826809 A^6 b^6) / (a^{19} b^5))^{1/6} - \sqrt{3} \cdot (B^6 a^6 - 78 A B^5 a^5 b + 2535 A^2 B^4 a^4 b^2 - 43940 A^3 B^3 a^3 b^3 + 428415 A^4 B^2 a^2 b^4 - 2227758 A^5 B a b^5 + 4826809 A^6 b^6) / (B^6 a^6 - 78 A B^5 a^5 b + 2535 A^2 B^4 a^4 b^2 - 43940 A^3 B^3 a^3 b^3 + 428415 A^4 B^2 a^2 b^4 - 2227758 A^5 B a b^5 + 4826809 A^6 b^6)) + 28 \sqrt{3} \cdot (a^3 b^2 x^7 + 2 a^4 b x^4 + a^5 x) \cdot (- (B^6 a^6 - 78 A B^5 a^5 b + 2535 A^2 B^4 a^4 b^2 - 43940 A^3 B^3 a^3 b^3 + 428415 A^4 B^2 a^2 b^4 - 2227758 A^5 B a b^5 + 4826809 A^6 b^6) / (a^{19} b^5))^{1/6} \cdot \arctan(1/50421 \cdot (2 \sqrt{3}) \sqrt{-282475249 \cdot (B^5 a^{21} b^4 - 65 A B^4 a^{20} b^5 + 1690 A^2 B^3 a^{19} b^6 - 21970 A^3 B^2 a^{18} b^7 + 142805 A^4 B a^{17} b^8 - 371293 A^5 a^{16} b^9) \sqrt{x} \cdot (- (B^6 a^6 - 78 A B^5 a^5 b + 2535 A^2 B^4 a^4 b^2 - 43940 A^3 B^3 a^3 b^3 + 428415 A^4 B^2 a^2 b^4 - 2227758 A^5 B a b^5 + 4826809 A^6 b^6) / (a^{19} b^5))^{5/6} + 282475249 \cdot (B^{10} a^{10} - 130 A B^9 a^9 b + 7605 A^2 B^8 a^8 b^2 - 263640 A^3 B^7 a^7 b^3 + 5997810 A^4 B^6 a^6 b^4 - 93565836 A^5 B^5 a^5 b^5 + 1013629890 A^6 B^4 a^4 b^6 - 7529822040 A^7 B^3 a^3 b^7 + 36707882445 A^8 B^2 a^2 b^8 - 106044993730 A^9 B a b^9 + 137858491849 A^{10} b^{10}) \cdot x - 282475249 \cdot (B^6 a^{19} b^3 - 78 A B^5 a^{18} b^4 + 2535 A^2 B^4 a^{17} b^5 - 43940 A^3 B^3 a^{16} b^6 + 428415 A^4 B^2 a^{15} b^7 - 2227758 A^5 B a^{14} b^8 + 4826809 A^6 a^{13} b^9) \cdot (- (B^6 a^6 - 78 A B^5 a^5 b + 2535 A^2 B^4 a^4 b^2 - 43940 A^3 B^3 a^3 b^3 + 428415 A^4 B^2 a^2 b^4 - 2227758 A^5 B a b^5 + 4826809 A^6 b^6) / (a^{19} b^5))^{2/3}) \cdot a^3 b \cdot (- (B^6 a^6 - 78 A B^5 a^5 b + 2535 A^2 B^4 a^4 b^2 - 43940 A^3 B^3 a^3 b^3 + 428415 A^4 B^2 a^2 b^4 - 2227758 A^5 B a b^5 + 4826809 A^6 b^6) / (a^{19} b^5))^{1/6} + 33614 \sqrt{3} \cdot (B^5 a^8 b - 65 A B^4 a^7 b^2 + 1690 A^2 B^3 a^6 b^3 - 21970 A^3 B^2 a^5 b^4 + 142805 A^4 B a^4 b^5 - 371293 A^5 a^3 b^6) \sqrt{x} \cdot (- (B^6 a^6 - 78 A B^5 a^5 b + 2535 A^2 B^4 a^4 b^2 - 43940 A^3 B^3 a^3 b^3 + 428415 A^4 B^2 a^2 b^4 - 2227758 A^5 B a b^5 + 4826809 A^6 b^6) / (a^{19} b^5))^{1/6} + 16807 \sqrt{3} \cdot (B^6 a^6 - 78 A B^5 a^5 b + 2535 A^2 B^4 a^4 b^2 - 43940 A^3 B^3 a^3 b^3 + 428415 A^4 B^2 a^2 b^4 - 2227758 A^5 B a b^5 + 4826809 A^6 b^6) / (B^6 a^6 - 78 A B^5 a^5 b + 2535 A^2 B^4 a^4 b^2 - 43940 A^3 B^3 a^3 b^3 + 428415 A^4 B^2 a^2 b^4 - 2227758 A^5 B a b^5 + 4826809 A^6 b^6)) - 14 \cdot (a^3 b^2 x^7 + 2 a^4 b x^4 + a^5 x) \cdot (- (B^6 a^6 - 78 A B^5 a^5 b + 2535 A^2 B^4 a^4 b^2 - 43940 A^3 B^3 a^3 b^3 + 428415 A^4 B^2 a^2 b^4 - 2227758 A^5 B a b^5 + 4826809 A^6 b^6) / (a^{19} b^5))^{1/6} \cdot \log(16807 a^{16} b^4 \cdot (- (B^6 a^6 - 78 A B^5 a^5 b + 2535 A^2 B^4 a^4 b^2 - 43940 A^3 B^3 a^3 b^3 + 428415 A^4 B^2 a^2 b^4 - 2227758 A^5 B a b^5 + 4826809 A^6 b^6) / (a^{19} b^5))^{5/6} - 16807 \cdot (B^5 a^5 - 65 A B^4 a^4 b + 1690 A^2 B^3 a^3 b^2 - 21970 A^3 B^2 a^2 b^3 + 142805 A^4 B a b^4 - 371293 A^5 b^5) \sqrt{x}) + 14 \cdot (a^3 b^2 x^7 + 2 a^4 b x^4 + a^5 x) \cdot (- (B^6 a^6 - 78 A B^5 a^5 b + 2535 A^2 B^4 a^4 b^2 - 43940 A^3 B^3 a^3 b^3 + 428415 A^4 B^2 a^2 b^4 - 2227758 A^5 B a b^5 + 4826809 A^6 b^6) / (a^{19} b^5))^{1/6} \cdot \log(-16807 a^{16} b^4 \cdot (- (B^6 a^6 - 78 A B^5 a^5 b + 2535 A^2 B^4 a^4 b^2 - 43940 A^3 B^3 a^3 b^3 + 428415 A^4 B^2 a^2 b^4 - 2227758 A^5 B a b^5 + 4826809 A^6 b^6) / (a^{19} b^5))^{5/6} - 16807 \cdot (B^5 a^5 - 65 A
\end{aligned}$$

$$\begin{aligned}
& *B^4a^4b + 1690A^2B^3a^3b^2 - 21970A^3B^2a^2b^3 + 142805A^4B^1a^1b^4 - 371293A^5b^5) \sqrt{x}) + 7(a^3b^2x^7 + 2a^4b^1x^4 + a^5x) * (- (B^6a^6 - 78A^1B^5a^5b + 2535A^2B^4a^4b^2 - 43940A^3B^3a^3b^3 + 428415A^4B^2a^2b^4 - 2227758A^5B^1a^1b^5 + 4826809A^6b^6) / (a^{19}b^5))^{(1/6)} \\
& * \log(282475249(B^5a^{21}b^4 - 65A^1B^4a^{20}b^5 + 1690A^2B^3a^{19}b^6 - 21970A^3B^2a^{18}b^7 + 142805A^4B^1a^{17}b^8 - 371293A^5a^{16}b^9) * \sqrt{x}) \\
& * (- (B^6a^6 - 78A^1B^5a^5b + 2535A^2B^4a^4b^2 - 43940A^3B^3a^3b^3 + 428415A^4B^2a^2b^4 - 2227758A^5B^1a^1b^5 + 4826809A^6b^6) / (a^{19}b^5))^{(5/6)} \\
& + 282475249(B^{10}a^{10} - 130A^1B^9a^9b + 7605A^2B^8a^8b^2 - 263640A^3B^7a^7b^3 + 5997810A^4B^6a^6b^4 - 93565836A^5B^5a^5b^5 + 1013629890A^6B^4a^4b^6 - 7529822040A^7B^3a^3b^7 + 36707882445A^8B^2a^2b^8 - 106044993730A^9B^1a^1b^9 + 137858491849A^{10}b^{10}) * x \\
& - 282475249(B^6a^{19}b^3 - 78A^1B^5a^{18}b^4 + 2535A^2B^4a^{17}b^5 - 43940A^3B^3a^{16}b^6 + 428415A^4B^2a^{15}b^7 - 2227758A^5B^1a^{14}b^8 + 4826809A^6a^{13}b^9) * (- (B^6a^6 - 78A^1B^5a^5b + 2535A^2B^4a^4b^2 - 43940A^3B^3a^3b^3 + 428415A^4B^2a^2b^4 - 2227758A^5B^1a^1b^5 + 4826809A^6b^6) / (a^{19}b^5))^{(2/3)} \\
& - 7(a^3b^2x^7 + 2a^4b^1x^4 + a^5x) * (- (B^6a^6 - 78A^1B^5a^5b + 2535A^2B^4a^4b^2 - 43940A^3B^3a^3b^3 + 428415A^4B^2a^2b^4 - 2227758A^5B^1a^1b^5 + 4826809A^6b^6) / (a^{19}b^5))^{(1/6)} \\
& * \log(-282475249(B^5a^{21}b^4 - 65A^1B^4a^{20}b^5 + 1690A^2B^3a^{19}b^6 - 21970A^3B^2a^{18}b^7 + 142805A^4B^1a^{17}b^8 - 371293A^5a^{16}b^9) * \sqrt{x}) \\
& * (- (B^6a^6 - 78A^1B^5a^5b + 2535A^2B^4a^4b^2 - 43940A^3B^3a^3b^3 + 428415A^4B^2a^2b^4 - 2227758A^5B^1a^1b^5 + 4826809A^6b^6) / (a^{19}b^5))^{(5/6)} \\
& + 282475249(B^{10}a^{10} - 130A^1B^9a^9b + 7605A^2B^8a^8b^2 - 263640A^3B^7a^7b^3 + 5997810A^4B^6a^6b^4 - 93565836A^5B^5a^5b^5 + 1013629890A^6B^4a^4b^6 - 7529822040A^7B^3a^3b^7 + 36707882445A^8B^2a^2b^8 - 106044993730A^9B^1a^1b^9 + 137858491849A^{10}b^{10}) * x \\
& - 282475249(B^6a^{19}b^3 - 78A^1B^5a^{18}b^4 + 2535A^2B^4a^{17}b^5 - 43940A^3B^3a^{16}b^6 + 428415A^4B^2a^{15}b^7 - 2227758A^5B^1a^{14}b^8 + 4826809A^6a^{13}b^9) * (- (B^6a^6 - 78A^1B^5a^5b + 2535A^2B^4a^4b^2 - 43940A^3B^3a^3b^3 + 428415A^4B^2a^2b^4 - 2227758A^5B^1a^1b^5 + 4826809A^6b^6) / (a^{19}b^5))^{(2/3)} \\
& + 12(7(B^1a^1b^1 - 13A^1b^2) * x^6 + 13(B^1a^2 - 13A^1a^1b^1) * x^3 - 72A^1a^2) * \sqrt{x}) / (a^3b^2x^7 + 2a^4b^1x^4 + a^5x)
\end{aligned}$$

giac [A] time = 0.48, size = 329, normalized size = 0.94

$$\frac{2A}{a^5\sqrt{c}} + \frac{7Bab^5 - 19Ab^2a^2 + 13Ba^2a^2 - 25Aab^2}{36(b^3+a)^{a^3}} - \frac{7\sqrt{5}\left((ab)^{\frac{1}{2}}Ba - 13(ab)^{\frac{1}{2}}Ab\right)\log\left(\sqrt{5}\sqrt{c}\left(\frac{c}{b}\right)^{\frac{1}{2}} + x + \left(\frac{c}{b}\right)^{\frac{1}{2}}\right)}{432a^{16}} + \frac{7\sqrt{5}\left((ab)^{\frac{1}{2}}Ba - 13(ab)^{\frac{1}{2}}Ab\right)\log\left(-\sqrt{5}\sqrt{c}\left(\frac{c}{b}\right)^{\frac{1}{2}} + x + \left(\frac{c}{b}\right)^{\frac{1}{2}}\right)}{432a^{16}} + \frac{7\left((ab)^{\frac{1}{2}}Ba - 13(ab)^{\frac{1}{2}}Ab\right)\arctan\left(\frac{\sqrt{5}\left(\frac{c}{b}\right)^{\frac{1}{2}} + \sqrt{c}}{\left(\frac{c}{b}\right)^{\frac{1}{2}}}\right)}{216a^{16}} + \frac{7\left((ab)^{\frac{1}{2}}Ba - 13(ab)^{\frac{1}{2}}Ab\right)\arctan\left(\frac{\sqrt{5}\left(\frac{c}{b}\right)^{\frac{1}{2}} - \sqrt{c}}{\left(\frac{c}{b}\right)^{\frac{1}{2}}}\right)}{216a^{16}} + \frac{7\left((ab)^{\frac{1}{2}}Ba - 13(ab)^{\frac{1}{2}}Ab\right)\arctan\left(\frac{\sqrt{c}}{\left(\frac{c}{b}\right)^{\frac{1}{2}}}\right)}{108a^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $-2A/(a^3\sqrt{x}) + 1/36(7B^1a^1b^1x^{11/2} - 19A^1b^2x^{11/2} + 13B^1a^2x^{5/2} - 25A^1a^1b^1x^{5/2}) / ((b^1x^3 + a)^2a^3) - 7/432\sqrt{3}((a^1b^5)^{5/6}B^1a - 13(a^1b^5)^{5/6}A^1b) * \log(\sqrt{3}\sqrt{x}(a/b)^{1/6} + x + (a/b))$

$$\begin{aligned} & \frac{1}{(1/3)} \frac{1}{(a^4 b^5)} + \frac{7}{432} \sqrt{3} \frac{((a b^5)^{5/6} B a - 13 (a b^5)^{5/6} A b)}{(a^4 b^5)} \\ & + \log(-\sqrt{3} \sqrt{x} (a/b)^{1/6} + x + (a/b)^{1/3}) \frac{1}{(a^4 b^5)} + \frac{7}{216} \frac{((a b^5)^{5/6} B a - 13 (a b^5)^{5/6} A b) \arctan(\sqrt{3} (a/b)^{1/6} + 2 \sqrt{x})}{(a/b)^{1/6}} \\ & \frac{1}{(a^4 b^5)} + \frac{7}{216} \frac{((a b^5)^{5/6} B a - 13 (a b^5)^{5/6} A b) \arctan(-(\sqrt{3} (a/b)^{1/6} - 2 \sqrt{x}) / (a/b)^{1/6})}{(a^4 b^5)} + \frac{7}{108} \\ & \frac{((a b^5)^{5/6} B a - 13 (a b^5)^{5/6} A b) \arctan(\sqrt{x} / (a/b)^{1/6})}{(a^4 b^5)} \end{aligned}$$

maple [A] time = 0.17, size = 441, normalized size = 1.26

$$\frac{19 A^2 b^5}{36 (b^3 + a^3)^2} - \frac{7 B a^2}{36 (b^3 + a^3)^2} - \frac{25 A b^3}{36 (b^3 + a^3)^2} - \frac{13 B^2}{36 (b^3 + a^3)^2} - \frac{91 A \arctan\left(\frac{2\sqrt{3}}{(b^3 + a^3)^{1/3}}\right)}{108 (b^3 + a^3)^2} - \frac{91 A \arctan\left(\frac{2\sqrt{3} + \sqrt{3}}{(b^3 + a^3)^{1/3}}\right)}{216 (b^3 + a^3)^2} - \frac{91 A \arctan\left(\frac{2\sqrt{3} - \sqrt{3}}{(b^3 + a^3)^{1/3}}\right)}{216 (b^3 + a^3)^2} - \frac{91 \sqrt{3} (b^3 + a^3) \operatorname{Im}\left(\sqrt{3} (b^3 + a^3)^{1/3} + (b^3 + a^3)^{1/3}\right)}{432 a^2} - \frac{91 \sqrt{3} (b^3 + a^3) \operatorname{Im}\left(\sqrt{3} (b^3 + a^3)^{1/3} - (b^3 + a^3)^{1/3}\right)}{432 a^2} - \frac{77 \arctan\left(\frac{2\sqrt{3}}{(b^3 + a^3)^{1/3}}\right)}{108 (b^3 + a^3)^2} - \frac{77 \arctan\left(\frac{2\sqrt{3} + \sqrt{3}}{(b^3 + a^3)^{1/3}}\right)}{216 (b^3 + a^3)^2} - \frac{77 \arctan\left(\frac{2\sqrt{3} - \sqrt{3}}{(b^3 + a^3)^{1/3}}\right)}{216 (b^3 + a^3)^2} - \frac{7 \sqrt{3} (b^3 + a^3) \operatorname{Im}\left(\sqrt{3} (b^3 + a^3)^{1/3} + (b^3 + a^3)^{1/3}\right)}{432 a^2} - \frac{7 \sqrt{3} (b^3 + a^3) \operatorname{Im}\left(\sqrt{3} (b^3 + a^3)^{1/3} - (b^3 + a^3)^{1/3}\right)}{432 a^2} - \frac{2 A}{27 \sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x)

[Out]
$$\begin{aligned} & -19/36/a^3/(b*x^3+a)^2*x^{11/2}*b^2*A+7/36/a^2/(b*x^3+a)^2*x^{11/2}*B*b-25/ \\ & 36/a^2/(b*x^3+a)^2*A*x^{5/2}*b+13/36/a/(b*x^3+a)^2*B*x^{5/2}-91/108/a^3*A/(\\ & a/b)^{1/6}*\arctan(1/(a/b)^{1/6}*x^{1/2})-91/432/a^4*A*b^3^{1/2}*(a/b)^{5/6} \\ & * \ln(-x+3^{1/2}*(a/b)^{1/6}*x^{1/2}-(a/b)^{1/3})-91/216/a^3*A/(a/b)^{1/6}*\arctan(2/ \\ & (a/b)^{1/6}*x^{1/2}-3^{1/2})+91/432/a^4*A*b^3^{1/2}*(a/b)^{5/6}*\ln(x+3^{1/2}*(a/b)^{1/6}*x^{1/2}+ \\ & (a/b)^{1/3})-91/216/a^3*A/(a/b)^{1/6}*\arctan(2/(a/b)^{1/6}*x^{1/2}+3^{1/2})+7/108/a^2*B/b/ \\ & (a/b)^{1/6}*\arctan(1/(a/b)^{1/6}*x^{1/2})+7/432/a^3*B*b^3^{1/2}*(a/b)^{5/6}*\ln(-x+3^{1/2}*(a/b)^{1/6}*x^{1/2} \\ & -(a/b)^{1/3})+7/216/a^2*B/b/(a/b)^{1/6}*\arctan(2/(a/b)^{1/6}*x^{1/2}-3^{1/2})-7/432/a^3*B*b^3^{1/2} \\ & *(a/b)^{5/6}*\ln(x+3^{1/2}*(a/b)^{1/6}*x^{1/2}+(a/b)^{1/3})+7/216/a^2*B/b/(a/b)^{1/6}*\arctan(2/(a/b)^{1/6} \\ & *x^{1/2}+3^{1/2})-2*A/a^3/x^{1/2} \end{aligned}$$

maxima [A] time = 1.29, size = 273, normalized size = 0.78

$$\frac{7(Bab - 13Ab^2)x^6 + 13(Ba^2 - 13Aab)x^3 - 72Aa^2}{36(a^3b^2x^2 + 2a^4bx^2 + a^5\sqrt{x})} - \frac{7(Ba - 13Ab) \left(\frac{\sqrt{3} \log\left(\sqrt{3}a^{\frac{1}{2}}b^{\frac{1}{2}}\sqrt{x+b^3x+a^3}\right)}{a^{\frac{5}{2}}b^{\frac{5}{2}}} - \frac{\sqrt{3} \log\left(-\sqrt{3}a^{\frac{1}{2}}b^{\frac{1}{2}}\sqrt{x+b^3x+a^3}\right)}{a^{\frac{5}{2}}b^{\frac{5}{2}}} - \frac{2 \arctan\left(\frac{\frac{1}{\sqrt{3}a^{\frac{1}{2}}b^{\frac{1}{2}}+2b^{\frac{1}{2}}\sqrt{x}}{\sqrt{a^3b^3}}}\right)}{\frac{2}{b^3}\sqrt{a^3b^3}} - \frac{2 \arctan\left(\frac{-\frac{1}{\sqrt{3}a^{\frac{1}{2}}b^{\frac{1}{2}}-2b^{\frac{1}{2}}\sqrt{x}}{\sqrt{a^3b^3}}}\right)}{\frac{2}{b^3}\sqrt{a^3b^3}} - \frac{4 \arctan\left(\frac{\frac{1}{\sqrt{3}a^{\frac{1}{2}}b^{\frac{1}{2}}}}{\sqrt{a^3b^3}}\right)}{\frac{2}{b^3}\sqrt{a^3b^3}} \right)}{432a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & 1/36*(7*(B*a*b - 13*A*b^2)*x^6 + 13*(B*a^2 - 13*A*a*b)*x^3 - 72*A*a^2)/(a^3 \\ & *b^2*x^{13/2} + 2*a^4*b*x^{7/2} + a^5*\sqrt{x}) - 7/432*(B*a - 13*A*b)*(\sqrt{3} \\ & * \log(\sqrt{3}*a^{1/6}*b^{1/6}*\sqrt{x} + b^{1/3}*x + a^{1/3})/(a^{1/6}*b^{5/6}) - \sqrt{3} \\ & * \log(-\sqrt{3}*a^{1/6}*b^{1/6}*\sqrt{x} + b^{1/3}*x + a^{1/3})/(a^{1/6}*b^{5/6}) - 2*\arctan((\sqrt{3}*a^{1/6}*b^{1/6} + 2*b^{1/3}*\sqrt{x}) \\ & / \sqrt{a^{1/3}*b^{1/3}}) / (b^{2/3}*\sqrt{a^{1/3}*b^{1/3}}) - 2*\arctan(-(\sqrt{3}*a^{1/6}*b^{1/6} - 2*b^{1/3}*\sqrt{x}) \\ & / \sqrt{a^{1/3}*b^{1/3}}) / (b^{2/3}*\sqrt{a^{1/3}*b^{1/3}}) \end{aligned}$$

$$(a^{(1/3)}*b^{(1/3)}) - 4*\arctan(b^{(1/3)}*\sqrt{x}/\sqrt{a^{(1/3)}*b^{(1/3)}})/(b^{(2/3)}*\sqrt{a^{(1/3)}*b^{(1/3)}})/a^3$$

mupad [B] time = 2.91, size = 1786, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^(3/2)*(a + b*x^3)^3), x)

[Out] (atan((((13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))*1i)/((-a)^(19/3)*b^(5/3)) + ((13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))*1i)/((-a)^(19/3)*b^(5/3)))/(((13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))))/((-a)^(19/3)*b^(5/3)) - ((13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))))/((-a)^(19/3)*b^(5/3)) - ((2*A)/a + (13*x^3*(13*A*b - B*a))/(36*a^2) + (7*b*x^6*(13*A*b - B*a))/(36*a^3))/((a^2*x^(1/2) + b^2*x^(13/2) + 2*a*b*x^(7/2)) + (atan((((3^(1/2)*1i)/2 - 1/2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5)))/(10077696*(-a)^(19/6)*b^(5/6))))*1i)/((-a)^(19/3)*b^(5/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5)))/(10077696*(-a)^(19/6)*b^(5/6))))*1i)/((-a)^(19/3)*b^(5/3)))/(((3^(1/2)*1i)/2 - 1/2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5)))/(10077696*(-a)^(19/6)*b^(5/6)))))/((-a)^(19/3)*b^(5/3)) - (((3^(1/2)*1i)/2 -

$$\frac{1}{2}^{2*(13*A*b - B*a)^2*(62019785528064*A^3*a^{21}*b^6 - 28229306112*B^3*a^{24}*b^3 + 1100942938368*A*B^2*a^{23}*b^4 - 14312258198784*A^2*B*a^{22}*b^5 + (343*x^{1/2}*((3^{1/2}*i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^{24}*b^6 + 829406312792064*B^2*a^{26}*b^4 - 21564564132593664*A*B*a^{25}*b^5)))/(10077696*(-a)^{(19/6)*b^{(5/6)}}))/((-a)^{(19/3)*b^{(5/3)}}))*((3^{1/2}*i)/2 - 1/2)*(13*A*b - B*a)*7i)/(108*(-a)^{(19/6)*b^{(5/6)}} + (\operatorname{atan}((((3^{1/2}*i)/2 + 1/2)^{2*(13*A*b - B*a)^2*(28229306112*B^3*a^{24}*b^3 - 62019785528064*A^3*a^{21}*b^6 - 1100942938368*A*B^2*a^{23}*b^4 + 14312258198784*A^2*B*a^{22}*b^5 + (343*x^{1/2}*((3^{1/2}*i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^{24}*b^6 + 829406312792064*B^2*a^{26}*b^4 - 21564564132593664*A*B*a^{25}*b^5)))/(10077696*(-a)^{(19/6)*b^{(5/6)}}))*i)/((-a)^{(19/3)*b^{(5/3)}} + (((3^{1/2}*i)/2 + 1/2)^{2*(13*A*b - B*a)^2*(62019785528064*A^3*a^{21}*b^6 - 28229306112*B^3*a^{24}*b^3 + 1100942938368*A*B^2*a^{23}*b^4 - 14312258198784*A^2*B*a^{22}*b^5 + (343*x^{1/2}*((3^{1/2}*i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^{24}*b^6 + 829406312792064*B^2*a^{26}*b^4 - 21564564132593664*A*B*a^{25}*b^5)))/(10077696*(-a)^{(19/6)*b^{(5/6)}}))*i)/((-a)^{(19/3)*b^{(5/3)}})/((((3^{1/2}*i)/2 + 1/2)^{2*(13*A*b - B*a)^2*(28229306112*B^3*a^{24}*b^3 - 62019785528064*A^3*a^{21}*b^6 - 1100942938368*A*B^2*a^{23}*b^4 + 14312258198784*A^2*B*a^{22}*b^5 + (343*x^{1/2}*((3^{1/2}*i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^{24}*b^6 + 829406312792064*B^2*a^{26}*b^4 - 21564564132593664*A*B*a^{25}*b^5)))/(10077696*(-a)^{(19/6)*b^{(5/6)}})))/((-a)^{(19/3)*b^{(5/3)}} - (((3^{1/2}*i)/2 + 1/2)^{2*(13*A*b - B*a)^2*(62019785528064*A^3*a^{21}*b^6 - 28229306112*B^3*a^{24}*b^3 + 1100942938368*A*B^2*a^{23}*b^4 - 14312258198784*A^2*B*a^{22}*b^5 + (343*x^{1/2}*((3^{1/2}*i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^{24}*b^6 + 829406312792064*B^2*a^{26}*b^4 - 21564564132593664*A*B*a^{25}*b^5)))/(10077696*(-a)^{(19/6)*b^{(5/6)}})))/((-a)^{(19/3)*b^{(5/3)}}))*((3^{1/2}*i)/2 + 1/2)*(13*A*b - B*a)*7i)/(108*(-a)^{(19/6)*b^{(5/6)}})$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**3,x)

[Out] Timed out

$$3.173 \quad \int \frac{A+Bx^3}{x^{5/2}(a+bx^3)^3} dx$$

Optimal. Leaf size=129

$$-\frac{(5Ab - aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}} + \frac{aB - 5Ab}{4a^3bx^{3/2}} + \frac{5Ab - aB}{12a^2bx^{3/2}(a + bx^3)} + \frac{Ab - aB}{6abx^{3/2}(a + bx^3)^2}$$

Rubi [A] time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 290, 325, 329, 275, 205}

$$\frac{5Ab - aB}{12a^2bx^{3/2}(a + bx^3)} - \frac{5Ab - aB}{4a^3bx^{3/2}} - \frac{(5Ab - aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}} + \frac{Ab - aB}{6abx^{3/2}(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3), x]

[Out] -(5*A*b - a*B)/(4*a^3*b*x^(3/2)) + (A*b - a*B)/(6*a*b*x^(3/2)*(a + b*x^3)^2) + (5*A*b - a*B)/(12*a^2*b*x^(3/2)*(a + b*x^3)) - ((5*A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(4*a^(7/2)*Sqrt[b])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 325

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^{5/2}(a + bx^3)^3} dx &= \frac{Ab - aB}{6abx^{3/2}(a + bx^3)^2} + \frac{\left(\frac{15Ab}{2} - \frac{3aB}{2}\right) \int \frac{1}{x^{5/2}(a+bx^3)^2} dx}{6ab} \\
&= \frac{Ab - aB}{6abx^{3/2}(a + bx^3)^2} + \frac{5Ab - aB}{12a^2bx^{3/2}(a + bx^3)} + \frac{(3(5Ab - aB)) \int \frac{1}{x^{5/2}(a+bx^3)} dx}{8a^2b} \\
&= -\frac{5Ab - aB}{4a^3bx^{3/2}} + \frac{Ab - aB}{6abx^{3/2}(a + bx^3)^2} + \frac{5Ab - aB}{12a^2bx^{3/2}(a + bx^3)} - \frac{(3(5Ab - aB)) \int \frac{\sqrt{x}}{a+bx^3} dx}{8a^3} \\
&= -\frac{5Ab - aB}{4a^3bx^{3/2}} + \frac{Ab - aB}{6abx^{3/2}(a + bx^3)^2} + \frac{5Ab - aB}{12a^2bx^{3/2}(a + bx^3)} - \frac{(3(5Ab - aB)) \text{Subst}\left(\int \frac{x^2}{a+bx^6} dx\right)}{4a^3} \\
&= -\frac{5Ab - aB}{4a^3bx^{3/2}} + \frac{Ab - aB}{6abx^{3/2}(a + bx^3)^2} + \frac{5Ab - aB}{12a^2bx^{3/2}(a + bx^3)} - \frac{(5Ab - aB) \text{Subst}\left(\int \frac{1}{a+bx^2} dx\right)}{4a^3} \\
&= -\frac{5Ab - aB}{4a^3bx^{3/2}} + \frac{Ab - aB}{6abx^{3/2}(a + bx^3)^2} + \frac{5Ab - aB}{12a^2bx^{3/2}(a + bx^3)} - \frac{(5Ab - aB) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 102, normalized size = 0.79

$$\frac{(aB - 5Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}} + \frac{a^2(5Bx^3 - 8A) + a(3bBx^6 - 25Abx^3) - 15Ab^2x^6}{12a^3x^{3/2}(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3), x]

[Out] (-15*A*b^2*x^6 + a^2*(-8*A + 5*B*x^3) + a*(-25*A*b*x^3 + 3*b*B*x^6))/(12*a^3*x^(3/2)*(a + b*x^3)^2) + ((-5*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(4*a^(7/2)*Sqrt[b])

IntegrateAlgebraic [A] time = 0.17, size = 102, normalized size = 0.79

$$\frac{(aB - 5Ab) \tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}} + \frac{-8a^2A + 5a^2Bx^3 - 25aAbx^3 + 3abBx^6 - 15Ab^2x^6}{12a^3x^{3/2}(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3), x]

[Out] $(-8*a^2*A - 25*a*A*b*x^3 + 5*a^2*B*x^3 - 15*A*b^2*x^6 + 3*a*b*B*x^6)/(12*a^3*x^{3/2}*(a + b*x^3)^2) + ((-5*A*b + a*B)*\text{ArcTan}[\sqrt{b}*x^{3/2}]/\sqrt{a}]/(4*a^{7/2}*\sqrt{b})$

fricas [A] time = 0.83, size = 347, normalized size = 2.69

$$\frac{3((Bab^2 - 5Ab^2)x^3 + 2(Ba^2b - 5Aab^2)x^2 + (Ba^2 - 5Aa^2b)x)\sqrt{-ab} \log\left(\frac{bx^2 + \sqrt{ab}x^{3/2} - a}{bx^3 + a}\right) + 2(3(Ba^2b^2 - 5Aab^2)x^6 - 8Aa^2b + 5(Ba^2b - 5Aa^2b^2)x^2)\sqrt{x} - 3((Bab^2 - 5Ab^2)x^3 + 2(Ba^2b - 5Aa^2b^2)x^2 + (Ba^2 - 5Aa^2b)x)\sqrt{ab} \arctan\left(\frac{\sqrt{ab}x^{3/2}}{a}\right) + (3(Ba^2b^2 - 5Aab^2)x^6 - 8Aa^2b + 5(Ba^2b - 5Aa^2b^2)x^2)\sqrt{x}}{24(a^3b^3x^6 + 2a^2b^2x^5 + a^3bx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $[1/24*(3*((B*a*b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 - 5*A*a^2*b)*x^2)*\sqrt{-a*b}*\log((b*x^3 + 2*\sqrt{-a*b})*x^{3/2} - a)/(b*x^3 + a) + 2*(3*(B*a^2*b^2 - 5*A*a*b^3)*x^6 - 8*A*a^3*b + 5*(B*a^3*b - 5*A*a^2*b^2)*x^3)*\sqrt{x}]/(a^4*b^3*x^8 + 2*a^5*b^2*x^5 + a^6*b*x^2), 1/12*(3*((B*a*b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 - 5*A*a^2*b)*x^2)*\sqrt{a*b}*\arctan(\sqrt{a*b}*x^{3/2}/a) + (3*(B*a^2*b^2 - 5*A*a*b^3)*x^6 - 8*A*a^3*b + 5*(B*a^3*b - 5*A*a^2*b^2)*x^3)*\sqrt{x}]/(a^4*b^3*x^8 + 2*a^5*b^2*x^5 + a^6*b*x^2)]$

giac [A] time = 0.18, size = 88, normalized size = 0.68

$$\frac{(Ba - 5Ab) \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{4\sqrt{ab}a^3} - \frac{2A}{3a^3x^2} + \frac{3Babx^9 - 7Ab^2x^9 + 5Ba^2x^3 - 9Aabx^3}{12(bx^3 + a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $1/4*(B*a - 5*A*b)*\arctan(b*x^{3/2}/\sqrt{a*b})/(\sqrt{a*b}*a^3) - 2/3*A/(a^3*x^{3/2}) + 1/12*(3*B*a*b*x^{9/2} - 7*A*b^2*x^{9/2} + 5*B*a^2*x^{3/2} - 9*A*a*b*x^{3/2})/((b*x^3 + a)^2*a^3)$

maple [A] time = 0.07, size = 133, normalized size = 1.03

$$-\frac{7Ab^2x^9}{12(bx^3 + a)^2a^3} + \frac{Bbx^9}{4(bx^3 + a)^2a^2} - \frac{3Abx^3}{4(bx^3 + a)^2a^2} + \frac{5Bx^3}{12(bx^3 + a)^2a} - \frac{5Ab \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{4\sqrt{ab}a^3} + \frac{B \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{4\sqrt{ab}a^2} - \frac{2A}{3a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x)`

[Out]
$$-7/12/a^3/(b*x^3+a)^2*x^(9/2)*b^2*A+1/4/a^2/(b*x^3+a)^2*x^(9/2)*B*b-3/4/a^2/(b*x^3+a)^2*A*x^(3/2)*b+5/12/a/(b*x^3+a)^2*B*x^(3/2)-5/4/a^3/(a*b)^(1/2)*a*\arctan(1/(a*b)^(1/2)*b*x^(3/2))*A*b+1/4/a^2/(a*b)^(1/2)*\arctan(1/(a*b)^(1/2))*b*x^(3/2)*B-2/3/a^3*A/x^(3/2)$$

maxima [A] time = 1.35, size = 100, normalized size = 0.78

$$\frac{3(Bab - 5Ab^2)x^6 + 5(Ba^2 - 5Aab)x^3 - 8Aa^2}{12\left(a^3b^2x^{\frac{15}{2}} + 2a^4bx^{\frac{9}{2}} + a^5x^{\frac{3}{2}}\right)} + \frac{(Ba - 5Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="maxima")`

[Out]
$$1/12*(3*(B*a*b - 5*A*b^2)*x^6 + 5*(B*a^2 - 5*A*a*b)*x^3 - 8*A*a^2)/(a^3*b^2*x^(15/2) + 2*a^4*b*x^(9/2) + a^5*x^(3/2)) + 1/4*(B*a - 5*A*b)*\arctan(b*x^(3/2)/\sqrt{a*b})/(\sqrt{a*b}*a^3)$$

mupad [B] time = 2.73, size = 163, normalized size = 1.26

$$\frac{\frac{2A}{3a} + \frac{5x^3(5Ab-Ba)}{12a^2} + \frac{bx^6(5Ab-Ba)}{4a^3}}{a^2x^{3/2} + b^2x^{15/2} + 2abx^{9/2}} - \frac{\operatorname{atan}\left(\frac{8a^{7/2}\sqrt{b}x^{3/2}(86400A^2a^9b^5 - 34560ABa^{10}b^4 + 3456B^2a^{11}b^3)}{(5Ab-Ba)(138240Aa^{13}b^4 - 27648Ba^{14}b^3)}\right)}{4a^{7/2}\sqrt{b}}(5Ab - Ba)}{4a^{7/2}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x)`

[Out]
$$-((2A)/(3a) + (5*x^3*(5*A*b - B*a))/(12*a^2) + (b*x^6*(5*A*b - B*a))/(4*a^3))/ (a^2*x^(3/2) + b^2*x^(15/2) + 2*a*b*x^(9/2)) - (\operatorname{atan}((8*a^(7/2)*b^(1/2)*x^(3/2)*(86400*A^2*a^9*b^5 + 3456*B^2*a^11*b^3 - 34560*A*B*a^10*b^4))/((5*A*b - B*a)*(138240*A*a^13*b^4 - 27648*B*a^14*b^3)))*(5*A*b - B*a))/(4*a^(7/2)*b^(1/2))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**3,x)`

[Out] Timed out

$$3.174 \quad \int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^3} dx$$

Optimal. Leaf size=351

$$\frac{11(17Ab - 5aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{23/6} \sqrt[6]{b}} - \frac{11(17Ab - 5aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{23/6} \sqrt[6]{b}} + \frac{11(17A}{$$

Rubi [A] time = 0.54, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {457, 290, 325, 329, 209, 634, 618, 204, 628, 205}

$$\frac{17Ab - 5aB}{36a^2 b^{1/2} (a + bx^3)} - \frac{11(17Ab - 5aB)}{180a^2 b^{5/2}} + \frac{11(17Ab - 5aB) \log\left(-\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{23/6} \sqrt[6]{b}} - \frac{11(17Ab - 5aB) \log\left(\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x} + \sqrt[3]{a} + \sqrt[3]{b} x\right)}{144\sqrt{3} a^{23/6} \sqrt[6]{b}} + \frac{11(17Ab - 5aB) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{a}}{\sqrt[6]{b}}\right)}{216a^{23/6} \sqrt[6]{b}} - \frac{11(17Ab - 5aB) \tan^{-1}\left(\frac{2\sqrt[6]{a}}{\sqrt[6]{b}} + \sqrt{3}\right)}{216a^{23/6} \sqrt[6]{b}} - \frac{11(17Ab - 5aB) \tan^{-1}\left(\frac{\sqrt[6]{a}}{\sqrt[6]{b}}\right)}{108a^{23/6} \sqrt[6]{b}} + \frac{Ab - aB}{6abx^{5/2} (a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3), x]

[Out] (-11*(17*A*b - 5*a*B))/(180*a^3*b*x^(5/2)) + (A*b - a*B)/(6*a*b*x^(5/2)*(a + b*x^3)^2) + (17*A*b - 5*a*B)/(36*a^2*b*x^(5/2)*(a + b*x^3)) + (11*(17*A*b - 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(23/6)*b^(1/6)) - (11*(17*A*b - 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(23/6)*b^(1/6)) - (11*(17*A*b - 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(23/6)*b^(1/6)) + (11*(17*A*b - 5*a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(23/6)*b^(1/6)) - (11*(17*A*b - 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(23/6)*b^(1/6))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 209

Int[((a_) + (b_.)*(x_)^(n_))(-1), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*

$(k - 1)\pi/n]x)/(r^2 - 2rs\cos[(2k - 1)\pi/n]x + s^2x^2), x] + \text{Int}[(r + s\cos[(2k - 1)\pi/n]x)/(r^2 + 2rs\cos[(2k - 1)\pi/n]x + s^2x^2), x]; (2r^2\text{Int}[1/(r^2 + s^2x^2), x])/(a^n) + \text{Dist}[(2r)/(a^n), \text{Sum}[u, \{k, 1, (n - 2)/4\}], x], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{IGtQ}[(n - 2)/4, 0] \&\& \text{PosQ}[a/b]$

Rule 290

$\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^p, x_Symbol] \rightarrow -\text{Simp}[(c \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} / (a \cdot c \cdot n \cdot (p+1)), x] + \text{Dist}[(m + n \cdot (p+1) + 1) / (a \cdot n \cdot (p+1)), \text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^{p+1}, x], x] /; \text{FreeQ}\{a, b, c, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 325

$\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^p, x_Symbol] \rightarrow \text{Simp}[(c \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} / (a \cdot c \cdot (m+1)), x] - \text{Dist}[(b \cdot (m + n \cdot (p+1) + 1)) / (a \cdot c^n \cdot (m+1)), \text{Int}[(c \cdot x)^{m+n} \cdot (a + b \cdot x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[m, -1] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 329

$\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^p, x_Symbol] \rightarrow \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{k \cdot (m+1) - 1} \cdot (a + (b \cdot x^{k \cdot n})) / c^n]^p, x], x, (c \cdot x)^{1/k}], x] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 457

$\text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^p \cdot ((c) + (d \cdot x)^n), x_Symbol] \rightarrow -\text{Simp}[(b \cdot c - a \cdot d) \cdot (e \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} / (a \cdot b \cdot e \cdot n \cdot (p+1)), x] - \text{Dist}[(a \cdot d \cdot (m+1) - b \cdot c \cdot (m + n \cdot (p+1) + 1)) / (a \cdot b \cdot n \cdot (p+1)), \text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^{p+1}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x] \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{LtQ}[p, -1] \&\& ((\text{!IntegerQ}[p + 1/2] \&\& \text{NeQ}[p, -5/4]) \|\| \text{!RationalQ}[m] \|\| (\text{IGtQ}[n, 0] \&\& \text{ILtQ}[p + 1/2, 0] \&\& \text{LeQ}[-1, m, -(n \cdot (p+1))]))$

Rule 618

$\text{Int}[(a + b \cdot x + c \cdot x^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4 \cdot a \cdot c - x^2, x], x], x, b + 2 \cdot c \cdot x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^{7/2}(a + bx^3)^3} dx &= \frac{Ab - aB}{6abx^{5/2}(a + bx^3)^2} + \frac{\left(\frac{17Ab}{2} - \frac{5aB}{2}\right) \int \frac{1}{x^{7/2}(a+bx^3)^2} dx}{6ab} \\
&= \frac{Ab - aB}{6abx^{5/2}(a + bx^3)^2} + \frac{17Ab - 5aB}{36a^2bx^{5/2}(a + bx^3)} + \frac{(11(17Ab - 5aB)) \int \frac{1}{x^{7/2}(a+bx^3)} dx}{72a^2b} \\
&= -\frac{11(17Ab - 5aB)}{180a^3bx^{5/2}} + \frac{Ab - aB}{6abx^{5/2}(a + bx^3)^2} + \frac{17Ab - 5aB}{36a^2bx^{5/2}(a + bx^3)} - \frac{(11(17Ab - 5aB)) \int \frac{1}{\sqrt{x}} dx}{72a^3} \\
&= -\frac{11(17Ab - 5aB)}{180a^3bx^{5/2}} + \frac{Ab - aB}{6abx^{5/2}(a + bx^3)^2} + \frac{17Ab - 5aB}{36a^2bx^{5/2}(a + bx^3)} - \frac{(11(17Ab - 5aB)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{x}} dx, x, a+bx^3\right)}{36a^3} \\
&= -\frac{11(17Ab - 5aB)}{180a^3bx^{5/2}} + \frac{Ab - aB}{6abx^{5/2}(a + bx^3)^2} + \frac{17Ab - 5aB}{36a^2bx^{5/2}(a + bx^3)} - \frac{(11(17Ab - 5aB)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{x}} dx, x, a+bx^3\right)}{36a^3} \\
&= -\frac{11(17Ab - 5aB)}{180a^3bx^{5/2}} + \frac{Ab - aB}{6abx^{5/2}(a + bx^3)^2} + \frac{17Ab - 5aB}{36a^2bx^{5/2}(a + bx^3)} - \frac{11(17Ab - 5aB) \tan^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{108a^{23/6}\sqrt[6]{b}} \\
&= -\frac{11(17Ab - 5aB)}{180a^3bx^{5/2}} + \frac{Ab - aB}{6abx^{5/2}(a + bx^3)^2} + \frac{17Ab - 5aB}{36a^2bx^{5/2}(a + bx^3)} - \frac{11(17Ab - 5aB) \tan^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{108a^{23/6}\sqrt[6]{b}} \\
&= -\frac{11(17Ab - 5aB)}{180a^3bx^{5/2}} + \frac{Ab - aB}{6abx^{5/2}(a + bx^3)^2} + \frac{17Ab - 5aB}{36a^2bx^{5/2}(a + bx^3)} + \frac{11(17Ab - 5aB) \tan^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{216a^{23/6}\sqrt[6]{b}}
\end{aligned}$$

Mathematica [C] time = 0.12, size = 96, normalized size = 0.27

$$\frac{a(a^2(85Bx^3 - 72A) + a(55bBx^6 - 289Abx^3) - 187Ab^2x^6)}{(a+bx^3)^2} + 55x^3(5aB - 17Ab) {}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; -\frac{bx^3}{a}\right)$$

$$180a^4x^{5/2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3), x]

[Out] $((a*(-187*A*b^2*x^6 + a^2*(-72*A + 85*B*x^3) + a*(-289*A*b*x^3 + 55*b*B*x^6)))/(a + b*x^3)^2 + 55*(-17*A*b + 5*a*B)*x^3 \text{Hypergeometric2F1}[1/6, 1, 7/6, -(b*x^3)/a])/(180*a^4*x^{(5/2)})$

IntegrateAlgebraic [A] time = 0.64, size = 222, normalized size = 0.63

$$\frac{11(5aB - 17Ab) \tan^{-1}\left(\frac{\sqrt[6]{b} \sqrt{x}}{\sqrt[9]{a}}\right)}{108a^{23/6} \sqrt[6]{b}} - \frac{11(5aB - 17Ab) \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b}x}{\sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}\right)}{216a^{23/6} \sqrt[6]{b}} + \frac{11(5aB - 17Ab) \tanh^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \sqrt[6]{b} \sqrt{x}}{\sqrt[3]{a} + \sqrt[3]{b}x}\right)}{72\sqrt{3} a^{23/6} \sqrt[6]{b}} + \frac{-72a^2A + 85a^2Bx^3 - 289aBx^3 + 55abBx^6 - 187Ab^2x^6}{180a^3x^{5/2} (a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3), x]

[Out] $(-72*a^2*A - 289*a*A*b*x^3 + 85*a^2*B*x^3 - 187*A*b^2*x^6 + 55*a*b*B*x^6)/(180*a^3*x^{(5/2)}*(a + b*x^3)^2) + (11*(-17*A*b + 5*a*B)*\text{ArcTan}[(b^{(1/6)}*\text{Sqrt}[x])/a^{(1/6)}])/(108*a^{(23/6)}*b^{(1/6)}) - (11*(-17*A*b + 5*a*B)*\text{ArcTan}[(a^{(1/3)} - b^{(1/3)}*x)/(a^{(1/6)}*b^{(1/6)}*\text{Sqrt}[x])])/(216*a^{(23/6)}*b^{(1/6)}) + (11*(-17*A*b + 5*a*B)*\text{ArcTanh}[(\text{Sqrt}[3]*a^{(1/6)}*b^{(1/6)}*\text{Sqrt}[x])/(a^{(1/3)} + b^{(1/3)}*x)])/(72*\text{Sqrt}[3]*a^{(23/6)}*b^{(1/6)})$

fricas [B] time = 1.33, size = 2690, normalized size = 7.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $-1/2160*(220*\text{sqrt}(3)*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^{23}*b))^{(1/6)}*\text{arctan}(1/3*(2*\text{sqrt}(3)*\text{sqrt}(a^8*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^{23}*b))^{(1/3)} + (25*B^2*a^2 - 170*A*B*a*b + 289*A^2*b^2)*x + (5*B*a^5 - 17*A*a^4*b)*\text{sqrt}(x)*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^{23}*b))^{(1/6)})*a^{19}*b*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^{23}*b))^{(5/6)} + 2*\text{sqrt}(3)*(5*B*a^{20}*b - 17*A*a^{19}*b^2)*\text{sqrt}(x)*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^{23}*b))^{(5/6)} - \text{sqrt}(3)*(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6))/(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 2413$

$$\begin{aligned}
& 7569A^6b^6)) + 220\sqrt{3}(a^3b^2x^9 + 2a^4bx^6 + a^5x^3)*(-(15625 \\
& *B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) \\
& / (a^{23}b))^{\frac{1}{6}} * \arctan\left(\frac{1}{3} * (2\sqrt{3})\sqrt{a^8 * (-(15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) / (a^{23}b))^{\frac{1}{3}}}\right) \\
& + (25B^2a^2 - 170AB^1ab + 289A^2b^2)*x - (5B^5a^5 - 17A^4b^4)*\sqrt{x} * (-(15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) / (a^{23}b))^{\frac{1}{6}}) \\
& * a^{19}b * (-(15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) / (a^{23}b))^{\frac{5}{6}} + 2\sqrt{3} \\
& * (5B^5a^{20}b - 17A^4b^{19}) * \sqrt{x} * (-(15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) / (a^{23}b))^{\frac{5}{6}} + \sqrt{3} \\
& * (15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) / (15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) \\
& - 55(a^3b^2x^9 + 2a^4bx^6 + a^5x^3) * (-(15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) / (a^{23}b))^{\frac{1}{6}} * \log(121a^8 * (-(15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) / (a^{23}b))^{\frac{1}{3}}) \\
& + 121 * (25B^2a^2 - 170AB^1ab + 289A^2b^2)*x + 121 * (5B^5a^5 - 17A^4b^4)*\sqrt{x} * (-(15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) / (a^{23}b))^{\frac{1}{6}}) \\
& + 55(a^3b^2x^9 + 2a^4bx^6 + a^5x^3) * (-(15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) / (a^{23}b))^{\frac{1}{6}} * \log(121a^8 * (-(15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) / (a^{23}b))^{\frac{1}{3}}) \\
& + 121 * (25B^2a^2 - 170AB^1ab + 289A^2b^2)*x - 121 * (5B^5a^5 - 17A^4b^4)*\sqrt{x} * (-(15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) / (a^{23}b))^{\frac{1}{6}}) \\
& + 110(a^3b^2x^9 + 2a^4bx^6 + a^5x^3) * (-(15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) / (a^{23}b))^{\frac{1}{6}} * \log(11a^4 * (-(15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) / (a^{23}b))^{\frac{1}{3}}) \\
& - 11 * (5B^5a^5 - 17A^4b^4)*\sqrt{x} - 110(a^3b^2x^9 + 2a^4bx^6 + a^5x^3) * (-(15625B^6a^6 - 318750AB^5a^5b + 2709375A^2B^4a^4b^2 - 12282500A^3B^3a^3b^3 + 31320375A^4B^2a^2b^4 - 42595710A^5B^1ab^5 + 24137569A^6b^6) / (a^{23}b))^{\frac{1}{6}}
\end{aligned}$$

$$0375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^{23}b)^{(1/6)}*\log(-11*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^{23}b))^{(1/6)} - 11*(5*B*a - 17*A*b)*\sqrt{x}) - 12*(11*(5*B*a*b - 17*A*b^2)*x^6 + 17*(5*B*a^2 - 17*A*a*b)*x^3 - 72*A*a^2)*\sqrt{x})/(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)$$

giac [A] time = 0.24, size = 334, normalized size = 0.95

$$\frac{11\sqrt{3}\left(5(a^6)^2 B a - 17(a^6)^2 A b\right) \log\left(\sqrt{3}\sqrt{\frac{a}{b}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - 11\sqrt{3}\left(5(a^6)^2 B a - 17(a^6)^2 A b\right) \log\left(-\sqrt{3}\sqrt{\frac{a}{b}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + 11\left(5(a^6)^2 B a - 17(a^6)^2 A b\right) \arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{3}} + \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - 11\left(5(a^6)^2 B a - 17(a^6)^2 A b\right) \arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{3}} - \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 11\left(5(a^6)^2 B a - 17(a^6)^2 A b\right) \arctan\left(\frac{x\sqrt{3}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + \frac{11 B a b^2 - 23 A b^2 a^2 + 17 B a^2 \sqrt{3} - 29 A a b \sqrt{3}}{36(b^3 + a)^3} - \frac{2 A}{5 a^2 \sqrt{3}}}{432 a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="giac")

[Out] 11/432*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b) - 11/432*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b) + 11/216*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^4*b) + 11/216*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^4*b) + 11/108*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^4*b) + 1/36*(11*B*a*b*x^(7/2) - 23*A*b^2*x^(7/2) + 17*B*a^2*sqrt(x) - 29*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^3) - 2/5*A/(a^3*x^(5/2))

maple [A] time = 0.17, size = 435, normalized size = 1.24

$$\frac{\frac{11 A a^2}{36(b^3+a)^2} - \frac{11 B a^2}{36(b^3+a)^2} - \frac{29 A a b \sqrt{3}}{36(b^3+a)^2} - \frac{11 B a b^2}{36(b^3+a)^2} + \frac{187 (b^2 A b \arctan\left(\frac{x}{b}\right) - \frac{2 A}{b})}{108 a^2} - \frac{187 (b^2 A b \arctan\left(\frac{2 x}{b}\right) - \frac{2 A}{b})}{216 a^2} - \frac{187 (b^2 A b \arctan\left(\frac{2 x}{b} + \sqrt{x}\right) - \frac{2 A}{b})}{216 a^2} - \frac{187 \sqrt{3} (b^2 A b (\sqrt{x} + \sqrt{3} (b^2 \sqrt{x} + (b^2)^{\frac{1}{3}})) - 187 \sqrt{3} (b^2 A b (\sqrt{x} - \sqrt{3} (b^2 \sqrt{x} - (b^2)^{\frac{1}{3}})) - \frac{2 A}{b}))}{432 a^2} + \frac{55 (b^2 A b \arctan\left(\frac{x}{b}\right) - \frac{2 A}{b})}{108 a^2} + \frac{55 (b^2 A b \arctan\left(\frac{2 x}{b} + \sqrt{x}\right) - \frac{2 A}{b})}{216 a^2} + \frac{55 (b^2 A b \arctan\left(\frac{2 x}{b} + \sqrt{x}\right) - \frac{2 A}{b})}{216 a^2} + \frac{55 \sqrt{3} (b^2 A b (\sqrt{x} + \sqrt{3} (b^2 \sqrt{x} + (b^2)^{\frac{1}{3}})) - 55 \sqrt{3} (b^2 A b (\sqrt{x} - \sqrt{3} (b^2 \sqrt{x} - (b^2)^{\frac{1}{3}})) - \frac{2 A}{b}))}{432 a^2} - \frac{2 A}{5 a^2 \sqrt{3}}}{432 a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x)

[Out] -23/36/a^3/(b*x^3+a)^2*x^(7/2)*b^2*A+11/36/a^2/(b*x^3+a)^2*x^(7/2)*B*b-29/36/a^2/(b*x^3+a)^2*A*x^(1/2)*b+17/36/a/(b*x^3+a)^2*B*x^(1/2)-187/108/a^4*A*b*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))+187/432/a^4*A*b^3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))-187/216/a^4*A*b*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))-187/432/a^4*A*b^3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-187/216/a^4*A*b*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))+55/108/a^3*B*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))-55/432/a^3*B^3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))+55/216/a^3*B*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))+55/432/a^3*B^3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+55/216/a^3*B*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))-2/5*A/a^3/x^(5/2)

maxima [A] time = 1.34, size = 346, normalized size = 0.99

$$\frac{11(5Bab - 17Ab^2)x^6 + 17(5Ba^2 - 17Aab)x^3 - 72Aa^2}{180(a^3b^2x^7 + 2a^4bx^{\frac{11}{2}} + a^5x^3)} + \frac{11 \left(\frac{\sqrt{3}(5Ba - 17Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x + b^{\frac{1}{3}}x + a^{\frac{1}{3}}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(5Ba - 17Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x + b^{\frac{1}{3}}x + a^{\frac{1}{3}}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4(5Ba^{\frac{4}{3}} - 17Ab^{\frac{4}{3}})\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}} + \frac{2(5Ba^{\frac{4}{3}}b^{\frac{1}{3}} - 17Aa^{\frac{1}{3}}b^{\frac{4}{3}})\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}} + \frac{2(5Ba^{\frac{4}{3}}b^{\frac{1}{3}} - 17Aa^{\frac{1}{3}}b^{\frac{4}{3}})\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} \right)}{432a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] 1/180*(11*(5*B*a*b - 17*A*b^2)*x^6 + 17*(5*B*a^2 - 17*A*a*b)*x^3 - 72*A*a^2)/(a^3*b^2*x^(17/2) + 2*a^4*b*x^(11/2) + a^5*x^(5/2)) + 11/432*(sqrt(3)*(5*B*a - 17*A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(5*B*a - 17*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(5*B*a*b^(1/3) - 17*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 17*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 17*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3)))/a^3

mupad [B] time = 2.96, size = 2109, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x)

[Out] - ((2*A)/(5*a) + (17*x^3*(17*A*b - 5*B*a))/(180*a^2) + (11*b*x^6*(17*A*b - 5*B*a))/(180*a^3))/(a^2*x^(5/2) + b^2*x^(17/2) + 2*a*b*x^(11/2)) - (atan(((x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) - (11*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7)))/(216*(-a)^(23/6)*b^(1/6)))*(17*A*b - 5*B*a)*11i)/(216*(-a)^(23/6)*b^(1/6)) + ((x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) + (11*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7)))/(216*(-a)^(23/6)*b^(1/6)))*(17*A*b - 5*B*a)*11i)/(216*(-a)^(23/6)*b^(1/6)))/((11*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) - (11*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5

$$\begin{aligned}
& + 132985945575014400*A*B^2*a^{21}*b^6 - 452152214955048960*A^2*B*a^{20}*b^7))/ \\
& (216*(-a)^{(23/6)}*b^{(1/6)})))*(17*A*b - 5*B*a))/(216*(-a)^{(23/6)}*b^{(1/6)} - (1 \\
& 1*(x^{(1/2)}*(443639472636450816*A^4*a^{15}*b^9 + 3319819810560000*B^4*a^{19}*b^5 \\
& + 230262702060441600*A^2*B^2*a^{17}*b^7 - 45149549423616000*A*B^3*a^{18}*b^6 - \\
& 521928791337000960*A^3*B*a^{16}*b^8) + (11*(17*A*b - 5*B*a)*(512439176949055 \\
& 488*A^3*a^{19}*b^8 - 13037837801472000*B^3*a^{22}*b^5 + 132985945575014400*A*B^ \\
& 2*a^{21}*b^6 - 452152214955048960*A^2*B*a^{20}*b^7))/(216*(-a)^{(23/6)}*b^{(1/6)})) \\
& *(17*A*b - 5*B*a))/(216*(-a)^{(23/6)}*b^{(1/6)})))*(17*A*b - 5*B*a)*11i)/(108*(\\
& -a)^{(23/6)}*b^{(1/6)} - (\operatorname{atan}((((3^{(1/2)}*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^{(1 \\
& /2)}*(443639472636450816*A^4*a^{15}*b^9 + 3319819810560000*B^4*a^{19}*b^5 + 2302 \\
& 62702060441600*A^2*B^2*a^{17}*b^7 - 45149549423616000*A*B^3*a^{18}*b^6 - 521928 \\
& 791337000960*A^3*B*a^{16}*b^8) - (11*((3^{(1/2)}*1i)/2 - 1/2)*(17*A*b - 5*B*a)* \\
& (512439176949055488*A^3*a^{19}*b^8 - 13037837801472000*B^3*a^{22}*b^5 + 1329859 \\
& 45575014400*A*B^2*a^{21}*b^6 - 452152214955048960*A^2*B*a^{20}*b^7))/(216*(-a)^ \\
& (23/6)*b^{(1/6)})))*11i)/(216*(-a)^{(23/6)}*b^{(1/6)} + (((3^{(1/2)}*1i)/2 - 1/2)*(\\
& 17*A*b - 5*B*a)*(x^{(1/2)}*(443639472636450816*A^4*a^{15}*b^9 + 331981981056000 \\
& 0*B^4*a^{19}*b^5 + 230262702060441600*A^2*B^2*a^{17}*b^7 - 45149549423616000*A* \\
& B^3*a^{18}*b^6 - 521928791337000960*A^3*B*a^{16}*b^8) + (11*((3^{(1/2)}*1i)/2 - 1 \\
& /2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^{19}*b^8 - 13037837801472000*B \\
& ^3*a^{22}*b^5 + 132985945575014400*A*B^2*a^{21}*b^6 - 452152214955048960*A^2*B* \\
& a^{20}*b^7))/(216*(-a)^{(23/6)}*b^{(1/6)})))*11i)/(216*(-a)^{(23/6)}*b^{(1/6)})))/((11* \\
& ((3^{(1/2)}*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^{(1/2)}*(443639472636450816*A^4*a^{ \\
& 15}*b^9 + 3319819810560000*B^4*a^{19}*b^5 + 230262702060441600*A^2*B^2*a^{17}*b^ \\
& 7 - 45149549423616000*A*B^3*a^{18}*b^6 - 521928791337000960*A^3*B*a^{16}*b^8) - \\
& (11*((3^{(1/2)}*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^{19}*b \\
& ^8 - 13037837801472000*B^3*a^{22}*b^5 + 132985945575014400*A*B^2*a^{21}*b^6 - 4 \\
& 52152214955048960*A^2*B*a^{20}*b^7))/(216*(-a)^{(23/6)}*b^{(1/6)})))/((216*(-a)^{(2 \\
& 3/6)}*b^{(1/6)} - (11*((3^{(1/2)}*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^{(1/2)}*(44363 \\
& 9472636450816*A^4*a^{15}*b^9 + 3319819810560000*B^4*a^{19}*b^5 + 23026270206044 \\
& 1600*A^2*B^2*a^{17}*b^7 - 45149549423616000*A*B^3*a^{18}*b^6 - 5219287913370009 \\
& 60*A^3*B*a^{16}*b^8) + (11*((3^{(1/2)}*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(512439176 \\
& 949055488*A^3*a^{19}*b^8 - 13037837801472000*B^3*a^{22}*b^5 + 13298594557501440 \\
& 0*A*B^2*a^{21}*b^6 - 452152214955048960*A^2*B*a^{20}*b^7))/(216*(-a)^{(23/6)}*b^{(\\
& 1/6)})))/((216*(-a)^{(23/6)}*b^{(1/6)})))*((3^{(1/2)}*1i)/2 - 1/2)*(17*A*b - 5*B*a) \\
& *11i)/(108*(-a)^{(23/6)}*b^{(1/6)} - (\operatorname{atan}((((3^{(1/2)}*1i)/2 + 1/2)*(17*A*b - \\
& 5*B*a)*(x^{(1/2)}*(443639472636450816*A^4*a^{15}*b^9 + 3319819810560000*B^4*a^{1 \\
& 9}*b^5 + 230262702060441600*A^2*B^2*a^{17}*b^7 - 45149549423616000*A*B^3*a^{18} \\
& b^6 - 521928791337000960*A^3*B*a^{16}*b^8) - (11*((3^{(1/2)}*1i)/2 + 1/2)*(17*A \\
& *b - 5*B*a)*(512439176949055488*A^3*a^{19}*b^8 - 13037837801472000*B^3*a^{22}*b \\
& ^5 + 132985945575014400*A*B^2*a^{21}*b^6 - 452152214955048960*A^2*B*a^{20}*b^7) \\
&)/(216*(-a)^{(23/6)}*b^{(1/6)})))*11i)/(216*(-a)^{(23/6)}*b^{(1/6)} + (((3^{(1/2)}*1i \\
&)/2 + 1/2)*(17*A*b - 5*B*a)*(x^{(1/2)}*(443639472636450816*A^4*a^{15}*b^9 + 331 \\
& 9819810560000*B^4*a^{19}*b^5 + 230262702060441600*A^2*B^2*a^{17}*b^7 - 45149549 \\
& 423616000*A*B^3*a^{18}*b^6 - 521928791337000960*A^3*B*a^{16}*b^8) + (11*((3^{(1/ \\
& 2)}*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^{19}*b^8 - 1303783
\end{aligned}$$

$$\begin{aligned} & 7801472000*B^3*a^{22}*b^5 + 132985945575014400*A*B^2*a^{21}*b^6 - 4521522149550 \\ & 48960*A^2*B*a^{20}*b^7)/(216*(-a)^{(23/6)}*b^{(1/6)}))^{11i}/(216*(-a)^{(23/6)}*b^{(1/6)}))^{11i} \\ & /((11*((3^{(1/2)}*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(x^{(1/2)}*(4436394726364 \\ & 50816*A^4*a^{15}*b^9 + 3319819810560000*B^4*a^{19}*b^5 + 230262702060441600*A^2 \\ & *B^2*a^{17}*b^7 - 45149549423616000*A*B^3*a^{18}*b^6 - 521928791337000960*A^3*B \\ & *a^{16}*b^8) - (11*((3^{(1/2)}*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(51243917694905548 \\ & 8*A^3*a^{19}*b^8 - 13037837801472000*B^3*a^{22}*b^5 + 132985945575014400*A*B^2* \\ & a^{21}*b^6 - 452152214955048960*A^2*B*a^{20}*b^7))/(216*(-a)^{(23/6)}*b^{(1/6)})))/ \\ & (216*(-a)^{(23/6)}*b^{(1/6)}) - (11*((3^{(1/2)}*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(x^{(1/2)} \\ & *(443639472636450816*A^4*a^{15}*b^9 + 3319819810560000*B^4*a^{19}*b^5 + 23 \\ & 0262702060441600*A^2*B^2*a^{17}*b^7 - 45149549423616000*A*B^3*a^{18}*b^6 - 5219 \\ & 28791337000960*A^3*B*a^{16}*b^8) + (11*((3^{(1/2)}*1i)/2 + 1/2)*(17*A*b - 5*B*a \\ &)*(512439176949055488*A^3*a^{19}*b^8 - 13037837801472000*B^3*a^{22}*b^5 + 13298 \\ & 5945575014400*A*B^2*a^{21}*b^6 - 452152214955048960*A^2*B*a^{20}*b^7))/(216*(-a \\ &)^{(23/6)}*b^{(1/6)})))/(216*(-a)^{(23/6)}*b^{(1/6)}))^{11i}/(108*(-a)^{(23/6)}*b^{(1/6)}) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**3,x)

[Out] Timed out

$$3.175 \quad \int x^8 \sqrt{a + bx^3} (A + Bx^3) dx$$

Optimal. Leaf size=103

$$\frac{2a^2 (a + bx^3)^{3/2} (Ab - aB)}{9b^4} + \frac{2 (a + bx^3)^{7/2} (Ab - 3aB)}{21b^4} - \frac{2a (a + bx^3)^{5/2} (2Ab - 3aB)}{15b^4} + \frac{2B (a + bx^3)^{9/2}}{27b^4}$$

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{2a^2 (a + bx^3)^{3/2} (Ab - aB)}{9b^4} + \frac{2 (a + bx^3)^{7/2} (Ab - 3aB)}{21b^4} - \frac{2a (a + bx^3)^{5/2} (2Ab - 3aB)}{15b^4} + \frac{2B (a + bx^3)^{9/2}}{27b^4}$$

Antiderivative was successfully verified.

[In] Int[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]

[Out] (2*a^2*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b*x^3)^(5/2))/(15*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(7/2))/(21*b^4) + (2*B*(a + b*x^3)^(9/2))/(27*b^4)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int x^8 \sqrt{a + bx^3} (A + Bx^3) dx &= \frac{1}{3} \text{Subst} \left(\int x^2 \sqrt{a + bx} (A + Bx) dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{a^2(-Ab + aB)\sqrt{a + bx}}{b^3} + \frac{a(-2Ab + 3aB)(a + bx)^{3/2}}{b^3} + \frac{(Ab - 3aB)(a + bx)^{5/2}}{b^3} \right) dx, x, x^3 \right) \\ &= \frac{2a^2(Ab - aB)(a + bx^3)^{3/2}}{9b^4} - \frac{2a(2Ab - 3aB)(a + bx^3)^{5/2}}{15b^4} + \frac{2(Ab - 3aB)(a + bx^3)^{7/2}}{21b^4} \end{aligned}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 0.73

$$\frac{2(a + bx^3)^{3/2} (-16a^3B + 24a^2b(A + Bx^3) - 6ab^2x^3(6A + 5Bx^3) + 5b^3x^6(9A + 7Bx^3))}{945b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^8*sqrt[a + b*x^3]*(A + B*x^3), x]

[Out] (2*(a + b*x^3)^(3/2)*(-16*a^3*B + 24*a^2*b*(A + B*x^3) - 6*a*b^2*x^3*(6*A + 5*B*x^3) + 5*b^3*x^6*(9*A + 7*B*x^3)))/(945*b^4)

IntegrateAlgebraic [A] time = 0.05, size = 80, normalized size = 0.78

$$\frac{2(a + bx^3)^{3/2} (16a^3B - 24a^2Ab - 24a^2bBx^3 + 36aAb^2x^3 + 30ab^2Bx^6 - 45Ab^3x^6 - 35b^3Bx^9)}{945b^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8*sqrt[a + b*x^3]*(A + B*x^3), x]

[Out] (-2*(a + b*x^3)^(3/2)*(-24*a^2*A*b + 16*a^3*B + 36*a*A*b^2*x^3 - 24*a^2*b*B*x^3 - 45*A*b^3*x^6 + 30*a*b^2*B*x^6 - 35*b^3*B*x^9))/(945*b^4)

fricas [A] time = 0.79, size = 99, normalized size = 0.96

$$\frac{2(35Bb^4x^{12} + 5(Bab^3 + 9Ab^4)x^9 - 3(2Ba^2b^2 - 3Aab^3)x^6 - 16Ba^4 + 24Aa^3b + 4(2Ba^3b - 3Aa^2b^2)x^3)\sqrt{bx^3 + a}}{945b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] 2/945*(35*B*b^4*x^12 + 5*(B*a*b^3 + 9*A*b^4)*x^9 - 3*(2*B*a^2*b^2 - 3*A*a*b^3)*x^6 - 16*B*a^4 + 24*A*a^3*b + 4*(2*B*a^3*b - 3*A*a^2*b^2)*x^3)*sqrt(b*x^3 + a)/b^4

giac [A] time = 0.16, size = 104, normalized size = 1.01

$$\frac{2 \left(35 (bx^3 + a)^{\frac{9}{2}} B - 135 (bx^3 + a)^{\frac{7}{2}} Ba + 189 (bx^3 + a)^{\frac{5}{2}} Ba^2 - 105 (bx^3 + a)^{\frac{3}{2}} Ba^3 + 45 (bx^3 + a)^{\frac{7}{2}} Ab - 126 (bx^3 + a)^{\frac{5}{2}} Aab + 105 (bx^3 + a)^{\frac{3}{2}} Aa^2 b \right)}{945 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] 2/945*(35*(b*x^3 + a)^(9/2)*B - 135*(b*x^3 + a)^(7/2)*B*a + 189*(b*x^3 + a)^(5/2)*B*a^2 - 105*(b*x^3 + a)^(3/2)*B*a^3 + 45*(b*x^3 + a)^(7/2)*A*b - 126*(b*x^3 + a)^(5/2)*A*a*b + 105*(b*x^3 + a)^(3/2)*A*a^2*b)/b^4

maple [A] time = 0.05, size = 77, normalized size = 0.75

$$\frac{2 (b x^3 + a)^{\frac{3}{2}} (35 B x^9 b^3 + 45 A b^3 x^6 - 30 B a b^2 x^6 - 36 A a b^2 x^3 + 24 B a^2 b x^3 + 24 A a^2 b - 16 B a^3)}{945 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x)

[Out] 2/945*(b*x^3+a)^(3/2)*(35*B*b^3*x^9+45*A*b^3*x^6-30*B*a*b^2*x^6-36*A*a*b^2*x^3+24*B*a^2*b*x^3+24*A*a^2*b-16*B*a^3)/b^4

maxima [A] time = 0.62, size = 118, normalized size = 1.15

$$\frac{2}{945} B \left(\frac{35 (bx^3 + a)^{\frac{9}{2}}}{b^4} - \frac{135 (bx^3 + a)^{\frac{7}{2}} a}{b^4} + \frac{189 (bx^3 + a)^{\frac{5}{2}} a^2}{b^4} - \frac{105 (bx^3 + a)^{\frac{3}{2}} a^3}{b^4} \right) + \frac{2}{315} A \left(\frac{15 (bx^3 + a)^{\frac{7}{2}}}{b^3} - \frac{42 (bx^3 + a)^{\frac{5}{2}} a}{b^3} + \frac{35 (bx^3 + a)^{\frac{3}{2}} a^2}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] 2/945*B*(35*(b*x^3 + a)^(9/2)/b^4 - 135*(b*x^3 + a)^(7/2)*a/b^4 + 189*(b*x^3 + a)^(5/2)*a^2/b^4 - 105*(b*x^3 + a)^(3/2)*a^3/b^4) + 2/315*A*(15*(b*x^3 + a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3 + a)^(3/2)*a^2/b^3)

mupad [B] time = 2.72, size = 154, normalized size = 1.50

$$\frac{2 B x^{12} \sqrt{b x^3 + a}}{27} + \frac{x^9 \sqrt{b x^3 + a} \left(2 A b + \frac{2 B a}{9} \right)}{21 b} + \frac{8 a^2 \left(2 A a - \frac{6 a \left(2 A b + \frac{2 B a}{9} \right)}{7 b} \right) \sqrt{b x^3 + a}}{45 b^3} + \frac{x^6 \left(2 A a - \frac{6 a \left(2 A b + \frac{2 B a}{9} \right)}{7 b} \right) \sqrt{b x^3 + a}}{15 b} - \frac{4 a x^3 \left(2 A a - \frac{6 a \left(2 A b + \frac{2 B a}{9} \right)}{7 b} \right) \sqrt{b x^3 + a}}{45 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(A + B*x^3)*(a + b*x^3)^(1/2),x)

```
[Out] (2*B*x^12*(a + b*x^3)^(1/2))/27 + (x^9*(a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/9
))/ (21*b) + (8*a^2*(2*A*a - (6*a*(2*A*b + (2*B*a)/9)))/(7*b))*(a + b*x^3)^(1
/2))/(45*b^3) + (x^6*(2*A*a - (6*a*(2*A*b + (2*B*a)/9)))/(7*b))*(a + b*x^3)^(
1/2))/(15*b) - (4*a*x^3*(2*A*a - (6*a*(2*A*b + (2*B*a)/9)))/(7*b))*(a + b*x
^3)^(1/2))/(45*b^2)
```

sympy [A] time = 4.17, size = 219, normalized size = 2.13

$$\begin{cases} \frac{16Aa^3\sqrt{a+bx^3}}{315b^3} - \frac{8Aa^2x^3\sqrt{a+bx^3}}{315b^2} + \frac{2Aax^6\sqrt{a+bx^3}}{105b} + \frac{2Ax^9\sqrt{a+bx^3}}{21} - \frac{32Ba^4\sqrt{a+bx^3}}{945b^4} + \frac{16Ba^3x^3\sqrt{a+bx^3}}{945b^3} - \frac{4Ba^2x^6\sqrt{a+bx^3}}{315b^2} + \frac{2Bax^9\sqrt{a+bx^3}}{189b} + \frac{2Bx^{12}\sqrt{a+bx^3}}{27} & \text{for } b \neq 0 \\ \sqrt{a} \left(\frac{Ax^9}{9} + \frac{Bx^{12}}{12} \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((16*A*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*A*a**2*x**3*sqrt(a + b
*x**3)/(315*b**2) + 2*A*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*A*x**9*sqrt(a +
b*x**3)/21 - 32*B*a**4*sqrt(a + b*x**3)/(945*b**4) + 16*B*a**3*x**3*sqrt(a
+ b*x**3)/(945*b**3) - 4*B*a**2*x**6*sqrt(a + b*x**3)/(315*b**2) + 2*B*a*x
**9*sqrt(a + b*x**3)/(189*b) + 2*B*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (s
qrt(a)*(A*x**9/9 + B*x**12/12), True))
```

$$3.176 \quad \int x^5 \sqrt{a + bx^3} (A + Bx^3) dx$$

Optimal. Leaf size=73

$$\frac{2(a + bx^3)^{5/2} (Ab - 2aB)}{15b^3} - \frac{2a(a + bx^3)^{3/2} (Ab - aB)}{9b^3} + \frac{2B(a + bx^3)^{7/2}}{21b^3}$$

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{2(a + bx^3)^{5/2} (Ab - 2aB)}{15b^3} - \frac{2a(a + bx^3)^{3/2} (Ab - aB)}{9b^3} + \frac{2B(a + bx^3)^{7/2}}{21b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]

[Out] (-2*a*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)^(5/2))/(15*b^3) + (2*B*(a + b*x^3)^(7/2))/(21*b^3)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int x^5 \sqrt{a + bx^3} (A + Bx^3) dx &= \frac{1}{3} \text{Subst} \left(\int x \sqrt{a + bx} (A + Bx) dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{a(-Ab + aB)\sqrt{a + bx}}{b^2} + \frac{(Ab - 2aB)(a + bx)^{3/2}}{b^2} + \frac{B(a + bx)^{5/2}}{b^2} \right) dx \right) \\ &= -\frac{2a(Ab - aB)(a + bx^3)^{3/2}}{9b^3} + \frac{2(Ab - 2aB)(a + bx^3)^{5/2}}{15b^3} + \frac{2B(a + bx^3)^{7/2}}{21b^3} \end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.78

$$\frac{2(a + bx^3)^{3/2} (8a^2B - 2ab(7A + 6Bx^3) + 3b^2x^3(7A + 5Bx^3))}{315b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*Sqrt[a + b*x^3]*(A + B*x^3), x]

[Out] (2*(a + b*x^3)^(3/2)*(8*a^2*B + 3*b^2*x^3*(7*A + 5*B*x^3) - 2*a*b*(7*A + 6*B*x^3)))/(315*b^3)

IntegrateAlgebraic [A] time = 0.04, size = 56, normalized size = 0.77

$$\frac{2(a + bx^3)^{3/2} (8a^2B - 14aAb - 12abBx^3 + 21Ab^2x^3 + 15b^2Bx^6)}{315b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*Sqrt[a + b*x^3]*(A + B*x^3), x]

[Out] (2*(a + b*x^3)^(3/2)*(-14*a*A*b + 8*a^2*B + 21*A*b^2*x^3 - 12*a*b*B*x^3 + 15*b^2*B*x^6))/(315*b^3)

fricas [A] time = 0.83, size = 75, normalized size = 1.03

$$\frac{2(15Bb^3x^9 + 3(Bab^2 + 7Ab^3)x^6 + 8Ba^3 - 14Aa^2b - (4Ba^2b - 7Aab^2)x^3)\sqrt{bx^3 + a}}{315b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] 2/315*(15*B*b^3*x^9 + 3*(B*a*b^2 + 7*A*b^3)*x^6 + 8*B*a^3 - 14*A*a^2*b - (4*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^3

giac [A] time = 0.18, size = 73, normalized size = 1.00

$$\frac{2 \left(15 (bx^3 + a)^{\frac{7}{2}} B - 42 (bx^3 + a)^{\frac{5}{2}} Ba + 35 (bx^3 + a)^{\frac{3}{2}} Ba^2 + 21 (bx^3 + a)^{\frac{5}{2}} Ab - 35 (bx^3 + a)^{\frac{3}{2}} Aab \right)}{315 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] 2/315*(15*(b*x^3 + a)^(7/2)*B - 42*(b*x^3 + a)^(5/2)*B*a + 35*(b*x^3 + a)^(3/2)*B*a^2 + 21*(b*x^3 + a)^(5/2)*A*b - 35*(b*x^3 + a)^(3/2)*A*a*b)/b^3

maple [A] time = 0.04, size = 53, normalized size = 0.73

$$\frac{2 (bx^3 + a)^{\frac{3}{2}} (-15B b^2 x^6 - 21A b^2 x^3 + 12Bab x^3 + 14Aab - 8B a^2)}{315 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x)

[Out] -2/315*(b*x^3+a)^(3/2)*(-15*B*b^2*x^6-21*A*b^2*x^3+12*B*a*b*x^3+14*A*a*b-8*B*a^2)/b^3

maxima [A] time = 0.51, size = 84, normalized size = 1.15

$$\frac{2}{315} B \left(\frac{15 (bx^3 + a)^{\frac{7}{2}}}{b^3} - \frac{42 (bx^3 + a)^{\frac{5}{2}} a}{b^3} + \frac{35 (bx^3 + a)^{\frac{3}{2}} a^2}{b^3} \right) + \frac{2}{45} A \left(\frac{3 (bx^3 + a)^{\frac{5}{2}}}{b^2} - \frac{5 (bx^3 + a)^{\frac{3}{2}} a}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] 2/315*B*(15*(b*x^3 + a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3 + a)^(3/2)*a^2/b^3) + 2/45*A*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/2)*a/b^2)

mupad [B] time = 2.66, size = 114, normalized size = 1.56

$$\frac{2Bx^9\sqrt{bx^3+a}}{21} + \frac{x^6\sqrt{bx^3+a}\left(2Ab+\frac{2Ba}{7}\right)}{15b} - \frac{2a\left(2Aa-\frac{4a\left(2Ab+\frac{2Ba}{7}\right)}{5b}\right)\sqrt{bx^3+a}}{9b^2} + \frac{x^3\left(2Aa-\frac{4a\left(2Ab+\frac{2Ba}{7}\right)}{5b}\right)\sqrt{bx^3+a}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(A + B*x^3)*(a + b*x^3)^(1/2), x)`

[Out] $(2*B*x^9*(a + b*x^3)^{(1/2)})/21 + (x^6*(a + b*x^3)^{(1/2)}*(2*A*b + (2*B*a)/7))/((15*b) - (2*a*(2*A*a - (4*a*(2*A*b + (2*B*a)/7)))/(5*b))*(a + b*x^3)^{(1/2)})/(9*b^2) + (x^3*(2*A*a - (4*a*(2*A*b + (2*B*a)/7)))/(5*b))*(a + b*x^3)^{(1/2)})/(9*b)$

sympy [A] time = 1.82, size = 168, normalized size = 2.30

$$\begin{cases} -\frac{4Aa^2\sqrt{a+bx^3}}{45b^2} + \frac{2Aax^3\sqrt{a+bx^3}}{45b} + \frac{2Ax^6\sqrt{a+bx^3}}{15} + \frac{16Ba^3\sqrt{a+bx^3}}{315b^3} - \frac{8Ba^2x^3\sqrt{a+bx^3}}{315b^2} + \frac{2Bax^6\sqrt{a+bx^3}}{105b} + \frac{2Bx^9\sqrt{a+bx^3}}{21} & \text{for } b \neq 0 \\ \sqrt{a} \left(\frac{Ax^6}{6} + \frac{Bx^9}{9} \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(B*x**3+A)*(b*x**3+a)**(1/2), x)`

[Out] `Piecewise((-4*A*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*A*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*A*x**6*sqrt(a + b*x**3)/15 + 16*B*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*B*a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*B*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*B*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*(A*x**6/6 + B*x**9/9), True))`

$$3.177 \quad \int x^2 \sqrt{a + bx^3} (A + Bx^3) dx$$

Optimal. Leaf size=46

$$\frac{2(a + bx^3)^{3/2} (Ab - aB)}{9b^2} + \frac{2B(a + bx^3)^{5/2}}{15b^2}$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {444, 43}

$$\frac{2(a + bx^3)^{3/2} (Ab - aB)}{9b^2} + \frac{2B(a + bx^3)^{5/2}}{15b^2}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]

[Out] (2*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^2) + (2*B*(a + b*x^3)^(5/2))/(15*b^2)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sqrt{a + bx^3} (A + Bx^3) dx &= \frac{1}{3} \text{Subst} \left(\int \sqrt{a + bx} (A + Bx) dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{(Ab - aB)\sqrt{a + bx}}{b} + \frac{B(a + bx)^{3/2}}{b} \right) dx, x, x^3 \right) \\
&= \frac{2(Ab - aB)(a + bx^3)^{3/2}}{9b^2} + \frac{2B(a + bx^3)^{5/2}}{15b^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.74

$$\frac{2(a + bx^3)^{3/2}(-2aB + 5Ab + 3bBx^3)}{45b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a + b*x^3]*(A + B*x^3), x]

[Out] (2*(a + b*x^3)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^3))/(45*b^2)

IntegrateAlgebraic [A] time = 0.03, size = 34, normalized size = 0.74

$$\frac{2(a + bx^3)^{3/2}(-2aB + 5Ab + 3bBx^3)}{45b^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*Sqrt[a + b*x^3]*(A + B*x^3), x]

[Out] (2*(a + b*x^3)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^3))/(45*b^2)

fricas [A] time = 0.71, size = 50, normalized size = 1.09

$$\frac{2(3Bb^2x^6 + (Bab + 5Ab^2)x^3 - 2Ba^2 + 5Aab)\sqrt{bx^3 + a}}{45b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] 2/45*(3*B*b^2*x^6 + (B*a*b + 5*A*b^2)*x^3 - 2*B*a^2 + 5*A*a*b)*sqrt(b*x^3 + a)/b^2

giac [A] time = 0.15, size = 44, normalized size = 0.96

$$\frac{2 \left(3 (bx^3 + a)^{\frac{5}{2}} B - 5 (bx^3 + a)^{\frac{3}{2}} Ba + 5 (bx^3 + a)^{\frac{3}{2}} Ab \right)}{45 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] 2/45*(3*(b*x^3 + a)^(5/2)*B - 5*(b*x^3 + a)^(3/2)*B*a + 5*(b*x^3 + a)^(3/2)*A*b)/b^2

maple [A] time = 0.04, size = 31, normalized size = 0.67

$$\frac{2 (bx^3 + a)^{\frac{3}{2}} (3Bbx^3 + 5Ab - 2Ba)}{45b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x)

[Out] 2/45*(b*x^3+a)^(3/2)*(3*B*b*x^3+5*A*b-2*B*a)/b^2

maxima [A] time = 0.59, size = 49, normalized size = 1.07

$$\frac{2}{45} B \left(\frac{3 (bx^3 + a)^{\frac{5}{2}}}{b^2} - \frac{5 (bx^3 + a)^{\frac{3}{2}} a}{b^2} \right) + \frac{2 (bx^3 + a)^{\frac{3}{2}} A}{9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] 2/45*B*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/2)*a/b^2) + 2/9*(b*x^3 + a)^(3/2)*A/b

mupad [B] time = 2.60, size = 44, normalized size = 0.96

$$\frac{6 B (bx^3 + a)^{5/2} + 10 A b (bx^3 + a)^{3/2} - 10 B a (bx^3 + a)^{3/2}}{45 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(A + B*x^3)*(a + b*x^3)^(1/2),x)

[Out] $(6*B*(a + b*x^3)^{(5/2)} + 10*A*b*(a + b*x^3)^{(3/2)} - 10*B*a*(a + b*x^3)^{(3/2)})/(45*b^2)$

sympy [A] time = 0.74, size = 117, normalized size = 2.54

$$\begin{cases} \frac{2Aa\sqrt{a+bx^3}}{9b} + \frac{2Ax^3\sqrt{a+bx^3}}{9} - \frac{4Ba^2\sqrt{a+bx^3}}{45b^2} + \frac{2Bax^3\sqrt{a+bx^3}}{45b} + \frac{2Bx^6\sqrt{a+bx^3}}{15} & \text{for } b \neq 0 \\ \sqrt{a} \left(\frac{Ax^3}{3} + \frac{Bx^6}{6} \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(B*x**3+A)*(b*x**3+a)**(1/2),x)`

[Out] `Piecewise((2*A*a*sqrt(a + b*x**3)/(9*b) + 2*A*x**3*sqrt(a + b*x**3)/9 - 4*B*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*B*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*B*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*(A*x**3/3 + B*x**6/6), True)`

$$3.178 \quad \int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x} dx$$

Optimal. Leaf size=64

$$\frac{2}{3}A\sqrt{a+bx^3} - \frac{2}{3}\sqrt{a}A \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) + \frac{2B(a+bx^3)^{3/2}}{9b}$$

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 80, 50, 63, 208}

$$\frac{2}{3}A\sqrt{a+bx^3} - \frac{2}{3}\sqrt{a}A \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) + \frac{2B(a+bx^3)^{3/2}}{9b}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]

[Out] (2*A*Sqrt[a + b*x^3])/3 + (2*B*(a + b*x^3)^(3/2))/(9*b) - (2*Sqrt[a]*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

$n + p + 2$), $\text{Int}[(c + d*x)^n*(e + f*x)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, n, p\}, x] \&\& \text{NeQ}[n + p + 2, 0]$

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$ $\text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b]$

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + bx^3} (A + Bx^3)}{x} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{a + bx} (A + Bx)}{x} dx, x, x^3 \right) \\ &= \frac{2B(a + bx^3)^{3/2}}{9b} + \frac{1}{3} A \text{Subst} \left(\int \frac{\sqrt{a + bx}}{x} dx, x, x^3 \right) \\ &= \frac{2}{3} A \sqrt{a + bx^3} + \frac{2B(a + bx^3)^{3/2}}{9b} + \frac{1}{3} (aA) \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx}} dx, x, x^3 \right) \\ &= \frac{2}{3} A \sqrt{a + bx^3} + \frac{2B(a + bx^3)^{3/2}}{9b} + \frac{(2aA) \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + bx^3} \right)}{3b} \\ &= \frac{2}{3} A \sqrt{a + bx^3} + \frac{2B(a + bx^3)^{3/2}}{9b} - \frac{2}{3} \sqrt{a} A \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right) \end{aligned}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 0.94

$$\frac{2}{9} \left(\frac{\sqrt{a + bx^3} (B(a + bx^3) + 3Ab)}{b} - 3\sqrt{a} A \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]

[Out] $(2*((\text{Sqrt}[a + b*x^3]*(3*A*b + B*(a + b*x^3)))/b - 3*\text{Sqrt}[a]*A*\text{ArcTanh}[\text{Sqrt}[a + b*x^3]/\text{Sqrt}[a]]))/9$

IntegrateAlgebraic [A] time = 0.05, size = 61, normalized size = 0.95

$$\frac{2\sqrt{a + bx^3} (aB + 3Ab + bBx^3)}{9b} - \frac{2}{3}\sqrt{a} A \tanh^{-1}\left(\frac{\sqrt{a + bx^3}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]

[Out] $(2*\text{Sqrt}[a + b*x^3]*(3*A*b + a*B + b*B*x^3))/(9*b) - (2*\text{Sqrt}[a]*A*\text{ArcTanh}[\text{Sqrt}[a + b*x^3]/\text{Sqrt}[a]])/3$

fricas [A] time = 0.77, size = 125, normalized size = 1.95

$$\left[\frac{3A\sqrt{a}b \log\left(\frac{bx^3 - 2\sqrt{bx^3+a}\sqrt{a} + 2a}{x^3}\right) + 2(Bbx^3 + Ba + 3Ab)\sqrt{bx^3+a}}{9b}, \frac{2\left(3A\sqrt{-a}b \arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) + (Bbx^3 + Ba + 3Ab)\sqrt{bx^3+a}\right)}{9b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="fricas")

[Out] $[1/9*(3*A*\text{sqrt}(a)*b*\log((b*x^3 - 2*\text{sqrt}(b*x^3 + a)*\text{sqrt}(a) + 2*a)/x^3) + 2*(B*b*x^3 + B*a + 3*A*b)*\text{sqrt}(b*x^3 + a))/b, 2/9*(3*A*\text{sqrt}(-a)*b*\arctan(\text{sqrt}(b*x^3 + a)*\text{sqrt}(-a)/a) + (B*b*x^3 + B*a + 3*A*b)*\text{sqrt}(b*x^3 + a))/b]$

giac [A] time = 0.15, size = 61, normalized size = 0.95

$$\frac{2Aa \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}} + \frac{2\left((bx^3 + a)^{\frac{3}{2}}Bb^2 + 3\sqrt{bx^3 + a}Ab^3\right)}{9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="giac")

[Out] $2/3*A*a*\arctan(\text{sqrt}(b*x^3 + a)/\text{sqrt}(-a))/\text{sqrt}(-a) + 2/9*((b*x^3 + a)^{(3/2)}*B*b^2 + 3*\text{sqrt}(b*x^3 + a)*A*b^3)/b^3$

maple [A] time = 0.05, size = 50, normalized size = 0.78

$$\left(-\frac{2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3} + \frac{2\sqrt{bx^3+a}}{3} \right) A + \frac{2(bx^3 + a)^{\frac{3}{2}} B}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^3+A)*(b*x^3+a)^(1/2)/x,x)`

[Out] $2/9*B*(b*x^3+a)^{(3/2)}/b+A*(-2/3*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})*a^{(1/2)}+2/3*(b*x^3+a)^{(1/2)})$

maxima [A] time = 1.20, size = 67, normalized size = 1.05

$$\frac{1}{3} \left(\sqrt{a} \log \left(\frac{\sqrt{bx^3 + a} - \sqrt{a}}{\sqrt{bx^3 + a} + \sqrt{a}} \right) + 2 \sqrt{bx^3 + a} \right) A + \frac{2 (bx^3 + a)^{\frac{3}{2}} B}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="maxima")`

[Out] $1/3*(\operatorname{sqrt}(a)*\log((\operatorname{sqrt}(b*x^3 + a) - \operatorname{sqrt}(a))/(\operatorname{sqrt}(b*x^3 + a) + \operatorname{sqrt}(a)))) + 2*\operatorname{sqrt}(b*x^3 + a)*A + 2/9*(b*x^3 + a)^{(3/2)}*B/b$

mupad [B] time = 2.71, size = 80, normalized size = 1.25

$$\frac{2Bx^3\sqrt{bx^3+a}}{9} + \frac{\sqrt{bx^3+a}\left(2Ab + \frac{2Ba}{3}\right)}{3b} + \frac{A\sqrt{a}\ln\left(\frac{(\sqrt{bx^3+a}-\sqrt{a})^3(\sqrt{bx^3+a}+\sqrt{a})}{x^6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^(1/2))/x,x)`

[Out] $(2*B*x^3*(a + b*x^3)^{(1/2)})/9 + ((a + b*x^3)^{(1/2)}*(2*A*b + (2*B*a)/3))/(3*b) + (A*a^{(1/2)}*\log((((a + b*x^3)^{(1/2)} - a^{(1/2)})^3*((a + b*x^3)^{(1/2)} + a^{(1/2)}))/x^6))/3$

sympy [A] time = 25.83, size = 76, normalized size = 1.19

$$\frac{A \left(-\frac{2a \operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}} - 2\sqrt{a+bx^3} \right)}{3} - \frac{B \left(\begin{cases} -\sqrt{a}x^3 & \text{for } b = 0 \\ -\frac{2(a+bx^3)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x,x)`

[Out] $-A*(-2*a*\operatorname{atan}(\operatorname{sqrt}(a + b*x**3)/\operatorname{sqrt}(-a))/\operatorname{sqrt}(-a) - 2*\operatorname{sqrt}(a + b*x**3))/3 - B*\operatorname{Piecewise}((- \operatorname{sqrt}(a)*x**3, \operatorname{Eq}(b, 0)), (-2*(a + b*x**3)**(3/2)/(3*b), \operatorname{True}))/3$

$$3.179 \quad \int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4} dx$$

Optimal. Leaf size=84

$$\frac{\sqrt{a+bx^3}(2aB+Ab)}{3a} - \frac{(2aB+Ab) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{A(a+bx^3)^{3/2}}{3ax^3}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 78, 50, 63, 208}

$$\frac{\sqrt{a+bx^3}(2aB+Ab)}{3a} - \frac{(2aB+Ab) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{A(a+bx^3)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]

[Out] ((A*b + 2*a*B)*Sqrt[a + b*x^3])/(3*a) - (A*(a + b*x^3)^(3/2))/(3*a*x^3) - (A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]/(3*Sqrt[a])

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```


$f*(p + 1)*(c*f - d*e)), x] - \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + bx^3} (A + Bx^3)}{x^4} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{a + bx} (A + Bx)}{x^2} dx, x, x^3 \right) \\ &= -\frac{A(a + bx^3)^{3/2}}{3ax^3} + \frac{(Ab + 2aB) \text{Subst} \left(\int \frac{\sqrt{a+bx}}{x} dx, x, x^3 \right)}{6a} \\ &= \frac{(Ab + 2aB)\sqrt{a + bx^3}}{3a} - \frac{A(a + bx^3)^{3/2}}{3ax^3} + \frac{1}{6}(Ab + 2aB) \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx}} dx, x, x^3 \right) \\ &= \frac{(Ab + 2aB)\sqrt{a + bx^3}}{3a} - \frac{A(a + bx^3)^{3/2}}{3ax^3} + \frac{(Ab + 2aB) \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + bx^3} \right)}{3b} \\ &= \frac{(Ab + 2aB)\sqrt{a + bx^3}}{3a} - \frac{A(a + bx^3)^{3/2}}{3ax^3} - \frac{(Ab + 2aB) \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{3\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.75

$$\frac{1}{3} \left(\frac{\sqrt{a + bx^3} (2Bx^3 - A)}{x^3} - \frac{(2aB + Ab) \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]

[Out] ((Sqrt[a + b*x^3]*(-A + 2*B*x^3))/x^3 - ((A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/Sqrt[a])/3

IntegrateAlgebraic [A] time = 0.11, size = 65, normalized size = 0.77

$$\frac{\sqrt{a + bx^3} (2Bx^3 - A)}{3x^3} + \frac{(-2aB - Ab) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]

[Out] (Sqrt[a + b*x^3]*(-A + 2*B*x^3))/(3*x^3) + (((-A*b) - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a])

fricas [A] time = 0.90, size = 143, normalized size = 1.70

$$\left[\frac{(2Ba + Ab)\sqrt{a}x^3 \log\left(\frac{bx^3 - 2\sqrt{bx^3+a}\sqrt{a} + 2a}{x^3}\right) + 2(2Bax^3 - Aa)\sqrt{bx^3+a} (2Ba + Ab)\sqrt{-a}x^3 \arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) + (2Bax^3 - Aa)\sqrt{bx^3+a}}{6ax^3}, \frac{(2Ba + Ab)\sqrt{-a}x^3 \arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) + (2Bax^3 - Aa)\sqrt{bx^3+a}}{3ax^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")

[Out] [1/6*((2*B*a + A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(2*B*a*x^3 - A*a)*sqrt(b*x^3 + a))/(a*x^3), 1/3*((2*B*a + A*b)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (2*B*a*x^3 - A*a)*sqrt(b*x^3 + a))/(a*x^3)]

giac [A] time = 0.17, size = 68, normalized size = 0.81

$$\frac{2\sqrt{bx^3+a}Bb + \frac{(2Bab+Ab^2)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx^3+a}Ab}{x^3}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="giac")

[Out] 1/3*(2*sqrt(b*x^3 + a)*B*b + (2*B*a*b + A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x^3 + a)*A*b/x^3)/b

maple [A] time = 0.05, size = 72, normalized size = 0.86

$$\left(-\frac{b \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{\sqrt{bx^3+a}}{3x^3} \right) A + \left(-\frac{2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3} + \frac{2\sqrt{bx^3+a}}{3} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x)

[Out] A*(-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/3*(b*x^3+a)^(1/2)/x^3)+B*(-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/3*(b*x^3+a)^(1/2))

maxima [A] time = 1.26, size = 107, normalized size = 1.27

$$\frac{1}{6} \left(\frac{b \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\sqrt{bx^3+a}}{x^3} \right) A + \frac{1}{3} \left(\sqrt{a} \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 2\sqrt{bx^3+a} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")

[Out] 1/6*(b*log((sqrt(b*x^3+a)-sqrt(a))/(sqrt(b*x^3+a)+sqrt(a)))/sqrt(a)-2*sqrt(b*x^3+a)/x^3)*A+1/3*(sqrt(a)*log((sqrt(b*x^3+a)-sqrt(a))/(sqrt(b*x^3+a)+sqrt(a)))+2*sqrt(b*x^3+a))*B

mupad [B] time = 2.93, size = 76, normalized size = 0.90

$$\frac{2B\sqrt{bx^3+a}}{3} - \frac{A\sqrt{bx^3+a}}{3x^3} + \frac{\ln\left(\frac{(\sqrt{bx^3+a}-\sqrt{a})^3(\sqrt{bx^3+a}+\sqrt{a})}{x^6}\right)}{3\sqrt{a}} \left(\frac{Ab}{2} + Ba\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A+B*x^3)*(a+b*x^3)^(1/2))/x^4,x)

[Out] (2*B*(a+b*x^3)^(1/2))/3 - (A*(a+b*x^3)^(1/2))/(3*x^3) + (log((((a+b*x^3)^(1/2)-a^(1/2))^3*((a+b*x^3)^(1/2)+a^(1/2))))/x^6)*((A*b)/2+B*a)/(3*a^(1/2))

sympy [A] time = 43.66, size = 134, normalized size = 1.60

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}-\frac{2B\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3}+\frac{2Ba}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2B\sqrt{b}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**4,x)

[Out] -A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - A*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)) - 2*B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + 2*B*a/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*B*sqrt(b)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1))

$$3.180 \quad \int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7} dx$$

Optimal. Leaf size=88

$$\frac{b(Ab - 4aB) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}} + \frac{\sqrt{a+bx^3}(Ab - 4aB)}{12ax^3} - \frac{A(a+bx^3)^{3/2}}{6ax^6}$$

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 78, 47, 63, 208}

$$\frac{b(Ab - 4aB) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}} + \frac{\sqrt{a+bx^3}(Ab - 4aB)}{12ax^3} - \frac{A(a+bx^3)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]

[Out] ((A*b - 4*a*B)*Sqrt[a + b*x^3])/(12*a*x^3) - (A*(a + b*x^3)^(3/2))/(6*a*x^6) + (b*(A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2))

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```

*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

```

Rule 208

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 446

```

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3} (A+Bx^3)}{x^7} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{a+bx} (A+Bx)}{x^3} dx, x, x^3 \right) \\
&= -\frac{A(a+bx^3)^{3/2}}{6ax^6} + \frac{\left(-\frac{Ab}{2} + 2aB\right) \text{Subst} \left(\int \frac{\sqrt{a+bx}}{x^2} dx, x, x^3 \right)}{6a} \\
&= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A(a+bx^3)^{3/2}}{6ax^6} - \frac{(b(Ab-4aB)) \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx}} dx, x, x^3 \right)}{24a} \\
&= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A(a+bx^3)^{3/2}}{6ax^6} - \frac{(Ab-4aB) \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+bx^3} \right)}{12a} \\
&= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A(a+bx^3)^{3/2}}{6ax^6} + \frac{b(Ab-4aB) \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{12a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 93, normalized size = 1.06

$$\frac{-\left(a+bx^3\right)\left(2a\left(A+2Bx^3\right)+Abx^3\right)-bx^6\sqrt{\frac{bx^3}{a}+1}\left(4aB-Ab\right)\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)}{12ax^6\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]

[Out] $(-(a + b*x^3)*(A*b*x^3 + 2*a*(A + 2*B*x^3))) - b*(-(A*b) + 4*a*B)*x^6*\text{Sqrt}[1 + (b*x^3)/a]*\text{ArcTanh}[\text{Sqrt}[1 + (b*x^3)/a]]/(12*a*x^6*\text{Sqrt}[a + b*x^3])$

IntegrateAlgebraic [A] time = 0.12, size = 79, normalized size = 0.90

$$\frac{(Ab^2 - 4abB) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}} + \frac{\sqrt{a+bx^3}(-2aA - 4aBx^3 - Abx^3)}{12ax^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]

[Out] $(\text{Sqrt}[a + b*x^3]*(-2*a*A - A*b*x^3 - 4*a*B*x^3))/(12*a*x^6) + ((A*b^2 - 4*a*b*B)*\text{ArcTanh}[\text{Sqrt}[a + b*x^3]/\text{Sqrt}[a]])/(12*a^{(3/2)})$

fricas [A] time = 0.76, size = 172, normalized size = 1.95

$$\left[\frac{(4Bab - Ab^2)\sqrt{a}x^6 \log\left(\frac{bx^3 + 2\sqrt{bx^3+a}\sqrt{a+2a}}{x^3}\right) + 2((4Ba^2 + Aab)x^3 + 2Aa^2)\sqrt{bx^3+a}}{24a^2x^6}, \frac{(4Bab - Ab^2)\sqrt{-a}x^6 \arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) - ((4Ba^2 + Aab)x^3 + 2Aa^2)\sqrt{bx^3+a}}{12a^2x^6} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")

[Out] $[-1/24*((4*B*a*b - A*b^2)*\text{sqrt}(a)*x^6*\log((b*x^3 + 2*\text{sqrt}(b*x^3 + a))*\text{sqrt}(a) + 2*a)/x^3) + 2*((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*\text{sqrt}(b*x^3 + a))/(a^2*x^6), 1/12*((4*B*a*b - A*b^2)*\text{sqrt}(-a)*x^6*\arctan(\text{sqrt}(b*x^3 + a)*\text{sqrt}(-a)/a) - ((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*\text{sqrt}(b*x^3 + a))/(a^2*x^6)]$

giac [A] time = 0.20, size = 120, normalized size = 1.36

$$\frac{(4Bab^2 - Ab^3) \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{4(bx^3+a)^{\frac{3}{2}} Bab^2 - 4\sqrt{bx^3+a} Ba^2 b^2 + (bx^3+a)^{\frac{3}{2}} Ab^3 + \sqrt{bx^3+a} Ab^3}{ab^2x^6}$$

12b

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="giac")

[Out] $1/12*((4*B*a*b^2 - A*b^3)*\arctan(\text{sqrt}(b*x^3 + a)/\text{sqrt}(-a))/(\text{sqrt}(-a)*a) - (4*(b*x^3 + a)^{(3/2)}*B*a*b^2 - 4*\text{sqrt}(b*x^3 + a)*B*a^2*b^2 + (b*x^3 + a)^{(3/2)}*A*b^3 + \text{sqrt}(b*x^3 + a)*A*a*b^3)/(a*b^2*x^6))/b$

maple [A] time = 0.05, size = 96, normalized size = 1.09

$$\left(\frac{b^2 \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{12a^{\frac{3}{2}}} - \frac{\sqrt{bx^3+a} b}{12ax^3} - \frac{\sqrt{bx^3+a}}{6x^6} \right) A + \left(-\frac{b \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{\sqrt{bx^3+a}}{3x^3} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x)`

[Out] $B*(-1/3*b*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})/a^{(1/2)}-1/3*(b*x^3+a)^{(1/2)}/x^3)+A*(1/12*b^2*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})/a^{(3/2)}-1/6*(b*x^3+a)^{(1/2)}/x^6-1/12*(b*x^3+a)^{(1/2)}/a*b/x^3)$

maxima [B] time = 1.32, size = 158, normalized size = 1.80

$$-\frac{1}{24} \left(\frac{b^2 \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\left((bx^3+a)^{\frac{3}{2}}b^2 + \sqrt{bx^3+a}ab^2\right)}{(bx^3+a)^2a - 2(bx^3+a)a^2 + a^3} \right) A + \frac{1}{6} \left(\frac{b \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\sqrt{bx^3+a}}{x^3} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")`

[Out] $-1/24*(b^2*\log((\operatorname{sqrt}(b*x^3+a)-\operatorname{sqrt}(a))/(\operatorname{sqrt}(b*x^3+a)+\operatorname{sqrt}(a))))/a^{(3/2)}+2*((b*x^3+a)^{(3/2)}*b^2+\operatorname{sqrt}(b*x^3+a)*a*b^2)/((b*x^3+a)^2*a-2*(b*x^3+a)*a^2+a^3)*A+1/6*(b*\log((\operatorname{sqrt}(b*x^3+a)-\operatorname{sqrt}(a))/(\operatorname{sqrt}(b*x^3+a)+\operatorname{sqrt}(a))))/\operatorname{sqrt}(a)-2*\operatorname{sqrt}(b*x^3+a)/x^3)*B$

mupad [B] time = 3.12, size = 93, normalized size = 1.06

$$\frac{b \ln\left(\frac{(\sqrt{bx^3+a}-\sqrt{a})(\sqrt{bx^3+a}+\sqrt{a})^3}{x^6}\right) (Ab - 4Ba)}{24a^{3/2}} - \frac{(4Ba^2 + Aba) \sqrt{bx^3+a}}{12a^2x^3} - \frac{A \sqrt{bx^3+a}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A+B*x^3)*(a+b*x^3)^(1/2))/x^7,x)`

[Out] $(b*\log((((a+b*x^3)^{(1/2)}-a^{(1/2)})*((a+b*x^3)^{(1/2)}+a^{(1/2)})^3)/x^6)*(A*b-4*B*a))/(24*a^{(3/2)})-((4*B*a^2+A*a*b)*(a+b*x^3)^{(1/2)})/(12*a^2*x^3)-(A*(a+b*x^3)^{(1/2)})/(6*x^6)$

sympy [B] time = 129.14, size = 160, normalized size = 1.82

$$-\frac{Aa}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{A\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{Ab^{\frac{3}{2}}}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{Ab^2 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{Bb \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**7,x)

[Out] $-A*a/(6*\sqrt{b}*x^{15/2}*\sqrt{a/(b*x^3)+1}) - A*\sqrt{b}/(4*x^{9/2}*\sqrt{a/(b*x^3)+1}) - A*b^{3/2}/(12*a*x^{3/2}*\sqrt{a/(b*x^3)+1}) + A*b*2*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x^{3/2}))/12*a^{3/2} - B*\sqrt{b}*\sqrt{a/(b*x^3)+1}/(3*x^{3/2}) - B*b*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x^{3/2}))/3*\sqrt{a}$

$$\mathbf{3.181} \quad \int x^8 (a + bx^3)^{3/2} (A + Bx^3) dx$$

Optimal. Leaf size=103

$$\frac{2a^2 (a + bx^3)^{5/2} (Ab - aB)}{15b^4} + \frac{2 (a + bx^3)^{9/2} (Ab - 3aB)}{27b^4} - \frac{2a (a + bx^3)^{7/2} (2Ab - 3aB)}{21b^4} + \frac{2B (a + bx^3)^{11/2}}{33b^4}$$

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{2a^2 (a + bx^3)^{5/2} (Ab - aB)}{15b^4} + \frac{2 (a + bx^3)^{9/2} (Ab - 3aB)}{27b^4} - \frac{2a (a + bx^3)^{7/2} (2Ab - 3aB)}{21b^4} + \frac{2B (a + bx^3)^{11/2}}{33b^4}$$

Antiderivative was successfully verified.

[In] Int[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]

[Out] (2*a^2*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b*x^3)^(7/2))/(21*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(9/2))/(27*b^4) + (2*B*(a + b*x^3)^(11/2))/(33*b^4)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^8 (a + bx^3)^{3/2} (A + Bx^3) dx &= \frac{1}{3} \text{Subst} \left(\int x^2 (a + bx)^{3/2} (A + Bx) dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{a^2(-Ab + aB)(a + bx)^{3/2}}{b^3} + \frac{a(-2Ab + 3aB)(a + bx)^{5/2}}{b^3} + \frac{(Ab - aB)(a + bx)^{7/2}}{b^3} \right) dx, x, x^3 \right) \\
&= \frac{2a^2(Ab - aB)(a + bx^3)^{5/2}}{15b^4} - \frac{2a(2Ab - 3aB)(a + bx^3)^{7/2}}{21b^4} + \frac{2(Ab - 3aB)(a + bx^3)^{9/2}}{27b^4}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 78, normalized size = 0.76

$$\frac{2(a + bx^3)^{5/2} (-48a^3B + 8a^2b(11A + 15Bx^3) - 10ab^2x^3(22A + 21Bx^3) + 35b^3x^6(11A + 9Bx^3))}{10395b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^8*(a + b*x^3)^(3/2)*(A + B*x^3), x]

[Out] (2*(a + b*x^3)^(5/2)*(-48*a^3*B + 35*b^3*x^6*(11*A + 9*B*x^3) + 8*a^2*b*(11*A + 15*B*x^3) - 10*a*b^2*x^3*(22*A + 21*B*x^3)))/(10395*b^4)

IntegrateAlgebraic [A] time = 0.06, size = 80, normalized size = 0.78

$$\frac{2(a + bx^3)^{5/2} (48a^3B - 88a^2Ab - 120a^2bBx^3 + 220aAb^2x^3 + 210ab^2Bx^6 - 385Ab^3x^6 - 315b^3Bx^9)}{10395b^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8*(a + b*x^3)^(3/2)*(A + B*x^3), x]

[Out] (-2*(a + b*x^3)^(5/2)*(-88*a^2*A*b + 48*a^3*B + 220*a*A*b^2*x^3 - 120*a^2*b*B*x^3 - 385*A*b^3*x^6 + 210*a*b^2*B*x^6 - 315*b^3*B*x^9))/(10395*b^4)

fricas [A] time = 1.11, size = 124, normalized size = 1.20

$$\frac{2(315Bb^5x^{15} + 35(12Bab^4 + 11Ab^5)x^{12} + 5(3Ba^2b^3 + 110Aab^4)x^9 - 3(6Ba^3b^2 - 11Aa^2b^3)x^6 - 48Ba^5 + 88Aa^4b + 4(6Ba^4b - 11Aa^3b^2)x^3)\sqrt{bx^3 + a}}{10395b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A), x, algorithm="fricas")

[Out] 2/10395*(315*B*b^5*x^15 + 35*(12*B*a*b^4 + 11*A*b^5)*x^12 + 5*(3*B*a^2*b^3 + 110*A*a*b^4)*x^9 - 3*(6*B*a^3*b^2 - 11*A*a^2*b^3)*x^6 - 48*B*a^5 + 88*A*a^4*b + 4*(6*B*a^4*b - 11*A*a^3*b^2)*x^3)*sqrt(b*x^3 + a)/b^4

giac [A] time = 0.16, size = 104, normalized size = 1.01

$$\frac{2 \left(315 (bx^3 + a)^{\frac{11}{2}} B - 1155 (bx^3 + a)^{\frac{9}{2}} Ba + 1485 (bx^3 + a)^{\frac{7}{2}} Ba^2 - 693 (bx^3 + a)^{\frac{5}{2}} Ba^3 + 385 (bx^3 + a)^{\frac{9}{2}} Ab - 990 (bx^3 + a)^{\frac{7}{2}} Aab + 693 (bx^3 + a)^{\frac{5}{2}} Aa^2 b \right)}{10395 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")

[Out] 2/10395*(315*(b*x^3 + a)^(11/2)*B - 1155*(b*x^3 + a)^(9/2)*B*a + 1485*(b*x^3 + a)^(7/2)*B*a^2 - 693*(b*x^3 + a)^(5/2)*B*a^3 + 385*(b*x^3 + a)^(9/2)*A*b - 990*(b*x^3 + a)^(7/2)*A*a*b + 693*(b*x^3 + a)^(5/2)*A*a^2*b)/b^4

maple [A] time = 0.05, size = 77, normalized size = 0.75

$$\frac{2 (bx^3 + a)^{\frac{5}{2}} (315Bx^9b^3 + 385Ab^3x^6 - 210Bab^2x^6 - 220Aab^2x^3 + 120Ba^2bx^3 + 88Aa^2b - 48Ba^3)}{10395b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x)

[Out] 2/10395*(b*x^3+a)^(5/2)*(315*B*b^3*x^9+385*A*b^3*x^6-210*B*a*b^2*x^6-220*A*a*b^2*x^3+120*B*a^2*b*x^3+88*A*a^2*b-48*B*a^3)/b^4

maxima [A] time = 0.61, size = 118, normalized size = 1.15

$$\frac{2}{945} \left(\frac{35 (bx^3 + a)^{\frac{9}{2}}}{b^3} - \frac{90 (bx^3 + a)^{\frac{7}{2}} a}{b^3} + \frac{63 (bx^3 + a)^{\frac{5}{2}} a^2}{b^3} \right) A + \frac{2}{3465} \left(\frac{105 (bx^3 + a)^{\frac{11}{2}}}{b^4} - \frac{385 (bx^3 + a)^{\frac{9}{2}} a}{b^4} + \frac{495 (bx^3 + a)^{\frac{7}{2}} a^2}{b^4} - \frac{231 (bx^3 + a)^{\frac{5}{2}} a^3}{b^4} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")

[Out] 2/945*(35*(b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 + a)^(5/2)*a^2/b^3)*A + 2/3465*(105*(b*x^3 + a)^(11/2)/b^4 - 385*(b*x^3 + a)^(9/2)*a/b^4 + 495*(b*x^3 + a)^(7/2)*a^2/b^4 - 231*(b*x^3 + a)^(5/2)*a^3/b^4)*B

mupad [B] time = 2.65, size = 206, normalized size = 2.00

$$\frac{20Aa^9\sqrt{bx^3+a}}{189} + \frac{2Abx^{12}\sqrt{bx^3+a}}{27} + \frac{8Bax^{12}\sqrt{bx^3+a}}{99} + \frac{2Bbx^{15}\sqrt{bx^3+a}}{33} + \frac{16Aa^4\sqrt{bx^3+a}}{945b^3} - \frac{32Ba^5\sqrt{bx^3+a}}{3465b^4} - \frac{8Aa^3x^3\sqrt{bx^3+a}}{945b^2} + \frac{2Aa^2x^6\sqrt{bx^3+a}}{315b} + \frac{16Ba^4x^3\sqrt{bx^3+a}}{3465b^3} - \frac{4Ba^3x^6\sqrt{bx^3+a}}{1155b^2} + \frac{2Ba^2x^9\sqrt{bx^3+a}}{693b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(A + B*x^3)*(a + b*x^3)^(3/2),x)

[Out] $(20Aax^9(a + bx^3)^{1/2})/189 + (2Abx^{12}(a + bx^3)^{1/2})/27 + (8Bax^{12}(a + bx^3)^{1/2})/99 + (2Bbx^{15}(a + bx^3)^{1/2})/33 + (16Aa^4(a + bx^3)^{1/2})/(945b^3) - (32Ba^5(a + bx^3)^{1/2})/(3465b^4) - (8Aa^3x^3(a + bx^3)^{1/2})/(945b^2) + (2Aa^2x^6(a + bx^3)^{1/2})/(315b) + (16Ba^4x^3(a + bx^3)^{1/2})/(3465b^3) - (4Ba^3x^6(a + bx^3)^{1/2})/(1155b^2) + (2Ba^2x^9(a + bx^3)^{1/2})/(693b)$

sympy [A] time = 8.44, size = 267, normalized size = 2.59

$$\begin{cases} \frac{16Aa^4\sqrt{a+bx^3}}{945b^3} - \frac{8Aa^3x^3\sqrt{a+bx^3}}{945b^2} + \frac{2Aa^2x^6\sqrt{a+bx^3}}{315b} + \frac{20Aax^9\sqrt{a+bx^3}}{189} + \frac{2Abx^{12}\sqrt{a+bx^3}}{27} - \frac{32Ba^5\sqrt{a+bx^3}}{3465b^4} + \frac{16Ba^4x^3\sqrt{a+bx^3}}{3465b^3} - \frac{4Ba^3x^6\sqrt{a+bx^3}}{1155b^2} + \frac{2Ba^2x^9\sqrt{a+bx^3}}{693b} + \frac{8Bax^{12}\sqrt{a+bx^3}}{99} + \frac{2Bbx^{15}\sqrt{a+bx^3}}{33} & \text{for } b \neq 0 \\ a^{\frac{3}{2}} \left(\frac{Ax^9}{9} + \frac{Bx^{12}}{12} \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(b*x**3+a)**(3/2)*(B*x**3+A), x)

[Out] Piecewise((16Aa**4*sqrt(a + b*x**3)/(945*b**3) - 8Aa**3*x**3*sqrt(a + b*x**3)/(945*b**2) + 2Aa**2*x**6*sqrt(a + b*x**3)/(315*b) + 20Aa*x**9*sqrt(a + b*x**3)/189 + 2Ab*x**12*sqrt(a + b*x**3)/27 - 32Ba**5*sqrt(a + b*x**3)/(3465*b**4) + 16Ba**4*x**3*sqrt(a + b*x**3)/(3465*b**3) - 4Ba**3*x**6*sqrt(a + b*x**3)/(1155*b**2) + 2Ba**2*x**9*sqrt(a + b*x**3)/(693*b) + 8B*a*x**12*sqrt(a + b*x**3)/99 + 2*B*b*x**15*sqrt(a + b*x**3)/33, Ne(b, 0)), (a**(3/2)*(A*x**9/9 + B*x**12/12), True))

$$3.182 \quad \int x^5 (a + bx^3)^{3/2} (A + Bx^3) dx$$

Optimal. Leaf size=73

$$\frac{2(a + bx^3)^{7/2} (Ab - 2aB)}{21b^3} - \frac{2a(a + bx^3)^{5/2} (Ab - aB)}{15b^3} + \frac{2B(a + bx^3)^{9/2}}{27b^3}$$

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{2(a + bx^3)^{7/2} (Ab - 2aB)}{21b^3} - \frac{2a(a + bx^3)^{5/2} (Ab - aB)}{15b^3} + \frac{2B(a + bx^3)^{9/2}}{27b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]

[Out] (-2*a*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)^(7/2))/(21*b^3) + (2*B*(a + b*x^3)^(9/2))/(27*b^3)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int x^5 (a + bx^3)^{3/2} (A + Bx^3) dx &= \frac{1}{3} \text{Subst} \left(\int x(a + bx)^{3/2} (A + Bx) dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{a(-Ab + aB)(a + bx)^{3/2}}{b^2} + \frac{(Ab - 2aB)(a + bx)^{5/2}}{b^2} + \frac{B(a + bx)^{7/2}}{b^2} \right) dx \right) \\ &= -\frac{2a(Ab - aB)(a + bx^3)^{5/2}}{15b^3} + \frac{2(Ab - 2aB)(a + bx^3)^{7/2}}{21b^3} + \frac{2B(a + bx^3)^{9/2}}{27b^3} \end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.78

$$\frac{2(a + bx^3)^{5/2} (8a^2B - 2ab(9A + 10Bx^3) + 5b^2x^3(9A + 7Bx^3))}{945b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(a + b*x^3)^(3/2)*(A + B*x^3), x]

[Out] (2*(a + b*x^3)^(5/2)*(8*a^2*B + 5*b^2*x^3*(9*A + 7*B*x^3) - 2*a*b*(9*A + 10*B*x^3)))/(945*b^3)

IntegrateAlgebraic [A] time = 0.04, size = 56, normalized size = 0.77

$$\frac{2(a + bx^3)^{5/2} (8a^2B - 18aAb - 20abBx^3 + 45Ab^2x^3 + 35b^2Bx^6)}{945b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*(a + b*x^3)^(3/2)*(A + B*x^3), x]

[Out] (2*(a + b*x^3)^(5/2)*(-18*a*A*b + 8*a^2*B + 45*A*b^2*x^3 - 20*a*b*B*x^3 + 35*b^2*B*x^6))/(945*b^3)

fricas [A] time = 0.82, size = 99, normalized size = 1.36

$$\frac{2(35Bb^4x^{12} + 5(10Bab^3 + 9Ab^4)x^9 + 3(Ba^2b^2 + 24Aab^3)x^6 + 8Ba^4 - 18Aa^3b - (4Ba^3b - 9Aa^2b^2)x^3)\sqrt{bx^3 + a}}{945b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A), x, algorithm="fricas")

[Out] 2/945*(35*B*b^4*x^12 + 5*(10*B*a*b^3 + 9*A*b^4)*x^9 + 3*(B*a^2*b^2 + 24*A*a*b^3)*x^6 + 8*B*a^4 - 18*A*a^3*b - (4*B*a^3*b - 9*A*a^2*b^2)*x^3)*sqrt(b*x^3 + a)/b^3

giac [A] time = 0.18, size = 73, normalized size = 1.00

$$\frac{2 \left(35 (bx^3 + a)^{\frac{9}{2}} B - 90 (bx^3 + a)^{\frac{7}{2}} Ba + 63 (bx^3 + a)^{\frac{5}{2}} Ba^2 + 45 (bx^3 + a)^{\frac{7}{2}} Ab - 63 (bx^3 + a)^{\frac{5}{2}} Aab \right)}{945 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")

[Out] 2/945*(35*(b*x^3 + a)^(9/2)*B - 90*(b*x^3 + a)^(7/2)*B*a + 63*(b*x^3 + a)^(5/2)*B*a^2 + 45*(b*x^3 + a)^(7/2)*A*b - 63*(b*x^3 + a)^(5/2)*A*a*b)/b^3

maple [A] time = 0.05, size = 53, normalized size = 0.73

$$\frac{2 (bx^3 + a)^{\frac{5}{2}} (-35B b^2 x^6 - 45A b^2 x^3 + 20Bab x^3 + 18Aab - 8B a^2)}{945 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x)

[Out] -2/945*(b*x^3+a)^(5/2)*(-35*B*b^2*x^6-45*A*b^2*x^3+20*B*a*b*x^3+18*A*a*b-8*B*a^2)/b^3

maxima [A] time = 0.61, size = 84, normalized size = 1.15

$$\frac{2}{105} \left(\frac{5 (bx^3 + a)^{\frac{7}{2}}}{b^2} - \frac{7 (bx^3 + a)^{\frac{5}{2}} a}{b^2} \right) A + \frac{2}{945} \left(\frac{35 (bx^3 + a)^{\frac{9}{2}}}{b^3} - \frac{90 (bx^3 + a)^{\frac{7}{2}} a}{b^3} + \frac{63 (bx^3 + a)^{\frac{5}{2}} a^2}{b^3} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")

[Out] 2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a)^(5/2)*a/b^2)*A + 2/945*(35*(b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 + a)^(5/2)*a^2/b^3)*B

mupad [B] time = 2.72, size = 211, normalized size = 2.89

$$\frac{x^6 \sqrt{bx^3 + a} \left(2B a^2 + 4A a b - \frac{6a(2Aa^2 + 20Bab)}{7b} \right)}{15b} - \frac{2a \left(2A a^2 - \frac{4a \left(2B a^2 + 4A a b - \frac{6a(2Aa^2 + 20Bab)}{7b} \right)}{5b} \right) \sqrt{bx^3 + a}}{9b^2} + \frac{2B b x^{12} \sqrt{bx^3 + a}}{27} + \frac{x^3 \left(2A a^2 - \frac{4a \left(2B a^2 + 4A a b - \frac{6a(2Aa^2 + 20Bab)}{7b} \right)}{5b} \right) \sqrt{bx^3 + a}}{9b} + \frac{x^9 \left(2A b^2 + \frac{20Bab}{9} \right) \sqrt{bx^3 + a}}{21b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(A + B*x^3)*(a + b*x^3)^(3/2),x)`

[Out] $(x^6*(a + b*x^3)^{(1/2)}*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2 + (20*B*a*b)/9)))/(7*b)))/(15*b) - (2*a*(2*A*a^2 - (4*a*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2 + (20*B*a*b)/9)))/(7*b)))/(5*b))*(a + b*x^3)^{(1/2))/(9*b^2) + (2*B*b*x^{12}*(a + b*x^3)^{(1/2)))/27 + (x^3*(2*A*a^2 - (4*a*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2 + (20*B*a*b)/9)))/(7*b)))/(5*b))*(a + b*x^3)^{(1/2))/(9*b) + (x^9*(2*A*b^2 + (20*B*a*b)/9)*(a + b*x^3)^{(1/2)))/(21*b)}$

sympy [A] time = 4.75, size = 216, normalized size = 2.96

$$\begin{cases} -\frac{4Aa^3\sqrt{a+bx^3}}{105b^2} + \frac{2Aa^2x^3\sqrt{a+bx^3}}{105b} + \frac{16Aax^6\sqrt{a+bx^3}}{105} + \frac{2Abx^9\sqrt{a+bx^3}}{21} + \frac{16Ba^4\sqrt{a+bx^3}}{945b^3} - \frac{8Ba^3x^3\sqrt{a+bx^3}}{945b^2} + \frac{2Ba^2x^6\sqrt{a+bx^3}}{315b} + \frac{20Bax^9\sqrt{a+bx^3}}{189} + \frac{2Bbx^{12}\sqrt{a+bx^3}}{27} & \text{for } b \neq 0 \\ a^{\frac{3}{2}} \left(\frac{Ax^6}{6} + \frac{Bx^9}{9} \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b*x**3+a)**(3/2)*(B*x**3+A),x)`

[Out] `Piecewise((-4*A*a**3*sqrt(a + b*x**3)/(105*b**2) + 2*A*a**2*x**3*sqrt(a + b*x**3)/(105*b) + 16*A*a*x**6*sqrt(a + b*x**3)/105 + 2*A*b*x**9*sqrt(a + b*x**3)/21 + 16*B*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*B*a**3*x**3*sqrt(a + b*x**3)/(945*b**2) + 2*B*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*B*a*x**9*sqrt(a + b*x**3)/189 + 2*B*b*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (a**(3/2)*(A*x**6/6 + B*x**9/9), True))`

$$3.183 \quad \int x^2 (a + bx^3)^{3/2} (A + Bx^3) dx$$

Optimal. Leaf size=46

$$\frac{2(a + bx^3)^{5/2} (Ab - aB)}{15b^2} + \frac{2B(a + bx^3)^{7/2}}{21b^2}$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {444, 43}

$$\frac{2(a + bx^3)^{5/2} (Ab - aB)}{15b^2} + \frac{2B(a + bx^3)^{7/2}}{21b^2}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]

[Out] (2*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^2) + (2*B*(a + b*x^3)^(7/2))/(21*b^2)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rubi steps

$$\begin{aligned} \int x^2 (a + bx^3)^{3/2} (A + Bx^3) dx &= \frac{1}{3} \text{Subst} \left(\int (a + bx)^{3/2} (A + Bx) dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{(Ab - aB)(a + bx)^{3/2}}{b} + \frac{B(a + bx)^{5/2}}{b} \right) dx, x, x^3 \right) \\ &= \frac{2(Ab - aB)(a + bx^3)^{5/2}}{15b^2} + \frac{2B(a + bx^3)^{7/2}}{21b^2} \end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.74

$$\frac{2(a + bx^3)^{5/2} (-2aB + 7Ab + 5bBx^3)}{105b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x^3)^(3/2)*(A + B*x^3), x]

[Out] (2*(a + b*x^3)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^3))/(105*b^2)

IntegrateAlgebraic [A] time = 0.03, size = 34, normalized size = 0.74

$$\frac{2(a + bx^3)^{5/2} (-2aB + 7Ab + 5bBx^3)}{105b^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*(a + b*x^3)^(3/2)*(A + B*x^3), x]

[Out] (2*(a + b*x^3)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^3))/(105*b^2)

fricas [A] time = 1.02, size = 73, normalized size = 1.59

$$\frac{2(5Bb^3x^9 + (8Bab^2 + 7Ab^3)x^6 - 2Ba^3 + 7Aa^2b + (Ba^2b + 14Aab^2)x^3)\sqrt{bx^3 + a}}{105b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A), x, algorithm="fricas")

[Out] 2/105*(5*B*b^3*x^9 + (8*B*a*b^2 + 7*A*b^3)*x^6 - 2*B*a^3 + 7*A*a^2*b + (B*a^2*b + 14*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^2

giac [A] time = 0.17, size = 44, normalized size = 0.96

$$\frac{2 \left(5 (bx^3 + a)^{\frac{7}{2}} B - 7 (bx^3 + a)^{\frac{5}{2}} Ba + 7 (bx^3 + a)^{\frac{5}{2}} Ab \right)}{105 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")

[Out] 2/105*(5*(b*x^3 + a)^(7/2)*B - 7*(b*x^3 + a)^(5/2)*B*a + 7*(b*x^3 + a)^(5/2)*A*b)/b^2

maple [A] time = 0.05, size = 31, normalized size = 0.67

$$\frac{2 (bx^3 + a)^{\frac{5}{2}} (5Bbx^3 + 7Ab - 2Ba)}{105b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x)

[Out] 2/105*(b*x^3+a)^(5/2)*(5*B*b*x^3+7*A*b-2*B*a)/b^2

maxima [A] time = 0.49, size = 49, normalized size = 1.07

$$\frac{2 (bx^3 + a)^{\frac{5}{2}} A}{15 b} + \frac{2}{105} \left(\frac{5 (bx^3 + a)^{\frac{7}{2}}}{b^2} - \frac{7 (bx^3 + a)^{\frac{5}{2}} a}{b^2} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")

[Out] 2/15*(b*x^3 + a)^(5/2)*A/b + 2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a)^(5/2)*a/b^2)*B

mupad [B] time = 3.35, size = 150, normalized size = 3.26

$$\frac{\left(2Aa^2 - \frac{2a \left(2Ba^2 + 4Aab - \frac{4a(2Ab^2 + \frac{16Bab}{7})}{5b} \right)}{3b} \right) \sqrt{bx^3 + a}}{3b} + \frac{x^3 \sqrt{bx^3 + a} \left(2Ba^2 + 4Aab - \frac{4a(2Ab^2 + \frac{16Bab}{7})}{5b} \right)}{9b} + \frac{2Bbx^9 \sqrt{bx^3 + a}}{21} + \frac{x^6 \left(2Ab^2 + \frac{16Bab}{7} \right) \sqrt{bx^3 + a}}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(A + B*x^3)*(a + b*x^3)^(3/2),x)

```
[Out] ((2*A*a^2 - (2*a*(2*B*a^2 + 4*A*a*b - (4*a*(2*A*b^2 + (16*B*a*b)/7)))/(5*b))
)/(3*b))*(a + b*x^3)^(1/2))/(3*b) + (x^3*(a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a
*b - (4*a*(2*A*b^2 + (16*B*a*b)/7)))/(5*b)))/(9*b) + (2*B*b*x^9*(a + b*x^3)^(
1/2))/21 + (x^6*(2*A*b^2 + (16*B*a*b)/7)*(a + b*x^3)^(1/2))/(15*b)
```

sympy [A] time = 2.52, size = 165, normalized size = 3.59

$$\begin{cases} \frac{2Aa^2\sqrt{a+bx^3}}{15b} + \frac{4Aax^3\sqrt{a+bx^3}}{15} + \frac{2Abx^6\sqrt{a+bx^3}}{15} - \frac{4Ba^3\sqrt{a+bx^3}}{105b^2} + \frac{2Ba^2x^3\sqrt{a+bx^3}}{105b} + \frac{16Bax^6\sqrt{a+bx^3}}{105} + \frac{2Bbx^9\sqrt{a+bx^3}}{21} & \text{for } b \neq 0 \\ a^{\frac{3}{2}} \left(\frac{Ax^3}{3} + \frac{Bx^6}{6} \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(3/2)*(B*x**3+A), x)
```

```
[Out] Piecewise((2*A*a**2*sqrt(a + b*x**3)/(15*b) + 4*A*a*x**3*sqrt(a + b*x**3)/1
5 + 2*A*b*x**6*sqrt(a + b*x**3)/15 - 4*B*a**3*sqrt(a + b*x**3)/(105*b**2) +
2*B*a**2*x**3*sqrt(a + b*x**3)/(105*b) + 16*B*a*x**6*sqrt(a + b*x**3)/105
+ 2*B*b*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (a**(3/2)*(A*x**3/3 + B*x**6/6
), True))
```

$$3.184 \quad \int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x} dx$$

Optimal. Leaf size=81

$$-\frac{2}{3}a^{3/2}A \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) + \frac{2}{9}A(a+bx^3)^{3/2} + \frac{2}{3}aA\sqrt{a+bx^3} + \frac{2B(a+bx^3)^{5/2}}{15b}$$

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 80, 50, 63, 208}

$$-\frac{2}{3}a^{3/2}A \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) + \frac{2}{9}A(a+bx^3)^{3/2} + \frac{2}{3}aA\sqrt{a+bx^3} + \frac{2B(a+bx^3)^{5/2}}{15b}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]

[Out] (2*a*A*Sqrt[a + b*x^3])/3 + (2*A*(a + b*x^3)^(3/2))/9 + (2*B*(a + b*x^3)^(5/2))/(15*b) - (2*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

$n + p + 2$), $\text{Int}[(c + d*x)^n*(e + f*x)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, n, p\}, x]$ && $\text{NeQ}[n + p + 2, 0]$

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$ $\text{FreeQ}\{a, b\}, x]$ && $\text{NegQ}[a/b]$

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] :> \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, c, d, m, n, p, q\}, x]$ && $\text{NeQ}[b*c - a*d, 0]$ && $\text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^{3/2} (A + Bx^3)}{x} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^{3/2} (A + Bx)}{x} dx, x, x^3 \right) \\ &= \frac{2B(a + bx^3)^{5/2}}{15b} + \frac{1}{3} A \text{Subst} \left(\int \frac{(a + bx)^{3/2}}{x} dx, x, x^3 \right) \\ &= \frac{2}{9} A (a + bx^3)^{3/2} + \frac{2B(a + bx^3)^{5/2}}{15b} + \frac{1}{3} (aA) \text{Subst} \left(\int \frac{\sqrt{a + bx}}{x} dx, x, x^3 \right) \\ &= \frac{2}{3} aA \sqrt{a + bx^3} + \frac{2}{9} A (a + bx^3)^{3/2} + \frac{2B(a + bx^3)^{5/2}}{15b} + \frac{1}{3} (a^2 A) \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx}} dx, x, x^3 \right) \\ &= \frac{2}{3} aA \sqrt{a + bx^3} + \frac{2}{9} A (a + bx^3)^{3/2} + \frac{2B(a + bx^3)^{5/2}}{15b} + \frac{(2a^2 A) \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, x^3 \right)}{3b} \\ &= \frac{2}{3} aA \sqrt{a + bx^3} + \frac{2}{9} A (a + bx^3)^{3/2} + \frac{2B(a + bx^3)^{5/2}}{15b} - \frac{2}{3} a^{3/2} A \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right) \end{aligned}$$

Mathematica [A] time = 0.08, size = 80, normalized size = 0.99

$$\frac{2 \left(-15a^{3/2} Ab \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right) + 5Ab (a + bx^3)^{3/2} + 15aAb \sqrt{a + bx^3} + 3B (a + bx^3)^{5/2} \right)}{45b}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]

[Out] (2*(15*a*A*b*Sqrt[a + b*x^3] + 5*A*b*(a + b*x^3)^(3/2) + 3*B*(a + b*x^3)^(5/2) - 15*a^(3/2)*A*b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(45*b)

IntegrateAlgebraic [A] time = 0.07, size = 85, normalized size = 1.05

$$\frac{2\sqrt{a + bx^3} (3a^2B + 20aAb + 6abBx^3 + 5Ab^2x^3 + 3b^2Bx^6)}{45b} - \frac{2}{3}a^{3/2}A \tanh^{-1}\left(\frac{\sqrt{a + bx^3}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]

[Out] (2*Sqrt[a + b*x^3]*(20*a*A*b + 3*a^2*B + 5*A*b^2*x^3 + 6*a*b*B*x^3 + 3*b^2*B*x^6))/(45*b) - (2*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3

fricas [A] time = 0.82, size = 172, normalized size = 2.12

$$\left[\frac{15 A a^2 b \log\left(\frac{bx^3 - 2\sqrt{bx^3+a}\sqrt{a+2a}}{x^3}\right) + 2(3 B b^2 x^6 + (6 B a b + 5 A b^2)x^3 + 3 B a^2 + 20 A a b)\sqrt{bx^3+a}}{45 b}, \frac{2(15 A \sqrt{-a} b \arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) + (3 B b^2 x^6 + (6 B a b + 5 A b^2)x^3 + 3 B a^2 + 20 A a b)\sqrt{bx^3+a})}{45 b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="fricas")

[Out] [1/45*(15*A*a^(3/2)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*B*b^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3 + a))/b, 2/45*(15*A*sqrt(-a)*a*b*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*B*b^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3 + a))/b]

giac [A] time = 0.16, size = 80, normalized size = 0.99

$$\frac{2 A a^2 \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3 \sqrt{-a}} + \frac{2\left(3(bx^3+a)^{\frac{5}{2}} B b^4 + 5(bx^3+a)^{\frac{3}{2}} A b^5 + 15 \sqrt{bx^3+a} A a b^5\right)}{45 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="giac")

[Out] 2/3*A*a^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/45*(3*(b*x^3 + a)^(5/2)*B*b^4 + 5*(b*x^3 + a)^(3/2)*A*b^5 + 15*sqrt(b*x^3 + a)*A*a*b^5)/b^5

maple [A] time = 0.05, size = 66, normalized size = 0.81

$$\left(\frac{2\sqrt{bx^3+a} bx^3}{9} - \frac{2a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3} + \frac{8\sqrt{bx^3+a} a}{9} \right) A + \frac{2(bx^3+a)^{\frac{5}{2}} B}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^(3/2)*(B*x^3+A)/x,x)`

[Out] $2/15*B*(b*x^3+a)^{(5/2)}/b+A*(2/9*(b*x^3+a)^{(1/2)}*b*x^3+8/9*(b*x^3+a)^{(1/2)}*a-2/3*a^{(3/2)}*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)}))$

maxima [A] time = 1.26, size = 80, normalized size = 0.99

$$\frac{2(bx^3+a)^{\frac{5}{2}}B}{15b} + \frac{1}{9} \left(3a^{\frac{3}{2}} \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 2(bx^3+a)^{\frac{3}{2}} + 6\sqrt{bx^3+a}a \right) A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="maxima")`

[Out] $2/15*(b*x^3+a)^{(5/2)}*B/b + 1/9*(3*a^{(3/2)}*\log((\operatorname{sqrt}(b*x^3+a)-\operatorname{sqrt}(a))/(\operatorname{sqrt}(b*x^3+a)+\operatorname{sqrt}(a)))) + 2*(b*x^3+a)^{(3/2)} + 6*\operatorname{sqrt}(b*x^3+a)*a)*A$

mupad [B] time = 2.79, size = 131, normalized size = 1.62

$$\frac{Aa^{3/2} \ln\left(\frac{(\sqrt{bx^3+a}-\sqrt{a})^3(\sqrt{bx^3+a}+\sqrt{a})}{x^6}\right)}{3} + \frac{\sqrt{bx^3+a} \left(2Ba^2+4Aab-\frac{2a(2Ab^2+\frac{12Bab}{5})}{3b}\right)}{3b} + \frac{2Bbx^6\sqrt{bx^3+a}}{15} + \frac{x^3\left(2Ab^2+\frac{12Bab}{5}\right)\sqrt{bx^3+a}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A+B*x^3)*(a+b*x^3)^(3/2))/x,x)`

[Out] $(A*a^{(3/2)}*\log((((a+b*x^3)^{(1/2)}-a^{(1/2)})^3*((a+b*x^3)^{(1/2)}+a^{(1/2)})))/x^6)/3 + ((a+b*x^3)^{(1/2)}*(2*B*a^2+4*A*a*b-(2*a*(2*A*b^2+(12*B*a*b)/5)))/(3*b)))/(3*b) + (2*B*b*x^6*(a+b*x^3)^{(1/2)})/15 + (x^3*(2*A*b^2+(12*B*a*b)/5)*(a+b*x^3)^{(1/2)})/(9*b)$

sympy [A] time = 66.50, size = 82, normalized size = 1.01

$$\frac{2Aa^2 \operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3\sqrt{-a}} + \frac{2Aa\sqrt{a+bx^3}}{3} + \frac{2A(a+bx^3)^{\frac{3}{2}}}{9} + \frac{2B(a+bx^3)^{\frac{5}{2}}}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x,x)
```

```
[Out] 2*A*a**2*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*sqrt(-a)) + 2*A*a*sqrt(a + b*x*  
*3)/3 + 2*A*(a + b*x**3)**(3/2)/9 + 2*B*(a + b*x**3)**(5/2)/(15*b)
```

$$3.185 \quad \int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4} dx$$

Optimal. Leaf size=110

$$\frac{(a+bx^3)^{3/2}(2aB+3Ab)}{9a} + \frac{1}{3}\sqrt{a+bx^3}(2aB+3Ab) - \frac{1}{3}\sqrt{a}(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{A(a+bx^3)^{5/2}}{3ax^3}$$

Rubi [A] time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 78, 50, 63, 208}

$$\frac{(a+bx^3)^{3/2}(2aB+3Ab)}{9a} + \frac{1}{3}\sqrt{a+bx^3}(2aB+3Ab) - \frac{1}{3}\sqrt{a}(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{A(a+bx^3)^{5/2}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4, x]

[Out] ((3*A*b + 2*a*B)*Sqrt[a + b*x^3])/3 + ((3*A*b + 2*a*B)*(a + b*x^3)^(3/2))/(9*a) - (A*(a + b*x^3)^(5/2))/(3*a*x^3) - (Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(

$f*(p + 1)*(c*f - d*e)), x] - \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

$\text{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}], x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^{3/2} (A + Bx^3)}{x^4} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^{3/2} (A + Bx)}{x^2} dx, x, x^3 \right) \\ &= -\frac{A(a + bx^3)^{5/2}}{3ax^3} + \frac{\left(\frac{3Ab}{2} + aB\right) \text{Subst} \left(\int \frac{(a+bx)^{3/2}}{x} dx, x, x^3 \right)}{3a} \\ &= \frac{(3Ab + 2aB)(a + bx^3)^{3/2}}{9a} - \frac{A(a + bx^3)^{5/2}}{3ax^3} + \frac{1}{6}(3Ab + 2aB) \text{Subst} \left(\int \frac{\sqrt{a + bx}}{x} dx, x, x^3 \right) \\ &= \frac{1}{3}(3Ab + 2aB)\sqrt{a + bx^3} + \frac{(3Ab + 2aB)(a + bx^3)^{3/2}}{9a} - \frac{A(a + bx^3)^{5/2}}{3ax^3} + \frac{1}{6}(a(3Ab + 2aB) \sqrt{a + bx^3} - A(a + bx^3)^{5/2}) \\ &= \frac{1}{3}(3Ab + 2aB)\sqrt{a + bx^3} + \frac{(3Ab + 2aB)(a + bx^3)^{3/2}}{9a} - \frac{A(a + bx^3)^{5/2}}{3ax^3} + \frac{(a(3Ab + 2aB) \sqrt{a + bx^3} - A(a + bx^3)^{5/2})}{6} \\ &= \frac{1}{3}(3Ab + 2aB)\sqrt{a + bx^3} + \frac{(3Ab + 2aB)(a + bx^3)^{3/2}}{9a} - \frac{A(a + bx^3)^{5/2}}{3ax^3} - \frac{1}{3}\sqrt{a} (3Ab + 2aB) \end{aligned}$$

Mathematica [A] time = 0.09, size = 80, normalized size = 0.73

$$\frac{1}{9} \left(\frac{\sqrt{a+bx^3} (-3aA + 8aBx^3 + 6Abx^3 + 2bBx^6)}{x^3} - 3\sqrt{a} (2aB + 3Ab) \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]

[Out] ((Sqrt[a + b*x^3]*(-3*a*A + 6*A*b*x^3 + 8*a*B*x^3 + 2*b*B*x^6))/x^3 - 3*Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/9

IntegrateAlgebraic [A] time = 0.11, size = 85, normalized size = 0.77

$$\frac{1}{3} (-2a^{3/2}B - 3\sqrt{a}Ab) \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right) + \frac{\sqrt{a+bx^3} (-3aA + 8aBx^3 + 6Abx^3 + 2bBx^6)}{9x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]

[Out] (Sqrt[a + b*x^3]*(-3*a*A + 6*A*b*x^3 + 8*a*B*x^3 + 2*b*B*x^6))/(9*x^3) + ((-3*Sqrt[a]*A*b - 2*a^(3/2)*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3

fricas [A] time = 0.57, size = 169, normalized size = 1.54

$$\left[\frac{3(2Ba + 3Ab)\sqrt{a}x^3 \log\left(\frac{bx^3 - 2\sqrt{bx^3+a}\sqrt{a}}{x^3}\right) + 2(2Bbx^6 + 2(4Ba + 3Ab)x^3 - 3Aa)\sqrt{bx^3+a} - 3(2Ba + 3Ab)\sqrt{-a}x^3 \arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) + (2Bbx^6 + 2(4Ba + 3Ab)x^3 - 3Aa)\sqrt{bx^3+a}}{18x^3}, \frac{3(2Ba + 3Ab)\sqrt{-a}x^3 \arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) + (2Bbx^6 + 2(4Ba + 3Ab)x^3 - 3Aa)\sqrt{bx^3+a}}{9x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="fricas")

[Out] [1/18*(3*(2*B*a + 3*A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 + a))/x^3, 1/9*(3*(2*B*a + 3*A*b)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 + a))/x^3]

giac [A] time = 0.20, size = 103, normalized size = 0.94

$$\frac{2(bx^3 + a)^{\frac{3}{2}}Bb + 6\sqrt{bx^3 + a}Bab + 6\sqrt{bx^3 + a}Ab^2 + \frac{3(2Ba^2b + 3Aab^2)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{3\sqrt{bx^3+a}Aab}{x^3}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="giac")

[Out] $\frac{1}{9}*(2*(b*x^3 + a)^{(3/2)}*B*b + 6*\sqrt{b*x^3 + a}*B*a*b + 6*\sqrt{b*x^3 + a}*A*b^2 + 3*(2*B*a^2*b + 3*A*a*b^2)*\arctan(\sqrt{b*x^3 + a}/\sqrt{-a})/\sqrt{-a} - 3*\sqrt{b*x^3 + a}*A*a*b/x^3)/b$

maple [A] time = 0.05, size = 101, normalized size = 0.92

$$\left(-\sqrt{a} b \operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)+\frac{2 \sqrt{b x^3+a} b}{3}-\frac{\sqrt{b x^3+a} a}{3 x^3}\right) A+\left(\frac{2 \sqrt{b x^3+a} b x^3}{9}-\frac{2 a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{3}+\frac{8 \sqrt{b x^3+a} a}{9}\right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x)

[Out] $A*(-1/3*(b*x^3+a)^{(1/2)}*a/x^3+2/3*(b*x^3+a)^{(1/2)}*b-b*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})*a^{(1/2)})+B*(2/9*(b*x^3+a)^{(1/2)}*b*x^3+8/9*(b*x^3+a)^{(1/2)}*a-2/3*a^{(3/2)}*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)}))$

maxima [A] time = 1.44, size = 134, normalized size = 1.22

$$\frac{1}{6}\left(3 \sqrt{a} b \log \left(\frac{\sqrt{b x^3+a}-\sqrt{a}}{\sqrt{b x^3+a}+\sqrt{a}}\right)+4 \sqrt{b x^3+a} b-\frac{2 \sqrt{b x^3+a} a}{x^3}\right) A+\frac{1}{9}\left(3 a^{\frac{3}{2}} \log \left(\frac{\sqrt{b x^3+a}-\sqrt{a}}{\sqrt{b x^3+a}+\sqrt{a}}\right)+2\left(b x^3+a\right)^{\frac{3}{2}}+6 \sqrt{b x^3+a} a\right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="maxima")

[Out] $\frac{1}{6}*(3*\sqrt{a}*b*\log((\sqrt{b*x^3 + a} - \sqrt{a})/(\sqrt{b*x^3 + a} + \sqrt{a}))) + 4*\sqrt{b*x^3 + a}*b - 2*\sqrt{b*x^3 + a}*a/x^3)*A + \frac{1}{9}*(3*a^{(3/2)}*\log((\sqrt{b*x^3 + a} - \sqrt{a})/(\sqrt{b*x^3 + a} + \sqrt{a}))) + 2*(b*x^3 + a)^{(3/2)} + 6*\sqrt{b*x^3 + a}*a)*B$

mupad [B] time = 3.38, size = 111, normalized size = 1.01

$$\frac{\ln\left(\frac{(\sqrt{b x^3+a}-\sqrt{a})^3(\sqrt{b x^3+a}+\sqrt{a})}{x^6}\right)}{3}(3 A b+2 B a) \sqrt{\frac{a}{4}}+\frac{\left(2 A b^2+\frac{8 B a b}{3}\right) \sqrt{b x^3+a}}{3 b}-\frac{A a \sqrt{b x^3+a}}{3 x^3}+\frac{2 B b x^3 \sqrt{b x^3+a}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^4,x)

[Out] $(\log(((a + b*x^3)^{(1/2)} - a^{(1/2)})^3*((a + b*x^3)^{(1/2)} + a^{(1/2)}))/x^6)*(3*A*b + 2*B*a)*(a/4)^{(1/2)}/3 + ((2*A*b^2 + (8*B*a*b)/3)*(a + b*x^3)^{(1/2)})/(3*b) - (A*a*(a + b*x^3)^{(1/2)})/(3*x^3) + (2*B*b*x^3*(a + b*x^3)^{(1/2)})/9$

sympy [A] time = 58.40, size = 223, normalized size = 2.03

$$-A\sqrt{a}b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) - \frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2Aa\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2Ab^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} - \frac{2Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{2Ba^2}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2Ba\sqrt{b}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + Bb \begin{cases} \frac{\sqrt{a}x^{\frac{3}{2}}}{3} & \text{for } b = 0 \\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**4,x)

[Out] $-A\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) - Aa\sqrt{b}\sqrt{\frac{a}{bx^3}+1}/(3x^{\frac{3}{2}}) + 2Aa\sqrt{b}/(3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}) + 2Ab^{\frac{3}{2}}x^{\frac{3}{2}}/(3\sqrt{\frac{a}{bx^3}+1}) - 2Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)/3 + 2Ba^2/(3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}) + 2Ba\sqrt{b}x^{\frac{3}{2}}/(3\sqrt{\frac{a}{bx^3}+1}) + Bb\operatorname{Piecewise}\left(\left(\frac{\sqrt{a}x^{\frac{3}{2}}}{3}\right), \operatorname{Eq}(b, 0)\right), \left(\frac{2(a+bx^3)^{\frac{3}{2}}}{9b}\right), \operatorname{True}\right)$

$$3.186 \quad \int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7} dx$$

Optimal. Leaf size=115

$$-\frac{(a+bx^3)^{3/2}(4aB+Ab)}{12ax^3} + \frac{b\sqrt{a+bx^3}(4aB+Ab)}{4a} - \frac{b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{A(a+bx^3)^{5/2}}{6ax^6}$$

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {446, 78, 47, 50, 63, 208}

$$-\frac{(a+bx^3)^{3/2}(4aB+Ab)}{12ax^3} + \frac{b\sqrt{a+bx^3}(4aB+Ab)}{4a} - \frac{b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{A(a+bx^3)^{5/2}}{6ax^6}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]

[Out] (b*(A*b + 4*a*B)*Sqrt[a + b*x^3])/(4*a) - ((A*b + 4*a*B)*(a + b*x^3)^(3/2))/(12*a*x^3) - (A*(a + b*x^3)^(5/2))/(6*a*x^6) - (b*(A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a])

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63


```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_
.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^{3/2} (A + Bx^3)}{x^7} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^{3/2} (A + Bx)}{x^3} dx, x, x^3 \right) \\
&= -\frac{A(a + bx^3)^{5/2}}{6ax^6} + \frac{(Ab + 4aB) \text{Subst} \left(\int \frac{(a+bx)^{3/2}}{x^2} dx, x, x^3 \right)}{12a} \\
&= -\frac{(Ab + 4aB)(a + bx^3)^{3/2}}{12ax^3} - \frac{A(a + bx^3)^{5/2}}{6ax^6} + \frac{(b(Ab + 4aB)) \text{Subst} \left(\int \frac{\sqrt{a+bx}}{x} dx, x, x^3 \right)}{8a} \\
&= \frac{b(Ab + 4aB)\sqrt{a + bx^3}}{4a} - \frac{(Ab + 4aB)(a + bx^3)^{3/2}}{12ax^3} - \frac{A(a + bx^3)^{5/2}}{6ax^6} + \frac{1}{8}(b(Ab + 4aB)) \ln \left| \frac{\sqrt{a+bx^3} + x}{\sqrt{a+bx^3} - x} \right| \\
&= \frac{b(Ab + 4aB)\sqrt{a + bx^3}}{4a} - \frac{(Ab + 4aB)(a + bx^3)^{3/2}}{12ax^3} - \frac{A(a + bx^3)^{5/2}}{6ax^6} + \frac{1}{4}(Ab + 4aB) \ln \left| \frac{\sqrt{a+bx^3} + x}{\sqrt{a+bx^3} - x} \right| \\
&= \frac{b(Ab + 4aB)\sqrt{a + bx^3}}{4a} - \frac{(Ab + 4aB)(a + bx^3)^{3/2}}{12ax^3} - \frac{A(a + bx^3)^{5/2}}{6ax^6} - \frac{b(Ab + 4aB)}{4} \ln \left| \frac{\sqrt{a+bx^3} + x}{\sqrt{a+bx^3} - x} \right|
\end{aligned}$$

Mathematica [C] time = 0.04, size = 59, normalized size = 0.51

$$\frac{(a + bx^3)^{5/2} \left(bx^6(4aB + Ab) {}_2F_1 \left(2, \frac{5}{2}; \frac{7}{2}; \frac{bx^3}{a} + 1 \right) - 5a^2A \right)}{30a^3x^6}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7, x]

[Out] ((a + b*x^3)^(5/2)*(-5*a^2*A + b*(A*b + 4*a*B)*x^6*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a]))/(30*a^3*x^6)

IntegrateAlgebraic [A] time = 0.15, size = 84, normalized size = 0.73

$$\frac{(-4abB - Ab^2) \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{4\sqrt{a}} + \frac{\sqrt{a + bx^3} (-2aA - 4aBx^3 - 5Abx^3 + 8bBx^6)}{12x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7, x]

[Out] $(\sqrt{a + b*x^3}*(-2*a*A - 5*A*b*x^3 - 4*a*B*x^3 + 8*b*B*x^6))/(12*x^6) + ((-A*b^2) - 4*a*b*B)*\text{ArcTanh}[\sqrt{a + b*x^3}/\sqrt{a}]/(4*\sqrt{a})$

fricas [A] time = 0.89, size = 191, normalized size = 1.66

$$\frac{3(4Bab + Ab^2)\sqrt{a} \log\left(\frac{bx^3 - 2\sqrt{bx^3+a}\sqrt{a} + 2a}{x^3}\right) + 2(8Babx^6 - (4Ba^2 + 5Aab)x^3 - 2Aa^2)\sqrt{bx^3+a}}{24ax^6}, \frac{3(4Bab + Ab^2)\sqrt{-a} \arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) + (8Babx^6 - (4Ba^2 + 5Aab)x^3 - 2Aa^2)\sqrt{bx^3+a}}{12ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="fricas")`

[Out] $[1/24*(3*(4*B*a*b + A*b^2)*\sqrt{a})*x^6*\log((b*x^3 - 2*\sqrt{a}*\sqrt{b*x^3 + a})*\sqrt{a} + 2*a)/x^3) + 2*(8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*\sqrt{b*x^3 + a}/(a*x^6), 1/12*(3*(4*B*a*b + A*b^2)*\sqrt{-a})*x^6*\arctan(\sqrt{b*x^3 + a}*\sqrt{-a}/a) + (8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*\sqrt{b*x^3 + a}/(a*x^6)]$

giac [A] time = 0.18, size = 131, normalized size = 1.14

$$\frac{8\sqrt{bx^3+a}Bb^2 + \frac{3(4Bab^2+Ab^3)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{4(bx^3+a)^{\frac{3}{2}}Bab^2 - 4\sqrt{bx^3+a}Ba^2b^2 + 5(bx^3+a)^{\frac{3}{2}}Ab^3 - 3\sqrt{bx^3+a}Aab^3}{b^2x^6}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="giac")`

[Out] $1/12*(8*\sqrt{b*x^3 + a}*B*b^2 + 3*(4*B*a*b^2 + A*b^3)*\arctan(\sqrt{b*x^3 + a}/\sqrt{-a}))/\sqrt{-a} - (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*\sqrt{b*x^3 + a}*B*a^2*b^2 + 5*(b*x^3 + a)^(3/2)*A*b^3 - 3*\sqrt{b*x^3 + a}*A*a*b^3)/(b^2*x^6))/b$

maple [A] time = 0.05, size = 107, normalized size = 0.93

$$\left(\frac{b^2 \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{5\sqrt{bx^3+a}b}{12x^3} - \frac{\sqrt{bx^3+a}a}{6x^6}\right)A + \left(-\sqrt{a}b \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right) + \frac{2\sqrt{bx^3+a}b}{3} - \frac{\sqrt{bx^3+a}a}{3x^3}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x)`

[Out] $B*(-1/3*(b*x^3+a)^(1/2)*a/x^3 + 2/3*(b*x^3+a)^(1/2)*b - b*\operatorname{arctanh}((b*x^3+a)^(1/2)/a^(1/2)))*a^(1/2) + A*(-1/6*a*(b*x^3+a)^(1/2)/x^6 - 5/12*(b*x^3+a)^(1/2)*b/x^3 - 1/4*b^2*\operatorname{arctanh}((b*x^3+a)^(1/2)/a^(1/2)))/a^(1/2)$

maxima [A] time = 1.31, size = 171, normalized size = 1.49

$$\frac{1}{24} \left(\frac{3b^2 \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\left(5(bx^3+a)^{\frac{3}{2}}b^2 - 3\sqrt{bx^3+a}ab^2\right)}{(bx^3+a)^2 - 2(bx^3+a)a + a^2} \right) A + \frac{1}{6} \left(3\sqrt{a}b \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 4\sqrt{bx^3+a}b - \frac{2\sqrt{bx^3+aa}}{x^3} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="maxima")

[Out] 1/24*(3*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a) - 2*(5*(b*x^3 + a)^(3/2)*b^2 - 3*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2 - 2*(b*x^3 + a)*a + a^2))*A + 1/6*(3*sqrt(a)*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) + 4*sqrt(b*x^3 + a)*b - 2*sqrt(b*x^3 + a)*a/x^3)*B

mupad [B] time = 3.47, size = 110, normalized size = 0.96

$$\frac{2Bb\sqrt{bx^3+a}}{3} - \frac{\sqrt{bx^3+a}(4Ba^3+5Aba^2)}{12a^2x^3} - \frac{Aa\sqrt{bx^3+a}}{6x^6} + \frac{b \ln\left(\frac{(\sqrt{bx^3+a}-\sqrt{a})^3(\sqrt{bx^3+a}+\sqrt{a})}{x^6}\right)}{8\sqrt{a}} (Ab+4Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^7,x)

[Out] (2*B*b*(a + b*x^3)^(1/2))/3 - ((a + b*x^3)^(1/2)*(4*B*a^3 + 5*A*a^2*b))/(12*a^2*x^3) - (A*a*(a + b*x^3)^(1/2))/(6*x^6) + (b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2))))/x^6*(A*b + 4*B*a))/(8*a^(1/2))

sympy [B] time = 153.11, size = 243, normalized size = 2.11

$$-\frac{Aa^2}{6\sqrt{bx^2}\sqrt{\frac{a}{bx^3}+1}} - \frac{Aa\sqrt{b}}{4x^2\sqrt{\frac{a}{bx^3}+1}} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{Ab^{\frac{3}{2}}}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{Ab^2 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^{\frac{3}{2}}}}\right)}{4\sqrt{a}} - B\sqrt{a}b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^{\frac{3}{2}}}}\right) - \frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2Ba\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2Bb^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**7,x)

[Out] -A*a**2/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*a*sqrt(b)/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - A*b**(3/2)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - A*b**(3/2)/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - A*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a)) - B*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - B*a*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*B*a*sqrt(b)/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*B*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1))

$$3.187 \quad \int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=103

$$\frac{2a^2\sqrt{a+bx^3}(Ab-aB)}{3b^4} + \frac{2(a+bx^3)^{5/2}(Ab-3aB)}{15b^4} - \frac{2a(a+bx^3)^{3/2}(2Ab-3aB)}{9b^4} + \frac{2B(a+bx^3)^{7/2}}{21b^4}$$

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{2a^2\sqrt{a+bx^3}(Ab-aB)}{3b^4} + \frac{2(a+bx^3)^{5/2}(Ab-3aB)}{15b^4} - \frac{2a(a+bx^3)^{3/2}(2Ab-3aB)}{9b^4} + \frac{2B(a+bx^3)^{7/2}}{21b^4}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(A + B*x^3))/Sqrt[a + b*x^3], x]

[Out] (2*a^2*(A*b - a*B)*Sqrt[a + b*x^3])/((3*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b*x^3)^(3/2))/(9*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(5/2))/(15*b^4) + (2*B*(a + b*x^3)^(7/2))/(21*b^4)

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{x^8 (A + Bx^3)}{\sqrt{a + bx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (A + Bx)}{\sqrt{a + bx}} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{a^2(-Ab + aB)}{b^3 \sqrt{a + bx}} + \frac{a(-2Ab + 3aB)\sqrt{a + bx}}{b^3} + \frac{(Ab - 3aB)(a + bx)^{3/2}}{b^3} + \frac{B(a + bx)^{5/2}}{b^3} \right) dx, x, x^3 \right) \\
&= \frac{2a^2(Ab - aB)\sqrt{a + bx^3}}{3b^4} - \frac{2a(2Ab - 3aB)(a + bx^3)^{3/2}}{9b^4} + \frac{2(Ab - 3aB)(a + bx^3)^{5/2}}{15b^4} + \frac{2B(a + bx^3)^{7/2}}{21b^4}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 78, normalized size = 0.76

$$\frac{2\sqrt{a + bx^3} (-48a^3B + 8a^2b(7A + 3Bx^3) - 2ab^2x^3(14A + 9Bx^3) + 3b^3x^6(7A + 5Bx^3))}{315b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(A + B*x^3))/Sqrt[a + b*x^3], x]

[Out] (2*Sqrt[a + b*x^3]*(-48*a^3*B + 8*a^2*b*(7*A + 3*B*x^3) + 3*b^3*x^6*(7*A + 5*B*x^3) - 2*a*b^2*x^3*(14*A + 9*B*x^3)))/(315*b^4)

IntegrateAlgebraic [A] time = 0.05, size = 80, normalized size = 0.78

$$\frac{2\sqrt{a + bx^3} (48a^3B - 56a^2Ab - 24a^2bBx^3 + 28aAb^2x^3 + 18ab^2Bx^6 - 21Ab^3x^6 - 15b^3Bx^9)}{315b^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8*(A + B*x^3))/Sqrt[a + b*x^3], x]

[Out] (-2*Sqrt[a + b*x^3]*(-56*a^2*A*b + 48*a^3*B + 28*a*A*b^2*x^3 - 24*a^2*b*B*x^3 - 21*A*b^3*x^6 + 18*a*b^2*B*x^6 - 15*b^3*B*x^9))/(315*b^4)

fricas [A] time = 0.92, size = 76, normalized size = 0.74

$$\frac{2(15Bb^3x^9 - 3(6Bab^2 - 7Ab^3)x^6 - 48Ba^3 + 56Aa^2b + 4(6Ba^2b - 7Aab^2)x^3)\sqrt{bx^3 + a}}{315b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] 2/315*(15*B*b^3*x^9 - 3*(6*B*a*b^2 - 7*A*b^3)*x^6 - 48*B*a^3 + 56*A*a^2*b + 4*(6*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^4

giac [A] time = 0.16, size = 101, normalized size = 0.98

$$\frac{2(Ba^3 - Aa^2b)\sqrt{bx^3 + a}}{3b^4} + \frac{2\left(15(bx^3 + a)^{\frac{7}{2}}B - 63(bx^3 + a)^{\frac{5}{2}}Ba + 105(bx^3 + a)^{\frac{3}{2}}Ba^2 + 21(bx^3 + a)^{\frac{5}{2}}Ab - 70(bx^3 + a)^{\frac{3}{2}}Aab\right)}{315b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] $-2/3*(B*a^3 - A*a^2*b)*\text{sqrt}(b*x^3 + a)/b^4 + 2/315*(15*(b*x^3 + a)^{(7/2)}*B - 63*(b*x^3 + a)^{(5/2)}*B*a + 105*(b*x^3 + a)^{(3/2)}*B*a^2 + 21*(b*x^3 + a)^{(5/2)}*A*b - 70*(b*x^3 + a)^{(3/2)}*A*a*b)/b^4$

maple [A] time = 0.04, size = 77, normalized size = 0.75

$$\frac{2\sqrt{bx^3 + a} (15Bx^9b^3 + 21A b^3x^6 - 18Ba b^2x^6 - 28Aa b^2x^3 + 24B a^2b x^3 + 56A a^2b - 48B a^3)}{315b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x)

[Out] $2/315*(b*x^3+a)^{(1/2)}*(15*B*b^3*x^9+21*A*b^3*x^6-18*B*a*b^2*x^6-28*A*a*b^2*x^3+24*B*a^2*b*x^3+56*A*a^2*b-48*B*a^3)/b^4$

maxima [A] time = 0.59, size = 118, normalized size = 1.15

$$\frac{2}{105}B\left(\frac{5(bx^3 + a)^{\frac{7}{2}}}{b^4} - \frac{21(bx^3 + a)^{\frac{5}{2}}a}{b^4} + \frac{35(bx^3 + a)^{\frac{3}{2}}a^2}{b^4} - \frac{35\sqrt{bx^3 + a}a^3}{b^4}\right) + \frac{2}{45}A\left(\frac{3(bx^3 + a)^{\frac{5}{2}}}{b^3} - \frac{10(bx^3 + a)^{\frac{3}{2}}a}{b^3} + \frac{15\sqrt{bx^3 + a}a^2}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] $2/105*B*(5*(b*x^3 + a)^{(7/2)}/b^4 - 21*(b*x^3 + a)^{(5/2)}*a/b^4 + 35*(b*x^3 + a)^{(3/2)}*a^2/b^4 - 35*\text{sqrt}(b*x^3 + a)*a^3/b^4) + 2/45*A*(3*(b*x^3 + a)^{(5/2)}/b^3 - 10*(b*x^3 + a)^{(3/2)}*a/b^3 + 15*\text{sqrt}(b*x^3 + a)*a^2/b^3)$

mupad [B] time = 2.68, size = 104, normalized size = 1.01

$$\frac{8a^2\sqrt{bx^3 + a}\left(2A - \frac{12Ba}{7b}\right)}{45b^3} + \frac{x^6\sqrt{bx^3 + a}\left(2A - \frac{12Ba}{7b}\right)}{15b} + \frac{2Bx^9\sqrt{bx^3 + a}}{21b} - \frac{4ax^3\sqrt{bx^3 + a}\left(2A - \frac{12Ba}{7b}\right)}{45b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8*(A + B*x^3))/(a + b*x^3)^(1/2),x)

[Out] $(8*a^2*(a + b*x^3)^{(1/2)}*(2*A - (12*B*a)/(7*b)))/(45*b^3) + (x^6*(a + b*x^3)^{(1/2)}*(2*A - (12*B*a)/(7*b)))/(15*b) + (2*B*x^9*(a + b*x^3)^{(1/2)})/(21*b) - (4*a*x^3*(a + b*x^3)^{(1/2)}*(2*A - (12*B*a)/(7*b)))/(45*b^2)$

sympy [A] time = 3.44, size = 175, normalized size = 1.70

$$\begin{cases} \frac{16Aa^2\sqrt{a+bx^3}}{45b^3} - \frac{8Aax^3\sqrt{a+bx^3}}{45b^2} + \frac{2Ax^6\sqrt{a+bx^3}}{15b} - \frac{32Ba^3\sqrt{a+bx^3}}{105b^4} + \frac{16Ba^2x^3\sqrt{a+bx^3}}{105b^3} - \frac{4Bax^6\sqrt{a+bx^3}}{35b^2} + \frac{2Bx^9\sqrt{a+bx^3}}{21b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^9}{9} + \frac{Bx^{12}}{12}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(B*x**3+A)/(b*x**3+a)**(1/2),x)`

[Out] `Piecewise((16*A*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*A*a*x**3*sqrt(a + b*x**3)/(45*b**2) + 2*A*x**6*sqrt(a + b*x**3)/(15*b) - 32*B*a**3*sqrt(a + b*x**3)/(105*b**4) + 16*B*a**2*x**3*sqrt(a + b*x**3)/(105*b**3) - 4*B*a*x**6*sqrt(a + b*x**3)/(35*b**2) + 2*B*x**9*sqrt(a + b*x**3)/(21*b), Ne(b, 0)), ((A*x**9/9 + B*x**12/12)/sqrt(a), True))`

$$3.188 \quad \int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=73

$$\frac{2(a+bx^3)^{3/2}(Ab-2aB)}{9b^3} - \frac{2a\sqrt{a+bx^3}(Ab-aB)}{3b^3} + \frac{2B(a+bx^3)^{5/2}}{15b^3}$$

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{2(a+bx^3)^{3/2}(Ab-2aB)}{9b^3} - \frac{2a\sqrt{a+bx^3}(Ab-aB)}{3b^3} + \frac{2B(a+bx^3)^{5/2}}{15b^3}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(A + B*x^3))/Sqrt[a + b*x^3], x]

[Out] (-2*a*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)^(3/2))/(9*b^3) + (2*B*(a + b*x^3)^(5/2))/(15*b^3)

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (A + Bx^3)}{\sqrt{a + bx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(A + Bx)}{\sqrt{a + bx}} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{a(-Ab + aB)}{b^2 \sqrt{a + bx}} + \frac{(Ab - 2aB)\sqrt{a + bx}}{b^2} + \frac{B(a + bx)^{3/2}}{b^2} \right) dx, x, x^3 \right) \\
&= -\frac{2a(Ab - aB)\sqrt{a + bx^3}}{3b^3} + \frac{2(Ab - 2aB)(a + bx^3)^{3/2}}{9b^3} + \frac{2B(a + bx^3)^{5/2}}{15b^3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 56, normalized size = 0.77

$$\frac{2\sqrt{a + bx^3} (8a^2B - 2ab(5A + 2Bx^3) + b^2x^3(5A + 3Bx^3))}{45b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(A + B*x^3))/Sqrt[a + b*x^3], x]

[Out] (2*Sqrt[a + b*x^3]*(8*a^2*B - 2*a*b*(5*A + 2*B*x^3) + b^2*x^3*(5*A + 3*B*x^3)))/(45*b^3)

IntegrateAlgebraic [A] time = 0.04, size = 56, normalized size = 0.77

$$\frac{2\sqrt{a + bx^3} (8a^2B - 10aAb - 4abBx^3 + 5Ab^2x^3 + 3b^2Bx^6)}{45b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(A + B*x^3))/Sqrt[a + b*x^3], x]

[Out] (2*Sqrt[a + b*x^3]*(-10*a*A*b + 8*a^2*B + 5*A*b^2*x^3 - 4*a*b*B*x^3 + 3*b^2*B*x^6))/(45*b^3)

fricas [A] time = 0.81, size = 52, normalized size = 0.71

$$\frac{2(3Bb^2x^6 - (4Bab - 5Ab^2)x^3 + 8Ba^2 - 10Aab)\sqrt{bx^3 + a}}{45b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] 2/45*(3*B*b^2*x^6 - (4*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 10*A*a*b)*sqrt(b*x^3 + a)/b^3

giac [A] time = 0.16, size = 70, normalized size = 0.96

$$\frac{2\sqrt{bx^3+a}(Ba^2 - Aab)}{3b^3} + \frac{2\left(3(bx^3+a)^{\frac{5}{2}}B - 10(bx^3+a)^{\frac{3}{2}}Ba + 5(bx^3+a)^{\frac{3}{2}}Ab\right)}{45b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] 2/3*sqrt(b*x^3 + a)*(B*a^2 - A*a*b)/b^3 + 2/45*(3*(b*x^3 + a)^(5/2)*B - 10*(b*x^3 + a)^(3/2)*B*a + 5*(b*x^3 + a)^(3/2)*A*b)/b^3

maple [A] time = 0.05, size = 53, normalized size = 0.73

$$\frac{2\sqrt{bx^3+a}(-3Bb^2x^6 - 5Ab^2x^3 + 4Babx^3 + 10Aab - 8Ba^2)}{45b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x)

[Out] -2/45*(b*x^3+a)^(1/2)*(-3*B*b^2*x^6-5*A*b^2*x^3+4*B*a*b*x^3+10*A*a*b-8*B*a^2)/b^3

maxima [A] time = 0.45, size = 83, normalized size = 1.14

$$\frac{2}{45}B\left(\frac{3(bx^3+a)^{\frac{5}{2}}}{b^3} - \frac{10(bx^3+a)^{\frac{3}{2}}a}{b^3} + \frac{15\sqrt{bx^3+a}a^2}{b^3}\right) + \frac{2}{9}A\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^2} - \frac{3\sqrt{bx^3+a}a}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] 2/45*B*(3*(b*x^3 + a)^(5/2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x^3 + a)*a^2/b^3) + 2/9*A*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2)

mupad [B] time = 2.65, size = 52, normalized size = 0.71

$$\frac{2\sqrt{bx^3+a}(8Ba^2 - 4Babx^3 - 10Aab + 3Bb^2x^6 + 5Ab^2x^3)}{45b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*(A + B*x^3))/(a + b*x^3)^(1/2),x)

[Out] $(2*(a + b*x^3)^{(1/2)}*(8*B*a^2 + 5*A*b^2*x^3 + 3*B*b^2*x^6 - 10*A*a*b - 4*B*a*b*x^3))/(45*b^3)$

sympy [A] time = 1.82, size = 124, normalized size = 1.70

$$\begin{cases} -\frac{4Aa\sqrt{a+bx^3}}{9b^2} + \frac{2Ax^3\sqrt{a+bx^3}}{9b} + \frac{16Ba^2\sqrt{a+bx^3}}{45b^3} - \frac{8Bax^3\sqrt{a+bx^3}}{45b^2} + \frac{2Bx^6\sqrt{a+bx^3}}{15b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^6}{6} + \frac{Bx^9}{9}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(B*x**3+A)/(b*x**3+a)**(1/2), x)`

[Out] `Piecewise((-4*A*a*sqrt(a + b*x**3)/(9*b**2) + 2*A*x**3*sqrt(a + b*x**3)/(9*b) + 16*B*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*B*a*x**3*sqrt(a + b*x**3)/(45*b**2) + 2*B*x**6*sqrt(a + b*x**3)/(15*b), Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/sqrt(a), True))`

$$3.189 \quad \int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=46

$$\frac{2\sqrt{a+bx^3}(Ab-aB)}{3b^2} + \frac{2B(a+bx^3)^{3/2}}{9b^2}$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {444, 43}

$$\frac{2\sqrt{a+bx^3}(Ab-aB)}{3b^2} + \frac{2B(a+bx^3)^{3/2}}{9b^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]

[Out] (2*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^2) + (2*B*(a + b*x^3)^(3/2))/(9*b^2)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (A + Bx^3)}{\sqrt{a + bx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{\sqrt{a + bx}} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{Ab - aB}{b\sqrt{a + bx}} + \frac{B\sqrt{a + bx}}{b} \right) dx, x, x^3 \right) \\
&= \frac{2(Ab - aB)\sqrt{a + bx^3}}{3b^2} + \frac{2B(a + bx^3)^{3/2}}{9b^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.72

$$\frac{2\sqrt{a + bx^3} (-2aB + 3Ab + bBx^3)}{9b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x^3))/Sqrt[a + b*x^3], x]

[Out] (2*Sqrt[a + b*x^3]*(3*A*b - 2*a*B + b*B*x^3))/(9*b^2)

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 0.72

$$\frac{2\sqrt{a + bx^3} (-2aB + 3Ab + bBx^3)}{9b^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(A + B*x^3))/Sqrt[a + b*x^3], x]

[Out] (2*Sqrt[a + b*x^3]*(3*A*b - 2*a*B + b*B*x^3))/(9*b^2)

fricas [A] time = 1.11, size = 29, normalized size = 0.63

$$\frac{2(Bbx^3 - 2Ba + 3Ab)\sqrt{bx^3 + a}}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] 2/9*(B*b*x^3 - 2*B*a + 3*A*b)*sqrt(b*x^3 + a)/b^2

giac [A] time = 0.16, size = 38, normalized size = 0.83

$$\frac{2(bx^3 + a)^{\frac{3}{2}}B}{9b^2} - \frac{2\sqrt{bx^3 + a}(Ba - Ab)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] 2/9*(b*x^3 + a)^(3/2)*B/b^2 - 2/3*sqrt(b*x^3 + a)*(B*a - A*b)/b^2

maple [A] time = 0.04, size = 30, normalized size = 0.65

$$\frac{2\sqrt{bx^3 + a} (Bbx^3 + 3Ab - 2Ba)}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x)

[Out] 2/9*(b*x^3+a)^(1/2)*(B*b*x^3+3*A*b-2*B*a)/b^2

maxima [A] time = 0.46, size = 48, normalized size = 1.04

$$\frac{2}{9}B \left(\frac{(bx^3 + a)^{\frac{3}{2}}}{b^2} - \frac{3\sqrt{bx^3 + a}a}{b^2} \right) + \frac{2\sqrt{bx^3 + a}A}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2) + 2/3*sqrt(b*x^3 + a)*A/b

mupad [B] time = 2.60, size = 29, normalized size = 0.63

$$\frac{2\sqrt{bx^3 + a} (Bbx^3 + 3Ab - 2Ba)}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(1/2),x)

[Out] (2*(a + b*x^3)^(1/2)*(3*A*b - 2*B*a + B*b*x^3))/(9*b^2)

sympy [A] time = 0.98, size = 75, normalized size = 1.63

$$\begin{cases} \frac{2A\sqrt{a+bx^3}}{3b} - \frac{4Ba\sqrt{a+bx^3}}{9b^2} + \frac{2Bx^3\sqrt{a+bx^3}}{9b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3} + \frac{Bx^6}{6}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(B*x**3+A)/(b*x**3+a)**(1/2),x)`

[Out] `Piecewise((2*A*sqrt(a + b*x**3)/(3*b) - 4*B*a*sqrt(a + b*x**3)/(9*b**2) + 2*B*x**3*sqrt(a + b*x**3)/(9*b), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/sqrt(a), True))`

$$3.190 \quad \int \frac{A+Bx^3}{x\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=48

$$\frac{2B\sqrt{a+bx^3}}{3b} - \frac{2A \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {446, 80, 63, 208}

$$\frac{2B\sqrt{a+bx^3}}{3b} - \frac{2A \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x*sqrt[a + b*x^3]), x]

[Out] (2*B*sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/sqrt[a]])/(3*sqrt[a])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx^3}{x\sqrt{a + bx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x\sqrt{a + bx}} dx, x, x^3 \right) \\
 &= \frac{2B\sqrt{a + bx^3}}{3b} + \frac{1}{3} A \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx}} dx, x, x^3 \right) \\
 &= \frac{2B\sqrt{a + bx^3}}{3b} + \frac{(2A) \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + bx^3} \right)}{3b} \\
 &= \frac{2B\sqrt{a + bx^3}}{3b} - \frac{2A \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right)}{3\sqrt{a}}
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 1.00

$$\frac{2B\sqrt{a + bx^3}}{3b} - \frac{2A \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*Sqrt[a + b*x^3]), x]
```

```
[Out] (2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqr
t[a])
```

IntegrateAlgebraic [A] time = 0.04, size = 48, normalized size = 1.00

$$\frac{2B\sqrt{a + bx^3}}{3b} - \frac{2A \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x*Sqrt[a + b*x^3]), x]
```

[Out] $(2*B*\sqrt{a + b*x^3})/(3*b) - (2*A*\text{ArcTanh}[\sqrt{a + b*x^3}/\sqrt{a}])/(3*\sqrt{a})$

fricas [A] time = 1.01, size = 105, normalized size = 2.19

$$\left[\frac{A\sqrt{a} b \log\left(\frac{bx^3 - 2\sqrt{bx^3+a}\sqrt{a} + 2a}{x^3}\right) + 2\sqrt{bx^3+a} Ba}{3ab}, \frac{2\left(A\sqrt{-a} b \arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) + \sqrt{bx^3+a} Ba\right)}{3ab} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="fricas")`

[Out] $[1/3*(A*\sqrt{a}*b*\log((b*x^3 - 2*\sqrt{b*x^3 + a})*\sqrt{a} + 2*a)/x^3) + 2*\sqrt{b*x^3 + a}*B*a)/(a*b), 2/3*(A*\sqrt{-a}*b*\arctan(\sqrt{b*x^3 + a}*\sqrt{-a})/a) + \sqrt{b*x^3 + a}*B*a)/(a*b)]$

giac [A] time = 0.19, size = 40, normalized size = 0.83

$$\frac{2 A \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3 \sqrt{-a}} + \frac{2 \sqrt{bx^3+a} B}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="giac")`

[Out] $2/3*A*\arctan(\sqrt{b*x^3 + a}/\sqrt{-a})/\sqrt{-a} + 2/3*\sqrt{b*x^3 + a}*B/b$

maple [A] time = 0.04, size = 37, normalized size = 0.77

$$-\frac{2A \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}} + \frac{2\sqrt{bx^3+a} B}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^3+A)/x/(b*x^3+a)^(1/2),x)`

[Out] $-2/3*A*\operatorname{arctanh}((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2) + 2/3*B*(b*x^3+a)^(1/2)/b$

maxima [A] time = 1.08, size = 54, normalized size = 1.12

$$\frac{A \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3 \sqrt{a}} + \frac{2 \sqrt{bx^3+a} B}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] 1/3*A*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a) + 2/3*sqrt(b*x^3 + a)*B/b

mupad [B] time = 2.72, size = 57, normalized size = 1.19

$$\frac{2B\sqrt{bx^3+a}}{3b} + \frac{A \ln\left(\frac{(\sqrt{bx^3+a}-\sqrt{a})^3(\sqrt{bx^3+a}+\sqrt{a})}{x^6}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x*(a + b*x^3)^(1/2)),x)

[Out] (2*B*(a + b*x^3)^(1/2))/(3*b) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2))))/x^6))/(3*a^(1/2))

sympy [A] time = 11.27, size = 65, normalized size = 1.35

$$\frac{2A \operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\sqrt{a+bx^3}}\right)}{3a\sqrt{-\frac{1}{a}}} - \frac{B \begin{cases} -\frac{x^3}{\sqrt{a}} & \text{for } b = 0 \\ -\frac{2\sqrt{a+bx^3}}{b} & \text{otherwise} \end{cases}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x/(b*x**3+a)**(1/2),x)

[Out] 2*A*atan(1/(sqrt(-1/a)*sqrt(a + b*x**3)))/(3*a*sqrt(-1/a)) - B*Piecewise((-x**3/sqrt(a), Eq(b, 0)), (-2*sqrt(a + b*x**3)/b, True))/3

$$3.191 \quad \int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=58

$$\frac{(Ab - 2aB) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{A\sqrt{a+bx^3}}{3ax^3}$$

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {446, 78, 63, 208}

$$\frac{(Ab - 2aB) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{A\sqrt{a+bx^3}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]

[Out] -(A*Sqrt[a + b*x^3])/(3*a*x^3) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1))]/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^4 \sqrt{a + bx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x^2 \sqrt{a + bx}} dx, x, x^3 \right) \\
&= -\frac{A\sqrt{a + bx^3}}{3ax^3} + \frac{\left(-\frac{Ab}{2} + aB\right) \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx}} dx, x, x^3 \right)}{3a} \\
&= -\frac{A\sqrt{a + bx^3}}{3ax^3} + \frac{\left(2\left(-\frac{Ab}{2} + aB\right)\right) \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + bx^3} \right)}{3ab} \\
&= -\frac{A\sqrt{a + bx^3}}{3ax^3} + \frac{(Ab - 2aB) \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{3a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 0.98

$$\frac{1}{3} \left(\frac{(Ab - 2aB) \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{a^{3/2}} - \frac{A\sqrt{a + bx^3}}{ax^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]), x]

[Out] (-((A*Sqrt[a + b*x^3])/(a*x^3)) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/a^(3/2))/3

IntegrateAlgebraic [A] time = 0.07, size = 58, normalized size = 1.00

$$\frac{(Ab - 2aB) \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{3a^{3/2}} - \frac{A\sqrt{a + bx^3}}{3ax^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]

[Out] $-1/3*(A*\text{Sqrt}[a + b*x^3])/(a*x^3) + ((A*b - 2*a*B)*\text{ArcTanh}[\text{Sqrt}[a + b*x^3]/\text{Sqrt}[a]])/(3*a^{(3/2)})$

fricas [A] time = 1.51, size = 126, normalized size = 2.17

$$\left[\frac{(2Ba - Ab)\sqrt{a}x^3 \log\left(\frac{bx^3 + 2\sqrt{bx^3+a}\sqrt{a} + 2a}{x^3}\right) + 2\sqrt{bx^3+a}Aa}{6a^2x^3}, \frac{(2Ba - Ab)\sqrt{-a}x^3 \arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) - \sqrt{bx^3+a}Aa}{3a^2x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] $[-1/6*((2*B*a - A*b)*\text{sqrt}(a)*x^3*\log((b*x^3 + 2*\text{sqrt}(b*x^3 + a)*\text{sqrt}(a) + 2*a)/x^3) + 2*\text{sqrt}(b*x^3 + a)*A*a)/(a^2*x^3), 1/3*((2*B*a - A*b)*\text{sqrt}(-a)*x^3*\arctan(\text{sqrt}(b*x^3 + a)*\text{sqrt}(-a)/a) - \text{sqrt}(b*x^3 + a)*A*a)/(a^2*x^3)]$

giac [A] time = 0.16, size = 62, normalized size = 1.07

$$\frac{(2Bab - Ab^2) \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right) - \frac{\sqrt{bx^3+a} Ab}{ax^3}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] $1/3*((2*B*a*b - A*b^2)*\arctan(\text{sqrt}(b*x^3 + a)/\text{sqrt}(-a))/(\text{sqrt}(-a)*a) - \text{sqrt}(b*x^3 + a)*A*b/(a*x^3))/b$

maple [A] time = 0.05, size = 62, normalized size = 1.07

$$-\frac{2B \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}} + \left(\frac{b \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}} - \frac{\sqrt{bx^3+a}}{3ax^3} \right) A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x)

[Out] $A*(1/3*b*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})/a^{(3/2)} - 1/3*(b*x^3+a)^{(1/2)}/a/x^3) - 2/3*B*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})/a^{(1/2)}$

maxima [B] time = 1.17, size = 109, normalized size = 1.88

$$-\frac{1}{6}A \left(\frac{2\sqrt{bx^3+a}b}{(bx^3+a)a-a^2} + \frac{b \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} \right) + \frac{B \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] -1/6*A*(2*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) + b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)) + 1/3*B*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)

mupad [B] time = 2.89, size = 67, normalized size = 1.16

$$\frac{\ln\left(\frac{(\sqrt{bx^3+a}-\sqrt{a})(\sqrt{bx^3+a}+\sqrt{a})^3}{x^6}\right)(Ab-2Ba)}{6a^{3/2}} - \frac{A\sqrt{bx^3+a}}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^4*(a + b*x^3)^(1/2)),x)

[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*(A*b - 2*B*a))/(6*a^(3/2)) - (A*(a + b*x^3)^(1/2))/(3*a*x^3)

sympy [A] time = 31.83, size = 80, normalized size = 1.38

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}} + \frac{Ab \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^{\frac{3}{2}}}}\right)}{3a^{\frac{3}{2}}} - \frac{2B \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^{\frac{3}{2}}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(1/2),x)

[Out] -A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*x**(3/2)) + A*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*a**(3/2)) - 2*B*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))

$$3.192 \quad \int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=90

$$-\frac{b(3Ab - 4aB) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}} + \frac{\sqrt{a+bx^3}(3Ab - 4aB)}{12a^2x^3} - \frac{A\sqrt{a+bx^3}}{6ax^6}$$

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 78, 51, 63, 208}

$$\frac{\sqrt{a+bx^3}(3Ab - 4aB)}{12a^2x^3} - \frac{b(3Ab - 4aB) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}} - \frac{A\sqrt{a+bx^3}}{6ax^6}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^7*sqrt[a + b*x^3]), x]

[Out] -(A*sqrt[a + b*x^3])/(6*a*x^6) + ((3*A*b - 4*a*B)*sqrt[a + b*x^3])/(12*a^2*x^3) - (b*(3*A*b - 4*a*B)*ArcTanh[sqrt[a + b*x^3]/sqrt[a]])/(12*a^(5/2))

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f

$(p + 1)))/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x],$
 $x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x\} \&\& \text{LtQ}[p, -1] \&\& (!\text{LtQ}[n, -1] || \text{Int}$
 $\text{egerQ}[p] || !(\text{IntegerQ}[n] || !(\text{EqQ}[e, 0] || !(\text{EqQ}[c, 0] || \text{LtQ}[p, n]))))))$

Rule 208

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/$
 $\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b]$

Rule 446

$\text{Int}[(x_)^{(m_.)}*((a_.) + (b_.)*(x_)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_)^{(n_.)})^{(q_.$
 $)}, x_Symbol] :> \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p$
 $*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x\} \&\& \text{NeQ}[$
 $b*c - a*d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^3}{x^7\sqrt{a + bx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x^3\sqrt{a + bx}} dx, x, x^3 \right) \\ &= -\frac{A\sqrt{a + bx^3}}{6ax^6} + \frac{\left(-\frac{3Ab}{2} + 2aB\right) \text{Subst} \left(\int \frac{1}{x^2\sqrt{a + bx}} dx, x, x^3 \right)}{6a} \\ &= -\frac{A\sqrt{a + bx^3}}{6ax^6} + \frac{(3Ab - 4aB)\sqrt{a + bx^3}}{12a^2x^3} + \frac{(b(3Ab - 4aB)) \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx}} dx, x, x^3 \right)}{24a^2} \\ &= -\frac{A\sqrt{a + bx^3}}{6ax^6} + \frac{(3Ab - 4aB)\sqrt{a + bx^3}}{12a^2x^3} + \frac{(3Ab - 4aB) \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + bx^3} \right)}{12a^2} \\ &= -\frac{A\sqrt{a + bx^3}}{6ax^6} + \frac{(3Ab - 4aB)\sqrt{a + bx^3}}{12a^2x^3} - \frac{b(3Ab - 4aB) \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right)}{12a^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.29, size = 81, normalized size = 0.90

$$\frac{\sqrt{a + bx^3} \left(b \left(2aB - \frac{3Ab}{2} \right) \left(\frac{\tanh^{-1} \left(\sqrt{\frac{bx^3}{a} + 1} \right)}{\sqrt{\frac{bx^3}{a} + 1}} - \frac{a}{bx^3} \right) - \frac{a^2A}{x^6} \right)}{6a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]), x]

[Out] (Sqrt[a + b*x^3]*(-(a^2*A)/x^6) + b*((-3*A*b)/2 + 2*a*B)*(-(a/(b*x^3)) + ArcTanh[Sqrt[1 + (b*x^3)/a]]/Sqrt[1 + (b*x^3)/a]))/(6*a^3)

IntegrateAlgebraic [A] time = 0.11, size = 80, normalized size = 0.89

$$\frac{(4abB - 3Ab^2) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}} + \frac{\sqrt{a+bx^3} (-2aA - 4aBx^3 + 3Abx^3)}{12a^2x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]), x]

[Out] (Sqrt[a + b*x^3]*(-2*a*A + 3*A*b*x^3 - 4*a*B*x^3))/(12*a^2*x^6) + ((-3*A*b^2 + 4*a*b*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(5/2))

fricas [A] time = 1.08, size = 173, normalized size = 1.92

$$\left[\frac{(4Bab - 3Ab^2)\sqrt{a}x^6 \log\left(\frac{bx^3 - 2\sqrt{bx^3+a}\sqrt{a+2a}}{x^3}\right) + 2((4Ba^2 - 3Aab)x^3 + 2Aa^2)\sqrt{bx^3+a}}{24a^3x^6}, \frac{(4Bab - 3Ab^2)\sqrt{-a}x^6 \arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) + ((4Ba^2 - 3Aab)x^3 + 2Aa^2)\sqrt{bx^3+a}}{12a^3x^6} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] [-1/24*((4*B*a*b - 3*A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^3*x^6), -1/12*((4*B*a*b - 3*A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + ((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^3*x^6)]

giac [A] time = 0.19, size = 121, normalized size = 1.34

$$\frac{(4Bab^2 - 3Ab^3) \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{4(bx^3+a)^{\frac{3}{2}}Bab^2 - 4\sqrt{bx^3+a}Ba^2b^2 - 3(bx^3+a)^{\frac{3}{2}}Ab^3 + 5\sqrt{bx^3+a}Aab^3}{a^2b^2x^6}$$

12 b

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2), x, algorithm="giac")

[Out] -1/12*((4*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 - 3*(b*x^3 + a)^(3/2)*A*b^3 + 5*sqrt(b*x^3 + a)*A*a*b^3)/(a^2*b^2*x^6))/b

maple [A] time = 0.08, size = 102, normalized size = 1.13

$$\left(\frac{b^2 \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}} + \frac{\sqrt{bx^3+a} b}{4a^2 x^3} - \frac{\sqrt{bx^3+a}}{6a x^6} \right) A + \left(\frac{b \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}} - \frac{\sqrt{bx^3+a}}{3a x^3} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x)`

[Out] $B*(1/3*b*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})/a^{(3/2)}-1/3*(b*x^3+a)^{(1/2)}/a/x^3)+A*(-1/4*b^2*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})/a^{(5/2)}-1/6*(b*x^3+a)^{(1/2)}/a/x^6+1/4*(b*x^3+a)^{(1/2)}/a^2*b/x^3)$

maxima [B] time = 1.20, size = 178, normalized size = 1.98

$$-\frac{1}{6} B \left(\frac{2\sqrt{bx^3+a} b}{(bx^3+a)a-a^2} + \frac{b \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} \right) + \frac{1}{24} A \left(\frac{3b^2 \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2\left(3(bx^3+a)^{\frac{3}{2}}b^2-5\sqrt{bx^3+a}ab^2\right)}{(bx^3+a)^2a^2-2(bx^3+a)a^3+a^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] $-1/6*B*(2*\operatorname{sqrt}(b*x^3+a)*b/((b*x^3+a)*a-a^2)+b*\log((\operatorname{sqrt}(b*x^3+a)-\operatorname{sqrt}(a))/(\operatorname{sqrt}(b*x^3+a)+\operatorname{sqrt}(a))))/a^{(3/2)}+1/24*A*(3*b^2*\log((\operatorname{sqrt}(b*x^3+a)-\operatorname{sqrt}(a))/(\operatorname{sqrt}(b*x^3+a)+\operatorname{sqrt}(a))))/a^{(5/2)}+2*(3*(b*x^3+a)^{(3/2)}*b^2-5*\operatorname{sqrt}(b*x^3+a)*a*b^2)/((b*x^3+a)^2*a^2-2*(b*x^3+a)*a^3+a^4))$

mupad [B] time = 2.99, size = 95, normalized size = 1.06

$$\frac{\sqrt{bx^3+a} (3Ab-4Ba)}{12a^2x^3} - \frac{A\sqrt{bx^3+a}}{6ax^6} + \frac{b \ln\left(\frac{(\sqrt{bx^3+a}-\sqrt{a})^3(\sqrt{bx^3+a}+\sqrt{a})}{x^6}\right) (3Ab-4Ba)}{24a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*x^3)/(x^7*(a+b*x^3)^(1/2)),x)`

[Out] $((a+b*x^3)^{(1/2)}*(3*A*b-4*B*a))/(12*a^2*x^3)-(A*(a+b*x^3)^{(1/2)})/(6*a*x^6)+(b*\log((((a+b*x^3)^{(1/2)}-a^{(1/2)})^3*((a+b*x^3)^{(1/2)}+a^{(1/2)})))/x^6)*(3*A*b-4*B*a)/(24*a^{(5/2)})$

sympy [B] time = 71.18, size = 163, normalized size = 1.81

$$-\frac{A}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{A\sqrt{b}}{12ax^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{Ab^{\frac{3}{2}}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{Ab^2 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4a^{\frac{5}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}} + \frac{Bb \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**(1/2), x)

[Out] $-A/(6*\sqrt{b}*x^{(15/2)}*\sqrt{a/(b*x^{(3)}+1)}) + A*\sqrt{b}/(12*a*x^{(9/2)}*\sqrt{a/(b*x^{(3)}+1)}) + A*b^{(3/2)}/(4*a^{(5/2)}*x^{(3/2)}*\sqrt{a/(b*x^{(3)}+1)}) - A*b^{(3/2)}*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x^{(3/2)}))/(4*a^{(5/2)}) - B*\sqrt{b}*x^{(3/2)}*\sqrt{a/(b*x^{(3)}+1)}/(3*a*x^{(3/2)}) + B*b*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x^{(3/2)}))/(3*a^{(3/2)})$

$$3.193 \quad \int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=103

$$-\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}} + \frac{2(a+bx^3)^{3/2}(Ab-3aB)}{9b^4} - \frac{2a\sqrt{a+bx^3}(2Ab-3aB)}{3b^4} + \frac{2B(a+bx^3)^{5/2}}{15b^4}$$

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$-\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}} + \frac{2(a+bx^3)^{3/2}(Ab-3aB)}{9b^4} - \frac{2a\sqrt{a+bx^3}(2Ab-3aB)}{3b^4} + \frac{2B(a+bx^3)^{5/2}}{15b^4}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2), x]

[Out] (-2*a^2*(A*b - a*B))/(3*b^4*Sqrt[a + b*x^3]) - (2*a*(2*A*b - 3*a*B)*Sqrt[a + b*x^3])/(3*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(3/2))/(9*b^4) + (2*B*(a + b*x^3)^(5/2))/(15*b^4)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8 (A + Bx^3)}{(a + bx^3)^{3/2}} dx = \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (A + Bx)}{(a + bx)^{3/2}} dx, x, x^3 \right)$$

$$= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{a^2(-Ab + aB)}{b^3(a + bx)^{3/2}} + \frac{a(-2Ab + 3aB)}{b^3 \sqrt{a + bx}} + \frac{(Ab - 3aB)\sqrt{a + bx}}{b^3} + \frac{B(a + bx)^{3/2}}{b^3} \right) dx, x, x^3 \right)$$

$$= -\frac{2a^2(Ab - aB)}{3b^4 \sqrt{a + bx^3}} - \frac{2a(2Ab - 3aB)\sqrt{a + bx^3}}{3b^4} + \frac{2(Ab - 3aB)(a + bx^3)^{3/2}}{9b^4} + \frac{2B(a + bx^3)^{5/2}}{15b^4}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.75

$$\frac{2(48a^3B - 8a^2b(5A - 3Bx^3) - 2ab^2x^3(10A + 3Bx^3) + b^3x^6(5A + 3Bx^3))}{45b^4\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2), x]

[Out] (2*(48*a^3*B - 8*a^2*b*(5*A - 3*B*x^3) + b^3*x^6*(5*A + 3*B*x^3) - 2*a*b^2*x^3*(10*A + 3*B*x^3)))/(45*b^4*Sqrt[a + b*x^3])

IntegrateAlgebraic [A] time = 0.06, size = 80, normalized size = 0.78

$$\frac{2(48a^3B - 40a^2Ab + 24a^2bBx^3 - 20aAb^2x^3 - 6ab^2Bx^6 + 5Ab^3x^6 + 3b^3Bx^9)}{45b^4\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2), x]

[Out] (2*(-40*a^2*A*b + 48*a^3*B - 20*a*A*b^2*x^3 + 24*a^2*b*B*x^3 + 5*A*b^3*x^6 - 6*a*b^2*B*x^6 + 3*b^3*B*x^9))/(45*b^4*Sqrt[a + b*x^3])

fricas [A] time = 1.04, size = 88, normalized size = 0.85

$$\frac{2(3Bb^3x^9 - (6Bab^2 - 5Ab^3)x^6 + 48Ba^3 - 40Aa^2b + 4(6Ba^2b - 5Aab^2)x^3)\sqrt{bx^3 + a}}{45(b^5x^3 + ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2), x, algorithm="fricas")

[Out] $\frac{2}{45} \cdot (3Bb^3x^9 - (6Ba^2b - 5A^2b^3)x^6 + 48Ba^3 - 40A^2b) + 4 \cdot (6Ba^2b - 5A^2b^2)x^3 \cdot \sqrt{bx^3 + a} / (b^5x^3 + ab^4)$

giac [A] time = 0.17, size = 114, normalized size = 1.11

$$\frac{2(Ba^3 - Aa^2b)}{3\sqrt{bx^3 + ab^4}} + \frac{2\left(3(bx^3 + a)^{\frac{5}{2}}Bb^{16} - 15(bx^3 + a)^{\frac{3}{2}}Bab^{16} + 45\sqrt{bx^3 + a}Ba^2b^{16} + 5(bx^3 + a)^{\frac{3}{2}}Ab^{17} - 30\sqrt{bx^3 + a}Aab^{17}\right)}{45b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")`

[Out] $\frac{2}{3} \cdot (Ba^3 - A^2b) / (\sqrt{bx^3 + a} \cdot b^4) + \frac{2}{45} \cdot (3(bx^3 + a)^{5/2} \cdot Bb^{16} - 15(bx^3 + a)^{3/2} \cdot B^2ab^{16} + 45\sqrt{bx^3 + a} \cdot B^2a^2b^{16} + 5(bx^3 + a)^{3/2} \cdot A^2b^{17} - 30\sqrt{bx^3 + a} \cdot A^2ab^{17}) / b^{20}$

maple [A] time = 0.04, size = 77, normalized size = 0.75

$$\frac{2(-3Bx^9b^3 - 5Ab^3x^6 + 6Bab^2x^6 + 20Aab^2x^3 - 24Ba^2bx^3 + 40Aa^2b - 48Ba^3)}{45\sqrt{bx^3 + ab^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x)`

[Out] $\frac{-2}{45} \cdot (bx^3 + a)^{1/2} \cdot (-3Bb^3x^9 - 5A^2b^3x^6 + 6B^2ab^2x^6 + 20A^2ab^2x^3 - 24B^2a^2bx^3 + 40A^2a^2b - 48B^2a^3) / b^4$

maxima [A] time = 0.59, size = 116, normalized size = 1.13

$$\frac{2}{15} B \left(\frac{(bx^3 + a)^{\frac{5}{2}}}{b^4} - \frac{5(bx^3 + a)^{\frac{3}{2}}a}{b^4} + \frac{15\sqrt{bx^3 + a}a^2}{b^4} + \frac{5a^3}{\sqrt{bx^3 + ab^4}} \right) + \frac{2}{9} A \left(\frac{(bx^3 + a)^{\frac{3}{2}}}{b^3} - \frac{6\sqrt{bx^3 + a}a}{b^3} - \frac{3a^2}{\sqrt{bx^3 + ab^3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")`

[Out] $\frac{2}{15} \cdot B \cdot ((bx^3 + a)^{5/2} / b^4 - 5 \cdot (bx^3 + a)^{3/2} \cdot a / b^4 + 15 \cdot \sqrt{bx^3 + a} \cdot a^2 / b^4 + 5 \cdot a^3 / (\sqrt{bx^3 + a} \cdot b^4)) + \frac{2}{9} \cdot A \cdot ((bx^3 + a)^{3/2} / b^3 - 6 \cdot \sqrt{bx^3 + a} \cdot a / b^3 - 3 \cdot a^2 / (\sqrt{bx^3 + a} \cdot b^3))$

mupad [B] time = 2.77, size = 152, normalized size = 1.48

$$\frac{\sqrt{bx^3 + a} \left(\frac{2(Ba^2 - Aab)}{b^3} - \frac{2a \left(\frac{2(Ab^2 - Bab)}{b^3} - \frac{8Ba}{5b^2} \right)}{3b} \right)}{3b} + \frac{x^3 \sqrt{bx^3 + a} \left(\frac{2(Ab^2 - Bab)}{b^3} - \frac{8Ba}{5b^2} \right)}{9b} - \frac{a^2 \left(\frac{2A}{3b} - \frac{2Ba}{3b^2} \right)}{b^2 \sqrt{bx^3 + a}} + \frac{2Bx^6 \sqrt{bx^3 + a}}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^8*(A + B*x^3))/(a + b*x^3)^(3/2), x)`

[Out] $((a + b*x^3)^{(1/2)}*((2*(B*a^2 - A*a*b))/b^3 - (2*a*((2*(A*b^2 - B*a*b))/b^3 - (8*B*a)/(5*b^2)))/(3*b)))/(3*b) + (x^3*(a + b*x^3)^{(1/2)}*((2*(A*b^2 - B*a*b))/b^3 - (8*B*a)/(5*b^2)))/(9*b) - (a^2*((2*A)/(3*b) - (2*B*a)/(3*b^2)))/(b^2*(a + b*x^3)^{(1/2)}) + (2*B*x^6*(a + b*x^3)^{(1/2)})/(15*b^2)$

sympy [A] time = 3.90, size = 175, normalized size = 1.70

$$\begin{cases} \frac{16Aa^2}{9b^3\sqrt{a+bx^3}} - \frac{8Aax^3}{9b^2\sqrt{a+bx^3}} + \frac{2Ax^6}{9b\sqrt{a+bx^3}} + \frac{32Ba^3}{15b^4\sqrt{a+bx^3}} + \frac{16Ba^2x^3}{15b^3\sqrt{a+bx^3}} - \frac{4Bax^6}{15b^2\sqrt{a+bx^3}} + \frac{2Bx^9}{15b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{Ax^9}{9} + \frac{Bx^{12}}{12} & \text{otherwise} \\ a^{\frac{3}{2}} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(B*x**3+A)/(b*x**3+a)**(3/2), x)`

[Out] `Piecewise((-16*A*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*A*a*x**3/(9*b**2*sqrt(a + b*x**3)) + 2*A*x**6/(9*b*sqrt(a + b*x**3)) + 32*B*a**3/(15*b**4*sqrt(a + b*x**3)) + 16*B*a**2*x**3/(15*b**3*sqrt(a + b*x**3)) - 4*B*a*x**6/(15*b**2*sqrt(a + b*x**3)) + 2*B*x**9/(15*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**9/9 + B*x**12/12)/a**(3/2), True))`

$$3.194 \quad \int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=73

$$\frac{2\sqrt{a+bx^3}(Ab-2aB)}{3b^3} + \frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}} + \frac{2B(a+bx^3)^{3/2}}{9b^3}$$

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{2\sqrt{a+bx^3}(Ab-2aB)}{3b^3} + \frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}} + \frac{2B(a+bx^3)^{3/2}}{9b^3}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]

[Out] (2*a*(A*b - a*B))/(3*b^3*Sqrt[a + b*x^3]) + (2*(A*b - 2*a*B)*Sqrt[a + b*x^3])/ (3*b^3) + (2*B*(a + b*x^3)^(3/2))/(9*b^3)

Rule 77

```
Int[((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (A + Bx^3)}{(a + bx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(A + Bx)}{(a + bx)^{3/2}} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{a(-Ab + aB)}{b^2(a + bx)^{3/2}} + \frac{Ab - 2aB}{b^2 \sqrt{a + bx}} + \frac{B\sqrt{a + bx}}{b^2} \right) dx, x, x^3 \right) \\
&= \frac{2a(Ab - aB)}{3b^3 \sqrt{a + bx^3}} + \frac{2(Ab - 2aB)\sqrt{a + bx^3}}{3b^3} + \frac{2B(a + bx^3)^{3/2}}{9b^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.75

$$\frac{2(-8a^2B + a(6Ab - 4bBx^3) + b^2x^3(3A + Bx^3))}{9b^3\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2), x]

[Out] (2*(-8*a^2*B + b^2*x^3*(3*A + B*x^3) + a*(6*A*b - 4*b*B*x^3)))/(9*b^3*Sqrt[a + b*x^3])

IntegrateAlgebraic [A] time = 0.05, size = 56, normalized size = 0.77

$$\frac{2(8a^2B - 6aAb + 4abBx^3 - 3Ab^2x^3 - b^2Bx^6)}{9b^3\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2), x]

[Out] (-2*(-6*a*A*b + 8*a^2*B - 3*A*b^2*x^3 + 4*a*b*B*x^3 - b^2*B*x^6))/(9*b^3*Sqrt[a + b*x^3])

fricas [A] time = 0.98, size = 63, normalized size = 0.86

$$\frac{2(Bb^2x^6 - (4Bab - 3Ab^2)x^3 - 8Ba^2 + 6Aab)\sqrt{bx^3 + a}}{9(b^4x^3 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2), x, algorithm="fricas")

[Out] $\frac{2}{9} * (B * b^2 * x^6 - (4 * B * a * b - 3 * A * b^2) * x^3 - 8 * B * a^2 + 6 * A * a * b) * \sqrt{b * x^3 + a} / (b^4 * x^3 + a * b^3)$

giac [A] time = 0.18, size = 77, normalized size = 1.05

$$-\frac{2(Ba^2 - Aab)}{3\sqrt{bx^3 + ab^3}} + \frac{2\left((bx^3 + a)^{\frac{3}{2}}Bb^6 - 6\sqrt{bx^3 + a}Bab^6 + 3\sqrt{bx^3 + a}Ab^7\right)}{9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")`

[Out] $-\frac{2}{3} * (B * a^2 - A * a * b) / (\sqrt{b * x^3 + a} * b^3) + \frac{2}{9} * ((b * x^3 + a)^{\frac{3}{2}} * B * b^6 - 6 * \sqrt{b * x^3 + a} * B * a * b^6 + 3 * \sqrt{b * x^3 + a} * A * b^7) / b^9$

maple [A] time = 0.05, size = 52, normalized size = 0.71

$$\frac{\frac{2}{9} B b^2 x^6 + \frac{2}{3} A b^2 x^3 - \frac{8}{9} B a b x^3 + \frac{4}{3} A a b - \frac{16}{9} B a^2}{\sqrt{b x^3 + a} b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x)`

[Out] $\frac{2}{9} / (b * x^3 + a)^{\frac{1}{2}} * (B * b^2 * x^6 + 3 * A * b^2 * x^3 - 4 * B * a * b * x^3 + 6 * A * a * b - 8 * B * a^2) / b^3$

maxima [A] time = 0.65, size = 81, normalized size = 1.11

$$\frac{2}{9} B \left(\frac{(bx^3 + a)^{\frac{3}{2}}}{b^3} - \frac{6\sqrt{bx^3 + a}a}{b^3} - \frac{3a^2}{\sqrt{bx^3 + a}b^3} \right) + \frac{2}{3} A \left(\frac{\sqrt{bx^3 + a}}{b^2} + \frac{a}{\sqrt{bx^3 + a}b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")`

[Out] $\frac{2}{9} * B * ((b * x^3 + a)^{\frac{3}{2}} / b^3 - 6 * \sqrt{b * x^3 + a} * a / b^3 - 3 * a^2 / (\sqrt{b * x^3 + a} * b^3)) + \frac{2}{3} * A * (\sqrt{b * x^3 + a} / b^2 + a / (\sqrt{b * x^3 + a} * b^2))$

mupad [B] time = 2.68, size = 60, normalized size = 0.82

$$\frac{2B(bx^3 + a)^2 - 6Ba^2 + 6Ab(bx^3 + a) - 12Ba(bx^3 + a) + 6Aab}{9b^3\sqrt{bx^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(A + B*x^3))/(a + b*x^3)^(3/2), x)`

[Out] $(2*B*(a + b*x^3)^2 - 6*B*a^2 + 6*A*b*(a + b*x^3) - 12*B*a*(a + b*x^3) + 6*A*a*b)/(9*b^3*(a + b*x^3)^(1/2))$

sympy [A] time = 1.84, size = 124, normalized size = 1.70

$$\begin{cases} \frac{4Aa}{3b^2\sqrt{a+bx^3}} + \frac{2Ax^3}{3b\sqrt{a+bx^3}} - \frac{16Ba^2}{9b^3\sqrt{a+bx^3}} - \frac{8Bax^3}{9b^2\sqrt{a+bx^3}} + \frac{2Bx^6}{9b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^6}{6} + \frac{Bx^9}{9}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(B*x**3+A)/(b*x**3+a)**(3/2), x)`

[Out] `Piecewise((4*A*a/(3*b**2*sqrt(a + b*x**3)) + 2*A*x**3/(3*b*sqrt(a + b*x**3)) - 16*B*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*B*a*x**3/(9*b**2*sqrt(a + b*x**3)) + 2*B*x**6/(9*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/a**(3/2), True))`

$$3.195 \quad \int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=46

$$\frac{2B\sqrt{a+bx^3}}{3b^2} - \frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {444, 43}

$$\frac{2B\sqrt{a+bx^3}}{3b^2} - \frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]

[Out] (-2*(A*b - a*B))/(3*b^2*Sqrt[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/(3*b^2)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A+Bx}{(a+bx)^{3/2}} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{Ab-aB}{b(a+bx)^{3/2}} + \frac{B}{b\sqrt{a+bx}} \right) dx, x, x^3 \right) \\
&= -\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}} + \frac{2B\sqrt{a+bx^3}}{3b^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.72

$$\frac{2(2aB - Ab + bBx^3)}{3b^2\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2), x]

[Out] (2*(-(A*b) + 2*a*B + b*B*x^3))/(3*b^2*Sqrt[a + b*x^3])

IntegrateAlgebraic [A] time = 0.04, size = 33, normalized size = 0.72

$$\frac{2(2aB - Ab + bBx^3)}{3b^2\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2), x]

[Out] (2*(-(A*b) + 2*a*B + b*B*x^3))/(3*b^2*Sqrt[a + b*x^3])

fricas [A] time = 0.55, size = 41, normalized size = 0.89

$$\frac{2(Bbx^3 + 2Ba - Ab)\sqrt{bx^3 + a}}{3(b^3x^3 + ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2), x, algorithm="fricas")

[Out] 2/3*(B*b*x^3 + 2*B*a - A*b)*sqrt(b*x^3 + a)/(b^3*x^3 + a*b^2)

giac [A] time = 0.18, size = 38, normalized size = 0.83

$$\frac{2\sqrt{bx^3+a}B}{3b^2} + \frac{2(Ba-Ab)}{3\sqrt{bx^3+a}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] 2/3*sqrt(b*x^3 + a)*B/b^2 + 2/3*(B*a - A*b)/(sqrt(b*x^3 + a)*b^2)

maple [A] time = 0.05, size = 30, normalized size = 0.65

$$-\frac{2(-Bbx^3 + Ab - 2Ba)}{3\sqrt{bx^3 + a}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x)

[Out] -2/3/(b*x^3+a)^(1/2)*(-B*b*x^3+A*b-2*B*a)/b^2

maxima [A] time = 0.56, size = 47, normalized size = 1.02

$$\frac{2}{3}B\left(\frac{\sqrt{bx^3+a}}{b^2} + \frac{a}{\sqrt{bx^3+a}b^2}\right) - \frac{2A}{3\sqrt{bx^3+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] 2/3*B*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2)) - 2/3*A/(sqrt(b*x^3 + a)*b)

mupad [B] time = 2.61, size = 33, normalized size = 0.72

$$\frac{2Ba - 2Ab + 2B(bx^3 + a)}{3b^2\sqrt{bx^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x)

[Out] (2*B*a - 2*A*b + 2*B*(a + b*x^3))/(3*b^2*(a + b*x^3)^(1/2))

sympy [A] time = 1.08, size = 75, normalized size = 1.63

$$\begin{cases} -\frac{2A}{3b\sqrt{a+bx^3}} + \frac{4Ba}{3b^2\sqrt{a+bx^3}} + \frac{2Bx^3}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3} + \frac{Bx^6}{6}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(3/2), x)

[Out] Piecewise((-2*A/(3*b*sqrt(a + b*x**3)) + 4*B*a/(3*b**2*sqrt(a + b*x**3)) + 2*B*x**3/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/a**(3/2), True))

$$3.196 \quad \int \frac{A+Bx^3}{x(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=58

$$\frac{2(Ab - aB)}{3ab\sqrt{a + bx^3}} - \frac{2A \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {446, 78, 63, 208}

$$\frac{2(Ab - aB)}{3ab\sqrt{a + bx^3}} - \frac{2A \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]

[Out] (2*(A*b - a*B))/(3*a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx^3}{x(a + bx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x(a + bx)^{3/2}} dx, x, x^3 \right) \\
 &= \frac{2(Ab - aB)}{3ab\sqrt{a + bx^3}} + \frac{A \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx}} dx, x, x^3 \right)}{3a} \\
 &= \frac{2(Ab - aB)}{3ab\sqrt{a + bx^3}} + \frac{(2A) \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + bx^3} \right)}{3ab} \\
 &= \frac{2(Ab - aB)}{3ab\sqrt{a + bx^3}} - \frac{2A \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{3a^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 58, normalized size = 1.00

$$\frac{1}{3} \left(\frac{2(Ab - aB)}{ab\sqrt{a + bx^3}} - \frac{2A \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{a^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^(3/2)), x]

[Out] ((2*(A*b - a*B))/(a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]))/a^(3/2))/3

IntegrateAlgebraic [A] time = 0.07, size = 58, normalized size = 1.00

$$\frac{2A \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{3a^{3/2}} - \frac{2(aB - Ab)}{3ab\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]

[Out] (-2*(-(A*b) + a*B))/(3*a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2))

fricas [A] time = 1.07, size = 170, normalized size = 2.93

$$\left[\frac{(Ab^2x^3 + Aab)\sqrt{a} \log\left(\frac{bx^3 - 2\sqrt{bx^3+a}\sqrt{a} + 2a}{x^3}\right) - 2\sqrt{bx^3+a}(Ba^2 - Aab)}{3(a^2b^2x^3 + a^3b)}, \frac{2\left((Ab^2x^3 + Aab)\sqrt{-a} \arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) - \sqrt{bx^3+a}(Ba^2 - Aab)\right)}{3(a^2b^2x^3 + a^3b)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] [1/3*((A*b^2*x^3 + A*a*b)*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) - 2*sqrt(b*x^3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b), 2/3*((A*b^2*x^3 + A*a*b)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) - sqrt(b*x^3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b)]

giac [A] time = 0.16, size = 53, normalized size = 0.91

$$\frac{2A \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}a} - \frac{2(Ba - Ab)}{3\sqrt{bx^3+a}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - 2/3*(B*a - A*b)/(sqrt(b*x^3 + a)*a*b)

maple [A] time = 0.05, size = 57, normalized size = 0.98

$$\left(-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}} + \frac{2}{3\sqrt{\left(x^3 + \frac{a}{b}\right)ba}} \right) A - \frac{2B}{3\sqrt{bx^3+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x/(b*x^3+a)^(3/2),x)

[Out] -2/3*B/b/(b*x^3+a)^(1/2)+A*(2/3/a/((x^3+a/b)*b)^(1/2)-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2)))/a^(3/2))

maxima [A] time = 1.35, size = 70, normalized size = 1.21

$$\frac{1}{3} A \left(\frac{\log \left(\frac{\sqrt{bx^3+a} - \sqrt{a}}{\sqrt{bx^3+a} + \sqrt{a}} \right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{bx^3+a} a} \right) - \frac{2B}{3 \sqrt{bx^3+a} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] 1/3*A*(log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2) + 2/(sqrt(b*x^3 + a)*a)) - 2/3*B/(sqrt(b*x^3 + a)*b)

mupad [B] time = 2.77, size = 65, normalized size = 1.12

$$\frac{\frac{2A}{3a} - \frac{2B}{3b}}{\sqrt{bx^3+a}} + \frac{A \ln \left(\frac{(\sqrt{bx^3+a} - \sqrt{a})^3 (\sqrt{bx^3+a} + \sqrt{a})}{x^6} \right)}{3a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x*(a + b*x^3)^(3/2)),x)

[Out] ((2*A)/(3*a) - (2*B)/(3*b))/(a + b*x^3)^(1/2) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(3/2))

sympy [A] time = 20.82, size = 56, normalized size = 0.97

$$\frac{2A \operatorname{atan} \left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}} \right)}{3a\sqrt{-a}} - \frac{2(-Ab + Ba)}{3ab\sqrt{a + bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x/(b*x**3+a)**(3/2),x)

[Out] 2*A*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*a*sqrt(-a)) - 2*(-A*b + B*a)/(3*a*b*sqrt(a + b*x**3))

$$3.197 \quad \int \frac{A+Bx^3}{x^4(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=86

$$\frac{(3Ab - 2aB) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}} + \frac{2aB - 3Ab}{3a^2\sqrt{a+bx^3}} - \frac{A}{3ax^3\sqrt{a+bx^3}}$$

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 78, 51, 63, 208}

$$-\frac{3Ab - 2aB}{3a^2\sqrt{a+bx^3}} + \frac{(3Ab - 2aB) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}} - \frac{A}{3ax^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]

[Out] -(3*A*b - 2*a*B)/(3*a^2*Sqrt[a + b*x^3]) - A/(3*a*x^3*Sqrt[a + b*x^3]) + ((3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]/(3*a^(5/2)))

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx^3}{x^4(a + bx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x^2(a + bx)^{3/2}} dx, x, x^3 \right) \\
 &= -\frac{A}{3ax^3\sqrt{a + bx^3}} + \frac{\left(-\frac{3Ab}{2} + aB\right) \text{Subst} \left(\int \frac{1}{x(a + bx)^{3/2}} dx, x, x^3 \right)}{3a} \\
 &= -\frac{3Ab - 2aB}{3a^2\sqrt{a + bx^3}} - \frac{A}{3ax^3\sqrt{a + bx^3}} - \frac{(3Ab - 2aB) \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx}} dx, x, x^3 \right)}{6a^2} \\
 &= -\frac{3Ab - 2aB}{3a^2\sqrt{a + bx^3}} - \frac{A}{3ax^3\sqrt{a + bx^3}} - \frac{(3Ab - 2aB) \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + bx^3} \right)}{3a^2b} \\
 &= -\frac{3Ab - 2aB}{3a^2\sqrt{a + bx^3}} - \frac{A}{3ax^3\sqrt{a + bx^3}} + \frac{(3Ab - 2aB) \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right)}{3a^{5/2}}
 \end{aligned}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.66

$$\frac{x^3(2aB - 3Ab) {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^3}{a} + 1 \right) - aA}{3a^2x^3\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)), x]

[Out] $(-(aA) + (-3A*b + 2A*B)*x^3 \text{Hypergeometric2F1}[-1/2, 1, 1/2, 1 + (b*x^3)/a]) / (3*a^2*x^3*\text{Sqrt}[a + b*x^3])$

IntegrateAlgebraic [A] time = 0.13, size = 77, normalized size = 0.90

$$\frac{(3Ab - 2aB) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}} + \frac{-aA + 2aBx^3 - 3Abx^3}{3a^2x^3\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)), x]

[Out] $(-(aA) - 3A*b*x^3 + 2A*B*x^3) / (3*a^2*x^3*\text{Sqrt}[a + b*x^3]) + ((3A*b - 2A*B)*\text{ArcTanh}[\text{Sqrt}[a + b*x^3]/\text{Sqrt}[a]]) / (3*a^{(5/2)})$

fricas [A] time = 0.96, size = 233, normalized size = 2.71

$$\left[\frac{\left((2Bab - 3Ab^2)x^6 + (2Ba^2 - 3Aab)x^3 \right) \sqrt{a} \log\left(\frac{bx^3 + 2\sqrt{bx^3+a}\sqrt{a} + 2a}{x^3}\right) - 2\left((2Ba^2 - 3Aab)x^3 - Aa^2 \right) \sqrt{bx^3+a}}{6(a^3bx^6 + a^4x^3)}, \frac{\left((2Bab - 3Ab^2)x^6 + (2Ba^2 - 3Aab)x^3 \right) \sqrt{-a} \arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) + \left((2Ba^2 - 3Aab)x^3 - Aa^2 \right) \sqrt{bx^3+a}}{3(a^3bx^6 + a^4x^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2), x, algorithm="fricas")

[Out] $[-1/6 * (((2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3) * \text{sqrt}(a) * \log((b*x^3 + 2*\text{sqrt}(b*x^3 + a)*\text{sqrt}(a) + 2*a)/x^3) - 2 * ((2*B*a^2 - 3*A*a*b)*x^3 - A*a^2) * \text{sqrt}(b*x^3 + a)) / (a^3*b*x^6 + a^4*x^3), 1/3 * (((2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3) * \text{sqrt}(-a) * \arctan(\text{sqrt}(b*x^3 + a)*\text{sqrt}(-a)/a) + ((2*B*a^2 - 3*A*a*b)*x^3 - A*a^2) * \text{sqrt}(b*x^3 + a)) / (a^3*b*x^6 + a^4*x^3)]$

giac [A] time = 0.17, size = 99, normalized size = 1.15

$$\frac{(2Ba - 3Ab) \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}a^2} + \frac{2(bx^3 + a)Ba - 2Ba^2 - 3(bx^3 + a)Ab + 2Aab}{3\left((bx^3 + a)^{\frac{3}{2}} - \sqrt{bx^3 + a}a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2), x, algorithm="giac")

[Out] $\frac{1}{3}*(2*B*a - 3*A*b)*\arctan(\sqrt{b*x^3 + a}/\sqrt{-a})/(\sqrt{-a}*a^2) + \frac{1}{3}*(2*(b*x^3 + a)*B*a - 2*B*a^2 - 3*(b*x^3 + a)*A*b + 2*A*a*b)/(((b*x^3 + a)^{(3/2)} - \sqrt{b*x^3 + a})*a^2)$

maple [A] time = 0.06, size = 100, normalized size = 1.16

$$\left(\frac{b \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} - \frac{2b}{3\sqrt{\left(x^3 + \frac{a}{b}\right)ba^2}} - \frac{\sqrt{bx^3+a}}{3a^2x^3} \right) A + \left(-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}} + \frac{2}{3\sqrt{\left(x^3 + \frac{a}{b}\right)ba}} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((B*x^3+A)/x^4/(b*x^3+a)^{(3/2)}, x)$

[Out] $A*(-1/3*(b*x^3+a)^{(1/2)}/a^2/x^3-2/3/((x^3+a/b)*b)^{(1/2)}/a^2*b+b*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})/a^{(5/2)})+B*(2/3/((x^3+a/b)*b)^{(1/2)}/a-2/3*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})/a^{(3/2)})$

maxima [B] time = 1.39, size = 144, normalized size = 1.67

$$-\frac{1}{6}A \left(\frac{2(3(bx^3+a)b-2ab)}{(bx^3+a)^{\frac{3}{2}}a^2-\sqrt{bx^3+a}a^3} + \frac{3b \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}} \right) + \frac{1}{3}B \left(\frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{bx^3+a}a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((B*x^3+A)/x^4/(b*x^3+a)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out] $-1/6*A*(2*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^{(3/2)}*a^2 - \sqrt{b*x^3 + a})*a^3) + 3*b*\log((\sqrt{b*x^3 + a} - \sqrt{a})/(\sqrt{b*x^3 + a} + \sqrt{a}))/a^{(5/2)} + 1/3*B*(\log((\sqrt{b*x^3 + a} - \sqrt{a})/(\sqrt{b*x^3 + a} + \sqrt{a}))/a^{(3/2)} + 2/(\sqrt{b*x^3 + a}*a))$

mupad [B] time = 2.93, size = 131, normalized size = 1.52

$$\frac{\ln\left(\frac{(\sqrt{bx^3+a}-\sqrt{a})(\sqrt{bx^3+a}+\sqrt{a})^3}{x^6}\right)}{6a^{5/2}} - \frac{(3Ab-2Ba)}{\frac{2Ba^2-3Aab}{2a^3} - \frac{a\left(\frac{Ab^2}{3a^3} + \frac{5b(2Ba^2-3Aab)}{6a^4}\right)}{\sqrt{bx^3+a}}} - \frac{A\sqrt{bx^3+a}}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((A + B*x^3)/(x^4*(a + b*x^3)^{(3/2)}), x)$

[Out] $(\log(\frac{((a + b*x^3)^{1/2} - a^{1/2}) * ((a + b*x^3)^{1/2} + a^{1/2})^3}{x^6}) * (3*A*b - 2*B*a)) / (6*a^{5/2}) - ((2*B*a^2 - 3*A*a*b) / (2*a^3) - (a * ((A*b^2) / (3*a^3) + (5*b * (2*B*a^2 - 3*A*a*b)) / (6*a^4))) / b) / (a + b*x^3)^{1/2} - (A * (a + b*x^3)^{1/2}) / (3*a^2*x^3)$

sympy [B] time = 79.79, size = 264, normalized size = 3.07

$$A \left(-\frac{1}{3a\sqrt{b}x^2\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}}{a^2x^2\sqrt{\frac{a}{bx^3}+1}} + \frac{b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{a^2} \right) + B \left(\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^2+3a^2bx^3} + \frac{a^3 \log\left(\frac{bx^3}{a}\right)}{3a^2+3a^2bx^3} - \frac{2a^3 \log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^2+3a^2bx^3} + \frac{a^2bx^3 \log\left(\frac{bx^3}{a}\right)}{3a^2+3a^2bx^3} - \frac{2a^2bx^3 \log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^2+3a^2bx^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(3/2), x)

[Out] $A * (-1 / (3*a*\sqrt{b}*x^{9/2}*\sqrt{a/(b*x^3) + 1})) - \sqrt{b} / (a^2*x^{3/2}*\sqrt{a/(b*x^3) + 1}) + b*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x^{3/2})) / a^{5/2} + B * (2*a^3*\sqrt{1 + b*x^3/a} / (3*a^{9/2} + 3*a^{7/2}*b*x^3) + a^3*\log(b*x^3/a) / (3*a^{9/2} + 3*a^{7/2}*b*x^3) - 2*a^3*\log(\sqrt{1 + b*x^3/a} + 1) / (3*a^{9/2} + 3*a^{7/2}*b*x^3) + a^2*b*x^3*\log(b*x^3/a) / (3*a^{9/2} + 3*a^{7/2}*b*x^3) - 2*a^2*b*x^3*\log(\sqrt{1 + b*x^3/a} + 1) / (3*a^{9/2} + 3*a^{7/2}*b*x^3))$

$$3.198 \quad \int \frac{A+Bx^3}{x^7(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=118

$$-\frac{b(5Ab - 4aB) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}} + \frac{b(5Ab - 4aB)}{4a^3\sqrt{a+bx^3}} + \frac{5Ab - 4aB}{12a^2x^3\sqrt{a+bx^3}} - \frac{A}{6ax^6\sqrt{a+bx^3}}$$

Rubi [A] time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 78, 51, 63, 208}

$$\frac{\sqrt{a+bx^3}(5Ab - 4aB)}{4a^3x^3} - \frac{5Ab - 4aB}{6a^2x^3\sqrt{a+bx^3}} - \frac{b(5Ab - 4aB) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}} - \frac{A}{6ax^6\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)), x]

[Out] -A/(6*a*x^6*Sqrt[a + b*x^3]) - (5*A*b - 4*a*B)/(6*a^2*x^3*Sqrt[a + b*x^3]) + ((5*A*b - 4*a*B)*Sqrt[a + b*x^3])/(4*a^3*x^3) - (b*(5*A*b - 4*a*B)*ArcTan[h[Sqrt[a + b*x^3]/Sqrt[a]]]/(4*a^(7/2)))

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx^3}{x^7(a + bx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x^3(a + bx)^{3/2}} dx, x, x^3 \right) \\
 &= -\frac{A}{6ax^6\sqrt{a + bx^3}} + \frac{\left(-\frac{5Ab}{2} + 2aB\right) \text{Subst} \left(\int \frac{1}{x^2(a+bx)^{3/2}} dx, x, x^3 \right)}{6a} \\
 &= -\frac{A}{6ax^6\sqrt{a + bx^3}} - \frac{5Ab - 4aB}{6a^2x^3\sqrt{a + bx^3}} - \frac{(5Ab - 4aB) \text{Subst} \left(\int \frac{1}{x^2\sqrt{a+bx}} dx, x, x^3 \right)}{4a^2} \\
 &= -\frac{A}{6ax^6\sqrt{a + bx^3}} - \frac{5Ab - 4aB}{6a^2x^3\sqrt{a + bx^3}} + \frac{(5Ab - 4aB)\sqrt{a + bx^3}}{4a^3x^3} + \frac{(b(5Ab - 4aB)) \text{Subst} \left(\int \frac{1}{x} dx, x, x^3 \right)}{8a^3} \\
 &= -\frac{A}{6ax^6\sqrt{a + bx^3}} - \frac{5Ab - 4aB}{6a^2x^3\sqrt{a + bx^3}} + \frac{(5Ab - 4aB)\sqrt{a + bx^3}}{4a^3x^3} + \frac{(5Ab - 4aB) \text{Subst} \left(\int \frac{1}{x} dx, x, x^3 \right)}{4a^3} \\
 &= -\frac{A}{6ax^6\sqrt{a + bx^3}} - \frac{5Ab - 4aB}{6a^2x^3\sqrt{a + bx^3}} + \frac{(5Ab - 4aB)\sqrt{a + bx^3}}{4a^3x^3} - \frac{b(5Ab - 4aB) \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{x} \right)}{4a^{7/2}}
 \end{aligned}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.51

$$\frac{bx^6(5Ab - 4aB) {}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; \frac{bx^3}{a} + 1\right) - a^2A}{6a^3x^6\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)), x]

[Out] $(-(a^2A) + b(5A*b - 4a*B)*x^6\text{Hypergeometric2F1}[-1/2, 2, 1/2, 1 + (b*x^3)/a]) / (6*a^3*x^6*\text{Sqrt}[a + b*x^3])$

IntegrateAlgebraic [A] time = 0.16, size = 102, normalized size = 0.86

$$\frac{(4abB - 5Ab^2) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}} + \frac{-2a^2A - 4a^2Bx^3 + 5aAbx^3 - 12abBx^6 + 15Ab^2x^6}{12a^3x^6\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)), x]

[Out] $(-2*a^2*A + 5*a*A*b*x^3 - 4*a^2*B*x^3 + 15*A*b^2*x^6 - 12*a*b*B*x^6) / (12*a^3*x^6*\text{Sqrt}[a + b*x^3]) + ((-5*A*b^2 + 4*a*b*B)*\text{ArcTanh}[\text{Sqrt}[a + b*x^3]/\text{Sqrt}[a]]) / (4*a^{(7/2)})$

fricas [A] time = 0.93, size = 289, normalized size = 2.45

$$\left[\frac{3((4Bab^2 - 5Ab^3)x^3 + (4Ba^2b - 5Aab^2)x^6) \sqrt{a} \log\left(\frac{bx^3 - 2\sqrt{bx^3+a} + \sqrt{a}}{24(a^3bx^3 + a^3x^6)}\right) + 2(3(4Ba^2b - 5Aab^2)x^6 + 2Aa^3 + (4Ba^3 - 5Aa^2b)x^3)\sqrt{bx^3+a} - 3((4Bab^2 - 5Ab^3)x^3 + (4Ba^2b - 5Aab^2)x^6)\sqrt{-a} \arctan\left(\frac{\sqrt{bx^3+a}}{a}\right) + (3(4Ba^2b - 5Aab^2)x^6 + 2Aa^3 + (4Ba^3 - 5Aa^2b)x^3)\sqrt{bx^3+a}}{12(a^3bx^3 + a^3x^6)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2), x, algorithm="fricas")

[Out] $[-1/24*(3*((4B*a*b^2 - 5A*b^3)*x^9 + (4B*a^2*b - 5A*a*b^2)*x^6)*\text{sqrt}(a) * \log((b*x^3 - 2*\text{sqrt}(b*x^3 + a))*\text{sqrt}(a) + 2*a)/x^3) + 2*(3*(4B*a^2*b - 5A*a*b^2)*x^6 + 2*A*a^3 + (4B*a^3 - 5A*a^2*b)*x^3)*\text{sqrt}(b*x^3 + a)) / (a^4*b*x^9 + a^5*x^6), -1/12*(3*((4B*a*b^2 - 5A*b^3)*x^9 + (4B*a^2*b - 5A*a*b^2)*x^6)*\text{sqrt}(-a)*\arctan(\text{sqrt}(b*x^3 + a)*\text{sqrt}(-a)/a) + (3*(4B*a^2*b - 5A*a*b^2)*x^6 + 2*A*a^3 + (4B*a^3 - 5A*a^2*b)*x^3)*\text{sqrt}(b*x^3 + a)) / (a^4*b*x^9 + a^5*x^6)]$

giac [A] time = 0.18, size = 137, normalized size = 1.16

$$\frac{(4Bab - 5Ab^2) \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{4\sqrt{-a}a^3} - \frac{2(Bab - Ab^2)}{3\sqrt{bx^3+a}a^3} - \frac{4(bx^3+a)^{\frac{3}{2}}Bab - 4\sqrt{bx^3+a}Ba^2b - 7(bx^3+a)^{\frac{3}{2}}Ab^2 + 9\sqrt{bx^3+a}Aab^2}{12a^3b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] $-1/4*(4*B*a*b - 5*A*b^2)*\arctan(\sqrt{b*x^3 + a}/\sqrt{-a})/(\sqrt{-a}*a^3) - 2/3*(B*a*b - A*b^2)/(\sqrt{b*x^3 + a}*a^3) - 1/12*(4*(b*x^3 + a)^(3/2)*B*a*b - 4*\sqrt{b*x^3 + a}*B*a^2*b - 7*(b*x^3 + a)^(3/2)*A*b^2 + 9*\sqrt{b*x^3 + a}*A*a*b^2)/(a^3*b^2*x^6)$

maple [A] time = 0.05, size = 141, normalized size = 1.19

$$\left(-\frac{5b^2 \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}} + \frac{2b^2}{3\sqrt{\left(x^3 + \frac{a}{b}\right)b a^3}} + \frac{7\sqrt{bx^3+a} b}{12a^3x^3} - \frac{\sqrt{bx^3+a}}{6a^2x^6} \right) A + \left(\frac{b \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} - \frac{2b}{3\sqrt{\left(x^3 + \frac{a}{b}\right)b a^2}} - \frac{\sqrt{bx^3+a}}{3a^2x^3} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x)

[Out] $B*(-1/3*(b*x^3+a)^(1/2)/a^2/x^3-2/3/((x^3+a/b)*b)^(1/2)/a^2*b+b*\operatorname{arctanh}((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2))+A*(-1/6*(b*x^3+a)^(1/2)/a^2/x^6+7/12*(b*x^3+a)^(1/2)/a^3*b/x^3+2/3/((x^3+a/b)*b)^(1/2)/a^3*b^2-5/4*b^2*\operatorname{arctanh}((b*x^3+a)^(1/2)/a^(1/2))/a^(7/2))$

maxima [B] time = 1.34, size = 215, normalized size = 1.82

$$\frac{1}{24} A \left(\frac{2(15(bx^3+a)^2b^2 - 25(bx^3+a)ab^2 + 8a^2b^2)}{(bx^3+a)^{\frac{5}{2}}a^3 - 2(bx^3+a)^{\frac{3}{2}}a^4 + \sqrt{bx^3+a}a^5} + \frac{15b^2 \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}} \right) - \frac{1}{6} B \left(\frac{2(3(bx^3+a)b - 2ab)}{(bx^3+a)^{\frac{3}{2}}a^2 - \sqrt{bx^3+a}a^3} + \frac{3b \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] $1/24*A*(2*(15*(b*x^3 + a)^2*b^2 - 25*(b*x^3 + a)*a*b^2 + 8*a^2*b^2)/((b*x^3 + a)^(5/2)*a^3 - 2*(b*x^3 + a)^(3/2)*a^4 + \sqrt{b*x^3 + a}*a^5) + 15*b^2*1 \operatorname{og}((\sqrt{b*x^3 + a} - \sqrt{a})/(\sqrt{b*x^3 + a} + \sqrt{a}))/a^(7/2)) - 1/6*B*(2*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - \sqrt{b*x^3 + a}*a^3) + 3*b*\log((\sqrt{b*x^3 + a} - \sqrt{a})/(\sqrt{b*x^3 + a} + \sqrt{a}))/a^(5/2))$

mapad [B] time = 3.18, size = 167, normalized size = 1.42

$$\frac{b \ln\left(\frac{(\sqrt{bx^3+a}-\sqrt{a})^3(\sqrt{bx^3+a}+\sqrt{a})}{x^6}\right) (5Ab-4Ba)}{8a^{7/2}} - \frac{(4Ba^2-7Aab)\sqrt{bx^3+a}}{12a^4x^3} - \frac{A\sqrt{bx^3+a}}{6a^2x^6} - \frac{a\left(\frac{7Ab^3-4Bab^2-5b^2(5Ab-4Ba)}{8a^4}\right)}{b\sqrt{bx^3+a}} + \frac{3b(5Ab-4Ba)}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x)`

[Out] $(b \cdot \log(((a + b \cdot x^3)^{1/2} - a^{1/2})^3 \cdot ((a + b \cdot x^3)^{1/2} + a^{1/2}))) / x^6$
 $\cdot (5 \cdot A \cdot b - 4 \cdot B \cdot a) / (8 \cdot a^{7/2}) - ((4 \cdot B \cdot a^2 - 7 \cdot A \cdot a \cdot b) \cdot (a + b \cdot x^3)^{1/2}) / (12$
 $\cdot a^4 \cdot x^3) - (A \cdot (a + b \cdot x^3)^{1/2}) / (6 \cdot a^2 \cdot x^6) - ((a \cdot ((7 \cdot A \cdot b^3 - 4 \cdot B \cdot a \cdot b^2) /$
 $(12 \cdot a^4) - (5 \cdot b^2 \cdot (5 \cdot A \cdot b - 4 \cdot B \cdot a)) / (8 \cdot a^4))) / b + (3 \cdot b \cdot (5 \cdot A \cdot b - 4 \cdot B \cdot a) / (8 \cdot a$
 $\cdot 3)) / (a + b \cdot x^3)^{1/2}$

sympy [A] time = 179.01, size = 192, normalized size = 1.63

$$A \left(-\frac{1}{6a\sqrt{b}x^{15}\sqrt{\frac{a}{bx^3}+1}} + \frac{5\sqrt{b}}{12a^2x^9\sqrt{\frac{a}{bx^3}+1}} + \frac{5b^{\frac{3}{2}}}{4a^3x^3\sqrt{\frac{a}{bx^3}+1}} - \frac{5b^2 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4a^{\frac{7}{2}}} \right) + B \left(-\frac{1}{3a\sqrt{b}x^9\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}}{a^2x^3\sqrt{\frac{a}{bx^3}+1}} + \frac{b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)/x**7/(b*x**3+a)**(3/2),x)`

[Out] $A \cdot (-1 / (6 \cdot a \cdot \sqrt{b} \cdot x^{15/2} \cdot \sqrt{a / (b \cdot x^3) + 1})) + 5 \cdot \sqrt{b} / (12 \cdot a^{9/2} \cdot x^{9/2} \cdot \sqrt{a / (b \cdot x^3) + 1})$
 $+ 5 \cdot b^{3/2} / (4 \cdot a^{7/2} \cdot x^{3/2} \cdot \sqrt{a / (b \cdot x^3) + 1}) - 5 \cdot b^{3/2} \cdot \operatorname{asinh}(\sqrt{a} / (\sqrt{b} \cdot x^{3/2})) / (4 \cdot a^{7/2}) + B \cdot (-1 / (3 \cdot a \cdot \sqrt{b} \cdot x^{9/2} \cdot \sqrt{a / (b \cdot x^3) + 1}))$
 $- \sqrt{b} / (a^{5/2} \cdot x^{3/2} \cdot \sqrt{a / (b \cdot x^3) + 1}) + b \cdot \operatorname{asinh}(\sqrt{a} / (\sqrt{b} \cdot x^{3/2})) / a^{5/2}$

$$3.199 \quad \int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}} dx$$

Optimal. Leaf size=103

$$-\frac{2a^2(Ab - aB)}{9b^4(a + bx^3)^{3/2}} + \frac{2a(2Ab - 3aB)}{3b^4\sqrt{a + bx^3}} + \frac{2\sqrt{a + bx^3}(Ab - 3aB)}{3b^4} + \frac{2B(a + bx^3)^{3/2}}{9b^4}$$

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$-\frac{2a^2(Ab - aB)}{9b^4(a + bx^3)^{3/2}} + \frac{2a(2Ab - 3aB)}{3b^4\sqrt{a + bx^3}} + \frac{2\sqrt{a + bx^3}(Ab - 3aB)}{3b^4} + \frac{2B(a + bx^3)^{3/2}}{9b^4}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2), x]

[Out] (-2*a^2*(A*b - a*B))/(9*b^4*(a + b*x^3)^(3/2)) + (2*a*(2*A*b - 3*a*B))/(3*b^4*Sqrt[a + b*x^3]) + (2*(A*b - 3*a*B)*Sqrt[a + b*x^3])/(3*b^4) + (2*B*(a + b*x^3)^(3/2))/(9*b^4)

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0]) || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\int \frac{x^8 (A + Bx^3)}{(a + bx^3)^{5/2}} dx = \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (A + Bx)}{(a + bx)^{5/2}} dx, x, x^3 \right)$$

$$= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{a^2(-Ab + aB)}{b^3(a + bx)^{5/2}} + \frac{a(-2Ab + 3aB)}{b^3(a + bx)^{3/2}} + \frac{Ab - 3aB}{b^3 \sqrt{a + bx}} + \frac{B\sqrt{a + bx}}{b^3} \right) dx, x, x^3 \right)$$

$$= -\frac{2a^2(Ab - aB)}{9b^4 (a + bx^3)^{3/2}} + \frac{2a(2Ab - 3aB)}{3b^4 \sqrt{a + bx^3}} + \frac{2(Ab - 3aB)\sqrt{a + bx^3}}{3b^4} + \frac{2B(a + bx^3)^{3/2}}{9b^4}$$

Mathematica [A] time = 0.07, size = 73, normalized size = 0.71

$$\frac{2(-16a^3B + 8a^2b(A - 3Bx^3) - 6ab^2x^3(Bx^3 - 2A) + b^3x^6(3A + Bx^3))}{9b^4(a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2), x]

[Out] (2*(-16*a^3*B + 8*a^2*b*(A - 3*B*x^3) - 6*a*b^2*x^3*(-2*A + B*x^3) + b^3*x^6*(3*A + B*x^3)))/(9*b^4*(a + b*x^3)^(3/2))

IntegrateAlgebraic [A] time = 0.06, size = 80, normalized size = 0.78

$$\frac{2(16a^3B - 8a^2Ab + 24a^2bBx^3 - 12aAb^2x^3 + 6ab^2Bx^6 - 3Ab^3x^6 - b^3Bx^9)}{9b^4(a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2), x]

[Out] (-2*(-8*a^2*A*b + 16*a^3*B - 12*a*A*b^2*x^3 + 24*a^2*b*B*x^3 - 3*A*b^3*x^6 + 6*a*b^2*B*x^6 - b^3*B*x^9))/(9*b^4*(a + b*x^3)^(3/2))

fricas [A] time = 0.84, size = 98, normalized size = 0.95

$$\frac{2(Bb^3x^9 - 3(2Bab^2 - Ab^3)x^6 - 16Ba^3 + 8Aa^2b - 12(2Ba^2b - Aab^2)x^3)\sqrt{bx^3 + a}}{9(b^6x^6 + 2ab^5x^3 + a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")

[Out] $\frac{2}{9} * (B * b^3 * x^9 - 3 * (2 * B * a * b^2 - A * b^3) * x^6 - 16 * B * a^3 + 8 * A * a^2 * b - 12 * (2 * B * a^2 * b - A * a * b^2) * x^3) * \text{sqrt}(b * x^3 + a) / (b^6 * x^6 + 2 * a * b^5 * x^3 + a^2 * b^4)$

giac [A] time = 0.17, size = 104, normalized size = 1.01

$$\frac{2(9(bx^3+a)Ba^2 - Ba^3 - 6(bx^3+a)Aab + Aa^2b)}{9(bx^3+a)^{\frac{3}{2}}b^4} + \frac{2\left((bx^3+a)^{\frac{3}{2}}Bb^8 - 9\sqrt{bx^3+a}Bab^8 + 3\sqrt{bx^3+a}Ab^9\right)}{9b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")

[Out] $\frac{-2}{9} * (9 * (b * x^3 + a) * B * a^2 - B * a^3 - 6 * (b * x^3 + a) * A * a * b + A * a^2 * b) / ((b * x^3 + a)^{(3/2)} * b^4) + \frac{2}{9} * ((b * x^3 + a)^{(3/2)} * B * b^8 - 9 * \text{sqrt}(b * x^3 + a) * B * a * b^8 + 3 * \text{sqrt}(b * x^3 + a) * A * b^9) / b^{12}$

maple [A] time = 0.05, size = 76, normalized size = 0.74

$$\frac{\frac{2}{9} B x^9 b^3 + \frac{2}{3} A b^3 x^6 - \frac{4}{3} B a b^2 x^6 + \frac{8}{3} A a b^2 x^3 - \frac{16}{3} B a^2 b x^3 + \frac{16}{9} A a^2 b - \frac{32}{9} B a^3}{(b x^3 + a)^{\frac{3}{2}} b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x)

[Out] $\frac{2}{9} * (B * (b * x^3 + a)^{(3/2)} * (B * b^3 * x^9 + 3 * A * b^3 * x^6 - 6 * B * a * b^2 * x^6 + 12 * A * a * b^2 * x^3 - 24 * B * a^2 * b * x^3 + 8 * A * a^2 * b - 16 * B * a^3) / b^4$

maxima [A] time = 0.59, size = 116, normalized size = 1.13

$$\frac{2}{9} B \left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^4} - \frac{9\sqrt{bx^3+a}a}{b^4} - \frac{9a^2}{\sqrt{bx^3+a}b^4} + \frac{a^3}{(bx^3+a)^{\frac{3}{2}}b^4} \right) + \frac{2}{9} A \left(\frac{3\sqrt{bx^3+a}}{b^3} + \frac{6a}{\sqrt{bx^3+a}b^3} - \frac{a^2}{(bx^3+a)^{\frac{3}{2}}b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")

[Out] $\frac{2}{9} * B * ((b * x^3 + a)^{(3/2)} / b^4 - 9 * \text{sqrt}(b * x^3 + a) * a / b^4 - 9 * a^2 / (\text{sqrt}(b * x^3 + a) * b^4) + a^3 / ((b * x^3 + a)^{(3/2)} * b^4)) + \frac{2}{9} * A * (3 * \text{sqrt}(b * x^3 + a) / b^3 + 6 * a / (\text{sqrt}(b * x^3 + a) * b^3) - a^2 / ((b * x^3 + a)^{(3/2)} * b^3))$

mupad [B] time = 2.80, size = 145, normalized size = 1.41

$$\frac{\sqrt{bx^3+a} \left(\frac{2(Ab-2Ba)}{b^3} - \frac{4Ba}{3b^3} \right)}{3b} - \frac{\frac{2Ba^2-2Aab}{3b^4} - \frac{a \left(\frac{2Ab^2-2Bab}{3b^4} - \frac{2Ba}{3b^3} \right)}{b}}{\sqrt{bx^3+a}} - \frac{a^2 \left(\frac{2A}{9b} - \frac{2Ba}{9b^2} \right)}{b^2 (bx^3+a)^{3/2}} + \frac{2Bx^3 \sqrt{bx^3+a}}{9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^8*(A + B*x^3))/(a + b*x^3)^(5/2), x)`

[Out] $((a + b*x^3)^{(1/2)} * ((2*(A*b - 2*B*a))/b^3 - (4*B*a)/(3*b^3))) / (3*b) - ((2*B*a^2 - 2*A*a*b) / (3*b^4) - (a * ((2*A*b^2 - 2*B*a*b) / (3*b^4) - (2*B*a) / (3*b^3))) / b) / (a + b*x^3)^{(1/2)} - (a^2 * ((2*A) / (9*b) - (2*B*a) / (9*b^2))) / (b^2 * (a + b*x^3)^{(3/2)}) + (2*B*x^3 * (a + b*x^3)^{(1/2)}) / (9*b^3)$

sympy [A] time = 5.18, size = 338, normalized size = 3.28

$$\begin{cases} \frac{16Aa^2b}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} + \frac{24Aab^2x^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} + \frac{6Ab^3x^6}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} - \frac{32Ba^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} - \frac{48Ba^2bx^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} - \frac{12Bab^2x^6}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} + \frac{2Bb^3x^9}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{Ax^9}{9} + \frac{Bx^{12}}{12} & \text{otherwise} \\ a^{\frac{5}{2}} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(B*x**3+A)/(b*x**3+a)**(5/2), x)`

[Out] `Piecewise((16*A*a**2*b/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*x**3)) + 24*A*a*b**2*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*x**3)) + 6*A*b**3*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*x**3)) - 32*B*a**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*x**3)) - 48*B*a**2*b*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*x**3)) - 12*B*a*b**2*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*x**3)) + 2*B*b**3*x**9/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**9/9 + B*x**12/12)/a**(5/2), True))`

$$3.200 \quad \int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}} dx$$

Optimal. Leaf size=73

$$-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}} + \frac{2a(Ab-aB)}{9b^3(a+bx^3)^{3/2}} + \frac{2B\sqrt{a+bx^3}}{3b^3}$$

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}} + \frac{2a(Ab-aB)}{9b^3(a+bx^3)^{3/2}} + \frac{2B\sqrt{a+bx^3}}{3b^3}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2), x]

[Out] (2*a*(A*b - a*B))/(9*b^3*(a + b*x^3)^(3/2)) - (2*(A*b - 2*a*B))/(3*b^3*Sqrt[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/(3*b^3)

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^5 (A + Bx^3)}{(a + bx^3)^{5/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(A + Bx)}{(a + bx)^{5/2}} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{a(-Ab + aB)}{b^2(a + bx)^{5/2}} + \frac{Ab - 2aB}{b^2(a + bx)^{3/2}} + \frac{B}{b^2 \sqrt{a + bx}} \right) dx, x, x^3 \right) \\ &= \frac{2a(Ab - aB)}{9b^3 (a + bx^3)^{3/2}} - \frac{2(Ab - 2aB)}{3b^3 \sqrt{a + bx^3}} + \frac{2B\sqrt{a + bx^3}}{3b^3} \end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.74

$$\frac{16a^2B - 4ab(A - 6Bx^3) + 6b^2x^3(Bx^3 - A)}{9b^3(a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2), x]

[Out] (16*a^2*B - 4*a*b*(A - 6*B*x^3) + 6*b^2*x^3*(-A + B*x^3))/(9*b^3*(a + b*x^3)^(3/2))

IntegrateAlgebraic [A] time = 0.05, size = 56, normalized size = 0.77

$$\frac{2(8a^2B - 2aAb + 12abBx^3 - 3Ab^2x^3 + 3b^2Bx^6)}{9b^3(a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2), x]

[Out] (2*(-2*a*A*b + 8*a^2*B - 3*A*b^2*x^3 + 12*a*b*B*x^3 + 3*b^2*B*x^6))/(9*b^3*(a + b*x^3)^(3/2))

fricas [A] time = 0.71, size = 75, normalized size = 1.03

$$\frac{2(3Bb^2x^6 + 3(4Bab - Ab^2)x^3 + 8Ba^2 - 2Aab)\sqrt{bx^3 + a}}{9(b^5x^6 + 2ab^4x^3 + a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2), x, algorithm="fricas")

[Out] $\frac{2}{9} \cdot (3Bb^2x^6 + 3(4Ba^2b - Ab^2)x^3 + 8Ba^2 - 2A^2ab) \sqrt{bx^3 + a} / (b^5x^6 + 2ab^4x^3 + a^2b^3)$

giac [A] time = 0.20, size = 63, normalized size = 0.86

$$\frac{2\sqrt{bx^3+a}B}{3b^3} + \frac{2(6(bx^3+a)Ba - Ba^2 - 3(bx^3+a)Ab + Aab)}{9(bx^3+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")`

[Out] $\frac{2}{3} \sqrt{bx^3 + a} B / b^3 + \frac{2}{9} (6(bx^3 + a)Ba - Ba^2 - 3(bx^3 + a)A^2b + A^2ab) / ((bx^3 + a)^{3/2} b^3)$

maple [A] time = 0.05, size = 53, normalized size = 0.73

$$\frac{2(-3Bb^2x^6 + 3Aa^2x^3 - 12Babx^3 + 2Aab - 8Ba^2)}{9(bx^3+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x)`

[Out] $-2/9 / (bx^3+a)^{3/2} * (-3Bb^2x^6 + 3Aa^2x^3 - 12Babx^3 + 2Aa^2b - 8Ba^2) / b^3$

maxima [A] time = 0.49, size = 84, normalized size = 1.15

$$\frac{2}{9} B \left(\frac{3\sqrt{bx^3+a}}{b^3} + \frac{6a}{\sqrt{bx^3+a}b^3} - \frac{a^2}{(bx^3+a)^{\frac{3}{2}}b^3} \right) - \frac{2}{9} A \left(\frac{3}{\sqrt{bx^3+a}b^2} - \frac{a}{(bx^3+a)^{\frac{3}{2}}b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")`

[Out] $\frac{2}{9} B (3\sqrt{bx^3+a}/b^3 + 6a/(\sqrt{bx^3+a}b^3) - a^2/((bx^3+a)^{3/2}b^3)) - \frac{2}{9} A (3/(\sqrt{bx^3+a}b^2) - a/((bx^3+a)^{3/2}b^2))$

mupad [B] time = 2.76, size = 60, normalized size = 0.82

$$\frac{6B(bx^3+a)^2 - 2Ba^2 - 6Ab(bx^3+a) + 12Ba(bx^3+a) + 2Aab}{9b^3(bx^3+a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(A + B*x^3))/(a + b*x^3)^(5/2), x)`

[Out] $(6*B*(a + b*x^3)^2 - 2*B*a^2 - 6*A*b*(a + b*x^3) + 12*B*a*(a + b*x^3) + 2*A*a*b)/(9*b^3*(a + b*x^3)^{(3/2)})$

sympy [A] time = 2.42, size = 240, normalized size = 3.29

$$\begin{cases} -\frac{4Aab}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} - \frac{6Ab^2x^3}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} + \frac{16Ba^2}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} + \frac{24Babx^3}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} + \frac{6Bb^2x^6}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{Ax^6 + Bx^9}{\frac{5}{a^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(B*x**3+A)/(b*x**3+a)**(5/2), x)`

[Out] `Piecewise((-4*A*a*b/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3)) - 6*A*b**2*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3)) + 16*B*a**2/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3)) + 24*B*a*b*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3)) + 6*B*b**2*x**6/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/a**(5/2), True))`

$$3.201 \quad \int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}} dx$$

Optimal. Leaf size=46

$$-\frac{2(Ab - aB)}{9b^2 (a + bx^3)^{3/2}} - \frac{2B}{3b^2 \sqrt{a + bx^3}}$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {444, 43}

$$-\frac{2(Ab - aB)}{9b^2 (a + bx^3)^{3/2}} - \frac{2B}{3b^2 \sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2), x]

[Out] (-2*(A*b - a*B))/(9*b^2*(a + b*x^3)^(3/2)) - (2*B)/(3*b^2*sqrt[a + b*x^3])

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (A + Bx^3)}{(a + bx^3)^{5/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{(a + bx)^{5/2}} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{Ab - aB}{b(a + bx)^{5/2}} + \frac{B}{b(a + bx)^{3/2}} \right) dx, x, x^3 \right) \\
&= -\frac{2(Ab - aB)}{9b^2 (a + bx^3)^{3/2}} - \frac{2B}{3b^2 \sqrt{a + bx^3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.72

$$-\frac{2(2aB + Ab + 3bBx^3)}{9b^2 (a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2), x]

[Out] (-2*(A*b + 2*a*B + 3*b*B*x^3))/(9*b^2*(a + b*x^3)^(3/2))

IntegrateAlgebraic [A] time = 0.04, size = 33, normalized size = 0.72

$$-\frac{2(2aB + Ab + 3bBx^3)}{9b^2 (a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2), x]

[Out] (-2*(A*b + 2*a*B + 3*b*B*x^3))/(9*b^2*(a + b*x^3)^(3/2))

fricas [A] time = 0.73, size = 52, normalized size = 1.13

$$-\frac{2(3Bbx^3 + 2Ba + Ab)\sqrt{bx^3 + a}}{9(b^4x^6 + 2ab^3x^3 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2), x, algorithm="fricas")

[Out] -2/9*(3*B*b*x^3 + 2*B*a + A*b)*sqrt(b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)

giac [A] time = 0.16, size = 32, normalized size = 0.70

$$-\frac{2(3(bx^3 + a)B - Ba + Ab)}{9(bx^3 + a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")

[Out] -2/9*(3*(b*x^3 + a)*B - B*a + A*b)/((b*x^3 + a)^(3/2)*b^2)

maple [A] time = 0.05, size = 30, normalized size = 0.65

$$-\frac{2(3Bbx^3 + Ab + 2Ba)}{9(bx^3 + a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x)

[Out] -2/9/(b*x^3+a)^(3/2)*(3*B*b*x^3+A*b+2*B*a)/b^2

maxima [A] time = 0.50, size = 49, normalized size = 1.07

$$-\frac{2}{9}B\left(\frac{3}{\sqrt{bx^3 + a}b^2} - \frac{a}{(bx^3 + a)^{\frac{3}{2}}b^2}\right) - \frac{2A}{9(bx^3 + a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")

[Out] -2/9*B*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2)) - 2/9*A/((b*x^3 + a)^(3/2)*b)

mupad [B] time = 2.68, size = 33, normalized size = 0.72

$$-\frac{2Ab - 2Ba + 6B(bx^3 + a)}{9b^2(bx^3 + a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x)

[Out] $-(2* A * b - 2* B * a + 6* B * (a + b * x^3)) / (9 * b^2 * (a + b * x^3)^{(3/2)})$

sympy [A] time = 1.34, size = 144, normalized size = 3.13

$$\begin{cases} -\frac{2Ab}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} - \frac{4Ba}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} - \frac{6Bbx^3}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3} + \frac{Bx^6}{6}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(B*x**3+A)/(b*x**3+a)**(5/2), x)`

[Out] `Piecewise((-2*A*b/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)) - 4*B*a/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)) - 6*B*b*x**3/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/a**(5/2), True))`

$$3.202 \quad \int \frac{A+Bx^3}{x(a+bx^3)^{5/2}} dx$$

Optimal. Leaf size=77

$$-\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}} + \frac{2A}{3a^2\sqrt{a+bx^3}} + \frac{2(Ab-aB)}{9ab(a+bx^3)^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 78, 51, 63, 208}

$$\frac{2A}{3a^2\sqrt{a+bx^3}} - \frac{2A \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}} + \frac{2(Ab-aB)}{9ab(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]

[Out] (2*(A*b - a*B))/(9*a*b*(a + b*x^3)^(3/2)) + (2*A)/(3*a^2*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx^3}{x(a + bx^3)^{5/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x(a + bx)^{5/2}} dx, x, x^3 \right) \\
 &= \frac{2(Ab - aB)}{9ab(a + bx^3)^{3/2}} + \frac{A \text{Subst} \left(\int \frac{1}{x(a + bx)^{3/2}} dx, x, x^3 \right)}{3a} \\
 &= \frac{2(Ab - aB)}{9ab(a + bx^3)^{3/2}} + \frac{2A}{3a^2 \sqrt{a + bx^3}} + \frac{A \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx}} dx, x, x^3 \right)}{3a^2} \\
 &= \frac{2(Ab - aB)}{9ab(a + bx^3)^{3/2}} + \frac{2A}{3a^2 \sqrt{a + bx^3}} + \frac{(2A) \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + bx^3} \right)}{3a^2 b} \\
 &= \frac{2(Ab - aB)}{9ab(a + bx^3)^{3/2}} + \frac{2A}{3a^2 \sqrt{a + bx^3}} - \frac{2A \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right)}{3a^{5/2}}
 \end{aligned}$$

Mathematica [C] time = 0.05, size = 62, normalized size = 0.81

$$\frac{2a(Ab - aB) + 6Ab(a + bx^3) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^3}{a} + 1\right)}{9a^2b(a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^(5/2)), x]

[Out] (2*a*(A*b - a*B) + 6*A*b*(a + b*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^3)/a])/(9*a^2*b*(a + b*x^3)^(3/2))

IntegrateAlgebraic [A] time = 0.09, size = 70, normalized size = 0.91

$$-\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}} - \frac{2(a^2B - 4aAb - 3Ab^2x^3)}{9a^2b(a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)^(5/2)), x]

[Out] (-2*(-4*a*A*b + a^2*B - 3*A*b^2*x^3))/(9*a^2*b*(a + b*x^3)^(3/2)) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))

fricas [A] time = 0.56, size = 243, normalized size = 3.16

$$\left[\frac{3(Ab^3x^6 + 2Aab^2x^3 + Aa^2b)\sqrt{a} \log\left(\frac{bx^3 - 2\sqrt{bx^3+a}\sqrt{a+2a}}{x^3}\right) + 2(3Aab^2x^3 - Ba^3 + 4Aa^2b)\sqrt{bx^3+a}}{9(a^3b^3x^6 + 2a^4b^2x^3 + a^5b)}, \frac{2\left(3(Ab^3x^6 + 2Aab^2x^3 + Aa^2b)\sqrt{-a} \arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) + (3Aab^2x^3 - Ba^3 + 4Aa^2b)\sqrt{bx^3+a}\right)}{9(a^3b^3x^6 + 2a^4b^2x^3 + a^5b)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2), x, algorithm="fricas")

[Out] [1/9*(3*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*A*a*b^2*x^3 - B*a^3 + 4*A*a^2*b)*sqrt(b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b), 2/9*(3*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*A*a*b^2*x^3 - B*a^3 + 4*A*a^2*b)*sqrt(b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)]

giac [A] time = 0.17, size = 67, normalized size = 0.87

$$\frac{2A \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}a^2} - \frac{2(Ba^2 - 3(bx^3 + a)Ab - Aab)}{9(bx^3 + a)^{3/2}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="giac")

[Out] $\frac{2}{3}A \arctan(\sqrt{bx^3+a}/\sqrt{-a})/(\sqrt{-a}a^2) - \frac{2}{9}(B a^2 - 3(bx^3+a)A b - A a b)/((bx^3+a)^{3/2}a^2 b)$

maple [A] time = 0.09, size = 85, normalized size = 1.10

$$\left(-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}} + \frac{2}{3\sqrt{\left(x^3+\frac{a}{b}\right)ba^2}} + \frac{2\sqrt{bx^3+a}}{9\left(x^3+\frac{a}{b}\right)^2 ab^2} \right) A - \frac{2B}{9(bx^3+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x/(b*x^3+a)^(5/2),x)

[Out] $-\frac{2}{9}B/b/(bx^3+a)^{3/2} + A \left(\frac{2}{9} \frac{1}{a/b^2} (bx^3+a)^{1/2} / (x^3+a/b)^{2+2/3} a^2 / ((x^3+a/b)b)^{1/2} - \frac{2}{3} \frac{1}{a^{5/2}} \operatorname{arctanh}\left(\frac{(bx^3+a)^{1/2}}{a^{1/2}}\right) \right)$

maxima [A] time = 1.17, size = 81, normalized size = 1.05

$$\frac{1}{9}A \left(\frac{3 \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2(3bx^3+4a)}{(bx^3+a)^{\frac{3}{2}}a^2} \right) - \frac{2B}{9(bx^3+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="maxima")

[Out] $\frac{1}{9}A \left(3 \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) / (a^{5/2}) + \frac{2(3bx^3+4a)}{(bx^3+a)^{3/2}a^2} \right) - \frac{2}{9}B / ((bx^3+a)^{3/2}b)$

mupad [B] time = 2.78, size = 80, normalized size = 1.04

$$\frac{\frac{2A}{9a} - \frac{2B}{9b}}{(bx^3+a)^{3/2}} + \frac{2A}{3a^2 \sqrt{bx^3+a}} + \frac{A \ln\left(\frac{(\sqrt{bx^3+a}-\sqrt{a})^3 (\sqrt{bx^3+a}+\sqrt{a})}{x^6}\right)}{3a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x*(a + b*x^3)^(5/2)),x)

```
[Out] ((2*A)/(9*a) - (2*B)/(9*b))/(a + b*x^3)^(3/2) + (2*A)/(3*a^2*(a + b*x^3)^(1/2)) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(5/2))
```

sympy [A] time = 38.43, size = 76, normalized size = 0.99

$$\frac{2A}{3a^2\sqrt{a+bx^3}} + \frac{2A \operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3a^2\sqrt{-a}} - \frac{2(-Ab+Ba)}{9ab(a+bx^3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(5/2),x)
```

```
[Out] 2*A/(3*a**2*sqrt(a + b*x**3)) + 2*A*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*a**2*sqrt(-a)) - 2*(-A*b + B*a)/(9*a*b*(a + b*x**3)**(3/2))
```


$$3.203 \quad \int \frac{A+Bx^3}{x^4(a+bx^3)^{5/2}} dx$$

Optimal. Leaf size=113

$$\frac{(5Ab - 2aB) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}} - \frac{5Ab - 2aB}{3a^3\sqrt{a+bx^3}} + \frac{2aB - 5Ab}{9a^2(a+bx^3)^{3/2}} - \frac{A}{3ax^3(a+bx^3)^{3/2}}$$

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 78, 51, 63, 208}

$$-\frac{5Ab - 2aB}{3a^3\sqrt{a+bx^3}} - \frac{5Ab - 2aB}{9a^2(a+bx^3)^{3/2}} + \frac{(5Ab - 2aB) \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}} - \frac{A}{3ax^3(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)), x]

[Out] -(5*A*b - 2*a*B)/(9*a^2*(a + b*x^3)^(3/2)) - A/(3*a*x^3*(a + b*x^3)^(3/2)) - (5*A*b - 2*a*B)/(3*a^3*Sqrt[a + b*x^3]) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(7/2))

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x^4 (a + bx^3)^{5/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{A + Bx}{x^2 (a + bx)^{5/2}} dx, x, x^3 \right) \\
&= -\frac{A}{3ax^3 (a + bx^3)^{3/2}} + \frac{\left(-\frac{5Ab}{2} + aB\right) \text{Subst} \left(\int \frac{1}{x(a+bx)^{5/2}} dx, x, x^3 \right)}{3a} \\
&= -\frac{5Ab - 2aB}{9a^2 (a + bx^3)^{3/2}} - \frac{A}{3ax^3 (a + bx^3)^{3/2}} - \frac{(5Ab - 2aB) \text{Subst} \left(\int \frac{1}{x(a+bx)^{3/2}} dx, x, x^3 \right)}{6a^2} \\
&= -\frac{5Ab - 2aB}{9a^2 (a + bx^3)^{3/2}} - \frac{A}{3ax^3 (a + bx^3)^{3/2}} - \frac{5Ab - 2aB}{3a^3 \sqrt{a + bx^3}} - \frac{(5Ab - 2aB) \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx}} dx, x, x^3 \right)}{6a^3} \\
&= -\frac{5Ab - 2aB}{9a^2 (a + bx^3)^{3/2}} - \frac{A}{3ax^3 (a + bx^3)^{3/2}} - \frac{5Ab - 2aB}{3a^3 \sqrt{a + bx^3}} - \frac{(5Ab - 2aB) \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, x^3 \right)}{3a^3 b} \\
&= -\frac{5Ab - 2aB}{9a^2 (a + bx^3)^{3/2}} - \frac{A}{3ax^3 (a + bx^3)^{3/2}} - \frac{5Ab - 2aB}{3a^3 \sqrt{a + bx^3}} + \frac{(5Ab - 2aB) \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{3a^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.50

$$\frac{x^3(2aB - 5Ab) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{bx^3}{a} + 1\right) - 3aA}{9a^2 x^3 (a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)), x]

[Out] (-3*a*A + (-5*A*b + 2*a*B)*x^3*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*x^3)/a])/(9*a^2*x^3*(a + b*x^3)^(3/2))

IntegrateAlgebraic [A] time = 0.14, size = 99, normalized size = 0.88

$$\frac{(5Ab - 2aB) \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{3a^{7/2}} + \frac{-3a^2A + 8a^2Bx^3 - 20aAbx^3 + 6abBx^6 - 15Ab^2x^6}{9a^3 x^3 (a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]

[Out] $(-3a^2A - 20aAbx^3 + 8a^2Bx^3 - 15Ab^2x^6 + 6a^2Bx^6)/(9a^3x^3(a + b^2x^3)^{3/2}) + ((5Ab - 2a^2B) \operatorname{ArcTanh}[\sqrt{a + b^2x^3}/\sqrt{a}])/(3a^{7/2})$

fricas [A] time = 0.75, size = 351, normalized size = 3.11

$$\frac{3((2Ba^2 - 5Ab^2)x^6 + 2(2Ba^2b - 5Aab^2)x^4 + (2Ba^2 - 5Aab^2)x^2)\sqrt{a} \log\left(\frac{bx^3 + a}{\sqrt{a}}\right) - 2(3(2Ba^2b - 5Aab^2)x^6 - 3Aa^3 + 4(2Ba^2 - 5Aab^2)x^2)\sqrt{bx^3 + a} - 3((2Ba^2 - 5Ab^2)x^6 + 2(2Ba^2b - 5Aab^2)x^4 + (2Ba^2 - 5Aab^2)x^2)\sqrt{-a} \operatorname{arctan}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{-a}}\right) + (3(2Ba^2b - 5Aab^2)x^6 - 3Aa^3 + 4(2Ba^2 - 5Aab^2)x^2)\sqrt{bx^3 + a}}{18(a^2b^2x^3 + 2a^2bx^3 + a^2x^3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="fricas")

[Out] $[-1/18*(3*((2Ba^2b - 5Aab^2)x^9 + 2*(2Ba^2b - 5Aa^2b^2)x^6 + (2Ba^3 - 5Aa^2b^2)x^3)*\sqrt{a}*\log((bx^3 + 2*\sqrt{bx^3 + a})*\sqrt{a} + 2a)/x^3) - 2*(3*(2Ba^2b - 5Aa^2b^2)x^6 - 3Aa^3 + 4*(2Ba^3 - 5Aa^2b^2)x^3)*\sqrt{bx^3 + a}]/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3), 1/9*(3*((2Ba^2b - 5Aab^2)x^9 + 2*(2Ba^2b - 5Aa^2b^2)x^6 + (2Ba^3 - 5Aa^2b^2)x^3)*\sqrt{-a}*\operatorname{arctan}(\sqrt{bx^3 + a}*\sqrt{-a}/a) + (3*(2Ba^2b - 5Aa^2b^2)x^6 - 3Aa^3 + 4*(2Ba^3 - 5Aa^2b^2)x^3)*\sqrt{bx^3 + a}]/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)]$

giac [A] time = 0.19, size = 101, normalized size = 0.89

$$\frac{(2Ba - 5Ab) \operatorname{arctan}\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}a^3} + \frac{2(3(bx^3+a)Ba + Ba^2 - 6(bx^3+a)Ab - Aab)}{9(bx^3+a)^2a^3} - \frac{\sqrt{bx^3+a}A}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="giac")

[Out] $1/3*(2Ba - 5Ab)*\operatorname{arctan}(\sqrt{bx^3 + a}/\sqrt{-a})/(\sqrt{-a}*a^3) + 2/9*(3*(bx^3 + a)*Ba + Ba^2 - 6*(bx^3 + a)*Ab - Aa*b)/((bx^3 + a)^{3/2}*a^3) - 1/3*\sqrt{bx^3 + a}*A/(a^3*x^3)$

maple [A] time = 0.09, size = 157, normalized size = 1.39

$$\left(\frac{5b \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3a^2} - \frac{4b}{3\sqrt{\left(x^3 + \frac{a}{b}\right)b}a^3} - \frac{2\sqrt{bx^3+a}}{9\left(x^3 + \frac{a}{b}\right)^2a^2b} - \frac{\sqrt{bx^3+a}}{3a^3x^3} \right) A + \left(-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3a^2} + \frac{2}{3\sqrt{\left(x^3 + \frac{a}{b}\right)b}a^2} + \frac{2\sqrt{bx^3+a}}{9\left(x^3 + \frac{a}{b}\right)^2ab^2} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x)`

[Out] $A*(-1/3/a^3*(b*x^3+a)^{(1/2)}/x^3-2/9/a^2/b*(b*x^3+a)^{(1/2)}/(x^3+a/b)^{2-4/3*b/a^3}/((x^3+a/b)*b)^{(1/2)}+5/3*b/a^{(7/2)}*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)}))+B*(2/9*(b*x^3+a)^{(1/2)}/(x^3+a/b)^2/a/b^2+2/3/((x^3+a/b)*b)^{(1/2)}/a^{2-2/3/a^{(5/2)}}*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)}))$

maxima [A] time = 1.25, size = 170, normalized size = 1.50

$$-\frac{1}{18}A\left(\frac{2\left(15(bx^3+a)^2b-10(bx^3+a)ab-2a^2b\right)}{(bx^3+a)^{\frac{5}{2}}a^3-(bx^3+a)^{\frac{3}{2}}a^4}+\frac{15b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)+\frac{1}{9}B\left(\frac{3\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2(3bx^3+4a)}{(bx^3+a)^{\frac{3}{2}}a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="maxima")`

[Out] $-1/18*A*(2*(15*(b*x^3+a)^2*b-10*(b*x^3+a)*a*b-2*a^2*b)/((b*x^3+a)^{(5/2)}*a^3-(b*x^3+a)^{(3/2)}*a^4)+15*b*\log((\operatorname{sqrt}(b*x^3+a)-\operatorname{sqrt}(a))/(\operatorname{sqrt}(b*x^3+a)+\operatorname{sqrt}(a)))/a^{(7/2)}+1/9*B*(3*\log((\operatorname{sqrt}(b*x^3+a)-\operatorname{sqrt}(a))/(\operatorname{sqrt}(b*x^3+a)+\operatorname{sqrt}(a)))/a^{(5/2)}+2*(3*b*x^3+4*a)/((b*x^3+a)^{(3/2)}*a^2))$

mupad [B] time = 2.97, size = 198, normalized size = 1.75

$$\ln\left(\frac{\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{x^6}\right)\left(\frac{\sqrt{bx^3+a}+\sqrt{a}}{x^6}\right)^3}{6a^{7/2}}\right)(5Ab-2Ba)-\frac{2Ba^2-5Aab}{2a^4}-\frac{a\left(\frac{Ab^2}{3a^4}+\frac{5b(2Ba^2-5Aab)}{6a^5}\right)}{\sqrt{bx^3+a}}-\frac{2Ba^3-5Aa^2b}{4a^4}-\frac{a\left(\frac{13b(2Ba^3-5Aa^2b)}{36a^5}+\frac{Ab^2}{3a^3}\right)}{(bx^3+a)^{3/2}}-\frac{A\sqrt{bx^3+a}}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*x^3)/(x^4*(a+b*x^3)^(5/2)),x)`

[Out] $(\log(((a+b*x^3)^{(1/2)}-a^{(1/2)})*((a+b*x^3)^{(1/2)}+a^{(1/2)})^3)/x^6)*(5*A*b-2*B*a)/(6*a^{(7/2)})-((2*B*a^2-5*A*a*b)/(2*a^4)-(a*((A*b^2)/(3*a^4)+(5*b*(2*B*a^2-5*A*a*b))/(6*a^5)))/b)/(a+b*x^3)^{(1/2)}-((2*B*a^3-5*A*a^2*b)/(4*a^4)-(a*((13*b*(2*B*a^3-5*A*a^2*b))/(36*a^5)+(A*b^2)/(3*a^3)))/b)/(a+b*x^3)^{(3/2)}-(A*(a+b*x^3)^{(1/2)})/(3*a^3*x^3)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)/x**4/(b*x**3+a)**(5/2),x)`

[Out] Timed out

$$3.204 \quad \int \frac{x^8 \sqrt{c+dx^3}}{4c+dx^3} dx$$

Optimal. Leaf size=97

$$-\frac{32c^{5/2} \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3} + \frac{32c^2\sqrt{c+dx^3}}{3d^3} - \frac{10c(c+dx^3)^{3/2}}{9d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3}$$

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {446, 88, 50, 63, 203}

$$\frac{32c^2\sqrt{c+dx^3}}{3d^3} - \frac{32c^{5/2} \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3} - \frac{10c(c+dx^3)^{3/2}}{9d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

[In] Int[(x^8*sqrt[c + d*x^3])/(4*c + d*x^3),x]

[Out] (32*c^2*sqrt[c + d*x^3])/(3*d^3) - (10*c*(c + d*x^3)^(3/2))/(9*d^3) + (2*(c + d*x^3)^(5/2))/(15*d^3) - (32*c^(5/2)*ArcTan[sqrt[c + d*x^3]/(sqrt[3]*sqrt[c])])/(sqrt[3]*d^3)

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

$x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x] \&\& \text{IntegersQ}[m, n] \&\& (\text{IntegerQ}[p] \parallel (\text{GtQ}[m, 0] \&\& \text{GeQ}[n, -1]))$

Rule 203

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTan}[(\text{Rt}[b, 2] \cdot x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2] \cdot \text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rule 446

$\text{Int}[(x_)^{(m_ \cdot)} \cdot ((a_ + (b_ \cdot)(x_)^{(n_)})^{(p_ \cdot)}) \cdot ((c_ + (d_ \cdot)(x_)^{(n_)})^{(q_ \cdot)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) \cdot (a + b \cdot x)^p \cdot (c + d \cdot x)^q}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned}
 \int \frac{x^8 \sqrt{c + dx^3}}{4c + dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 \sqrt{c + dx}}{4c + dx} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{5c \sqrt{c + dx}}{d^2} + \frac{(c + dx)^{3/2}}{d^2} + \frac{16c^2 \sqrt{c + dx}}{d^2(4c + dx)} \right) dx, x, x^3 \right) \\
 &= -\frac{10c (c + dx^3)^{3/2}}{9d^3} + \frac{2 (c + dx^3)^{5/2}}{15d^3} + \frac{(16c^2) \text{Subst} \left(\int \frac{\sqrt{c + dx}}{4c + dx} dx, x, x^3 \right)}{3d^2} \\
 &= \frac{32c^2 \sqrt{c + dx^3}}{3d^3} - \frac{10c (c + dx^3)^{3/2}}{9d^3} + \frac{2 (c + dx^3)^{5/2}}{15d^3} - \frac{(16c^3) \text{Subst} \left(\int \frac{1}{\sqrt{c + dx}(4c + dx)} dx, x, x^3 \right)}{d^2} \\
 &= \frac{32c^2 \sqrt{c + dx^3}}{3d^3} - \frac{10c (c + dx^3)^{3/2}}{9d^3} + \frac{2 (c + dx^3)^{5/2}}{15d^3} - \frac{(32c^3) \text{Subst} \left(\int \frac{1}{3c + x^2} dx, x, \sqrt{c + dx^3} \right)}{d^3} \\
 &= \frac{32c^2 \sqrt{c + dx^3}}{3d^3} - \frac{10c (c + dx^3)^{3/2}}{9d^3} + \frac{2 (c + dx^3)^{5/2}}{15d^3} - \frac{32c^{5/2} \tan^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{3} \sqrt{c}} \right)}{\sqrt{3} d^3}
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 77, normalized size = 0.79

$$\frac{2\sqrt{c + dx^3} (218c^2 - 19cdx^3 + 3d^2x^6) - 480\sqrt{3} c^{5/2} \tan^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{3} \sqrt{c}} \right)}{45d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3), x]

[Out] (2*Sqrt[c + d*x^3]*(218*c^2 - 19*c*d*x^3 + 3*d^2*x^6) - 480*Sqrt[3]*c^(5/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(45*d^3)

IntegrateAlgebraic [A] time = 0.07, size = 78, normalized size = 0.80

$$\frac{2\sqrt{c+dx^3}(218c^2-19cdx^3+3d^2x^6)}{45d^3} - \frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3), x]

[Out] (2*Sqrt[c + d*x^3]*(218*c^2 - 19*c*d*x^3 + 3*d^2*x^6))/(45*d^3) - (32*c^(5/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^3)

fricas [A] time = 1.15, size = 156, normalized size = 1.61

$$\left[\frac{2\left(120\sqrt{3}\sqrt{-c}c^2\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right) + (3d^2x^6 - 19cdx^3 + 218c^2)\sqrt{dx^3+c}\right)}{45d^3}, -\frac{2\left(240\sqrt{3}c^{5/2}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right) - (3d^2x^6 - 19cdx^3 + 218c^2)\sqrt{dx^3+c}\right)}{45d^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c), x, algorithm="fricas")

[Out] [2/45*(120*sqrt(3)*sqrt(-c)*c^2*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) + (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^3 + c))/d^3, -2/45*(240*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^3 + c))/d^3]

giac [A] time = 0.17, size = 82, normalized size = 0.85

$$-\frac{32\sqrt{3}c^{5/2}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3d^3} + \frac{2\left(3(dx^3+c)^{5/2}d^{12} - 25(dx^3+c)^{3/2}cd^{12} + 240\sqrt{dx^3+c}c^2d^{12}\right)}{45d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c), x, algorithm="giac")

[Out] -32/3*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^3 + 2/45*(3*(d*x^3 + c)^(5/2)*d^12 - 25*(d*x^3 + c)^(3/2)*c*d^12 + 240*sqrt(d*x^3 + c)*c^2*d^12)/d^15

maple [C] time = 4.46, size = 506, normalized size = 5.22

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c), x)`

[Out] $\frac{1}{d^2} \left(\frac{d(2/15x^6(d^2x^3+c)^{1/2}) + 2/45c/dx^3(d^2x^3+c)^{1/2} - 4/45c^2(d^2x^3+c)^{1/2}/d^2 - 8/9c/d(d^2x^3+c)^{3/2} + 16c^2/d^2(2/3(d^2x^3+c)^{1/2}/d + 1/3I/d^3 \sum_{k=1}^2 (d^2x^3+c)^{1/2} \sum_{l=1}^2 ((-c*d^2)^{1/3})^{k+l} (1/2 * I * d * (2*x + 1/d * (-I * 3^{1/2}) * (-c*d^2)^{1/3} + (-c*d^2)^{1/3})) / ((-c*d^2)^{1/3})^{k+l} (1/2) * (d * (x - 1/d * (-c*d^2)^{1/3}) / (-3 * (-c*d^2)^{1/3} + I * 3^{1/2} * (-c*d^2)^{1/3}))^{k+l} * (-1/2 * I * d * (2*x + 1/d * (I * 3^{1/2}) * (-c*d^2)^{1/3} + (-c*d^2)^{1/3})) / ((-c*d^2)^{1/3})^{k+l} / (d^2x^3+c)^{1/2}} \right) * (I * (-c*d^2)^{1/3} * \alpha * 3^{1/2} * d - I * 3^{1/2} * (-c*d^2)^{2/3} + 2 * \alpha^2 * d^2 - (-c*d^2)^{1/3} * \alpha * d - (-c*d^2)^{2/3} * \text{EllipticPi}(1/3 * 3^{1/2} * (I * (x + 1/2/d * (-c*d^2)^{1/3} - 1/2 * I * 3^{1/2}/d * (-c*d^2)^{1/3})) * 3^{1/2} * d / (-c*d^2)^{1/3})^{1/2}, 1/6/d * (2 * I * (-c*d^2)^{1/3} * 3^{1/2} * \alpha^2 * d - I * (-c*d^2)^{2/3} * 3^{1/2}) * \alpha + I * 3^{1/2} * c * d - 3 * (-c*d^2)^{2/3} * \alpha - 3 * c * d) / c, (I * 3^{1/2}/d * (-c*d^2)^{1/3} / (-3/2/d * (-c*d^2)^{1/3} + 1/2 * I * 3^{1/2}/d * (-c*d^2)^{1/3}))^{1/2}, \alpha = \text{RootOf}(_Z^3 * d + 4 * c))$

maxima [A] time = 1.15, size = 69, normalized size = 0.71

$$\frac{2 \left(240 \sqrt{3} c^{\frac{5}{2}} \arctan \left(\frac{\sqrt{3} \sqrt{dx^3+c}}{3\sqrt{c}} \right) - 3 (dx^3+c)^{\frac{5}{2}} + 25 (dx^3+c)^{\frac{3}{2}} c - 240 \sqrt{dx^3+c} c^2 \right)}{45 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c), x, algorithm="maxima")`

[Out] $-2/45 * (240 * \text{sqrt}(3) * c^{5/2} * \arctan(1/3 * \text{sqrt}(3) * \text{sqrt}(d*x^3+c) / \text{sqrt}(c)) - 3 * (d*x^3+c)^{5/2} + 25 * (d*x^3+c)^{3/2} * c - 240 * \text{sqrt}(d*x^3+c) * c^2) / d^3$

mupad [B] time = 4.54, size = 109, normalized size = 1.12

$$\frac{436 c^2 \sqrt{d x^3 + c}}{45 d^3} + \frac{2 x^6 \sqrt{d x^3 + c}}{15 d} - \frac{38 c x^3 \sqrt{d x^3 + c}}{45 d^2} + \frac{\sqrt{3} c^{5/2} \ln \left(\frac{2 \sqrt{3} c - \sqrt{3} d x^3 + \sqrt{c} \sqrt{d x^3 + c} 6i}{d x^3 + 4 c} \right)}{3 d^3} 16i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^8*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)`

[Out] $(436*c^2*(c + d*x^3)^{(1/2)})/(45*d^3) + (2*x^6*(c + d*x^3)^{(1/2)})/(15*d) - (38*c*x^3*(c + d*x^3)^{(1/2)})/(45*d^2) + (3^{(1/2)}*c^{(5/2)}*\log((2*3^{(1/2)}*c + c^{(1/2)}*(c + d*x^3)^{(1/2)}*6i - 3^{(1/2)}*d*x^3)/(4*c + d*x^3))*16i)/(3*d^3)$

sympy [A] time = 38.27, size = 85, normalized size = 0.88

$$\frac{2 \left(-\frac{16\sqrt{3}c^{\frac{5}{2}} \operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3} + \frac{16c^2\sqrt{c+dx^3}}{3} - \frac{5c(c+dx^3)^{\frac{3}{2}}}{9} + \frac{(c+dx^3)^{\frac{5}{2}}}{15} \right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)`

[Out] $2*(-16*\sqrt{3}*c^{(5/2)}*\operatorname{atan}(\sqrt{3}*\sqrt{c + d*x**3}/(3*\sqrt{c}))/3 + 16*c**2*\sqrt{c + d*x**3}/3 - 5*c*(c + d*x**3)**(3/2)/9 + (c + d*x**3)**(5/2)/15)/d**3$

$$3.205 \quad \int \frac{x^5 \sqrt{c+dx^3}}{4c+dx^3} dx$$

Optimal. Leaf size=76

$$\frac{8c^{3/2} \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2} - \frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2(c+dx^3)^{3/2}}{9d^2}$$

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {446, 80, 50, 63, 203}

$$\frac{8c^{3/2} \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2} - \frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2(c+dx^3)^{3/2}}{9d^2}$$

Antiderivative was successfully verified.

[In] Int[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3), x]

[Out] (-8*c*Sqrt[c + d*x^3])/(3*d^2) + (2*(c + d*x^3)^(3/2))/(9*d^2) + (8*c^(3/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^2)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p

+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5 \sqrt{c + dx^3}}{4c + dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x \sqrt{c + dx}}{4c + dx} dx, x, x^3 \right) \\
 &= \frac{2(c + dx^3)^{3/2}}{9d^2} - \frac{(4c) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{4c+dx} dx, x, x^3 \right)}{3d} \\
 &= -\frac{8c\sqrt{c + dx^3}}{3d^2} + \frac{2(c + dx^3)^{3/2}}{9d^2} + \frac{(4c^2) \text{Subst} \left(\int \frac{1}{\sqrt{c+dx}(4c+dx)} dx, x, x^3 \right)}{d} \\
 &= -\frac{8c\sqrt{c + dx^3}}{3d^2} + \frac{2(c + dx^3)^{3/2}}{9d^2} + \frac{(8c^2) \text{Subst} \left(\int \frac{1}{3c+x^2} dx, x, \sqrt{c + dx^3} \right)}{d^2} \\
 &= -\frac{8c\sqrt{c + dx^3}}{3d^2} + \frac{2(c + dx^3)^{3/2}}{9d^2} + \frac{8c^{3/2} \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}} \right)}{\sqrt{3}d^2}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.86

$$\frac{24\sqrt{3}c^{3/2} \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}} \right) + 2(dx^3 - 11c)\sqrt{c + dx^3}}{9d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]

[Out] (2*(-11*c + d*x^3)*Sqrt[c + d*x^3] + 24*Sqrt[3]*c^(3/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(9*d^2)

IntegrateAlgebraic [A] time = 0.05, size = 67, normalized size = 0.88

$$\frac{8c^{3/2} \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2} - \frac{2(11c - dx^3)\sqrt{c+dx^3}}{9d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]

[Out] (-2*(11*c - d*x^3)*Sqrt[c + d*x^3])/(9*d^2) + (8*c^(3/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^2)

fricas [A] time = 0.93, size = 129, normalized size = 1.70

$$\left[\frac{2\left(6\sqrt{3}\sqrt{-c}c\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\sqrt{dx^3+c}(dx^3-11c)\right)}{9d^2}, \frac{2\left(12\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\sqrt{dx^3+c}(dx^3-11c)\right)}{9d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")

[Out] [2/9*(6*sqrt(3)*sqrt(-c)*c*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 11*c))/d^2, 2/9*(12*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt(d*x^3 + c)*(d*x^3 - 11*c))/d^2]

giac [A] time = 0.19, size = 64, normalized size = 0.84

$$\frac{8\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3d^2} + \frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^4-12\sqrt{dx^3+c}cd^4\right)}{9d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")

[Out] 8/3*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^2 + 2/9*(d*x^3 + c)^(3/2)*d^4 - 12*sqrt(d*x^3 + c)*c*d^4/d^6

maple [C] time = 0.17, size = 446, normalized size = 5.87

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c), x)`

[Out]
$$\frac{2}{9} \frac{(d x^3 + c)^{3/2}}{d^2 - 4 c/d} + \frac{2}{3} \frac{(d x^3 + c)^{1/2}}{d} + \frac{1}{3} \frac{I}{d^3} 2^{1/2} \sum \left(\frac{(-c d^2)^{1/3} (1/2 I (2 x + (-I 3^{1/2}) (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d)}{(-c d^2)^{1/3} d^{1/2} ((x - (-c d^2)^{1/3} / d) / (-3 (-c d^2)^{1/3} + I 3^{1/2} (-c d^2)^{1/3}))} \right) + \dots$$

maxima [A] time = 1.30, size = 53, normalized size = 0.70

$$\frac{2 \left(12 \sqrt{3} c^{\frac{3}{2}} \arctan \left(\frac{\sqrt{3} \sqrt{d x^3 + c}}{3 \sqrt{c}} \right) + (d x^3 + c)^{\frac{3}{2}} - 12 \sqrt{d x^3 + c} c \right)}{9 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c), x, algorithm="maxima")`

[Out]
$$\frac{2}{9} \frac{(12 \sqrt{3} c^{\frac{3}{2}} \arctan(1/3 \sqrt{3} \sqrt{d x^3 + c}) / \sqrt{c}) + (d x^3 + c)^{\frac{3}{2}} - 12 \sqrt{d x^3 + c} c}{d^2}$$

mupad [B] time = 4.28, size = 88, normalized size = 1.16

$$\frac{2 x^3 \sqrt{d x^3 + c}}{9 d} - \frac{22 c \sqrt{d x^3 + c}}{9 d^2} + \frac{\sqrt{3} c^{3/2} \ln \left(\frac{\sqrt{3} d x^3 - 2 \sqrt{3} c + \sqrt{c} \sqrt{d x^3 + c}}{d x^3 + 4 c} \right)}{3 d^2} 4i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)`

[Out] $(2x^3(c + dx^3)^{1/2})/(9d) - (22c(c + dx^3)^{1/2})/(9d^2) + (3^{1/2}c^{3/2}\log((c^{1/2}(c + dx^3)^{1/2}6i - 23^{1/2}c + 3^{1/2}dx^3)/(4c + dx^3))4i)/(3d^2)$

sympy [A] time = 16.96, size = 68, normalized size = 0.89

$$\frac{2 \left(\frac{4\sqrt{3}c^{\frac{3}{2}} \operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3} - \frac{4c\sqrt{c+dx^3}}{3} + \frac{(c+dx^3)^{\frac{3}{2}}}{9} \right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)`

[Out] $2*(4*\sqrt{3}*c^{3/2}*\operatorname{atan}(\sqrt{3}*\sqrt{c + d*x**3}/(3*\sqrt{c}))/3 - 4*c*\sqrt{c + d*x**3}/3 + (c + d*x**3)^{3/2}/9)/d**2$

$$3.206 \quad \int \frac{x^2 \sqrt{c+dx^3}}{4c+dx^3} dx$$

Optimal. Leaf size=57

$$\frac{2\sqrt{c+dx^3}}{3d} - \frac{2\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d}$$

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {444, 50, 63, 203}

$$\frac{2\sqrt{c+dx^3}}{3d} - \frac{2\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]

[Out] (2*Sqrt[c + d*x^3])/(3*d) - (2*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d)

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```


Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2 \sqrt{c + dx^3}}{4c + dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c + dx}}{4c + dx} dx, x, x^3 \right) \\
 &= \frac{2\sqrt{c + dx^3}}{3d} - c \text{Subst} \left(\int \frac{1}{\sqrt{c + dx}(4c + dx)} dx, x, x^3 \right) \\
 &= \frac{2\sqrt{c + dx^3}}{3d} - \frac{(2c) \text{Subst} \left(\int \frac{1}{3c + x^2} dx, x, \sqrt{c + dx^3} \right)}{d} \\
 &= \frac{2\sqrt{c + dx^3}}{3d} - \frac{2\sqrt{c} \tan^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{3} \sqrt{c}} \right)}{\sqrt{3} d}
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.95

$$\frac{2 \left(\sqrt{c + dx^3} - \sqrt{3} \sqrt{c} \tan^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{3} \sqrt{c}} \right) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]

[Out] (2*(Sqrt[c + d*x^3] - Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]))/(3*d)

IntegrateAlgebraic [A] time = 0.05, size = 57, normalized size = 1.00

$$\frac{2\sqrt{c + dx^3}}{3d} - \frac{2\sqrt{c} \tan^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{3} \sqrt{c}} \right)}{\sqrt{3} d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]

[Out] (2*Sqrt[c + d*x^3])/(3*d) - (2*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d)

fricas [A] time = 0.78, size = 110, normalized size = 1.93

$$\left[\frac{\sqrt{3} \sqrt{-c} \log\left(\frac{dx^3 - 2\sqrt{3} \sqrt{dx^3+c} \sqrt{-c} - 2c}{dx^3+4c}\right) + 2\sqrt{dx^3+c}}{3d}, -\frac{2\left(\sqrt{3} \sqrt{c} \arctan\left(\frac{\sqrt{3} \sqrt{dx^3+c}}{3\sqrt{c}}\right) - \sqrt{dx^3+c}\right)}{3d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")

[Out] [1/3*(sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(dx^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) + 2*sqrt(dx^3 + c))/d, -2/3*(sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(dx^3 + c)/sqrt(c)) - sqrt(dx^3 + c))/d]

giac [A] time = 0.16, size = 44, normalized size = 0.77

$$-\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3d} + \frac{2\sqrt{dx^3+c}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")

[Out] -2/3*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(dx^3 + c)/sqrt(c))/d + 2/3*sqrt(dx^3 + c)/d

maple [C] time = 0.15, size = 425, normalized size = 7.46

$$\frac{\sqrt{-c} \sqrt{\frac{\sqrt{3} \sqrt{dx^3+c}}{3\sqrt{c}}}}{3d} + \frac{\sqrt{dx^3+c}}{3d} + \frac{\sqrt{3} \sqrt{c} \arctan\left(\frac{\sqrt{3} \sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)

[Out] 2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3))/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)

$\frac{1}{3} * _alpha * d - I * 3^{(1/2)} * (-c * d^2)^{(2/3)} - (-c * d^2)^{(2/3)} * \text{EllipticPi}(1/3 * 3^{(1/2)} * (I * (x + 1/2 * (-c * d^2)^{(1/3)} / d - 1/2 * I * 3^{(1/2)} * (-c * d^2)^{(1/3)} / d) * 3^{(1/2)} / (-c * d^2)^{(1/3)} * d)^{(1/2)}, 1/6 * (2 * I * (-c * d^2)^{(1/3)} * 3^{(1/2)} * _alpha^2 * d + I * 3^{(1/2)} * c * d - 3 * c * d - I * (-c * d^2)^{(2/3)} * 3^{(1/2)} * _alpha - 3 * (-c * d^2)^{(2/3)} * _alpha) / c / d, (I * 3^{(1/2)} * (-c * d^2)^{(1/3)} / (-3/2 * (-c * d^2)^{(1/3)} / d + 1/2 * I * 3^{(1/2)} * (-c * d^2)^{(1/3)} / d) / d)^{(1/2)}, _alpha = \text{RootOf}(_Z^3 * d + 4 * c))$

maxima [A] time = 1.21, size = 42, normalized size = 0.74

$$\frac{2 \left(\sqrt{3} \sqrt{c} \arctan \left(\frac{\sqrt{3} \sqrt{dx^3 + c}}{3 \sqrt{c}} \right) - \sqrt{dx^3 + c} \right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")

[Out] -2/3*(sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - sqrt(d*x^3 + c))/d

mupad [B] time = 3.87, size = 71, normalized size = 1.25

$$\frac{2 \sqrt{dx^3 + c}}{3d} + \frac{\sqrt{3} \sqrt{c} \ln \left(\frac{2 \sqrt{3} c - \sqrt{3} dx^3 + \sqrt{c} \sqrt{dx^3 + c} 6i}{dx^3 + 4c} \right) 1i}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)

[Out] (2*(c + d*x^3)^(1/2))/(3*d) + (3^(1/2)*c^(1/2)*log((2*3^(1/2)*c + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(3*d)

sympy [A] time = 6.10, size = 51, normalized size = 0.89

$$\frac{2 \left(-\frac{\sqrt{3} \sqrt{c} \operatorname{atan} \left(\frac{\sqrt{3} \sqrt{c+dx^3}}{3 \sqrt{c}} \right)}{3} + \frac{\sqrt{c+dx^3}}{3} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)

[Out] 2*(-sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/3 + sqrt(c + d*x**3)/3)/d

$$3.207 \quad \int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)} dx$$

Optimal. Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {446, 83, 63, 208, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]

[Out] ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(2*Sqrt[3]*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(6*Sqrt[c])

Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_)^(p_.))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt
[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c+dx}}{x(4c+dx)} dx, x, x^3 \right) \\
 &= \frac{1}{12} \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right) + \frac{1}{4} d \text{Subst} \left(\int \frac{1}{\sqrt{c+dx}(4c+dx)} dx, x, x^3 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{3c+x^2} dx, x, \sqrt{c+dx^3} \right) + \frac{\text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{6d} \\
 &= \frac{\tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}} \right)}{2\sqrt{3}\sqrt{c}} - \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{6\sqrt{c}}
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 0.91

$$\frac{\sqrt{3} \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}} \right) - \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)), x]

[Out] (Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(6*Sqrt[c])

IntegrateAlgebraic [A] time = 0.05, size = 65, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]

[Out] ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(2*Sqrt[3]*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(6*Sqrt[c])

fricas [A] time = 0.82, size = 147, normalized size = 2.26

$$\left[\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right) + \sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{12c}, -\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right) - 2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)}{12c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="fricas")

[Out] [1/12*(2*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(dx^3 + c)/sqrt(c)) + sqrt(c)*log((dx^3 - 2*sqrt(dx^3 + c)*sqrt(c) + 2*c)/x^3))/c, -1/12*(sqrt(3)*sqrt(-c)*log((dx^3 - 2*sqrt(3)*sqrt(dx^3 + c)*sqrt(-c) - 2*c)/(dx^3 + 4*c)) - 2*sqrt(-c)*arctan(sqrt(dx^3 + c)*sqrt(-c)/c))/c]

giac [A] time = 0.16, size = 50, normalized size = 0.77

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{6\sqrt{c}} + \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="giac")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(dx^3 + c)/sqrt(c))/sqrt(c) + 1/6*arctan(sqrt(dx^3 + c)/sqrt(-c))/sqrt(-c)

maple [C] time = 0.19, size = 468, normalized size = 7.20

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x)`

[Out]
$$-1/4/c*d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*\sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))+1/4/c*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{(dx^3 + 4c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="maxima")`

[Out] `integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x), x)`

mupad [B] time = 4.66, size = 93, normalized size = 1.43

$$\frac{\ln\left(\frac{\left(\sqrt{dx^3+c}-\sqrt{c}\right)^3\left(\sqrt{dx^3+c}+\sqrt{c}\right)}{x^6}\right)}{12\sqrt{c}} + \frac{\sqrt{3}\ln\left(\frac{\sqrt{3}dx^3-2\sqrt{3}c+\sqrt{c}\sqrt{dx^3+c}6i}{dx^3+4c}\right)1i}{12\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)^(1/2)/(x*(4*c + d*x^3)),x)`

[Out]
$$\log\left(\frac{\left(\left(c + d*x^3\right)^{1/2} - c^{1/2}\right)^3\left(\left(c + d*x^3\right)^{1/2} + c^{1/2}\right)}{x^6}\right) / \left(12*c^{1/2}\right) + \left(3^{1/2}\right)*\log\left(\frac{c^{1/2}\left(c + d*x^3\right)^{1/2}*6i - 2*3^{1/2}*c + 3^{1/2}*d*x^3}{4*c + d*x^3}\right) / \left(12*c^{1/2}\right)$$

sympy [A] time = 13.28, size = 66, normalized size = 1.02

$$\frac{2 \left(\frac{d \operatorname{atan} \left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}} \right)}{12\sqrt{-c}} + \frac{\sqrt{3} d \operatorname{atan} \left(\frac{\sqrt{3} \sqrt{c+dx^3}}{3\sqrt{c}} \right)}{12\sqrt{c}} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c)**(1/2)/x/(d*x**3+4*c),x)

[Out] 2*(d*atan(sqrt(c + d*x**3)/sqrt(-c))/(12*sqrt(-c)) + sqrt(3)*d*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(12*sqrt(c)))/d

$$3.208 \quad \int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)} dx$$

Optimal. Leaf size=88

$$-\frac{d \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}} - \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}} - \frac{\sqrt{c+dx^3}}{12cx^3}$$

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {446, 99, 156, 63, 208, 203}

$$-\frac{d \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}} - \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}} - \frac{\sqrt{c+dx^3}}{12cx^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]

[Out] -Sqrt[c + d*x^3]/(12*c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])]/(24*c^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 99

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 156

Int[((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +

$f*x)^p/(a + b*x), x], x] - \text{Dist}[(d*g - c*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(c + d*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x]$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])]$

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b]$

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n) - 1}*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{c + dx^3}}{x^4(4c + dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c + dx}}{x^2(4c + dx)} dx, x, x^3 \right) \\
 &= -\frac{\sqrt{c + dx^3}}{12cx^3} + \frac{\text{Subst} \left(\int \frac{cd - \frac{d^2x}{2}}{x\sqrt{c+dx}(4c+dx)} dx, x, x^3 \right)}{12c} \\
 &= -\frac{\sqrt{c + dx^3}}{12cx^3} + \frac{d \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{48c} - \frac{d^2 \text{Subst} \left(\int \frac{1}{\sqrt{c+dx}(4c+dx)} dx, x, x^3 \right)}{16c} \\
 &= -\frac{\sqrt{c + dx^3}}{12cx^3} + \frac{\text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{24c} - \frac{d \text{Subst} \left(\int \frac{1}{3c+x^2} dx, x, \sqrt{c + dx^3} \right)}{8c} \\
 &= -\frac{\sqrt{c + dx^3}}{12cx^3} - \frac{d \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}} \right)}{8\sqrt{3}c^{3/2}} - \frac{d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{24c^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 1.00

$$-\frac{d \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}} - \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}} - \frac{\sqrt{c+dx^3}}{12cx^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]

[Out] -1/12*Sqrt[c + d*x^3]/(c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2))

IntegrateAlgebraic [A] time = 0.11, size = 88, normalized size = 1.00

$$-\frac{d \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}} - \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}} - \frac{\sqrt{c+dx^3}}{12cx^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]

[Out] -1/12*Sqrt[c + d*x^3]/(c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2))

fricas [A] time = 0.95, size = 194, normalized size = 2.20

$$\left[\frac{2\sqrt{3}\sqrt{c}dx^3 \arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right) - \sqrt{c}dx^3 \log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) + 4\sqrt{dx^3+c}c}{48c^2x^3}, \frac{\sqrt{3}\sqrt{-c}dx^3 \log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right) - 2\sqrt{-c}dx^3 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + 4\sqrt{dx^3+c}c}{48c^2x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="fricas")

[Out] [-1/48*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 4*sqrt(d*x^3 + c)*c)/(c^2*x^3), -1/48*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) - 2*sqrt(-c)*d*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]

giac [A] time = 0.17, size = 72, normalized size = 0.82

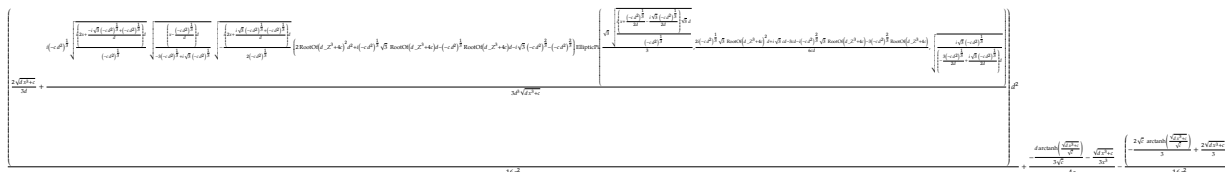
$$-\frac{\sqrt{3}d \arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{24c^{\frac{3}{2}}} + \frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{24\sqrt{-c}c} - \frac{\sqrt{dx^3+c}}{12cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(3)*d*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/24*
d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/12*sqrt(d*x^3 + c)/(c*x
^3)
```

maple [C] time = 0.19, size = 511, normalized size = 5.81



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x)
```

```
[Out] 1/4/c*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1
/2))+1/16*d^2/c^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/
3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)
*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
)*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^
2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*
_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c))-1/16/c^2*d*(2/3*(d*
x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{(dx^3 + 4c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^4), x)
```

mupad [B] time = 4.86, size = 113, normalized size = 1.28

$$\frac{d \ln \left(\frac{(\sqrt{dx^3+c}-\sqrt{c})^3 (\sqrt{dx^3+c}+\sqrt{c})}{x^6} \right)}{48 c^{3/2}} - \frac{\sqrt{dx^3+c}}{12 c x^3} + \frac{\sqrt{3} d \ln \left(\frac{2\sqrt{3}c - \sqrt{3}dx^3 + \sqrt{c}\sqrt{dx^3+c} 6i}{dx^3+4c} \right)}{48 c^{3/2}} 1i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)^(1/2)/(x^4*(4*c + d*x^3)), x)`

[Out] `(d*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))))/x^6)/ (48*c^(3/2)) - (c + d*x^3)^(1/2)/(12*c*x^3) + (3^(1/2)*d*log((2*3^(1/2)*c + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(48*c^(3/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x^4 (4c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)**(1/2)/x**4/(d*x**3+4*c), x)`

[Out] `Integral(sqrt(c + d*x**3)/(x**4*(4*c + d*x**3)), x)`

$$3.209 \quad \int \frac{x^8}{\sqrt{c+dx^3}(4c+dx^3)} dx$$

Optimal. Leaf size=78

$$\frac{32c^{3/2} \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^3} - \frac{10c\sqrt{c+dx^3}}{3d^3} + \frac{2(c+dx^3)^{3/2}}{9d^3}$$

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {446, 88, 63, 203}

$$\frac{32c^{3/2} \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^3} - \frac{10c\sqrt{c+dx^3}}{3d^3} + \frac{2(c+dx^3)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (-10*c*Sqrt[c + d*x^3])/(3*d^3) + (2*(c + d*x^3)^(3/2))/(9*d^3) + (32*c^(3/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^8}{\sqrt{c+dx^3}(4c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{\sqrt{c+dx}(4c+dx)} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{5c}{d^2\sqrt{c+dx}} + \frac{\sqrt{c+dx}}{d^2} + \frac{16c^2}{d^2\sqrt{c+dx}(4c+dx)} \right) dx, x, x^3 \right) \\
 &= -\frac{10c\sqrt{c+dx^3}}{3d^3} + \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{(16c^2) \text{Subst} \left(\int \frac{1}{\sqrt{c+dx}(4c+dx)} dx, x, x^3 \right)}{3d^2} \\
 &= -\frac{10c\sqrt{c+dx^3}}{3d^3} + \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{(32c^2) \text{Subst} \left(\int \frac{1}{3c+x^2} dx, x, \sqrt{c+dx^3} \right)}{3d^3} \\
 &= -\frac{10c\sqrt{c+dx^3}}{3d^3} + \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{32c^{3/2} \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}} \right)}{3\sqrt{3}d^3}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 0.83

$$\frac{32\sqrt{3}c^{3/2} \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}} \right) + 2(dx^3 - 14c)\sqrt{c+dx^3}}{9d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)), x]

[Out] (2*(-14*c + d*x^3)*Sqrt[c + d*x^3] + 32*Sqrt[3]*c^(3/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(9*d^3)

IntegrateAlgebraic [A] time = 0.06, size = 69, normalized size = 0.88

$$\frac{32c^{3/2} \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}} \right)}{3\sqrt{3}d^3} - \frac{2(14c - dx^3)\sqrt{c+dx^3}}{9d^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] $(-2*(14*c - d*x^3)*\text{Sqrt}[c + d*x^3])/(9*d^3) + (32*c^{(3/2)}*\text{ArcTan}[\text{Sqrt}[c + d*x^3]/(\text{Sqrt}[3]*\text{Sqrt}[c])])/(3*\text{Sqrt}[3]*d^3)$

fricas [A] time = 0.81, size = 129, normalized size = 1.65

$$\left[\frac{2 \left(8 \sqrt{3} \sqrt{-c} c \log \left(\frac{dx^3 + 2 \sqrt{3} \sqrt{dx^3 + c} \sqrt{-c} - 2c}{dx^3 + 4c} \right) + \sqrt{dx^3 + c} (dx^3 - 14c) \right)}{9 d^3}, \frac{2 \left(16 \sqrt{3} c^{\frac{3}{2}} \arctan \left(\frac{\sqrt{3} \sqrt{dx^3 + c}}{3 \sqrt{c}} \right) + \sqrt{dx^3 + c} (dx^3 - 14c) \right)}{9 d^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] $[2/9*(8*\text{sqrt}(3)*\text{sqrt}(-c)*c*\log((d*x^3 + 2*\text{sqrt}(3)*\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c) - 2*c)/(d*x^3 + 4*c)) + \text{sqrt}(d*x^3 + c)*(d*x^3 - 14*c))/d^3, 2/9*(16*\text{sqrt}(3)*c^{(3/2)}*\arctan(1/3*\text{sqrt}(3)*\text{sqrt}(d*x^3 + c)/\text{sqrt}(c)) + \text{sqrt}(d*x^3 + c)*(d*x^3 - 14*c))/d^3]$

giac [A] time = 0.16, size = 64, normalized size = 0.82

$$\frac{32 \sqrt{3} c^{\frac{3}{2}} \arctan \left(\frac{\sqrt{3} \sqrt{dx^3 + c}}{3 \sqrt{c}} \right)}{9 d^3} + \frac{2 \left((dx^3 + c)^{\frac{3}{2}} d^6 - 15 \sqrt{dx^3 + c} c d^6 \right)}{9 d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] $32/9*\text{sqrt}(3)*c^{(3/2)}*\arctan(1/3*\text{sqrt}(3)*\text{sqrt}(d*x^3 + c)/\text{sqrt}(c))/d^3 + 2/9*((d*x^3 + c)^{(3/2)}*d^6 - 15*\text{sqrt}(d*x^3 + c)*c*d^6)/d^9$

maple [C] time = 0.24, size = 467, normalized size = 5.99

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)

[Out] $1/d^2*(d*(2/9/d*x^3*(d*x^3+c)^{(1/2)}-4/9*c*(d*x^3+c)^{(1/2)}/d^2)-8/3*c*(d*x^3+c)^{(1/2)}/d)-16/9*I*c/d^5*2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)})*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}))$

/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c))

maxima [A] time = 1.34, size = 53, normalized size = 0.68

$$\frac{2 \left(16 \sqrt{3} c^{\frac{3}{2}} \arctan \left(\frac{\sqrt{3} \sqrt{dx^3+c}}{3 \sqrt{c}} \right) + (dx^3+c)^{\frac{3}{2}} - 15 \sqrt{dx^3+c} c \right)}{9 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] 2/9*(16*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3+c)/sqrt(c)) + (d*x^3+c)^(3/2) - 15*sqrt(d*x^3+c)*c)/d^3

mupad [B] time = 5.38, size = 88, normalized size = 1.13

$$\frac{2 x^3 \sqrt{d x^3+c}}{9 d^2} - \frac{28 c \sqrt{d x^3+c}}{9 d^3} + \frac{\sqrt{3} c^{3/2} \ln \left(\frac{\sqrt{3} d x^3-2 \sqrt{3} c+\sqrt{c} \sqrt{d x^3+c} 6 i}{d x^3+4 c} \right) 16 i}{9 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((c+d*x^3)^(1/2)*(4*c+d*x^3)),x)

[Out] (2*x^3*(c+d*x^3)^(1/2))/(9*d^2) - (28*c*(c+d*x^3)^(1/2))/(9*d^3) + (3^(1/2)*c^(3/2)*log((c^(1/2)*(c+d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/(4*c+d*x^3))*16i)/(9*d^3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{c+dx^3}(4c+dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)

[Out] Integral(x**8/(sqrt(c+d*x**3)*(4*c+d*x**3)), x)

$$3.210 \quad \int \frac{x^5}{\sqrt{c+dx^3}(4c+dx^3)} dx$$

Optimal. Leaf size=59

$$\frac{2\sqrt{c+dx^3}}{3d^2} - \frac{8\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^2}$$

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {446, 80, 63, 203}

$$\frac{2\sqrt{c+dx^3}}{3d^2} - \frac{8\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^2}$$

Antiderivative was successfully verified.

[In] Int[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] (2*Sqrt[c + d*x^3])/(3*d^2) - (8*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^2)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{\sqrt{c + dx^3} (4c + dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{\sqrt{c + dx} (4c + dx)} dx, x, x^3 \right) \\
 &= \frac{2\sqrt{c + dx^3}}{3d^2} - \frac{(4c) \text{Subst} \left(\int \frac{1}{\sqrt{c + dx} (4c + dx)} dx, x, x^3 \right)}{3d} \\
 &= \frac{2\sqrt{c + dx^3}}{3d^2} - \frac{(8c) \text{Subst} \left(\int \frac{1}{3c + x^2} dx, x, \sqrt{c + dx^3} \right)}{3d^2} \\
 &= \frac{2\sqrt{c + dx^3}}{3d^2} - \frac{8\sqrt{c} \tan^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{3} \sqrt{c}} \right)}{3\sqrt{3} d^2}
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 0.95

$$\frac{6\sqrt{c + dx^3} - 8\sqrt{3} \sqrt{c} \tan^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{3} \sqrt{c}} \right)}{9d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] (6*Sqrt[c + d*x^3] - 8*Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(9*d^2)

IntegrateAlgebraic [A] time = 0.05, size = 59, normalized size = 1.00

$$\frac{2\sqrt{c + dx^3}}{3d^2} - \frac{8\sqrt{c} \tan^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{3} \sqrt{c}} \right)}{3\sqrt{3} d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] (2*Sqrt[c + d*x^3])/(3*d^2) - (8*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^2)

fricas [A] time = 0.85, size = 112, normalized size = 1.90

$$\left[\frac{2 \left(2 \sqrt{3} \sqrt{-c} \log \left(\frac{dx^3 - 2 \sqrt{3} \sqrt{dx^3 + c} \sqrt{-c} - 2c}{dx^3 + 4c} \right) + 3 \sqrt{dx^3 + c} \right)}{9 d^2}, - \frac{2 \left(4 \sqrt{3} \sqrt{c} \arctan \left(\frac{\sqrt{3} \sqrt{dx^3 + c}}{3 \sqrt{c}} \right) - 3 \sqrt{dx^3 + c} \right)}{9 d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [2/9*(2*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) + 3*sqrt(d*x^3 + c))/d^2, -2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*sqrt(d*x^3 + c))/d^2]

giac [A] time = 0.16, size = 49, normalized size = 0.83

$$- \frac{2 \left(\frac{4 \sqrt{3} \sqrt{c} \arctan \left(\frac{\sqrt{3} \sqrt{dx^3 + c}}{3 \sqrt{c}} \right)}{d} - \frac{3 \sqrt{dx^3 + c}}{d} \right)}{9 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] -2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d - 3*sqrt(d*x^3 + c)/d

maple [C] time = 0.18, size = 425, normalized size = 7.20

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)

[Out] 2/3*(d*x^3+c)^(1/2)/d^2+4/9*I/d^4*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)

$$\frac{1}{(dx^3+c)^{1/2}} \cdot (2 \sqrt[3]{d} + I \sqrt[3]{-cd^2})^{1/3} \cdot 3^{1/2} \sqrt[3]{d} - (-cd^2)^{1/3} \sqrt[3]{d} - I \sqrt[3]{d} \cdot (-cd^2)^{2/3} - (-cd^2)^{2/3} \cdot \text{EllipticPi}\left(\frac{1}{3} \sqrt[3]{d} \cdot (I \sqrt[3]{d} + (-cd^2)^{1/3}) / d - \frac{1}{2} I \sqrt[3]{d} \cdot (-cd^2)^{1/3} / d\right) \cdot 3^{1/2} / (-cd^2)^{1/3} \cdot d^{1/2}, \frac{1}{6} (2 I \sqrt[3]{d} + I \sqrt[3]{d}) \cdot 3^{1/2} \sqrt[3]{d} + I \sqrt[3]{d} \cdot 3^{1/2} \cdot d - 3 \sqrt[3]{d} - I \sqrt[3]{d} \cdot (-cd^2)^{2/3} \cdot 3^{1/2} \sqrt[3]{d} - 3 \sqrt[3]{d} \cdot (-cd^2)^{2/3} \sqrt[3]{d} / c / d, (I \sqrt[3]{d} \cdot (-cd^2)^{1/3} / (-3/2 \sqrt[3]{d} + I \sqrt[3]{d} \cdot (-cd^2)^{1/3} / d) / d)^{1/2}, \sqrt[3]{d} = \text{RootOf}(_Z^3 d + 4c))$$

maxima [A] time = 1.46, size = 43, normalized size = 0.73

$$\frac{2 \left(4 \sqrt{3} \sqrt{c} \arctan \left(\frac{\sqrt{3} \sqrt{dx^3+c}}{3 \sqrt{c}} \right) - 3 \sqrt{dx^3+c} \right)}{9 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] $-2/9 * (4 * \text{sqrt}(3) * \text{sqrt}(c) * \arctan(1/3 * \text{sqrt}(3) * \text{sqrt}(d * x^3 + c) / \text{sqrt}(c)) - 3 * \text{sqrt}(d * x^3 + c)) / d^2$

mupad [B] time = 4.86, size = 71, normalized size = 1.20

$$\frac{2 \sqrt{d x^3 + c}}{3 d^2} + \frac{\sqrt{3} \sqrt{c} \ln \left(\frac{2 \sqrt{3} c - \sqrt{3} d x^3 + \sqrt{c} \sqrt{d x^3 + c}}{d x^3 + 4 c} \right) 4i}{9 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)

[Out] $(2 * (c + d * x^3)^{1/2}) / (3 * d^2) + (3^{1/2} * c^{1/2} * \log((2 * 3^{1/2} * c + c^{1/2}) * (c + d * x^3)^{1/2} * 6i - 3^{1/2} * d * x^3) / (4 * c + d * x^3)) * 4i) / (9 * d^2)$

sympy [A] time = 15.58, size = 65, normalized size = 1.10

$$\begin{cases} \frac{2 \left(-\frac{4 \sqrt{3} \sqrt{c} \operatorname{atan} \left(\frac{\sqrt{3} \sqrt{c+dx^3}}{3 \sqrt{c}} \right)}{9d} + \frac{\sqrt{c+dx^3}}{3d} \right)}{d} & \text{for } d \neq 0 \\ \frac{x^6}{24c^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)

[Out] $\text{Piecewise}((2 * (-4 * \text{sqrt}(3) * \text{sqrt}(c) * \operatorname{atan}(\text{sqrt}(3) * \text{sqrt}(c + d * x^3) / (3 * \text{sqrt}(c)))) / (9 * d) + \text{sqrt}(c + d * x^3) / (3 * d)) / d, \text{Ne}(d, 0)), (x^6 / (24 * c^{3/2}), \text{True}))$

$$3.211 \quad \int \frac{x^2}{\sqrt{c+dx^3}(4c+dx^3)} dx$$

Optimal. Leaf size=40

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3} \sqrt{c}} \right)}{3\sqrt{3} \sqrt{c} d}$$

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {444, 63, 203}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3} \sqrt{c}} \right)}{3\sqrt{3} \sqrt{c} d}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{\sqrt{c+dx^3}(4c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt{c+dx}(4c+dx)} dx, x, x^3 \right) \\
 &= \frac{2 \text{Subst} \left(\int \frac{1}{3c+x^2} dx, x, \sqrt{c+dx^3} \right)}{3d} \\
 &= \frac{2 \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}} \right)}{3\sqrt{3}\sqrt{c}d}
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}} \right)}{3\sqrt{3}\sqrt{c}d}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)

IntegrateAlgebraic [A] time = 0.04, size = 40, normalized size = 1.00

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}} \right)}{3\sqrt{3}\sqrt{c}d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)

fricas [A] time = 0.72, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{3}\sqrt{-c} \log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)}{9cd}, \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [-1/9*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c))/(c*d), 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)]

giac [A] time = 0.18, size = 29, normalized size = 0.72

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)

maple [C] time = 0.18, size = 413, normalized size = 10.32

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)

[Out] -1/9*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2), 1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d, (I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)), _alpha=RootOf(_Z^3*d+4*c))

maxima [A] time = 1.24, size = 29, normalized size = 0.72

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)

mupad [B] time = 5.21, size = 56, normalized size = 1.40

$$\frac{\sqrt{3} \ln\left(\frac{\sqrt{3} dx^3 - 2\sqrt{3} c + \sqrt{c} \sqrt{dx^3 + c}}{2dx^3 + 8c}\right) 1i}{9\sqrt{c} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)

[Out] (3^(1/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/(8*c + 2*d*x^3))*1i)/(9*c^(1/2)*d)

sympy [A] time = 10.09, size = 37, normalized size = 0.92

$$\frac{2\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3} \sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)

[Out] 2*sqrt(3)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(9*sqrt(c)*d)

$$3.212 \quad \int \frac{1}{x\sqrt{c+dx^3}(4c+dx^3)} dx$$

Optimal. Leaf size=65

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {446, 86, 63, 208, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] -ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(6*Sqrt[3]*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(6*c^(3/2))

Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_)^(p_))/((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)),
x_Symbol] := Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
p}, x] && !IntegerQ[p]
```

Rule 203

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt
[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x\sqrt{c+dx^3}(4c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}(4c+dx)} dx, x, x^3 \right) \\
 &= \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{12c} - \frac{d \text{Subst} \left(\int \frac{1}{\sqrt{c+dx}(4c+dx)} dx, x, x^3 \right)}{12c} \\
 &= -\frac{\text{Subst} \left(\int \frac{1}{3c+x^2} dx, x, \sqrt{c+dx^3} \right)}{6c} + \frac{\text{Subst} \left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{6cd} \\
 &= -\frac{\tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}} \right)}{6\sqrt{3}c^{3/2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{6c^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 0.91

$$-\frac{\sqrt{3} \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}} \right) + 3 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{18c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] -1/18*(sqrt[3]*ArcTan[sqrt[c + d*x^3]/(sqrt[3]*sqrt[c])]) + 3*ArcTanh[sqrt[c + d*x^3]/sqrt[c]])/c^(3/2)

IntegrateAlgebraic [A] time = 0.05, size = 65, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] -1/6*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(Sqrt[3]*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(6*c^(3/2))

fricas [A] time = 0.84, size = 148, normalized size = 2.28

$$\left[\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right) - 3\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{36c^2}, -\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right) - 6\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)}{36c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [-1/36*(2*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, -1/36*(sqrt(3)*sqrt(-c)*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) - 6*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/c^2]

giac [A] time = 0.17, size = 53, normalized size = 0.82

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{18c^{3/2}} + \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/6*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)

maple [C] time = 0.19, size = 433, normalized size = 6.66

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{6c^{3/2}} - \frac{\operatorname{arctan}\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{18c^{3/2}} + \frac{\operatorname{arctan}\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)`

[Out] $\frac{1}{36} \frac{I}{c^2 d^2} 2^{1/2} \sum \left((-c d^2)^{1/3} \left(\frac{1}{2} I (2x + (-I 3^{1/2}) (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d \right) / (-c d^2)^{1/3} d^{1/2} \left(\frac{x - (-c d^2)^{1/3} / d}{-3 (-c d^2)^{1/3} + I 3^{1/2} (-c d^2)^{1/3}} \right) d^{1/2} \left(-\frac{1}{2} I (2x + (I 3^{1/2}) (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d \right) / (-c d^2)^{1/3} d^{1/2} / (d x^3 + c)^{1/2} \left(2 * _alpha^2 d^2 + I (-c d^2)^{1/3} 3^{1/2} * _alpha d - (-c d^2)^{1/3} * _alpha d - I 3^{1/2} (-c d^2)^{2/3} - (-c d^2)^{2/3} \right) * \text{EllipticPi} \left(\frac{1}{3} 3^{1/2} (I (x + 1/2 (-c d^2)^{1/3} / d - 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) 3^{1/2} / (-c d^2)^{1/3} d)^{1/2} \right), \frac{1}{6} (2 I (-c d^2)^{1/3} 3^{1/2} * _alpha^2 d + I 3^{1/2} c d - 3 c d - I (-c d^2)^{2/3} 3^{1/2} * _alpha - 3 (-c d^2)^{2/3} * _alpha) / c d, (I 3^{1/2} (-c d^2)^{1/3} / (-3/2 (-c d^2)^{1/3} / d + 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) / d)^{1/2} \right), _alpha = \text{RootOf}(_Z^3 d + 4 c) - 1/6 * \text{arctanh}((d x^3 + c)^{1/2} / c^{1/2}) / c^{3/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx^3 + 4c)\sqrt{dx^3 + cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x), x)`

mupad [B] time = 5.51, size = 94, normalized size = 1.45

$$\frac{\ln \left(\frac{\left(\sqrt{dx^3+c} - \sqrt{c} \right)^3 \left(\sqrt{dx^3+c} + \sqrt{c} \right)}{x^6} \right)}{12 c^{3/2}} + \frac{\sqrt{3} \ln \left(\frac{2 \sqrt{3} c - \sqrt{3} dx^3 + \sqrt{c} \sqrt{dx^3+c} 6i}{dx^3+4c} \right) 1i}{36 c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)`

[Out] $\log \left(\frac{((c + d x^3)^{1/2} - c^{1/2})^3 ((c + d x^3)^{1/2} + c^{1/2})}{x^6} \right) / (12 c^{3/2}) + (3^{1/2} * \log((2 * 3^{1/2} * c + c^{1/2} * (c + d x^3)^{1/2}) * 6i - 3^{1/2} * d x^3) / (4 c + d x^3)) * 1i) / (36 c^{3/2})$

sympy [A] time = 12.25, size = 63, normalized size = 0.97

$$\frac{\text{atan} \left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}} \right)}{6c\sqrt{-c}} - \frac{\sqrt{3} \text{atan} \left(\frac{\sqrt{3} \sqrt{c+dx^3}}{3\sqrt{c}} \right)}{18c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**3)/sqrt(-c))/(6*c*sqrt(-c)) - sqrt(3)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(18*c**(3/2))
```

$$3.213 \quad \int \frac{1}{x^4 \sqrt{c+dx^3} (4c+dx^3)} dx$$

Optimal. Leaf size=88

$$\frac{d \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3} \sqrt{c}} \right)}{24\sqrt{3} c^{5/2}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{8c^{5/2}} - \frac{\sqrt{c+dx^3}}{12c^2 x^3}$$

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {446, 103, 156, 63, 208, 203}

$$-\frac{\sqrt{c+dx^3}}{12c^2 x^3} + \frac{d \tan^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{3} \sqrt{c}} \right)}{24\sqrt{3} c^{5/2}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{8c^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] -Sqrt[c + d*x^3]/(12*c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +

$f*x)^p/(a + b*x), x], x] - \text{Dist}[(d*g - c*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(c + d*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x]$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])]$

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b]$

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n) - 1}*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^4 \sqrt{c + dx^3} (4c + dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2 \sqrt{c + dx} (4c + dx)} dx, x, x^3 \right) \\
 &= -\frac{\sqrt{c + dx^3}}{12c^2 x^3} - \frac{\text{Subst} \left(\int \frac{3cd + \frac{d^2 x}{2}}{x \sqrt{c + dx} (4c + dx)} dx, x, x^3 \right)}{12c^2} \\
 &= -\frac{\sqrt{c + dx^3}}{12c^2 x^3} - \frac{d \text{Subst} \left(\int \frac{1}{x \sqrt{c + dx}} dx, x, x^3 \right)}{16c^2} + \frac{d^2 \text{Subst} \left(\int \frac{1}{\sqrt{c + dx} (4c + dx)} dx, x, x^3 \right)}{48c^2} \\
 &= -\frac{\sqrt{c + dx^3}}{12c^2 x^3} - \frac{\text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{8c^2} + \frac{d \text{Subst} \left(\int \frac{1}{3c + x^2} dx, x, \sqrt{c + dx^3} \right)}{24c^2} \\
 &= -\frac{\sqrt{c + dx^3}}{12c^2 x^3} + \frac{d \tan^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{3} \sqrt{c}} \right)}{24\sqrt{3}c^{5/2}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{8c^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 1.00

$$\frac{d \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}} - \frac{\sqrt{c+dx^3}}{12c^2x^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] -1/12*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2))

IntegrateAlgebraic [A] time = 0.09, size = 88, normalized size = 1.00

$$\frac{d \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}} - \frac{\sqrt{c+dx^3}}{12c^2x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] -1/12*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2))

fricas [A] time = 0.65, size = 194, normalized size = 2.20

$$\left[\frac{2\sqrt{3}\sqrt{c}dx^3 \arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right) + 9\sqrt{c}dx^3 \log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) - 12\sqrt{dx^3+c}c}{144c^3x^3}, -\frac{\sqrt{3}\sqrt{-c}dx^3 \log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right) + 18\sqrt{-c}dx^3 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + 12\sqrt{dx^3+c}c}{144c^3x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/144*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 12*sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/144*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) + 18*sqrt(-c)*d*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]

giac [A] time = 0.16, size = 72, normalized size = 0.82

$$\frac{\sqrt{3}d \arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{72c^{\frac{5}{2}}} - \frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{8\sqrt{-c}c^2} - \frac{\sqrt{dx^3+c}}{12c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] 1/72*sqrt(3)*d*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(5/2) - 1/8*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/12*sqrt(d*x^3 + c)/(c^2*x^3)

maple [C] time = 0.23, size = 477, normalized size = 5.42

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)

[Out] 1/4/c*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-1/144*I/d/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2),_alpha=RootOf(_Z^3*d+4*c))+1/24*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx^3 + 4c)\sqrt{dx^3 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^4), x)

mupad [B] time = 5.72, size = 112, normalized size = 1.27

$$\frac{d \ln \left(\frac{(\sqrt{dx^3+c}-\sqrt{c})(\sqrt{dx^3+c}+\sqrt{c})^3}{x^6} \right)}{16c^{5/2}} - \frac{\sqrt{dx^3+c}}{12c^2x^3} + \frac{\sqrt{3} d \ln \left(\frac{\sqrt{3} dx^3-2\sqrt{3}c+\sqrt{c}\sqrt{dx^3+c}6i}{dx^3+4c} \right)}{144c^{5/2}} 1i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)`

[Out] `(d*log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)/
(16*c^(5/2)) - (c + d*x^3)^(1/2)/(12*c^2*x^3) + (3^(1/2)*d*log((c^(1/2)*(
c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(144*
c^(5/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt{c + dx^3} (4c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)`

[Out] `Integral(1/(x**4*sqrt(c + d*x**3)*(4*c + d*x**3)), x)`

$$3.214 \quad \int \frac{x}{\sqrt{c+dx^3}(4c+dx^3)} dx$$

Optimal. Leaf size=206

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{dx}\right)}{\sqrt{c+dx^3}}\right)}{3^{2/3}\sqrt{3}c^{5/6}d^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3^{2/3}\sqrt{3}c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{dx}\right)}{\sqrt{c+dx^3}}\right)}{3^{2/3}c^{5/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9^{2/3}c^{5/6}d^{2/3}}$$

Rubi [A] time = 0.03, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {484}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{dx}\right)}{\sqrt{c+dx^3}}\right)}{3^{2/3}\sqrt{3}c^{5/6}d^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3^{2/3}\sqrt{3}c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{dx}\right)}{\sqrt{c+dx^3}}\right)}{3^{2/3}c^{5/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9^{2/3}c^{5/6}d^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] -ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3*2^(2/3)*c^(5/6)*d^(2/3)) + ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(9*2^(2/3)*c^(5/6)*d^(2/3))

Rule 484

Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> With[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d, 0] && PosQ[c]

Rubi steps

$$\int \frac{x}{\sqrt{c+dx^3} (4c+dx^3)} dx = -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{dx}\right)}{\sqrt{c+dx^3}}\right)}{3^{2/3}\sqrt{3}c^{5/6}d^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3^{2/3}\sqrt{3}c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{dx}\right)}{\sqrt{c+dx^3}}\right)}{3^{2/3}c^{5/6}d^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3^{2/3}\sqrt{3}c^{5/6}d^{2/3}}$$

Mathematica [C] time = 0.03, size = 67, normalized size = 0.33

$$\frac{x^2 \sqrt{\frac{c+dx^3}{c}} F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; -\frac{dx^3}{c}, -\frac{dx^3}{4c}\right)}{8c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)), x]

[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(8*c*Sqrt[c + d*x^3])

IntegrateAlgebraic [F] time = 15.36, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c+dx^3} (4c+dx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)), x]

[Out] Defer[IntegrateAlgebraic][x/(Sqrt[c + d*x^3]*(4*c + d*x^3)), x]

fricas [B] time = 2.37, size = 2274, normalized size = 11.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2), x, algorithm="fricas")

[Out] 1/9*sqrt(3)*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*arctan(-1/3*(3*(sqrt(3)*sqrt(1/3)*c^3*d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^4))^(1/6) + 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 + 4*c^5*d^3)*(-1/(c^5*d^4))^(5/6))*sqrt(d*x^3 + c) + (2*sqrt(3)*(1/2)^(1/3)*(c^2*d^2*x^3 + c^3*d)*(-1/(c^5*d^4))^(1/3) + sqrt(3)*(d*x^4 + c*x) + 3*(sqrt(3)*sqrt(1/3)*c^3*d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^4))^(1/6) - 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 - 2*c^5*d^3)*(-1/(c^5*d^4))^(5/6))

$$\begin{aligned}
&/6))\sqrt{d*x^3 + c)}\sqrt{(d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^{(2/3)} \\
&)*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^{(2/3)} + 12*(1/ \\
&2)^{(1/3)}*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^{(1/3)} \\
&+ 12*(648*(1/432)^{(5/6)}*c^5*d^5*x^5*(-1/(c^5*d^4))^{(5/6)} - \sqrt{1/3}*(c^3* \\
&d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*\sqrt{-1/(c^5*d^4)} - (1/432)^{(1/6)}*(c \\
&*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^{(1/6)})\sqrt{d*x^3 + c} \\
&)/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3))/(d*x^4 + c*x)) + 1/9*\sqrt{3}*(1/432)^{(1/6)}*(-1/(c^5*d^4))^{(1/6)}*\arctan(-1/3*(\sqrt{3}*\sqrt{1/3} \\
&*c^3*d^2*x*\sqrt{-1/(c^5*d^4)} + 2*\sqrt{3}*(1/432)^{(1/6)}*c*d*x^2*(-1/(c^5*d^ \\
&4))^{(1/6)} + 24*\sqrt{3}*(1/432)^{(5/6)}*(c^4*d^4*x^3 + 4*c^5*d^3)*(-1/(c^5*d^4 \\
&))^{(5/6)})\sqrt{d*x^3 + c} - (2*\sqrt{3}*(1/2)^{(1/3)}*(c^2*d^2*x^3 + c^3*d)*(- \\
&1/(c^5*d^4))^{(1/3)} + \sqrt{3}*(d*x^4 + c*x) - 3*(\sqrt{3}*\sqrt{1/3}*c^3*d^2*x \\
&*\sqrt{-1/(c^5*d^4)} + 2*\sqrt{3}*(1/432)^{(1/6)}*c*d*x^2*(-1/(c^5*d^4))^{(1/6)} \\
&- 24*\sqrt{3}*(1/432)^{(5/6)}*(c^4*d^4*x^3 - 2*c^5*d^3)*(-1/(c^5*d^4))^{(5/6)})* \\
&\sqrt{d*x^3 + c)}\sqrt{(d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^{(2/3)}*(c^ \\
&4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^{(2/3)} + 12*(1/2)^{(1 \\
&/3)}*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^{(1/3)} - 12 \\
&*(648*(1/432)^{(5/6)}*c^5*d^5*x^5*(-1/(c^5*d^4))^{(5/6)} - \sqrt{1/3}*(c^3*d^4*x \\
&^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*\sqrt{-1/(c^5*d^4)} - (1/432)^{(1/6)}*(c*d^3*x \\
&x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^{(1/6)})\sqrt{d*x^3 + c))/(d^ \\
&3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3))/(d*x^4 + c*x)) - 1/36*(1/43 \\
&2)^{(1/6)}*(-1/(c^5*d^4))^{(1/6)}*\log((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/ \\
&2)^{(2/3)}*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^{(2/3)} + 12* \\
&(1/2)^{(1/3)}*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^{(1/3)} \\
&- 12*(648*(1/432)^{(5/6)}*c^5*d^5*x^5*(-1/(c^5*d^4))^{(5/6)} - \sqrt{1/3}*(c \\
&^3*d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*\sqrt{-1/(c^5*d^4)} - (1/432)^{(1/6)} \\
&)*(c*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^{(1/6)})\sqrt{d*x^3 + \\
&c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 1/36*(1/432)^{(\\
&1/6)}*(-1/(c^5*d^4))^{(1/6)}*\log((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^{(\\
&2/3)}*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^{(2/3)} + 12* \\
&(1/2)^{(1/3)}*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^{(1 \\
&/3)} - 12*(648*(1/432)^{(5/6)}*c^5*d^5*x^5*(-1/(c^5*d^4))^{(5/6)} - \sqrt{1/3}*(c \\
&^3*d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*\sqrt{-1/(c^5*d^4)} - (1/432)^{(1/6)} \\
&)*(c*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^{(1/6)})\sqrt{d*x^3 + \\
&c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 1/18*(1/432)^{(1/6)} \\
&)*(-1/(c^5*d^4))^{(1/6)}*\log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 + \\
&48*(1/2)^{(2/3)}*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x)*(-1/(c^5*d^4))^{(2 \\
&/3)} + 12*(1/2)^{(1/3)}*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5 \\
&*d^4))^{(1/3)} + 6*(1296*(1/432)^{(5/6)}*c^5*d^5*x^5*(-1/(c^5*d^4))^{(5/6)} + \sqrt{ \\
&t(1/3}*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^3 - 16*c^5*d^2)*\sqrt{-1/(c^5*d^4)} + 2 \\
&*(1/432)^{(1/6)}*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^{(1/6 \\
&))\sqrt{d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 1/1 \\
&8*(1/432)^{(1/6)}*(-1/(c^5*d^4))^{(1/6)}*\log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d \\
&*x^3 - 32*c^3 + 48*(1/2)^{(2/3)}*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x)*(- \\
&1/(c^5*d^4))^{(2/3)} + 12*(1/2)^{(1/3)}*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^
\end{aligned}$$

$$2*x^2)*(-1/(c^5*d^4))^(1/3) - 6*(1296*(1/432))^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) + \text{sqrt}(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^3 - 16*c^5*d^2)*\text{sqrt}(-1/(c^5*d^4)) + 2*(1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*\text{sqrt}(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3))$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx^3 + 4c)\sqrt{dx^3 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)

maple [C] time = 0.18, size = 416, normalized size = 2.02

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)

[Out]
$$-1/9*I/d^3/c^2^{(1/2)}*\text{sum}(1/_\text{alpha}*(-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)})/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_\text{alpha}^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_\text{alpha}*d-(-c*d^2)^{(1/3)}*_\text{alpha}*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)})/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)})/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, 1/6*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_\text{alpha}^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_\text{alpha}-3*(-c*d^2)^{(2/3)}*_\text{alpha})/c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)})/(-3/2*(-c*d^2)^{(1/3)})/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)})/d/d)^{(1/2)}, _\text{alpha}=\text{RootOf}(_Z^3*d+4*c))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx^3 + 4c)\sqrt{dx^3 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)

mupad [B] time = 25.80, size = 453, normalized size = 2.20

$$\frac{\sqrt{314928} \operatorname{arctan}\left(\frac{\sqrt{314928} \sqrt{c+d x^3}}{2916(-c)^{5/6}}\right) + \sqrt{314928} \operatorname{arctan}\left(\frac{\sqrt{314928} \sqrt{c+d x^3}}{2916(-c)^{5/6}}\right)}{2916(-c)^{5/6}} + \frac{\sqrt{314928} \operatorname{arctan}\left(\frac{\sqrt{314928} \sqrt{c+d x^3}}{2916(-c)^{5/6}}\right) + \sqrt{314928} \operatorname{arctan}\left(\frac{\sqrt{314928} \sqrt{c+d x^3}}{2916(-c)^{5/6}}\right)}{2916(-c)^{5/6}} + \frac{\sqrt{314928} \operatorname{arctan}\left(\frac{\sqrt{314928} \sqrt{c+d x^3}}{2916(-c)^{5/6}}\right) + \sqrt{314928} \operatorname{arctan}\left(\frac{\sqrt{314928} \sqrt{c+d x^3}}{2916(-c)^{5/6}}\right)}{2916(-c)^{5/6}} + \frac{\sqrt{314928} \operatorname{arctan}\left(\frac{\sqrt{314928} \sqrt{c+d x^3}}{2916(-c)^{5/6}}\right) + \sqrt{314928} \operatorname{arctan}\left(\frac{\sqrt{314928} \sqrt{c+d x^3}}{2916(-c)^{5/6}}\right)}{2916(-c)^{5/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)

[Out] (3^(1/2)*314928^(1/3)*log(((c + d*x^3)^(1/2) + 3^(1/2)*(-c)^(1/2) - 2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(54*(c + d*x^3)^(1/2) - 54*3^(1/2)*(-c)^(1/2) + 54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)/(d^(1/3)*x - 2^(2/3)*(-c)^(1/3))^6)/(2916*(-c)^(5/6)*d^(2/3)) + (3^(1/2)*314928^(1/3)*log(((2*3^(1/2)*(-c)^(1/2) - 2*(c + d*x^3)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*3i + 2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(108*(c + d*x^3)^(1/2) + 108*3^(1/2)*(-c)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*162i + 54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x))/(2*d^(1/3)*x + 2^(2/3)*(-c)^(1/3) - 2^(2/3)*3^(1/2)*(-c)^(1/3)*1i)^6)*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(2916*(-c)^(5/6)*d^(2/3)) + (3^(1/2)*314928^(1/3)*log(((2*(c + d*x^3)^(1/2) + 2*3^(1/2)*(-c)^(1/2) - 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*3i + 2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(108*(c + d*x^3)^(1/2) - 108*3^(1/2)*(-c)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*162i - 54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x))/(2*d^(1/3)*x + 2^(2/3)*(-c)^(1/3) + 2^(2/3)*3^(1/2)*(-c)^(1/3)*1i)^6)*((3^(1/2)*1i)/2 + 1/2)^(1/2)*1i)/(2916*(-c)^(5/6)*d^(2/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c + dx^3} (4c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)

[Out] Integral(x/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)

$$3.215 \quad \int \frac{x}{\sqrt{1-x^3}(4-x^3)} dx$$

Optimal. Leaf size=127

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}x)}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3}} + \frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9 \cdot 2^{2/3}}$$

Rubi [A] time = 0.02, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {484}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}x)}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3}} + \frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]

[Out] -ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(3*2^(2/3)*Sqrt[3]) + ArcTan[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/Sqrt[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))

Rule 484

Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> With[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d, 0] && PosQ[c]

Rubi steps

$$\int \frac{x}{\sqrt{1-x^3}(4-x^3)} dx = -\frac{\tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}x)}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3}} + \frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9 \cdot 2^{2/3}}$$

Mathematica [C] time = 0.03, size = 28, normalized size = 0.22

$$\frac{1}{8}x^2F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; x^3, \frac{x^3}{4}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]

[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8

IntegrateAlgebraic [F] time = 20.88, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{1-x^3}(4-x^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(Sqrt[1 - x^3]*(4 - x^3)),x]

[Out] Defer[IntegrateAlgebraic][x/(Sqrt[1 - x^3]*(4 - x^3)), x]

fricas [B] time = 1.26, size = 1191, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/31104*432^{(5/6)}*\text{sqrt}(3)*\log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^{(2/3)}*(x^8 - 5*x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^{(5/6)}*\text{sqrt}(3)*(x^7 - 26*x^4 + 16*x) - 216*432^{(1/6)}*\text{sqrt}(3)*(7*x^5 - 4*x^2))*\text{sqrt}(-x^3 + 1) + 3888*2^{(1/3)}*(x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) - 1/31104*432^{(5/6)}*\text{sqrt}(3)*\log(36*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^{(2/3)}*(x^8 - 5*x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^{(5/6)}*\text{sqrt}(3)*(x^7 - 26*x^4 + 16*x) - 216*432^{(1/6)}*\text{sqrt}(3)*(7*x^5 - 4*x^2))*\text{sqrt}(-x^3 + 1) + 3888*2^{(1/3)}*(x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^{(5/6)}*\text{sqrt}(3)*\log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^{(2/3)}*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6 - 2592*x^3 - 432^{(5/6)}*\text{sqrt}(3)*(x^7 - 26*x^4 + 16*x) - 216*432^{(1/6)}*\text{sqrt}(3)*(7*x^5 - 4*x^2))*\text{sqrt}(-x^3 + 1) + 3888*2^{(1/3)}*(x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) - 1/1944*432^{(5/6)}*\text{arctan}(1/216*\text{sqrt}(-x^3 + 1)*(72*432^{(1/6)}*x^2 + 432^{(5/6)}*x + 72*\text{sqrt}(3))/(2*x^3 - 1)) + 1/3888*432^{(5/6)}*\text{arctan}(-1/648*(6*\text{sqrt}(-x^3 + 1)*(432^{(5/6)}*(x^4 + 2*x) - 36*\text{sqrt}(3)*(x^3 - 4 \end{aligned}$$

) + 18*432^(1/6)*(x^5 + 8*x^2)) + (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sqrt(3)*2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) - sqrt(-x^3 + 1)*(432^(5/6)*(2*x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2))) * sqrt((36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4)) + 1/3888*432^(5/6)*arctan(-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4) + 18*432^(1/6)*(x^5 + 8*x^2)) - (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sqrt(3)*2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) + sqrt(-x^3 + 1)*(432^(5/6)*(2*x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2))) * sqrt((36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{(x^3 - 4)\sqrt{-x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)

maple [C] time = 0.80, size = 164, normalized size = 1.29

$$\frac{i\sqrt{(2x+1-i\sqrt{3})}\sqrt{\frac{x-1}{i\sqrt{3-3}}}\sqrt{\frac{i(2x+1+i\sqrt{3})}{2}}(-2\text{RootOf}(_Z^2-4)^2+\text{RootOf}(_Z^2-4)+1+i\sqrt{3}(-\text{RootOf}(_Z^2-4)+1))\text{RootOf}(_Z^2-4)^2\text{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{x+1-i\sqrt{3}}{3}}\sqrt{5}}{3},\frac{i\sqrt{3}\text{RootOf}(_Z^2-4)^2}{3}+\frac{\text{RootOf}(_Z^2-4)}{2}+\frac{i\sqrt{3}\text{RootOf}(_Z^2-4)}{6}-\frac{1}{2}+\frac{i\sqrt{3}}{6},\sqrt{\frac{i\sqrt{3}}{\frac{1}{2}+i\sqrt{3}}}\right)}{36\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^3+4)/(-x^3+1)^(1/2),x)

[Out] 1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(2*x+1-I*3^(1/2)))^(1/2)*((x-1)/(I*3^(1/2)-3))^(1/2)*(-1/2*I*(2*x+1+I*3^(1/2)))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha^2+_alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2)))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*3^(1/2)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^3-4))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{(x^3 - 4)\sqrt{-x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")

[Out] -integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)

mupad [B] time = 3.42, size = 653, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x/((1 - x^3)^(1/2)*(x^3 - 4)),x)

[Out]
$$-\frac{2^{1/3} \left((3^{1/2} i)/2 + 3/2 \right) (x^3 - 1)^{1/2} \left(-(x - (3^{1/2} i)/2 + 1/2) / \left((3^{1/2} i)/2 - 3/2 \right)^{1/2} \left((x + (3^{1/2} i)/2 + 1/2) / \left((3^{1/2} i)/2 + 3/2 \right) \right)^{1/2} \operatorname{ellipticPi} \left(-\left((3^{1/2} i)/2 + 3/2 \right) / \left(2^{2/3} - 1 \right), \operatorname{asin} \left(-(x - 1) / \left((3^{1/2} i)/2 + 3/2 \right) \right)^{1/2} \right) - \left((3^{1/2} i)/2 + 3/2 \right) / \left((3^{1/2} i)/2 - 3/2 \right)}{3 \left(1 - x^3 \right)^{1/2} \left(2^{2/3} - 1 \right) \left(\left((3^{1/2} i)/2 - 1/2 \right) \left((3^{1/2} i)/2 + 1/2 \right) - x \left(\left((3^{1/2} i)/2 - 1/2 \right) \left((3^{1/2} i)/2 + 1/2 \right) + 1 \right) + x^3)^{1/2}} - \frac{2^{1/3} \left((3^{1/2} i)/2 + 3/2 \right) (x^3 - 1)^{1/2} \left(-(x - (3^{1/2} i)/2 + 1/2) / \left((3^{1/2} i)/2 - 3/2 \right)^{1/2} \left((x + (3^{1/2} i)/2 + 1/2) / \left((3^{1/2} i)/2 + 3/2 \right) \right)^{1/2} \operatorname{ellipticPi} \left(\left((3^{1/2} i)/2 + 3/2 \right) / \left(2^{2/3} \right) \left(\left((3^{1/2} i)/2 + 1/2 \right) + 1 \right), \operatorname{asin} \left(-(x - 1) / \left((3^{1/2} i)/2 + 3/2 \right) \right)^{1/2} \right) - \left((3^{1/2} i)/2 + 3/2 \right) / \left((3^{1/2} i)/2 - 3/2 \right)}{3 \left((3^{1/2} i)/2 + 1/2 \right) \left(1 - x^3 \right)^{1/2} \left(2^{2/3} \right) \left(\left((3^{1/2} i)/2 + 1/2 \right) + 1 \right) \left(\left((3^{1/2} i)/2 - 1/2 \right) \left((3^{1/2} i)/2 + 1/2 \right) - x \left(\left((3^{1/2} i)/2 - 1/2 \right) \left((3^{1/2} i)/2 + 1/2 \right) + 1 \right) + x^3)^{1/2}} - \frac{2^{1/3} \left((3^{1/2} i)/2 + 3/2 \right) (x^3 - 1)^{1/2} \left(-(x - (3^{1/2} i)/2 + 1/2) / \left((3^{1/2} i)/2 - 3/2 \right)^{1/2} \left((x + (3^{1/2} i)/2 + 1/2) / \left((3^{1/2} i)/2 + 3/2 \right) \right)^{1/2} \operatorname{ellipticPi} \left(-\left((3^{1/2} i)/2 + 3/2 \right) / \left(2^{2/3} \right) \left(\left((3^{1/2} i)/2 - 1/2 \right) - 1 \right), \operatorname{asin} \left(-(x - 1) / \left((3^{1/2} i)/2 + 3/2 \right) \right)^{1/2} \right) - \left((3^{1/2} i)/2 + 3/2 \right) / \left(\left((3^{1/2} i)/2 - 3/2 \right) \right)}{3 \left((3^{1/2} i)/2 - 1/2 \right) \left(1 - x^3 \right)^{1/2} \left(2^{2/3} \right) \left(\left((3^{1/2} i)/2 - 1/2 \right) - 1 \right) \left(\left((3^{1/2} i)/2 - 1/2 \right) \left((3^{1/2} i)/2 + 1/2 \right) - x \left(\left((3^{1/2} i)/2 - 1/2 \right) \left((3^{1/2} i)/2 + 1/2 \right) + 1 \right) + x^3)^{1/2}}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^3 \sqrt{1-x^3} - 4 \sqrt{1-x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)

[Out] -Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)

$$3.216 \quad \int \frac{x^{11} \sqrt{c+dx^3}}{8c-dx^3} dx$$

Optimal. Leaf size=111

$$\frac{1024c^{7/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4} - \frac{1024c^3 \sqrt{c+dx^3}}{3d^4} - \frac{38c^2 (c+dx^3)^{3/2}}{3d^4} - \frac{4c (c+dx^3)^{5/2}}{5d^4} - \frac{2 (c+dx^3)^{7/2}}{21d^4}$$

Rubi [A] time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 88, 50, 63, 206}

$$-\frac{1024c^3 \sqrt{c+dx^3}}{3d^4} - \frac{38c^2 (c+dx^3)^{3/2}}{3d^4} + \frac{1024c^{7/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4} - \frac{4c (c+dx^3)^{5/2}}{5d^4} - \frac{2 (c+dx^3)^{7/2}}{21d^4}$$

Antiderivative was successfully verified.

[In] Int[(x^11*sqrt[c + d*x^3])/(8*c - d*x^3), x]

[Out] (-1024*c^3*sqrt[c + d*x^3])/(3*d^4) - (38*c^2*(c + d*x^3)^(3/2))/(3*d^4) - (4*c*(c + d*x^3)^(5/2))/(5*d^4) - (2*(c + d*x^3)^(7/2))/(21*d^4) + (1024*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*sqrt[c])])/d^4

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p], x]

$x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x\} \&\& \text{IntegersQ}[m, n] \&\& (\text{IntegerQ}[p] \parallel (\text{GtQ}[m, 0] \&\& \text{GeQ}[n, -1]))$

Rule 206

$\text{Int}[(a_ + (b_ \cdot (x_)^2)^{-1}), x_ \text{Symbol}] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[(\text{Rt}[-b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 446

$\text{Int}[(x_)^{(m_)} \cdot ((a_ + (b_ \cdot (x_)^{(n_)})^{(p_)} \cdot ((c_ + (d_ \cdot (x_)^{(n_)})^{(q_)}), x_ \text{Symbol}] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) \cdot (a + b \cdot x)^p \cdot (c + d \cdot x)^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x\} \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11} \sqrt{c + dx^3}}{8c - dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3 \sqrt{c + dx}}{8c - dx} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{57c^2 \sqrt{c + dx}}{d^3} + \frac{512c^3 \sqrt{c + dx}}{d^3(8c - dx)} - \frac{6c(c + dx)^{3/2}}{d^3} - \frac{(c + dx)^{5/2}}{d^3} \right) dx, x, x^3 \right) \\
 &= -\frac{38c^2 (c + dx^3)^{3/2}}{3d^4} - \frac{4c (c + dx^3)^{5/2}}{5d^4} - \frac{2(c + dx^3)^{7/2}}{21d^4} + \frac{(512c^3) \text{Subst} \left(\int \frac{\sqrt{c + dx}}{8c - dx} dx, x, x^3 \right)}{3d^3} \\
 &= -\frac{1024c^3 \sqrt{c + dx^3}}{3d^4} - \frac{38c^2 (c + dx^3)^{3/2}}{3d^4} - \frac{4c (c + dx^3)^{5/2}}{5d^4} - \frac{2(c + dx^3)^{7/2}}{21d^4} + \frac{(1536c^4) \text{Subst} \left(\int \frac{1}{8c - dx} dx, x, x^3 \right)}{3d^3} \\
 &= -\frac{1024c^3 \sqrt{c + dx^3}}{3d^4} - \frac{38c^2 (c + dx^3)^{3/2}}{3d^4} - \frac{4c (c + dx^3)^{5/2}}{5d^4} - \frac{2(c + dx^3)^{7/2}}{21d^4} + \frac{(3072c^4) \text{Subst} \left(\int \frac{1}{8c - dx} dx, x, x^3 \right)}{3d^3} \\
 &= -\frac{1024c^3 \sqrt{c + dx^3}}{3d^4} - \frac{38c^2 (c + dx^3)^{3/2}}{3d^4} - \frac{4c (c + dx^3)^{5/2}}{5d^4} - \frac{2(c + dx^3)^{7/2}}{21d^4} + \frac{1024c^{7/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{d^4}
 \end{aligned}$$

Mathematica [A] time = 0.11, size = 81, normalized size = 0.73

$$\frac{107520c^{7/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right) - 2\sqrt{c + dx^3} (18632c^3 + 764c^2 dx^3 + 57cd^2 x^6 + 5d^3 x^9)}{105d^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3), x]

[Out] (-2*Sqrt[c + d*x^3]*(18632*c^3 + 764*c^2*d*x^3 + 57*c*d^2*x^6 + 5*d^3*x^9) + 107520*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(105*d^4)

IntegrateAlgebraic [A] time = 0.07, size = 82, normalized size = 0.74

$$\frac{1024c^{7/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4} - \frac{2\sqrt{c+dx^3} (18632c^3 + 764c^2dx^3 + 57cd^2x^6 + 5d^3x^9)}{105d^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3), x]

[Out] (-2*Sqrt[c + d*x^3]*(18632*c^3 + 764*c^2*d*x^3 + 57*c*d^2*x^6 + 5*d^3*x^9) / (105*d^4) + (1024*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]) / d^4

fricas [A] time = 0.90, size = 169, normalized size = 1.52

$$\left[\frac{2 \left(26880 c^{\frac{7}{2}} \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) - (5d^3x^9 + 57cd^2x^6 + 764c^2dx^3 + 18632c^3)\sqrt{dx^3+c} \right)}{105d^4}, \frac{2 \left(53760 \sqrt{-c} c^3 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + (5d^3x^9 + 57cd^2x^6 + 764c^2dx^3 + 18632c^3)\sqrt{dx^3+c} \right)}{105d^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c), x, algorithm="fricas")

[Out] [2/105*(26880*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*x^3 + c))/d^4, -2/105*(53760*sqrt(-c)*c^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*x^3 + c))/d^4]

giac [A] time = 0.16, size = 100, normalized size = 0.90

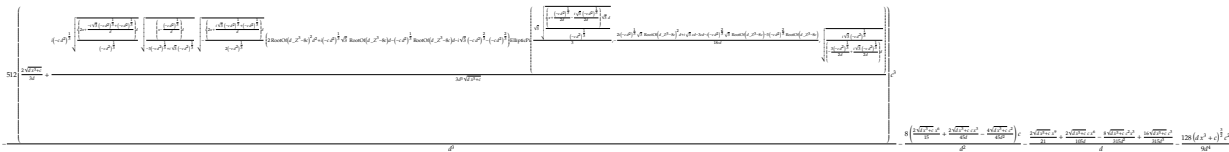
$$\frac{1024c^4 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^4} - \frac{2 \left(5(dx^3+c)^{\frac{7}{2}}d^{24} + 42(dx^3+c)^{\frac{5}{2}}cd^{24} + 665(dx^3+c)^{\frac{3}{2}}c^2d^{24} + 17920\sqrt{dx^3+c}c^3d^{24} \right)}{105d^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c), x, algorithm="giac")

[Out] -1024*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/105*(5*(d*x^3 + c)^(7/2)*d^24 + 42*(d*x^3 + c)^(5/2)*c*d^24 + 665*(d*x^3 + c)^(3/2)*c^2*d^24 + 17920*sqrt(d*x^3 + c)*c^3*d^24)/d^28

maple [C] time = 0.35, size = 582, normalized size = 5.24



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c), x)`

[Out]
$$-1/d*(2/21*x^9*(d*x^3+c)^{(1/2)}+2/105*c/d*x^6*(d*x^3+c)^{(1/2)}-8/315*c^2/d^2*x^3*(d*x^3+c)^{(1/2)}+16/315*c^3*(d*x^3+c)^{(1/2)}/d^3-8*c/d^2*(2/15*(d*x^3+c)^{(1/2)}*x^6+2/45*(d*x^3+c)^{(1/2)}*c/d*x^3-4/45*(d*x^3+c)^{(1/2)}*c^2/d^2)-128/9*c^2*(d*x^3+c)^{(3/2)}/d^4-512*c^3/d^3*(2/3*(d*x^3+c)^{(1/2)}/d+1/3*I/d^3*2^{(1/2)}*sum((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}), _alpha=RootOf(_Z^3*d-8*c))$$

maxima [A] time = 1.31, size = 96, normalized size = 0.86

$$\frac{2 \left(26880 c^7 \log \left(\frac{\sqrt{dx^3+c-3\sqrt{c}}}{\sqrt{dx^3+c+3\sqrt{c}}} \right) + 5 (dx^3+c)^{7/2} + 42 (dx^3+c)^{5/2} c + 665 (dx^3+c)^{3/2} c^2 + 17920 \sqrt{dx^3+c} c^3 \right)}{105 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c), x, algorithm="maxima")`

[Out]
$$-2/105*(26880*c^{(7/2)}*\log((\text{sqrt}(d*x^3+c)-3*\text{sqrt}(c))/(\text{sqrt}(d*x^3+c)+3*\text{sqrt}(c))))+5*(d*x^3+c)^{(7/2)}+42*(d*x^3+c)^{(5/2)}*c+665*(d*x^3+c)^{(3/2)}*c^2+17920*\text{sqrt}(d*x^3+c)*c^3)/d^4$$

mupad [B] time = 3.51, size = 118, normalized size = 1.06

$$\frac{512 c^{7/2} \ln \left(\frac{10c+dx^3+6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3} \right)}{d^4} - \frac{37264 c^3 \sqrt{dx^3+c}}{105 d^4} - \frac{2 x^9 \sqrt{dx^3+c}}{21 d} - \frac{38 c x^6 \sqrt{dx^3+c}}{35 d^2} - \frac{1528 c^2 x^3 \sqrt{dx^3+c}}{105 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^11*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)`

[Out] $(512*c^{(7/2)}*\log((10*c + d*x^3 + 6*c^{(1/2)}*(c + d*x^3)^{(1/2)})/(8*c - d*x^3)))/d^4 - (37264*c^3*(c + d*x^3)^{(1/2)})/(105*d^4) - (2*x^9*(c + d*x^3)^{(1/2)})/(21*d) - (38*c*x^6*(c + d*x^3)^{(1/2)})/(35*d^2) - (1528*c^2*x^3*(c + d*x^3)^{(1/2)})/(105*d^3)$

sympy [A] time = 60.27, size = 99, normalized size = 0.89

$$2 \left(\frac{512c^4 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{512c^3 \sqrt{c+dx^3}}{3} - \frac{19c^2(c+dx^3)^{\frac{3}{2}}}{3} - \frac{2c(c+dx^3)^{\frac{5}{2}}}{5} - \frac{(c+dx^3)^{\frac{7}{2}}}{21} \right) \frac{1}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)`

[Out] $2*(-512*c**4*\operatorname{atan}(\operatorname{sqrt}(c + d*x**3)/(3*\operatorname{sqrt}(-c)))/\operatorname{sqrt}(-c) - 512*c**3*\operatorname{sqrt}(c + d*x**3)/3 - 19*c**2*(c + d*x**3)**(3/2)/3 - 2*c*(c + d*x**3)**(5/2)/5 - (c + d*x**3)**(7/2)/21)/d**4$

$$3.217 \quad \int \frac{x^8 \sqrt{c+dx^3}}{8c-dx^3} dx$$

Optimal. Leaf size=90

$$\frac{128c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3} - \frac{128c^2 \sqrt{c+dx^3}}{3d^3} - \frac{14c(c+dx^3)^{3/2}}{9d^3} - \frac{2(c+dx^3)^{5/2}}{15d^3}$$

Rubi [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 88, 50, 63, 206}

$$-\frac{128c^2 \sqrt{c+dx^3}}{3d^3} + \frac{128c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3} - \frac{14c(c+dx^3)^{3/2}}{9d^3} - \frac{2(c+dx^3)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

[In] Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]

[Out] (-128*c^2*Sqrt[c + d*x^3])/(3*d^3) - (14*c*(c + d*x^3)^(3/2))/(9*d^3) - (2*(c + d*x^3)^(5/2))/(15*d^3) + (128*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

$x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x\} \&\& \text{IntegersQ}[m, n] \&\& (\text{IntegerQ}[p] \parallel (\text{GtQ}[m, 0] \&\& \text{GeQ}[n, -1]))$

Rule 206

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(1 \cdot \text{ArcTanh}[(\text{Rt}[-b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 446

$\text{Int}[(x_)^{(m_ \cdot)} \cdot ((a_ + (b_ \cdot)(x_)^{(n_)})^{(p_ \cdot)}) \cdot ((c_ + (d_ \cdot)(x_)^{(n_)})^{(q_ \cdot)}), x_Symbol] :> \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) \cdot (a + b \cdot x)^p \cdot (c + d \cdot x)^q}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x\} \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{x^8 \sqrt{c+dx^3}}{8c-dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 \sqrt{c+dx}}{8c-dx} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{7c\sqrt{c+dx}}{d^2} + \frac{64c^2\sqrt{c+dx}}{d^2(8c-dx)} - \frac{(c+dx)^{3/2}}{d^2} \right) dx, x, x^3 \right) \\ &= -\frac{14c(c+dx^3)^{3/2}}{9d^3} - \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{(64c^2) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{8c-dx} dx, x, x^3 \right)}{3d^2} \\ &= -\frac{128c^2\sqrt{c+dx^3}}{3d^3} - \frac{14c(c+dx^3)^{3/2}}{9d^3} - \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{(192c^3) \text{Subst} \left(\int \frac{1}{(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{d^2} \\ &= -\frac{128c^2\sqrt{c+dx^3}}{3d^3} - \frac{14c(c+dx^3)^{3/2}}{9d^3} - \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{(384c^3) \text{Subst} \left(\int \frac{1}{9c-x^2} dx, x, \sqrt{c+dx^3} \right)}{d^3} \\ &= -\frac{128c^2\sqrt{c+dx^3}}{3d^3} - \frac{14c(c+dx^3)^{3/2}}{9d^3} - \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{128c^{5/2} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{d^3} \end{aligned}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 0.78

$$\frac{5760c^{5/2} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) - 2\sqrt{c+dx^3} (998c^2 + 41cdx^3 + 3d^2x^6)}{45d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*sqrt[c + d*x^3])/(8*c - d*x^3),x]

[Out] (-2*sqrt[c + d*x^3]*(998*c^2 + 41*c*d*x^3 + 3*d^2*x^6) + 5760*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*sqrt[c])])/(45*d^3)

IntegrateAlgebraic [A] time = 0.06, size = 71, normalized size = 0.79

$$\frac{128c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3} - \frac{2\sqrt{c+dx^3} (998c^2 + 41cdx^3 + 3d^2x^6)}{45d^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8*sqrt[c + d*x^3])/(8*c - d*x^3),x]

[Out] (-2*sqrt[c + d*x^3]*(998*c^2 + 41*c*d*x^3 + 3*d^2*x^6))/(45*d^3) + (128*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*sqrt[c])])/d^3

fricas [A] time = 0.75, size = 147, normalized size = 1.63

$$\left[\frac{2 \left(1440 c^2 \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) - (3d^2x^6 + 41cdx^3 + 998c^2)\sqrt{dx^3+c} \right)}{45d^3}, - \frac{2 \left(2880 \sqrt{-c} c^2 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + (3d^2x^6 + 41cdx^3 + 998c^2)\sqrt{dx^3+c} \right)}{45d^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")

[Out] [2/45*(1440*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - (3*d^2*x^6 + 41*c*d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3, -2/45*(2880*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^2*x^6 + 41*c*d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3]

giac [A] time = 0.17, size = 83, normalized size = 0.92

$$-\frac{128c^3 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^3} - \frac{2\left(3(dx^3+c)^{\frac{5}{2}}d^{12} + 35(dx^3+c)^{\frac{3}{2}}cd^{12} + 960\sqrt{dx^3+c}c^2d^{12}\right)}{45d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")

[Out] -128*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/45*(3*(d*x^3 + c)^(5/2)*d^12 + 35*(d*x^3 + c)^(3/2)*c*d^12 + 960*sqrt(d*x^3 + c)*c^2*d^12)/d^15

maple [C] time = 0.16, size = 507, normalized size = 5.63



Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^8*(d*x^3+c)^{(1/2)}/(-d*x^3+8*c), x)$

[Out]
$$-1/d^2*((2/15*(d*x^3+c)^{(1/2)}*x^6+2/45*(d*x^3+c)^{(1/2)}*c/d*x^3-4/45*(d*x^3+c)^{(1/2)}*c^2/d^2)*d+16/9*(d*x^3+c)^{(3/2)}*c/d-64*c^2/d^2*(2/3*(d*x^3+c)^{(1/2)}/d+1/3*I/d^3*2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}), _alpha = \text{RootOf}(Z^3*d-8*c))$$

maxima [A] time = 1.21, size = 82, normalized size = 0.91

$$\frac{2 \left(1440 c^2 \log \left(\frac{\sqrt{dx^3+c} - 3\sqrt{c}}{\sqrt{dx^3+c} + 3\sqrt{c}} \right) + 3 (dx^3 + c)^{5/2} + 35 (dx^3 + c)^{3/2} c + 960 \sqrt{dx^3 + c} c^2 \right)}{45 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^8*(d*x^3+c)^{(1/2)}/(-d*x^3+8*c), x, \text{algorithm}="maxima")$

[Out]
$$-2/45*(1440*c^2*\log((\text{sqrt}(d*x^3 + c) - 3*\text{sqrt}(c))/(\text{sqrt}(d*x^3 + c) + 3*\text{sqrt}(c))) + 3*(d*x^3 + c)^{(5/2)} + 35*(d*x^3 + c)^{(3/2)}*c + 960*\text{sqrt}(d*x^3 + c)*c^2)/d^3$$

mupad [B] time = 3.40, size = 98, normalized size = 1.09

$$\frac{64 c^{5/2} \ln \left(\frac{10 c + d x^3 + 6 \sqrt{c} \sqrt{d x^3 + c}}{8 c - d x^3} \right)}{d^3} - \frac{1996 c^2 \sqrt{d x^3 + c}}{45 d^3} - \frac{2 x^6 \sqrt{d x^3 + c}}{15 d} - \frac{82 c x^3 \sqrt{d x^3 + c}}{45 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^8*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)`

[Out] $(64*c^{5/2}*\log((10*c + d*x^3 + 6*c^{1/2}*(c + d*x^3)^{1/2}))/ (8*c - d*x^3)) /d^3 - (1996*c^2*(c + d*x^3)^{1/2})/(45*d^3) - (2*x^6*(c + d*x^3)^{1/2})/(15*d) - (82*c*x^3*(c + d*x^3)^{1/2})/(45*d^2)$

sympy [A] time = 30.23, size = 82, normalized size = 0.91

$$\frac{2 \left(-\frac{64c^3 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{64c^2\sqrt{c+dx^3}}{3} - \frac{7c(c+dx^3)^{\frac{3}{2}}}{9} - \frac{(c+dx^3)^{\frac{5}{2}}}{15} \right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)`

[Out] $2*(-64*c**3*\operatorname{atan}(\operatorname{sqrt}(c + d*x**3)/(3*\operatorname{sqrt}(-c)))/\operatorname{sqrt}(-c) - 64*c**2*\operatorname{sqrt}(c + d*x**3)/3 - 7*c*(c + d*x**3)**(3/2)/9 - (c + d*x**3)**(5/2)/15)/d**3$

$$3.218 \quad \int \frac{x^5 \sqrt{c+dx^3}}{8c-dx^3} dx$$

Optimal. Leaf size=69

$$\frac{16c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2} - \frac{16c\sqrt{c+dx^3}}{3d^2} - \frac{2(c+dx^3)^{3/2}}{9d^2}$$

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 80, 50, 63, 206}

$$\frac{16c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2} - \frac{16c\sqrt{c+dx^3}}{3d^2} - \frac{2(c+dx^3)^{3/2}}{9d^2}$$

Antiderivative was successfully verified.

[In] Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3), x]

[Out] (-16*c*Sqrt[c + d*x^3])/(3*d^2) - (2*(c + d*x^3)^(3/2))/(9*d^2) + (16*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(

$n + p + 2$), $\text{Int}[(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n, p\}, x] \ \&\& \ \text{NeQ}[n + p + 2, 0]$

Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])]$

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{x^5 \sqrt{c + dx^3}}{8c - dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x \sqrt{c + dx}}{8c - dx} dx, x, x^3 \right) \\ &= -\frac{2(c + dx^3)^{3/2}}{9d^2} + \frac{(8c) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{8c-dx} dx, x, x^3 \right)}{3d} \\ &= -\frac{16c\sqrt{c + dx^3}}{3d^2} - \frac{2(c + dx^3)^{3/2}}{9d^2} + \frac{(24c^2) \text{Subst} \left(\int \frac{1}{(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{d} \\ &= -\frac{16c\sqrt{c + dx^3}}{3d^2} - \frac{2(c + dx^3)^{3/2}}{9d^2} + \frac{(48c^2) \text{Subst} \left(\int \frac{1}{9c-x^2} dx, x, \sqrt{c + dx^3} \right)}{d^2} \\ &= -\frac{16c\sqrt{c + dx^3}}{3d^2} - \frac{2(c + dx^3)^{3/2}}{9d^2} + \frac{16c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{d^2} \end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.84

$$\frac{144c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) - 2\sqrt{c + dx^3} (25c + dx^3)}{9d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]

[Out] (-2*Sqrt[c + d*x^3]*(25*c + d*x^3) + 144*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2)

IntegrateAlgebraic [A] time = 0.06, size = 59, normalized size = 0.86

$$\frac{16c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2} - \frac{2\sqrt{c+dx^3}(25c+dx^3)}{9d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]

[Out] (-2*Sqrt[c + d*x^3]*(25*c + d*x^3))/(9*d^2) + (16*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2

fricas [A] time = 0.99, size = 121, normalized size = 1.75

$$\left[\frac{2\left(36c^{\frac{3}{2}} \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) - (dx^3+25c)\sqrt{dx^3+c}\right)}{9d^2}, -\frac{2\left(72\sqrt{-c}c \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + (dx^3+25c)\sqrt{dx^3+c}\right)}{9d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")

[Out] [2/9*(36*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2, -2/9*(72*sqrt(-c)*c*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2]

giac [A] time = 0.21, size = 65, normalized size = 0.94

$$-\frac{16c^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^2} - \frac{2\left((dx^3+c)^{\frac{3}{2}}d^4 + 24\sqrt{dx^3+c}cd^4\right)}{9d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")

[Out] -16*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/9*((d*x^3 + c)^(3/2)*d^4 + 24*sqrt(d*x^3 + c)*c*d^4)/d^6

maple [C] time = 0.18, size = 446, normalized size = 6.46

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)`

[Out]
$$-2/9*(d*x^3+c)^{(3/2)}/d^2-8*c/d*(2/3*(d*x^3+c)^{(1/2)}/d+1/3*I/d^3*2^{(1/2)}*\text{sum}(((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},-1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}),_alpha=\text{RootOf}(_Z^3*d-8*c))$$

maxima [A] time = 1.30, size = 66, normalized size = 0.96

$$\frac{2 \left(36 c^3 \log \left(\frac{\sqrt{dx^3+c} - 3\sqrt{c}}{\sqrt{dx^3+c} + 3\sqrt{c}} \right) + (dx^3 + c)^{3/2} + 24 \sqrt{dx^3 + c} c \right)}{9 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")`

[Out]
$$-2/9*(36*c^{(3/2)}*\log((\text{sqrt}(d*x^3 + c) - 3*\text{sqrt}(c))/(\text{sqrt}(d*x^3 + c) + 3*\text{sqrt}(c)))) + (d*x^3 + c)^{(3/2)} + 24*\text{sqrt}(d*x^3 + c)*c)/d^2$$

mupad [B] time = 3.51, size = 78, normalized size = 1.13

$$\frac{8 c^{3/2} \ln \left(\frac{10 c + d x^3 + 6 \sqrt{c} \sqrt{d x^3 + c}}{8 c - d x^3} \right)}{d^2} - \frac{50 c \sqrt{d x^3 + c}}{9 d^2} - \frac{2 x^3 \sqrt{d x^3 + c}}{9 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)`

[Out] $(8c^{3/2} \log((10c + dx^3 + 6c^{1/2})(c + dx^3)^{1/2}) / (8c - dx^3)) / d^2 - (50c(c + dx^3)^{1/2}) / (9d^2) - (2x^3(c + dx^3)^{1/2}) / (9d)$

sympy [A] time = 14.91, size = 65, normalized size = 0.94

$$\frac{2 \left(-\frac{8c^2 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{8c\sqrt{c+dx^3}}{3} - \frac{(c+dx^3)^{3/2}}{9} \right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)`

[Out] $2*(-8c**2*\operatorname{atan}(\operatorname{sqrt}(c + d*x**3)/(3*\operatorname{sqrt}(-c)))/\operatorname{sqrt}(-c) - 8*c*\operatorname{sqrt}(c + d*x**3)/3 - (c + d*x**3)**(3/2)/9)/d**2$

$$3.219 \quad \int \frac{x^2 \sqrt{c+dx^3}}{8c-dx^3} dx$$

Optimal. Leaf size=50

$$\frac{2\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d} - \frac{2\sqrt{c+dx^3}}{3d}$$

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {444, 50, 63, 206}

$$\frac{2\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d} - \frac{2\sqrt{c+dx^3}}{3d}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]

[Out] (-2*Sqrt[c + d*x^3])/(3*d) + (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2 \sqrt{c + dx^3}}{8c - dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c + dx}}{8c - dx} dx, x, x^3 \right) \\
 &= -\frac{2\sqrt{c + dx^3}}{3d} + (3c) \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right) \\
 &= -\frac{2\sqrt{c + dx^3}}{3d} + \frac{(6c) \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{d} \\
 &= -\frac{2\sqrt{c + dx^3}}{3d} + \frac{2\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{d}
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.94

$$\frac{2 \left(\sqrt{c + dx^3} - 3\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3), x]

[Out] (-2*(Sqrt[c + d*x^3] - 3*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(3*d)

IntegrateAlgebraic [A] time = 0.04, size = 50, normalized size = 1.00

$$\frac{2\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{d} - \frac{2\sqrt{c + dx^3}}{3d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3), x]

[Out] $(-2*\sqrt{c + d*x^3})/(3*d) + (2*\sqrt{c}*\text{ArcTanh}[\sqrt{c + d*x^3}/(3*\sqrt{c})])/d$

fricas [A] time = 0.82, size = 101, normalized size = 2.02

$$\left[\frac{3\sqrt{c} \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) - 2\sqrt{dx^3+c}}{3d}, -\frac{2\left(3\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + \sqrt{dx^3+c}\right)}{3d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")`

[Out] $[1/3*(3*\sqrt{c}*\log((d*x^3 + 6*\sqrt{d*x^3 + c})*\sqrt{c} + 10*c)/(d*x^3 - 8*c)) - 2*\sqrt{d*x^3 + c}]/d, -2/3*(3*\sqrt{-c}*\arctan(1/3*\sqrt{d*x^3 + c}*\sqrt{-c}/c) + \sqrt{d*x^3 + c})/d]$

giac [A] time = 0.16, size = 43, normalized size = 0.86

$$-\frac{2c \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d} - \frac{2\sqrt{dx^3+c}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")`

[Out] $-2*c*\arctan(1/3*\sqrt{d*x^3 + c}/\sqrt{-c})/(\sqrt{-c}*d) - 2/3*\sqrt{d*x^3 + c}/d$

maple [C] time = 0.15, size = 425, normalized size = 8.50

The image shows a complex mathematical expression from the Maple CAS. It consists of several terms involving square roots of polynomials, logarithmic functions, and elliptic integrals. The expression is highly nested and difficult to read in its current form.

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)`

[Out] $-2/3*(d*x^3+c)^(1/2)/d - 1/3*I/d^3*2^(1/2)*\sum(((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3))/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*\text{EllipticPi}(1/3*3^(1$

$/2) * (I * (x + 1/2 * (-c * d^2)^{1/3}) / d - 1/2 * I * 3^{1/2} * (-c * d^2)^{1/3} / d) * 3^{1/2} / (-c * d^2)^{1/3} * d)^{1/2}, -1/18 * (2 * I * (-c * d^2)^{1/3} * 3^{1/2} * _alpha^2 * d + I * 3^{1/2} * c * d - 3 * c * d - I * (-c * d^2)^{2/3} * 3^{1/2} * _alpha - 3 * (-c * d^2)^{2/3} * _alpha) / c / d, (I * 3^{1/2} * (-c * d^2)^{1/3} / (-3/2 * (-c * d^2)^{1/3} / d + 1/2 * I * 3^{1/2} * (-c * d^2)^{1/3} / d)^{1/2}), _alpha = \text{RootOf}(_Z^3 * d - 8 * c))$

maxima [A] time = 1.15, size = 56, normalized size = 1.12

$$\frac{3 \sqrt{c} \log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right) + 2 \sqrt{dx^3+c}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")

[Out] -1/3*(3*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c))) + 2*sqrt(d*x^3 + c))/d

mupad [B] time = 3.50, size = 59, normalized size = 1.18

$$\frac{\sqrt{c} \ln\left(\frac{10c+dx^3+6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{d} - \frac{2\sqrt{dx^3+c}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)

[Out] (c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/d - (2*(c + d*x^3)^(1/2))/(3*d)

sympy [A] time = 5.11, size = 46, normalized size = 0.92

$$\frac{2 \left(-\frac{c \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{\sqrt{c+dx^3}}{3} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)

[Out] 2*(-c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - sqrt(c + d*x**3)/3)/d

$$3.220 \quad \int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)} dx$$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 83, 63, 208, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(4*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(12*Sqrt[c])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_)^(p_.))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```


Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c+dx}}{x(8c-dx)} dx, x, x^3 \right) \\
 &= \frac{1}{24} \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right) + \frac{1}{8} (3d) \text{Subst} \left(\int \frac{1}{(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right) \\
 &= \frac{3}{4} \text{Subst} \left(\int \frac{1}{9c-x^2} dx, x, \sqrt{c+dx^3} \right) + \frac{\text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{12d} \\
 &= \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{4\sqrt{c}} - \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{12\sqrt{c}}
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.91

$$\frac{3 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) - \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{12\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)), x]

[Out] (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(12*Sqrt[c])

IntegrateAlgebraic [A] time = 0.04, size = 58, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]

[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(4*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(12*Sqrt[c])

fricas [A] time = 0.91, size = 138, normalized size = 2.38

$$\left[\frac{3\sqrt{c} \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + \sqrt{c} \log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{24c}, \frac{\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - 3\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)}{12c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="fricas")

[Out] [1/24*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, 1/12*(sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c))/c]

giac [A] time = 0.16, size = 48, normalized size = 0.83

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="giac")

[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c)

maple [C] time = 0.16, size = 468, normalized size = 8.07

The Maple output is a complex expression enclosed in large square brackets. It consists of several terms, each involving square roots and arctan functions. The terms are arranged in a sequence, with some terms being more complex than others. The overall structure is a sum of these terms, with some terms being multiplied by constants or other functions. The expression is highly detailed and difficult to transcribe exactly, but it represents the antiderivative found by Maple for the given integral.

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x)`

[Out]
$$-1/8/c*d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*\text{sum}((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*\text{EllipticPi}(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=\text{RootOf}(_Z^3*d-8*c)))+1/8/c*(2/3*(d*x^3+c)^(1/2)-2/3*\text{arctanh}((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{dx^3+c}}{(dx^3-8c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="maxima")`

[Out] `-integrate(sqrt(d*x^3+c)/((d*x^3-8*c)*x),x)`

mupad [B] time = 4.69, size = 125, normalized size = 2.16

$$\frac{\ln\left(\frac{\left(\sqrt{dx^3+c}-\sqrt{c}\right)^3\left(\sqrt{dx^3+c}+\sqrt{c}\right)\left(6c+dx^3+6\sqrt{c}\sqrt{dx^3+c}\right)^3\left(24c^2-24c^{3/2}\sqrt{dx^3+c}+d^2x^6-20cdx^3\right)^3}{x^{15}\left(8c-dx^3\right)^3\left(24c-dx^3\right)^3}\right)}{24\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c+d*x^3)^(1/2)/(x*(8*c-d*x^3)),x)`

[Out]
$$\log\left(\frac{\left(\left(c+d*x^3\right)^{1/2}-c^{1/2}\right)^3\left(\left(c+d*x^3\right)^{1/2}+c^{1/2}\right)\left(6*c+d*x^3+6*c^{1/2}\left(c+d*x^3\right)^{1/2}\right)^3\left(24*c^2-24*c^{3/2}\left(c+d*x^3\right)^{1/2}+d^2*x^6-20*c*d*x^3\right)^3}{x^{15}\left(8*c-d*x^3\right)^3\left(24*c-d*x^3\right)^3}\right)/(24*c^{1/2})$$

sympy [A] time = 8.22, size = 60, normalized size = 1.03

$$\frac{2 \left(-\frac{d \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{8\sqrt{-c}} + \frac{d \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24\sqrt{-c}} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c),x)

[Out] 2*(-d*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(8*sqrt(-c)) + d*atan(sqrt(c + d*x**3)/sqrt(-c))/(24*sqrt(-c)))/d

$$3.221 \quad \int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)} dx$$

Optimal. Leaf size=81

$$\frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}} - \frac{5d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}} - \frac{\sqrt{c+dx^3}}{24cx^3}$$

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {446, 99, 156, 63, 208, 206}

$$\frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}} - \frac{5d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}} - \frac{\sqrt{c+dx^3}}{24cx^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]

[Out] -Sqrt[c + d*x^3]/(24*c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])]/(96*c^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 99

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 156

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +

$f*x)^p/(a + b*x), x], x] - \text{Dist}[(d*g - c*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(c + d*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x]$

Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])]$

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b]$

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n) - 1}*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{c + dx^3}}{x^4(8c - dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c + dx}}{x^2(8c - dx)} dx, x, x^3 \right) \\
 &= -\frac{\sqrt{c + dx^3}}{24cx^3} + \frac{\text{Subst} \left(\int \frac{5cd + \frac{d^2x}{2}}{x(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{24c} \\
 &= -\frac{\sqrt{c + dx^3}}{24cx^3} + \frac{(5d) \text{Subst} \left(\int \frac{1}{x\sqrt{c + dx}} dx, x, x^3 \right)}{192c} + \frac{(3d^2) \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{64c} \\
 &= -\frac{\sqrt{c + dx^3}}{24cx^3} + \frac{5 \text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{96c} + \frac{(3d) \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{32c} \\
 &= -\frac{\sqrt{c + dx^3}}{24cx^3} + \frac{d \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{32c^{3/2}} - \frac{5d \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{96c^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 1.00

$$\frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}} - \frac{5d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}} - \frac{\sqrt{c+dx^3}}{24cx^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]

[Out] -1/24*Sqrt[c + d*x^3]/(c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*2*c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))

IntegrateAlgebraic [A] time = 0.08, size = 81, normalized size = 1.00

$$\frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}} - \frac{5d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}} - \frac{\sqrt{c+dx^3}}{24cx^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]

[Out] -1/24*Sqrt[c + d*x^3]/(c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*2*c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))

fricas [A] time = 0.76, size = 186, normalized size = 2.30

$$\left[\frac{3\sqrt{c}dx^3 \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + 5\sqrt{c}dx^3 \log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) - 8\sqrt{dx^3+c}c - 5\sqrt{-c}dx^3 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - 3\sqrt{-c}dx^3 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - 4\sqrt{dx^3+c}c}{192c^2x^3}, \frac{3\sqrt{c}dx^3 \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + 5\sqrt{c}dx^3 \log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) - 8\sqrt{dx^3+c}c - 5\sqrt{-c}dx^3 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - 3\sqrt{-c}dx^3 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - 4\sqrt{dx^3+c}c}{96c^2x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")

[Out] [1/192*(3*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 5*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*sqrt(d*x^3 + c)*c)/(c^2*x^3), 1/96*(5*sqrt(-c)*d*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]

giac [A] time = 0.17, size = 73, normalized size = 0.90

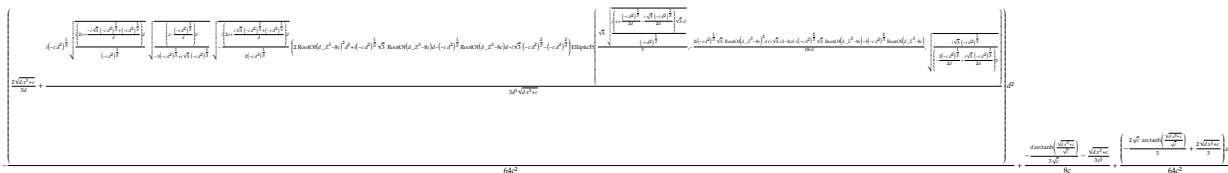
$$\frac{5d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c} - \frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{32\sqrt{-c}c} - \frac{\sqrt{dx^3+c}}{24cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")

[Out] $\frac{5}{96}d \arctan\left(\frac{\sqrt{d x^3 + c}}{\sqrt{-c}}\right) / (\sqrt{-c} * c) - \frac{1}{32}d \arctan\left(\frac{1}{3} \sqrt{\frac{d x^3 + c}{-c}}\right) / (\sqrt{-c} * c) - \frac{1}{24} \sqrt{d x^3 + c} / (c x^3)$

maple [C] time = 0.18, size = 511, normalized size = 6.31



Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x)

[Out] $\frac{1}{8} \frac{1}{c} \left(-\frac{1}{3} (d x^3 + c)^{1/2} / x^3 - \frac{1}{3} d \operatorname{arctanh}\left(\frac{(d x^3 + c)^{1/2}}{c^{1/2}}\right) / c^{1/2} \right) - \frac{1}{64} d^2 / c^2 \left(\frac{2}{3} (d x^3 + c)^{1/2} / d + \frac{1}{3} I / d^3 \right)^{2/3} \sum \left((-c d^2)^{1/3} \left(\frac{1}{3} (2 x + (-I)^{1/2} (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d \right) / (-c d^2)^{1/3} \right) * d^{1/2} * \left(\frac{x - (-c d^2)^{1/3} / d}{-3 (-c d^2)^{1/3} + I 3^{1/2} (-c d^2)^{1/3}} \right) * d^{1/2} * \left(-\frac{1}{2} I (2 x + I 3^{1/2} (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d \right) / (-c d^2)^{1/3} * d^{1/2} / (d x^3 + c)^{1/2} * (2 \alpha^2 d^2 + I (-c d^2)^{1/3} 3^{1/2} * \alpha d - (-c d^2)^{1/3} * \alpha d - I 3^{1/2} (-c d^2)^{2/3} - (-c d^2)^{2/3}) * \operatorname{EllipticPi}\left(\frac{1}{3} 3^{1/2} (I (x + 1/2 (-c d^2)^{1/3} / d - 1/2 I 3^{1/2} (-c d^2)^{1/3}) / d) * 3^{1/2} / (-c d^2)^{1/3} * d^{1/2}, -\frac{1}{18} (2 I (-c d^2)^{1/3} 3^{1/2} * \alpha^2 d + I 3^{1/2} * c d - 3 c d - I (-c d^2)^{2/3} 3^{1/2} * \alpha - 3 (-c d^2)^{2/3}) * \alpha / c d, (I 3^{1/2} (-c d^2)^{1/3} / (-3/2 (-c d^2)^{1/3} / d + 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) / d)^{1/2}, \alpha = \operatorname{RootOf}(_Z^3 d - 8 c)\right) + \frac{1}{64} \frac{1}{c^2} d \left(\frac{2}{3} (d x^3 + c)^{1/2} - \frac{2}{3} \operatorname{arctanh}\left(\frac{(d x^3 + c)^{1/2}}{c^{1/2}}\right) * c^{1/2} \right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sqrt{d x^3 + c}}{(d x^3 - 8 c) x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")

[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^4), x)

mupad [B] time = 3.75, size = 69, normalized size = 0.85

$$\frac{d \operatorname{atanh}\left(\frac{c \sqrt{d x^3 + c}}{3 \sqrt{c^3}}\right)}{32 \sqrt{c^3}} - \frac{5 d \operatorname{atanh}\left(\frac{c \sqrt{d x^3 + c}}{\sqrt{c^3}}\right)}{96 \sqrt{c^3}} - \frac{\sqrt{d x^3 + c}}{24 c x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(8*c - d*x^3)), x)
```

```
[Out] (d*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(32*(c^3)^(1/2)) - (5*d*atanh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)))/(96*(c^3)^(1/2)) - (c + d*x^3)^(1/2)/(24*c*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c + dx^3}}{-8cx^4 + dx^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c), x)
```

```
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x)
```

$$3.222 \quad \int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)} dx$$

Optimal. Leaf size=107

$$\frac{d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}} + \frac{d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{\sqrt{c+dx^3}}{48cx^6}$$

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {446, 99, 151, 156, 63, 208, 206}

$$\frac{d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}} + \frac{d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{\sqrt{c+dx^3}}{48cx^6}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]

[Out] -Sqrt[c + d*x^3]/(48*c*x^6) - (d*Sqrt[c + d*x^3])/(64*c^2*x^3) + (d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(5/2)) + (d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])])/(256*c^(5/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 99

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n + p + 2)*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 151

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

```

Rule 156

```

Int[((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 446

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c+dx}}{x^3(8c-dx)} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c+dx^3}}{48cx^6} + \frac{\text{Subst} \left(\int \frac{6cd+\frac{3d^2x}{2}}{x^2(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{48c} \\
&= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{\text{Subst} \left(\int \frac{6c^2d^2-3cd^3x}{x(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{384c^3} \\
&= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{d^2 \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{512c^2} + \frac{(3d^3) \text{Subst} \left(\int \frac{1}{(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{512c^2} \\
&= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{d \text{Subst} \left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{256c^2} + \frac{(3d^2) \text{Subst} \left(\int \frac{1}{9c-x^2} dx, x, \sqrt{c+dx^3} \right)}{256c^2} \\
&= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} + \frac{d^2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{256c^{5/2}} + \frac{d^2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{256c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 96, normalized size = 0.90

$$\frac{3d^2x^6 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) + 3d^2x^6 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right) - 4\sqrt{c} \sqrt{c+dx^3} (4c + 3dx^3)}{768c^{5/2}x^6}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)), x]

[Out] (-4*Sqrt[c]*Sqrt[c + d*x^3]*(4*c + 3*d*x^3) + 3*d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + 3*d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(5/2)*x^6)

IntegrateAlgebraic [A] time = 0.14, size = 95, normalized size = 0.89

$$\frac{d^2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{256c^{5/2}} + \frac{d^2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{256c^{5/2}} + \frac{(-4c - 3dx^3) \sqrt{c+dx^3}}{192c^2x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]

[Out] $\frac{((-4*c - 3*d*x^3)*\text{Sqrt}[c + d*x^3])/(192*c^2*x^6) + (d^2*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/(3*\text{Sqrt}[c])])/(256*c^{(5/2)}) + (d^2*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/\text{Sqrt}[c])/(256*c^{(5/2)})}$

fricas [A] time = 0.80, size = 188, normalized size = 1.76

$$\left[\frac{3\sqrt{c}d^2x^6 \log\left(\frac{d^2x^6+24cdx^3+8(dx^3+4c)\sqrt{dx^3+c}\sqrt{c+32c^2}}{dx^6-8cx^3}\right) - 8(3cdx^3+4c^2)\sqrt{dx^3+c}}{1536c^3x^6}, -\frac{3\sqrt{-c}d^2x^6 \arctan\left(\frac{(dx^3+4c)\sqrt{dx^3+c}\sqrt{-c}}{4(cd x^3+c^2)}\right) + 4(3cdx^3+4c^2)\sqrt{dx^3+c}}{768c^3x^6} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")

[Out] $[1/1536*(3*\text{sqrt}(c)*d^2*x^6*\log((d^2*x^6 + 24*c*d*x^3 + 8*(d*x^3 + 4*c))*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 32*c^2)/(d*x^6 - 8*c*x^3)) - 8*(3*c*d*x^3 + 4*c^2)*\text{sqrt}(d*x^3 + c)/(c^3*x^6), -1/768*(3*\text{sqrt}(-c)*d^2*x^6*\arctan(1/4*(d*x^3 + 4*c)*\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/(c*d*x^3 + c^2)) + 4*(3*c*d*x^3 + 4*c^2)*\text{sqrt}(d*x^3 + c)/(c^3*x^6)]$

giac [A] time = 0.18, size = 100, normalized size = 0.93

$$\frac{d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{256\sqrt{-c}c^2} - \frac{d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{256\sqrt{-c}c^2} - \frac{3(dx^3+c)^{\frac{3}{2}}d^2 + \sqrt{dx^3+c}cd^2}{192c^2d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")

[Out] $-1/256*d^2*\arctan(\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*c^2) - 1/256*d^2*\arctan(1/3*\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*c^2) - 1/192*(3*(d*x^3 + c)^{(3/2)}*d^2 + \text{sqrt}(d*x^3 + c)*c*d^2)/(c^2*d^2*x^6)$

maple [C] time = 0.19, size = 574, normalized size = 5.36

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x)

```
[Out] 1/64/c^2*d*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))
/c^(1/2))+1/8/c*(-1/6*(d*x^3+c)^(1/2)/x^6-1/12*d*(d*x^3+c)^(1/2)/c/x^3+1/12
*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-1/512/c^3*d^3*(2/3*(d*x^3+c)
^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3*(-c*d^2)^(1/3)+I*3^(1/2))*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/
2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2
)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d
-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(
1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c))+1/512/c^3*d^2*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d
*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{dx^3+c}}{(dx^3-8c)x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^7), x)
```

mupad [B] time = 3.91, size = 83, normalized size = 0.78

$$\frac{d^2 \operatorname{atanh}\left(\frac{d^4 \sqrt{dx^3+c}}{2048 c^{7/2} \left(\frac{d^4}{2048 c^3} + \frac{d^5 x^3}{8192 c^4}\right)}\right)}{256 c^{5/2}} - \frac{\sqrt{dx^3+c}}{192 c x^6} - \frac{(dx^3+c)^{3/2}}{64 c^2 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^7*(8*c - d*x^3)),x)
```

```
[Out] (d^2*atanh((d^4*(c + d*x^3)^(1/2))/(2048*c^(7/2)*(d^4/(2048*c^3) + (d^5*x^3
)/(8192*c^4)))))/(256*c^(5/2)) - (c + d*x^3)^(1/2)/(192*c*x^6) - (c + d*x^3
)^(3/2)/(64*c^2*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^7+dx^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c),x)
```

```
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**7 + d*x**10), x)
```

$$3.223 \quad \int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3} dx$$

Optimal. Leaf size=130

$$\frac{9216c^{9/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4} - \frac{3072c^4\sqrt{c+dx^3}}{d^4} - \frac{1024c^3(c+dx^3)^{3/2}}{9d^4} - \frac{38c^2(c+dx^3)^{5/2}}{5d^4} - \frac{4c(c+dx^3)^{7/2}}{7d^4} - \frac{2(c+dx^3)^{9/2}}{27d^4}$$

Rubi [A] time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 88, 50, 63, 206}

$$-\frac{3072c^4\sqrt{c+dx^3}}{d^4} - \frac{1024c^3(c+dx^3)^{3/2}}{9d^4} - \frac{38c^2(c+dx^3)^{5/2}}{5d^4} + \frac{9216c^{9/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4} - \frac{4c(c+dx^3)^{7/2}}{7d^4} - \frac{2(c+dx^3)^{9/2}}{27d^4}$$

Antiderivative was successfully verified.

[In] Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3), x]

[Out] (-3072*c^4*sqrt[c + d*x^3])/d^4 - (1024*c^3*(c + d*x^3)^(3/2))/(9*d^4) - (38*c^2*(c + d*x^3)^(5/2))/(5*d^4) - (4*c*(c + d*x^3)^(7/2))/(7*d^4) - (2*(c + d*x^3)^(9/2))/(27*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3]/(3*sqrt[c])])/d^4

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```


$x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x] \&\& \text{IntegersQ}[m, n] \&\& (\text{IntegerQ}[p] \parallel (\text{GtQ}[m, 0] \&\& \text{GeQ}[n, -1]))$

Rule 206

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(1 \cdot \text{ArcTanh}[(\text{Rt}[-b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 446

$\text{Int}[(x_)^{(m_ \cdot)} \cdot ((a_ + (b_ \cdot)(x_)^{(n_ \cdot)})^{(p_ \cdot)}) \cdot ((c_ + (d_ \cdot)(x_)^{(n_ \cdot)})^{(q_ \cdot)}), x_Symbol] :> \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) \cdot (a + b \cdot x)^p \cdot (c + d \cdot x)^q}, x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11} (c + dx^3)^{3/2}}{8c - dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3 (c + dx)^{3/2}}{8c - dx} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{57c^2 (c + dx)^{3/2}}{d^3} + \frac{512c^3 (c + dx)^{3/2}}{d^3 (8c - dx)} - \frac{6c (c + dx)^{5/2}}{d^3} - \frac{(c + dx)^{7/2}}{d^3} \right) dx, x, x^3 \right) \\
 &= -\frac{38c^2 (c + dx^3)^{5/2}}{5d^4} - \frac{4c (c + dx^3)^{7/2}}{7d^4} - \frac{2 (c + dx^3)^{9/2}}{27d^4} + \frac{(512c^3) \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{8c - dx} dx, x, x^3 \right)}{3d^3} \\
 &= -\frac{1024c^3 (c + dx^3)^{3/2}}{9d^4} - \frac{38c^2 (c + dx^3)^{5/2}}{5d^4} - \frac{4c (c + dx^3)^{7/2}}{7d^4} - \frac{2 (c + dx^3)^{9/2}}{27d^4} + \frac{(1536c^4)}{27d^4} \\
 &= -\frac{3072c^4 \sqrt{c + dx^3}}{d^4} - \frac{1024c^3 (c + dx^3)^{3/2}}{9d^4} - \frac{38c^2 (c + dx^3)^{5/2}}{5d^4} - \frac{4c (c + dx^3)^{7/2}}{7d^4} - \frac{2 (c + dx^3)^{9/2}}{27d^4} \\
 &= -\frac{3072c^4 \sqrt{c + dx^3}}{d^4} - \frac{1024c^3 (c + dx^3)^{3/2}}{9d^4} - \frac{38c^2 (c + dx^3)^{5/2}}{5d^4} - \frac{4c (c + dx^3)^{7/2}}{7d^4} - \frac{2 (c + dx^3)^{9/2}}{27d^4} \\
 &= -\frac{3072c^4 \sqrt{c + dx^3}}{d^4} - \frac{1024c^3 (c + dx^3)^{3/2}}{9d^4} - \frac{38c^2 (c + dx^3)^{5/2}}{5d^4} - \frac{4c (c + dx^3)^{7/2}}{7d^4} - \frac{2 (c + dx^3)^{9/2}}{27d^4}
 \end{aligned}$$

Mathematica [A] time = 0.13, size = 93, normalized size = 0.72

$$\frac{9216c^{9/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4} - \frac{2\sqrt{c+dx^3} (1509176c^4 + 61892c^3dx^3 + 4611c^2d^2x^6 + 410cd^3x^9 + 35d^4x^{12})}{945d^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3), x]

[Out] (-2*Sqrt[c + d*x^3]*(1509176*c^4 + 61892*c^3*d*x^3 + 4611*c^2*d^2*x^6 + 410*c*d^3*x^9 + 35*d^4*x^12))/(945*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4

IntegrateAlgebraic [A] time = 0.07, size = 93, normalized size = 0.72

$$\frac{9216c^{9/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4} - \frac{2\sqrt{c+dx^3} (1509176c^4 + 61892c^3dx^3 + 4611c^2d^2x^6 + 410cd^3x^9 + 35d^4x^{12})}{945d^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3), x]

[Out] (-2*Sqrt[c + d*x^3]*(1509176*c^4 + 61892*c^3*d*x^3 + 4611*c^2*d^2*x^6 + 410*c*d^3*x^9 + 35*d^4*x^12))/(945*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4

fricas [A] time = 0.55, size = 191, normalized size = 1.47

$$\left[\frac{2 \left(2177280 c^{\frac{9}{2}} \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c+10c}}{dx^3-8c}\right) - (35d^4x^{12} + 410cd^3x^9 + 4611c^2d^2x^6 + 61892c^3dx^3 + 1509176c^4)\sqrt{dx^3+c} \right)}{945d^4}, - \frac{2 \left(4354560\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + (35d^4x^{12} + 410cd^3x^9 + 4611c^2d^2x^6 + 61892c^3dx^3 + 1509176c^4)\sqrt{dx^3+c} \right)}{945d^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c), x, algorithm="fricas")

[Out] [2/945*(2177280*c^(9/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2*d^2*x^6 + 61892*c^3*d*x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4, -2/945*(4354560*sqrt(-c)*c^4*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2*d^2*x^6 + 61892*c^3*d*x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4]

giac [A] time = 0.17, size = 117, normalized size = 0.90

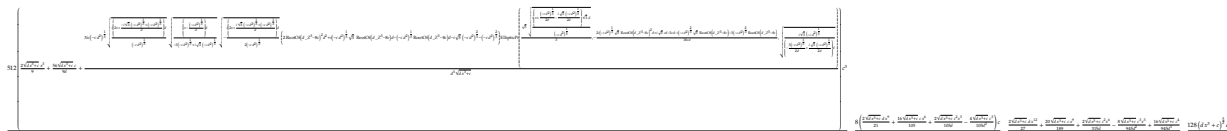
$$\frac{9216c^5 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{\sqrt{-c}d^4} - \frac{2 \left(35(dx^3+c)^{\frac{9}{2}}d^{32} + 270(dx^3+c)^{\frac{7}{2}}cd^{32} + 3591(dx^3+c)^{\frac{5}{2}}c^2d^{32} + 53760(dx^3+c)^{\frac{3}{2}}c^3d^{32} + 1451520\sqrt{dx^3+c}c^4d^{32} \right)}{945d^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(d*x³+c)^(3/2)/(-d*x³+8*c),x, algorithm="giac")

[Out]
$$\frac{-9216*c^5*\arctan(1/3*\sqrt{d*x^3+c})/\sqrt{-c}}{\sqrt{-c}*d^4} - \frac{2}{945}*(35*(d*x^3+c)^{9/2}*d^32 + 270*(d*x^3+c)^{7/2}*c*d^32 + 3591*(d*x^3+c)^{5/2}*c^2*d^32 + 53760*(d*x^3+c)^{3/2}*c^3*d^32 + 1451520*\sqrt{d*x^3+c}*c^4*d^32)/d^36$$

maple [C] time = 0.25, size = 634, normalized size = 4.88



Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹*(d*x³+c)^(3/2)/(-d*x³+8*c),x)

[Out]
$$\begin{aligned} & -1/d*(2/27*d*x^{12}*(d*x^3+c)^{1/2}+20/189*c*x^9*(d*x^3+c)^{1/2}+2/315*c^2/d*x^6*(d*x^3+c)^{1/2}-8/945*c^3/d^2*x^3*(d*x^3+c)^{1/2}+16/945*c^4/d^3*(d*x^3+c)^{1/2}) \\ & -8*c/d^2*(2/21*d*x^9*(d*x^3+c)^{1/2}+16/105*c*x^6*(d*x^3+c)^{1/2}+2/105*c^2/d*x^3*(d*x^3+c)^{1/2}-4/105*c^3/d^2*(d*x^3+c)^{1/2})-128/15*c^2*(d*x^3+c)^{5/2}/d^4 \\ & -512*c^3/d^3*(2/9*x^3*(d*x^3+c)^{1/2}+56/9*(d*x^3+c)^{1/2})*c/d+3*I*c/d^3*2^{1/2}*sum((-c*d^2)^{1/3}*(1/2*I*(2*x+(-I*3^{1/2})*(-c*d^2)^{1/3}+(-c*d^2)^{1/3}))/d \\ & /(-c*d^2)^{1/3}*d)^{1/2}*((x-(-c*d^2)^{1/3}/d)/(-3*(-c*d^2)^{1/3}+I*3^{1/2}*(-c*d^2)^{1/3})*d)^{1/2}*(-1/2*I*(2*x+(I*3^{1/2})*(-c*d^2)^{1/3}+(-c*d^2)^{1/3}))/d \\ & /(-c*d^2)^{1/3}*d)^{1/2}/(d*x^3+c)^{1/2}*(2*_alpha^2*d^2+I*(-c*d^2)^{1/3}*3^{1/2}*_alpha*d-(-c*d^2)^{1/3}*_alpha*d-I*3^{1/2}*(-c*d^2)^{2/3}-(-c*d^2)^{2/3})*EllipticPi(1/3*3^{1/2}*(I*(x+1/2*(-c*d^2)^{1/3}/d-1/2*I*3^{1/2}*(-c*d^2)^{1/3}/d)*3^{1/2}/(-c*d^2)^{1/3}*d)^{1/2},-1/18*(2*I*(-c*d^2)^{1/3}*3^{1/2}*_alpha^2*d+I*3^{1/2}*c*d-3*c*d-I*(-c*d^2)^{2/3})*3^{1/2}*_alpha-3*(-c*d^2)^{2/3}*_alpha)/c/d, (I*3^{1/2}*(-c*d^2)^{1/3}/(-3/2*(-c*d^2)^{1/3}/d+1/2*I*3^{1/2}*(-c*d^2)^{1/3}/d)/d)^{1/2}), _alpha=RootOf(_Z^3*d-8*c)) \end{aligned}$$

maxima [A] time = 1.29, size = 110, normalized size = 0.85

$$\frac{2\left(2177280c^9\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c+3\sqrt{c}}}\right)+35(dx^3+c)^{\frac{9}{2}}+270(dx^3+c)^{\frac{7}{2}}c+3591(dx^3+c)^{\frac{5}{2}}c^2+53760(dx^3+c)^{\frac{3}{2}}c^3+1451520\sqrt{dx^3+c}c^4\right)}{945d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(d*x³+c)^(3/2)/(-d*x³+8*c),x, algorithm="maxima")

[Out]
$$-2/945*(2177280*c^9*\log((\sqrt{d*x^3+c}-3*\sqrt{c})/(\sqrt{d*x^3+c}+3*\sqrt{c}))) + 35*(d*x^3+c)^{9/2} + 270*(d*x^3+c)^{7/2}*c + 3591*(d*x^3+c)^{5/2}*c^2 + 53760*(d*x^3+c)^{3/2}*c^3 + 1451520*\sqrt{d*x^3+c}*c^4/d^4$$

$$(3 + c)^{5/2} * c^2 + 53760 * (d * x^3 + c)^{3/2} * c^3 + 1451520 * \sqrt{d * x^3 + c} * c^4) / d^4$$

mupad [B] time = 3.53, size = 135, normalized size = 1.04

$$\frac{4608 c^{9/2} \ln\left(\frac{10c + dx^3 + 6\sqrt{c} \sqrt{dx^3 + c}}{8c - dx^3}\right)}{d^4} - \frac{2x^{12} \sqrt{dx^3 + c}}{27} - \frac{3018352 c^4 \sqrt{dx^3 + c}}{945 d^4} - \frac{164 c x^9 \sqrt{dx^3 + c}}{189 d} - \frac{123784 c^3 x^3 \sqrt{dx^3 + c}}{945 d^3} - \frac{3074 c^2 x^6 \sqrt{dx^3 + c}}{315 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x¹¹*(c + d*x³)^(3/2))/(8*c - d*x³),x)

[Out] (4608*c^(9/2)*log((10*c + d*x³ + 6*c^(1/2)*(c + d*x³)^(1/2))/(8*c - d*x³)))/d⁴ - (2*x¹²*(c + d*x³)^(1/2))/27 - (3018352*c⁴*(c + d*x³)^(1/2))/(945*d⁴) - (164*c*x⁹*(c + d*x³)^(1/2))/(189*d) - (123784*c³*x³*(c + d*x³)^(1/2))/(945*d³) - (3074*c²*x⁶*(c + d*x³)^(1/2))/(315*d²)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(d*x³+c)^(3/2)/(-d*x³+8*c),x)

[Out] Timed out

$$3.224 \quad \int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3} dx$$

Optimal. Leaf size=109

$$\frac{1152c^{7/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3} - \frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2(c+dx^3)^{3/2}}{9d^3} - \frac{14c(c+dx^3)^{5/2}}{15d^3} - \frac{2(c+dx^3)^{7/2}}{21d^3}$$

Rubi [A] time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 88, 50, 63, 206}

$$-\frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2(c+dx^3)^{3/2}}{9d^3} + \frac{1152c^{7/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3} - \frac{14c(c+dx^3)^{5/2}}{15d^3} - \frac{2(c+dx^3)^{7/2}}{21d^3}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3), x]

[Out] (-384*c^3*Sqrt[c + d*x^3])/d^3 - (128*c^2*(c + d*x^3)^(3/2))/(9*d^3) - (14*c*(c + d*x^3)^(5/2))/(15*d^3) - (2*(c + d*x^3)^(7/2))/(21*d^3) + (1152*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x]]

$x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x\} \&\& \text{IntegersQ}\{m, n\} \&\& (\text{IntegerQ}\{p\} \parallel (\text{GtQ}\{m, 0\} \&\& \text{GeQ}\{n, -1\}))$

Rule 206

$\text{Int}[(a_ + (b_ \cdot (x_)^2)^{-1}), x_ \text{Symbol}] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[(\text{Rt}[-b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}\{a/b\} \&\& (\text{GtQ}\{a, 0\} \parallel \text{LtQ}\{b, 0\})$

Rule 446

$\text{Int}[(x_)^{(m_ \cdot)} \cdot ((a_ + (b_ \cdot (x_)^{(n_)})^{(p_ \cdot)} \cdot ((c_ + (d_ \cdot (x_)^{(n_)})^{(q_ \cdot)}), x_ \text{Symbol}] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) \cdot (a + b \cdot x)^p \cdot (c + d \cdot x)^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x\} \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned}
 \int \frac{x^8 (c + dx^3)^{3/2}}{8c - dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (c + dx)^{3/2}}{8c - dx} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{7c(c + dx)^{3/2}}{d^2} + \frac{64c^2(c + dx)^{3/2}}{d^2(8c - dx)} - \frac{(c + dx)^{5/2}}{d^2} \right) dx, x, x^3 \right) \\
 &= -\frac{14c(c + dx^3)^{5/2}}{15d^3} - \frac{2(c + dx^3)^{7/2}}{21d^3} + \frac{(64c^2) \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{8c - dx} dx, x, x^3 \right)}{3d^2} \\
 &= -\frac{128c^2(c + dx^3)^{3/2}}{9d^3} - \frac{14c(c + dx^3)^{5/2}}{15d^3} - \frac{2(c + dx^3)^{7/2}}{21d^3} + \frac{(192c^3) \text{Subst} \left(\int \frac{\sqrt{c + dx}}{8c - dx} dx, x, x^3 \right)}{d^2} \\
 &= -\frac{384c^3 \sqrt{c + dx^3}}{d^3} - \frac{128c^2(c + dx^3)^{3/2}}{9d^3} - \frac{14c(c + dx^3)^{5/2}}{15d^3} - \frac{2(c + dx^3)^{7/2}}{21d^3} + \frac{(1728c^4) \text{Subst} \left(\int \frac{1}{8c - dx} dx, x, x^3 \right)}{d} \\
 &= -\frac{384c^3 \sqrt{c + dx^3}}{d^3} - \frac{128c^2(c + dx^3)^{3/2}}{9d^3} - \frac{14c(c + dx^3)^{5/2}}{15d^3} - \frac{2(c + dx^3)^{7/2}}{21d^3} + \frac{(3456c^4) \text{Subst} \left(\int \frac{1}{8c - dx} dx, x, x^3 \right)}{d} \\
 &= -\frac{384c^3 \sqrt{c + dx^3}}{d^3} - \frac{128c^2(c + dx^3)^{3/2}}{9d^3} - \frac{14c(c + dx^3)^{5/2}}{15d^3} - \frac{2(c + dx^3)^{7/2}}{21d^3} + \frac{1152c^{7/2} \tan^{-1} \left(\frac{\sqrt{c + dx}}{8c - dx} \right)}{d}
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 81, normalized size = 0.74

$$\frac{362880c^{7/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 2\sqrt{c+dx^3} (62882c^3 + 2579c^2dx^3 + 192cd^2x^6 + 15d^3x^9)}{315d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3), x]

[Out] (-2*Sqrt[c + d*x^3]*(62882*c^3 + 2579*c^2*d*x^3 + 192*c*d^2*x^6 + 15*d^3*x^9) + 362880*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(315*d^3)

IntegrateAlgebraic [A] time = 0.08, size = 82, normalized size = 0.75

$$\frac{1152c^{7/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3} - \frac{2\sqrt{c+dx^3} (62882c^3 + 2579c^2dx^3 + 192cd^2x^6 + 15d^3x^9)}{315d^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3), x]

[Out] (-2*Sqrt[c + d*x^3]*(62882*c^3 + 2579*c^2*d*x^3 + 192*c*d^2*x^6 + 15*d^3*x^9))/(315*d^3) + (1152*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3

fricas [A] time = 0.96, size = 169, normalized size = 1.55

$$\left[\frac{2 \left(90720 c^{\frac{7}{2}} \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) - (15d^3x^9 + 192cd^2x^6 + 2579c^2dx^3 + 62882c^3)\sqrt{dx^3+c} \right)}{315d^3}, \frac{2 \left(181440 \sqrt{-c} c^3 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + (15d^3x^9 + 192cd^2x^6 + 2579c^2dx^3 + 62882c^3)\sqrt{dx^3+c} \right)}{315d^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c), x, algorithm="fricas")

[Out] [2/315*(90720*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt(d*x^3 + c))/d^3, -2/315*(181440*sqrt(-c)*c^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt(d*x^3 + c))/d^3]

giac [A] time = 0.16, size = 100, normalized size = 0.92

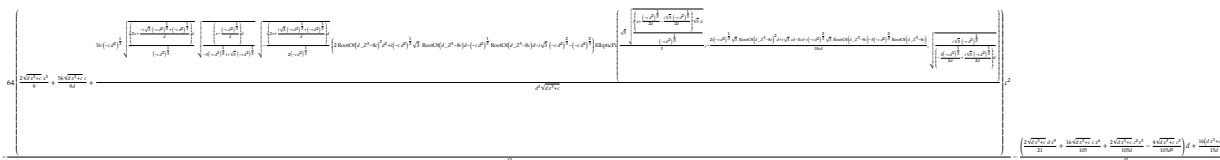
$$\frac{1152c^4 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^3} - \frac{2 \left(15(dx^3+c)^{\frac{7}{2}}d^{18} + 147(dx^3+c)^{\frac{5}{2}}cd^{18} + 2240(dx^3+c)^{\frac{3}{2}}c^2d^{18} + 60480\sqrt{dx^3+c}c^3d^{18} \right)}{315d^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")

[Out] $-1152*c^4*\arctan(1/3*\sqrt{d*x^3 + c}/\sqrt{-c})/(\sqrt{-c}*d^3) - 2/315*(15*(d*x^3 + c)^{(7/2)}*d^{18} + 147*(d*x^3 + c)^{(5/2)}*c*d^{18} + 2240*(d*x^3 + c)^{(3/2)}*c^2*d^{18} + 60480*\sqrt{d*x^3 + c}*c^3*d^{18})/d^{21}$

maple [C] time = 0.16, size = 541, normalized size = 4.96



Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)

[Out] $-1/d^2*(d*(2/21*(d*x^3+c)^{(1/2)}*d*x^9+16/105*(d*x^3+c)^{(1/2)}*c*x^6+2/105*(d*x^3+c)^{(1/2)}*c^2/d*x^3-4/105*(d*x^3+c)^{(1/2)}*c^3/d^2)+16/15*c/d*(d*x^3+c)^{(5/2)})-64*c^2/d^2*(2/9*(d*x^3+c)^{(1/2)}*x^3+56/9*(d*x^3+c)^{(1/2)}*c/d+3*I*c/d^3*2^{(1/2)}*\sum((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)}))/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)}))/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},-1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)},_alpha=RootOf(_Z^3*d-8*c)))$

maxima [A] time = 1.34, size = 96, normalized size = 0.88

$$\frac{2 \left(90720 c^2 \log \left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}} \right) + 15 (dx^3+c)^{\frac{7}{2}} + 147 (dx^3+c)^{\frac{5}{2}} c + 2240 (dx^3+c)^{\frac{3}{2}} c^2 + 60480 \sqrt{dx^3+c} c^3 \right)}{315 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")

[Out] $-2/315*(90720*c^2*\log((\sqrt{d*x^3 + c} - 3*\sqrt{c})/(\sqrt{d*x^3 + c} + 3*\sqrt{c}))) + 15*(d*x^3 + c)^{(7/2)} + 147*(d*x^3 + c)^{(5/2)}*c + 2240*(d*x^3 + c)^{(3/2)}*c^2 + 60480*\sqrt{d*x^3 + c}*c^3/d^3$

mupad [B] time = 3.50, size = 115, normalized size = 1.06

$$\frac{576c^{7/2} \ln\left(\frac{10c+dx^3+6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{d^3} - \frac{2x^9\sqrt{dx^3+c}}{21} - \frac{125764c^3\sqrt{dx^3+c}}{315d^3} - \frac{128cx^6\sqrt{dx^3+c}}{105d} - \frac{5158c^2x^3\sqrt{dx^3+c}}{315d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)`

[Out] $(576*c^{(7/2)}*\log((10*c + d*x^3 + 6*c^{(1/2)}*(c + d*x^3)^{(1/2)})/(8*c - d*x^3)))/d^3 - (2*x^9*(c + d*x^3)^{(1/2)})/21 - (125764*c^3*(c + d*x^3)^{(1/2)})/(315*d^3) - (128*c*x^6*(c + d*x^3)^{(1/2)})/(105*d) - (5158*c^2*x^3*(c + d*x^3)^{(1/2)})/(315*d^2)$

sympy [A] time = 108.79, size = 110, normalized size = 1.01

$$-\frac{1152c^4 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^3\sqrt{-c}} - \frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2(c+dx^3)^{\frac{3}{2}}}{9d^3} - \frac{14c(c+dx^3)^{\frac{5}{2}}}{15d^3} - \frac{2(c+dx^3)^{\frac{7}{2}}}{21d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c), x)`

[Out] $-1152*c**4*\operatorname{atan}(\operatorname{sqrt}(c + d*x**3)/(3*\operatorname{sqrt}(-c)))/(d**3*\operatorname{sqrt}(-c)) - 384*c**3*\operatorname{sqrt}(c + d*x**3)/d**3 - 128*c**2*(c + d*x**3)**(3/2)/(9*d**3) - 14*c*(c + d*x**3)**(5/2)/(15*d**3) - 2*(c + d*x**3)**(7/2)/(21*d**3)$

$$3.225 \quad \int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3} dx$$

Optimal. Leaf size=88

$$\frac{144c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2} - \frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c(c+dx^3)^{3/2}}{9d^2} - \frac{2(c+dx^3)^{5/2}}{15d^2}$$

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 80, 50, 63, 206}

$$-\frac{48c^2\sqrt{c+dx^3}}{d^2} + \frac{144c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2} - \frac{16c(c+dx^3)^{3/2}}{9d^2} - \frac{2(c+dx^3)^{5/2}}{15d^2}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3), x]

[Out] (-48*c^2*sqrt[c + d*x^3])/d^2 - (16*c*(c + d*x^3)^(3/2))/(9*d^2) - (2*(c + d*x^3)^(5/2))/(15*d^2) + (144*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*sqrt[c])])/d^2

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5 (c + dx^3)^{3/2}}{8c - dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(c + dx)^{3/2}}{8c - dx} dx, x, x^3 \right) \\
 &= -\frac{2(c + dx^3)^{5/2}}{15d^2} + \frac{(8c) \text{Subst} \left(\int \frac{(c+dx)^{3/2}}{8c-dx} dx, x, x^3 \right)}{3d} \\
 &= -\frac{16c(c + dx^3)^{3/2}}{9d^2} - \frac{2(c + dx^3)^{5/2}}{15d^2} + \frac{(24c^2) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{8c-dx} dx, x, x^3 \right)}{d} \\
 &= -\frac{48c^2\sqrt{c + dx^3}}{d^2} - \frac{16c(c + dx^3)^{3/2}}{9d^2} - \frac{2(c + dx^3)^{5/2}}{15d^2} + \frac{(216c^3) \text{Subst} \left(\int \frac{1}{(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{d} \\
 &= -\frac{48c^2\sqrt{c + dx^3}}{d^2} - \frac{16c(c + dx^3)^{3/2}}{9d^2} - \frac{2(c + dx^3)^{5/2}}{15d^2} + \frac{(432c^3) \text{Subst} \left(\int \frac{1}{9c-x^2} dx, x, \sqrt{c + dx^3} \right)}{d^2} \\
 &= -\frac{48c^2\sqrt{c + dx^3}}{d^2} - \frac{16c(c + dx^3)^{3/2}}{9d^2} - \frac{2(c + dx^3)^{5/2}}{15d^2} + \frac{144c^{5/2} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{d^2}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.80

$$\frac{6480c^{5/2} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) - 2\sqrt{c + dx^3} (1123c^2 + 46cdx^3 + 3d^2x^6)}{45d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3), x]

[Out] (-2*Sqrt[c + d*x^3]*(1123*c^2 + 46*c*d*x^3 + 3*d^2*x^6) + 6480*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^2)

IntegrateAlgebraic [A] time = 0.07, size = 71, normalized size = 0.81

$$\frac{144c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2} - \frac{2\sqrt{c+dx^3}(1123c^2 + 46cdx^3 + 3d^2x^6)}{45d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3), x]

[Out] (-2*Sqrt[c + d*x^3]*(1123*c^2 + 46*c*d*x^3 + 3*d^2*x^6))/(45*d^2) + (144*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2

fricas [A] time = 0.63, size = 147, normalized size = 1.67

$$\left[\frac{2\left(1620c^{\frac{5}{2}} \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) - (3d^2x^6 + 46cdx^3 + 1123c^2)\sqrt{dx^3+c}\right)}{45d^2}, -\frac{2\left(3240\sqrt{-c}c^2 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + (3d^2x^6 + 46cdx^3 + 1123c^2)\sqrt{dx^3+c}\right)}{45d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c), x, algorithm="fricas")

[Out] [2/45*(1620*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - (3*d^2*x^6 + 46*c*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2, -2/45*(3240*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^2*x^6 + 46*c*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2]

giac [A] time = 0.18, size = 83, normalized size = 0.94

$$\frac{144c^3 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^2} - \frac{2\left(3(dx^3+c)^{\frac{5}{2}}d^8 + 40(dx^3+c)^{\frac{3}{2}}cd^8 + 1080\sqrt{dx^3+c}c^2d^8\right)}{45d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c), x, algorithm="giac")

[Out] -144*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/45*(3*(d*x^3 + c)^(5/2)*d^8 + 40*(d*x^3 + c)^(3/2)*c*d^8 + 1080*sqrt(d*x^3 + c)*c^2*d^8)/d^10

maple [C] time = 0.18, size = 462, normalized size = 5.25

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^5*(d*x^3+c)^{(3/2)} / (-d*x^3+8*c), x)$

[Out]
$$-2/15*(d*x^3+c)^{(5/2)}/d^2-8*c/d*(2/9*(d*x^3+c)^{(1/2)}*x^3+56/9*(d*x^3+c)^{(1/2)}*c/d+3*I*c/d^3*2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=\text{RootOf}(Z^3*d-8*c)))$$

maxima [A] time = 1.44, size = 82, normalized size = 0.93

$$\frac{2 \left(1620 c^{\frac{5}{2}} \log \left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}} \right) + 3 (dx^3+c)^{\frac{5}{2}} + 40 (dx^3+c)^{\frac{3}{2}} c + 1080 \sqrt{dx^3+c} c^2 \right)}{45 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^5*(d*x^3+c)^{(3/2)} / (-d*x^3+8*c), x, \text{algorithm}="maxima")$

[Out]
$$-2/45*(1620*c^{(5/2)}*\log((\text{sqrt}(d*x^3+c)-3*\text{sqrt}(c))/(\text{sqrt}(d*x^3+c)+3*\text{sqrt}(c))))+3*(d*x^3+c)^{(5/2)}+40*(d*x^3+c)^{(3/2)}*c+1080*\text{sqrt}(d*x^3+c)*c^2)/d^2$$

mupad [B] time = 3.52, size = 95, normalized size = 1.08

$$\frac{72 c^{5/2} \ln \left(\frac{10 c+d x^3+6 \sqrt{c} \sqrt{d x^3+c}}{8 c-d x^3} \right)}{d^2} - \frac{2 x^6 \sqrt{d x^3+c}}{15} - \frac{2246 c^2 \sqrt{d x^3+c}}{45 d^2} - \frac{92 c x^3 \sqrt{d x^3+c}}{45 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)`

[Out] $(72*c^{5/2}*\log((10*c + d*x^3 + 6*c^{1/2}*(c + d*x^3)^{1/2}))/ (8*c - d*x^3)) /d^2 - (2*x^6*(c + d*x^3)^{1/2})/15 - (2246*c^2*(c + d*x^3)^{1/2})/(45*d^2) - (92*c*x^3*(c + d*x^3)^{1/2})/(45*d)$

sympy [A] time = 62.74, size = 90, normalized size = 1.02

$$-\frac{144c^3 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^2\sqrt{-c}} - \frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c(c+dx^3)^{\frac{3}{2}}}{9d^2} - \frac{2(c+dx^3)^{\frac{5}{2}}}{15d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)`

[Out] $-144*c**3*\operatorname{atan}(\operatorname{sqrt}(c + d*x**3)/(3*\operatorname{sqrt}(-c)))/(d**2*\operatorname{sqrt}(-c)) - 48*c**2*\operatorname{sqrt}(c + d*x**3)/d**2 - 16*c*(c + d*x**3)**(3/2)/(9*d**2) - 2*(c + d*x**3)**(5/2)/(15*d**2)$

$$3.226 \quad \int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3} dx$$

Optimal. Leaf size=67

$$\frac{18c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d} - \frac{6c\sqrt{c+dx^3}}{d} - \frac{2(c+dx^3)^{3/2}}{9d}$$

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {444, 50, 63, 206}

$$\frac{18c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d} - \frac{6c\sqrt{c+dx^3}}{d} - \frac{2(c+dx^3)^{3/2}}{9d}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3), x]

[Out] (-6*c*Sqrt[c + d*x^3])/d - (2*(c + d*x^3)^(3/2))/(9*d) + (18*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^(p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2 (c + dx^3)^{3/2}}{8c - dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{8c - dx} dx, x, x^3 \right) \\
 &= -\frac{2(c + dx^3)^{3/2}}{9d} + (3c) \text{Subst} \left(\int \frac{\sqrt{c + dx}}{8c - dx} dx, x, x^3 \right) \\
 &= -\frac{6c\sqrt{c + dx^3}}{d} - \frac{2(c + dx^3)^{3/2}}{9d} + (27c^2) \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right) \\
 &= -\frac{6c\sqrt{c + dx^3}}{d} - \frac{2(c + dx^3)^{3/2}}{9d} + \frac{(54c^2) \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{d} \\
 &= -\frac{6c\sqrt{c + dx^3}}{d} - \frac{2(c + dx^3)^{3/2}}{9d} + \frac{18c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{d}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.87

$$\frac{162c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right) - 2\sqrt{c + dx^3} (28c + dx^3)}{9d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3), x]

[Out] (-2*Sqrt[c + d*x^3]*(28*c + d*x^3) + 162*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d)

IntegrateAlgebraic [A] time = 0.06, size = 59, normalized size = 0.88

$$\frac{18c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d} - \frac{2\sqrt{c+dx^3}(28c+dx^3)}{9d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3), x]

[Out] (-2*Sqrt[c + d*x^3]*(28*c + d*x^3))/(9*d) + (18*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d

fricas [A] time = 0.58, size = 121, normalized size = 1.81

$$\left[\frac{81c^3 \log\left(\frac{dx^3 + 6\sqrt{dx^3+c}\sqrt{c+10c}}{dx^3-8c}\right) - 2(dx^3 + 28c)\sqrt{dx^3+c}}{9d}, -\frac{2\left(81\sqrt{-c}c \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + (dx^3 + 28c)\sqrt{dx^3+c}\right)}{9d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c), x, algorithm="fricas")

[Out] [1/9*(81*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 2*(d*x^3 + 28*c)*sqrt(d*x^3 + c))/d, -2/9*(81*sqrt(-c)*c*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 28*c)*sqrt(d*x^3 + c))/d]

giac [A] time = 0.16, size = 65, normalized size = 0.97

$$-\frac{18c^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d} - \frac{2\left((dx^3+c)^2 d^2 + 27\sqrt{dx^3+c}cd^2\right)}{9d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c), x, algorithm="giac")

[Out] -18*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/9*((d*x^3 + c)^(3/2)*d^2 + 27*sqrt(d*x^3 + c)*c*d^2)/d^3

maple [C] time = 0.16, size = 441, normalized size = 6.58

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)`

[Out]
$$-2/9*(d*x^3+c)^{(1/2)}*x^3-56/9*(d*x^3+c)^{(1/2)}*c/d-3*I*c/d^3*2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)}))/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)}))/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},-1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}),_alpha=\text{RootOf}(_Z^3*d-8*c))$$

maxima [A] time = 1.39, size = 68, normalized size = 1.01

$$\frac{81 c^{\frac{3}{2}} \log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right) + 2(dx^3+c)^{\frac{3}{2}} + 54\sqrt{dx^3+c}c}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")`

[Out]
$$-1/9*(81*c^{(3/2)}*\log((\text{sqrt}(d*x^3+c)-3*\text{sqrt}(c))/(\text{sqrt}(d*x^3+c)+3*\text{sqrt}(c))))+2*(d*x^3+c)^{(3/2)}+54*\text{sqrt}(d*x^3+c)*c)/d$$

mupad [B] time = 3.45, size = 75, normalized size = 1.12

$$\frac{9c^{3/2} \ln\left(\frac{10c+dx^3+6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{d} - \frac{56c\sqrt{dx^3+c}}{9d} - \frac{2x^3\sqrt{dx^3+c}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(c+d*x^3)^(3/2))/(8*c-d*x^3),x)`

[Out]
$$(9*c^{(3/2)}*\log((10*c+d*x^3+6*c^{(1/2)}*(c+d*x^3)^{(1/2)})/(8*c-d*x^3)))/d - (56*c*(c+d*x^3)^{(1/2)})/(9*d) - (2*x^3*(c+d*x^3)^{(1/2)})/9$$

sympy [A] time = 29.25, size = 65, normalized size = 0.97

$$-\frac{18c^2 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d\sqrt{-c}} - \frac{6c\sqrt{c+dx^3}}{d} - \frac{2(c+dx^3)^{\frac{3}{2}}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -18*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d*sqrt(-c)) - 6*c*sqrt(c + d*  
x**3)/d - 2*(c + d*x**3)**(3/2)/(9*d)
```

$$3.227 \quad \int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)} dx$$

Optimal. Leaf size=73

$$-\frac{2}{3}\sqrt{c+dx^3} + \frac{9}{4}\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \frac{1}{12}\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {446, 84, 156, 63, 208, 206}

$$-\frac{2}{3}\sqrt{c+dx^3} + \frac{9}{4}\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \frac{1}{12}\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]

[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 84

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), Int[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]

Rule 156

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx^3)^{3/2}}{x(8c - dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{x(8c - dx)} dx, x, x^3 \right) \\
 &= -\frac{2}{3} \sqrt{c + dx^3} - \frac{\text{Subst} \left(\int \frac{-c^2d - 10cd^2x}{x(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{3d} \\
 &= -\frac{2}{3} \sqrt{c + dx^3} + \frac{1}{24}c \text{Subst} \left(\int \frac{1}{x\sqrt{c + dx}} dx, x, x^3 \right) + \frac{1}{8}(27cd) \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right) \\
 &= -\frac{2}{3} \sqrt{c + dx^3} + \frac{1}{4}(27c) \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right) + \frac{c \text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{12d} \\
 &= -\frac{2}{3} \sqrt{c + dx^3} + \frac{9}{4} \sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right) - \frac{1}{12} \sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 73, normalized size = 1.00

$$-\frac{2}{3} \sqrt{c + dx^3} + \frac{9}{4} \sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right) - \frac{1}{12} \sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]

[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12

IntegrateAlgebraic [A] time = 0.05, size = 73, normalized size = 1.00

$$-\frac{2}{3}\sqrt{c+dx^3} + \frac{9}{4}\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \frac{1}{12}\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]

[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12

fricas [A] time = 0.85, size = 152, normalized size = 2.08

$$\left[\frac{9}{8}\sqrt{c} \log\left(\frac{dx^3 + 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c}\right) + \frac{1}{24}\sqrt{c} \log\left(\frac{dx^3 - 2\sqrt{dx^3+c}\sqrt{c} + 2c}{x^3}\right) - \frac{2}{3}\sqrt{dx^3+c}, \frac{1}{12}\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - \frac{9}{4}\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - \frac{2}{3}\sqrt{dx^3+c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="fricas")

[Out] [9/8*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 1/24*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2/3*sqrt(d*x^3 + c), 1/12*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 9/4*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 2/3*sqrt(d*x^3 + c)]

giac [A] time = 0.18, size = 61, normalized size = 0.84

$$\frac{c \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}} - \frac{9c \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{4\sqrt{-c}} - \frac{2}{3}\sqrt{dx^3+c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="giac")

[Out] 1/12*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/4*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 2/3*sqrt(d*x^3 + c)

maple [C] time = 0.19, size = 500, normalized size = 6.85

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x)`

[Out]
$$-1/8/c*d*(2/9*(d*x^3+c)^{(1/2)}*x^3+56/9*(d*x^3+c)^{(1/2)}*c/d+3*I*c/d^3*2^{(1/2)})*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},-1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)},_alpha=\text{RootOf}(_Z^3*d-8*c))) + 1/8/c*(2/9*d*x^3*(d*x^3+c)^{(1/2)}+8/9*c*(d*x^3+c)^{(1/2)}-2/3*c^{(3/2)}*\text{arctanh}((d*x^3+c)^{(1/2)}/c^{(1/2)}))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(dx^3 + c)^{\frac{3}{2}}}{(dx^3 - 8c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="maxima")`

[Out] `-integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x), x)`

mupad [B] time = 5.89, size = 89, normalized size = 1.22

$$\frac{\sqrt{c} \ln \left(\frac{(\sqrt{dx^3+c}-\sqrt{c})^3 (\sqrt{dx^3+c}+\sqrt{c}) (10c+dx^3+6\sqrt{c}\sqrt{dx^3+c})^{27}}{x^6 (8c-dx^3)^{27}} \right)}{24} - \frac{2\sqrt{dx^3+c}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x)`

[Out]
$$(c^{(1/2)}*\log((((c + d*x^3)^{(1/2)} - c^{(1/2)})^3*((c + d*x^3)^{(1/2)} + c^{(1/2)})*(10*c + d*x^3 + 6*c^{(1/2)}*(c + d*x^3)^{(1/2)})^{27}/(x^6*(8*c - d*x^3)^{27}))))/24 - (2*(c + d*x^3)^{(1/2)})/3$$

sympy [A] time = 24.74, size = 73, normalized size = 1.00

$$-\frac{9c \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{4\sqrt{-c}} + \frac{c \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12\sqrt{-c}} - \frac{2\sqrt{c+dx^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c),x)

[Out] -9*c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(4*sqrt(-c)) + c*atan(sqrt(c + d*x**3)/sqrt(-c))/(12*sqrt(-c)) - 2*sqrt(c + d*x**3)/3

$$3.228 \quad \int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)} dx$$

Optimal. Leaf size=78

$$-\frac{\sqrt{c+dx^3}}{24x^3} + \frac{9d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {446, 98, 156, 63, 208, 206}

$$-\frac{\sqrt{c+dx^3}}{24x^3} + \frac{9d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)), x]

[Out] -Sqrt[c + d*x^3]/(24*x^3) + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])]/(96*Sqrt[c])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 98

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^3)^{3/2}}{x^4(8c - dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{x^2(8c - dx)} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c + dx^3}}{24x^3} - \frac{\text{Subst} \left(\int \frac{-13c^2d - \frac{17}{2}cd^2x}{x(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{24c} \\
&= -\frac{\sqrt{c + dx^3}}{24x^3} + \frac{1}{192}(13d) \text{Subst} \left(\int \frac{1}{x\sqrt{c + dx}} dx, x, x^3 \right) + \frac{1}{64}(27d^2) \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c + dx^3}}{24x^3} + \frac{13}{96} \text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right) + \frac{1}{32}(27d) \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right) \\
&= -\frac{\sqrt{c + dx^3}}{24x^3} + \frac{9d \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{32\sqrt{c}} - \frac{13d \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{96\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 1.00

$$-\frac{\sqrt{c+dx^3}}{24x^3} + \frac{9d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)), x]

[Out] -1/24*sqrt[c + d*x^3]/x^3 + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*sqrt[c])])/(32*sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/sqrt[c])]/(96*sqrt[c])

IntegrateAlgebraic [A] time = 0.08, size = 78, normalized size = 1.00

$$-\frac{\sqrt{c+dx^3}}{24x^3} + \frac{9d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)), x]

[Out] -1/24*sqrt[c + d*x^3]/x^3 + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*sqrt[c])])/(32*sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/sqrt[c])]/(96*sqrt[c])

fricas [A] time = 0.86, size = 186, normalized size = 2.38

$$\left[\frac{27\sqrt{c}dx^3 \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + 13\sqrt{c}dx^3 \log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) - 8\sqrt{dx^3+c}c}{192cx^3}, \frac{13\sqrt{-c}dx^3 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - 27\sqrt{-c}dx^3 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - 4\sqrt{dx^3+c}c}{96cx^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c), x, algorithm="fricas")

[Out] [1/192*(27*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 13*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*sqrt(d*x^3 + c)*c)/(c*x^3), 1/96*(13*sqrt(-c)*d*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 27*sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*sqrt(d*x^3 + c)*c)/(c*x^3)]

giac [A] time = 0.17, size = 64, normalized size = 0.82

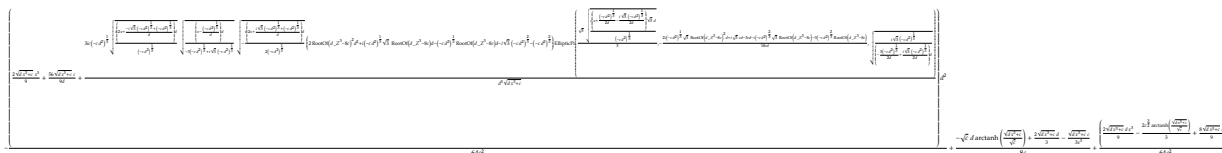
$$\frac{13d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}} - \frac{9d \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{32\sqrt{-c}} - \frac{\sqrt{dx^3+c}}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")

[Out] 13/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/32*d*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/24*sqrt(d*x^3 + c)/x^3

maple [C] time = 0.26, size = 556, normalized size = 7.13



Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x)

[Out] 1/8/c*(-1/3*c*(d*x^3+c)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/2)*d*arctanh((d*x^3+c)^(1/2)/c^(1/2)))-1/64*d^2/c^2*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*d*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x^3+c)^(1/2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(dx^3 + c)^{\frac{3}{2}}}{(dx^3 - 8c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")

[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^4), x)

mupad [B] time = 3.52, size = 56, normalized size = 0.72

$$\frac{9d \operatorname{atanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d \operatorname{atanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{96\sqrt{c}} - \frac{\sqrt{dx^3+c}}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)), x)`

[Out] $(9*d*\operatorname{atanh}((c + d*x^3)^{(1/2)}/(3*c^{(1/2)})))/(32*c^{(1/2)}) - (13*d*\operatorname{atanh}((c + d*x^3)^{(1/2)}/c^{(1/2)}))/(96*c^{(1/2)}) - (c + d*x^3)^{(1/2)}/(24*x^3)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c\sqrt{c+dx^3}}{-8cx^4+dx^7} dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^4+dx^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c), x)`

[Out] $-\operatorname{Integral}(c*\operatorname{sqrt}(c + d*x**3)/(-8*c*x**4 + d*x**7), x) - \operatorname{Integral}(d*x**3*\operatorname{sqrt}(c + d*x**3)/(-8*c*x**4 + d*x**7), x)$

$$3.229 \quad \int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)} dx$$

Optimal. Leaf size=104

$$\frac{9d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}} - \frac{37d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}} - \frac{11d\sqrt{c+dx^3}}{192cx^3} - \frac{\sqrt{c+dx^3}}{48x^6}$$

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {446, 98, 151, 156, 63, 208, 206}

$$\frac{9d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}} - \frac{37d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}} - \frac{11d\sqrt{c+dx^3}}{192cx^3} - \frac{\sqrt{c+dx^3}}{48x^6}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]

[Out] -Sqrt[c + d*x^3]/(48*x^6) - (11*d*Sqrt[c + d*x^3])/(192*c*x^3) + (9*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(3/2)) - (37*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 98

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 151

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

```

Rule 156

```

Int[((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 446

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^3)^{3/2}}{x^7(8c - dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{x^3(8c - dx)} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c + dx^3}}{48x^6} - \frac{\text{Subst} \left(\int \frac{-22c^2d - \frac{35}{2}cd^2x}{x^2(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{48c} \\
&= -\frac{\sqrt{c + dx^3}}{48x^6} - \frac{11d\sqrt{c + dx^3}}{192cx^3} + \frac{\text{Subst} \left(\int \frac{74c^3d^2 + 11c^2d^3x}{x(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{384c^3} \\
&= -\frac{\sqrt{c + dx^3}}{48x^6} - \frac{11d\sqrt{c + dx^3}}{192cx^3} + \frac{(37d^2) \text{Subst} \left(\int \frac{1}{x\sqrt{c + dx}} dx, x, x^3 \right)}{1536c} + \frac{(27d^3) \text{Subst} \left(\int \frac{1}{(8c - dx)} dx, x, x^3 \right)}{512c} \\
&= -\frac{\sqrt{c + dx^3}}{48x^6} - \frac{11d\sqrt{c + dx^3}}{192cx^3} + \frac{(37d) \text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{768c} + \frac{(27d^2) \text{Subst} \left(\int \frac{1}{9c - dx} dx, x, x^3 \right)}{512c} \\
&= -\frac{\sqrt{c + dx^3}}{48x^6} - \frac{11d\sqrt{c + dx^3}}{192cx^3} + \frac{9d^2 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{256c^{3/2}} - \frac{37d^2 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{768c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 96, normalized size = 0.92

$$\frac{27d^2x^6 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right) - 37d^2x^6 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right) - 4\sqrt{c} \sqrt{c + dx^3} (4c + 11dx^3)}{768c^{3/2}x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)), x]

[Out] (-4*Sqrt[c]*Sqrt[c + d*x^3]*(4*c + 11*d*x^3) + 27*d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 37*d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(3/2)*x^6)

IntegrateAlgebraic [A] time = 0.11, size = 95, normalized size = 0.91

$$\frac{9d^2 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{256c^{3/2}} - \frac{37d^2 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{768c^{3/2}} + \frac{(-4c - 11dx^3) \sqrt{c + dx^3}}{192cx^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]

[Out] ((-4*c - 11*d*x^3)*Sqrt[c + d*x^3])/(192*c*x^6) + (9*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(3/2)) - (37*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(3/2))

fricas [A] time = 0.87, size = 218, normalized size = 2.10

$$\frac{27\sqrt{c}d^2x^6\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+37\sqrt{c}d^2x^6\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-8(11cdx^3+4c^2)\sqrt{dx^3+c}-37\sqrt{-c}d^2x^6\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-27\sqrt{-c}d^2x^6\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-4(11cdx^3+4c^2)\sqrt{dx^3+c}}{1536c^2x^6}, \frac{37\sqrt{-c}d^2x^6\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-27\sqrt{-c}d^2x^6\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-4(11cdx^3+4c^2)\sqrt{dx^3+c}}{768c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")

[Out] [1/1536*(27*sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 37*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^2*x^6), 1/768*(37*sqrt(-c)*d^2*x^6*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 27*sqrt(-c)*d^2*x^6*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^2*x^6)]

giac [A] time = 0.19, size = 101, normalized size = 0.97

$$\frac{37d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{768\sqrt{-c}c} - \frac{9d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{256\sqrt{-c}c} - \frac{11(dx^3+c)^{\frac{3}{2}}d^2-7\sqrt{dx^3+c}cd^2}{192cd^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")

[Out] 37/768*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 9/256*d^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/192*(11*(d*x^3 + c)^(3/2)*d^2 - 7*sqrt(d*x^3 + c)*c*d^2)/(c*d^2*x^6)

maple [C] time = 0.24, size = 617, normalized size = 5.93

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x)

```
[Out] 1/64/c^2*d*(-1/3*(d*x^3+c)^(1/2)*c/x^3+2/3*(d*x^3+c)^(1/2)*d-c^(1/2)*d*arctanh((d*x^3+c)^(1/2)/c^(1/2)))+1/8/c*(-1/6*c*(d*x^3+c)^(1/2)/x^6-5/12*d*(d*x^3+c)^(1/2)/x^3-1/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))-1/512/c^3*d^3*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2))*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/512/c^3*d^2*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x^3+c)^(1/2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(dx^3 + c)^{\frac{3}{2}}}{(dx^3 - 8c)x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^7), x)
```

mupad [B] time = 3.74, size = 87, normalized size = 0.84

$$\frac{7\sqrt{dx^3+c}}{192x^6} - \frac{37d^2 \operatorname{atanh}\left(\frac{c\sqrt{dx^3+c}}{\sqrt{c^3}}\right)}{768\sqrt{c^3}} + \frac{9d^2 \operatorname{atanh}\left(\frac{c\sqrt{dx^3+c}}{3\sqrt{c^3}}\right)}{256\sqrt{c^3}} - \frac{11(dx^3+c)^{3/2}}{192cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x)
```

```
[Out] (7*(c + d*x^3)^(1/2))/(192*x^6) - (37*d^2*atanh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)))/(768*(c^3)^(1/2)) + (9*d^2*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(256*(c^3)^(1/2)) - (11*(c + d*x^3)^(3/2))/(192*c*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c),x)
```

```
[Out] Timed out
```

$$3.230 \quad \int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}} dx$$

Optimal. Leaf size=90

$$\frac{1024c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4} - \frac{38c^2\sqrt{c+dx^3}}{d^4} - \frac{4c(c+dx^3)^{3/2}}{3d^4} - \frac{2(c+dx^3)^{5/2}}{15d^4}$$

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {446, 88, 63, 206}

$$-\frac{38c^2\sqrt{c+dx^3}}{d^4} + \frac{1024c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4} - \frac{4c(c+dx^3)^{3/2}}{3d^4} - \frac{2(c+dx^3)^{5/2}}{15d^4}$$

Antiderivative was successfully verified.

[In] Int[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (-38*c^2*Sqrt[c + d*x^3])/d^4 - (4*c*(c + d*x^3)^(3/2))/(3*d^4) - (2*(c + d*x^3)^(5/2))/(15*d^4) + (1024*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^4)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11}}{(8c - dx^3)\sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{57c^2}{d^3\sqrt{c + dx}} + \frac{512c^3}{d^3(8c - dx)\sqrt{c + dx}} - \frac{6c\sqrt{c + dx}}{d^3} - \frac{(c + dx)^{3/2}}{d^3} \right) dx, x, x^3 \right) \\
 &= -\frac{38c^2\sqrt{c + dx^3}}{d^4} - \frac{4c(c + dx^3)^{3/2}}{3d^4} - \frac{2(c + dx^3)^{5/2}}{15d^4} + \frac{(512c^3) \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{3d^3} \\
 &= -\frac{38c^2\sqrt{c + dx^3}}{d^4} - \frac{4c(c + dx^3)^{3/2}}{3d^4} - \frac{2(c + dx^3)^{5/2}}{15d^4} + \frac{(1024c^3) \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, x^3 \right)}{3d^4} \\
 &= -\frac{38c^2\sqrt{c + dx^3}}{d^4} - \frac{4c(c + dx^3)^{3/2}}{3d^4} - \frac{2(c + dx^3)^{5/2}}{15d^4} + \frac{1024c^{5/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{9d^4}
 \end{aligned}$$

Mathematica [A] time = 0.11, size = 69, normalized size = 0.77

$$\frac{5120c^{5/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right) - 6\sqrt{c + dx^3} (296c^2 + 12cdx^3 + d^2x^6)}{45d^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (-6*Sqrt[c + d*x^3]*(296*c^2 + 12*c*d*x^3 + d^2*x^6) + 5120*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^4)

IntegrateAlgebraic [A] time = 0.06, size = 72, normalized size = 0.80

$$\frac{1024c^{5/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{9d^4} - \frac{2\sqrt{c + dx^3} (296c^2 + 12cdx^3 + d^2x^6)}{15d^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (-2*Sqrt[c + d*x^3]*(296*c^2 + 12*c*d*x^3 + d^2*x^6))/(15*d^4) + (1024*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^4)

fricas [A] time = 0.92, size = 146, normalized size = 1.62

$$\left[\frac{2 \left(1280 c^{\frac{5}{2}} \log \left(\frac{dx^3 + 6 \sqrt{dx^3 + c} \sqrt{c} + 10c}{dx^3 - 8c} \right) - 3 (d^2 x^6 + 12 cd x^3 + 296 c^2) \sqrt{dx^3 + c} \right)}{45 d^4}, - \frac{2 \left(2560 \sqrt{-c} c^2 \arctan \left(\frac{\sqrt{dx^3 + c} \sqrt{-c}}{3c} \right) + 3 (d^2 x^6 + 12 cd x^3 + 296 c^2) \sqrt{dx^3 + c} \right)}{45 d^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [2/45*(1280*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 3*(d^2*x^6 + 12*c*d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4, -2/45*(2560*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(d^2*x^6 + 12*c*d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4]

giac [A] time = 0.16, size = 82, normalized size = 0.91

$$-\frac{1024 c^3 \arctan \left(\frac{\sqrt{dx^3 + c}}{3 \sqrt{-c}} \right)}{9 \sqrt{-c} d^4} - \frac{2 \left((dx^3 + c)^{\frac{5}{2}} d^{16} + 10 (dx^3 + c)^{\frac{3}{2}} cd^{16} + 285 \sqrt{dx^3 + c} c^2 d^{16} \right)}{15 d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] -1024/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/15*((d*x^3 + c)^(5/2)*d^16 + 10*(d*x^3 + c)^(3/2)*c*d^16 + 285*sqrt(d*x^3 + c)*c^2*d^16)/d^20

maple [C] time = 0.28, size = 528, normalized size = 5.87

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)

[Out] -1/d*(2/15/d*x^6*(d*x^3+c)^(1/2)-8/45*c/d^2*x^3*(d*x^3+c)^(1/2)+16/45*c^2*(d*x^3+c)^(1/2)/d^3-8*c/d^2*(2/9*(d*x^3+c)^(1/2)/d*x^3-4/9*(d*x^3+c)^(1/2)*

$c/d^2) - 128/3 * c^2 * (d*x^3+c)^{(1/2)}/d^4 - 512/27 * I * c^2/d^6 * 2^{(1/2)} * \text{sum}((-c*d^2)^{(1/3)} * (1/2 * I * (2*x + (-I*3^{(1/2)} * (-c*d^2)^{(1/3)} + (-c*d^2)^{(1/3)})/d) / (-c*d^2)^{(1/3)} * d)^{(1/2)} * ((x - (-c*d^2)^{(1/3)}/d) / (-3 * (-c*d^2)^{(1/3)} + I*3^{(1/2)} * (-c*d^2)^{(1/3)}) * d)^{(1/2)} * (-1/2 * I * (2*x + (I*3^{(1/2)} * (-c*d^2)^{(1/3)} + (-c*d^2)^{(1/3)})/d) / (-c*d^2)^{(1/3)} * d)^{(1/2)} / (d*x^3+c)^{(1/2)} * (2*_alpha^2*d^2 + I * (-c*d^2)^{(1/3)} * 3^{(1/2)} *_alpha * d - (-c*d^2)^{(1/3)} *_alpha * d - I*3^{(1/2)} * (-c*d^2)^{(2/3)} - (-c*d^2)^{(2/3)}) * \text{EllipticPi}(1/3 * 3^{(1/2)} * (I * (x + 1/2 * (-c*d^2)^{(1/3)}/d - 1/2 * I*3^{(1/2)} * (-c*d^2)^{(1/3)}/d) * 3^{(1/2)} / (-c*d^2)^{(1/3)} * d)^{(1/2)}, -1/18 * (2 * I * (-c*d^2)^{(1/3)} * 3^{(1/2)} *_alpha^2 * d + I*3^{(1/2)} * c * d - 3 * c * d - I * (-c*d^2)^{(2/3)} * 3^{(1/2)} *_alpha - 3 * (-c*d^2)^{(2/3)} *_alpha) / c / d, (I*3^{(1/2)} * (-c*d^2)^{(1/3)} / (-3/2 * (-c*d^2)^{(1/3)} / d + 1/2 * I*3^{(1/2)} * (-c*d^2)^{(1/3)} / d) / d)^{(1/2)}, _alpha = \text{RootOf}(_Z^3 * d - 8 * c))$

maxima [A] time = 1.26, size = 82, normalized size = 0.91

$$\frac{2 \left(1280 c^{\frac{5}{2}} \log \left(\frac{\sqrt{dx^3+c} - 3\sqrt{c}}{\sqrt{dx^3+c} + 3\sqrt{c}} \right) + 3 (dx^3 + c)^{\frac{5}{2}} + 30 (dx^3 + c)^{\frac{3}{2}} c + 855 \sqrt{dx^3 + c} c^2 \right)}{45 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] -2/45*(1280*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 30*(d*x^3 + c)^(3/2)*c + 855*sqrt(d*x^3 + c)*c^2)/d^4

mupad [B] time = 3.22, size = 98, normalized size = 1.09

$$\frac{512 c^{5/2} \ln \left(\frac{10c + dx^3 + 6\sqrt{c} \sqrt{dx^3+c}}{8c - dx^3} \right)}{9 d^4} - \frac{592 c^2 \sqrt{dx^3 + c}}{15 d^4} - \frac{2 x^6 \sqrt{dx^3 + c}}{15 d^2} - \frac{8 c x^3 \sqrt{dx^3 + c}}{5 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)

[Out] (512*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^4) - (592*c^2*(c + d*x^3)^(1/2))/(15*d^4) - (2*x^6*(c + d*x^3)^(1/2))/(15*d^2) - (8*c*x^3*(c + d*x^3)^(1/2))/(5*d^3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{11}}{-8c\sqrt{c + dx^3} + dx^3\sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**11/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```


$$3.231 \quad \int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}} dx$$

Optimal. Leaf size=71

$$\frac{128c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3} - \frac{14c\sqrt{c+dx^3}}{3d^3} - \frac{2(c+dx^3)^{3/2}}{9d^3}$$

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {446, 88, 63, 206}

$$\frac{128c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3} - \frac{14c\sqrt{c+dx^3}}{3d^3} - \frac{2(c+dx^3)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

[In] Int[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (-14*c*Sqrt[c + d*x^3]/(3*d^3) - (2*(c + d*x^3)^(3/2))/(9*d^3) + (128*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^3)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(8c - dx^3)\sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{7c}{d^2\sqrt{c + dx}} + \frac{64c^2}{d^2(8c - dx)\sqrt{c + dx}} - \frac{\sqrt{c + dx}}{d^2} \right) dx, x, x^3 \right) \\
&= -\frac{14c\sqrt{c + dx^3}}{3d^3} - \frac{2(c + dx^3)^{3/2}}{9d^3} + \frac{(64c^2) \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{3d^2} \\
&= -\frac{14c\sqrt{c + dx^3}}{3d^3} - \frac{2(c + dx^3)^{3/2}}{9d^3} + \frac{(128c^2) \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{3d^3} \\
&= -\frac{14c\sqrt{c + dx^3}}{3d^3} - \frac{2(c + dx^3)^{3/2}}{9d^3} + \frac{128c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{9d^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 58, normalized size = 0.82

$$\frac{128c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right) - 2\sqrt{c + dx^3} (22c + dx^3)}{9d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (-2*Sqrt[c + d*x^3]*(22*c + d*x^3) + 128*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^3)

IntegrateAlgebraic [A] time = 0.06, size = 61, normalized size = 0.86

$$\frac{128c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{9d^3} - \frac{2\sqrt{c + dx^3} (22c + dx^3)}{9d^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] $(-2*\text{Sqrt}[c + d*x^3]*(22*c + d*x^3)/(9*d^3) + (128*c^{(3/2)}*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/(3*\text{Sqrt}[c])])/(9*d^3)$

fricas [A] time = 1.10, size = 121, normalized size = 1.70

$$\left[\frac{2 \left(32 c^{\frac{3}{2}} \log \left(\frac{dx^3 + 6 \sqrt{dx^3 + c} \sqrt{c} + 10c}{dx^3 - 8c} \right) - (dx^3 + 22c) \sqrt{dx^3 + c} \right)}{9 d^3}, - \frac{2 \left(64 \sqrt{-c} c \arctan \left(\frac{\sqrt{dx^3 + c} \sqrt{-c}}{3c} \right) + (dx^3 + 22c) \sqrt{dx^3 + c} \right)}{9 d^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] $[2/9*(32*c^{(3/2)}*\log((d*x^3 + 6*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 10*c)/(d*x^3 - 8*c)) - (d*x^3 + 22*c)*\text{sqrt}(d*x^3 + c))/d^3, -2/9*(64*\text{sqrt}(-c)*c*\arctan(1/3*\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + (d*x^3 + 22*c)*\text{sqrt}(d*x^3 + c))/d^3]$

giac [A] time = 0.19, size = 65, normalized size = 0.92

$$-\frac{128 c^2 \arctan \left(\frac{\sqrt{dx^3+c}}{3 \sqrt{-c}} \right)}{9 \sqrt{-c} d^3} - \frac{2 \left((dx^3 + c)^{\frac{3}{2}} d^6 + 21 \sqrt{dx^3 + c} c d^6 \right)}{9 d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] $-128/9*c^2*\arctan(1/3*\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*d^3) - 2/9*((d*x^3 + c)^{(3/2)}*d^6 + 21*\text{sqrt}(d*x^3 + c)*c*d^6)/d^9$

maple [C] time = 0.16, size = 468, normalized size = 6.59

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)

[Out] $-1/d^2*((2/9*(d*x^3+c)^{(1/2)}/d*x^3-4/9*(d*x^3+c)^{(1/2)}*c/d^2)*d+16/3*(d*x^3+c)^{(1/2)}*c/d)-64/27*I*c/d^5*2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(1/2)*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)$

$$\begin{aligned} & \frac{(-1/3)/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)})*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},-1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2))},_alpha=RootOf(_Z^3*d-8*c)) \end{aligned}$$

maxima [A] time = 1.31, size = 66, normalized size = 0.93

$$-\frac{2\left(32c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}+21\sqrt{dx^3+c}c\right)}{9d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] -2/9*(32*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c))) + (d*x^3 + c)^(3/2) + 21*sqrt(d*x^3 + c)*c)/d^3

mupad [B] time = 3.39, size = 78, normalized size = 1.10

$$\frac{64c^{3/2}\ln\left(\frac{10c+dx^3+6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{9d^3}-\frac{44c\sqrt{dx^3+c}}{9d^3}-\frac{2x^3\sqrt{dx^3+c}}{9d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)

[Out] (64*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^3) - (44*c*(c + d*x^3)^(1/2))/(9*d^3) - (2*x^3*(c + d*x^3)^(1/2))/(9*d^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^8}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)

[Out] -Integral(x**8/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)

$$3.232 \quad \int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}} dx$$

Optimal. Leaf size=52

$$\frac{16\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2} - \frac{2\sqrt{c+dx^3}}{3d^2}$$

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {446, 80, 63, 206}

$$\frac{16\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2} - \frac{2\sqrt{c+dx^3}}{3d^2}$$

Antiderivative was successfully verified.

[In] Int[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (-2*Sqrt[c + d*x^3]/(3*d^2) + (16*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(8c - dx^3)\sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right) \\ &= -\frac{2\sqrt{c + dx^3}}{3d^2} + \frac{(8c) \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{3d} \\ &= -\frac{2\sqrt{c + dx^3}}{3d^2} + \frac{(16c) \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{3d^2} \\ &= -\frac{2\sqrt{c + dx^3}}{3d^2} + \frac{16\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{9d^2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.94

$$\frac{2 \left(3\sqrt{c + dx^3} - 8\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right) \right)}{9d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]), x]

[Out] (-2*(3*Sqrt[c + d*x^3] - 8*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(9*d^2)

IntegrateAlgebraic [A] time = 0.05, size = 52, normalized size = 1.00

$$\frac{16\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{9d^2} - \frac{2\sqrt{c + dx^3}}{3d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] $(-2*\text{Sqrt}[c + d*x^3])/(3*d^2) + (16*\text{Sqrt}[c]*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/(3*\text{Sqrt}[c])])/(9*d^2)$

fricas [A] time = 0.65, size = 103, normalized size = 1.98

$$\left[\frac{2 \left(4 \sqrt{c} \log \left(\frac{dx^3 + 6 \sqrt{dx^3 + c} \sqrt{c} + 10c}{dx^3 - 8c} \right) - 3 \sqrt{dx^3 + c} \right)}{9 d^2}, - \frac{2 \left(8 \sqrt{-c} \arctan \left(\frac{\sqrt{dx^3 + c} \sqrt{-c}}{3c} \right) + 3 \sqrt{dx^3 + c} \right)}{9 d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] $[2/9*(4*\text{sqrt}(c)*\log((d*x^3 + 6*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 10*c)/(d*x^3 - 8*c)) - 3*\text{sqrt}(d*x^3 + c))/d^2, -2/9*(8*\text{sqrt}(-c)*\arctan(1/3*\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + 3*\text{sqrt}(d*x^3 + c))/d^2]$

giac [A] time = 0.16, size = 48, normalized size = 0.92

$$-\frac{2 \left(\frac{8c \arctan \left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}} \right)}{\sqrt{-c}d} + \frac{3\sqrt{dx^3+c}}{d} \right)}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] $-2/9*(8*c*\arctan(1/3*\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*d) + 3*\text{sqrt}(d*x^3 + c)/d)/d$

maple [C] time = 0.17, size = 425, normalized size = 8.17

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)

[Out] $-2/3*(d*x^3+c)^(1/2)/d^2 - 8/27*I/d^4*2^(1/2)*\text{sum}((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)$

$2)/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},-1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)),_alpha=RootOf(_Z^3*d-8*c))$

maxima [A] time = 1.40, size = 56, normalized size = 1.08

$$\frac{2 \left(4 \sqrt{c} \log \left(\frac{\sqrt{dx^3+c} - 3\sqrt{c}}{\sqrt{dx^3+c} + 3\sqrt{c}} \right) + 3 \sqrt{dx^3+c} \right)}{9d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] -2/9*(4*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c))) + 3*sqrt(d*x^3 + c))/d^2

mupad [B] time = 3.27, size = 60, normalized size = 1.15

$$\frac{8\sqrt{c} \ln\left(\frac{10c+dx^3+6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{9d^2} - \frac{2\sqrt{dx^3+c}}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)

[Out] (8*c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^2) - (2*(c + d*x^3)^(1/2))/(3*d^2)

sympy [A] time = 16.35, size = 61, normalized size = 1.17

$$\begin{cases} \frac{2 \left(\frac{8c \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right) - \sqrt{c+dx^3}}{9d\sqrt{-c}} - \frac{\sqrt{c+dx^3}}{3d} \right)}{d} & \text{for } d \neq 0 \\ \frac{x^6}{48c^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)

[Out] Piecewise((2*(-8*c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*d*sqrt(-c)) - sqrt(c + d*x**3)/(3*d))/d, Ne(d, 0)), (x**6/(48*c**(3/2)), True))

$$3.233 \quad \int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}} dx$$

Optimal. Leaf size=33

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {444, 63, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

Antiderivative was successfully verified.

[In] Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(8c - dx^3)\sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right) \\ &= \frac{2 \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{3d} \\ &= \frac{2 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{9\sqrt{c}d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{9\sqrt{c}d}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 1.00

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{9\sqrt{c}d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)

fricas [A] time = 0.84, size = 78, normalized size = 2.36

$$\left[\frac{\log \left(\frac{dx^3 + 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c} \right)}{9\sqrt{c}d}, -\frac{2\sqrt{-c} \arctan \left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c} \right)}{9cd} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/9*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))/(sqrt(c)*d), -2/9*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c)/(c*d)]

giac [A] time = 0.16, size = 27, normalized size = 0.82

$$\frac{2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] -2/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d)

maple [C] time = 0.20, size = 413, normalized size = 12.52

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)

[Out]
$$\frac{-1/27 * I/d^3/c * 2^{(1/2)} * \sum((-c*d^2)^{(1/3)} * (1/2 * I * (2*x + (-I*3^{(1/2)} * (-c*d^2)^{(1/3)} + (-c*d^2)^{(1/3)})) / d) / (-c*d^2)^{(1/3)} * d)^{(1/2)} * ((x - (-c*d^2)^{(1/3)} / d) / (-3 * (-c*d^2)^{(1/3)} + I*3^{(1/2)} * (-c*d^2)^{(1/3)})) * d)^{(1/2)} * (-1/2 * I * (2*x + (I*3^{(1/2)} * (-c*d^2)^{(1/3)} + (-c*d^2)^{(1/3)})) / d) / (-c*d^2)^{(1/3)} * d)^{(1/2)} / (d*x^3+c)^{(1/2)} * (2 * _alpha^2 * d^2 + I * (-c*d^2)^{(1/3)} * 3^{(1/2)} * _alpha * d - (-c*d^2)^{(1/3)} * _alpha * d - I * 3^{(1/2)} * (-c*d^2)^{(2/3)} - (-c*d^2)^{(2/3)}) * \text{EllipticPi}(1/3 * 3^{(1/2)} * (I * (x + 1/2 * (-c*d^2)^{(1/3)} / d - 1/2 * I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / d) * 3^{(1/2)} / (-c*d^2)^{(1/3)} * d)^{(1/2)}, -1/18 * (2 * I * (-c*d^2)^{(1/3)} * 3^{(1/2)} * _alpha^2 * d + I * 3^{(1/2)} * c * d - 3 * c * d - I * (-c*d^2)^{(2/3)} * 3^{(1/2)} * _alpha - 3 * (-c*d^2)^{(2/3)} * _alpha) / c / d, (I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / (-3/2 * (-c*d^2)^{(1/3)} / d + 1/2 * I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / d) / d)^{(1/2)}, _alpha = \text{RootOf}(_Z^3 * d - 8 * c))$$

maxima [A] time = 1.21, size = 42, normalized size = 1.27

$$\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{9\sqrt{c}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] -1/9*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/(sqrt(c)*d)

mupad [B] time = 3.23, size = 45, normalized size = 1.36

$$\frac{\ln\left(\frac{10c+dx^3+6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{9\sqrt{c}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)

[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(9*c^(1/2)*d)

sympy [A] time = 11.20, size = 32, normalized size = 0.97

$$-\frac{2 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9d\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)

[Out] -2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*d*sqrt(-c))

$$3.234 \quad \int \frac{1}{x(8c-dx^3)\sqrt{c+dx^3}} dx$$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 86, 63, 208, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(36*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(12*c^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 86

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && !IntegerQ[p]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 446

$\text{Int}[(x_)^{(m_)} \cdot ((a_) + (b_ \cdot)(x_)^{(n_)})^{(p_)} \cdot ((c_) + (d_ \cdot)(x_)^{(n_)})^{(q_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) \cdot (a + b \cdot x)^p \cdot (c + d \cdot x)^q}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x(8c - dx^3)\sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{24c} + \frac{d \text{Subst} \left(\int \frac{1}{(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{24c} \\ &= \frac{\text{Subst} \left(\int \frac{1}{9c-x^2} dx, x, \sqrt{c + dx^3} \right)}{12c} + \frac{\text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{12cd} \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{36c^{3/2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{12c^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.88

$$\frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) - 3 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{36c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(36*c^(3/2))

IntegrateAlgebraic [A] time = 0.05, size = 58, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(36*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(12*c^(3/2))

fricas [A] time = 0.89, size = 139, normalized size = 2.40

$$\left[\frac{\sqrt{c} \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + 3\sqrt{c} \log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{72c^2}, \frac{3\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - \sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)}{36c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/72*(sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 3*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, 1/36*(3*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c))/c^2]

giac [A] time = 0.17, size = 54, normalized size = 0.93

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}c} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{36\sqrt{-c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/36*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)

maple [C] time = 0.17, size = 433, normalized size = 7.47

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{12c^2} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{216c^2\sqrt{dx^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)`

[Out]
$$-1/216*I/c^2/d^2*2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},-1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)},_alpha=\text{RootOf}(_Z^3*d-8*c))-1/12*\text{arctanh}((d*x^3+c)^{(1/2)}/c^{(1/2)})/c^{(3/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3+c}(dx^3-8c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")`

[Out] `-integrate(1/(sqrt(d*x^3+c)*(d*x^3-8*c)*x),x)`

mupad [B] time = 3.28, size = 47, normalized size = 0.81

$$\frac{3 \operatorname{atanh}\left(\frac{c\sqrt{dx^3+c}}{\sqrt{c^3}}\right) - \operatorname{atanh}\left(\frac{c\sqrt{dx^3+c}}{3\sqrt{c^3}}\right)}{36\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(c+d*x^3)^(1/2)*(8*c-d*x^3)),x)`

[Out]
$$-(3*\operatorname{atanh}((c*(c+d*x^3)^{(1/2)})/(c^3)^{(1/2)}) - \operatorname{atanh}((c*(c+d*x^3)^{(1/2)})/(3*(c^3)^{(1/2)})))/(36*(c^3)^{(1/2)})$$

sympy [A] time = 12.22, size = 58, normalized size = 1.00

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{36c\sqrt{-c}} + \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12c\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(36*c*sqrt(-c)) + atan(sqrt(c + d*x**3)/sqrt(-c))/(12*c*sqrt(-c))
```

$$3.235 \quad \int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}} dx$$

Optimal. Leaf size=81

$$\frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}} - \frac{\sqrt{c+dx^3}}{24c^2x^3}$$

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {446, 103, 156, 63, 208, 206}

$$-\frac{\sqrt{c+dx^3}}{24c^2x^3} + \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] -Sqrt[c + d*x^3]/(24*c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(288*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +

$f*x)^p/(a + b*x), x], x] - \text{Dist}[(d*g - c*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(c + d*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x]$

Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x^4(8c - dx^3)\sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right) \\ &= -\frac{\sqrt{c + dx^3}}{24c^2x^3} - \frac{\text{Subst} \left(\int \frac{3cd - \frac{d^2x}{2}}{x(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{24c^2} \\ &= -\frac{\sqrt{c + dx^3}}{24c^2x^3} - \frac{d \text{Subst} \left(\int \frac{1}{x\sqrt{c + dx}} dx, x, x^3 \right)}{64c^2} + \frac{d^2 \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{192c^2} \\ &= -\frac{\sqrt{c + dx^3}}{24c^2x^3} - \frac{\text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{32c^2} + \frac{d \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{96c^2} \\ &= -\frac{\sqrt{c + dx^3}}{24c^2x^3} + \frac{d \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{288c^{5/2}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{32c^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 1.00

$$\frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}} - \frac{\sqrt{c+dx^3}}{24c^2x^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] -1/24*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(288*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))

IntegrateAlgebraic [A] time = 0.08, size = 81, normalized size = 1.00

$$\frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}} - \frac{\sqrt{c+dx^3}}{24c^2x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] -1/24*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(288*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))

fricas [A] time = 0.80, size = 184, normalized size = 2.27

$$\left[\frac{\sqrt{c} dx^3 \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + 9\sqrt{c} dx^3 \log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) - 24\sqrt{dx^3+c} - 9\sqrt{-c} dx^3 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + \sqrt{-c} dx^3 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 12\sqrt{dx^3+c}}{576c^3x^3}, -\frac{\sqrt{c} dx^3 \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + 9\sqrt{c} dx^3 \log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) - 24\sqrt{dx^3+c} - 9\sqrt{-c} dx^3 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + \sqrt{-c} dx^3 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 12\sqrt{dx^3+c}}{288c^3x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/576*(sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/288*(9*sqrt(-c)*d*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]

giac [A] time = 0.16, size = 73, normalized size = 0.90

$$-\frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{32\sqrt{-c}c^2} - \frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{288\sqrt{-c}c^2} - \frac{\sqrt{dx^3+c}}{24c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] $-1/32*d*\arctan(\sqrt{d*x^3 + c}/\sqrt{-c})/(\sqrt{-c}*c^2) - 1/288*d*\arctan(1/3*\sqrt{d*x^3 + c}/\sqrt{-c})/(\sqrt{-c}*c^2) - 1/24*\sqrt{d*x^3 + c}/(c^2*x^3)$

maple [C] time = 0.19, size = 477, normalized size = 5.89

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)

[Out] $1/8/c*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*\operatorname{arctanh}((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-1/1728*I/d/c^3*2^(1/2)*\sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3))/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*\operatorname{EllipticPi}(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=\operatorname{RootOf}(_Z^3*d-8*c))-1/96*d*\operatorname{arctanh}((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^4), x)

mupad [B] time = 3.42, size = 73, normalized size = 0.90

$$\frac{d \operatorname{atanh}\left(\frac{c^2 \sqrt{dx^3+c}}{\sqrt{c^5}}\right)}{32 \sqrt{c^5}} + \frac{d \operatorname{atanh}\left(\frac{c^2 \sqrt{dx^3+c}}{3 \sqrt{c^5}}\right)}{288 \sqrt{c^5}} - \frac{\sqrt{dx^3 + c}}{24 c^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(c + d*x^3)^(1/2))*(8*c - d*x^3)),x)`

[Out] $(d*\operatorname{atanh}((c^2*(c + d*x^3)^{1/2})/(c^5)^{1/2}))/((32*(c^5)^{1/2})) + (d*\operatorname{atanh}((c^2*(c + d*x^3)^{1/2})/(3*(c^5)^{1/2})))/(288*(c^5)^{1/2}) - (c + d*x^3)^{1/2}/(24*c^2*x^3)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^4\sqrt{c + dx^3} + dx^7\sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)`

[Out] `-Integral(1/(-8*c*x**4*sqrt(c + d*x**3) + d*x**7*sqrt(c + d*x**3)), x)`

$$3.236 \quad \int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}} dx$$

Optimal. Leaf size=107

$$\frac{d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}} - \frac{7d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} - \frac{\sqrt{c+dx^3}}{48c^2x^6}$$

Rubi [A] time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {446, 103, 151, 156, 63, 208, 206}

$$\frac{d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}} - \frac{7d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} - \frac{\sqrt{c+dx^3}}{48c^2x^6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] -Sqrt[c + d*x^3]/(48*c^2*x^6) + (5*d*Sqrt[c + d*x^3])/(192*c^3*x^3) + (d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2304*c^(7/2)) - (7*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])/(256*c^(7/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*m, 2*p])

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^3(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6} - \frac{\text{Subst} \left(\int \frac{10cd-\frac{3d^2x}{2}}{x^2(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{48c^2} \\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{\text{Subst} \left(\int \frac{42c^2d^2-5cd^3x}{x(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{384c^4} \\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{(7d^2) \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{512c^3} + \frac{d^3 \text{Subst} \left(\int \frac{1}{x^2\sqrt{c+dx}} dx, x, x^3 \right)}{512c^3} \\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{(7d) \text{Subst} \left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{256c^3} + \frac{d^2 \text{Subst} \left(\int \frac{1}{x^2\sqrt{c+dx}} dx, x, x^3 \right)}{512c^3} \\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{d^2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{2304c^{7/2}} - \frac{7d^2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{256c^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 95, normalized size = 0.89

$$\frac{d^2x^6 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) - 63d^2x^6 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right) + 12\sqrt{c} \sqrt{c+dx^3} (5dx^3 - 4c)}{2304c^{7/2}x^6}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (12*Sqrt[c]*Sqrt[c + d*x^3]*(-4*c + 5*d*x^3) + d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 63*d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2304*c^(7/2)*x^6)

IntegrateAlgebraic [A] time = 0.14, size = 95, normalized size = 0.89

$$\frac{d^2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{2304c^{7/2}} - \frac{7d^2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{256c^{7/2}} + \frac{\sqrt{c+dx^3} (5dx^3 - 4c)}{192c^3x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (Sqrt[c + d*x^3]*(-4*c + 5*d*x^3))/(192*c^3*x^6) + (d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2304*c^(7/2)) - (7*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(256*c^(7/2))

fricas [A] time = 0.81, size = 217, normalized size = 2.03

$$\left[\frac{\sqrt{c} d^2 x^6 \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + 63\sqrt{c} d^2 x^6 \log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) + 24(5cdx^3-4c^2)\sqrt{dx^3+c}}{4608c^4x^6}, \frac{63\sqrt{-c} d^2 x^6 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - \sqrt{-c} d^2 x^6 \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 12(5cdx^3-4c^2)\sqrt{dx^3+c}}{2304c^4x^6} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/4608*(sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 63*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^4*x^6), 1/2304*(63*sqrt(-c)*d^2*x^6*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(-c)*d^2*x^6*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^4*x^6)]

giac [A] time = 0.18, size = 101, normalized size = 0.94

$$\frac{7 d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{256 \sqrt{-c} c^3} - \frac{d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3 \sqrt{-c}}\right)}{2304 \sqrt{-c} c^3} + \frac{5 (dx^3 + c)^{\frac{3}{2}} d^2 - 9 \sqrt{dx^3 + c} c d^2}{192 c^3 d^2 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] 7/256*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/2304*d^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1/192*(5*(d*x^3 + c)^(3/2)*d^2 - 9*sqrt(d*x^3 + c)*c*d^2)/(c^3*d^2*x^6)

maple [C] time = 0.21, size = 540, normalized size = 5.05

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)

[Out] 1/64/c^2*d*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2)))/c^(3/2))+1/8/c*(-1/6*(d*x^3+c)^(1/2)/c/x^6+1/4*d*(d*x^3+c)^(1/2)/c^2/x^3

$$-1/4*d^2*\operatorname{arctanh}((d*x^3+c)^{(1/2)}/c^{(1/2)})/c^{(5/2)}-1/13824*I/c^4*2^{(1/2)}*\operatorname{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\operatorname{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},-1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}),_alpha=\operatorname{RootOf}(_Z^3*d-8*c))-1/768*d^2*\operatorname{arctanh}((d*x^3+c)^{(1/2)}/c^{(1/2)})/c^{(7/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3+c}(dx^3-8c)x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^7), x)

mupad [B] time = 3.50, size = 94, normalized size = 0.88

$$\frac{d^2 \operatorname{atanh}\left(\frac{c^3 \sqrt{dx^3+c}}{3 \sqrt{c^7}}\right)}{2304 \sqrt{c^7}} - \frac{7 d^2 \operatorname{atanh}\left(\frac{c^3 \sqrt{dx^3+c}}{\sqrt{c^7}}\right)}{256 \sqrt{c^7}} - \frac{3 \sqrt{dx^3+c}}{64 c^2 x^6} + \frac{5 (dx^3+c)^{3/2}}{192 c^3 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^7*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)

[Out] $(d^2*\operatorname{atanh}((c^3*(c + d*x^3)^{(1/2)})/(3*(c^7)^{(1/2)})))/(2304*(c^7)^{(1/2)}) - (7*d^2*\operatorname{atanh}((c^3*(c + d*x^3)^{(1/2)})/(c^7)^{(1/2)}))/(256*(c^7)^{(1/2)}) - (3*(c + d*x^3)^{(1/2)})/(64*c^2*x^6) + (5*(c + d*x^3)^{(3/2)})/(192*c^3*x^6)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^7\sqrt{c+dx^3}+dx^{10}\sqrt{c+dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)

[Out] -Integral(1/(-8*c*x**7*sqrt(c + d*x**3) + d*x**10*sqrt(c + d*x**3)), x)

$$3.237 \quad \int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}} dx$$

Optimal. Leaf size=141

$$\frac{\tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{c} (\sqrt[3]{c} + \sqrt[3]{dx^3})}{\sqrt{c+dx^3}}\right)}{6\sqrt{3} c^{5/6} d^{2/3}} + \frac{\tanh^{-1}\left(\frac{(\sqrt[3]{c} + \sqrt[3]{dx^3})^2}{3 \sqrt[6]{c} \sqrt{c+dx^3}}\right)}{18c^{5/6} d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6} d^{2/3}}$$

Rubi [A] time = 0.41, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{c} (\sqrt[3]{c} + \sqrt[3]{dx^3})}{\sqrt{c+dx^3}}\right)}{6\sqrt{3} c^{5/6} d^{2/3}} + \frac{\tanh^{-1}\left(\frac{(\sqrt[3]{c} + \sqrt[3]{dx^3})^2}{3 \sqrt[6]{c} \sqrt{c+dx^3}}\right)}{18c^{5/6} d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6} d^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] -ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/(6*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(18*c^(5/6)*d^(2/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(18*c^(5/6)*d^(2/3))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 486

Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] := With[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]), x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^2*x^2)*Sqrt[c + d*x^3]), x], x))] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[8*b*c + a*d, 0]

Rule 2138

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rule 2145

Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := Dist[-2*g*h, Subst[Int[1/(2*e*h - (b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x}{(8c - dx^3)\sqrt{c + dx^3}} dx &= -\frac{\int \frac{2\sqrt[3]{c}d^{2/3} - 2dx - \frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4 + \frac{2\sqrt[3]{d}x}{\sqrt[3]{c}} + \frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}} dx}{12cd} + \frac{\int \frac{1 + \frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2 - \frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}} dx}{12c^{2/3}\sqrt[3]{d}} - \frac{\sqrt[3]{d} \int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}} dx}{4\sqrt[3]{c}} \\
&= \frac{\text{Subst}\left(\int \frac{1}{9-cx^2} dx, x, \frac{\left(1 + \frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{6\sqrt[3]{c}d^{2/3}} - \frac{\sqrt[3]{d} \text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}} dx, x, x^3\right)}{12\sqrt[3]{c}} + \frac{d^{4/3} \text{Subst}\left(\int \frac{1}{9-cx^2} dx, x, \sqrt{c+dx^3}\right)}{6\sqrt[3]{c}d^{2/3}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c} + \sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c} + \sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{9-cx^2} dx, x, \sqrt{c+dx^3}\right)}{6\sqrt[3]{c}d^{2/3}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c} + \sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c} + \sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 67, normalized size = 0.48

$$\frac{x^2 \sqrt{\frac{c+dx^3}{c}} F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; -\frac{dx^3}{c}, \frac{dx^3}{8c}\right)}{16c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(16*c*Sqrt[c + d*x^3])

IntegrateAlgebraic [F] time = 15.45, size = 0, normalized size = 0.00

$$\int \frac{x}{(8c - dx^3)\sqrt{c + dx^3}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] Defer[IntegrateAlgebraic][x/((8*c - d*x^3)*Sqrt[c + d*x^3]), x]

fricas [B] time = 2.47, size = 2459, normalized size = 17.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/54*\sqrt{3}*(1/(c^5*d^4))^{1/6}*\arctan(1/9*((9*\sqrt{3})*c*d^2*x^5*(1/(c^5*d^4))^{1/6} - \sqrt{3}*(c^4*d^5*x^6 - 40*c^5*d^4*x^3 - 32*c^6*d^3)*(1/(c^5*d^4))^{5/6} + 3*\sqrt{3}*(5*c^3*d^3*x^4 + 8*c^4*d^2*x)*\sqrt{1/(c^5*d^4)}))*\sqrt{d*x^3 + c} \\ & + (18*\sqrt{3}*(c^4*d^4*x^5 + c^5*d^3*x^2)*(1/(c^5*d^4))^{2/3} + 12*\sqrt{3}*(c^2*d^3*x^6 - c^3*d^2*x^3 - 2*c^4*d)*(1/(c^5*d^4))^{1/3} + 3*\sqrt{3}*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) + \sqrt{d*x^3 + c}*(\sqrt{3}*(c^4*d^5*x^6 + 32*c^5*d^4*x^3 + 40*c^6*d^3)*(1/(c^5*d^4))^{5/6} + 3*\sqrt{3}*(7*c^3*d^3*x^4 + 4*c^4*d^2*x)*\sqrt{1/(c^5*d^4)} + 9*\sqrt{3}*(c*d^2*x^5 + 2*c^2*d*x^2)*(1/(c^5*d^4))^{1/6}))*\sqrt{(d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^{2/3} + 6*\sqrt{d*x^3 + c}*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^{5/6} - 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*\sqrt{1/(c^5*d^4)} - (c*d^3*x^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^{1/6}) + 18*(c^2*d^4*x^8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^{1/3})/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x) - 1/54*\sqrt{3}*(1/(c^5*d^4))^{1/6}*\arctan(1/9*((9*\sqrt{3})*c*d^2*x^5*(1/(c^5*d^4))^{1/6} - \sqrt{3}*(c^4*d^5*x^6 - 40*c^5*d^4*x^3 - 32*c^6*d^3)*(1/(c^5*d^4))^{5/6} + 3*\sqrt{3}*(5*c^3*d^3*x^4 + 8*c^4*d^2*x)*\sqrt{1/(c^5*d^4)}))*\sqrt{d*x^3 + c} - (18*\sqrt{3}*(c^4*d^4*x^5 + c^5*d^3*x^2)*(1/(c^5*d^4))^{2/3} + 12*\sqrt{3}*(c^2*d^3*x^6 - c^3*d^2*x^3 - 2*c^4*d)*(1/(c^5*d^4))^{1/3} + 3*\sqrt{3}*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - \sqrt{d*x^3 + c}*(\sqrt{3}*(c^4*d^5*x^6 + 32*c^5*d^4*x^3 + 40*c^6*d^3)*(1/(c^5*d^4))^{5/6} + 3*\sqrt{3}*(7*c^3*d^3*x^4 + 4*c^4*d^2*x)*\sqrt{1/(c^5*d^4)} + 9*\sqrt{3}*(c*d^2*x^5 + 2*c^2*d*x^2)*(1/(c^5*d^4))^{1/6}))*\sqrt{(d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^{2/3} - 6*\sqrt{d*x^3 + c}*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^{5/6} - 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*\sqrt{1/(c^5*d^4)} - (c*d^3*x^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^{1/6}) + 18*(c^2*d^4*x^8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^{1/3})/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x) + 1/108*(1/(c^5*d^4))^{1/6}*\log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x)*(1/(c^5*d^4))^{2/3} + 6*\sqrt{d*x^3 + c}*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2)*(1/(c^5*d^4))^{5/6} + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*\sqrt{1/(c^5*d^4)} + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x)*(1/(c^5*d^4))^{1/6}) + 18*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1/(c^5*d^4))^{1/3})/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 1/108*(1/(c^5*d^4))^{1/6}*\log((d^3*x^9 + 318 \end{aligned}$$

```
*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4
+ 32*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 +
32*c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 6
4*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x)*(
1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1
/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3) - 1/
216*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 108
8*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3
) + 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) -
4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x
^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 +
20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x
^6 + 192*c^2*d*x^3 - 512*c^3)) + 1/216*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 2
76*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4
- 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 +
c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) - 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^
5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c
^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^
5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\sqrt{dx^3 + c}(dx^3 - 8c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] integrate(-x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)

maple [C] time = 0.16, size = 416, normalized size = 2.95

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)

[Out]
$$-1/27*I/d^3/c^2^{(1/2)}*\sum(1/_alpha*(-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)})*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)})/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)})*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+$$

$1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}$
 $*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-$
 $I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)}*(-c$
 $*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}$
 $), _alpha=RootOf(_Z^3*d-8*c))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{dx^3 + c}(dx^3 - 8c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] -integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)

mupad [B] time = 40.22, size = 272, normalized size = 1.93

$$\frac{\ln\left(\frac{(\sqrt{dx^3+c}+\sqrt{c})(\sqrt{dx^3+c}-\sqrt{c}+2c^{1/6}d^{1/3}x)^3}{x^3(d^{1/3}x-2c^{1/3})^3}\right)}{54c^{5/6}d^{2/3}} + \frac{\sqrt{2}\ln\left(\frac{(\sqrt{dx^3+c}-\sqrt{c})(-\sqrt{3}c^{1/6}d^{1/3}x+\sqrt{dx^3+c}1i+\sqrt{c}1i+c^{1/6}d^{1/3}x1i)^3}{x^3(d^{1/3}x+c^{1/3}-\sqrt{3}c^{1/3}1i)^3}\right)\sqrt{-1+\sqrt{3}1i}}{108c^{5/6}d^{2/3}} + \frac{\sqrt{2}\ln\left(\frac{(\sqrt{dx^3+c}+\sqrt{c})(\sqrt{3}c^{1/6}d^{1/3}x-\sqrt{dx^3+c}1i+\sqrt{c}1i+c^{1/6}d^{1/3}x1i)^3}{x^3(d^{1/3}x+c^{1/3}+\sqrt{3}c^{1/3}1i)^3}\right)\sqrt{1+\sqrt{3}1i}}{108c^{5/6}d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)

[Out] log((((c + d*x^3)^(1/2) + c^(1/2))*((c + d*x^3)^(1/2) - c^(1/2) + 2*c^(1/6)*d^(1/3)*x)^3)/(x^3*(d^(1/3)*x - 2*c^(1/3))^3))/(54*c^(5/6)*d^(2/3)) + (2^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2)*1i + c^(1/2)*1i + c^(1/6)*d^(1/3)*x*1i - 3^(1/2)*c^(1/6)*d^(1/3)*x)^3)/(x^3*(d^(1/3)*x - 3^(1/2)*c^(1/3)*1i + c^(1/3))^3))*(3^(1/2)*1i - 1)^(1/2))/(108*c^(5/6)*d^(2/3)) + (2^(1/2)*log((((c + d*x^3)^(1/2) + c^(1/2))*((c + d*x^3)^(1/2)*1i - (c + d*x^3)^(1/2)*1i + c^(1/6)*d^(1/3)*x*1i + 3^(1/2)*c^(1/6)*d^(1/3)*x)^3)/(x^3*(3^(1/2)*c^(1/3)*1i + d^(1/3)*x + c^(1/3))^3))*(3^(1/2)*1i + 1)^(1/2)*1i)/(108*c^(5/6)*d^(2/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-8c\sqrt{c + dx^3} + dx^3\sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)

[Out] -Integral(x/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)

$$3.238 \quad \int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=90

$$\frac{1024c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4} + \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{4c\sqrt{c+dx^3}}{d^4} - \frac{2(c+dx^3)^{3/2}}{9d^4}$$

Rubi [A] time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 87, 43, 63, 206}

$$\frac{2c^2}{27d^4\sqrt{c+dx^3}} + \frac{1024c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4} - \frac{4c\sqrt{c+dx^3}}{d^4} - \frac{2(c+dx^3)^{3/2}}{9d^4}$$

Antiderivative was successfully verified.

[In] Int[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (2*c^2)/(27*d^4*Sqrt[c + d*x^3]) - (4*c*Sqrt[c + d*x^3])/d^4 - (2*(c + d*x^3)^(3/2))/(9*d^4) + (1024*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^4)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 87

Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,

f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11}}{(8c - dx^3)(c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3}{(8c - dx)(c + dx)^{3/2}} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{c^2}{9d^3(c + dx)^{3/2}} - \frac{7c}{d^3\sqrt{c + dx}} - \frac{x}{d^2\sqrt{c + dx}} + \frac{512c^2}{9d^3(8c - dx)\sqrt{c + dx}} \right) dx, x, x^3 \right) \\
 &= \frac{2c^2}{27d^4\sqrt{c + dx^3}} - \frac{14c\sqrt{c + dx^3}}{3d^4} + \frac{(512c^2) \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{27d^3} \\
 &= \frac{2c^2}{27d^4\sqrt{c + dx^3}} - \frac{14c\sqrt{c + dx^3}}{3d^4} + \frac{(1024c^2) \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{27d^4} \\
 &= \frac{2c^2}{27d^4\sqrt{c + dx^3}} - \frac{4c\sqrt{c + dx^3}}{d^4} - \frac{2(c + dx^3)^{3/2}}{9d^4} + \frac{1024c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{81d^4}
 \end{aligned}$$

Mathematica [C] time = 0.06, size = 66, normalized size = 0.73

$$\frac{2 \left(512c^2 {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3 + c}{9c} \right) - 456c^2 + 60cdx^3 + 3d^2x^6 \right)}{27d^4\sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] $(-2*(-456*c^2 + 60*c*d*x^3 + 3*d^2*x^6 + 512*c^2*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)])))/(27*d^4*\sqrt{c + d*x^3})$

IntegrateAlgebraic [A] time = 0.08, size = 73, normalized size = 0.81

$$\frac{1024c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4} - \frac{2(56c^2 + 60cdx^3 + 3d^2x^6)}{27d^4\sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] $(-2*(56*c^2 + 60*c*d*x^3 + 3*d^2*x^6))/(27*d^4*\sqrt{c + d*x^3}) + (1024*c^(3/2)*ArcTanh[\sqrt{c + d*x^3}/(3*\sqrt{c})])/(81*d^4)$

fricas [A] time = 0.73, size = 189, normalized size = 2.10

$$\left[\frac{2\left(256(cdx^3 + c^2)\sqrt{c} \log\left(\frac{dx^3 + 6\sqrt{dx^3 + c}\sqrt{c} + 10c}{dx^3 - 8c}\right) - 3(3d^2x^6 + 60cdx^3 + 56c^2)\sqrt{dx^3 + c}\right)}{81(d^5x^3 + cd^4)}, - \frac{2\left(512(cdx^3 + c^2)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3 + c}\sqrt{-c}}{3c}\right) + 3(3d^2x^6 + 60cdx^3 + 56c^2)\sqrt{dx^3 + c}\right)}{81(d^5x^3 + cd^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] $[2/81*(256*(c*d*x^3 + c^2)*\sqrt{c}*\log((d*x^3 + 6*\sqrt{d*x^3 + c})*\sqrt{c} + 10*c)/(d*x^3 - 8*c)) - 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*\sqrt{d*x^3 + c}]/(d^5*x^3 + c*d^4), -2/81*(512*(c*d*x^3 + c^2)*\sqrt{-c}*\arctan(1/3*\sqrt{d*x^3 + c})*\sqrt{-c}/c) + 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*\sqrt{d*x^3 + c}]/(d^5*x^3 + c*d^4)]$

giac [A] time = 0.20, size = 82, normalized size = 0.91

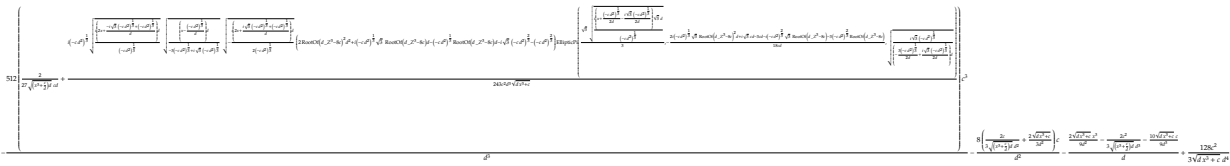
$$-\frac{1024c^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}d^4} + \frac{2c^2}{27\sqrt{dx^3 + c}d^4} - \frac{2\left((dx^3 + c)^{\frac{3}{2}}d^8 + 18\sqrt{dx^3 + c}cd^8\right)}{9d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $-1024/81*c^2*\arctan(1/3*\sqrt{d*x^3 + c}/\sqrt{-c})/(\sqrt{-c}*d^4) + 2/27*c^2/(\sqrt{d*x^3 + c}*d^4) - 2/9*((d*x^3 + c)^(3/2)*d^8 + 18*\sqrt{d*x^3 + c}*c*d^8)/d^{12}$

maple [C] time = 0.30, size = 560, normalized size = 6.22



Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{11}/(-d*x^3+8*c)/(d*x^3+c)^{(3/2)}, x)$

[Out]
$$\begin{aligned} & -1/d*(-2/3/d^3*c^2/((x^3+c/d)*d)^{(1/2)}+2/9/d^2*x^3*(d*x^3+c)^{(1/2)}-10/9*c*(d*x^3+c)^{(1/2)}/d^3)-8*c/d^2*(2/3/d^2*c/((x^3+c/d)*d)^{(1/2)}+2/3*(d*x^3+c)^{(1/2)}/d^2)+128/3*c^2/d^4/(d*x^3+c)^{(1/2)}-512*c^3/d^3*(2/27/d/c/((x^3+c/d)*d)^{(1/2)}+1/243*I/c^2/d^3*2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)})/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}), _alpha=RootOf(_Z^3*d-8*c)) \end{aligned}$$

maxima [A] time = 1.25, size = 82, normalized size = 0.91

$$\frac{2 \left(256 c^3 \log \left(\frac{\sqrt{dx^3+c} - 3\sqrt{c}}{\sqrt{dx^3+c} + 3\sqrt{c}} \right) + 9 (dx^3 + c)^{\frac{3}{2}} + 162 \sqrt{dx^3 + c} c - \frac{3c^2}{\sqrt{dx^3+c}} \right)}{81 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{11}/(-d*x^3+8*c)/(d*x^3+c)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out]
$$\begin{aligned} & -2/81*(256*c^{(3/2)}*\log((\text{sqrt}(d*x^3 + c) - 3*\text{sqrt}(c))/(\text{sqrt}(d*x^3 + c) + 3*\text{sqrt}(c))) + 9*(d*x^3 + c)^{(3/2)} + 162*\text{sqrt}(d*x^3 + c)*c - 3*c^2/\text{sqrt}(d*x^3 + c))/d^4 \end{aligned}$$

mupad [B] time = 3.78, size = 95, normalized size = 1.06

$$\frac{512 c^{3/2} \ln \left(\frac{10c+dx^3+6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3} \right)}{81 d^4} - \frac{38c\sqrt{dx^3+c}}{9 d^4} + \frac{2c^2}{27 d^4 \sqrt{dx^3+c}} - \frac{2x^3\sqrt{dx^3+c}}{9 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (512*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*d^4) - (38*c*(c + d*x^3)^(1/2))/(9*d^4) + (2*c^2)/(27*d^4*(c + d*x^3)^(1/2)) - (2*x^3*(c + d*x^3)^(1/2))/(9*d^3)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

$$3.239 \quad \int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=71

$$-\frac{2c}{27d^3\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{128\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {446, 87, 63, 206}

$$-\frac{2c}{27d^3\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{128\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

[In] Int[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (-2*c)/(27*d^3*Sqrt[c + d*x^3]) - (2*Sqrt[c + d*x^3])/(3*d^3) + (128*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 87

Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.))/((a_.) + (b_.)*(x_)), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^8}{(8c - dx^3)(c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(8c - dx)(c + dx)^{3/2}} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c}{9d^2(c + dx)^{3/2}} - \frac{1}{d^2\sqrt{c + dx}} + \frac{64c}{9d^2(8c - dx)\sqrt{c + dx}} \right) dx, x, x^3 \right) \\
 &= -\frac{2c}{27d^3\sqrt{c + dx^3}} - \frac{2\sqrt{c + dx^3}}{3d^3} + \frac{(64c) \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{27d^2} \\
 &= -\frac{2c}{27d^3\sqrt{c + dx^3}} - \frac{2\sqrt{c + dx^3}}{3d^3} + \frac{(128c) \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{27d^3} \\
 &= -\frac{2c}{27d^3\sqrt{c + dx^3}} - \frac{2\sqrt{c + dx^3}}{3d^3} + \frac{128\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{81d^3}
 \end{aligned}$$

Mathematica [C] time = 0.05, size = 53, normalized size = 0.75

$$\frac{2 \left(64c {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3 + c}{9c} \right) - 54c + 9dx^3 \right)}{27d^3\sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (-2*(-54*c + 9*d*x^3 + 64*c*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)]))/(27*d^3*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.07, size = 62, normalized size = 0.87

$$\frac{128\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{81d^3} - \frac{2(10c + 9dx^3)}{27d^3\sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] $(-2*(10*c + 9*d*x^3))/(27*d^3*\text{Sqrt}[c + d*x^3]) + (128*\text{Sqrt}[c]*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/(3*\text{Sqrt}[c])])/(81*d^3)$

fricas [A] time = 0.62, size = 161, normalized size = 2.27

$$\left[\frac{2 \left(32 (dx^3 + c) \sqrt{c} \log \left(\frac{dx^3 + 6 \sqrt{dx^3 + c} \sqrt{c} + 10c}{dx^3 - 8c} \right) - 3 (9 dx^3 + 10c) \sqrt{dx^3 + c} \right)}{81 (d^4 x^3 + cd^3)}, - \frac{2 \left(64 (dx^3 + c) \sqrt{-c} \arctan \left(\frac{\sqrt{dx^3 + c} \sqrt{-c}}{3c} \right) + 3 (9 dx^3 + 10c) \sqrt{dx^3 + c} \right)}{81 (d^4 x^3 + cd^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] $[2/81*(32*(d*x^3 + c)*\text{sqrt}(c)*\log((d*x^3 + 6*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 10*c)/(d*x^3 - 8*c)) - 3*(9*d*x^3 + 10*c)*\text{sqrt}(d*x^3 + c))/(d^4*x^3 + c*d^3), -2/81*(64*(d*x^3 + c)*\text{sqrt}(-c)*\arctan(1/3*\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + 3*(9*d*x^3 + 10*c)*\text{sqrt}(d*x^3 + c))/(d^4*x^3 + c*d^3)]$

giac [A] time = 0.17, size = 58, normalized size = 0.82

$$-\frac{128 c \arctan \left(\frac{\sqrt{dx^3 + c}}{3 \sqrt{-c}} \right)}{81 \sqrt{-c} d^3} - \frac{2 \sqrt{dx^3 + c}}{3 d^3} - \frac{2 c}{27 \sqrt{dx^3 + c} d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $-128/81*c*\arctan(1/3*\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*d^3) - 2/3*\text{sqrt}(d*x^3 + c)/d^3 - 2/27*c/(\text{sqrt}(d*x^3 + c)*d^3)$

maple [C] time = 0.17, size = 501, normalized size = 7.06

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)

```
[Out] -1/d^2*(d*(2/3/((x^3+c/d)*d)^(1/2)*c/d^2+2/3*(d*x^3+c)^(1/2)/d^2)-16/3*c/d/
(d*x^3+c)^(1/2))-64*c^2/d^2*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2
^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(
2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8
*c)))
```

maxima [A] time = 1.12, size = 68, normalized size = 0.96

$$\frac{2 \left(32 \sqrt{c} \log \left(\frac{\sqrt{dx^3+c} - 3\sqrt{c}}{\sqrt{dx^3+c} + 3\sqrt{c}} \right) + 27 \sqrt{dx^3+c} + \frac{3c}{\sqrt{dx^3+c}} \right)}{81 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/81*(32*sqrt(c)*log((sqrt(d*x^3+c)-3*sqrt(c))/(sqrt(d*x^3+c)+3*sqrt(c)))
+27*sqrt(d*x^3+c)+3*c/sqrt(d*x^3+c))/d^3
```

mupad [B] time = 3.71, size = 75, normalized size = 1.06

$$\frac{64 \sqrt{c} \ln \left(\frac{10c+dx^3+6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3} \right)}{81 d^3} - \frac{2c}{27 d^3 \sqrt{dx^3+c}} - \frac{2 \sqrt{dx^3+c}}{3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c+d*x^3)^(3/2)*(8*c-d*x^3)),x)
```

```
[Out] (64*c^(1/2)*log((10*c+d*x^3+6*c^(1/2)*(c+d*x^3)^(1/2))/(8*c-d*x^3))
)/(81*d^3)-(2*c)/(27*d^3*(c+d*x^3)^(1/2))-(2*(c+d*x^3)^(1/2))/(3*d^
3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

$$3.240 \quad \int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=52

$$\frac{2}{27d^2\sqrt{c+dx^3}} + \frac{16 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {446, 78, 63, 206}

$$\frac{2}{27d^2\sqrt{c+dx^3}} + \frac{16 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

Antiderivative was successfully verified.

[In] Int[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] 2/(27*d^2*Sqrt[c + d*x^3]) + (16*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*Sqrt[c]*d^2)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(8c - dx^3)(c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{(8c - dx)(c + dx)^{3/2}} dx, x, x^3 \right) \\ &= \frac{2}{27d^2 \sqrt{c + dx^3}} + \frac{8 \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{27d} \\ &= \frac{2}{27d^2 \sqrt{c + dx^3}} + \frac{16 \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{27d^2} \\ &= \frac{2}{27d^2 \sqrt{c + dx^3}} + \frac{16 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{81\sqrt{c} d^2} \end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.94

$$\frac{2 \left(\frac{3}{\sqrt{c + dx^3}} + \frac{8 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{\sqrt{c}} \right)}{81d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)), x]

[Out] (2*(3/Sqrt[c + d*x^3] + (8*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))/(81*d^2)

IntegrateAlgebraic [A] time = 0.05, size = 52, normalized size = 1.00

$$\frac{2}{27d^2 \sqrt{c + dx^3}} + \frac{16 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{81\sqrt{c} d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] $2/(27*d^2*\text{Sqrt}[c + d*x^3]) + (16*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/(3*\text{Sqrt}[c])])/(81*\text{Sqrt}[c]*d^2)$

fricas [A] time = 0.51, size = 149, normalized size = 2.87

$$\left[\frac{2 \left(4 (dx^3 + c) \sqrt{c} \log \left(\frac{dx^3 + 6 \sqrt{dx^3 + c} \sqrt{c} + 10c}{dx^3 - 8c} \right) + 3 \sqrt{dx^3 + c} \right)}{81 (cd^3x^3 + c^2d^2)}, \frac{2 \left(8 (dx^3 + c) \sqrt{-c} \arctan \left(\frac{\sqrt{dx^3 + c} \sqrt{-c}}{3c} \right) - 3 \sqrt{dx^3 + c} \right)}{81 (cd^3x^3 + c^2d^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] $[2/81*(4*(d*x^3 + c)*\text{sqrt}(c)*\log((d*x^3 + 6*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 10*c)/(d*x^3 - 8*c)) + 3*\text{sqrt}(d*x^3 + c)*c)/(c*d^3*x^3 + c^2*d^2), -2/81*(8*(d*x^3 + c)*\text{sqrt}(-c)*\arctan(1/3*\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) - 3*\text{sqrt}(d*x^3 + c)*c)/(c*d^3*x^3 + c^2*d^2)]$

giac [A] time = 0.17, size = 47, normalized size = 0.90

$$\frac{2 \left(\frac{8 \arctan \left(\frac{\sqrt{dx^3+c}}{3 \sqrt{-c}} \right)}{\sqrt{-c} d} - \frac{3}{\sqrt{dx^3+cd}} \right)}{81 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $-2/81*(8*\arctan(1/3*\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*d) - 3/(\text{sqrt}(d*x^3 + c)*d))/d$

maple [C] time = 0.18, size = 456, normalized size = 8.77

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)

```
[Out] 2/3/d^2/(d*x^3+c)^(1/2)-8*c/d*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3
*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2
)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d
-8*c))
```

maxima [A] time = 1.21, size = 56, normalized size = 1.08

$$-\frac{2 \left(\frac{4 \log \left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}} \right)}{\sqrt{c}} - \frac{3}{\sqrt{dx^3+c}} \right)}{81 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/81*(4*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/s
qrt(c) - 3/sqrt(d*x^3 + c))/d^2
```

mupad [B] time = 3.68, size = 60, normalized size = 1.15

$$\frac{2}{27 d^2 \sqrt{d x^3 + c}} + \frac{8 \ln \left(\frac{10 c + d x^3 + 6 \sqrt{c} \sqrt{d x^3 + c}}{8 c - d x^3} \right)}{81 \sqrt{c} d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] 2/(27*d^2*(c + d*x^3)^(1/2)) + (8*log(((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)
^(1/2))/(8*c - d*x^3)))/(81*c^(1/2)*d^2)
```

sympy [A] time = 27.57, size = 58, normalized size = 1.12

$$\begin{cases} \frac{2 \left(\frac{1}{27d \sqrt{c+dx^3}} - \frac{8 \operatorname{atan} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}} \right)}{81d \sqrt{-c}} \right)}{d} & \text{for } d \neq 0 \\ \frac{x^6}{48c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)`

[Out] `Piecewise((2*(1/(27*d*sqrt(c + d*x**3)) - 8*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(81*d*sqrt(-c)))/d, Ne(d, 0)), (x**6/(48*c**(5/2)), True))`

$$3.241 \quad \int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=55

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d} - \frac{2}{27cd\sqrt{c+dx^3}}$$

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {444, 51, 63, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d} - \frac{2}{27cd\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] -2/(27*c*d*Sqrt[c + d*x^3]) + (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*c^(3/2)*d)

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

Rule 444

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(8c - dx^3)(c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(8c - dx)(c + dx)^{3/2}} dx, x, x^3 \right) \\ &= -\frac{2}{27cd\sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{27c} \\ &= -\frac{2}{27cd\sqrt{c + dx^3}} + \frac{2 \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{27cd} \\ &= -\frac{2}{27cd\sqrt{c + dx^3}} + \frac{2 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{81c^{3/2}d} \end{aligned}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.78

$$\frac{{}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3 + c}{9c} \right)}{27cd\sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)])/(27*c*d*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.05, size = 55, normalized size = 1.00

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{81c^{3/2}d} - \frac{2}{27cd\sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] $-2/(27*c*d*\text{Sqrt}[c + d*x^3]) + (2*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/(3*\text{Sqrt}[c])])/(81*c^{3/2}*d)$

fricas [A] time = 0.72, size = 147, normalized size = 2.67

$$\left[\frac{(dx^3 + c)\sqrt{c} \log\left(\frac{dx^3 + 6\sqrt{dx^3 + c}\sqrt{c} + 10c}{dx^3 - 8c}\right) - 6\sqrt{dx^3 + c}c}{81(c^2d^2x^3 + c^3d)}, -\frac{2\left((dx^3 + c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3 + c}\sqrt{-c}}{3c}\right) + 3\sqrt{dx^3 + c}c\right)}{81(c^2d^2x^3 + c^3d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] $[1/81*((d*x^3 + c)*\text{sqrt}(c)*\log((d*x^3 + 6*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 10*c)/(d*x^3 - 8*c)) - 6*\text{sqrt}(d*x^3 + c)*c)/(c^2*d^2*x^3 + c^3*d), -2/81*((d*x^3 + c)*\text{sqrt}(-c)*\arctan(1/3*\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + 3*\text{sqrt}(d*x^3 + c)*c)/(c^2*d^2*x^3 + c^3*d)]$

giac [A] time = 0.18, size = 48, normalized size = 0.87

$$-\frac{2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}cd} - \frac{2}{27\sqrt{dx^3+c}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $-2/81*\arctan(1/3*\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*c*d) - 2/27/(\text{sqrt}(d*x^3 + c)*c*d)$

maple [C] time = 0.21, size = 435, normalized size = 7.91

$$\frac{2}{27\sqrt{(d^3 + 3)d}cd} - \frac{2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}cd} - \frac{2}{27\sqrt{dx^3+c}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)

[Out] $-2/27/((x^3+c/d)*d)^{1/2}/c/d - 1/243*I/c^2/d^3*2^{1/2}*sum((-c*d^2)^{1/3}*(1/2*I*(2*x+(-I*3^{1/2})*(-c*d^2)^{1/3})+(-c*d^2)^{1/3})/d)/(-c*d^2)^{1/3}*d)^{1/2}$

$$\frac{1}{2} * ((x - (-c*d^2)^{(1/3)} / d) / (-3 * (-c*d^2)^{(1/3)} + I * 3^{(1/2)} * (-c*d^2)^{(1/3)}) * d)^{(1/2)} * (-1/2 * I * (2*x + (I * 3^{(1/2)} * (-c*d^2)^{(1/3)} + (-c*d^2)^{(1/3)}) / d) / (-c*d^2)^{(1/3)} * d)^{(1/2)} / (d*x^3 + c)^{(1/2)} * (2 * _alpha^2 * d^2 + I * (-c*d^2)^{(1/3)} * 3^{(1/2)} * _alpha * d - (-c*d^2)^{(1/3)} * _alpha * d - I * 3^{(1/2)} * (-c*d^2)^{(2/3)} - (-c*d^2)^{(2/3)}) * \text{EllipticPi}(1/3 * 3^{(1/2)} * (I * (x + 1/2 * (-c*d^2)^{(1/3)} / d - 1/2 * I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / d) * 3^{(1/2)} / (-c*d^2)^{(1/3)} * d)^{(1/2)}, -1/18 * (2 * I * (-c*d^2)^{(1/3)} * 3^{(1/2)} * _alpha^2 * d + I * 3^{(1/2)} * c * d - 3 * c * d - I * (-c*d^2)^{(2/3)} * 3^{(1/2)} * _alpha - 3 * (-c*d^2)^{(2/3)} * _alpha) / c / d, (I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / (-3/2 * (-c*d^2)^{(1/3)} / d + 1/2 * I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / d) / d)^{(1/2)}, _alpha = \text{RootOf}(_Z^3 * d - 8 * c))$$

maxima [A] time = 1.23, size = 58, normalized size = 1.05

$$-\frac{\log\left(\frac{\sqrt{dx^3+c-3\sqrt{c}}}{\sqrt{dx^3+c+3\sqrt{c}}}\right) + \frac{6}{\sqrt{dx^3+c}}}{81d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")

[Out] -1/81*(log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(3/2) + 6/(sqrt(d*x^3 + c)*c))/d

mupad [B] time = 3.63, size = 63, normalized size = 1.15

$$\frac{\ln\left(\frac{10c+dx^3+6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{81c^{3/2}d} - \frac{2}{27cd\sqrt{dx^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)

[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(81*c^(3/2)*d) - 2/(27*c*d*(c + d*x^3)^(1/2))

sympy [A] time = 25.53, size = 51, normalized size = 0.93

$$-\frac{2}{27cd\sqrt{c+dx^3}} - \frac{2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81cd\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)

[Out] -2/(27*c*d*sqrt(c + d*x**3)) - 2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(81*c*d*sqrt(-c))

$$3.242 \quad \int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=76

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}} + \frac{2}{27c^2\sqrt{c+dx^3}}$$

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {446, 85, 156, 63, 208, 206}

$$\frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] 2/(27*c^2*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(324*c^(5/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(12*c^(5/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 85

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)), x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[p, -1]

Rule 156

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c

+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(8c - dx^3)(c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(8c - dx)(c + dx)^{3/2}} dx, x, x^3 \right) \\
 &= \frac{2}{27c^2 \sqrt{c + dx^3}} - \frac{\text{Subst} \left(\int \frac{-9cd + d^2x}{x(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{27c^2 d} \\
 &= \frac{2}{27c^2 \sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{c + dx}} dx, x, x^3 \right)}{24c^2} + \frac{d \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{216c^2} \\
 &= \frac{2}{27c^2 \sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{108c^2} + \frac{\text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{12c^2 d} \\
 &= \frac{2}{27c^2 \sqrt{c + dx^3}} + \frac{\tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{324c^{5/2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{12c^{5/2}}
 \end{aligned}$$

Mathematica [C] time = 0.03, size = 63, normalized size = 0.83

$$\frac{{}_9F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3}{c} + 1\right) - {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3+c}{9c}\right)}{108c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)), x]

[Out] (-Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)] + 9*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(108*c^2*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.06, size = 76, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}} + \frac{2}{27c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)), x]

[Out] 2/(27*c^2*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(324*c^(5/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(12*c^(5/2))

fricas [A] time = 0.66, size = 213, normalized size = 2.80

$$\left[\frac{(dx^3+c)\sqrt{c} \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + 27(dx^3+c)\sqrt{c} \log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) + 48\sqrt{dx^3+c}c - 27(dx^3+c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - (dx^3+c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 24\sqrt{dx^3+c}c}{648(c^3dx^3+c^4)}, \frac{27(dx^3+c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - (dx^3+c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 24\sqrt{dx^3+c}c}{324(c^3dx^3+c^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2), x, algorithm="fricas")

[Out] [1/648*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 27*(d*x^3 + c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 48*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c^4), 1/324*(27*(d*x^3 + c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (d*x^3 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c^4)]

giac [A] time = 0.16, size = 68, normalized size = 0.89

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}c^2} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{324\sqrt{-c}c^2} + \frac{2}{27\sqrt{dx^3+c}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/324*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) + 2/27/(sqrt(d*x^3 + c)*c^2)

maple [C] time = 0.19, size = 485, normalized size = 6.38

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)

[Out]
$$-1/8/c*d*(2/27/((x^3+c/d)*d)^{(1/2)}/c/d+1/243*I/c^2/d^3*2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=\text{RootOf}(_Z^3*d-8*c)))+1/8/c*(2/3/c/((x^3+c/d)*d)^{(1/2)}-2/3*\text{arctanh}((d*x^3+c)^{(1/2)}/c^{(1/2)})/c^{(3/2)})$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(dx^3 + c)^{\frac{3}{2}}(dx^3 - 8c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")

[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x), x)

mupad [B] time = 3.66, size = 68, normalized size = 0.89

$$\frac{2}{27 c^2 \sqrt{d x^3 + c}} - \frac{\operatorname{atanh}\left(\frac{c^2 \sqrt{d x^3 + c}}{\sqrt{c^5}}\right)}{12 \sqrt{c^5}} + \frac{\operatorname{atanh}\left(\frac{c^2 \sqrt{d x^3 + c}}{3 \sqrt{c^5}}\right)}{324 \sqrt{c^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)`

[Out] $2/(27*c^2*(c + d*x^3)^{(1/2)}) - \operatorname{atanh}((c^2*(c + d*x^3)^{(1/2)})/(c^5)^{(1/2)})/(12*(c^5)^{(1/2)}) + \operatorname{atanh}((c^2*(c + d*x^3)^{(1/2)})/(3*(c^5)^{(1/2)}))/(324*(c^5)^{(1/2)})$

sympy [A] time = 17.02, size = 78, normalized size = 1.03

$$\frac{2}{27c^2\sqrt{c + dx^3}} - \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{324c^2\sqrt{-c}} + \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12c^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)`

[Out] $2/(27*c**2*\sqrt{c + d*x**3}) - \operatorname{atan}(\sqrt{c + d*x**3}/(3*\sqrt{-c}))/ (324*c**2*\sqrt{-c}) + \operatorname{atan}(\sqrt{c + d*x**3}/\sqrt{-c})/(12*c**2*\sqrt{-c})$

$$3.243 \quad \int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=100

$$\frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}} + \frac{11d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}} - \frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}}$$

Rubi [A] time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {446, 103, 152, 156, 63, 208, 206}

$$-\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}} + \frac{11d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (-25*d)/(216*c^3*Sqrt[c + d*x^3]) - 1/(24*c^2*x^3*Sqrt[c + d*x^3]) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2592*c^(7/2)) + (11*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])]/(96*c^(7/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 152

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[2*m, 2*n, 2*p]

```

Rule 156

```

Int[((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

```

Rule 206

```

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

```

Rule 208

```

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 446

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2(8c-dx)(c+dx)^{3/2}} dx, x, x^3 \right) \\
&= -\frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{\text{Subst} \left(\int \frac{11cd-\frac{3d^2x}{2}}{x(8c-dx)(c+dx)^{3/2}} dx, x, x^3 \right)}{24c^2} \\
&= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{\text{Subst} \left(\int \frac{\frac{99c^2d^2}{2}-\frac{25}{4}cd^3x}{x(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{108c^4d} \\
&= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{(11d) \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{192c^3} + \frac{d^2 S}{192c^3} \\
&= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{11 \text{Subst} \left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{96c^3} + \frac{d^2 S}{192c^3} \\
&= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{2592c^{7/2}} + \frac{11d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{96c^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 77, normalized size = 0.77

$$\frac{-dx^3 {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3+c}{9c}\right) - 99dx^3 {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3}{c} + 1\right) - 36c}{864c^3x^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)), x]

[Out] (-36*c - d*x^3*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)] - 99*d*x^3*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(864*c^3*x^3*sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.10, size = 91, normalized size = 0.91

$$\frac{d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{2592c^{7/2}} + \frac{11d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{96c^{7/2}} + \frac{-9c - 25dx^3}{216c^3x^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] $(-9*c - 25*d*x^3)/(216*c^3*x^3*\text{Sqrt}[c + d*x^3]) + (d*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]]/(3*\text{Sqrt}[c]))/(2592*c^{(7/2)}) + (11*d*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/\text{Sqrt}[c]])/(96*c^{(7/2)})$

fricas [A] time = 0.66, size = 272, normalized size = 2.72

$$\frac{(d^2x^6 + cdx^3)\sqrt{c} \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c+10c}}{dx^3-8c}\right) + 297(d^2x^6 + cdx^3)\sqrt{c} \log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c+2c}}{x^3}\right) - 24(25cdx^3 + 9c^2)\sqrt{dx^3+c} - 297(d^2x^6 + cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + (d^2x^6 + cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 12(25cdx^3 + 9c^2)\sqrt{dx^3+c}}{5184(c^4dx^6 + c^5x^3)}, \frac{297(d^2x^6 + cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + (d^2x^6 + cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 12(25cdx^3 + 9c^2)\sqrt{dx^3+c}}{2592(c^4dx^6 + c^5x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] $[1/5184*((d^2*x^6 + c*d*x^3)*\text{sqrt}(c)*\log((d*x^3 + 6*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 10*c)/(d*x^3 - 8*c)) + 297*(d^2*x^6 + c*d*x^3)*\text{sqrt}(c)*\log((d*x^3 + 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 2*c)/x^3) - 24*(25*c*d*x^3 + 9*c^2)*\text{sqrt}(d*x^3 + c))/(c^4*d*x^6 + c^5*x^3), -1/2592*(297*(d^2*x^6 + c*d*x^3)*\text{sqrt}(-c)*\arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + (d^2*x^6 + c*d*x^3)*\text{sqrt}(-c)*\arctan(1/3*\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + 12*(25*c*d*x^3 + 9*c^2)*\text{sqrt}(d*x^3 + c))/(c^4*d*x^6 + c^5*x^3)]$

giac [A] time = 0.18, size = 100, normalized size = 1.00

$$\frac{11 d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96 \sqrt{-c} c^3} - \frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{3 \sqrt{-c}}\right)}{2592 \sqrt{-c} c^3} - \frac{25 (dx^3 + c) d - 16 cd}{216 \left((dx^3 + c)^2 - \sqrt{dx^3 + c} c \right) c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $-11/96*d*\arctan(\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*c^3) - 1/2592*d*\arctan(1/3*\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*c^3) - 1/216*(25*(d*x^3 + c)*d - 16*c*d)/(((d*x^3 + c)^(3/2) - \text{sqrt}(d*x^3 + c)*c)*c^3)$

maple [C] time = 0.19, size = 549, normalized size = 5.49

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)`

[Out] $\frac{1}{8} \frac{1}{c} \left(-\frac{1}{3} (d x^3 + c)^{1/2} / c^2 / x^3 - \frac{2}{3} d / c^2 / ((x^3 + c/d) d)^{1/2} + d \operatorname{arctanh} \left(\frac{(d x^3 + c)^{1/2} / c^{1/2}}{c^{5/2}} \right) - \frac{1}{64} \frac{1}{c^2} d^2 \frac{(2/27 / ((x^3 + c/d) d)^{1/2} / c / d + 1/243 I / c^2 / d^3)^{2^{1/2}} \sum((-c d^2)^{1/3} (1/2 I (2 x + (-I 3^{1/2}) (-c d^2)^{1/3} + (-c d^2)^{1/3})) / d) / (-c d^2)^{1/3} d)^{1/2} * ((x - (-c d^2)^{1/3}) / d) / (-3 * (-c d^2)^{1/3} + I 3^{1/2} * (-c d^2)^{1/3}) * d)^{1/2} * (-1/2 I (2 x + (I 3^{1/2}) * (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d) / (-c d^2)^{1/3} d)^{1/2} / (d x^3 + c)^{1/2} \right) * (2 * \alpha^2 d^2 + I (-c d^2)^{1/3} 3^{1/2} * \alpha d - (-c d^2)^{1/3} * \alpha d - I 3^{1/2} * (-c d^2)^{2/3} - (-c d^2)^{2/3}) * \operatorname{EllipticPi} \left(\frac{1}{3} 3^{1/2} * (I * (x + 1/2 * (-c d^2)^{1/3} / d - 1/2 I 3^{1/2} * (-c d^2)^{1/3} / d) * 3^{1/2} / (-c d^2)^{1/3} d)^{1/2}, -1/18 * (2 I * (-c d^2)^{1/3} * 3^{1/2} * \alpha^2 d + I 3^{1/2} * c d - 3 * c d - I * (-c d^2)^{2/3} * 3^{1/2} * \alpha - 3 * (-c d^2)^{2/3} * \alpha) / c / d, (I 3^{1/2} * (-c d^2)^{1/3} / (-3/2 * (-c d^2)^{1/3} / d + 1/2 I 3^{1/2} * (-c d^2)^{1/3} / d) / d)^{1/2} \right), \alpha = \operatorname{RootOf}(_Z^3 d - 8 c)) + \frac{1}{64} \frac{1}{c^2} d^2 \frac{(2/3 / ((x^3 + c/d) d)^{1/2} / c - 2/3 \operatorname{arctanh} \left(\frac{(d x^3 + c)^{1/2} / c^{1/2}}{c^{3/2}} \right))}{c^{3/2}}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")`

[Out] `-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^4), x)`

mupad [B] time = 3.80, size = 88, normalized size = 0.88

$$\frac{11 d \operatorname{atanh} \left(\frac{c^3 \sqrt{d x^3 + c}}{\sqrt{c^7}} \right)}{96 \sqrt{c^7}} - \frac{25 d}{216 c^3 \sqrt{d x^3 + c}} + \frac{d \operatorname{atanh} \left(\frac{c^3 \sqrt{d x^3 + c}}{3 \sqrt{c^7}} \right)}{2592 \sqrt{c^7}} - \frac{1}{24 c^2 x^3 \sqrt{d x^3 + c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)`

[Out] $(11 * d * \operatorname{atanh}((c^3 * (c + d * x^3)^{1/2}) / (c^7)^{1/2})) / (96 * (c^7)^{1/2}) - (25 * d) / (216 * c^3 * (c + d * x^3)^{1/2}) + (d * \operatorname{atanh}((c^3 * (c + d * x^3)^{1/2}) / (3 * (c^7)^{1/2}))) / (2592 * (c^7)^{1/2}) - 1 / (24 * c^2 * x^3 * (c + d * x^3)^{1/2})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^2x^4\sqrt{c+dx^3} - 7cdx^7\sqrt{c+dx^3} + d^2x^{10}\sqrt{c+dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(1/(-8*c**2*x**4*sqrt(c + d*x**3) - 7*c*d*x**7*sqrt(c + d*x**3) +  
d**2*x**10*sqrt(c + d*x**3)), x)
```

$$3.244 \quad \int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=128

$$\frac{d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}} - \frac{109d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}} + \frac{245d^2}{1728c^4\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}}$$

Rubi [A] time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {446, 103, 151, 152, 156, 63, 208, 206}

$$\frac{245d^2}{1728c^4\sqrt{c+dx^3}} + \frac{d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}} - \frac{109d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (245*d^2)/(1728*c^4*Sqrt[c + d*x^3]) - 1/(48*c^2*x^6*Sqrt[c + d*x^3]) + (3*d)/(64*c^3*x^3*Sqrt[c + d*x^3]) + (d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(20736*c^(9/2)) - (109*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(9/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 151


```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7 (8c - dx^3)(c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^3 (8c - dx)(c + dx)^{3/2}} dx, x, x^3 \right) \\
&= -\frac{1}{48c^2 x^6 \sqrt{c + dx^3}} - \frac{\text{Subst} \left(\int \frac{18cd - \frac{5d^2 x}{2}}{x^2 (8c - dx)(c + dx)^{3/2}} dx, x, x^3 \right)}{48c^2} \\
&= -\frac{1}{48c^2 x^6 \sqrt{c + dx^3}} + \frac{3d}{64c^3 x^3 \sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{218c^2 d^2 - 27cd^3 x}{x(8c - dx)(c + dx)^{3/2}} dx, x, x^3 \right)}{384c^4} \\
&= \frac{245d^2}{1728c^4 \sqrt{c + dx^3}} - \frac{1}{48c^2 x^6 \sqrt{c + dx^3}} + \frac{3d}{64c^3 x^3 \sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{981c^3 d^3 - \frac{243}{2}}{x(8c - dx)\sqrt{c + dx^3}} dx, x, x^3 \right)}{1728c^6} \\
&= \frac{245d^2}{1728c^4 \sqrt{c + dx^3}} - \frac{1}{48c^2 x^6 \sqrt{c + dx^3}} + \frac{3d}{64c^3 x^3 \sqrt{c + dx^3}} + \frac{(109d^2) \text{Subst} \left(\int \frac{1}{x} dx, x, x^3 \right)}{1536c^6} \\
&= \frac{245d^2}{1728c^4 \sqrt{c + dx^3}} - \frac{1}{48c^2 x^6 \sqrt{c + dx^3}} + \frac{3d}{64c^3 x^3 \sqrt{c + dx^3}} + \frac{(109d) \text{Subst} \left(\int \frac{-\frac{c}{d}}{-\frac{c}{d}} dx, x, x^3 \right)}{768c^6} \\
&= \frac{245d^2}{1728c^4 \sqrt{c + dx^3}} - \frac{1}{48c^2 x^6 \sqrt{c + dx^3}} + \frac{3d}{64c^3 x^3 \sqrt{c + dx^3}} + \frac{d^2 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{20736c^{9/2}}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 91, normalized size = 0.71

$$\frac{-d^2 x^6 {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3 + c}{9c} \right) + 981d^2 x^6 {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3}{c} + 1 \right) + 36c(9dx^3 - 4c)}{6912c^4 x^6 \sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (36*c*(-4*c + 9*d*x^3) - d^2*x^6*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)] + 981*d^2*x^6*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(6912*c^4*x^6*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.14, size = 106, normalized size = 0.83

$$\frac{d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}} - \frac{109d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}} + \frac{-36c^2 + 81cdx^3 + 245d^2x^6}{1728c^4x^6\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] $(-36*c^2 + 81*c*d*x^3 + 245*d^2*x^6)/(1728*c^4*x^6*\text{Sqrt}[c + d*x^3]) + (d^2*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/(3*\text{Sqrt}[c])])/(20736*c^{(9/2)}) - (109*d^2*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/\text{Sqrt}[c]])/(768*c^{(9/2)})$

fricas [A] time = 0.79, size = 303, normalized size = 2.37

$$\frac{(d^3x^9 + cd^2x^6)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + 2943(d^3x^9 + cd^2x^6)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{c}\right) + 24(245cd^2x^6 + 81c^2dx^3 - 36c^2)\sqrt{dx^3+c} + 2943(d^3x^9 + cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{c}}{c}\right) - (d^3x^9 + cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{c}}{3c}\right) + 12(245cd^2x^6 + 81c^2dx^3 - 36c^2)\sqrt{dx^3+c}}{41472(c^5dx^9 + c^6x^6)}, \frac{2943(d^3x^9 + cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{c}}{c}\right) - (d^3x^9 + cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{c}}{3c}\right) + 12(245cd^2x^6 + 81c^2dx^3 - 36c^2)\sqrt{dx^3+c}}{20736(c^5dx^9 + c^6x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] $[1/41472*((d^3*x^9 + c*d^2*x^6)*\text{sqrt}(c)*\log((d*x^3 + 6*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 10*c)/(d*x^3 - 8*c)) + 2943*(d^3*x^9 + c*d^2*x^6)*\text{sqrt}(c)*\log((d*x^3 - 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 2*c)/x^3) + 24*(245*c*d^2*x^6 + 81*c^2*d*x^3 - 36*c^3)*\text{sqrt}(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6), 1/20736*(2943*(d^3*x^9 + c*d^2*x^6)*\text{sqrt}(-c)*\arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) - (d^3*x^9 + c*d^2*x^6)*\text{sqrt}(-c)*\arctan(1/3*\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + 12*(245*c*d^2*x^6 + 81*c^2*d*x^3 - 36*c^3)*\text{sqrt}(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6)]$

giac [A] time = 0.16, size = 118, normalized size = 0.92

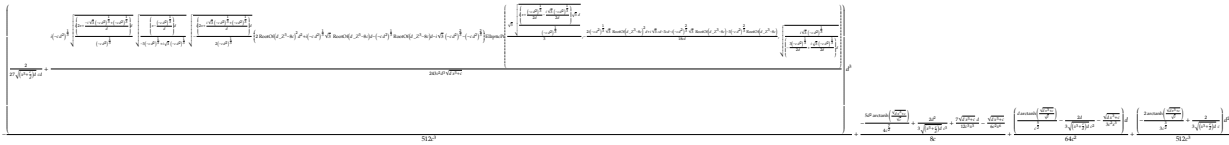
$$\frac{109d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{768\sqrt{-c}c^4} - \frac{d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{20736\sqrt{-c}c^4} + \frac{2d^2}{27\sqrt{dx^3+c}c^4} + \frac{13(dx^3+c)^{\frac{3}{2}}d^2 - 17\sqrt{dx^3+c}cd^2}{192c^4d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $109/768*d^2*\arctan(\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*c^4) - 1/20736*d^2*\arctan(1/3*\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*c^4) + 2/27*d^2/(\text{sqrt}(d*x^3 + c)*c^4) + 1/192*(13*(d*x^3 + c)^{(3/2)}*d^2 - 17*\text{sqrt}(d*x^3 + c)*c*d^2)/(c^4*d^2*x^6)$

maple [C] time = 0.22, size = 636, normalized size = 4.97



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2), x)`

[Out] $\frac{1}{64} \frac{d^2}{c^2} \left(-\frac{1}{3} (d x^3 + c)^{1/2} / c^2 x^3 - \frac{2}{3} \frac{((x^3 + c/d) d)^{1/2}}{c^2 d} + \operatorname{arctanh}\left(\frac{(d x^3 + c)^{1/2}}{c^{1/2}}\right) / c^{5/2} \right) + \frac{1}{8} \frac{d}{c} \left(-\frac{1}{6} (d x^3 + c)^{1/2} / c^2 x^6 + \frac{7}{12} \frac{d (d x^3 + c)^{1/2}}{c^3 x^3} + \frac{2}{3} \frac{d^2}{c^3} \frac{((x^3 + c/d) d)^{1/2}}{(x^3 + c/d) d} - \frac{5}{4} \frac{d^2}{c^2} \operatorname{arctanh}\left(\frac{(d x^3 + c)^{1/2}}{c^{1/2}}\right) / c^{7/2} \right) - \frac{1}{512} \frac{d^3}{c^3} \frac{(2/27) ((x^3 + c/d) d)^{1/2}}{c/d + 1/243} \frac{I/c^2/d^3 \cdot 2^{1/2} \sum((-c d^2)^{1/3} (1/2 I (2 x + (-I)^{3/2}) (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d) / (-c d^2)^{1/3} d^{1/2} ((x - (-c d^2)^{1/3}) / d) / (-3 (-c d^2)^{1/3} + I^{3/2} (-c d^2)^{1/3}) d^{1/2} (-1/2 I (2 x + I^{3/2}) (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d) / (-c d^2)^{1/3} d^{1/2} / (d x^3 + c)^{1/2} (2 \alpha^2 d^2 + I (-c d^2)^{1/3} 3^{1/2} \alpha d - (-c d^2)^{1/3} \alpha d - I^{3/2} (-c d^2)^{2/3} - (-c d^2)^{2/3}) \operatorname{EllipticPi}(1/3, 3^{1/2} (I (x + 1/2 (-c d^2)^{1/3} / d - 1/2 I^{3/2} (-c d^2)^{1/3} / d) 3^{1/2} / (-c d^2)^{1/3} d)^{1/2}, -1/18 (2 I (-c d^2)^{1/3} 3^{1/2} \alpha^2 d + I^{3/2} c d - 3 c d - I (-c d^2)^{2/3} 3^{1/2} \alpha - 3 (-c d^2)^{2/3} \alpha) / c d, (I^{3/2} (-c d^2)^{1/3} / (-3/2 (-c d^2)^{1/3} / d + 1/2 I^{3/2} (-c d^2)^{1/3} / d) / d)^{1/2}}, \alpha = \operatorname{RootOf}(_Z^3 d - 8 c)) + \frac{1}{512} \frac{d^2}{c^3} \frac{(2/3) ((x^3 + c/d) d)^{1/2}}{c^2} - \frac{2}{3} \operatorname{arctanh}\left(\frac{(d x^3 + c)^{1/2}}{c^{1/2}}\right) / c^{3/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{(d x^3 + c)^{3/2} (d x^3 - 8 c) x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2), x, algorithm="maxima")`

[Out] `-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^7), x)`

mupad [B] time = 4.03, size = 112, normalized size = 0.88

$$\frac{245 d^2}{1728 c^4 \sqrt{d x^3 + c}} - \frac{109 d^2 \operatorname{atanh}\left(\frac{c^4 \sqrt{d x^3 + c}}{\sqrt{c^9}}\right)}{768 \sqrt{c^9}} + \frac{d^2 \operatorname{atanh}\left(\frac{c^4 \sqrt{d x^3 + c}}{3 \sqrt{c^9}}\right)}{20736 \sqrt{c^9}} - \frac{1}{48 c^2 x^6 \sqrt{d x^3 + c}} + \frac{3 d}{64 c^3 x^3 \sqrt{d x^3 + c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^7*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)`

[Out] $(245*d^2)/(1728*c^4*(c + d*x^3)^{(1/2)}) - (109*d^2*\operatorname{atanh}((c^4*(c + d*x^3)^{(1/2)})/(c^9)^{(1/2)}))/(768*(c^9)^{(1/2)}) + (d^2*\operatorname{atanh}((c^4*(c + d*x^3)^{(1/2)})/(3*(c^9)^{(1/2)})))/(20736*(c^9)^{(1/2)}) - 1/(48*c^2*x^6*(c + d*x^3)^{(1/2)}) + (3*d)/(64*c^3*x^3*(c + d*x^3)^{(1/2)})$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)`

[Out] Timed out

$$3.245 \quad \int \frac{x}{\sqrt{a+bx^3} (2(5+3\sqrt{3})a+bx^3)} dx$$

Optimal. Leaf size=318

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[6]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{(1-\sqrt{3})\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}((1+\sqrt{3})\sqrt[3]{a}-2\sqrt[3]{bx})}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Rubi [A] time = 0.06, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {487}

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[6]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{(1-\sqrt{3})\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}((1+\sqrt{3})\sqrt[3]{a}-2\sqrt[3]{bx})}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]

[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*(1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x)/(Sqrt[2]*Sqrt[a + b*x^3])]/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)))

Rule 487

Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> With[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))]/(3*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x] - Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(6*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x]]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^3} (2(5+3\sqrt{3})a+bx^3)} dx = -\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[6]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{(1-\sqrt{3})\sqrt{a}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Mathematica [C] time = 0.10, size = 83, normalized size = 0.26

$$\frac{x^2 \sqrt{\frac{bx^3}{a}} + {}_1F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; -\frac{bx^3}{a}, -\frac{bx^3}{6\sqrt{3}a+10a}\right)}{(12\sqrt{3}a+20a)\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)), x]

[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]/((20*a + 12*Sqrt[3]*a)*Sqrt[a + b*x^3]))

IntegrateAlgebraic [F] time = 31.27, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^3} (2(5+3\sqrt{3})a+bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)), x]

[Out] Defer[IntegrateAlgebraic][x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.39, size = 538, normalized size = 1.69



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)/
3)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/
b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*
b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(
1/2)*(4*3^(1/2)*_alpha^2*b^2-6*_alpha^2*b^2-3*I*(-a*b^2)^(1/3)*3^(1/2)*_alp
ha*b-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^
2)^(1/3)*_alpha*b+3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(
-a*b^2)^(2/3)+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(
1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6
*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-4*I*(-a*b^2)^(1/3)*_alpha^2*b+I*3^(
1/2)*a*b+2*3^(1/2)*a*b-2*I*a*b-3*a*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+2*3^(1
/2)*(-a*b^2)^(2/3)*_alpha+2*I*(-a*b^2)^(2/3)*_alpha-3*(-a*b^2)^(2/3)*_alpha
)/a/b,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b+6*3^(1/2)*a+10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^3 + 2a(3\sqrt{3} + 5))\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(3*sqrt(3)+5))*sqrt(b*x^3+a),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{bx^3 + a} (bx^3 + 2a(3\sqrt{3} + 5))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((a + b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)`

[Out] `int(x/((a + b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bx^3} (10a + 6\sqrt{3}a + bx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**3+2*a*(5+3*3**(1/2)))/(b*x**3+a)**(1/2), x)`

[Out] `Integral(x/(sqrt(a + b*x**3)*(10*a + 6*sqrt(3)*a + b*x**3)), x)`

$$3.246 \quad \int \frac{x}{\sqrt{a-bx^3} (2(5+3\sqrt{3})a-bx^3)} dx$$

Optimal. Leaf size=324

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{bx}\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{(1-\sqrt{3})\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{bx}\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Rubi [A] time = 0.07, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {487}

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{bx}\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{(1-\sqrt{3})\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{bx}\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left((1+\sqrt{3})\sqrt[3]{a}+2\sqrt[3]{bx}\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]

[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))

Rule 487

Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] := With[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x] - Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x]]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]

Rubi steps

$$\int \frac{x}{\sqrt{a-bx^3} (2(5+3\sqrt{3})a-bx^3)} dx = -\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[6]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{(1-\sqrt{3})\sqrt{a}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Mathematica [C] time = 0.08, size = 83, normalized size = 0.26

$$\frac{x^2 \sqrt{1 - \frac{bx^3}{a}} F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; \frac{bx^3}{a}, \frac{bx^3}{6\sqrt{3}a+10a}\right)}{(12\sqrt{3}a + 20a) \sqrt{a - bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]

[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[a - b*x^3])

IntegrateAlgebraic [F] time = 31.26, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a-bx^3} (2(5+3\sqrt{3})a-bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]

[Out] Defer[IntegrateAlgebraic][x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-b*x^3+2*a*(5+3*sqrt(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

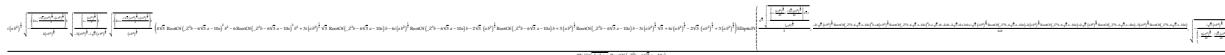
Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.41, size = 509, normalized size = 1.57



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*(2*x+(I*3^(1/2))*(a*
b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(-
3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3))*b)^(1/2)*(1/2*I*(2*x+(-I*3^(1/2)*(
a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)/(-b*x^3+a)^(1/2)*(4*3
^(1/2)*_alpha^2*b^2-6*_alpha^2*b^2+3*I*(a*b^2)^(1/3)*3^(1/2)*_alpha*b-6*I*(
a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(1/3)*_alp
ha*b-3*I*(a*b^2)^(2/3)*3^(1/2)+6*I*(a*b^2)^(2/3)-2*3^(1/2)*(a*b^2)^(2/3)+3*
(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2),1/6*(-2*I*3^(1/2)*(a*b^
2)^(1/3)*_alpha^2*b+4*I*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b-2*I*a*b-2*3^
(1/2)*a*b+3*a*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*I*(a*b^2)^(2/3)*_alpha+2*3
^(1/2)*(a*b^2)^(2/3)*_alpha-3*(a*b^2)^(2/3)*_alpha)/a/b,(-I*3^(1/2)*(a*b^2)
^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2)),_alph
a=RootOf(_Z^3*b-6*3^(1/2)*a-10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{(bx^3 - 2a(3\sqrt{3} + 5))\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x}{\sqrt{a - bx^3} (bx^3 - 2a(3\sqrt{3} + 5))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-x/((a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)`

[Out] `-int(x/((a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-6\sqrt{3}a\sqrt{a-bx^3} - 10a\sqrt{a-bx^3} + bx^3\sqrt{a-bx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-b*x**3+2*a*(5+3*3**(1/2)))/(-b*x**3+a)**(1/2), x)`

[Out] `-Integral(x/(-6*sqrt(3)*a*sqrt(a - b*x**3) - 10*a*sqrt(a - b*x**3) + b*x**3*sqrt(a - b*x**3)), x)`

$$3.247 \quad \int \frac{x}{\sqrt{-a+bx^3} (-2(5+3\sqrt{3})a+bx^3)} dx$$

Optimal. Leaf size=328

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}((1+\sqrt{3})\sqrt[3]{a}+2\sqrt[3]{bx})}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Rubi [A] time = 0.07, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {488}

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}((1+\sqrt{3})\sqrt[3]{a}+2\sqrt[3]{bx})}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[6]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{(1-\sqrt{3})\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]

[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])]/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])]/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])]/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])]/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)))

Rule 488

Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> With[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))]/(3*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1 + r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)), x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(6*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]

Rubi steps

$$\int \frac{x}{\sqrt{-a + bx^3} (-2(5 + 3\sqrt{3})a + bx^3)} dx = \frac{(2 - \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3}(1 - \sqrt{3}) \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{bx})}{\sqrt{2} \sqrt{-a + bx^3}} \right)}{6\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}} + \frac{(2 - \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3} \sqrt[6]{a}}{\sqrt{2} \sqrt{-a + bx^3}} \right)}{3\sqrt{2} \sqrt[4]{3} a^{5/6}}$$

Mathematica [C] time = 0.12, size = 85, normalized size = 0.26

$$\frac{x^2 \sqrt{1 - \frac{bx^3}{a}} F_1 \left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; \frac{bx^3}{a}, \frac{bx^3}{6\sqrt{3}a + 10a} \right)}{(12\sqrt{3}a + 20a) \sqrt{bx^3 - a}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]

[Out] -((x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a + b*x^3]))

IntegrateAlgebraic [F] time = 31.29, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a + bx^3} (-2(5 + 3\sqrt{3})a + bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]

[Out] Defer[IntegrateAlgebraic][x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.41, size = 510, normalized size = 1.55



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a^2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*(2*x+(I*3^(1/2))*(a
*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(
-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3))*b)^(1/2)*(1/2*I*(2*x+(-I*3^(1/2)*
(a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)/(b*x^3-a)^(1/2)*(4*3
^(1/2)*_alpha^2*b^2-6*_alpha^2*b^2+3*I*(a*b^2)^(1/3)*3^(1/2)*_alpha*b-6*I*(
a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(1/3)*_alp
ha*b-3*I*(a*b^2)^(2/3)*3^(1/2)+6*I*(a*b^2)^(2/3)-2*3^(1/2)*(a*b^2)^(2/3)+3*
(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2),1/6*(-2*I*3^(1/2)*(a*b^
2)^(1/3)*_alpha^2*b+4*I*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b-2*I*a*b-2*3^
(1/2)*a*b+3*a*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*I*(a*b^2)^(2/3)*_alpha+2*3
^(1/2)*(a*b^2)^(2/3)*_alpha-3*(a*b^2)^(2/3)*_alpha)/a/b,(-I*3^(1/2)*(a*b^2)
^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2)),_alph
a=RootOf(_Z^3*b-6*3^(1/2)*a-10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^3 - 2a(3\sqrt{3} + 5))\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3-2*a*(3*sqrt(3)+5))*sqrt(b*x^3-a),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 - a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{bx^3 - a} (bx^3 - 2a(3\sqrt{3} + 5))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((b*x^3 - a)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)`

[Out] `int(x/((b*x^3 - a)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a + bx^3} (-6\sqrt{3}a - 10a + bx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**3-2*a*(5+3*3**(1/2)))/(b*x**3-a)**(1/2), x)`

[Out] `Integral(x/(sqrt(-a + b*x**3)*(-6*sqrt(3)*a - 10*a + b*x**3)), x)`

$$3.248 \quad \int \frac{x}{\sqrt{-a-bx^3}(-2(5+3\sqrt{3})a-bx^3)} dx$$

Optimal. Leaf size=330

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3} \sqrt[6]{a}((1+\sqrt{3})\sqrt[3]{a}-2\sqrt[3]{bx})}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Rubi [A] time = 0.07, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {488}

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3} \sqrt[6]{a}((1+\sqrt{3})\sqrt[3]{a}-2\sqrt[3]{bx})}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[6]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{(1-\sqrt{3})\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]

[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])]/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])]/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])]/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])]/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)))

Rule 488

Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> With[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))]/(3*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1 + r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)), x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(6*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]

Rubi steps

$$\int \frac{x}{\sqrt{-a - bx^3} (-2(5 + 3\sqrt{3})a - bx^3)} dx = \frac{(2 - \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3} \sqrt[6]{a} ((1 + \sqrt{3}) \sqrt[3]{a} - 2 \sqrt[3]{bx})}{\sqrt{2} \sqrt{-a - bx^3}} \right)}{3\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}} + \frac{(2 - \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3} (1 - \sqrt{3}) \sqrt[6]{a} ((1 + \sqrt{3}) \sqrt[3]{a} - 2 \sqrt[3]{bx})}{\sqrt{2} \sqrt{-a - bx^3}} \right)}{6\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}}$$

Mathematica [C] time = 0.11, size = 87, normalized size = 0.26

$$\frac{x^2 \sqrt{\frac{bx^3}{a}} + {}_1F_1 \left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; -\frac{bx^3}{a}, -\frac{bx^3}{6\sqrt{3}a + 10a} \right)}{(12\sqrt{3}a + 20a) \sqrt{-a - bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]

[Out] -((x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a - b*x^3]))

IntegrateAlgebraic [F] time = 31.17, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a - bx^3} (-2(5 + 3\sqrt{3})a - bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]

[Out] Defer[IntegrateAlgebraic][x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.36, size = 541, normalized size = 1.64



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3))-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(-b*x^3-a)^(1/2)*(4*3^(1/2)*_alpha^2*b^2-6*_alpha^2*b^2-3*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(1/3)*_alpha*b+3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-4*I*(-a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b+2*3^(1/2)*a*b-2*I*a*b-3*a*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+2*I*(-a*b^2)^(2/3)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/a/b,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b+6*3^(1/2)*a+10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{(bx^3 + 2a(3\sqrt{3} + 5))\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 - a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{-bx^3 - a} (bx^3 + 2a(3\sqrt{3} + 5))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-x/((- a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))),x)`

[Out] `int(-x/((- a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{10a\sqrt{-a - bx^3} + 6\sqrt{3}a\sqrt{-a - bx^3} + bx^3\sqrt{-a - bx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-b*x**3-2*a*(5+3*3**(1/2)))/(-b*x**3-a)**(1/2),x)`

[Out] `-Integral(x/(10*a*sqrt(-a - b*x**3) + 6*sqrt(3)*a*sqrt(-a - b*x**3) + b*x**3*sqrt(-a - b*x**3)), x)`

$$3.249 \quad \int \frac{x}{\sqrt{a+bx^3} (2(5-3\sqrt{3})a+bx^3)} dx$$

Optimal. Leaf size=310

$$\frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3} \sqrt[6]{a} \left((1-\sqrt{3}) \sqrt[3]{a} - 2 \sqrt[3]{b} x \right)}{\sqrt{2} \sqrt{a+bx^3}} \right)}{3\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}} - \frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3} (1+\sqrt{3}) \sqrt[6]{a} \left(\sqrt[3]{a} + \sqrt[3]{b} x \right)}{\sqrt{2} \sqrt{a+bx^3}} \right)}{6\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}} + \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{\sqrt[4]{3} (1-\sqrt{3}) \sqrt[6]{a} \left(\sqrt[3]{a} + \sqrt[3]{b} x \right)}{\sqrt{2} \sqrt{a+bx^3}} \right)}{2\sqrt{2} 3^{3/4} a^{5/6} b^{2/3}}$$

Rubi [A] time = 0.05, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {487}

$$\frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3} \sqrt[6]{a} \left((1-\sqrt{3}) \sqrt[3]{a} - 2 \sqrt[3]{b} x \right)}{\sqrt{2} \sqrt{a+bx^3}} \right)}{3\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}} - \frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3} (1+\sqrt{3}) \sqrt[6]{a} \left(\sqrt[3]{a} + \sqrt[3]{b} x \right)}{\sqrt{2} \sqrt{a+bx^3}} \right)}{6\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}} + \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{\sqrt[4]{3} (1-\sqrt{3}) \sqrt[6]{a} \left(\sqrt[3]{a} + \sqrt[3]{b} x \right)}{\sqrt{2} \sqrt{a+bx^3}} \right)}{2\sqrt{2} 3^{3/4} a^{5/6} b^{2/3}} + \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{(1+\sqrt{3}) \sqrt{a+bx^3}}{\sqrt{2} 3^{3/4} \sqrt{a}} \right)}{3\sqrt{2} 3^{3/4} a^{5/6} b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)), x]

[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])]/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)))

Rule 487

Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> With[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))]/(3*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x] - Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(6*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x]]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^3} (2(5-3\sqrt{3})a+bx^3)} dx = -\frac{(2+\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3} \sqrt[6]{a} ((1-\sqrt{3})\sqrt[3]{a}-2\sqrt[3]{bx})}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{(2+\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[3]{a}}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Mathematica [C] time = 0.15, size = 83, normalized size = 0.27

$$\frac{x^2 \sqrt{\frac{bx^3}{a}} + {}_1F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right)}{(20a-12\sqrt{3}a)\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)), x]

[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))])/(20*a - 12*Sqrt[3]*a)*Sqrt[a + b*x^3])

IntegrateAlgebraic [F] time = 31.16, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^3} (2(5-3\sqrt{3})a+bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)), x]

[Out] Defer[IntegrateAlgebraic][x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

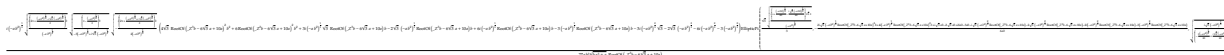
Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.42, size = 538, normalized size = 1.74



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a^2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)
)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b
)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b
^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1
/2)*(4*3^(1/2)*_alpha^2*b^2+6*_alpha^2*b^2+3*I*(-a*b^2)^(1/3)*3^(1/2)*_alph
a*b-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^2
)^(1/3)*_alpha*b-3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-
a*b^2)^(2/3)-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1
/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6*
(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b+4*I*(-a*b^2)^(1/3)*_alpha^2*b+I*3^(1
/2)*a*b-2*3^(1/2)*a*b+2*I*a*b-3*a*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha-2*3^(1
/2)*(-a*b^2)^(2/3)*_alpha-2*I*(-a*b^2)^(2/3)*_alpha-3*(-a*b^2)^(2/3)*_alpha
/a/b,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b-6*3^(1/2)*a+10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^3 - 2a(3\sqrt{3} - 5))\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{bx^3 + a} (bx^3 - 2a(3\sqrt{3} - 5))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((a + b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))), x)`

[Out] `int(x/((a + b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bx^3} (-6\sqrt{3}a + 10a + bx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**3+2*a*(5-3*3**(1/2)))/(b*x**3+a)**(1/2), x)`

[Out] `Integral(x/(sqrt(a + b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)`

$$3.250 \quad \int \frac{x}{\sqrt{a-bx^3} (2(5-3\sqrt{3})a-bx^3)} dx$$

Optimal. Leaf size=316

$$\frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3} (1 + \sqrt{3}) \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{b} x)}{\sqrt{2} \sqrt{a-bx^3}} \right)}{6\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}} - \frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3} \sqrt[6]{a} ((1 - \sqrt{3}) \sqrt[3]{a} + 2 \sqrt[3]{b} x)}{\sqrt{2} \sqrt{a-bx^3}} \right)}{3\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}} + \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{\sqrt[4]{3} (1 - \sqrt{3}) \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{b} x)}{\sqrt{2} \sqrt{a-bx^3}} \right)}{2\sqrt{2} 3^{3/4} a^{5/6} b^{2/3}}$$

Rubi [A] time = 0.05, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {487}

$$\frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3} (1 + \sqrt{3}) \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{b} x)}{\sqrt{2} \sqrt{a-bx^3}} \right)}{6\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}} - \frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3} \sqrt[6]{a} ((1 - \sqrt{3}) \sqrt[3]{a} + 2 \sqrt[3]{b} x)}{\sqrt{2} \sqrt{a-bx^3}} \right)}{3\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}} + \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{\sqrt[4]{3} (1 - \sqrt{3}) \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{b} x)}{\sqrt{2} \sqrt{a-bx^3}} \right)}{2\sqrt{2} 3^{3/4} a^{5/6} b^{2/3}} + \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{(1 + \sqrt{3}) \sqrt{a-bx^3}}{\sqrt{2} 3^{3/4} \sqrt{a}} \right)}{3\sqrt{2} 3^{3/4} a^{5/6} b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]

[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])]/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])]/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])]/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])]/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)))

Rule 487

Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] := With[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))]/(3*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x] - Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])]/(6*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x]]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]

Rubi steps

$$\int \frac{x}{\sqrt{a-bx^3} (2(5-3\sqrt{3})a-bx^3)} dx = -\frac{(2+\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[6]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{(2+\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Mathematica [C] time = 0.10, size = 83, normalized size = 0.26

$$\frac{x^2 \sqrt{1 - \frac{bx^3}{a}} F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right)}{(20a - 12\sqrt{3}a) \sqrt{a - bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]

[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[a - b*x^3])

IntegrateAlgebraic [F] time = 31.17, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a-bx^3} (2(5-3\sqrt{3})a-bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]

[Out] Defer[IntegrateAlgebraic][x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.37, size = 509, normalized size = 1.61



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a^2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*(2*x+(I*3^(1/2)*(a
*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(
-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3))*b)^(1/2)*(1/2*I*(2*x+(-I*3^(1/2)*
(a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)/(-b*x^3+a)^(1/2)*(4*
3^(1/2)*_alpha^2*b^2+6*_alpha^2*b^2-3*I*(a*b^2)^(1/3)*3^(1/2)*_alpha*b-2*3^(
1/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-3*(a*b^2)^(1/3)*_al
pha*b+3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)-3
*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(
1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2),1/6*(-2*I*3^(1/2)*(a*b
^2)^(1/3)*_alpha^2*b-4*I*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b+2*3^(1/2)*a
*b+2*I*a*b+3*a*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*(a*b^2)^(2/3)*_al
pha+2*I*(a*b^2)^(2/3)*_alpha-3*(a*b^2)^(2/3)*_alpha)/a/b,(-I*3^(1/2)*(a*b^2
)^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2)),_alp
ha=RootOf(_Z^3*b+6*3^(1/2)*a-10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{(bx^3 + 2a(3\sqrt{3} - 5))\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{a - bx^3} (bx^3 + 2a(3\sqrt{3} - 5))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-x/((a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)`

[Out] `int(-x/((a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-10a\sqrt{a - bx^3} + 6\sqrt{3}a\sqrt{a - bx^3} + bx^3\sqrt{a - bx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-b*x**3+2*a*(5-3*3**(1/2)))/(-b*x**3+a)**(1/2), x)`

[Out] `-Integral(x/(-10*a*sqrt(a - b*x**3) + 6*sqrt(3)*a*sqrt(a - b*x**3) + b*x**3*sqrt(a - b*x**3)), x)`

$$3.251 \quad \int \frac{x}{(2(5-3\sqrt{3})a-bx^3)\sqrt{-a+bx^3}} dx$$

Optimal. Leaf size=320

$$\frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3}(1-\sqrt{3}) \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{bx^3})}{\sqrt{2} \sqrt{bx^3-a}} \right)}{2\sqrt{2} 3^{3/4} a^{5/6} b^{2/3}} - \frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{(1+\sqrt{3}) \sqrt{bx^3-a}}{\sqrt{2} 3^{3/4} \sqrt{a}} \right)}{3\sqrt{2} 3^{3/4} a^{5/6} b^{2/3}} - \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{\sqrt[4]{3}(1+\sqrt{3}) \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{bx^3})}{\sqrt{2} \sqrt{bx^3-a}} \right)}{6\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}}$$

Rubi [A] time = 0.05, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {488}

$$\frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3}(1-\sqrt{3}) \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{bx^3})}{\sqrt{2} \sqrt{bx^3-a}} \right)}{2\sqrt{2} 3^{3/4} a^{5/6} b^{2/3}} - \frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{(1+\sqrt{3}) \sqrt{bx^3-a}}{\sqrt{2} 3^{3/4} \sqrt{a}} \right)}{3\sqrt{2} 3^{3/4} a^{5/6} b^{2/3}} - \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{\sqrt[4]{3}(1+\sqrt{3}) \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{bx^3})}{\sqrt{2} \sqrt{bx^3-a}} \right)}{6\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}} - \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{\sqrt[4]{3} \sqrt[6]{a} ((1-\sqrt{3}) \sqrt[3]{a} + 2 \sqrt[3]{bx^3})}{\sqrt{2} \sqrt{bx^3-a}} \right)}{3\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]

[Out] ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])]/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])]/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))

Rule 488

Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> With[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1 + r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)), x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]

Rubi steps

$$\int \frac{x}{(2(5-3\sqrt{3})a-bx^3)\sqrt{-a+bx^3}} dx = \frac{(2+\sqrt{3})\tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2+\sqrt{3})\tan^{-1}\left(\frac{(1+\sqrt{3})\sqrt[6]{a}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Mathematica [C] time = 0.11, size = 84, normalized size = 0.26

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{(20a-12\sqrt{3}a)\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/((2*(5 - 3*Sqrt[3]))*a - b*x^3)*Sqrt[-a + b*x^3],x]

[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]/((20*a - 12*Sqrt[3]*a)*Sqrt[-a + b*x^3]))

IntegrateAlgebraic [F] time = 31.23, size = 0, normalized size = 0.00

$$\int \frac{x}{(2(5-3\sqrt{3})a-bx^3)\sqrt{-a+bx^3}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/((2*(5 - 3*Sqrt[3]))*a - b*x^3)*Sqrt[-a + b*x^3]),x]

[Out] Defer[IntegrateAlgebraic][x/((2*(5 - 3*Sqrt[3]))*a - b*x^3)*Sqrt[-a + b*x^3]), x]

fricas [B] time = 22.76, size = 5060, normalized size = 15.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] -1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan(1/3*(3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1978*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 - 1142*a^5*b^3)))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-

$$\begin{aligned}
& -((1351\sqrt{3} + 2340)/(a^5b^4))^{(1/6)} - (6(1/9)^{(1/3)}(7a^2b^2x^3 - \\
& 7a^3b - 4\sqrt{3}(a^2b^2x^3 - a^3b)) \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(1/3)} - \sqrt{3}(bx^4 - ax) + 3\sqrt{3}(bx^3 - a) \cdot (108(1/1944)^{(5/6)}(265a^4b^4x^3 + 1448a^5b^3 - \sqrt{3}(153a^4b^4x^3 + 836a^5b^3)) \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(5/6)} - \sqrt{1/6}(41\sqrt{3}a^3b^2x - 71a^3b^2x) \cdot \sqrt{-((1351\sqrt{3} + 2340)/(a^5b^4))} - (1/1944)^{(1/6)}(5\sqrt{3}ax^2 - 9a^2bx^2) \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(1/6)})) \cdot \sqrt{(b^4x^{12} - 100a^2b^3x^9 + 240a^2b^2x^6 - 832a^3bx^3 + 448a^4 - 6(1/9)^{(2/3)}(1545a^4b^6x^{10} - 12492a^5b^5x^7 - 10512a^6b^4x^4 - 2112a^7b^3x - 4\sqrt{3}(223a^4b^6x^{10} - 1803a^5b^5x^7 - 1518a^6b^4x^4 - 304a^7b^3x) \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(2/3)} + 6(1/9)^{(1/3)}(26a^2b^5x^{11} + 498a^3b^4x^8 + 384a^4b^3x^5 + 64a^5b^2x^2 - 3\sqrt{3}(5a^2b^5x^{11} + 96a^3b^4x^8 + 72a^4b^3x^5 + 16a^5b^2x^2)) \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(1/3)} + 32\sqrt{3}(a^2b^3x^9 + 6a^2b^2x^6 - 15a^3bx^3 + 8a^4) + 2\sqrt{3}(bx^3 - a) \cdot (1944(1/1944)^{(5/6)}(3691a^5b^6x^8 + 2896a^6b^5x^5 + 568a^7b^4x^2 - \sqrt{3}(2131a^5b^6x^8 + 1672a^6b^5x^5 + 328a^7b^4x^2)) \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(5/6)} - 2\sqrt{1/6}(123a^3b^5x^9 + 5112a^4b^4x^6 + 3960a^5b^3x^3 + 768a^6b^2 - \sqrt{3}(71a^3b^5x^9 + 2952a^4b^4x^6 + 2280a^5b^3x^3 + 448a^6b^2)) \cdot \sqrt{-((1351\sqrt{3} + 2340)/(a^5b^4))} - 3(1/1944)^{(1/6)}(5a^2b^4x^{10} - 12a^2b^3x^7 - 72a^3b^2x^4 + 160a^4bx - 3\sqrt{3}(a^2b^4x^{10} - 4a^2b^3x^7 + 8a^3b^2x^4 - 32a^4bx) \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(1/6)})))/(b^4x^{12} - 40a^2b^3x^9 + 384a^2b^2x^6 + 320a^3bx^3 + 64a^4)))/(bx^4 - ax) - 1/6\sqrt{3}(1/1944)^{(1/6)} \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(1/6)} \cdot \arctan(1/3(3\sqrt{3}(bx^3 - a) \cdot (108(1/1944)^{(5/6)}(265a^4b^4x^3 - 1978a^5b^3 - \sqrt{3}(153a^4b^4x^3 - 1142a^5b^3)) \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(5/6)} + \sqrt{1/6}(41\sqrt{3}a^3b^2x - 71a^3b^2x) \cdot \sqrt{-((1351\sqrt{3} + 2340)/(a^5b^4))} + (1/1944)^{(1/6)}(5\sqrt{3}ax^2 - 9a^2bx^2) \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(1/6)})) + (6(1/9)^{(1/3)}(7a^2b^2x^3 - 7a^3b - 4\sqrt{3}(a^2b^2x^3 - a^3b)) \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(1/3)} - \sqrt{3}(bx^4 - ax) - 3\sqrt{3}(bx^3 - a) \cdot (108(1/1944)^{(5/6)}(265a^4b^4x^3 + 1448a^5b^3 - \sqrt{3}(153a^4b^4x^3 + 836a^5b^3)) \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(5/6)} - \sqrt{1/6}(41\sqrt{3}a^3b^2x - 71a^3b^2x) \cdot \sqrt{-((1351\sqrt{3} + 2340)/(a^5b^4))} - (1/1944)^{(1/6)}(5\sqrt{3}ax^2 - 9a^2bx^2) \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(1/6)})) \cdot \sqrt{(b^4x^{12} - 100a^2b^3x^9 + 240a^2b^2x^6 - 832a^3bx^3 + 448a^4 - 6(1/9)^{(2/3)}(1545a^4b^6x^{10} - 12492a^5b^5x^7 - 10512a^6b^4x^4 - 2112a^7b^3x - 4\sqrt{3}(223a^4b^6x^{10} - 1803a^5b^5x^7 - 1518a^6b^4x^4 - 304a^7b^3x) \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(2/3)} + 6(1/9)^{(1/3)}(26a^2b^5x^{11} + 498a^3b^4x^8 + 384a^4b^3x^5 + 64a^5b^2x^2 - 3\sqrt{3}(5a^2b^5x^{11} + 96a^3b^4x^8 + 72a^4b^3x^5 + 16a^5b^2x^2)) \cdot (-((1351\sqrt{3} + 2340)/(a^5b^4))^{(1/3)} + 32\sqrt{3}(a^2b^3x^9 + 6a^2b^2x^6 - 15a^3bx^3 + 8a^4) - 2\sqrt{3}(bx^3 - a) \cdot (1944(1/1944)^{(5/6)}(3691a^5b^6x^8 + 2896a^6b^5x^5 + 568a^7b^4x^2 - \sqrt{3}(2131a^5b^6x^8 + 1672a^6b^5x
\end{aligned}$$

$$\begin{aligned}
&^5 + 328a^7b^4x^2)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{(5/6)} - 2\sqrt{1/6} * (123a^3b^5x^9 + 5112a^4b^4x^6 + 3960a^5b^3x^3 + 768a^6b^2 - \sqrt{3} * (71a^3b^5x^9 + 2952a^4b^4x^6 + 2280a^5b^3x^3 + 448a^6b^2)) * \sqrt{-(1351\sqrt{3} + 2340) / (a^5b^4)} - 3 * (1/1944)^{(1/6)} * (5a^4b^4x^{10} - 12a^2b^3x^7 - 72a^3b^2x^4 + 160a^4b^2x - 3\sqrt{3} * (a^4b^4x^{10} - 4a^2b^3x^7 + 8a^3b^2x^4 - 32a^4b^2x)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{(1/6)}) / (b^4x^{12} - 40a^3b^3x^9 + 384a^2b^2x^6 + 320a^3b^2x^3 + 64a^4) / (b^4x^4 - a^4) + 1/12 * (1/1944)^{(1/6)} * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{(1/6)} * \log((b^4x^{12} + 68a^3b^3x^9 + 168a^2b^2x^6 - 544a^3b^2x^3 + 64a^4 + 6 * (1/9)^{(2/3)} * (2799a^4b^6x^{10} + 11556a^5b^5x^7 + 7776a^6b^4x^4 + 1440a^7b^3x - 8\sqrt{3} * (202a^4b^6x^{10} + 834a^5b^5x^7 + 561a^6b^4x^4 + 104a^7b^3x)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{(2/3)} + 6 * (1/9)^{(1/3)} * (26a^2b^5x^{11} + 498a^3b^4x^8 + 384a^4b^3x^5 + 64a^5b^2x^2 - 3\sqrt{3} * (5a^2b^5x^{11} + 96a^3b^4x^8 + 72a^4b^3x^5 + 16a^5b^2x^2)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{(1/3)} - 64\sqrt{3} * (a^3b^3x^9 - 3a^2b^2x^6 + 3a^3b^2x^3 - a^4) + 2\sqrt{3} * (b^3x^3 - a) * (1944 * (1/1944))^{(5/6)} * (3691a^5b^6x^8 + 2896a^6b^5x^5 + 568a^7b^4x^2 - \sqrt{3} * (2131a^5b^6x^8 + 1672a^6b^5x^5 + 328a^7b^4x^2)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{(5/6)} + 4\sqrt{1/6} * (168a^3b^5x^9 + 1845a^4b^4x^6 + 1368a^5b^3x^3 + 264a^6b^2 - \sqrt{3} * (97a^3b^5x^9 + 1065a^4b^4x^6 + 792a^5b^3x^3 + 152a^6b^2)) * \sqrt{-(1351\sqrt{3} + 2340) / (a^5b^4)} + 3 * (1/1944)^{(1/6)} * (5a^4b^4x^{10} + 216a^2b^3x^7 + 120a^3b^2x^4 + 64a^4b^2x - 3\sqrt{3} * (a^4b^4x^{10} + 40a^2b^3x^7 + 40a^3b^2x^4)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{(1/6)}) / (b^4x^{12} - 40a^3b^3x^9 + 384a^2b^2x^6 + 320a^3b^2x^3 + 64a^4) - 1/12 * (1/1944)^{(1/6)} * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{(1/6)} * \log((b^4x^{12} + 68a^3b^3x^9 + 168a^2b^2x^6 - 544a^3b^2x^3 + 64a^4 + 6 * (1/9)^{(2/3)} * (2799a^4b^6x^{10} + 11556a^5b^5x^7 + 7776a^6b^4x^4 + 1440a^7b^3x - 8\sqrt{3} * (202a^4b^6x^{10} + 834a^5b^5x^7 + 561a^6b^4x^4 + 104a^7b^3x)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{(2/3)} + 6 * (1/9)^{(1/3)} * (26a^2b^5x^{11} + 498a^3b^4x^8 + 384a^4b^3x^5 + 64a^5b^2x^2 - 3\sqrt{3} * (5a^2b^5x^{11} + 96a^3b^4x^8 + 72a^4b^3x^5 + 16a^5b^2x^2)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{(1/3)} - 64\sqrt{3} * (a^3b^3x^9 - 3a^2b^2x^6 + 3a^3b^2x^3 - a^4) - 2\sqrt{3} * (b^3x^3 - a) * (1944 * (1/1944))^{(5/6)} * (3691a^5b^6x^8 + 2896a^6b^5x^5 + 568a^7b^4x^2 - \sqrt{3} * (2131a^5b^6x^8 + 1672a^6b^5x^5 + 328a^7b^4x^2)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{(5/6)} + 4\sqrt{1/6} * (168a^3b^5x^9 + 1845a^4b^4x^6 + 1368a^5b^3x^3 + 264a^6b^2 - \sqrt{3} * (97a^3b^5x^9 + 1065a^4b^4x^6 + 792a^5b^3x^3 + 152a^6b^2)) * \sqrt{-(1351\sqrt{3} + 2340) / (a^5b^4)} + 3 * (1/1944)^{(1/6)} * (5a^4b^4x^{10} + 216a^2b^3x^7 + 120a^3b^2x^4 + 64a^4b^2x - 3\sqrt{3} * (a^4b^4x^{10} + 40a^2b^3x^7 + 40a^3b^2x^4)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{(1/6)}) / (b^4x^{12} - 40a^3b^3x^9 + 384a^2b^2x^6 + 320a^3b^2x^3 + 64a^4) - 1/24 * (1/1944)^{(1/6)} * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{(1/6)} * \log((b^4x^{12} - 100a^3b^3x^9 + 240a^2b^2x^6 - 832a^3b^2x^3 + 448a^4 - 6 * (1/9)^{(2/3)} * (1545a^4b^6x^{10} - 12492a^5b^5x^7 - 10512a^6b^4x^4 - 2112a^7b^3x - 4\sqrt{3} * (223a^4b^6x^{10} - 1803a^
\end{aligned}$$

$$\begin{aligned}
& 5*b^5*x^7 - 1518*a^6*b^4*x^4 - 304*a^7*b^3*x)) * (-(1351*\sqrt{3} + 2340)/(a^5 \\
& *b^4))^{(2/3)} + 6*(1/9)^{(1/3)}*(26*a^2*b^5*x^{11} + 498*a^3*b^4*x^8 + 384*a^4*b \\
& ^3*x^5 + 64*a^5*b^2*x^2 - 3*\sqrt{3}*(5*a^2*b^5*x^{11} + 96*a^3*b^4*x^8 + 72*a \\
& ^4*b^3*x^5 + 16*a^5*b^2*x^2)) * (-(1351*\sqrt{3} + 2340)/(a^5*b^4))^{(1/3)} + 32 \\
& *\sqrt{3}*(a*b^3*x^9 + 6*a^2*b^2*x^6 - 15*a^3*b*x^3 + 8*a^4) + 2*\sqrt{b*x^3 \\
& - a}*(1944*(1/1944)^{(5/6)}*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^ \\
& 4*x^2 - \sqrt{3}*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2)) * (- \\
& (1351*\sqrt{3} + 2340)/(a^5*b^4))^{(5/6)} - 2*\sqrt{1/6}*(123*a^3*b^5*x^9 + 511 \\
& 2*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^2 - \sqrt{3}*(71*a^3*b^5*x^9 + \\
& 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6*b^2))*\sqrt{-(1351*\sqrt{3} + 2 \\
& 340)/(a^5*b^4))} - 3*(1/1944)^{(1/6)}*(5*a*b^4*x^{10} - 12*a^2*b^3*x^7 - 72*a^3* \\
& b^2*x^4 + 160*a^4*b*x - 3*\sqrt{3}*(a*b^4*x^{10} - 4*a^2*b^3*x^7 + 8*a^3*b^2*x \\
& ^4 - 32*a^4*b*x)) * (-(1351*\sqrt{3} + 2340)/(a^5*b^4))^{(1/6))} / (b^4*x^{12} - 40 \\
& *a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) + 1/24*(1/1944)^{(1/ \\
& 6)} * (-(1351*\sqrt{3} + 2340)/(a^5*b^4))^{(1/6)} * \log((b^4*x^{12} - 100*a*b^3*x^9 + \\
& 240*a^2*b^2*x^6 - 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^{(2/3)}*(1545*a^4*b^6*x^ \\
& 10 - 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 - 2112*a^7*b^3*x - 4*\sqrt{3}*(22 \\
& 3*a^4*b^6*x^{10} - 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 - 304*a^7*b^3*x)) * (-(1 \\
& 351*\sqrt{3} + 2340)/(a^5*b^4))^{(2/3)} + 6*(1/9)^{(1/3)}*(26*a^2*b^5*x^{11} + 498 \\
& *a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2 - 3*\sqrt{3}*(5*a^2*b^5*x^{11} \\
& + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2*x^2)) * (-(1351*\sqrt{3} + 234 \\
& 0)/(a^5*b^4))^{(1/3)} + 32*\sqrt{3}*(a*b^3*x^9 + 6*a^2*b^2*x^6 - 15*a^3*b*x^3 \\
& + 8*a^4) - 2*\sqrt{b*x^3 - a}*(1944*(1/1944)^{(5/6)}*(3691*a^5*b^6*x^8 + 2896* \\
& a^6*b^5*x^5 + 568*a^7*b^4*x^2 - \sqrt{3}*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x^ \\
& 5 + 328*a^7*b^4*x^2)) * (-(1351*\sqrt{3} + 2340)/(a^5*b^4))^{(5/6)} - 2*\sqrt{1/6} \\
&)*(123*a^3*b^5*x^9 + 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^2 - sq \\
& rt(3)*(71*a^3*b^5*x^9 + 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6*b^2)) \\
& *\sqrt{-(1351*\sqrt{3} + 2340)/(a^5*b^4))} - 3*(1/1944)^{(1/6)}*(5*a*b^4*x^{10} - \\
& 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 + 160*a^4*b*x - 3*\sqrt{3}*(a*b^4*x^{10} - 4*a \\
& ^2*b^3*x^7 + 8*a^3*b^2*x^4 - 32*a^4*b*x)) * (-(1351*\sqrt{3} + 2340)/(a^5*b^4) \\
&)^{(1/6))} / (b^4*x^{12} - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a \\
& ^4))
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value

maple [C] time = 0.36, size = 510, normalized size = 1.59



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2), x)`

[Out]
$$-1/27*I/b^3/a^2^{(1/2)}*\text{sum}(1/_\text{alpha}*(a*b^2)^{(1/3)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(a*b^2)^{(1/3)}+(a*b^2)^{(1/3)})/b)/(a*b^2)^{(1/3)}*b)^{(1/2)}*((x-(a*b^2)^{(1/3)}/b)/(-3*(a*b^2)^{(1/3)}-I*3^{(1/2)}*(a*b^2)^{(1/3)})*b)^{(1/2)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(a*b^2)^{(1/3)}+(a*b^2)^{(1/3)})/b)/(a*b^2)^{(1/3)}*b)^{(1/2)}/(b*x^3-a)^{(1/2)}*(4*3^{(1/2)}*_\text{alpha}^2*b^2+6*_\text{alpha}^2*b^2-3*I*(a*b^2)^{(1/3)}*3^{(1/2)}*_\text{alpha}*b-2*3^{(1/2)}*(a*b^2)^{(1/3)}*_\text{alpha}*b-6*I*(a*b^2)^{(1/3)}*_\text{alpha}*b-3*(a*b^2)^{(1/3)}*_\text{alpha}*b+3*I*(a*b^2)^{(2/3)}*3^{(1/2)}-2*3^{(1/2)}*(a*b^2)^{(2/3)}+6*I*(a*b^2)^{(2/3)}-3*(a*b^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(-I*(x+1/2*(a*b^2)^{(1/3)}/b+1/2*I*3^{(1/2)}*(a*b^2)^{(1/3)}/b)*3^{(1/2)}/(a*b^2)^{(1/3)}*b)^{(1/2)}, 1/6*(-2*I*3^{(1/2)}*(a*b^2)^{(1/3)}*_\text{alpha}^2*b-4*I*(a*b^2)^{(1/3)}*_\text{alpha}^2*b+I*3^{(1/2)}*a*b+2*3^{(1/2)}*a*b+2*I*a*b+3*a*b+I*3^{(1/2)}*(a*b^2)^{(2/3)}*_\text{alpha}-2*3^{(1/2)}*(a*b^2)^{(2/3)}*_\text{alpha}+2*I*(a*b^2)^{(2/3)}*_\text{alpha}-3*(a*b^2)^{(2/3)}*_\text{alpha})/a/b, (-I*3^{(1/2)}*(a*b^2)^{(1/3)}/(-3/2*(a*b^2)^{(1/3)}/b-1/2*I*3^{(1/2)}*(a*b^2)^{(1/3)}/b)/b)^{(1/2)}), _\text{alpha}=\text{RootOf}(_Z^3*b+6*3^{(1/2)}*a-10*a))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{(bx^3 + 2a(3\sqrt{3} - 5))\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2), x, algorithm="maxima")`

[Out] `-integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 - a)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{bx^3 - a} (bx^3 + 2a(3\sqrt{3} - 5))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-x/((b*x^3 - a)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)`

[Out] `int(-x/((b*x^3 - a)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-10a\sqrt{-a + bx^3} + 6\sqrt{3}a\sqrt{-a + bx^3} + bx^3\sqrt{-a + bx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-b*x**3+2*a*(5-3*3**(1/2)))/(b*x**3-a)**(1/2),x)

[Out] -Integral(x/(-10*a*sqrt(-a + b*x**3) + 6*sqrt(3)*a*sqrt(-a + b*x**3) + b*x**3*sqrt(-a + b*x**3)), x)

$$3.252 \quad \int \frac{x}{\sqrt{-a-bx^3} (2(5-3\sqrt{3})a+bx^3)} dx$$

Optimal. Leaf size=322

$$\frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3} (1-\sqrt{3}) \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt{2} \sqrt{-a-bx^3}} \right)}{2\sqrt{2} 3^{3/4} a^{5/6} b^{2/3}} - \frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{(1+\sqrt{3}) \sqrt{-a-bx^3}}{\sqrt{2} 3^{3/4} \sqrt{a}} \right)}{3\sqrt{2} 3^{3/4} a^{5/6} b^{2/3}} - \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{\sqrt[4]{3} \sqrt[6]{a} ((1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})}{\sqrt{2} \sqrt{-a-bx^3}} \right)}{3\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}}$$

Rubi [A] time = 0.05, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {488}

$$\frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3} (1-\sqrt{3}) \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt{2} \sqrt{-a-bx^3}} \right)}{2\sqrt{2} 3^{3/4} a^{5/6} b^{2/3}} - \frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{(1+\sqrt{3}) \sqrt{-a-bx^3}}{\sqrt{2} 3^{3/4} \sqrt{a}} \right)}{3\sqrt{2} 3^{3/4} a^{5/6} b^{2/3}} - \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{\sqrt[4]{3} \sqrt[6]{a} ((1-\sqrt{3}) \sqrt[3]{a} - 2\sqrt[3]{bx})}{\sqrt{2} \sqrt{-a-bx^3}} \right)}{3\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}} - \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{\sqrt[4]{3} (1+\sqrt{3}) \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt{2} \sqrt{-a-bx^3}} \right)}{6\sqrt{2} \sqrt[4]{3} a^{5/6} b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]

[Out] ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])]/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))

Rule 488

Int[(x_)/(Sqrt[(a_) + (b_)*(x_)^3]*((c_) + (d_)*(x_)^3)), x_Symbol] :> With[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1 + r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)), x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x]]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]

Rubi steps

$$\int \frac{x}{\sqrt{-a-bx^3} (2(5-3\sqrt{3})a+bx^3)} dx = \frac{(2+\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2+\sqrt{3}) \tan^{-1}\left(\frac{(1+\sqrt{3})\sqrt{-a}}{\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Mathematica [C] time = 0.13, size = 86, normalized size = 0.27

$$\frac{x^2 \sqrt{\frac{bx^3}{a}} + {}_1F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right)}{(20a-12\sqrt{3}a)\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]

[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a - b*x^3])

IntegrateAlgebraic [F] time = 31.15, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a-bx^3} (2(5-3\sqrt{3})a+bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]

[Out] Defer[IntegrateAlgebraic][x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)), x]

fricas [B] time = 21.05, size = 5060, normalized size = 15.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] -1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan(-1/3*(3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1978*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 + 1142*a^5*b^3)))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)

$$\begin{aligned}
& *(- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(1/6)} + (6(1/9)^{(1/3)} (7a^2 b^2 x^3 + 7a^3 b - 4\sqrt{3}(a^2 b^2 x^3 + a^3 b)) * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(1/3)} + \sqrt{3}(b x^4 + a x) + 3\sqrt{3}(-b x^3 - a) * (108(1/1944)^{(5/6)} * (265 a^4 b^4 x^3 - 1448 a^5 b^3 - \sqrt{3}(153 a^4 b^4 x^3 - 836 a^5 b^3))) \\
& * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(5/6)} - \sqrt{1/6} (41 \sqrt{3} a^3 b^2 x - 71 a^3 b^2 x) * \sqrt{-(1351 \sqrt{3} + 2340) / (a^5 b^4)} + (1/1944)^{(1/6)} * (5 \sqrt{3} a b x^2 - 9 a b x^2) * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(1/6)}) * \sqrt{(b^4 x^{12} + 100 a b^3 x^9 + 240 a^2 b^2 x^6 + 832 a^3 b x^3 + 448 a^4 - 6(1/9)^{(2/3)} * (1545 a^4 b^6 x^{10} + 12492 a^5 b^5 x^7 - 10512 a^6 b^4 x^4 + 2112 a^7 b^3 x - 4 \sqrt{3} * (223 a^4 b^6 x^{10} + 1803 a^5 b^5 x^7 - 1518 a^6 b^4 x^4 + 304 a^7 b^3 x)) * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(2/3)} - 6(1/9)^{(1/3)} * (26 a^2 b^5 x^{11} - 498 a^3 b^4 x^8 + 384 a^4 b^3 x^5 - 64 a^5 b^2 x^2 - 3 \sqrt{3} * (5 a^2 b^5 x^{11} - 96 a^3 b^4 x^8 + 72 a^4 b^3 x^5 - 16 a^5 b^2 x^2)) * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(1/3)} - 32 \sqrt{3} * (a b^3 x^9 - 6 a^2 b^2 x^6 - 15 a^3 b x^3 - 8 a^4) + 2 \sqrt{-(b x^3 - a)} * (1944(1/1944)^{(5/6)} * (3691 a^5 b^6 x^8 - 2896 a^6 b^5 x^5 + 568 a^7 b^4 x^2 - \sqrt{3} * (2131 a^5 b^6 x^8 - 1672 a^6 b^5 x^5 + 328 a^7 b^4 x^2)) * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(5/6)} + 2 \sqrt{1/6} * (123 a^3 b^5 x^9 - 5112 a^4 b^4 x^6 + 3960 a^5 b^3 x^3 - 768 a^6 b^2 - \sqrt{3} * (71 a^3 b^5 x^9 - 2952 a^4 b^4 x^6 + 2280 a^5 b^3 x^3 - 448 a^6 b^2)) * \sqrt{-(1351 \sqrt{3} + 2340) / (a^5 b^4)} - 3(1/1944)^{(1/6)} * (5 a b^4 x^{10} + 12 a^2 b^3 x^7 - 72 a^3 b^2 x^4 - 160 a^4 b x - 3 \sqrt{3} * (a b^4 x^{10} + 4 a^2 b^3 x^7 + 8 a^3 b^2 x^4 + 32 a^4 b x)) * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(1/6))} / (b^4 x^{12} + 40 a b^3 x^9 + 384 a^2 b^2 x^6 - 320 a^3 b x^3 + 64 a^4)) / (b x^4 + a x) - 1/6 \sqrt{3} * (1/1944)^{(1/6)} * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(1/6)} * \arctan(-1/3 * (3 \sqrt{3}(-b x^3 - a) * (108(1/1944)^{(5/6)} * (265 a^4 b^4 x^3 + 1978 a^5 b^3 - \sqrt{3}(153 a^4 b^4 x^3 + 1142 a^5 b^3)) * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(5/6)} + \sqrt{1/6} * (41 \sqrt{3} a^3 b^2 x - 71 a^3 b^2 x) * \sqrt{-(1351 \sqrt{3} + 2340) / (a^5 b^4)} - (1/1944)^{(1/6)} * (5 \sqrt{3} a b x^2 - 9 a b x^2) * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(1/6)})) - (6(1/9)^{(1/3)} (7a^2 b^2 x^3 + 7a^3 b - 4\sqrt{3}(a^2 b^2 x^3 + a^3 b)) * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(1/3)} + \sqrt{3}(b x^4 + a x) - 3\sqrt{3}(-b x^3 - a) * (108(1/1944)^{(5/6)} * (265 a^4 b^4 x^3 - 1448 a^5 b^3 - \sqrt{3}(153 a^4 b^4 x^3 - 836 a^5 b^3))) * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(5/6)} - \sqrt{1/6} (41 \sqrt{3} a^3 b^2 x - 71 a^3 b^2 x) * \sqrt{-(1351 \sqrt{3} + 2340) / (a^5 b^4)} + (1/1944)^{(1/6)} * (5 \sqrt{3} a b x^2 - 9 a b x^2) * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(1/6)})) * \sqrt{(b^4 x^{12} + 100 a b^3 x^9 + 240 a^2 b^2 x^6 + 832 a^3 b x^3 + 448 a^4 - 6(1/9)^{(2/3)} * (1545 a^4 b^6 x^{10} + 12492 a^5 b^5 x^7 - 10512 a^6 b^4 x^4 + 2112 a^7 b^3 x - 4 \sqrt{3} * (223 a^4 b^6 x^{10} + 1803 a^5 b^5 x^7 - 1518 a^6 b^4 x^4 + 304 a^7 b^3 x)) * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(2/3)} - 6(1/9)^{(1/3)} * (26 a^2 b^5 x^{11} - 498 a^3 b^4 x^8 + 384 a^4 b^3 x^5 - 64 a^5 b^2 x^2 - 3 \sqrt{3} * (5 a^2 b^5 x^{11} - 96 a^3 b^4 x^8 + 72 a^4 b^3 x^5 - 16 a^5 b^2 x^2)) * (- (1351 \sqrt{3} + 2340) / (a^5 b^4))^{(1/3)} - 32 \sqrt{3} * (a b^3 x^9 - 6 a^2 b^2 x^6 - 15 a^3 b x^3 - 8 a^4) - 2 \sqrt{-(b x^3 - a)} * (1944(1/1944)^{(5/6)} * (3691 a^5 b^6 x^8 - 2896 a^6 b^5 x^5 + 568 a^7 b^4 x^2 - \sqrt{3} * (2131 a^5 b^6 x^8 - 1672 a
\end{aligned}$$

$$\begin{aligned}
& ^6b^5x^5 + 328a^7b^4x^2)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{5/6} + 2 \\
& * \sqrt{1/6} * (123a^3b^5x^9 - 5112a^4b^4x^6 + 3960a^5b^3x^3 - 768a^6 \\
& * b^2 - \sqrt{3}) * (71a^3b^5x^9 - 2952a^4b^4x^6 + 2280a^5b^3x^3 - 448a^6 \\
& * b^2)) * \sqrt{-(1351\sqrt{3} + 2340) / (a^5b^4))} - 3 * (1/1944)^{1/6} * (5a * b^4 \\
& * x^{10} + 12a^2b^3x^7 - 72a^3b^2x^4 - 160a^4b * x - 3\sqrt{3}) * (a * b^4x^{10} \\
& + 4a^2b^3x^7 + 8a^3b^2x^4 + 32a^4b * x)) * (- (1351\sqrt{3} + 2340) / \\
& (a^5b^4))^{1/6} / (b^4x^{12} + 40a * b^3x^9 + 384a^2b^2x^6 - 320a^3b * x^3 + 64a^4)) / (b * x^4 + a * x)) - 1/24 * (1/1944)^{1/6} * (- (1351\sqrt{3} + 2340) \\
& / (a^5b^4))^{1/6} * \log((b^4x^{12} + 100a * b^3x^9 + 240a^2b^2x^6 + 832a^3 \\
& * b * x^3 + 448a^4 - 6 * (1/9)^{2/3} * (1545a^4b^6x^{10} + 12492a^5b^5x^7 - 1 \\
& 0512a^6b^4x^4 + 2112a^7b^3x - 4\sqrt{3}) * (223a^4b^6x^{10} + 1803a^5 \\
& b^5x^7 - 1518a^6b^4x^4 + 304a^7b^3x)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{2/3} - 6 * (1/9)^{1/3} * (26a^2b^5x^{11} - 498a^3b^4x^8 + 384a^4b^3 \\
& * x^5 - 64a^5b^2x^2 - 3\sqrt{3}) * (5a^2b^5x^{11} - 96a^3b^4x^8 + 72a^4 \\
& * b^3x^5 - 16a^5b^2x^2)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{1/3} - 32 * s \\
& \text{qrt}(3) * (a * b^3x^9 - 6a^2b^2x^6 - 15a^3b * x^3 - 8a^4) + 2 * \sqrt{-b * x^3 - \\
& a} * (1944 * (1/1944)^{5/6} * (3691a^5b^6x^8 - 2896a^6b^5x^5 + 568a^7b^4 \\
& * x^2 - \sqrt{3}) * (2131a^5b^6x^8 - 1672a^6b^5x^5 + 328a^7b^4x^2)) * (- (\\
& 1351\sqrt{3} + 2340) / (a^5b^4))^{5/6} + 2 * \sqrt{1/6} * (123a^3b^5x^9 - 5112 \\
& * a^4b^4x^6 + 3960a^5b^3x^3 - 768a^6b^2 - \sqrt{3}) * (71a^3b^5x^9 - 2 \\
& 952a^4b^4x^6 + 2280a^5b^3x^3 - 448a^6b^2)) * \sqrt{-(1351\sqrt{3} + 23 \\
& 40) / (a^5b^4))} - 3 * (1/1944)^{1/6} * (5a * b^4x^{10} + 12a^2b^3x^7 - 72a^3b \\
& ^2x^4 - 160a^4b * x - 3\sqrt{3}) * (a * b^4x^{10} + 4a^2b^3x^7 + 8a^3b^2x^ \\
& 4 + 32a^4b * x)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{1/6} / (b^4x^{12} + 40 * \\
& a * b^3x^9 + 384a^2b^2x^6 - 320a^3b * x^3 + 64a^4)) + 1/24 * (1/1944)^{1/6} \\
& * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{1/6} * \log((b^4x^{12} + 100a * b^3x^9 + \\
& 240a^2b^2x^6 + 832a^3b * x^3 + 448a^4 - 6 * (1/9)^{2/3} * (1545a^4b^6x^{1 \\
& 0 + 12492a^5b^5x^7 - 10512a^6b^4x^4 + 2112a^7b^3x - 4\sqrt{3}) * (223 \\
& * a^4b^6x^{10} + 1803a^5b^5x^7 - 1518a^6b^4x^4 + 304a^7b^3x)) * (- (13 \\
& 51\sqrt{3} + 2340) / (a^5b^4))^{2/3} - 6 * (1/9)^{1/3} * (26a^2b^5x^{11} - 498 * \\
& a^3b^4x^8 + 384a^4b^3x^5 - 64a^5b^2x^2 - 3\sqrt{3}) * (5a^2b^5x^{11} \\
& - 96a^3b^4x^8 + 72a^4b^3x^5 - 16a^5b^2x^2)) * (- (1351\sqrt{3} + 2340 \\
&) / (a^5b^4))^{1/3} - 32 * \sqrt{3} * (a * b^3x^9 - 6a^2b^2x^6 - 15a^3b * x^3 - \\
& 8a^4) - 2 * \sqrt{-b * x^3 - a} * (1944 * (1/1944)^{5/6} * (3691a^5b^6x^8 - 2896 * \\
& a^6b^5x^5 + 568a^7b^4x^2 - \sqrt{3}) * (2131a^5b^6x^8 - 1672a^6b^5x^ \\
& 5 + 328a^7b^4x^2)) * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{5/6} + 2 * \sqrt{1/6} \\
& * (123a^3b^5x^9 - 5112a^4b^4x^6 + 3960a^5b^3x^3 - 768a^6b^2 - sq \\
& \text{rt}(3) * (71a^3b^5x^9 - 2952a^4b^4x^6 + 2280a^5b^3x^3 - 448a^6b^2)) \\
& * \sqrt{-(1351\sqrt{3} + 2340) / (a^5b^4))} - 3 * (1/1944)^{1/6} * (5a * b^4x^{10} + \\
& 12a^2b^3x^7 - 72a^3b^2x^4 - 160a^4b * x - 3\sqrt{3}) * (a * b^4x^{10} + 4a \\
& ^2b^3x^7 + 8a^3b^2x^4 + 32a^4b * x)) * (- (1351\sqrt{3} + 2340) / (a^5b^4) \\
&)^{1/6} / (b^4x^{12} + 40a * b^3x^9 + 384a^2b^2x^6 - 320a^3b * x^3 + 64a \\
& ^4)) + 1/12 * (1/1944)^{1/6} * (- (1351\sqrt{3} + 2340) / (a^5b^4))^{1/6} * \log(-(b \\
& ^4x^{12} - 68a * b^3x^9 + 168a^2b^2x^6 + 544a^3b * x^3 + 64a^4 + 6 * (1/9) \\
& ^{2/3} * (2799a^4b^6x^{10} - 11556a^5b^5x^7 + 7776a^6b^4x^4 - 1440a^7
\end{aligned}$$

$$\begin{aligned}
& *b^3*x - 8*\sqrt{3}*(202*a^4*b^6*x^{10} - 834*a^5*b^5*x^7 + 561*a^6*b^4*x^4 - \\
& 104*a^7*b^3*x)*(-(1351*\sqrt{3}) + 2340)/(a^5*b^4))^{(2/3)} - 6*(1/9)^{(1/3)}*(2 \\
& 6*a^2*b^5*x^{11} - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*\sqrt{3} \\
& t(3)*(5*a^2*b^5*x^{11} - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(- \\
& (1351*\sqrt{3}) + 2340)/(a^5*b^4))^{(1/3)} + 64*\sqrt{3}*(a*b^3*x^9 + 3*a^2*b^2 \\
& *x^6 + 3*a^3*b*x^3 + a^4) + 2*\sqrt{-b*x^3 - a}*(1944*(1/1944)^{(5/6)}*(3691*a \\
& ^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - \sqrt{3}*(2131*a^5*b^6*x^8 \\
& - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*\sqrt{3}) + 2340)/(a^5*b^4))^{(\\
& (5/6)} - 4*\sqrt{1/6}*(168*a^3*b^5*x^9 - 1845*a^4*b^4*x^6 + 1368*a^5*b^3*x^3 \\
& - 264*a^6*b^2 - \sqrt{3}*(97*a^3*b^5*x^9 - 1065*a^4*b^4*x^6 + 792*a^5*b^3*x^ \\
& 3 - 152*a^6*b^2))*\sqrt{-(1351*\sqrt{3}) + 2340)/(a^5*b^4)} + 3*(1/1944)^{(1/6)} \\
& *(5*a*b^4*x^{10} - 216*a^2*b^3*x^7 + 120*a^3*b^2*x^4 - 64*a^4*b*x - 3*\sqrt{3} \\
& *(a*b^4*x^{10} - 40*a^2*b^3*x^7 + 40*a^3*b^2*x^4))*(-(1351*\sqrt{3}) + 2340)/(a \\
& ^5*b^4))^{(1/6)))/(b^4*x^{12} + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 \\
& + 64*a^4) - 1/12*(1/1944)^{(1/6)}*(-(1351*\sqrt{3}) + 2340)/(a^5*b^4))^{(1/6)}* \\
& \log(-(b^4*x^{12} - 68*a*b^3*x^9 + 168*a^2*b^2*x^6 + 544*a^3*b*x^3 + 64*a^4 + \\
& 6*(1/9)^{(2/3)}*(2799*a^4*b^6*x^{10} - 11556*a^5*b^5*x^7 + 7776*a^6*b^4*x^4 - 1 \\
& 440*a^7*b^3*x - 8*\sqrt{3}*(202*a^4*b^6*x^{10} - 834*a^5*b^5*x^7 + 561*a^6*b^4 \\
& *x^4 - 104*a^7*b^3*x))*(-(1351*\sqrt{3}) + 2340)/(a^5*b^4))^{(2/3)} - 6*(1/9)^{(\\
& 1/3)}*(26*a^2*b^5*x^{11} - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 \\
& - 3*\sqrt{3}*(5*a^2*b^5*x^{11} - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x \\
& ^2))*(-(1351*\sqrt{3}) + 2340)/(a^5*b^4))^{(1/3)} + 64*\sqrt{3}*(a*b^3*x^9 + 3* \\
& a^2*b^2*x^6 + 3*a^3*b*x^3 + a^4) - 2*\sqrt{-b*x^3 - a}*(1944*(1/1944)^{(5/6)}* \\
& (3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - \sqrt{3}*(2131*a^5*b \\
& ^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*\sqrt{3}) + 2340)/(a^5 \\
& *b^4))^{(5/6)} - 4*\sqrt{1/6}*(168*a^3*b^5*x^9 - 1845*a^4*b^4*x^6 + 1368*a^5*b \\
& ^3*x^3 - 264*a^6*b^2 - \sqrt{3}*(97*a^3*b^5*x^9 - 1065*a^4*b^4*x^6 + 792*a^5 \\
& *b^3*x^3 - 152*a^6*b^2))*\sqrt{-(1351*\sqrt{3}) + 2340)/(a^5*b^4)} + 3*(1/1944 \\
&)^{(1/6)}*(5*a*b^4*x^{10} - 216*a^2*b^3*x^7 + 120*a^3*b^2*x^4 - 64*a^4*b*x - 3* \\
& \sqrt{3}*(a*b^4*x^{10} - 40*a^2*b^3*x^7 + 40*a^3*b^2*x^4))*(-(1351*\sqrt{3}) + 2 \\
& 340)/(a^5*b^4))^{(1/6)))/(b^4*x^{12} + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^ \\
& 3*b*x^3 + 64*a^4)
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value

maple [C] time = 0.39, size = 541, normalized size = 1.68



Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x/(b*x^3+2*a*(5-3*3^{(1/2)}))/(-b*x^3-a)^{(1/2)}, x)$

[Out] $\frac{1}{27} \frac{I}{b^3} \frac{a^{2(1/2)} \sum(1/_alpha * (-a*b^2)^{(1/3)} * (1/2 * I * (2*x + ((-a*b^2)^{(1/3)} - I * 3^{(1/2)} * (-a*b^2)^{(1/3)})) / b) / (-a*b^2)^{(1/3)} * b)^{(1/2)} * ((x - (-a*b^2)^{(1/3)}) / b) / (-3 * (-a*b^2)^{(1/3)} + I * 3^{(1/2)} * (-a*b^2)^{(1/3)}) * b)^{(1/2)} * (-1/2 * I * (2*x + ((-a*b^2)^{(1/3)} + I * 3^{(1/2)} * (-a*b^2)^{(1/3)})) / b) / (-a*b^2)^{(1/3)} * b)^{(1/2)} / (-b*x^3 - a)^{(1/2)} * (4*3^{(1/2)} * _alpha^2 * b^2 + 6 * _alpha^2 * b^2 + 3 * I * (-a*b^2)^{(1/3)} * 3^{(1/2)} * _alpha * b - 2*3^{(1/2)} * (-a*b^2)^{(1/3)} * _alpha * b + 6 * I * (-a*b^2)^{(1/3)} * _alpha * b - 3 * (-a*b^2)^{(1/3)} * _alpha * b - 3 * I * (-a*b^2)^{(2/3)} * 3^{(1/2)} - 2*3^{(1/2)} * (-a*b^2)^{(2/3)} - 6 * I * (-a*b^2)^{(2/3)} - 3 * (-a*b^2)^{(2/3)}) * \text{EllipticPi}(1/3 * 3^{(1/2)} * (I * (x + 1/2 * (-a*b^2)^{(1/3)}) / b - 1/2 * I * 3^{(1/2)} * (-a*b^2)^{(1/3)}) / b) * 3^{(1/2)} / (-a*b^2)^{(1/3)} * b)^{(1/2)}, -1/6 * (2 * I * 3^{(1/2)} * (-a*b^2)^{(1/3)} * _alpha^2 * b + 4 * I * (-a*b^2)^{(1/3)} * _alpha^2 * b + I * 3^{(1/2)} * a * b - 2 * 3^{(1/2)} * a * b + 2 * I * a * b - 3 * a * b - I * 3^{(1/2)} * (-a*b^2)^{(2/3)} * _alpha - 2 * 3^{(1/2)} * (-a*b^2)^{(2/3)} * _alpha - 3 * (-a*b^2)^{(2/3)} * _alpha) / a / b, (I * 3^{(1/2)} * (-a*b^2)^{(1/3)} / (-3/2 * (-a*b^2)^{(1/3)}) / b + 1/2 * I * 3^{(1/2)} * (-a*b^2)^{(1/3)} / b) / b)^{(1/2)}, _alpha = \text{RootOf}(_Z^3 * b - 6 * 3^{(1/2)} * a + 10 * a))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^3 - 2a(3\sqrt{3} - 5))\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x/(b*x^3+2*a*(5-3*3^{(1/2)}))/(-b*x^3-a)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(x/((b*x^3 - 2*a*(3*\text{sqrt}(3) - 5))*\text{sqrt}(-b*x^3 - a)), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{-bx^3 - a} (bx^3 - 2a(3\sqrt{3} - 5))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x/((-a - b*x^3)^{(1/2)}*(b*x^3 - 2*a*(3*3^{(1/2)} - 5))), x)$

[Out] $\text{int}(x/((-a - b*x^3)^{(1/2)}*(b*x^3 - 2*a*(3*3^{(1/2)} - 5))), x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a - bx^3} (-6\sqrt{3}a + 10a + bx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**3+2*a*(5-3*3**(1/2)))/(-b*x**3-a)**(1/2), x)

[Out] Integral(x/(sqrt(-a - b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)

$$3.253 \quad \int \frac{x^8 \sqrt{c+dx^3}}{a+bx^3} dx$$

Optimal. Leaf size=125

$$-\frac{2a^2 \sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}} + \frac{2a^2 \sqrt{c+dx^3}}{3b^3} - \frac{2(c+dx^3)^{3/2}(ad+bc)}{9b^2 d^2} + \frac{2(c+dx^3)^{5/2}}{15bd^2}$$

Rubi [A] time = 0.13, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 88, 50, 63, 208}

$$\frac{2a^2 \sqrt{c+dx^3}}{3b^3} - \frac{2a^2 \sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}} - \frac{2(c+dx^3)^{3/2}(ad+bc)}{9b^2 d^2} + \frac{2(c+dx^3)^{5/2}}{15bd^2}$$

Antiderivative was successfully verified.

[In] Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]

[Out] (2*a^2*Sqrt[c + d*x^3])/(3*b^3) - (2*(b*c + a*d)*(c + d*x^3)^(3/2))/(9*b^2*d^2) + (2*(c + d*x^3)^(5/2))/(15*b*d^2) - (2*a^2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

$x)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{IntegersQ}[m, n] \ \&\& \ (\text{IntegerQ}[p] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{GeQ}[n, -1]))$

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \ :> \ \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] \ :> \ \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned}
 \int \frac{x^8 \sqrt{c + dx^3}}{a + bx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 \sqrt{c + dx}}{a + bx} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{(-bc - ad)\sqrt{c + dx}}{b^2 d} + \frac{a^2 \sqrt{c + dx}}{b^2(a + bx)} + \frac{(c + dx)^{3/2}}{bd} \right) dx, x, x^3 \right) \\
 &= -\frac{2(bc + ad)(c + dx^3)^{3/2}}{9b^2 d^2} + \frac{2(c + dx^3)^{5/2}}{15bd^2} + \frac{a^2 \text{Subst} \left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^3 \right)}{3b^2} \\
 &= \frac{2a^2 \sqrt{c + dx^3}}{3b^3} - \frac{2(bc + ad)(c + dx^3)^{3/2}}{9b^2 d^2} + \frac{2(c + dx^3)^{5/2}}{15bd^2} + \frac{(a^2(bc - ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3b^3} \\
 &= \frac{2a^2 \sqrt{c + dx^3}}{3b^3} - \frac{2(bc + ad)(c + dx^3)^{3/2}}{9b^2 d^2} + \frac{2(c + dx^3)^{5/2}}{15bd^2} + \frac{(2a^2(bc - ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx}{d}} dx, x, x^3 \right)}{3b^3 d} \\
 &= \frac{2a^2 \sqrt{c + dx^3}}{3b^3} - \frac{2(bc + ad)(c + dx^3)^{3/2}}{9b^2 d^2} + \frac{2(c + dx^3)^{5/2}}{15bd^2} - \frac{2a^2 \sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{7/2}}
 \end{aligned}$$

Mathematica [A] time = 0.31, size = 121, normalized size = 0.97

$$\frac{2\sqrt{c + dx^3} (15a^2 d^2 - 5abd(c + dx^3) + b^2(-2c^2 + cdx^3 + 3d^2 x^6))}{45b^3 d^2} - \frac{2a^2 \sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*sqrt[c + d*x^3])/(a + b*x^3),x]

[Out] (2*sqrt[c + d*x^3]*(15*a^2*d^2 - 5*a*b*d*(c + d*x^3) + b^2*(-2*c^2 + c*d*x^3 + 3*d^2*x^6)))/(45*b^3*d^2) - (2*a^2*sqrt[b*c - a*d]*ArcTanh[(sqrt[b]*sqrt[c + d*x^3])/sqrt[b*c - a*d]])/(3*b^(7/2))

IntegrateAlgebraic [A] time = 0.16, size = 139, normalized size = 1.11

$$\frac{2a^2\sqrt{ad-bc}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{7/2}} + \frac{2\sqrt{c+dx^3}(15a^2d^2 - 5abcd - 5abd^2x^3 - 2b^2c^2 + b^2cdx^3 + 3b^2d^2x^6)}{45b^3d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8*sqrt[c + d*x^3])/(a + b*x^3),x]

[Out] (2*sqrt[c + d*x^3]*(-2*b^2*c^2 - 5*a*b*c*d + 15*a^2*d^2 + b^2*c*d*x^3 - 5*a*b*d^2*x^3 + 3*b^2*d^2*x^6))/(45*b^3*d^2) + (2*a^2*sqrt[-(b*c) + a*d]*ArcTan[(sqrt[b]*sqrt[-(b*c) + a*d]*sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(7/2))

fricas [A] time = 0.70, size = 280, normalized size = 2.24

$$\frac{15a^2d^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bt^3+2kc-ad-2\sqrt{dx^3+c}\sqrt{\frac{bc-ad}{b}}}{b^3+a}\right)+2(3b^2d^2x^6-2b^2c^2-5abcd+15a^2d^2+(b^2cd-5abd^2)x^3)\sqrt{dx^3+c}}{45b^3d^2} - \frac{2\left(15a^2d^2\sqrt{\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\sqrt{\frac{bc-ad}{b}}}{bc-ad}\right)-(3b^2d^2x^6-2b^2c^2-5abcd+15a^2d^2+(b^2cd-5abd^2)x^3)\sqrt{dx^3+c}\right)}{45b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")

[Out] [1/45*(15*a^2*d^2*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(3*b^2*d^2*x^6 - 2*b^2*c^2 - 5*a*b*c*d + 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c)/(b^3*d^2), -2/45*(15*a^2*d^2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (3*b^2*d^2*x^6 - 2*b^2*c^2 - 5*a*b*c*d + 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c)/(b^3*d^2)]

giac [A] time = 0.17, size = 139, normalized size = 1.11

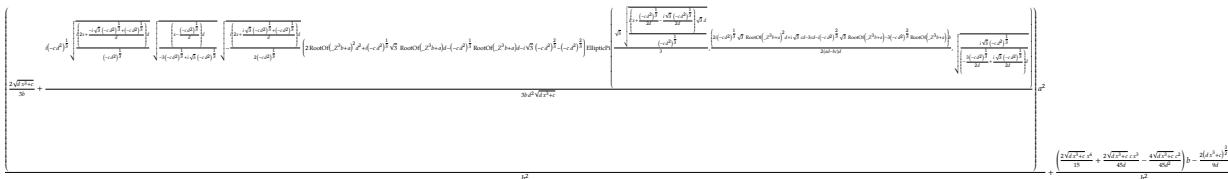
$$\frac{2(a^2bc - a^3d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^3} + \frac{2\left(3(dx^3+c)^{\frac{5}{2}}b^4d^8 - 5(dx^3+c)^{\frac{3}{2}}b^4cd^8 - 5(dx^3+c)^{\frac{3}{2}}ab^3d^9 + 15\sqrt{dx^3+c}a^2b^2d^{10}\right)}{45b^5d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{2}{3}(a^2bc - a^3d)\arctan(\sqrt{dx^3 + c})b/\sqrt{-b^2c + a^2bd})/(\sqrt{-b^2c + a^2bd}b^3) + \frac{2}{45}(3(dx^3 + c)^{5/2}b^4d^8 - 5(dx^3 + c)^{3/2}b^4cd^8 - 5(dx^3 + c)^{3/2}a^2b^3d^9 + 15\sqrt{dx^3 + c}a^2b^2d^{10})/(b^5d^{10})$

maple [C] time = 0.42, size = 514, normalized size = 4.11



Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (x^8(dx^3+c)^{1/2}/(bx^3+a), x)$

[Out] $\frac{1}{b^2}(b(2/15(dx^3+c)^{1/2}x^6+2/45(dx^3+c)^{1/2}c/dx^3-4/45(dx^3+c)^{1/2}c^2/d^2)-2/9a/d(dx^3+c)^{3/2})+a^2/b^2(2/3(dx^3+c)^{1/2}/b+1/3I/b/d^2)^{1/2}\sum((-cd^2)^{1/3}(1/2I(2x+(-I)^{3/2}(-cd^2)^{1/3})+(-cd^2)^{1/3})/d)/(-cd^2)^{1/3}d^{1/2}((x-(-cd^2)^{1/3})/d)/(-3(-cd^2)^{1/3}+I)^{3/2}(-cd^2)^{1/3}d^{1/2}(-1/2I(2x+I)^{3/2}(-cd^2)^{1/3}+(-cd^2)^{1/3})/d)/(-cd^2)^{1/3}d^{1/2}/(dx^3+c)^{1/2}(2\alpha^2d^2+I(-cd^2)^{1/3}3^{1/2}\alpha d-(-cd^2)^{1/3}\alpha d-I)^{3/2}(-cd^2)^{2/3}-(-cd^2)^{2/3})\text{EllipticPi}(1/33^{1/2}(I(x+1/2(-cd^2)^{1/3})/d-1/2I)^{3/2}(-cd^2)^{1/3}d^{1/2}), 1/2b/d(2I(-cd^2)^{1/3}3^{1/2}\alpha^2d+I)^{3/2}cd-3cd-I(-cd^2)^{2/3}3^{1/2}\alpha-3(-cd^2)^{2/3}\alpha)/(a-d-bc), (I)^{3/2}(-cd^2)^{1/3}/(-3/2(-cd^2)^{1/3}/d+1/2I)^{3/2}(-cd^2)^{1/3}/d)^{1/2}, \alpha=\text{RootOf}(_Z^3+b+a))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^8(dx^3+c)^{1/2}/(bx^3+a), x, \text{algorithm}=\text{"maxima"})$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more details)Is a*d-b*c positive or negative?

mupad [B] time = 6.17, size = 176, normalized size = 1.41

$$\frac{2a^2\sqrt{dx^3+c}}{3b^3} + \frac{2(dx^3+c)^{5/2}}{15bd^2} - \frac{2a(dx^3+c)^{3/2}}{9b^2d} - \frac{2c(dx^3+c)^{3/2}}{9bd^2} + \frac{a^2 \ln\left(\frac{a^2d^2(1+b^2c^2-2\sqrt{b}\sqrt{dx^3+c}(ad-bc)^{3/2}-abd^2x^3+1+b^2cdx^3-1-bcd3i)}{2bx^3+2a}\right)}{3b^{7/2}} \sqrt{ad-bc} \text{li}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^8*(c + d*x^3)^(1/2))/(a + b*x^3),x)`

[Out] $(2*a^2*(c + d*x^3)^{(1/2)})/(3*b^3) + (2*(c + d*x^3)^{(5/2)})/(15*b*d^2) - (2*a*(c + d*x^3)^{(3/2)})/(9*b^2*d) - (2*c*(c + d*x^3)^{(3/2)})/(9*b*d^2) + (a^2*\log((a^2*d^2*i + b^2*c^2*2i - 2*b^{(1/2)}*(c + d*x^3)^{(1/2)}*(a*d - b*c)^{(3/2)} - a*b*d^2*x^3*1i + b^2*c*d*x^3*1i - a*b*c*d*3i)/(2*a + 2*b*x^3))*(a*d - b*c)^{(1/2)*1i})/(3*b^{(7/2)})$

sympy [A] time = 30.27, size = 128, normalized size = 1.02

$$2 \left(\frac{a^2 d^3 \sqrt{c+dx^3}}{3b^3} - \frac{a^2 d^3 (ad-bc) \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4 \sqrt{\frac{ad-bc}{b}}} + \frac{d(c+dx^3)^{\frac{5}{2}}}{15b} + \frac{(c+dx^3)^{\frac{3}{2}}(-ad^2-bcd)}{9b^2} \right) \frac{1}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a),x)`

[Out] $2*(a**2*d**3*\sqrt{c + d*x**3})/(3*b**3) - a**2*d**3*(a*d - b*c)*\operatorname{atan}(\sqrt{c + d*x**3}/\sqrt{(a*d - b*c)/b})/(3*b**4*\sqrt{(a*d - b*c)/b}) + d*(c + d*x**3)**(5/2)/(15*b) + (c + d*x**3)**(3/2)*(-a*d**2 - b*c*d)/(9*b**2)/d**3$

$$3.254 \quad \int \frac{x^5 \sqrt{c+dx^3}}{a+bx^3} dx$$

Optimal. Leaf size=93

$$\frac{2a\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}} - \frac{2a\sqrt{c+dx^3}}{3b^2} + \frac{2(c+dx^3)^{3/2}}{9bd}$$

Rubi [A] time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 80, 50, 63, 208}

$$-\frac{2a\sqrt{c+dx^3}}{3b^2} + \frac{2a\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}} + \frac{2(c+dx^3)^{3/2}}{9bd}$$

Antiderivative was successfully verified.

[In] Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]

[Out] (-2*a*Sqrt[c + d*x^3])/(3*b^2) + (2*(c + d*x^3)^(3/2))/(9*b*d) + (2*a*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

$n + p + 2$), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5 \sqrt{c + dx^3}}{a + bx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x \sqrt{c + dx}}{a + bx} dx, x, x^3 \right) \\
 &= \frac{2(c + dx^3)^{3/2}}{9bd} - \frac{a \text{Subst} \left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^3 \right)}{3b} \\
 &= -\frac{2a\sqrt{c + dx^3}}{3b^2} + \frac{2(c + dx^3)^{3/2}}{9bd} - \frac{(a(bc - ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3b^2} \\
 &= -\frac{2a\sqrt{c + dx^3}}{3b^2} + \frac{2(c + dx^3)^{3/2}}{9bd} - \frac{(2a(bc - ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3b^2 d} \\
 &= -\frac{2a\sqrt{c + dx^3}}{3b^2} + \frac{2(c + dx^3)^{3/2}}{9bd} + \frac{2a\sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 88, normalized size = 0.95

$$\frac{2a\sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{5/2}} + \frac{2\sqrt{c + dx^3} (b(c + dx^3) - 3ad)}{9b^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3), x]

[Out] $(2\sqrt{c + dx^3}(-3ad + b(c + dx^3)))/(9b^2d) + (2a\sqrt{bc - ad})\text{ArcTanh}[\sqrt{b}\sqrt{c + dx^3}]/\sqrt{bc - ad}]/(3b^{5/2})$

IntegrateAlgebraic [A] time = 0.12, size = 100, normalized size = 1.08

$$-\frac{2a\sqrt{ad - bc} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{5/2}} - \frac{2\sqrt{c + dx^3} (3ad - bc - bdx^3)}{9b^2d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*sqrt[c + d*x^3])/(a + b*x^3), x]

[Out] $(-2\sqrt{c + dx^3}(-(b*c) + 3*a*d - b*d*x^3))/(9*b^2*d) - (2*a*\sqrt{-(b*c) + a*d})\text{ArcTan}[(\sqrt{b}*\sqrt{-(b*c) + a*d}*\sqrt{c + d*x^3})/(b*c - a*d)]/(3*b^{(5/2)})$

fricas [A] time = 0.55, size = 195, normalized size = 2.10

$$\left[\frac{3ad\sqrt{\frac{bc-ad}{b}} \log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right) + 2(bdx^3+bc-3ad)\sqrt{dx^3+c}}{9b^2d}, \frac{2\left(3ad\sqrt{-\frac{bc-ad}{b}} \arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right) + (bdx^3+bc-3ad)\sqrt{dx^3+c}\right)}{9b^2d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a), x, algorithm="fricas")

[Out] $[1/9*(3*a*d*\sqrt{(b*c - a*d)/b})*\log((b*d*x^3 + 2*b*c - a*d + 2*\sqrt{d*x^3 + c})*b*\sqrt{(b*c - a*d)/b})/(b*x^3 + a) + 2*(b*d*x^3 + b*c - 3*a*d)*\sqrt{d*x^3 + c})/(b^2*d), 2/9*(3*a*d*\sqrt{-(b*c - a*d)/b})*\arctan(-\sqrt{d*x^3 + c})*b*\sqrt{-(b*c - a*d)/b}/(b*c - a*d) + (b*d*x^3 + b*c - 3*a*d)*\sqrt{d*x^3 + c})/(b^2*d)]$

giac [A] time = 0.16, size = 96, normalized size = 1.03

$$-\frac{2(abc - a^2d) \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^2} + \frac{2\left((dx^3+c)^{\frac{3}{2}}b^2d^2 - 3\sqrt{dx^3+c}abd^3\right)}{9b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a), x, algorithm="giac")

[Out] $-2/3*(a*b*c - a^2*d)*\arctan(\sqrt{d*x^3 + c}*b/\sqrt{-b^2*c + a*b*d})/(\sqrt{-b^2*c + a*b*d}*b^2) + 2/9*((d*x^3 + c)^{(3/2)}*b^2*d^2 - 3*\sqrt{d*x^3 + c}*a*b*d^3)/(b^3*d^3)$

maple [C] time = 0.24, size = 458, normalized size = 4.92

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a), x)`

[Out]
$$\frac{2}{9} \frac{(d x^3 + c)^{3/2}}{b d - a} - \frac{a}{b} \frac{(2/3 (d x^3 + c)^{1/2} / b + 1/3 I / b / d^2 2^{1/2} \sum((-c d^2)^{1/3} (1/2 I (2 x + (-I 3^{1/2} (-c d^2)^{1/3} + (-c d^2)^{1/3})) / d) / (-c d^2)^{1/3} d)^{1/2} ((x - (-c d^2)^{1/3} / d) / (-3 (-c d^2)^{1/3} + I 3^{1/2} (-c d^2)^{1/3})) d)^{1/2} (-1/2 I (2 x + (I 3^{1/2} (-c d^2)^{1/3} + (-c d^2)^{1/3})) / d) / (-c d^2)^{1/3} d)^{1/2} / (d x^3 + c)^{1/2} (2 \alpha^2 d^2 + I (-c d^2)^{1/3} 3^{1/2} \alpha d - (-c d^2)^{1/3} \alpha d - I 3^{1/2} (-c d^2)^{2/3} - (-c d^2)^{2/3}) \text{EllipticPi}(1/3 3^{1/2} (I (x + 1/2 (-c d^2)^{1/3} / d - 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) 3^{1/2} / (-c d^2)^{1/3} d)^{1/2}, 1/2 (2 I (-c d^2)^{1/3} 3^{1/2} \alpha^2 d + I 3^{1/2} c d - 3 c d - I (-c d^2)^{2/3} 3^{1/2} \alpha - 3 (-c d^2)^{2/3} \alpha) / (a d - b c) b / d, (I 3^{1/2} (-c d^2)^{1/3} / (-3/2 (-c d^2)^{1/3} / d + 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) / d)^{1/2}}, \alpha = \text{RootOf}(_Z^3 b + a))$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a), x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more details) Is a*d-b*c positive or negative?

mupad [B] time = 6.06, size = 136, normalized size = 1.46

$$\frac{2(d x^3 + c)^{3/2}}{9 b d} - \frac{2 a \sqrt{d x^3 + c}}{3 b^2} + \frac{a \ln \left(\frac{a^2 d^2 l_1 + b^2 c^2 2 i + 2 \sqrt{b} \sqrt{d x^3 + c} (a d - b c)^{3/2} - a b d^2 x^3 l_1 + b^2 c d x^3 l_1 - a b c d 3 i}{2 b x^3 + 2 a} \right) \sqrt{a d - b c} l_1}{3 b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(c + d*x^3)^(1/2))/(a + b*x^3), x)`

```
[Out] (2*(c + d*x^3)^(3/2))/(9*b*d) - (2*a*(c + d*x^3)^(1/2))/(3*b^2) + (a*log((a^2*d^2*1i + b^2*c^2*2i + 2*b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2) - a*b*d^2*x^3*1i + b^2*c*d*x^3*1i - a*b*c*d*3i)/(2*a + 2*b*x^3))*(a*d - b*c)^(1/2)*1i)/(3*b^(5/2))
```

sympy [A] time = 14.67, size = 95, normalized size = 1.02

$$\frac{2 \left(-\frac{ad^2\sqrt{c+dx^3}}{3b^2} + \frac{ad^2(ad-bc) \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}} + \frac{d(c+dx^3)^{\frac{3}{2}}}{9b} \right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a), x)
```

```
[Out] 2*(-a*d**2*sqrt(c + d*x**3)/(3*b**2) + a*d**2*(a*d - b*c)*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**3*sqrt((a*d - b*c)/b)) + d*(c + d*x**3)**(3/2)/(9*b))/d**2
```

$$3.255 \quad \int \frac{x^2 \sqrt{c+dx^3}}{a+bx^3} dx$$

Optimal. Leaf size=70

$$\frac{2\sqrt{c+dx^3}}{3b} - \frac{2\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {444, 50, 63, 208}

$$\frac{2\sqrt{c+dx^3}}{3b} - \frac{2\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]

[Out] (2*Sqrt[c + d*x^3])/(3*b) - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2))

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2 \sqrt{c + dx^3}}{a + bx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c + dx}}{a + bx} dx, x, x^3 \right) \\
 &= \frac{2\sqrt{c + dx^3}}{3b} + \frac{(bc - ad) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3b} \\
 &= \frac{2\sqrt{c + dx^3}}{3b} + \frac{(2(bc - ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3bd} \\
 &= \frac{2\sqrt{c + dx^3}}{3b} - \frac{2\sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^3}}{\sqrt{bc - ad}} \right)}{3b^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 1.00

$$\frac{1}{3} \left(\frac{2\sqrt{c + dx^3}}{b} - \frac{2\sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^3}}{\sqrt{bc - ad}} \right)}{b^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3), x]

[Out] ((2*Sqrt[c + d*x^3])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2))/3

IntegrateAlgebraic [A] time = 0.07, size = 80, normalized size = 1.14

$$\frac{2\sqrt{ad - bc} \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^3} \sqrt{ad - bc}}{bc - ad} \right)}{3b^{3/2}} + \frac{2\sqrt{c + dx^3}}{3b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*sqrt[c + d*x^3])/(a + b*x^3), x]

[Out] (2*sqrt[c + d*x^3])/(3*b) + (2*sqrt[-(b*c) + a*d]*ArcTan[(sqrt[b]*sqrt[-(b*c) + a*d]*sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(3/2))

fricas [A] time = 0.64, size = 156, normalized size = 2.23

$$\left[\frac{\sqrt{\frac{bc-ad}{b}} \log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right) + 2\sqrt{dx^3+c}}{3b}, -\frac{2\left(\sqrt{-\frac{bc-ad}{b}} \arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right) - \sqrt{dx^3+c}\right)}{3b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a), x, algorithm="fricas")

[Out] [1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)/b, -2/3*(sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqrt(d*x^3 + c)/b]

giac [A] time = 0.16, size = 66, normalized size = 0.94

$$\frac{2(bc - ad) \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b} + \frac{2\sqrt{dx^3+c}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a), x, algorithm="giac")

[Out] 2/3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b) + 2/3*sqrt(d*x^3 + c)/b

maple [C] time = 0.23, size = 434, normalized size = 6.20

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a), x)

[Out] 2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d^(1/2)*((x-(-c


```
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c
*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)
*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3+b+a)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
details)Is a*d-b*c positive or negative?
```

mupad [B] time = 6.16, size = 82, normalized size = 1.17

$$\frac{2\sqrt{dx^3+c}}{3b} + \frac{\ln\left(\frac{ad-2bc-bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}2i}{bx^3+a}\right)\sqrt{ad-bc}}{3b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b) + (log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)
*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(3*b^(3
/2))
```

sympy [A] time = 6.35, size = 68, normalized size = 0.97

$$\frac{2\left(\frac{d\sqrt{c+dx^3}}{3b} - \frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x**3)/(3*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b)))/(3*b**2*sqrt((a*d - b*c)/b))/d
```

$$3.256 \quad \int \frac{\sqrt{c+dx^3}}{x(a+bx^3)} dx$$

Optimal. Leaf size=85

$$\frac{2\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}} - \frac{2\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}$$

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 83, 63, 208}

$$\frac{2\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}} - \frac{2\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)), x]

[Out] (-2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(3*a) + (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]/(3*a*Sqrt[b]))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 83

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{c+dx^3}}{x(a+bx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c+dx}}{x(a+bx)} dx, x, x^3 \right) \\
 &= \frac{c \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right) - (bc-ad) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3a} \\
 &= \frac{(2c) \text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c+dx^3} \right) - (2(bc-ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{3ad} \\
 &= -\frac{2\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a} + \frac{2\sqrt{bc-ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3a\sqrt{b}}
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 0.95

$$\frac{2 \left(\frac{\sqrt{bc-ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{\sqrt{b}} - \sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right) \right)}{3a}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]

[Out] (2*(-(Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]) + (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b]))/(3*a)

IntegrateAlgebraic [A] time = 0.09, size = 95, normalized size = 1.12

$$-\frac{2\sqrt{ad-bc} \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad} \right)}{3a\sqrt{b}} - \frac{2\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]

[Out] $(-2*\text{Sqrt}[-(b*c) + a*d]*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[-(b*c) + a*d]*\text{Sqrt}[c + d*x^3])/ (b*c - a*d)])/(3*a*\text{Sqrt}[b]) - (2*\text{Sqrt}[c]*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/\text{Sqrt}[c]])/(3*a)$

fricas [A] time = 0.54, size = 383, normalized size = 4.51

$$\left[\frac{\sqrt{\frac{bc-ad}{b}} \log\left(\frac{bdx^3+2bc-ad+2\sqrt{bd^3+c}b\sqrt{\frac{bc-ad}{b}}}{bd^3+a}\right) + \sqrt{c} \log\left(\frac{dx^3-2\sqrt{bd^3+c}\sqrt{c}+2c}{x^3}\right)}{3a}, \frac{2\sqrt{\frac{bc-ad}{b}} \arctan\left(\frac{\sqrt{bd^3+c}b\sqrt{\frac{bc-ad}{b}}}{bc-ad}\right) + \sqrt{c} \log\left(\frac{dx^3-2\sqrt{bd^3+c}\sqrt{c}+2c}{x^3}\right)}{3a}, \frac{2\sqrt{-c} \arctan\left(\frac{\sqrt{bd^3+c}\sqrt{c}}{c}\right) + \sqrt{\frac{bc-ad}{b}} \log\left(\frac{bdx^3+2bc-ad+2\sqrt{bd^3+c}b\sqrt{\frac{bc-ad}{b}}}{bd^3+a}\right)}{3a}, \frac{2\left(\sqrt{\frac{bc-ad}{b}} \arctan\left(\frac{\sqrt{bd^3+c}b\sqrt{\frac{bc-ad}{b}}}{bc-ad}\right) + \sqrt{-c} \arctan\left(\frac{\sqrt{bd^3+c}\sqrt{c}}{c}\right)\right)}{3a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="fricas")

[Out] $[1/3*(\text{sqrt}((b*c - a*d)/b)*\log((b*d*x^3 + 2*b*c - a*d + 2*\text{sqrt}(d*x^3 + c)*b*\text{sqrt}((b*c - a*d)/b))/(b*x^3 + a)) + \text{sqrt}(c)*\log((d*x^3 - 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 2*c)/x^3))/a, 1/3*(2*\text{sqrt}(-(b*c - a*d)/b)*\arctan(-\text{sqrt}(d*x^3 + c)*b*\text{sqrt}(-(b*c - a*d)/b)/(b*c - a*d)) + \text{sqrt}(c)*\log((d*x^3 - 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 2*c)/x^3))/a, 1/3*(2*\text{sqrt}(-c)*\arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + \text{sqrt}((b*c - a*d)/b)*\log((b*d*x^3 + 2*b*c - a*d + 2*\text{sqrt}(d*x^3 + c)*b*\text{sqrt}((b*c - a*d)/b))/(b*x^3 + a)))/a, 2/3*(\text{sqrt}(-(b*c - a*d)/b)*\arctan(-\text{sqrt}(d*x^3 + c)*b*\text{sqrt}(-(b*c - a*d)/b)/(b*c - a*d)) + \text{sqrt}(-c)*\arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c))/a]$

giac [A] time = 0.17, size = 79, normalized size = 0.93

$$-\frac{2(bc-ad)\arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a} + \frac{2c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="giac")

[Out] $-2/3*(b*c - a*d)*\arctan(\text{sqrt}(d*x^3 + c)*b/\text{sqrt}(-b^2*c + a*b*d))/(\text{sqrt}(-b^2*c + a*b*d)*a) + 2/3*c*\arctan(\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(a*\text{sqrt}(-c))$

maple [C] time = 0.26, size = 476, normalized size = 5.60

$$\left[\frac{2\sqrt{\frac{bc-ad}{b}} \arctan\left(\frac{\sqrt{bd^3+c}b\sqrt{\frac{bc-ad}{b}}}{bc-ad}\right) + \sqrt{c} \log\left(\frac{dx^3-2\sqrt{bd^3+c}\sqrt{c}+2c}{x^3}\right)}{3a}, \frac{2\sqrt{-c} \arctan\left(\frac{\sqrt{bd^3+c}\sqrt{c}}{c}\right) + \sqrt{\frac{bc-ad}{b}} \log\left(\frac{bdx^3+2bc-ad+2\sqrt{bd^3+c}b\sqrt{\frac{bc-ad}{b}}}{bd^3+a}\right)}{3a}, \frac{2\left(\sqrt{\frac{bc-ad}{b}} \arctan\left(\frac{\sqrt{bd^3+c}b\sqrt{\frac{bc-ad}{b}}}{bc-ad}\right) + \sqrt{-c} \arctan\left(\frac{\sqrt{bd^3+c}\sqrt{c}}{c}\right)\right)}{3a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(1/2)/x/(b*x^3+a),x)

[Out]
$$-1/a*b*(2/3*(d*x^3+c)^{(1/2)}/b+1/3*I/b/d^2*2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)},_alpha=\text{RootOf}(_Z^3*b+a)))+1/a*(2/3*(d*x^3+c)^{(1/2)}-2/3*\text{arctanh}((d*x^3+c)^{(1/2)}/c^{(1/2)})*c^{(1/2)})$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{(bx^3+a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x), x)

mupad [B] time = 7.94, size = 114, normalized size = 1.34

$$\frac{\sqrt{c} \ln\left(\frac{(\sqrt{dx^3+c}-\sqrt{c})^3(\sqrt{dx^3+c}+\sqrt{c})}{x^6}\right)}{3a} + \frac{\ln\left(\frac{2bc-ad+bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}2i}{bx^3+a}\right)\sqrt{ad-bc}1i}{3a\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^3)^(1/2)/(x*(a + b*x^3)),x)

[Out]
$$(c^{(1/2)}*\log((((c + d*x^3)^{(1/2)} - c^{(1/2)})^3*((c + d*x^3)^{(1/2)} + c^{(1/2)})/x^6))/(3*a) + (\log((2*b*c - a*d + b^{(1/2)}*(c + d*x^3)^{(1/2)}*(a*d - b*c)^{(1/2)}*2i + b*d*x^3)/(a + b*x^3))*(a*d - b*c)^{(1/2)}*1i)/(3*a*b^{(1/2)})$$

sympy [A] time = 12.25, size = 85, normalized size = 1.00

$$\frac{2 \left(\frac{cd \operatorname{atan} \left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}} \right)}{3a\sqrt{-c}} + \frac{d(ad-bc) \operatorname{atan} \left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}} \right)}{3ab\sqrt{\frac{ad-bc}{b}}} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c)**(1/2)/x/(b*x**3+a), x)

[Out] 2*(c*d*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c)) + d*(a*d - b*c)*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*b*sqrt((a*d - b*c)/b)))/d

$$3.257 \quad \int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)} dx$$

Optimal. Leaf size=115

$$\frac{(2bc - ad) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{2\sqrt{b}\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2} - \frac{\sqrt{c+dx^3}}{3ax^3}$$

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc - ad) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{2\sqrt{b}\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2} - \frac{\sqrt{c+dx^3}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]

[Out] -Sqrt[c + d*x^3]/(3*a*x^3) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(3*a^2*Sqrt[c]) - (2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 99

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 156


```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[(((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(-2bc+ad) - \frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3a} \\
&= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{(b(bc-ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3a^2} - \frac{(2bc-ad) \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{6a^2} \\
&= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{(2b(bc-ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{3a^2d} - \frac{(2bc-ad) \text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{dx^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{3a^2d} \\
&= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{(2bc-ad) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^2\sqrt{c}} - \frac{2\sqrt{b}\sqrt{bc-ad} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3a^2}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 107, normalized size = 0.93

$$\frac{\frac{(2bc-ad) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{\sqrt{c}} - 2\sqrt{b}\sqrt{bc-ad} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}} \right) - \frac{a\sqrt{c+dx^3}}{x^3}}{3a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]

[Out] (-((a*Sqrt[c + d*x^3])/x^3) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c] - 2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]/(3*a^2)

IntegrateAlgebraic [A] time = 0.26, size = 125, normalized size = 1.09

$$\frac{2\sqrt{b}\sqrt{ad-bc}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^2} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{\sqrt{c+dx^3}}{3ax^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]

[Out] -1/3*Sqrt[c + d*x^3]/(a*x^3) + (2*Sqrt[b]*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)]/(3*a^2) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2*Sqrt[c])

fricas [A] time = 0.52, size = 513, normalized size = 4.46

$$\frac{2\sqrt{b^2c-abd}\arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2} - \frac{(2bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a^2\sqrt{-c}} - \frac{\sqrt{dx^3+c}}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="fricas")

[Out] [1/6*(2*sqrt(b^2*c - a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - (2*b*c - a*d)*sqrt(c)*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), 1/6*(4*sqrt(-b^2*c + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), -1/3*((2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(b^2*c - a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), 1/3*(2*sqrt(-b^2*c + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3)]

giac [A] time = 0.19, size = 107, normalized size = 0.93

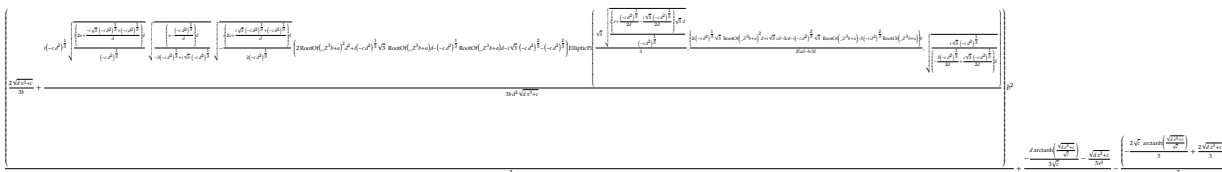
$$\frac{2(b^2c - abd)\arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2} - \frac{(2bc - ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a^2\sqrt{-c}} - \frac{\sqrt{dx^3+c}}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{2}{3}*(b^2*c - a*b*d)*\arctan(\sqrt{d*x^3 + c}*b/\sqrt{-b^2*c + a*b*d})/(\sqrt{-b^2*c + a*b*d}*a^2) - \frac{1}{3}*(2*b*c - a*d)*\arctan(\sqrt{d*x^3 + c}/\sqrt{-c})/(a^2*\sqrt{-c}) - \frac{1}{3}*\sqrt{d*x^3 + c}/(a*x^3)$

maple [C] time = 0.25, size = 518, normalized size = 4.50



Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x)

[Out] $\frac{1}{a}*(-\frac{1}{3}*(d*x^3+c)^{(1/2)}/x^3 - \frac{1}{3}*d*\operatorname{arctanh}((d*x^3+c)^{(1/2)}/c^{(1/2)})/c^{(1/2)}) + \frac{1}{a^2*b^2}*(\frac{2}{3}*(d*x^3+c)^{(1/2)}/b + \frac{1}{3}*I/b/d^2*2^{(1/2)}*\sum((-c*d^2)^{(1/3)}*(\frac{1}{2}*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-\frac{1}{2}*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d - (-c*d^2)^{(1/3)}*_alpha*d - I*3^{(1/2)}*(-c*d^2)^{(2/3)} - (-c*d^2)^{(2/3)})*\operatorname{EllipticPi}(\frac{1}{3}*3^{(1/2)}*(I*(x+\frac{1}{2}*(-c*d^2)^{(1/3)}/d - \frac{1}{2}*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, \frac{1}{2}*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d + I*3^{(1/2)}*c*d - 3*c*d - I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha - 3*(-c*d^2)^{(2/3)}*_alpha)/(a*d - b*c)*b/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d + \frac{1}{2}*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha = \operatorname{RootOf}(_Z^3*b+a)) - \frac{1}{a^2*b}*(\frac{2}{3}*(d*x^3+c)^{(1/2)} - \frac{2}{3}*\operatorname{arctanh}((d*x^3+c)^{(1/2)}/c^{(1/2)})*c^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{(bx^3 + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^4), x)

mupad [B] time = 5.13, size = 137, normalized size = 1.19

$$\frac{\ln\left(\frac{ad-2bc+2\sqrt{dx^3+c}\sqrt{b^2c-abd-bdx^3}}{bx^3+a}\right)\sqrt{b^2c-abd}}{3a^2} - \frac{\sqrt{dx^3+c}}{3ax^3} + \frac{\ln\left(\frac{(\sqrt{dx^3+c}-\sqrt{c})^3(\sqrt{dx^3+c}+\sqrt{c})}{x^6}\right)(ad-2bc)}{6a^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^3)^(1/2)/(x^4*(a + b*x^3)),x)

[Out] (log((a*d - 2*b*c + 2*(c + d*x^3)^(1/2)*(b^2*c - a*b*d)^(1/2) - b*d*x^3)/(a + b*x^3))*(b^2*c - a*b*d)^(1/2))/(3*a^2) - (c + d*x^3)^(1/2)/(3*a*x^3) + (log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))))/x^6)*(a*d - 2*b*c))/(6*a^2*c^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x^4(a + bx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a),x)

[Out] Integral(sqrt(c + d*x**3)/(x**4*(a + b*x**3)), x)

$$3.258 \quad \int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3} dx$$

Optimal. Leaf size=154

$$\frac{2a^2(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}} + \frac{2a^2\sqrt{c+dx^3}(bc-ad)}{3b^4} + \frac{2a^2(c+dx^3)^{3/2}}{9b^3} - \frac{2(c+dx^3)^{5/2}(ad+bc)}{15b^2d^2} + \frac{2(c+dx^3)^{7/2}}{21bd^2}$$

Rubi [A] time = 0.15, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 88, 50, 63, 208}

$$\frac{2a^2(c+dx^3)^{3/2}}{9b^3} + \frac{2a^2\sqrt{c+dx^3}(bc-ad)}{3b^4} - \frac{2a^2(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}} - \frac{2(c+dx^3)^{5/2}(ad+bc)}{15b^2d^2} + \frac{2(c+dx^3)^{7/2}}{21bd^2}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3), x]

[Out] (2*a^2*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^4) + (2*a^2*(c + d*x^3)^(3/2))/(9*b^3) - (2*(b*c + a*d)*(c + d*x^3)^(5/2))/(15*b^2*d^2) + (2*(c + d*x^3)^(7/2))/(21*b*d^2) - (2*a^2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(9/2))

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p], x]

$x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x\} \&\& \text{IntegersQ}[m, n] \&\& (\text{IntegerQ}[p] \parallel (\text{GtQ}[m, 0] \&\& \text{GeQ}[n, -1]))$

Rule 208

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b]$

Rule 446

$\text{Int}[(x_)^{(m_)} \cdot ((a_ + (b_ \cdot)(x_)^{(n_)})^{(p_)} \cdot ((c_ + (d_ \cdot)(x_)^{(n_)})^{(q_)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) \cdot (a + b \cdot x)^p \cdot (c + d \cdot x)^q}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x\} \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned}
 \int \frac{x^8 (c + dx^3)^{3/2}}{a + bx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (c + dx)^{3/2}}{a + bx} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{(-bc - ad)(c + dx)^{3/2}}{b^2 d} + \frac{a^2 (c + dx)^{3/2}}{b^2 (a + bx)} + \frac{(c + dx)^{5/2}}{bd} \right) dx, x, x^3 \right) \\
 &= -\frac{2(bc + ad)(c + dx^3)^{5/2}}{15b^2 d^2} + \frac{2(c + dx^3)^{7/2}}{21bd^2} + \frac{a^2 \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{a + bx} dx, x, x^3 \right)}{3b^2} \\
 &= \frac{2a^2 (c + dx^3)^{3/2}}{9b^3} - \frac{2(bc + ad)(c + dx^3)^{5/2}}{15b^2 d^2} + \frac{2(c + dx^3)^{7/2}}{21bd^2} + \frac{(a^2(bc - ad)) \text{Subst} \left(\int \frac{\sqrt{c + dx}}{a + bx} dx, x, x^3 \right)}{3b^3} \\
 &= \frac{2a^2(bc - ad)\sqrt{c + dx^3}}{3b^4} + \frac{2a^2 (c + dx^3)^{3/2}}{9b^3} - \frac{2(bc + ad)(c + dx^3)^{5/2}}{15b^2 d^2} + \frac{2(c + dx^3)^{7/2}}{21bd^2} + \frac{(a^2(bc - ad)) \text{Subst} \left(\int \frac{\sqrt{c + dx}}{a + bx} dx, x, x^3 \right)}{3b^3} \\
 &= \frac{2a^2(bc - ad)\sqrt{c + dx^3}}{3b^4} + \frac{2a^2 (c + dx^3)^{3/2}}{9b^3} - \frac{2(bc + ad)(c + dx^3)^{5/2}}{15b^2 d^2} + \frac{2(c + dx^3)^{7/2}}{21bd^2} + \frac{(a^2(bc - ad)) \text{Subst} \left(\int \frac{\sqrt{c + dx}}{a + bx} dx, x, x^3 \right)}{3b^3} \\
 &= \frac{2a^2(bc - ad)\sqrt{c + dx^3}}{3b^4} + \frac{2a^2 (c + dx^3)^{3/2}}{9b^3} - \frac{2(bc + ad)(c + dx^3)^{5/2}}{15b^2 d^2} + \frac{2(c + dx^3)^{7/2}}{21bd^2} - \frac{2a^2 (c + dx^3)^{3/2}}{9b^3}
 \end{aligned}$$

Mathematica [A] time = 0.24, size = 145, normalized size = 0.94

$$\frac{2 \left(105a^2(bc - ad) \left(\frac{\sqrt{c+dx^3}}{b} - \frac{\sqrt{bc-ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{b^{3/2}} \right) + 35a^2 (c + dx^3)^{3/2} - \frac{21b(c+dx^3)^{5/2} (ad+bc)}{d^2} + \frac{15b^2(c+dx^3)^{7/2}}{d^2} \right)}{315b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3), x]

[Out] (2*(35*a^2*(c + d*x^3)^(3/2) - (21*b*(b*c + a*d)*(c + d*x^3)^(5/2))/d^2 + (15*b^2*(c + d*x^3)^(7/2))/d^2 + 105*a^2*(b*c - a*d)*(Sqrt[c + d*x^3]/b - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2)))/(315*b^3)

IntegrateAlgebraic [A] time = 0.29, size = 195, normalized size = 1.27

$$\frac{2a^2(ad - bc)^{3/2} \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad} \right)}{3b^{9/2}} - \frac{2\sqrt{c+dx^3} (105a^3d^3 - 140a^2bcd^2 - 35a^2bd^3x^3 + 21ab^2c^2d + 42ab^2cd^2x^3 + 21ab^2d^3x^6 + 6b^3c^3 - 3b^3c^2dx^3 - 24b^3cd^2x^6 - 15b^3d^3x^9)}{315b^4d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3), x]

[Out] (-2*Sqrt[c + d*x^3]*(6*b^3*c^3 + 21*a*b^2*c^2*d - 140*a^2*b*c*d^2 + 105*a^3*d^3 - 3*b^3*c^2*d*x^3 + 42*a*b^2*c*d^2*x^3 - 35*a^2*b*d^3*x^3 - 24*b^3*c*d^2*x^6 + 21*a*b^2*d^3*x^6 - 15*b^3*d^3*x^9))/(315*b^4*d^2) - (2*a^2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(9/2))

fricas [A] time = 0.92, size = 410, normalized size = 2.66

$$\frac{105 \left(d^3 b c^3 - a^3 d^3 \right) \sqrt{\frac{bc-ad}{b}} \log \left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad} \right) - 2 \left(15 b^3 c^2 d^2 - 7 a b^2 c^2 d^3 \right) x^6 - 6 b^3 c^3 - 21 a b^2 c^2 d^2 + 140 a^2 b c d^2 - 105 a^3 d^3 + (3 b^3 c^2 d - 42 a b^2 c^2 d^2 + 35 a^2 b d^3) x^3 \sqrt{d x^3 + c}}{315 b^4 d^2} - \frac{2 \left(105 \left(d^3 b c^3 - a^3 d^3 \right) \sqrt{\frac{bc-ad}{b}} \operatorname{arctan} \left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad} \right) - (15 b^3 c^2 d^2 - 7 a b^2 c^2 d^3) x^6 - 6 b^3 c^3 - 21 a b^2 c^2 d^2 + 140 a^2 b c d^2 - 105 a^3 d^3 + (3 b^3 c^2 d - 42 a b^2 c^2 d^2 + 35 a^2 b d^3) x^3 \sqrt{d x^3 + c} \right)}{315 b^4 d^2}}{315 b^4 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a), x, algorithm="fricas")

[Out] [-1/315*(105*(a^2*b*c*d^2 - a^3*d^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(15*b^3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c^3 - 21*a*b^2*c^2*d + 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*b^2*c*d^2 + 35*a^2*b*d^3)*x^3)*sqrt(d*x^3 + c)/(b^4*d^2), -2/315*(105*(a^2*b*c*d^2 - a^3*d^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (15*b^3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c^3 - 21*a*b^2*c^2*d + 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*b^2*c*d^2 + 35*a^2*b*d^3)*x^3)*sqrt(d*x^3 + c)/(b^4*d^2)]

giac [A] time = 0.17, size = 193, normalized size = 1.25

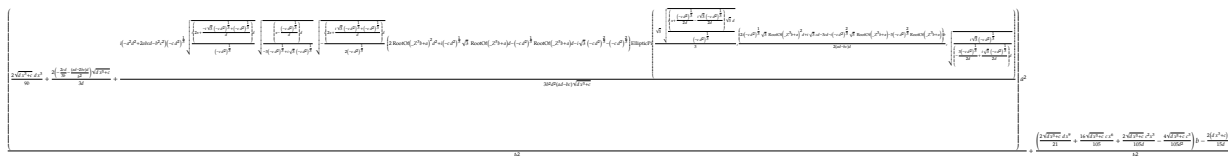
$$\frac{2(a^2b^2c^2 - 2a^3bcd + a^4d^2) \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^4} + \frac{2\left(15(dx^3+c)^{\frac{7}{2}}b^6d^{12} - 21(dx^3+c)^{\frac{5}{2}}b^6cd^{12} - 21(dx^3+c)^{\frac{5}{2}}ab^5d^{13} + 35(dx^3+c)^{\frac{3}{2}}a^2b^4d^{14} + 105\sqrt{dx^3+c}a^2b^4cd^{14} - 105\sqrt{dx^3+c}a^3b^3d^{15}\right)}{315b^7d^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{2}{3}*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*\arctan(\sqrt{d*x^3 + c}*b/\sqrt{-b^2*c + a*b*d})/(\sqrt{-b^2*c + a*b*d}*b^4) + \frac{2}{315}*(15*(d*x^3 + c)^{(7/2)}*b^6*d^{12} - 21*(d*x^3 + c)^{(5/2)}*b^6*c*d^{12} - 21*(d*x^3 + c)^{(5/2)}*a*b^5*d^{13} + 35*(d*x^3 + c)^{(3/2)}*a^2*b^4*d^{14} + 105*\sqrt{d*x^3 + c}*a^2*b^4*c*d^{14} - 105*\sqrt{d*x^3 + c}*a^3*b^3*d^{15})/(b^7*d^{14})$

maple [C] time = 0.36, size = 605, normalized size = 3.93



Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x)

[Out] $\frac{1}{b^2}*(b*(\frac{2}{21}*(d*x^3+c)^{(1/2)}*d*x^9+16/105*(d*x^3+c)^{(1/2)}*c*x^6+2/105*(d*x^3+c)^{(1/2)}*c^2/d*x^3-4/105*(d*x^3+c)^{(1/2)}*c^3/d^2)-2/15*a/d*(d*x^3+c)^{(5/2)})+a^2/b^2*(2/9*d/b*x^3*(d*x^3+c)^{(1/2)}+2/3*(-d*(a*d-2*b*c)/b^2-2/3*d/b*c)/d*(d*x^3+c)^{(1/2)}+1/3*I/b^2/d^2*2^{(1/2)}*\sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)}))/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)}))/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)},_alpha=RootOf(Z^3+b*a)))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 6.12, size = 330, normalized size = 2.14

$$\frac{2dx^9\sqrt{dx^3+c}}{21b} - \frac{\left(2i\left(\frac{2}{7b} + \frac{4\left(\frac{a^2-2cd}{b}\right)}{3d}\right) + 2i\left(\frac{2i}{7b} + \frac{2i\left(\frac{a^2-2cd}{b}\right)}{3d} + \frac{4\left(\frac{2a^2-16cd}{5d}\right)}{3d}\right)\right)\sqrt{dx^3+c}}{3d} + \frac{x^3\sqrt{dx^3+c}\left(\frac{2c^2}{b} + \frac{2a\left(\frac{a^2-2cd}{b}\right)}{b} + \frac{4\left(\frac{2a^2-16cd}{5d}\right)}{5d}\right)}{9d} - \frac{x^6\sqrt{dx^3+c}\left(\frac{2a^2}{b^2} - \frac{16cd}{7b}\right)}{15d} + \frac{a^2\ln\left(\frac{d^2d^2+2d^2d-2ab^2d^2+2cd^3-3ab^2cd-\sqrt{d^3+c}(ad-bc)^{3/2}}{b^3+a}\right)(ad-bc)^{3/2}}{3b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x)

[Out] $(2*d*x^9*(c + d*x^3)^{(1/2)})/(21*b) - (((2*a*(c^2/b + (a*((a*d^2)/b^2 - (2*c*d)/b))/b) + (2*c*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b) + (4*c*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(5*d)))/(3*d) * (c + d*x^3)^{(1/2)})/(3*d) + (x^3*(c + d*x^3)^{(1/2)}*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b) + (4*c*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(5*d)))/(9*d) - (x^6*(c + d*x^3)^{(1/2)}*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(15*d) + (a^2*log((a^2*d^2 + 2*b^2*c^2 - b^(1/2)*(c + d*x^3)^{(1/2)}*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*b^(9/2))$

sympy [A] time = 129.92, size = 153, normalized size = 0.99

$$\frac{2a^2(c+dx^3)^{\frac{3}{2}}}{9b^3} + \frac{2a^2(ad-bc)^2 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^5\sqrt{\frac{ad-bc}{b}}} + \frac{2(c+dx^3)^{\frac{7}{2}}}{21bd^2} + \frac{(c+dx^3)^{\frac{5}{2}}(-2ad-2bc)}{15b^2d^2} + \frac{\sqrt{c+dx^3}(-2a^3d+2a^2bc)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a),x)

[Out] $2*a**2*(c + d*x**3)**(3/2)/(9*b**3) + 2*a**2*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**5*sqrt((a*d - b*c)/b)) + 2*(c + d*x**3)**(7/2)/(21*b*d**2) + (c + d*x**3)**(5/2)*(-2*a*d - 2*b*c)/(15*b**2*d**2) + sqrt(c + d*x**3)*(-2*a**3*d + 2*a**2*b*c)/(3*b**4)$

$$3.259 \quad \int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3} dx$$

Optimal. Leaf size=120

$$\frac{2a(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}} - \frac{2a\sqrt{c+dx^3}(bc-ad)}{3b^3} - \frac{2a(c+dx^3)^{3/2}}{9b^2} + \frac{2(c+dx^3)^{5/2}}{15bd}$$

Rubi [A] time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 80, 50, 63, 208}

$$-\frac{2a(c+dx^3)^{3/2}}{9b^2} - \frac{2a\sqrt{c+dx^3}(bc-ad)}{3b^3} + \frac{2a(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}} + \frac{2(c+dx^3)^{5/2}}{15bd}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]

[Out] (-2*a*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^3) - (2*a*(c + d*x^3)^(3/2))/(9*b^2) + (2*(c + d*x^3)^(5/2))/(15*b*d) + (2*a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5 (c + dx^3)^{3/2}}{a + bx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(c + dx)^{3/2}}{a + bx} dx, x, x^3 \right) \\
 &= \frac{2(c + dx^3)^{5/2}}{15bd} - \frac{a \text{Subst} \left(\int \frac{(c+dx)^{3/2}}{a+bx} dx, x, x^3 \right)}{3b} \\
 &= -\frac{2a(c + dx^3)^{3/2}}{9b^2} + \frac{2(c + dx^3)^{5/2}}{15bd} - \frac{(a(bc - ad)) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^3 \right)}{3b^2} \\
 &= -\frac{2a(bc - ad)\sqrt{c + dx^3}}{3b^3} - \frac{2a(c + dx^3)^{3/2}}{9b^2} + \frac{2(c + dx^3)^{5/2}}{15bd} - \frac{(a(bc - ad)^2) \text{Subst} \left(\int \frac{1}{(a+bx)^2} dx, x, x^3 \right)}{3b^3} \\
 &= -\frac{2a(bc - ad)\sqrt{c + dx^3}}{3b^3} - \frac{2a(c + dx^3)^{3/2}}{9b^2} + \frac{2(c + dx^3)^{5/2}}{15bd} - \frac{(2a(bc - ad)^2) \text{Subst} \left(\int \frac{1}{a-bx} dx, x, x^3 \right)}{3b^3d} \\
 &= -\frac{2a(bc - ad)\sqrt{c + dx^3}}{3b^3} - \frac{2a(c + dx^3)^{3/2}}{9b^2} + \frac{2(c + dx^3)^{5/2}}{15bd} + \frac{2a(bc - ad)^{3/2} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{7/2}}
 \end{aligned}$$

Mathematica [A] time = 0.12, size = 111, normalized size = 0.92

$$\frac{2\sqrt{c + dx^3} \left(15a^2d^2 - 5abd(4c + dx^3) + 3b^2(c + dx^3)^2 \right)}{45b^3d} + \frac{2a(bc - ad)^{3/2} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3), x]

[Out] (2*sqrt[c + d*x^3]*(15*a^2*d^2 + 3*b^2*(c + d*x^3)^2 - 5*a*b*d*(4*c + d*x^3)))/(45*b^3*d) + (2*a*(b*c - a*d)^(3/2)*ArcTanh[(sqrt[b]*sqrt[c + d*x^3])/sqrt[b*c - a*d]])/(3*b^(7/2))

IntegrateAlgebraic [A] time = 0.17, size = 138, normalized size = 1.15

$$\frac{2\sqrt{c + dx^3} (15a^2d^2 - 20abcd - 5abd^2x^3 + 3b^2c^2 + 6b^2cdx^3 + 3b^2d^2x^6)}{45b^3d} + \frac{2a(ad - bc)^{3/2} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3), x]

[Out] (2*sqrt[c + d*x^3]*(3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + 6*b^2*c*d*x^3 - 5*a*b*d^2*x^3 + 3*b^2*d^2*x^6))/(45*b^3*d) + (2*a*(-(b*c) + a*d)^(3/2)*ArcTan[(sqrt[b]*sqrt[-(b*c) + a*d]*sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(7/2))

fricas [A] time = 0.78, size = 297, normalized size = 2.48

$$\left[\frac{15(abcd - a^2d^2)\sqrt{\frac{bc-ad}{b}} \log\left(\frac{bd^3+2bc-ad-2\sqrt{dx^3+c}\sqrt{\frac{bc-ad}{b}}}{b^3+a}\right) - 2(3b^2d^2x^6 + 3b^2c^2 - 20abcd + 15a^2d^2 + (6b^2cd - 5abd^2)x^3)\sqrt{dx^3+c}}{45b^3d}, \frac{2\left(15(abcd - a^2d^2)\sqrt{\frac{bc-ad}{b}} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{\frac{bc-ad}{b}}}{bc-ad}\right) + (3b^2d^2x^6 + 3b^2c^2 - 20abcd + 15a^2d^2 + (6b^2cd - 5abd^2)x^3)\sqrt{dx^3+c}\right)}{45b^3d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a), x, algorithm="fricas")

[Out] [-1/45*(15*(a*b*c*d - a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b)))/(b*x^3 + a) - 2*(3*b^2*d^2*x^6 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c)/(b^3*d), 2/45*(15*(a*b*c*d - a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d) + (3*b^2*d^2*x^6 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^3*d)]

giac [A] time = 0.19, size = 151, normalized size = 1.26

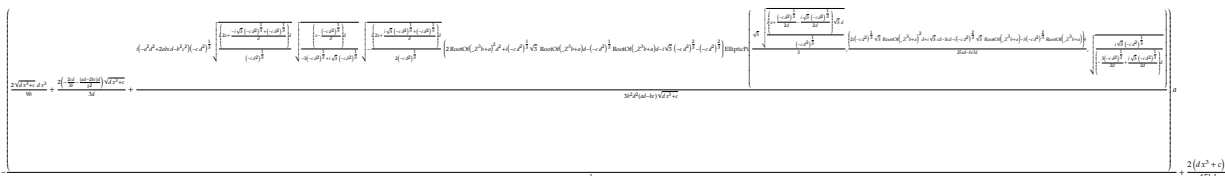
$$-\frac{2(ab^2c^2 - 2a^2bcd + a^3d^2) \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^3} + \frac{2\left(3(dx^3+c)^{\frac{5}{2}}b^4d^4 - 5(dx^3+c)^{\frac{3}{2}}ab^3d^5 - 15\sqrt{dx^3+c}ab^3cd^5 + 15\sqrt{dx^3+c}a^2b^2d^6\right)}{45b^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a), x, algorithm="giac")

```
[Out] -2/3*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d^4 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^5 - 15*sqrt(d*x^3 + c)*a*b^3*c*d^5 + 15*sqrt(d*x^3 + c)*a^2*b^2*d^6)/(b^5*d^5)
```

maple [C] time = 0.32, size = 531, normalized size = 4.42



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a), x)
```

```
[Out] 2/15*(d*x^3+c)^(5/2)/b/d-a/b*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2), 1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d, (I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)), _alpha=RootOf(_Z^3*b+a))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a), x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?
```

mupad [B] time = 6.13, size = 215, normalized size = 1.79

$$\frac{\sqrt{dx^3+c} \left(\frac{2c^2}{b} + \frac{2a \left(\frac{a^2}{b^2} - \frac{2cd}{b} \right)}{b} + \frac{2c \left(\frac{2ad^2}{b^2} - \frac{12cd}{5b} \right)}{3d} \right)}{3d} + \frac{2dx^6\sqrt{dx^3+c}}{15b} - \frac{x^3\sqrt{dx^3+c} \left(\frac{2ad^2}{b^2} - \frac{12cd}{5b} \right)}{9d} + \frac{a \ln \left(\frac{a^2d^2+2b^2c^2-abd^2x^3+b^2cdx^3-3abcd+\sqrt{b}\sqrt{dx^3+c}(ad-bc)^{3/2}2i}{bx^3+a} \right)}{3b^{7/2}} (ad-bc)^{3/2}i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x)`

[Out] $((c + d*x^3)^{(1/2)}*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (2*c*((2*a*d^2)/b^2 - (12*c*d)/(5*b)))/(3*d)))/(3*d) + (2*d*x^6*(c + d*x^3)^{(1/2)})/(15*b) - (x^3*(c + d*x^3)^{(1/2)}*((2*a*d^2)/b^2 - (12*c*d)/(5*b)))/(9*d) + (a*\log((a^2*d^2 + 2*b^2*c^2 + b^{(1/2)}*(c + d*x^3)^{(1/2)}*(a*d - b*c)^{(3/2)*2} - a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^{(3/2)*1})/(3*b^{(7/2)})$

sympy [A] time = 70.60, size = 116, normalized size = 0.97

$$-\frac{2a(c + dx^3)^{\frac{3}{2}}}{9b^2} - \frac{2a(ad - bc)^2 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4 \sqrt{\frac{ad-bc}{b}}} + \frac{2(c + dx^3)^{\frac{5}{2}}}{15bd} + \frac{\sqrt{c + dx^3} (2a^2d - 2abc)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a),x)`

[Out] $-2*a*(c + d*x**3)**(3/2)/(9*b**2) - 2*a*(a*d - b*c)**2*\operatorname{atan}(\operatorname{sqrt}(c + d*x**3)/\operatorname{sqrt}((a*d - b*c)/b))/(3*b**4*\operatorname{sqrt}((a*d - b*c)/b)) + 2*(c + d*x**3)**(5/2)/(15*b*d) + \operatorname{sqrt}(c + d*x**3)*(2*a**2*d - 2*a*b*c)/(3*b**3)$

$$3.260 \quad \int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3} dx$$

Optimal. Leaf size=96

$$-\frac{2(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}} + \frac{2\sqrt{c+dx^3}(bc-ad)}{3b^2} + \frac{2(c+dx^3)^{3/2}}{9b}$$

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {444, 50, 63, 208}

$$\frac{2\sqrt{c+dx^3}(bc-ad)}{3b^2} - \frac{2(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}} + \frac{2(c+dx^3)^{3/2}}{9b}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3), x]

[Out] (2*(b*c - a*d)*Sqrt[c + d*x^3])/((3*b^2) + (2*(c + d*x^3)^(3/2))/(9*b) - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (c + dx^3)^{3/2}}{a + bx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{a + bx} dx, x, x^3 \right) \\
&= \frac{2(c + dx^3)^{3/2}}{9b} + \frac{(bc - ad) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^3 \right)}{3b} \\
&= \frac{2(bc - ad)\sqrt{c + dx^3}}{3b^2} + \frac{2(c + dx^3)^{3/2}}{9b} + \frac{(bc - ad)^2 \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3b^2} \\
&= \frac{2(bc - ad)\sqrt{c + dx^3}}{3b^2} + \frac{2(c + dx^3)^{3/2}}{9b} + \frac{(2(bc - ad)^2) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3b^2 d} \\
&= \frac{2(bc - ad)\sqrt{c + dx^3}}{3b^2} + \frac{2(c + dx^3)^{3/2}}{9b} - \frac{2(bc - ad)^{3/2} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 85, normalized size = 0.89

$$\frac{2\sqrt{c + dx^3} (-3ad + 4bc + bdx^3)}{9b^2} - \frac{2(bc - ad)^{3/2} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3), x]

[Out] (2*Sqrt[c + d*x^3]*(4*b*c - 3*a*d + b*d*x^3))/(9*b^2) - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))

IntegrateAlgebraic [A] time = 0.17, size = 95, normalized size = 0.99

$$\frac{2\sqrt{c + dx^3} (-3ad + 4bc + bdx^3)}{9b^2} - \frac{2(ad - bc)^{3/2} \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad} \right)}{3b^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3), x]

[Out] (2*sqrt(c + d*x^3)*(4*b*c - 3*a*d + b*d*x^3))/(9*b^2) - (2*(-(b*c) + a*d)^(3/2)*ArcTan[(sqrt[b]*sqrt[-(b*c) + a*d]*sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(5/2))

fricas [A] time = 0.69, size = 204, normalized size = 2.12

$$\left[\frac{3(bc-ad)\sqrt{\frac{bc-ad}{b}} \log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right) - 2(bdx^3+4bc-3ad)\sqrt{dx^3+c}}{9b^2}, -\frac{2\left(3(bc-ad)\sqrt{\frac{bc-ad}{b}} \arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bc-ad}\right) - (bdx^3+4bc-3ad)\sqrt{dx^3+c}\right)}{9b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a), x, algorithm="fricas")

[Out] [-1/9*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(b*d*x^3 + 4*b*c - 3*a*d)*sqrt(d*x^3 + c)/b^2, -2/9*(3*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (b*d*x^3 + 4*b*c - 3*a*d)*sqrt(d*x^3 + c)/b^2]

giac [A] time = 0.17, size = 113, normalized size = 1.18

$$\frac{2(b^2c^2 - 2abcd + a^2d^2) \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^2} + \frac{2\left((dx^3+c)^{\frac{3}{2}}b^2 + 3\sqrt{dx^3+c}b^2c - 3\sqrt{dx^3+c}abd\right)}{9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a), x, algorithm="giac")

[Out] 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2 + 3*sqrt(d*x^3 + c)*b^2*c - 3*sqrt(d*x^3 + c)*a*b*d)/b^3

maple [C] time = 0.25, size = 507, normalized size = 5.28

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a), x)

```
[Out] 2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.91, size = 143, normalized size = 1.49

$$\frac{2dx^3\sqrt{dx^3+c}}{9b} - \frac{\sqrt{dx^3+c}\left(\frac{2ad^2}{b^2} - \frac{8cd}{3b}\right)}{3d} + \frac{\ln\left(\frac{a^2d^2+2b^2c^2-ad^2x^3+b^2cdx^3-3abcd-\sqrt{b}\sqrt{dx^3+c}(ad-bc)^{3/2}2i}{bx^3+a}\right)}{3b^{5/2}}(ad-bc)^{3/2}1i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c+d*x^3)^(3/2))/(a+b*x^3),x)
```

```
[Out] (log((a^2*d^2+2*b^2*c^2-b^(1/2)*(c+d*x^3)^(1/2)*(a*d-b*c)^(3/2)*2i-a*b*d^2*x^3+b^2*c*d*x^3-3*a*b*c*d)/(a+b*x^3))*(a*d-b*c)^(3/2)*1i)/(3*b^(5/2))-((c+d*x^3)^(1/2)*((2*a*d^2)/b^2-(8*c*d)/(3*b)))/(3*d)+(2*d*x^3*(c+d*x^3)^(1/2))/(9*b)
```

sympy [A] time = 35.29, size = 90, normalized size = 0.94

$$\frac{2(c+dx^3)^{\frac{3}{2}}}{9b} + \frac{\sqrt{c+dx^3}(-2ad+2bc)}{3b^2} + \frac{2(ad-bc)^2 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] 2*(c + d*x**3)**(3/2)/(9*b) + sqrt(c + d*x**3)*(-2*a*d + 2*b*c)/(3*b**2) +  
2*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**3*sqrt((a  
*d - b*c)/b))
```

$$3.261 \quad \int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)} dx$$

Optimal. Leaf size=104

$$\frac{2(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}} - \frac{2c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a} + \frac{2d\sqrt{c+dx^3}}{3b}$$

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 84, 156, 63, 208}

$$\frac{2(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}} - \frac{2c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a} + \frac{2d\sqrt{c+dx^3}}{3b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]

[Out] (2*d*Sqrt[c + d*x^3])/(3*b) - (2*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a) + (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a*b^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 84

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), Int[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]

Rule 156

Int((((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c

+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx^3)^{3/2}}{x(a + bx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{x(a + bx)} dx, x, x^3 \right) \\
 &= \frac{2d\sqrt{c + dx^3}}{3b} + \frac{\text{Subst} \left(\int \frac{bc^2 + d(2bc - ad)x}{x(a + bx)\sqrt{c + dx}} dx, x, x^3 \right)}{3b} \\
 &= \frac{2d\sqrt{c + dx^3}}{3b} + \frac{c^2 \text{Subst} \left(\int \frac{1}{x\sqrt{c + dx}} dx, x, x^3 \right)}{3a} - \frac{(bc - ad)^2 \text{Subst} \left(\int \frac{1}{(a + bx)\sqrt{c + dx}} dx, x, x^3 \right)}{3ab} \\
 &= \frac{2d\sqrt{c + dx^3}}{3b} + \frac{(2c^2) \text{Subst} \left(\int \frac{1}{-\frac{c}{a} + \frac{x^2}{a}} dx, x, \sqrt{c + dx^3} \right)}{3ad} - \frac{(2(bc - ad)^2) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{a} + \frac{bx^2}{a}} dx, x, \sqrt{c + dx^3} \right)}{3abd} \\
 &= \frac{2d\sqrt{c + dx^3}}{3b} - \frac{2c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{3a} + \frac{2(bc - ad)^{3/2} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c + dx^3}}{\sqrt{bc - ad}} \right)}{3ab^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 105, normalized size = 1.01

$$\frac{2 \left(a\sqrt{b}d\sqrt{c + dx^3} + (bc - ad)^{3/2} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c + dx^3}}{\sqrt{bc - ad}} \right) - b^{3/2}c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right) \right)}{3ab^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)), x]

[Out] $(2*(a*\text{Sqrt}[b]*d*\text{Sqrt}[c + d*x^3] - b^{(3/2)}*c^{(3/2)}*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/\text{Sqrt}[c]]) + (b*c - a*d)^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[c + d*x^3])/\text{Sqrt}[b*c - a*d]])/(3*a*b^{(3/2)})$

IntegrateAlgebraic [A] time = 0.23, size = 114, normalized size = 1.10

$$\frac{2(ad - bc)^{3/2} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3ab^{3/2}} - \frac{2c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a} + \frac{2d\sqrt{c+dx^3}}{3b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]

[Out] $(2*d*\text{Sqrt}[c + d*x^3])/(3*b) + (2*(-(b*c) + a*d)^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[-(b*c) + a*d]*\text{Sqrt}[c + d*x^3])/(b*c - a*d)])/(3*a*b^{(3/2)}) - (2*c^{(3/2)}*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/\text{Sqrt}[c]])/(3*a)$

fricas [A] time = 1.00, size = 486, normalized size = 4.67

$$\frac{b^2 \log\left(\frac{a^2 b \sqrt{c^2 d^2 + c^2 d} + 2 \sqrt{d^3 + c} ad - (bc - ad) \sqrt{\frac{c^2 d}{a}} \log\left(\frac{a^2 b \sqrt{c^2 d^2 + c^2 d} + 2 \sqrt{d^3 + c} ad + 2 (bc - ad) \sqrt{\frac{c^2 d}{a}} \arctan\left(\frac{\sqrt{c^2 d^2 + c^2 d}}{a d}\right)}{a^2 b}\right)}{3 ab} + b^2 \log\left(\frac{a^2 b \sqrt{c^2 d^2 + c^2 d} + 2 \sqrt{d^3 + c} ad + 2 (bc - ad) \sqrt{\frac{c^2 d}{a}} \arctan\left(\frac{\sqrt{c^2 d^2 + c^2 d}}{a d}\right)}{a^2 b}\right)}{3 ab} + 2 \sqrt{c} \arctan\left(\frac{\sqrt{c^2 d^2 + c^2 d}}{a d}\right) + 2 \sqrt{d^3 + c} ad - (bc - ad) \sqrt{\frac{c^2 d}{a}} \log\left(\frac{a^2 b \sqrt{c^2 d^2 + c^2 d} + 2 \sqrt{d^3 + c} ad + 2 (bc - ad) \sqrt{\frac{c^2 d}{a}} \arctan\left(\frac{\sqrt{c^2 d^2 + c^2 d}}{a d}\right)}{a^2 b}\right)}{3 ab} + 2 \sqrt{c} \arctan\left(\frac{\sqrt{c^2 d^2 + c^2 d}}{a d}\right) + \sqrt{d^3 + c} ad + (bc - ad) \sqrt{\frac{c^2 d}{a}} \arctan\left(\frac{\sqrt{c^2 d^2 + c^2 d}}{a d}\right)}{3 ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="fricas")

[Out] $[1/3*(b*c^{(3/2)}*\log((d*x^3 - 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 2*c)/x^3) + 2*\text{sqrt}(d*x^3 + c)*a*d - (b*c - a*d)*\text{sqrt}((b*c - a*d)/b)*\log((b*d*x^3 + 2*b*c - a*d - 2*\text{sqrt}(d*x^3 + c)*b*\text{sqrt}((b*c - a*d)/b)))/(b*x^3 + a))/(a*b), 1/3*(b*c^{(3/2)}*\log((d*x^3 - 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 2*c)/x^3) + 2*\text{sqrt}(d*x^3 + c)*a*d + 2*(b*c - a*d)*\text{sqrt}(-(b*c - a*d)/b)*\arctan(-\text{sqrt}(d*x^3 + c)*b*\text{sqrt}(-(b*c - a*d)/b))/(b*c - a*d))/(a*b), 1/3*(2*b*\text{sqrt}(-c)*c*\arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + 2*\text{sqrt}(d*x^3 + c)*a*d - (b*c - a*d)*\text{sqrt}((b*c - a*d)/b)*\log((b*d*x^3 + 2*b*c - a*d - 2*\text{sqrt}(d*x^3 + c)*b*\text{sqrt}((b*c - a*d)/b)))/(b*x^3 + a))/(a*b), 2/3*(b*\text{sqrt}(-c)*c*\arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + \text{sqrt}(d*x^3 + c)*a*d + (b*c - a*d)*\text{sqrt}(-(b*c - a*d)/b)*\arctan(-\text{sqrt}(d*x^3 + c)*b*\text{sqrt}(-(b*c - a*d)/b))/(b*c - a*d))/(a*b)]$

giac [A] time = 0.18, size = 112, normalized size = 1.08

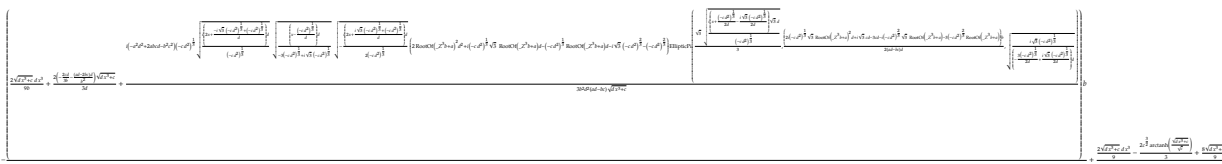
$$\frac{2c^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a\sqrt{-c}} + \frac{2\sqrt{dx^3+cd}}{3b} - \frac{2(b^2c^2 - 2abcd + a^2d^2) \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{2}{3}c^2 \arctan\left(\frac{\sqrt{d x^3 + c}}{\sqrt{-c}}\right) / (a \sqrt{-c}) + \frac{2}{3} \sqrt{d x^3 + c} * d / b - \frac{2}{3} (b^2 c^2 - 2 a b c d + a^2 d^2) \arctan\left(\frac{\sqrt{d x^3 + c} * b / \sqrt{-b^2 c + a b d}}{\sqrt{-b^2 c + a b d}}\right) / (\sqrt{-b^2 c + a b d} * a * b)$

maple [C] time = 0.26, size = 565, normalized size = 5.43



Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(3/2)/x/(b*x^3+a),x)

[Out] $-1/a*b*(2/9*(d*x^3+c)^{(1/2)}/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^{(1/2)}/d+1/3*I/b^2/d^2*2^{(1/2)}*\text{sum}((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)}*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)},_alpha=\text{RootOf}(_Z^3*b+a))+1/a*(2/9*d*x^3*(d*x^3+c)^{(1/2)}+8/9*c*(d*x^3+c)^{(1/2)}-2/3*c^{(3/2)}*\text{arctanh}((d*x^3+c)^{(1/2)}/c^{(1/2)}))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{\frac{3}{2}}}{(bx^3 + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="maxima")

[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x), x)

mupad [B] time = 7.88, size = 155, normalized size = 1.49

$$\frac{c^{3/2} \ln\left(\frac{(\sqrt{dx^3+c}-\sqrt{c})^3(\sqrt{dx^3+c}+\sqrt{c})}{x^6}\right)}{3a} + \frac{2d\sqrt{dx^3+c}}{3b} + \frac{\ln\left(\frac{a^2d^2+2b^2c^2-abd^2x^3+b^2cdx^3-3abcd+\sqrt{b}\sqrt{dx^3+c}(ad-bc)^{3/2}2i}{bx^3+a}\right)(ad-bc)^{3/2}1i}{3ab^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^3)^(3/2)/(x*(a + b*x^3)),x)

[Out] (c^(3/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6))/(3*a) + (2*d*(c + d*x^3)^(1/2))/(3*b) + (log((a^2*d^2 + 2*b^2*c^2 + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*a*b^(3/2))

sympy [A] time = 39.29, size = 102, normalized size = 0.98

$$\frac{2d\sqrt{c+dx^3}}{3b} + \frac{2c^2 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}} - \frac{2(ad-bc)^2 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab^2\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c)**(3/2)/x/(b*x**3+a),x)

[Out] 2*d*sqrt(c + d*x**3)/(3*b) + 2*c**2*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c)) - 2*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*b**2*sqrt((a*d - b*c)/b))

$$3.262 \quad \int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)} dx$$

Optimal. Leaf size=116

$$-\frac{2(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}} + \frac{\sqrt{c}(2bc-3ad) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} - \frac{c\sqrt{c+dx^3}}{3ax^3}$$

Rubi [A] time = 0.15, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 98, 156, 63, 208}

$$-\frac{2(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}} + \frac{\sqrt{c}(2bc-3ad) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} - \frac{c\sqrt{c+dx^3}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)), x]

[Out] -(c*Sqrt[c + d*x^3])/(3*a*x^3) + (Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2) - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*Sqrt[b])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 98

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^3)^{3/2}}{x^4(a + bx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{x^2(a + bx)} dx, x, x^3 \right) \\
&= -\frac{c\sqrt{c + dx^3}}{3ax^3} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}c(2bc - 3ad) + \frac{1}{2}d(bc - 2ad)x}{x(a + bx)\sqrt{c + dx}} dx, x, x^3 \right)}{3a} \\
&= -\frac{c\sqrt{c + dx^3}}{3ax^3} - \frac{(c(2bc - 3ad)) \text{Subst} \left(\int \frac{1}{x\sqrt{c + dx}} dx, x, x^3 \right)}{6a^2} + \frac{(bc - ad)^2 \text{Subst} \left(\int \frac{1}{(a + bx)\sqrt{c + dx}} dx, x, x^3 \right)}{3a^2} \\
&= -\frac{c\sqrt{c + dx^3}}{3ax^3} - \frac{(c(2bc - 3ad)) \text{Subst} \left(\int \frac{1}{\frac{-c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3a^2d} + \frac{(2(bc - ad)^2) \text{Subst} \left(\int \frac{1}{a + bx} dx, x, \sqrt{c + dx^3} \right)}{3a^2} \\
&= -\frac{c\sqrt{c + dx^3}}{3ax^3} + \frac{\sqrt{c}(2bc - 3ad) \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{3a^2} - \frac{2(bc - ad)^{3/2} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c + dx^3}}{\sqrt{bc - ad}} \right)}{3a^2\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 108, normalized size = 0.93

$$\frac{-\frac{2(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}} + \sqrt{c}(2bc-3ad) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) - \frac{ac\sqrt{c+dx^3}}{x^3}}{3a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)), x]

[Out] $-\left(\frac{a*c*\text{Sqrt}[c + d*x^3]}{x^3}\right) + \text{Sqrt}[c]*(2*b*c - 3*a*d)*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/\text{Sqrt}[c]] - \frac{(2*(b*c - a*d)^{3/2}*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[c + d*x^3])/\text{Sqrt}[b*c - a*d]])}{\text{Sqrt}[b]}/(3*a^2)$

IntegrateAlgebraic [A] time = 0.25, size = 130, normalized size = 1.12

$$\frac{(2bc^{3/2} - 3a\sqrt{c}d) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} - \frac{2(ad - bc)^{3/2} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^2\sqrt{b}} - \frac{c\sqrt{c+dx^3}}{3ax^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)), x]

[Out] $-1/3*(c*\text{Sqrt}[c + d*x^3])/(a*x^3) - (2*(-(b*c) + a*d)^{3/2}*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[-(b*c) + a*d]*\text{Sqrt}[c + d*x^3])/(b*c - a*d)])/(3*a^2*\text{Sqrt}[b]) + ((2*b*c)^{3/2} - 3*a*\text{Sqrt}[c]*d)*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/\text{Sqrt}[c]]/(3*a^2)$

fricas [A] time = 0.88, size = 538, normalized size = 4.64

$$\frac{2(b^2c - ad^2)\sqrt{c}\log\left(\frac{\sqrt{b^2c - ad^2}\sqrt{c+dx^3}}{2ax}\right) + (2b^2c - 3ad^2)\sqrt{c}\log\left(\frac{d\sqrt{c+dx^3}}{2ax}\right) + 2\sqrt{c}\sqrt{c+dx^3} + (2b^2c - ad^2)\sqrt{c}\arctan\left(\frac{\sqrt{b^2c - ad^2}}{2ax}\right) + (2b^2c - 3ad^2)\sqrt{c}\log\left(\frac{d\sqrt{c+dx^3}}{2ax}\right) + 2\sqrt{c}\sqrt{c+dx^3} - (2b^2c - 3ad^2)\sqrt{c}\arctan\left(\frac{\sqrt{b^2c - ad^2}}{2ax}\right) + (2b^2c - ad^2)\sqrt{c}\log\left(\frac{\sqrt{b^2c - ad^2}\sqrt{c+dx^3}}{2ax}\right) + \sqrt{c}\sqrt{c+dx^3}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a), x, algorithm="fricas")

[Out] $[-1/6*(2*(b*c - a*d)*x^3*\text{sqrt}((b*c - a*d)/b)*\log((b*d*x^3 + 2*b*c - a*d + 2*\text{sqrt}(d*x^3 + c)*b*\text{sqrt}((b*c - a*d)/b))/(b*x^3 + a)) + (2*b*c - 3*a*d)*\text{sqrt}(c)*x^3*\log((d*x^3 - 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 2*c)/x^3) + 2*\text{sqrt}(d*x^3 + c)*a*c)/(a^2*x^3), -1/6*(4*(b*c - a*d)*x^3*\text{sqrt}(-(b*c - a*d)/b)*\arctan(-\text{sqrt}(d*x^3 + c)*b*\text{sqrt}(-(b*c - a*d)/b))/(b*c - a*d) + (2*b*c - 3*a*d)*\text{sqrt}(c)*x^3*\log((d*x^3 - 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 2*c)/x^3) + 2*\text{sqrt}(d*x^3 + c)*a*c)/(a^2*x^3), -1/3*((2*b*c - 3*a*d)*\text{sqrt}(-c)*x^3*\arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + (b*c - a*d)*x^3*\text{sqrt}((b*c - a*d)/b)*\log((b*d*x^3 + 2*b*c - a*d + 2*\text{sqrt}(d*x^3 + c)*b*\text{sqrt}((b*c - a*d)/b))/(b*x^3 + a)) + \text{sqrt}(d*x^3 + c)*a*c)/(a^2*x^3), -1/3*(2*(b*c - a*d)*x^3*\text{sqrt}(-(b*c - a*d)/b)*\arctan(-\text{sqrt}(\text{sqrt}(d*x^3 + c)*b*\text{sqrt}(-(b*c - a*d)/b))/(b*c - a*d)) + (2*b*c - 3*a*d)*\text{sqrt}(c)*x^3*\log((d*x^3 - 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 2*c)/x^3) + 2*\text{sqrt}(d*x^3 + c)*a*c)/(a^2*x^3)]$

$d*x^3 + c)*b*\sqrt{-(b*c - a*d)/b}/(b*c - a*d)) + (2*b*c - 3*a*d)*\sqrt{-c}*x^3*\arctan(\sqrt{d*x^3 + c}*\sqrt{-c}/c) + \sqrt{d*x^3 + c}*a*c/(a^2*x^3)]$

giac [A] time = 0.17, size = 121, normalized size = 1.04

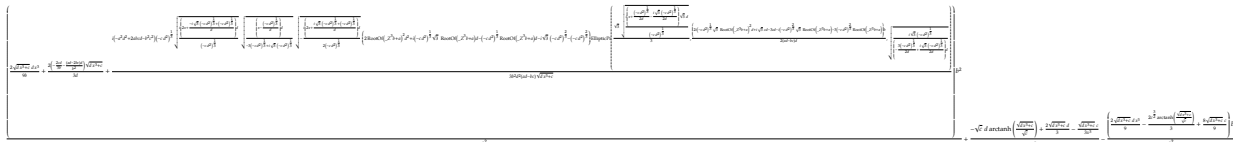
$$\frac{2(b^2c^2 - 2abcd + a^2d^2) \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right) - (2bc^2 - 3acd) \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right) - \frac{\sqrt{dx^3+c}c}{3ax^3}}{3\sqrt{-b^2c+abd}a^2} - \frac{(2bc^2 - 3acd) \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right) - \frac{\sqrt{dx^3+c}c}{3ax^3}}{3a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{2}{3}*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\arctan(\sqrt{d*x^3 + c}*b/\sqrt{-b^2*c + a*b*d})/(\sqrt{-b^2*c + a*b*d}*a^2) - \frac{1}{3}*(2*b*c^2 - 3*a*c*d)*\arctan(\sqrt{d*x^3 + c}/\sqrt{-c})/(a^2*\sqrt{-c}) - \frac{1}{3}*\sqrt{d*x^3 + c}*c/(a*x^3)$

maple [C] time = 0.28, size = 620, normalized size = 5.34



Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x)

[Out] $\frac{1}{a}*(-\frac{1}{3}*c*(d*x^3+c)^{(1/2)}/x^3+\frac{2}{3}*d*(d*x^3+c)^{(1/2)}-c^{(1/2)}*d*\operatorname{arctanh}((d*x^3+c)^{(1/2)}/c^{(1/2)}))+\frac{1}{a^2*b^2}*(\frac{2}{9}*(d*x^3+c)^{(1/2)}/b*d*x^3+\frac{2}{3}*(-\frac{2}{3}/b*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^{(1/2)}/d+\frac{1}{3}*I/b^2/d^2*2^{(1/2)}*\sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^{(1/3)}*(\frac{1}{2}*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-\frac{1}{2}*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\operatorname{EllipticPi}(\frac{1}{3}*3^{(1/2)}*(I*(x+\frac{1}{2}*(-c*d^2)^{(1/3)}/d-\frac{1}{2}*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},\frac{1}{2}*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-\frac{3}{2}*(-c*d^2)^{(1/3)}/d+\frac{1}{2}*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}),_alpha=\operatorname{RootOf}(Z^3*b+a))-1/a^2*b*(\frac{2}{9}*(d*x^3+c)^{(1/2)}*d*x^3+\frac{8}{9}*(d*x^3+c)^{(1/2)}*c-\frac{2}{3}*c^{(3/2)}*\operatorname{arctanh}((d*x^3+c)^{(1/2)}/c^{(1/2)}))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{\frac{3}{2}}}{(bx^3 + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="maxima")

[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^4), x)

mupad [B] time = 9.52, size = 167, normalized size = 1.44

$$\frac{\sqrt{c} \ln\left(\frac{(\sqrt{dx^3+c}-\sqrt{c})^3(\sqrt{dx^3+c}+\sqrt{c})}{x^6}\right) (3ad-2bc)}{6a^2} - \frac{c\sqrt{dx^3+c}}{3ax^3} + \frac{\ln\left(\frac{a^2d^2+2b^2c^2-abd^2x^3+b^2cdx^3-3abcd-\sqrt{b}\sqrt{dx^3+c}(ad-bc)^{3/2}2i}{bx^3+a}\right) (ad-bc)^{3/2} 1i}{3a^2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x)

[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)*(3*a*d - 2*b*c)/(6*a^2) - (c*(c + d*x^3)^(1/2))/(3*a*x^3) + (log((a^2*d^2 + 2*b^2*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*a^2*b^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^3)^{\frac{3}{2}}}{x^4(a + bx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a),x)

[Out] Integral((c + d*x**3)**(3/2)/(x**4*(a + b*x**3)), x)

$$3.263 \quad \int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}} dx$$

Optimal. Leaf size=104

$$-\frac{2a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}} - \frac{2\sqrt{c+dx^3}(ad+bc)}{3b^2d^2} + \frac{2(c+dx^3)^{3/2}}{9bd^2}$$

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 88, 63, 208}

$$-\frac{2a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}} - \frac{2\sqrt{c+dx^3}(ad+bc)}{3b^2d^2} + \frac{2(c+dx^3)^{3/2}}{9bd^2}$$

Antiderivative was successfully verified.

[In] Int[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]

[Out] (-2*(b*c + a*d)*Sqrt[c + d*x^3])/(3*b^2*d^2) + (2*(c + d*x^3)^(3/2))/(9*b*d^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^8}{(a + bx^3)\sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(a + bx)\sqrt{c + dx}} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{-bc - ad}{b^2 d \sqrt{c + dx}} + \frac{a^2}{b^2 (a + bx) \sqrt{c + dx}} + \frac{\sqrt{c + dx}}{bd} \right) dx, x, x^3 \right) \\
 &= -\frac{2(bc + ad)\sqrt{c + dx^3}}{3b^2 d^2} + \frac{2(c + dx^3)^{3/2}}{9bd^2} + \frac{a^2 \text{Subst} \left(\int \frac{1}{(a + bx)\sqrt{c + dx}} dx, x, x^3 \right)}{3b^2} \\
 &= -\frac{2(bc + ad)\sqrt{c + dx^3}}{3b^2 d^2} + \frac{2(c + dx^3)^{3/2}}{9bd^2} + \frac{(2a^2) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3b^2 d} \\
 &= -\frac{2(bc + ad)\sqrt{c + dx^3}}{3b^2 d^2} + \frac{2(c + dx^3)^{3/2}}{9bd^2} - \frac{2a^2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^3}}{\sqrt{bc - ad}} \right)}{3b^{5/2} \sqrt{bc - ad}}
 \end{aligned}$$

Mathematica [A] time = 0.22, size = 91, normalized size = 0.88

$$\frac{2\sqrt{c + dx^3} (-3ad - 2bc + bdx^3)}{9b^2 d^2} - \frac{2a^2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^3}}{\sqrt{bc - ad}} \right)}{3b^{5/2} \sqrt{bc - ad}}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((a + b*x^3)*Sqrt[c + d*x^3]), x]

[Out] (2*Sqrt[c + d*x^3]*(-2*b*c - 3*a*d + b*d*x^3))/(9*b^2*d^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])

IntegrateAlgebraic [A] time = 0.19, size = 102, normalized size = 0.98

$$\frac{2a^2 \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^3} \sqrt{ad - bc}}{bc - ad} \right)}{3b^{5/2} \sqrt{ad - bc}} - \frac{2\sqrt{c + dx^3} (3ad + 2bc - bdx^3)}{9b^2 d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]

[Out] $(-2\sqrt{c + dx^3}(2bc + 3ad - bdx^3))/(9b^2d^2) - (2a^2\text{ArcTan}[\sqrt{b}\sqrt{-(bc) + ad}\sqrt{c + dx^3}]/(bc - ad))/(3b^{5/2}\sqrt{-(bc) + ad})$

fricas [A] time = 0.87, size = 289, normalized size = 2.78

$$\frac{3\sqrt{b^2c - abd}a^2d^2 \log\left(\frac{bdx^3 + 2bc - ad - 2\sqrt{dx^3 + c}\sqrt{b^2c - abd}}{bx^3 + a}\right) - 2(2b^3c^2 + ab^2cd - 3a^2bd^2 - (b^3cd - ab^2d^2)x^3)\sqrt{dx^3 + c}}{9(b^4cd^2 - ab^3d^3)} + \frac{2\left(3\sqrt{-b^2c + abd}a^2d^2 \arctan\left(\frac{\sqrt{dx^3 + c}\sqrt{-b^2c + abd}}{bdx^3 + bc}\right) - (2b^3c^2 + ab^2cd - 3a^2bd^2 - (b^3cd - ab^2d^2)x^3)\sqrt{dx^3 + c}\right)}{9(b^4cd^2 - ab^3d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] $[1/9*(3*\sqrt{b^2*c - a*b*d})*a^2*d^2*\log((b*d*x^3 + 2*b*c - a*d - 2*\sqrt{d*x^3 + c})*\sqrt{b^2*c - a*b*d})/(b*x^3 + a) - 2*(2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^3)*\sqrt{d*x^3 + c})/(b^4*c*d^2 - a*b^3*d^3), 2/9*(3*\sqrt{-b^2*c + a*b*d})*a^2*d^2*\arctan(\sqrt{d*x^3 + c}*\sqrt{-b^2*c + a*b*d})/(b*d*x^3 + b*c) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^3)*\sqrt{d*x^3 + c})/(b^4*c*d^2 - a*b^3*d^3)]$

giac [A] time = 0.17, size = 106, normalized size = 1.02

$$\frac{2a^2 \arctan\left(\frac{\sqrt{dx^3 + cb}}{\sqrt{-b^2c + abd}}\right)}{3\sqrt{-b^2c + abd}b^2} + \frac{2\left(\left(dx^3 + c\right)^{\frac{3}{2}}b^2d^4 - 3\sqrt{dx^3 + c}b^2cd^4 - 3\sqrt{dx^3 + c}abd^5\right)}{9b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] $2/3*a^2*\arctan(\sqrt{d*x^3 + c}*b/\sqrt{-b^2*c + a*b*d})/(\sqrt{-b^2*c + a*b*d})*b^2 + 2/9*((d*x^3 + c)^{(3/2)}*b^2*d^4 - 3*\sqrt{d*x^3 + c}*b^2*c*d^4 - 3*\sqrt{d*x^3 + c}*a*b*d^5)/(b^3*d^6)$

maple [C] time = 0.34, size = 488, normalized size = 4.69

$$\frac{a^2(-c)^{\frac{3}{2}}\sqrt{\frac{b^2c + abd}{(-c)^2}}\sqrt{\frac{bdx^3 + 2bc - ad - 2\sqrt{dx^3 + c}\sqrt{b^2c - abd}}{bx^3 + a}}}{3\sqrt{b^2c - abd}b^2} + \frac{2\left(\left(dx^3 + c\right)^{\frac{3}{2}}b^2d^4 - 3\sqrt{dx^3 + c}b^2cd^4 - 3\sqrt{dx^3 + c}abd^5\right)}{9b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x)


```
[Out] 1/b^2*(b*(2/9*(d*x^3+c)^(1/2)/d*x^3-4/9*(d*x^3+c)^(1/2)*c/d^2)-2/3*a/d*(d*x^3+c)^(1/2))-1/3*I*a^2/b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2))*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.43, size = 121, normalized size = 1.16

$$\frac{2x^3\sqrt{dx^3+c}}{9bd} - \frac{\left(\frac{2a}{b^2} + \frac{4c}{3bd}\right)\sqrt{dx^3+c}}{3d} + \frac{a^2 \ln\left(\frac{2bc-ad+bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}2i}{bx^3+a}\right)1i}{3b^{5/2}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*b*d) - (((2*a)/b^2 + (4*c)/(3*b*d))*(c + d*x^3)^(1/2))/(3*d) + (a^2*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*b^(5/2)*(a*d - b*c)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a + bx^3)\sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

$$3.264 \quad \int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}} dx$$

Optimal. Leaf size=74

$$\frac{2a \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}} + \frac{2\sqrt{c+dx^3}}{3bd}$$

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 80, 63, 208}

$$\frac{2a \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}} + \frac{2\sqrt{c+dx^3}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]

[Out] (2*Sqrt[c + d*x^3])/(3*b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^(n)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(a + bx^3)\sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{(a + bx)\sqrt{c + dx}} dx, x, x^3 \right) \\ &= \frac{2\sqrt{c + dx^3}}{3bd} - \frac{a \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3b} \\ &= \frac{2\sqrt{c + dx^3}}{3bd} - \frac{(2a) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3bd} \\ &= \frac{2\sqrt{c + dx^3}}{3bd} + \frac{2a \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{3/2}\sqrt{bc-ad}} \end{aligned}$$

Mathematica [A] time = 0.12, size = 74, normalized size = 1.00

$$\frac{2 \left(\frac{a \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{\sqrt{bc-ad}} + \frac{\sqrt{b} \sqrt{c+dx^3}}{d} \right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^3)*Sqrt[c + d*x^3]), x]

[Out] (2*((Sqrt[b]*Sqrt[c + d*x^3])/d + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d]))/(3*b^(3/2))

IntegrateAlgebraic [A] time = 0.08, size = 84, normalized size = 1.14

$$\frac{2a \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad} \right)}{3b^{3/2}\sqrt{ad-bc}} + \frac{2\sqrt{c + dx^3}}{3bd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]

[Out] $(2\sqrt{c + dx^3})/(3bd) + (2a\text{ArcTan}[\sqrt{b}\sqrt{-(bc) + ad}]\sqrt{c + dx^3})/(b^2c - ad^2)/(3b^{3/2}\sqrt{-(bc) + ad})$

fricas [A] time = 0.88, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c - abd} \operatorname{ad} \log\left(\frac{bdx^3 + 2bc - ad + 2\sqrt{dx^3 + c}\sqrt{b^2c - abd}}{bx^3 + a}\right) + 2\sqrt{dx^3 + c}(b^2c - abd)}{3(b^3cd - ab^2d^2)}, -\frac{2\left(\sqrt{-b^2c + abd} \operatorname{ad} \arctan\left(\frac{\sqrt{dx^3 + c}\sqrt{-b^2c + abd}}{bdx^3 + bc}\right) - \sqrt{dx^3 + c}(b^2c - abd)\right)}{3(b^3cd - ab^2d^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] $[1/3(\sqrt{b^2c - a^2bd})\operatorname{ad} \log((bdx^3 + 2b^2c - ad + 2\sqrt{dx^3 + c})\sqrt{b^2c - a^2bd})/(b^2x^3 + a) + 2\sqrt{dx^3 + c}(b^2c - a^2bd)/(b^3cd - a^2b^2d^2), -2/3(\sqrt{-b^2c + a^2bd})\operatorname{ad} \arctan(\sqrt{dx^3 + c}\sqrt{-b^2c + a^2bd})/(bdx^3 + b^2c) - \sqrt{dx^3 + c}(b^2c - a^2bd)/(b^3cd - a^2b^2d^2)]$

giac [A] time = 0.17, size = 64, normalized size = 0.86

$$\frac{2\left(\frac{\operatorname{ad} \arctan\left(\frac{\sqrt{dx^3 + c}b}{\sqrt{-b^2c + abd}b}\right) - \frac{\sqrt{dx^3 + c}}{b}}{\sqrt{-b^2c + abd}b}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] $-2/3(a^2d \arctan(\sqrt{dx^3 + c}b/\sqrt{-b^2c + a^2bd})/(\sqrt{-b^2c + a^2bd}) - \sqrt{dx^3 + c}/b)/d$

maple [C] time = 0.23, size = 448, normalized size = 6.05

$$\frac{\frac{\sqrt{dx^3 + c} \operatorname{arctan}\left(\frac{\sqrt{dx^3 + c}b}{\sqrt{-b^2c + abd}b}\right) - \frac{\sqrt{dx^3 + c}}{b}}{\sqrt{-b^2c + abd}b}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x)

[Out] $2/3(d\sqrt{dx^3 + c})/b/d + 1/3I a/b/d^2 \sum(1/(a^2d - b^2c) * (-c*d^2)^{1/3}) * (1/2I * (2*x + (-I * 3^{1/2}) * (-c*d^2)^{1/3}) + (-c*d^2)^{1/3})/d / (-c*d^2)^{1/3} *$

```
d^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2
)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_
_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_
_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.10, size = 86, normalized size = 1.16

$$\frac{2\sqrt{dx^3+c}}{3bd} + \frac{a \ln\left(\frac{ad-2bc-bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}2i}{bx^3+a}\right)}{3b^{3/2}\sqrt{ad-bc}} 1i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b*d) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(
1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*b^(3/2)*(a*d - b*c
)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + bx^3)\sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

$$3.265 \quad \int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}} dx$$

Optimal. Leaf size=51

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {444, 63, 208}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]

[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*Sqrt[b*c - a*d])

Rule 63

Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 444

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right) \\ &= \frac{2 \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{3d} \\ &= -\frac{2 \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3\sqrt{b}\sqrt{bc-ad}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$-\frac{2 \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]

[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]/(3*Sqrt[b]*Sqrt[b*c - a*d]))

IntegrateAlgebraic [A] time = 0.05, size = 61, normalized size = 1.20

$$-\frac{2 \tan^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad} \right)}{3\sqrt{b}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]

[Out] (-2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)]/(3*Sqrt[b]*Sqrt[-(b*c) + a*d]))

fricas [A] time = 0.85, size = 130, normalized size = 2.55

$$\left[\frac{\log \left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a} \right)}{3\sqrt{b^2c-abd}}, \frac{2\sqrt{-b^2c+abd} \arctan \left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc} \right)}{3(b^2c-abd)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a))/sqrt(b^2*c - a*b*d), 2/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c))/(b^2*c - a*b*d)]

giac [A] time = 0.16, size = 40, normalized size = 0.78

$$\frac{2 \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] 2/3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)

maple [C] time = 0.28, size = 426, normalized size = 8.35

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x)

[Out] $-1/3*I/d^2*2^{(1/2)}*\sum(1/(a*d-b*c)*(-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)})/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, 1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=RootOf(_Z^3*b+a))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 5.89, size = 70, normalized size = 1.37

$$\frac{\ln\left(\frac{ad - bc + 2\sqrt{dx^3+c}\sqrt{abd-b^2c-bdx^3}}{bx^3+a}\right)}{3\sqrt{abd-b^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x^3)*(c + d*x^3)^(1/2)),x)

[Out] (log((a*d - b*c + 2*(c + d*x^3)^(1/2)*(a*b*d - b^2*c)^(1/2) - b*d*x^3)/(a + b*x^3)))/(3*(a*b*d - b^2*c)^(1/2))

sympy [A] time = 10.26, size = 39, normalized size = 0.76

$$\frac{2 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**3+a)/(d*x**3+c)**(1/2),x)

[Out] 2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b*c)/b))

$$3.266 \quad \int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}} dx$$

Optimal. Leaf size=85

$$\frac{2\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 86, 63, 208}

$$\frac{2\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]

[Out] (-2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(3*a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]/(3*a*Sqrt[b*c - a*d]))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 86

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && !IntegerQ[p]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}} dx &= \frac{1}{3} \text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}} dx, x, x^3\right) \\
 &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3\right) - b \text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3\right)}{3a} \\
 &= \frac{2 \text{Subst}\left(\int \frac{1}{-\frac{c}{a} + \frac{x^2}{a}} dx, x, \sqrt{c+dx^3}\right) - (2b) \text{Subst}\left(\int \frac{1}{a - \frac{bc}{a} + \frac{bx^2}{a}} dx, x, \sqrt{c+dx^3}\right)}{3ad} \\
 &= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}} + \frac{2\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 81, normalized size = 0.95

$$\frac{2 \left(\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}} \right)}{3a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]), x]

[Out] (2*(-(ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]/Sqrt[b*c - a*d]))/(3*a)

IntegrateAlgebraic [A] time = 0.12, size = 95, normalized size = 1.12

$$\frac{2\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a\sqrt{ad-bc}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]

[Out] (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)]/(3*a*Sqrt[-(b*c) + a*d]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a*Sqrt[c])

fricas [A] time = 0.46, size = 431, normalized size = 5.07

$$\left(\frac{c\sqrt{\frac{c}{b-cd}} \log\left(\frac{bdx^3+2bc-ad+2\sqrt{bd^2+c}(b-cd)\sqrt{\frac{c}{b-cd}}}{bd^2+a}\right) + \sqrt{c} \log\left(\frac{bd^2-2\sqrt{bd^2+c}d}{d^2}\right)}{3ac}, \frac{2c\sqrt{\frac{c}{b-cd}} \arctan\left(\frac{\sqrt{bd^2+c}(b-cd)\sqrt{\frac{c}{b-cd}}}{bd^2+bc}\right) + \sqrt{c} \log\left(\frac{bd^2-2\sqrt{bd^2+c}d}{d^2}\right)}{3ac}, \frac{c\sqrt{\frac{c}{b-cd}} \log\left(\frac{bdx^3+2bc-ad+2\sqrt{bd^2+c}(b-cd)\sqrt{\frac{c}{b-cd}}}{bd^2+a}\right) + 2\sqrt{-c} \arctan\left(\frac{\sqrt{bd^2+c}\sqrt{c}}{c}\right)}{3ac}, \frac{2\left(c\sqrt{\frac{c}{b-cd}} \arctan\left(\frac{\sqrt{bd^2+c}(b-cd)\sqrt{\frac{c}{b-cd}}}{bd^2+bc}\right) + \sqrt{-c} \arctan\left(\frac{\sqrt{bd^2+c}\sqrt{c}}{c}\right)\right)}{3ac} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/3*(c*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(2*c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^3 + b*c)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(c*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/(a*c), 2/3*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^3 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/(a*c)]

giac [A] time = 0.17, size = 71, normalized size = 0.84

$$-\frac{2b \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a} + \frac{2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] -2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c))

maple [C] time = 0.24, size = 453, normalized size = 5.33

$$\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{-c}}\right)}{3 a \sqrt{-c}} + \frac{2 b \operatorname{arctan}\left(\frac{\sqrt{d x^3+c} b}{\sqrt{-b^2 c+a b d}}\right)}{3 a b \sqrt{-b^2 c+a b d}} + \frac{2 \operatorname{arctan}\left(\frac{\sqrt{d x^3+c}}{\sqrt{-c}}\right)}{3 a \sqrt{-c}} + \frac{2 \operatorname{arctan}\left(\frac{\sqrt{d x^3+c} b}{\sqrt{-b^2 c+a b d}}\right)}{3 a b \sqrt{-b^2 c+a b d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x)

```
[Out] 1/3*I/a*b/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1
/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+
c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_
_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*
3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/
a/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)\sqrt{dx^3 + cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x), x)
```

mupad [B] time = 7.31, size = 114, normalized size = 1.34

$$\frac{\ln\left(\frac{(\sqrt{dx^3+c}-\sqrt{c})^3(\sqrt{dx^3+c}+\sqrt{c})}{x^6}\right)}{3a\sqrt{c}} + \frac{\sqrt{b} \ln\left(\frac{ad-2bc-bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}2i}{bx^3+a}\right)}{3a\sqrt{ad-bc}} 1i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a*c^(1/2)) + (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d -
b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*a*(a*d - b*c)^(1/2))
```

sympy [A] time = 19.39, size = 70, normalized size = 0.82

$$-\frac{2 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}} + \frac{2 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] -2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)) + 2  
*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c))
```

$$3.267 \quad \int \frac{1}{x^4(a+bx^3)\sqrt{c+dx^3}} dx$$

Optimal. Leaf size=117

$$-\frac{2b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}} + \frac{(ad+2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} - \frac{\sqrt{c+dx^3}}{3acx^3}$$

Rubi [A] time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 103, 156, 63, 208}

$$-\frac{2b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}} + \frac{(ad+2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} - \frac{\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]

[Out] -Sqrt[c + d*x^3]/(3*a*c*x^3) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(3*a^2*c^(3/2)) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]/(3*a^2*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156


```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(a+bx^3)\sqrt{c+dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c+dx^3}}{3acx^3} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(2bc+ad) + \frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3ac} \\
&= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{b^2 \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3a^2} - \frac{(2bc+ad) \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{6a^2c} \\
&= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{(2b^2) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{3a^2d} - \frac{(2bc+ad) \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{6a^2c} \\
&= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{(2bc+ad) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^2c^{3/2}} - \frac{2b^{3/2} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3a^2\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 151, normalized size = 1.29

$$\frac{2b^{3/2}\sqrt{bc-ad} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3a^2(ad-bc)} + \frac{2b \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^2\sqrt{c}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3ac^{3/2}} - \frac{\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]

[Out] $-\frac{1}{3}\sqrt{c + dx^3}/(a^2c^2) + \frac{2b\operatorname{ArcTanh}[\sqrt{c + dx^3}/\sqrt{c}]}{(3a^2c^2)\sqrt{c}} + \frac{d\operatorname{ArcTanh}[\sqrt{c + dx^3}/\sqrt{c}]}{(3a^2c^2)\sqrt{c}} + \frac{2b^{3/2}\sqrt{bc - ad}\operatorname{ArcTanh}[(\sqrt{b}\sqrt{c + dx^3})/\sqrt{bc - ad}]}{(3a^2c^2)\sqrt{bc - ad}} + \frac{2b^{3/2}\sqrt{bc - ad}\operatorname{ArcTanh}[(\sqrt{b}\sqrt{c + dx^3})/\sqrt{bc - ad}]}{(3a^2c^2)\sqrt{bc - ad}}$

IntegrateAlgebraic [A] time = 0.23, size = 127, normalized size = 1.09

$$-\frac{2b^{3/2} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^2\sqrt{ad-bc}} + \frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} - \frac{\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]

[Out] $-\frac{1}{3}\sqrt{c + dx^3}/(a^2c^2) - \frac{2b^{3/2}\operatorname{ArcTan}[(\sqrt{b}\sqrt{c + dx^3})/\sqrt{bc - ad}]}{(3a^2c^2)\sqrt{bc - ad}} + \frac{((2b^2c + a^2d)\sqrt{c + dx^3})/\sqrt{bc - ad}}{(3a^2c^2)\sqrt{bc - ad}} + \frac{((2b^2c + a^2d)\sqrt{c + dx^3})/\sqrt{bc - ad}}{(3a^2c^2)\sqrt{bc - ad}}$

fricas [A] time = 0.49, size = 565, normalized size = 4.83

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{6}(2b^2c^2x^3\sqrt{b/(bc - ad)})\log((bdx^3 + 2b^2c - ad - 2\sqrt{d(x^3 + c)}(bx^3 + c))\sqrt{b/(bc - ad)})/(bx^3 + a) + (2b^2c + a^2d)\sqrt{c}x^3\log((dx^3 + 2\sqrt{d(x^3 + c)}\sqrt{c} + 2c)/x^3) - 2\sqrt{d(x^3 + c)}(bx^3 + c)\sqrt{c}/(a^2c^2x^3) - \frac{1}{6}(4b^2c^2x^3\sqrt{-b/(bc - ad)})\arctan(\sqrt{d(x^3 + c)}(bx^3 + c)\sqrt{-b/(bc - ad)})/(bdx^3 + bc) - (2b^2c + a^2d)\sqrt{c}x^3\log((dx^3 + 2\sqrt{d(x^3 + c)}\sqrt{c} + 2c)/x^3) + 2\sqrt{d(x^3 + c)}(bx^3 + c)\sqrt{c}/(a^2c^2x^3) + \frac{1}{3}(b^2c^2x^3\sqrt{b/(bc - ad)})\log((bdx^3 + 2b^2c - ad - 2\sqrt{d(x^3 + c)}(bx^3 + c))\sqrt{b/(bc - ad)})/(bx^3 + a) - (2b^2c + a^2d)\sqrt{-c}x^3\arctan(\sqrt{d(x^3 + c)}\sqrt{-c}/c) - \sqrt{d(x^3 + c)}(bx^3 + c)\sqrt{c}/(a^2c^2x^3) - \frac{1}{3}(2b^2c^2x^3\sqrt{-b/(bc - ad)})\arctan(-\sqrt{d(x^3 + c)}(bx^3 + c)\sqrt{-b/(bc - ad)})/(bdx^3 + bc) + (2b^2c + a^2d)\sqrt{-c}x^3\arctan(\sqrt{d(x^3 + c)}\sqrt{-c}/c) + \sqrt{d(x^3 + c)}(bx^3 + c)\sqrt{c}/(a^2c^2x^3)$

giac [A] time = 0.19, size = 104, normalized size = 0.89

$$\frac{2 b^2 \arctan\left(\frac{\sqrt{d x^3+c b}}{\sqrt{-b^2 c+a b d}}\right)}{3 \sqrt{-b^2 c+a b d} a^2}-\frac{(2 b c+a d) \arctan\left(\frac{\sqrt{d x^3+c}}{\sqrt{-c}}\right)}{3 a^2 \sqrt{-c} c}-\frac{\sqrt{d x^3+c}}{3 a c x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] 2/3*b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) - 1/3*(2*b*c + a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c) - 1/3*sqrt(d*x^3 + c)/(a*c*x^3)

maple [C] time = 0.25, size = 498, normalized size = 4.26

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x)

[Out] 1/a*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-1/3*I/a^2*b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2),_alpha=RootOf(_Z^3*b+a))+2/3/a^2*b*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b x^3 + a) \sqrt{d x^3 + c} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^4), x)

mupad [B] time = 8.42, size = 142, normalized size = 1.21

$$\frac{\ln\left(\frac{(\sqrt{dx^3+c}-\sqrt{c})(\sqrt{dx^3+c}+\sqrt{c})^3}{x^6}\right)(ad+2bc)}{6a^2c^{3/2}} - \frac{\sqrt{dx^3+c}}{3acx^3} + \frac{b^{3/2}\ln\left(\frac{2bc-ad+bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}2i}{bx^3+a}\right)1i}{3a^2\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a + b*x^3)*(c + d*x^3)^(1/2)),x)

[Out] (log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(a*d + 2*b*c))/(6*a^2*c^(3/2)) - (c + d*x^3)^(1/2)/(3*a*c*x^3) + (b^(3/2)*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*a^2*(a*d - b*c)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + bx^3) \sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(1/2),x)

[Out] Integral(1/(x**4*(a + b*x**3)*sqrt(c + d*x**3)), x)

$$3.268 \quad \int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=107

$$-\frac{2a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}} + \frac{2c^2}{3d^2\sqrt{c+dx^3}(bc-ad)} + \frac{2\sqrt{c+dx^3}}{3bd^2}$$

Rubi [A] time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 87, 63, 208}

$$-\frac{2a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}} + \frac{2c^2}{3d^2\sqrt{c+dx^3}(bc-ad)} + \frac{2\sqrt{c+dx^3}}{3bd^2}$$

Antiderivative was successfully verified.

[In] Int[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (2*c^2)/(3*d^2*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(3*b*d^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 87

Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x_)), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(a + bx^3)(c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(a + bx)(c + dx)^{3/2}} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c^2}{d(-bc + ad)(c + dx)^{3/2}} + \frac{1}{bd\sqrt{c + dx}} + \frac{a^2}{b(bc - ad)(a + bx)\sqrt{c + dx}} \right) dx, x, x^3 \right) \\
&= \frac{2c^2}{3d^2(bc - ad)\sqrt{c + dx^3}} + \frac{2\sqrt{c + dx^3}}{3bd^2} + \frac{a^2 \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3b(bc - ad)} \\
&= \frac{2c^2}{3d^2(bc - ad)\sqrt{c + dx^3}} + \frac{2\sqrt{c + dx^3}}{3bd^2} + \frac{(2a^2) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3bd(bc - ad)} \\
&= \frac{2c^2}{3d^2(bc - ad)\sqrt{c + dx^3}} + \frac{2\sqrt{c + dx^3}}{3bd^2} - \frac{2a^2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{3/2}(bc - ad)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 100, normalized size = 0.93

$$\frac{2 \left(-a^2 d^2 {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b(dx^3+c)}{bc-ad} \right) + a^2 d^2 + abd(c + dx^3) + b^2(-c)(2c + dx^3) \right)}{3b^2 d^2 \sqrt{c + dx^3} (ad - bc)}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (2*(a^2*d^2 + a*b*d*(c + d*x^3) - b^2*c*(2*c + d*x^3) - a^2*d^2*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d]])/(3*b^2*d^2*(-(b*c) + a*d)*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.21, size = 124, normalized size = 1.16

$$\frac{2a^2 \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{3/2}(ad-bc)^{3/2}} - \frac{2(-acd - ad^2x^3 + 2bc^2 + bcdx^3)}{3bd^2\sqrt{c+dx^3}(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]

[Out] $(-2*(2*b*c^2 - a*c*d + b*c*d*x^3 - a*d^2*x^3))/(3*b*d^2*(-(b*c) + a*d)*\text{Sqrt}[c + d*x^3]) + (2*a^2*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[-(b*c) + a*d]*\text{Sqrt}[c + d*x^3])/(b*c - a*d)])/(3*b^(3/2)*(-(b*c) + a*d)^(3/2))$

fricas [B] time = 0.80, size = 440, normalized size = 4.11

$$\left[\frac{(a^2d^3x^3 + a^2cd^2)\sqrt{b^2c - abd} \log\left(\frac{bd^3x^3 + a^2d^2\sqrt{b^2c - abd}}{bd^3x^3 + a^2d^2}\right) - 2(2b^3c^3 - 3ab^2c^2d + a^2bcd^2 + (b^3c^2d - 2ab^2cd^2 + a^2bd^3)x^3)\sqrt{dx^3 + c}}{3(b^4c^3d^2 - 2ab^3c^2d^3 + a^2b^2cd^4 + (b^4c^2d^3 - 2ab^3cd^4 + a^2b^2d^5)x^3)}, \frac{2((a^2d^3x^3 + a^2cd^2)\sqrt{-b^2c + abd} \arctan\left(\frac{\sqrt{dx^3 + c}\sqrt{-b^2c + abd}}{bd^3x^3 + a^2d^2}\right) + (2b^3c^3 - 3ab^2c^2d + a^2bcd^2 + (b^3c^2d - 2ab^2cd^2 + a^2bd^3)x^3)\sqrt{dx^3 + c}}{3(b^4c^3d^2 - 2ab^3c^2d^3 + a^2b^2cd^4 + (b^4c^2d^3 - 2ab^3cd^4 + a^2b^2d^5)x^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] $[-1/3*((a^2*d^3*x^3 + a^2*c*d^2)*\text{sqrt}(b^2*c - a*b*d)*\log((b*d*x^3 + 2*b*c - a*d + 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^3 - 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^3)*\text{sqrt}(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b^2*c*d^4 + (b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3), 2/3*((a^2*d^3*x^3 + a^2*c*d^2)*\text{sqrt}(-b^2*c + a*b*d)*\arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (2*b^3*c^3 - 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^3)*\text{sqrt}(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b^2*c*d^4 + (b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3)]$

giac [A] time = 0.18, size = 103, normalized size = 0.96

$$\frac{2a^2 \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3(b^2c - abd)\sqrt{-b^2c + abd}} + \frac{2c^2}{3(bcd^2 - ad^3)\sqrt{dx^3 + c}} + \frac{2\sqrt{dx^3 + c}}{3bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $2/3*a^2*\arctan(\text{sqrt}(d*x^3 + c)*b/\text{sqrt}(-b^2*c + a*b*d))/((b^2*c - a*b*d)*\text{sqrt}(-b^2*c + a*b*d)) + 2/3*c^2/((b*c*d^2 - a*d^3)*\text{sqrt}(d*x^3 + c)) + 2/3*\text{sqrt}(d*x^3 + c)/(b*d^2)$

maple [C] time = 0.34, size = 527, normalized size = 4.93

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x)`

[Out] $\frac{1}{b^2} \left(\frac{b \left(\frac{2}{3} \left((x^3+c/d) d \right)^{1/2} \right) c/d^2 + \frac{2}{3} (d x^3+c)^{1/2} / d^2 + \frac{2}{3} a/d}{(d x^3+c)^{1/2}} + a^2/b^2 \left(-\frac{2}{3} / (a d - b c) / \left((x^3+c/d) d \right)^{1/2} - \frac{1}{3} I b/d^2 \right)^{1/2} \right) \sum \left(\frac{1}{(-a d + b c) / (a d - b c) * (-c d^2)^{1/3}} \right) \left(\frac{1}{2} I * (2 x + (-I 3^{1/2}) * (-c d^2)^{1/3}) + (-c d^2)^{1/3} \right) / d / (-c d^2)^{1/3} * d^{1/2} * \left(\frac{x - (-c d^2)^{1/3}}{d} / (-3 * (-c d^2)^{1/3} + I 3^{1/2} * (-c d^2)^{1/3}) * d \right)^{1/2} * \left(-\frac{1}{2} I * (2 x + (I 3^{1/2}) * (-c d^2)^{1/3}) + (-c d^2)^{1/3} \right) / d / (-c d^2)^{1/3} * d^{1/2} / (d x^3+c)^{1/2} \right) * \left(2 * \alpha^2 d^2 + I * (-c d^2)^{1/3} * 3^{1/2} * \alpha * d - (-c d^2)^{1/3} * \alpha * d - I 3^{1/2} * (-c d^2)^{2/3} - (-c d^2)^{2/3} \right) * \text{EllipticPi} \left(\frac{1}{3} * 3^{1/2} * (I * (x + 1/2 * (-c d^2)^{1/3}) / d - 1/2 * I 3^{1/2} * (-c d^2)^{1/3} / d) * 3^{1/2} / (-c d^2)^{1/3} * d \right)^{1/2}, \frac{1}{2} * (2 * I * (-c d^2)^{1/3} * 3^{1/2} * \alpha^2 d + I 3^{1/2} * c * d - 3 * c * d - I * (-c d^2)^{2/3} * 3^{1/2} * \alpha - 3 * (-c d^2)^{2/3} * \alpha) / (a d - b c) * b/d, (I 3^{1/2} * (-c d^2)^{1/3} / (-3/2 * (-c d^2)^{1/3} / d + 1/2 * I 3^{1/2} * (-c d^2)^{1/3} / d) / d)^{1/2} \right), \alpha = \text{RootOf}(_Z^3 + b + a))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details) Is a*d-b*c positive or negative?

mupad [B] time = 6.46, size = 115, normalized size = 1.07

$$\frac{2 \sqrt{d x^3 + c}}{3 b d^2} - \frac{2 c^2}{3 d^2 \sqrt{d x^3 + c} (a d - b c)} + \frac{a^2 \ln \left(\frac{a d - 2 b c - b d x^3 + \sqrt{b} \sqrt{d x^3 + c} \sqrt{a d - b c}}{b x^3 + a} \right)}{3 b^{3/2} (a d - b c)^{3/2}} + \frac{2 i}{3 b^{3/2} (a d - b c)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x)`

[Out] $(2*(c + d*x^3)^{(1/2)})/(3*b*d^2) - (2*c^2)/(3*d^2*(c + d*x^3)^{(1/2)}*(a*d - b*c)) + (a^2*\log((a*d - 2*b*c + b^{(1/2)}*(c + d*x^3)^{(1/2)}*(a*d - b*c)^{(1/2)}*2i - b*d*x^3)/(a + b*x^3)*1i)/(3*b^{(3/2)}*(a*d - b*c)^{(3/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a + bx^3)(c + dx^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8/(b*x**3+a)/(d*x**3+c)**(3/2),x)`

[Out] `Integral(x**8/((a + b*x**3)*(c + d*x**3)**(3/2)), x)`

$$3.269 \quad \int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=82

$$\frac{2a \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}} - \frac{2c}{3d\sqrt{c+dx^3}(bc-ad)}$$

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 78, 63, 208}

$$\frac{2a \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}} - \frac{2c}{3d\sqrt{c+dx^3}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (-2*c)/(3*d*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(a + bx^3)(c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{(a + bx)(c + dx)^{3/2}} dx, x, x^3 \right) \\ &= -\frac{2c}{3d(bc - ad)\sqrt{c + dx^3}} - \frac{a \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3(bc - ad)} \\ &= -\frac{2c}{3d(bc - ad)\sqrt{c + dx^3}} - \frac{(2a) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3d(bc - ad)} \\ &= -\frac{2c}{3d(bc - ad)\sqrt{c + dx^3}} + \frac{2a \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3\sqrt{b} (bc - ad)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.12, size = 88, normalized size = 1.07

$$\frac{2 \left(\frac{c(ad-bc)}{d\sqrt{c+dx^3}} + \frac{a\sqrt{bc-ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{\sqrt{b}} \right)}{3(bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (2*((c*(-(b*c) + a*d))/(d*Sqrt[c + d*x^3]) + (a*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b]))/(3*(b*c - a*d)^2)

IntegrateAlgebraic [A] time = 0.15, size = 92, normalized size = 1.12

$$\frac{2c}{3d\sqrt{c + dx^3} (ad - bc)} - \frac{2a \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad} \right)}{3\sqrt{b} (ad - bc)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]

[Out] $(2*c)/(3*d*(-(b*c) + a*d)*\text{Sqrt}[c + d*x^3]) - (2*a*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[-(b*c) + a*d]*\text{Sqrt}[c + d*x^3])/(b*c - a*d)])/(3*\text{Sqrt}[b]*(-(b*c) + a*d)^(3/2))$

fricas [B] time = 0.81, size = 326, normalized size = 3.98

$$\left[\frac{(ad^2x^3 + acd)\sqrt{b^2c - abd} \log\left(\frac{bdx^3 + 2bc - ad - 2\sqrt{dx^3 + c}\sqrt{b^2c - abd}}{bx^3 + a}\right) + 2(b^2c^2 - abcd)\sqrt{dx^3 + c}}{3(b^3c^3d - 2ab^2c^2d^2 + a^2bcd^3 + (b^3c^2d^2 - 2ab^2cd^3 + a^2bd^4)x^3)} - \frac{2\left((ad^2x^3 + acd)\sqrt{-b^2c + abd} \arctan\left(\frac{\sqrt{dx^3 + c}\sqrt{-b^2c + abd}}{bdx^3 + bc}\right) + (b^2c^2 - abcd)\sqrt{dx^3 + c}\right)}{3(b^3c^3d - 2ab^2c^2d^2 + a^2bcd^3 + (b^3c^2d^2 - 2ab^2cd^3 + a^2bd^4)x^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] $[-1/3*((a*d^2*x^3 + a*c*d)*\text{sqrt}(b^2*c - a*b*d)*\log((b*d*x^3 + 2*b*c - a*d - 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(b^2*c^2 - a*b*c*d)*\text{sqrt}(d*x^3 + c)/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3), -2/3*((a*d^2*x^3 + a*c*d)*\text{sqrt}(-b^2*c + a*b*d)*\arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (b^2*c^2 - a*b*c*d)*\text{sqrt}(d*x^3 + c)/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)]$

giac [A] time = 0.19, size = 78, normalized size = 0.95

$$\frac{2\left(\frac{ad \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)} + \frac{c}{\sqrt{dx^3+c}(bc-ad)}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $-2/3*(a*d*\arctan(\text{sqrt}(d*x^3 + c)*b/\text{sqrt}(-b^2*c + a*b*d))/(\text{sqrt}(-b^2*c + a*b*d)*(b*c - a*d)) + c/(\text{sqrt}(d*x^3 + c)*(b*c - a*d)))/d$

maple [C] time = 0.27, size = 487, normalized size = 5.94

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x)`

[Out]
$$-2/3/b/d/(d*x^3+c)^{(1/2)}-a/b*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^{(1/2)}-1/3*I*b/d^{2*2^{(1/2)}}*\sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)})*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)})/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)})*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)*d})^{(1/2)},1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)),_alpha=RootOf(_Z^3*b+a))$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 5.99, size = 94, normalized size = 1.15

$$\frac{2c}{3d\sqrt{dx^3+c}(ad-bc)} + \frac{a \ln\left(\frac{2bc-ad+bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}2i}{bx^3+a}\right)}{3\sqrt{b}(ad-bc)^{3/2}} 1i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x)`

[Out]
$$(2*c)/(3*d*(c + d*x^3)^{(1/2)}*(a*d - b*c)) + (a*\log((2*b*c - a*d + b^{(1/2)}*(c + d*x^3)^{(1/2)}*(a*d - b*c)^{(1/2)}*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*b^{(1/2)}*(a*d - b*c)^{(3/2)})$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + bx^3)(c + dx^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

$$3.270 \quad \int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=77

$$\frac{2}{3\sqrt{c+dx^3}(bc-ad)} - \frac{2\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {444, 51, 63, 208}

$$\frac{2}{3\sqrt{c+dx^3}(bc-ad)} - \frac{2\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]

[Out] 2/(3*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*(b*c - a*d)^(3/2))

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a + bx^3)(c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(a + bx)(c + dx)^{3/2}} dx, x, x^3 \right) \\ &= \frac{2}{3(bc - ad)\sqrt{c + dx^3}} + \frac{b \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3(bc - ad)} \\ &= \frac{2}{3(bc - ad)\sqrt{c + dx^3}} + \frac{(2b) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3d(bc - ad)} \\ &= \frac{2}{3(bc - ad)\sqrt{c + dx^3}} - \frac{2\sqrt{b} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3(bc - ad)^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.68

$$\frac{{}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b(dx^3+c)}{bc-ad} \right)}{3\sqrt{c + dx^3} (ad - bc)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)]/(3*(-(b*c) + a*d)*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.09, size = 109, normalized size = 1.42

$$\frac{\frac{2}{3} \frac{2\sqrt{b} \sqrt{c+dx^3} \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad} \right)}{3\sqrt{ad-bc}}}{bc\sqrt{c + dx^3} - ad\sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (2/3 - (2*sqrt[b]*sqrt[c + d*x^3]*ArcTan[(sqrt[b]*sqrt[-(b*c) + a*d]*sqrt[c + d*x^3])/(b*c - a*d)])/(3*sqrt[-(b*c) + a*d]))/(b*c*sqrt[c + d*x^3] - a*d*sqrt[c + d*x^3])

fricas [A] time = 0.70, size = 236, normalized size = 3.06

$$\left[\frac{(dx^3 + c)\sqrt{\frac{b}{bc-ad}} \log\left(\frac{bdx^3 + 2bc - ad + 2\sqrt{dx^3 + c}(bc - ad)\sqrt{\frac{b}{bc-ad}}}{bx^3 + a}\right) - 2\sqrt{dx^3 + c}}{3((bcd - ad^2)x^3 + bc^2 - acd)}, -\frac{2\left((dx^3 + c)\sqrt{-\frac{b}{bc-ad}} \arctan\left(-\frac{\sqrt{dx^3 + c}(bc - ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3 + bc}\right) - \sqrt{dx^3 + c}\right)}{3((bcd - ad^2)x^3 + bc^2 - acd)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] [-1/3*((d*x^3 + c)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) - 2*sqrt(d*x^3 + c))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d), -2/3*((d*x^3 + c)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^3 + b*c)) - sqrt(d*x^3 + c))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)]

giac [A] time = 0.19, size = 73, normalized size = 0.95

$$\frac{2b \arctan\left(\frac{\sqrt{dx^3 + cb}}{\sqrt{-b^2c + abd}}\right)}{3\sqrt{-b^2c + abd}(bc - ad)} + \frac{2}{3\sqrt{dx^3 + c}(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] 2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c - a*d)) + 2/3/(sqrt(d*x^3 + c)*(b*c - a*d))

maple [C] time = 0.23, size = 463, normalized size = 6.01

$$\frac{\frac{\frac{\sqrt{\frac{b}{bc-ad}} \log\left(\frac{bdx^3 + 2bc - ad + 2\sqrt{dx^3 + c}(bc - ad)\sqrt{\frac{b}{bc-ad}}}{bx^3 + a}\right) - 2\sqrt{dx^3 + c}}{3((bcd - ad^2)x^3 + bc^2 - acd)}}{\sqrt{\frac{b}{bc-ad}}} - \frac{2\left(\sqrt{-\frac{b}{bc-ad}} \arctan\left(-\frac{\sqrt{dx^3 + c}(bc - ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3 + bc}\right) - \sqrt{dx^3 + c}\right)}{3((bcd - ad^2)x^3 + bc^2 - acd)}}{3((bcd - ad^2)x^3 + bc^2 - acd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x)

```
[Out] -2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2))*(-c*d^2)^(1/3)*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3))-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2))*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2))*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2))*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.85, size = 89, normalized size = 1.16

$$-\frac{2}{3\sqrt{dx^3+c}(ad-bc)} + \frac{\sqrt{b} \ln\left(\frac{ad-2bc-bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}}{bx^3+a}\right) 1i}{3(ad-bc)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*(a*d - b*c)^(3/2)) - 2/(3*(c + d*x^3)^(1/2)*(a*d - b*c))
```

sympy [A] time = 32.78, size = 66, normalized size = 0.86

$$-\frac{2}{3\sqrt{c+dx^3}(ad-bc)} - \frac{2 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3\sqrt{\frac{ad-bc}{b}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] -2/(3*sqrt(c + d*x**3)*(a*d - b*c)) - 2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*sqrt((a*d - b*c)/b)*(a*d - b*c))
```

$$3.271 \quad \int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=114

$$\frac{2b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}} - \frac{2d}{3c\sqrt{c+dx^3}(bc-ad)} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}}$$

Rubi [A] time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 85, 156, 63, 208}

$$\frac{2b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}} - \frac{2d}{3c\sqrt{c+dx^3}(bc-ad)} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*d)/(3*c*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a*c^(3/2)) + (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

$f*x)^p/(a + b*x), x], x] - \text{Dist}[(d*g - c*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(c + d*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x]$

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a + bx^3)(c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(a + bx)(c + dx)^{3/2}} dx, x, x^3 \right) \\ &= -\frac{2d}{3c(bc - ad)\sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{bc - ad - bdx}{x(a + bx)\sqrt{c + dx}} dx, x, x^3 \right)}{3c(bc - ad)} \\ &= -\frac{2d}{3c(bc - ad)\sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{c + dx}} dx, x, x^3 \right)}{3ac} - \frac{b^2 \text{Subst} \left(\int \frac{1}{(a + bx)\sqrt{c + dx}} dx, x, x^3 \right)}{3a(bc - ad)} \\ &= -\frac{2d}{3c(bc - ad)\sqrt{c + dx^3}} + \frac{2 \text{Subst} \left(\int \frac{1}{\frac{-c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3acd} - \frac{(2b^2) \text{Subst} \left(\int \frac{1}{(a + bx)\sqrt{c + dx}} dx, x, x^3 \right)}{3a(bc - ad)} \\ &= -\frac{2d}{3c(bc - ad)\sqrt{c + dx^3}} - \frac{2 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{3ac^{3/2}} + \frac{2b^{3/2} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c + dx^3}}{\sqrt{bc - ad}} \right)}{3a(bc - ad)^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.05, size = 89, normalized size = 0.78

$$\frac{2 \left(bc {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b(dx^3 + c)}{bc - ad} \right) + (ad - bc) {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3}{c} + 1 \right) \right)}{3ac\sqrt{c + dx^3}(bc - ad)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]

[Out] $(-2*(b*c*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)] + (-b*c) + a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(3*a*c*(b*c - a*d)*Sqrt[c + d*x^3])$

IntegrateAlgebraic [A] time = 0.29, size = 124, normalized size = 1.09

$$-\frac{2b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad}\right)}{3a(ad-bc)^{3/2}} - \frac{2d}{3c\sqrt{c+dx^3}(bc-ad)} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]

[Out] $(-2*d)/(3*c*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*b^{(3/2)}*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*a*(-(b*c) + a*d)^{(3/2)}) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a*c^{(3/2)})$

fricas [B] time = 0.58, size = 790, normalized size = 6.93

$\frac{2 \sqrt{c+dx^3} \sqrt{bc-ad} \sqrt{ad-bc}}{3a(ad-bc)^{3/2}} - \frac{2d}{3c\sqrt{c+dx^3}(bc-ad)} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] $[-1/3*(2*\sqrt{d*x^3 + c}*a*c*d + (b*c^2*d*x^3 + b*c^3)*\sqrt{b/(b*c - a*d)})*\log((b*d*x^3 + 2*b*c - a*d - 2*\sqrt{d*x^3 + c}*(b*c - a*d)*\sqrt{b/(b*c - a*d)})/(b*x^3 + a)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*\sqrt{c}*\log((d*x^3 - 2*\sqrt{d*x^3 + c}*\sqrt{c} + 2*c)/x^3)/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^3), -1/3*(2*\sqrt{d*x^3 + c}*a*c*d - 2*(b*c^2*d*x^3 + b*c^3)*\sqrt{-b/(b*c - a*d)})*\arctan(-\sqrt{d*x^3 + c}*(b*c - a*d)*\sqrt{-b/(b*c - a*d)})/(b*d*x^3 + b*c) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*\sqrt{c}*\log((d*x^3 - 2*\sqrt{d*x^3 + c}*\sqrt{c} + 2*c)/x^3)/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^3), -1/3*(2*\sqrt{d*x^3 + c}*a*c*d - 2*((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*\sqrt{-c})*\arctan(\sqrt{d*x^3 + c}*\sqrt{-c}/c) + (b*c^2*d*x^3 + b*c^3)*\sqrt{b/(b*c - a*d)}*\log((b*d*x^3 + 2*b*c - a*d - 2*\sqrt{d*x^3 + c}*(b*c - a*d)*\sqrt{b/(b*c - a*d)})/(b*x^3 + a))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^3), -2/3*(\sqrt{d*x^3 + c}*a*c*d - (b*c^2*d*x^3 + b*c^3)*\sqrt{-b/(b*c - a*d)})*\arctan(-\sqrt{d*x^3 + c}*(b*c - a*d)*\sqrt{-b/(b*c - a*d)})/(b*d*x^3 + b*c) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*\sqrt{-c}*\arctan(\sqrt{d*x^3 + c}*\sqrt{-c}/c)/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^3)]$

giac [A] time = 0.17, size = 111, normalized size = 0.97

$$\frac{2b^2 \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3(abc-a^2d)\sqrt{-b^2c+abd}} - \frac{2d}{3\sqrt{dx^3+c}(bc^2-acd)} + \frac{2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a\sqrt{-c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $-2/3*b^2*\arctan(\sqrt{d*x^3+c}*b/\sqrt{-b^2*c+a*b*d})/((a*b*c-a^2*d)*\sqrt{-b^2*c+a*b*d}) - 2/3*d/(\sqrt{d*x^3+c}*(b*c^2-a*c*d)) + 2/3*\arctan(\sqrt{d*x^3+c}/\sqrt{-c})/(a*\sqrt{-c}*c)$

maple [C] time = 0.29, size = 512, normalized size = 4.49

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x)

[Out] $-1/a*b*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^{(1/2)} - 1/3*I*b/d^{2*2}^{(1/2)}*\sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, 1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=\text{RootOf}(_Z^3*b+a))+1/a*(2/3/((x^3+c/d)*d)^{(1/2)}/c-2/3*\text{arctanh}((d*x^3+c)^{(1/2)}/c^{(1/2)})/c^{(3/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x), x)

mupad [B] time = 8.44, size = 139, normalized size = 1.22

$$\frac{\ln\left(\frac{(\sqrt{dx^3+c}-\sqrt{c})^3(\sqrt{dx^3+c}+\sqrt{c})}{x^6}\right)}{3ac^{3/2}} + \frac{2d}{3c\sqrt{dx^3+c}(ad-bc)} + \frac{b^{3/2} \ln\left(\frac{2bc-ad+bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}2i}{bx^3+a}\right)}{3a(ad-bc)^{3/2}} 1i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x)

[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3*a*c^(3/2)) + (2*d)/(3*c*(c + d*x^3)^(1/2)*(a*d - b*c)) + (b^(3/2)*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*a*(a*d - b*c)^(3/2))

sympy [A] time = 24.63, size = 104, normalized size = 0.91

$$\frac{2d}{3c\sqrt{c+dx^3}(ad-bc)} + \frac{2b \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}(ad-bc)} + \frac{2 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3ac\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**3+a)/(d*x**3+c)**(3/2),x)

[Out] 2*d/(3*c*sqrt(c + d*x**3)*(a*d - b*c)) + 2*b*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)*(a*d - b*c)) + 2*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*c*sqrt(-c))

$$3.272 \quad \int \frac{1}{x^4(a+bx^3)(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=158

$$\frac{2b^{5/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}} + \frac{(3ad+2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}} - \frac{d(bc-3ad)}{3ac^2\sqrt{c+dx^3}(bc-ad)} - \frac{1}{3acx^3\sqrt{c+dx^3}}$$

Rubi [A] time = 0.22, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 103, 152, 156, 63, 208}

$$\frac{2b^{5/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}} + \frac{(3ad+2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}} - \frac{d(bc-3ad)}{3ac^2\sqrt{c+dx^3}(bc-ad)} - \frac{1}{3acx^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]

[Out] -(d*(b*c - 3*a*d))/(3*a*c^2*(b*c - a*d)*Sqrt[c + d*x^3]) - 1/(3*a*c*x^3*Sqrt[c + d*x^3]) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2*c^(5/2)) - (2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 152

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2*n, 2*p]

```

Rule 156

```

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 446

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (a + bx^3)(c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2(a + bx)(c + dx)^{3/2}} dx, x, x^3 \right) \\
&= -\frac{1}{3acx^3\sqrt{c + dx^3}} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(2bc+3ad) + \frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}} dx, x, x^3 \right)}{3ac} \\
&= -\frac{d(bc - 3ad)}{3ac^2(bc - ad)\sqrt{c + dx^3}} - \frac{1}{3acx^3\sqrt{c + dx^3}} + \frac{2 \text{Subst} \left(\int \frac{-\frac{1}{4}(bc-ad)(2bc+3ad) - \frac{1}{4}bd}{x(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3ac^2(bc - ad)} \\
&= -\frac{d(bc - 3ad)}{3ac^2(bc - ad)\sqrt{c + dx^3}} - \frac{1}{3acx^3\sqrt{c + dx^3}} + \frac{b^3 \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3a^2(bc - ad)} \\
&= -\frac{d(bc - 3ad)}{3ac^2(bc - ad)\sqrt{c + dx^3}} - \frac{1}{3acx^3\sqrt{c + dx^3}} + \frac{(2b^3) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, x^3 \right)}{3a^2d(bc - ad)} \\
&= -\frac{d(bc - 3ad)}{3ac^2(bc - ad)\sqrt{c + dx^3}} - \frac{1}{3acx^3\sqrt{c + dx^3}} + \frac{(2bc + 3ad) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^2c^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 117, normalized size = 0.74

$$\frac{2b^2c^2x^3 {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b(dx^3+c)}{bc-ad}\right) + (ad - bc) \left(x^3(3ad + 2bc) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3}{c} + 1\right) + ac\right)}{3a^2c^2x^3\sqrt{c + dx^3}(bc - ad)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (2*b^2*c^2*x^3*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)] + (-b*c) + a*d)*(a*c + (2*b*c + 3*a*d)*x^3*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c]))/(3*a^2*c^2*(b*c - a*d)*x^3*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.39, size = 165, normalized size = 1.04

$$\frac{2b^{5/2} \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad} \right)}{3a^2(ad - bc)^{3/2}} + \frac{(3ad + 2bc) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^2c^{5/2}} + \frac{-acd - 3ad^2x^3 + bc^2 + bcdx^3}{3ac^2x^3\sqrt{c + dx^3}(ad - bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]

[Out] $(b*c^2 - a*c*d + b*c*d*x^3 - 3*a*d^2*x^3)/(3*a*c^2*(-(b*c) + a*d)*x^3*\text{Sqrt}[c + d*x^3]) + (2*b^{(5/2)}*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[-(b*c) + a*d]*\text{Sqrt}[c + d*x^3])/(b*c - a*d)])/(3*a^2*(-(b*c) + a*d)^{(3/2)}) + ((2*b*c + 3*a*d)*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/\text{Sqrt}[c]])/(3*a^2*c^{(5/2)})$

fricas [B] time = 0.87, size = 1120, normalized size = 7.09

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] $[-1/6*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*\text{sqrt}(b/(b*c - a*d))*\log((b*d*x^3 + 2*b*c - a*d + 2*\text{sqrt}(d*x^3 + c))*(b*c - a*d)*\text{sqrt}(b/(b*c - a*d)))/(b*x^3 + a) - ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*\text{sqrt}(c)*\log((d*x^3 + 2*\text{sqrt}(d*x^3 + c))*\text{sqrt}(c) + 2*c)/x^3) + 2*(a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*\text{sqrt}(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/6*(4*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*\text{sqrt}(-b/(b*c - a*d))*\text{arctan}(-\text{sqrt}(d*x^3 + c)*(b*c - a*d)*\text{sqrt}(-b/(b*c - a*d)))/(b*d*x^3 + b*c)) - ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*\text{sqrt}(c)*\log((d*x^3 + 2*\text{sqrt}(d*x^3 + c))*\text{sqrt}(c) + 2*c)/x^3) + 2*(a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*\text{sqrt}(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/3*((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*\text{sqrt}(-c)*\text{arctan}(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + (b^2*c^3*d*x^6 + b^2*c^4*x^3)*\text{sqrt}(b/(b*c - a*d))*\log((b*d*x^3 + 2*b*c - a*d + 2*\text{sqrt}(d*x^3 + c))*(b*c - a*d)*\text{sqrt}(b/(b*c - a*d)))/(b*x^3 + a)) + (a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*\text{sqrt}(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/3*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*\text{sqrt}(-b/(b*c - a*d))*\text{arctan}(-\text{sqrt}(d*x^3 + c)*(b*c - a*d)*\text{sqrt}(-b/(b*c - a*d)))/(b*d*x^3 + b*c)) + ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*\text{sqrt}(-c)*\text{arctan}(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + (a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*\text{sqrt}(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3)]$

giac [A] time = 0.17, size = 173, normalized size = 1.09

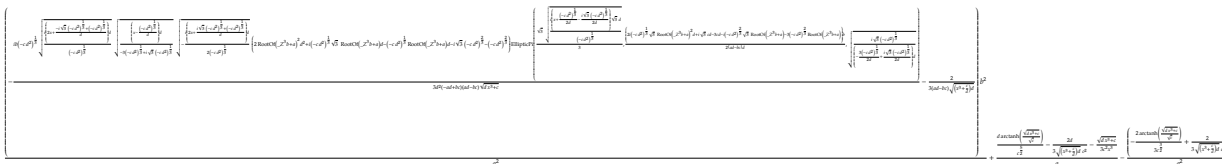
$$\frac{2b^3 \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3(a^2bc - a^3d)\sqrt{-b^2c+abd}} - \frac{(dx^3+c)bcd - 3(dx^3+c)ad^2 + 2acd^2}{3(abc^3 - a^2c^2d)\left((dx^3+c)^{\frac{3}{2}} - \sqrt{dx^3+cc}\right)} - \frac{(2bc + 3ad) \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a^2\sqrt{-c}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $\frac{2}{3}b^3 \arctan\left(\frac{\sqrt{d x^3 + c} b}{\sqrt{-b^2 c + a b d}}\right) / \left((a^2 b c - a^3 d) \sqrt{-b^2 c + a b d}\right) - \frac{1}{3} \left(\frac{(d x^3 + c) b c d - 3(d x^3 + c) a d^2 + 2 a^2 c d^2}{(a b c^3 - a^2 c^2 d) (d x^3 + c)^{3/2} - \sqrt{d x^3 + c} c}\right) - \frac{1}{3} (2 b c + 3 a d) \arctan\left(\frac{\sqrt{d x^3 + c}}{\sqrt{-c}}\right) / (a^2 \sqrt{-c} c^2)$

maple [C] time = 0.41, size = 575, normalized size = 3.64



Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x)

[Out] $\frac{1}{a} \left(-\frac{1}{3} (d x^3 + c)^{1/2} / c^2 x^3 - \frac{2}{3} / ((x^3 + c/d) d)^{1/2} / c^2 d + d \operatorname{arctanh}\left(\frac{d x^3 + c}{c}\right)^{1/2} / c^{5/2} \right) + \frac{1}{a^2 b^2} \left(-\frac{2}{3} / (a d - b c) / ((x^3 + c/d) d)^{1/2} - \frac{1}{3} I b / d^2 \sum \frac{1}{(-a d + b c) (a d - b c)} (-c d^2)^{1/3} (1/2 I (2 x + (-I 3^{1/2}) (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d) / (-c d^2)^{1/3} d^{1/2} \right) \left((x - (-c d^2)^{1/3} / d) / (-3 (-c d^2)^{1/3} + I 3^{1/2} (-c d^2)^{1/3}) d^{1/2} \right) \left(-1/2 I (2 x + (I 3^{1/2}) (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d \right) / (-c d^2)^{1/3} d^{1/2} \left(\frac{1}{2} / (d x^3 + c)^{1/2} \left(2 \alpha^2 d^2 + I (-c d^2)^{1/3} 3^{1/2} \alpha d - (-c d^2)^{1/3} \alpha d - I 3^{1/2} (-c d^2)^{2/3} - (-c d^2)^{2/3} \right) \operatorname{EllipticPi}\left(\frac{1}{3} 3^{1/2} (I (x + 1/2 (-c d^2)^{1/3} / d - 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) 3^{1/2}) / (-c d^2)^{1/3} d^{1/2}, \frac{1}{2} (2 I (-c d^2)^{1/3} 3^{1/2} \alpha^2 d + I 3^{1/2} c d - 3 c d - I (-c d^2)^{2/3} 3^{1/2} \alpha - 3 (-c d^2)^{2/3} \alpha) / (a d - b c) b / d, (I 3^{1/2} (-c d^2)^{1/3} / (-3/2 (-c d^2)^{1/3} / d + 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) / d \right)^{1/2} \right) - \frac{1}{a^2 b} \left(\frac{2}{3} / ((x^3 + c/d) d)^{1/2} / c - \frac{2}{3} \operatorname{arctanh}\left(\frac{d x^3 + c}{c}\right)^{1/2} / c^{3/2} \right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b x^3 + a) (d x^3 + c)^{\frac{3}{2}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^4), x)

mupad [B] time = 10.47, size = 597, normalized size = 3.78

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x)`

[Out]
$$\left(\frac{\log\left(\frac{(c + dx^3)^{1/2} - c^{1/2}}{(c + dx^3)^{1/2} + c^{1/2}}\right)^3}{x^6} \cdot (3ad + 2bc) \right) / (6a^2c^{5/2}) - (c + dx^3)^{1/2} / (3ac^2x^3) - \left(\frac{c \left((c + dx^3)^{1/2} - c^{1/2} \right) \left((3a^2d^4 + 15b^2c^2d^2 + 24ab^2cd^3) / (8a^3c^5) + (c + dx^3)^{1/2} \right)}{8a^3c^5} + \frac{c \left((c + dx^3)^{1/2} - c^{1/2} \right) \left((3b^2d^4) / (8a^3c^5) + (b^2d^4(5ad - 3bc)) / (8a^3c^4(bc^2 - acd)) \right)}{8a^3c^5} - \frac{b^2d^4(ad + 2bc)(5ad - 3bc)}{4a^3c^5(bc^2 - acd)} \right) / d - \left(\frac{3b^2d^3(ad + 2bc)}{4a^3c^5} + \frac{d(5ad - 3bc)(3a^2d^4 + 15b^2c^2d^2 + 24ab^2cd^3)}{24a^3c^5(bc^2 - acd)} \right) / d - \left(\frac{d^2(5ad - 3bc)(6a^2d^2 + 3b^2c^2 + 14ab^2cd)}{12a^3c^4(bc^2 - acd)} \right) / d - \left(\frac{d(6a^2d^2 + 3b^2c^2 + 14ab^2cd)}{4a^3c^4} + \frac{d^2(5ad - 3bc)(13ad + 18bc)}{24a^2c^3(bc^2 - acd)} \right) / d + \left(\frac{d(13ad + 18bc)}{8a^2c^3} - \frac{d(3ad + 2bc)(5ad - 3bc)}{6a^2c^2(bc^2 - acd)} \right) / d - \frac{(3ad + 2bc) / (2a^2c^2)}{(c + dx^3)^{1/2}} + \frac{b^{5/2} \log\left(\frac{ad - 2bc + b^{1/2}(c + dx^3)^{1/2}(ad - bc)^{1/2} + b^2dx^3}{(a + bx^3)}\right)}{3a^2(ad - bc)^{3/2}}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + bx^3) (c + dx^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(3/2),x)`

[Out] `Integral(1/(x**4*(a + b*x**3)*(c + d*x**3)**(3/2)), x)`

$$3.273 \quad \int \frac{x^{11} \sqrt{c+dx^3}}{(8c-dx^3)^2} dx$$

Optimal. Leaf size=117

$$-\frac{3968c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4} + \frac{2c\sqrt{c+dx^3}(1146c+47dx^3)}{15d^4} + \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d(8c-dx^3)}$$

Rubi [A] time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {446, 97, 153, 147, 63, 206}

$$-\frac{3968c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4} + \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{2c\sqrt{c+dx^3}(1146c+47dx^3)}{15d^4} + \frac{x^9\sqrt{c+dx^3}}{3d(8c-dx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]

[Out] (7*x^6*Sqrt[c + d*x^3])/(15*d^2) + (x^9*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)) + (2*c*Sqrt[c + d*x^3]*(1146*c + 47*d*x^3))/(15*d^4) - (3968*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^4)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 97

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 147

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_Symbol] := -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]

```

Rule 153

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))
^(p_.)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rule 446

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3\sqrt{c+dx}}{(8c-dx)^2} dx, x, x^3 \right) \\
&= \frac{x^9\sqrt{c+dx^3}}{3d(8c-dx^3)} - \frac{\text{Subst} \left(\int \frac{x^2 \left(3c + \frac{7dx}{2}\right)}{(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{3d} \\
&= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d(8c-dx^3)} + \frac{2 \text{Subst} \left(\int \frac{x \left(-56c^2d - \frac{141}{2}cd^2x\right)}{(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{15d^3} \\
&= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d(8c-dx^3)} + \frac{2c\sqrt{c+dx^3}(1146c+47dx^3)}{15d^4} - \frac{(1984c^3) \text{Subst} \left(\int \frac{1}{(8c-dx)} dx, x, x^3 \right)}{3d^3} \\
&= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d(8c-dx^3)} + \frac{2c\sqrt{c+dx^3}(1146c+47dx^3)}{15d^4} - \frac{(3968c^3) \text{Subst} \left(\int \frac{1}{9c-x^2} dx, x, x^3 \right)}{3d^4} \\
&= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d(8c-dx^3)} + \frac{2c\sqrt{c+dx^3}(1146c+47dx^3)}{15d^4} - \frac{3968c^{5/2} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{9d^4}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 101, normalized size = 0.86

$$\frac{19840c^{5/2}(8c-dx^3) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) + 6\sqrt{c+dx^3}(-9168c^3 + 770c^2dx^3 + 19cd^2x^6 + d^3x^9)}{45d^4(dx^3-8c)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2, x]

[Out] (6*Sqrt[c + d*x^3]*(-9168*c^3 + 770*c^2*d*x^3 + 19*c*d^2*x^6 + d^3*x^9) + 19840*c^(5/2)*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^4*(-8*c + d*x^3))

IntegrateAlgebraic [A] time = 0.12, size = 95, normalized size = 0.81

$$-\frac{3968c^{5/2} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{9d^4} - \frac{2\sqrt{c+dx^3}(9168c^3 - 770c^2dx^3 - 19cd^2x^6 - d^3x^9)}{15d^4(dx^3-8c)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]

[Out] (-2*Sqrt[c + d*x^3]*(9168*c^3 - 770*c^2*d*x^3 - 19*c*d^2*x^6 - d^3*x^9))/(15*d^4*(-8*c + d*x^3)) - (3968*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^4)

fricas [A] time = 0.64, size = 219, normalized size = 1.87

$$\left[\frac{2 \left(4960 (c^2 dx^3 - 8c^3) \sqrt{c} \log \left(\frac{dx^3 - 6\sqrt{dx^3+c}\sqrt{c+10c}}{dx^3-8c} \right) + 3 (d^3 x^9 + 19cd^2 x^6 + 770c^2 dx^3 - 9168c^3) \sqrt{dx^3+c} \right)}{45 (d^3 x^3 - 8cd^4)}, \frac{2 \left(9920 (c^2 dx^3 - 8c^3) \sqrt{-c} \arctan \left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c} \right) + 3 (d^3 x^9 + 19cd^2 x^6 + 770c^2 dx^3 - 9168c^3) \sqrt{dx^3+c} \right)}{45 (d^3 x^3 - 8cd^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] [2/45*(4960*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 3*(d^3*x^9 + 19*c*d^2*x^6 + 770*c^2*d*x^3 - 9168*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/45*(9920*(c^2*d*x^3 - 8*c^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(d^3*x^9 + 19*c*d^2*x^6 + 770*c^2*d*x^3 - 9168*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]

giac [A] time = 0.16, size = 110, normalized size = 0.94

$$\frac{3968c^3 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d^4} - \frac{512\sqrt{dx^3+c}c^3}{3(dx^3-8c)d^4} + \frac{2\left(\left(dx^3+c\right)^{\frac{5}{2}}d^{16} + 25\left(dx^3+c\right)^{\frac{3}{2}}cd^{16} + 960\sqrt{dx^3+c}c^2d^{16}\right)}{15d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] 3968/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/3*sqrt(d*x^3 + c)*c^3/((d*x^3 - 8*c)*d^4) + 2/15*((d*x^3 + c)^(5/2)*d^16 + 25*(d*x^3 + c)^(3/2)*c*d^16 + 960*sqrt(d*x^3 + c)*c^2*d^16)/d^20

maple [C] time = 0.28, size = 952, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)

[Out] 1/d^3*(d*(2/15*(d*x^3+c)^(1/2)*x^6+2/45*(d*x^3+c)^(1/2)*c/d*x^3-4/45*(d*x^3+c)^(1/2)*c^2/d^2)+32/9*c/d*(d*x^3+c)^(3/2))+512*c^3/d^3*(-1/3/d*(d*x^3+c)^(1/2)/(d*x^3-8*c)+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3)^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3))^(1/3)+(-c*d^2)^(1/3))/d)

$$2)^{(1/3)/d}/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)}*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=RootOf(_Z^3*d-8*c)))+192*c^2/d^3*(2/3*(d*x^3+c)^{(1/2)}/d+1/3*I/d^3*2^{(1/2)}*sum((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)}*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=RootOf(_Z^3*d-8*c))$$

maxima [A] time = 1.24, size = 107, normalized size = 0.91

$$\frac{2 \left(4960 c^2 \log \left(\frac{\sqrt{dx^3+c} - 3\sqrt{c}}{\sqrt{dx^3+c} + 3\sqrt{c}} \right) + 3 (dx^3 + c)^{\frac{5}{2}} + 75 (dx^3 + c)^{\frac{3}{2}} c + 2880 \sqrt{dx^3 + c} c^2 - \frac{3840 \sqrt{dx^3+c} c^3}{dx^3-8c} \right)}{45 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")

[Out] 2/45*(4960*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 75*(d*x^3 + c)^(3/2)*c + 2880*sqrt(d*x^3 + c)*c^2 - 3840*sqrt(d*x^3 + c)*c^3/(d*x^3 - 8*c))/d^4

mupad [B] time = 4.09, size = 127, normalized size = 1.09

$$\frac{1984 c^{5/2} \ln \left(\frac{10c+dx^3-6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3} \right)}{9 d^4} + \frac{1972 c^2 \sqrt{dx^3+c}}{15 d^4} + \frac{2 x^6 \sqrt{dx^3+c}}{15 d^2} + \frac{18 c x^3 \sqrt{dx^3+c}}{5 d^3} + \frac{512 c^3 \sqrt{dx^3+c}}{3 d^4 (8c-dx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^11*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)

[Out] (1984*c^(5/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^4) + (1972*c^2*(c + d*x^3)^(1/2))/(15*d^4) + (2*x^6*(c + d*x^3)^(1/2))/(15*d^2) + (18*c*x^3*(c + d*x^3)^(1/2))/(5*d^3) + (512*c^3*(c + d*x^3)^(1/2))/(3*d^4*(8*c - d*x^3))

$$\frac{1}{2})/(15*d^2) + (18*c*x^3*(c + d*x^3)^{(1/2)})/(5*d^3) + (512*c^3*(c + d*x^3)^{(1/2)})/(3*d^4*(8*c - d*x^3))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)

[Out] Timed out

$$3.274 \quad \int \frac{x^8 \sqrt{c+dx^3}}{(8c-dx^3)^2} dx$$

Optimal. Leaf size=102

$$-\frac{352c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3} + \frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)} + \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2(c+dx^3)^{3/2}}{9d^3}$$

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {446, 89, 80, 50, 63, 206}

$$-\frac{352c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3} + \frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)} + \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2(c+dx^3)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

[In] Int[(x^8*sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]

[Out] (352*c*sqrt[c + d*x^3])/(27*d^3) + (2*(c + d*x^3)^(3/2))/(9*d^3) + (64*c*(c + d*x^3)^(3/2))/(27*d^3*(8*c - d*x^3)) - (352*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*sqrt[c])])/(9*d^3)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^(2)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8 \sqrt{c + dx^3}}{(8c - dx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 \sqrt{c + dx}}{(8c - dx)^2} dx, x, x^3 \right) \\
&= \frac{64c (c + dx^3)^{3/2}}{27d^3 (8c - dx^3)} - \frac{\text{Subst} \left(\int \frac{\sqrt{c+dx} (104c^2d + 9cd^2x)}{8c-dx} dx, x, x^3 \right)}{27cd^3} \\
&= \frac{2 (c + dx^3)^{3/2}}{9d^3} + \frac{64c (c + dx^3)^{3/2}}{27d^3 (8c - dx^3)} - \frac{(176c) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{8c-dx} dx, x, x^3 \right)}{27d^2} \\
&= \frac{352c\sqrt{c + dx^3}}{27d^3} + \frac{2 (c + dx^3)^{3/2}}{9d^3} + \frac{64c (c + dx^3)^{3/2}}{27d^3 (8c - dx^3)} - \frac{(176c^2) \text{Subst} \left(\int \frac{1}{(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{3d^2} \\
&= \frac{352c\sqrt{c + dx^3}}{27d^3} + \frac{2 (c + dx^3)^{3/2}}{9d^3} + \frac{64c (c + dx^3)^{3/2}}{27d^3 (8c - dx^3)} - \frac{(352c^2) \text{Subst} \left(\int \frac{1}{9c-x^2} dx, x, \sqrt{c + dx^3} \right)}{3d^3} \\
&= \frac{352c\sqrt{c + dx^3}}{27d^3} + \frac{2 (c + dx^3)^{3/2}}{9d^3} + \frac{64c (c + dx^3)^{3/2}}{27d^3 (8c - dx^3)} - \frac{352c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{9d^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 90, normalized size = 0.88

$$\frac{352c^{3/2} (8c - dx^3) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) + 2\sqrt{c + dx^3} (-488c^2 + 41cdx^3 + d^2x^6)}{9d^3 (dx^3 - 8c)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]

[Out] (2*sqrt[c + d*x^3]*(-488*c^2 + 41*c*d*x^3 + d^2*x^6) + 352*c^(3/2)*(8*c - d*x^3)*ArcTanh[sqrt[c + d*x^3]/(3*sqrt[c])])/(9*d^3*(-8*c + d*x^3))

IntegrateAlgebraic [A] time = 0.08, size = 84, normalized size = 0.82

$$-\frac{352c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{9d^3} - \frac{2\sqrt{c + dx^3} (488c^2 - 41cdx^3 - d^2x^6)}{9d^3 (dx^3 - 8c)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8*sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]

[Out] (-2*sqrt[c + d*x^3]*(488*c^2 - 41*c*d*x^3 - d^2*x^6))/(9*d^3*(-8*c + d*x^3)) - (352*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*sqrt[c])])/(9*d^3)

fricas [A] time = 0.68, size = 191, normalized size = 1.87

$$\left[\frac{2 \left(88 (cdx^3 - 8c^2) \sqrt{c} \log \left(\frac{dx^3 - 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c} \right) + (d^2x^6 + 41cdx^3 - 488c^2) \sqrt{dx^3+c} \right)}{9(d^4x^3 - 8cd^3)}, \frac{2 \left(176 (cdx^3 - 8c^2) \sqrt{-c} \arctan \left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c} \right) + (d^2x^6 + 41cdx^3 - 488c^2) \sqrt{dx^3+c} \right)}{9(d^4x^3 - 8cd^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] [2/9*(88*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3), 2/9*(176*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]

giac [A] time = 0.18, size = 93, normalized size = 0.91

$$\frac{352c^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d^3} - \frac{64\sqrt{dx^3+c}c^2}{3(dx^3-8c)d^3} + \frac{2\left((dx^3+c)^{\frac{3}{2}}d^6 + 48\sqrt{dx^3+c}cd^6\right)}{9d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] 352/9*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 64/3*sqrt(d*x^3 + c)*c^2/((d*x^3 - 8*c)*d^3) + 2/9*((d*x^3 + c)^(3/2)*d^6 + 48*sqrt(d*x^3 + c)*c*d^6)/d^9

maple [C] time = 0.21, size = 892, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)

[Out] 2/9*(d*x^3+c)^(3/2)/d^3+64*c^2/d^2*(-1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)

$$\begin{aligned}
 & *(-c*d^2)^{(2/3)} - (-c*d^2)^{(2/3)} * \text{EllipticPi}\left(\frac{1}{3} * 3^{(1/2)} * (I * (x + 1/2 * (-c*d^2)^{(1/3)} / d - 1/2 * I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / d) * 3^{(1/2)} / (-c*d^2)^{(1/3)} * d)^{(1/2)}, -1/18 * (2 * I * (-c*d^2)^{(1/3)} * 3^{(1/2)} * _alpha^2 * d + I * 3^{(1/2)} * c * d - 3 * c * d - I * (-c*d^2)^{(2/3)} * 3^{(1/2)} * _alpha - 3 * (-c*d^2)^{(2/3)} * _alpha) / c / d, (I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / (-3/2 * (-c*d^2)^{(1/3)} / d + 1/2 * I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / d) / d)^{(1/2)}\right), _alpha = \text{RootOf}(_Z^3 * d - 8 * c)) \\
 & + 16 * c / d^2 * (2/3 * (d * x^3 + c)^{(1/2)} / d + 1/3 * I / d^3 * 2^{(1/2)} * \text{sum}((-c*d^2)^{(1/3)} * (1/2 * I * (2 * x + (-I * 3^{(1/2)} * (-c*d^2)^{(1/3)} + (-c*d^2)^{(1/3)}) / d) / (-c*d^2)^{(1/3)} * d)^{(1/2)} * ((x - (-c*d^2)^{(1/3)} / d) / (-3 * (-c*d^2)^{(1/3)} + I * 3^{(1/2)} * (-c*d^2)^{(1/3)}) * d)^{(1/2)} * (-1/2 * I * (2 * x + (I * 3^{(1/2)} * (-c*d^2)^{(1/3)} + (-c*d^2)^{(1/3)}) / d) / (-c*d^2)^{(1/3)} * d)^{(1/2)} / (d * x^3 + c)^{(1/2)} * (2 * _alpha^2 * d^2 + I * (-c*d^2)^{(1/3)} * 3^{(1/2)} * _alpha * d - (-c*d^2)^{(1/3)} * _alpha * d - I * 3^{(1/2)} * (-c*d^2)^{(2/3)} - (-c*d^2)^{(2/3)}) * \text{EllipticPi}\left(\frac{1}{3} * 3^{(1/2)} * (I * (x + 1/2 * (-c*d^2)^{(1/3)} / d - 1/2 * I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / d) * 3^{(1/2)} / (-c*d^2)^{(1/3)} * d)^{(1/2)}, -1/18 * (2 * I * (-c*d^2)^{(1/3)} * 3^{(1/2)} * _alpha^2 * d + I * 3^{(1/2)} * c * d - 3 * c * d - I * (-c*d^2)^{(2/3)} * 3^{(1/2)} * _alpha - 3 * (-c*d^2)^{(2/3)} * _alpha) / c / d, (I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / (-3/2 * (-c*d^2)^{(1/3)} / d + 1/2 * I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / d) / d)^{(1/2)}\right), _alpha = \text{RootOf}(_Z^3 * d - 8 * c))
 \end{aligned}$$

maxima [A] time = 1.22, size = 91, normalized size = 0.89

$$\frac{2 \left(88 c^{\frac{3}{2}} \log \left(\frac{\sqrt{d x^3 + c} - 3 \sqrt{c}}{\sqrt{d x^3 + c} + 3 \sqrt{c}} \right) + (d x^3 + c)^{\frac{3}{2}} + 48 \sqrt{d x^3 + c} c - \frac{96 \sqrt{d x^3 + c} c^2}{d x^3 - 8 c} \right)}{9 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")

[Out] 2/9*(88*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c))) + (d*x^3 + c)^(3/2) + 48*sqrt(d*x^3 + c)*c - 96*sqrt(d*x^3 + c)*c^2/(d*x^3 - 8*c))/d^3

mupad [B] time = 4.01, size = 107, normalized size = 1.05

$$\frac{98 c \sqrt{d x^3 + c}}{9 d^3} + \frac{176 c^{3/2} \ln \left(\frac{10 c + d x^3 - 6 \sqrt{c} \sqrt{d x^3 + c}}{8 c - d x^3} \right)}{9 d^3} + \frac{2 x^3 \sqrt{d x^3 + c}}{9 d^2} + \frac{64 c^2 \sqrt{d x^3 + c}}{3 d^3 (8 c - d x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)

[Out] (98*c*(c + d*x^3)^(1/2))/(9*d^3) + (176*c^(3/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^3) + (2*x^3*(c + d*x^3)^(1/2))/(9*d^2) + (64*c^2*(c + d*x^3)^(1/2))/(3*d^3*(8*c - d*x^3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 \sqrt{c + dx^3}}{(-8c + dx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)

[Out] Integral(x**8*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)

$$3.275 \quad \int \frac{x^5 \sqrt{c+dx^3}}{(8c-dx^3)^2} dx$$

Optimal. Leaf size=82

$$\frac{8(c+dx^3)^{3/2}}{27d^2(8c-dx^3)} + \frac{26\sqrt{c+dx^3}}{27d^2} - \frac{26\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 78, 50, 63, 206}

$$\frac{8(c+dx^3)^{3/2}}{27d^2(8c-dx^3)} + \frac{26\sqrt{c+dx^3}}{27d^2} - \frac{26\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

Antiderivative was successfully verified.

[In] Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]

[Out] (26*Sqrt[c + d*x^3])/(27*d^2) + (8*(c + d*x^3)^(3/2))/(27*d^2*(8*c - d*x^3)) - (26*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(

$f*(p + 1)*(c*f - d*e)), x] - \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

Rule 206

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 446

$\text{Int}[(x_)^{(m_.)*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)*((c_) + (d_.)*(x_)^{(n_.)})^{(q_.)}}, x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q}, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5 \sqrt{c + dx^3}}{(8c - dx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x \sqrt{c + dx}}{(8c - dx)^2} dx, x, x^3 \right) \\
 &= \frac{8(c + dx^3)^{3/2}}{27d^2(8c - dx^3)} - \frac{13 \text{Subst} \left(\int \frac{\sqrt{c+dx}}{8c-dx} dx, x, x^3 \right)}{27d} \\
 &= \frac{26\sqrt{c + dx^3}}{27d^2} + \frac{8(c + dx^3)^{3/2}}{27d^2(8c - dx^3)} - \frac{(13c) \text{Subst} \left(\int \frac{1}{(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{3d} \\
 &= \frac{26\sqrt{c + dx^3}}{27d^2} + \frac{8(c + dx^3)^{3/2}}{27d^2(8c - dx^3)} - \frac{(26c) \text{Subst} \left(\int \frac{1}{9c-x^2} dx, x, \sqrt{c + dx^3} \right)}{3d^2} \\
 &= \frac{26\sqrt{c + dx^3}}{27d^2} + \frac{8(c + dx^3)^{3/2}}{27d^2(8c - dx^3)} - \frac{26\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{9d^2}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.96

$$\frac{6\sqrt{c+dx^3}(dx^3-12c) + 26\sqrt{c}(8c-dx^3)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2(dx^3-8c)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]

[Out] (6*(-12*c + d*x^3)*Sqrt[c + d*x^3] + 26*Sqrt[c]*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2*(-8*c + d*x^3))

IntegrateAlgebraic [A] time = 0.07, size = 73, normalized size = 0.89

$$-\frac{2\sqrt{c+dx^3}(12c-dx^3)}{3d^2(dx^3-8c)} - \frac{26\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]

[Out] (-2*(12*c - d*x^3)*Sqrt[c + d*x^3])/(3*d^2*(-8*c + d*x^3)) - (26*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2)

fricas [A] time = 0.47, size = 165, normalized size = 2.01

$$\left[\frac{13(dx^3-8c)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c+10c}}{dx^3-8c}\right) + 6\sqrt{dx^3+c}(dx^3-12c)}{9(d^3x^3-8cd^2)}, \frac{2\left(13(dx^3-8c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 3\sqrt{dx^3+c}(dx^3-12c)\right)}{9(d^3x^3-8cd^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] [1/9*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 6*sqrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2), 2/9*(13*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*sqrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2)]

giac [A] time = 0.16, size = 69, normalized size = 0.84

$$\frac{26c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d^2} + \frac{2\sqrt{dx^3+c}}{3d^2} - \frac{8\sqrt{dx^3+c}c}{3(dx^3-8c)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] 26/9*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) + 2/3*sqrt(d*x^3 + c)/d^2 - 8/3*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d^2)

maple [C] time = 0.17, size = 874, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)

[Out] 8*c/d*(-1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2),_alpha=RootOf(_Z^3*d-8*c))+1/d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2),_alpha=RootOf(_Z^3*d-8*c)))

maxima [A] time = 1.25, size = 79, normalized size = 0.96

$$\frac{13\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+6\sqrt{dx^3+c}-\frac{24\sqrt{dx^3+c}}{dx^3-8c}}{9d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")

[Out] $1/9*(13*\sqrt{c}*\log((\sqrt{d*x^3 + c}) - 3*\sqrt{c}))/(\sqrt{d*x^3 + c}) + 3*\sqrt{c}) + 6*\sqrt{d*x^3 + c} - 24*\sqrt{d*x^3 + c}*c/(d*x^3 - 8*c))/d^2$

mupad [B] time = 3.99, size = 87, normalized size = 1.06

$$\frac{2\sqrt{dx^3+c}}{3d^2} + \frac{13\sqrt{c}\ln\left(\frac{10c+dx^3-6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{9d^2} + \frac{8c\sqrt{dx^3+c}}{3d^2(8c-dx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)`

[Out] $(2*(c + d*x^3)^{(1/2)})/(3*d^2) + (13*c^{(1/2)}*\log((10*c + d*x^3 - 6*c^{(1/2)}*(c + d*x^3)^{(1/2)})/(8*c - d*x^3)))/(9*d^2) + (8*c*(c + d*x^3)^{(1/2)})/(3*d^2*(8*c - d*x^3))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \sqrt{c + dx^3}}{(-8c + dx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)`

[Out] `Integral(x**5*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)`

$$3.276 \quad \int \frac{x^2 \sqrt{c+dx^3}}{(8c-dx^3)^2} dx$$

Optimal. Leaf size=64

$$\frac{\sqrt{c+dx^3}}{3d(8c-dx^3)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {444, 47, 63, 206}

$$\frac{\sqrt{c+dx^3}}{3d(8c-dx^3)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]

[Out] Sqrt[c + d*x^3]/(3*d*(8*c - d*x^3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(9*Sqrt[c]*d)

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```


Q[a, 0] || LtQ[b, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2 \sqrt{c + dx^3}}{(8c - dx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c + dx}}{(8c - dx)^2} dx, x, x^3 \right) \\ &= \frac{\sqrt{c + dx^3}}{3d(8c - dx^3)} - \frac{1}{6} \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right) \\ &= \frac{\sqrt{c + dx^3}}{3d(8c - dx^3)} - \frac{\text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{3d} \\ &= \frac{\sqrt{c + dx^3}}{3d(8c - dx^3)} - \frac{\tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{9\sqrt{c}d} \end{aligned}$$

Mathematica [A] time = 0.06, size = 61, normalized size = 0.95

$$\frac{\frac{3\sqrt{c+dx^3}}{8c-dx^3} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}}{9d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]

[Out] ((3*Sqrt[c + d*x^3])/(8*c - d*x^3) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c])/(9*d)

IntegrateAlgebraic [A] time = 0.06, size = 63, normalized size = 0.98

$$-\frac{\sqrt{c + dx^3}}{3d(dx^3 - 8c)} - \frac{\tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{9\sqrt{c}d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]

[Out] -1/3*sqrt[c + d*x^3]/(d*(-8*c + d*x^3)) - ArcTanh[Sqrt[c + d*x^3]/(3*sqrt[c])]/(9*sqrt[c]*d)

fricas [A] time = 0.49, size = 149, normalized size = 2.33

$$\left[\frac{(dx^3 - 8c)\sqrt{c} \log\left(\frac{dx^3 - 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c}\right) - 6\sqrt{dx^3+c}c (dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - 3\sqrt{dx^3+c}c}{18(cd^2x^3 - 8c^2d)}, \frac{(dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - 3\sqrt{dx^3+c}c}{9(cd^2x^3 - 8c^2d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] [1/18*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c*d^2*x^3 - 8*c^2*d), 1/9*((d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(d*x^3 + c)*c)/(c*d^2*x^3 - 8*c^2*d)]

giac [A] time = 0.16, size = 53, normalized size = 0.83

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d} - \frac{\sqrt{dx^3+c}}{3(dx^3-8c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] 1/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 1/3*sqrt(d*x^3 + c)/((d*x^3 - 8*c)*d)

maple [C] time = 0.18, size = 439, normalized size = 6.86

$$\frac{\sqrt{dx^3+c}}{3(dx^3-8c)d} + \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d} - \frac{\sqrt{dx^3+c}}{3(dx^3-8c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)

[Out] -1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)

$$\begin{aligned} &^{(1/2)} * ((x - (-c*d^2)^{(1/3)})/d) / (-3*(-c*d^2)^{(1/3)} + I*3^{(1/2)}*(-c*d^2)^{(1/3)}) * d \\ &)^{(1/2)} * (-1/2*I*(2*x + (I*3^{(1/2)}*(-c*d^2)^{(1/3)} + (-c*d^2)^{(1/3)})/d) / (-c*d^2)^{(1/3)} * d)^{(1/2)} / (d*x^3 + c)^{(1/2)} * (2*_alpha^2*d^2 + I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha * d - (-c*d^2)^{(1/3)}*_alpha * d - I*3^{(1/2)}*(-c*d^2)^{(2/3)} - (-c*d^2)^{(2/3)}) * \text{EllipticPi} \\ &(1/3*3^{(1/2)}*(I*(x + 1/2*(-c*d^2)^{(1/3)})/d - 1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)})/d) * 3^{(1/2)} / (-c*d^2)^{(1/3)} * d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2 * d + I*3^{(1/2)}*c*d - 3*c*d - I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha - 3*(-c*d^2)^{(2/3)}*_alpha) / c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}) / (-3/2*(-c*d^2)^{(1/3)})/d + 1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)})/d) / d)^{(1/2)}, _alpha = \text{RootOf}(_Z^3*d - 8*c)) \end{aligned}$$

maxima [A] time = 1.39, size = 66, normalized size = 1.03

$$\frac{\log\left(\frac{\sqrt{dx^3+c-3\sqrt{c}}}{\sqrt{dx^3+c+3\sqrt{c}}}\right) - \frac{6\sqrt{dx^3+c}}{dx^3-8c}}{\sqrt{c}} \frac{1}{18d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")

[Out] 1/18*(log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/sqrt(c) - 6*sqrt(d*x^3 + c)/(d*x^3 - 8*c))/d

mupad [B] time = 3.93, size = 72, normalized size = 1.12

$$\frac{\ln\left(\frac{10c+dx^3-6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{18\sqrt{c}d} + \frac{\sqrt{dx^3+c}}{3d(8c-dx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)

[Out] log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(18*c^(1/2)*d) + (c + d*x^3)^(1/2)/(3*d*(8*c - d*x^3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{c+dx^3}}{(-8c+dx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)

[Out] Integral(x**2*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)

$$3.277 \quad \int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2} dx$$

Optimal. Leaf size=88

$$\frac{5 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}} + \frac{\sqrt{c+dx^3}}{24c(8c-dx^3)}$$

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {446, 99, 156, 63, 208, 206}

$$\frac{5 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}} + \frac{\sqrt{c+dx^3}}{24c(8c-dx^3)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]

[Out] Sqrt[c + d*x^3]/(24*c*(8*c - d*x^3)) + (5*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])))/(288*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 99

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/(m + 1)*(b*e - a*f), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n + p + 2)*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c+dx}}{x(8c-dx)^2} dx, x, x^3 \right) \\
&= \frac{\sqrt{c+dx^3}}{24c(8c-dx^3)} - \frac{\text{Subst} \left(\int \frac{-c-\frac{dx}{2}}{x(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{24c} \\
&= \frac{\sqrt{c+dx^3}}{24c(8c-dx^3)} + \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{192c} + \frac{(5d) \text{Subst} \left(\int \frac{1}{(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{192c} \\
&= \frac{\sqrt{c+dx^3}}{24c(8c-dx^3)} + \frac{5 \text{Subst} \left(\int \frac{1}{9c-x^2} dx, x, \sqrt{c+dx^3} \right)}{96c} + \frac{\text{Subst} \left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{96cd} \\
&= \frac{\sqrt{c+dx^3}}{24c(8c-dx^3)} + \frac{5 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{288c^{3/2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{96c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 102, normalized size = 1.16

$$\frac{12\sqrt{c}\sqrt{c+dx^3} + 5(8c-dx^3)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 3(dx^3-8c)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{288c^{3/2}(8c-dx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2), x]

[Out] (12*Sqrt[c]*Sqrt[c + d*x^3] + 5*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + 3*(-8*c + d*x^3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(288*c^(3/2)*(8*c - d*x^3))

IntegrateAlgebraic [A] time = 0.08, size = 88, normalized size = 1.00

$$\frac{5 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{288c^{3/2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{96c^{3/2}} + \frac{\sqrt{c+dx^3}}{24c(8c-dx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2), x]

[Out] Sqrt[c + d*x^3]/(24*c*(8*c - d*x^3)) + (5*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(288*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(3/2))

fricas [A] time = 0.53, size = 226, normalized size = 2.57

$$\left[\frac{5(dx^3 - 8c)\sqrt{c} \log\left(\frac{dx^3 + 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c}\right) + 3(dx^3 - 8c)\sqrt{c} \log\left(\frac{dx^3 - 2\sqrt{dx^3+c}\sqrt{c} + 2c}{x^3}\right) - 24\sqrt{dx^3+c} c - 3(dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - 5(dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - 12\sqrt{dx^3+c} c}{576(c^2 dx^3 - 8c^3)}, \frac{5(dx^3 - 8c)\sqrt{c} \log\left(\frac{dx^3 + 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c}\right) + 3(dx^3 - 8c)\sqrt{c} \log\left(\frac{dx^3 - 2\sqrt{dx^3+c}\sqrt{c} + 2c}{x^3}\right) - 24\sqrt{dx^3+c} c - 3(dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - 5(dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - 12\sqrt{dx^3+c} c}{288(c^2 dx^3 - 8c^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] [1/576*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 3*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^2*d*x^3 - 8*c^3), 1/288*(3*(d*x^3 - 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 5*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x^3 + c)*c)/(c^2*d*x^3 - 8*c^3)]

giac [A] time = 0.16, size = 79, normalized size = 0.90

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c} - \frac{5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{288\sqrt{-c}c} - \frac{\sqrt{dx^3+c}}{24(dx^3-8c)c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 5/288*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/24*sqrt(d*x^3 + c)/((d*x^3 - 8*c)*c)

maple [C] time = 0.18, size = 912, normalized size = 10.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x)

[Out] 1/8/c*d*(-1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2))*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2

/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))-1/64/c^2*d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c)^(1/2))*c^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{(dx^3 - 8c)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x), x)

mupad [B] time = 3.98, size = 76, normalized size = 0.86

$$\frac{5 \operatorname{atanh}\left(\frac{c \sqrt{dx^3+c}}{3 \sqrt{c^3}}\right)}{288 \sqrt{c^3}} - \frac{\operatorname{atanh}\left(\frac{c \sqrt{dx^3+c}}{\sqrt{c^3}}\right)}{96 \sqrt{c^3}} + \frac{\sqrt{dx^3+c}}{8c(24c-3dx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^3)^(1/2)/(x*(8*c - d*x^3)^2),x)

[Out] (5*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(288*(c^3)^(1/2)) - atanh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2))/(96*(c^3)^(1/2)) + (c + d*x^3)^(1/2)/(8*c*(24*c - 3*d*x^3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x(-8c + dx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c)**2,x)

[Out] Integral(sqrt(c + d*x**3)/(x*(-8*c + d*x**3)**2), x)

$$3.278 \quad \int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)^2} dx$$

Optimal. Leaf size=124

$$\frac{7d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}} - \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}} + \frac{d\sqrt{c+dx^3}}{96c^2(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24cx^3(8c-dx^3)}$$

Rubi [A] time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {446, 99, 151, 156, 63, 208, 206}

$$\frac{d\sqrt{c+dx^3}}{96c^2(8c-dx^3)} + \frac{7d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}} - \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}} - \frac{\sqrt{c+dx^3}}{24cx^3(8c-dx^3)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2), x]

[Out] (d*Sqrt[c + d*x^3])/(96*c^2*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*c*x^3*(8*c - d*x^3)) + (7*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1152*c^(5/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])]/(128*c^(5/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 99

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/(m + 1)*(b*e - a*f), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n + p + 2)*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 151

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

```

Rule 156

```

Int[((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 446

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c+dx}}{x^2(8c-dx)^2} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c+dx^3}}{24cx^3(8c-dx^3)} + \frac{\text{Subst} \left(\int \frac{6cd+\frac{3d^2x}{2}}{x(8c-dx)^2\sqrt{c+dx}} dx, x, x^3 \right)}{24c} \\
&= \frac{d\sqrt{c+dx^3}}{96c^2(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24cx^3(8c-dx^3)} - \frac{\text{Subst} \left(\int \frac{-54c^2d^2-9cd^3x}{x(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{1728c^3d} \\
&= \frac{d\sqrt{c+dx^3}}{96c^2(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24cx^3(8c-dx^3)} + \frac{d \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{256c^2} + \frac{(7d^2) \text{Subst} \left(\int \frac{1}{(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{768c^2} \\
&= \frac{d\sqrt{c+dx^3}}{96c^2(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24cx^3(8c-dx^3)} + \frac{\text{Subst} \left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{128c^2} + \frac{(7d) \text{Subst} \left(\int \frac{1}{(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{768c^2} \\
&= \frac{d\sqrt{c+dx^3}}{96c^2(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24cx^3(8c-dx^3)} + \frac{7d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{1152c^{5/2}} - \frac{d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{128c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 97, normalized size = 0.78

$$\frac{7d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) - 9d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right) + \frac{12\sqrt{c} \sqrt{c+dx^3} (4c-dx^3)}{dx^6-8cx^3}}{1152c^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2), x]

[Out] ((12*Sqrt[c]*(4*c - d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 7*d*ArcTan h[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(1152*c^(5/2))

IntegrateAlgebraic [A] time = 0.12, size = 102, normalized size = 0.82

$$\frac{7d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) - d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right) + \frac{\sqrt{c+dx^3} (dx^3 - 4c)}{96c^2x^3(8c-dx^3)}}{1152c^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2), x]

[Out] $((-4*c + d*x^3)*\text{Sqrt}[c + d*x^3])/(96*c^2*x^3*(8*c - d*x^3)) + (7*d*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/(3*\text{Sqrt}[c])])/(1152*c^{(5/2)}) - (d*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/\text{Sqrt}[c])/(128*c^{(5/2)})$

fricas [A] time = 0.52, size = 278, normalized size = 2.24

$$\left[\frac{7(d^2x^6 - 8cdx^3)\sqrt{c} \log\left(\frac{d^3+6\sqrt{d^3+c}\sqrt{c}+10c}{d^3-8c}\right) + 9(d^2x^6 - 8cdx^3)\sqrt{c} \log\left(\frac{d^3-2\sqrt{d^3+c}\sqrt{c}+2c}{x^3}\right) - 24(cd^3 - 4c^2)\sqrt{d^3+c} - 9(d^2x^6 - 8cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{d^3+c}\sqrt{-c}}{c}\right) - 7(d^2x^6 - 8cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{d^3+c}\sqrt{-c}}{3c}\right) - 12(cd^3 - 4c^2)\sqrt{d^3+c}}{2304(c^3dx^6 - 8c^4x^3)}, \frac{-7(d^2x^6 - 8cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{d^3+c}\sqrt{-c}}{c}\right) - 7(d^2x^6 - 8cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{d^3+c}\sqrt{-c}}{3c}\right) - 12(cd^3 - 4c^2)\sqrt{d^3+c}}{1152(c^3dx^6 - 8c^4x^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] $[1/2304*(7*(d^2*x^6 - 8*c*d*x^3)*\text{sqrt}(c)*\log((d*x^3 + 6*\text{sqrt}(d*x^3 + c))*\text{sqrt}(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^2*x^6 - 8*c*d*x^3)*\text{sqrt}(c)*\log((d*x^3 - 2*\text{sqrt}(d*x^3 + c))*\text{sqrt}(c) + 2*c)/x^3) - 24*(c*d*x^3 - 4*c^2)*\text{sqrt}(d*x^3 + c))/(c^3*d*x^6 - 8*c^4*x^3), 1/1152*(9*(d^2*x^6 - 8*c*d*x^3)*\text{sqrt}(-c)*\arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) - 7*(d^2*x^6 - 8*c*d*x^3)*\text{sqrt}(-c)*\arctan(1/3*\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) - 12*(c*d*x^3 - 4*c^2)*\text{sqrt}(d*x^3 + c))/(c^3*d*x^6 - 8*c^4*x^3)]$

giac [A] time = 0.17, size = 113, normalized size = 0.91

$$\frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{128 \sqrt{-c} c^2} - \frac{7 d \arctan\left(\frac{\sqrt{dx^3+c}}{3 \sqrt{-c}}\right)}{1152 \sqrt{-c} c^2} - \frac{(dx^3 + c)^{\frac{3}{2}} d - 5 \sqrt{dx^3 + c} cd}{96 \left((dx^3 + c)^2 - 10 (dx^3 + c) c + 9 c^2 \right) c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] $1/128*d*\arctan(\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*c^2) - 7/1152*d*\arctan(1/3*\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*c^2) - 1/96*((d*x^3 + c)^{(3/2)}*d - 5*\text{sqrt}(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c^2)$

maple [C] time = 0.21, size = 957, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x)

[Out] $1/64/c^2*d^2*(-1/3*(d*x^3+c)^{(1/2)})/(d*x^3-8*c)/d+1/54*I/d^3/c^2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)})*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c$

```

*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c
*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^
2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-
c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c
^2*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)
)-1/256/c^3*d^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)
*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d
)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*
d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)
^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*
_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*(2/3*(d
*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{(dx^3 - 8c)^2 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^4), x)

mupad [B] time = 4.21, size = 117, normalized size = 0.94

$$\frac{\frac{5d\sqrt{dx^3+c}}{32c} - \frac{d(dx^3+c)^{3/2}}{32c^2}}{3(dx^3+c)^2 - 30c(dx^3+c) + 27c^2} + \frac{d \left(\operatorname{atanh}\left(\frac{c^2\sqrt{dx^3+c}}{\sqrt{c^5}}\right) \operatorname{li} - \frac{\operatorname{atanh}\left(\frac{c^2\sqrt{dx^3+c}}{3\sqrt{c^5}}\right) 7i}{9} \operatorname{li} \right)}{128\sqrt{c^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^3)^(1/2)/(x^4*(8*c - d*x^3)^2),x)

```
[Out] ((5*d*(c + d*x^3)^(1/2))/(32*c) - (d*(c + d*x^3)^(3/2))/(32*c^2))/(3*(c + d
*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) + (d*(atanh((c^2*(c + d*x^3)^(1/2))/(c
^5)^(1/2))*1i - (atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))*7i)/9)*1i)/
(128*(c^5)^(1/2))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

$$3.279 \quad \int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)^2} dx$$

Optimal. Leaf size=164

$$\frac{23d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}} - \frac{d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}} + \frac{5d^2\sqrt{c+dx^3}}{1536c^3(8c-dx^3)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48cx^6(8c-dx^3)}$$

Rubi [A] time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {446, 99, 151, 156, 63, 208, 206}

$$\frac{5d^2\sqrt{c+dx^3}}{1536c^3(8c-dx^3)} + \frac{23d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}} - \frac{d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48cx^6(8c-dx^3)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2), x]

[Out] (5*d^2*Sqrt[c + d*x^3])/(1536*c^3*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*c*x^6*(8*c - d*x^3)) - (7*d*Sqrt[c + d*x^3])/(384*c^2*x^3*(8*c - d*x^3)) + (23*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(18432*c^(7/2)) - (d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])/(2048*c^(7/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 99

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n + p + 2)*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c+dx}}{x^3(8c-dx)^2} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c+dx^3}}{48cx^6(8c-dx^3)} + \frac{\text{Subst} \left(\int \frac{7cd+\frac{5d^2x}{2}}{x^2(8c-dx)^2\sqrt{c+dx}} dx, x, x^3 \right)}{48c} \\
&= -\frac{\sqrt{c+dx^3}}{48cx^6(8c-dx^3)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3(8c-dx^3)} - \frac{\text{Subst} \left(\int \frac{-6c^2d^2-\frac{21}{2}cd^3x}{x(8c-dx)^2\sqrt{c+dx}} dx, x, x^3 \right)}{384c^3} \\
&= \frac{5d^2\sqrt{c+dx^3}}{1536c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48cx^6(8c-dx^3)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3(8c-dx^3)} + \frac{\text{Subst} \left(\int \frac{54c^3d^3+45c^2d^4x}{x(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{27648c^5d} \\
&= \frac{5d^2\sqrt{c+dx^3}}{1536c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48cx^6(8c-dx^3)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3(8c-dx^3)} + \frac{d^2 \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{4096c^3} \\
&= \frac{5d^2\sqrt{c+dx^3}}{1536c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48cx^6(8c-dx^3)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3(8c-dx^3)} + \frac{d \text{Subst} \left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{2048c^3} \\
&= \frac{5d^2\sqrt{c+dx^3}}{1536c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48cx^6(8c-dx^3)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3(8c-dx^3)} + \frac{23d^2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{18432c^{7/2}} - \dots
\end{aligned}$$

Mathematica [A] time = 0.21, size = 112, normalized size = 0.68

$$\frac{12\sqrt{c}\sqrt{c+dx^3}(32c^2+28cdx^3-5d^2x^6)}{dx^9-8cx^6} + 23d^2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) - 9d^2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{18432c^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2), x]

[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 28*c*d*x^3 - 5*d^2*x^6))/(-8*c*x^6 + d*x^9) + 23*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(18432*c^(7/2))

IntegrateAlgebraic [A] time = 0.17, size = 118, normalized size = 0.72

$$\frac{23d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}} - \frac{d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}} + \frac{\sqrt{c+dx^3}(-32c^2 - 28cdx^3 + 5d^2x^6)}{1536c^3x^6(8c - dx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2), x]

[Out] (Sqrt[c + d*x^3]*(-32*c^2 - 28*c*d*x^3 + 5*d^2*x^6))/(1536*c^3*x^6*(8*c - d*x^3)) + (23*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(18432*c^(7/2)) - (d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])]/(2048*c^(7/2))

fricas [A] time = 0.47, size = 310, normalized size = 1.89

$$\frac{23(d^3x^9 - 8cd^2x^6)\sqrt{c} \log\left(\frac{d^3+6\sqrt{d^3+c}\sqrt{c}+10c}{d^3-8c}\right) + 9(d^3x^9 - 8cd^2x^6)\sqrt{c} \log\left(\frac{d^3-2\sqrt{d^3+c}\sqrt{c}+2c}{d^3}\right) - 24(5cd^2x^6 - 28c^2dx^3 - 32c^2)\sqrt{dx^3+c} - 9(d^3x^9 - 8cd^2x^6)\sqrt{-c} \arctan\left(\frac{\sqrt{d^3+c}\sqrt{c}}{c}\right) - 23(d^3x^9 - 8cd^2x^6)\sqrt{-c} \arctan\left(\frac{\sqrt{d^3+c}\sqrt{c}}{3c}\right) - 12(5cd^2x^6 - 28c^2dx^3 - 32c^2)\sqrt{dx^3+c}}{36864(c^4dx^9 - 8c^5x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] [1/36864*(23*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d^2*x^6 - 28*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6), 1/18432*(9*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 23*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*(5*c*d^2*x^6 - 28*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6)]

giac [A] time = 0.18, size = 105, normalized size = 0.64

$$\frac{d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\sqrt{-c}c^3} - \frac{23d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{18432\sqrt{-c}c^3} - \frac{\sqrt{dx^3+c}d^2}{1536(dx^3-8c)c^3} - \frac{(dx^3+c)^{\frac{3}{2}}}{384c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] 1/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 23/18432*d^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/1536*sqrt(d*x^3 + c)*d^2/((d*x^3 - 8*c)*c^3) - 1/384*(d*x^3 + c)^(3/2)/(c^3*x^6)

maple [C] time = 0.19, size = 1020, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/512/c^3*d^3*(-1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum(
(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d
^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3
)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(
-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/256
/c^3*d*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(
1/2))+1/64/c^2*(-1/6*(d*x^3+c)^(1/2)/x^6-1/12*(d*x^3+c)^(1/2)/c*d/x^3+1/12*
d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-3/4096/c^4*d^3*(2/3*(d*x^3+c)
^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/
2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2
)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d
-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(
1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c))+3/4096/c^4*d^2*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((
d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{(dx^3 - 8c)^2 x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^7), x)
```

mupad [B] time = 4.45, size = 154, normalized size = 0.94

$$\frac{\frac{d^2 \sqrt{dx^3+c}}{512c} - \frac{19d^2(dx^3+c)^{3/2}}{256c^2} + \frac{5d^2(dx^3+c)^{5/2}}{512c^3}}{33c(dx^3+c)^2 - 57c^2(dx^3+c) - 3(dx^3+c)^3 + 27c^3} + \frac{d^2 \left(\operatorname{atanh}\left(\frac{c^3 \sqrt{dx^3+c}}{\sqrt{c^7}}\right) 1i - \frac{\operatorname{atanh}\left(\frac{c^3 \sqrt{dx^3+c}}{3\sqrt{c^7}}\right) 23i}{9} \right) 1i}{2048 \sqrt{c^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)^(1/2)/(x^7*(8*c - d*x^3)^2), x)`

[Out] `((d^2*(c + d*x^3)^(1/2))/(512*c) - (19*d^2*(c + d*x^3)^(3/2))/(256*c^2) + (5*d^2*(c + d*x^3)^(5/2))/(512*c^3))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3) - 3*(c + d*x^3)^3 + 27*c^3) + (d^2*(atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2))*1i - (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*23i)/9)*1i)/(2048*(c^7)^(1/2))`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c)**2, x)`

[Out] Timed out

$$3.280 \quad \int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$$

Optimal. Leaf size=134

$$-\frac{4992c^{7/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4} + \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{2c(c+dx^3)^{3/2}(694c+51dx^3)}{21d^4} + \frac{3x^6(c+dx^3)^{3/2}}{7d^2} + \frac{x^9(c+dx^3)^{3/2}}{3d(8c-dx^3)}$$

Rubi [A] time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {446, 97, 153, 147, 50, 63, 206}

$$\frac{1664c^3\sqrt{c+dx^3}}{d^4} - \frac{4992c^{7/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4} + \frac{3x^6(c+dx^3)^{3/2}}{7d^2} + \frac{2c(c+dx^3)^{3/2}(694c+51dx^3)}{21d^4} + \frac{x^9(c+dx^3)^{3/2}}{3d(8c-dx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] (1664*c^3*Sqrt[c + d*x^3])/d^4 + (3*x^6*(c + d*x^3)^(3/2))/(7*d^2) + (x^9*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) + (2*c*(c + d*x^3)^(3/2)*(694*c + 51*d*x^3))/(21*d^4) - (4992*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))*((g_.) + (h_.)*(x_)), x_Symbol] := -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m + 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p + 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11} (c + dx^3)^{3/2}}{(8c - dx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3 (c + dx)^{3/2}}{(8c - dx)^2} dx, x, x^3 \right) \\
&= \frac{x^9 (c + dx^3)^{3/2}}{3d(8c - dx^3)} - \frac{\text{Subst} \left(\int \frac{x^2 \sqrt{c+dx} \left(3c + \frac{9dx}{2}\right)}{8c-dx} dx, x, x^3 \right)}{3d} \\
&= \frac{3x^6 (c + dx^3)^{3/2}}{7d^2} + \frac{x^9 (c + dx^3)^{3/2}}{3d(8c - dx^3)} + \frac{2 \text{Subst} \left(\int \frac{x \sqrt{c+dx} \left(-72c^2d - \frac{255}{2}cd^2x\right)}{8c-dx} dx, x, x^3 \right)}{21d^3} \\
&= \frac{3x^6 (c + dx^3)^{3/2}}{7d^2} + \frac{x^9 (c + dx^3)^{3/2}}{3d(8c - dx^3)} + \frac{2c (c + dx^3)^{3/2} (694c + 51dx^3)}{21d^4} - \frac{(832c^3) \text{Subst} \left(\int \frac{x \sqrt{c+dx} \left(-72c^2d - \frac{255}{2}cd^2x\right)}{8c-dx} dx, x, x^3 \right)}{d^3} \\
&= \frac{1664c^3 \sqrt{c + dx^3}}{d^4} + \frac{3x^6 (c + dx^3)^{3/2}}{7d^2} + \frac{x^9 (c + dx^3)^{3/2}}{3d(8c - dx^3)} + \frac{2c (c + dx^3)^{3/2} (694c + 51dx^3)}{21d^4} \\
&= \frac{1664c^3 \sqrt{c + dx^3}}{d^4} + \frac{3x^6 (c + dx^3)^{3/2}}{7d^2} + \frac{x^9 (c + dx^3)^{3/2}}{3d(8c - dx^3)} + \frac{2c (c + dx^3)^{3/2} (694c + 51dx^3)}{21d^4} \\
&= \frac{1664c^3 \sqrt{c + dx^3}}{d^4} + \frac{3x^6 (c + dx^3)^{3/2}}{7d^2} + \frac{x^9 (c + dx^3)^{3/2}}{3d(8c - dx^3)} + \frac{2c (c + dx^3)^{3/2} (694c + 51dx^3)}{21d^4}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 0.83

$$\frac{2 \left(52416c^{7/2} (8c - dx^3) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) + \sqrt{c + dx^3} (-145328c^4 + 12206c^3dx^3 + 301c^2d^2x^6 + 16cd^3x^9 + d^4x^{12}) \right)}{21d^4(dx^3 - 8c)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] (2*(Sqrt[c + d*x^3]*(-145328*c^4 + 12206*c^3*d*x^3 + 301*c^2*d^2*x^6 + 16*c*d^3*x^9 + d^4*x^12) + 52416*c^(7/2)*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(21*d^4*(-8*c + d*x^3))

IntegrateAlgebraic [A] time = 0.10, size = 104, normalized size = 0.78

$$\frac{4992c^{7/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4} - \frac{2\sqrt{c+dx^3} (145328c^4 - 12206c^3dx^3 - 301c^2d^2x^6 - 16cd^3x^9 - d^4x^{12})}{21d^4(dx^3 - 8c)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] (-2*Sqrt[c + d*x^3]*(145328*c^4 - 12206*c^3*d*x^3 - 301*c^2*d^2*x^6 - 16*c*d^3*x^9 - d^4*x^12))/(21*d^4*(-8*c + d*x^3)) - (4992*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4

fricas [A] time = 0.44, size = 239, normalized size = 1.78

$$\frac{2\left(26208(c^3dx^3 - 8c^4)\sqrt{c} \log\left(\frac{dx^3 - 6\sqrt{dx^3+c}\sqrt{c+10c}}{dx^3 - 8c}\right) + (d^4x^{12} + 16cd^3x^9 + 301c^2d^2x^6 + 12206c^3dx^3 - 145328c^4)\sqrt{dx^3+c}\right)}{21(d^5x^3 - 8cd^4)} - \frac{2\left(52416(c^3dx^3 - 8c^4)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + (d^4x^{12} + 16cd^3x^9 + 301c^2d^2x^6 + 12206c^3dx^3 - 145328c^4)\sqrt{dx^3+c}\right)}{21(d^5x^3 - 8cd^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] [2/21*(26208*(c^3*d*x^3 - 8*c^4)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + (d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6 + 12206*c^3*d*x^3 - 145328*c^4)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/21*(52416*(c^3*d*x^3 - 8*c^4)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6 + 12206*c^3*d*x^3 - 145328*c^4)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]

giac [A] time = 0.17, size = 127, normalized size = 0.95

$$\frac{4992c^4 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^4} - \frac{1536\sqrt{dx^3+c}c^4}{(dx^3-8c)d^4} + \frac{2\left(\left(dx^3+c\right)^{\frac{7}{2}}d^{24} + 21\left(dx^3+c\right)^{\frac{5}{2}}cd^{24} + 448\left(dx^3+c\right)^{\frac{3}{2}}c^2d^{24} + 15680\sqrt{dx^3+c}c^3d^{24}\right)}{21d^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] 4992*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 1536*sqrt(d*x^3 + c)*c^4/((d*x^3 - 8*c)*d^4) + 2/21*((d*x^3 + c)^(7/2)*d^24 + 21*(d*x^3 + c)^(5/2)*c*d^24 + 448*(d*x^3 + c)^(3/2)*c^2*d^24 + 15680*sqrt(d*x^3 + c)*c^3*d^24)/d^28

maple [C] time = 0.28, size = 998, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)`

[Out] $\frac{1}{d^3} \left(\frac{d}{21} (d x^3 + c)^{1/2} d x^9 + \frac{16}{105} (d x^3 + c)^{1/2} c x^6 + \frac{2}{105} (d x^3 + c)^{1/2} c^2 / d x^3 - \frac{4}{105} (d x^3 + c)^{1/2} c^3 / d^2 + \frac{32}{15} c / d (d x^3 + c)^{5/2} \right) + 512 c^3 / d^3 (-3 c / d (d x^3 + c)^{1/2} / (d x^3 - 8 c) + 2/3 (d x^3 + c)^{1/2} / d + 1/2 I / d^3 2^{1/2} \sum((-c d^2)^{1/3} (1/2 I (2 x + (-I 3^{1/2}) (-c d^2)^{1/3}) + (-c d^2)^{1/3}) / d) / (-c d^2)^{1/3} d)^{1/2} ((x - (-c d^2)^{1/3} / d) / (-3 (-c d^2)^{1/3} + I 3^{1/2} (-c d^2)^{1/3}) d)^{1/2} (-1/2 I (2 x + (I 3^{1/2}) (-c d^2)^{1/3}) + (-c d^2)^{1/3}) / d) / (-c d^2)^{1/3} d)^{1/2} / (d x^3 + c)^{1/2} (2 \alpha^2 d^2 + I (-c d^2)^{1/3} 3^{1/2} \alpha d - (-c d^2)^{1/3} \alpha d - I 3^{1/2} (-c d^2)^{2/3} - (-c d^2)^{2/3}) \text{EllipticPi}(1/3 3^{1/2} (I (x + 1/2 (-c d^2)^{1/3} / d - 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) 3^{1/2} / (-c d^2)^{1/3} d)^{1/2}, -1/18 (2 I (-c d^2)^{1/3} 3^{1/2} \alpha^2 d + I 3^{1/2} c d - 3 c d - I (-c d^2)^{2/3} 3^{1/2} \alpha - 3 (-c d^2)^{2/3} \alpha) / c d, (I 3^{1/2} (-c d^2)^{1/3} / (-3/2 (-c d^2)^{1/3} / d + 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) / d)^{1/2}, \alpha = \text{RootOf}(_Z^3 d - 8 c)) \right) + 192 c^2 / d^3 (2/9 (d x^3 + c)^{1/2} x^3 + 56/9 (d x^3 + c)^{1/2} c / d + 3 I c / d^3 2^{1/2} \sum((-c d^2)^{1/3} (1/2 I (2 x + (-I 3^{1/2}) (-c d^2)^{1/3}) + (-c d^2)^{1/3}) / d) / (-c d^2)^{1/3} d)^{1/2} ((x - (-c d^2)^{1/3} / d) / (-3 (-c d^2)^{1/3} + I 3^{1/2} (-c d^2)^{1/3}) d)^{1/2} (-1/2 I (2 x + (I 3^{1/2}) (-c d^2)^{1/3}) + (-c d^2)^{1/3}) / d) / (-c d^2)^{1/3} d)^{1/2} / (d x^3 + c)^{1/2} (2 \alpha^2 d^2 + I (-c d^2)^{1/3} 3^{1/2} \alpha d - (-c d^2)^{1/3} \alpha d - I 3^{1/2} (-c d^2)^{2/3} - (-c d^2)^{2/3}) \text{EllipticPi}(1/3 3^{1/2} (I (x + 1/2 (-c d^2)^{1/3} / d - 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) 3^{1/2} / (-c d^2)^{1/3} d)^{1/2}, -1/18 (2 I (-c d^2)^{1/3} 3^{1/2} \alpha^2 d + I 3^{1/2} c d - 3 c d - I (-c d^2)^{2/3} 3^{1/2} \alpha - 3 (-c d^2)^{2/3} \alpha) / c d, (I 3^{1/2} (-c d^2)^{1/3} / (-3/2 (-c d^2)^{1/3} / d + 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) / d)^{1/2}, \alpha = \text{RootOf}(_Z^3 d - 8 c)) \right)$

maxima [A] time = 1.18, size = 119, normalized size = 0.89

$$\frac{2 \left(26208 c^{\frac{7}{2}} \log \left(\frac{\sqrt{d x^3 + c} - 3 \sqrt{c}}{\sqrt{d x^3 + c} + 3 \sqrt{c}} \right) + (d x^3 + c)^{\frac{7}{2}} + 21 (d x^3 + c)^{\frac{5}{2}} c + 448 (d x^3 + c)^{\frac{3}{2}} c^2 + 15680 \sqrt{d x^3 + c} c^3 - \frac{16128 \sqrt{d x^3 + c} c^4}{d x^3 - 8 c} \right)}{21 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")`

[Out] $\frac{2}{21} (26208 c^{7/2} \log((\sqrt{d x^3 + c} - 3 \sqrt{c}) / (\sqrt{d x^3 + c} + 3 \sqrt{c})) + (d x^3 + c)^{7/2} + 21 (d x^3 + c)^{5/2} c + 448 (d x^3 + c)^{3/2} c^2 + 15680 \sqrt{d x^3 + c} c^3 - 16128 \sqrt{d x^3 + c} c^4 / (d x^3 - 8 c)) / d^4$

mupad [B] time = 4.10, size = 147, normalized size = 1.10

$$\frac{2496 c^{7/2} \ln\left(\frac{10c+dx^3-6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{d^4} + \frac{32300 c^3 \sqrt{dx^3+c}}{21 d^4} + \frac{2x^9 \sqrt{dx^3+c}}{21 d} + \frac{16 c x^6 \sqrt{dx^3+c}}{7 d^2} + \frac{986 c^2 x^3 \sqrt{dx^3+c}}{21 d^3} + \frac{1536 c^4 \sqrt{dx^3+c}}{d^4 (8c-dx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x¹¹*(c + d*x³)^(3/2))/(8*c - d*x³)²,x)

[Out] (2496*c^(7/2)*log((10*c + d*x³ - 6*c^(1/2)*(c + d*x³)^(1/2))/(8*c - d*x³)))/d⁴ + (32300*c³*(c + d*x³)^(1/2))/(21*d⁴) + (2*x⁹*(c + d*x³)^(1/2))/(21*d) + (16*c*x⁶*(c + d*x³)^(1/2))/(7*d²) + (986*c²*x³*(c + d*x³)^(1/2))/(21*d³) + (1536*c⁴*(c + d*x³)^(1/2))/(d⁴*(8*c - d*x³))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)

[Out] Timed out

$$3.281 \quad \int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$$

Optimal. Leaf size=119

$$-\frac{480c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3} + \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} + \frac{160c(c+dx^3)^{3/2}}{27d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3}$$

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {446, 89, 80, 50, 63, 206}

$$\frac{160c^2\sqrt{c+dx^3}}{d^3} - \frac{480c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3} + \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} + \frac{160c(c+dx^3)^{3/2}}{27d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] (160*c^2*Sqrt[c + d*x^3])/d^3 + (160*c*(c + d*x^3)^(3/2))/(27*d^3) + (2*(c + d*x^3)^(5/2))/(15*d^3) + (64*c*(c + d*x^3)^(5/2))/(27*d^3*(8*c - d*x^3)) - (480*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^(2)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8 (c + dx^3)^{3/2}}{(8c - dx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (c + dx)^{3/2}}{(8c - dx)^2} dx, x, x^3 \right) \\
&= \frac{64c (c + dx^3)^{5/2}}{27d^3 (8c - dx^3)} - \frac{\text{Subst} \left(\int \frac{(c+dx)^{3/2} (168c^2 d + 9cd^2 x)}{8c - dx} dx, x, x^3 \right)}{27cd^3} \\
&= \frac{2 (c + dx^3)^{5/2}}{15d^3} + \frac{64c (c + dx^3)^{5/2}}{27d^3 (8c - dx^3)} - \frac{(80c) \text{Subst} \left(\int \frac{(c+dx)^{3/2}}{8c - dx} dx, x, x^3 \right)}{9d^2} \\
&= \frac{160c (c + dx^3)^{3/2}}{27d^3} + \frac{2 (c + dx^3)^{5/2}}{15d^3} + \frac{64c (c + dx^3)^{5/2}}{27d^3 (8c - dx^3)} - \frac{(80c^2) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{8c - dx} dx, x, x^3 \right)}{d^2} \\
&= \frac{160c^2 \sqrt{c + dx^3}}{d^3} + \frac{160c (c + dx^3)^{3/2}}{27d^3} + \frac{2 (c + dx^3)^{5/2}}{15d^3} + \frac{64c (c + dx^3)^{5/2}}{27d^3 (8c - dx^3)} - \frac{(720c^3) \text{Subst}}{d^2} \\
&= \frac{160c^2 \sqrt{c + dx^3}}{d^3} + \frac{160c (c + dx^3)^{3/2}}{27d^3} + \frac{2 (c + dx^3)^{5/2}}{15d^3} + \frac{64c (c + dx^3)^{5/2}}{27d^3 (8c - dx^3)} - \frac{(1440c^3) \text{Subst}}{d^2} \\
&= \frac{160c^2 \sqrt{c + dx^3}}{d^3} + \frac{160c (c + dx^3)^{3/2}}{27d^3} + \frac{2 (c + dx^3)^{5/2}}{15d^3} + \frac{64c (c + dx^3)^{5/2}}{27d^3 (8c - dx^3)} - \frac{480c^{5/2} \tanh^{-1}}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 102, normalized size = 0.86

$$\frac{21600c^{5/2} (8c - dx^3) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) + 2\sqrt{c + dx^3} (-29944c^3 + 2515c^2 dx^3 + 62cd^2 x^6 + 3d^3 x^9)}{45d^3 (dx^3 - 8c)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] (2*Sqrt[c + d*x^3]*(-29944*c^3 + 2515*c^2*d*x^3 + 62*c*d^2*x^6 + 3*d^3*x^9) + 21600*c^(5/2)*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^3*(-8*c + d*x^3))

IntegrateAlgebraic [A] time = 0.09, size = 93, normalized size = 0.78

$$\frac{480c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3} - \frac{2\sqrt{c+dx^3} (29944c^3 - 2515c^2dx^3 - 62cd^2x^6 - 3d^3x^9)}{45d^3(dx^3 - 8c)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] (-2*sqrt[c + d*x^3]*(29944*c^3 - 2515*c^2*d*x^3 - 62*c*d^2*x^6 - 3*d^3*x^9))/(45*d^3*(-8*c + d*x^3)) - (480*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*sqrt[c])])/d^3

fricas [A] time = 0.44, size = 219, normalized size = 1.84

$$\left[\frac{2 \left(5400 (c^2 dx^3 - 8c^3) \sqrt{c} \log \left(\frac{dx^3 - 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c} \right) + (3d^3x^9 + 62cd^2x^6 + 2515c^2dx^3 - 29944c^3) \sqrt{dx^3+c} \right)}{45(dx^3 - 8cd^3)}, \frac{2 \left(10800 (c^2 dx^3 - 8c^3) \sqrt{-c} \arctan \left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c} \right) + (3d^3x^9 + 62cd^2x^6 + 2515c^2dx^3 - 29944c^3) \sqrt{dx^3+c} \right)}{45(dx^3 - 8cd^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] [2/45*(5400*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + (3*d^3*x^9 + 62*c*d^2*x^6 + 2515*c^2*d*x^3 - 29944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3), 2/45*(10800*(c^2*d*x^3 - 8*c^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^3*x^9 + 62*c*d^2*x^6 + 2515*c^2*d*x^3 - 29944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]

giac [A] time = 0.20, size = 111, normalized size = 0.93

$$\frac{480c^3 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^3} - \frac{192\sqrt{dx^3+c}c^3}{(dx^3-8c)d^3} + \frac{2\left(3(dx^3+c)^{\frac{5}{2}}d^{12} + 80(dx^3+c)^{\frac{3}{2}}cd^{12} + 3120\sqrt{dx^3+c}c^2d^{12}\right)}{45d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] 480*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 192*sqrt(d*x^3 + c)*c^3/((d*x^3 - 8*c)*d^3) + 2/45*(3*(d*x^3 + c)^(5/2)*d^12 + 80*(d*x^3 + c)^(3/2)*c*d^12 + 3120*sqrt(d*x^3 + c)*c^2*d^12)/d^15

maple [C] time = 0.19, size = 920, normalized size = 7.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^8*(d*x^3+c)^{(3/2)} / (-d*x^3+8*c)^2, x)$

[Out] $2/15*(d*x^3+c)^{(5/2)}/d^3+64*c^2/d^2*(-3*(d*x^3+c)^{(1/2)}/(d*x^3-8*c)*c/d+2/3*(d*x^3+c)^{(1/2)}/d+1/2*I/d^3*2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=\text{RootOf}(_Z^3*d-8*c)))+16*c/d^2*(2/9*(d*x^3+c)^{(1/2)}*x^3+56/9*(d*x^3+c)^{(1/2)}*c/d+3*I*c/d^3*2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=\text{RootOf}(_Z^3*d-8*c))$

maxima [A] time = 1.39, size = 107, normalized size = 0.90

$$\frac{2 \left(5400 c^2 \log \left(\frac{\sqrt{dx^3+c} - 3\sqrt{c}}{\sqrt{dx^3+c} + 3\sqrt{c}} \right) + 3 (dx^3 + c)^{\frac{5}{2}} + 80 (dx^3 + c)^{\frac{3}{2}} c + 3120 \sqrt{dx^3 + c} c^2 - \frac{4320 \sqrt{dx^3+c} c^3}{dx^3-8c} \right)}{45 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^8*(d*x^3+c)^{(3/2)} / (-d*x^3+8*c)^2, x, \text{algorithm}="maxima")$

[Out] $2/45*(5400*c^2*\log((\text{sqrt}(d*x^3 + c) - 3*\text{sqrt}(c))/(\text{sqrt}(d*x^3 + c) + 3*\text{sqrt}(c)))) + 3*(d*x^3 + c)^{(5/2)} + 80*(d*x^3 + c)^{(3/2)}*c + 3120*\text{sqrt}(d*x^3 + c)*c^2 - 4320*\text{sqrt}(d*x^3 + c)*c^3/(d*x^3 - 8*c))/d^3$

mupad [B] time = 4.05, size = 127, normalized size = 1.07

$$\frac{240 c^2 \ln \left(\frac{10c+d x^3-6\sqrt{c}\sqrt{d x^3+c}}{8c-d x^3} \right)}{d^3} + \frac{6406 c^2 \sqrt{d x^3+c}}{45 d^3} + \frac{2 x^6 \sqrt{d x^3+c}}{15 d} + \frac{172 c x^3 \sqrt{d x^3+c}}{45 d^2} + \frac{192 c^3 \sqrt{d x^3+c}}{d^3 (8c-d x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (240*c^(5/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/d^3 + (6406*c^2*(c + d*x^3)^(1/2))/(45*d^3) + (2*x^6*(c + d*x^3)^(1/2))/(15*d) + (172*c*x^3*(c + d*x^3)^(1/2))/(45*d^2) + (192*c^3*(c + d*x^3)^(1/2))/(d^3*(8*c - d*x^3))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

$$3.282 \quad \int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$$

Optimal. Leaf size=97

$$-\frac{42c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2} + \frac{8(c+dx^3)^{5/2}}{27d^2(8c-dx^3)} + \frac{14(c+dx^3)^{3/2}}{27d^2} + \frac{14c\sqrt{c+dx^3}}{d^2}$$

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 78, 50, 63, 206}

$$-\frac{42c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2} + \frac{8(c+dx^3)^{5/2}}{27d^2(8c-dx^3)} + \frac{14(c+dx^3)^{3/2}}{27d^2} + \frac{14c\sqrt{c+dx^3}}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] (14*c*Sqrt[c + d*x^3])/d^2 + (14*(c + d*x^3)^(3/2))/(27*d^2) + (8*(c + d*x^3)^(5/2))/(27*d^2*(8*c - d*x^3)) - (42*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (c + dx^3)^{3/2}}{(8c - dx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(c + dx)^{3/2}}{(8c - dx)^2} dx, x, x^3 \right) \\
&= \frac{8(c + dx^3)^{5/2}}{27d^2(8c - dx^3)} - \frac{7 \text{Subst} \left(\int \frac{(c+dx)^{3/2}}{8c-dx} dx, x, x^3 \right)}{9d} \\
&= \frac{14(c + dx^3)^{3/2}}{27d^2} + \frac{8(c + dx^3)^{5/2}}{27d^2(8c - dx^3)} - \frac{(7c) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{8c-dx} dx, x, x^3 \right)}{d} \\
&= \frac{14c\sqrt{c + dx^3}}{d^2} + \frac{14(c + dx^3)^{3/2}}{27d^2} + \frac{8(c + dx^3)^{5/2}}{27d^2(8c - dx^3)} - \frac{(63c^2) \text{Subst} \left(\int \frac{1}{(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{d} \\
&= \frac{14c\sqrt{c + dx^3}}{d^2} + \frac{14(c + dx^3)^{3/2}}{27d^2} + \frac{8(c + dx^3)^{5/2}}{27d^2(8c - dx^3)} - \frac{(126c^2) \text{Subst} \left(\int \frac{1}{9c-x^2} dx, x, \sqrt{c + dx^3} \right)}{d^2} \\
&= \frac{14c\sqrt{c + dx^3}}{d^2} + \frac{14(c + dx^3)^{3/2}}{27d^2} + \frac{8(c + dx^3)^{5/2}}{27d^2(8c - dx^3)} - \frac{42c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 90, normalized size = 0.93

$$\frac{378c^{3/2}(8c - dx^3) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) + 2\sqrt{c + dx^3} (-524c^2 + 44cdx^3 + d^2x^6)}{9d^2(dx^3 - 8c)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] (2*Sqrt[c + d*x^3]*(-524*c^2 + 44*c*d*x^3 + d^2*x^6) + 378*c^(3/2)*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2*(-8*c + d*x^3))

IntegrateAlgebraic [A] time = 0.08, size = 82, normalized size = 0.85

$$\frac{42c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{d^2} - \frac{2\sqrt{c + dx^3} (524c^2 - 44cdx^3 - d^2x^6)}{9d^2(dx^3 - 8c)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] $(-2*\sqrt{c + d*x^3}*(524*c^2 - 44*c*d*x^3 - d^2*x^6))/(9*d^2*(-8*c + d*x^3)) - (42*c^{(3/2)}*ArcTanh[\sqrt{c + d*x^3}/(3*\sqrt{c})])/d^2$

fricas [A] time = 0.45, size = 192, normalized size = 1.98

$$\left[\frac{189(cdx^3 - 8c^2)\sqrt{c} \log\left(\frac{dx^3 - 6\sqrt{dx^3+c}\sqrt{c+10c}}{dx^3 - 8c}\right) + 2(d^2x^6 + 44cdx^3 - 524c^2)\sqrt{dx^3+c}}{9(d^3x^3 - 8cd^2)}, \frac{2\left(189(cdx^3 - 8c^2)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + (d^2x^6 + 44cdx^3 - 524c^2)\sqrt{dx^3+c}\right)}{9(d^3x^3 - 8cd^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] $[1/9*(189*(c*d*x^3 - 8*c^2)*\sqrt{c}*\log((d*x^3 - 6*\sqrt{d*x^3 + c})*\sqrt{c} + 10*c)/(d*x^3 - 8*c)) + 2*(d^2*x^6 + 44*c*d*x^3 - 524*c^2)*\sqrt{d*x^3 + c}]/(d^3*x^3 - 8*c*d^2), 2/9*(189*(c*d*x^3 - 8*c^2)*\sqrt{-c}*\arctan(1/3*\sqrt{d*x^3 + c})*\sqrt{-c}/c) + (d^2*x^6 + 44*c*d*x^3 - 524*c^2)*\sqrt{d*x^3 + c}]/(d^3*x^3 - 8*c*d^2)]$

giac [A] time = 0.17, size = 93, normalized size = 0.96

$$\frac{42c^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^2} - \frac{24\sqrt{dx^3+c}c^2}{(dx^3-8c)d^2} + \frac{2\left((dx^3+c)^{\frac{3}{2}}d^4 + 51\sqrt{dx^3+c}cd^4\right)}{9d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] $42*c^2*\arctan(1/3*\sqrt{d*x^3 + c}/\sqrt{-c})/(\sqrt{-c}*d^2) - 24*\sqrt{d*x^3 + c}*c^2/((d*x^3 - 8*c)*d^2) + 2/9*((d*x^3 + c)^(3/2)*d^4 + 51*\sqrt{d*x^3 + c}*c*d^4)/d^6$

maple [C] time = 0.17, size = 902, normalized size = 9.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)

[Out] $8*c/d*(-3*(d*x^3+c)^(1/2)/(d*x^3-8*c)*c/d+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3*2^(1/2)*\sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(1/3))$

$(2/3) - (-c*d^2)^{(2/3)} * \text{EllipticPi}(1/3 * 3^{(1/2)} * (I * (x + 1/2 * (-c*d^2)^{(1/3)} / d - 1/2 * I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / d) * 3^{(1/2)} / (-c*d^2)^{(1/3)} * d)^{(1/2)}, -1/18 * (2 * I * (-c*d^2)^{(1/3)} * 3^{(1/2)} * _alpha^2 * d + I * 3^{(1/2)} * c * d - 3 * c * d - I * (-c*d^2)^{(2/3)} * 3^{(1/2)} * _alpha - 3 * (-c*d^2)^{(2/3)} * _alpha) / c / d, (I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / (-3/2 * (-c*d^2)^{(1/3)} / d + 1/2 * I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / d) / d)^{(1/2)}, _alpha = \text{RootOf}(_Z^3 * d - 8 * c)) + 1/d * (2/9 * (d*x^3 + c)^{(1/2)} * x^3 + 56/9 * (d*x^3 + c)^{(1/2)} * c / d + 3 * I * c / d^3 * 2^{(1/2)} * \text{sum}((-c*d^2)^{(1/3)} * (1/2 * I * (2*x + (-I * 3^{(1/2)} * (-c*d^2)^{(1/3)} + (-c*d^2)^{(1/3)}) / d) / (-c*d^2)^{(1/3)} * d)^{(1/2)} * ((x - (-c*d^2)^{(1/3)} / d) / (-3 * (-c*d^2)^{(1/3)} + I * 3^{(1/2)} * (-c*d^2)^{(1/3)}) * d)^{(1/2)} * (-1/2 * I * (2*x + (I * 3^{(1/2)} * (-c*d^2)^{(1/3)} + (-c*d^2)^{(1/3)}) / d) / (-c*d^2)^{(1/3)} * d)^{(1/2)} / (d*x^3 + c)^{(1/2)} * (2 * _alpha^2 * d^2 + I * (-c*d^2)^{(1/3)} * 3^{(1/2)} * _alpha * d - (-c*d^2)^{(1/3)} * _alpha * d - I * 3^{(1/2)} * (-c*d^2)^{(2/3)} - (-c*d^2)^{(2/3)}) * \text{EllipticPi}(1/3 * 3^{(1/2)} * (I * (x + 1/2 * (-c*d^2)^{(1/3)} / d - 1/2 * I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / d) * 3^{(1/2)} / (-c*d^2)^{(1/3)} * d)^{(1/2)}, -1/18 * (2 * I * (-c*d^2)^{(1/3)} * 3^{(1/2)} * _alpha^2 * d + I * 3^{(1/2)} * c * d - 3 * c * d - I * (-c*d^2)^{(2/3)} * 3^{(1/2)} * _alpha - 3 * (-c*d^2)^{(2/3)} * _alpha) / c / d, (I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / (-3/2 * (-c*d^2)^{(1/3)} / d + 1/2 * I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / d) / d)^{(1/2)}, _alpha = \text{RootOf}(_Z^3 * d - 8 * c))$

maxima [A] time = 1.33, size = 93, normalized size = 0.96

$$\frac{189 c^{\frac{3}{2}} \log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right) + 2(dx^3+c)^{\frac{3}{2}} + 102\sqrt{dx^3+c}c - \frac{216\sqrt{dx^3+c}c^2}{dx^3-8c}}{9d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")

[Out] 1/9*(189*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c))) + 2*(d*x^3 + c)^(3/2) + 102*sqrt(d*x^3 + c)*c - 216*sqrt(d*x^3 + c)*c^2/(d*x^3 - 8*c))/d^2

mupad [B] time = 4.04, size = 107, normalized size = 1.10

$$\frac{104c\sqrt{dx^3+c}}{9d^2} + \frac{21c^{3/2}\ln\left(\frac{10c+dx^3-6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{d^2} + \frac{2x^3\sqrt{dx^3+c}}{9d} + \frac{24c^2\sqrt{dx^3+c}}{d^2(8c-dx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)

[Out] (104*c*(c + d*x^3)^(1/2))/(9*d^2) + (21*c^(3/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/d^2 + (2*x^3*(c + d*x^3)^(1/2))/(9*d) + (24*c^2*(c + d*x^3)^(1/2))/(d^2*(8*c - d*x^3))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)

[Out] Timed out

$$3.283 \quad \int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$$

Optimal. Leaf size=77

$$\frac{(c+dx^3)^{3/2}}{3d(8c-dx^3)} + \frac{\sqrt{c+dx^3}}{d} - \frac{3\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {444, 47, 50, 63, 206}

$$\frac{(c+dx^3)^{3/2}}{3d(8c-dx^3)} + \frac{\sqrt{c+dx^3}}{d} - \frac{3\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] Sqrt[c + d*x^3]/d + (c + d*x^3)^(3/2)/(3*d*(8*c - d*x^3)) - (3*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63


```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (c + dx^3)^{3/2}}{(8c - dx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{(8c - dx)^2} dx, x, x^3 \right) \\
&= \frac{(c + dx^3)^{3/2}}{3d(8c - dx^3)} - \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{c + dx}}{8c - dx} dx, x, x^3 \right) \\
&= \frac{\sqrt{c + dx^3}}{d} + \frac{(c + dx^3)^{3/2}}{3d(8c - dx^3)} - \frac{1}{2}(9c) \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right) \\
&= \frac{\sqrt{c + dx^3}}{d} + \frac{(c + dx^3)^{3/2}}{3d(8c - dx^3)} - \frac{(9c) \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{d} \\
&= \frac{\sqrt{c + dx^3}}{d} + \frac{(c + dx^3)^{3/2}}{3d(8c - dx^3)} - \frac{3\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{d}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.56

$$\frac{2(c + dx^3)^{5/2} {}_2F_1\left(2, \frac{5}{2}; \frac{7}{2}; \frac{dx^3+c}{9c}\right)}{1215c^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] (2*(c + d*x^3)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (c + d*x^3)/(9*c)])/(1215*c^2*d)

IntegrateAlgebraic [A] time = 0.07, size = 71, normalized size = 0.92

$$-\frac{\sqrt{c + dx^3} (25c - 2dx^3)}{3d(dx^3 - 8c)} - \frac{3\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] -1/3*((25*c - 2*d*x^3)*Sqrt[c + d*x^3])/(d*(-8*c + d*x^3)) - (3*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d

fricas [A] time = 0.44, size = 162, normalized size = 2.10

$$\left[\frac{9(dx^3 - 8c)\sqrt{c} \log\left(\frac{dx^3 - 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c}\right) + 2(2dx^3 - 25c)\sqrt{dx^3+c} - 9(dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + (2dx^3 - 25c)\sqrt{dx^3+c}}{6(d^2x^3 - 8cd)}, \frac{9(dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + (2dx^3 - 25c)\sqrt{dx^3+c}}{3(d^2x^3 - 8cd)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] [1/6*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 2*(2*d*x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d), 1/3*(9*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (2*d*x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d)]

giac [A] time = 0.17, size = 69, normalized size = 0.90

$$\frac{3c \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d} + \frac{2\sqrt{dx^3+c}}{3d} - \frac{3\sqrt{dx^3+c}c}{(dx^3-8c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] $3*c*\arctan(1/3*\sqrt{d*x^3 + c}/\sqrt{-c})/(\sqrt{-c}*d) + 2/3*\sqrt{d*x^3 + c}/d - 3*\sqrt{d*x^3 + c}*c/((d*x^3 - 8*c)*d)$

maple [C] time = 0.18, size = 451, normalized size = 5.86

$$\frac{\frac{3\sqrt{d^2x^3+c}}{(d^2-8c)d} - \frac{2\sqrt{d^2x^3+c}}{3d}}{2d\sqrt{d^2x^3+c}} \cdot \left(\frac{\sqrt{\frac{d^2x^3+c}{-c}}}{\sqrt{-c}} \sqrt{\frac{d^2x^3+c}{-c}} \sqrt{\frac{d^2x^3+c}{-c}} \right) \left(2\operatorname{RootOf}(d^2x^3-8c)^2 \sqrt{-c} \sqrt{\operatorname{RootOf}(d^2x^3-8c)} d - (-c)^2 \operatorname{RootOf}(d^2x^3-8c) d - \sqrt{5} \sqrt{-c} \sqrt{-c} \sqrt{-c} \right) \operatorname{EllipticF} \left(\frac{\sqrt{\frac{d^2x^3+c}{-c}}}{\sqrt{-c}} \sqrt{\frac{d^2x^3+c}{-c}} \sqrt{\frac{d^2x^3+c}{-c}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)

[Out] $-3*(d*x^3+c)^{(1/2)}/(d*x^3-8*c)*c/d+2/3*(d*x^3+c)^{(1/2)}/d+1/2*I/d^3*2^{(1/2)*}\operatorname{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)}*\operatorname{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)})/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=\operatorname{RootOf}(_Z^3*d-8*c))$

maxima [A] time = 1.31, size = 79, normalized size = 1.03

$$\frac{9\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+4\sqrt{dx^3+c}-\frac{18\sqrt{dx^3+c}}{dx^3-8c}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")

[Out] $1/6*(9*\sqrt{c}*\log((\sqrt{d*x^3 + c} - 3*\sqrt{c})/(\sqrt{d*x^3 + c} + 3*\sqrt{c}))) + 4*\sqrt{d*x^3 + c} - 18*\sqrt{d*x^3 + c}*c/(d*x^3 - 8*c))/d$

mupad [B] time = 3.99, size = 87, normalized size = 1.13

$$\frac{2\sqrt{dx^3+c}}{3d} + \frac{3\sqrt{c}\ln\left(\frac{10c+dx^3-6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{2d} + \frac{3c\sqrt{dx^3+c}}{d(8c-dx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d) + (3*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c +
d*x^3)^(1/2))/(8*c - d*x^3)))/(2*d) + (3*c*(c + d*x^3)^(1/2))/(d*(8*c - d*
x^3))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

$$3.284 \quad \int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2} dx$$

Optimal. Leaf size=85

$$\frac{3\sqrt{c+dx^3}}{8(8c-dx^3)} - \frac{3 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {446, 98, 156, 63, 208, 206}

$$\frac{3\sqrt{c+dx^3}}{8(8c-dx^3)} - \frac{3 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2), x]

[Out] (3*sqrt[c + d*x^3])/(8*(8*c - d*x^3)) - (3*ArcTanh[Sqrt[c + d*x^3]/(3*sqrt[c])])/(32*sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/sqrt[c]]/(96*sqrt[c])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 98

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^3)^{3/2}}{x(8c - dx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{x(8c - dx)^2} dx, x, x^3 \right) \\
&= \frac{3\sqrt{c + dx^3}}{8(8c - dx^3)} - \frac{\text{Subst} \left(\int \frac{-c^2 d + \frac{7}{2} cd^2 x}{x(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{24cd} \\
&= \frac{3\sqrt{c + dx^3}}{8(8c - dx^3)} + \frac{1}{192} \text{Subst} \left(\int \frac{1}{x\sqrt{c + dx}} dx, x, x^3 \right) - \frac{1}{64} (9d) \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right) \\
&= \frac{3\sqrt{c + dx^3}}{8(8c - dx^3)} - \frac{9}{32} \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right) + \frac{\text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{96d} \\
&= \frac{3\sqrt{c + dx^3}}{8(8c - dx^3)} - \frac{3 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{32\sqrt{c}} - \frac{\tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{96\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 100, normalized size = 1.18

$$\frac{36\sqrt{c}\sqrt{c + dx^3} + (9dx^3 - 72c) \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right) + (dx^3 - 8c) \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{96\sqrt{c}(8c - dx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2), x]

[Out] (36*sqrt[c]*sqrt[c + d*x^3] + (-72*c + 9*d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*sqrt[c])] + (-8*c + d*x^3)*ArcTanh[Sqrt[c + d*x^3]/sqrt[c]]/(96*sqrt[c]*(8*c - d*x^3))

IntegrateAlgebraic [A] time = 0.08, size = 85, normalized size = 1.00

$$\frac{3\sqrt{c + dx^3}}{8(8c - dx^3)} - \frac{3 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{32\sqrt{c}} - \frac{\tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2), x]

[Out] $(3\sqrt{c + dx^3})/(8(8c - dx^3)) - (3\text{ArcTanh}[\sqrt{c + dx^3}/(3\sqrt{c})])/(32\sqrt{c}) - \text{ArcTanh}[\sqrt{c + dx^3}/\sqrt{c}]/(96\sqrt{c})$

fricas [A] time = 0.46, size = 220, normalized size = 2.59

$$\left[\frac{9(dx^3 - 8c)\sqrt{c} \log\left(\frac{dx^3 - 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c}\right) + (dx^3 - 8c)\sqrt{c} \log\left(\frac{dx^3 - 2\sqrt{dx^3+c}\sqrt{c} + 2c}{x^3}\right) - 72\sqrt{dx^3+c} (dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + 9(dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - 36\sqrt{dx^3+c}c}{192(cd^3 - 8c^2)}, \frac{9(dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + 9(dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - 36\sqrt{dx^3+c}c}{96(cd^3 - 8c^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx^3+c)^(3/2)/x/(-dx^3+8*c)^2,x, algorithm="fricas")

[Out] $[1/192*(9*(dx^3 - 8*c)*\sqrt{c}*\log((dx^3 - 6*\sqrt{dx^3 + c})*\sqrt{c} + 10*c)/(dx^3 - 8*c)) + (dx^3 - 8*c)*\sqrt{c}*\log((dx^3 - 2*\sqrt{dx^3 + c})*\sqrt{c} + 2*c)/x^3 - 72*\sqrt{dx^3 + c}*c/(c*dx^3 - 8*c^2), 1/96*((dx^3 - 8*c)*\sqrt{-c}*\arctan(\sqrt{dx^3 + c}*\sqrt{-c}/c) + 9*(dx^3 - 8*c)*\sqrt{-c}*\arctan(1/3*\sqrt{dx^3 + c}*\sqrt{-c}/c) - 36*\sqrt{dx^3 + c}*c/(c*dx^3 - 8*c^2)]$

giac [A] time = 0.17, size = 70, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}} + \frac{3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{32\sqrt{-c}} - \frac{3\sqrt{dx^3+c}}{8(dx^3-8c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx^3+c)^(3/2)/x/(-dx^3+8*c)^2,x, algorithm="giac")

[Out] $1/96*\arctan(\sqrt{dx^3 + c}/\sqrt{-c})/\sqrt{-c} + 3/32*\arctan(1/3*\sqrt{dx^3 + c}/\sqrt{-c})/\sqrt{-c} - 3/8*\sqrt{dx^3 + c}/(dx^3 - 8*c)$

maple [C] time = 0.17, size = 956, normalized size = 11.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((dx^3+c)^(3/2)/x/(-dx^3+8*c)^2,x)

[Out] $1/8/c*d*(-3*(dx^3+c)^(1/2)/(dx^3-8*c)*c/d+2/3*(dx^3+c)^(1/2)/d+1/2*I/d^3*2^(1/2)*\sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2))*(-c*d^2)^(1/3)*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(dx^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2))*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2))*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2), -1/18*(2*I*($

$$-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=RootOf(_Z^3*d-8*c))) - 1/64/c^2*d*(2/9*(d*x^3+c)^{(1/2)}*x^3+56/9*(d*x^3+c)^{(1/2)}*c/d+3*I*c/d^3*2^{(1/2)}*sum((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)}/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)}/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=RootOf(_Z^3*d-8*c))) + 1/64/c^2*(2/9*(d*x^3+c)^{(1/2)}*d*x^3+8/9*(d*x^3+c)^{(1/2)}*c-2/3*c^(3/2)*arctanh((d*x^3+c)^{(1/2)}/c^(1/2)))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{\frac{3}{2}}}{(dx^3 - 8c)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")

[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x), x)

mupad [B] time = 4.72, size = 101, normalized size = 1.19

$$\frac{3\sqrt{dx^3+c}}{8(8c-dx^3)} + \frac{\ln\left(\frac{(\sqrt{dx^3+c}-\sqrt{c})^3(\sqrt{dx^3+c}+\sqrt{c})(10c+dx^3-6\sqrt{c}\sqrt{dx^3+c})^9}{x^6(8c-dx^3)^9}\right)}{192\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2), x)

[Out] $(3*(c + d*x^3)^{(1/2)})/(8*(8*c - d*x^3)) + \log(((c + d*x^3)^{(1/2)} - c^{(1/2)})^3*((c + d*x^3)^{(1/2)} + c^{(1/2)})*(10*c + d*x^3 - 6*c^{(1/2)}*(c + d*x^3)^{(1/2)}))^9/(x^6*(8*c - d*x^3)^9)/(192*c^{(1/2)})$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

$$3.285 \quad \int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2} dx$$

Optimal. Leaf size=121

$$\frac{3d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}} - \frac{7d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}} + \frac{5d\sqrt{c+dx^3}}{96c(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24x^3(8c-dx^3)}$$

Rubi [A] time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {446, 98, 151, 156, 63, 208, 206}

$$\frac{3d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}} - \frac{7d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}} + \frac{5d\sqrt{c+dx^3}}{96c(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24x^3(8c-dx^3)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2), x]

[Out] (5*d*Sqrt[c + d*x^3]/(96*c*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*x^3*(8*c - d*x^3)) + (3*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(128*c^(3/2)) - (7*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])/(384*c^(3/2)))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 98

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^3)^{3/2}}{x^4 (8c - dx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{x^2 (8c - dx)^2} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c + dx^3}}{24x^3 (8c - dx^3)} - \frac{\text{Subst} \left(\int \frac{-14c^2 d - \frac{19}{2} cd^2 x}{x(8c - dx)^2 \sqrt{c + dx}} dx, x, x^3 \right)}{24c} \\
&= \frac{5d\sqrt{c + dx^3}}{96c (8c - dx^3)} - \frac{\sqrt{c + dx^3}}{24x^3 (8c - dx^3)} + \frac{\text{Subst} \left(\int \frac{126c^3 d^2 + 45c^2 d^3 x}{x(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{1728c^3 d} \\
&= \frac{5d\sqrt{c + dx^3}}{96c (8c - dx^3)} - \frac{\sqrt{c + dx^3}}{24x^3 (8c - dx^3)} + \frac{(7d) \text{Subst} \left(\int \frac{1}{x\sqrt{c + dx}} dx, x, x^3 \right)}{768c} + \frac{(9d^2) \text{Subst} \left(\int \frac{1}{x^2 \sqrt{c + dx}} dx, x, x^3 \right)}{2} \\
&= \frac{5d\sqrt{c + dx^3}}{96c (8c - dx^3)} - \frac{\sqrt{c + dx^3}}{24x^3 (8c - dx^3)} + \frac{7 \text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{384c} + \frac{(9d) \text{Subst} \left(\int \frac{1}{x^2 \sqrt{c + dx}} dx, x, x^3 \right)}{2} \\
&= \frac{5d\sqrt{c + dx^3}}{96c (8c - dx^3)} - \frac{\sqrt{c + dx^3}}{24x^3 (8c - dx^3)} + \frac{3d \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{128c^{3/2}} - \frac{7d \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{384c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 97, normalized size = 0.80

$$\frac{9d \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right) - 7d \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right) + \frac{4\sqrt{c} \sqrt{c + dx^3} (4c - 5dx^3)}{dx^6 - 8cx^3}}{384c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2), x]

[Out] ((4*sqrt[c]*(4*c - 5*d*x^3)*sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 9*d*ArcTanh[Sqrt[c + d*x^3]/(3*sqrt[c])] - 7*d*ArcTanh[Sqrt[c + d*x^3]/sqrt[c]])/(384*c^(3/2))

IntegrateAlgebraic [A] time = 0.12, size = 103, normalized size = 0.85

$$\frac{3d \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{128c^{3/2}} - \frac{7d \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{384c^{3/2}} + \frac{\sqrt{c + dx^3} (5dx^3 - 4c)}{96cx^3 (8c - dx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2), x]

[Out] (Sqrt[c + d*x^3]*(-4*c + 5*d*x^3))/(96*c*x^3*(8*c - d*x^3)) + (3*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(128*c^(3/2)) - (7*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(384*c^(3/2))

fricas [A] time = 0.45, size = 280, normalized size = 2.31

$$\frac{9(d^2x^6 - 8cdx^3)\sqrt{c} \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c+10c}}{dx^3-8c}\right) + 7(d^2x^6 - 8cdx^3)\sqrt{c} \log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c+2c}}{x^3}\right) - 8(5cdx^3 - 4c^2)\sqrt{dx^3+c} - 7(d^2x^6 - 8cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - 9(d^2x^6 - 8cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - 4(5cdx^3 - 4c^2)\sqrt{dx^3+c}}{768(c^2dx^6 - 8c^3x^3)}, \frac{7(d^2x^6 - 8cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - 9(d^2x^6 - 8cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - 4(5cdx^3 - 4c^2)\sqrt{dx^3+c}}{384(c^2dx^6 - 8c^3x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] [1/768*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^2*d*x^6 - 8*c^3*x^3), 1/384*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^2*d*x^6 - 8*c^3*x^3)]

giac [A] time = 0.19, size = 114, normalized size = 0.94

$$\frac{7d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\sqrt{-c}c} - \frac{3d \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{128\sqrt{-c}c} - \frac{5(dx^3+c)^{\frac{3}{2}}d - 9\sqrt{dx^3+c}cd}{96\left((dx^3+c)^2 - 10(dx^3+c)c + 9c^2\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] 7/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 3/128*d*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/96*(5*(d*x^3 + c)^(3/2)*d - 9*sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c)

maple [C] time = 0.19, size = 1014, normalized size = 8.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x)

[Out] 1/64/c^2*d^2*(-3*(d*x^3+c)^(1/2)/(d*x^3-8*c)*c/d+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3))+(-c

$d^2)^{(1/3))/d)/(-c*d^2)^{(1/3)*d)^{(1/2)*((x-(-c*d^2)^{(1/3))/d)/(-3*(-c*d^2)^{(1/3)+I*3^{(1/2)*(-c*d^2)^{(1/3))*d)^{(1/2)*(-1/2*I*(2*x+(I*3^{(1/2)*(-c*d^2)^{(1/3)+(-c*d^2)^{(1/3))/d)/(-c*d^2)^{(1/3)*d)^{(1/2)/(d*x^3+c)^{(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)*3^{(1/2)*_alpha*d-(-c*d^2)^{(1/3)*_alpha*d-I*3^{(1/2)*(-c*d^2)^{(2/3)-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)*(I*(x+1/2*(-c*d^2)^{(1/3)/d-1/2*I*3^{(1/2)*(-c*d^2)^{(1/3)/d)*3^{(1/2)/(-c*d^2)^{(1/3)*d)^{(1/2)},-1/18*(2*I*(-c*d^2)^{(1/3)*3^{(1/2)*_alpha^2*d+I*3^{(1/2)*c*d-3*c*d-I*(-c*d^2)^{(2/3)*3^{(1/2)*_alpha-3*(-c*d^2)^{(2/3)*_alpha)/c/d,(I*3^{(1/2)*(-c*d^2)^{(1/3)/(-3/2*(-c*d^2)^{(1/3)/d+1/2*I*3^{(1/2)*(-c*d^2)^{(1/3)/d)/d)^{(1/2)}),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-1/3*(d*x^3+c)^{(1/2)*c/x^3+2/3*(d*x^3+c)^{(1/2)*d-c^{(1/2)*d*arctanh((d*x^3+c)^{(1/2)/c^{(1/2)})})-1/256/c^3*d^2*(2/9*(d*x^3+c)^{(1/2)*x^3+56/9*(d*x^3+c)^{(1/2)*c/d+3*I*c/d^3*2^{(1/2)*sum((-c*d^2)^{(1/3)*(1/2*I*(2*x+(-I*3^{(1/2)*(-c*d^2)^{(1/3)+(-c*d^2)^{(1/3))/d)/(-c*d^2)^{(1/3)*d)^{(1/2)*(x-(-c*d^2)^{(1/3)/d)/(-3*(-c*d^2)^{(1/3)+I*3^{(1/2)*(-c*d^2)^{(1/3))*d)^{(1/2)*(-1/2*I*(2*x+(I*3^{(1/2)*(-c*d^2)^{(1/3)+(-c*d^2)^{(1/3))/d)/(-c*d^2)^{(1/3)*d)^{(1/2)/(d*x^3+c)^{(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)*3^{(1/2)*_alpha*d-(-c*d^2)^{(1/3)*_alpha*d-I*3^{(1/2)*(-c*d^2)^{(2/3)-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)*(I*(x+1/2*(-c*d^2)^{(1/3)/d-1/2*I*3^{(1/2)*(-c*d^2)^{(1/3)/d)*3^{(1/2)/(-c*d^2)^{(1/3)*d)^{(1/2)},-1/18*(2*I*(-c*d^2)^{(1/3)*3^{(1/2)*_alpha^2*d+I*3^{(1/2)*c*d-3*c*d-I*(-c*d^2)^{(2/3)*3^{(1/2)*_alpha-3*(-c*d^2)^{(2/3)*_alpha)/c/d,(I*3^{(1/2)*(-c*d^2)^{(1/3)/(-3/2*(-c*d^2)^{(1/3)/d+1/2*I*3^{(1/2)*(-c*d^2)^{(1/3)/d)/d)^{(1/2)}),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d^2*(2/9*(d*x^3+c)^{(1/2)*d*x^3+8/9*(d*x^3+c)^{(1/2)*c-2/3*c^{(3/2)*arctanh((d*x^3+c)^{(1/2)/c^{(1/2)})})})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{\frac{3}{2}}}{(dx^3 - 8c)^2 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")

[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^4), x)

mupad [B] time = 4.24, size = 110, normalized size = 0.91

$$\frac{\frac{9d\sqrt{dx^3+c}}{32} - \frac{5d(dx^3+c)^{3/2}}{32c}}{3(dx^3+c)^2 - 30c(dx^3+c) + 27c^2} + \frac{d \left(\operatorname{atanh} \left(\frac{c\sqrt{dx^3+c}}{\sqrt{c^3}} \right) 1i - \frac{\operatorname{atanh} \left(\frac{c\sqrt{dx^3+c}}{3\sqrt{c^3}} \right) 9i}{7} \right) 7i}{384\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x)
```

```
[Out] ((9*d*(c + d*x^3)^(1/2))/32 - (5*d*(c + d*x^3)^(3/2))/(32*c))/(3*(c + d*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) + (d*(atanh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)))*1i - (atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2)))*9i)/7)*7i)/(384*(c^3)^(1/2))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```


$$3.286 \quad \int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2} dx$$

Optimal. Leaf size=161

$$\frac{15d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}} - \frac{17d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}} + \frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)} - \frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)}$$

Rubi [A] time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {446, 98, 151, 156, 63, 208, 206}

$$\frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)} + \frac{15d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}} - \frac{17d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}} - \frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2), x]

[Out] (7*d^2*Sqrt[c + d*x^3])/(512*c^2*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*x^6*(8*c - d*x^3)) - (23*d*Sqrt[c + d*x^3])/(384*c*x^3*(8*c - d*x^3)) + (15*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2048*c^(5/2)) - (17*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])/(2048*c^(5/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 98

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^3)^{3/2}}{x^7 (8c - dx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{x^3 (8c - dx)^2} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c + dx^3}}{48x^6 (8c - dx^3)} - \frac{\text{Subst} \left(\int \frac{-23c^2 d - \frac{37}{2} cd^2 x}{x^2 (8c - dx)^2 \sqrt{c + dx}} dx, x, x^3 \right)}{48c} \\
&= -\frac{\sqrt{c + dx^3}}{48x^6 (8c - dx^3)} - \frac{23d\sqrt{c + dx^3}}{384cx^3 (8c - dx^3)} + \frac{\text{Subst} \left(\int \frac{102c^3 d^2 + \frac{69}{2} c^2 d^3 x}{x (8c - dx)^2 \sqrt{c + dx}} dx, x, x^3 \right)}{384c^3} \\
&= \frac{7d^2\sqrt{c + dx^3}}{512c^2 (8c - dx^3)} - \frac{\sqrt{c + dx^3}}{48x^6 (8c - dx^3)} - \frac{23d\sqrt{c + dx^3}}{384cx^3 (8c - dx^3)} - \frac{\text{Subst} \left(\int \frac{-918c^4 d^3 - 189c^3 d^4 x}{x (8c - dx) \sqrt{c + dx}} dx, x, x^3 \right)}{27648c^5 d} \\
&= \frac{7d^2\sqrt{c + dx^3}}{512c^2 (8c - dx^3)} - \frac{\sqrt{c + dx^3}}{48x^6 (8c - dx^3)} - \frac{23d\sqrt{c + dx^3}}{384cx^3 (8c - dx^3)} + \frac{(17d^2) \text{Subst} \left(\int \frac{1}{x\sqrt{c + dx}} dx, x, x^3 \right)}{4096c^2} \\
&= \frac{7d^2\sqrt{c + dx^3}}{512c^2 (8c - dx^3)} - \frac{\sqrt{c + dx^3}}{48x^6 (8c - dx^3)} - \frac{23d\sqrt{c + dx^3}}{384cx^3 (8c - dx^3)} + \frac{(17d) \text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, x^3 \right)}{2048c^2} \\
&= \frac{7d^2\sqrt{c + dx^3}}{512c^2 (8c - dx^3)} - \frac{\sqrt{c + dx^3}}{48x^6 (8c - dx^3)} - \frac{23d\sqrt{c + dx^3}}{384cx^3 (8c - dx^3)} + \frac{15d^2 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{2048c^{5/2}} - \frac{17d}{2048c^2}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 112, normalized size = 0.70

$$\frac{4\sqrt{c}\sqrt{c+dx^3}(32c^2+92cdx^3-21d^2x^6)}{dx^9-8cx^6} + 45d^2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) - 51d^2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)$$

$$6144c^{5/2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2), x]

[Out] ((4*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 92*c*d*x^3 - 21*d^2*x^6))/(-8*c*x^6 + d*x^9) + 45*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 51*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(6144*c^(5/2))

IntegrateAlgebraic [A] time = 0.14, size = 118, normalized size = 0.73

$$\frac{15d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}} - \frac{17d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}} + \frac{\sqrt{c+dx^3}(-32c^2 - 92cdx^3 + 21d^2x^6)}{1536c^2x^6(8c - dx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2), x]

[Out] (Sqrt[c + d*x^3]*(-32*c^2 - 92*c*d*x^3 + 21*d^2*x^6))/(1536*c^2*x^6*(8*c - d*x^3)) + (15*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2048*c^(5/2)) - (17*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])]/(2048*c^(5/2))

fricas [A] time = 0.47, size = 310, normalized size = 1.93

$$\frac{45(d^3x^9 - 8cd^2x^6)\sqrt{c}\log\left(\frac{d^3x^6 + \sqrt{d^3x^3 + c}\sqrt{c} + 10c}{d^3 - 8c}\right) + 51(d^3x^9 - 8cd^2x^6)\sqrt{c}\log\left(\frac{d^3x^6 + \sqrt{d^3x^3 + c}\sqrt{c} + 2c}{d^3 - 8c}\right) - 8(21cd^2x^6 - 92c^2dx^3 - 32c^3)\sqrt{d^3x^3 + c} - 51(d^3x^9 - 8cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{d^3x^3 + c}\sqrt{-c}}{c}\right) - 45(d^3x^9 - 8cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{d^3x^3 + c}\sqrt{-c}}{3c}\right) - 4(21cd^2x^6 - 92c^2dx^3 - 32c^3)\sqrt{d^3x^3 + c}}{12288(c^3dx^9 - 8c^4x^6)}, \frac{51(d^3x^9 - 8cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{d^3x^3 + c}\sqrt{-c}}{c}\right) - 45(d^3x^9 - 8cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{d^3x^3 + c}\sqrt{-c}}{3c}\right) - 4(21cd^2x^6 - 92c^2dx^3 - 32c^3)\sqrt{d^3x^3 + c}}{6144(c^3dx^9 - 8c^4x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] [1/12288*(45*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 51*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(21*c*d^2*x^6 - 92*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6), 1/6144*(51*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 45*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(21*c*d^2*x^6 - 92*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6)]

giac [A] time = 0.18, size = 129, normalized size = 0.80

$$\frac{17d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\sqrt{-c}c^2} - \frac{15d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{2048\sqrt{-c}c^2} - \frac{3\sqrt{dx^3+c}d^2}{512(dx^3-8c)c^2} - \frac{3(dx^3+c)^{\frac{3}{2}}d^2 - 2\sqrt{dx^3+c}cd^2}{384c^2d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] 17/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 15/2048*d^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 3/512*sqrt(d*x^3 + c)*d^2/((d*x^3 - 8*c)*c^2) - 1/384*(3*(d*x^3 + c)^(3/2)*d^2 - 2*sqrt(d*x^3 + c)*c*d^2)/(c^2*d^2*x^6)

maple [C] time = 0.19, size = 1075, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x)`

[Out] $\frac{1}{512} \frac{c^3 d^3 (-3(d^2 x^3 + c)^{1/2})}{(d^2 x^3 - 8c) c/d + 2/3 (d^2 x^3 + c)^{1/2} / d + 1/2} \cdot I/d^3 \cdot 2^{1/2} \cdot \sum \left(\frac{(-c d^2)^{1/3} (1/2 I (2x + (-I 3^{1/2}) (-c d^2)^{1/3}) + (-c d^2)^{1/3}) / d}{(-c d^2)^{1/3} d^{1/2}} \cdot \frac{(x - (-c d^2)^{1/3}) / d}{(-3(-c d^2)^{1/3} + I 3^{1/2} (-c d^2)^{1/3}) d^{1/2}} \cdot \frac{(-1/2 I (2x + (I 3^{1/2}) (-c d^2)^{1/3}) + (-c d^2)^{1/3}) / d}{(-c d^2)^{1/3} d^{1/2}} \cdot \frac{(d^2 x^3 + c)^{1/2} (2 \alpha^2 d^2 + I (-c d^2)^{1/3} 3^{1/2} \alpha d - (-c d^2)^{1/3} \alpha d - I 3^{1/2} (-c d^2)^{2/3} - (-c d^2)^{2/3}) \cdot \text{EllipticPi}(1/3, 3^{1/2} (I(x + 1/2 (-c d^2)^{1/3}) / d - 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) 3^{1/2} / (-c d^2)^{1/3} d^{1/2}, -1/18 (2 I (-c d^2)^{1/3} 3^{1/2} \alpha^2 d + I 3^{1/2} c d - 3 c d - I (-c d^2)^{2/3}) 3^{1/2} \alpha - 3 (-c d^2)^{2/3} \alpha) / c/d, (I 3^{1/2} (-c d^2)^{1/3} / (-3/2 (-c d^2)^{1/3} / d + 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) / d)^{1/2} \right), \alpha = \text{RootOf}(_Z^3 d - 8c)) + \frac{1}{256} \frac{c^3 d^3 (-1/3 (d^2 x^3 + c)^{1/2} c / x^3 + 2/3 (d^2 x^3 + c)^{1/2} d - c^{1/2} d \cdot \text{arctanh}((d^2 x^3 + c)^{1/2} / c^{1/2})) + 1/64 c^2 (-1/6 c (d^2 x^3 + c)^{1/2} / x^6 - 5/12 d (d^2 x^3 + c)^{1/2} / x^3 - 1/4 d^2 \cdot \text{arctanh}((d^2 x^3 + c)^{1/2} / c^{1/2})) / c^{1/2} - 3/4096 c^4 d^3 (2/9 (d^2 x^3 + c)^{1/2} x^3 + 56/9 (d^2 x^3 + c)^{1/2} c/d + 3 I c/d^3 \cdot 2^{1/2} \cdot \sum \left(\frac{(-c d^2)^{1/3} (1/2 I (2x + (-I 3^{1/2}) (-c d^2)^{1/3}) + (-c d^2)^{1/3}) / d}{(-c d^2)^{1/3} d^{1/2}} \cdot \frac{(x - (-c d^2)^{1/3}) / d}{(-3(-c d^2)^{1/3} + I 3^{1/2} (-c d^2)^{1/3}) d^{1/2}} \cdot \frac{(-1/2 I (2x + (I 3^{1/2}) (-c d^2)^{1/3}) + (-c d^2)^{1/3}) / d}{(-c d^2)^{1/3} d^{1/2}} \cdot \frac{(d^2 x^3 + c)^{1/2} (2 \alpha^2 d^2 + I (-c d^2)^{1/3} 3^{1/2} \alpha d - (-c d^2)^{1/3} \alpha d - I 3^{1/2} (-c d^2)^{2/3} - (-c d^2)^{2/3}) \cdot \text{EllipticPi}(1/3, 3^{1/2} (I(x + 1/2 (-c d^2)^{1/3}) / d - 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) 3^{1/2} / (-c d^2)^{1/3} d^{1/2}, -1/18 (2 I (-c d^2)^{1/3} 3^{1/2} \alpha^2 d + I 3^{1/2} c d - 3 c d - I (-c d^2)^{2/3}) 3^{1/2} \alpha - 3 (-c d^2)^{2/3} \alpha) / c/d, (I 3^{1/2} (-c d^2)^{1/3} / (-3/2 (-c d^2)^{1/3} / d + 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) / d)^{1/2} \right), \alpha = \text{RootOf}(_Z^3 d - 8c)) + \frac{3}{4096} \frac{c^4 d^2 (2/9 (d^2 x^3 + c)^{1/2} d x^3 + 8/9 (d^2 x^3 + c)^{1/2} c - 2/3 c^{3/2} \cdot \text{arctanh}((d^2 x^3 + c)^{1/2} / c^{1/2}))}{c^{1/2}}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{\frac{3}{2}}}{(dx^3 - 8c)^2 x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")`

[Out] `integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^7), x)`

mupad [B] time = 4.62, size = 151, normalized size = 0.94

$$\frac{\frac{81d^2\sqrt{dx^3+c}}{512} - \frac{67d^2(dx^3+c)^{3/2}}{256c} + \frac{21d^2(dx^3+c)^{5/2}}{512c^2}}{33c(dx^3+c)^2 - 57c^2(dx^3+c) - 3(dx^3+c)^3 + 27c^3} + \frac{d^2 \left(\operatorname{atanh}\left(\frac{c^2\sqrt{dx^3+c}}{\sqrt{c^5}}\right) 1i - \frac{\operatorname{atanh}\left(\frac{c^2\sqrt{dx^3+c}}{3\sqrt{c^5}}\right) 15i}{17} \right) 17i}{2048\sqrt{c^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2), x)`

[Out] `((81*d^2*(c + d*x^3)^(1/2))/512 - (67*d^2*(c + d*x^3)^(3/2))/(256*c) + (21*d^2*(c + d*x^3)^(5/2))/(512*c^2))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3) - 3*(c + d*x^3)^3 + 27*c^3) + (d^2*(atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))*1i - (atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))*15i)/17)*17i)/(2048*(c^5)^(1/2))`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c)**2, x)`

[Out] Timed out

$$3.287 \quad \int \frac{x^{11}}{(8c-dx^3)^2 \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=95

$$-\frac{2944c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4} + \frac{2\sqrt{c+dx^3}(170c+7dx^3)}{27d^4} + \frac{8x^6\sqrt{c+dx^3}}{27d^2(8c-dx^3)}$$

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 98, 147, 63, 206}

$$-\frac{2944c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4} + \frac{8x^6\sqrt{c+dx^3}}{27d^2(8c-dx^3)} + \frac{2\sqrt{c+dx^3}(170c+7dx^3)}{27d^4}$$

Antiderivative was successfully verified.

[In] Int[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (8*x^6*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) + (2*Sqrt[c + d*x^3]*(170*c + 7*d*x^3))/(27*d^4) - (2944*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^4)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 98

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_Symbol] := -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3)
) + d^2*e*g*(m + n + 2)*(m + n + 3))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{(8c - dx^3)^2 \sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3}{(8c - dx)^2 \sqrt{c + dx}} dx, x, x^3 \right) \\
&= \frac{8x^6 \sqrt{c + dx^3}}{27d^2 (8c - dx^3)} - \frac{\text{Subst} \left(\int \frac{x(16c^2 + 21cdx)}{(8c - dx) \sqrt{c + dx}} dx, x, x^3 \right)}{27cd^2} \\
&= \frac{8x^6 \sqrt{c + dx^3}}{27d^2 (8c - dx^3)} + \frac{2\sqrt{c + dx^3} (170c + 7dx^3)}{27d^4} - \frac{(1472c^2) \text{Subst} \left(\int \frac{1}{(8c - dx) \sqrt{c + dx}} dx, x, x^3 \right)}{27d^3} \\
&= \frac{8x^6 \sqrt{c + dx^3}}{27d^2 (8c - dx^3)} + \frac{2\sqrt{c + dx^3} (170c + 7dx^3)}{27d^4} - \frac{(2944c^2) \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{27d^4} \\
&= \frac{8x^6 \sqrt{c + dx^3}}{27d^2 (8c - dx^3)} + \frac{2\sqrt{c + dx^3} (170c + 7dx^3)}{27d^4} - \frac{2944c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{81d^4}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.96

$$\frac{2944c^{3/2} (8c - dx^3) \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right) + 6\sqrt{c + dx^3} (-1360c^2 + 114cdx^3 + 3d^2x^6)}{81d^4 (dx^3 - 8c)}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (6*Sqrt[c + d*x^3]*(-1360*c^2 + 114*c*d*x^3 + 3*d^2*x^6) + 2944*c^(3/2)*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^4*(-8*c + d*x^3))

IntegrateAlgebraic [A] time = 0.09, size = 84, normalized size = 0.88

$$-\frac{2944c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{81d^4} - \frac{2\sqrt{c + dx^3} (1360c^2 - 114cdx^3 - 3d^2x^6)}{27d^4 (dx^3 - 8c)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] $(-2\sqrt{c + dx^3} \cdot (1360c^2 - 114cdx^3 - 3d^2x^6)) / (27d^4(-8c + dx^3)) - (2944c^{3/2} \cdot \text{ArcTanh}[\sqrt{c + dx^3} / (3\sqrt{c})]) / (81d^4)$

fricas [A] time = 0.44, size = 195, normalized size = 2.05

$$\left[\frac{2 \left(736 (cdx^3 - 8c^2) \sqrt{c} \log \left(\frac{dx^3 - 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c} \right) + 3(3d^2x^6 + 114cdx^3 - 1360c^2) \sqrt{dx^3+c} \right)}{81(d^5x^3 - 8cd^4)}, \frac{2 \left(1472 (cdx^3 - 8c^2) \sqrt{-c} \arctan \left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c} \right) + 3(3d^2x^6 + 114cdx^3 - 1360c^2) \sqrt{dx^3+c} \right)}{81(d^5x^3 - 8cd^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")`

[Out] $[2/81 \cdot (736 \cdot (cdx^3 - 8c^2) \cdot \sqrt{c}) \cdot \log((dx^3 - 6\sqrt{dx^3+c}) \cdot \sqrt{c} + 10c) / (dx^3 - 8c) + 3 \cdot (3d^2x^6 + 114cdx^3 - 1360c^2) \cdot \sqrt{dx^3+c} / (d^5x^3 - 8cd^4), 2/81 \cdot (1472 \cdot (cdx^3 - 8c^2) \cdot \sqrt{-c}) \cdot \arctan(1/3 \cdot \sqrt{dx^3+c} \cdot \sqrt{-c} / c) + 3 \cdot (3d^2x^6 + 114cdx^3 - 1360c^2) \cdot \sqrt{dx^3+c} / (d^5x^3 - 8cd^4)]$

giac [A] time = 0.19, size = 93, normalized size = 0.98

$$\frac{2944c^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}d^4} - \frac{512\sqrt{dx^3+c}c^2}{27(dx^3-8c)d^4} + \frac{2\left((dx^3+c)^{\frac{3}{2}}d^8 + 45\sqrt{dx^3+c}cd^8\right)}{9d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")`

[Out] $2944/81 \cdot c^2 \cdot \arctan(1/3 \cdot \sqrt{dx^3+c} / \sqrt{-c}) / (\sqrt{-c} \cdot d^4) - 512/27 \cdot \sqrt{dx^3+c} \cdot c^2 / ((dx^3 - 8c) \cdot d^4) + 2/9 \cdot ((dx^3+c)^{3/2} \cdot d^8 + 45 \cdot \sqrt{dx^3+c} \cdot cd^8) / d^{12}$

maple [C] time = 0.30, size = 916, normalized size = 9.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)`

[Out] $1/d^3 \cdot ((2/9 \cdot (dx^3+c)^{1/2} / dx^3 - 4/9 \cdot (dx^3+c)^{1/2} \cdot c / d^2) \cdot d + 32/3 \cdot (dx^3+c)^{1/2} \cdot c / d) + 512 \cdot c^3 / d^3 \cdot (-1/27 \cdot c / d \cdot (dx^3+c)^{1/2} / (dx^3-8c) - 1/486 \cdot I / c^2 / d^3 \cdot 2^{1/2} \cdot \sum((-cd^2)^{1/3} \cdot (1/2 \cdot I \cdot (2x + (-I \cdot 3^{1/2}) \cdot (-cd^2)^{1/3}) + (-cd^2)^{1/3}) / d) / (-cd^2)^{1/3} \cdot d)^{1/2} \cdot ((x - (-cd^2)^{1/3}) / d) / (-3 \cdot (-cd^2)^{1/3} + I \cdot 3^{1/2}) \cdot (-cd^2)^{1/3} \cdot d)^{1/2} \cdot (-1/2 \cdot I \cdot (2x + (I \cdot 3^{1/2}) \cdot (-cd^2)^{1/3}) + (-cd^2)^{1/3}) / d) / (-cd^2)^{1/3} \cdot d)^{1/2} / (dx^3+c)^{1/2} \cdot (2 \cdot \alpha^2 \cdot d^2 + I \cdot (-cd^2)^{1/3} \cdot 3^{1/2} \cdot \alpha \cdot d - (-cd^2)^{1/3} \cdot \alpha \cdot d - I \cdot 3^{1/2}) \cdot (-$

$c*d^2)^{(2/3)} - (-c*d^2)^{(2/3)} * \text{EllipticPi}(1/3*3^{(1/2)} * (I*(x+1/2*(-c*d^2)^{(1/3)})/d - 1/2*I*3^{(1/2)} * (-c*d^2)^{(1/3)}/d) * 3^{(1/2)}/(-c*d^2)^{(1/3)} * d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)} * 3^{(1/2)} * _alpha^2*d + I*3^{(1/2)} * c*d - 3*c*d - I*(-c*d^2)^{(2/3)} * 3^{(1/2)} * _alpha - 3*(-c*d^2)^{(2/3)} * _alpha)/c/d, (I*3^{(1/2)} * (-c*d^2)^{(1/3)}/(-3/2 * (-c*d^2)^{(1/3)}/d + 1/2*I*3^{(1/2)} * (-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha = \text{RootOf}(_Z^3*d - 8*c)) + 64/9 * I * c/d^6 * 2^{(1/2)} * \text{sum}((-c*d^2)^{(1/3)} * (1/2 * I * (2*x + (-I*3^{(1/2)} * (-c*d^2)^{(1/3)} + (-c*d^2)^{(1/3)})/d) / (-c*d^2)^{(1/3)} * d)^{(1/2)} * ((x - (-c*d^2)^{(1/3)}/d) / (-3 * (-c*d^2)^{(1/3)} + I*3^{(1/2)} * (-c*d^2)^{(1/3)}) * d)^{(1/2)} * (-1/2 * I * (2*x + I*3^{(1/2)} * (-c*d^2)^{(1/3)} + (-c*d^2)^{(1/3)})/d) / (-c*d^2)^{(1/3)} * d)^{(1/2)} / (d*x^3 + c)^{(1/2)} * (2 * _alpha^2*d^2 + I*(-c*d^2)^{(1/3)} * 3^{(1/2)} * _alpha*d - (-c*d^2)^{(1/3)} * _alpha*d - I*3^{(1/2)} * (-c*d^2)^{(2/3)} - (-c*d^2)^{(2/3)}) * \text{EllipticPi}(1/3*3^{(1/2)} * (I*(x+1/2*(-c*d^2)^{(1/3)})/d - 1/2*I*3^{(1/2)} * (-c*d^2)^{(1/3)}/d) * 3^{(1/2)}/(-c*d^2)^{(1/3)} * d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)} * 3^{(1/2)} * _alpha^2*d + I*3^{(1/2)} * c*d - 3*c*d - I*(-c*d^2)^{(2/3)} * 3^{(1/2)} * _alpha - 3*(-c*d^2)^{(2/3)} * _alpha)/c/d, (I*3^{(1/2)} * (-c*d^2)^{(1/3)}/(-3/2 * (-c*d^2)^{(1/3)}/d + 1/2*I*3^{(1/2)} * (-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha = \text{RootOf}(_Z^3*d - 8*c))$

maxima [A] time = 1.34, size = 93, normalized size = 0.98

$$\frac{2 \left(736 c^{\frac{3}{2}} \log \left(\frac{\sqrt{dx^3+c} - 3\sqrt{c}}{\sqrt{dx^3+c} + 3\sqrt{c}} \right) + 9(dx^3+c)^{\frac{3}{2}} + 405\sqrt{dx^3+c}c - \frac{768\sqrt{dx^3+c^2}}{dx^3-8c} \right)}{81d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] 2/81*(736*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c))) + 9*(d*x^3 + c)^(3/2) + 405*sqrt(d*x^3 + c)*c - 768*sqrt(d*x^3 + c)*c^2/(d*x^3 - 8*c))/d^4

mupad [B] time = 4.06, size = 107, normalized size = 1.13

$$\frac{92c\sqrt{dx^3+c}}{9d^4} + \frac{1472c^{3/2} \ln\left(\frac{10c+dx^3-6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{81d^4} + \frac{2x^3\sqrt{dx^3+c}}{9d^3} + \frac{512c^2\sqrt{dx^3+c}}{27d^4(8c-dx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)

[Out] (92*c*(c + d*x^3)^(1/2))/(9*d^4) + (1472*c^(3/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*d^4) + (2*x^3*(c + d*x^3)^(1/2))/(9*d^3) + (512*c^2*(c + d*x^3)^(1/2))/(27*d^4*(8*c - d*x^3))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.288 \quad \int \frac{x^8}{(8c-dx^3)^2 \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=83

$$\frac{64c\sqrt{c+dx^3}}{27d^3(8c-dx^3)} + \frac{2\sqrt{c+dx^3}}{3d^3} - \frac{224\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 89, 80, 63, 206}

$$\frac{64c\sqrt{c+dx^3}}{27d^3(8c-dx^3)} + \frac{2\sqrt{c+dx^3}}{3d^3} - \frac{224\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

[In] Int[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (2*Sqrt[c + d*x^3]/(3*d^3) + (64*c*Sqrt[c + d*x^3]/(27*d^3*(8*c - d*x^3)) - (224*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 89

Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))

)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(-p_.)*((c_) + (d_.)*(x_)^(n_.))^(-q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^8}{(8c - dx^3)^2 \sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(8c - dx)^2 \sqrt{c + dx}} dx, x, x^3 \right) \\
 &= \frac{64c\sqrt{c + dx^3}}{27d^3(8c - dx^3)} - \frac{\text{Subst} \left(\int \frac{40c^2d + 9cd^2x}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{27cd^3} \\
 &= \frac{2\sqrt{c + dx^3}}{3d^3} + \frac{64c\sqrt{c + dx^3}}{27d^3(8c - dx^3)} - \frac{(112c) \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{27d^2} \\
 &= \frac{2\sqrt{c + dx^3}}{3d^3} + \frac{64c\sqrt{c + dx^3}}{27d^3(8c - dx^3)} - \frac{(224c) \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{27d^3} \\
 &= \frac{2\sqrt{c + dx^3}}{3d^3} + \frac{64c\sqrt{c + dx^3}}{27d^3(8c - dx^3)} - \frac{224\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{81d^3}
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 82, normalized size = 0.99

$$-\frac{64c\sqrt{c+dx^3}}{27d^3(dx^3-8c)} + \frac{2\sqrt{c+dx^3}}{3d^3} - \frac{224\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (2*Sqrt[c + d*x^3])/(3*d^3) - (64*c*Sqrt[c + d*x^3])/(27*d^3*(-8*c + d*x^3)) - (224*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)

IntegrateAlgebraic [A] time = 0.07, size = 73, normalized size = 0.88

$$-\frac{2\sqrt{c+dx^3}(104c-9dx^3)}{27d^3(dx^3-8c)} - \frac{224\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (-2*(104*c - 9*d*x^3)*Sqrt[c + d*x^3])/(27*d^3*(-8*c + d*x^3)) - (224*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)

fricas [A] time = 0.43, size = 167, normalized size = 2.01

$$\left[\frac{2\left(56(dx^3-8c)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c+10c}}{dx^3-8c}\right) + 3(9dx^3-104c)\sqrt{dx^3+c}\right)}{81(d^4x^3-8cd^3)}, \frac{2\left(112(dx^3-8c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 3(9dx^3-104c)\sqrt{dx^3+c}\right)}{81(d^4x^3-8cd^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-d*x^3+8*c)^(2/(d*x^3+c)^(1/2)),x, algorithm="fricas")

[Out] [2/81*(56*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3), 2/81*(112*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]

giac [A] time = 0.17, size = 69, normalized size = 0.83

$$\frac{224c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}d^3} + \frac{2\sqrt{dx^3+c}}{3d^3} - \frac{64\sqrt{dx^3+c}c}{27(dx^3-8c)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] 224/81*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) + 2/3*sqrt(d*x^3 + c)/d^3 - 64/27*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d^3)

maple [C] time = 0.20, size = 874, normalized size = 10.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)

[Out]
$$\frac{2/3*(d*x^3+c)^{(1/2)}/d^3+64*c^2/d^2*(-1/27*(d*x^3+c)^{(1/2)}/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^{(1/2)}*\sum((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},-1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)},_alpha=RootOf(_Z^3*d-8*c)))+16/27*I/d^5*2^{(1/2)}*\sum((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},-1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)},_alpha=RootOf(_Z^3*d-8*c))$$

maxima [A] time = 1.56, size = 79, normalized size = 0.95

$$\frac{2 \left(56 \sqrt{c} \log \left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}} \right) + 27 \sqrt{dx^3+c} - \frac{96 \sqrt{dx^3+cc}}{dx^3-8c} \right)}{81 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] $2/81*(56*\sqrt{c})*\log((\sqrt{d*x^3 + c} - 3*\sqrt{c})/(\sqrt{d*x^3 + c} + 3*\sqrt{c})) + 27*\sqrt{d*x^3 + c} - 96*\sqrt{d*x^3 + c}*c/(d*x^3 - 8*c))/d^3$

mupad [B] time = 4.00, size = 87, normalized size = 1.05

$$\frac{2\sqrt{dx^3+c}}{3d^3} + \frac{112\sqrt{c}\ln\left(\frac{10c+dx^3-6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{81d^3} + \frac{64c\sqrt{dx^3+c}}{27d^3(8c-dx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)`

[Out] $(2*(c + d*x^3)^{(1/2)})/(3*d^3) + (112*c^{(1/2)}*\log((10*c + d*x^3 - 6*c^{(1/2)}*(c + d*x^3)^{(1/2)})/(8*c - d*x^3)))/(81*d^3) + (64*c*(c + d*x^3)^{(1/2)})/(27*d^3*(8*c - d*x^3))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(-8c + dx^3)^2 \sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2), x)`

[Out] `Integral(x**8/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)`

$$3.289 \quad \int \frac{x^5}{(8c-dx^3)^2 \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=64

$$\frac{8\sqrt{c+dx^3}}{27d^2(8c-dx^3)} - \frac{10 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {446, 78, 63, 206}

$$\frac{8\sqrt{c+dx^3}}{27d^2(8c-dx^3)} - \frac{10 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

Antiderivative was successfully verified.

[In] Int[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (8*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*Sqrt[c]*d^2)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p *(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(8c - dx^3)^2 \sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{(8c - dx)^2 \sqrt{c + dx}} dx, x, x^3 \right) \\ &= \frac{8\sqrt{c + dx^3}}{27d^2 (8c - dx^3)} - \frac{5 \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{27d} \\ &= \frac{8\sqrt{c + dx^3}}{27d^2 (8c - dx^3)} - \frac{10 \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{27d^2} \\ &= \frac{8\sqrt{c + dx^3}}{27d^2 (8c - dx^3)} - \frac{10 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{81\sqrt{c} d^2} \end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.98

$$-\frac{8\sqrt{c + dx^3}}{27d^2 (dx^3 - 8c)} - \frac{10 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{81\sqrt{c} d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (-8*Sqrt[c + d*x^3])/(27*d^2*(-8*c + d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*Sqrt[c]*d^2)

IntegrateAlgebraic [A] time = 0.06, size = 63, normalized size = 0.98

$$-\frac{8\sqrt{c + dx^3}}{27d^2 (dx^3 - 8c)} - \frac{10 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{81\sqrt{c} d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (-8*Sqrt[c + d*x^3])/(27*d^2*(-8*c + d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*Sqrt[c]*d^2)

fricas [A] time = 0.47, size = 155, normalized size = 2.42

$$\left[\frac{5(dx^3 - 8c)\sqrt{c} \log\left(\frac{dx^3 - 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c}\right) - 24\sqrt{dx^3+c}c}{81(cd^3x^3 - 8c^2d^2)}, \frac{2\left(5(dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - 12\sqrt{dx^3+c}c\right)}{81(cd^3x^3 - 8c^2d^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/81*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 24*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2), 2/81*(5*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2)]

giac [A] time = 0.16, size = 58, normalized size = 0.91

$$\frac{2 \left(\frac{5 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d} - \frac{12\sqrt{dx^3+c}}{(dx^3-8c)d} \right)}{81d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] 2/81*(5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 12*sqrt(d*x^3 + c)/((d*x^3 - 8*c)*d))/d

maple [C] time = 0.17, size = 861, normalized size = 13.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)

[Out] 8*c/d*(-1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))

/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/27*I/d^4/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))

maxima [A] time = 1.32, size = 67, normalized size = 1.05

$$\frac{5 \log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right) - \frac{24\sqrt{dx^3+c}}{dx^3-8c}}{81d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] 1/81*(5*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/sqrt(c) - 24*sqrt(d*x^3 + c)/(d*x^3 - 8*c))/d^2

mupad [B] time = 4.01, size = 72, normalized size = 1.12

$$\frac{5 \ln\left(\frac{10c+dx^3-6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{81\sqrt{c}d^2} + \frac{8\sqrt{dx^3+c}}{27d^2(8c-dx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)

[Out] (5*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*c^(1/2)*d^2) + (8*(c + d*x^3)^(1/2))/(27*d^2*(8*c - d*x^3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(-8c + dx^3)^2 \sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2), x)

[Out] Integral(x**5/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)

$$3.290 \quad \int \frac{x^2}{(8c-dx^3)^2 \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=67

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d} + \frac{\sqrt{c+dx^3}}{27cd(8c-dx^3)}$$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {444, 51, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d} + \frac{\sqrt{c+dx^3}}{27cd(8c-dx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] Sqrt[c + d*x^3]/(27*c*d*(8*c - d*x^3)) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(81*c^(3/2)*d)

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(8c - dx^3)^2 \sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(8c - dx)^2 \sqrt{c + dx}} dx, x, x^3 \right) \\ &= \frac{\sqrt{c + dx^3}}{27cd(8c - dx^3)} + \frac{\text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{54c} \\ &= \frac{\sqrt{c + dx^3}}{27cd(8c - dx^3)} + \frac{\text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{27cd} \\ &= \frac{\sqrt{c + dx^3}}{27cd(8c - dx^3)} + \frac{\tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{81c^{3/2}d} \end{aligned}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.96

$$\frac{\frac{3\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3} + \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] ((3*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqr
t[c])))/(81*c^(3/2)*d)

IntegrateAlgebraic [A] time = 0.06, size = 67, normalized size = 1.00

$$\frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{81c^{3/2}d} + \frac{\sqrt{c + dx^3}}{27cd(8c - dx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] Sqrt[c + d*x^3]/(27*c*d*(8*c - d*x^3)) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(81*c^(3/2)*d)

fricas [A] time = 0.44, size = 153, normalized size = 2.28

$$\left[\frac{(dx^3 - 8c)\sqrt{c} \log\left(\frac{dx^3 + 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c}\right) - 6\sqrt{dx^3+c}c}{162(c^2d^2x^3 - 8c^3d)}, \frac{(dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 3\sqrt{dx^3+c}c}{81(c^2d^2x^3 - 8c^3d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/162*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 - 8*c^3*d), -1/81*((d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 - 8*c^3*d)]

giac [A] time = 0.16, size = 59, normalized size = 0.88

$$-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}cd} - \frac{\sqrt{dx^3+c}}{27(dx^3-8c)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] -1/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 1/27*sqrt(d*x^3 + c)/((d*x^3 - 8*c)*c*d)

maple [C] time = 0.19, size = 442, normalized size = 6.60

$$\frac{\sqrt{dx^3+c}}{27(dx^3-8c)d} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)

[Out] -1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1

$$\frac{1}{3}d)^{1/2} * ((x - (-c*d^2)^{1/3}/d) / (-3*(-c*d^2)^{1/3} + I*3^{1/2}*(-c*d^2)^{1/3})) * d)^{1/2} * (-1/2*I*(2*x + (I*3^{1/2}*(-c*d^2)^{1/3} + (-c*d^2)^{1/3})/d) / (-c*d^2)^{1/3} * d)^{1/2} / (d*x^3 + c)^{1/2} * (2*_alpha^2*d^2 + I*(-c*d^2)^{1/3} * 3^{1/2} *_alpha*d - (-c*d^2)^{1/3} *_alpha*d - I*3^{1/2}*(-c*d^2)^{2/3} - (-c*d^2)^{2/3}) * \text{EllipticPi}(1/3*3^{1/2} * (I*(x + 1/2*(-c*d^2)^{1/3}/d - 1/2*I*3^{1/2}*(-c*d^2)^{1/3}/d) * 3^{1/2} / (-c*d^2)^{1/3} * d)^{1/2}, -1/18*(2*I*(-c*d^2)^{1/3} * 3^{1/2} *_alpha^2*d + I*3^{1/2} * c*d - 3*c*d - I*(-c*d^2)^{2/3} * 3^{1/2} *_alpha - 3*(-c*d^2)^{2/3} *_alpha) / c/d, (I*3^{1/2}*(-c*d^2)^{1/3} / (-3/2*(-c*d^2)^{1/3}/d + 1/2*I*3^{1/2}*(-c*d^2)^{1/3}/d) / d)^{1/2}), _alpha = \text{RootOf}(_Z^3*d - 8*c))$$

maxima [A] time = 1.39, size = 72, normalized size = 1.07

$$\frac{\frac{6\sqrt{dx^3+c}}{(dx^3+c)c-9c^2} + \frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{162d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2), x, algorithm="maxima")

[Out] -1/162*(6*sqrt(d*x^3 + c)/((d*x^3 + c)*c - 9*c^2) + log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c))))/c^(3/2))/d

mapad [B] time = 3.98, size = 75, normalized size = 1.12

$$\frac{\ln\left(\frac{10c+dx^3+6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{162c^{3/2}d} + \frac{\sqrt{dx^3+c}}{27cd(8c-dx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)

[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(162*c^(3/2)*d) + (c + d*x^3)^(1/2)/(27*c*d*(8*c - d*x^3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-8c + dx^3)^2 \sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2), x)

[Out] Integral(x**2/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)

$$3.291 \quad \int \frac{1}{x(8c-dx^3)^2 \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=88

$$\frac{13 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}} + \frac{\sqrt{c+dx^3}}{216c^2(8c-dx^3)}$$

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {446, 103, 156, 63, 208, 206}

$$\frac{\sqrt{c+dx^3}}{216c^2(8c-dx^3)} + \frac{13 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] Sqrt[c + d*x^3]/(216*c^2*(8*c - d*x^3)) + (13*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2592*c^(5/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(5/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(8c - dx^3)^2 \sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(8c - dx)^2 \sqrt{c + dx}} dx, x, x^3 \right) \\
&= \frac{\sqrt{c + dx^3}}{216c^2 (8c - dx^3)} - \frac{\text{Subst} \left(\int \frac{-9cd - \frac{d^2x}{2}}{x(8c - dx) \sqrt{c + dx}} dx, x, x^3 \right)}{216c^2 d} \\
&= \frac{\sqrt{c + dx^3}}{216c^2 (8c - dx^3)} + \frac{\text{Subst} \left(\int \frac{1}{x \sqrt{c + dx}} dx, x, x^3 \right)}{192c^2} + \frac{(13d) \text{Subst} \left(\int \frac{1}{(8c - dx) \sqrt{c + dx}} dx, x, x^3 \right)}{1728c^2} \\
&= \frac{\sqrt{c + dx^3}}{216c^2 (8c - dx^3)} + \frac{13 \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{864c^2} + \frac{\text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{96c^2 d} \\
&= \frac{\sqrt{c + dx^3}}{216c^2 (8c - dx^3)} + \frac{13 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{2592c^{5/2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{96c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 83, normalized size = 0.94

$$\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3} + 13 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) - 27 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{2592c^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + 13*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 27*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2592*c^(5/2))

IntegrateAlgebraic [A] time = 0.09, size = 88, normalized size = 1.00

$$\frac{13 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{2592c^{5/2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{96c^{5/2}} + \frac{\sqrt{c + dx^3}}{216c^2 (8c - dx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] $\text{Sqrt}[c + d*x^3]/(216*c^2*(8*c - d*x^3)) + (13*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/(3*\text{Sqrt}[c])])/(2592*c^{(5/2)}) - \text{ArcTanh}[\text{Sqrt}[c + d*x^3]/\text{Sqrt}[c]]/(96*c^{(5/2)})$

fricas [A] time = 0.47, size = 226, normalized size = 2.57

$$\left[\frac{13(dx^3 - 8c)\sqrt{c} \log\left(\frac{dx^2 + 6\sqrt{dx^3+c}\sqrt{c+10c}}{dx^3-8c}\right) + 27(dx^3 - 8c)\sqrt{c} \log\left(\frac{dx^2 - 2\sqrt{dx^3+c}\sqrt{c+2c}}{x^3}\right) - 24\sqrt{dx^3+c} \cdot 27(dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - 13(dx^3 - 8c)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - 12\sqrt{dx^3+c}c}{5184(c^3dx^3 - 8c^4)}, \frac{2592(c^3dx^3 - 8c^4)}{2592(c^3dx^3 - 8c^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")`

[Out] $[1/5184*(13*(d*x^3 - 8*c)*\text{sqrt}(c)*\log((d*x^3 + 6*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 10*c)/(d*x^3 - 8*c)) + 27*(d*x^3 - 8*c)*\text{sqrt}(c)*\log((d*x^3 - 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 2*c)/x^3) - 24*\text{sqrt}(d*x^3 + c)*c)/(c^3*d*x^3 - 8*c^4), 1/2592*(27*(d*x^3 - 8*c)*\text{sqrt}(-c)*\arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) - 13*(d*x^3 - 8*c)*\text{sqrt}(-c)*\arctan(1/3*\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) - 12*\text{sqrt}(d*x^3 + c)*c)/(c^3*d*x^3 - 8*c^4)]$

giac [A] time = 0.19, size = 79, normalized size = 0.90

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c^2} - \frac{13\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{2592\sqrt{-c}c^2} - \frac{\sqrt{dx^3+c}}{216(dx^3-8c)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")`

[Out] $1/96*\arctan(\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*c^2) - 13/2592*\arctan(1/3*\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*c^2) - 1/216*\text{sqrt}(d*x^3 + c)/((d*x^3 - 8*c)*c^2)$

maple [C] time = 0.19, size = 880, normalized size = 10.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)`

[Out] $1/8/c*d*(-1/27*(d*x^3+c)^{(1/2)}/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*$

$(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)*(-c*d^2)^{(1/3)}/d)/d)^{(1/2))}, _alpha=RootOf(_Z^3*d-8*c)))-1/172*8*I/c^3/d^2*2^{(1/2)*sum((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)}))/d)/(-c*d^2)^{(1/3)*d)^{(1/2)*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)+I*3^{(1/2)*(-c*d^2)^{(1/3)})*d)^{(1/2)*(-1/2*I*(2*x+(I*3^{(1/2)*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)}))/d)/(-c*d^2)^{(1/3)*d)^{(1/2)/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)*I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)*(-c*d^2)^{(1/3)}/d)/d)^{(1/2))}, _alpha=RootOf(_Z^3*d-8*c))-1/96*arctanh((d*x^3+c)^{(1/2)}/c^{(1/2)})/c^{(5/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3 + c} (dx^3 - 8c)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x), x)

mupad [B] time = 4.01, size = 80, normalized size = 0.91

$$\frac{13 \operatorname{atanh}\left(\frac{c^2 \sqrt{dx^3+c}}{3 \sqrt{c^5}}\right)}{2592 \sqrt{c^5}} - \frac{\operatorname{atanh}\left(\frac{c^2 \sqrt{dx^3+c}}{\sqrt{c^5}}\right)}{96 \sqrt{c^5}} + \frac{\sqrt{dx^3 + c}}{72 c^2 (24c - 3dx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)

[Out] (13*atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2))))/(2592*(c^5)^(1/2)) - atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))/(96*(c^5)^(1/2)) + (c + d*x^3)^(1/2)/(72*c^2*(24*c - 3*d*x^3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(-8c + dx^3)^2 \sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```


$$3.292 \quad \int \frac{1}{x^4(8c-dx^3)^2 \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=124

$$\frac{11d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}} + \frac{5d\sqrt{c+dx^3}}{864c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)}$$

Rubi [A] time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {446, 103, 151, 156, 63, 208, 206}

$$\frac{5d\sqrt{c+dx^3}}{864c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)} + \frac{11d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (5*d*Sqrt[c + d*x^3]/(864*c^3*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*c^2*x^3*(8*c - d*x^3)) + (11*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(10368*c^(7/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])/(384*c^(7/2)))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(8c-dx^3)^2\sqrt{c+dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2(8c-dx)^2\sqrt{c+dx}} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)} - \frac{\text{Subst} \left(\int \frac{2cd-\frac{3d^2x}{2}}{x(8c-dx)^2\sqrt{c+dx}} dx, x, x^3 \right)}{24c^2} \\
&= \frac{5d\sqrt{c+dx^3}}{864c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)} + \frac{\text{Subst} \left(\int \frac{-18c^2d^2+5cd^3x}{x(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{1728c^4d} \\
&= \frac{5d\sqrt{c+dx^3}}{864c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)} - \frac{d \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{768c^3} + \frac{(11d^2)}{1728c^4d} \\
&= \frac{5d\sqrt{c+dx^3}}{864c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)} - \frac{\text{Subst} \left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{384c^3} + \frac{(11d^2)}{1728c^4d} \\
&= \frac{5d\sqrt{c+dx^3}}{864c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)} + \frac{11d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{10368c^{7/2}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{384c^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 97, normalized size = 0.78

$$\frac{11d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) + 27d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right) + \frac{12\sqrt{c} \sqrt{c+dx^3} (36c-5dx^3)}{dx^6-8cx^3}}{10368c^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] ((12*Sqrt[c]*(36*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 11*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + 27*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(10368*c^(7/2))

IntegrateAlgebraic [A] time = 0.13, size = 103, normalized size = 0.83

$$\frac{11d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{10368c^{7/2}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{384c^{7/2}} + \frac{\sqrt{c+dx^3} (5dx^3 - 36c)}{864c^3x^3(8c-dx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (Sqrt[c + d*x^3]*(-36*c + 5*d*x^3))/(864*c^3*x^3*(8*c - d*x^3)) + (11*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(10368*c^(7/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])]/(384*c^(7/2))

fricas [A] time = 0.45, size = 280, normalized size = 2.26

$$\frac{11(d^2x^6 - 8cdx^3)\sqrt{c} \log\left(\frac{dx^2 + 6\sqrt{dx^3+c}\sqrt{c+10c}}{dx^2-8c}\right) + 27(d^2x^6 - 8cdx^3)\sqrt{c} \log\left(\frac{dx^2 + 2\sqrt{dx^3+c}\sqrt{c+2c}}{x^3}\right) - 24(5cdx^3 - 36c^2)\sqrt{dx^3+c} - 27(d^2x^6 - 8cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + 11(d^2x^6 - 8cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 12(5cdx^3 - 36c^2)\sqrt{dx^3+c}}{20736(c^4dx^6 - 8c^5x^3)}, \dots, \frac{27(d^2x^6 - 8cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + 11(d^2x^6 - 8cdx^3)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 12(5cdx^3 - 36c^2)\sqrt{dx^3+c}}{10368(c^4dx^6 - 8c^5x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/20736*(11*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 27*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d*x^3 - 36*c^2)*sqrt(d*x^3 + c))/(c^4*d*x^6 - 8*c^5*x^3), -1/10368*(27*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 11*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(5*c*d*x^3 - 36*c^2)*sqrt(d*x^3 + c))/(c^4*d*x^6 - 8*c^5*x^3)]

giac [A] time = 0.16, size = 114, normalized size = 0.92

$$-\frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384 \sqrt{-c} c^3} - \frac{11 d \arctan\left(\frac{\sqrt{dx^3+c}}{3 \sqrt{-c}}\right)}{10368 \sqrt{-c} c^3} - \frac{5(dx^3+c)^{\frac{3}{2}}d - 41\sqrt{dx^3+c}cd}{864\left((dx^3+c)^2 - 10(dx^3+c)c + 9c^2\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] -1/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 11/10368*d*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/864*(5*(d*x^3 + c)^(3/2)*d - 41*sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c^3)

maple [C] time = 0.20, size = 926, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)

```
[Out] 1/64/c^2*d^2*(-1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^(1/2)
*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/
d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/
2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-
(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+
1/64/c^2*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))
/c^(3/2))-1/6912*I/c^4/d^2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/
3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c
)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_a
lpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*
d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-
c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/
2)),_alpha=RootOf(_Z^3*d-8*c))-1/384*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(
7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}(dx^3-8c)^2 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3+c)*(d*x^3-8*c)^2*x^4),x)
```

mupad [B] time = 4.11, size = 117, normalized size = 0.94

$$\frac{\frac{41 d \sqrt{d x^3+c}}{288 c^2} - \frac{5 d (d x^3+c)^{3/2}}{288 c^3}}{3 (d x^3+c)^2 - 30 c (d x^3+c) + 27 c^2} - \frac{d \left(\operatorname{atanh} \left(\frac{c^3 \sqrt{d x^3+c}}{\sqrt{c^7}} \right) \operatorname{li} + \frac{\operatorname{atanh} \left(\frac{c^3 \sqrt{d x^3+c}}{3 \sqrt{c^7}} \right) \operatorname{li}}{27} \right) \operatorname{li}}{384 \sqrt{c^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(1/2))*(8*c - d*x^3)^2), x)
```

```
[Out] ((41*d*(c + d*x^3)^(1/2))/(288*c^2) - (5*d*(c + d*x^3)^(3/2))/(288*c^3))/(3
*(c + d*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) - (d*(atanh((c^3*(c + d*x^3)^(1
/2)))/(c^7)^(1/2))*1i + (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*11i)
/27)*1i)/(384*(c^7)^(1/2))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{x^4 (-8c + dx^3)^2 \sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2), x)
```

```
[Out] Integral(1/(x**4*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

$$3.293 \quad \int \frac{1}{x^7(8c-dx^3)^2 \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=164

$$\frac{31d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}} - \frac{19d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}} - \frac{35d^2 \sqrt{c+dx^3}}{13824c^4(8c-dx^3)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48c^2x^6(8c-dx^3)}$$

Rubi [A] time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {446, 103, 151, 156, 63, 208, 206}

$$-\frac{35d^2 \sqrt{c+dx^3}}{13824c^4(8c-dx^3)} + \frac{31d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}} - \frac{19d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48c^2x^6(8c-dx^3)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (-35*d^2*Sqrt[c + d*x^3])/((13824*c^4*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*c^2*x^6*(8*c - d*x^3)) + (3*d*Sqrt[c + d*x^3]/(128*c^3*x^3*(8*c - d*x^3)) + (31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(165888*c^(9/2)) - (19*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])/(6144*c^(9/2)))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7 (8c - dx^3)^2 \sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^3 (8c - dx)^2 \sqrt{c + dx}} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c + dx^3}}{48c^2 x^6 (8c - dx^3)} - \frac{\text{Subst} \left(\int \frac{9cd - \frac{5d^2 x}{2}}{x^2 (8c - dx)^2 \sqrt{c + dx}} dx, x, x^3 \right)}{48c^2} \\
&= -\frac{\sqrt{c + dx^3}}{48c^2 x^6 (8c - dx^3)} + \frac{3d\sqrt{c + dx^3}}{128c^3 x^3 (8c - dx^3)} + \frac{\text{Subst} \left(\int \frac{38c^2 d^2 - \frac{27}{2} cd^3 x}{x(8c - dx)^2 \sqrt{c + dx}} dx, x, x^3 \right)}{384c^4} \\
&= -\frac{35d^2 \sqrt{c + dx^3}}{13824c^4 (8c - dx^3)} - \frac{\sqrt{c + dx^3}}{48c^2 x^6 (8c - dx^3)} + \frac{3d\sqrt{c + dx^3}}{128c^3 x^3 (8c - dx^3)} - \frac{\text{Subst} \left(\int \frac{-3}{x} dx, x, x^3 \right)}{384c^4} \\
&= -\frac{35d^2 \sqrt{c + dx^3}}{13824c^4 (8c - dx^3)} - \frac{\sqrt{c + dx^3}}{48c^2 x^6 (8c - dx^3)} + \frac{3d\sqrt{c + dx^3}}{128c^3 x^3 (8c - dx^3)} + \frac{(19d^2) \text{Subst} \left(\int \frac{-3}{x} dx, x, x^3 \right)}{384c^4} \\
&= -\frac{35d^2 \sqrt{c + dx^3}}{13824c^4 (8c - dx^3)} - \frac{\sqrt{c + dx^3}}{48c^2 x^6 (8c - dx^3)} + \frac{3d\sqrt{c + dx^3}}{128c^3 x^3 (8c - dx^3)} + \frac{(19d^2) \text{Subst} \left(\int \frac{-3}{x} dx, x, x^3 \right)}{384c^4} \\
&= -\frac{35d^2 \sqrt{c + dx^3}}{13824c^4 (8c - dx^3)} - \frac{\sqrt{c + dx^3}}{48c^2 x^6 (8c - dx^3)} + \frac{3d\sqrt{c + dx^3}}{128c^3 x^3 (8c - dx^3)} + \frac{(19d^2) \text{Subst} \left(\int \frac{-3}{x} dx, x, x^3 \right)}{384c^4} \\
&= -\frac{35d^2 \sqrt{c + dx^3}}{13824c^4 (8c - dx^3)} - \frac{\sqrt{c + dx^3}}{48c^2 x^6 (8c - dx^3)} + \frac{3d\sqrt{c + dx^3}}{128c^3 x^3 (8c - dx^3)} + \frac{31d^2 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{165888c^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 112, normalized size = 0.68

$$\frac{12\sqrt{c}\sqrt{c+dx^3}(288c^2-324cdx^3+35d^2x^6)}{dx^9-8cx^6} + 31d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 513d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

$$165888c^{9/2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3]*(288*c^2 - 324*c*d*x^3 + 35*d^2*x^6))/(-8*c*x^6 + d*x^9) + 31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 513*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(165888*c^(9/2))

IntegrateAlgebraic [A] time = 0.15, size = 118, normalized size = 0.72

$$\frac{31d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}} - \frac{19d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}} + \frac{\sqrt{c+dx^3}(-288c^2 + 324cdx^3 - 35d^2x^6)}{13824c^4x^6(8c - dx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (Sqrt[c + d*x^3]*(-288*c^2 + 324*c*d*x^3 - 35*d^2*x^6))/(13824*c^4*x^6*(8*c - d*x^3)) + (31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(165888*c^(9/2)) - (19*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(6144*c^(9/2))

fricas [A] time = 0.65, size = 310, normalized size = 1.89

$$\frac{31(d^3x^9 - 8cd^2x^6)\sqrt{c} \log\left(\frac{d^3x^9 - 8cd^2x^6 - 324c^2dx^3 + 288c^3}{d^3 - 8c}\right) + 513(d^3x^9 - 8cd^2x^6)\sqrt{c} \log\left(\frac{d^3 - 2\sqrt{d^3x^3 + c}\sqrt{c}}{d^3 - 8c}\right) + 24(35cd^2x^6 - 324c^2dx^3 + 288c^3)\sqrt{d^3x^3 + c}}{331776(c^5dx^9 - 8c^6x^6)} - \frac{513(d^3x^9 - 8cd^2x^6)\sqrt{-c} \arctan\left(\frac{\sqrt{d^3x^3 + c}\sqrt{-c}}{c}\right) - 31(d^3x^9 - 8cd^2x^6)\sqrt{-c} \arctan\left(\frac{\sqrt{d^3x^3 + c}}{3c}\right) + 12(35cd^2x^6 - 324c^2dx^3 + 288c^3)\sqrt{d^3x^3 + c}}{165888(c^5dx^9 - 8c^6x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/331776*(31*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c))*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 513*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c))*sqrt(c) + 2*c)/x^3) + 24*(35*c*d^2*x^6 - 324*c^2*d*x^3 + 288*c^3)*sqrt(d*x^3 + c)/(c^5*d*x^9 - 8*c^6*x^6), 1/165888*(513*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 31*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(35*c*d^2*x^6 - 324*c^2*d*x^3 + 288*c^3)*sqrt(d*x^3 + c)/(c^5*d*x^9 - 8*c^6*x^6)]

giac [A] time = 0.18, size = 128, normalized size = 0.78

$$\frac{19d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6144\sqrt{-c}c^4} - \frac{31d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{165888\sqrt{-c}c^4} - \frac{\sqrt{dx^3+c}d^2}{13824(dx^3-8c)c^4} + \frac{(dx^3+c)^{\frac{3}{2}}d^2 - 2\sqrt{dx^3+c}cd^2}{384c^4d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] 19/6144*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 31/165888*d^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/13824*sqrt(d*x^3 + c)*d^2/((d*x^3 - 8*c)*c^4) + 1/384*((d*x^3 + c)^(3/2)*d^2 - 2*sqrt(d*x^3 + c)*c*d^2)/(c^4*d^2*x^6)

maple [C] time = 0.20, size = 989, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2), x)`

[Out]
$$\frac{1}{512} \frac{1}{c^3} d^3 (-1/27 (d x^3 + c)^{1/2} / (d x^3 - 8c) / c / d - 1/486 I / c^2 / d^3 2^{1/2}) \sum \left((-c d^2)^{1/3} (1/2 I (2x + (-I 3^{1/2}) (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d) / (-c d^2)^{1/3} d^{1/2} \right) \left((x - (-c d^2)^{1/3} / d) / (-3 (-c d^2)^{1/3} + I 3^{1/2} (-c d^2)^{1/3}) d^{1/2} \right) \left(-1/2 I (2x + (I 3^{1/2}) (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d \right) / (-c d^2)^{1/3} d^{1/2} / (d x^3 + c)^{1/2} (2 \alpha^2 d^2 + I (-c d^2)^{1/3} 3^{1/2} \alpha d - (-c d^2)^{1/3} \alpha d - I 3^{1/2} (-c d^2)^{2/3} - (-c d^2)^{2/3}) \text{EllipticPi} \left(1/3 3^{1/2} (I (x + 1/2 (-c d^2)^{1/3} / d - 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) 3^{1/2} / (-c d^2)^{1/3} d^{1/2} \right) / (-c d^2)^{1/3} d^{1/2}, -1/18 (2 I (-c d^2)^{1/3} 3^{1/2} \alpha^2 d + I 3^{1/2} c d - 3 c d - I (-c d^2)^{2/3} 3^{1/2} \alpha - 3 (-c d^2)^{2/3} \alpha) / c / d, (I 3^{1/2} (-c d^2)^{1/3} / (-3/2 (-c d^2)^{1/3} / d + 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) / d)^{1/2} \right), \alpha = \text{RootOf}(_Z^3 d - 8c)) + 1/256 c^3 d (-1/3 (d x^3 + c)^{1/2} / c / x^3 + 1/3 d \text{arctanh}((d x^3 + c)^{1/2} / c^{1/2}) / c^{3/2}) + 1/64 c^2 (-1/6 (d x^3 + c)^{1/2} / c / x^6 + 1/4 (d x^3 + c)^{1/2} / c^2 d / x^3 - 1/4 d^2 \text{arctanh}((d x^3 + c)^{1/2} / c^{1/2}) / c^{5/2}) - 1/36864 I / c^5 2^{1/2} \sum \left((-c d^2)^{1/3} (1/2 I (2x + (-I 3^{1/2}) (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d) / (-c d^2)^{1/3} d^{1/2} \right) \left((x - (-c d^2)^{1/3} / d) / (-3 (-c d^2)^{1/3} + I 3^{1/2} (-c d^2)^{1/3}) d^{1/2} \right) \left(-1/2 I (2x + (I 3^{1/2}) (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d \right) / (-c d^2)^{1/3} d^{1/2} / (d x^3 + c)^{1/2} (2 \alpha^2 d^2 + I (-c d^2)^{1/3} 3^{1/2} \alpha d - (-c d^2)^{1/3} \alpha d - I 3^{1/2} (-c d^2)^{2/3} - (-c d^2)^{2/3}) \text{EllipticPi} \left(1/3 3^{1/2} (I (x + 1/2 (-c d^2)^{1/3} / d - 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) 3^{1/2} / (-c d^2)^{1/3} d^{1/2} \right) / (-c d^2)^{1/3} d^{1/2}, -1/18 (2 I (-c d^2)^{1/3} 3^{1/2} \alpha^2 d + I 3^{1/2} c d - 3 c d - I (-c d^2)^{2/3} 3^{1/2} \alpha - 3 (-c d^2)^{2/3} \alpha) / c / d, (I 3^{1/2} (-c d^2)^{1/3} / (-3/2 (-c d^2)^{1/3} / d + 1/2 I 3^{1/2} (-c d^2)^{1/3} / d) / d)^{1/2} \right), \alpha = \text{RootOf}(_Z^3 d - 8c)) - 1/2048 d^2 \text{arctanh}((d x^3 + c)^{1/2} / c^{1/2}) / c^{9/2}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3 + c} (dx^3 - 8c)^2 x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2), x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^7), x)`

mupad [B] time = 4.37, size = 155, normalized size = 0.95

$$\frac{\frac{647 d^2 \sqrt{d x^3+c}}{4608 c^2} - \frac{197 d^2 (d x^3+c)^{3/2}}{2304 c^3} + \frac{35 d^2 (d x^3+c)^{5/2}}{4608 c^4}}{33 c (d x^3+c)^2 - 57 c^2 (d x^3+c) - 3 (d x^3+c)^3 + 27 c^3} + \frac{d^2 \left(\operatorname{atanh}\left(\frac{c^4 \sqrt{d x^3+c}}{\sqrt{c^9}}\right) 19i - \frac{\operatorname{atanh}\left(\frac{c^4 \sqrt{d x^3+c}}{3 \sqrt{c^9}}\right) 31i}{513} \right) 19i}{6144 \sqrt{c^9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^7*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)`

[Out] $(d^2 * (\operatorname{atanh}((c^4 * (c + d * x^3)^{1/2}) / (c^9)^{1/2})) * 19i - (\operatorname{atanh}((c^4 * (c + d * x^3)^{1/2}) / (3 * (c^9)^{1/2})) * 31i) / 513) * 19i) / (6144 * (c^9)^{1/2}) - ((647 * d^2 * (c + d * x^3)^{1/2}) / (4608 * c^2) - (197 * d^2 * (c + d * x^3)^{3/2}) / (2304 * c^3) + (35 * d^2 * (c + d * x^3)^{5/2}) / (4608 * c^4)) / (33 * c * (c + d * x^3)^2 - 57 * c^2 * (c + d * x^3) - 3 * (c + d * x^3)^3 + 27 * c^3)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 (-8c + dx^3)^2 \sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2), x)`

[Out] `Integral(1/(x**7*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)`

$$3.294 \quad \int \frac{x^{11}}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=95

$$\frac{2(38c + 39dx^3)}{81d^4\sqrt{c + dx^3}} - \frac{640\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4} + \frac{8x^6}{27d^2(8c - dx^3)\sqrt{c + dx^3}}$$

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 98, 146, 63, 206}

$$\frac{8x^6}{27d^2(8c - dx^3)\sqrt{c + dx^3}} + \frac{2(38c + 39dx^3)}{81d^4\sqrt{c + dx^3}} - \frac{640\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}$$

Antiderivative was successfully verified.

[In] Int[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] (8*x^6)/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (2*(38*c + 39*d*x^3))/(81*d^4*Sqrt[c + d*x^3]) - (640*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*d^4)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 98

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 146

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
+ 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2)
- c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rule 446

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{(8c - dx^3)^2 (c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3}{(8c - dx)^2 (c + dx)^{3/2}} dx, x, x^3 \right) \\
&= \frac{8x^6}{27d^2 (8c - dx^3) \sqrt{c + dx^3}} - \frac{\text{Subst} \left(\int \frac{x(16c^2 + 13cdx)}{(8c - dx)(c + dx)^{3/2}} dx, x, x^3 \right)}{27cd^2} \\
&= \frac{8x^6}{27d^2 (8c - dx^3) \sqrt{c + dx^3}} + \frac{2(38c + 39dx^3)}{81d^4 \sqrt{c + dx^3}} - \frac{(320c) \text{Subst} \left(\int \frac{1}{(8c - dx)\sqrt{c + dx}} dx, x, x^3 \right)}{81d^3} \\
&= \frac{8x^6}{27d^2 (8c - dx^3) \sqrt{c + dx^3}} + \frac{2(38c + 39dx^3)}{81d^4 \sqrt{c + dx^3}} - \frac{(640c) \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{81d^4} \\
&= \frac{8x^6}{27d^2 (8c - dx^3) \sqrt{c + dx^3}} + \frac{2(38c + 39dx^3)}{81d^4 \sqrt{c + dx^3}} - \frac{640\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{243d^4}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 88, normalized size = 0.93

$$\frac{6(752c^2 - 198cdx^3 + 9d^2x^6) - 640c(8c - dx^3) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3 + c}{9c}\right)}{81d^4(dx^3 - 8c)\sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)), x]

[Out] (6*(752*c^2 - 198*c*d*x^3 + 9*d^2*x^6) - 640*c*(8*c - d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)])/(81*d^4*(-8*c + d*x^3)*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.09, size = 127, normalized size = 1.34

$$\frac{\left(\frac{5120c^{3/2}}{243d^4} - \frac{640\sqrt{c}x^3}{243d^3}\right)\sqrt{c + dx^3} \tanh^{-1}\left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}}\right) - \frac{608c^2}{81d^4} - \frac{548cx^3}{81d^3} + \frac{2x^6}{3d^2}}{dx^3\sqrt{c + dx^3} - 8c\sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)), x]

[Out] $\left(\frac{-608c^2}{81d^4} - \frac{548cx^3}{81d^3} + \frac{2x^6}{3d^2} + \frac{(5120c^3/2)}{(243d^4)} - \frac{640\sqrt{c}x^3}{243d^3}\right)\sqrt{c+d^2x^3} \operatorname{ArcTanh}\left[\frac{\sqrt{c+d^2x^3}}{3\sqrt{c}}\right] - \frac{8c\sqrt{c+d^2x^3} + d^3x^3\sqrt{c+d^2x^3}}{(-8c\sqrt{c+d^2x^3} + d^3x^3\sqrt{c+d^2x^3})}$

fricas [A] time = 0.76, size = 233, normalized size = 2.45

$$\left[\frac{2\left(160(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{c} \log\left(\frac{dx^3 - 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c}\right) + 3(27d^2x^6 - 274cdx^3 - 304c^2)\sqrt{dx^3+c}\right)}{243(d^6x^6 - 7cd^5x^3 - 8c^2d^4)}, \frac{2\left(320(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 3(27d^2x^6 - 274cdx^3 - 304c^2)\sqrt{dx^3+c}\right)}{243(d^6x^6 - 7cd^5x^3 - 8c^2d^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")`

[Out] $\left[\frac{2}{243} \cdot (160 \cdot (d^2x^6 - 7cdx^3 - 8c^2) \cdot \sqrt{c}) \cdot \log\left(\frac{dx^3 - 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c}\right) + 3 \cdot (27d^2x^6 - 274cdx^3 - 304c^2) \cdot \sqrt{dx^3+c} \right] / (d^6x^6 - 7cd^5x^3 - 8c^2d^4), \frac{2}{243} \cdot (320 \cdot (d^2x^6 - 7cdx^3 - 8c^2) \cdot \sqrt{-c}) \cdot \arctan\left(\frac{1}{3} \cdot \sqrt{dx^3+c}\sqrt{-c}/c\right) + 3 \cdot (27d^2x^6 - 274cdx^3 - 304c^2) \cdot \sqrt{dx^3+c} \right] / (d^6x^6 - 7cd^5x^3 - 8c^2d^4)$

giac [A] time = 0.19, size = 88, normalized size = 0.93

$$\frac{640c \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}d^4} + \frac{2\sqrt{dx^3+c}}{3d^4} - \frac{2(85(dx^3+c)c + 3c^2)}{81\left(\left(dx^3+c\right)^{\frac{3}{2}} - 9\sqrt{dx^3+c}c\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")`

[Out] $640/243 \cdot c \cdot \arctan\left(\frac{1}{3} \cdot \sqrt{dx^3+c}/\sqrt{-c}\right) / (\sqrt{-c} \cdot d^4) + \frac{2}{3} \cdot \sqrt{dx^3+c} / d^4 - \frac{2}{81} \cdot (85 \cdot (dx^3+c) \cdot c + 3 \cdot c^2) / \left(\left(dx^3+c\right)^{\frac{3}{2}} - 9 \cdot \sqrt{dx^3+c} \cdot c\right) \cdot d^4$

maple [C] time = 0.29, size = 970, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)`

[Out] $\frac{1}{d^3} \cdot \left(\frac{2}{3} \cdot \left(\frac{x^3+c}{d} \right) \cdot d \right)^{\frac{1}{2}} \cdot c / d^2 + \frac{2}{3} \cdot (dx^3+c)^{\frac{1}{2}} / d^2 \cdot d - \frac{32}{3} \cdot (dx^3+c)^{\frac{1}{2}} \cdot c / d + \frac{512c^3}{d^3} \cdot \left(-\frac{1}{243} \cdot \frac{d}{c^2} \cdot (dx^3+c)^{\frac{1}{2}} / (dx^3-8c) - \frac{2}{243} \cdot \frac{d}{c^2} \cdot \left(\frac{x^3+c}{d} \right) \cdot d \right)^{\frac{1}{2}} - \frac{1}{1458} \cdot \frac{I}{d^3} \cdot \frac{c^3 \cdot 2^{\frac{1}{2}}}{c^3} \cdot \sum\left(\frac{-c \cdot d^2}{d}\right)^{\frac{1}{3}} \cdot \left(\frac{1}{2} \cdot I \cdot (2x + (-I \cdot 3^{\frac{1}{2}}) \cdot (-c \cdot d^2)^{\frac{1}{3}} + (-c \cdot d^2)^{\frac{1}{3}}) / d\right) / (-c \cdot d^2)^{\frac{1}{3}} \cdot d \right)^{\frac{1}{2}} \cdot \left(\frac{x - (-c \cdot d^2)^{\frac{1}{3}}}{d} \right) / \left(-3 \cdot (-c \cdot d^2)^{\frac{1}{3}} + I \cdot 3^{\frac{1}{2}} \cdot (-c \cdot d^2)^{\frac{1}{3}} \right) \cdot d \right)^{\frac{1}{2}}$

$$\frac{1}{2} * (-\frac{1}{2} * I * (2 * x + (I * 3^{1/2}) * (-c * d^2)^{1/3} + (-c * d^2)^{1/3}) / d) / (-c * d^2)^{1/3} * d^{1/2} / (d * x^3 + c)^{1/2} * (2 * _alpha^2 * d^2 + I * (-c * d^2)^{1/3} * 3^{1/2} * _alpha * d - (-c * d^2)^{1/3} * _alpha * d - I * 3^{1/2} * (-c * d^2)^{2/3} - (-c * d^2)^{2/3}) * \text{EllipticPi}(1/3 * 3^{1/2} * (I * (x + 1/2 * (-c * d^2)^{1/3}) / d - 1/2 * I * 3^{1/2} * (-c * d^2)^{1/3}) / d) * 3^{1/2} / (-c * d^2)^{1/3} * d^{1/2}, -1/18 * (2 * I * (-c * d^2)^{1/3} * 3^{1/2} * _alpha^2 * d + I * 3^{1/2} * c * d - 3 * c * d - I * (-c * d^2)^{2/3} * 3^{1/2} * _alpha - 3 * (-c * d^2)^{2/3} * _alpha) / c / d, (I * 3^{1/2} * (-c * d^2)^{1/3} / (-3/2 * (-c * d^2)^{1/3} / d + 1/2 * I * 3^{1/2} * (-c * d^2)^{1/3} / d) / d)^{1/2}, _alpha = \text{RootOf}(_Z^3 * d - 8 * c)) + 192 * c^2 / d^3 * (2/27 / ((x^3 + c/d) * d)^{1/2} / c / d + 1/243 * I / c^2 / d^3 * 2^{1/2} * \text{sum}((-c * d^2)^{1/3} * (1/2 * I * (2 * x + (-I * 3^{1/2}) * (-c * d^2)^{1/3} + (-c * d^2)^{1/3}) / d) / (-c * d^2)^{1/3} * d^{1/2} * ((x - (-c * d^2)^{1/3}) / d) / (-3 * (-c * d^2)^{1/3} + I * 3^{1/2} * (-c * d^2)^{1/3}) * d)^{1/2} * (-1/2 * I * (2 * x + (I * 3^{1/2}) * (-c * d^2)^{1/3} + (-c * d^2)^{1/3}) / d) / (-c * d^2)^{1/3} * d^{1/2} / (d * x^3 + c)^{1/2} * (2 * _alpha^2 * d^2 + I * (-c * d^2)^{1/3} * 3^{1/2} * _alpha * d - (-c * d^2)^{1/3} * _alpha * d - I * 3^{1/2} * (-c * d^2)^{2/3} - (-c * d^2)^{2/3}) * \text{EllipticPi}(1/3 * 3^{1/2} * (I * (x + 1/2 * (-c * d^2)^{1/3}) / d - 1/2 * I * 3^{1/2} * (-c * d^2)^{1/3}) / d) * 3^{1/2} / (-c * d^2)^{1/3} * d^{1/2}, -1/18 * (2 * I * (-c * d^2)^{1/3} * 3^{1/2} * _alpha^2 * d + I * 3^{1/2} * c * d - 3 * c * d - I * (-c * d^2)^{2/3} * 3^{1/2} * _alpha - 3 * (-c * d^2)^{2/3} * _alpha) / c / d, (I * 3^{1/2} * (-c * d^2)^{1/3} / (-3/2 * (-c * d^2)^{1/3} / d + 1/2 * I * 3^{1/2} * (-c * d^2)^{1/3} / d) / d)^{1/2}, _alpha = \text{RootOf}(_Z^3 * d - 8 * c))$$

maxima [A] time = 1.21, size = 98, normalized size = 1.03

$$\frac{2 \left(160 \sqrt{c} \log \left(\frac{\sqrt{dx^3+c} - 3\sqrt{c}}{\sqrt{dx^3+c} + 3\sqrt{c}} \right) + 81 \sqrt{dx^3+c} - \frac{3(85(dx^3+c)c + 3c^2)}{(dx^3+c)^2 - 9\sqrt{dx^3+c}} \right)}{243 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")

[Out] 2/243*(160*sqrt(c)*log((sqrt(d*x^3+c)-3*sqrt(c))/(sqrt(d*x^3+c)+3*sqrt(c)))+81*sqrt(d*x^3+c)-3*(85*(d*x^3+c)*c+3*c^2)/((d*x^3+c)^(3/2)-9*sqrt(d*x^3+c)*c))/d^4

mupad [B] time = 4.38, size = 111, normalized size = 1.17

$$\frac{2 \sqrt{dx^3+c}}{3 d^4} + \frac{320 \sqrt{c} \ln \left(\frac{10c + dx^3 - 6\sqrt{c} \sqrt{dx^3+c}}{8c - dx^3} \right)}{243 d^4} + \frac{\sqrt{dx^3+c} \left(\frac{176c^2}{81 d^4} + \frac{170c x^3}{81 d^3} \right)}{8c^2 + 7c d x^3 - d^2 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/((c+d*x^3)^(3/2)*(8*c-d*x^3)^2),x)

[Out] (2*(c+d*x^3)^(1/2))/(3*d^4) + (320*c^(1/2)*log((10*c+d*x^3-6*c^(1/2)*(c+d*x^3)^(1/2))/(8*c-d*x^3)))/(243*d^4) + ((c+d*x^3)^(1/2)*((176*c^2)/(81*d^4) + (170*c*x^3)/(81*d^3)))/(8*c^2 - d^2*x^6 + 7*c*d*x^3)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2), x)

[Out] Timed out

$$3.295 \quad \int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=83

$$\frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{22}{81d^3\sqrt{c+dx^3}} - \frac{32 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}d^3}$$

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 89, 78, 63, 206}

$$\frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{22}{81d^3\sqrt{c+dx^3}} - \frac{32 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}d^3}$$

Antiderivative was successfully verified.

[In] Int[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)), x]

[Out] -22/(81*d^3*Sqrt[c + d*x^3]) + (64*c)/(27*d^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) - (32*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*Sqrt[c]*d^3)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 89

Int[((a_.) + (b_.)*(x_))^(2*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))

)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^8}{(8c - dx^3)^2 (c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(8c - dx)^2 (c + dx)^{3/2}} dx, x, x^3 \right) \\
 &= \frac{64c}{27d^3 (8c - dx^3) \sqrt{c + dx^3}} - \frac{\text{Subst} \left(\int \frac{-24c^2d + 9cd^2x}{(8c - dx)(c + dx)^{3/2}} dx, x, x^3 \right)}{27cd^3} \\
 &= -\frac{22}{81d^3 \sqrt{c + dx^3}} + \frac{64c}{27d^3 (8c - dx^3) \sqrt{c + dx^3}} - \frac{16 \text{Subst} \left(\int \frac{1}{(8c - dx) \sqrt{c + dx}} dx, x, x^3 \right)}{81d^2} \\
 &= -\frac{22}{81d^3 \sqrt{c + dx^3}} + \frac{64c}{27d^3 (8c - dx^3) \sqrt{c + dx^3}} - \frac{32 \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{81d^3} \\
 &= -\frac{22}{81d^3 \sqrt{c + dx^3}} + \frac{64c}{27d^3 (8c - dx^3) \sqrt{c + dx^3}} - \frac{32 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{243\sqrt{c}d^3}
 \end{aligned}$$

Mathematica [A] time = 0.11, size = 71, normalized size = 0.86

$$\frac{2 \left(\frac{3(8c+11dx^3)}{(8c-dx^3)\sqrt{c+dx^3}} - \frac{16 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}} \right)}{243d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] (2*((3*(8*c + 11*d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) - (16*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))/(243*d^3)

IntegrateAlgebraic [A] time = 0.08, size = 115, normalized size = 1.39

$$\frac{\frac{16c}{81d^3} + \left(\frac{32x^3}{243\sqrt{c}d^2} - \frac{256\sqrt{c}}{243d^3} \right) \sqrt{c+dx^3} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + \frac{22x^3}{81d^2}}{8c\sqrt{c+dx^3} - dx^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] ((16*c)/(81*d^3) + (22*x^3)/(81*d^2) + ((-256*Sqrt[c])/(243*d^3) + (32*x^3)/(243*Sqrt[c]*d^2))*Sqrt[c + d*x^3]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(8*c*Sqrt[c + d*x^3] - d*x^3*Sqrt[c + d*x^3])

fricas [A] time = 0.71, size = 223, normalized size = 2.69

$$\left[\frac{2 \left(8(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{c} \log\left(\frac{dx^3 - 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c}\right) - 3(11cdx^3 + 8c^2)\sqrt{dx^3+c} \right)}{243(cd^5x^6 - 7c^2d^4x^3 - 8c^3d^3)}, \frac{2 \left(16(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) - 3(11cdx^3 + 8c^2)\sqrt{dx^3+c} \right)}{243(cd^5x^6 - 7c^2d^4x^3 - 8c^3d^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] [2/243*(8*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 3*(11*c*d*x^3 + 8*c^2)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3), 2/243*(16*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*(11*c*d*x^3 + 8*c^2)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)]

giac [A] time = 0.17, size = 67, normalized size = 0.81

$$\frac{32 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}d^3} - \frac{2(11dx^3+8c)}{81\left((dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] 32/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/81*(11*d*x^3 + 8*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*d^3)

maple [C] time = 0.18, size = 926, normalized size = 11.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)

[Out] -2/3/d^3/(d*x^3+c)^(1/2)+64*c^2/d^2*(-1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)^(1/2)/c^2/d-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))) +16*c/d^2*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))

maxima [A] time = 1.41, size = 81, normalized size = 0.98

$$\frac{2 \left(\frac{8 \log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}} - \frac{3(11dx^3+8c)}{(dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}} \right)}{243 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")

[Out] 2/243*(8*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/sqrt(c) - 3*(11*d*x^3 + 8*c)/((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c))/d^3

mupad [B] time = 4.30, size = 94, normalized size = 1.13

$$\frac{16 \ln\left(\frac{10c+dx^3-6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{243\sqrt{c}d^3} + \frac{\sqrt{dx^3+c}\left(\frac{16c}{81d^3} + \frac{22x^3}{81d^2}\right)}{8c^2 + 7cdx^3 - d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)

[Out] (16*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(243*c^(1/2)*d^3) + ((c + d*x^3)^(1/2)*((16*c)/(81*d^3) + (22*x^3)/(81*d^2)))/(8*c^2 - d^2*x^6 + 7*c*d*x^3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(-8c + dx^3)^2 (c + dx^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)

[Out] Integral(x**8/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)

$$3.296 \quad \int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=85

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2} + \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{2}{81cd^2\sqrt{c+dx^3}}$$

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 78, 51, 63, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2} + \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{2}{81cd^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -2/(81*c*d^2*Sqrt[c + d*x^3]) + 8/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*c^(3/2)*d^2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
```


$f*(p + 1)*(c*f - d*e), x] - \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] :> \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(8c - dx^3)^2 (c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{(8c - dx)^2 (c + dx)^{3/2}} dx, x, x^3 \right) \\ &= \frac{8}{27d^2 (8c - dx^3) \sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{1}{(8c - dx)(c + dx)^{3/2}} dx, x, x^3 \right)}{9d} \\ &= -\frac{2}{81cd^2 \sqrt{c + dx^3}} + \frac{8}{27d^2 (8c - dx^3) \sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{1}{(8c - dx) \sqrt{c + dx}} dx, x, x^3 \right)}{81cd} \\ &= -\frac{2}{81cd^2 \sqrt{c + dx^3}} + \frac{8}{27d^2 (8c - dx^3) \sqrt{c + dx^3}} + \frac{2 \text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{81cd^2} \\ &= -\frac{2}{81cd^2 \sqrt{c + dx^3}} + \frac{8}{27d^2 (8c - dx^3) \sqrt{c + dx^3}} + \frac{2 \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{243c^{3/2}d^2} \end{aligned}$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.80

$$\frac{2 \left((dx^3 - 8c) {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3 + c}{9c} \right) + 12c \right)}{81cd^2 (dx^3 - 8c) \sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] (-2*(12*c + (-8*c + d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)])))/(81*c*d^2*(-8*c + d*x^3)*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.08, size = 117, normalized size = 1.38

$$\frac{\left(\frac{16}{243\sqrt{c}d^2} - \frac{2x^3}{243c^{3/2}d} \right) \sqrt{c + dx^3} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) + \frac{2x^3}{81cd} + \frac{8}{81d^2}}{8c\sqrt{c + dx^3} - dx^3\sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] (8/(81*d^2) + (2*x^3)/(81*c*d) + (16/(243*Sqrt[c]*d^2) - (2*x^3)/(243*c^(3/2)*d))*Sqrt[c + d*x^3]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(8*c*Sqrt[c + d*x^3] - d*x^3*Sqrt[c + d*x^3])

fricas [A] time = 0.71, size = 223, normalized size = 2.62

$$\left[\frac{(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{c} \log\left(\frac{dx^3 + 6\sqrt{dx^3+c}\sqrt{c} + 10c}{dx^3 - 8c}\right) - 6(cd^2x^3 + 4c^2)\sqrt{dx^3+c}}{243(c^2d^4x^6 - 7c^3d^3x^3 - 8c^4d^2)}, -\frac{2((d^2x^6 - 7cdx^3 - 8c^2)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 3(cd^2x^3 + 4c^2)\sqrt{dx^3+c})}{243(c^2d^4x^6 - 7c^3d^3x^3 - 8c^4d^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] [1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c))*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2), -2/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)]

giac [A] time = 0.19, size = 76, normalized size = 0.89

$$\frac{2 \left(\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{\sqrt{-c}cd} + \frac{3(dx^3+4c)}{\left((dx^3+c)^2-9\sqrt{dx^3+c}c\right)cd} \right)}{243d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $-2/243*(\arctan(1/3*\sqrt{d*x^3 + c})/\sqrt{-c})/(\sqrt{-c}*c*d) + 3*(d*x^3 + 4*c)/(((d*x^3 + c)^(3/2) - 9*\sqrt{d*x^3 + c})*c*d)/d$

maple [C] time = 0.18, size = 908, normalized size = 10.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)

[Out] $8*c/d*(-1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)^(1/2)/c^2/d-1/1458*I/d^3/c^3*2^(1/2)*\sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3))/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/d*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*\sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3))/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))$

maxima [A] time = 1.23, size = 83, normalized size = 0.98

$$\frac{\frac{6(dx^3+4c)}{(dx^3+c)^2 c - 9\sqrt{dx^3+c}c^2} + \frac{\log\left(\frac{\sqrt{dx^3+c-3\sqrt{c}}}{\sqrt{dx^3+c+3\sqrt{c}}}\right)}{c^2}}{243d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")

[Out] $-1/243*(6*(d*x^3 + 4*c)/((d*x^3 + c)^(3/2)*c - 9*\sqrt{d*x^3 + c}*c^2) + \log((\sqrt{d*x^3 + c} - 3*\sqrt{c})/(\sqrt{d*x^3 + c} + 3*\sqrt{c}))/c^(3/2))/d^2$

mupad [B] time = 4.26, size = 96, normalized size = 1.13

$$\frac{\left(\frac{8}{81d^2} + \frac{2x^3}{81cd}\right) \sqrt{dx^3 + c}}{8c^2 + 7cdx^3 - d^2x^6} + \frac{\ln\left(\frac{10c+dx^3+6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{243c^{3/2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)

[Out] $((8/(81*d^2) + (2*x^3)/(81*c*d))*(c + d*x^3)^(1/2))/(8*c^2 - d^2*x^6 + 7*c*d*x^3) + \log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(243*c^(3/2)*d^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(-8c + dx^3)^2 (c + dx^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)

[Out] Integral(x**5/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)

$$3.297 \quad \int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d} - \frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd(8c-dx^3)\sqrt{c+dx^3}}$$

Rubi [A] time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {444, 51, 63, 206}

$$-\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d} + \frac{1}{27cd(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)), x]

[Out] -1/(81*c^2*d*Sqrt[c + d*x^3]) + 1/(27*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(243*c^(5/2)*d)

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{(8c - dx^3)^2 (c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(8c - dx)^2 (c + dx)^{3/2}} dx, x, x^3 \right) \\
 &= \frac{1}{27cd (8c - dx^3) \sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{1}{(8c - dx)(c + dx)^{3/2}} dx, x, x^3 \right)}{18c} \\
 &= -\frac{1}{81c^2 d \sqrt{c + dx^3}} + \frac{1}{27cd (8c - dx^3) \sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{1}{(8c - dx) \sqrt{c + dx}} dx, x, x^3 \right)}{162c^2} \\
 &= -\frac{1}{81c^2 d \sqrt{c + dx^3}} + \frac{1}{27cd (8c - dx^3) \sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{1}{9c - x^2} dx, x, \sqrt{c + dx^3} \right)}{81c^2 d} \\
 &= -\frac{1}{81c^2 d \sqrt{c + dx^3}} + \frac{1}{27cd (8c - dx^3) \sqrt{c + dx^3}} + \frac{\tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{3\sqrt{c}} \right)}{243c^{5/2} d}
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.49

$$\frac{{}_2F_1 \left(-\frac{1}{2}, 2; \frac{1}{2}; \frac{dx^3 + c}{9c} \right)}{243c^2 d \sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] (-2*Hypergeometric2F1[-1/2, 2, 1/2, (c + d*x^3)/(9*c)])/(243*c^2*d*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.10, size = 76, normalized size = 0.86

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d} + \frac{dx^3 - 5c}{81c^2d(8c - dx^3)\sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] (-5*c + d*x^3)/(81*c^2*d*(8*c - d*x^3)*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(243*c^(5/2)*d)

fricas [A] time = 0.59, size = 219, normalized size = 2.49

$$\left[\frac{(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{c} \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c+10c}}{dx^3-8c}\right) - 6(cdx^3 - 5c^2)\sqrt{dx^3+c}}{486(c^3d^3x^6 - 7c^4d^2x^3 - 8c^5d)}, \frac{(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 3(cdx^3 - 5c^2)\sqrt{dx^3+c}}{243(c^3d^3x^6 - 7c^4d^2x^3 - 8c^5d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] [1/486*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c))*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 - 5*c^2)*sqrt(d*x^3 + c))/(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d), -1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(c*d*x^3 - 5*c^2)*sqrt(d*x^3 + c))/(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)]

giac [A] time = 0.16, size = 72, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}c^2d} - \frac{dx^3 - 5c}{81\left(\left(dx^3 + c\right)^2 - 9\sqrt{dx^3 + c}c\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] -1/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2*d) - 1/81*(d*x^3 - 5*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^2*d)

maple [C] time = 0.19, size = 463, normalized size = 5.26

The Maple output is highly complex, involving nested radicals and RootOf functions. It includes terms like $\frac{\sqrt{c^2+dx^3}}{243(d^3x^6-8c^2d)}$ and $\frac{2}{243\sqrt{c^2+dx^3}}$, along with arctan and sqrt functions of various expressions involving d , c , and x .

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)`

[Out]
$$-1/243*(d*x^3+c)^{(1/2)}/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)^{(1/2)}/c^2/d-1/1458*I/d^3/c^3*2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, -1/18*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/c/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=\text{RootOf}(_Z^3*d-8*c))$$

maxima [A] time = 1.27, size = 85, normalized size = 0.97

$$\frac{\frac{6(dx^3-5c)}{(dx^3+c)^2c^2-9\sqrt{dx^3+c}c^3} + \frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^2}}{486d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")`

[Out]
$$-1/486*(6*(d*x^3 - 5*c)/((d*x^3 + c)^{(3/2)}*c^2 - 9*\text{sqrt}(d*x^3 + c)*c^3) + \log((\text{sqrt}(d*x^3 + c) - 3*\text{sqrt}(c))/(\text{sqrt}(d*x^3 + c) + 3*\text{sqrt}(c)))/c^{(5/2)})/d$$

mupad [B] time = 4.26, size = 97, normalized size = 1.10

$$\frac{\ln\left(\frac{10c+dx^3+6\sqrt{c}\sqrt{dx^3+c}}{8c-dx^3}\right)}{486c^{5/2}d} - \frac{\left(\frac{5}{81cd} - \frac{x^3}{81c^2}\right)\sqrt{dx^3+c}}{8c^2+7cdx^3-d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)`

[Out]
$$\log((10*c + d*x^3 + 6*c^{(1/2)}*(c + d*x^3)^{(1/2)})/(8*c - d*x^3))/(486*c^{(5/2)}*d) - ((5/(81*c*d) - x^3/(81*c^2))*(c + d*x^3)^{(1/2)})/(8*c^2 - d^2*x^6 + 7*c*d*x^3)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-8c + dx^3)^2 (c + dx^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2), x)

[Out] Integral(x**2/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)

$$3.298 \quad \int \frac{1}{x(8c-dx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=106

$$\frac{7 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}} + \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2(8c-dx^3)\sqrt{c+dx^3}}$$

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {446, 103, 152, 156, 63, 208, 206}

$$\frac{1}{216c^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{7 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] 5/(648*c^3*Sqrt[c + d*x^3]) + 1/(216*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(7776*c^(7/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(7/2))

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[2*m, 2*n, 2*p]

```

Rule 156

```

Int[((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

```

Rule 206

```

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

```

Rule 208

```

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 446

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(8c-dx^3)^2(c+dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(8c-dx)^2(c+dx)^{3/2}} dx, x, x^3 \right) \\
&= \frac{1}{216c^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{\text{Subst} \left(\int \frac{-9cd-\frac{3d^2x}{2}}{x(8c-dx)(c+dx)^{3/2}} dx, x, x^3 \right)}{216c^2d} \\
&= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{\text{Subst} \left(\int \frac{-\frac{81}{2}c^2d^2+\frac{15}{4}cd^3x}{x(8c-dx)\sqrt{c+dx}} dx, x, x^3 \right)}{972c^4d^2} \\
&= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{192c^3} + \dots \\
&= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{7 \text{Subst} \left(\int \frac{1}{9c-x^2} dx, x, \sqrt{c+dx^3} \right)}{2592c^3} \\
&= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{7 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{7776c^{7/2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right)}{96c^3}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 97, normalized size = 0.92

$$\frac{(7dx^3 - 56c) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3+c}{9c}\right) + 27(8c-dx^3) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3}{c} + 1\right) + 12c}{2592c^3(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)), x]

[Out] (12*c + (-56*c + 7*d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)] + 27*(8*c - d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(2592*c^3*(8*c - d*x^3)*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.10, size = 163, normalized size = 1.54

$$\frac{\sqrt{c+dx^3} \left(\frac{7}{972c^{5/2}} - \frac{7dx^3}{7776c^{7/2}} \right) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}} \right) + \sqrt{c+dx^3} \left(\frac{dx^3}{96c^{7/2}} - \frac{1}{12c^{5/2}} \right) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right) - \frac{5dx^3}{648c^3} + \frac{43}{648c^2}}{8c\sqrt{c+dx^3} - dx^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] $\frac{43}{648c^2} - \frac{5dx^3}{648c^3} + \frac{\sqrt{c+dx^3}}{7776c^{5/2}} - \frac{7}{972c^{5/2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right] + \frac{\sqrt{c+dx^3}}{96c^{5/2}} - \frac{7}{972c^{5/2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right]}{(8c\sqrt{c+dx^3} - dx^3\sqrt{c+dx^3})}$

fricas [A] time = 0.66, size = 316, normalized size = 2.98

$$\frac{7(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{c} \log\left(\frac{dx^3 + \sqrt{dx^3+c}\sqrt{-c}}{dx^3+c}\right) + 81(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{c} \log\left(\frac{dx^3 - 2\sqrt{dx^3+c}\sqrt{-c}}{dx^3+c}\right) + 24(5cdx^3 - 43c^2)\sqrt{dx^3+c} + 81(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - 7(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{-c} \arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 12(5cdx^3 - 43c^2)\sqrt{dx^3+c}}{15552(c^4d^2x^6 - 7c^5dx^3 - 8c^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] $\frac{1}{15552} \frac{7(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{c} \log((dx^3 + 6\sqrt{dx^3+c})\sqrt{c} + 10c)/(dx^3 - 8c) + 81(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{c} \log((dx^3 - 2\sqrt{dx^3+c})\sqrt{c} + 2c)/x^3 + 24(5cdx^3 - 43c^2)\sqrt{dx^3+c}/(c^4d^2x^6 - 7c^5dx^3 - 8c^6) + 1/7776(81(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{-c} \arctan(\sqrt{dx^3+c}\sqrt{-c}/c) - 7(d^2x^6 - 7cdx^3 - 8c^2)\sqrt{-c} \arctan(1/3\sqrt{dx^3+c}\sqrt{-c}/c) + 12(5cdx^3 - 43c^2)\sqrt{dx^3+c})/(c^4d^2x^6 - 7c^5dx^3 - 8c^6)}$

giac [A] time = 0.17, size = 93, normalized size = 0.88

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c^3} - \frac{7\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{7776\sqrt{-c}c^3} + \frac{5dx^3 - 43c}{648\left(\left(dx^3+c\right)^{\frac{3}{2}} - 9\sqrt{dx^3+c}c\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $\frac{1}{96} \frac{\arctan(\sqrt{dx^3+c}/\sqrt{-c})/(\sqrt{-c}c^3) - 7/7776 \arctan(1/3\sqrt{dx^3+c}/\sqrt{-c})/(\sqrt{-c}c^3) + 1/648(5dx^3 - 43c)/((dx^3+c)^{3/2} - 9\sqrt{dx^3+c}c)c^3}$

maple [C] time = 0.17, size = 953, normalized size = 8.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)

```
[Out] 1/8/c*d*(-1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)^(1/2)
/c^2/d-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)
)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)
^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_al
pha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-
c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
),_alpha=RootOf(_Z^3*d-8*c))-1/64/c^2*d*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/2
43*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-
c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_
alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(
1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^
2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
,-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=R
ootOf(_Z^3*d-8*c))+1/64/c^2*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*arctanh((d*x^3+
c)^(1/2)/c^(1/2))/c^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx^3 + c)^{\frac{3}{2}}(dx^3 - 8c)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x), x)
```

mupad [B] time = 4.33, size = 101, normalized size = 0.95

$$-\frac{\frac{5(dx^3+c)}{216c^3} - \frac{2}{9c^2}}{27c\sqrt{dx^3+c} - 3(dx^3+c)^{3/2}} + \frac{\left(\operatorname{atanh}\left(\frac{c^3\sqrt{dx^3+c}}{\sqrt{c^7}}\right) \operatorname{li} - \frac{\operatorname{atanh}\left(\frac{c^3\sqrt{dx^3+c}}{3\sqrt{c^7}}\right) 7i}{81} \right) \operatorname{li}}{96\sqrt{c^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)`

[Out] $((\operatorname{atanh}((c^3*(c + d*x^3)^{1/2}))/c^7)^{1/2})*1i - (\operatorname{atanh}((c^3*(c + d*x^3)^{1/2}))/3*(c^7)^{1/2}))*7i)/81)*1i)/(96*(c^7)^{1/2}) - ((5*(c + d*x^3))/(216*c^3) - 2/(9*c^2))/(27*c*(c + d*x^3)^{1/2} - 3*(c + d*x^3)^{3/2})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(-8c + dx^3)^2 (c + dx^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)`

[Out] `Integral(1/(x*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)`

$$3.299 \quad \int \frac{1}{x^4(8c-dx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=143

$$\frac{5d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}} + \frac{5d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}} - \frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3(8c-dx^3)\sqrt{c}}$$

Rubi [A] time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {446, 103, 151, 152, 156, 63, 208, 206}

$$\frac{5d}{864c^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{35d}{2592c^4\sqrt{c+dx^3}} - \frac{1}{24c^2x^3(8c-dx^3)\sqrt{c+dx^3}} + \frac{5d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}} + \frac{5d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] (-35*d)/(2592*c^4*Sqrt[c + d*x^3]) + (5*d)/(864*c^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) - 1/(24*c^2*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (5*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(31104*c^(9/2)) + (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c])]/(384*c^(9/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 151


```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (8c - dx^3)^2 (c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2 (8c - dx)^2 (c + dx)^{3/2}} dx, x, x^3 \right) \\
&= -\frac{1}{24c^2 x^3 (8c - dx^3) \sqrt{c + dx^3}} - \frac{\text{Subst} \left(\int \frac{10cd - \frac{5d^2 x}{2}}{x(8c - dx)^2 (c + dx)^{3/2}} dx, x, x^3 \right)}{24c^2} \\
&= \frac{5d}{864c^3 (8c - dx^3) \sqrt{c + dx^3}} - \frac{1}{24c^2 x^3 (8c - dx^3) \sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{-90c^2 d^2}{x(8c - dx)} dx, x, x^3 \right)}{172} \\
&= -\frac{35d}{2592c^4 \sqrt{c + dx^3}} + \frac{5d}{864c^3 (8c - dx^3) \sqrt{c + dx^3}} - \frac{1}{24c^2 x^3 (8c - dx^3) \sqrt{c + dx^3}} \\
&= -\frac{35d}{2592c^4 \sqrt{c + dx^3}} + \frac{5d}{864c^3 (8c - dx^3) \sqrt{c + dx^3}} - \frac{1}{24c^2 x^3 (8c - dx^3) \sqrt{c + dx^3}} \\
&= -\frac{35d}{2592c^4 \sqrt{c + dx^3}} + \frac{5d}{864c^3 (8c - dx^3) \sqrt{c + dx^3}} - \frac{1}{24c^2 x^3 (8c - dx^3) \sqrt{c + dx^3}} \\
&= -\frac{35d}{2592c^4 \sqrt{c + dx^3}} + \frac{5d}{864c^3 (8c - dx^3) \sqrt{c + dx^3}} - \frac{1}{24c^2 x^3 (8c - dx^3) \sqrt{c + dx^3}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 117, normalized size = 0.82

$$\frac{5dx^3 (dx^3 - 8c) {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3 + c}{9c} \right) + 135dx^3 (dx^3 - 8c) {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3}{c} + 1 \right) + 12c (5dx^3 - 36c)}{10368c^4 x^3 (8c - dx^3) \sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] (12*c*(-36*c + 5*d*x^3) + 5*d*x^3*(-8*c + d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)] + 135*d*x^3*(-8*c + d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(10368*c^4*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.15, size = 114, normalized size = 0.80

$$\frac{5d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}} + \frac{5d \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}} + \frac{-108c^2 - 265cdx^3 + 35d^2x^6}{2592c^4x^3(8c - dx^3)\sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] $(-108*c^2 - 265*c*d*x^3 + 35*d^2*x^6)/(2592*c^4*x^3*(8*c - d*x^3)*\text{Sqrt}[c + d*x^3]) + (5*d*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/(3*\text{Sqrt}[c])])/(31104*c^{(9/2)}) + (5*d*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/\text{Sqrt}[c])/(384*c^{(9/2)})$

fricas [A] time = 0.78, size = 368, normalized size = 2.57

$$\frac{5(d^3x^9 - 7cd^2x^6 - 8c^2dx^3)\sqrt{c}\log\left(\frac{\sqrt{c+dx^3}\sqrt{c}}{3}\right) + 405(d^3x^9 - 7cd^2x^6 - 8c^2dx^3)\sqrt{c}\log\left(\frac{\sqrt{c+dx^3}\sqrt{c}}{c}\right) - 24(35cd^2x^3 - 108c^2)\sqrt{dx^3+c} - 405(d^3x^9 - 7cd^2x^6 - 8c^2dx^3)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}}{c}\right) + 5(d^3x^9 - 7cd^2x^6 - 8c^2dx^3)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}}{3c}\right) + 12(35cd^2x^3 - 265c^2dx^3 - 108c^2)\sqrt{dx^3+c}}{62208(d^3x^9 - 7cd^2x^6 - 8c^2dx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] $[1/62208*(5*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x^3)*\text{sqrt}(c)*\log((d*x^3 + 6*\text{sqrt}(d*x^3 + c))*\text{sqrt}(c) + 10*c)/(d*x^3 - 8*c)) + 405*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x^3)*\text{sqrt}(c)*\log((d*x^3 + 2*\text{sqrt}(d*x^3 + c))*\text{sqrt}(c) + 2*c)/x^3) - 24*(35*c*d^2*x^3 - 265*c^2*d*x^3 - 108*c^3)*\text{sqrt}(d*x^3 + c)/(c^5*d^2*x^9 - 7*c^6*d*x^6 - 8*c^7*x^3), -1/31104*(405*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x^3)*\text{sqrt}(-c)*\arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + 5*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x^3)*\text{sqrt}(-c)*\arctan(1/3*\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + 12*(35*c*d^2*x^3 - 265*c^2*d*x^3 - 108*c^3)*\text{sqrt}(d*x^3 + c)/(c^5*d^2*x^9 - 7*c^6*d*x^6 - 8*c^7*x^3)]$

giac [A] time = 0.17, size = 129, normalized size = 0.90

$$\frac{5d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\sqrt{-c}c^4} - \frac{5d \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{31104\sqrt{-c}c^4} - \frac{35(dx^3+c)^2d - 335(dx^3+c)cd + 192c^2d}{2592\left(\left(dx^3+c\right)^{\frac{5}{2}} - 10\left(dx^3+c\right)^{\frac{3}{2}}c + 9\sqrt{dx^3+c}c^2\right)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $-5/384*d*\text{arctan}(\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*c^4) - 5/31104*d*\text{arctan}(1/3*\text{sqrt}(d*x^3 + c)/\text{sqrt}(-c))/(\text{sqrt}(-c)*c^4) - 1/2592*(35*(d*x^3 + c)^2*d$

$- 335*(d*x^3 + c)*c*d + 192*c^2*d)/(((d*x^3 + c)^{(5/2)} - 10*(d*x^3 + c)^{(3/2)})*c + 9*\sqrt{d*x^3 + c}*c^2)*c^4)$

maple [C] time = 0.20, size = 1019, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)`

[Out] $\frac{1}{64} \frac{1}{c^2 d^2} (-1/243 (d x^3 + c)^{1/2} / (d x^3 - 8 c) / c^2 / d - 2/243 / ((x^3 + c/d) * d)^{1/2} / c^2 / d - 1/1458 * I / d^3 / c^3 * 2^{1/2} * \text{sum}((-c*d^2)^{1/3} * (1/2 * I * (2*x + (-I * 3^{1/2} * (-c*d^2)^{1/3} + (-c*d^2)^{1/3}) / d) / (-c*d^2)^{1/3} * d)^{1/2} * ((x - (-c*d^2)^{1/3}) / d) / (-3 * (-c*d^2)^{1/3} + I * 3^{1/2} * (-c*d^2)^{1/3}) * d)^{1/2} * (-1/2 * I * (2*x + (I * 3^{1/2} * (-c*d^2)^{1/3} + (-c*d^2)^{1/3}) / d) / (-c*d^2)^{1/3} * d)^{1/2} / (d * x^3 + c)^{1/2} * (2 * \alpha^2 * d^2 + I * (-c*d^2)^{1/3} * 3^{1/2} * \alpha * d - (-c*d^2)^{1/3} * \alpha * d - I * 3^{1/2} * (-c*d^2)^{2/3} - (-c*d^2)^{2/3}) * \text{EllipticPi}(1/3 * 3^{1/2} * (I * (x + 1/2 * (-c*d^2)^{1/3}) / d - 1/2 * I * 3^{1/2} * (-c*d^2)^{1/3} / d) * 3^{1/2} / (-c*d^2)^{1/3} * d)^{1/2}, -1/18 * (2 * I * (-c*d^2)^{1/3} * 3^{1/2} * \alpha^2 * d + I * 3^{1/2} * c * d - 3 * c * d - I * (-c*d^2)^{2/3} * 3^{1/2} * \alpha - 3 * (-c*d^2)^{2/3} * \alpha) / c / d, (I * 3^{1/2} * (-c*d^2)^{1/3} / (-3/2 * (-c*d^2)^{1/3} / d + 1/2 * I * 3^{1/2} * (-c*d^2)^{1/3} / d) / d)^{1/2}), \alpha = \text{RootOf}(_Z^3 * d - 8 * c)) + 1/64 * c^2 * (-1/3 * (d * x^3 + c)^{1/2} / c^2 / x^3 - 2/3 / ((x^3 + c/d) * d)^{1/2} / c^2 * d + d * \text{arctanh}((d * x^3 + c)^{1/2} / c^{1/2}) / c^{5/2}) - 1/256 * c^3 * d^2 * (2/27 / ((x^3 + c/d) * d)^{1/2} / c / d + 1/243 * I / c^2 / d^3 * 2^{1/2} * \text{sum}((-c*d^2)^{1/3} * (1/2 * I * (2*x + (-I * 3^{1/2} * (-c*d^2)^{1/3} + (-c*d^2)^{1/3}) / d) / (-c*d^2)^{1/3} * d)^{1/2} * ((x - (-c*d^2)^{1/3}) / d) / (-3 * (-c*d^2)^{1/3} + I * 3^{1/2} * (-c*d^2)^{1/3}) * d)^{1/2} * (-1/2 * I * (2*x + (I * 3^{1/2} * (-c*d^2)^{1/3} + (-c*d^2)^{1/3}) / d) / (-c*d^2)^{1/3} * d)^{1/2} / (d * x^3 + c)^{1/2} * (2 * \alpha^2 * d^2 + I * (-c*d^2)^{1/3} * 3^{1/2} * \alpha * d - (-c*d^2)^{1/3} * \alpha * d - I * 3^{1/2} * (-c*d^2)^{2/3} - (-c*d^2)^{2/3}) * \text{EllipticPi}(1/3 * 3^{1/2} * (I * (x + 1/2 * (-c*d^2)^{1/3}) / d - 1/2 * I * 3^{1/2} * (-c*d^2)^{1/3} / d) * 3^{1/2} / (-c*d^2)^{1/3} * d)^{1/2}, -1/18 * (2 * I * (-c*d^2)^{1/3} * 3^{1/2} * \alpha^2 * d + I * 3^{1/2} * c * d - 3 * c * d - I * (-c*d^2)^{2/3} * 3^{1/2} * \alpha - 3 * (-c*d^2)^{2/3} * \alpha) / c / d, (I * 3^{1/2} * (-c*d^2)^{1/3} / (-3/2 * (-c*d^2)^{1/3} / d + 1/2 * I * 3^{1/2} * (-c*d^2)^{1/3} / d) / d)^{1/2}), \alpha = \text{RootOf}(_Z^3 * d - 8 * c)) + 1/256 * c^3 * d * (2/3 / ((x^3 + c/d) * d)^{1/2} / c - 2/3 * \text{arctanh}((d * x^3 + c)^{1/2} / c^{1/2}) / c^{3/2})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8c)^2 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")`

[Out] `integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^4), x)`

mupad [B] time = 4.56, size = 133, normalized size = 0.93

$$\frac{\frac{2d}{9c^2} + \frac{35d(dx^3+c)^2}{864c^4} - \frac{335d(dx^3+c)}{864c^3}}{3(dx^3+c)^{5/2} - 30c(dx^3+c)^{3/2} + 27c^2\sqrt{dx^3+c}} - \frac{d \left(\operatorname{atanh} \left(\frac{c^4 \sqrt{dx^3+c}}{\sqrt{c^9}} \right) 1i + \frac{\operatorname{atanh} \left(\frac{c^4 \sqrt{dx^3+c}}{3\sqrt{c^9}} \right) 1i}{81} \right) 5i}{384\sqrt{c^9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)`

[Out] $-\left(\frac{2d}{9c^2} + \frac{35d(c + dx^3)^2}{864c^4} - \frac{335d(c + dx^3)}{864c^3}\right) / \left(3(c + dx^3)^{5/2} - 30c(c + dx^3)^{3/2} + 27c^2(c + dx^3)^{1/2}\right) - \left(d \operatorname{atanh}\left(\frac{c^4(c + dx^3)^{1/2}}{c^9}\right) 1i + \operatorname{atanh}\left(\frac{c^4(c + dx^3)^{1/2}}{3c^9}\right) 1i\right) / (81) 5i / (384(c^9)^{1/2})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(-8c + dx^3)^2(c + dx^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2), x)`

[Out] `Integral(1/(x**4*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)`

$$3.300 \quad \int \frac{1}{x^7(8c-dx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=185

$$\frac{13d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{497664c^{11/2}} - \frac{33d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{11/2}} + \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4(8c-dx^3)\sqrt{c+dx^3}} + \frac{17d}{384c^3x^3(8c-dx^3)\sqrt{c+dx^3}}$$

Rubi [A] time = 0.16, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {446, 103, 151, 152, 156, 63, 208, 206}

$$-\frac{71d^2}{13824c^4(8c-dx^3)\sqrt{c+dx^3}} + \frac{665d^2}{41472c^5\sqrt{c+dx^3}} + \frac{13d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{497664c^{11/2}} - \frac{33d^2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{11/2}} + \frac{17d}{384c^3x^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] (665*d^2)/(41472*c^5*Sqrt[c + d*x^3]) - (71*d^2)/(13824*c^4*(8*c - d*x^3)*Sqrt[c + d*x^3]) - 1/(48*c^2*x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (17*d)/(384*c^3*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (13*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(497664*c^(11/2)) - (33*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2048*c^(11/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n)*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n)*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7 (8c - dx^3)^2 (c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^3 (8c - dx)^2 (c + dx)^{3/2}} dx, x, x^3 \right) \\
&= -\frac{1}{48c^2 x^6 (8c - dx^3) \sqrt{c + dx^3}} - \frac{\text{Subst} \left(\int \frac{17cd - \frac{7d^2 x}{2}}{x^2 (8c - dx)^2 (c + dx)^{3/2}} dx, x, x^3 \right)}{48c^2} \\
&= -\frac{1}{48c^2 x^6 (8c - dx^3) \sqrt{c + dx^3}} + \frac{17d}{384c^3 x^3 (8c - dx^3) \sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{19}{x(8c - dx)^2} dx, x, x^3 \right)}{384c^3 x^3 (8c - dx^3)^2} \\
&= -\frac{71d^2}{13824c^4 (8c - dx^3) \sqrt{c + dx^3}} - \frac{1}{48c^2 x^6 (8c - dx^3) \sqrt{c + dx^3}} + \frac{1}{384c^3 x^3 (8c - dx^3)^2} \\
&= \frac{665d^2}{41472c^5 \sqrt{c + dx^3}} - \frac{71d^2}{13824c^4 (8c - dx^3) \sqrt{c + dx^3}} - \frac{1}{48c^2 x^6 (8c - dx^3) \sqrt{c + dx^3}} \\
&= \frac{665d^2}{41472c^5 \sqrt{c + dx^3}} - \frac{71d^2}{13824c^4 (8c - dx^3) \sqrt{c + dx^3}} - \frac{1}{48c^2 x^6 (8c - dx^3) \sqrt{c + dx^3}} \\
&= \frac{665d^2}{41472c^5 \sqrt{c + dx^3}} - \frac{71d^2}{13824c^4 (8c - dx^3) \sqrt{c + dx^3}} - \frac{1}{48c^2 x^6 (8c - dx^3) \sqrt{c + dx^3}} \\
&= \frac{665d^2}{41472c^5 \sqrt{c + dx^3}} - \frac{71d^2}{13824c^4 (8c - dx^3) \sqrt{c + dx^3}} - \frac{1}{48c^2 x^6 (8c - dx^3) \sqrt{c + dx^3}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 135, normalized size = 0.73

$$\frac{13d^2 x^6 (dx^3 - 8c) {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3 + c}{9c} \right) - 3 \left(4c (288c^2 - 612cdx^3 + 71d^2 x^6) + 891d^2 x^6 (dx^3 - 8c) {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3}{c} + 1 \right) \right)}{165888c^5 x^6 (8c - dx^3) \sqrt{c + dx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)), x]

[Out] $(13*d^2*x^6*(-8*c + d*x^3)*\text{Hypergeometric2F1}[-1/2, 1, 1/2, (c + d*x^3)/(9*c)] - 3*(4*c*(288*c^2 - 612*c*d*x^3 + 71*d^2*x^6) + 891*d^2*x^6*(-8*c + d*x^3)*\text{Hypergeometric2F1}[-1/2, 1, 1/2, 1 + (d*x^3)/c]))/(165888*c^5*x^6*(8*c - d*x^3)*\text{Sqrt}[c + d*x^3])$

IntegrateAlgebraic [A] time = 0.19, size = 208, normalized size = 1.12

$$\frac{x^6\sqrt{c+dx^3}\left(\frac{13d^2}{62208c^{9/2}} - \frac{13d^3x^3}{497664c^{11/2}}\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + x^6\sqrt{c+dx^3}\left(\frac{33d^3x^3}{2048c^{11/2}} - \frac{33d^2}{256c^{9/2}}\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) - \frac{665d^3x^9}{41472c^5} + \frac{5107d^2x^6}{41472c^4} + \frac{17dx^3}{384c^3} - \frac{1}{48c^2}}{x^6(8c\sqrt{c+dx^3} - dx^3\sqrt{c+dx^3})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] $(-1/48*1/c^2 + (17*d*x^3)/(384*c^3) + (5107*d^2*x^6)/(41472*c^4) - (665*d^3*x^9)/(41472*c^5) + x^6*\text{Sqrt}[c + d*x^3]*((13*d^2)/(62208*c^{(9/2)}) - (13*d^3*x^3)/(497664*c^{(11/2)}))*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/(3*\text{Sqrt}[c])] + x^6*\text{Sqrt}[c + d*x^3]*((-33*d^2)/(256*c^{(9/2)}) + (33*d^3*x^3)/(2048*c^{(11/2)}))*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/\text{Sqrt}[c]])/x^6*(8*c*\text{Sqrt}[c + d*x^3] - d*x^3*\text{Sqrt}[c + d*x^3])$

fricas [A] time = 0.90, size = 398, normalized size = 2.15

$$\frac{13(d^{11}x^{12} - 7cd^9x^9 - 8c^2d^7x^6)\sqrt{c}\log\left(\frac{d^2x\sqrt{c^2+d^2x^2}}{2cd^2x^2} + 8019(d^{11}x^{12} - 7cd^9x^9 - 8c^2d^7x^6)\sqrt{c}\log\left(\frac{d^2x\sqrt{c^2+d^2x^2}}{2cd^2x^2} + 24(665cd^3x^9 - 5107c^2d^2x^6 - 1836c^3d^3x^3 + 864c^4)\sqrt{dx^3+c} + 8019(d^{11}x^{12} - 7cd^9x^9 - 8c^2d^7x^6)\sqrt{c}\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right) - 13(d^{11}x^{12} - 7cd^9x^9 - 8c^2d^7x^6)\sqrt{c}\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right) + 12(665cd^3x^9 - 5107c^2d^2x^6 - 1836c^3d^3x^3 + 864c^4)\sqrt{dx^3+c}\right)}{995328(d^{11}x^{12} - 7cd^9x^9 - 8c^2d^7x^6)} + \frac{497664(d^{11}x^{12} - 7cd^9x^9 - 8c^2d^7x^6)}{497664(d^{11}x^{12} - 7cd^9x^9 - 8c^2d^7x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] $[1/995328*(13*(d^4*x^{12} - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*\text{sqrt}(c)*\log((d*x^3 + 6*\text{sqrt}(d*x^3 + c))*\text{sqrt}(c) + 10*c)/(d*x^3 - 8*c)) + 8019*(d^4*x^{12} - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*\text{sqrt}(c)*\log((d*x^3 - 2*\text{sqrt}(d*x^3 + c))*\text{sqrt}(c) + 2*c)/x^3) + 24*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836*c^3*d^3*x^3 + 864*c^4)*\text{sqrt}(d*x^3 + c)/(c^6*d^2*x^{12} - 7*c^7*d*x^9 - 8*c^8*x^6), 1/497664*(8019*(d^4*x^{12} - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*\text{sqrt}(-c)*\arctan(\text{sqrt}(d*x^3 + c))*\text{sqrt}(-c)/c - 13*(d^4*x^{12} - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*\text{sqrt}(-c)*\arctan(1/3*\text{sqrt}(d*x^3 + c))*\text{sqrt}(-c)/c + 12*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836*c^3*d^3*x^3 + 864*c^4)*\text{sqrt}(d*x^3 + c))/(c^6*d^2*x^{12} - 7*c^7*d*x^9 - 8*c^8*x^6)]$

giac [A] time = 0.18, size = 149, normalized size = 0.81

$$\frac{33d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\sqrt{-c}c^5} - \frac{13d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{497664\sqrt{-c}c^5} + \frac{341(dx^3+c)d^2 - 3072cd^2}{41472\left(\left(dx^3+c\right)^{\frac{3}{2}} - 9\sqrt{dx^3+c}c\right)c^5} + \frac{3(dx^3+c)^{\frac{3}{2}}d^2 - 4\sqrt{dx^3+c}cd^2}{384c^5d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 33/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) - 13/497664*d^2
*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) + 1/41472*(341*(d*x^3
+ c)*d^2 - 3072*c*d^2)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^5) + 1/
384*(3*(d*x^3 + c)^(3/2)*d^2 - 4*sqrt(d*x^3 + c)*c*d^2)/(c^5*d^2*x^6)
```

maple [C] time = 0.21, size = 1106, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/512/c^3*d^3*(-1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)
^(1/2)/c^2/d-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2
)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2
*x+I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*
x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/
3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d
-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/256/c^3*d*(-1/3*(d*x^3+c)^(1/2)/c^2/
x^3-2/3/((x^3+c/d)*d)^(1/2)/c^2*d+d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2
))+1/64/c^2*(-1/6*(d*x^3+c)^(1/2)/c^2/x^6+7/12*(d*x^3+c)^(1/2)/c^3*d/x^3+2/
3/((x^3+c/d)*d)^(1/2)/c^3*d^2-5/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7
/2))-3/4096/c^4*d^3*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*s
um((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)
)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)
*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+3/
4096/c^4*d^2*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2)
)/c^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx^3 + c)^{\frac{3}{2}}(dx^3 - 8c)^2 x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^7), x)

mupad [B] time = 4.76, size = 171, normalized size = 0.92

$$\frac{\frac{2d^2}{9c^2} - \frac{10373d^2(dx^3+c)}{13824c^3} + \frac{3551d^2(dx^3+c)^2}{6912c^4} - \frac{665d^2(dx^3+c)^3}{13824c^5}}{33c(dx^3+c)^{5/2} - 3(dx^3+c)^{7/2} + 27c^3\sqrt{dx^3+c} - 57c^2(dx^3+c)^{3/2}} + \frac{d^2 \left(\operatorname{atanh}\left(\frac{c^5\sqrt{dx^3+c}}{\sqrt{c^{11}}}\right) 1i - \frac{\operatorname{atanh}\left(\frac{c^5\sqrt{dx^3+c}}{3\sqrt{c^{11}}}\right) 13i}{8019} \right) 33i}{2048\sqrt{c^{11}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^7*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)

[Out] ((2*d^2)/(9*c^2) - (10373*d^2*(c + d*x^3))/(13824*c^3) + (3551*d^2*(c + d*x^3)^2)/(6912*c^4) - (665*d^2*(c + d*x^3)^3)/(13824*c^5))/(33*c*(c + d*x^3)^(5/2) - 3*(c + d*x^3)^(7/2) + 27*c^3*(c + d*x^3)^(1/2) - 57*c^2*(c + d*x^3)^(3/2)) + (d^2*(atanh((c^5*(c + d*x^3)^(1/2))/(c^11)^(1/2))*1i - (atanh((c^5*(c + d*x^3)^(1/2))/(3*(c^11)^(1/2)))*13i)/8019)*33i)/(2048*(c^11)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7(-8c + dx^3)^2(c + dx^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)

[Out] Integral(1/(x**7*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)

$$3.301 \quad \int \frac{x^8 \sqrt{c+dx^3}}{(a+bx^3)^2} dx$$

Optimal. Leaf size=161

$$\frac{a^2 (c + dx^3)^{3/2}}{3b^2 (a + bx^3) (bc - ad)} + \frac{a(4bc - 5ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{7/2} \sqrt{bc - ad}} - \frac{a\sqrt{c + dx^3} (4bc - 5ad)}{3b^3 (bc - ad)} + \frac{2 (c + dx^3)^{3/2}}{9b^2 d}$$

Rubi [A] time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 89, 80, 50, 63, 208}

$$\frac{a^2 (c + dx^3)^{3/2}}{3b^2 (a + bx^3) (bc - ad)} - \frac{a\sqrt{c + dx^3} (4bc - 5ad)}{3b^3 (bc - ad)} + \frac{a(4bc - 5ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{7/2} \sqrt{bc - ad}} + \frac{2 (c + dx^3)^{3/2}}{9b^2 d}$$

Antiderivative was successfully verified.

[In] Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]

[Out] -(a*(4*b*c - 5*a*d)*Sqrt[c + d*x^3])/(3*b^3*(b*c - a*d)) + (2*(c + d*x^3)^(3/2))/(9*b^2*d) - (a^2*(c + d*x^3)^(3/2))/(3*b^2*(b*c - a*d)*(a + b*x^3)) + (a*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2)*Sqrt[b*c - a*d])

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8 \sqrt{c+dx^3}}{(a+bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 \sqrt{c+dx}}{(a+bx)^2} dx, x, x^3 \right) \\
&= -\frac{a^2 (c+dx^3)^{3/2}}{3b^2(bc-ad)(a+bx^3)} + \frac{\text{Subst} \left(\int \frac{\sqrt{c+dx} \left(-\frac{1}{2}a(2bc-3ad)+b(bc-ad)x \right)}{a+bx} dx, x, x^3 \right)}{3b^2(bc-ad)} \\
&= \frac{2(c+dx^3)^{3/2}}{9b^2d} - \frac{a^2 (c+dx^3)^{3/2}}{3b^2(bc-ad)(a+bx^3)} - \frac{(a(4bc-5ad)) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^3 \right)}{6b^2(bc-ad)} \\
&= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2(c+dx^3)^{3/2}}{9b^2d} - \frac{a^2 (c+dx^3)^{3/2}}{3b^2(bc-ad)(a+bx^3)} - \frac{(a(4bc-5ad)) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^3 \right)}{6b^2(bc-ad)} \\
&= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2(c+dx^3)^{3/2}}{9b^2d} - \frac{a^2 (c+dx^3)^{3/2}}{3b^2(bc-ad)(a+bx^3)} - \frac{(a(4bc-5ad)) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^3 \right)}{6b^2(bc-ad)} \\
&= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2(c+dx^3)^{3/2}}{9b^2d} - \frac{a^2 (c+dx^3)^{3/2}}{3b^2(bc-ad)(a+bx^3)} + \frac{a(4bc-5ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{7/2} \sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 147, normalized size = 0.91

$$\frac{-\frac{a^2(c+dx^3)^{3/2}}{a+bx^3} + \frac{a(5ad-4bc) \left(\sqrt{b} \sqrt{c+dx^3} - \sqrt{bc-ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right) \right)}{b^{3/2}} + \frac{2(c+dx^3)^{3/2}(bc-ad)}{3d}}{3b^2(bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2, x]

[Out] ((2*(b*c - a*d)*(c + d*x^3)^(3/2))/(3*d) - (a^2*(c + d*x^3)^(3/2))/(a + b*x^3) + (a*(-4*b*c + 5*a*d)*(Sqrt[b]*Sqrt[c + d*x^3] - Sqrt[b*c - a*d]*ArcTan[h[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]])/b^(3/2))/(3*b^2*(b*c - a*d))

IntegrateAlgebraic [A] time = 0.27, size = 142, normalized size = 0.88

$$\frac{(4abc - 5a^2d) \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad} \right)}{3b^{7/2} \sqrt{ad-bc}} + \frac{\sqrt{c+dx^3} (-15a^2d + 2abc - 10abdx^3 + 2b^2cx^3 + 2b^2dx^6)}{9b^3d(a+bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8*sqrt[c + d*x^3])/(a + b*x^3)^2,x]

[Out] (sqrt[c + d*x^3]*(2*a*b*c - 15*a^2*d + 2*b^2*c*x^3 - 10*a*b*d*x^3 + 2*b^2*d*x^6))/(9*b^3*d*(a + b*x^3)) + ((4*a*b*c - 5*a^2*d)*ArcTan[(sqrt[b]*sqrt[-(b*c) + a*d]*sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(7/2)*sqrt[-(b*c) + a*d])

fricas [A] time = 0.92, size = 469, normalized size = 2.91

$$\frac{3(4a^2bd - 5a^2d^2 + (4a^2bd - 5a^2d^2)^2)^{\sqrt{c-dx^3}} - abd \log\left(\frac{(a^2bd - ab^2d^2)\sqrt{c-dx^3}}{a^2bd}\right) - 2(2(a^2bd - ab^2d^2)^2 + 2ab^2d^2 - 17a^2bd^2 + 15a^2bd^2 + 2(b^2c^2 - 6ab^2cd + 5a^2bd^2)^2)\sqrt{c-dx^3} + c - 3(4a^2bd - 5a^2d^2 + (4a^2bd - 5a^2d^2)^2)^{\sqrt{-b^2c+abd}} \arctan\left(\frac{\sqrt{c-dx^3}}{\sqrt{-b^2c+abd}}\right) - 2(2(a^2bd - ab^2d^2)^2 + 2ab^2d^2 - 17a^2bd^2 + 15a^2bd^2 + 2(b^2c^2 - 6ab^2cd + 5a^2bd^2)^2)\sqrt{c-dx^3}}{18(a^2bd - ab^2d^2 + (b^2c^2 - 6ab^2cd + 5a^2bd^2)^2) \sqrt{c-dx^3} - 9(a^2bd - ab^2d^2 + (b^2c^2 - 6ab^2cd + 5a^2bd^2)^2) \sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [-1/18*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17*a^2*b^2*c*d + 15*a^3*b*d^2 + 2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c)/(a*b^5*c*d - a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3), -1/9*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17*a^2*b^2*c*d + 15*a^3*b*d^2 + 2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c)/(a*b^5*c*d - a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3)]

giac [A] time = 0.18, size = 136, normalized size = 0.84

$$\frac{\sqrt{dx^3 + c} a^2 d}{3((dx^3 + c)b - bc + ad)b^3} - \frac{(4abc - 5a^2d) \arctan\left(\frac{\sqrt{dx^3 + c} b}{\sqrt{-b^2c + abd}}\right)}{3\sqrt{-b^2c + abd} b^3} + \frac{2\left((dx^3 + c)^{\frac{3}{2}} b^4 d^2 - 6\sqrt{dx^3 + c} ab^3 d^3\right)}{9b^6 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/3*sqrt(d*x^3 + c)*a^2*d/(((d*x^3 + c)*b - b*c + a*d)*b^3) - 1/3*(4*a*b*c - 5*a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/9*((d*x^3 + c)^(3/2)*b^4*d^2 - 6*sqrt(d*x^3 + c)*a*b^3*d^3)/(b^6*d^3)

maple [C] time = 0.36, size = 917, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)

[Out] $\frac{2}{9} \frac{(d x^3 + c)^{3/2}}{b^2 d - 2 a b} \frac{2/3 (d x^3 + c)^{1/2}}{b + 1/3 I/b/d^2} \sum \left(\frac{(-c d^2)^{1/3} (1/2 I (2 x + (-I)^{1/2} (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d)}{(-c d^2)^{1/3} d^{1/2}} \frac{(x - (-c d^2)^{1/3} / d)}{(-3 (-c d^2)^{1/3} + I)^{1/2}} \frac{(-c d^2)^{1/3} d^{1/2}}{(-1/2 I (2 x + I^{1/2} (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d)} \frac{(-c d^2)^{1/3} d^{1/2}}{(d x^3 + c)^{1/2}} \frac{2 \alpha^2 d^2 + I (-c d^2)^{1/3} 3^{1/2} \alpha d - (-c d^2)^{1/3} \alpha d - I^{1/2} (-c d^2)^{2/3} - (-c d^2)^{2/3}}{\text{EllipticPi}\left(\frac{1}{3} 3^{1/2} (I (x + 1/2 (-c d^2)^{1/3} / d - 1/2 I^{1/2} (-c d^2)^{1/3} / d) 3^{1/2} / (-c d^2)^{1/3} d^{1/2}, 1/2 (2 I (-c d^2)^{1/3} 3^{1/2} \alpha^2 d + I^{1/2} c d - 3 c d - I (-c d^2)^{2/3} 3^{1/2} \alpha - 3 (-c d^2)^{2/3} \alpha) / (a d - b c) b / d, (I^{1/2} (-c d^2)^{1/3} / (-3/2 (-c d^2)^{1/3} / d + 1/2 I^{1/2} (-c d^2)^{1/3} / d) / d)^{1/2}\right), \alpha = \text{RootOf}(_Z^3 b + a)} + a^2 / b^2 \frac{(-1/3 (d x^3 + c)^{1/2} / b / (b x^3 + a) - 1/6 I / d / b^2)^{1/2} \sum \left(\frac{1}{(a d - b c) (-c d^2)^{1/3} (1/2 I (2 x + (-I)^{1/2} (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d)} \frac{(-c d^2)^{1/3} d^{1/2}}{(-c d^2)^{1/3} d^{1/2}} \frac{(x - (-c d^2)^{1/3} / d)}{(-3 (-c d^2)^{1/3} + I)^{1/2}} \frac{(-c d^2)^{1/3} d^{1/2}}{(-1/2 I (2 x + I^{1/2} (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d)} \frac{(-c d^2)^{1/3} d^{1/2}}{(d x^3 + c)^{1/2}} \frac{2 \alpha^2 d^2 + I (-c d^2)^{1/3} 3^{1/2} \alpha d - (-c d^2)^{1/3} \alpha d - I^{1/2} (-c d^2)^{2/3} - (-c d^2)^{2/3}}{\text{EllipticPi}\left(\frac{1}{3} 3^{1/2} (I (x + 1/2 (-c d^2)^{1/3} / d - 1/2 I^{1/2} (-c d^2)^{1/3} / d) 3^{1/2} / (-c d^2)^{1/3} d^{1/2}, 1/2 (2 I (-c d^2)^{1/3} 3^{1/2} \alpha^2 d + I^{1/2} c d - 3 c d - I (-c d^2)^{2/3} 3^{1/2} \alpha - 3 (-c d^2)^{2/3} \alpha) / (a d - b c) b / d, (I^{1/2} (-c d^2)^{1/3} / (-3/2 (-c d^2)^{1/3} / d + 1/2 I^{1/2} (-c d^2)^{1/3} / d) / d)^{1/2}\right), \alpha = \text{RootOf}(_Z^3 b + a)} \right)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details) Is a*d-b*c positive or negative?

mupad [B] time = 6.84, size = 202, normalized size = 1.25

$$\frac{2x^3 \sqrt{dx^3+c}}{9b^2} - \frac{\sqrt{dx^3+c} \left(\frac{4c}{3b^2} - \frac{2b^2c-2abd}{b^4} + \frac{2ad}{b^3} \right)}{3d} + \frac{a^2 \left(\frac{2ad}{3(2b^2c-2abd)} - \frac{2bc}{3(2b^2c-2abd)} \right) \sqrt{dx^3+c}}{b^2(bx^3+a)} + \frac{a \ln \left(\frac{2bc-ad+bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}}{bx^3+a} \right) (5ad-4bc)}{6b^{7/2}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)


```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*b^2) - ((c + d*x^3)^(1/2)*((4*c)/(3*b^2) - (2*
b^2*c - 2*a*b*d)/b^4 + (2*a*d)/b^3))/(3*d) + (a*log((2*b*c - a*d + b^(1/2)*
(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d - 4*b
*c)*1i)/(6*b^(7/2)*(a*d - b*c)^(1/2)) + (a^2*((2*a*d)/(3*(2*b^2*c - 2*a*b*d
))) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d)))*(c + d*x^3)^(1/2))/(b^2*(a + b*x^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

$$3.302 \quad \int \frac{x^5 \sqrt{c+dx^3}}{(a+bx^3)^2} dx$$

Optimal. Leaf size=136

$$-\frac{(2bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^3}(2bc-3ad)}{3b^2(bc-ad)} + \frac{a(c+dx^3)^{3/2}}{3b(a+bx^3)(bc-ad)}$$

Rubi [A] time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 78, 50, 63, 208}

$$\frac{\sqrt{c+dx^3}(2bc-3ad)}{3b^2(bc-ad)} - \frac{(2bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}} + \frac{a(c+dx^3)^{3/2}}{3b(a+bx^3)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]

[Out] ((2*b*c - 3*a*d)*Sqrt[c + d*x^3])/(3*b^2*(b*c - a*d)) + (a*(c + d*x^3)^(3/2))/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^5 \sqrt{c + dx^3}}{(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x \sqrt{c + dx}}{(a + bx)^2} dx, x, x^3 \right) \\
 &= \frac{a(c + dx^3)^{3/2}}{3b(bc - ad)(a + bx^3)} + \frac{(2bc - 3ad) \text{Subst} \left(\int \frac{\sqrt{c + dx}}{a + bx} dx, x, x^3 \right)}{6b(bc - ad)} \\
 &= \frac{(2bc - 3ad) \sqrt{c + dx^3}}{3b^2(bc - ad)} + \frac{a(c + dx^3)^{3/2}}{3b(bc - ad)(a + bx^3)} + \frac{(2bc - 3ad) \text{Subst} \left(\int \frac{1}{(a + bx) \sqrt{c + dx}} dx, x, x^3 \right)}{6b^2} \\
 &= \frac{(2bc - 3ad) \sqrt{c + dx^3}}{3b^2(bc - ad)} + \frac{a(c + dx^3)^{3/2}}{3b(bc - ad)(a + bx^3)} + \frac{(2bc - 3ad) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3b^2 d} \\
 &= \frac{(2bc - 3ad) \sqrt{c + dx^3}}{3b^2(bc - ad)} + \frac{a(c + dx^3)^{3/2}}{3b(bc - ad)(a + bx^3)} - \frac{(2bc - 3ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^3}}{\sqrt{bc - ad}} \right)}{3b^{5/2} \sqrt{bc - ad}}
 \end{aligned}$$

Mathematica [A] time = 0.15, size = 117, normalized size = 0.86

$$\frac{(2bc-3ad)\left(\sqrt{b}\sqrt{c+dx^3}-\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)\right)}{b^{3/2}} + \frac{a(c+dx^3)^{3/2}}{a+bx^3}$$

$$3b(bc-ad)$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]

[Out] ((a*(c + d*x^3)^(3/2))/(a + b*x^3) + ((2*b*c - 3*a*d)*(Sqrt[b]*Sqrt[c + d*x^3] - Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]))/b^(3/2))/(3*b*(b*c - a*d))

IntegrateAlgebraic [A] time = 0.20, size = 108, normalized size = 0.79

$$\frac{(3ad - 2bc) \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{5/2}\sqrt{ad-bc}} + \frac{(3a + 2bx^3)\sqrt{c+dx^3}}{3b^2(a+bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]

[Out] ((3*a + 2*b*x^3)*Sqrt[c + d*x^3])/(3*b^2*(a + b*x^3)) + ((-2*b*c + 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(5/2)*Sqrt[-(b*c) + a*d])

fricas [A] time = 0.63, size = 334, normalized size = 2.46

$$\frac{\left(\frac{(2b^2c - 3abd)x^3 + 2abc - 3a^2d\sqrt{b^2c - abd} \log\left(\frac{bbs^2 + 2bc - ad + \sqrt{d^3x^3 + c}\sqrt{b^2c - abd}}{bs^2 + a}\right) - 2(3ab^2c - 3a^2bd + 2(b^3c - ab^2d)x^3)\sqrt{d^3x^3 + c}}{6(ab^4c - a^2b^3d + (b^5c - ab^4d)x^3)}\right) - \frac{\left(\frac{(2b^2c - 3abd)x^3 + 2abc - 3a^2d\sqrt{-b^2c + abd} \arctan\left(\frac{\sqrt{d^3x^3 + c}\sqrt{-b^2c + abd}}{bs^2 + bc}\right) + (3ab^2c - 3a^2bd + 2(b^3c - ab^2d)x^3)\sqrt{d^3x^3 + c}}{3(ab^4c - a^2b^3d + (b^5c - ab^4d)x^3)}\right)}{3(ab^4c - a^2b^3d + (b^5c - ab^4d)x^3)}\right)}{3(ab^4c - a^2b^3d + (b^5c - ab^4d)x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [-1/6*(((2*b^2*c - 3*a*b*d)*x^3 + 2*a*b*c - 3*a^2*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3), 1/3*(((2*b^2*c - 3*a*b*d)*x^3 + 2*a*b*c - 3*a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3)]

giac [A] time = 0.17, size = 102, normalized size = 0.75

$$\frac{\sqrt{dx^3 + c} ad}{3 \left((dx^3 + c)b - bc + ad \right) b^2} + \frac{(2bc - 3ad) \arctan\left(\frac{\sqrt{dx^3 + c} b}{\sqrt{-b^2c + abd}}\right)}{3 \sqrt{-b^2c + abd} b^2} + \frac{2 \sqrt{dx^3 + c}}{3 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/3*sqrt(d*x^3 + c)*a*d/(((d*x^3 + c)*b - b*c + a*d)*b^2) + 1/3*(2*b*c - 3*a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) + 2/3*sqrt(d*x^3 + c)/b^2

maple [C] time = 0.26, size = 897, normalized size = 6.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)

[Out] 1/b*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-a/b*(-1/3*(d*x^3+c)^(1/2)/(b*x^3+a)/b-1/6*I/d/b*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 6.09, size = 152, normalized size = 1.12

$$\frac{2\sqrt{dx^3+c}}{3b^2} - \frac{a\left(\frac{2ad}{3(2b^2c-2abd)} - \frac{2bc}{3(2b^2c-2abd)}\right)\sqrt{dx^3+c}}{b(bx^3+a)} + \frac{\ln\left(\frac{ad-2bc-bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}}{bx^3+a}\right)(3ad-2bc)}{6b^{5/2}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)

[Out] (2*(c + d*x^3)^(1/2))/(3*b^2) + (log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 2*b*c)*1i)/(6*b^(5/2)*(a*d - b*c)^(1/2)) - (a*((2*a*d)/(3*(2*b^2*c - 2*a*b*d)) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d)))*(c + d*x^3)^(1/2))/(b*(a + b*x^3))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)

[Out] Timed out

$$3.303 \quad \int \frac{x^2 \sqrt{c+dx^3}}{(a+bx^3)^2} dx$$

Optimal. Leaf size=80

$$-\frac{d \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^3}}{3b(a+bx^3)}$$

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {444, 47, 63, 208}

$$-\frac{d \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^3}}{3b(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]

[Out] -Sqrt[c + d*x^3]/(3*b*(a + b*x^3)) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2 \sqrt{c + dx^3}}{(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c + dx}}{(a + bx)^2} dx, x, x^3 \right) \\ &= -\frac{\sqrt{c + dx^3}}{3b(a + bx^3)} + \frac{d \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{6b} \\ &= -\frac{\sqrt{c + dx^3}}{3b(a + bx^3)} + \frac{\text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3b} \\ &= -\frac{\sqrt{c + dx^3}}{3b(a + bx^3)} - \frac{d \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{3/2} \sqrt{bc - ad}} \end{aligned}$$

Mathematica [A] time = 0.11, size = 80, normalized size = 1.00

$$\frac{d \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{ad-bc}} \right)}{3b^{3/2} \sqrt{ad - bc}} - \frac{\sqrt{c + dx^3}}{3b(a + bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(b*(a + b*x^3)) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/
Sqrt[-(b*c) + a*d]])/(3*b^(3/2)*Sqrt[-(b*c) + a*d])
```


IntegrateAlgebraic [A] time = 0.18, size = 90, normalized size = 1.12

$$\frac{d \tan^{-1}\left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad}\right)}{3b^{3/2}\sqrt{ad-bc}} - \frac{\sqrt{c+dx^3}}{3b(a+bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]

[Out] $-1/3*\text{Sqrt}[c + d*x^3]/(b*(a + b*x^3)) - (d*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[-(b*c) + a*d])*\text{Sqrt}[c + d*x^3])/(b*c - a*d)]/(3*b^(3/2)*\text{Sqrt}[-(b*c) + a*d])$

fricas [A] time = 0.81, size = 255, normalized size = 3.19

$$\left[\frac{(bdx^3 + ad)\sqrt{b^2c - abd} \log\left(\frac{bdx^3 + 2bc - ad - 2\sqrt{dx^3 + c}\sqrt{b^2c - abd}}{bx^3 + a}\right) - 2\sqrt{dx^3 + c}(b^2c - abd) (bdx^3 + ad)\sqrt{-b^2c + abd} \arctan\left(\frac{\sqrt{dx^3 + c}\sqrt{-b^2c + abd}}{bdx^3 + bc}\right) - \sqrt{dx^3 + c}(b^2c - abd)}{6(ab^3c - a^2b^2d + (b^4c - ab^3d)x^3)}, \frac{(bdx^3 + ad)\sqrt{-b^2c + abd} \arctan\left(\frac{\sqrt{dx^3 + c}\sqrt{-b^2c + abd}}{bdx^3 + bc}\right) - \sqrt{dx^3 + c}(b^2c - abd)}{3(ab^3c - a^2b^2d + (b^4c - ab^3d)x^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] $[1/6*((b*d*x^3 + a*d)*\text{sqrt}(b^2*c - a*b*d)*\log((b*d*x^3 + 2*b*c - a*d - 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(b^2*c - a*b*d))/(b*x^3 + a)) - 2*\text{sqrt}(d*x^3 + c)*(b^2*c - a*b*d))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^3), 1/3*((b*d*x^3 + a*d)*\text{sqrt}(-b^2*c + a*b*d)*\arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - \text{sqrt}(d*x^3 + c)*(b^2*c - a*b*d))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^3)]$

giac [A] time = 0.20, size = 79, normalized size = 0.99

$$\frac{d \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b} - \frac{\sqrt{dx^3+cd}}{3((dx^3+c)b-bc+ad)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")

[Out] $1/3*d*\arctan(\text{sqrt}(d*x^3 + c)*b/\text{sqrt}(-b^2*c + a*b*d))/(\text{sqrt}(-b^2*c + a*b*d)*b) - 1/3*\text{sqrt}(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*b)$

maple [C] time = 0.27, size = 453, normalized size = 5.66

The Maple output shows a highly complex antiderivative involving nested square roots, logarithms, and elliptic integrals. The expression is divided into two main parts, each enclosed in large square brackets. The first part contains several nested square roots and a logarithmic term. The second part contains a similar structure with nested square roots and an elliptic integral function. The entire expression is divided by a denominator that is a product of a square root and a linear term in x^3.

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)`

[Out]
$$-1/3*(d*x^3+c)^{(1/2)}/(b*x^3+a)/b-1/6*I/d/b*2^{(1/2)}*\text{sum}(1/(a*d-b*c)*(-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)),_alpha=\text{RootOf}(_Z^3*b+a)$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 5.69, size = 125, normalized size = 1.56

$$\frac{\left(\frac{2ad}{3(2b^2c-2abd)} - \frac{2bc}{3(2b^2c-2abd)}\right) \sqrt{dx^3+c}}{bx^3+a} + \frac{d \ln\left(\frac{2bc-ad+bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}}{bx^3+a}\right) 1i}{6b^{3/2}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)`

[Out]
$$\left(\frac{(2*a*d)/(3*(2*b^2*c - 2*a*b*d)) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d))}{(a + b*x^3)} + \frac{(d*\log((2*b*c - a*d + b^{(1/2)}*(c + d*x^3)^{(1/2)}*(a*d - b*c)^{(1/2)}*2i + b*d*x^3)/(a + b*x^3))*1i}{(6*b^{(3/2)}*(a*d - b*c)^{(1/2)})}\right)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{c + dx^3}}{(a + bx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

$$3.304 \quad \int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2} dx$$

Optimal. Leaf size=121

$$\frac{(2bc - ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3a^2 \sqrt{b} \sqrt{bc - ad}} - \frac{2\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^2} + \frac{\sqrt{c + dx^3}}{3a(a + bx^3)}$$

Rubi [A] time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc - ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3a^2 \sqrt{b} \sqrt{bc - ad}} - \frac{2\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^2} + \frac{\sqrt{c + dx^3}}{3a(a + bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]/(3*a*(a + b*x^3)) - (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*Sqrt[b]*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^(n), x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/(m + 1)*(b*e - a*f), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(
m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
+ n + p + 2)*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c+dx}}{x(a+bx)^2} dx, x, x^3 \right) \\
&= \frac{\sqrt{c+dx^3}}{3a(a+bx^3)} - \frac{\text{Subst} \left(\int \frac{-c-\frac{dx}{2}}{x(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3a} \\
&= \frac{\sqrt{c+dx^3}}{3a(a+bx^3)} + \frac{c \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{3a^2} - \frac{(2bc-ad) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{6a^2} \\
&= \frac{\sqrt{c+dx^3}}{3a(a+bx^3)} + \frac{(2c) \text{Subst} \left(\int \frac{1}{-\frac{c}{a} + \frac{x^2}{a}} dx, x, \sqrt{c+dx^3} \right)}{3a^2d} - \frac{(2bc-ad) \text{Subst} \left(\int \frac{1}{a-\frac{bc}{a} + \frac{bx^2}{a}} dx, x, \sqrt{c+dx^3} \right)}{3a^2d} \\
&= \frac{\sqrt{c+dx^3}}{3a(a+bx^3)} - \frac{2\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^2} + \frac{(2bc-ad) \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3a^2\sqrt{b}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 112, normalized size = 0.93

$$\frac{\frac{a\sqrt{c+dx^3}}{a+bx^3} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}} - 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2), x]

[Out] ((a*Sqrt[c + d*x^3])/(a + b*x^3) - 2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a*d]))/(3*a^2)

IntegrateAlgebraic [A] time = 0.51, size = 131, normalized size = 1.08

$$\frac{(2bc - ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^2\sqrt{b}\sqrt{ad-bc}} - \frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{\sqrt{c+dx^3}}{3a(a+bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2), x]

[Out] Sqrt[c + d*x^3]/(3*a*(a + b*x^3)) + ((2*b*c - a*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*a^2*Sqrt[b]*Sqrt[-(b*c) + a*d]) - (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)

fricas [B] time = 0.73, size = 856, normalized size = 7.07

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [-1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - (a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), 1/6*(4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-c)*arctan(sqrt(c)

$$d*x^3 + c)*\sqrt{-c}/c) - ((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*\sqrt{b^2*c - a*b*d}*\log((b*d*x^3 + 2*b*c - a*d - 2*\sqrt{d*x^3 + c})*\sqrt{b^2*c - a*b*d})/(b*x^3 + a)) + 2*(a*b^2*c - a^2*b*d)*\sqrt{d*x^3 + c})/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*\sqrt{-b^2*c + a*b*d}*\arctan(\sqrt{d*x^3 + c}*\sqrt{-b^2*c + a*b*d}/(b*d*x^3 + b*c)) - 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*\sqrt{-c}*\arctan(\sqrt{d*x^3 + c}*\sqrt{-c}/c) - (a*b^2*c - a^2*b*d)*\sqrt{d*x^3 + c})/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3)]$$

giac [A] time = 0.17, size = 114, normalized size = 0.94

$$\frac{\sqrt{dx^3 + cd}}{3((dx^3 + c)b - bc + ad)a} - \frac{(2bc - ad) \arctan\left(\frac{\sqrt{dx^3 + cb}}{\sqrt{-b^2c + abd}}\right)}{3\sqrt{-b^2c + abd}a^2} + \frac{2c \arctan\left(\frac{\sqrt{dx^3 + c}}{\sqrt{-c}}\right)}{3a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*a) - 1/3*(2*b*c - a*d)*a*rctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) + 2/3*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c))

maple [C] time = 0.28, size = 934, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x)

[Out]
$$-1/a^2*b*(2/3*(d*x^3+c)^{(1/2)}/b+1/3*I/b/d^2*2^{(1/2)}*\sum((-c*d^2)^{(1/3)}*(1/2)*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{Elliptic Pi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)*d)^{(1/2)},1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}),_alpha=\text{RootOf}(_Z^3*b+a)))-1/a*b*(-1/3*(d*x^3+c)^{(1/2)}/(b*x^3+a)/b-1/6*I/d/b*2^{(1/2)}*\sum(1/(a*d-b*c)*(-c*d^2)^{(1/3)}*(1/2)*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*$$

$d - (-c*d^2)^{(1/3)} * _alpha*d - I*3^{(1/2)} * (-c*d^2)^{(2/3)} - (-c*d^2)^{(2/3)} * \text{EllipticPi}(1/3*3^{(1/2)} * (I*(x+1/2*(-c*d^2)^{(1/3)}/d - 1/2*I*3^{(1/2)} * (-c*d^2)^{(1/3)}/d) * 3^{(1/2)} / (-c*d^2)^{(1/3)} * d)^{(1/2)}, 1/2*(2*I*(-c*d^2)^{(1/3)} * 3^{(1/2)} * _alpha^2*d + I*3^{(1/2)} * c*d - 3*c*d - I*(-c*d^2)^{(2/3)} * 3^{(1/2)} * _alpha - 3*(-c*d^2)^{(2/3)} * _alpha) / (a*d - b*c) * b/d, (I*3^{(1/2)} * (-c*d^2)^{(1/3)} / (-3/2*(-c*d^2)^{(1/3)}/d + 1/2*I*3^{(1/2)} * (-c*d^2)^{(1/3)}/d) / d)^{(1/2)}, _alpha = \text{RootOf}(_Z^3*b+a)) + 1/a^2 * (2/3*(d*x^3+c)^{(1/2)} - 2/3*\text{arctanh}((d*x^3+c)^{(1/2)}/c)^{(1/2)}) * c^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{(bx^3 + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x), x)

mupad [B] time = 8.28, size = 182, normalized size = 1.50

$$\frac{\sqrt{c} \ln\left(\frac{(\sqrt{dx^3+c}-\sqrt{c})^3(\sqrt{dx^3+c}+\sqrt{c})}{x^6}\right)}{3a^2} - \frac{\left(\frac{bd}{3(b^2c-abd)} - \frac{b^2c}{3a(b^2c-abd)}\right)\sqrt{dx^3+c}}{bx^3+a} + \frac{\ln\left(\frac{2bc-ad+bdx^3+\sqrt{dx^3+c}\sqrt{abd-b^2c}2i}{bx^3+a}\right)(ad-2bc)1i}{6a^2\sqrt{abd-b^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^3)^(1/2)/(x*(a + b*x^3)^2),x)

[Out] $(c^{(1/2)} * \log((((c + d*x^3)^{(1/2)} - c^{(1/2)})^3 * ((c + d*x^3)^{(1/2)} + c^{(1/2)}) / x^6)) / (3*a^2) - (((b*d) / (3*(b^2*c - a*b*d)) - (b^2*c) / (3*a*(b^2*c - a*b*d))) * (c + d*x^3)^{(1/2)}) / (a + b*x^3) + (\log((2*b*c - a*d + (c + d*x^3)^{(1/2)} * (a*b*d - b^2*c)^{(1/2)} * 2i + b*d*x^3) / (a + b*x^3)) * (a*d - 2*b*c) * 1i) / (6*a^2 * (a*b*d - b^2*c)^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x(a + bx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c)**(1/2)/x/(b*x**3+a)**2,x)

[Out] Integral(sqrt(c + d*x**3)/(x*(a + b*x**3)**2), x)

$$3.305 \quad \int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2} dx$$

Optimal. Leaf size=161

$$\frac{\sqrt{b}(4bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}} + \frac{(4bc-ad) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}} - \frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)}$$

Rubi [A] time = 0.22, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 99, 151, 156, 63, 208}

$$\frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{\sqrt{b}(4bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}} + \frac{(4bc-ad) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}} - \frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2), x]

[Out] (-2*b*Sqrt[c + d*x^3])/(3*a^2*(a + b*x^3)) - Sqrt[c + d*x^3]/(3*a*x^3*(a + b*x^3)) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(3*a^3*Sqrt[c]) - (Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 99

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)^2} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(-4bc+ad) - \frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}} dx, x, x^3 \right)}{3a} \\
&= -\frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{\text{Subst} \left(\int \frac{-\frac{1}{2}(bc-ad)(4bc-ad) - bd(bc-ad)x}{x(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3a^2(bc-ad)} \\
&= -\frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{(b(4bc-3ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{6a^3} - \frac{(4bc-ad)}{3a^2} \\
&= -\frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{(b(4bc-3ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{3a^3d} - \frac{(4bc-ad)}{3a^2} \\
&= -\frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{(4bc-ad) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^3\sqrt{c}} - \frac{\sqrt{b}(4bc-3ad) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3a^3\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 190, normalized size = 1.18

$$\frac{\sqrt{c} \left(a(a+2bx^3)\sqrt{c+dx^3}(bc-ad) + \sqrt{b}x^3(a+bx^3)(4bc-3ad)\sqrt{bc-ad} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}} \right) - x^3(a+bx^3)(a^2d^2 - 5abcd + 4b^2c^2) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right) \right)}{3a^3\sqrt{c}x^3(a+bx^3)(ad-bc)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2), x]

[Out] (-(4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*x^3*(a + b*x^3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]) + Sqrt[c]*(a*(b*c - a*d)*(a + 2*b*x^3)*Sqrt[c + d*x^3] + Sqrt[b]*(4*b*c - 3*a*d)*Sqrt[b*c - a*d]*x^3*(a + b*x^3)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*Sqrt[c]*(-(b*c) + a*d)*x^3*(a + b*x^3))

IntegrateAlgebraic [A] time = 0.54, size = 157, normalized size = 0.98

$$\frac{(3a\sqrt{b}d - 4b^{3/2}c) \tan^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad} \right)}{3a^3\sqrt{ad-bc}} + \frac{(4bc-ad) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^3\sqrt{c}} + \frac{(-a-2bx^3)\sqrt{c+dx^3}}{3a^2x^3(a+bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]

[Out]
$$\frac{((-a - 2*b*x^3)*\text{Sqrt}[c + d*x^3])/(3*a^2*x^3*(a + b*x^3)) + ((-4*b^{(3/2)}*c + 3*a*\text{Sqrt}[b]*d)*\text{ArcTan}[\text{Sqrt}[b]*\text{Sqrt}[-(b*c) + a*d]*\text{Sqrt}[c + d*x^3])/(b*c - a*d)}{(3*a^3*\text{Sqrt}[-(b*c) + a*d])} + \frac{((4*b*c - a*d)*\text{ArcTanh}[\text{Sqrt}[c + d*x^3]/\text{Sqrt}[c]])}{(3*a^3*\text{Sqrt}[c])}$$

fricas [A] time = 1.11, size = 870, normalized size = 5.40

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/6*((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*\text{sqrt}(b/(b*c - a*d)) * \log((b*d*x^3 + 2*b*c - a*d + 2*\text{sqrt}(d*x^3 + c)*(b*c - a*d)*\text{sqrt}(b/(b*c - a*d)))/(b*x^3 + a)) \\ & + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*\text{sqrt}(c) * \log((d*x^3 - 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 2*c)/x^3) + 2*(2*a*b*c*x^3 + a^2*c)*\text{sqrt}(d*x^3 + c) \\ & / (a^3*b*c*x^6 + a^4*c*x^3), -1/6*(2*((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*\text{sqrt}(-b/(b*c - a*d)) * \arctan(-\text{sqrt}(d*x^3 + c)*(b*c - a*d)*\text{sqrt}(-b/(b*c - a*d)))/(b*d*x^3 + b*c)) \\ & + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*\text{sqrt}(c) * \log((d*x^3 - 2*\text{sqrt}(d*x^3 + c)*\text{sqrt}(c) + 2*c)/x^3) + 2*(2*a*b*c*x^3 + a^2*c)*\text{sqrt}(d*x^3 + c) \\ & / (a^3*b*c*x^6 + a^4*c*x^3), -1/6*(2*((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*\text{sqrt}(-c) * \arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) \\ & + ((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*\text{sqrt}(b/(b*c - a*d)) * \log((b*d*x^3 + 2*b*c - a*d + 2*\text{sqrt}(d*x^3 + c)*(b*c - a*d)*\text{sqrt}(b/(b*c - a*d)))/(b*x^3 + a)) \\ & + 2*(2*a*b*c*x^3 + a^2*c)*\text{sqrt}(d*x^3 + c) / (a^3*b*c*x^6 + a^4*c*x^3), -1/3 * ((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*\text{sqrt}(-b/(b*c - a*d)) * \arctan(-\text{sqrt}(d*x^3 + c)*(b*c - a*d)*\text{sqrt}(-b/(b*c - a*d)))/(b*d*x^3 + b*c)) \\ & + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*\text{sqrt}(-c) * \arctan(\text{sqrt}(d*x^3 + c)*\text{sqrt}(-c)/c) + (2*a*b*c*x^3 + a^2*c)*\text{sqrt}(d*x^3 + c) / (a^3*b*c*x^6 + a^4*c*x^3)] \end{aligned}$$

giac [A] time = 0.22, size = 183, normalized size = 1.14

$$\frac{(4b^2c - 3abd) \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^3} - \frac{(4bc - ad) \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a^3\sqrt{-c}} - \frac{2(dx^3+c)^{\frac{3}{2}}bd - 2\sqrt{dx^3+c}bcd + \sqrt{dx^3+c}ad^2}{3\left((dx^3+c)^2b - 2(dx^3+c)bc + bc^2 + (dx^3+c)ad - acd\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")

```
[Out] 1/3*(4*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3) - 1/3*(4*b*c - a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)) - 1/3*(2*(d*x^3 + c)^(3/2)*b*d - 2*sqrt(d*x^3 + c)*b*c*d + sqrt(d*x^3 + c)*a*d^2)/(((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*d)*a^2)
```

maple [C] time = 0.26, size = 978, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x)
```

```
[Out] 1/a^2*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+2/a^3*b^2*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*b^2*(-1/3*(d*x^3+c)^(1/2)/(b*x^3+a)/b-1/6*I/d/b^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-2*b/a^3*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{(bx^3 + a)^2 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^4), x)

mupad [B] time = 9.69, size = 438, normalized size = 2.72

$$\frac{\left(\frac{\frac{d^2 x^6 + 2 d^2 a x^3 + c^2}{2 x^3} - \frac{2 d^2 a x^3 + c^2}{2 x^3} \frac{d^2 x^6 + 2 d^2 a x^3 + c^2}{2 x^3}}{b^2 x^6 + 2 a b x^3 + a^2} - \frac{b^2 d^2 x^6 + 2 a b d^2 x^3 + a^2 d^2}{2 d^2 x^3} \right) \sqrt{d x^3 + c}}{b x^3 + a} - \frac{\sqrt{d x^3 + c}}{3 a^2 x^3} + \frac{\ln \left(\frac{(\sqrt{d x^3 + c} - \sqrt{c})^3 (\sqrt{d x^3 + c} + \sqrt{c})}{x^6} \right) (a d - 4 b c)}{6 a^3 \sqrt{c}} + \frac{\sqrt{b} \ln \left(\frac{a d - 2 b c - b d x^3 + \sqrt{b} \sqrt{d x^3 + c} \sqrt{a d - b c}}{x^2 + a} \right) (3 a d - 4 b c)}{6 a^3 \sqrt{a d - b c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^3)^(1/2)/(x^4*(a + b*x^3)^2), x)

[Out] (((a*((a*((a*((b^2*d^2)/(2*a^3*c^2) - (b^2*d^2*(3*a*d - 4*b*c))/(6*a^2*c^2*(a^2*d - a*b*c)) + (b^2*d*(2*a*d - b*c)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^2*d - a*b*c)))))/b - (b*d*(2*a*d - b*c))/(2*a^3*c^2) + (b*(3*a*d - 4*b*c)*(2*b^2*c^2 - a^2*d^2 + 2*a*b*c*d))/(6*a^3*c^2*(a^2*d - a*b*c)))))/b - (2*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(2*a^3*c^2) + (b*(a*d - 4*b*c)*(3*a*d - 4*b*c))/(6*a^2*c*(a^2*d - a*b*c))))/b - (a*d - 4*b*c)/(2*a^2*c))*(c + d*x^3)^(1/2)/(a + b*x^3) - (c + d*x^3)^(1/2)/(3*a^2*x^3) + (log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)*(a*d - 4*b*c))/(6*a^3*c^(1/2)) + (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 4*b*c)*1i)/(6*a^3*(a*d - b*c)^(1/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a)**2,x)

[Out] Timed out

$$3.306 \quad \int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2} dx$$

Optimal. Leaf size=189

$$\frac{a^2(c+dx^3)^{5/2}}{3b^2(a+bx^3)(bc-ad)} + \frac{a(4bc-7ad)\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}} - \frac{a\sqrt{c+dx^3}(4bc-7ad)}{3b^4} - \frac{a(c+dx^3)^{3/2}(4bc-7ad)}{9b^3(bc-ad)}$$

Rubi [A] time = 0.24, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 89, 80, 50, 63, 208}

$$\frac{a^2(c+dx^3)^{5/2}}{3b^2(a+bx^3)(bc-ad)} - \frac{a(c+dx^3)^{3/2}(4bc-7ad)}{9b^3(bc-ad)} - \frac{a\sqrt{c+dx^3}(4bc-7ad)}{3b^4} + \frac{a(4bc-7ad)\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}} + \frac{2(c+dx^3)^{5/2}}{15b^2d}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]

[Out] -(a*(4*b*c - 7*a*d)*Sqrt[c + d*x^3])/(3*b^4) - (a*(4*b*c - 7*a*d)*(c + d*x^3)^(3/2))/(9*b^3*(b*c - a*d)) + (2*(c + d*x^3)^(5/2))/(15*b^2*d) - (a^2*(c + d*x^3)^(5/2))/(3*b^2*(b*c - a*d)*(a + b*x^3)) + (a*(4*b*c - 7*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(9/2))

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8 (c + dx^3)^{3/2}}{(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (c + dx)^{3/2}}{(a + bx)^2} dx, x, x^3 \right) \\
&= -\frac{a^2 (c + dx^3)^{5/2}}{3b^2(bc - ad)(a + bx^3)} + \frac{\text{Subst} \left(\int \frac{(c+dx)^{3/2} \left(-\frac{1}{2}a(2bc-5ad) + b(bc-ad)x \right)}{a+bx} dx, x, x^3 \right)}{3b^2(bc - ad)} \\
&= \frac{2(c + dx^3)^{5/2}}{15b^2d} - \frac{a^2 (c + dx^3)^{5/2}}{3b^2(bc - ad)(a + bx^3)} - \frac{(a(4bc - 7ad)) \text{Subst} \left(\int \frac{(c+dx)^{3/2}}{a+bx} dx, x, x^3 \right)}{6b^2(bc - ad)} \\
&= -\frac{a(4bc - 7ad)(c + dx^3)^{3/2}}{9b^3(bc - ad)} + \frac{2(c + dx^3)^{5/2}}{15b^2d} - \frac{a^2 (c + dx^3)^{5/2}}{3b^2(bc - ad)(a + bx^3)} - \frac{(a(4bc - 7ad)) \text{Subst} \left(\int \frac{(c+dx)^{3/2}}{a+bx} dx, x, x^3 \right)}{6b^2(bc - ad)} \\
&= -\frac{a(4bc - 7ad)\sqrt{c + dx^3}}{3b^4} - \frac{a(4bc - 7ad)(c + dx^3)^{3/2}}{9b^3(bc - ad)} + \frac{2(c + dx^3)^{5/2}}{15b^2d} - \frac{a^2 (c + dx^3)^{5/2}}{3b^2(bc - ad)(a + bx^3)} \\
&= -\frac{a(4bc - 7ad)\sqrt{c + dx^3}}{3b^4} - \frac{a(4bc - 7ad)(c + dx^3)^{3/2}}{9b^3(bc - ad)} + \frac{2(c + dx^3)^{5/2}}{15b^2d} - \frac{a^2 (c + dx^3)^{5/2}}{3b^2(bc - ad)(a + bx^3)} \\
&= -\frac{a(4bc - 7ad)\sqrt{c + dx^3}}{3b^4} - \frac{a(4bc - 7ad)(c + dx^3)^{3/2}}{9b^3(bc - ad)} + \frac{2(c + dx^3)^{5/2}}{15b^2d} - \frac{a^2 (c + dx^3)^{5/2}}{3b^2(bc - ad)(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 162, normalized size = 0.86

$$\frac{\sqrt{c + dx^3} \left(105a^3d^2 + 5a^2bd(14dx^3 - 19c) + 2ab^2(3c^2 - 34cdx^3 - 7d^2x^6) + 6b^3x^3(c + dx^3)^2 \right)}{45b^4d(a + bx^3)} + \frac{a(4bc - 7ad)\sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c + dx^3}}{\sqrt{bc - ad}} \right)}{3b^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]

[Out] (Sqrt[c + d*x^3]*(105*a^3*d^2 + 6*b^3*x^3*(c + d*x^3)^2 + 5*a^2*b*d*(-19*c + 14*d*x^3) + 2*a*b^2*(3*c^2 - 34*c*d*x^3 - 7*d^2*x^6)))/(45*b^4*d*(a + b*x^3)) + (a*(4*b*c - 7*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(9/2))

IntegrateAlgebraic [A] time = 0.29, size = 211, normalized size = 1.12

$$\frac{(7a^3d^2 - 11a^2bcd + 4ab^2c^2) \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{9/2}\sqrt{ad-bc}} + \frac{\sqrt{c+dx^3}(105a^3d^2 - 95a^2bcd + 70a^2bd^2x^3 + 6ab^2c^2 - 68ab^2cdx^3 - 14ab^2d^2x^6 + 6b^3c^2x^3 + 12b^3cdx^6 + 6b^3d^2x^9)}{45b^4d(a+bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]

[Out] (Sqrt[c + d*x^3]*(6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3*d^2 + 6*b^3*c^2*x^3 - 68*a*b^2*c*d*x^3 + 70*a^2*b*d^2*x^3 + 12*b^3*c*d*x^6 - 14*a*b^2*d^2*x^6 + 6*b^3*d^2*x^9))/(45*b^4*d*(a + b*x^3)) + ((4*a*b^2*c^2 - 11*a^2*b*c*d + 7*a^3*d^2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d])*Sqrt[c + d*x^3]]/(b*c - a*d))/(3*b^(9/2)*Sqrt[-(b*c) + a*d])

fricas [A] time = 0.69, size = 443, normalized size = 2.34

$$\frac{15(4a^3bcd - 7a^2d^2 + (4ab^2cd - 7a^2bd^2)x^3)\sqrt{\frac{c+dx^3}{b}} \log\left(\frac{(b^2dx^3 + 2b^2c - ad - 2\sqrt{c+dx^3})\sqrt{b}}{b^2dx^3 + 2b^2c - ad}\right) - 2(6b^3d^2 + 2(6b^3d^2 + 6ab^2c - 95a^2bcd + 105a^3d^2 + 2(3b^3c^2 - 34a^2b^2cd + 35a^2bd^2)x^3)\sqrt{c+dx^3})\sqrt{\frac{c+dx^3}{b}} \arctan\left(\frac{\sqrt{c+dx^3}}{b}\right) + (6b^3d^2 + 2(6b^3d^2 - 7a^2bd^2)x^3 + 6ab^2c^2 - 95a^2bcd + 105a^3d^2 + 2(3b^3c^2 - 34a^2b^2cd + 35a^2bd^2)x^3)\sqrt{c+dx^3}}{90(b^2a^2 + a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [-1/90*(15*(4*a^2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(6*b^3*d^2*x^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)*x^6 + 6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*c*d + 35*a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^5*d*x^3 + a*b^4*d), 1/45*(15*(4*a^2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (6*b^3*d^2*x^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)*x^6 + 6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*c*d + 35*a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^5*d*x^3 + a*b^4*d)]

giac [A] time = 0.18, size = 211, normalized size = 1.12

$$\frac{(4ab^2c^2 - 11a^2bcd + 7a^3d^2) \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^4} - \frac{\sqrt{dx^3+c}a^2bcd - \sqrt{dx^3+c}a^3d^2}{3((dx^3+c)b - bc + ad)b^4} + \frac{2\left(3(dx^3+c)^{\frac{5}{2}}b^8d^4 - 10(dx^3+c)^{\frac{3}{2}}ab^7d^5 - 30\sqrt{dx^3+c}ab^7cd^5 + 45\sqrt{dx^3+c}a^2b^6d^6\right)}{45b^{10}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/3*(4*a*b^2*c^2 - 11*a^2*b*c*d + 7*a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) - 1/3*(sqrt(d*x^3 + c)*a^2*b*c*d - sqrt(d*x^3 + c)*a^3*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^4) + 2/45*(3*(

$$d*x^3 + c)^{(5/2)}*b^8*d^4 - 10*(d*x^3 + c)^{(3/2)}*a*b^7*d^5 - 30*\text{sqrt}(d*x^3 + c)*a*b^7*c*d^5 + 45*\text{sqrt}(d*x^3 + c)*a^2*b^6*d^6)/(b^{10}*d^5)$$

maple [C] time = 0.37, size = 1003, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)`

[Out]
$$\frac{2}{15} \frac{(d*x^3+c)^{(5/2)}}{b^2/d-2*a/b^2} \frac{(2/9*(d*x^3+c)^{(1/2)}/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^{(1/2)}/d+1/3*I/b^2/d^2*2^{(1/2)}*\text{sum}((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3))}/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3))}/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, 1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=\text{RootOf}(_Z^3*b+a)))+a^2/b^2*(1/3*(a*d-b*c)/b^2*(d*x^3+c)^{(1/2)}/(b*x^3+a)+2/3*d*(d*x^3+c)^{(1/2)}/b^2+1/2*I/d/b^2*2^{(1/2)}*\text{sum}((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3))}/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3))}/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, 1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=\text{RootOf}(_Z^3*b+a))$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 7.75, size = 331, normalized size = 1.75

$$\frac{\sqrt{dx^3+c} \left(\frac{2(ad-bd^2)}{b^4} + \frac{2 \left(\frac{2d(d-2b)}{3d} + \frac{2d^2}{b^3} + \frac{8cd}{5b^2} \right)}{3d} + \frac{2 \left(\frac{d(d-2b)}{b^3} + \frac{c^2}{b^3} \right)}{b} \right)}{15b^2} + \frac{x^3 \sqrt{dx^3+c}}{9d} - \frac{x^3 \sqrt{dx^3+c} \left(\frac{2d(d-2b)}{3d} + \frac{2d^2}{b^3} + \frac{8cd}{5b^2} \right)}{9d} - \frac{d^2 \left(\frac{2b^2}{3(2b^2-2abd)} + \frac{4bd}{3(2b^2-2abd)} \right) \sqrt{dx^3+c}}{b^2(bx^3+a)} + \frac{d \ln \left(\frac{xd-2bc-bd^2+\sqrt{dx^3+c}\sqrt{dx^3+c}}{bx^3+a} \right) \sqrt{ad-bc} (7ad-4bc)}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)

[Out] ((c + d*x^3)^(1/2)*((2*(a*d - b*c)^2)/b^4 + (2*c*((2*d*(a*d - 2*b*c))/b^3 + (2*a*d^2)/b^3 + (8*c*d)/(5*b^2)))/(3*d) + (2*a*((d*(a*d - 2*b*c))/b^3 + (a*d^2)/b^3))/b)/(3*d) + (2*d*x^6*(c + d*x^3)^(1/2))/(15*b^2) - (x^3*(c + d*x^3)^(1/2)*((2*d*(a*d - 2*b*c))/b^3 + (2*a*d^2)/b^3 + (8*c*d)/(5*b^2)))/(9*d) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(7*a*d - 4*b*c)*1i)/(6*b^(9/2)) - (a^2*((2*b*c^2)/(3*(2*b^2*c - 2*a*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*a*b*d))))/b)*(c + d*x^3)^(1/2))/(b^2*(a + b*x^3))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)

[Out] Timed out

$$3.307 \quad \int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2} dx$$

Optimal. Leaf size=163

$$\frac{(2bc - 5ad)\sqrt{bc - ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}} + \frac{\sqrt{c+dx^3}(2bc - 5ad)}{3b^3} + \frac{(c+dx^3)^{3/2}(2bc - 5ad)}{9b^2(bc - ad)} + \frac{a(c+dx^3)^{5/2}}{3b(a+bx^3)(bc - ad)}$$

Rubi [A] time = 0.14, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 78, 50, 63, 208}

$$\frac{(c+dx^3)^{3/2}(2bc - 5ad)}{9b^2(bc - ad)} + \frac{\sqrt{c+dx^3}(2bc - 5ad)}{3b^3} - \frac{(2bc - 5ad)\sqrt{bc - ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}} + \frac{a(c+dx^3)^{5/2}}{3b(a+bx^3)(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]

[Out] ((2*b*c - 5*a*d)*Sqrt[c + d*x^3])/(3*b^3) + ((2*b*c - 5*a*d)*(c + d*x^3)^(3/2))/(9*b^2*(b*c - a*d)) + (a*(c + d*x^3)^(5/2))/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 446

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (c + dx^3)^{3/2}}{(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(c + dx)^{3/2}}{(a + bx)^2} dx, x, x^3 \right) \\
&= \frac{a(c + dx^3)^{5/2}}{3b(bc - ad)(a + bx^3)} + \frac{(2bc - 5ad) \text{Subst} \left(\int \frac{(c+dx)^{3/2}}{a+bx} dx, x, x^3 \right)}{6b(bc - ad)} \\
&= \frac{(2bc - 5ad)(c + dx^3)^{3/2}}{9b^2(bc - ad)} + \frac{a(c + dx^3)^{5/2}}{3b(bc - ad)(a + bx^3)} + \frac{(2bc - 5ad) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^3 \right)}{6b^2} \\
&= \frac{(2bc - 5ad)\sqrt{c + dx^3}}{3b^3} + \frac{(2bc - 5ad)(c + dx^3)^{3/2}}{9b^2(bc - ad)} + \frac{a(c + dx^3)^{5/2}}{3b(bc - ad)(a + bx^3)} + \frac{((2bc - 5ad) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^3 \right))}{6b^2} \\
&= \frac{(2bc - 5ad)\sqrt{c + dx^3}}{3b^3} + \frac{(2bc - 5ad)(c + dx^3)^{3/2}}{9b^2(bc - ad)} + \frac{a(c + dx^3)^{5/2}}{3b(bc - ad)(a + bx^3)} + \frac{((2bc - 5ad) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^3 \right))}{6b^2} \\
&= \frac{(2bc - 5ad)\sqrt{c + dx^3}}{3b^3} + \frac{(2bc - 5ad)(c + dx^3)^{3/2}}{9b^2(bc - ad)} + \frac{a(c + dx^3)^{5/2}}{3b(bc - ad)(a + bx^3)} - \frac{((2bc - 5ad) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^3 \right))}{6b^2}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 125, normalized size = 0.77

$$\frac{\sqrt{c + dx^3} (-15a^2d + ab(11c - 10dx^3) + 2b^2x^3(4c + dx^3))}{9b^3(a + bx^3)} - \frac{(2bc - 5ad)\sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2, x]

[Out] (Sqrt[c + d*x^3]*(-15*a^2*d + a*b*(11*c - 10*d*x^3) + 2*b^2*x^3*(4*c + d*x^3)))/(9*b^3*(a + b*x^3)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))

IntegrateAlgebraic [A] time = 0.29, size = 150, normalized size = 0.92

$$\frac{(-5a^2d^2 + 7abcd - 2b^2c^2) \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad} \right)}{3b^{7/2}\sqrt{ad-bc}} + \frac{\sqrt{c + dx^3} (-15a^2d + 11abc - 10abdx^3 + 8b^2cx^3 + 2b^2dx^6)}{9b^3(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]

[Out] (Sqrt[c + d*x^3]*(11*a*b*c - 15*a^2*d + 8*b^2*c*x^3 - 10*a*b*d*x^3 + 2*b^2*d*x^6))/(9*b^3*(a + b*x^3)) + ((-2*b^2*c^2 + 7*a*b*c*d - 5*a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(7/2)*Sqrt[-(b*c) + a*d])

fricas [A] time = 1.10, size = 314, normalized size = 1.93

$$\frac{3 \left((2b^2c - 5abd)x^3 + 2abc - 5a^2d \right) \sqrt{\frac{bc-ad}{b}} \log\left(\frac{bdx^3 + 2bc - ad + 2\sqrt{d^3+c} \sqrt{\frac{bc-ad}{b}}}{bx^3+a} \right) - 2(2b^2dx^6 + 2(4b^2c - 5abd)x^3 + 11abc - 15a^2d)\sqrt{dx^3+c} - 3 \left((2b^2c - 5abd)x^3 + 2abc - 5a^2d \right) \sqrt{\frac{bc-ad}{b}} \arctan\left(\frac{\sqrt{d^3+c} \sqrt{\frac{bc-ad}{b}}}{bc-ad} \right) - (2b^2dx^6 + 2(4b^2c - 5abd)x^3 + 11abc - 15a^2d)\sqrt{dx^3+c}}{18(b^4x^3 + ab^3) \sqrt{\frac{bc-ad}{b}} \sqrt{dx^3+c} + 9(b^4x^3 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [-1/18*(3*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 11*a*b*c - 15*a^2*d)*sqrt(d*x^3 + c)/(b^4*x^3 + a*b^3), -1/9*(3*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 11*a*b*c - 15*a^2*d)*sqrt(d*x^3 + c)/(b^4*x^3 + a*b^3)]

giac [A] time = 0.19, size = 173, normalized size = 1.06

$$\frac{(2b^2c^2 - 7abcd + 5a^2d^2) \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right) + \frac{\sqrt{dx^3+c}abcd - \sqrt{dx^3+c}a^2d^2}{3((dx^3+c)b - bc + ad)b^3} + \frac{2\left((dx^3+c)^{\frac{3}{2}}b^4 + 3\sqrt{dx^3+c}b^4c - 6\sqrt{dx^3+c}ab^3d\right)}{9b^6}}{3\sqrt{-b^2c+abd}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/3*(2*b^2*c^2 - 7*a*b*c*d + 5*a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/3*(sqrt(d*x^3 + c)*a*b*c*d - sqrt(d*x^3 + c)*a^2*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^3) + 2/9*((d*x^3 + c)^(3/2)*b^4 + 3*sqrt(d*x^3 + c)*b^4*c - 6*sqrt(d*x^3 + c)*a*b^3*d)/b^6

maple [C] time = 0.27, size = 983, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)


```
[Out] 1/b*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+
c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)
*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-
c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
)-a/b*(1/3*(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)/b^2+2/3*(d*x^3+c)^(1/2)/b^2*
d+1/2*I/d/b^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(
-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*
_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(
1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d
^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
,1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(
2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
,_alpha=RootOf(_Z^3*b+a))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
details)Is a*d-b*c positive or negative?
```

mupad [B] time = 7.38, size = 229, normalized size = 1.40

$$\frac{2dx^3\sqrt{dx^3+c}}{9b^2} - \frac{\sqrt{dx^3+c}\left(\frac{2d(ad-2bc)}{b^3} + \frac{2ad^2}{b^3} + \frac{4cd}{3b^2}\right)}{3d} + \frac{a\left(\frac{2b^2c^2}{3(2b^2c-2abd)} + \frac{a\left(\frac{2a^2d^2}{3(2b^2c-2abd)} - \frac{4bcd}{3(2b^2c-2abd)}\right)}{b}\right)\sqrt{dx^3+c}}{b(bx^3+a)} + \frac{\ln\left(\frac{2bc-ad+bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}}{bx^3+a}\right)\sqrt{ad-bc}}{6b^{7/2}}(5ad-2bc)11$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*d*x^3*(c + d*x^3)^(1/2))/(9*b^2) - ((c + d*x^3)^(1/2)*((2*d*(a*d - 2*b*c
))/b^3 + (2*a*d^2)/b^3 + (4*c*d)/(3*b^2)))/(3*d) + (log((2*b*c - a*d + b^(1
/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - b
*c)^(1/2)*(5*a*d - 2*b*c)*1i)/(6*b^(7/2)) + (a*((2*b*c^2)/(3*(2*b^2*c - 2*a
*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*
a*b*d))))/b)*(c + d*x^3)^(1/2))/(b*(a + b*x^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

$$3.308 \quad \int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2} dx$$

Optimal. Leaf size=94

$$-\frac{d\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}} - \frac{(c+dx^3)^{3/2}}{3b(a+bx^3)} + \frac{d\sqrt{c+dx^3}}{b^2}$$

Rubi [A] time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {444, 47, 50, 63, 208}

$$-\frac{d\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}} - \frac{(c+dx^3)^{3/2}}{3b(a+bx^3)} + \frac{d\sqrt{c+dx^3}}{b^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]

[Out] (d*Sqrt[c + d*x^3])/b^2 - (c + d*x^3)^(3/2)/(3*b*(a + b*x^3)) - (d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(5/2)

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (c + dx^3)^{3/2}}{(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{(a + bx)^2} dx, x, x^3 \right) \\
&= -\frac{(c + dx^3)^{3/2}}{3b(a + bx^3)} + \frac{d \text{Subst} \left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^3 \right)}{2b} \\
&= \frac{d\sqrt{c + dx^3}}{b^2} - \frac{(c + dx^3)^{3/2}}{3b(a + bx^3)} + \frac{(d(bc - ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{2b^2} \\
&= \frac{d\sqrt{c + dx^3}}{b^2} - \frac{(c + dx^3)^{3/2}}{3b(a + bx^3)} + \frac{(bc - ad) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{b^2} \\
&= \frac{d\sqrt{c + dx^3}}{b^2} - \frac{(c + dx^3)^{3/2}}{3b(a + bx^3)} - \frac{d\sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.57

$$\frac{2d(c + dx^3)^{5/2} {}_2F_1\left(2, \frac{5}{2}; \frac{7}{2}; -\frac{b(dx^3+c)}{ad-bc}\right)}{15(ad - bc)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]

[Out] (2*d*(c + d*x^3)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*(c + d*x^3))/(-(b*c) + a*d))]/(15*(-(b*c) + a*d)^2)

IntegrateAlgebraic [A] time = 0.19, size = 103, normalized size = 1.10

$$\frac{d\sqrt{ad - bc} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{b^{5/2}} + \frac{\sqrt{c + dx^3} (3ad - bc + 2bdx^3)}{3b^2 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]

[Out] (Sqrt[c + d*x^3]*(-(b*c) + 3*a*d + 2*b*d*x^3))/(3*b^2*(a + b*x^3)) + (d*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/b^(5/2)

fricas [A] time = 0.90, size = 234, normalized size = 2.49

$$\left[\frac{3(bdx^3 + ad)\sqrt{\frac{bc-ad}{b}} \log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right) + 2(2bdx^3 - bc + 3ad)\sqrt{dx^3+c}}{6(b^3x^3 + ab^2)}, \frac{3(bdx^3 + ad)\sqrt{\frac{bc-ad}{b}} \arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bc-ad}\right) - (2bdx^3 - bc + 3ad)\sqrt{dx^3+c}}{3(b^3x^3 + ab^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [1/6*(3*(b*d*x^3 + a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(2*b*d*x^3 - b*c + 3*a*d)*sqrt(d*x^3 + c)/(b^3*x^3 + a*b^2), -1/3*(3*(b*d*x^3 + a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (2*b*d*x^3 - b*c + 3*a*d)*sqrt(d*x^3 + c)/(b^3*x^3 + a*b^2)]

giac [A] time = 0.17, size = 122, normalized size = 1.30

$$\frac{2\sqrt{dx^3 + c}d}{3b^2} + \frac{(bcd - ad^2) \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2} - \frac{\sqrt{dx^3 + c}bcd - \sqrt{dx^3 + c}ad^2}{3((dx^3 + c)b - bc + ad)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")

[Out] $\frac{2}{3}\sqrt{d*x^3 + c}*d/b^2 + (b*c*d - a*d^2)*\arctan(\sqrt{d*x^3 + c}*b/\sqrt{-b^2*c + a*b*d})/(\sqrt{-b^2*c + a*b*d}*b^2) - 1/3*(\sqrt{d*x^3 + c}*b*c*d - \sqrt{d*x^3 + c}*a*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^2)$

maple [C] time = 0.27, size = 466, normalized size = 4.96

$$\frac{2\sqrt{d^3+c}d}{3b^2} + \frac{(b^2cd - ad^2)\arctan\left(\frac{b\sqrt{d^3+c}}{\sqrt{-b^2c+abd}}\right)}{b^2\sqrt{-b^2c+abd}} - \frac{(d^2cd - ad^3)\sqrt{d^3+c}}{3b^2(d^2b + a)^2} + \frac{(d^2cd - ad^3)\sqrt{d^3+c}}{3b^2(d^2b + a)^2} + \frac{(d^2cd - ad^3)\sqrt{d^3+c}}{3b^2(d^2b + a)^2} + \frac{(d^2cd - ad^3)\sqrt{d^3+c}}{3b^2(d^2b + a)^2} + \frac{(d^2cd - ad^3)\sqrt{d^3+c}}{3b^2(d^2b + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)

[Out] $\frac{1}{3}(ad-bc)(d^2x^3+c)^{1/2}/(b^2x^3+a)/b^2 + \frac{2}{3}(d^2x^3+c)^{1/2}/b^2 + \frac{1}{2}I/d/b^2 + \frac{1}{2}(-cd^2)^{1/3}\sum(((-cd^2)^{1/3})^{1/2}I*(2x+(-I)^{1/2}(-cd^2)^{1/3}+(-cd^2)^{1/3})/d)/(-cd^2)^{1/3}d^{1/2}((x-(-cd^2)^{1/3}/d)/(-3(-cd^2)^{1/3}+I)^{1/2}(-cd^2)^{1/3}d)^{1/2}(-1/2I(2x+(I)^{1/2}(-cd^2)^{1/3}+(-cd^2)^{1/3})/d)/(-cd^2)^{1/3}d^{1/2}/(d^2x^3+c)^{1/2}(2\alpha^2d^2+I(-cd^2)^{1/3}3^{1/2}\alpha d-(-cd^2)^{1/3}\alpha d-I^3^{1/2}(-cd^2)^{2/3}-(-cd^2)^{2/3})\text{EllipticPi}(1/33^{1/2}(I(x+1/2(-cd^2)^{1/3}/d-1/2I^3^{1/2}(-cd^2)^{1/3})/d)3^{1/2}/(-cd^2)^{1/3}d)^{1/2}, 1/2(2I(-cd^2)^{1/3}3^{1/2}\alpha^2d+I^3^{1/2}cd-3cd-I(-cd^2)^{2/3}3^{1/2}\alpha-3(-cd^2)^{2/3}\alpha)/(ad-bc)b/d, (I^3^{1/2}(-cd^2)^{1/3})/(-3/2(-cd^2)^{1/3}/d+1/2I^3^{1/2}(-cd^2)^{1/3}/d)/d^{1/2}), \alpha = \text{RootOf}(Z^3+b+a)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more details) Is a*d-b*c positive or negative?

mupad [B] time = 7.35, size = 170, normalized size = 1.81

$$\frac{2d\sqrt{d^3+c}}{3b^2} - \frac{\left(\frac{2bc^2}{3(2b^2c-2abd)} + \frac{a\left(\frac{2ad^2}{3(2b^2c-2abd)} - \frac{4bcd}{3(2b^2c-2abd)}\right)}{b}\right)\sqrt{d^3+c}}{bx^3+a} + \frac{d \ln\left(\frac{ad-2bc-bdx^3+\sqrt{b}\sqrt{d^3+c}\sqrt{ad-bc}2i}{bx^3+a}\right)\sqrt{ad-bc}}{2b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)`

[Out]
$$\frac{2*d*(c + d*x^3)^{(1/2)}}{3*b^2} - \left(\frac{(2*b*c^2)/(3*(2*b^2*c - 2*a*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*a*b*d))))}{b} \right) * (c + d*x^3)^{(1/2)} / (a + b*x^3) + (d*\log((a*d - 2*b*c + b^{(1/2)}*(c + d*x^3)^{(1/2)}*(a*d - b*c)^{(1/2)*2i - b*d*x^3}/(a + b*x^3))*(a*d - b*c)^{(1/2)*1i}) / (2*b^{(5/2)})$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)`

[Out] Timed out

$$3.309 \quad \int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2} dx$$

Optimal. Leaf size=131

$$\frac{\sqrt{bc-ad}(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}} - \frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{\sqrt{c+dx^3}(bc-ad)}{3ab(a+bx^3)}$$

Rubi [A] time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 98, 156, 63, 208}

$$\frac{\sqrt{bc-ad}(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}} - \frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{\sqrt{c+dx^3}(bc-ad)}{3ab(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2), x]

[Out] ((b*c - a*d)*Sqrt[c + d*x^3])/(3*a*b*(a + b*x^3)) - (2*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2) + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*b^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 98

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 156

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx^3)^{3/2}}{x(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{x(a + bx)^2} dx, x, x^3 \right) \\
 &= \frac{(bc - ad)\sqrt{c + dx^3}}{3ab(a + bx^3)} + \frac{\text{Subst} \left(\int \frac{bc^2 + \frac{1}{2}d(bc + ad)x}{x(a + bx)\sqrt{c + dx}} dx, x, x^3 \right)}{3ab} \\
 &= \frac{(bc - ad)\sqrt{c + dx^3}}{3ab(a + bx^3)} + \frac{c^2 \text{Subst} \left(\int \frac{1}{x\sqrt{c + dx}} dx, x, x^3 \right)}{3a^2} - \frac{((bc - ad)(2bc + ad)) \text{Subst} \left(\int \frac{1}{(a + bx)} dx, x, x^3 \right)}{6a^2b} \\
 &= \frac{(bc - ad)\sqrt{c + dx^3}}{3ab(a + bx^3)} + \frac{(2c^2) \text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3a^2d} - \frac{((bc - ad)(2bc + ad)) \text{Subst} \left(\int \frac{1}{(a + bx)} dx, x, x^3 \right)}{6a^2b} \\
 &= \frac{(bc - ad)\sqrt{c + dx^3}}{3ab(a + bx^3)} - \frac{2c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c + dx^3}}{\sqrt{c}} \right)}{3a^2} + \frac{\sqrt{bc - ad} (2bc + ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^3}}{\sqrt{bc - ad}} \right)}{3a^2b^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.19, size = 122, normalized size = 0.93

$$\frac{\frac{\sqrt{bc-ad}(ad+2bc) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}} + \frac{a\sqrt{c+dx^3}(bc-ad)}{b(a+bx^3)} - 2c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2), x]

[Out] ((a*(b*c - a*d)*Sqrt[c + d*x^3])/(b*(a + b*x^3)) - 2*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]] + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2))/(3*a^2)

IntegrateAlgebraic [A] time = 0.31, size = 156, normalized size = 1.19

$$\frac{(-a^2d^2 - abcd + 2b^2c^2) \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^2b^{3/2}\sqrt{ad-bc}} - \frac{2c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{\sqrt{c+dx^3}(bc-ad)}{3ab(a+bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2), x]

[Out] ((b*c - a*d)*Sqrt[c + d*x^3])/(3*a*b*(a + b*x^3)) + ((2*b^2*c^2 - a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*a^2*b^(3/2)*Sqrt[-(b*c) + a*d]) - (2*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)

fricas [A] time = 1.07, size = 686, normalized size = 5.24

$$\frac{\left(\frac{\sqrt{bc-ad}(ad+2bc) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}} + \frac{a\sqrt{c+dx^3}(bc-ad)}{b(a+bx^3)} - 2c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [1/6*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(b^2*c*x^3 + a*b*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/3*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b^2*c*x^3 + a*b*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/6*(4*(b^2*c*x^3 + a*b*c)*sqrt(-c)*arc tan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)

$\sqrt{(b*c - a*d)/b} * \log((b*d*x^3 + 2*b*c - a*d + 2*\sqrt{d*x^3 + c}) * b * \sqrt{(b*c - a*d)/b}) / (b*x^3 + a) + 2*\sqrt{d*x^3 + c} * (a*b*c - a^2*d) / (a^2*b^2*x^3 + a^3*b), 1/3*((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*\sqrt{-(b*c - a*d)/b} * \arctan(-\sqrt{d*x^3 + c}) * b * \sqrt{-(b*c - a*d)/b} / (b*c - a*d) + 2*(b^2*c*x^3 + a*b*c)*\sqrt{-c} * \arctan(\sqrt{d*x^3 + c}) * \sqrt{-c}/c + \sqrt{d*x^3 + c} * (a*b*c - a^2*d) / (a^2*b^2*x^3 + a^3*b)]$

giac [A] time = 0.17, size = 155, normalized size = 1.18

$$\frac{2c^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a^2\sqrt{-c}} - \frac{(2b^2c^2 - abcd - a^2d^2) \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2b} + \frac{\sqrt{dx^3+c}bcd - \sqrt{dx^3+c}ad^2}{3((dx^3+c)b - bc + ad)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="giac")

[Out] $\frac{2}{3}c^2 \arctan(\sqrt{d*x^3 + c}/\sqrt{-c}) / (a^2\sqrt{-c}) - \frac{1}{3}(2b^2c^2 - a*b*c*d - a^2*d^2) \arctan(\sqrt{d*x^3 + c}) * b / \sqrt{-b^2*c + a*b*d} / (\sqrt{-b^2*c + a*b*d} * a^2*b) + \frac{1}{3}(\sqrt{d*x^3 + c}) * b * c * d - \sqrt{d*x^3 + c} * a * d^2 / ((d*x^3 + c) * b - b*c + a*d) * a * b$

maple [C] time = 0.27, size = 1036, normalized size = 7.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x)

[Out] $-1/a^2*b*(2/9*(d*x^3+c)^{(1/2)}/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^{(1/2)}/d+1/3*I/b^2/d^2*2^{(1/2)}*\sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, 1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)}, _alpha=RootOf(_Z^3*b+a))-1/a*b*(1/3*(a*d-b*c)*(d*x^3+c)^{(1/2)}/(b*x^3+a)/b^2+2/3*(d*x^3+c)^{(1/2)}/b^2*d+1/2*I/d/b^2*2^{(1/2)}*\sum((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}$

/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x^3+c)^(1/2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{\frac{3}{2}}}{(bx^3 + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="maxima")

[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x), x)

mupad [B] time = 9.14, size = 214, normalized size = 1.63

$$\frac{c^{3/2} \ln\left(\frac{(\sqrt{dx^3+c}-\sqrt{c})^3(\sqrt{dx^3+c}+\sqrt{c})}{x^6}\right)}{3a^2} + \frac{\sqrt{dx^3+c} \left(\frac{a \left(\frac{bd^2}{3(b^2c-abd)} - \frac{2b^2cd}{3a(b^2c-abd)} \right) + \frac{b^2c^2}{3a(b^2c-abd)}}{bx^3+a} \right)}{bx^3+a} + \frac{\ln\left(\frac{2bc-ad+bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}2i}{bx^3+a}\right) \sqrt{ad-bc} (ad+2bc) i}{6a^2b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x)

[Out] (c^(3/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6))/(3*a^2) + (((c + d*x^3)^(1/2)*((a*((b*d^2)/(3*(b^2*c - a*b*d)) - (2*b^2*c*d)/(3*a*(b^2*c - a*b*d))))/b + (b^2*c^2)/(3*a*(b^2*c - a*b*d))))/(a + b*x^3) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(a*d + 2*b*c)*i)/(6*a^2*b^(3/2)))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c)**(3/2)/x/(b*x**3+a)**2,x)

[Out] Timed out

$$3.310 \quad \int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2} dx$$

Optimal. Leaf size=170

$$\frac{\sqrt{c}(4bc-3ad) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3} - \frac{\sqrt{bc-ad}(4bc-ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{b}} - \frac{\sqrt{c+dx^3}(2bc-ad)}{3a^2(a+bx^3)} - \frac{c\sqrt{c+dx^3}}{3ax^3(a+bx^3)}$$

Rubi [A] time = 0.26, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 98, 151, 156, 63, 208}

$$-\frac{\sqrt{c+dx^3}(2bc-ad)}{3a^2(a+bx^3)} + \frac{\sqrt{c}(4bc-3ad) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3} - \frac{\sqrt{bc-ad}(4bc-ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{b}} - \frac{c\sqrt{c+dx^3}}{3ax^3(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2), x]

[Out] -((2*b*c - a*d)*Sqrt[c + d*x^3])/(3*a^2*(a + b*x^3)) - (c*Sqrt[c + d*x^3])/(3*a*x^3*(a + b*x^3)) + (Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3) - (Sqrt[b*c - a*d]*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*Sqrt[b])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 98

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^3)^{3/2}}{x^4 (a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(c + dx)^{3/2}}{x^2 (a + bx)^2} dx, x, x^3 \right) \\
&= -\frac{c\sqrt{c + dx^3}}{3ax^3 (a + bx^3)} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}c(4bc-3ad) + \frac{1}{2}d(3bc-2ad)x}{x(a+bx)^2 \sqrt{c+dx}} dx, x, x^3 \right)}{3a} \\
&= -\frac{(2bc - ad)\sqrt{c + dx^3}}{3a^2 (a + bx^3)} - \frac{c\sqrt{c + dx^3}}{3ax^3 (a + bx^3)} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}c(4bc-3ad)(bc-ad) + \frac{1}{2}d(bc-ad)(2bc-ad)x}{x(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3a^2(bc - ad)} \\
&= -\frac{(2bc - ad)\sqrt{c + dx^3}}{3a^2 (a + bx^3)} - \frac{c\sqrt{c + dx^3}}{3ax^3 (a + bx^3)} - \frac{(c(4bc - 3ad)) \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{6a^3} + \frac{((b - ad)) \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{6a^3} \\
&= -\frac{(2bc - ad)\sqrt{c + dx^3}}{3a^2 (a + bx^3)} - \frac{c\sqrt{c + dx^3}}{3ax^3 (a + bx^3)} - \frac{(c(4bc - 3ad)) \text{Subst} \left(\int \frac{1}{-\frac{c}{a} + \frac{x^2}{a}} dx, x, \sqrt{c + dx^3} \right)}{3a^3 d} \\
&= -\frac{(2bc - ad)\sqrt{c + dx^3}}{3a^2 (a + bx^3)} - \frac{c\sqrt{c + dx^3}}{3ax^3 (a + bx^3)} + \frac{\sqrt{c} (4bc - 3ad) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^3} - \frac{\sqrt{bc - ad} (4bc - ad)}{3a^3}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 142, normalized size = 0.84

$$\frac{\frac{a\sqrt{c+dx^3}(-ac+adx^3-2bcx^3)}{x^3(a+bx^3)} + \sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) - \frac{\sqrt{bc-ad}(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}}}{3a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2), x]

[Out] ((a*Sqrt[c + d*x^3]*(-a*c) - 2*b*c*x^3 + a*d*x^3))/(x^3*(a + b*x^3)) + Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]] - (Sqrt[b*c - a*d]*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b])/(3*a^3)

IntegrateAlgebraic [A] time = 0.68, size = 179, normalized size = 1.05

$$\frac{(4bc^{3/2} - 3a\sqrt{c}d)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3} + \frac{\sqrt{c+dx^3}(-ac+adx^3-2bcx^3)}{3a^2x^3(a+bx^3)} + \frac{(-a^2d^2 + 5abcd - 4b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^3\sqrt{b}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]

[Out] (Sqrt[c + d*x^3]*(-(a*c) - 2*b*c*x^3 + a*d*x^3))/(3*a^2*x^3*(a + b*x^3)) + ((-4*b^2*c^2 + 5*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)]/(3*a^3*Sqrt[b]*Sqrt[-(b*c) + a*d]) + ((4*b*c^(3/2) - 3*a*Sqrt[c]*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3)

fricas [A] time = 1.18, size = 838, normalized size = 4.93

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [-1/6*(((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1/6*(2*((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1/6*(2*((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a) + 2*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1/3*(((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3)]

giac [A] time = 0.22, size = 216, normalized size = 1.27

$$\frac{(4b^2c^2 - 5abcd + a^2d^2) \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^3} - \frac{(4bc^2 - 3acd) \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a^3\sqrt{-c}} - \frac{2(dx^3+c)^{\frac{3}{2}}bcd - 2\sqrt{dx^3+c}bc^2d - (dx^3+c)^{\frac{3}{2}}ad^2 + 2\sqrt{dx^3+c}acd^2}{3\left((dx^3+c)^2b - 2(dx^3+c)bc + bc^2 + (dx^3+c)ad - acd\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/3*(4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3) - 1/3*(4*b*c^2 - 3*a*c*d)*arctan(sqrt(c)

$(d*x^3 + c)/\sqrt{-c})/(a^3*\sqrt{-c}) - 1/3*(2*(d*x^3 + c)^{(3/2)}*b*c*d - 2*\sqrt{d*x^3 + c}*b*c^2*d - (d*x^3 + c)^{(3/2)}*a*d^2 + 2*\sqrt{d*x^3 + c}*a*c*d^2)/(((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*d)*a^2)$

maple [C] time = 0.25, size = 1093, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x)`

[Out] $1/a^2*(-1/3*(d*x^3+c)^{(1/2)}*c/x^3+2/3*(d*x^3+c)^{(1/2)}*d-c^{(1/2)}*d*\operatorname{arctanh}((d*x^3+c)^{(1/2)}/c^{(1/2)}))+2/a^3*b^2*(2/9*(d*x^3+c)^{(1/2)}/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^{(1/2)}/d+1/3*I/b^2/d^2*2^{(1/2)}*\sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)}))/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)}))/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\operatorname{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)},_alpha=\operatorname{RootOf}(_Z^3*b+a)))+1/a^2*b^2*(1/3*(a*d-b*c)*(d*x^3+c)^{(1/2)}/(b*x^3+a)/b^2+2/3*(d*x^3+c)^{(1/2)}/b^2*d+1/2*I/d/b^2*2^{(1/2)}*\sum((-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)}))/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)}/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)}))/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\operatorname{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)}/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)},1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d,(I*3^{(1/2)}*(-c*d^2)^{(1/3)}/(-3/2*(-c*d^2)^{(1/3)}/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)}/d)/d)^{(1/2)},_alpha=\operatorname{RootOf}(_Z^3*b+a)))-2*b/a^3*(2/9*(d*x^3+c)^{(1/2)}*d*x^3+8/9*(d*x^3+c)^{(1/2)}*c-2/3*c^{(3/2)}*\operatorname{arctanh}((d*x^3+c)^{(1/2)}/c^{(1/2)}))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{\frac{3}{2}}}{(bx^3 + a)^2 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")

[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^4), x)

mupad [B] time = 10.82, size = 531, normalized size = 3.12

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x)

[Out] $(c^{1/2}) \cdot \log\left(\frac{(c + d \cdot x^3)^{1/2} - c^{1/2}}{(c + d \cdot x^3)^{1/2} + c^{1/2}}\right) \cdot \frac{1}{x^6} \cdot \frac{(3 \cdot a \cdot d - 4 \cdot b \cdot c)}{(6 \cdot a^3)} - \frac{c \cdot (c + d \cdot x^3)^{1/2}}{(3 \cdot a^2 \cdot x^3)} - \frac{(c + d \cdot x^3)^{1/2} \cdot ((3 \cdot a \cdot d - 4 \cdot b \cdot c) / (2 \cdot a^2) - (a \cdot ((a \cdot ((b \cdot d^2 \cdot (a \cdot d + b \cdot c)) / (a^3 \cdot c^2) - (a \cdot ((b^2 \cdot d^3) / (2 \cdot a^3 \cdot c^2) - (b^2 \cdot d^3 \cdot (3 \cdot a \cdot d - 4 \cdot b \cdot c)) / (6 \cdot a^2 \cdot c^2 \cdot (a^2 \cdot d - a \cdot b \cdot c))) / b + (b \cdot (3 \cdot a \cdot d - 4 \cdot b \cdot c) \cdot (a^2 \cdot d^3 - b^2 \cdot c^2 \cdot d + 4 \cdot a \cdot b \cdot c \cdot d^2)) / (6 \cdot a^3 \cdot c^2 \cdot (a^2 \cdot d - a \cdot b \cdot c)))) / b - (a^2 \cdot d^3 - b^2 \cdot c^2 \cdot d + 4 \cdot a \cdot b \cdot c \cdot d^2) / (2 \cdot a^3 \cdot c^2) + (b \cdot (3 \cdot a \cdot d - 4 \cdot b \cdot c) \cdot (2 \cdot b^2 \cdot c^3 - 4 \cdot a^2 \cdot c \cdot d^2 + 2 \cdot a \cdot b \cdot c^2 \cdot d)) / (6 \cdot a^3 \cdot c^2 \cdot (a^2 \cdot d - a \cdot b \cdot c))) / b - (2 \cdot b^2 \cdot c^3 - 4 \cdot a^2 \cdot c \cdot d^2 + 2 \cdot a \cdot b \cdot c^2 \cdot d) / (2 \cdot a^3 \cdot c^2) + (b \cdot (3 \cdot a \cdot d - 4 \cdot b \cdot c)^2) / (6 \cdot a^2 \cdot (a^2 \cdot d - a \cdot b \cdot c))) / b) / (a + b \cdot x^3) + \log\left(\frac{(2 \cdot b \cdot c - a \cdot d + b^{1/2}) \cdot (c + d \cdot x^3)^{1/2} \cdot (a \cdot d - b \cdot c)^{1/2} \cdot 2i + b \cdot d \cdot x^3}{(a + b \cdot x^3)}\right) \cdot \frac{1}{(6 \cdot a^3 \cdot b^{1/2})} \cdot (a \cdot d - b \cdot c)^{1/2} \cdot (a \cdot d - 4 \cdot b \cdot c) \cdot 1i$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a)**2,x)

[Out] Timed out

$$3.311 \quad \int \frac{x^8}{(a+bx^3)^2 \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=123

$$-\frac{a^2 \sqrt{c+dx^3}}{3b^2 (a+bx^3)(bc-ad)} + \frac{a(4bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}} + \frac{2\sqrt{c+dx^3}}{3b^2 d}$$

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2 \sqrt{c+dx^3}}{3b^2 (a+bx^3)(bc-ad)} + \frac{a(4bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}} + \frac{2\sqrt{c+dx^3}}{3b^2 d}$$

Antiderivative was successfully verified.

[In] Int[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (2*Sqrt[c + d*x^3]/(3*b^2*d) - (a^2*Sqrt[c + d*x^3])/(3*b^2*(b*c - a*d)*(a + b*x^3)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 89

Int[((a_.) + (b_.)*(x_))^(2)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))

)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^8}{(a + bx^3)^2 \sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(a + bx)^2 \sqrt{c + dx}} dx, x, x^3 \right) \\
 &= -\frac{a^2 \sqrt{c + dx^3}}{3b^2(bc - ad)(a + bx^3)} + \frac{\text{Subst} \left(\int \frac{-\frac{1}{2}a(2bc - ad) + b(bc - ad)x}{(a + bx)\sqrt{c + dx}} dx, x, x^3 \right)}{3b^2(bc - ad)} \\
 &= \frac{2\sqrt{c + dx^3}}{3b^2d} - \frac{a^2 \sqrt{c + dx^3}}{3b^2(bc - ad)(a + bx^3)} - \frac{(a(4bc - 3ad)) \text{Subst} \left(\int \frac{1}{(a + bx)\sqrt{c + dx}} dx, x, x^3 \right)}{6b^2(bc - ad)} \\
 &= \frac{2\sqrt{c + dx^3}}{3b^2d} - \frac{a^2 \sqrt{c + dx^3}}{3b^2(bc - ad)(a + bx^3)} - \frac{(a(4bc - 3ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3b^2d(bc - ad)} \\
 &= \frac{2\sqrt{c + dx^3}}{3b^2d} - \frac{a^2 \sqrt{c + dx^3}}{3b^2(bc - ad)(a + bx^3)} + \frac{a(4bc - 3ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^3}}{\sqrt{bc - ad}} \right)}{3b^{5/2}(bc - ad)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.28, size = 107, normalized size = 0.87

$$\frac{1}{3} \left(\frac{\sqrt{c+dx^3} \left(\frac{a^2}{(a+bx^3)(ad-bc)} + \frac{2}{d} \right)}{b^2} + \frac{a(4bc-3ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{b^{5/2}(bc-ad)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] ((Sqrt[c + d*x^3]*(2/d + a^2/((-b*c) + a*d)*(a + b*x^3)))/b^2 + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c - a*d)^(3/2))/3

IntegrateAlgebraic [A] time = 0.31, size = 143, normalized size = 1.16

$$\frac{(3a^2d - 4abc) \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad} \right)}{3b^{5/2}(ad-bc)^{3/2}} - \frac{\sqrt{c+dx^3} (3a^2d - 2abc + 2abdx^3 - 2b^2cx^3)}{3b^2d(a+bx^3)(bc-ad)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] -1/3*(Sqrt[c + d*x^3]*(-2*a*b*c + 3*a^2*d - 2*b^2*c*x^3 + 2*a*b*d*x^3))/(b^2*d*(b*c - a*d)*(a + b*x^3)) + ((-4*a*b*c + 3*a^2*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(5/2)*(-(b*c) + a*d)^(3/2))

fricas [B] time = 1.14, size = 475, normalized size = 3.86

$$\frac{(4a^2bd - 3a^2d^2 + (4ab^2cd - 3a^2bd^2)x^2)\sqrt{bc-ad} \log\left(\frac{bx^2+2bx+a}{bx^2+2bx+a}\right) + 2(2ab^2c^2 - 5a^2b^2cd + 3a^2bd^2 + 2(b^2c^2 - 2ab^2cd + a^2b^2d^2))\sqrt{ad^3+c}}{6(ab^2cd - 2a^2b^2cd^2 + a^2b^2d^3) + (b^2cd - 2ab^2cd^2 + a^2b^2d^3)x^2} - \frac{(4a^2bd - 3a^2d^2 + (4ab^2cd - 3a^2bd^2)x^2)\sqrt{-bc+abd} \arctan\left(\frac{\sqrt{bc-ad}\sqrt{c+dx^3}}{bc-ad}\right) - (2ab^2c^2 - 5a^2b^2cd + 3a^2bd^2 + 2(b^2c^2 - 2ab^2cd + a^2b^2d^2))\sqrt{ad^3+c}}{3(ab^2cd - 2a^2b^2cd^2 + a^2b^2d^3) + (b^2cd - 2ab^2cd^2 + a^2b^2d^3)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/6*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3), -1/3*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b

$\sqrt{dx^3 + c}$)²*x³)/((a*b⁵*c²*d - 2*a²*b⁴*c*d² + a³*b³*d³ + (b⁶*c²*d - 2*a*b⁵*c*d² + a²*b⁴*d³)*x³)]

giac [A] time = 0.23, size = 134, normalized size = 1.09

$$\frac{\sqrt{dx^3 + c} a^2 d}{3 (b^3 c - a b^2 d) ((dx^3 + c) b - b c + a d)} - \frac{(4 a b c - 3 a^2 d) \arctan\left(\frac{\sqrt{dx^3 + c} b}{\sqrt{-b^2 c + a b d}}\right)}{3 (b^3 c - a b^2 d) \sqrt{-b^2 c + a b d}} + \frac{2 \sqrt{dx^3 + c}}{3 b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁸/(b*x³+a)²/(d*x³+c)^(1/2),x, algorithm="giac")

[Out] -1/3*sqrt(d*x³ + c)*a²*d/((b³*c - a*b²*d)*((d*x³ + c)*b - b*c + a*d)) - 1/3*(4*a*b*c - 3*a²*d)*arctan(sqrt(d*x³ + c)*b/sqrt(-b²*c + a*b*d))/((b³*c - a*b²*d)*sqrt(-b²*c + a*b*d)) + 2/3*sqrt(d*x³ + c)/(b²*d)

maple [C] time = 0.42, size = 911, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x⁸/(b*x³+a)²/(d*x³+c)^(1/2),x)

[Out] 2/3*(d*x³+c)^(1/2)/b²/d+2/3*I*a/b²/d²*2^(1/2)*sum(1/(a*d-b*c)*(-c*d²)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d²)^(1/3)+(-c*d²)^(1/3))/d)/(-c*d²)^(1/3)*d)^(1/2)*((x-(-c*d²)^(1/3)/d)/(-3*(-c*d²)^(1/3)+I*3^(1/2)*(-c*d²)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d²)^(1/3)+(-c*d²)^(1/3))/d)/(-c*d²)^(1/3)*d)^(1/2)/(d*x³+c)^(1/2)*(2*_alpha²*d²+I*(-c*d²)^(1/3)*3^(1/2)*_alpha*d-(-c*d²)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d²)^(2/3)-(-c*d²)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d²)^(1/3)/d-1/2*I*3^(1/2)*(-c*d²)^(1/3))/d)*3^(1/2)/(-c*d²)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d²)^(1/3)*3^(1/2)*_alpha²*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d²)^(2/3)*3^(1/2)*_alpha-3*(-c*d²)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d²)^(1/3)/(-3/2*(-c*d²)^(1/3)/d+1/2*I*3^(1/2)*(-c*d²)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z³*b+a))+a²/b²*(1/3/(a*d-b*c)*(d*x³+c)^(1/2)/(b*x³+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)²*(-c*d²)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d²)^(1/3)+(-c*d²)^(1/3))/d)/(-c*d²)^(1/3)*d)^(1/2)*((x-(-c*d²)^(1/3)/d)/(-3*(-c*d²)^(1/3)+I*3^(1/2)*(-c*d²)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d²)^(1/3)+(-c*d²)^(1/3))/d)/(-c*d²)^(1/3)*d)^(1/2)/(d*x³+c)^(1/2)*(2*_alpha²*d²+I*(-c*d²)^(1/3)*3^(1/2)*_alpha*d-(-c*d²)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d²)^(2/3)-(-c*d²)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d²)^(1/3)/d-1/2*I*3^(1/2)*(-c*d²)^(1/3))/d)*3^(1/2)/(-c*d²)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d²)^(1/3)*3^(1/2)*_alpha²*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d²)^(2/3)*3^(1/2)*_alpha-3*(-c*d²)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d²)^(1/3)/(-3/2*(-c*d²)^(1/3)/d+1/2*I*3^(1/2)*(-c*d²)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z³*b+a)))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 7.29, size = 160, normalized size = 1.30

$$\frac{2\sqrt{dx^3+c}(2b^2c-2abd)}{3d(2b^4c-2ab^3d)} - \frac{2a^2\sqrt{dx^3+c}}{3b(bx^3+a)(2b^2c-2abd)} + \frac{a \ln\left(\frac{ad-2bc-bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}}{bx^3+a}\right)}{6b^{5/2}(ad-bc)^{3/2}} (3ad-4bc) \operatorname{li}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)

[Out] (2*(c + d*x^3)^(1/2)*(2*b^2*c - 2*a*b*d))/(3*d*(2*b^4*c - 2*a*b^3*d)) - (2*a^2*(c + d*x^3)^(1/2))/(3*b*(a + b*x^3)*(2*b^2*c - 2*a*b*d)) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 4*b*c)*1i)/(6*b^(5/2)*(a*d - b*c)^(3/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)

[Out] Timed out

$$3.312 \quad \int \frac{x^5}{(a+bx^3)^2 \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^3}}{3b(a+bx^3)(bc-ad)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^3}}{3b(a+bx^3)(bc-ad)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (a*Sqrt[c + d*x^3])/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)^(n_))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{(a + bx^3)^2 \sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{(a + bx)^2 \sqrt{c + dx}} dx, x, x^3 \right) \\
 &= \frac{a\sqrt{c + dx^3}}{3b(bc - ad)(a + bx^3)} + \frac{(2bc - ad) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{6b(bc - ad)} \\
 &= \frac{a\sqrt{c + dx^3}}{3b(bc - ad)(a + bx^3)} + \frac{(2bc - ad) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3bd(bc - ad)} \\
 &= \frac{a\sqrt{c + dx^3}}{3b(bc - ad)(a + bx^3)} - \frac{(2bc - ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^3}}{\sqrt{bc - ad}} \right)}{3b^{3/2}(bc - ad)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^3}}{(a+bx^3)(bc-ad)} + \frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{3b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] ((a*Sqrt[b]*Sqrt[c + d*x^3])/((b*c - a*d)*(a + b*x^3)) + ((-2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(3*b^(3/2))

IntegrateAlgebraic [A] time = 0.16, size = 109, normalized size = 1.10

$$\frac{(2bc - ad) \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3} \sqrt{ad-bc}}{bc-ad} \right)}{3b^{3/2}(ad - bc)^{3/2}} + \frac{a\sqrt{c + dx^3}}{3b(a + bx^3)(bc - ad)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (a*Sqrt[c + d*x^3])/(3*b*(b*c - a*d)*(a + b*x^3)) + ((2*b*c - a*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)]/(3*b^(3/2)*(-(b*c) + a*d)^(3/2))

fricas [A] time = 1.11, size = 348, normalized size = 3.52

$$\left[\frac{\left((2b^2c - abd)x^3 + 2abc - a^2d \right) \sqrt{b^2c - abd} \log \left(\frac{bdx^3 + 2bc - ad - 2\sqrt{dx^3+c} \sqrt{b^2c - abd}}{bx^3 + a} \right) + 2(ab^2c - a^2bd) \sqrt{dx^3 + c} + ((2b^2c - abd)x^3 + 2abc - a^2d) \sqrt{-b^2c + abd} \arctan \left(\frac{\sqrt{dx^3+c} \sqrt{-b^2c + abd}}{bdx^3 + bc} \right) + (ab^2c - a^2bd) \sqrt{dx^3 + c}}{6(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2 + (b^5c^2 - 2ab^4cd + a^2b^3d^2)x^3)}, \frac{3(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2 + (b^5c^2 - 2ab^4cd + a^2b^3d^2)x^3)}{3(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2 + (b^5c^2 - 2ab^4cd + a^2b^3d^2)x^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^3), 1/3*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^3)]

giac [A] time = 0.17, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^3+c} ad^2}{(b^2c-abd)((dx^3+c)b-bc+ad)} + \frac{(2bcd-ad^2) \arctan\left(\frac{\sqrt{dx^3+c} b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] 1/3*(sqrt(d*x^3 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^3 + c)*b - b*c + a*d)) + (2*b*c*d - a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sqrt(-b^2*c + a*b*d))/d

maple [C] time = 0.27, size = 892, normalized size = 9.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^5/(b*x^3+a)^2/(d*x^3+c)^{(1/2)}, x)$

[Out]
$$-1/3*I/b/d^2*2^{(1/2)}*\text{sum}(1/(a*d-b*c)*(-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)})/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)})/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)})/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, 1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)})/(-3/2*(-c*d^2)^{(1/3)})/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)})/d)^{(1/2)}), _alpha=\text{RootOf}(_Z^3*b+a))-a/b*(1/3/(a*d-b*c)*(d*x^3+c)^{(1/2)}/(b*x^3+a)-1/6*I/d*2^{(1/2)}*\text{sum}(1/(a*d-b*c)^2*(-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)})/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)})/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)})/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, 1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)})/(-3/2*(-c*d^2)^{(1/3)})/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)})/d)^{(1/2)}), _alpha=\text{RootOf}(_Z^3*b+a)))$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^5/(b*x^3+a)^2/(d*x^3+c)^{(1/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 6.85, size = 111, normalized size = 1.12

$$\frac{2a\sqrt{dx^3+c}}{3(bx^3+a)(2b^2c-2abd)} + \frac{\ln\left(\frac{2bc-ad+bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}2i}{bx^3+a}\right)(ad-2bc)1i}{6b^{3/2}(ad-bc)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)

[Out] (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - 2*b*c)*1i)/(6*b^(3/2)*(a*d - b*c)^(3/2)) + (2*a*(c + d*x^3)^(1/2))/(3*(a + b*x^3)*(2*b^2*c - 2*a*b*d))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)

[Out] Timed out

$$3.313 \quad \int \frac{x^2}{(a+bx^3)^2 \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=87

$$\frac{d \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3\sqrt{b} (bc-ad)^{3/2}} - \frac{\sqrt{c+dx^3}}{3(a+bx^3)(bc-ad)}$$

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {444, 51, 63, 208}

$$\frac{d \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3\sqrt{b} (bc-ad)^{3/2}} - \frac{\sqrt{c+dx^3}}{3(a+bx^3)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] -Sqrt[c + d*x^3]/(3*(b*c - a*d)*(a + b*x^3)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(3/2))

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 444

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]`

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{(a + bx^3)^2 \sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(a + bx)^2 \sqrt{c + dx}} dx, x, x^3 \right) \\
 &= -\frac{\sqrt{c + dx^3}}{3(bc - ad)(a + bx^3)} - \frac{d \text{Subst} \left(\int \frac{1}{(a + bx) \sqrt{c + dx}} dx, x, x^3 \right)}{6(bc - ad)} \\
 &= -\frac{\sqrt{c + dx^3}}{3(bc - ad)(a + bx^3)} - \frac{\text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^3} \right)}{3(bc - ad)} \\
 &= -\frac{\sqrt{c + dx^3}}{3(bc - ad)(a + bx^3)} + \frac{d \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^3}}{\sqrt{bc - ad}} \right)}{3\sqrt{b}(bc - ad)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 85, normalized size = 0.98

$$\frac{1}{3} \left(\frac{\sqrt{c + dx^3}}{(a + bx^3)(ad - bc)} + \frac{d \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^3}}{\sqrt{ad - bc}} \right)}{\sqrt{b}(ad - bc)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (Sqrt[c + d*x^3]/((-b*c) + a*d)*(a + b*x^3) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/3

IntegrateAlgebraic [A] time = 0.11, size = 97, normalized size = 1.11

$$\frac{\sqrt{c+dx^3}}{3(a+bx^3)(bc-ad)} - \frac{d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3\sqrt{b}(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] -1/3*Sqrt[c + d*x^3]/((b*c - a*d)*(a + b*x^3)) - (d*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)]/(3*Sqrt[b]*(-(b*c) + a*d)^(3/2)))

fricas [B] time = 0.89, size = 302, normalized size = 3.47

$$\left[\frac{(bdx^3 + ad)\sqrt{b^2c - abd} \log\left(\frac{bdx^3 + 2bc - ad - 2\sqrt{dx^3 + c}\sqrt{b^2c - abd}}{bx^3 + a}\right) + 2\sqrt{dx^3 + c}(b^2c - abd)}{6(ab^3c^2 - 2a^2b^2cd + a^3bd^2 + (b^4c^2 - 2ab^3cd + a^2b^2d^2)x^3)}, - \frac{(bdx^3 + ad)\sqrt{-b^2c + abd} \arctan\left(\frac{\sqrt{dx^3 + c}\sqrt{-b^2c + abd}}{bdx^3 + bc}\right) + \sqrt{dx^3 + c}(b^2c - abd)}{3(ab^3c^2 - 2a^2b^2cd + a^3bd^2 + (b^4c^2 - 2ab^3cd + a^2b^2d^2)x^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [-1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^3), -1/3*((b*d*x^3 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^3)]

giac [A] time = 0.16, size = 93, normalized size = 1.07

$$\frac{d \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^3+cd}}{3((dx^3+c)b-bc+ad)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] -1/3*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c - a*d)) - 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*(b*c - a*d))

maple [C] time = 0.26, size = 457, normalized size = 5.25

The image shows a complex mathematical expression from Maple CAS. It features multiple nested square roots, with the innermost terms involving $\sqrt[3]{-c*d^2}$. The expression is divided by a denominator $3(ad-bc)\sqrt{d^2+c}$. The numerator is a sum of several terms, including products of square roots and an elliptic function EllipticPi with arguments involving $\sqrt[3]{-c*d^2}$ and d .

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2), x)`

[Out]
$$\frac{1}{3} \frac{1}{(a*d-b*c) * (d*x^3+c)^{(1/2)} / (b*x^3+a) - 1/6 * I / d^{2*(1/2)} * \text{sum}(1 / (a*d-b*c)^{2*(1/2)} * (-c*d^2)^{(1/3)} * (1/2 * I * (2*x + (-I * 3^{(1/2)} * (-c*d^2)^{(1/3)} + (-c*d^2)^{(1/3)}) / d) / (-c*d^2)^{(1/3)} * d)^{(1/2)} * ((x - (-c*d^2)^{(1/3)} / d) / (-3 * (-c*d^2)^{(1/3)} + I * 3^{(1/2)} * (-c*d^2)^{(1/3)}) * d)^{(1/2)} * (-1/2 * I * (2*x + (I * 3^{(1/2)} * (-c*d^2)^{(1/3)} + (-c*d^2)^{(1/3)}) / d) / (-c*d^2)^{(1/3)} * d)^{(1/2)} / (d*x^3+c)^{(1/2)} * (2 * \alpha^2 * d^2 + I * (-c*d^2)^{(1/3)} * 3^{(1/2)} * \alpha * d - (-c*d^2)^{(1/3)} * \alpha * d - I * 3^{(1/2)} * (-c*d^2)^{(2/3)} - (-c*d^2)^{(2/3)}) * \text{EllipticPi}(1/3 * 3^{(1/2)} * (I * (x + 1/2 * (-c*d^2)^{(1/3)} / d - 1/2 * I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / d) * 3^{(1/2)} / (-c*d^2)^{(1/3)} * d)^{(1/2)}, 1/2 * (2 * I * (-c*d^2)^{(1/3)} * 3^{(1/2)} * \alpha^2 * d + I * 3^{(1/2)} * c * d - 3 * c * d - I * (-c*d^2)^{(2/3)} * 3^{(1/2)} * \alpha - 3 * (-c*d^2)^{(2/3)} * \alpha) / (a*d-b*c) * b / d, (I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / (-3/2 * (-c*d^2)^{(1/3)} / d + 1/2 * I * 3^{(1/2)} * (-c*d^2)^{(1/3)} / d) / d)^{(1/2)}, \alpha = \text{RootOf}(_Z^3 * b + a)}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2), x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more details)Is a*d-b*c positive or negative?

mupad [B] time = 6.37, size = 104, normalized size = 1.20

$$\frac{2b\sqrt{dx^3+c}}{3(bx^3+a)(2b^2c-2abd)} + \frac{d \ln\left(\frac{2bc-ad+bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}}{bx^3+a}\right) 1i}{6\sqrt{b}(ad-bc)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)`

[Out]
$$\frac{(d * \log((2 * b * c - a * d + b^{(1/2)} * (c + d * x^3)^{(1/2)} * (a * d - b * c)^{(1/2)} * 2i + b * d * x^3) / (a + b * x^3)) * 1i) / (6 * b^{(1/2)} * (a * d - b * c)^{(3/2)}) - (2 * b * (c + d * x^3)^{(1/2)}) / (3 * (a + b * x^3) * (2 * b^2 * c - 2 * a * b * d))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)

[Out] Timed out

$$3.314 \quad \int \frac{1}{x(a+bx^3)^2 \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=132

$$\frac{\sqrt{b}(2bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{b\sqrt{c+dx^3}}{3a(a+bx^3)(bc-ad)}$$

Rubi [A] time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}(2bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{b\sqrt{c+dx^3}}{3a(a+bx^3)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (b*Sqrt[c + d*x^3])/(3*a*(b*c - a*d)*(a + b*x^3)) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(3*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]/(3*a^2*(b*c - a*d)^(3/2)))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[(((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx^3)^2 \sqrt{c+dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(a+bx)^2 \sqrt{c+dx}} dx, x, x^3 \right) \\
&= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)(a+bx^3)} + \frac{\text{Subst} \left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3a(bc-ad)} \\
&= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)(a+bx^3)} + \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{3a^2} - \frac{(b(2bc-3ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{6a^2(bc-ad)} \\
&= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)(a+bx^3)} + \frac{2 \text{Subst} \left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}} dx, x, \sqrt{c+dx^3} \right)}{3a^2d} - \frac{(b(2bc-3ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{6a^2(bc-ad)} \\
&= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)(a+bx^3)} - \frac{2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad) \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}} \right)}{3a^2(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^3}}{(a+bx^3)(bc-ad)} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] ((a*b*Sqrt[c + d*x^3])/((b*c - a*d)*(a + b*x^3)) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(3*a^2)

IntegrateAlgebraic [A] time = 0.31, size = 146, normalized size = 1.11

$$\frac{(3a\sqrt{b}d - 2b^{3/2}c)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^2(ad-bc)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{b\sqrt{c+dx^3}}{3a(a+bx^3)(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] -1/3*(b*Sqrt[c + d*x^3])/(a*(-(b*c) + a*d)*(a + b*x^3)) + ((-2*b^(3/2)*c + 3*a*Sqrt[b]*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)]/(3*a^2*(-(b*c) + a*d)^(3/2)) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2*Sqrt[c])

fricas [A] time = 0.93, size = 862, normalized size = 6.53

1/3*(b*Sqrt[c + d*x^3])/(a*(-(b*c) + a*d)*(a + b*x^3)) + ((-2*b^(3/2)*c + 3*a*Sqrt[b]*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)]/(3*a^2*(-(b*c) + a*d)^(3/2)) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2*Sqrt[c])

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/6*(2*sqrt(d*x^3 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sqrt(d*x^3 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^3 + b*c)) + ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3)

$*b*c*d)*x^3)$, $1/6*(2*\sqrt{d*x^3 + c})*a*b*c + 4*((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*\sqrt{-c}*\arctan(\sqrt{d*x^3 + c}*\sqrt{-c}/c) + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*\sqrt{b/(b*c - a*d)}*\log((b*d*x^3 + 2*b*c - a*d + 2*\sqrt{d*x^3 + c})*(b*c - a*d)*\sqrt{b/(b*c - a*d)})/(b*x^3 + a)))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3)$, $1/3*(\sqrt{d*x^3 + c})*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*\sqrt{-b/(b*c - a*d)}*\arctan(-\sqrt{d*x^3 + c}*(b*c - a*d)*\sqrt{-b/(b*c - a*d)})/(b*d*x^3 + b*c)) + 2*((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*\sqrt{-c}*\arctan(\sqrt{d*x^3 + c}*\sqrt{-c}/c))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3)]$

giac [A] time = 0.16, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^3 + c}bd}{3(abc - a^2d)((dx^3 + c)b - bc + ad)} - \frac{(2b^2c - 3abd) \arctan\left(\frac{\sqrt{dx^3 + c}b}{\sqrt{-b^2c + abd}}\right)}{3(a^2bc - a^3d)\sqrt{-b^2c + abd}} + \frac{2 \arctan\left(\frac{\sqrt{dx^3 + c}}{\sqrt{-c}}\right)}{3a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] $1/3*\sqrt{d*x^3 + c}*b*d/((a*b*c - a^2*d)*((d*x^3 + c)*b - b*c + a*d)) - 1/3*(2*b^2*c - 3*a*b*d)*\arctan(\sqrt{d*x^3 + c}*b/\sqrt{-b^2*c + a*b*d})/((a^2*b*c - a^3*d)*\sqrt{-b^2*c + a*b*d}) + 2/3*\arctan(\sqrt{d*x^3 + c}/\sqrt{-c})/(a^2*\sqrt{-c})$

maple [C] time = 0.28, size = 915, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)

[Out] $1/3*I/a^2*b/d^2*2^(1/2)*\sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3))/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3+b+a))-1/a*b*(1/3/(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*\sum(1/(a*d-b*c)^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3))/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*$

$$I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2) / (d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2), 1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d, (I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)), _alpha=RootOf(_Z^3*b+a))-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(1/2)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^2 \sqrt{dx^3 + cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2), x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x), x)

mupad [B] time = 9.65, size = 162, normalized size = 1.23

$$\frac{\ln\left(\frac{(\sqrt{dx^3+c}-\sqrt{c})^3(\sqrt{dx^3+c}+\sqrt{c})}{x^6}\right)}{3a^2\sqrt{c}} + \frac{b^2\sqrt{dx^3+c}}{3a(bx^3+a)(b^2c-abd)} + \frac{\sqrt{b}\ln\left(\frac{ad-2bc-bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}2i}{bx^3+a}\right)(3ad-2bc)1i}{6a^2(ad-bc)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^3)^2*(c + d*x^3)^(1/2)), x)

[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3*a^2*c^(1/2)) + (b^2*(c + d*x^3)^(1/2))/(3*a*(a + b*x^3)*(b^2*c - a*b*d)) + (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 2*b*c)*1i)/(6*a^2*(a*d - b*c)^(3/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + bx^3)^2 \sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(1/2), x)

[Out] Integral(1/(x*(a + b*x**3)**2*sqrt(c + d*x**3)), x)

$$3.315 \quad \int \frac{1}{x^4(a+bx^3)^2 \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=185

$$\frac{b^{3/2}(4bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}} + \frac{(ad + 4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}} - \frac{b\sqrt{c+dx^3}(2bc-ad)}{3a^2c(a+bx^3)(bc-ad)} - \frac{\sqrt{c+dx^3}}{3acx^3(a+bx^3)}$$

Rubi [A] time = 0.24, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 103, 151, 156, 63, 208}

$$\frac{b^{3/2}(4bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}} + \frac{(ad + 4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}} - \frac{b\sqrt{c+dx^3}(2bc-ad)}{3a^2c(a+bx^3)(bc-ad)} - \frac{\sqrt{c+dx^3}}{3acx^3(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^3])/(3*a^2*c*(b*c - a*d)*(a + b*x^3)) - Sqrt[c + d*x^3]/(3*a*c*x^3*(a + b*x^3)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 151

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

```

Rule 156

```

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 446

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (a + bx^3)^2 \sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2 (a + bx)^2 \sqrt{c + dx}} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c + dx^3}}{3acx^3 (a + bx^3)} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(4bc+ad) + \frac{3bdx}{2}}{x(a+bx)^2 \sqrt{c+dx}} dx, x, x^3 \right)}{3ac} \\
&= -\frac{b(2bc - ad)\sqrt{c + dx^3}}{3a^2c(bc - ad)(a + bx^3)} - \frac{\sqrt{c + dx^3}}{3acx^3 (a + bx^3)} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad) + \frac{1}{2}bd(2bc-)}{x(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3a^2c(bc - ad)} \\
&= -\frac{b(2bc - ad)\sqrt{c + dx^3}}{3a^2c(bc - ad)(a + bx^3)} - \frac{\sqrt{c + dx^3}}{3acx^3 (a + bx^3)} + \frac{(b^2(4bc - 5ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{6a^3(bc - ad)} \\
&= -\frac{b(2bc - ad)\sqrt{c + dx^3}}{3a^2c(bc - ad)(a + bx^3)} - \frac{\sqrt{c + dx^3}}{3acx^3 (a + bx^3)} + \frac{(b^2(4bc - 5ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^3}{d}} dx, x, x^3 \right)}{3a^3d(bc - ad)} \\
&= -\frac{b(2bc - ad)\sqrt{c + dx^3}}{3a^2c(bc - ad)(a + bx^3)} - \frac{\sqrt{c + dx^3}}{3acx^3 (a + bx^3)} + \frac{(4bc + ad) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^3c^{3/2}} - \frac{b^3}{3a^3c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.61, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^3}(a^2d+ab(dx^3-c)-2b^2cx^3)}{x^3(a+bx^3)(bc-ad)} + \frac{b^{3/2}c(5ad-4bc) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{(ad+4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^3c}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]), x]

[Out] ((a*Sqrt[c + d*x^3]*(a^2*d - 2*b^2*c*x^3 + a*b*(-c + d*x^3)))/((b*c - a*d)*x^3*(a + b*x^3)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c] + (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(3*a^3*c)

IntegrateAlgebraic [A] time = 0.64, size = 187, normalized size = 1.01

$$\frac{(4b^{5/2}c - 5ab^{3/2}d) \tan^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad} \right)}{3a^3(ad-bc)^{3/2}} + \frac{(ad+4bc) \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^3c^{3/2}} + \frac{\sqrt{c+dx^3}(-a^2d+abc-abdx^3+2b^2cx^3)}{3a^2cx^3(a+bx^3)(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (Sqrt[c + d*x^3]*(a*b*c - a^2*d + 2*b^2*c*x^3 - a*b*d*x^3))/(3*a^2*c*(-(b*c) + a*d)*x^3*(a + b*x^3)) + ((4*b^(5/2)*c - 5*a*b^(3/2)*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)]/(3*a^3*(-(b*c) + a*d)^(3/2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3*c^(3/2))

fricas [A] time = 0.91, size = 1236, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/6*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^3 + b*c)) - ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/3*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^3 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3)]

giac [A] time = 0.17, size = 257, normalized size = 1.39

$$\frac{(4b^3c - 5ab^2d) \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3(a^3bc - a^4d)\sqrt{-b^2c+abd}} - \frac{2(dx^3+c)^{\frac{3}{2}}b^2cd - 2\sqrt{dx^3+c}b^2c^2d - (dx^3+c)^{\frac{3}{2}}abd^2 + 2\sqrt{dx^3+c}abcd^2 - \sqrt{dx^3+c}a^2d^3}{3(a^2bc^2 - a^3cd)\left((dx^3+c)^2b - 2(dx^3+c)bc + bc^2 + (dx^3+c)ad - acd\right)} - \frac{(4bc + ad) \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a^3\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")`

[Out] $\frac{1}{3} \cdot (4 \cdot b^3 \cdot c - 5 \cdot a \cdot b^2 \cdot d) \cdot \arctan\left(\frac{\sqrt{d \cdot x^3 + c} \cdot b}{\sqrt{-b^2 \cdot c + a \cdot b \cdot d}}\right) / \left((a^3 \cdot b \cdot c - a^4 \cdot d) \cdot \sqrt{-b^2 \cdot c + a \cdot b \cdot d} \right) - \frac{1}{3} \cdot (2 \cdot (d \cdot x^3 + c)^{(3/2)} \cdot b^2 \cdot c \cdot d - 2 \cdot \sqrt{d \cdot x^3 + c} \cdot b^2 \cdot c^2 \cdot d - (d \cdot x^3 + c)^{(3/2)} \cdot a \cdot b \cdot d^2 + 2 \cdot \sqrt{d \cdot x^3 + c} \cdot a \cdot b \cdot c \cdot d^2 - \sqrt{d \cdot x^3 + c} \cdot a^2 \cdot d^3) / \left((a^2 \cdot b \cdot c^2 - a^3 \cdot c \cdot d) \cdot ((d \cdot x^3 + c)^{2 \cdot b} - 2 \cdot (d \cdot x^3 + c) \cdot b \cdot c + b \cdot c^2 + (d \cdot x^3 + c) \cdot a \cdot d - a \cdot c \cdot d) \right) - \frac{1}{3} \cdot (4 \cdot b \cdot c + a \cdot d) \cdot \arctan\left(\frac{\sqrt{d \cdot x^3 + c}}{\sqrt{-c}}\right) / (a^3 \cdot \sqrt{-c} \cdot c)$

maple [C] time = 0.29, size = 961, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)`

[Out] $\frac{1}{a^2} \cdot \left(-\frac{1}{3} \cdot (d \cdot x^3 + c)^{(1/2)} / c \cdot x^3 + \frac{1}{3} \cdot d \cdot \operatorname{arctanh}\left(\frac{(d \cdot x^3 + c)^{(1/2)}}{c^{(1/2)}}\right) / c^{(3/2)} - \frac{2}{3} \cdot I / a^3 \cdot b^2 / d^2 \cdot 2^{(1/2)} \cdot \sum\left(\frac{1}{(a \cdot d - b \cdot c)} \cdot (-c \cdot d^2)^{(1/3)} \cdot \left(\frac{1}{2} \cdot I \cdot (2 \cdot x + (-I \cdot 3^{(1/2)} \cdot (-c \cdot d^2)^{(1/3)} + (-c \cdot d^2)^{(1/3)}) / d\right) / (-c \cdot d^2)^{(1/3)} \cdot d\right)^{(1/2)} \cdot \left(\frac{x - (-c \cdot d^2)^{(1/3)} / d}{(-3 \cdot (-c \cdot d^2)^{(1/3)} + I \cdot 3^{(1/2)} \cdot (-c \cdot d^2)^{(1/3)}) \cdot d}\right)^{(1/2)} \cdot \left(-\frac{1}{2} \cdot I \cdot (2 \cdot x + (I \cdot 3^{(1/2)} \cdot (-c \cdot d^2)^{(1/3)} + (-c \cdot d^2)^{(1/3)}) / d) / (-c \cdot d^2)^{(1/3)} \cdot d\right)^{(1/2)} / (d \cdot x^3 + c)^{(1/2)} \cdot (2 \cdot \alpha^2 \cdot d^2 + I \cdot (-c \cdot d^2)^{(1/3)} \cdot 3^{(1/2)} \cdot \alpha \cdot d - (-c \cdot d^2)^{(1/3)} \cdot \alpha \cdot d - I \cdot 3^{(1/2)} \cdot (-c \cdot d^2)^{(2/3)} - (-c \cdot d^2)^{(2/3)}) \cdot \operatorname{EllipticPi}\left(\frac{1}{3} \cdot 3^{(1/2)} \cdot (I \cdot (x + 1/2 \cdot (-c \cdot d^2)^{(1/3)} / d - 1/2 \cdot I \cdot 3^{(1/2)} \cdot (-c \cdot d^2)^{(1/3)} / d) \cdot 3^{(1/2)} / (-c \cdot d^2)^{(1/3)} \cdot d\right)^{(1/2)}, \frac{1}{2} \cdot (2 \cdot I \cdot (-c \cdot d^2)^{(1/3)} \cdot 3^{(1/2)} \cdot \alpha^2 \cdot d + I \cdot 3^{(1/2)} \cdot c \cdot d - 3 \cdot c \cdot d - I \cdot (-c \cdot d^2)^{(2/3)} \cdot 3^{(1/2)} \cdot \alpha - 3 \cdot (-c \cdot d^2)^{(2/3)} \cdot \alpha) / (a \cdot d - b \cdot c) \cdot b / d, (I \cdot 3^{(1/2)} \cdot (-c \cdot d^2)^{(1/3)} / (-3/2 \cdot (-c \cdot d^2)^{(1/3)} / d + 1/2 \cdot I \cdot 3^{(1/2)} \cdot (-c \cdot d^2)^{(1/3)} / d) / d\right)^{(1/2)}, \alpha = \operatorname{RootOf}(_Z^3 \cdot b + a) \right) + \frac{1}{a^2} \cdot b^2 \cdot \left(\frac{1}{3} / (a \cdot d - b \cdot c) \cdot (d \cdot x^3 + c)^{(1/2)} / (b \cdot x^3 + a) - \frac{1}{6} \cdot I / d^2 \cdot 2^{(1/2)} \cdot \sum\left(\frac{1}{(a \cdot d - b \cdot c)} \cdot (-c \cdot d^2)^{(1/3)} \cdot \left(\frac{1}{2} \cdot I \cdot (2 \cdot x + (-I \cdot 3^{(1/2)} \cdot (-c \cdot d^2)^{(1/3)} + (-c \cdot d^2)^{(1/3)}) / d\right) / (-c \cdot d^2)^{(1/3)} \cdot d\right)^{(1/2)} \cdot \left(\frac{x - (-c \cdot d^2)^{(1/3)} / d}{(-3 \cdot (-c \cdot d^2)^{(1/3)} + I \cdot 3^{(1/2)} \cdot (-c \cdot d^2)^{(1/3)}) \cdot d}\right)^{(1/2)} \cdot \left(-\frac{1}{2} \cdot I \cdot (2 \cdot x + (I \cdot 3^{(1/2)} \cdot (-c \cdot d^2)^{(1/3)} + (-c \cdot d^2)^{(1/3)}) / d) / (-c \cdot d^2)^{(1/3)} \cdot d\right)^{(1/2)} / (d \cdot x^3 + c)^{(1/2)} \cdot (2 \cdot \alpha^2 \cdot d^2 + I \cdot (-c \cdot d^2)^{(1/3)} \cdot 3^{(1/2)} \cdot \alpha \cdot d - (-c \cdot d^2)^{(1/3)} \cdot \alpha \cdot d - I \cdot 3^{(1/2)} \cdot (-c \cdot d^2)^{(2/3)} - (-c \cdot d^2)^{(2/3)}) \cdot \operatorname{EllipticPi}\left(\frac{1}{3} \cdot 3^{(1/2)} \cdot (I \cdot (x + 1/2 \cdot (-c \cdot d^2)^{(1/3)} / d - 1/2 \cdot I \cdot 3^{(1/2)} \cdot (-c \cdot d^2)^{(1/3)} / d) \cdot 3^{(1/2)} / (-c \cdot d^2)^{(1/3)} \cdot d\right)^{(1/2)}, \frac{1}{2} \cdot (2 \cdot I \cdot (-c \cdot d^2)^{(1/3)} \cdot 3^{(1/2)} \cdot \alpha^2 \cdot d + I \cdot 3^{(1/2)} \cdot c \cdot d - 3 \cdot c \cdot d - I \cdot (-c \cdot d^2)^{(2/3)} \cdot 3^{(1/2)} \cdot \alpha - 3 \cdot (-c \cdot d^2)^{(2/3)} \cdot \alpha) / (a \cdot d - b \cdot c) \cdot b / d, (I \cdot 3^{(1/2)} \cdot (-c \cdot d^2)^{(1/3)} / (-3/2 \cdot (-c \cdot d^2)^{(1/3)} / d + 1/2 \cdot I \cdot 3^{(1/2)} \cdot (-c \cdot d^2)^{(1/3)} / d) / d\right)^{(1/2)}, \alpha = \operatorname{RootOf}(_Z^3 \cdot b + a) \right) + \frac{4}{3} \cdot b / a^3 \cdot \operatorname{arctanh}\left(\frac{(d \cdot x^3 + c)^{(1/2)}}{c^{(1/2)}}\right) / c^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^2 \sqrt{dx^3 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c))*x^4), x)

mupad [B] time = 11.55, size = 355, normalized size = 1.92

$$\frac{\sqrt{dx^3+c} \left(\frac{d^2+4bca}{2a^3c^2} - \frac{a \left(\frac{2c^2+2adb}{2a^3c^2} - \frac{d \left(\frac{d^2}{2a^3c^2} + \frac{b(2c^2-2adb)(3ad-4bc)}{6a^3c^2(dx+bc)} \right) - \frac{d^2d(3ad-4bc)}{6a^3c^2(dx+bc)} \right)}{b} \right)}{bx^3+a} - \frac{\sqrt{dx^3+c}}{3a^2cx^3} + \frac{\ln \left(\frac{(\sqrt{dx^3+c}-\sqrt{c})(\sqrt{dx^3+c}+\sqrt{c})^3}{x^6} \right) (ad+4bc)}{6a^3c^{3/2}} + \frac{b^{3/2} \ln \left(\frac{2bc-ad+bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}}{bx^3+a} \right) (5ad-4bc)}{6a^3(ad-bc)^{3/2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)

[Out] ((c + d*x^3)^(1/2)*((a^2*d + 4*a*b*c)/(2*a^3*c^2) - (a*((2*b^2*c + 2*a*b*d)/(2*a^3*c^2) - (a*((b^2*d)/(2*a^3*c^2) + (b*(2*b^2*c + 2*a*b*d)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^2*d - a*b*c)) - (b^2*d*(3*a*d - 4*b*c))/(6*a^2*c^2*(a^2*d - a*b*c)))))/b + (b*(a^2*d + 4*a*b*c)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^2*d - a*b*c)))/b)/(a + b*x^3) - (c + d*x^3)^(1/2)/(3*a^2*c*x^3) + (log((((c + d*x^3)^(1/2) - c^(1/2))*(c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(a*d + 4*b*c))/(6*a^3*c^(3/2)) + (b^(3/2)*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d - 4*b*c)*1i)/(6*a^3*(a*d - b*c)^(3/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + bx^3)^2 \sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)

[Out] Integral(1/(x**4*(a + b*x**3)**2*sqrt(c + d*x**3)), x)

$$3.316 \quad \int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=150

$$\frac{-a^2d^2 - 2b^2c^2}{3b^2d\sqrt{c+dx^3}(bc-ad)^2} - \frac{a^2}{3b^2(a+bx^3)\sqrt{c+dx^3}(bc-ad)} + \frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

Rubi [A] time = 0.19, antiderivative size = 149, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 89, 78, 63, 208}

$$-\frac{a^2d^2 + 2b^2c^2}{3b^2d\sqrt{c+dx^3}(bc-ad)^2} - \frac{a^2}{3b^2(a+bx^3)\sqrt{c+dx^3}(bc-ad)} + \frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] -(2*b^2*c^2 + a^2*d^2)/(3*b^2*d*(b*c - a*d)^2*Sqrt[c + d*x^3]) - a^2/(3*b^2*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) + (a*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(5/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^(n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 89

```
Int[((a_.) + (b_.)*(x_))^(2*((c_.) + (d_.)*(x_))^(n_.))*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^8}{(a + bx^3)^2 (c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(a + bx)^2 (c + dx)^{3/2}} dx, x, x^3 \right) \\
 &= -\frac{a^2}{3b^2(bc - ad)(a + bx^3)\sqrt{c + dx^3}} + \frac{\text{Subst} \left(\int \frac{-\frac{1}{2}a(2bc + ad) + b(bc - ad)x}{(a + bx)(c + dx)^{3/2}} dx, x, x^3 \right)}{3b^2(bc - ad)} \\
 &= -\frac{2b^2c^2 + a^2d^2}{3b^2d(bc - ad)^2\sqrt{c + dx^3}} - \frac{a^2}{3b^2(bc - ad)(a + bx^3)\sqrt{c + dx^3}} - \frac{(a(4bc - ad)) \text{Sul}}{6} \\
 &= -\frac{2b^2c^2 + a^2d^2}{3b^2d(bc - ad)^2\sqrt{c + dx^3}} - \frac{a^2}{3b^2(bc - ad)(a + bx^3)\sqrt{c + dx^3}} - \frac{(a(4bc - ad)) \text{Sul}}{6} \\
 &= -\frac{2b^2c^2 + a^2d^2}{3b^2d(bc - ad)^2\sqrt{c + dx^3}} - \frac{a^2}{3b^2(bc - ad)(a + bx^3)\sqrt{c + dx^3}} + \frac{a(4bc - ad) \tanh}{3b^{3/2}(bc - ad)\sqrt{c + dx^3}}
 \end{aligned}$$

Mathematica [A] time = 0.30, size = 134, normalized size = 0.89

$$\frac{a(4bc-ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}} - \frac{\sqrt{b}(a^2d(c+dx^3)+2abc^2+2b^2c^2x^3)}{d(a+bx^3)\sqrt{c+dx^3}(bc-ad)^2}}{3b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x]

[Out] (-((Sqrt[b]*(2*a*b*c^2 + 2*b^2*c^2*x^3 + a^2*d*(c + d*x^3)))/(d*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3])) + (a*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2))/(3*b^(3/2))

IntegrateAlgebraic [A] time = 0.25, size = 151, normalized size = 1.01

$$\frac{(4abc - a^2d) \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{3/2}(ad - bc)^{5/2}} + \frac{-a^2cd - a^2d^2x^3 - 2abc^2 - 2b^2c^2x^3}{3bd(a + bx^3)\sqrt{c + dx^3}(bc - ad)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x]

[Out] (-2*a*b*c^2 - a^2*c*d - 2*b^2*c^2*x^3 - a^2*d^2*x^3)/(3*b*d*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3]) + ((4*a*b*c - a^2*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(3/2)*(-(b*c) + a*d)^(5/2))

fricas [B] time = 0.92, size = 746, normalized size = 4.97

$$\frac{\left(\frac{(4abc - a^2d) \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{3/2}(ad - bc)^{5/2}} + \frac{-a^2cd - a^2d^2x^3 - 2abc^2 - 2b^2c^2x^3}{3bd(a + bx^3)\sqrt{c + dx^3}(bc - ad)^2}\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2), x, algorithm="fricas")

[Out] [-1/6*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(2*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*a*b^3*c^2*d + a^2*b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*d - 3*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 - 3*a*b^5*c^2*d^3 + 3*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b^5*c^3*d^2 + 2*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^3), -1/3*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(b^2*c - a*b*d) + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(b^2*c - a*b*d)]

$$\begin{aligned}
 & -b^2c + a*b*d) * \arctan(\sqrt{d*x^3 + c} * \sqrt{-b^2c + a*b*d}) / (b*d*x^3 + b*c) \\
 &) + (2*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*a*b^3*c^2*d \\
 & + a^2*b^2*c*d^2 - a^3*b*d^3) * x^3) * \sqrt{d*x^3 + c} / (a*b^5*c^4*d - 3*a^2*b^4 \\
 & 4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 - 3*a*b^5*c^2* \\
 & d^3 + 3*a^2*b^4*c*d^4 - a^3*b^3*d^5) * x^6 + (b^6*c^4*d - 2*a*b^5*c^3*d^2 + 2 \\
 & *a^3*b^3*c*d^4 - a^4*b^2*d^5) * x^3]
 \end{aligned}$$

giac [A] time = 0.19, size = 195, normalized size = 1.30

$$\frac{(4abc - a^2d) \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3(b^3c^2 - 2ab^2cd + a^2bd^2)\sqrt{-b^2c + abd}} - \frac{2(dx^3 + c)b^2c^2 - 2b^2c^3 + 2abc^2d + (dx^3 + c)a^2d^2}{3(b^3c^2d - 2ab^2cd^2 + a^2bd^3)\left((dx^3 + c)^{\frac{3}{2}}b - \sqrt{dx^3 + c}bc + \sqrt{dx^3 + c}ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] -1/3*(4*a*b*c - a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3 + c)*b^2*c^2 - 2*b^2*c^3 + 2*a*b*c^2*d + (d*x^3 + c)*a^2*d^2)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + sqrt(d*x^3 + c)*a*d))

maple [C] time = 0.36, size = 978, normalized size = 6.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)

[Out] -2/3/b^2/d/(d*x^3+c)^(1/2)-2*a/b^2*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3))/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*I*(x+1/2*(-c*d^2)^(1/3))/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+a^2/b^2*(-1/3*b/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*d/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)+1/2*I*b/d*2^(1/2)*sum(1/(a*d-b*c)^3*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3))/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)

$$2)^{(1/3)+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)*d)^{(1/2)/(d*x^3+c)^{(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)*3^{(1/2)*_alpha*d-(-c*d^2)^{(1/3)*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)-(-c*d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)*(I*(x+1/2*(-c*d^2)^{(1/3)/d-1/2*I*3^{(1/2)*(-c*d^2)^{(1/3)/d)*3^{(1/2)/(-c*d^2)^{(1/3)*d)^{(1/2)},1/2*(2*I*(-c*d^2)^{(1/3)*3^{(1/2)*_alpha^2*d+I*3^{(1/2)*c*d-3*c*d-I*(-c*d^2)^{(2/3)*3^{(1/2)*_alpha-3*(-c*d^2)^{(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^{(1/2)*(-c*d^2)^{(1/3)/(-3/2*(-c*d^2)^{(1/3)/d+1/2*I*3^{(1/2)*(-c*d^2)^{(1/3)/d)/d)^{(1/2)}),_alpha=RootOf(_Z^3*b+a))}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 7.90, size = 367, normalized size = 2.45

$$\frac{\sqrt{dx^3+c} \left(x^3 \left(\frac{\frac{3bd(ad+b)c-bd(ad+2bc)}{3(a^2b^3-2a^2c^2+b^3c^2d)} \frac{bd(ad+b)c}{a^2b^3-2a^2c^2+b^3c^2d} \right)^{(ad+b)c} + \frac{abcd}{a^2b^3-2a^2c^2+b^3c^2d} \right) + \frac{ac \left(\frac{3bd(ad+b)c-bd(ad+2bc)}{3(a^2b^3-2a^2c^2+b^3c^2d)} \frac{bd(ad+b)c}{a^2b^3-2a^2c^2+b^3c^2d} \right)}{bd} \right)}{bdx^6+(ad+bc)x^3+ac} + \frac{a \ln \left(\frac{2bc-ad+bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}}{bx^3+a} \right)}{6b^{3/2}(ad-bc)^{5/2}} (ad-4bc) 1i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)

[Out] ((c + d*x^3)^(1/2)*(x^3*(((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2))*(a*d + b*c))/(b*d) + (a*b*c*d)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (a*c*((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)))/(b*d)))/(a*c + x^3*(a*d + b*c) + b*d*x^6) + (a*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - 4*b*c)*1i)/(6*b^(3/2)*(a*d - b*c)^(5/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)

[Out] Timed out

$$3.317 \quad \int \frac{x^5}{(a+bx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=134

$$\frac{a}{3b(a+bx^3)\sqrt{c+dx^3}(bc-ad)} + \frac{ad+2bc}{3b\sqrt{c+dx^3}(bc-ad)^2} - \frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{5/2}}$$

Rubi [A] time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 78, 51, 63, 208}

$$\frac{a}{3b(a+bx^3)\sqrt{c+dx^3}(bc-ad)} + \frac{ad+2bc}{3b\sqrt{c+dx^3}(bc-ad)^2} - \frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*b*c + a*d)/(3*b*(b*c - a*d)^2*Sqrt[c + d*x^3]) + a/(3*b*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) - ((2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{(a + bx^3)^2 (c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{(a + bx)^2 (c + dx)^{3/2}} dx, x, x^3 \right) \\
 &= \frac{a}{3b(bc - ad)(a + bx^3)\sqrt{c + dx^3}} + \frac{(2bc + ad) \text{Subst} \left(\int \frac{1}{(a + bx)(c + dx)^{3/2}} dx, x, x^3 \right)}{6b(bc - ad)} \\
 &= \frac{2bc + ad}{3b(bc - ad)^2 \sqrt{c + dx^3}} + \frac{a}{3b(bc - ad)(a + bx^3)\sqrt{c + dx^3}} + \frac{(2bc + ad) \text{Subst} \left(\int \frac{1}{(a + bx)(c + dx)^{3/2}} dx, x, x^3 \right)}{6b(bc - ad)} \\
 &= \frac{2bc + ad}{3b(bc - ad)^2 \sqrt{c + dx^3}} + \frac{a}{3b(bc - ad)(a + bx^3)\sqrt{c + dx^3}} + \frac{(2bc + ad) \text{Subst} \left(\int \frac{1}{(a + bx)(c + dx)^{3/2}} dx, x, x^3 \right)}{3d} \\
 &= \frac{2bc + ad}{3b(bc - ad)^2 \sqrt{c + dx^3}} + \frac{a}{3b(bc - ad)(a + bx^3)\sqrt{c + dx^3}} - \frac{(2bc + ad) \tanh^{-1} \left(\frac{x}{\sqrt{c + dx^3}} \right)}{3\sqrt{b}(bc - ad)}
 \end{aligned}$$

Mathematica [C] time = 0.03, size = 91, normalized size = 0.68

$$\frac{(a + bx^3)(ad + 2bc) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b(dx^3+c)}{bc-ad}\right) + a(bc - ad)}{3b(a + bx^3)\sqrt{c + dx^3}(bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x]

[Out] (a*(b*c - a*d) + (2*b*c + a*d)*(a + b*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)]/(3*b*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.26, size = 123, normalized size = 0.92

$$\frac{3ac + adx^3 + 2bcx^3}{3(a + bx^3)\sqrt{c + dx^3}(bc - ad)^2} + \frac{(-ad - 2bc) \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3\sqrt{b}(ad - bc)^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x]

[Out] (3*a*c + 2*b*c*x^3 + a*d*x^3)/(3*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3]) + ((-2*b*c - a*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)]/(3*Sqrt[b]*(-(b*c) + a*d)^(5/2)))

fricas [B] time = 1.42, size = 630, normalized size = 4.70

$$\frac{\left(\frac{(2b^2cd + abd^2)c^2 + 2abc^2 + a^2cd + (2b^2c^2 + 3abcd + a^2d^2)c^2\sqrt{bc - ad} \log\left(\frac{bd^2x^3 + ad^2\sqrt{bc - ad}}{2bc - ad}\right) + 2(3ad^2c^2 - 3a^2bcd + (2b^2c^2 - ab^2d - a^2bd^2)c^2)\sqrt{d^3 + c}}{6(ab^4c - 3a^2b^2cd + 3a^2b^2cd^2 - a^2bd^3)c^2 + (b^5cd - 3ab^2cd^2 + 3a^2b^2cd^2 - a^2bd^3)c^2} + (b^5c^4 - 2ab^4cd + 2a^2b^3cd^2 - a^2bd^3)c^2\right)}{3(ab^4c - 3a^2b^2cd + 3a^2b^2cd^2 - a^2bd^3)c^2 + (b^5c^4 - 2ab^4cd + 2a^2b^3cd^2 - a^2bd^3)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2), x, algorithm="fricas")

[Out] [1/6*(((2*b^2*c*d + a*b*d^2)*x^6 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(3*a*b^2*c^2 - 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c^4 - 2*a*b^4*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^3), 1/3*(((2*b^2*c*d + a*b*d^2)*x^6 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^3)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a*b^2*c^2

$$- 3a^2bc^2d + (2b^3c^2 - ab^2cd - a^2bd^2)x^3) \sqrt{dx^3 + c} / ($$

$$ab^4c^4 - 3a^2b^3c^3d + 3a^3b^2c^2d^2 - a^4b^2cd^3 + (b^5c^3d$$

$$- 3ab^4c^2d^2 + 3a^2b^3c^2d^3 - a^3b^2d^4)x^6 + (b^5c^4 - 2ab^4$$

$$c^3d + 2a^3b^2cd^3 - a^4bd^4)x^3)]$$

giac [A] time = 0.20, size = 181, normalized size = 1.35

$$\frac{(2bcd+ad^2) \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2abcd+a^2d^2)\sqrt{-b^2c+abd}} + \frac{2(dx^3+c)bcd-2bc^2d+(dx^3+c)ad^2+2acd^2}{(b^2c^2-2abcd+a^2d^2)\left((dx^3+c)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}ad\right)}$$

$3d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $1/3 * ((2*b*c*d + a*d^2) * \arctan(\sqrt{d*x^3 + c} * b / \sqrt{-b^2*c + a*b*d})) / ((b^2*c^2 - 2*a*b*c*d + a^2*d^2) * \sqrt{-b^2*c + a*b*d}) + (2*(d*x^3 + c) * b*c*d - 2*b*c^2*d + (d*x^3 + c) * a*d^2 + 2*a*c*d^2) / ((b^2*c^2 - 2*a*b*c*d + a^2*d^2) * ((d*x^3 + c)^{3/2} * b - \sqrt{d*x^3 + c} * b*c + \sqrt{d*x^3 + c} * a*d)) / d$

maple [C] time = 0.27, size = 958, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)

[Out] $1/b * (-2/3 / (a*d - b*c) / ((x^3 + c/d) * d)^{1/2} - 1/3 * I * b / d^2 * 2^{1/2} * \sum(1 / (-a*d + b*c) / (a*d - b*c) * (-c*d^2)^{1/3} * (1/2 * I * (2*x + (-I * 3^{1/2}) * (-c*d^2)^{1/3} + (-c*d^2)^{1/3})) / d) / (-c*d^2)^{1/3} * d)^{1/2} * ((x - (-c*d^2)^{1/3} / d) / (-3 * (-c*d^2)^{1/3} + I * 3^{1/2} * (-c*d^2)^{1/3}) * d)^{1/2} * (-1/2 * I * (2*x + (I * 3^{1/2}) * (-c*d^2)^{1/3} + (-c*d^2)^{1/3}) / d) / (-c*d^2)^{1/3} * d)^{1/2} / (d*x^3 + c)^{1/2} * (2 * _alpha^2 * d^2 + I * (-c*d^2)^{1/3} * 3^{1/2} * _alpha * d - (-c*d^2)^{1/3} * _alpha * d - I * 3^{1/2} * (-c*d^2)^{2/3} - (-c*d^2)^{2/3}) * \text{EllipticPi}(1/3 * 3^{1/2} * (I * (x + 1/2 * (-c*d^2)^{1/3} / d - 1/2 * I * 3^{1/2} * (-c*d^2)^{1/3} / d) * 3^{1/2} / (-c*d^2)^{1/3} * d)^{1/2}, 1/2 * (2 * I * (-c*d^2)^{1/3} * 3^{1/2} * _alpha^2 * d + I * 3^{1/2} * c * d - 3 * c * d - I * (-c*d^2)^{2/3} * 3^{1/2} * _alpha - 3 * (-c*d^2)^{2/3} * _alpha) / (a*d - b*c) * b / d, (I * 3^{1/2} * (-c*d^2)^{1/3} / (-3/2 * (-c*d^2)^{1/3} / d + 1/2 * I * 3^{1/2} * (-c*d^2)^{1/3} / d) / d)^{1/2}), _alpha = \text{RootOf}(_Z^3 * b + a)) - a/b * (-1/3 / (a*d - b*c)^2 * (d*x^3 + c)^{1/2} / (b*x^3 + a) * b - 2/3 / (a*d - b*c)^2 / ((x^3 + c/d) * d)^{1/2} * d + 1/2 * I * b / d^2 * 2^{1/2} * \sum(1 / (a*d - b*c) * (-c*d^2)^{1/3} * (1/2 * I * (2*x + (-I * 3^{1/2}) * (-c*d^2)^{1/3} + (-c*d^2)^{1/3})) / d) / (-c*d^2)^{1/3} * d)^{1/2} * ((x - (-c*d^2)^{1/3} / d) / (-3 * (-c*d^2)^{1/3} + I * 3^{1/2} * (-c*d^2)^{1/3}) * d)^{1/2} * (-1/2 * I * (2*x + (I * 3^{1/2}) * (-c*d^2)^{1/3} + (-c*d^2)^{1/3}) / d) / (-c*d^2)^{1/3} * d)^{1/2} / (d*x^3 + c)^{1/2} * (2 * _alpha^2 * d^2 + I * (-c*d^2)^{1/3} * 3^{1/2} * _alpha * d - (-c*d^2)^{1/3} * _alpha * d - I * 3^{1/2} * (-c*d^2)^{2/3} - (-c*d^2)^{2/3}) * \text{EL}$

```

lipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_
alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
details)Is a*d-b*c positive or negative?
```

mupad [B] time = 7.70, size = 247, normalized size = 1.84

$$\frac{\sqrt{dx^3+c} \left(x^3 \left(\frac{3bd(ad+bc)-bd(ad+2bc)}{3(a^2bd^3-2ab^2cd^2+b^3c^2d)} - \frac{bd(ad+bc)}{a^2bd^3-2ab^2cd^2+b^3c^2d} \right) - \frac{abcd}{a^2bd^3-2ab^2cd^2+b^3c^2d} \right)}{bdx^6+(ad+bc)x^3+ac} + \frac{\ln\left(\frac{2bc-ad+bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}}{bx^3+a}\right)}{6\sqrt{b}(ad-bc)^{5/2}} (ad+2bc) \operatorname{li}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^
3)/(a + b*x^3))*(a*d + 2*b*c)*1i)/(6*b^(1/2)*(a*d - b*c)^(5/2)) - ((c + d*x
^3)^(1/2)*(x^3*((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 + b^3
*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^2*d - 2*a*b
^2*c*d^2)) - (a*b*c*d)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2))/(a*c + x^3
*(a*d + b*c) + b*d*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

$$3.318 \quad \int \frac{x^2}{(a+bx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=108

$$-\frac{d}{\sqrt{c+dx^3}(bc-ad)^2} - \frac{1}{3(a+bx^3)\sqrt{c+dx^3}(bc-ad)} + \frac{\sqrt{b}d \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {444, 51, 63, 208}

$$-\frac{d}{\sqrt{c+dx^3}(bc-ad)^2} - \frac{1}{3(a+bx^3)\sqrt{c+dx^3}(bc-ad)} + \frac{\sqrt{b}d \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] -(d/((b*c - a*d)^2*Sqrt[c + d*x^3])) - 1/(3*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2)

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 444

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]`

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{(a + bx^3)^2 (c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(a + bx)^2 (c + dx)^{3/2}} dx, x, x^3 \right) \\
 &= -\frac{1}{3(bc - ad)(a + bx^3)\sqrt{c + dx^3}} - \frac{d \text{Subst} \left(\int \frac{1}{(a + bx)(c + dx)^{3/2}} dx, x, x^3 \right)}{2(bc - ad)} \\
 &= -\frac{d}{(bc - ad)^2 \sqrt{c + dx^3}} - \frac{1}{3(bc - ad)(a + bx^3)\sqrt{c + dx^3}} - \frac{(bd) \text{Subst} \left(\int \frac{1}{(a + bx)\sqrt{c + dx}} dx, x, x^3 \right)}{2(bc - ad)^2} \\
 &= -\frac{d}{(bc - ad)^2 \sqrt{c + dx^3}} - \frac{1}{3(bc - ad)(a + bx^3)\sqrt{c + dx^3}} - \frac{b \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, x^3 \right)}{(bc - ad)^2} \\
 &= -\frac{d}{(bc - ad)^2 \sqrt{c + dx^3}} - \frac{1}{3(bc - ad)(a + bx^3)\sqrt{c + dx^3}} + \frac{\sqrt{b} d \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^3}}{\sqrt{bc - ad}} \right)}{(bc - ad)^{5/2}}
 \end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.50

$$\frac{2d {}_2F_1 \left(-\frac{1}{2}, 2; \frac{1}{2}; -\frac{b(dx^3+c)}{ad-bc} \right)}{3\sqrt{c + dx^3} (ad - bc)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] (-2*d*Hypergeometric2F1[-1/2, 2, 1/2, -((b*(c + d*x^3))/(-b*c) + a*d)))/(3*(-(b*c) + a*d)^2*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.24, size = 110, normalized size = 1.02

$$\frac{-2ad - bc - 3bdx^3}{3(a + bx^3)\sqrt{c + dx^3}(bc - ad)^2} + \frac{\sqrt{b}d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{(ad - bc)^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] $(-(b*c) - 2*a*d - 3*b*d*x^3)/(3*(b*c - a*d)^2*(a + b*x^3)*\text{Sqrt}[c + d*x^3]) + (\text{Sqrt}[b]*d*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[-(b*c) + a*d]*\text{Sqrt}[c + d*x^3])/(b*c - a*d)])/(-(b*c) + a*d)^{(5/2)}$

fricas [B] time = 1.47, size = 450, normalized size = 4.17

$$\frac{3(bd^2x^6 + (bcd + ad^2)x^3 + acd)\sqrt{\frac{b}{bc-ad}} \log\left(\frac{bdx^3 + bc - ad + 2\sqrt{dx^3+c}\sqrt{\frac{b}{bc-ad}}}{bdx^3 + a}\right) - 2(3bdx^3 + bc + 2ad)\sqrt{dx^3+c} - 3(bd^2x^6 + (bcd + ad^2)x^3 + acd)\sqrt{\frac{b}{bc-ad}} \arctan\left(\frac{-\sqrt{dx^3+c}\sqrt{\frac{b}{bc-ad}}}{bdx^3 + bc}\right) - (3bdx^3 + bc + 2ad)\sqrt{dx^3+c}}{6((b^3c^2d - 2ab^2cd^2 + a^2bd^3)x^6 + ab^2c^3 - 2a^2bc^2d + a^3cd^2 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] $[1/6*(3*(b*d^2*x^6 + (b*c*d + a*d^2)*x^3 + a*c*d)*\text{sqrt}(b/(b*c - a*d))*\log((b*d*x^3 + 2*b*c - a*d + 2*\text{sqrt}(d*x^3 + c)*(b*c - a*d)*\text{sqrt}(b/(b*c - a*d)))/(b*x^3 + a)) - 2*(3*b*d*x^3 + b*c + 2*a*d)*\text{sqrt}(d*x^3 + c))/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^3), 1/3*(3*(b*d^2*x^6 + (b*c*d + a*d^2)*x^3 + a*c*d)*\text{sqrt}(-b/(b*c - a*d))*\arctan(-\text{sqrt}(d*x^3 + c)*(b*c - a*d)*\text{sqrt}(-b/(b*c - a*d)))/(b*d*x^3 + b*c)) - (3*b*d*x^3 + b*c + 2*a*d)*\text{sqrt}(d*x^3 + c))/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^3)]$

giac [A] time = 0.21, size = 153, normalized size = 1.42

$$\frac{bd \arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2 - 2abcd + a^2d^2)\sqrt{-b^2c + abd}} - \frac{3(dx^3 + c)bd - 2bcd + 2ad^2}{3(b^2c^2 - 2abcd + a^2d^2)\left((dx^3 + c)^2b - \sqrt{dx^3 + c}bc + \sqrt{dx^3 + c}ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out]
$$-b*d*\arctan(\sqrt{d*x^3+c})*b/\sqrt{-b^2*c+a*b*d})/((b^2*c^2-2*a*b*c*d+a^2*d^2)*\sqrt{-b^2*c+a*b*d})-1/3*(3*(d*x^3+c)*b*d-2*b*c*d+2*a*d^2)/((b^2*c^2-2*a*b*c*d+a^2*d^2)*((d*x^3+c)^{(3/2)}*b-\sqrt{d*x^3+c}*b*c+\sqrt{d*x^3+c}*a*d))$$

maple [C] time = 0.25, size = 485, normalized size = 4.49

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2/(b*x^3+a)^2/(d*x^3+c)^{(3/2)}, x)$

[Out]
$$-1/3/(a*d-b*c)^2*(d*x^3+c)^{(1/2)}/(b*x^3+a)*b-2/3/(a*d-b*c)^2/((x^3+c/d)*d)^{(1/2)}*d+1/2*I*b/d^2*(1/2)*\sum(1/(a*d-b*c)^3*(-c*d^2)^{(1/3)}*(1/2*I*(2*x+(-I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}*((x-(-c*d^2)^{(1/3)})/d)/(-3*(-c*d^2)^{(1/3)}+I*3^{(1/2)}*(-c*d^2)^{(1/3)})*d)^{(1/2)}*(-1/2*I*(2*x+(I*3^{(1/2)}*(-c*d^2)^{(1/3)}+(-c*d^2)^{(1/3)})/d)/(-c*d^2)^{(1/3)}*d)^{(1/2)}/(d*x^3+c)^{(1/2)}*(2*_alpha^2*d^2+I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha*d-(-c*d^2)^{(1/3)}*_alpha*d-I*3^{(1/2)}*(-c*d^2)^{(2/3)}-(-c*d^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2*(-c*d^2)^{(1/3)})/d-1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)})/d)*3^{(1/2)}/(-c*d^2)^{(1/3)}*d)^{(1/2)}, 1/2*(2*I*(-c*d^2)^{(1/3)}*3^{(1/2)}*_alpha^2*d+I*3^{(1/2)}*c*d-3*c*d-I*(-c*d^2)^{(2/3)}*3^{(1/2)}*_alpha-3*(-c*d^2)^{(2/3)}*_alpha)/(a*d-b*c)*b/d, (I*3^{(1/2)}*(-c*d^2)^{(1/3)})/(-3/2*(-c*d^2)^{(1/3)})/d+1/2*I*3^{(1/2)}*(-c*d^2)^{(1/3)})/d)^{(1/2)}, _alpha=\text{RootOf}(_Z^3*b+a)$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2/(b*x^3+a)^2/(d*x^3+c)^{(3/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more details)Is a*d-b*c positive or negative?

mupad [B] time = 7.43, size = 199, normalized size = 1.84

$$\frac{\left(\frac{3bd(ad+bc)-bd(ad+2bc)}{3(a^2bd^3-2ab^2cd^2+b^3c^2d)} + \frac{b^2d^2x^3}{a^2bd^3-2ab^2cd^2+b^3c^2d}\right)\sqrt{dx^3+c}}{bdx^6+(ad+bc)x^3+ac} + \frac{\sqrt{b}d\ln\left(\frac{ad-2bc-bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}2i}{bx^3+a}\right)}{2(ad-bc)^{5/2}} \text{li}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)`

[Out] $(b^{1/2}*d*\log((a*d - 2*b*c + b^{1/2}*(c + d*x^3)^{1/2}*(a*d - b*c)^{1/2}*2i - b*d*x^3)/(a + b*x^3))*1i)/(2*(a*d - b*c)^{5/2}) - (((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (b^2*d^2*x^3)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2))*(c + d*x^3)^{1/2})/(a*c + x^3*(a*d + b*c) + b*d*x^6)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)`

[Out] Timed out

$$3.319 \quad \int \frac{1}{x(a+bx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=172

$$\frac{b^{3/2}(2bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right) - 2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2(bc - ad)^{5/2}} + \frac{b}{3a(a + bx^3)\sqrt{c + dx^3}(bc - ad)} + \frac{d(2ad + bc)}{3ac\sqrt{c + dx^3}(bc - ad)}$$

Rubi [A] time = 0.22, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 103, 152, 156, 63, 208}

$$\frac{b^{3/2}(2bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right) - 2 \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2(bc - ad)^{5/2}} + \frac{b}{3a(a + bx^3)\sqrt{c + dx^3}(bc - ad)} + \frac{d(2ad + bc)}{3ac\sqrt{c + dx^3}(bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] (d*(b*c + 2*a*d))/(3*a*c*(b*c - a*d)^2*Sqrt[c + d*x^3]) + b/(3*a*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2*c^(3/2)) + (b^(3/2)*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*(b*c - a*d)^(5/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 152

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2*n, 2*p]

```

Rule 156

```

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 446

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx^3)^2(c+dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(a+bx)^2(c+dx)^{3/2}} dx, x, x^3 \right) \\
&= \frac{b}{3a(bc-ad)(a+bx^3)\sqrt{c+dx^3}} + \frac{\text{Subst} \left(\int \frac{bc-ad+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}} dx, x, x^3 \right)}{3a(bc-ad)} \\
&= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{2 \text{Subst} \left(\int \frac{-\frac{1}{2}(bc-ad)}{x(a+bx)\sqrt{c+dx}} dx, x, x^3 \right)}{3ac} \\
&= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)(a+bx^3)\sqrt{c+dx^3}} + \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^3 \right)}{3a^2c} \\
&= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)(a+bx^3)\sqrt{c+dx^3}} + \frac{2 \text{Subst} \left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}} dx, x, x^3 \right)}{3a^2c} \\
&= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^2c^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.13, size = 123, normalized size = 0.72

$$\frac{\frac{b(2bc-5ad) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b(dx^3+c)}{bc-ad}\right)}{a(ad-bc)} + \left(\frac{2b}{a} - \frac{2d}{c}\right) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3}{c} + 1\right) + \frac{b}{a+bx^3}}{3a\sqrt{c+dx^3}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)), x]

[Out] (b/(a + b*x^3) + (b*(2*b*c - 5*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)]/(a*(-(b*c) + a*d)) + ((2*b)/a - (2*d)/c)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(3*a*(b*c - a*d)*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.72, size = 183, normalized size = 1.06

$$\frac{(2b^{5/2}c - 5ab^{3/2}d) \tan^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad} \right)}{3a^2(ad-bc)^{5/2}} + \frac{2a^2d^2 + 2abd^2x^3 + b^2c^2 + b^2cdx^3}{3ac(a+bx^3)\sqrt{c+dx^3}(ad-bc)^2} - \frac{2 \tanh^{-1} \left(\frac{\sqrt{c+dx^3}}{\sqrt{c}} \right)}{3a^2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (b^2*c^2 + 2*a^2*d^2 + b^2*c*d*x^3 + 2*a*b*d^2*x^3)/(3*a*c*(-(b*c) + a*d)^2
*(a + b*x^3)*Sqrt[c + d*x^3]) + ((2*b^(5/2)*c - 5*a*b^(3/2)*d)*ArcTan[(Sqrt
[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)]/(3*a^2*(-(b*c) + a*d)
^(5/2)) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2*c^(3/2))
```

fricas [B] time = 1.66, size = 1819, normalized size = 10.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^6 +
(2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(b/(b*c - a*d))*log
((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d))
)/(b*x^3 + a)) - 2*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d
^3)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*
b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))
/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3
*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 +
a^5*c^2*d^3)*x^3), 1/3*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*
b^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(
-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b
*d*x^3 + b*c)) + ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 -
2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3
)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + (a*b^2*
c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/(a^
3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^
2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5
*c^2*d^3)*x^3), 1/6*(4*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a
^3*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (2*a*b^2*c^4 - 5
*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c
^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d
- 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a*b
^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/
(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3
*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 +
a^5*c^2*d^3)*x^3), 1/3*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b
^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*
```

$$d*x^3 + b*c)) + 2*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^3)*\sqrt{-c}*\arctan(\sqrt{d*x^3 + c}*\sqrt{-c}/c) + (a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*\sqrt{d*x^3 + c})/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^3)$$

giac [A] time = 0.19, size = 226, normalized size = 1.31

$$\frac{(2b^3c - 5ab^2d)\arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3(a^2b^2c^2 - 2a^3bcd + a^4d^2)\sqrt{-b^2c+abd}} + \frac{(dx^3+c)b^2cd + 2(dx^3+c)abd^2 - 2abcd^2 + 2a^2d^3}{3(ab^2c^3 - 2a^2bc^2d + a^3cd^2)\left((dx^3+c)^{\frac{3}{2}}b - \sqrt{dx^3+c}bc + \sqrt{dx^3+c}ad\right)} + \frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $-1/3*(2*b^3*c - 5*a*b^2*d)*\arctan(\sqrt{d*x^3 + c}*b/\sqrt{-b^2*c + a*b*d})/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*\sqrt{-b^2*c + a*b*d}) + 1/3*((d*x^3 + c)*b^2*c*d + 2*(d*x^3 + c)*a*b*d^2 - 2*a*b*c*d^2 + 2*a^2*d^3)/((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*((d*x^3 + c)^(3/2)*b - \sqrt{d*x^3 + c}*b*c + \sqrt{d*x^3 + c}*a*d)) + 2/3*\arctan(\sqrt{d*x^3 + c}/\sqrt{-c})/(a^2*\sqrt{-c}*c)$

maple [C] time = 0.27, size = 1002, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)

[Out] $-1/a^2*b*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*\sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-1/a*b*(-1/3/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)*b-2/3/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)*d+1/2*I*b/d*2^(1/2)*\sum(1/(a*d-b*c)^3*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2))*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)$

$$\begin{aligned} & \left(\frac{1}{3} d \right)^{1/2} \left(\frac{x - (-c d^2)^{1/3} / d}{-3 (-c d^2)^{1/3} + I 3^{1/2} (-c d^2)^{1/3}} \right) \left(\frac{1}{3} d \right)^{1/2} \left(\frac{-1/2 I (2 x + I 3^{1/2} (-c d^2)^{1/3} + (-c d^2)^{1/3}) / d}{(-c d^2)^{1/3} d^{1/2} / (d x^3 + c)^{1/2} * (2 \alpha^2 d^2 + I (-c d^2)^{1/3} * 3^{1/2} \alpha d - (-c d^2)^{1/3} \alpha d - I 3^{1/2} (-c d^2)^{2/3} - (-c d^2)^{2/3}) * \text{EllipticPi}(1/3 * 3^{1/2} * (I (x + 1/2 * (-c d^2)^{1/3} / d - 1/2 * I 3^{1/2} * (-c d^2)^{1/3} / d) * 3^{1/2} / (-c d^2)^{1/3} * d)^{1/2}, 1/2 * (2 * I * (-c d^2)^{1/3} * 3^{1/2} * \alpha^2 d + I 3^{1/2} * c d - 3 * c d - I (-c d^2)^{2/3} * 3^{1/2} * \alpha - 3 * (-c d^2)^{2/3} * \alpha) / (a d - b c) * b / d, (I 3^{1/2} * (-c d^2)^{1/3} / (-3/2 * (-c d^2)^{1/3} / d + 1/2 * I 3^{1/2} * (-c d^2)^{1/3} / d) / d)^{1/2}} \right), \alpha = \text{RootOf}(_Z^3 + b + a)) + 1/a^2 * (2/3 / ((x^3 + c/d) * d)^{1/2} / c - 2/3 * \text{arctanh}((d x^3 + c)^{1/2} / c^{1/2}) / c^{3/2}) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b x^3 + a)^2 (d x^3 + c)^{3/2} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x), x)

mupad [B] time = 12.18, size = 288, normalized size = 1.67

$$\ln \left(\frac{(\sqrt{d x^3 + c} - \sqrt{c})^3 (\sqrt{d x^3 + c} + \sqrt{c})}{x^6} \right) + \frac{\left(\frac{(2 a d + b c)^4 + (2 a d + b c)^2 ((a d + 2 b c)(2 a d + b c) - 9 a b c d)}{9 a c (2 a d + b c)^2 (a^2 d^2 - 2 a b c d + b^2 c^2)} + \frac{b d x^3 (2 a d + b c)}{3 a c (a^2 d^2 - 2 a b c d + b^2 c^2)} \right) \sqrt{d x^3 + c}}{3 a^2 c^{3/2} b d x^6 + (a d + b c) x^3 + a c} + \frac{b^{3/2} \ln \left(\frac{2 b c - a d + b d x^3 + \sqrt{b} \sqrt{d x^3 + c} \sqrt{a d - b c}}{b x^3 + a} \right) (5 a d - 2 b c)}{6 a^2 (a d - b c)^{5/2}} 11$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)

[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3*a^2*c^(3/2)) + (((2*a*d + b*c)^4 + (2*a*d + b*c)^2*((a*d + 2*b*c)*(2*a*d + b*c) - 9*a*b*c*d))/(9*a*c*(2*a*d + b*c)^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^3*(2*a*d + b*c))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))*(c + d*x^3)^(1/2)/(a*c + x^3*(a*d + b*c) + b*d*x^6) + (b^(3/2)*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d - 2*b*c)*1i)/(6*a^2*(a*d - b*c)^(5/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x (a + b x^3)^2 (c + d x^3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

$$3.320 \quad \int \frac{1}{x^4(a+bx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal. Leaf size=241

$$\frac{b^{5/2}(4bc - 7ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{5/2}} + \frac{(3ad + 4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}} - \frac{d(3a^2d^2 - 2abcd + 2b^2c^2)}{3a^2c^2\sqrt{c+dx^3}(bc-ad)^2} - \frac{b(2bc-ad)}{3a^2c(a+bx^3)\sqrt{c+dx^3}(bc-ad)} - \frac{1}{3acx^3(a+bx^3)\sqrt{c+dx^3}}$$

Rubi [A] time = 0.36, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 103, 151, 152, 156, 63, 208}

$$\frac{d(3a^2d^2 - 2abcd + 2b^2c^2)}{3a^2c^2\sqrt{c+dx^3}(bc-ad)^2} - \frac{b^{5/2}(4bc - 7ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{5/2}} + \frac{(3ad + 4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}} - \frac{b(2bc-ad)}{3a^2c(a+bx^3)\sqrt{c+dx^3}(bc-ad)} - \frac{1}{3acx^3(a+bx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)), x]

[Out] -(d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2))/(3*a^2*c^2*(b*c - a*d)^2*Sqrt[c + d*x^3]) - (b*(2*b*c - a*d))/(3*a^2*c*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) - 1/(3*a*c*x^3*(a + b*x^3)*Sqrt[c + d*x^3]) + ((4*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3*c^(5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*(b*c - a*d)^(5/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*m, 2*p])

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (a + bx^3)^2 (c + dx^3)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2 (a + bx)^2 (c + dx)^{3/2}} dx, x, x^3 \right) \\
&= -\frac{1}{3acx^3 (a + bx^3) \sqrt{c + dx^3}} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(4bc+3ad) + \frac{5bdx}{2}}{x(a+bx)^2(c+dx)^{3/2}} dx, x, x^3 \right)}{3ac} \\
&= -\frac{b(2bc - ad)}{3a^2c(bc - ad) (a + bx^3) \sqrt{c + dx^3}} - \frac{1}{3acx^3 (a + bx^3) \sqrt{c + dx^3}} - \frac{\text{Subst} \left(\int \frac{1}{x^2 (a + bx)^2 (c + dx)^{3/2}} dx, x, x^3 \right)}{3ac} \\
&= -\frac{d(2b^2c^2 - 2abcd + 3a^2d^2)}{3a^2c^2(bc - ad)^2 \sqrt{c + dx^3}} - \frac{b(2bc - ad)}{3a^2c(bc - ad) (a + bx^3) \sqrt{c + dx^3}} - \frac{\text{Subst} \left(\int \frac{1}{x^2 (a + bx)^2 (c + dx)^{3/2}} dx, x, x^3 \right)}{3ac} \\
&= -\frac{d(2b^2c^2 - 2abcd + 3a^2d^2)}{3a^2c^2(bc - ad)^2 \sqrt{c + dx^3}} - \frac{b(2bc - ad)}{3a^2c(bc - ad) (a + bx^3) \sqrt{c + dx^3}} - \frac{\text{Subst} \left(\int \frac{1}{x^2 (a + bx)^2 (c + dx)^{3/2}} dx, x, x^3 \right)}{3ac} \\
&= -\frac{d(2b^2c^2 - 2abcd + 3a^2d^2)}{3a^2c^2(bc - ad)^2 \sqrt{c + dx^3}} - \frac{b(2bc - ad)}{3a^2c(bc - ad) (a + bx^3) \sqrt{c + dx^3}} - \frac{\text{Subst} \left(\int \frac{1}{x^2 (a + bx)^2 (c + dx)^{3/2}} dx, x, x^3 \right)}{3ac} \\
&= -\frac{d(2b^2c^2 - 2abcd + 3a^2d^2)}{3a^2c^2(bc - ad)^2 \sqrt{c + dx^3}} - \frac{b(2bc - ad)}{3a^2c(bc - ad) (a + bx^3) \sqrt{c + dx^3}} - \frac{\text{Subst} \left(\int \frac{1}{x^2 (a + bx)^2 (c + dx)^{3/2}} dx, x, x^3 \right)}{3ac} \\
&= -\frac{d(2b^2c^2 - 2abcd + 3a^2d^2)}{3a^2c^2(bc - ad)^2 \sqrt{c + dx^3}} - \frac{b(2bc - ad)}{3a^2c(bc - ad) (a + bx^3) \sqrt{c + dx^3}} - \frac{\text{Subst} \left(\int \frac{1}{x^2 (a + bx)^2 (c + dx)^{3/2}} dx, x, x^3 \right)}{3ac}
\end{aligned}$$

Mathematica [C] time = 0.16, size = 189, normalized size = 0.78

$$\frac{b^2c^2x^3(a + bx^3)(4bc - 7ad) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b(dx^3+c)}{bc-ad}\right) - (ad - bc)\left(x^3(a + bx^3)(3a^2d^2 + abcd - 4b^2c^2) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{dx^3}{c} + 1\right) + ac(a^2d + ab(dx^3 - c) - 2b^2cx^3)\right)}{3a^3c^2x^3(a + bx^3)\sqrt{c + dx^3}(bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)), x]

[Out] (b^2*c^2*(4*b*c - 7*a*d)*x^3*(a + b*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)] - ((b*c) + a*d)*(a*c*(a^2*d - 2*b^2*c*x^3 + a*b*(-c + d*x^3)) + (-4*b^2*c^2 + a*b*c*d + 3*a^2*d^2)*x^3*(a + b*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(3*a^3*c^2*(b*c - a*d)^2*x^3*(a + b*x^3)*Sqrt[c + d*x^3])

IntegrateAlgebraic [A] time = 0.95, size = 272, normalized size = 1.13

$$\frac{(7ab^{5/2}d - 4b^{7/2}c) \tan^{-1}\left(\frac{\sqrt{b}\sqrt{cdx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^3(ad-bc)^{5/2}} + \frac{(3ad+4bc) \tanh^{-1}\left(\frac{\sqrt{cdx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}} + \frac{-a^3cd^2 - 3a^2d^3x^3 + 2a^2bcd + a^2bcd^2x^3 - 3a^2bd^3x^6 - ab^2c^3 + ab^2c^2dx^3 + 2ab^2cd^2x^6 - 2b^3c^3x^3 - 2b^3c^2dx^6}{3a^2c^2x^3(a+bx^3)\sqrt{c+dx^3}(ad-bc)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out]
$$\begin{aligned} &(-a^2b^2c^3) + 2a^2b^2c^2d - a^3c^2d^2 - 2b^3c^2d^2x^3 + a^2b^2c^2d^2x^3 \\ &+ a^2b^2c^2d^2x^3 - 3a^3d^3x^3 - 2b^3c^2d^2x^6 + 2a^2b^2c^2d^2x^6 - \\ &3a^2b^2d^3x^6)/(3a^2c^2(-b^2c + a^2d)^2x^3(a + b^2x^3)\sqrt{c + d^2x^3}) \\ &+ ((-4b^{7/2}c + 7a^2b^{5/2}d)\operatorname{ArcTan}[\sqrt{b}\sqrt{-(b^2c + a^2d)}\sqrt{c + d^2x^3}]/(b^2c - a^2d))/ \\ &((3a^3(-b^2c + a^2d)^{5/2}) + ((4b^2c + 3a^2d)\operatorname{ArcTanh}[\sqrt{c + d^2x^3}/\sqrt{c}])/(3a^3c^{5/2})) \end{aligned}$$

fricas [B] time = 2.00, size = 2384, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} &[-1/6(((4b^4c^4d - 7a^2b^3c^3d^2)x^9 + (4b^4c^5 - 3a^2b^3c^4d - 7a^2b^2c^3d^2)x^6 + (4a^2b^3c^5 - 7a^2b^2c^4d)x^3)\sqrt{b/(b^2c - a^2d)}) \\ &\log((b^2d^2x^3 + 2b^2c - a^2d + 2\sqrt{d^2x^3 + c})(b^2c - a^2d)\sqrt{b/(b^2c - a^2d)}) \\ &)/((b^2d^2x^3 + a^2d) - ((4b^4c^3d - 5a^2b^3c^2d^2 - 2a^2b^2c^2d^3 + 3a^3b^2d^4)x^9 + (4b^4c^4 - a^2b^3c^3d - 7a^2b^2c^2d^2 + a^3b^2c^2d^3 + 3a^4d^4)x^6 + (4a^2b^3c^4 - 5a^2b^2c^3d - 2a^3b^2c^2d^2 + 3a^4c^2d^3)x^3)\sqrt{c}) \\ &\log((d^2x^3 + 2\sqrt{d^2x^3 + c})\sqrt{c} + 2c)/x^3) + 2(a^2b^2c^4 - 2a^3b^2c^3d + a^4c^2d^2 + (2a^2b^3c^3d - 2a^2b^2c^2d^2 + 3a^3b^2c^2d^3)x^6 + (2a^2b^3c^4 - a^2b^2c^3d - a^3b^2c^2d^2 + 3a^4c^2d^3)x^3)\sqrt{d^2x^3 + c}) \\ &/((a^3b^3c^5d - 2a^4b^2c^4d^2 + a^5b^2c^3d^3)x^9 + (a^3b^3c^6 - a^4b^2c^5d - a^5b^2c^4d^2 + a^6c^3d^3)x^6 + (a^4b^2c^6 - 2a^5b^2c^5d + a^6c^4d^2)x^3), -1/6(2 \\ &(((4b^4c^4d - 7a^2b^3c^3d^2)x^9 + (4b^4c^5 - 3a^2b^3c^4d - 7a^2b^2c^3d^2)x^6 + (4a^2b^3c^5 - 7a^2b^2c^4d)x^3)\sqrt{-b/(b^2c - a^2d)}) \\ &\operatorname{arctan}(-\sqrt{d^2x^3 + c})(b^2c - a^2d)\sqrt{-b/(b^2c - a^2d)})/(b^2d^2x^3 + b^2c)) \\ &- ((4b^4c^3d - 5a^2b^3c^2d^2 - 2a^2b^2c^2d^3 + 3a^3b^2d^4)x^9 + (4b^4c^4 - a^2b^3c^3d - 7a^2b^2c^2d^2 + a^3b^2c^2d^3 + 3a^4d^4)x^6 + (4a^2b^3c^4 - 5a^2b^2c^3d - 2a^3b^2c^2d^2 + 3a^4c^2d^3)x^3)\sqrt{c}) \\ &\log(((d^2x^3 + 2\sqrt{d^2x^3 + c})\sqrt{c} + 2c)/x^3) + 2(a^2b^2c^4 - 2a^3b^2c^3d + a^4c^2d^2 + (2a^2b^3c^3d - 2a^2b^2c^2d^2 + 3a^3b^2c^2d^3)x^6 + (2a^2b^3c^4 - a^2b^2c^3d - a^3b^2c^2d^2 + 3a^4c^2d^3)x^3)\sqrt{d^2x^3 + c}) \\ &/((a^3b^3c^5d - 2a^4b^2c^4d^2 + a^5b^2c^3d^3)x^9 + (a^3b^3c^6 - a^4b^2c^5d - a^5b^2c^4d^2 + a^6c^3d^3)x^6 + (a^4b^2c^6 - 2a^5b^2c^5d + a^6c^4d^2)x^3) \end{aligned}$$

$$\begin{aligned} &^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3), -1/6*(2*((4*b^4*c^3*d - 5*a*b^3 \\ &*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - \\ &7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2 \\ &*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c \\ &)*sqrt(-c)/c) + ((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3 \\ &*c^4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt \\ &(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)* \\ &sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^ \\ &2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*c \\ &c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(d*x^3 + c))/((\\ &a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4 \\ &*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d \\ &d + a^6*c^4*d^2)*x^3), -1/3*((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c \\ &c^5 - 3*a*b^3*c^4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4 \\ &*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(\\ &b*c - a*d))/(b*d*x^3 + b*c)) + ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c \\ &*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a \\ &^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2* \\ &d^2 + 3*a^4*c*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a^2* \\ &b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 \\ &+ 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4 \\ &*c*d^3)*x^3)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c \\ &^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x \\ &^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3)] \end{aligned}$$

giac [A] time = 0.22, size = 367, normalized size = 1.52

$$\frac{(4b^4c - 7ab^3d) \arctan\left(\frac{\sqrt{d^3+c}b}{\sqrt{-b^2c+abd}}\right) - 2(d^3+c)^2 b^3 c^2 d - 2(d^3+c) b^3 c^2 d - 2(d^3+c)^2 ab^2 c d^2 + 3(d^3+c) ab^2 c^2 d^2 + 3(d^3+c)^2 a^2 b d^3 - 7(d^3+c) a^2 b c d^3 + 2a^2 b c^2 d^3 + 3(d^3+c) a^3 d^4 - 2a^3 c d^4 - \frac{(4bc+3ad) \arctan\left(\frac{\sqrt{d^3+c}}{\sqrt{-c}}\right)}{3a^3 \sqrt{-c}^2}}{3(a^2 b^2 c^2 - 2a^4 b c d + a^2 d^2) \sqrt{-b^2 c + a b d}} \frac{2(d^3+c)^2 b^3 c^2 d - 2(d^3+c) b^3 c^2 d - 2(d^3+c)^2 ab^2 c d^2 + 3(d^3+c) ab^2 c^2 d^2 + 3(d^3+c)^2 a^2 b d^3 - 7(d^3+c) a^2 b c d^3 + 2a^2 b c^2 d^3 + 3(d^3+c) a^3 d^4 - 2a^3 c d^4}{3(a^2 b^2 c^2 - 2a^4 b c d + a^2 d^2) \left((d^3+c)^{\frac{5}{2}} b - 2(d^3+c)^{\frac{3}{2}} b c + \sqrt{d^3+c} b c^2 + (d^3+c)^{\frac{3}{2}} a d - \sqrt{d^3+c} a c d \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] $\frac{1}{3}*(4*b^4*c - 7*a*b^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*sqrt(-b^2*c + a*b*d)) - \frac{1}{3}*(2*(d*x^3 + c)^2*b^3*c^2*d - 2*(d*x^3 + c)*b^3*c^3*d - 2*(d*x^3 + c)^2*a*b^2*c*d^2 + 3*(d*x^3 + c)*a*b^2*c^2*d^2 + 3*(d*x^3 + c)^2*a^2*b*d^3 - 7*(d*x^3 + c)*a^2*b*c*d^3 + 2*a^2*b*c^2*d^3 + 3*(d*x^3 + c)*a^3*d^4 - 2*a^3*c*d^4)/((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(d*x^3 + c)^{(5/2)}*b - 2*(d*x^3 + c)^{(3/2)}*b*c + sqrt(d*x^3 + c)*b*c^2 + (d*x^3 + c)^{(3/2)}*a*d - sqrt(d*x^3 + c)*a*c*d)) - \frac{1}{3}*(4*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)*c^2)$

maple [C] time = 0.25, size = 1067, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)`

[Out]
$$\frac{1}{a^2} \frac{(-1/3(d^2x^3+c)^{1/2}/c^2/x^3-2/3((x^3+c/d)*d)^{1/2}/c^2*d+d*\operatorname{arctanh}((d^2x^3+c)^{1/2}/c^{1/2}))/c^{5/2}}{+2/a^3*b^2*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^{1/2}-1/3*I*b/d^2*2^{1/2}*\sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^{1/3}*(1/2*I*(2*x+(-I*3^{1/2})*(-c*d^2)^{1/3}+(-c*d^2)^{1/3}))/d)/(-c*d^2)^{1/3}*d)^{1/2}*((x-(-c*d^2)^{1/3}/d)/(-3*(-c*d^2)^{1/3}+I*3^{1/2})*(-c*d^2)^{1/3})*d)^{1/2}*(-1/2*I*(2*x+(I*3^{1/2})*(-c*d^2)^{1/3}+(-c*d^2)^{1/3}))/d)/(-c*d^2)^{1/3}*d)^{1/2}}{(d^2x^3+c)^{1/2}*(2*_alpha^2*d^2+I*(-c*d^2)^{1/3}*3^{1/2}*_alpha*d-(-c*d^2)^{1/3}*_alpha*d-I*3^{1/2}*(-c*d^2)^{2/3}-(-c*d^2)^{2/3})*\operatorname{EllipticPi}(1/3*3^{1/2}*(I*(x+1/2*(-c*d^2)^{1/3}/d-1/2*I*3^{1/2}*(-c*d^2)^{1/3}/d)*3^{1/2}/(-c*d^2)^{1/3}*d)^{1/2},1/2*(2*I*(-c*d^2)^{1/3}*3^{1/2}*_alpha^2*d+I*3^{1/2})*c*d-3*c*d-I*(-c*d^2)^{2/3}*3^{1/2}*_alpha-3*(-c*d^2)^{2/3}*_alpha)/(a*d-b*c)*b/d,(I*3^{1/2}*(-c*d^2)^{1/3}/(-3/2*(-c*d^2)^{1/3}/d+1/2*I*3^{1/2}*(-c*d^2)^{1/3}/d)/d)^{1/2}},_alpha=\operatorname{RootOf}(_Z^3*b+a)))+1/a^2*b^2*(-1/3/(a*d-b*c)^2*(d^2x^3+c)^{1/2}/(b*x^3+a)*b-2/3/(a*d-b*c)^2/((x^3+c/d)*d)^{1/2}*d+1/2*I*b/d^2*2^{1/2}*\sum(1/(a*d-b*c)^3*(-c*d^2)^{1/3}*(1/2*I*(2*x+(-I*3^{1/2})*(-c*d^2)^{1/3}+(-c*d^2)^{1/3}))/d)/(-c*d^2)^{1/3}*d)^{1/2}*((x-(-c*d^2)^{1/3}/d)/(-3*(-c*d^2)^{1/3}+I*3^{1/2})*(-c*d^2)^{1/3})*d)^{1/2}*(-1/2*I*(2*x+(I*3^{1/2})*(-c*d^2)^{1/3}+(-c*d^2)^{1/3}))/d)/(-c*d^2)^{1/3}*d)^{1/2}}{(d^2x^3+c)^{1/2}*(2*_alpha^2*d^2+I*(-c*d^2)^{1/3}*3^{1/2}*_alpha*d-(-c*d^2)^{1/3}*_alpha*d-I*3^{1/2}*(-c*d^2)^{2/3}-(-c*d^2)^{2/3})*\operatorname{EllipticPi}(1/3*3^{1/2}*(I*(x+1/2*(-c*d^2)^{1/3}/d-1/2*I*3^{1/2}*(-c*d^2)^{1/3}/d)*3^{1/2}/(-c*d^2)^{1/3}*d)^{1/2},1/2*(2*I*(-c*d^2)^{1/3}*3^{1/2}*_alpha^2*d+I*3^{1/2})*c*d-3*c*d-I*(-c*d^2)^{2/3}*3^{1/2}*_alpha-3*(-c*d^2)^{2/3}*_alpha)/(a*d-b*c)*b/d,(I*3^{1/2}*(-c*d^2)^{1/3}/(-3/2*(-c*d^2)^{1/3}/d+1/2*I*3^{1/2}*(-c*d^2)^{1/3}/d)/d)^{1/2}},_alpha=\operatorname{RootOf}(_Z^3*b+a)))-2*b/a^3*(2/3/((x^3+c/d)*d)^{1/2}/c-2/3*\operatorname{arctanh}((d^2x^3+c)^{1/2}/c^{1/2}))/c^{3/2}}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^2 (dx^3 + c)^{\frac{3}{2}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^4), x)`

mupad [B] time = 19.63, size = 18847, normalized size = 78.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^4*(a + b*x^3)^2*(c + d*x^3)^{(3/2)}),x)$

[Out] $(2*b*\log(1/x^6))/(3*a^3*c^{(3/2)}) - (c + d*x^3)^{(1/2)}/(3*a^2*c^2*x^3) + (d*\log(1/x^6))/(2*a^2*c^{(5/2)}) + (2*b*\log(c^{(3/2)}*(c + d*x^3)^{(1/2)} - c^{(1/2)}*(c + d*x^3)^{(3/2)} + d^2*x^6 + 2*c*d*x^3 + 3*c^{(1/2)}*d*x^3*(c + d*x^3)^{(1/2)}))/(3*a^3*c^{(3/2)}) + (d*\log(c^{(3/2)}*(c + d*x^3)^{(1/2)} - c^{(1/2)}*(c + d*x^3)^{(3/2)} + d^2*x^6 + 2*c*d*x^3 + 3*c^{(1/2)}*d*x^3*(c + d*x^3)^{(1/2)}))/(2*a^2*c^{(5/2)}) - (b^7*c^9*x^4*(c + d*x^3)^{(1/2)})/(2*(2*a^9*c^6*d^5*x + 2*a^9*c^5*d^6*x^4 + a^5*b^4*c^9*d^2*x^4 + a^6*b^3*c^8*d^3*x^4 - 3*a^7*b^2*c^7*d^4*x^4 + a^5*b^4*c^8*d^3*x^7 - 3*a^7*b^2*c^6*d^5*x^7 - 3*a^8*b*c^7*d^4*x + a^6*b^3*c^9*d^2*x - a^8*b*c^6*d^5*x^4 + 2*a^8*b*c^5*d^6*x^7)) - (5*a^9*d^7*x^4*(c + d*x^3)^{(1/2)})/(4*(a^6*b^5*c^9*x + a^5*b^6*c^9*x^4 - 3*a^7*b^4*c^7*d^2*x^4 - a^8*b^3*c^6*d^3*x^4 + 2*a^9*b^2*c^5*d^4*x^4 - 3*a^7*b^4*c^6*d^3*x^7 + 2*a^8*b^3*c^5*d^4*x^7 - 3*a^8*b^3*c^7*d^2*x + 2*a^9*b^2*c^6*d^3*x + a^6*b^5*c^8*d*x^4 + a^5*b^6*c^8*d*x^7)) + (3*a^2*d^2*x*(c + d*x^3)^{(1/2)})/(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7) + (2*b^2*c^2*x*(c + d*x^3)^{(1/2)})/(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7) - (b^{(7/2)}*c*\log((a^6*b^{(15/2)}*c^{10*36i})/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)})) - (a^7*b^{(13/2)}*c^9*d*198i)/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) + (a^{12}*b^{(3/2)}*c^4*d^6*18i)/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) - (a^{11}*b^{(5/2)}*c^5*d^5*126i)/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) + (a^{10}*b^{(7/2)}*c^6*d^4*360i)/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) - (a^9*b^{(9/2)}*c^7*d^3*540i)/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) + (a^8*b^{(11/2)}*c^8*d^2*450i)/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) + (a^6*b^{(15/2)}*c^9*d*x^3*18i)/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) - (a^{11}*b^{(5/2)}*c^4*d^6*x^3*18i)/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) + (a^{10}*b^{(7/2)}*c^5*d^5*x^3*90i)/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) - (a^9*b^{(9/2)}*c^6*d^4*x^3*180i)/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) + (a^8*b^{(11/2)}*c^7*d^3*x^3*180i)/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) - (a^7*b^{(13/2)}*c^8*d^2*x^3*90i)/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) + (36*a^6*b^7*c^9*(c + d*x^3)^{(1/2)}*(a*d - b*c)^{(1/2)})/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) + (360*a^8*b^5*c^7*d^2*(c + d*x^3)^{(1/2)}*(a*d - b*c)^{(1/2)})/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) - (360*a^9*b^4*c^6*d^3*(c + d*x^3)^{(1/2)}*(a*d - b*c)^{(1/2)})/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) + (180*a^10*b^3*c^5*d^4*(c + d*x^3)^{(1/2)}*(a*d - b*c)^{(1/2)})/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) - (36*a^11*b^2*c^4*d^5*(c + d*x^3)^{(1/2)}*(a*d - b*c)^{(1/2)})/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) - (180*a^7*b^6*c^8*d*(c + d*x^3)^{(1/2)}*(a*d - b*c)^{(1/2)})/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) + (b^{(5/2)}*d*\log((a^6*b^{(15/2)}*c^{10*36i})/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)})) - (a^7*b^{(13/2)}*c^9*d*198i)/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) + (a^{12}*b^{(3/2)}*c^4*d^6*18i)/(a*(a*d - b*c)^{(1/2)} + b*x^3*(a*d - b*c)^{(1/2)}) - (a^{11}*b^{(5/2)}$

$$\begin{aligned}
&) * c^5 * d^5 * 126i) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) + (a^{10} * b^{(7/2)} * c^6 * d^4 * 360i) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) - (a^9 * b^{(9/2)} * c^7 * d^3 * 540i) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) + (a^8 * b^{(11/2)} * c^8 * d^2 * 450i) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) + (a^6 * b^{(15/2)} * c^9 * d * x^3 * 18i) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) - (a^{11} * b^{(5/2)} * c^4 * d^6 * x^3 * 18i) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) + (a^{10} * b^{(7/2)} * c^5 * d^5 * x^3 * 90i) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) - (a^9 * b^{(9/2)} * c^6 * d^4 * x^3 * 180i) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) + (a^8 * b^{(11/2)} * c^7 * d^3 * x^3 * 180i) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) - (a^7 * b^{(13/2)} * c^8 * d^2 * x^3 * 90i) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) + (36 * a^6 * b^7 * c^9 * (c + d * x^3)^{(1/2)} * (a * d - b * c)^{(1/2)}) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) + (360 * a^8 * b^5 * c^7 * d^2 * (c + d * x^3)^{(1/2)} * (a * d - b * c)^{(1/2)}) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) - (360 * a^9 * b^4 * c^6 * d^3 * (c + d * x^3)^{(1/2)} * (a * d - b * c)^{(1/2)}) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) + (180 * a^{10} * b^3 * c^5 * d^4 * (c + d * x^3)^{(1/2)} * (a * d - b * c)^{(1/2)}) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) - (36 * a^{11} * b^2 * c^4 * d^5 * (c + d * x^3)^{(1/2)} * (a * d - b * c)^{(1/2)}) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) - (180 * a^7 * b^6 * c^8 * d * (c + d * x^3)^{(1/2)} * (a * d - b * c)^{(1/2)}) / (a * (a * d - b * c)^{(1/2)} + b * x^3 * (a * d - b * c)^{(1/2)}) * 7i) / (6 * a^2 * (a * d - b * c)^{(5/2)}) + (5 * a^4 * d^4 * x^4 * (c + d * x^3)^{(1/2)}) / (2 * (a^4 * b^2 * c^6 * x + a^3 * b^3 * c^6 * x^4 + 2 * a^5 * b * c^5 * d * x + 2 * a^4 * b^2 * c^4 * d^2 * x^7 + 3 * a^4 * b^2 * c^5 * d * x^4 + 2 * a^5 * b * c^4 * d^2 * x^4 + a^3 * b^3 * c^5 * d * x^7)) - (65 * a^3 * d^3 * x^4 * (c + d * x^3)^{(1/2)}) / (24 * (a^3 * b^2 * c^5 * x^4 + 2 * a^5 * c^3 * d^2 * x^4 + a^4 * b * c^5 * x + 2 * a^5 * c^4 * d * x + 3 * a^4 * b * c^4 * d * x^4 + a^3 * b^2 * c^4 * d * x^7 + 2 * a^4 * b * c^3 * d^2 * x^7)) - (8 * b^3 * c^3 * x^4 * (c + d * x^3)^{(1/2)}) / (3 * (a^3 * b^2 * c^5 * x^4 + 2 * a^5 * c^3 * d^2 * x^4 + a^4 * b * c^5 * x + 2 * a^5 * c^4 * d * x + 3 * a^4 * b * c^4 * d * x^4 + a^3 * b^2 * c^4 * d * x^7 + 2 * a^4 * b * c^3 * d^2 * x^7)) + (14 * b^3 * c^4 * x^4 * (c + d * x^3)^{(1/2)}) / (a^3 * b^2 * c^6 * x^4 + 2 * a^5 * c^4 * d^2 * x^4 + a^4 * b * c^6 * x + 2 * a^5 * c^5 * d * x + 3 * a^4 * b * c^5 * d * x^4 + a^3 * b^2 * c^5 * d * x^7 + 2 * a^4 * b * c^4 * d^2 * x^7) - (5 * a^7 * c^2 * d^5 * x * (c + d * x^3)^{(1/2)}) / (2 * (a^5 * b^4 * c^9 * x + a^4 * b^5 * c^9 * x^4 - 3 * a^6 * b^3 * c^7 * d^2 * x^4 - a^7 * b^2 * c^6 * d^3 * x^4 - 3 * a^6 * b^3 * c^6 * d^3 * x^7 + 2 * a^7 * b^2 * c^5 * d^4 * x^7 + 2 * a^8 * b * c^6 * d^3 * x - 3 * a^7 * b^2 * c^7 * d^2 * x + a^5 * b^4 * c^8 * d * x^4 + 2 * a^8 * b * c^5 * d^4 * x^4 + a^4 * b^5 * c^8 * d * x^7)) - (5 * a^7 * c * d^6 * x^4 * (c + d * x^3)^{(1/2)}) / (2 * (a^5 * b^4 * c^9 * x + a^4 * b^5 * c^9 * x^4 - 3 * a^6 * b^3 * c^7 * d^2 * x^4 - a^7 * b^2 * c^6 * d^3 * x^4 - 3 * a^6 * b^3 * c^6 * d^3 * x^7 + 2 * a^7 * b^2 * c^5 * d^4 * x^7 + 2 * a^8 * b * c^6 * d^3 * x - 3 * a^7 * b^2 * c^7 * d^2 * x + a^5 * b^4 * c^8 * d * x^4 + 2 * a^8 * b * c^5 * d^4 * x^4 + a^4 * b^5 * c^8 * d * x^7)) - (3 * a^8 * c^2 * d^5 * x * (c + d * x^3)^{(1/2)}) / (8 * (a^6 * b^4 * c^9 * x + a^5 * b^5 * c^9 * x^4 - 3 * a^7 * b^3 * c^7 * d^2 * x^4 - a^8 * b^2 * c^6 * d^3 * x^4 - 3 * a^7 * b^3 * c^6 * d^3 * x^7 + 2 * a^8 * b^2 * c^5 * d^4 * x^7 + 2 * a^9 * b * c^6 * d^3 * x - 3 * a^8 * b^2 * c^7 * d^2 * x + a^6 * b^4 * c^8 * d * x^4 + 2 * a^9 * b * c^5 * d^4 * x^4 + a^5 * b^5 * c^8 * d * x^7)) - (3 * a^8 * c * d^6 * x^4 * (c + d * x^3)^{(1/2)}) / (8 * (a^6 * b^4 * c^9 * x + a^5 * b^5 * c^9 * x^4 - 3 * a^7 * b^3 * c^7 * d^2 * x^4 - a^8 * b^2 * c^6 * d^3 * x^4 - 3 * a^7 * b^3 * c^6 * d^3 * x^7 + 2 * a^8 * b^2 * c^5 * d^4 * x^7 + 2 * a^9 * b * c^6 * d^3 * x - 3 * a^8 * b^2 * c^7 * d^2 * x + a^6 * b^4 * c^8 * d * x^4 + 2 * a^9 * b * c^5 * d^4 * x^4 + a^5 * b^5 * c^8 * d * x^7)) + (23 * a^9 * c^3 * d^5 * x * (c + d * x^3)^{(1/2)}) / (8 * (a^7 * b^4 * c^10 * x + a^6 * b^5 * c^10 * x^4 - 3 * a^8 * b^3 * c^8 * d^2 * x^4 - a^9 * b^2 * c^7 * d^3 * x^4 - 3 * a^8 * b^3 *
\end{aligned}$$

$$\begin{aligned}
& c^7 d^3 x^7 + 2 a^9 b^2 c^6 d^4 x^7 + 2 a^{10} b c^7 d^3 x - 3 a^9 b^2 c^8 d^2 x + a^7 b^4 c^9 d x^4 + 2 a^{10} b c^6 d^4 x^4 + a^6 b^5 c^9 d x^7) - (a b^6 c^9 x (c + d x^3)^{(1/2)}) / (2 (2 a^9 c^6 d^5 x + 2 a^9 c^5 d^6 x^4 + a^5 b^4 c^9 d^2 x^4 + a^6 b^3 c^8 d^3 x^4 - 3 a^7 b^2 c^7 d^4 x^4 + a^5 b^4 c^8 d^3 x^7 - 3 a^7 b^2 c^6 d^5 x^7 - 3 a^8 b c^7 d^4 x + a^6 b^3 c^9 d^2 x - a^8 b c^6 d^5 x^4 + 2 a^8 b c^5 d^6 x^7)) + (3 a^2 b^6 c^9 x^4 (c + d x^3)^{(1/2)}) / (4 (2 a^{10} c^7 d^4 x + 2 a^{10} c^6 d^5 x^4 + a^7 b^3 c^9 d^2 x^4 - 3 a^8 b^2 c^8 d^3 x^4 + a^6 b^4 c^9 d^2 x^7 - 3 a^8 b^2 c^7 d^4 x^7 + a^7 b^3 c^10 d x - 3 a^9 b c^8 d^3 x + a^6 b^4 c^10 d x^4 - a^9 b c^7 d^4 x^4 + 2 a^9 b c^6 d^5 x^7)) - (5 a^6 c^2 d^3 x (c + d x^3)^{(1/2)}) / (2 (a^6 b^2 c^7 x + a^5 b^3 c^7 x^4 + 2 a^7 b c^6 d x + 2 a^6 b^2 c^5 d^2 x^7 + 3 a^6 b^2 c^6 d x^4 + 2 a^7 b c^5 d^2 x^4 + a^5 b^3 c^6 d x^7)) - (5 a^6 c d^4 x^4 (c + d x^3)^{(1/2)}) / (2 (a^6 b^2 c^7 x + a^5 b^3 c^7 x^4 + 2 a^7 b c^6 d x + 2 a^6 b^2 c^5 d^2 x^7 + 3 a^6 b^2 c^6 d x^4 + 2 a^7 b c^5 d^2 x^4 + a^5 b^3 c^6 d x^7)) + (4 a^2 b^4 c^6 x (c + d x^3)^{(1/2)}) / (a^5 b^3 c^8 x + 2 a^8 c^5 d^3 x + a^4 b^4 c^8 x^4 + 2 a^8 c^4 d^4 x^4 - 3 a^6 b^2 c^6 d^2 x^4 - 3 a^6 b^2 c^5 d^3 x^7 - 3 a^7 b c^6 d^2 x + a^5 b^3 c^7 d x^4 - a^7 b c^5 d^3 x^4 + a^4 b^4 c^7 d x^7 + 2 a^7 b c^4 d^4 x^7) + (8 a b^5 c^6 x^4 (c + d x^3)^{(1/2)}) / (3 (a^5 b^3 c^8 x + 2 a^8 c^5 d^3 x + a^4 b^4 c^8 x^4 + 2 a^8 c^4 d^4 x^4 - 3 a^6 b^2 c^6 d^2 x^4 - 3 a^6 b^2 c^5 d^3 x^7 - 3 a^7 b c^6 d^2 x + a^5 b^3 c^7 d x^4 - a^7 b c^5 d^3 x^4 + a^4 b^4 c^7 d x^7 + 2 a^7 b c^4 d^4 x^7)) - (14 a^2 b^4 c^7 x (c + d x^3)^{(1/2)}) / (a^5 b^3 c^9 x + 2 a^8 c^6 d^3 x + a^4 b^4 c^9 x^4 + 2 a^8 c^5 d^4 x^4 - 3 a^6 b^2 c^7 d^2 x^4 - 3 a^6 b^2 c^6 d^3 x^7 - 3 a^7 b c^7 d^2 x + a^5 b^3 c^8 d x^4 - a^7 b c^6 d^3 x^4 + a^4 b^4 c^8 d x^7 + 2 a^7 b c^5 d^4 x^7) - (14 a b^5 c^7 x^4 (c + d x^3)^{(1/2)}) / (a^5 b^3 c^9 x + 2 a^8 c^6 d^3 x + a^4 b^4 c^9 x^4 + 2 a^8 c^5 d^4 x^4 - 3 a^6 b^2 c^7 d^2 x^4 - 3 a^6 b^2 c^6 d^3 x^7 - 3 a^7 b c^7 d^2 x + a^5 b^3 c^8 d x^4 - a^7 b c^6 d^3 x^4 + a^4 b^4 c^8 d x^7 + 2 a^7 b c^5 d^4 x^7) + (a^3 b^4 c^7 x (c + d x^3)^{(1/2)}) / (8 (a^6 b^3 c^9 x + 2 a^9 c^6 d^3 x + a^5 b^4 c^9 x^4 + 2 a^9 c^5 d^4 x^4 - 3 a^7 b^2 c^7 d^2 x^4 - 3 a^7 b^2 c^6 d^3 x^7 - 3 a^8 b c^7 d^2 x + a^6 b^3 c^8 d x^4 - a^8 b c^6 d^3 x^4 + a^5 b^4 c^8 d x^7 + 2 a^8 b c^5 d^4 x^7)) + (269 a^4 b^4 c^8 x (c + d x^3)^{(1/2)}) / (24 (a^7 b^3 c^10 x + 2 a^{10} c^7 d^3 x + a^6 b^4 c^10 x^4 + 2 a^{10} c^6 d^4 x^4 - 3 a^8 b^2 c^8 d^2 x^4 - 3 a^8 b^2 c^7 d^3 x^7 - 3 a^9 b c^8 d^2 x + a^7 b^3 c^9 d x^4 - a^9 b c^7 d^3 x^4 + a^6 b^4 c^9 d x^7 + 2 a^9 b c^6 d^4 x^7)) + (65 a^6 c^2 d^4 x (c + d x^3)^{(1/2)}) / (8 (a^5 b^3 c^8 x + 2 a^8 c^5 d^3 x + a^4 b^4 c^8 x^4 + 2 a^8 c^4 d^4 x^4 - 3 a^6 b^2 c^6 d^2 x^4 - 3 a^6 b^2 c^5 d^3 x^7 - 3 a^7 b c^6 d^2 x + a^5 b^3 c^7 d x^4 - a^7 b c^5 d^3 x^4 + a^4 b^4 c^8 d x^7 + 2 a^7 b c^5 d^4 x^7)) + (65 a^6 c d^5 x^4 (c + d x^3)^{(1/2)}) / (24 (a^5 b^3 c^8 x + 2 a^8 c^5 d^3 x + a^4 b^4 c^8 x^4 + 2 a^8 c^4 d^4 x^4 - 3 a^6 b^2 c^6 d^2 x^4 - 3 a^6 b^2 c^5 d^3 x^7 - 3 a^7 b c^6 d^2 x + a^5 b^3 c^7 d x^4 - a^7 b c^5 d^3 x^4 + a^4 b^4 c^8 d x^7 + 2 a^7 b c^5 d^4 x^7)) - (41 a^6 c^3 d^4 x (c + d x^3)^{(1/2)}) / (3 (a^5 b^3 c^9 x + 2 a^8 c^6 d^3 x + a^4 b^4 c^9 x^4 + 2 a^8 c^5 d^4 x^4 - 3 a^6 b^2 c^7 d^2 x^4 - 3 a^6 b^2 c^6 d^3 x^7 - 3 a^7 b c^7 d^2 x + a^5 b^3 c^8 d x^4 - a^7 b
\end{aligned}$$

$$\begin{aligned}
& b^6c^3d^3x^4 + a^4b^4c^8d^3x^7 + 2a^7b^3c^5d^4x^7) - (5a^7c^3d^4x^4 \\
& x^4(c + d^3x^3)^{(1/2)}) / (8(a^6b^3c^9x + 2a^9c^6d^3x + a^5b^4c^9x^4 \\
& + 2a^9c^5d^4x^4 - 3a^7b^2c^7d^2x^4 - 3a^7b^2c^6d^3x^7 - 3a^8 \\
& *b^3c^7d^2x + a^6b^3c^8d^3x^4 - a^8b^3c^6d^3x^4 + a^5b^4c^8d^3x^7 + \\
& 2a^8b^3c^5d^4x^7)) + (47a^8c^4d^4x^4(c + d^3x^3)^{(1/2)}) / (6(a^7b^3c^4 \\
& 10x + 2a^10c^7d^3x + a^6b^4c^10x^4 + 2a^10c^6d^4x^4 - 3a^8b^2 \\
& *c^8d^2x^4 - 3a^8b^2c^7d^3x^7 - 3a^9b^3c^8d^2x + a^7b^3c^9d^3x^4 \\
& - a^9b^3c^7d^3x^4 + a^6b^4c^9d^3x^7 + 2a^9b^3c^6d^4x^7)) - (34a^8 \\
& *b^2c^5x^4(c + d^3x^3)^{(1/2)}) / (3(a^5b^2c^7x^4 + 2a^7c^5d^2x^4 + a^6 \\
& *b^3c^7x + 2a^7c^6d^2x + 3a^6b^3c^6d^2x^4 + a^5b^2c^6d^2x^7 + 2a^6b^3 \\
& c^5d^2x^7)) + (47a^3c^2d^2x^4(c + d^3x^3)^{(1/2)}) / (3(a^3b^2c^6x^4 + \\
& 2a^5c^4d^2x^4 + a^4b^3c^6x + 2a^5c^5d^2x + 3a^4b^3c^5d^2x^4 + a^3b^2 \\
& c^5d^2x^7 + 2a^4b^3c^4d^2x^7)) + (38a^3c^3d^3x^4(c + d^3x^3)^{(1/2)}) \\
& / (3(a^3b^2c^6x^4 + 2a^5c^4d^2x^4 + a^4b^3c^6x + 2a^5c^5d^2x + 3a^4 \\
& b^3c^5d^2x^4 + a^3b^2c^5d^2x^7 + 2a^4b^3c^4d^2x^7)) - (257a^5c^3d^2 \\
& x^4(c + d^3x^3)^{(1/2)}) / (24(a^5b^2c^7x^4 + 2a^7c^5d^2x^4 + a^6b^3c^7 \\
& x + 2a^7c^6d^2x + 3a^6b^3c^6d^2x^4 + a^5b^2c^6d^2x^7 + 2a^6b^3c^5d^2 \\
& x^7)) - (5a^9c^3d^6x^4(c + d^3x^3)^{(1/2)}) / (4(a^6b^5c^9x + a^5b^6c^9 \\
& x^4 - 3a^7b^4c^7d^2x^4 - a^8b^3c^6d^3x^4 + 2a^9b^2c^5d^4x^4 - 3a^7b^4 \\
& c^6d^3x^7 + 2a^8b^3c^5d^4x^7 - 3a^8b^3c^7d^2x + 2a^9b^2c^6d^3x + a^6b^5 \\
& c^8d^3x^4 + a^5b^6c^8d^3x^7)) + (23a^9c^2d^6x^4(c + d^3x^3)^{(1/2)}) / (8(a^7 \\
& b^4c^10x + a^6b^5c^10x^4 - 3a^8b^3c^8d^2x^4 - a^9b^2c^7d^3x^4 - 3a^8b^3c^7 \\
& d^3x^7 + 2a^9b^2c^6d^4x^7 + 2a^10b^3c^7d^3x - 3a^9b^2c^8d^2x + a^7b^4c^9 \\
& d^3x^4 + 2a^10b^3c^6d^4x^4 + a^6b^5c^9d^3x^7)) + (a^2b^6c^10x^4(c + d^3x^3)^{(1/2)}) \\
& / (2(2a^10c^7d^5x + 2a^10c^6d^6x^4 + a^6b^4c^10d^2x^4 + a^7b^3c^9d^3x^4 - 3a^8 \\
& b^2c^8d^4x^4 + a^6b^4c^9d^3x^7 - 3a^8b^2c^7d^5x^7 - 3a^9b^3c^8d^4x + a^7b^3 \\
& c^10d^2x - a^9b^3c^7d^5x^4 + 2a^9b^3c^6d^6x^7)) + (a^2b^5c^7x^4(c + d^3x^3)^{(1/2)}) \\
& / (8(a^6b^3c^9x + 2a^9c^6d^3x + a^5b^4c^9x^4 + 2a^9c^5d^4x^4 - 3a^7b^2c^7d^2x^4 - 3a^7 \\
& b^2c^6d^3x^7 - 3a^8b^3c^7d^2x + a^6b^3c^8d^3x^4 - a^8b^3c^6d^3x^4 + a^5 \\
& b^4c^8d^3x^7 + 2a^8b^3c^5d^4x^7)) + (269a^3b^5c^8x^4(c + d^3x^3)^{(1/2)}) / (24(a^7 \\
& b^3c^10x + 2a^10c^7d^3x + a^6b^4c^10x^4 + 2a^10c^6d^4x^4 - 3a^8b^2c^8d^2x^4 - 3a^8 \\
& b^2c^7d^3x^7 - 3a^9b^3c^8d^2 \\
& x + a^7b^3c^9d^3x^4 - a^9b^3c^7d^3x^4 + a^6b^4c^9d^3x^7 + 2a^9b^3c^6d^4x^7)) - (26a^6c^2 \\
& d^5x^4(c + d^3x^3)^{(1/2)}) / (3(a^5b^3c^9x + 2a^8c^6d^3x + a^4b^4c^9x^4 + 2a^8c^5d^4x^4 - 3a^6 \\
& b^2c^7d^2x^4 - 3a^6b^2c^6d^3x^7 - 3a^7b^3c^7d^2x + a^5b^3c^8d^3x^4 - a^7b^3c^6 \\
& d^3x^4 + a^4b^4c^8d^3x^7 + 2a^7b^3c^5d^4x^7)) - (5a^7c^2d^5x^4(c + d^3x^3)^{(1/2)}) / (8(a^6 \\
& b^3c^9x + 2a^9c^6d^3x + a^5b^4c^9x^4 + 2a^9c^5d^4x^4 - 3a^7b^2c^7d^2x^4 - 3a^7b^2c^6 \\
& d^3x^7 - 3a^8b^3c^7d^2x + a^6b^3c^8d^3x^4 - a^8b^3c^6d^3x^4 + a^5b^4c^8d^3x^7 + 2a^8 \\
& b^3c^5d^4x^7))
\end{aligned}$$

$$\begin{aligned}
& *c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2* \\
& a^8*b*c^5*d^4*x^7) + (79*a^8*c^3*d^5*x^4*(c + d*x^3)^{(1/2)})/(12*(a^7*b^3*c \\
& ^{10}*x + 2*a^{10}*c^7*d^3*x + a^6*b^4*c^{10}*x^4 + 2*a^{10}*c^6*d^4*x^4 - 3*a^8*b^ \\
& 2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x \\
& ^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (34*a^ \\
& 2*b^3*c^5*x^4*(c + d*x^3)^{(1/2)})/(3*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + \\
& a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6 \\
& *b*c^5*d^2*x^7)) - (239*a^5*c^2*d^3*x^4*(c + d*x^3)^{(1/2)})/(24*(a^5*b^2*c^7 \\
& *x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 \\
& + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) - (3*a^2*b^5*c^8*x*(c + d*x^3)^ \\
& (1/2))/(4*(2*a^9*c^6*d^4*x + 2*a^9*c^5*d^5*x^4 + a^6*b^3*c^8*d^2*x^4 - 3*a^ \\
& 7*b^2*c^7*d^3*x^4 + a^5*b^4*c^8*d^2*x^7 - 3*a^7*b^2*c^6*d^4*x^7 + a^6*b^3*c \\
& ^9*d*x - 3*a^8*b*c^7*d^3*x + a^5*b^4*c^9*d*x^4 - a^8*b*c^6*d^4*x^4 + 2*a^8* \\
& b*c^5*d^5*x^7)) - (3*a*b^6*c^8*x^4*(c + d*x^3)^{(1/2)})/(4*(2*a^9*c^6*d^4*x + \\
& 2*a^9*c^5*d^5*x^4 + a^6*b^3*c^8*d^2*x^4 - 3*a^7*b^2*c^7*d^3*x^4 + a^5*b^4*c \\
& ^8*d^2*x^7 - 3*a^7*b^2*c^6*d^4*x^7 + a^6*b^3*c^9*d*x - 3*a^8*b*c^7*d^3*x + \\
& a^5*b^4*c^9*d*x^4 - a^8*b*c^6*d^4*x^4 + 2*a^8*b*c^5*d^5*x^7)) + (3*a^3*b^5 \\
& *c^9*x*(c + d*x^3)^{(1/2)})/(4*(2*a^{10}*c^7*d^4*x + 2*a^{10}*c^6*d^5*x^4 + a^7*b \\
& ^3*c^9*d^2*x^4 - 3*a^8*b^2*c^8*d^3*x^4 + a^6*b^4*c^9*d^2*x^7 - 3*a^8*b^2*c^ \\
& 7*d^4*x^7 + a^7*b^3*c^{10}*d*x - 3*a^9*b*c^8*d^3*x + a^6*b^4*c^{10}*d*x^4 - a^9 \\
& *b*c^7*d^4*x^4 + 2*a^9*b*c^6*d^5*x^7)) + (5*a^4*c*d^3*x*(c + d*x^3)^{(1/2)})/ \\
& (2*(a^4*b^2*c^6*x + a^3*b^3*c^6*x^4 + 2*a^5*b*c^5*d*x + 2*a^4*b^2*c^4*d^2*x \\
& ^7 + 3*a^4*b^2*c^5*d*x^4 + 2*a^5*b*c^4*d^2*x^4 + a^3*b^3*c^5*d*x^7)) - (8*a \\
& *b^4*c^5*x*(c + d*x^3)^{(1/2)})/(3*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4 \\
& *c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^ \\
& 7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6 \\
& *d*x^7 + 2*a^6*b*c^3*d^4*x^7)) - (5*a^5*c*d^4*x*(c + d*x^3)^{(1/2)})/(a^4*b^3 \\
& *c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2* \\
& c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 \\
& - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7) + (5*a^{10}*c \\
& ^2*d^6*x*(c + d*x^3)^{(1/2)})/(4*(a^7*b^5*c^{10}*x + a^6*b^6*c^{10}*x^4 - 3*a^8*b \\
& ^4*c^8*d^2*x^4 - a^9*b^3*c^7*d^3*x^4 + 2*a^{10}*b^2*c^6*d^4*x^4 - 3*a^8*b^4*c \\
& ^7*d^3*x^7 + 2*a^9*b^3*c^6*d^4*x^7 - 3*a^9*b^3*c^8*d^2*x + 2*a^{10}*b^2*c^7*d \\
& ^3*x + a^7*b^5*c^9*d*x^4 + a^6*b^6*c^9*d*x^7)) + (5*a^{10}*c*d^7*x^4*(c + d*x \\
& ^3)^{(1/2)})/(4*(a^7*b^5*c^{10}*x + a^6*b^6*c^{10}*x^4 - 3*a^8*b^4*c^8*d^2*x^4 - \\
& a^9*b^3*c^7*d^3*x^4 + 2*a^{10}*b^2*c^6*d^4*x^4 - 3*a^8*b^4*c^7*d^3*x^7 + 2*a^ \\
& 9*b^3*c^6*d^4*x^7 - 3*a^9*b^3*c^8*d^2*x + 2*a^{10}*b^2*c^7*d^3*x + a^7*b^5*c^ \\
& 9*d*x^4 + a^6*b^6*c^9*d*x^7)) - (11*a*b^2*c^3*x*(c + d*x^3)^{(1/2)})/(3*(a^3* \\
& b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4 \\
& *d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) + (14*a*b^2*c^4*x*(c + d \\
& *x^3)^{(1/2)})/(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5 \\
& *d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7) + (11*a \\
& *b*d^2*x^4*(c + d*x^3)^{(1/2)})/(2*(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3 \\
& *b*c^4*x + 2*a^4*c^3*d*x + 3*a^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b* \\
& c^2*d^2*x^7)) - (143*a^3*c*d^2*x*(c + d*x^3)^{(1/2)})/(24*(a^3*b^2*c^5*x^4 +
\end{aligned}$$

$$\begin{aligned}
& 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b \\
& \wedge 2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) + (22*b^2*c*d*x^4*(c + d*x^3)^{(1/2)})/(\\
& 3*(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a^ \\
& 3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7)) + (11*a^3*b^2*c^3 \\
& *d^2*x*(c + d*x^3)^{(1/2)})/(3*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7 \\
& *x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - \\
& 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x \\
& ^7 + 2*a^6*b*c^3*d^4*x^7)) - (31*a^4*b^2*c^4*d^2*x*(c + d*x^3)^{(1/2)})/(2*(a \\
& ^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^ \\
& 6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7 \\
& *d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) + (2 \\
& 8*a^2*b^4*c^5*d*x^4*(c + d*x^3)^{(1/2)})/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x \\
& + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c \\
& ^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^ \\
& 4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) + (29*a^5*b*c^2*d^4*x^4*(c + d*x^3) \\
& ^{(1/2)})/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d \\
& ^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x \\
& + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d \\
& ^4*x^7)) + (23*a^4*b^2*c^5*d^2*x*(c + d*x^3)^{(1/2)})/(3*(a^5*b^3*c^9*x + 2*a \\
& ^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 \\
& - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6 \\
& *d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) + (7*a^2*b^4*c^6*d*x^4 \\
& *(c + d*x^3)^{(1/2)})/(3*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + \\
& 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7* \\
& b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2 \\
& *a^7*b*c^5*d^4*x^7)) - (7*a^5*b*c^3*d^4*x^4*(c + d*x^3)^{(1/2)})/(4*(a^5*b^3* \\
& c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c \\
& ^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 \\
& - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (9*a^5*b^ \\
& 2*c^5*d^2*x*(c + d*x^3)^{(1/2)})/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^ \\
& 4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x \\
& ^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^ \\
& 8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (9*a^3*b^4*c^6*d*x^4*(c + d*x^3)^{(1/2)})/(\\
& 8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - \\
& 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3 \\
& *c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) \\
& - (9*a^6*b*c^3*d^4*x^4*(c + d*x^3)^{(1/2)})/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3 \\
& *x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^ \\
& 2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + \\
& a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (23*a^6*b^2*c^6*d^2*x*(c + d*x \\
& ^3)^{(1/2)})/(24*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^ \\
& 10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^ \\
& 8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9 \\
& *b*c^6*d^4*x^7)) - (31*a^4*b^4*c^7*d*x^4*(c + d*x^3)^{(1/2)})/(8*(a^7*b^3*c^1 \\
& 0*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*
\end{aligned}$$

$$\begin{aligned}
& c^8 d^2 x^4 - 3 a^8 b^2 c^7 d^3 x^7 - 3 a^9 b^3 c^8 d^2 x^4 + a^7 b^3 c^9 d x^4 \\
& - a^9 b^3 c^7 d^3 x^4 + a^6 b^4 c^9 d x^7 + 2 a^9 b^3 c^6 d^4 x^7) - (21 a^7 b^3 c^4 d^4 x^4 (c + d x^3)^{(1/2)}) / (8 (a^7 b^3 c^{10} x + 2 a^{10} c^7 d^3 x + a^6 b^4 c^{10} x^4 + 2 a^{10} c^6 d^4 x^4 - 3 a^8 b^2 c^8 d^2 x^4 - 3 a^8 b^2 c^7 d^3 x^7 - 3 a^9 b^3 c^8 d^2 x^4 + a^7 b^3 c^9 d x^4 - a^9 b^3 c^7 d^3 x^4 + a^6 b^4 c^9 d x^7 + 2 a^9 b^3 c^6 d^4 x^7)) + (209 a^2 b^3 c^2 d^2 x^4 (c + d x^3)^{(1/2)}) / (12 (a^3 b^2 c^6 x^4 + 2 a^5 c^4 d^2 x^4 + a^4 b^3 c^6 x + 2 a^5 c^5 d x^2 + 3 a^4 b^3 c^5 d x^4 + a^3 b^2 c^5 d x^7 + 2 a^4 b^3 c^4 d^2 x^7)) - (89 a^3 b^2 c^4 d x^4 (c + d x^3)^{(1/2)}) / (6 (a^5 b^2 c^7 x^4 + 2 a^7 c^5 d^2 x^4 + a^6 b^3 c^7 x + 2 a^7 c^6 d x + 3 a^6 b^3 c^6 d x^4 + a^5 b^2 c^6 d x^7 + 2 a^6 b^3 c^5 d^2 x^7)) - (109 a^4 b^3 c^3 d^2 x^4 (c + d x^3)^{(1/2)}) / (12 (a^5 b^2 c^7 x^4 + 2 a^7 c^5 d^2 x^4 + a^6 b^3 c^7 x + 2 a^7 c^6 d x + 3 a^6 b^3 c^6 d x^4 + a^5 b^2 c^6 d x^7 + 2 a^6 b^3 c^5 d^2 x^7)) - (33 a^2 b^3 c^2 d x^4 (c + d x^3)^{(1/2)}) / (2 (a^3 b^2 c^5 x^4 + 2 a^5 c^3 d^2 x^4 + a^4 b^3 c^5 x + 2 a^5 c^4 d x + 3 a^4 b^3 c^4 d x^4 + a^3 b^2 c^4 d x^7 + 2 a^4 b^3 c^3 d^2 x^7)) + (329 a^2 b^3 c^3 d x^4 (c + d x^3)^{(1/2)}) / (12 (a^3 b^2 c^6 x^4 + 2 a^5 c^4 d^2 x^4 + a^4 b^3 c^6 x + 2 a^5 c^5 d x^2 + 3 a^4 b^3 c^5 d x^4 + a^3 b^2 c^5 d x^7 + 2 a^4 b^3 c^4 d^2 x^7)) - (205 a^4 b^3 c^4 d x^4 (c + d x^3)^{(1/2)}) / (12 (a^5 b^2 c^7 x^4 + 2 a^7 c^5 d^2 x^4 + a^6 b^3 c^7 x + 2 a^7 c^6 d x + 3 a^6 b^3 c^6 d x^4 + a^5 b^2 c^6 d x^7 + 2 a^6 b^3 c^5 d^2 x^7)) + (6 a^2 b^3 c^3 d^2 x^4 (c + d x^3)^{(1/2)}) / (a^4 b^3 c^7 x + 2 a^7 c^4 d^3 x + a^3 b^4 c^7 x^4 + 2 a^7 c^3 d^4 x^4 - 3 a^5 b^2 c^5 d^2 x^4 - 3 a^5 b^2 c^4 d^3 x^7 - 3 a^6 b^3 c^5 d^2 x^4 + a^4 b^3 c^6 d x^4 - a^6 b^3 c^4 d^3 x^4 + a^3 b^4 c^6 d x^7 + 2 a^6 b^3 c^3 d^4 x^7) - (a^3 b^2 c^2 d^3 x^4 (c + d x^3)^{(1/2)}) / (a^4 b^3 c^7 x + 2 a^7 c^4 d^3 x + a^3 b^4 c^7 x^4 + 2 a^7 c^3 d^4 x^4 - 3 a^5 b^2 c^5 d^2 x^4 - 3 a^5 b^2 c^4 d^3 x^7 - 3 a^6 b^3 c^5 d^2 x^4 + a^4 b^3 c^6 d x^4 - a^6 b^3 c^4 d^3 x^4 + a^3 b^4 c^6 d x^7 + 2 a^6 b^3 c^3 d^4 x^7) - (19 a^3 b^3 c^4 d^2 x^4 (c + d x^3)^{(1/2)}) / (3 (a^5 b^3 c^8 x + 2 a^8 c^5 d^3 x + a^4 b^4 c^8 x^4 + 2 a^8 c^4 d^4 x^4 - 3 a^6 b^2 c^6 d^2 x^4 - 3 a^6 b^2 c^5 d^3 x^7 - 3 a^7 b^3 c^6 d^2 x^4 + a^5 b^3 c^7 d x^4 - a^7 b^3 c^5 d^3 x^4 + a^4 b^4 c^7 d x^7 + 2 a^7 b^3 c^4 d^4 x^7)) + (10 a^4 b^2 c^3 d^3 x^4 (c + d x^3)^{(1/2)}) / (3 (a^5 b^3 c^8 x + 2 a^8 c^5 d^3 x + a^4 b^4 c^8 x^4 + 2 a^8 c^4 d^4 x^4 - 3 a^6 b^2 c^6 d^2 x^4 - 3 a^6 b^2 c^5 d^3 x^7 - 3 a^7 b^3 c^6 d^2 x^4 + a^5 b^3 c^7 d x^4 - a^7 b^3 c^5 d^3 x^4 + a^4 b^4 c^7 d x^7 + 2 a^7 b^3 c^4 d^4 x^7)) - (a^3 b^3 c^5 d^2 x^4 (c + d x^3)^{(1/2)}) / (2 (a^5 b^3 c^9 x + 2 a^8 c^6 d^3 x + a^4 b^4 c^9 x^4 + 2 a^8 c^5 d^4 x^4 - 3 a^6 b^2 c^7 d^2 x^4 - 3 a^6 b^2 c^6 d^3 x^7 - 3 a^7 b^3 c^7 d^2 x^4 + a^5 b^3 c^8 d x^4 - a^7 b^3 c^6 d^3 x^4 + a^4 b^4 c^8 d x^7 + 2 a^7 b^3 c^5 d^4 x^7)) - (97 a^4 b^2 c^4 d^3 x^4 (c + d x^3)^{(1/2)}) / (6 (a^5 b^3 c^9 x + 2 a^8 c^6 d^3 x + a^4 b^4 c^9 x^4 + 2 a^8 c^5 d^4 x^4 - 3 a^6 b^2 c^7 d^2 x^4 - 3 a^6 b^2 c^6 d^3 x^7 - 3 a^7 b^3 c^7 d^2 x^4 + a^5 b^3 c^8 d x^4 - a^7 b^3 c^6 d^3 x^4 + a^4 b^4 c^8 d x^7 + 2 a^7 b^3 c^5 d^4 x^7)) - (9 a^4 b^3 c^5 d^2 x^4 (c + d x^3)^{(1/2)}) / (8 (a^6 b^3 c^9 x + 2 a^9 c^6 d^3 x + a^5 b^4 c^9 x^4 + 2 a^9 c^5 d^4 x^4 - 3 a^7 b^2 c^7 d^2 x^4 - 3 a^7 b^2 c^6 d^3 x^7 - 3 a^8 b^3 c^7 d^2 x^4 + a^6 b^3 c^8 d x^4 - a^8 b^3 c^6 d^3 x^4 + a^5 b^4 c^8 d x^7 + 2 a^8 b^3 c^5 d^4 x^7)) - (9 a^5 b^2 c^4 d^3 x^4
\end{aligned}$$

$$\begin{aligned}
& 4*(c + d*x^3)^{(1/2)})/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 \\
& + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8 \\
& *b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + \\
& 2*a^8*b*c^5*d^4*x^7)) + (31*a^5*b^3*c^6*d^2*x^4*(c + d*x^3)^{(1/2)})/(24*(a^7 \\
& *b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3* \\
& a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c \\
& ^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) + \\
& (415*a^6*b^2*c^5*d^3*x^4*(c + d*x^3)^{(1/2)})/(24*(a^7*b^3*c^10*x + 2*a^10*c^ \\
& 7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3 \\
& *a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^ \\
& 3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) + (11*a*b*c*d*x*(c + d*x^ \\
& 3)^{(1/2)})/(2*(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3 \\
& *d*x + 3*a^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7)) - (10* \\
& a^2*b^3*c^4*d*x*(c + d*x^3)^{(1/2)})/(3*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^ \\
& 3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d \\
& ^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^ \\
& 4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7)) - (19*a^4*b*c^2*d^3*x*(c + d*x^3)^{(1/2)} \\
&)/(6*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 \\
& - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4* \\
& b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7 \\
&)) - (8*a*b^4*c^4*d*x^4*(c + d*x^3)^{(1/2)})/(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x \\
& + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2* \\
& c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a \\
& ^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7) - (15*a^4*b*c*d^4*x^4*(c + d*x^3)^{(\\
& 1/2)})/(2*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4 \\
& *x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + \\
& a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4 \\
& *x^7)) + (12*a^3*b^3*c^5*d*x*(c + d*x^3)^{(1/2)})/(a^5*b^3*c^8*x + 2*a^8*c^5* \\
& d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6 \\
& *b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^ \\
& 4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7) + (51*a^5*b*c^3*d^3*x*(c + d*x \\
& ^3)^{(1/2)})/(4*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^ \\
& 4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2 \\
& *x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^ \\
& 4*d^4*x^7)) - (14*a^3*b^3*c^6*d*x*(c + d*x^3)^{(1/2)})/(3*(a^5*b^3*c^9*x + 2* \\
& a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 \\
& - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6 \\
& *d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (169*a^5*b*c^4*d^3* \\
& x*(c + d*x^3)^{(1/2)})/(12*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 \\
& + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^ \\
& 7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + \\
& 2*a^7*b*c^5*d^4*x^7)) - (9*a^4*b^3*c^6*d*x*(c + d*x^3)^{(1/2)})/(8*(a^6*b^3* \\
& c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c \\
& ^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 \\
& - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (9*a^6*b*
\end{aligned}$$


```

c^4*d^3*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*
c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7
- 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*
d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (7*a^5*b^3*c^7*d*x*(c + d*x^3)^(1/2))/(8*(a
^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 -
3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3
*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7))
+ (77*a^7*b*c^5*d^3*x*(c + d*x^3)^(1/2))/(8*(a^7*b^3*c^10*x + 2*a^10*c^7*d^
3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8
*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^
4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (41*a*b^2*c^2*d*x^4*(c + d*
x^3)^(1/2))/(3*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c
^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) - (7
1*a^2*b*c*d^2*x^4*(c + d*x^3)^(1/2))/(6*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^
4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2
*a^4*b*c^3*d^2*x^7)) + (133*a*b^2*c^3*d*x^4*(c + d*x^3)^(1/2))/(6*(a^3*b^2*
c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x
^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + bx^3)^2 (c + dx^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)

[Out] Integral(1/(x**4*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)

$$3.321 \quad \int \frac{x^5}{\sqrt{a+bx^3} \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=88

$$\frac{\sqrt{a+bx^3} \sqrt{c+dx^3}}{3bd} - \frac{(ad+bc) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}$$

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {446, 80, 63, 217, 206}

$$\frac{\sqrt{a+bx^3} \sqrt{c+dx^3}}{3bd} - \frac{(ad+bc) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]

[Out] (Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*b^(3/2)*d^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{\sqrt{a+bx}\sqrt{c+dx}} dx, x, x^3 \right) \\
 &= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad) \text{Subst} \left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}} dx, x, x^3 \right)}{6bd} \\
 &= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad) \text{Subst} \left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}} dx, x, \sqrt{a+bx^3} \right)}{3b^2d} \\
 &= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad) \text{Subst} \left(\int \frac{1}{1-\frac{dx^2}{b}} dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}} \right)}{3b^2d} \\
 &= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad) \tanh^{-1} \left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}} \right)}{3b^{3/2}d^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.27, size = 123, normalized size = 1.40

$$\frac{b\sqrt{d}\sqrt{a+bx^3}(c+dx^3) - \sqrt{bc-ad}(ad+bc)\sqrt{\frac{b(c+dx^3)}{bc-ad}} \sinh^{-1} \left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{bc-ad}} \right)}{3b^2d^{3/2}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]), x]

[Out] $(b\sqrt{d}\sqrt{a+bx^3})(c+dx^3) - \sqrt{bc-ad}(b^2c+ad^2)\sqrt{d}\sqrt{a+bx^3} / (b^2c-ad^2) \operatorname{ArcSinh}(\sqrt{d}\sqrt{a+bx^3}) / \sqrt{bc-ad} / (3b^2d^{3/2}\sqrt{c+dx^3})$

IntegrateAlgebraic [A] time = 1.30, size = 122, normalized size = 1.39

$$\frac{(-ad - bc) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{d}\sqrt{a+bx^3}}\right)}{3b^{3/2}d^{3/2}} - \frac{\sqrt{c+dx^3}(ad - bc)}{3bd\sqrt{a+bx^3}\left(\frac{b(c+dx^3)}{a+bx^3} - d\right)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]

[Out] $-1/3*((-b^2c+ad)\sqrt{c+dx^3})/(b^2d\sqrt{a+bx^3})(-d+(b^2c+ad^2)/(a+bx^3)) + ((-b^2c-ad)\operatorname{ArcTanh}(\sqrt{b}\sqrt{c+dx^3})/(\sqrt{d}\sqrt{a+bx^3}))/ (3b^2d^{3/2})$

fricas [A] time = 0.58, size = 256, normalized size = 2.91

$$\frac{4\sqrt{bx^3+a}\sqrt{dx^3+c}bd+(bc+ad)\sqrt{bd}\log\left(8b^2d^2x^6+b^2c^2+6abcd+a^2d^2+8(b^2cd+abd^2)x^3-4(2bdx^3+bc+ad)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{bd}\right)}{12b^2d^2} + \frac{2\sqrt{bx^3+a}\sqrt{dx^3+c}bd+(bc+ad)\sqrt{-bd}\arctan\left(\frac{(2bdx^3+bc+ad)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{-bd}}{2(b^2d^2x^6+abcd+(b^2cd+abd^2)x^3)}\right)}{6b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] $[1/12*(4*\sqrt{bx^3+a}*\sqrt{dx^3+c}*b*d + (b^2c+ad)*\sqrt{bd}*\log(8*b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^3 - 4*(2*b*d*x^3 + b^2*c + a*d)*\sqrt{bx^3+a}*\sqrt{dx^3+c}*\sqrt{bd}))/ (b^2*d^2), 1/6*(2*\sqrt{bx^3+a}*\sqrt{dx^3+c}*b*d + (b^2c+ad)*\sqrt{-bd}*\arctan(1/2*(2*b*d*x^3 + b^2*c + a*d)*\sqrt{bx^3+a}*\sqrt{dx^3+c}*\sqrt{-bd}) / (b^2*d^2*x^6 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^3)) / (b^2*d^2)]$

giac [A] time = 0.21, size = 104, normalized size = 1.18

$$\frac{(bc+ad)\log\left(-\sqrt{bx^3+a}\sqrt{bd}+\sqrt{b^2c+(bx^3+a)bd-abd}\right)}{\sqrt{bd}d} + \frac{\sqrt{bx^3+a}\sqrt{b^2c+(bx^3+a)bd-abd}}{bd}$$

$3|b|$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] $1/3*((b^2c+ad)*\log(\operatorname{abs}(-\sqrt{bx^3+a}*\sqrt{bd} + \sqrt{b^2c+(bx^3+a)bd-abd}))/(\sqrt{bd}*d) + \sqrt{bx^3+a}*\sqrt{b^2c+(bx^3+a)bd-abd}) / \operatorname{abs}(b)$

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{bx^3+a} \sqrt{dx^3+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)

[Out] int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c zero or nonzero?

mupad [B] time = 9.25, size = 283, normalized size = 3.22

$$\frac{\frac{(\sqrt{bx^3+a}-\sqrt{a})\left(\frac{2ad}{3}+\frac{2bc}{3}\right)}{d^3(\sqrt{dx^3+c}-\sqrt{c})} + \frac{(\sqrt{bx^3+a}-\sqrt{a})^3\left(\frac{2ad}{3}+\frac{2bc}{3}\right)}{bd^2(\sqrt{dx^3+c}-\sqrt{c})^3} - \frac{8\sqrt{a}\sqrt{c}(\sqrt{bx^3+a}-\sqrt{a})^2}{3d^2(\sqrt{dx^3+c}-\sqrt{c})^2}}{\frac{(\sqrt{bx^3+a}-\sqrt{a})^4}{(\sqrt{dx^3+c}-\sqrt{c})^4} + \frac{b^2}{d^2} - \frac{2b(\sqrt{bx^3+a}-\sqrt{a})^2}{d(\sqrt{dx^3+c}-\sqrt{c})^2}} - \frac{2 \operatorname{atanh}\left(\frac{\sqrt{d}(\sqrt{bx^3+a}-\sqrt{a})}{\sqrt{b}(\sqrt{dx^3+c}-\sqrt{c})}\right)(ad+bc)}{3b^{3/2}d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)

[Out] (((((a + b*x^3)^(1/2) - a^(1/2))*((2*a*d)/3 + (2*b*c)/3))/(d^3*((c + d*x^3)^(1/2) - c^(1/2))) + (((a + b*x^3)^(1/2) - a^(1/2))^3*((2*a*d)/3 + (2*b*c)/3))/(b*d^2*((c + d*x^3)^(1/2) - c^(1/2))^3) - (8*a^(1/2)*c^(1/2)*((a + b*x^3)^(1/2) - a^(1/2))^2)/(3*d^2*((c + d*x^3)^(1/2) - c^(1/2))^2))/(((a + b*x^3)^(1/2) - a^(1/2))^4/((c + d*x^3)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x^3)^(1/2) - a^(1/2))^2)/(d*((c + d*x^3)^(1/2) - c^(1/2))^2)) - (2*atanh((d^(1/2)*((a + b*x^3)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^3)^(1/2) - c^(1/2))))*(a*d + b*c))/(3*b^(3/2)*d^(3/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a + bx^3} \sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2), x)

[Out] Integral(x**5/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)

$$3.322 \quad \int \frac{x^2}{\sqrt{a+bx^3} \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=48

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^3}}{\sqrt{b} \sqrt{c+dx^3}} \right)}{3\sqrt{b} \sqrt{d}}$$

Rubi [A] time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {444, 63, 217, 206}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^3}}{\sqrt{b} \sqrt{c+dx^3}} \right)}{3\sqrt{b} \sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]

[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*Sqrt[b]*Sqrt[d])

Rule 63

Int[((a_) + (b_)*(x_)^(m))*((c_) + (d_)*(x_)^(n)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 444

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x

] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}} dx, x, x^3 \right) \\ &= \frac{2 \text{Subst} \left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}} dx, x, \sqrt{a+bx^3} \right)}{3b} \\ &= \frac{2 \text{Subst} \left(\int \frac{1}{1-\frac{dx^2}{b}} dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}} \right)}{3b} \\ &= \frac{2 \tanh^{-1} \left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}} \right)}{3\sqrt{b}\sqrt{d}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 85, normalized size = 1.77

$$\frac{2\sqrt{c+dx^3} \sinh^{-1} \left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{bc-ad}} \right)}{3\sqrt{d}\sqrt{bc-ad} \sqrt{\frac{b(c+dx^3)}{bc-ad}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]), x]

[Out] (2*Sqrt[c + d*x^3]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^3))/(b*c - a*d)])

IntegrateAlgebraic [A] time = 0.85, size = 48, normalized size = 1.00

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{d}\sqrt{a+bx^3}} \right)}{3\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]

[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/(Sqrt[d]*Sqrt[a + b*x^3])])/(3*Sqrt[b]*Sqrt[d])

fricas [B] time = 0.64, size = 194, normalized size = 4.04

$$\left[\frac{\sqrt{bd} \log\left(8b^2d^2x^6 + b^2c^2 + 6abcd + a^2d^2 + 8(b^2cd + abd^2)x^3 + 4(2bdx^3 + bc + ad)\sqrt{bx^3 + a}\sqrt{dx^3 + c}\sqrt{bd}\right)}{6bd}, -\frac{\sqrt{-bd} \arctan\left(\frac{(2bdx^3 + bc + ad)\sqrt{bx^3 + a}\sqrt{dx^3 + c}\sqrt{-bd}}{2(b^2d^2x^6 + abcd + (b^2cd + abd^2)x^3)}\right)}{3bd} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/6*sqrt(b*d)*log(8*b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^3 + 4*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(b*d))/(b*d), -1/3*sqrt(-b*d)*arctan(1/2*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-b*d)/(b^2*d^2*x^6 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^3))/(b*d)]

giac [A] time = 0.18, size = 54, normalized size = 1.12

$$\frac{2b \log\left(\left|-\sqrt{bx^3 + a}\sqrt{bd} + \sqrt{b^2c + (bx^3 + a)bd - abd}\right|\right)}{3\sqrt{bd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] -2/3*b*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d)))/(sqrt(b*d)*abs(b))

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx^3 + a}\sqrt{dx^3 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)

[Out] int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c zero or nonzero?

mupad [B] time = 5.04, size = 49, normalized size = 1.02

$$\frac{4 \operatorname{atan}\left(\frac{b(\sqrt{dx^3+c}-\sqrt{c})}{\sqrt{-bd}(\sqrt{bx^3+a}-\sqrt{a})}\right)}{3\sqrt{-bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)

[Out] -(4*atan((b*((c + d*x^3)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a + b*x^3)^(1/2) - a^(1/2)))))/(3*(-b*d)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + bx^3} \sqrt{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)

[Out] Integral(x**2/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)

$$3.323 \quad \int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}} dx$$

Optimal. Leaf size=48

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {446, 93, 208}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]

[Out] (-2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*Sqrt[a]*Sqrt[c])

Rule 93

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[(((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}} dx, x, x^3 \right) \\
&= \frac{2}{3} \text{Subst} \left(\int \frac{1}{-a+cx^2} dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}} \right) \\
&= -\frac{2 \tanh^{-1} \left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}} \right)}{3\sqrt{a}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.00

$$-\frac{2 \tanh^{-1} \left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}} \right)}{3\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]), x]

[Out] (-2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*Sqrt[a]*Sqrt[c])

IntegrateAlgebraic [A] time = 0.88, size = 48, normalized size = 1.00

$$-\frac{2 \tanh^{-1} \left(\frac{\sqrt{a}\sqrt{c+dx^3}}{\sqrt{c}\sqrt{a+bx^3}} \right)}{3\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]), x]

[Out] (-2*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^3])/(Sqrt[c]*Sqrt[a + b*x^3])])/(3*Sqrt[a]*Sqrt[c])

fricas [B] time = 0.65, size = 204, normalized size = 4.25

$$\left[\frac{\sqrt{ac} \log \left(\frac{(b^2c^2+6abcd+a^2d^2)x^6+8a^2c^2+8(abc^2+a^2cd)x^3-4((bc+ad)x^3+2ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac}}{x^6} \right)}{6ac}, \frac{\sqrt{-ac} \arctan \left(\frac{((bc+ad)x^3+2ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{-ac}}{2(abcdx^6+a^2c^2+(abc^2+a^2cd)x^3)} \right)}{3ac} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/6*sqrt(a*c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^3 - 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(a*c))/x^6)/(a*c), 1/3*sqrt(-a*c)*arctan(1/2*((b*c + a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*x^6 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^3))/(a*c)]

giac [B] time = 0.19, size = 89, normalized size = 1.85

$$\frac{2\sqrt{bd}b \arctan\left(\frac{b^2c+abd - \left(\sqrt{bx^3+a}\sqrt{bd} - \sqrt{b^2c+(bx^3+a)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{3\sqrt{-abcd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] -2/3*sqrt(b*d)*b*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*abs(b))

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)

[Out] int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c zero or nonzero?

mupad [B] time = 7.57, size = 136, normalized size = 2.83

$$\frac{\ln\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{dx^3+c}-\sqrt{c}}\right) - \ln\left(\frac{\left(\sqrt{c}\sqrt{bx^3+a}-\sqrt{a}\sqrt{dx^3+c}\right)\left(b\sqrt{c}-\frac{\sqrt{a}d(\sqrt{bx^3+a}-\sqrt{a})}{\sqrt{dx^3+c}-\sqrt{c}}\right)}{\sqrt{dx^3+c}-\sqrt{c}}\right)}{3\sqrt{a}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)`

[Out] `-(log(((a + b*x^3)^(1/2) - a^(1/2))/((c + d*x^3)^(1/2) - c^(1/2)))) - log(((c^(1/2)*(a + b*x^3)^(1/2) - a^(1/2)*(c + d*x^3)^(1/2))*(b*c^(1/2) - (a^(1/2)*d*((a + b*x^3)^(1/2) - a^(1/2)))/((c + d*x^3)^(1/2) - c^(1/2))))/(c + d*x^3)^(1/2) - c^(1/2)))/(3*a^(1/2)*c^(1/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)`

[Out] `Integral(1/(x*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)`

$$3.324 \quad \int \frac{1}{x^4 \sqrt{a+bx^3} \sqrt{c+dx^3}} dx$$

Optimal. Leaf size=91

$$\frac{(ad + bc) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^3}}{\sqrt{a} \sqrt{c+dx^3}} \right)}{3a^{3/2}c^{3/2}} - \frac{\sqrt{a + bx^3} \sqrt{c + dx^3}}{3acx^3}$$

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {446, 96, 93, 208}

$$\frac{(ad + bc) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^3}}{\sqrt{a} \sqrt{c+dx^3}} \right)}{3a^{3/2}c^{3/2}} - \frac{\sqrt{a + bx^3} \sqrt{c + dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*sqrt[a + b*x^3]*sqrt[c + d*x^3]),x]

[Out] -(sqrt[a + b*x^3]*sqrt[c + d*x^3])/((3*a*c*x^3) + ((b*c + a*d)*ArcTanh[(sqrt[c]*sqrt[a + b*x^3])/(sqrt[a]*sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^4 \sqrt{a + bx^3} \sqrt{c + dx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2 \sqrt{a + bx} \sqrt{c + dx}} dx, x, x^3 \right) \\ &= -\frac{\sqrt{a + bx^3} \sqrt{c + dx^3}}{3acx^3} - \frac{(bc + ad) \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx} \sqrt{c + dx}} dx, x, x^3 \right)}{6ac} \\ &= -\frac{\sqrt{a + bx^3} \sqrt{c + dx^3}}{3acx^3} - \frac{(bc + ad) \text{Subst} \left(\int \frac{1}{-a + cx^2} dx, x, \frac{\sqrt{a + bx^3}}{\sqrt{c + dx^3}} \right)}{3ac} \\ &= -\frac{\sqrt{a + bx^3} \sqrt{c + dx^3}}{3acx^3} + \frac{(bc + ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a + bx^3}}{\sqrt{a} \sqrt{c + dx^3}} \right)}{3a^{3/2} c^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.07, size = 91, normalized size = 1.00

$$\frac{(ad + bc) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a + bx^3}}{\sqrt{a} \sqrt{c + dx^3}} \right)}{3a^{3/2} c^{3/2}} - \frac{\sqrt{a + bx^3} \sqrt{c + dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]

[Out] -1/3*(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(a*c*x^3) + ((b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))

IntegrateAlgebraic [A] time = 1.65, size = 119, normalized size = 1.31

$$\frac{(ad + bc) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a + bx^3}}{\sqrt{a} \sqrt{c + dx^3}} \right)}{3a^{3/2} c^{3/2}} - \frac{\sqrt{a + bx^3} (ad - bc)}{3ac \sqrt{c + dx^3} \left(a - \frac{c(a + bx^3)}{c + dx^3} \right)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]

[Out] $-\frac{1}{3} \frac{(-bc + ad) \sqrt{a + bx^3}}{(c + dx^3)} + \frac{(bc + ad) \operatorname{ArcTanh}\left(\frac{\sqrt{c} \sqrt{a + bx^3}}{\sqrt{a} \sqrt{c + dx^3}}\right)}{3a^{3/2}c^{3/2}}$

fricas [A] time = 0.79, size = 278, normalized size = 3.05

$$\frac{\sqrt{ac}(bc+ad)x^3 \log\left(\frac{(b^2c^2+6abcd+a^2d^2)x^6+8a^2c^2+8(abc^2+a^2cd)x^3+4((bc+ad)x^3+2ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac}}{x^6}\right)-4\sqrt{bx^3+a}\sqrt{dx^3+c}ac-\sqrt{-ac}(bc+ad)x^3 \arctan\left(\frac{((bc+ad)x^3+2ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{-ac}}{2(abcdx^6+a^2c^2+(abc^2+a^2cd)x^3)}\right)+2\sqrt{bx^3+a}\sqrt{dx^3+c}ac}{12a^2c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{12} \frac{\sqrt{ac}(bc+ad)x^3 \log((b^2c^2+6ab^2cd+a^2d^2)x^6+8a^2c^2+8(a^2bc^2+a^2cd)x^3+4((bc+ad)x^3+2ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac})}{x^6} - 4 \frac{\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac}}{a^2c^2x^3} - \frac{1}{6} \frac{\sqrt{-ac}(bc+ad)x^3 \arctan(1/2((bc+ad)x^3+2ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{-ac})}{(ab^2cdx^6+a^2c^2+(abc^2+a^2cd)x^3)} + 2 \frac{\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac}}{a^2c^2x^3}$

giac [B] time = 0.24, size = 413, normalized size = 4.54

$$\frac{\sqrt{bd}b^4d \left(\frac{(bc+ad) \arctan\left(\frac{b^2c+a^2d-\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}ab^3cd} - \frac{2\left(b^3c^2-2ab^2cd+a^2bd^2-\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2 bc - \left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2 ad}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2 b^2c-2\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2 abd + \left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^4 ab^2cd}{3|b|} \right)}{3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{3} \frac{\sqrt{bd}b^4d \left((bc+ad) \arctan\left(\frac{b^2c+a^2d-\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}}{2\sqrt{-abcd}b}\right) - \frac{2\left(b^3c^2-2ab^2cd+a^2bd^2-\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2 bc - \left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2 ad}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2 b^2c-2\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2 abd + \left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^4 ab^2cd} \right)}{3|b|}}{\sqrt{bd}b^4d}$

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)`

[Out] `int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c zero or nonzero?

mupad [B] time = 10.77, size = 481, normalized size = 5.29

$$\frac{\frac{\sqrt{bx^3+a}-\sqrt{a}}{a^{3/2}c^{3/2}d\sqrt{dx^3+c-\sqrt{c}}}-\frac{b^2}{12acd}+\frac{(\sqrt{bx^3+a}-\sqrt{a})^2\left(\frac{d^2a^2}{12}-\frac{abd}{4}+\frac{b^2c^2}{12}\right)}{a^2c^2d\sqrt{dx^3+c-\sqrt{c}}}}{\frac{(\sqrt{bx^3+a}-\sqrt{a})^3}{(\sqrt{dx^3+c-\sqrt{c}})^3}+\frac{b(\sqrt{bx^3+a}-\sqrt{a})}{d\sqrt{dx^3+c-\sqrt{c}}}-\frac{(\sqrt{bx^3+a}-\sqrt{a})^2(ad+bc)}{\sqrt{a}\sqrt{c}d\sqrt{dx^3+c-\sqrt{c}}}}+\frac{\ln\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{dx^3+c-\sqrt{c}}}\right)(\sqrt{a}bc^{3/2}+a^{3/2}\sqrt{c}d)}{6a^2c^2}-\frac{\ln\left(\frac{\left(\sqrt{c}\sqrt{bx^3+a}-\sqrt{a}\sqrt{dx^3+c}\right)\left(b\sqrt{c}-\frac{\sqrt{a}d(\sqrt{bx^3+a}-\sqrt{a})}{\sqrt{dx^3+c-\sqrt{c}}}\right)}{\sqrt{dx^3+c-\sqrt{c}}}\right)(\sqrt{a}bc^{3/2}+a^{3/2}\sqrt{c}d)}{6a^2c^2}}{12ac\sqrt{dx^3+c-\sqrt{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(a+b*x^3)^(1/2)*(c+d*x^3)^(1/2)),x)`

[Out] `((((a+b*x^3)^(1/2)-a^(1/2))*((b^2*c)/12+(a*b*d)/12))/(a^(3/2)*c^(3/2)*d*((c+d*x^3)^(1/2)-c^(1/2)))-b^2/(12*a*c*d)+(((a+b*x^3)^(1/2)-a^(1/2))^2*((a^2*d^2)/12+(b^2*c^2)/12-(a*b*c*d)/4))/(a^2*c^2*d*((c+d*x^3)^(1/2)-c^(1/2))^2)/(((a+b*x^3)^(1/2)-a^(1/2))^3/((c+d*x^3)^(1/2)-c^(1/2))^3+(b*((a+b*x^3)^(1/2)-a^(1/2)))/(d*((c+d*x^3)^(1/2)-c^(1/2))))-(((a+b*x^3)^(1/2)-a^(1/2))^2*(a*d+b*c))/(a^(1/2)*c^(1/2)*d*((c+d*x^3)^(1/2)-c^(1/2))^2)+(log(((a+b*x^3)^(1/2)-a^(1/2))/(c+d*x^3)^(1/2)-c^(1/2)))*(a^(1/2)*b*c^(3/2)+a^(3/2)*c^(1/2)*d))/(6*a^2*c^2)-(log(((c^(1/2)*(a+b*x^3)^(1/2)-a^(1/2)*(c+d*x^3)^(1/2))*(b*c^(1/2)-(a^(1/2)*d*((a+b*x^3)^(1/2)-a^(1/2))))/(c+d*x^3)^(1/2)-c^(1/2))))/(c+d*x^3)^(1/2)-c^(1/2))*(a^(1/2)*b*c^(3/2)+a^(3/2)*c^(1/2)*d))/(6*a^2*c^2)-(d*((a+b*x^3)^(1/2)-a^(1/2)))/(12*a*c*((c+d*x^3)^(1/2)-c^(1/2)))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt{a+bx^3} \sqrt{c+dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.325 $\int (ex)^{7/2} \sqrt{a + bx^3} (A + Bx^3) dx$

Optimal. Leaf size=161

$$-\frac{a^2 e^{7/2} (2Ab - aB) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2} \sqrt{a+bx^3}}\right)}{24b^{5/2}} + \frac{ae^2 (ex)^{3/2} \sqrt{a+bx^3} (2Ab - aB)}{24b^2} + \frac{(ex)^{9/2} \sqrt{a+bx^3} (2Ab - aB)}{12be} + \frac{B(ex)^{9/2} (a + bx^3)^{3/2}}{9be}$$

Rubi [A] time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {459, 279, 321, 329, 275, 217, 206}

$$-\frac{a^2 e^{7/2} (2Ab - aB) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2} \sqrt{a+bx^3}}\right)}{24b^{5/2}} + \frac{ae^2 (ex)^{3/2} \sqrt{a+bx^3} (2Ab - aB)}{24b^2} + \frac{(ex)^{9/2} \sqrt{a+bx^3} (2Ab - aB)}{12be} + \frac{B(ex)^{9/2} (a + bx^3)^{3/2}}{9be}$$

Antiderivative was successfully verified.

[In] Int[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3), x]

[Out] (a*(2*A*b - a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*b^2) + ((2*A*b - a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(12*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(3/2))/(9*b*e) - (a^2*(2*A*b - a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3]])/(24*b^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 279

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p + 1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0]

tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k), x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
\int (ex)^{7/2} \sqrt{a+bx^3} (A+Bx^3) dx &= \frac{B(ex)^{9/2} (a+bx^3)^{3/2}}{9be} - \frac{\left(-9Ab + \frac{9aB}{2}\right) \int (ex)^{7/2} \sqrt{a+bx^3} dx}{9b} \\
&= \frac{(2Ab - aB)(ex)^{9/2} \sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2} (a+bx^3)^{3/2}}{9be} + \frac{(a(2Ab - aB)) \int \frac{(ex)^{7/2}}{\sqrt{a+bx^3}} dx}{8b} \\
&= \frac{a(2Ab - aB)e^2 (ex)^{3/2} \sqrt{a+bx^3}}{24b^2} + \frac{(2Ab - aB)(ex)^{9/2} \sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2} (a+bx^3)^{3/2}}{9be} \\
&= \frac{a(2Ab - aB)e^2 (ex)^{3/2} \sqrt{a+bx^3}}{24b^2} + \frac{(2Ab - aB)(ex)^{9/2} \sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2} (a+bx^3)^{3/2}}{9be} \\
&= \frac{a(2Ab - aB)e^2 (ex)^{3/2} \sqrt{a+bx^3}}{24b^2} + \frac{(2Ab - aB)(ex)^{9/2} \sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2} (a+bx^3)^{3/2}}{9be} \\
&= \frac{a(2Ab - aB)e^2 (ex)^{3/2} \sqrt{a+bx^3}}{24b^2} + \frac{(2Ab - aB)(ex)^{9/2} \sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2} (a+bx^3)^{3/2}}{9be} \\
&= \frac{a(2Ab - aB)e^2 (ex)^{3/2} \sqrt{a+bx^3}}{24b^2} + \frac{(2Ab - aB)(ex)^{9/2} \sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2} (a+bx^3)^{3/2}}{9be}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 145, normalized size = 0.90

$$\frac{e^3 \sqrt{ex} \sqrt{a+bx^3} \left(3a^{3/2} (aB - 2Ab) \sinh^{-1} \left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}} \right) + \sqrt{b} x^{3/2} \sqrt{\frac{bx^3}{a} + 1} (-3a^2 B + 2ab(3A + Bx^3) + 4b^2 x^3 (3A + 2Bx^3)) \right)}{72b^{5/2} \sqrt{x} \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3), x]

[Out] (e^3*Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(-3*a^2*B + 2*a*b*(3*A + B*x^3) + 4*b^2*x^3*(3*A + 2*B*x^3)) + 3*a^(3/2)*(-2*A*b + a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(72*b^(5/2)*Sqrt[x]*Sqrt[1 + (b*x^3)/a])

IntegrateAlgebraic [A] time = 0.60, size = 162, normalized size = 1.01

$$\frac{\sqrt{a+bx^3} (-3a^2 B e^6 (ex)^{3/2} + 6a A b e^6 (ex)^{3/2} + 2ab B e^3 (ex)^{9/2} + 12A b^2 e^3 (ex)^{9/2} + 8b^2 B (ex)^{15/2})}{72b^2 e^4} + \frac{e^5 \sqrt{\frac{b}{e^3}} (2a^2 A b - a^3 B) \log \left(\sqrt{a+bx^3} - \sqrt{\frac{b}{e^3}} (ex)^{3/2} \right)}{24b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3), x]

[Out] (Sqrt[a + b*x^3]*(6*a*A*b*e^6*(e*x)^(3/2) - 3*a^2*B*e^6*(e*x)^(3/2) + 12*A*b^2*e^3*(e*x)^(9/2) + 2*a*b*B*e^3*(e*x)^(9/2) + 8*b^2*B*(e*x)^(15/2)))/(72*b^2*e^4) + ((2*a^2*A*b - a^3*B)*Sqrt[b/e^3]*e^5*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(24*b^3)

fricas [A] time = 1.26, size = 295, normalized size = 1.83

$$\frac{3(Ba^3 - 2Aa^2b)^2 \sqrt{e} \log(-8B^2e^6 - 8abca^3 - a^2c + 4(2b^2x^4 + abx)\sqrt{bx^3 + a}\sqrt{cx}\sqrt{\frac{e}{b}}) - 4(8Bb^2e^3x^2 + 2(Bab + 6Ab^2)e^3x^4 - 3(Ba^2 - 2Aab)e^3)\sqrt{bx^3 + a}\sqrt{cx} - 3(Ba^3 - 2Aa^2b)^2 \sqrt{e} \arctan\left(\frac{2\sqrt{bx^3 + a}\sqrt{cx}\sqrt{\frac{e}{b}}}{2bx^3 + ae}\right) - 2(8Bb^2e^3x^2 + 2(Bab + 6Ab^2)e^3x^4 - 3(Ba^2 - 2Aab)e^3)\sqrt{bx^3 + a}\sqrt{cx}}{288b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] [-1/288*(3*(B*a^3 - 2*A*a^2*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(8*B*b^2*e^3*x^7 + 2*(B*a*b + 6*A*b^2)*e^3*x^4 - 3*(B*a^2 - 2*A*a*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/144*(3*(B*a^3 - 2*A*a^2*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(8*B*b^2*e^3*x^7 + 2*(B*a*b + 6*A*b^2)*e^3*x^4 - 3*(B*a^2 - 2*A*a*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]

giac [A] time = 0.75, size = 251, normalized size = 1.56

$$\frac{\frac{1}{12}\sqrt{bx^3e^4 + ae^4}\left(2x^3e^{-1} + \frac{ae^{-1}}{b}\right)Ax^{\frac{3}{2}}e^{\frac{3}{2}} + \frac{1}{72}\sqrt{bx^3e^4 + ae^4}\left(2\left(4x^3e^{-4} + \frac{ae^{-4}}{b}\right)x^3e^3 - \frac{3a^2e^{-1}}{b^2}\right)Bx^{\frac{3}{2}}e^{\frac{3}{2}} - \frac{(B^2a^6e^7 - 4ABa^5be^7 + 4A^2a^4b^2e^7)e^{\frac{1}{2}}\log\left(\left(-Ba^3x^{\frac{3}{2}}e^{\frac{11}{2}} - 2Aa^2bx^{\frac{3}{2}}e^{\frac{11}{2}}\right)\sqrt{e}e^{\frac{1}{2}} + \sqrt{B^2a^6e^{12} - 4ABa^5be^{12} + 4A^2a^4b^2e^{12}} + (Ba^3x^{\frac{3}{2}}e^{\frac{11}{2}} - 2Aa^2bx^{\frac{3}{2}}e^{\frac{11}{2}})^2\right)}{24b^2(-Ba^3e^3 + 2Aa^2be^3)}}{24b^2(-Ba^3e^3 + 2Aa^2be^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2), x, algorithm="giac")

[Out] 1/12*sqrt(b*x^3*e^4 + a*e^4)*(2*x^3*e^(-1) + a*e^(-1)/b)*A*x^(3/2)*e^(5/2) + 1/72*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*x^3*e^(-4) + a*e^(-4)/b)*x^3*e^3 - 3*a^2*e^(-1)/b^2)*B*x^(3/2)*e^(5/2) - 1/24*(B^2*a^6*e^7 - 4*A*B*a^5*b*e^7 + 4*A^2*a^4*b^2*e^7)*e^(-1/2)*log(abs(-(B*a^3*x^(3/2)*e^(11/2) - 2*A*a^2*b*x^(3/2)*e^(11/2))*sqrt(b)*e^(1/2) + sqrt(B^2*a^7*e^12 - 4*A*B*a^6*b*e^12 + 4*A^2*a^5*b^2*e^12 + (B*a^3*x^(3/2)*e^(11/2) - 2*A*a^2*b*x^(3/2)*e^(11/2))^2*b*e))) / (b^(5/2)*abs(-B*a^3*e^3 + 2*A*a^2*b*e^3))

maple [C] time = 8.02, size = 7293, normalized size = 45.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx^3 + A)\sqrt{bx^3 + a} (ex)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(7/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (Bx^3 + A) (ex)^{7/2} \sqrt{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(1/2),x)`

[Out] `int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(1/2), x)`

sympy [B] time = 130.86, size = 292, normalized size = 1.81

$$\frac{Aa^{\frac{3}{2}}e^{\frac{7}{2}}x^{\frac{3}{2}}}{12b\sqrt{1+\frac{bx^3}{a}}} + \frac{A\sqrt{a}e^{\frac{7}{2}}x^{\frac{9}{2}}}{4\sqrt{1+\frac{bx^3}{a}}} - \frac{Aa^2e^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{12b^{\frac{3}{2}}} + \frac{Abe^{\frac{7}{2}}x^{\frac{15}{2}}}{6\sqrt{a}\sqrt{1+\frac{bx^3}{a}}} - \frac{Ba^{\frac{5}{2}}e^{\frac{7}{2}}x^{\frac{3}{2}}}{24b^2\sqrt{1+\frac{bx^3}{a}}} - \frac{Ba^{\frac{3}{2}}e^{\frac{7}{2}}x^{\frac{9}{2}}}{72b\sqrt{1+\frac{bx^3}{a}}} + \frac{5B\sqrt{a}e^{\frac{7}{2}}x^{\frac{15}{2}}}{36\sqrt{1+\frac{bx^3}{a}}} + \frac{Ba^3e^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{24b^{\frac{5}{2}}} + \frac{Bbe^{\frac{7}{2}}x^{\frac{21}{2}}}{9\sqrt{a}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**(7/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)`

[Out] `A*a**(3/2)*e**(7/2)*x**(3/2)/(12*b*sqrt(1 + b*x**3/a)) + A*sqrt(a)*e**(7/2)*x**(9/2)/(4*sqrt(1 + b*x**3/a)) - A*a**2*e**(7/2)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(12*b**(3/2)) + A*b*e**(7/2)*x**(15/2)/(6*sqrt(a)*sqrt(1 + b*x**3/a)) - B*a**(5/2)*e**(7/2)*x**(3/2)/(24*b**2*sqrt(1 + b*x**3/a)) - B*a**(3/2)*e**(7/2)*x**(9/2)/(72*b*sqrt(1 + b*x**3/a)) + 5*B*sqrt(a)*e**(7/2)*x**(15/2)/(36*sqrt(1 + b*x**3/a)) + B*a**3*e**(7/2)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(24*b**(5/2)) + B*b*e**(7/2)*x**(21/2)/(9*sqrt(a)*sqrt(1 + b*x**3/a))`

3.326 $\int \sqrt{ex} \sqrt{a + bx^3} (A + Bx^3) dx$

Optimal. Leaf size=121

$$\frac{a\sqrt{e}(4Ab - aB) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}} + \frac{(ex)^{3/2}\sqrt{a+bx^3}(4Ab - aB)}{12be} + \frac{B(ex)^{3/2}(a+bx^3)^{3/2}}{6be}$$

Rubi [A] time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {459, 279, 329, 275, 217, 206}

$$\frac{a\sqrt{e}(4Ab - aB) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}} + \frac{(ex)^{3/2}\sqrt{a+bx^3}(4Ab - aB)}{12be} + \frac{B(ex)^{3/2}(a+bx^3)^{3/2}}{6be}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3), x]

[Out] ((4*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(12*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(6*b*e) + (a*(4*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(12*b^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 279

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p + 1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG

tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^(p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
 \int \sqrt{ex} \sqrt{a + bx^3} (A + Bx^3) dx &= \frac{B(ex)^{3/2} (a + bx^3)^{3/2}}{6be} - \frac{\left(-6Ab + \frac{3aB}{2}\right) \int \sqrt{ex} \sqrt{a + bx^3} dx}{6b} \\
 &= \frac{(4Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{12be} + \frac{B(ex)^{3/2} (a + bx^3)^{3/2}}{6be} + \frac{(a(4Ab - aB)) \int \frac{\sqrt{ex}}{\sqrt{a + bx^3}} dx}{8b} \\
 &= \frac{(4Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{12be} + \frac{B(ex)^{3/2} (a + bx^3)^{3/2}}{6be} + \frac{(a(4Ab - aB)) \text{Subst} \left(\int \frac{\sqrt{ex}}{\sqrt{a + bx^3}} dx \right)}{4be} \\
 &= \frac{(4Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{12be} + \frac{B(ex)^{3/2} (a + bx^3)^{3/2}}{6be} + \frac{(a(4Ab - aB)) \text{Subst} \left(\int \frac{\sqrt{ex}}{\sqrt{a + bx^3}} dx \right)}{12be} \\
 &= \frac{(4Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{12be} + \frac{B(ex)^{3/2} (a + bx^3)^{3/2}}{6be} + \frac{(a(4Ab - aB)) \text{Subst} \left(\int \frac{\sqrt{ex}}{\sqrt{a + bx^3}} dx \right)}{12be} \\
 &= \frac{(4Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{12be} + \frac{B(ex)^{3/2} (a + bx^3)^{3/2}}{6be} + \frac{a(4Ab - aB) \sqrt{e} \tanh^{-1} \left(\frac{\sqrt{ex} \sqrt{a + bx^3}}{\sqrt{a + bx^3}} \right)}{12b^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.15, size = 119, normalized size = 0.98

$$\frac{\sqrt{ex} \sqrt{a + bx^3} \left(\sqrt{b} x^{3/2} \sqrt{\frac{bx^3}{a} + 1} \left(B(a + 2bx^3) + 4Ab \right) - \sqrt{a} (aB - 4Ab) \sinh^{-1} \left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}} \right) \right)}{12b^{3/2} \sqrt{x} \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3), x]

[Out] (Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(4*A*b + B*(a + 2*b*x^3)) - Sqrt[a]*(-4*A*b + a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*b^(3/2)*Sqrt[x]*Sqrt[1 + (b*x^3)/a])

IntegrateAlgebraic [A] time = 0.52, size = 123, normalized size = 1.02

$$\frac{\sqrt{a + bx^3} (aBe^3(ex)^{3/2} + 4Abe^3(ex)^{3/2} + 2bB(ex)^{9/2})}{12be^4} - \frac{e^2 \sqrt{\frac{b}{e^3}} (4aAb - a^2B) \log \left(\sqrt{a + bx^3} - \sqrt{\frac{b}{e^3}} (ex)^{3/2} \right)}{12b^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3), x]

[Out] (Sqrt[a + b*x^3]*(4*A*b*e^3*(e*x)^(3/2) + a*B*e^3*(e*x)^(3/2) + 2*b*B*(e*x)^(9/2)))/(12*b*e^4) - ((4*a*A*b - a^2*B)*Sqrt[b/e^3]*e^2*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(12*b^2)

fricas [A] time = 1.16, size = 221, normalized size = 1.83

$$\left[\frac{(Ba^2 - 4Aab) \sqrt{\frac{e}{b}} \log \left(-8b^2ex^6 - 8abex^3 - a^2e - 4(2b^2x^4 + abx) \sqrt{bx^3 + a} \sqrt{ex} \sqrt{\frac{e}{b}} \right) - 4(2Bbx^4 + (Ba + 4Ab)x) \sqrt{bx^3 + a} \sqrt{ex} (Ba^2 - 4Aab) \sqrt{\frac{e}{b}} \arctan \left(\frac{2\sqrt{bx^3 + a} \sqrt{ex} \sqrt{\frac{e}{b}}}{2bx^3 + a} \right) + 2(2Bbx^4 + (Ba + 4Ab)x) \sqrt{bx^3 + a} \sqrt{ex}}{48b}, \frac{(Ba^2 - 4Aab) \sqrt{\frac{e}{b}} \arctan \left(\frac{2\sqrt{bx^3 + a} \sqrt{ex} \sqrt{\frac{e}{b}}}{2bx^3 + a} \right) + 2(2Bbx^4 + (Ba + 4Ab)x) \sqrt{bx^3 + a} \sqrt{ex}}{24b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] [-1/48*((B*a^2 - 4*A*a*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(2*B*b*x^4 + (B*a + 4*A*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b, 1/24*((B*a^2 - 4*A*a*b)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(2*B*b*x^4 + (B*a + 4*A*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b]

giac [A] time = 0.42, size = 137, normalized size = 1.13

$$\frac{1}{12} \sqrt{bx^3e^4 + ae^4} \left(2x^3e^{(-1)} + \frac{ae^{(-1)}}{b} \right) Bx^{\frac{3}{2}} e^{\left(-\frac{1}{2}\right)} + \frac{Ba^2e^{\frac{1}{2}} \log \left(-\sqrt{b} x^{\frac{3}{2}} e^2 + \sqrt{bx^3e^4 + ae^4} \right)}{12b^{\frac{3}{2}}} + \frac{1}{3} \left(\sqrt{bx^3e^4 + ae^4} x^{\frac{3}{2}} e^{\frac{3}{2}} - \frac{ae^{\frac{7}{2}} \log \left(-\sqrt{b} x^{\frac{3}{2}} e^2 + \sqrt{bx^3e^4 + ae^4} \right)}{\sqrt{b}} \right) Ae^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{12}\sqrt{b*x^3*e^4 + a*e^4}*(2*x^3*e^{-1} + a*e^{-1}/b)*B*x^{(3/2)}*e^{(-1/2)}$
 $+ \frac{1}{12}*B*a^2*e^{(1/2)}*\log(\text{abs}(-\sqrt{b}*x^{(3/2)}*e^2 + \sqrt{b*x^3*e^4 + a*e^4})))/b^{(3/2)} + \frac{1}{3}*(\sqrt{b*x^3*e^4 + a*e^4}*x^{(3/2)}*e^{(3/2)} - a*e^{(7/2)}*\log(\text{abs}(-\sqrt{b}*x^{(3/2)}*e^2 + \sqrt{b*x^3*e^4 + a*e^4}))/\sqrt{b})*A*e^{(-3)}$

maple [C] time = 1.09, size = 6858, normalized size = 56.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx^3 + A)\sqrt{bx^3 + a}\sqrt{ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (Bx^3 + A)\sqrt{ex}\sqrt{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(1/2),x)

[Out] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(1/2), x)

sympy [A] time = 9.67, size = 201, normalized size = 1.66

$$\frac{A\sqrt{a}(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e} + \frac{Aa\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}e^{\frac{3}{2}}}\right)}{3\sqrt{b}} + \frac{Ba^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{12be\sqrt{1+\frac{bx^3}{a}}} + \frac{B\sqrt{a}(ex)^{\frac{9}{2}}}{4e^4\sqrt{1+\frac{bx^3}{a}}} - \frac{Ba^2\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}e^{\frac{3}{2}}}\right)}{12b^{\frac{3}{2}}} + \frac{Bb(ex)^{\frac{15}{2}}}{6\sqrt{a}e^7\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)*(e*x)**(1/2)*(b*x**3+a)**(1/2),x)`

[Out] $A\sqrt{a}(e^x)^{3/2}\sqrt{1 + b x^3/a}/(3e) + A a \sqrt{e} \operatorname{asinh}(\sqrt{b} (e^x)^{3/2}/(\sqrt{a} e^{3/2}))/ (3\sqrt{b}) + B a^{3/2} (e^x)^{3/2}/(12 b e \sqrt{1 + b x^3/a}) + B \sqrt{a} (e^x)^{9/2}/(4 e^4 \sqrt{1 + b x^3/a}) - B a^2 \sqrt{e} \operatorname{asinh}(\sqrt{b} (e^x)^{3/2}/(\sqrt{a} e^{3/2}))/ (12 b^{3/2}) + B b (e^x)^{15/2}/(6 \sqrt{a} e^7 \sqrt{1 + b x^3/a})$

$$3.327 \quad \int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}} dx$$

Optimal. Leaf size=118

$$\frac{(aB + 2Ab) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}} + \frac{(ex)^{3/2}\sqrt{a+bx^3}(aB + 2Ab)}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {453, 279, 329, 275, 217, 206}

$$\frac{(ex)^{3/2}\sqrt{a+bx^3}(aB + 2Ab)}{3ae^4} + \frac{(aB + 2Ab) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2), x]

[Out] ((2*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*a*e^4) - (2*A*(a + b*x^3)^(3/2))/(3*a*e*(e*x)^(3/2)) + ((2*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3]])/(3*Sqrt[b]*e^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 279

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +

1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3} (A+Bx^3)}{(ex)^{5/2}} dx &= -\frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB) \int \sqrt{ex} \sqrt{a+bx^3} dx}{ae^3} \\
&= \frac{(2Ab+aB)(ex)^{3/2} \sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB) \int \frac{\sqrt{ex}}{\sqrt{a+bx^3}} dx}{2e^3} \\
&= \frac{(2Ab+aB)(ex)^{3/2} \sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB) \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}} dx, x, \frac{x}{\sqrt{e^3}} \right)}{e^4} \\
&= \frac{(2Ab+aB)(ex)^{3/2} \sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB) \operatorname{Subst} \left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}} dx, x, \frac{x}{\sqrt{e^3}} \right)}{3e^4} \\
&= \frac{(2Ab+aB)(ex)^{3/2} \sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB) \operatorname{Subst} \left(\int \frac{1}{1-\frac{bx^2}{e^3}} dx, x, \frac{x}{\sqrt{e^3}} \right)}{3e^4} \\
&= \frac{(2Ab+aB)(ex)^{3/2} \sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB) \tanh^{-1} \left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2} \sqrt{a+bx^3}} \right)}{3\sqrt{b}e^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 87, normalized size = 0.74

$$\frac{x\sqrt{a+bx^3} \left(\frac{x^{3/2}(aB+2Ab) \sinh^{-1} \left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}} \right) - 2A + Bx^3}{\sqrt{a} \sqrt{b} \sqrt{\frac{bx^3}{a} + 1}} \right)}{3(ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2), x]

[Out] (x*Sqrt[a + b*x^3]*(-2*A + B*x^3 + ((2*A*b + a*B)*x^(3/2)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*x^3)/a]))/(3*(e*x)^(5/2))

IntegrateAlgebraic [A] time = 0.56, size = 99, normalized size = 0.84

$$\frac{\sqrt{a+bx^3} (Be^3x^3 - 2Ae^3)}{3e^4(ex)^{3/2}} - \frac{\sqrt{\frac{b}{e^3}} (aB + 2Ab) \log \left(\sqrt{a+bx^3} - \sqrt{\frac{b}{e^3}} (ex)^{3/2} \right)}{3be}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2), x]

[Out] (Sqrt[a + b*x^3]*(-2*A*e^3 + B*e^3*x^3))/(3*e^4*(e*x)^(3/2)) - ((2*A*b + a*B)*Sqrt[b/e^3]*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(3*b*e)

fricas [A] time = 1.55, size = 207, normalized size = 1.75

$$\left[\frac{(Ba + 2Ab)\sqrt{be}x^2 \log\left(-8b^2ex^6 - 8abex^3 - a^2e - 4(2bx^4 + ax)\sqrt{bx^3 + a}\sqrt{be}\sqrt{ex}\right) + 4(Bbx^3 - 2Ab)\sqrt{bx^3 + a}\sqrt{ex}}{12be^3x^2}, -\frac{(Ba + 2Ab)\sqrt{-be}x^2 \arctan\left(\frac{2\sqrt{bx^3 + a}\sqrt{-be}\sqrt{ex}}{2bx^3 + ae}\right) - 2(Bbx^3 - 2Ab)\sqrt{bx^3 + a}\sqrt{ex}}{6be^3x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2), x, algorithm="fricas")

[Out] [1/12*((B*a + 2*A*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(B*b*x^3 - 2*A*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/6*((B*a + 2*A*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) - 2*(B*b*x^3 - 2*A*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{(ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2), x, algorithm="giac")

[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(5/2), x)

maple [C] time = 1.04, size = 6668, normalized size = 56.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2), x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{(ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^3 + A) \sqrt{bx^3 + a}}{(ex)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(5/2),x)

[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(5/2), x)

sympy [A] time = 9.72, size = 160, normalized size = 1.36

$$-\frac{2A\sqrt{a}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{2A\sqrt{b} \operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3e^{\frac{5}{2}}} - \frac{2Abx^{\frac{3}{2}}}{3\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{B\sqrt{a}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} + \frac{Ba \operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(5/2),x)

[Out] -2*A*sqrt(a)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(3*e**(5/2)) - 2*A*b*x**(3/2)/(3*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a)) + B*sqrt(a)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(3*sqrt(b)*e**(5/2))

$$3.328 \quad \int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}} dx$$

Optimal. Leaf size=79

$$-\frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} - \frac{2B\sqrt{a+bx^3}}{3x^{3/2}} + \frac{2}{3}\sqrt{b}B \tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {451, 277, 329, 275, 217, 206}

$$-\frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} - \frac{2B\sqrt{a+bx^3}}{3x^{3/2}} + \frac{2}{3}\sqrt{b}B \tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2), x]

[Out] (-2*B*Sqrt[a + b*x^3])/(3*x^(3/2)) - (2*A*(a + b*x^3)^(3/2))/(9*a*x^(9/2)) + (2*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/3

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 277

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[

$n, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !\text{ILtQ}[(m + n*p + n + 1)/n, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 329

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_*)})^{(p_*)}, x_Symbol] \ :> \ \text{With}[\{k = \text{Denominator}[m]\}, \ \text{Dist}[k/c, \ \text{Subst}[\text{Int}[x^{(k*(m+1)-1)}*(a + (b*x^{(k*n)})/c^n)^{p, x}], x, (c*x)^{(1/k)}], x]] \ ; \ \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{Fractio}[\text{ractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 451

$\text{Int}[(e_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_)^{(n_*)}), x_Symbol] \ :> \ \text{Simp}[(c*(e*x)^{(m+1)}*(a + b*x^n)^{(p+1)}/(a*e^{(m+1)}), x] + \text{Dist}[d/e^n, \ \text{Int}[(e*x)^{(m+n)}*(a + b*x^n)^p, x], x] \ ; \ \text{FreeQ}[\{a, b, c, d, e, m, n, p\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[m + n*(p+1) + 1, 0] \ \&\& \ (\text{IntegerQ}[n] \ || \ \text{GtQ}[e, 0]) \ \&\& \ ((\text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1]) \ || \ (\text{LtQ}[n, 0] \ \&\& \ \text{GtQ}[m + n, -1]))$

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a+bx^3} (A+Bx^3)}{x^{11/2}} dx &= -\frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + B \int \frac{\sqrt{a+bx^3}}{x^{5/2}} dx \\
 &= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + (bB) \int \frac{\sqrt{x}}{\sqrt{a+bx^3}} dx \\
 &= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + (2bB) \text{Subst} \left(\int \frac{x^2}{\sqrt{a+bx^6}} dx, x, \sqrt{x} \right) \\
 &= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + \frac{1}{3}(2bB) \text{Subst} \left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, x^{3/2} \right) \\
 &= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + \frac{1}{3}(2bB) \text{Subst} \left(\int \frac{1}{1-bx^2} dx, x, \frac{x^{3/2}}{\sqrt{a+bx^3}} \right) \\
 &= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + \frac{2}{3}\sqrt{b}B \tanh^{-1} \left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}} \right)
 \end{aligned}$$

Mathematica [A] time = 0.18, size = 87, normalized size = 1.10

$$\frac{2\sqrt{a+bx^3} \left(\frac{3\sqrt{a}\sqrt{b}B \sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right) - \frac{a(A+3Bx^3)+Abx^3}{x^{9/2}}}{\sqrt{\frac{bx^3}{a}+1}} \right)}{9a}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2), x]

[Out] (2*Sqrt[a + b*x^3]*(-(A*b*x^3 + a*(A + 3*B*x^3))/x^(9/2)) + (3*Sqrt[a]*Sqrt[b]*B*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]])/Sqrt[1 + (b*x^3)/a])/ (9*a)

IntegrateAlgebraic [A] time = 0.31, size = 75, normalized size = 0.95

$$\frac{2}{3}\sqrt{b}B \log\left(\sqrt{a+bx^3} + \sqrt{b}x^{3/2}\right) - \frac{2\sqrt{a+bx^3} (aA + 3aBx^3 + Abx^3)}{9ax^{9/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2), x]

[Out] (-2*Sqrt[a + b*x^3]*(a*A + A*b*x^3 + 3*a*B*x^3))/(9*a*x^(9/2)) + (2*Sqrt[b]*B*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/3

fricas [A] time = 0.95, size = 180, normalized size = 2.28

$$\left[\frac{3Ba\sqrt{b}x^5 \log\left(-8b^2x^6 - 8abx^3 - 4(2bx^4 + ax)\sqrt{bx^3 + a}\sqrt{x} - a^2\right) - 4((3Ba + Ab)x^3 + Aa)\sqrt{bx^3 + a}\sqrt{x}}{18ax^5}, \frac{3Ba\sqrt{-b}x^5 \arctan\left(\frac{2\sqrt{bx^3 + a}\sqrt{-bx^2}}{2bx^3 + a}\right) + 2((3Ba + Ab)x^3 + Aa)\sqrt{bx^3 + a}\sqrt{x}}{9ax^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2), x, algorithm="fricas")

[Out] [1/18*(3*B*a*sqrt(b)*x^5*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(x))/(a*x^5), -1/9*(3*B*a*sqrt(-b)*x^5*arctan(2*sqrt(b*x^3 + a)*sqrt(-b)*x^(3/2)/(2*b*x^3 + a)) + 2*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(x))/(a*x^5)]

giac [A] time = 0.25, size = 109, normalized size = 1.38

$$-\frac{2Bb \arctan\left(\frac{\sqrt{\frac{b+a}{x^3}}}{\sqrt{-b}}\right)}{3\sqrt{-b}} + \frac{2\left(3Bab \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + 3Ba\sqrt{-b}\sqrt{b} + A\sqrt{-b}b^{\frac{3}{2}}\right)}{9a\sqrt{-b}} - \frac{2\left(3Ba^3\sqrt{b + \frac{a}{x^3}} + Aa^2\left(b + \frac{a}{x^3}\right)^{\frac{3}{2}}\right)}{9a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="giac")

[Out]
$$-2/3*B*b*\arctan(\sqrt{b+a/x^3}/\sqrt{-b})/\sqrt{-b} + 2/9*(3*B*a*b*\arctan(\sqrt{b}/\sqrt{-b}) + 3*B*a*\sqrt{-b}*\sqrt{b} + A*\sqrt{-b}*b^{3/2})/(a*\sqrt{-b}) - 2/9*(3*B*a^3*\sqrt{b+a/x^3} + A*a^2*(b+a/x^3)^{3/2})/a^3$$

maple [C] time = 1.00, size = 3759, normalized size = 47.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x)

[Out]
$$\begin{aligned} & -2/9*(b*x^3+a)^{1/2}/x^{9/2}/b*(18*I*B*3^{1/2}*(-I*3^{1/2}-3)*x*b/(I*3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*((I*3^{1/2})*(-a*b^2)^{1/3}+2*b*x+(-a*b^2)^{1/3})/(1+I*3^{1/2})/(-b*x+(-a*b^2)^{1/3}))^{1/2}*((I*3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3})/(I*3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*EllipticF((-I*3^{1/2}-3)*x*b/(I*3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2},((I*3^{1/2}+3)*(I*3^{1/2}-1)/(1+I*3^{1/2})/(I*3^{1/2}-3))^{1/2}*(-a*b^2)^{2/3}*x^5*a-3*A*(x*(b*x^3+a))^{1/2}*(1/b^2*x*(-b*x+(-a*b^2)^{1/3})*(I*3^{1/2})*(-a*b^2)^{1/3}+2*b*x+(-a*b^2)^{1/3})*(I*3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3}))^{1/2}*x^3*b^2+I*A*(x*(b*x^3+a))^{1/2}*3^{1/2}*(1/b^2*x*(-b*x+(-a*b^2)^{1/3})*(I*3^{1/2})*(-a*b^2)^{1/3}+2*b*x+(-a*b^2)^{1/3})*(I*3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3}))^{1/2}*x^3*b^2-3*A*(x*(b*x^3+a))^{1/2}*(1/b^2*x*(-b*x+(-a*b^2)^{1/3})*(I*3^{1/2})*(-a*b^2)^{1/3}+2*b*x+(-a*b^2)^{1/3})*(I*3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3}))^{1/2}*a*b-36*I*B*3^{1/2}*(-I*3^{1/2}-3)*x*b/(I*3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*((I*3^{1/2})*(-a*b^2)^{1/3}+2*b*x+(-a*b^2)^{1/3})/(1+I*3^{1/2})/(-b*x+(-a*b^2)^{1/3}))^{1/2}*((I*3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3})/(I*3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*EllipticF((-I*3^{1/2}-3)*x*b/(I*3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2},((I*3^{1/2}+3)*(I*3^{1/2}-1)/(1+I*3^{1/2})/(I*3^{1/2}-3))^{1/2}*(-a*b^2)^{1/3}*x^6*a*b-18*I*B*3^{1/2}*(-I*3^{1/2}-3)*x*b/(I*3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*((I*3^{1/2})*(-a*b^2)^{1/3}+2*b*x+(-a*b^2)^{1/3})/(1+I*3^{1/2})/(-b*x+(-a*b^2)^{1/3}))^{1/2}*((I*3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3})/(I*3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*EllipticPi((-I*3^{1/2}-3)*x*b/(I*3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2},(I*3^{1/2}-1)/(I*3^{1/2}-3),((I*3^{1/2}+3)*(I*3^{1/2}-1)/(1+I*3^{1/2})/(I*3^{1/2}-3))^{1/2}*(-a*b^2)^{2/3}*x^5*a+I*A*(x*(b*x^3+a))^{1/2}*3^{1/2}*(1/b^2*x*(-b*x+(-a*b^2)^{1/3})*(I*3^{1/2})*(-a*b^2)^{1/3}+2*b*x+(-a*b^2)^{1/3})*(I*3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3}))^{1/2}*a*b-18*I*B*3^{1/2}*(-I*3^{1/2}-3)*x*b/(I*3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*((I*3^{1/2})*(-a*b^2)^{1/3}+2*b*x+(-a*b^2)^{1/3})/(1+I*3^{1/2})/(-b*x+(-a*b^2)^{1/3}))^{1/2}*((I*3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3})/(I*3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2} \end{aligned}$$

$$2)^{-3} * x * b / (I * 3^{(1/2)} - 1) / (-b * x + (-a * b^2)^{(1/3)})^{(1/2)} * ((I * 3^{(1/2)} * (-a * b^2)^{(1/3)} + 2 * b * x + (-a * b^2)^{(1/3)}) / (1 + I * 3^{(1/2)}) / (-b * x + (-a * b^2)^{(1/3)})^{(1/2)} * ((I * 3^{(1/2)} * (-a * b^2)^{(1/3)} - 2 * b * x - (-a * b^2)^{(1/3)}) / (I * 3^{(1/2)} - 1) / (-b * x + (-a * b^2)^{(1/3)})^{(1/2)} * \text{EllipticF}((-I * 3^{(1/2)} - 3) * x * b / (I * 3^{(1/2)} - 1) / (-b * x + (-a * b^2)^{(1/3)}))^{(1/2)}, ((I * 3^{(1/2)} + 3) * (I * 3^{(1/2)} - 1) / (1 + I * 3^{(1/2)}) / (I * 3^{(1/2)} - 3))^{(1/2)} * x^7 * a * b^2 - 9 * B * (x * (b * x^3 + a))^{(1/2)} * (1 / b^2 * x * (-b * x + (-a * b^2)^{(1/3)}) * (I * 3^{(1/2)} * (-a * b^2)^{(1/3)} + 2 * b * x + (-a * b^2)^{(1/3)}) * (I * 3^{(1/2)} * (-a * b^2)^{(1/3)} - 2 * b * x - (-a * b^2)^{(1/3)})^{(1/2)} * x^3 * a * b) / (x * (b * x^3 + a))^{(1/2)} / a / (I * 3^{(1/2)} - 3) / (1 / b^2 * x * (-b * x + (-a * b^2)^{(1/3)}) * (I * 3^{(1/2)} * (-a * b^2)^{(1/3)} + 2 * b * x + (-a * b^2)^{(1/3)}) * (I * 3^{(1/2)} * (-a * b^2)^{(1/3)} - 2 * b * x - (-a * b^2)^{(1/3)})^{(1/2)}$$

maxima [A] time = 1.22, size = 81, normalized size = 1.03

$$-\frac{1}{3} \left(\sqrt{b} \log \left(-\frac{\sqrt{b} - \frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b} + \frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}} \right) + \frac{2\sqrt{bx^3+a}}{x^{\frac{3}{2}}} \right) B - \frac{2(bx^3+a)^{\frac{3}{2}} A}{9ax^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="maxima")

[Out] -1/3*(sqrt(b)*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3 + a)/x^(3/2))) + 2*sqrt(b*x^3 + a)/x^(3/2))*B - 2/9*(b*x^3 + a)^(3/2)*A/(a*x^(9/2))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^3 + A) \sqrt{bx^3 + a}}{x^{11/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(11/2),x)

[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(11/2), x)

sympy [A] time = 61.86, size = 131, normalized size = 1.66

$$-\frac{2A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{9x^3} - \frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{9a} - \frac{2B\sqrt{a}}{3x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{2B\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{bx^3}}{\sqrt{a}}\right)}{3} - \frac{2Bbx^{\frac{3}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(11/2),x)
```

```
[Out] -2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(9*x**3) - 2*A*b**(3/2)*sqrt(a/(b*x**3) + 1)/(9*a) - 2*B*sqrt(a)/(3*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*B*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/3 - 2*B*b*x**(3/2)/(3*sqrt(a)*sqrt(1 + b*x**3/a))
```

$$3.329 \quad \int (ex)^{7/2} (a + bx^3)^{3/2} (A + Bx^3) dx$$

Optimal. Leaf size=201

$$\frac{a^3 e^{7/2} (8Ab - 3aB) \tanh^{-1} \left(\frac{\sqrt{b} (ex)^{3/2}}{e^{3/2} \sqrt{a+bx^3}} \right)}{192b^{5/2}} + \frac{a^2 e^2 (ex)^{3/2} \sqrt{a+bx^3} (8Ab - 3aB)}{192b^2} + \frac{(ex)^{9/2} (a + bx^3)^{3/2} (8Ab - 3aB)}{72be} + \frac{a(e)^{9/2} (a + bx^3)^{3/2} (8Ab - 3aB)}{72be} + \frac{B(e)^{9/2} (a + bx^3)^{5/2}}{12be}$$

Rubi [A] time = 0.14, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {459, 279, 321, 329, 275, 217, 206}

$$\frac{a^2 e^2 (ex)^{3/2} \sqrt{a+bx^3} (8Ab - 3aB)}{192b^2} - \frac{a^3 e^{7/2} (8Ab - 3aB) \tanh^{-1} \left(\frac{\sqrt{b} (ex)^{3/2}}{e^{3/2} \sqrt{a+bx^3}} \right)}{192b^{5/2}} + \frac{(ex)^{9/2} (a + bx^3)^{3/2} (8Ab - 3aB)}{72be} + \frac{a(e)^{9/2} \sqrt{a+bx^3} (8Ab - 3aB)}{96be} + \frac{B(e)^{9/2} (a + bx^3)^{5/2}}{12be}$$

Antiderivative was successfully verified.

[In] Int[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3), x]

[Out] (a^2*(8*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3]/(192*b^2) + (a*(8*A*b - 3*a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3]/(96*b*e) + ((8*A*b - 3*a*B)*(e*x)^(9/2)*(a + b*x^3)^(3/2))/(72*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(5/2))/(12*b*e) - (a^3*(8*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3]])/(192*b^(5/2)))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 279

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +

1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
\int (ex)^{7/2} (a + bx^3)^{3/2} (A + Bx^3) dx &= \frac{B(ex)^{9/2} (a + bx^3)^{5/2}}{12be} - \frac{\left(-12Ab + \frac{9aB}{2}\right) \int (ex)^{7/2} (a + bx^3)^{3/2} dx}{12b} \\
&= \frac{(8Ab - 3aB)(ex)^{9/2} (a + bx^3)^{3/2}}{72be} + \frac{B(ex)^{9/2} (a + bx^3)^{5/2}}{12be} + \frac{(a(8Ab - 3aB))}{72be} \\
&= \frac{a(8Ab - 3aB)(ex)^{9/2} \sqrt{a + bx^3}}{96be} + \frac{(8Ab - 3aB)(ex)^{9/2} (a + bx^3)^{3/2}}{72be} + \frac{B(ex)^{9/2}}{72be} \\
&= \frac{a^2(8Ab - 3aB)e^2(ex)^{3/2} \sqrt{a + bx^3}}{192b^2} + \frac{a(8Ab - 3aB)(ex)^{9/2} \sqrt{a + bx^3}}{96be} + \frac{(8Ab - 3aB)(ex)^{9/2}}{96be} \\
&= \frac{a^2(8Ab - 3aB)e^2(ex)^{3/2} \sqrt{a + bx^3}}{192b^2} + \frac{a(8Ab - 3aB)(ex)^{9/2} \sqrt{a + bx^3}}{96be} + \frac{(8Ab - 3aB)(ex)^{9/2}}{96be} \\
&= \frac{a^2(8Ab - 3aB)e^2(ex)^{3/2} \sqrt{a + bx^3}}{192b^2} + \frac{a(8Ab - 3aB)(ex)^{9/2} \sqrt{a + bx^3}}{96be} + \frac{(8Ab - 3aB)(ex)^{9/2}}{96be} \\
&= \frac{a^2(8Ab - 3aB)e^2(ex)^{3/2} \sqrt{a + bx^3}}{192b^2} + \frac{a(8Ab - 3aB)(ex)^{9/2} \sqrt{a + bx^3}}{96be} + \frac{(8Ab - 3aB)(ex)^{9/2}}{96be} \\
&= \frac{a^2(8Ab - 3aB)e^2(ex)^{3/2} \sqrt{a + bx^3}}{192b^2} + \frac{a(8Ab - 3aB)(ex)^{9/2} \sqrt{a + bx^3}}{96be} + \frac{(8Ab - 3aB)(ex)^{9/2}}{96be}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 167, normalized size = 0.83

$$\frac{e^3 \sqrt{ex} \sqrt{a + bx^3} \left(3a^{5/2}(3aB - 8Ab) \sinh^{-1} \left(\frac{\sqrt{bx^3/2}}{\sqrt{a}} \right) + \sqrt{b} x^{3/2} \sqrt{\frac{bx^3}{a} + 1} \left(-9a^3B + 6a^2b(4A + Bx^3) + 8ab^2x^3(14A + 9Bx^3) + 16b^3x^6(4A + 3Bx^3) \right) \right)}{576b^{5/2} \sqrt{x} \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3), x]

[Out] (e^3*Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(-9*a^3*B + 6*a^2*b*(4*A + B*x^3) + 16*b^3*x^6*(4*A + 3*B*x^3) + 8*a*b^2*x^3*(14*A + 9*B*x^3)) + 3*a^(5/2)*(-8*A*b + 3*a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(576*b^(5/2)*Sqrt[x]*Sqrt[1 + (b*x^3)/a])

$$B*a^4*x^{(3/2)}*e^{(11/2)} - 8*A*a^3*b*x^{(3/2)}*e^{(11/2)}*\text{sqrt}(b)*e^{(1/2)} + \text{sqrt}(9*B^2*a^9*e^{12} - 48*A*B*a^8*b*e^{12} + 64*A^2*a^7*b^2*e^{12} + (3*B*a^4*x^{(3/2)})*e^{(11/2)} - 8*A*a^3*b*x^{(3/2)}*e^{(11/2)})^2*b*e)))/(b^{(5/2)}*\text{abs}(-3*B*a^4*e^3 + 8*A*a^3*b*e^3))$$

maple [C] time = 1.14, size = 7705, normalized size = 38.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A), x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx^3 + A)(bx^3 + a)^{\frac{3}{2}} (ex)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A), x, algorithm="maxima")`

[Out] `integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(7/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (Bx^3 + A) (ex)^{7/2} (bx^3 + a)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(3/2), x)`

[Out] `int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**(7/2)*(b*x**3+a)**(3/2)*(B*x**3+A), x)`

[Out] Timed out

$$3.330 \quad \int \sqrt{ex} (a + bx^3)^{3/2} (A + Bx^3) dx$$

Optimal. Leaf size=161

$$\frac{a^2 \sqrt{e} (6Ab - aB) \tanh^{-1} \left(\frac{\sqrt{b} (ex)^{3/2}}{e^{3/2} \sqrt{a+bx^3}} \right)}{24b^{3/2}} + \frac{(ex)^{3/2} (a + bx^3)^{3/2} (6Ab - aB)}{36be} + \frac{a(ex)^{3/2} \sqrt{a + bx^3} (6Ab - aB)}{24be} + \frac{B(ex)^{3/2}}{9}$$

Rubi [A] time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {459, 279, 329, 275, 217, 206}

$$\frac{a^2 \sqrt{e} (6Ab - aB) \tanh^{-1} \left(\frac{\sqrt{b} (ex)^{3/2}}{e^{3/2} \sqrt{a+bx^3}} \right)}{24b^{3/2}} + \frac{(ex)^{3/2} (a + bx^3)^{3/2} (6Ab - aB)}{36be} + \frac{a(ex)^{3/2} \sqrt{a + bx^3} (6Ab - aB)}{24be} + \frac{B(ex)^{3/2} (a + bx^3)^{5/2}}{9be}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3), x]

[Out] (a*(6*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*b*e) + ((6*A*b - a*B)*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(36*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(5/2))/(9*b*e) + (a^2*(6*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3]])/(24*b^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 279

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +

1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{ex} (a + bx^3)^{3/2} (A + Bx^3) dx &= \frac{B(ex)^{3/2} (a + bx^3)^{5/2}}{9be} - \frac{\left(-9Ab + \frac{3aB}{2}\right) \int \sqrt{ex} (a + bx^3)^{3/2} dx}{9b} \\
&= \frac{(6Ab - aB)(ex)^{3/2} (a + bx^3)^{3/2}}{36be} + \frac{B(ex)^{3/2} (a + bx^3)^{5/2}}{9be} + \frac{(a(6Ab - aB)) \int \sqrt{ex} (a + bx^3)^{3/2} dx}{8b} \\
&= \frac{a(6Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{24be} + \frac{(6Ab - aB)(ex)^{3/2} (a + bx^3)^{3/2}}{36be} + \frac{B(ex)^{3/2} \int \sqrt{ex} (a + bx^3)^{3/2} dx}{9b} \\
&= \frac{a(6Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{24be} + \frac{(6Ab - aB)(ex)^{3/2} (a + bx^3)^{3/2}}{36be} + \frac{B(ex)^{3/2} \int \sqrt{ex} (a + bx^3)^{3/2} dx}{9b} \\
&= \frac{a(6Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{24be} + \frac{(6Ab - aB)(ex)^{3/2} (a + bx^3)^{3/2}}{36be} + \frac{B(ex)^{3/2} \int \sqrt{ex} (a + bx^3)^{3/2} dx}{9b} \\
&= \frac{a(6Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{24be} + \frac{(6Ab - aB)(ex)^{3/2} (a + bx^3)^{3/2}}{36be} + \frac{B(ex)^{3/2} \int \sqrt{ex} (a + bx^3)^{3/2} dx}{9b} \\
&= \frac{a(6Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{24be} + \frac{(6Ab - aB)(ex)^{3/2} (a + bx^3)^{3/2}}{36be} + \frac{B(ex)^{3/2} \int \sqrt{ex} (a + bx^3)^{3/2} dx}{9b}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 143, normalized size = 0.89

$$\frac{\sqrt{ex} \sqrt{a + bx^3} \left(\sqrt{b} x^{3/2} \sqrt{\frac{bx^3}{a} + 1} (3a^2B + 2ab(15A + 7Bx^3) + 4b^2x^3(3A + 2Bx^3)) - 3a^{3/2}(aB - 6Ab) \sinh^{-1} \left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}} \right) \right)}{72b^{3/2} \sqrt{x} \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3), x]

[Out] (Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(3*a^2*B + 4*b^2*x^3*(3*A + 2*B*x^3) + 2*a*b*(15*A + 7*B*x^3)) - 3*a^(3/2)*(-6*A*b + a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(72*b^(3/2)*Sqrt[x]*Sqrt[1 + (b*x^3)/a])

IntegrateAlgebraic [A] time = 0.71, size = 162, normalized size = 1.01

$$\frac{\sqrt{a + bx^3} (3a^2Be^6(ex)^{3/2} + 30aAbe^6(ex)^{3/2} + 14abBe^3(ex)^{9/2} + 12Ab^2e^3(ex)^{9/2} + 8b^2B(ex)^{15/2})}{72be^7} - \frac{e^2 \sqrt{\frac{b}{e^3}} (6a^2Ab - a^3B) \log \left(\sqrt{a + bx^3} - \sqrt{\frac{b}{e^3}} (ex)^{3/2} \right)}{24b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (Sqrt[a + b*x^3]*(30*a*A*b*e^6*(e*x)^(3/2) + 3*a^2*B*e^6*(e*x)^(3/2) + 12*A*b^2*e^3*(e*x)^(9/2) + 14*a*b*B*e^3*(e*x)^(9/2) + 8*b^2*B*(e*x)^(15/2)))/(7*2*b*e^7) - ((6*a^2*A*b - a^3*B)*Sqrt[b/e^3]*e^2*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(24*b^2)
```

fricas [A] time = 1.56, size = 273, normalized size = 1.70

$$\frac{3(Ba^3 - 6Aa^2b)\sqrt{\frac{e}{b}} \log\left(-8b^2e^6x^6 - 8a^2be^6x^3 - a^2e - 4(2b^2x^4 + abx)\sqrt{bx^3 + a}\sqrt{\frac{e}{b}}\right) - 4(8Bb^2x^2 + 2(7Bab + 6Aa^2b)x^4 + 3(Ba^2 + 10Aab)x)\sqrt{bx^3 + a}\sqrt{\frac{e}{b}}}{288b} - \frac{3(Ba^3 - 6Aa^2b)\sqrt{\frac{e}{b}} \arctan\left(\frac{2\sqrt{bx^3 + a}\sqrt{\frac{e}{b}}}{2bx^3 + ae}\right) + 2(8Bb^2x^2 + 2(7Bab + 6Aa^2b)x^4 + 3(Ba^2 + 10Aab)x)\sqrt{bx^3 + a}\sqrt{\frac{e}{b}}}{144b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(B*a^3 - 6*A*a^2*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(8*B*b^2*x^7 + 2*(7*B*a*b + 6*A*b^2)*x^4 + 3*(B*a^2 + 10*A*a*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b, 1/144*(3*(B*a^3 - 6*A*a^2*b)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(8*B*b^2*x^7 + 2*(7*B*a*b + 6*A*b^2)*x^4 + 3*(B*a^2 + 10*A*a*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,%%{-2,[1,0,0,4]%%}+%%{-2,[0,1,6,1]%%}+%%{-2,[0,1,0,1]%%},0,%%{1,[2,0,0,8]%%}+%%{2,[1,1,6,5]%%}+%%{-2,[1,1,0,5]%%}+%%{1,[0,2,12,2]%%}+%%{-2,[0,2,6,2]%%}+%%{1,[0,2,0,2]%%}] at parameters values [91,88.2886286299,-21,88]Warning, choosing root of [1,0,%%{-2,[1,0,0,4]%%}+%%{-2,[0,1,6,1]%%}+%%{-2,[0,1,0,1]%%},0,%%{1,[2,0,0,8]%%}+%%{2,[1,1,6,5]%%}+%%{-2,[1,1,0,5]%%}+%%{1,[0,2,12,2]%%}+%%{-2,[0,2,6,2]%%}+%%{1,[0,2,0,2]%%}] at parameters values [66,6.82230772497,-23,79]Warning, choosing root of [1,0,%%{-2,[1,0,0,4]%%}+%%{-2,[0,1,6,1]%%}+%%{-2,[0,1,0,1]%%},0,%%{1,[2,0,0,8]%%}+%%{2,[1,1,6,5]%%}+%%{-2,[1,1,0,5]%%}+%%{1,[0,2,12,2]%%}+%%{-2,[0,2,6,2]%%}+%%{1,[0,2,0,2]%%}] at parameters values [6,94.9264369817,-8,31]2*B*b*exp(1)/exp(3)*2*((17740800*b^10*exp(1)^4/638668800/b^10/exp(1)^11*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*
```

```

exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+4435200*b^9*exp(1)^7*a
/638668800/b^10/exp(1)^11)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt
t(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))-6652800*b^8*exp(1)^10*a^2/6386688
00/b^10/exp(1)^11)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(
1)^4+b*(x*exp(1))^3*exp(1))+2*A*b*exp(1)/exp(3)*2*(240*b^4*exp(1)/5760/b^4/
exp(1)^5*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x
*exp(1))*sqrt(x*exp(1))+120*b^3*exp(1)^4*a/5760/b^4/exp(1)^5)*sqrt(x*exp(1)
)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))+2*B*
a*exp(1)/exp(3)*2*(240*b^4*exp(1)/5760/b^4/exp(1)^5*sqrt(x*exp(1))*sqrt(x*e
xp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+120*b^3*
exp(1)^4*a/5760/b^4/exp(1)^5)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*
sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))+2*A*a*exp(1)/exp(3)/exp(1)/3*(1/2*sq
rt(x*exp(1))*x*exp(1)*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))-2*a*exp(1)^4/4
/sqrt(b*exp(1))*ln(abs(sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))-sqrt(b*exp(1)
))*sqrt(x*exp(1))*x*exp(1))))-1/2*exp(1)/4/exp(1)*exp(3)/b/exp(3)/abs(2*b*a^
2*A+B*a^3)/(4*exp(1)*b^2*a^4*A^2+4*exp(1)*b*B*a^5*A+exp(1)*B^2*a^6)^-1/3/sq
rt(b*exp(1))*ln(abs(sqrt(4*A^2*a^5*b^2*exp(1)^6+4*A*B*a^6*b*exp(1)^6+B^2*a^
7*exp(1)^6+9*b*(-B*a^3*exp(1)*sqrt(x*exp(1))*x*exp(1)/3-2*A*a^2*b*exp(1)*sq
rt(x*exp(1))*x*exp(1)/3)^2*exp(1))-sqrt(9*b*exp(1))*(-B*a^3*exp(1)*sqrt(x*e
xp(1))*x*exp(1)/3-2*A*a^2*b*exp(1)*sqrt(x*exp(1))*x*exp(1)/3)))

```

maple [C] time = 1.06, size = 7290, normalized size = 45.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b*x^3+a)^{(3/2)}*(B*x^3+A)*(e*x)^{(1/2)}, x)$

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}\sqrt{ex} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b*x^3+a)^{(3/2)}*(B*x^3+A)*(e*x)^{(1/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}((B*x^3 + A)*(b*x^3 + a)^{(3/2)}*\text{sqrt}(e*x), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (Bx^3 + A)\sqrt{ex}(bx^3 + a)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(3/2), x)`

[Out] `int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(3/2), x)`

sympy [B] time = 25.46, size = 335, normalized size = 2.08

$$\frac{Aa^{\frac{3}{2}}(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e} + \frac{Aa^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{12e\sqrt{1+\frac{bx^3}{a}}} + \frac{A\sqrt{a}b(ex)^{\frac{9}{2}}}{4e^4\sqrt{1+\frac{bx^3}{a}}} + \frac{Aa^2\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{ae^2}}\right)}{4\sqrt{b}} + \frac{Ab^2(ex)^{\frac{15}{2}}}{6\sqrt{a}e^7\sqrt{1+\frac{bx^3}{a}}} + \frac{Ba^{\frac{5}{2}}(ex)^{\frac{3}{2}}}{24be\sqrt{1+\frac{bx^3}{a}}} + \frac{17Ba^{\frac{3}{2}}(ex)^{\frac{9}{2}}}{72e^4\sqrt{1+\frac{bx^3}{a}}} + \frac{11B\sqrt{a}b(ex)^{\frac{15}{2}}}{36e^7\sqrt{1+\frac{bx^3}{a}}} - \frac{Ba^3\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{ae^2}}\right)}{24b^{\frac{3}{2}}} + \frac{Bb^2(ex)^{\frac{21}{2}}}{9\sqrt{a}e^{10}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(3/2)*(B*x**3+A)*(e*x)**(1/2), x)`

[Out] `A*a**(3/2)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*e) + A*a**(3/2)*(e*x)**(3/2)/(12*e*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b*(e*x)**(9/2)/(4*e**4*sqrt(1 + b*x**3/a)) + A*a**2*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(4*sqrt(b)) + A*b**2*(e*x)**(15/2)/(6*sqrt(a)*e**7*sqrt(1 + b*x**3/a)) + B*a**(5/2)*(e*x)**(3/2)/(24*b*e*sqrt(1 + b*x**3/a)) + 17*B*a**(3/2)*(e*x)**(9/2)/(72*e**4*sqrt(1 + b*x**3/a)) + 11*B*sqrt(a)*b*(e*x)**(15/2)/(36*e**7*sqrt(1 + b*x**3/a)) - B*a**3*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(24*b**(3/2)) + B*b**2*(e*x)**(21/2)/(9*sqrt(a)*e**10*sqrt(1 + b*x**3/a))`

$$3.331 \quad \int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}} dx$$

Optimal. Leaf size=152

$$\frac{a(aB + 4Ab) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{4\sqrt{b}e^{5/2}} + \frac{(ex)^{3/2}(a+bx^3)^{3/2}(aB+4Ab)}{6ae^4} + \frac{(ex)^{3/2}\sqrt{a+bx^3}(aB+4Ab)}{4e^4} - \frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}}$$

Rubi [A] time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {453, 279, 329, 275, 217, 206}

$$\frac{(ex)^{3/2}(a+bx^3)^{3/2}(aB+4Ab)}{6ae^4} + \frac{(ex)^{3/2}\sqrt{a+bx^3}(aB+4Ab)}{4e^4} + \frac{a(aB+4Ab) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{4\sqrt{b}e^{5/2}} - \frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2), x]

[Out] (((4*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(4*e^4) + (((4*A*b + a*B)*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(6*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(3*a*e*(e*x)^(3/2)) + (a*(4*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(4*Sqrt[b]*e^(5/2)))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 279

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +

1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 453

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^{3/2} (A + Bx^3)}{(ex)^{5/2}} dx &= -\frac{2A(a + bx^3)^{5/2}}{3ae(ex)^{3/2}} + \frac{(4Ab + aB) \int \sqrt{ex} (a + bx^3)^{3/2} dx}{ae^3} \\
&= \frac{(4Ab + aB)(ex)^{3/2} (a + bx^3)^{3/2}}{6ae^4} - \frac{2A(a + bx^3)^{5/2}}{3ae(ex)^{3/2}} + \frac{(3(4Ab + aB)) \int \sqrt{ex} \sqrt{a + bx^3}}{4e^3} \\
&= \frac{(4Ab + aB)(ex)^{3/2} \sqrt{a + bx^3}}{4e^4} + \frac{(4Ab + aB)(ex)^{3/2} (a + bx^3)^{3/2}}{6ae^4} - \frac{2A(a + bx^3)^{5/2}}{3ae(ex)^{3/2}} \\
&= \frac{(4Ab + aB)(ex)^{3/2} \sqrt{a + bx^3}}{4e^4} + \frac{(4Ab + aB)(ex)^{3/2} (a + bx^3)^{3/2}}{6ae^4} - \frac{2A(a + bx^3)^{5/2}}{3ae(ex)^{3/2}} \\
&= \frac{(4Ab + aB)(ex)^{3/2} \sqrt{a + bx^3}}{4e^4} + \frac{(4Ab + aB)(ex)^{3/2} (a + bx^3)^{3/2}}{6ae^4} - \frac{2A(a + bx^3)^{5/2}}{3ae(ex)^{3/2}} \\
&= \frac{(4Ab + aB)(ex)^{3/2} \sqrt{a + bx^3}}{4e^4} + \frac{(4Ab + aB)(ex)^{3/2} (a + bx^3)^{3/2}}{6ae^4} - \frac{2A(a + bx^3)^{5/2}}{3ae(ex)^{3/2}} \\
&= \frac{(4Ab + aB)(ex)^{3/2} \sqrt{a + bx^3}}{4e^4} + \frac{(4Ab + aB)(ex)^{3/2} (a + bx^3)^{3/2}}{6ae^4} - \frac{2A(a + bx^3)^{5/2}}{3ae(ex)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 126, normalized size = 0.83

$$\frac{x\sqrt{a + bx^3} \left(3\sqrt{a} x^{3/2} (aB + 4Ab) \sinh^{-1} \left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}} \right) + \sqrt{b} \sqrt{\frac{bx^3}{a} + 1} (-8aA + 5aBx^3 + 4Abx^3 + 2bBx^6) \right)}{12\sqrt{b} (ex)^{5/2} \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2), x]

[Out] (x*sqrt[a + b*x^3]*(sqrt[b]*sqrt[1 + (b*x^3)/a]*(-8*a*A + 4*A*b*x^3 + 5*a*B*x^3 + 2*b*B*x^6) + 3*sqrt[a]*(4*A*b + a*B)*x^(3/2)*ArcSinh[(sqrt[b]*x^(3/2))/sqrt[a]]))/(12*sqrt[b]*(e*x)^(5/2)*sqrt[1 + (b*x^3)/a])

IntegrateAlgebraic [A] time = 0.76, size = 125, normalized size = 0.82

$$\frac{\sqrt{a + bx^3} (-8aAe^6 + 5aBe^6x^3 + 4Abe^6x^3 + 2bBe^6x^6)}{12e^7(ex)^{3/2}} - \frac{\sqrt{\frac{b}{e^3}} (a^2B + 4aAb) \log\left(\sqrt{a + bx^3} - \sqrt{\frac{b}{e^3}} (ex)^{3/2}\right)}{4be}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2), x]

[Out] (Sqrt[a + b*x^3]*(-8*a*A*e^6 + 4*A*b*e^6*x^3 + 5*a*B*e^6*x^3 + 2*b*B*e^6*x^6))/(12*e^7*(e*x)^(3/2)) - ((4*a*A*b + a^2*B)*Sqrt[b/e^3]*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(4*b*e)

fricas [A] time = 1.45, size = 255, normalized size = 1.68

$$\left[\frac{3(Ba^2 + 4Aab)\sqrt{bc}x^2 \log\left(\frac{-8b^2ex^6 - 8abcx^3 - a^2e - 4(2bx^4 + ax)\sqrt{bx^3 + a}\sqrt{bc}\sqrt{ex}}{48b^3x^2}\right) + 4(2Bb^2x^6 + (5Bab + 4Ab^2)x^3 - 8Aab)\sqrt{bx^3 + a}\sqrt{ex}}{48b^3x^2}, \dots, \frac{3(Ba^2 + 4Aab)\sqrt{-bc}x^2 \arctan\left(\frac{2\sqrt{bx^3 + a}\sqrt{-bc}\sqrt{ex}}{2bx^3 + ax}\right) - 2(2Bb^2x^6 + (5Bab + 4Ab^2)x^3 - 8Aab)\sqrt{bx^3 + a}\sqrt{ex}}{24b^3x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2), x, algorithm="fricas")

[Out] [1/48*(3*(B*a^2 + 4*A*a*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(2*B*b^2*x^6 + (5*B*a*b + 4*A*b^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/24*(3*(B*a^2 + 4*A*a*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) - 2*(2*B*b^2*x^6 + (5*B*a*b + 4*A*b^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2), x, algorithm="giac")

[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(5/2), x)

maple [C] time = 1.18, size = 7108, normalized size = 46.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")`

[Out] `integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(5/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{3/2}}{(ex)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(5/2),x)`

[Out] `int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(5/2), x)`

sympy [B] time = 24.32, size = 289, normalized size = 1.90

$$-\frac{2Aa^{\frac{3}{2}}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{A\sqrt{a}bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} - \frac{2A\sqrt{a}bx^{\frac{3}{2}}}{3e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{Aa\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{e^{\frac{5}{2}}} + \frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} + \frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{B\sqrt{a}bx^{\frac{9}{2}}}{4e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4\sqrt{b}e^{\frac{5}{2}}} + \frac{Bb^2x^{\frac{15}{2}}}{6\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(5/2),x)`

[Out] `-2*A*a**(3/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 2*A*sqrt(a)*b*x**(3/2)/(3*e**(5/2)*sqrt(1 + b*x**3/a)) + A*a*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/e**(5/2) + B*a**(3/2)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(3/2)*x**(3/2)/(12*e**(5/2)*sqrt(1 + b*x**3/a)) + B*sqrt(a)*b*x**(9/2)/(4*e**(5/2)*sqrt(1 + b*x**3/a)) + B*a**2*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*sqrt(b)*e**(5/2)) + B*b**2*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a))`

$$3.332 \quad \int (ex)^{7/2} (a + bx^3)^{5/2} (A + Bx^3) dx$$

Optimal. Leaf size=241

$$-\frac{a^4 e^{7/2} (10Ab - 3aB) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{384b^{5/2}} + \frac{a^3 e^2 (ex)^{3/2} \sqrt{a+bx^3} (10Ab - 3aB)}{384b^2} + \frac{a^2 (ex)^{9/2} \sqrt{a+bx^3} (10Ab - 3aB)}{192be} + \dots$$

Rubi [A] time = 0.16, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {459, 279, 321, 329, 275, 217, 206}

$$\frac{a^3 e^2 (ex)^{3/2} \sqrt{a+bx^3} (10Ab - 3aB)}{384b^2} - \frac{a^4 e^{7/2} (10Ab - 3aB) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{384b^{5/2}} + \frac{a^2 (ex)^{9/2} \sqrt{a+bx^3} (10Ab - 3aB)}{192be} + \frac{(ex)^{9/2} (a+bx^3)^{5/2} (10Ab - 3aB)}{120be} + \frac{a(ex)^{9/2} (a+bx^3)^{3/2} (10Ab - 3aB)}{144be} + \frac{B(ex)^{9/2} (a+bx^3)^{7/2}}{15be}$$

Antiderivative was successfully verified.

[In] Int[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]

[Out] (a^3*(10*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(384*b^2) + (a^2*(10*A*b - 3*a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(192*b*e) + (a*(10*A*b - 3*a*B)*(e*x)^(9/2)*(a + b*x^3)^(3/2))/(144*b*e) + ((10*A*b - 3*a*B)*(e*x)^(9/2)*(a + b*x^3)^(5/2))/(120*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(7/2))/(15*b*e) - (a^4*(10*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3]])/(384*b^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 279

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +

1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
\int (ex)^{7/2} (a + bx^3)^{5/2} (A + Bx^3) dx &= \frac{B(ex)^{9/2} (a + bx^3)^{7/2}}{15be} - \frac{\left(-15Ab + \frac{9aB}{2}\right) \int (ex)^{7/2} (a + bx^3)^{5/2} dx}{15b} \\
&= \frac{(10Ab - 3aB)(ex)^{9/2} (a + bx^3)^{5/2}}{120be} + \frac{B(ex)^{9/2} (a + bx^3)^{7/2}}{15be} + \frac{(a(10Ab - 3aB))}{120be} \\
&= \frac{a(10Ab - 3aB)(ex)^{9/2} (a + bx^3)^{3/2}}{144be} + \frac{(10Ab - 3aB)(ex)^{9/2} (a + bx^3)^{5/2}}{120be} + \frac{B(ex)^{9/2} (a + bx^3)^{7/2}}{15be} \\
&= \frac{a^2(10Ab - 3aB)(ex)^{9/2} \sqrt{a + bx^3}}{192be} + \frac{a(10Ab - 3aB)(ex)^{9/2} (a + bx^3)^{3/2}}{144be} + \frac{B(ex)^{9/2} (a + bx^3)^{5/2}}{15be} \\
&= \frac{a^3(10Ab - 3aB)e^2 (ex)^{3/2} \sqrt{a + bx^3}}{384b^2} + \frac{a^2(10Ab - 3aB)(ex)^{9/2} \sqrt{a + bx^3}}{192be} + \frac{aB(ex)^{9/2} (a + bx^3)^{5/2}}{15be} \\
&= \frac{a^3(10Ab - 3aB)e^2 (ex)^{3/2} \sqrt{a + bx^3}}{384b^2} + \frac{a^2(10Ab - 3aB)(ex)^{9/2} \sqrt{a + bx^3}}{192be} + \frac{aB(ex)^{9/2} (a + bx^3)^{5/2}}{15be} \\
&= \frac{a^3(10Ab - 3aB)e^2 (ex)^{3/2} \sqrt{a + bx^3}}{384b^2} + \frac{a^2(10Ab - 3aB)(ex)^{9/2} \sqrt{a + bx^3}}{192be} + \frac{aB(ex)^{9/2} (a + bx^3)^{5/2}}{15be} \\
&= \frac{a^3(10Ab - 3aB)e^2 (ex)^{3/2} \sqrt{a + bx^3}}{384b^2} + \frac{a^2(10Ab - 3aB)(ex)^{9/2} \sqrt{a + bx^3}}{192be} + \frac{aB(ex)^{9/2} (a + bx^3)^{5/2}}{15be} \\
&= \frac{a^3(10Ab - 3aB)e^2 (ex)^{3/2} \sqrt{a + bx^3}}{384b^2} + \frac{a^2(10Ab - 3aB)(ex)^{9/2} \sqrt{a + bx^3}}{192be} + \frac{aB(ex)^{9/2} (a + bx^3)^{5/2}}{15be}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 188, normalized size = 0.78

$$\frac{e^3 \sqrt{ex} \sqrt{a + bx^3} \left(15a^{7/2} (3aB - 10Ab) \sinh^{-1} \left(\frac{\sqrt{bx^3}}{\sqrt{a}} \right) + \sqrt{b} x^{3/2} \sqrt{\frac{bx^3}{a} + 1} \left(-45a^4 B + 30a^3 b (5A + Bx^3) + 4a^2 b^2 x^3 (295A + 186Bx^3) + 16ab^3 x^6 (85A + 63Bx^3) + 96b^4 x^9 (5A + 4Bx^3) \right) \right)}{5760b^{5/2} \sqrt{x} \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3), x]

[Out] (e^3*Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(-45*a^4*B + 30*a^3*b*(5*A + B*x^3) + 96*b^4*x^9*(5*A + 4*B*x^3) + 16*a*b^3*x^6*(8

$5*A + 63*B*x^3) + 4*a^2*b^2*x^3*(295*A + 186*B*x^3) + 15*a^{7/2}*(-10*A*b + 3*a*B)*\text{ArcSinh}[(\text{Sqrt}[b]*x^{3/2})/\text{Sqrt}[a]])/(5760*b^{5/2}*\text{Sqrt}[x]*\text{Sqrt}[1 + (b*x^3)/a])$

IntegrateAlgebraic [A] time = 0.95, size = 238, normalized size = 0.99

$$\frac{e^5 \sqrt{\frac{b}{a}} (10a^4 Ab - 3a^5 B) \log\left(\sqrt{a + bx^3} - \sqrt{\frac{b}{a}} (cx)^{3/2}\right) + \sqrt{a + bx^3} (-45a^4 Bc^{12}(cx)^{3/2} + 150a^3 Abc^{12}(cx)^{3/2} + 30a^2 b Bc^9(cx)^{3/2} + 1180a^2 Ab^2 c^9(cx)^{3/2} + 744a^2 b^2 Bc^6(cx)^{3/2} + 1360a Ab^3 c^6(cx)^{3/2} + 1008ab^3 Bc^3(cx)^{3/2} + 480Ab^4 c^3(cx)^{3/2} + 384b^4 B(cx)^{3/2})}{384b^3} + \frac{\sqrt{a + bx^3} (-45a^4 Bc^{12}(cx)^{3/2} + 150a^3 Abc^{12}(cx)^{3/2} + 30a^2 b Bc^9(cx)^{3/2} + 1180a^2 Ab^2 c^9(cx)^{3/2} + 744a^2 b^2 Bc^6(cx)^{3/2} + 1360a Ab^3 c^6(cx)^{3/2} + 1008ab^3 Bc^3(cx)^{3/2} + 480Ab^4 c^3(cx)^{3/2} + 384b^4 B(cx)^{3/2})}{5760b^2 c^{10}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3), x]

[Out] $(\text{Sqrt}[a + b*x^3]*(150*a^3*A*b*e^{12}*(e*x)^{3/2} - 45*a^4*B*e^{12}*(e*x)^{3/2} + 1180*a^2*A*b^2*e^9*(e*x)^{9/2} + 30*a^3*b*B*e^9*(e*x)^{9/2} + 1360*a*A*b^3*e^6*(e*x)^{15/2} + 744*a^2*b^2*B*e^6*(e*x)^{15/2} + 480*A*b^4*e^3*(e*x)^{21/2} + 1008*a*b^3*B*e^3*(e*x)^{21/2} + 384*b^4*B*(e*x)^{27/2}))/ (5760*b^2*e^{10} + ((10*a^4*A*b - 3*a^5*B)*\text{Sqrt}[b/e^3]*e^5*\text{Log}[-(\text{Sqrt}[b/e^3]*(e*x)^{3/2}) + \text{Sqrt}[a + b*x^3]])/(384*b^3))$

fricas [A] time = 1.33, size = 409, normalized size = 1.70

$$\frac{15(3Ba^5 - 10Aa^4b)\sqrt{e} \log(-8b^2e^6x^6 - 8a^2be^3x^3 - a^2e + 4(2b^2x^4 + abx)\sqrt{e})\sqrt{e} - 4(384Bb^4e^3x^{13} + 48(21Bab^3 + 10Aab^4)e^3x^{10} + 8(93Ba^2b^2 + 170Aa^2b^3)e^3x^7 + 10(3Ba^3b + 118Aa^2b^2)e^3x^4 - 15(3Ba^4 - 10Aa^3b)e^3x)\sqrt{e}}{3840b^3} + \frac{15(3Ba^5 - 10Aa^4b)\sqrt{e} \arctan\left(\frac{\sqrt{a+bx^3}}{\sqrt{e}}\right) - 2(384Bb^4e^3x^{13} + 48(21Bab^3 + 10Aab^4)e^3x^{10} + 8(93Ba^2b^2 + 170Aa^2b^3)e^3x^7 + 10(3Ba^3b + 118Aa^2b^2)e^3x^4 - 15(3Ba^4 - 10Aa^3b)e^3x)\sqrt{e}}{11520b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A), x, algorithm="fricas")

[Out] $[-1/23040*(15*(3*B*a^5 - 10*A*a^4*b)*e^3*\text{sqrt}(e/b)*\log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*\text{sqrt}(b*x^3 + a)*\text{sqrt}(e*x)*\text{sqrt}(e/b)) - 4*(384*B*b^4*e^3*x^{13} + 48*(21*B*a*b^3 + 10*A*b^4)*e^3*x^{10} + 8*(93*B*a^2*b^2 + 170*A*a^2*b^3)*e^3*x^7 + 10*(3*B*a^3*b + 118*A*a^2*b^2)*e^3*x^4 - 15*(3*B*a^4 - 10*A*a^3*b)*e^3*x)*\text{sqrt}(b*x^3 + a)*\text{sqrt}(e*x))/b^2, -1/11520*(15*(3*B*a^5 - 10*A*a^4*b)*e^3*\text{sqrt}(-e/b)*\arctan(2*\text{sqrt}(b*x^3 + a)*\text{sqrt}(e*x)*b*x*\text{sqrt}(-e/b)/(2*b*e*x^3 + a*e)) - 2*(384*B*b^4*e^3*x^{13} + 48*(21*B*a*b^3 + 10*A*b^4)*e^3*x^{10} + 8*(93*B*a^2*b^2 + 170*A*a^2*b^3)*e^3*x^7 + 10*(3*B*a^3*b + 118*A*a^2*b^2)*e^3*x^4 - 15*(3*B*a^4 - 10*A*a^3*b)*e^3*x)*\text{sqrt}(b*x^3 + a)*\text{sqrt}(e*x))/b^2]$

giac [B] time = 1.49, size = 563, normalized size = 2.34

$$\frac{15(3Ba^5 - 10Aa^4b)\sqrt{e} \log(-8b^2e^6x^6 - 8a^2be^3x^3 - a^2e + 4(2b^2x^4 + abx)\sqrt{e})\sqrt{e} - 4(384Bb^4e^3x^{13} + 48(21Bab^3 + 10Aab^4)e^3x^{10} + 8(93Ba^2b^2 + 170Aa^2b^3)e^3x^7 + 10(3Ba^3b + 118Aa^2b^2)e^3x^4 - 15(3Ba^4 - 10Aa^3b)e^3x)\sqrt{e}}{3840b^3} + \frac{15(3Ba^5 - 10Aa^4b)\sqrt{e} \arctan\left(\frac{\sqrt{a+bx^3}}{\sqrt{e}}\right) - 2(384Bb^4e^3x^{13} + 48(21Bab^3 + 10Aab^4)e^3x^{10} + 8(93Ba^2b^2 + 170Aa^2b^3)e^3x^7 + 10(3Ba^3b + 118Aa^2b^2)e^3x^4 - 15(3Ba^4 - 10Aa^3b)e^3x)\sqrt{e}}{11520b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A), x, algorithm="giac")

[Out] $1/12*\text{sqrt}(b*x^3*e^4 + a*e^4)*(2*x^3*e^{-1} + a*e^{-1}/b)*A*a^2*x^{3/2}*e^{5/2} + 1/72*\text{sqrt}(b*x^3*e^4 + a*e^4)*(2*(4*x^3*e^{-4} + a*e^{-4}/b)*x^3*e^3 -$

$$3a^2e^{-1}/b^2) * B * a^2x^{3/2} * e^{5/2} + 1/36 * \sqrt{bx^3e^4 + ae^4} * (2 * (4x^3e^{-4} + ae^{-4}/b) * x^3e^3 - 3a^2e^{-1}/b^2) * A * a * bx^{3/2} * e^{5/2} + 1/288 * \sqrt{bx^3e^4 + ae^4} * (2 * (4 * (6x^3e^{-7} + ae^{-7}/b) * x^3e^3 - 5a^2e^{-4}/b^2) * x^3e^3 + 15a^3e^{-1}/b^3) * B * a * bx^{3/2} * e^{5/2} + 1/576 * \sqrt{bx^3e^4 + ae^4} * (2 * (4 * (6x^3e^{-7} + ae^{-7}/b) * x^3e^3 - 5a^2e^{-4}/b^2) * x^3e^3 + 15a^3e^{-1}/b^3) * A * b^2 * x^{3/2} * e^{5/2} + 1/5760 * \sqrt{bx^3e^4 + ae^4} * (2 * (4 * (6 * (8x^3e^{-10} + ae^{-10}/b) * x^3e^3 - 7a^2e^{-7}/b^2) * x^3e^3 + 35a^3e^{-4}/b^3) * x^3e^3 - 105a^4e^{-1}/b^4) * B * b^2 * x^{3/2} * e^{5/2} - 1/384 * (9B^2a^{10}e^7 - 60A * B * a^9 * b * e^7 + 100A^2 * a^8 * b^2 * e^7) * e^{-1/2} * \log(\text{abs}(- (3B * a^5 * x^{3/2}) * e^{11/2} - 10A * a^4 * b * x^{3/2}) * e^{11/2})) * \sqrt{b} * e^{1/2} + \sqrt{9B^2 * a^{11} * e^{12} - 60A * B * a^{10} * b * e^{12} + 100A^2 * a^9 * b^2 * e^{12} + (3B * a^5 * x^{3/2}) * e^{11/2} - 10A * a^4 * b * x^{3/2}) * e^{11/2}}^2 * b * e)) / (b^{5/2} * \text{abs}(-3B * a^5 * e^3 + 10A * a^4 * b * e^3))$$

maple [C] time = 1.06, size = 8117, normalized size = 33.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx^3 + A)(bx^3 + a)^{\frac{5}{2}} (ex)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")`

[Out] `integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(7/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (Bx^3 + A) (ex)^{7/2} (bx^3 + a)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(5/2),x)`

[Out] `int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(5/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] Timed out
```

3.333 $\int \sqrt{ex} (a + bx^3)^{5/2} (A + Bx^3) dx$

Optimal. Leaf size=201

$$\frac{5a^3\sqrt{e}(8Ab - aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{3/2}} + \frac{5a^2(ex)^{3/2}\sqrt{a+bx^3}(8Ab - aB)}{192be} + \frac{(ex)^{3/2}(a+bx^3)^{5/2}(8Ab - aB)}{72be} + \frac{5a(ex)^3}{192be}$$

Rubi [A] time = 0.13, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {459, 279, 329, 275, 217, 206}

$$\frac{5a^3\sqrt{e}(8Ab - aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{3/2}} + \frac{5a^2(ex)^{3/2}\sqrt{a+bx^3}(8Ab - aB)}{192be} + \frac{(ex)^{3/2}(a+bx^3)^{5/2}(8Ab - aB)}{72be} + \frac{5a(ex)^3}{288be} + \frac{B(ex)^{3/2}(a+bx^3)^{7/2}}{12be}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3), x]

[Out] (5*a^2*(8*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(192*b*e) + (5*a*(8*A*b - a*B)*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(288*b*e) + ((8*A*b - a*B)*(e*x)^(3/2)*(a + b*x^3)^(5/2))/(72*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(7/2))/(12*b*e) + (5*a^3*(8*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3]])/(192*b^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 279

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +

1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{ex} (a + bx^3)^{5/2} (A + Bx^3) dx &= \frac{B(ex)^{3/2} (a + bx^3)^{7/2}}{12be} - \frac{\left(-12Ab + \frac{3aB}{2}\right) \int \sqrt{ex} (a + bx^3)^{5/2} dx}{12b} \\
&= \frac{(8Ab - aB)(ex)^{3/2} (a + bx^3)^{5/2}}{72be} + \frac{B(ex)^{3/2} (a + bx^3)^{7/2}}{12be} + \frac{(5a(8Ab - aB)) \int \sqrt{ex} (a + bx^3)^{3/2} dx}{48b} \\
&= \frac{5a(8Ab - aB)(ex)^{3/2} (a + bx^3)^{3/2}}{288be} + \frac{(8Ab - aB)(ex)^{3/2} (a + bx^3)^{5/2}}{72be} + \frac{B(ex)^{3/2} (a + bx^3)^{7/2}}{12b} \\
&= \frac{5a^2(8Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{192be} + \frac{5a(8Ab - aB)(ex)^{3/2} (a + bx^3)^{3/2}}{288be} + \frac{(8Ab - aB)(ex)^{3/2} (a + bx^3)^{5/2}}{72be} \\
&= \frac{5a^2(8Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{192be} + \frac{5a(8Ab - aB)(ex)^{3/2} (a + bx^3)^{3/2}}{288be} + \frac{(8Ab - aB)(ex)^{3/2} (a + bx^3)^{5/2}}{72be} \\
&= \frac{5a^2(8Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{192be} + \frac{5a(8Ab - aB)(ex)^{3/2} (a + bx^3)^{3/2}}{288be} + \frac{(8Ab - aB)(ex)^{3/2} (a + bx^3)^{5/2}}{72be} \\
&= \frac{5a^2(8Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{192be} + \frac{5a(8Ab - aB)(ex)^{3/2} (a + bx^3)^{3/2}}{288be} + \frac{(8Ab - aB)(ex)^{3/2} (a + bx^3)^{5/2}}{72be} \\
&= \frac{5a^2(8Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{192be} + \frac{5a(8Ab - aB)(ex)^{3/2} (a + bx^3)^{3/2}}{288be} + \frac{(8Ab - aB)(ex)^{3/2} (a + bx^3)^{5/2}}{72be} \\
&= \frac{5a^2(8Ab - aB)(ex)^{3/2} \sqrt{a + bx^3}}{192be} + \frac{5a(8Ab - aB)(ex)^{3/2} (a + bx^3)^{3/2}}{288be} + \frac{(8Ab - aB)(ex)^{3/2} (a + bx^3)^{5/2}}{72be}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 146, normalized size = 0.73

$$\frac{x\sqrt{ex} \sqrt{a + bx^3} \left(\frac{(8Ab - aB) \left(15a^{5/2} \sinh^{-1} \left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}} \right) + \sqrt{b}x^{3/2} \sqrt{\frac{bx^3}{a} + 1} (33a^2 + 26abx^3 + 8b^2x^6) \right)}{48\sqrt{b}x^{3/2} \sqrt{\frac{bx^3}{a} + 1}} + B(a + bx^3)^3 \right)}{12b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3), x]

[Out] (x*Sqrt[e*x]*Sqrt[a + b*x^3]*(B*(a + b*x^3)^3 + ((8*A*b - a*B)*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(33*a^2 + 26*a*b*x^3 + 8*b^2*x^6) + 15*a^(5/2)*Arc

$\text{Sinh}(\left(\frac{\sqrt{b} \cdot x^{3/2}}{\sqrt{a}}\right) / (48 \cdot \sqrt{b} \cdot x^{3/2} \cdot \sqrt{1 + (b \cdot x^3)/a})) / (12 \cdot b)$

IntegrateAlgebraic [A] time = 0.77, size = 200, normalized size = 1.00

$$\frac{\sqrt{a+bx^3} (15a^3 B e^9 (ex)^{3/2} + 264a^2 A b e^9 (ex)^{3/2} + 118a^2 b B e^6 (ex)^{9/2} + 208a A b^2 e^6 (ex)^{9/2} + 136a b^2 B e^3 (ex)^{15/2} + 64A b^3 e^3 (ex)^{15/2} + 48b^3 B (ex)^{21/2})}{576b e^{10}} - \frac{5e^2 \sqrt{\frac{b}{a}} (8a^3 A b - a^4 B) \log\left(\sqrt{a+bx^3} - \sqrt{\frac{b}{a}} (ex)^{3/2}\right)}{192b^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[ex]*(a + b*x^3)^(5/2)*(A + B*x^3), x]

[Out] $(\sqrt{a + b x^3} * (264 * a^2 * A * b * e^9 * (e * x)^{3/2} + 15 * a^3 * B * e^9 * (e * x)^{3/2} + 208 * a * A * b^2 * e^6 * (e * x)^{9/2} + 118 * a^2 * b * B * e^6 * (e * x)^{9/2} + 64 * A * b^3 * e^3 * (e * x)^{15/2} + 136 * a * b^2 * B * e^3 * (e * x)^{15/2} + 48 * b^3 * B * (e * x)^{21/2})) / (576 * b * e^{10} - (5 * (8 * a^3 * A * b - a^4 * B) * \text{Sqrt}[b/e^3] * e^2 * \text{Log}[-(\text{Sqrt}[b/e^3] * (e * x)^{3/2}) + \text{Sqrt}[a + b * x^3]]) / (192 * b^2))$

fricas [A] time = 1.39, size = 323, normalized size = 1.61

$$\frac{15(Ba^4 - 8Aa^3b)\sqrt{\frac{b}{a}} \log\left(-8b^2 e^6 - 8a b e^6 e^{3/2} - a^2 e - 4(2b^2 x^4 + a b x) \sqrt{\frac{b}{a}} \sqrt{\frac{b}{a}}\right) - 4(48 B b^3 a^2 + 8(17 B a^2 + 8 A b^2) b^2 + 2(59 B a^2 b + 104 A a b^2) b + 3(5 B a^3 + 88 A a^2 b) x) \sqrt{\frac{b}{a}} \sqrt{\frac{b}{a}}}{2304 b} - \frac{15(Ba^4 - 8Aa^3b)\sqrt{\frac{b}{a}} \arctan\left(\frac{2\sqrt{\frac{b}{a}} \sqrt{\frac{b}{a}} \sqrt{\frac{b}{a}}}{2a e^{3/2}}\right) + 2(48 B b^3 a^2 + 8(17 B a^2 + 8 A b^2) b^2 + 2(59 B a^2 b + 104 A a b^2) b + 3(5 B a^3 + 88 A a^2 b) x) \sqrt{\frac{b}{a}} \sqrt{\frac{b}{a}}}{1152 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2), x, algorithm="fricas")

[Out] $[-1/2304 * (15 * (B * a^4 - 8 * A * a^3 * b) * \text{sqrt}(e/b) * \log(-8 * b^2 * e * x^6 - 8 * a * b * e * x^3 - a^2 * e - 4 * (2 * b^2 * x^4 + a * b * x) * \text{sqrt}(b * x^3 + a) * \text{sqrt}(e * x) * \text{sqrt}(e/b)) - 4 * (48 * B * b^3 * x^{10} + 8 * (17 * B * a * b^2 + 8 * A * b^3) * x^7 + 2 * (59 * B * a^2 * b + 104 * A * a * b^2) * x^4 + 3 * (5 * B * a^3 + 88 * A * a^2 * b) * x) * \text{sqrt}(b * x^3 + a) * \text{sqrt}(e * x)) / b, 1/1152 * (15 * (B * a^4 - 8 * A * a^3 * b) * \text{sqrt}(-e/b) * \arctan(2 * \text{sqrt}(b * x^3 + a) * \text{sqrt}(e * x) * b * x * \text{sqrt}(-e/b) / (2 * b * e * x^3 + a * e)) + 2 * (48 * B * b^3 * x^{10} + 8 * (17 * B * a * b^2 + 8 * A * b^3) * x^7 + 2 * (59 * B * a^2 * b + 104 * A * a * b^2) * x^4 + 3 * (5 * B * a^3 + 88 * A * a^2 * b) * x) * \text{sqrt}(b * x^3 + a) * \text{sqrt}(e * x)) / b]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2), x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of $[1, 0, \{\{-2, [1, 0, 0, 4]\}\} + \{\{-2, [0, 1, 6, 1]\}\} + \{\{-2, [0, 1, 0, 1]\}\}, 0, \{\{1, [2, 0, 0, 8]\}\} + \{\{2, [1, 1, 6, 5]\}\} + \{\{-2, [1, 1, 0, 5]\}\} + \{\{1, [0, 2, 12, 2]\}\} + \{\{-2, [0, 2, 6, 2]\}\} + \{\{1, [0, 2, 0, 2]\}\}]$ at paramet

ers values [91,88.2886286299,-21,88]Warning, choosing root of [1,0,%%{-2,[1,0,0,4]%%}+%%{-2,[0,1,6,1]%%}+%%{-2,[0,1,0,1]%%},0,%%{1,[2,0,0,8]%%}+%%{2,[1,1,6,5]%%}+%%{-2,[1,1,0,5]%%}+%%{1,[0,2,12,2]%%}+%%{-2,[0,2,6,2]%%}+%%{1,[0,2,0,2]%%}] at parameters values [66,6.82230772497,-23,79]Warning, choosing root of [1,0,%%{-2,[1,0,0,4]%%}+%%{-2,[0,1,6,1]%%}+%%{-2,[0,1,0,1]%%},0,%%{1,[2,0,0,8]%%}+%%{2,[1,1,6,5]%%}+%%{-2,[1,1,0,5]%%}+%%{1,[0,2,12,2]%%}+%%{-2,[0,2,6,2]%%}+%%{1,[0,2,0,2]%%}] at parameters values [6,94.9264369817,-8,31]2*B*b^2*exp(1)/exp(3)*2*(((26347216896000*b^16*exp(1)^7/1264666411008000/b^16/exp(1)^17*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+4391202816000*b^15*exp(1)^10*a/1264666411008000/b^16/exp(1)^17)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))-5489003520000*b^14*exp(1)^13*a^2/1264666411008000/b^16/exp(1)^17)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+8233505280000*b^13*exp(1)^16*a^3/1264666411008000/b^16/exp(1)^17)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))+2*A*b^2*exp(1)/exp(3)*2*((17740800*b^10*exp(1)^4/638668800/b^10/exp(1)^11*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+4435200*b^9*exp(1)^7*a/638668800/b^10/exp(1)^11)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))-6652800*b^8*exp(1)^10*a^2/638668800/b^10/exp(1)^11)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))+2*B*a^2*exp(1)/exp(3)*2*(240*b^4*exp(1)/5760/b^4/exp(1)^5*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+120*b^3*exp(1)^4*a/5760/b^4/exp(1)^5)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))+4*A*a*b*exp(1)/exp(3)*2*(240*b^4*exp(1)/5760/b^4/exp(1)^5*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+120*b^3*exp(1)^4*a/5760/b^4/exp(1)^5)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))+2*A*a^2*exp(1)/exp(3)/exp(1)/3*(1/2*sqrt(x*exp(1))*x*exp(1)*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))-2*a*exp(1)^4/4/sqrt(b*exp(1))*ln(abs(sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))-sqrt(b*exp(1))*sqrt(x*exp(1))*x*exp(1))))+4*B*a*b*exp(1)/exp(3)*2*((17740800*b^10*exp(1)^4/638668800/b^10/exp(1)^11*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+4435200*b^9*exp(1)^7*a/638668800/b^10/exp(1)^11)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))-6652800*b^8*exp(1)^10*a^2/638668800/b^10/exp(1)^11)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))-1/2*exp(1)/32/exp(1)*exp(3)/b/exp(3)/abs(24*b*a^3*A+5*B*a^4)/(576*exp(1)*b^2*a^6*A^2+240*exp(1)*b*B*a^7*A+25*exp(1)*B^2*a^8)^-1/3/sqrt(b*exp(1))*ln(abs(sqrt(576*A^2*a^7*b^2*exp(1)^6+240*A*B*a^8*b*exp(1)^6+25*B^2*a^9*exp(1)^6+9*b*(-5*B*a^4*exp(1))*sqrt(x*exp(1))*x*exp(1)/3-24*A*a^3*b*exp(1))*sqrt(x*exp(1))*x*exp(1)/3)^2*exp(1))-sqrt(9*b*exp(1))*(-5*B*a^4*exp(1))*sqrt(x*exp(1))*x*exp(1)/3-24*A*a^3*b*exp(1))*sqrt(x*exp(1))*x*exp(1)/3)))

maple [C] time = 1.10, size = 7702, normalized size = 38.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b*x^3+a)^{(5/2)}*(B*x^3+A)*(e*x)^{(1/2)}, x)$

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx^3 + A)(bx^3 + a)^{\frac{5}{2}}\sqrt{ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b*x^3+a)^{(5/2)}*(B*x^3+A)*(e*x)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}((B*x^3 + A)*(b*x^3 + a)^{(5/2)}*\text{sqrt}(e*x), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (Bx^3 + A)\sqrt{ex}(bx^3 + a)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B*x^3)*(e*x)^{(1/2)}*(a + b*x^3)^{(5/2)}, x)$

[Out] $\text{int}((A + B*x^3)*(e*x)^{(1/2)}*(a + b*x^3)^{(5/2)}, x)$

sympy [B] time = 56.28, size = 413, normalized size = 2.05

$$\frac{Aa^{\frac{5}{2}}(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e} + \frac{Aa^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{8e\sqrt{1+\frac{bx^3}{a}}} + \frac{35Aa^{\frac{3}{2}}b(ex)^{\frac{9}{2}}}{72e^4\sqrt{1+\frac{bx^3}{a}}} + \frac{17A\sqrt{a}b^2(ex)^{\frac{15}{2}}}{36e^7\sqrt{1+\frac{bx^3}{a}}} + \frac{5Aa^3\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}}\right)}{24\sqrt{b}} + \frac{Ab^3(ex)^{\frac{21}{2}}}{9\sqrt{a}e^{10}\sqrt{1+\frac{bx^3}{a}}} + \frac{5Ba^{\frac{7}{2}}(ex)^{\frac{3}{2}}}{192be\sqrt{1+\frac{bx^3}{a}}} + \frac{133Ba^{\frac{5}{2}}(ex)^{\frac{9}{2}}}{576e^4\sqrt{1+\frac{bx^3}{a}}} + \frac{127Ba^{\frac{3}{2}}b(ex)^{\frac{15}{2}}}{288e^7\sqrt{1+\frac{bx^3}{a}}} + \frac{23B\sqrt{a}b^2(ex)^{\frac{21}{2}}}{72e^{10}\sqrt{1+\frac{bx^3}{a}}} - \frac{5Ba^4\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}}\right)}{192b^{\frac{3}{2}}} + \frac{Bb^3(ex)^{\frac{27}{2}}}{12\sqrt{a}e^{13}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b*x**3+a)**(5/2)*(B*x**3+A)*(e*x)**(1/2), x)$

[Out] $A*a**(5/2)*(e*x)**(3/2)*\text{sqrt}(1 + b*x**3/a)/(3*e) + A*a**(5/2)*(e*x)**(3/2)/(8*e*\text{sqrt}(1 + b*x**3/a)) + 35*A*a**(3/2)*b*(e*x)**(9/2)/(72*e**4*\text{sqrt}(1 + b*x**3/a)) + 17*A*\text{sqrt}(a)*b**2*(e*x)**(15/2)/(36*e**7*\text{sqrt}(1 + b*x**3/a)) + 5*A*a**3*\text{sqrt}(e)*\text{asinh}(\text{sqrt}(b)*(e*x)**(3/2)/(\text{sqrt}(a)*e**(3/2)))/(24*\text{sqrt}(b)) + A*b**3*(e*x)**(21/2)/(9*\text{sqrt}(a)*e**10*\text{sqrt}(1 + b*x**3/a)) + 5*B*a**(7/2)*(e*x)**(3/2)/(192*b*e*\text{sqrt}(1 + b*x**3/a)) + 133*B*a**(5/2)*(e*x)**(9/2)/(576*e**4*\text{sqrt}(1 + b*x**3/a)) + 127*B*a**(3/2)*b*(e*x)**(15/2)/(288*e**7*\text{sqrt}(1 + b*x**3/a)) + 23*B*\text{sqrt}(a)*b**2*(e*x)**(21/2)/(72*e**10*\text{sqrt}(1 + b*x**3/a)) - 5*B*a**4*\text{sqrt}(e)*\text{asinh}(\text{sqrt}(b)*(e*x)**(3/2)/(\text{sqrt}(a)*e**(3/2)))/(192*b**(3/2)) + B*b**3*(e*x)**(27/2)/(12*\text{sqrt}(a)*e**13*\text{sqrt}(1 + b*x**3/a))$

$$3.334 \quad \int \frac{(a+bx^3)^{5/2} (A+Bx^3)}{(ex)^{5/2}} dx$$

Optimal. Leaf size=188

$$\frac{5a^2(aB + 6Ab) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24\sqrt{b}e^{5/2}} + \frac{(ex)^{3/2}(a+bx^3)^{5/2}(aB+6Ab)}{9ae^4} + \frac{5(ex)^{3/2}(a+bx^3)^{3/2}(aB+6Ab)}{36e^4} + \frac{5a(ex)^{3/2}}{36e^4}$$

Rubi [A] time = 0.13, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {453, 279, 329, 275, 217, 206}

$$\frac{5a^2(aB + 6Ab) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24\sqrt{b}e^{5/2}} + \frac{(ex)^{3/2}(a+bx^3)^{5/2}(aB+6Ab)}{9ae^4} + \frac{5(ex)^{3/2}(a+bx^3)^{3/2}(aB+6Ab)}{36e^4} + \frac{5a(ex)^{3/2}\sqrt{a+bx^3}(aB+6Ab)}{24e^4} - \frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2), x]

[Out] (5*a*(6*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*e^4) + (5*(6*A*b + a*B)*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(36*e^4) + ((6*A*b + a*B)*(e*x)^(3/2)*(a + b*x^3)^(5/2))/(9*a*e^4) - (2*A*(a + b*x^3)^(7/2))/(3*a*e*(e*x)^(3/2)) + (5*a^2*(6*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(24*Sqrt[b]*e^(5/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^{5/2} (A + Bx^3)}{(ex)^{5/2}} dx &= -\frac{2A(a + bx^3)^{7/2}}{3ae(ex)^{3/2}} + \frac{(6Ab + aB) \int \sqrt{ex} (a + bx^3)^{5/2} dx}{ae^3} \\
&= \frac{(6Ab + aB)(ex)^{3/2} (a + bx^3)^{5/2}}{9ae^4} - \frac{2A(a + bx^3)^{7/2}}{3ae(ex)^{3/2}} + \frac{(5(6Ab + aB)) \int \sqrt{ex} (a + bx^3)^{5/2} dx}{6e^3} \\
&= \frac{5(6Ab + aB)(ex)^{3/2} (a + bx^3)^{3/2}}{36e^4} + \frac{(6Ab + aB)(ex)^{3/2} (a + bx^3)^{5/2}}{9ae^4} - \frac{2A(a + bx^3)^{7/2}}{3ae(ex)^{3/2}} \\
&= \frac{5a(6Ab + aB)(ex)^{3/2} \sqrt{a + bx^3}}{24e^4} + \frac{5(6Ab + aB)(ex)^{3/2} (a + bx^3)^{3/2}}{36e^4} + \frac{(6Ab + aB)(ex)^{3/2} (a + bx^3)^{5/2}}{9ae^4} \\
&= \frac{5a(6Ab + aB)(ex)^{3/2} \sqrt{a + bx^3}}{24e^4} + \frac{5(6Ab + aB)(ex)^{3/2} (a + bx^3)^{3/2}}{36e^4} + \frac{(6Ab + aB)(ex)^{3/2} (a + bx^3)^{5/2}}{9ae^4} \\
&= \frac{5a(6Ab + aB)(ex)^{3/2} \sqrt{a + bx^3}}{24e^4} + \frac{5(6Ab + aB)(ex)^{3/2} (a + bx^3)^{3/2}}{36e^4} + \frac{(6Ab + aB)(ex)^{3/2} (a + bx^3)^{5/2}}{9ae^4} \\
&= \frac{5a(6Ab + aB)(ex)^{3/2} \sqrt{a + bx^3}}{24e^4} + \frac{5(6Ab + aB)(ex)^{3/2} (a + bx^3)^{3/2}}{36e^4} + \frac{(6Ab + aB)(ex)^{3/2} (a + bx^3)^{5/2}}{9ae^4} \\
&= \frac{5a(6Ab + aB)(ex)^{3/2} \sqrt{a + bx^3}}{24e^4} + \frac{5(6Ab + aB)(ex)^{3/2} (a + bx^3)^{3/2}}{36e^4} + \frac{(6Ab + aB)(ex)^{3/2} (a + bx^3)^{5/2}}{9ae^4}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 150, normalized size = 0.80

$$\frac{x\sqrt{a + bx^3} \left(15a^{3/2}x^{3/2}(aB + 6Ab) \sinh^{-1} \left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}} \right) + \sqrt{b} \sqrt{\frac{bx^3}{a} + 1} \left(a^2(33Bx^3 - 48A) + a(54Abx^3 + 26bBx^6) + 4b^2x^6(3A + 2Bx^3) \right) \right)}{72\sqrt{b}(ex)^{5/2}\sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2), x]

[Out] (x*Sqrt[a + b*x^3]*(Sqrt[b]*Sqrt[1 + (b*x^3)/a]*(4*b^2*x^6*(3*A + 2*B*x^3) + a^2*(-48*A + 33*B*x^3) + a*(54*A*b*x^3 + 26*b*B*x^6)) + 15*a^(3/2)*(6*A*b + a*B)*x^(3/2)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(72*Sqrt[b]*(e*x)^(5/2))*Sqrt[1 + (b*x^3)/a]

IntegrateAlgebraic [A] time = 0.85, size = 157, normalized size = 0.84

$$\frac{\sqrt{a+bx^3}(-48a^2Ae^9+33a^2Be^9x^3+54aAbe^9x^3+26abBe^9x^6+12Ab^2e^9x^6+8b^2Be^9x^9)}{72e^{10}(ex)^{3/2}} - \frac{5\sqrt{\frac{b}{e^3}}(a^3B+6a^2Ab)\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}(ex)^{3/2}\right)}{24be}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]

[Out] (Sqrt[a + b*x^3]*(-48*a^2*A*e^9 + 54*a*A*b*e^9*x^3 + 33*a^2*B*e^9*x^3 + 12*A*b^2*e^9*x^6 + 26*a*b*B*e^9*x^6 + 8*b^2*B*e^9*x^9))/(72*e^10*(e*x)^(3/2)) - (5*(6*a^2*A*b + a^3*B)*Sqrt[b/e^3]*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(24*b*e)

fricas [A] time = 1.28, size = 309, normalized size = 1.64

$$\frac{15(Ba^3+6Aa^2b)\sqrt{be}x^2\log(-8b^2ex^6-8a*be*x^3-a^2e-4(2bx^4+ax)\sqrt{be}+a\sqrt{be}\sqrt{ex})+4(8Bb^3x^9+2(13Bab^2+6Ab^3)x^6-48Aa^2b+3(11Ba^2b+18Aab^2)x^3)\sqrt{be}+a\sqrt{ex}}{288be^{1/2}} - \frac{15(Ba^3+6Aa^2b)\sqrt{-be}x^2\arctan\left(\frac{2\sqrt{be}\sqrt{ex}}{2be+ax}\right)-2(8Bb^3x^9+2(13Bab^2+6Ab^3)x^6-48Aa^2b+3(11Ba^2b+18Aab^2)x^3)\sqrt{be}+a\sqrt{ex}}{144be^{1/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")

[Out] [1/288*(15*(B*a^3 + 6*A*a^2*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(8*B*b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A*a^2*b + 3*(11*B*a^2*b + 18*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/144*(15*(B*a^3 + 6*A*a^2*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) - 2*(8*B*b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A*a^2*b + 3*(11*B*a^2*b + 18*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{5}{2}}}{(ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")

[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(5/2), x)

maple [C] time = 1.16, size = 7544, normalized size = 40.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{5}{2}}}{(ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")`

[Out] `integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(5/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{5/2}}{(ex)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(5/2),x)`

[Out] `int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(5/2), x)`

sympy [B] time = 63.94, size = 403, normalized size = 2.14

$$-\frac{2Aa^{\frac{5}{2}}}{3e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{2Aa^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} - \frac{7Aa^{\frac{3}{2}}bx^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{A\sqrt{a}b^{\frac{3}{2}}x^{\frac{3}{2}}}{4e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{5Aa^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{bx^3}}{\sqrt{a}}\right)}{4e^{\frac{5}{2}}} + \frac{Ab^{\frac{3}{2}}x^{\frac{15}{2}}}{6\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{Ba^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} + \frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}}{8e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{35Ba^{\frac{3}{2}}bx^{\frac{3}{2}}}{72e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{17B\sqrt{a}b^{\frac{3}{2}}x^{\frac{15}{2}}}{36e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{bx^3}}{\sqrt{a}}\right)}{24\sqrt{b}e^{\frac{5}{2}}} + \frac{Bb^{\frac{3}{2}}x^{\frac{21}{2}}}{9\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(5/2),x)`

[Out] `-2*A*a**(5/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*a**(3/2)*b*x**
(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 7*A*a**(3/2)*b*x**(3/2)/(12*e**(5/2)
) *sqrt(1 + b*x**3/a) + A*sqrt(a)*b**2*x**(9/2)/(4*e**(5/2)*sqrt(1 + b*x**3
/a)) + 5*A*a**2*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*e**(5/2)) + A*b*
*3*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a)) + B*a**(5/2)*x**(3/2)*
sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(5/2)*x**(3/2)/(8*e**(5/2)*sqrt(1 +
b*x**3/a)) + 35*B*a**(3/2)*b*x**(9/2)/(72*e**(5/2)*sqrt(1 + b*x**3/a)) + 17
*B*sqrt(a)*b**2*x**(15/2)/(36*e**(5/2)*sqrt(1 + b*x**3/a)) + 5*B*a**3*asinh
(sqrt(b)*x**(3/2)/sqrt(a))/(24*sqrt(b)*e**(5/2)) + B*b**3*x**(21/2)/(9*sqrt
(a)*e**(5/2)*sqrt(1 + b*x**3/a))`

$$3.335 \quad \int \frac{(ex)^{7/2}(A+Bx^3)}{\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=121

$$-\frac{ae^{7/2}(4Ab - 3aB) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}} + \frac{e^2(ex)^{3/2}\sqrt{a+bx^3}(4Ab - 3aB)}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

Rubi [A] time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 321, 329, 275, 217, 206}

$$\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}(4Ab - 3aB)}{12b^2} - \frac{ae^{7/2}(4Ab - 3aB) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

Antiderivative was successfully verified.

[In] Int[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3], x]

[Out] ((4*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(12*b^2) + (B*(e*x)^(9/2)*Sqrt[a + b*x^3])/(6*b*e) - (a*(4*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(12*b^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
+ 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{7/2} (A + Bx^3)}{\sqrt{a + bx^3}} dx &= \frac{B(ex)^{9/2} \sqrt{a + bx^3}}{6be} - \frac{\left(-6Ab + \frac{9aB}{2}\right) \int \frac{(ex)^{7/2}}{\sqrt{a+bx^3}} dx}{6b} \\
&= \frac{(4Ab - 3aB)e^2 (ex)^{3/2} \sqrt{a + bx^3}}{12b^2} + \frac{B(ex)^{9/2} \sqrt{a + bx^3}}{6be} - \frac{(a(4Ab - 3aB)e^3) \int \frac{\sqrt{ex}}{\sqrt{a+bx^3}} dx}{8b^2} \\
&= \frac{(4Ab - 3aB)e^2 (ex)^{3/2} \sqrt{a + bx^3}}{12b^2} + \frac{B(ex)^{9/2} \sqrt{a + bx^3}}{6be} - \frac{(a(4Ab - 3aB)e^2) \text{Subst} \left(\int \frac{1}{\sqrt{1-u^2}} du \right)}{4b^2} \\
&= \frac{(4Ab - 3aB)e^2 (ex)^{3/2} \sqrt{a + bx^3}}{12b^2} + \frac{B(ex)^{9/2} \sqrt{a + bx^3}}{6be} - \frac{(a(4Ab - 3aB)e^2) \text{Subst} \left(\int \frac{1}{\sqrt{1-u^2}} du \right)}{12b^2} \\
&= \frac{(4Ab - 3aB)e^2 (ex)^{3/2} \sqrt{a + bx^3}}{12b^2} + \frac{B(ex)^{9/2} \sqrt{a + bx^3}}{6be} - \frac{(a(4Ab - 3aB)e^2) \text{Subst} \left(\int \frac{1}{\sqrt{1-u^2}} du \right)}{12b^2} \\
&= \frac{(4Ab - 3aB)e^2 (ex)^{3/2} \sqrt{a + bx^3}}{12b^2} + \frac{B(ex)^{9/2} \sqrt{a + bx^3}}{6be} - \frac{a(4Ab - 3aB)e^{7/2} \tanh^{-1} \left(\frac{\sqrt{b}}{e^{3/2} \sqrt{a+bx^3}} \right)}{12b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 97, normalized size = 0.80

$$\frac{e^3 \sqrt{ex} \left(\sqrt{b} x^{3/2} \sqrt{a + bx^3} (-3aB + 4Ab + 2bBx^3) + a(3aB - 4Ab) \tanh^{-1} \left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a+bx^3}} \right) \right)}{12b^{5/2} \sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3], x]

[Out] (e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(4*A*b - 3*a*B + 2*b*B*x^3) + a*(-4*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/(12*b^(5/2)*Sqrt[x])

IntegrateAlgebraic [A] time = 1.06, size = 124, normalized size = 1.02

$$\frac{e^5 \sqrt{\frac{b}{e^3}} (4aAb - 3a^2B) \log \left(\sqrt{a + bx^3} - \sqrt{\frac{b}{e^3}} (ex)^{3/2} \right)}{12b^3} + \frac{\sqrt{a + bx^3} (-3aBe^3 (ex)^{3/2} + 4Abe^3 (ex)^{3/2} + 2bB(ex)^{9/2})}{12b^2 e}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic(((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3], x)

[Out] (Sqrt[a + b*x^3]*(4*A*b*e^3*(e*x)^(3/2) - 3*a*B*e^3*(e*x)^(3/2) + 2*b*B*(e*x)^(9/2)))/(12*b^2*e) + ((4*a*A*b - 3*a^2*B)*Sqrt[b/e^3]*e^5*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(12*b^3)

fricas [A] time = 1.44, size = 245, normalized size = 2.02

$$\left[\frac{(3Ba^2 - 4Aab)e^3 \sqrt{\frac{e}{b}} \log(-8b^2ex^6 - 8abex^3 - a^2e + 4(2b^2x^4 + abx)\sqrt{bx^3 + a}\sqrt{\frac{e}{b}}) - 4(2Bbe^3x^4 - (3Ba - 4Ab)e^3x)\sqrt{bx^3 + a}\sqrt{\frac{e}{b}}}{48b^2}, \frac{(3Ba^2 - 4Aab)e^3 \sqrt{\frac{e}{b}} \arctan\left(\frac{2\sqrt{bx^3 + a}\sqrt{\frac{e}{b}}\sqrt{\frac{e}{b}}}{2bex^3 + ae}\right) - 2(2Bbe^3x^4 - (3Ba - 4Ab)e^3x)\sqrt{bx^3 + a}\sqrt{\frac{e}{b}}}{24b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] [-1/48*((3*B*a^2 - 4*A*a*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(2*B*b*e^3*x^4 - (3*B*a - 4*A*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/24*((3*B*a^2 - 4*A*a*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(2*B*b*e^3*x^4 - (3*B*a - 4*A*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]

giac [A] time = 0.36, size = 114, normalized size = 0.94

$$\frac{1}{12} \sqrt{bx^3e^4 + ae^4} \left(\frac{2Bx^3e^{(-2)}}{b} - \frac{(3Bab^3e^5 - 4Ab^4e^5)e^{(-7)}}{b^5} \right) x^{\frac{3}{2}} e^{\frac{7}{2}} - \frac{(3Ba^2b^3e^9 - 4Aab^4e^9)e^{(-\frac{11}{2})} \log\left(\left| -\sqrt{b}x^{\frac{3}{2}}e^2 + \sqrt{bx^3e^4 + ae^4} \right|\right)}{12b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2), x, algorithm="giac")

[Out] 1/12*sqrt(b*x^3*e^4 + a*e^4)*(2*B*x^3*e^(-2))/b - (3*B*a*b^3*e^5 - 4*A*b^4*e^5)*e^(-7)/b^5*x^(3/2)*e^(7/2) - 1/12*(3*B*a^2*b^3*e^9 - 4*A*a*b^4*e^9)*e^(-11/2)*log(abs(-sqrt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/b^(11/2)

maple [C] time = 1.01, size = 6861, normalized size = 56.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2), x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)(ex)^{\frac{7}{2}}}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*(e*x)^(7/2)/sqrt(b*x^3 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^3 + A)(ex)^{7/2}}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(1/2),x)

[Out] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(1/2), x)

sympy [A] time = 95.30, size = 194, normalized size = 1.60

$$\frac{A\sqrt{a}e^{\frac{7}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3b} - \frac{Aae^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{bx^{\frac{3}{2}}}}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}} - \frac{Ba^{\frac{3}{2}}e^{\frac{7}{2}}x^{\frac{3}{2}}}{4b^2\sqrt{1+\frac{bx^3}{a}}} - \frac{B\sqrt{a}e^{\frac{7}{2}}x^{\frac{9}{2}}}{12b\sqrt{1+\frac{bx^3}{a}}} + \frac{Ba^2e^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{bx^{\frac{3}{2}}}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} + \frac{Be^{\frac{7}{2}}x^{\frac{15}{2}}}{6\sqrt{a}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)

[Out] A*sqrt(a)*e**(7/2)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*b) - A*a*e**(7/2)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(3*b**(3/2)) - B*a**(3/2)*e**(7/2)*x**(3/2)/(4*b**2*sqrt(1 + b*x**3/a)) - B*sqrt(a)*e**(7/2)*x**(9/2)/(12*b*sqrt(1 + b*x**3/a)) + B*a**2*e**(7/2)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*b**(5/2)) + B*e**(7/2)*x**(15/2)/(6*sqrt(a)*sqrt(1 + b*x**3/a))

$$3.336 \quad \int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=83

$$\frac{\sqrt{e}(2Ab - aB) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}} + \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {459, 329, 275, 217, 206}

$$\frac{\sqrt{e}(2Ab - aB) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}} + \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3], x]

[Out] (B*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*b*e) + ((2*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 329

Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F

ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ex} (A + Bx^3)}{\sqrt{a + bx^3}} dx &= \frac{B(ex)^{3/2} \sqrt{a + bx^3}}{3be} - \frac{\left(-3Ab + \frac{3aB}{2}\right) \int \frac{\sqrt{ex}}{\sqrt{a+bx^3}} dx}{3b} \\ &= \frac{B(ex)^{3/2} \sqrt{a + bx^3}}{3be} + \frac{(2Ab - aB) \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{a + \frac{bx^6}{e^3}}} dx, x, \sqrt{ex} \right)}{be} \\ &= \frac{B(ex)^{3/2} \sqrt{a + bx^3}}{3be} + \frac{(2Ab - aB) \operatorname{Subst} \left(\int \frac{1}{\sqrt{a + \frac{bx^2}{e^3}}} dx, x, (ex)^{3/2} \right)}{3be} \\ &= \frac{B(ex)^{3/2} \sqrt{a + bx^3}}{3be} + \frac{(2Ab - aB) \operatorname{Subst} \left(\int \frac{1}{1 - \frac{bx^2}{e^3}} dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}} \right)}{3be} \\ &= \frac{B(ex)^{3/2} \sqrt{a + bx^3}}{3be} + \frac{(2Ab - aB) \sqrt{e} \tanh^{-1} \left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2} \sqrt{a+bx^3}} \right)}{3b^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 0.94

$$\frac{\sqrt{ex} \left((2Ab - aB) \tanh^{-1} \left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a+bx^3}} \right) + \sqrt{b} Bx^{3/2} \sqrt{a + bx^3} \right)}{3b^{3/2} \sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3], x]

[Out] $(\text{Sqrt}[e*x] * (\text{Sqrt}[b] * B*x^{(3/2)} * \text{Sqrt}[a + b*x^3] + (2*A*b - a*B) * \text{ArcTanh}[(\text{Sqrt}[b]*x^{(3/2)})/\text{Sqrt}[a + b*x^3]])) / (3*b^{(3/2)} * \text{Sqrt}[x])$

IntegrateAlgebraic [A] time = 0.53, size = 89, normalized size = 1.07

$$\frac{B(ex)^{3/2} \sqrt{a + bx^3}}{3be} - \frac{e^2 \sqrt{\frac{b}{e^3}} (2Ab - aB) \log\left(\sqrt{a + bx^3} - \sqrt{\frac{b}{e^3}} (ex)^{3/2}\right)}{3b^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3], x]

[Out] $(B*(e*x)^{(3/2)} * \text{Sqrt}[a + b*x^3]) / (3*b*e) - ((2*A*b - a*B) * \text{Sqrt}[b/e^3] * e^2 * \text{Log}[-(\text{Sqrt}[b/e^3] * (e*x)^{(3/2)}) + \text{Sqrt}[a + b*x^3]]) / (3*b^2)$

fricas [A] time = 1.28, size = 184, normalized size = 2.22

$$\left[\frac{4 \sqrt{bx^3 + a} \sqrt{ex} Bx - (Ba - 2Ab) \sqrt{\frac{e}{b}} \log\left(-8b^2ex^6 - 8abex^3 - a^2e - 4(2b^2x^4 + abx) \sqrt{bx^3 + a} \sqrt{ex} \sqrt{\frac{e}{b}}\right)}{12b}, \frac{2 \sqrt{bx^3 + a} \sqrt{ex} Bx + (Ba - 2Ab) \sqrt{\frac{e}{b}} \arctan\left(\frac{2 \sqrt{bx^3 + a} \sqrt{ex} Bx \sqrt{\frac{e}{b}}}{2bex^3 + ae}\right)}{6b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] $[1/12*(4*\text{sqrt}(b*x^3 + a)*\text{sqrt}(e*x)*B*x - (B*a - 2*A*b)*\text{sqrt}(e/b)*\log(-8*b^2 * e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*\text{sqrt}(b*x^3 + a)*\text{sqrt}(e*x)*\text{sqrt}(e/b)))/b, 1/6*(2*\text{sqrt}(b*x^3 + a)*\text{sqrt}(e*x)*B*x + (B*a - 2*A*b)*\text{sqrt}(-e/b)*\arctan(2*\text{sqrt}(b*x^3 + a)*\text{sqrt}(e*x)*b*x*\text{sqrt}(-e/b)/(2*b*e*x^3 + a*e)))/b]$

giac [A] time = 0.36, size = 72, normalized size = 0.87

$$\frac{\sqrt{bx^3e^4 + ae^4} Bx^{\frac{3}{2}} e^{\left(-\frac{3}{2}\right)}}{3b} + \frac{(Bae^5 - 2Abe^5) e^{\left(-\frac{9}{2}\right)} \log\left(\left| -\sqrt{b} x^{\frac{3}{2}} e^2 + \sqrt{bx^3e^4 + ae^4} \right|\right)}{3b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2), x, algorithm="giac")

[Out] $1/3*\text{sqrt}(b*x^3*e^4 + a*e^4)*B*x^{(3/2)}*e^{(-3/2)}/b + 1/3*(B*a*e^5 - 2*A*b*e^5)*e^{(-9/2)}*\log(\text{abs}(-\text{sqrt}(b)*x^{(3/2)}*e^2 + \text{sqrt}(b*x^3*e^4 + a*e^4)))/b^{(3/2)}$

maple [C] time = 1.15, size = 6424, normalized size = 77.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)\sqrt{ex}}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*x^3 + A)*sqrt(e*x)/sqrt(b*x^3 + a), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^3 + A)\sqrt{ex}}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(1/2),x)`

[Out] `int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(1/2), x)`

sympy [A] time = 6.81, size = 107, normalized size = 1.29

$$\frac{2A\sqrt{e} \operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}e^{\frac{3}{2}}}\right)}{3\sqrt{b}} + \frac{B\sqrt{a}(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3be} - \frac{Ba\sqrt{e} \operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}e^{\frac{3}{2}}}\right)}{3b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(1/2),x)`

[Out] `2*A*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(3*sqrt(b)) + B*sqrt(a)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*b*e) - B*a*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(3*b**(3/2))`

$$3.337 \quad \int \frac{A+Bx^3}{(ex)^{5/2} \sqrt{a+bx^3}} dx$$

Optimal. Leaf size=75

$$\frac{2B \tanh^{-1} \left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2} \sqrt{a+bx^3}} \right)}{3\sqrt{b} e^{5/2}} - \frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {451, 329, 275, 217, 206}

$$\frac{2B \tanh^{-1} \left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2} \sqrt{a+bx^3}} \right)}{3\sqrt{b} e^{5/2}} - \frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]), x]

[Out] (-2*A*Sqrt[a + b*x^3]/(3*a*e*(e*x)^(3/2)) + (2*B*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*Sqrt[b]*e^(5/2)))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 329

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F

ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 451

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1]))

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx^3}{(ex)^{5/2}\sqrt{a + bx^3}} dx &= -\frac{2A\sqrt{a + bx^3}}{3ae(ex)^{3/2}} + \frac{B \int \frac{\sqrt{ex}}{\sqrt{a+bx^3}} dx}{e^3} \\
 &= -\frac{2A\sqrt{a + bx^3}}{3ae(ex)^{3/2}} + \frac{(2B) \text{Subst} \left(\int \frac{x^2}{\sqrt{a + \frac{bx^6}{e^3}}} dx, x, \sqrt{ex} \right)}{e^4} \\
 &= -\frac{2A\sqrt{a + bx^3}}{3ae(ex)^{3/2}} + \frac{(2B) \text{Subst} \left(\int \frac{1}{\sqrt{a + \frac{bx^2}{e^3}}} dx, x, (ex)^{3/2} \right)}{3e^4} \\
 &= -\frac{2A\sqrt{a + bx^3}}{3ae(ex)^{3/2}} + \frac{(2B) \text{Subst} \left(\int \frac{1}{1 - \frac{bx^2}{e^3}} dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}} \right)}{3e^4} \\
 &= -\frac{2A\sqrt{a + bx^3}}{3ae(ex)^{3/2}} + \frac{2B \tanh^{-1} \left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}} \right)}{3\sqrt{b}e^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 65, normalized size = 0.87

$$\frac{2x \left(\frac{Bx^{3/2} \tanh^{-1} \left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}} \right)}{\sqrt{b}} - \frac{A\sqrt{a+bx^3}}{a} \right)}{3(ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]), x]

[Out] (2*x*(-((A*Sqrt[a + b*x^3])/a) + (B*x^(3/2)*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/Sqrt[b])/(3*(e*x)^(5/2))

IntegrateAlgebraic [A] time = 0.79, size = 81, normalized size = 1.08

$$\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}} - \frac{2B\sqrt{\frac{b}{e^3}} \log\left(\sqrt{a+bx^3} - \sqrt{\frac{b}{e^3}}(ex)^{3/2}\right)}{3be}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]), x]

[Out] (-2*A*Sqrt[a + b*x^3]/(3*a*e*(e*x)^(3/2)) - (2*B*Sqrt[b/e^3]*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(3*b*e))

fricas [A] time = 1.19, size = 183, normalized size = 2.44

$$\left[\frac{\sqrt{be} B a x^2 \log\left(-8 b^2 e x^6 - 8 a b e x^3 - a^2 e - 4(2 b x^4 + a x) \sqrt{b x^3 + a} \sqrt{be} \sqrt{ex}\right) - 4 \sqrt{b x^3 + a} \sqrt{ex} A b}{6 a b e^3 x^2}, - \frac{\sqrt{-be} B a x^2 \arctan\left(\frac{2 \sqrt{b x^3 + a} \sqrt{-be} \sqrt{ex} x}{2 b e x^3 + a e}\right) + 2 \sqrt{b x^3 + a} \sqrt{ex} A b}{3 a b e^3 x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] [1/6*(sqrt(b*e)*B*a*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) - 4*sqrt(b*x^3 + a)*sqrt(e*x)*A*b)/(a*b*e^3*x^2), -1/3*(sqrt(-b*e)*B*a*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) + 2*sqrt(b*x^3 + a)*sqrt(e*x)*A*b)/(a*b*e^3*x^2)]

giac [A] time = 0.26, size = 110, normalized size = 1.47

$$-\frac{2}{3} \left(\left(\frac{B \arctan\left(\frac{\sqrt{be+\frac{ae}{x^3}}}{\sqrt{-be}}\right)}{\sqrt{-be}} + \frac{\sqrt{be+\frac{ae}{x^3}} A e^{(-1)}}{a} \right) e^{(-1)} - \frac{\left(B a \arctan\left(\frac{\sqrt{b} e^{\frac{1}{2}}}{\sqrt{-be}}\right) e + \sqrt{-be} A \sqrt{b} e^{\frac{1}{2}} \right) e^{(-2)}}{\sqrt{-be} a} \right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2), x, algorithm="giac")

[Out] -2/3*((B*arctan(sqrt(b*e + a*e/x^3)/sqrt(-b*e))/sqrt(-b*e) + sqrt(b*e + a*e/x^3)*A*e^(-1)/a)*e^(-1) - (B*a*arctan(sqrt(b)*e^(1/2)/sqrt(-b*e))*e + sqrt(-b*e)*A*sqrt(b)*e^(1/2))*e^(-2)/(sqrt(-b*e)*a))*e^(-1)

maple [C] time = 1.00, size = 3397, normalized size = 45.29

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((B*x^3+A)/(e*x)^{(5/2)}/(b*x^3+a)^{(1/2)}, x)$

[Out]
$$\begin{aligned} & -2/3*(b*x^3+a)^{(1/2)}/x/b^2*(6*I*B*3^{(1/2)}*(-(I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1) \\ & /(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}+2*b*x+(-a*b^2)^{(1/3)}) \\ & /((1+I*3^{(1/2)})/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}-2 \\ & *b*x-(-a*b^2)^{(1/3)})/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*EllipticF((\\ & -(I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}, ((I*3^{(1/2)}+3) \\ &)*(I*3^{(1/2)}-1)/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*x^4*a*b^2*e-6*I*B*3^{(1/2)} \\ & *(-(I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)} \\ & /2)*(-a*b^2)^{(1/3)}+2*b*x+(-a*b^2)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x+(-a*b^2)^{(1/3)}) \\ &)^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}-2*b*x-(-a*b^2)^{(1/3)})/(I*3^{(1/2)}-1)/(-b*x \\ & +(-a*b^2)^{(1/3)}))^{(1/2)}*EllipticPi((- (I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x \\ & +(-a*b^2)^{(1/3)}))^{(1/2)}, (I*3^{(1/2)}-1)/(I*3^{(1/2)}-3), ((I*3^{(1/2)}+3)*(I*3^{(1/2)} \\ & /2)-1)/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*x^4*a*b^2*e-12*I*B*3^{(1/2)}*(-(I*3 \\ & ^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^ \\ & 2)^{(1/3)}+2*b*x+(-a*b^2)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}* \\ & ((I*3^{(1/2)}*(-a*b^2)^{(1/3)}-2*b*x-(-a*b^2)^{(1/3)})/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2) \\ &)^{(1/3)}))^{(1/2)}*EllipticF((- (I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^ \\ & (1/3))^{(1/2)}, ((I*3^{(1/2)}+3)*(I*3^{(1/2)}-1)/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)} \\ &)*(-a*b^2)^{(1/3)}*x^3*a*b*e+12*I*B*3^{(1/2)}*(-(I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}- \\ & 1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}+2*b*x+(-a*b^2)^{(1/3)} \\ &)/(1+I*3^{(1/2)})/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)} \\ & -2*b*x-(-a*b^2)^{(1/3)})/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*EllipticP \\ & i((- (I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}, (I*3^{(1/2)} \\ & -1)/(I*3^{(1/2)}-3), ((I*3^{(1/2)}+3)*(I*3^{(1/2)}-1)/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3)) \\ & ^{(1/2)}*(-a*b^2)^{(1/3)}*x^3*a*b*e+6*I*B*3^{(1/2)}*(-(I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)} \\ & /2)-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}+2*b*x+(-a*b^2) \\ &)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)} \\ & -2*b*x-(-a*b^2)^{(1/3)})/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*Ellipt \\ & icF((- (I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}, ((I*3^{(1/2)} \\ & /2)+3)*(I*3^{(1/2)}-1)/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*(-a*b^2)^{(2/3)}*x^2 \\ & *a*e-6*I*B*3^{(1/2)}*(-(I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)})) \\ & ^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}+2*b*x+(-a*b^2)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x \\ & +(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}-2*b*x-(-a*b^2)^{(1/3)})/(I \\ & *3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*EllipticPi((- (I*3^{(1/2)}-3)*x*b/(I* \\ & 3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}, (I*3^{(1/2)}-1)/(I*3^{(1/2)}-3), ((I*3^{(1/2)} \\ & /2)+3)*(I*3^{(1/2)}-1)/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*(-a*b^2)^{(2/3)}*x^ \\ & 2*a*e-6*B*(-(I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((\\ & I*3^{(1/2)}*(-a*b^2)^{(1/3)}+2*b*x+(-a*b^2)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x+(-a*b^2) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{B \log \left(\frac{\sqrt{b} - \sqrt{bx^3+a}}{x^2} \right)}{3\sqrt{b}} - \frac{2(b\sqrt{e}x^4 + a\sqrt{e}x)A}{3\sqrt{bx^3+a}ae^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] B*integrate(sqrt(x)/sqrt(b*x^3 + a), x)/e^(5/2) - 2/3*(b*sqrt(e)*x^4 + a*sqrt(e)*x)*A/(sqrt(b*x^3 + a)*a*e^3*x^(5/2))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{Bx^3 + A}{(ex)^{5/2} \sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(1/2)),x)

[Out] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(1/2)), x)

sympy [A] time = 10.11, size = 60, normalized size = 0.80

$$-\frac{2A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ae^{\frac{5}{2}}} + \frac{2B \operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(1/2),x)

[Out] -2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*e**(5/2)) + 2*B*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(3*sqrt(b)*e**(5/2))

$$3.338 \quad \int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=120

$$\frac{e^{7/2}(2Ab - 3aB) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}} - \frac{e^2(ex)^{3/2}(2Ab - 3aB)}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

Rubi [A] time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {459, 288, 329, 275, 217, 206}

$$-\frac{e^2(ex)^{3/2}(2Ab - 3aB)}{3b^2\sqrt{a+bx^3}} + \frac{e^{7/2}(2Ab - 3aB) \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2), x]

[Out] -((2*A*b - 3*a*B)*e^2*(e*x)^(3/2))/(3*b^2*Sqrt[a + b*x^3]) + (B*(e*x)^(9/2))/(3*b*e*Sqrt[a + b*x^3]) + ((2*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^

$n*(m - n + 1)/(b*n*(p + 1)), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^{(p + 1)}, x], x]$
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
 LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_) + (b_*)*(x_)^{(n_)})^{(p_)}, x_Symbol] :> \text{With}[\{k =$
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
 n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
 ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

$\text{Int}[(e_*)*(x_)^{(m_*)}*((a_) + (b_*)*(x_)^{(n_)})^{(p_*)}*((c_) + (d_*)*(x_)^{(n$
 _)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{7/2} (A + Bx^3)}{(a + bx^3)^{3/2}} dx &= \frac{B(ex)^{9/2}}{3be\sqrt{a + bx^3}} - \frac{\left(-3Ab + \frac{9aB}{2}\right) \int \frac{(ex)^{7/2}}{(a+bx^3)^{3/2}} dx}{3b} \\
&= -\frac{(2Ab - 3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a + bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a + bx^3}} + \frac{\left((2Ab - 3aB)e^3\right) \int \frac{\sqrt{ex}}{\sqrt{a+bx^3}} dx}{2b^2} \\
&= -\frac{(2Ab - 3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a + bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a + bx^3}} + \frac{\left((2Ab - 3aB)e^2\right) \text{Subst} \left(\int \frac{x^2}{\sqrt{a + \frac{bx^6}{e^3}}} dx, x, \sqrt{ex} \right)}{b^2} \\
&= -\frac{(2Ab - 3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a + bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a + bx^3}} + \frac{\left((2Ab - 3aB)e^2\right) \text{Subst} \left(\int \frac{1}{\sqrt{a + \frac{bx^2}{e^3}}} dx, x, (ex) \right)}{3b^2} \\
&= -\frac{(2Ab - 3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a + bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a + bx^3}} + \frac{\left((2Ab - 3aB)e^2\right) \text{Subst} \left(\int \frac{1}{1 - \frac{bx^2}{e^3}} dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}} \right)}{3b^2} \\
&= -\frac{(2Ab - 3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a + bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a + bx^3}} + \frac{(2Ab - 3aB)e^{7/2} \tanh^{-1} \left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}} \right)}{3b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 109, normalized size = 0.91

$$\frac{e^3 \sqrt{ex} \left(\sqrt{b} x^{3/2} (3aB - 2Ab + bBx^3) - \sqrt{a} \sqrt{\frac{bx^3}{a} + 1} (3aB - 2Ab) \sinh^{-1} \left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}} \right) \right)}{3b^{5/2} \sqrt{x} \sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2), x]

[Out] (e^3*sqrt[e*x]*(sqrt[b]*x^(3/2)*(-2*A*b + 3*a*B + b*B*x^3) - sqrt[a]*(-2*A*b + 3*a*B)*sqrt[1 + (b*x^3)/a]*ArcSinh[(sqrt[b]*x^(3/2))/sqrt[a]])/(3*b^(5/2)*sqrt[x]*sqrt[a + b*x^3])

IntegrateAlgebraic [A] time = 1.76, size = 136, normalized size = 1.13

$$\frac{\sqrt{a + bx^3} (3aBe^5(ex)^{3/2} - 2Abe^5(ex)^{3/2} + bBe^2(ex)^{9/2})}{3b^2 (ae^3 + be^3x^3)} - \frac{e^5 \sqrt{\frac{b}{e^3}} (2Ab - 3aB) \log \left(\sqrt{a + bx^3} - \sqrt{\frac{b}{e^3}} (ex)^{3/2} \right)}{3b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]

[Out] (Sqrt[a + b*x^3]*(-2*A*b*e^5*(e*x)^(3/2) + 3*a*B*e^5*(e*x)^(3/2) + b*B*e^2*(e*x)^(9/2)))/(3*b^2*(a*e^3 + b*e^3*x^3)) - ((2*A*b - 3*a*B)*Sqrt[b/e^3]*e^5*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(3*b^3)

fricas [A] time = 1.92, size = 307, normalized size = 2.56

$$\frac{\left((3Bab - 2Ab^2)e^3x^3 + (3Ba^2 - 2Ab^2)e^3 \right) \sqrt{\frac{a}{b}} \log\left(\frac{-8b^2e^6x^3 - 8a^2e^3 - 4(2b^2x^4 + abx)\sqrt{bx^3 + a}\sqrt{\frac{a}{b}}}{12(b^3x^3 + ab^2)} \right) - 4(Bb^3x^4 + (3Ba - 2Ab)e^3x)\sqrt{bx^3 + a}\sqrt{\frac{a}{b}}}{6(b^3x^3 + ab^2)} + \frac{\left((3Bab - 2Ab^2)e^3x^3 + (3Ba^2 - 2Ab^2)e^3 \right) \sqrt{\frac{a}{b}} \arctan\left(\frac{2\sqrt{bx^3 + a}\sqrt{\frac{a}{b}}}{2bx^3 + ae^4} \right) + 2(Bb^3x^4 + (3Ba - 2Ab)e^3x)\sqrt{bx^3 + a}\sqrt{\frac{a}{b}}}{6(b^3x^3 + ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] [-1/12*(((3*B*a*b - 2*A*b^2)*e^3*x^3 + (3*B*a^2 - 2*A*a*b)*e^3)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(B*b*e^3*x^4 + (3*B*a - 2*A*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/(b^3*x^3 + a*b^2), 1/6*(((3*B*a*b - 2*A*b^2)*e^3*x^3 + (3*B*a^2 - 2*A*a*b)*e^3)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(B*b*e^3*x^4 + (3*B*a - 2*A*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/(b^3*x^3 + a*b^2)]

giac [A] time = 0.37, size = 107, normalized size = 0.89

$$\frac{\left(\frac{Bx^3e^4}{b} + \frac{3Bab^3e^4 - 2Ab^4e^4}{b^5} \right) x^{\frac{3}{2}} e^{\frac{3}{2}}}{3\sqrt{bx^3e^4 + ae^4}} + \frac{(3Bab^3e^4 - 2Ab^4e^4)e^{\left(-\frac{1}{2}\right)} \log\left(\left| -\sqrt{b}x^{\frac{3}{2}}e^2 + \sqrt{bx^3e^4 + ae^4} \right|\right)}{3b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] 1/3*(B*x^3*e^4/b + (3*B*a*b^3*e^4 - 2*A*b^4*e^4)/b^5)*x^(3/2)*e^(3/2)/sqrt(b*x^3*e^4 + a*e^4) + 1/3*(3*B*a*b^3*e^4 - 2*A*b^4*e^4)*e^(-1/2)*log(abs(-sqrt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/b^(11/2)

maple [C] time = 1.13, size = 7016, normalized size = 58.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)(ex)^{\frac{7}{2}}}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*(e*x)^(7/2)/(b*x^3 + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^3 + A)(ex)^{7/2}}{(bx^3 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(3/2),x)

[Out] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)

[Out] Timed out

$$3.339 \quad \int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=85

$$\frac{2(ex)^{3/2}(Ab - aB)}{3abe\sqrt{a + bx^3}} + \frac{2B\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {452, 329, 275, 217, 206}

$$\frac{2(ex)^{3/2}(Ab - aB)}{3abe\sqrt{a + bx^3}} + \frac{2B\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2), x]

[Out] (2*(A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*Sqrt[a + b*x^3]) + (2*B*Sqrt[e]*ArcTan h[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3]])/(3*b^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 329

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^

$n)^p, x], x, (c*x)^{(1/k)}, x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 452

$\text{Int}[(e_{.})*(x_{.})^{(m_{.})}*((a_{.}) + (b_{.})*(x_{.})^{(n_{.})})^{(p_{.})}*((c_{.}) + (d_{.})*(x_{.})^{(n_{.})}), x_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*(e*x)^{(m + 1)}*(a + b*x^n)^{(p + 1)}/(a*b*e*(m + 1)), x] + \text{Dist}[d/b, \text{Int}[(e*x)^m*(a + b*x^n)^{(p + 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, n, p\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[m + n*(p + 1) + 1, 0] \ \&\& \ \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ex} (A + Bx^3)}{(a + bx^3)^{3/2}} dx &= \frac{2(Ab - aB)(ex)^{3/2}}{3abe\sqrt{a + bx^3}} + \frac{B \int \frac{\sqrt{ex}}{\sqrt{a+bx^3}} dx}{b} \\ &= \frac{2(Ab - aB)(ex)^{3/2}}{3abe\sqrt{a + bx^3}} + \frac{(2B) \text{Subst} \left(\int \frac{x^2}{\sqrt{a + \frac{bx^6}{e^3}}} dx, x, \sqrt{ex} \right)}{be} \\ &= \frac{2(Ab - aB)(ex)^{3/2}}{3abe\sqrt{a + bx^3}} + \frac{(2B) \text{Subst} \left(\int \frac{1}{\sqrt{a + \frac{bx^2}{e^3}}} dx, x, (ex)^{3/2} \right)}{3be} \\ &= \frac{2(Ab - aB)(ex)^{3/2}}{3abe\sqrt{a + bx^3}} + \frac{(2B) \text{Subst} \left(\int \frac{1}{1 - \frac{bx^2}{e^3}} dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}} \right)}{3be} \\ &= \frac{2(Ab - aB)(ex)^{3/2}}{3abe\sqrt{a + bx^3}} + \frac{2B\sqrt{e} \tanh^{-1} \left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}} \right)}{3b^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 93, normalized size = 1.09

$$\frac{2\sqrt{ex} \left(a^{3/2} B \sqrt{\frac{bx^3}{a}} + 1 \sinh^{-1} \left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a}} \right) + \sqrt{b} x^{3/2} (Ab - aB) \right)}{3ab^{3/2} \sqrt{x} \sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2), x]

[Out] (2*Sqrt[e*x]*(Sqrt[b]*(A*b - a*B)*x^(3/2) + a^(3/2)*B*Sqrt[1 + (b*x^3)/a])*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a*b^(3/2)*Sqrt[x]*Sqrt[a + b*x^3])

IntegrateAlgebraic [A] time = 1.01, size = 107, normalized size = 1.26

$$\frac{2e^2(ex)^{3/2}\sqrt{a+bx^3}(aB-Ab)}{3ab(ae^3+be^3x^3)} - \frac{2Be^2\sqrt{\frac{b}{e^3}}\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}(ex)^{3/2}\right)}{3b^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2), x]

[Out] (-2*(-(A*b) + a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*a*b*(a*e^3 + b*e^3*x^3)) - (2*B*Sqrt[b/e^3]*e^2*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(3*b^2)

fricas [A] time = 0.86, size = 234, normalized size = 2.75

$$\left[\frac{4\sqrt{bx^3+a}(Ba-Ab)\sqrt{ex} - (Babx^3+Ba^2)\sqrt{\frac{e}{b}}\log\left(-8b^2ex^6-8abex^3-a^2e-4(2b^2x^4+abx)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)}{6(ab^2x^3+a^2b)}, \frac{2\sqrt{bx^3+a}(Ba-Ab)\sqrt{ex} + (Babx^3+Ba^2)\sqrt{\frac{e}{b}}\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}}{2bx^3+ae}\right)}{3(ab^2x^3+a^2b)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2), x, algorithm="fricas")

[Out] [-1/6*(4*sqrt(b*x^3 + a)*(B*a - A*b)*sqrt(e*x)*x - (B*a*b*x^3 + B*a^2)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)))/(a*b^2*x^3 + a^2*b), -1/3*(2*sqrt(b*x^3 + a)*(B*a - A*b)*sqrt(e*x)*x + (B*a*b*x^3 + B*a^2)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)))/(a*b^2*x^3 + a^2*b)]

giac [A] time = 0.62, size = 75, normalized size = 0.88

$$\frac{2Be^{\frac{1}{2}}\log\left(\left|-\sqrt{b}x^{\frac{3}{2}}e^2 + \sqrt{bx^3e^4 + ae^4}\right|\right)}{3b^{\frac{3}{2}}} - \frac{2(Bae - Abe)x^{\frac{3}{2}}e^{\frac{3}{2}}}{3\sqrt{bx^3e^4 + ae^4}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2), x, algorithm="giac")

[Out] -2/3*B*e^(1/2)*log(abs(-sqrt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/b^(3/2) - 2/3*(B*a*e - A*b*e)*x^(3/2)*e^(3/2)/(sqrt(b*x^3*e^4 + a*e^4)*a*b)

maple [C] time = 1.00, size = 3654, normalized size = 42.99

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((B*x^3+A)*(e*x)^{(1/2)}/(b*x^3+a)^{(3/2)}, x)$

[Out] $\frac{2}{3}*(e*x)^{(1/2)}/(b*x^3+a)^{(1/2)}/b^3*(-6*I*B*3^{(1/2)}*(-(I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}+2*b*x+(-a*b^2)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}-2*b*x-(-a*b^2)^{(1/3)})/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*E$
 $llipticF((-I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}, ((I*3^{(1/2)}+3)*(I*3^{(1/2)}-1)/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*(e*x*(b*x^3+a))^{(1/2)}*x^2*a*b^2-12*I*B*3^{(1/2)}*(-(I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}+2*b*x+(-a*b^2)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}-2*b*x-(-a*b^2)^{(1/3)})/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*EllipticPi((-I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}, (I*3^{(1/2)}-1)/(I*3^{(1/2)}-3), ((I*3^{(1/2)}+3)*(I*3^{(1/2)}-1)/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*(e*x*(b*x^3+a))^{(1/2)}*(-a*b^2)^{(1/3)}*x*a*b+12*I*B*3^{(1/2)}*(-(I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}+2*b*x+(-a*b^2)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}-2*b*x-(-a*b^2)^{(1/3)})/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*EllipticF((-I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}, ((I*3^{(1/2)}+3)*(I*3^{(1/2)}-1)/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*(e*x*(b*x^3+a))^{(1/2)}*(-a*b^2)^{(1/3)}*x*a*b-6*I*B*3^{(1/2)}*(-(I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}+2*b*x+(-a*b^2)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}-2*b*x-(-a*b^2)^{(1/3)})/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*EllipticF((-I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}, ((I*3^{(1/2)}+3)*(I*3^{(1/2)}-1)/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*(e*x*(b*x^3+a))^{(1/2)}*(-a*b^2)^{(2/3)}*a+6*I*B*3^{(1/2)}*(-(I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}+2*b*x+(-a*b^2)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}-2*b*x-(-a*b^2)^{(1/3)})/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*EllipticPi((-I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}, (I*3^{(1/2)}-1)/(I*3^{(1/2)}-3), ((I*3^{(1/2)}+3)*(I*3^{(1/2)}-1)/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*(e*x*(b*x^3+a))^{(1/2)}*x^2*a*b^2+I*A*3^{(1/2)}*(1/b^2*e*x*(-b*x+(-a*b^2)^{(1/3)})*(I*3^{(1/2)}*(-a*b^2)^{(1/3)}+2*b*x+(-a*b^2)^{(1/3)})*(I*3^{(1/2)}*(-a*b^2)^{(1/3)}-2*b*x-(-a*b^2)^{(1/3)}))^{(1/2)}*x^2*b^3+6*B*(-(I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}+2*b*x+(-a*b^2)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-a*b^2)^{(1/3)}-2*b*x-(-a*b^2)^{(1/3)})/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}*EllipticF((-I*3^{(1/2)}-3)*x*b/(I*3^{(1/2)}-1)/(-b*x+(-a*b^2)^{(1/3)}))^{(1/2)}, ((I*3^{(1/2)}+3)$

$$\begin{aligned}
&)*(I^3^{1/2}-1)/(1+I^3^{1/2})/(I^3^{1/2}-3))^{1/2}*(e^{*x}(b*x^3+a))^{1/2}*x \\
& ^2*a*b^2-6*B*(-(I^3^{1/2}-3)*x*b/(I^3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2} \\
& *((I^3^{1/2})*(-a*b^2)^{1/3}+2*b*x+(-a*b^2)^{1/3})/(1+I^3^{1/2})/(-b*x+(-a*b \\
& ^2)^{1/3}))^{1/2}*((I^3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3})/(I^3^{1/2} \\
& -1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*EllipticPi((-I^3^{1/2}-3)*x*b/(I^3^{1/2} \\
& -1)/(-b*x+(-a*b^2)^{1/3}))^{1/2},(I^3^{1/2}-1)/(I^3^{1/2}-3),((I^3^{1/2}+3) \\
& *(I^3^{1/2}-1)/(1+I^3^{1/2})/(I^3^{1/2}-3))^{1/2}*(e^{*x}(b*x^3+a))^{1/2}*x \\
& ^2*a*b^2-12*B*(-a*b^2)^{1/3}*(-(I^3^{1/2}-3)*x*b/(I^3^{1/2}-1)/(-b*x+(-a*b^ \\
& 2)^{1/3}))^{1/2}*((I^3^{1/2})*(-a*b^2)^{1/3}+2*b*x+(-a*b^2)^{1/3})/(1+I^3^{1/2} \\
&)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*((I^3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2) \\
& ^{1/3})/(I^3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*EllipticF((-I^3^{1/2}-3) \\
&)*x*b/(I^3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2},((I^3^{1/2}+3)*(I^3^{1/2}- \\
& 1)/(1+I^3^{1/2})/(I^3^{1/2}-3))^{1/2}*(e^{*x}(b*x^3+a))^{1/2}*x*a*b+12*B*(-a \\
& *b^2)^{1/3}*(-(I^3^{1/2}-3)*x*b/(I^3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}* \\
& ((I^3^{1/2})*(-a*b^2)^{1/3}+2*b*x+(-a*b^2)^{1/3})/(1+I^3^{1/2})/(-b*x+(-a*b^ \\
& 2)^{1/3}))^{1/2}*((I^3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3})/(I^3^{1/2} \\
& -1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*EllipticPi((-I^3^{1/2}-3)*x*b/(I^3^{1/2} \\
& -1)/(-b*x+(-a*b^2)^{1/3}))^{1/2},(I^3^{1/2}-1)/(I^3^{1/2}-3),((I^3^{1/2}+3) \\
& *(I^3^{1/2}-1)/(1+I^3^{1/2})/(I^3^{1/2}-3))^{1/2}*(e^{*x}(b*x^3+a))^{1/2}*x* \\
& a*b-I*B^3^{1/2}*(1/b^2*e^{*x}*(-b*x+(-a*b^2)^{1/3})*(I^3^{1/2})*(-a*b^2)^{1/3}+ \\
& 2*b*x+(-a*b^2)^{1/3})*(I^3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3}))^{1/2} \\
& *x^2*a*b^2+6*I*B^3^{1/2}*(-(I^3^{1/2}-3)*x*b/(I^3^{1/2}-1)/(-b*x+(-a*b^2)^ \\
& ^{1/3}))^{1/2}*((I^3^{1/2})*(-a*b^2)^{1/3}+2*b*x+(-a*b^2)^{1/3})/(1+I^3^{1/2} \\
&)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*((I^3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3} \\
&)/(I^3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*EllipticPi((-I^3^{1/2}-3)* \\
& x*b/(I^3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2},(I^3^{1/2}-1)/(I^3^{1/2}-3), \\
& ((I^3^{1/2}+3)*(I^3^{1/2}-1)/(1+I^3^{1/2})/(I^3^{1/2}-3))^{1/2}*(e^{*x}(b*x^ \\
& 3+a))^{1/2}*(-a*b^2)^{2/3}*a+6*B*(-a*b^2)^{2/3}*(-(I^3^{1/2}-3)*x*b/(I^3^{1/2} \\
& -1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*((I^3^{1/2})*(-a*b^2)^{1/3}+2*b*x+(-a*b^ \\
& 2)^{1/3})/(1+I^3^{1/2})/(-b*x+(-a*b^2)^{1/3}))^{1/2}*((I^3^{1/2})*(-a*b^2)^{1/3} \\
& -2*b*x-(-a*b^2)^{1/3})/(I^3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*Ellip \\
& ticF((-I^3^{1/2}-3)*x*b/(I^3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2},((I^3^{1/2} \\
& +3)*(I^3^{1/2}-1)/(1+I^3^{1/2})/(I^3^{1/2}-3))^{1/2}*(e^{*x}(b*x^3+a))^{1/2} \\
& *a-6*B*(-a*b^2)^{2/3}*(-(I^3^{1/2}-3)*x*b/(I^3^{1/2}-1)/(-b*x+(-a*b^2)^ \\
& ^{1/3}))^{1/2}*((I^3^{1/2})*(-a*b^2)^{1/3}+2*b*x+(-a*b^2)^{1/3})/(1+I^3^{1/2} \\
&)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*((I^3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3} \\
&)/(I^3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2}*EllipticPi((-I^3^{1/2}-3)* \\
& x*b/(I^3^{1/2}-1)/(-b*x+(-a*b^2)^{1/3}))^{1/2},(I^3^{1/2}-1)/(I^3^{1/2}-3), \\
& ((I^3^{1/2}+3)*(I^3^{1/2}-1)/(1+I^3^{1/2})/(I^3^{1/2}-3))^{1/2}*(e^{*x}(b*x^ \\
& 3+a))^{1/2}*a-3*A*(1/b^2*e^{*x}*(-b*x+(-a*b^2)^{1/3})*(I^3^{1/2})*(-a*b^2)^{1/3} \\
&)+2*b*x+(-a*b^2)^{1/3})*(I^3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3}))^{1/2} \\
& *x^2*b^3+3*B*(1/b^2*e^{*x}*(-b*x+(-a*b^2)^{1/3})*(I^3^{1/2})*(-a*b^2)^{1/3}+ \\
& 2*b*x+(-a*b^2)^{1/3})*(I^3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)^{1/3}))^{1/2} \\
& *x^2*a*b^2/x/(I^3^{1/2}-3)/(1/b^2*e^{*x}*(-b*x+(-a*b^2)^{1/3})*(I^3^{1/2})*(- \\
& a*b^2)^{1/3}+2*b*x+(-a*b^2)^{1/3})*(I^3^{1/2})*(-a*b^2)^{1/3}-2*b*x-(-a*b^2)
\end{aligned}$$

$$\sqrt[3]{(1/3))}^{\sqrt{1/2}}/a$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)\sqrt{ex}}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*sqrt(e*x)/(b*x^3 + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^3 + A)\sqrt{ex}}{(bx^3 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(3/2),x)

[Out] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(3/2), x)

sympy [A] time = 33.62, size = 95, normalized size = 1.12

$$\frac{2A\sqrt{e}x^{\frac{3}{2}}}{3a^{\frac{3}{2}}\sqrt{1 + \frac{bx^3}{a}}} + B \left(\frac{2\sqrt{e} \operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}} - \frac{2\sqrt{e}x^{\frac{3}{2}}}{3\sqrt{a}b\sqrt{1 + \frac{bx^3}{a}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(3/2),x)

[Out] 2*A*sqrt(e)*x**(3/2)/(3*a**(3/2)*sqrt(1 + b*x**3/a)) + B*(2*sqrt(e)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(3*b**(3/2)) - 2*sqrt(e)*x**(3/2)/(3*sqrt(a)*b*sqrt(1 + b*x**3/a))

$$3.340 \quad \int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=67

$$\frac{2(ex)^{3/2}(2Ab - aB)}{3a^2e^4\sqrt{a + bx^3}} - \frac{2A}{3ae(ex)^{3/2}\sqrt{a + bx^3}}$$

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {453, 264}

$$\frac{2(ex)^{3/2}(2Ab - aB)}{3a^2e^4\sqrt{a + bx^3}} - \frac{2A}{3ae(ex)^{3/2}\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)), x]

[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^3]) - (2*(2*A*b - a*B)*(e*x)^(3/2))/(3*a^2*e^4*Sqrt[a + b*x^3])

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\int \frac{A + Bx^3}{(ex)^{5/2} (a + bx^3)^{3/2}} dx = -\frac{2A}{3ae(ex)^{3/2}\sqrt{a + bx^3}} - \frac{(2Ab - aB) \int \frac{\sqrt{ex}}{(a+bx^3)^{3/2}} dx}{ae^3}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a + bx^3}} - \frac{2(2Ab - aB)(ex)^{3/2}}{3a^2e^4\sqrt{a + bx^3}}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.67

$$\frac{x(-2aA + 2aBx^3 - 4Abx^3)}{3a^2(ex)^{5/2}\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)), x]

[Out] (x*(-2*a*A - 4*A*b*x^3 + 2*a*B*x^3))/(3*a^2*(e*x)^(5/2)*Sqrt[a + b*x^3])

IntegrateAlgebraic [A] time = 1.04, size = 71, normalized size = 1.06

$$\frac{2\sqrt{a + bx^3} (-aAe^3 + aBe^3x^3 - 2Abe^3x^3)}{3a^2e(ex)^{3/2} (ae^3 + be^3x^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)), x]

[Out] (2*Sqrt[a + b*x^3]*(-(a*A*e^3) - 2*A*b*e^3*x^3 + a*B*e^3*x^3))/(3*a^2*e*(e*x)^(3/2)*(a*e^3 + b*e^3*x^3))

fricas [A] time = 0.65, size = 57, normalized size = 0.85

$$\frac{2((Ba - 2Ab)x^3 - Aa)\sqrt{bx^3 + a}\sqrt{ex}}{3(a^2be^3x^5 + a^3e^3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2), x, algorithm="fricas")

[Out] 2/3*((B*a - 2*A*b)*x^3 - A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^2*b*e^3*x^5 + a^3*e^3*x^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{3}{2}} (ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(5/2)), x)

maple [A] time = 0.05, size = 39, normalized size = 0.58

$$\frac{2(2Ax^3b - Bax^3 + Aa)x}{3\sqrt{bx^3 + a} (ex)^{\frac{5}{2}} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x)

[Out] -2/3*x*(2*A*b*x^3-B*a*x^3+A*a)/(b*x^3+a)^(1/2)/a^2/(e*x)^(5/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{3}{2}} (ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(5/2)), x)

mupad [B] time = 4.75, size = 70, normalized size = 1.04

$$\frac{\left(\frac{2A}{3abe^2} + \frac{x^3(4Ab-2Ba)}{3a^2be^2}\right)\sqrt{bx^3+a}}{x^4\sqrt{ex} + \frac{ax\sqrt{ex}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x)

[Out] -(((2*A)/(3*a*b*e^2) + (x^3*(4*A*b - 2*B*a))/(3*a^2*b*e^2))*(a + b*x^3)^(1/2))/(x^4*(e*x)^(1/2) + (a*x*(e*x)^(1/2))/b)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(3/2), x)

[Out] Timed out

$$3.341 \quad \int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}} dx$$

Optimal. Leaf size=114

$$\frac{2(ex)^{9/2}(Ab - aB)}{9abe(a + bx^3)^{3/2}} + \frac{2Be^{7/2} \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a + bx^3}}$$

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {452, 288, 329, 275, 217, 206}

$$\frac{2(ex)^{9/2}(Ab - aB)}{9abe(a + bx^3)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a + bx^3}} + \frac{2Be^{7/2} \tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2), x]

[Out] (2*(A*b - a*B)*(e*x)^(9/2))/(9*a*b*e*(a + b*x^3)^(3/2)) - (2*B*e^2*(e*x)^(3/2))/(3*b^2*sqrt[a + b*x^3]) + (2*B*e^(7/2)*ArcTanh[(sqrt[b]*(e*x)^(3/2))/(e^(3/2)*sqrt[a + b*x^3]])/(3*b^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{7/2} (A + Bx^3)}{(a + bx^3)^{5/2}} dx &= \frac{2(Ab - aB)(ex)^{9/2}}{9abe(a + bx^3)^{3/2}} + \frac{B \int \frac{(ex)^{7/2}}{(a+bx^3)^{3/2}} dx}{b} \\
&= \frac{2(Ab - aB)(ex)^{9/2}}{9abe(a + bx^3)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a + bx^3}} + \frac{(Be^3) \int \frac{\sqrt{ex}}{\sqrt{a+bx^3}} dx}{b^2} \\
&= \frac{2(Ab - aB)(ex)^{9/2}}{9abe(a + bx^3)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a + bx^3}} + \frac{(2Be^2) \text{Subst} \left(\int \frac{x^2}{\sqrt{a + \frac{bx^6}{e^3}}} dx, x, \sqrt{ex} \right)}{b^2} \\
&= \frac{2(Ab - aB)(ex)^{9/2}}{9abe(a + bx^3)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a + bx^3}} + \frac{(2Be^2) \text{Subst} \left(\int \frac{1}{\sqrt{a + \frac{bx^2}{e^3}}} dx, x, (ex)^{3/2} \right)}{3b^2} \\
&= \frac{2(Ab - aB)(ex)^{9/2}}{9abe(a + bx^3)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a + bx^3}} + \frac{(2Be^2) \text{Subst} \left(\int \frac{1}{1 - \frac{bx^2}{e^3}} dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}} \right)}{3b^2} \\
&= \frac{2(Ab - aB)(ex)^{9/2}}{9abe(a + bx^3)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a + bx^3}} + \frac{2Be^{7/2} \tanh^{-1} \left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}} \right)}{3b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.30, size = 119, normalized size = 1.04

$$\frac{2e^3\sqrt{ex} \left(3a^{3/2}B(a + bx^3) \sqrt{\frac{bx^3}{a} + 1} \sinh^{-1} \left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}} \right) + \sqrt{b}x^{3/2} (-3a^2B - 4abBx^3 + Ab^2x^3) \right)}{9ab^{5/2}\sqrt{x} (a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2), x]

[Out] (2*e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*(-3*a^2*B + A*b^2*x^3 - 4*a*b*B*x^3) + 3*a^(3/2)*B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(9*a*b^(5/2)*Sqrt[x]*(a + b*x^3)^(3/2))

IntegrateAlgebraic [A] time = 1.91, size = 137, normalized size = 1.20

$$\frac{2\sqrt{a+bx^3} \left(3a^2Be^8(ex)^{3/2} + 4abBe^5(ex)^{9/2} - Ab^2e^5(ex)^{9/2}\right)}{9ab^2 \left(ae^3 + be^3x^3\right)^2} - \frac{2Be^5\sqrt{\frac{b}{e^3}} \log\left(\sqrt{a+bx^3} - \sqrt{\frac{b}{e^3}}(ex)^{3/2}\right)}{3b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic(((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2), x)

[Out] (-2*sqrt[a + b*x^3]*(3*a^2*B*e^8*(e*x)^(3/2) - A*b^2*e^5*(e*x)^(9/2) + 4*a*b*B*e^5*(e*x)^(9/2))/(9*a*b^2*(a*e^3 + b*e^3*x^3)^2 - (2*B*sqrt[b/e^3]*e^5*Log[-(sqrt[b/e^3]*(e*x)^(3/2)) + sqrt[a + b*x^3]])/(3*b^3)

fricas [A] time = 0.84, size = 345, normalized size = 3.03

$$\frac{3(Bab^2e^3x^6 + 2Ba^2be^3x^3 + Ba^3e^3)\sqrt{\frac{e}{b}} \log\left(\frac{-8b^2ex^6 - 8abex^3 - a^2e - 4(2b^2x^4 + abx)\sqrt{bx^3 + a}\sqrt{ex}}{18(ab^4x^6 + 2a^2b^3x^3 + a^3b^2)}\right) - 4\left((4Bab - Ab^2)e^3x^4 + 3Ba^2e^3x\right)\sqrt{bx^3 + a}\sqrt{ex}}{9(ab^4x^6 + 2a^2b^3x^3 + a^3b^2)} + 2\left((4Bab - Ab^2)e^3x^4 + 3Ba^2e^3x\right)\sqrt{bx^3 + a}\sqrt{ex} \arctan\left(\frac{2\sqrt{bx^3 + a}\sqrt{ex}}{2be^3x^3 + ae}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2), x, algorithm="fricas")

[Out] [1/18*(3*(B*a*b^2*e^3*x^6 + 2*B*a^2*b*e^3*x^3 + B*a^3*e^3)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*((4*B*a*b - A*b^2)*e^3*x^4 + 3*B*a^2*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2), -1/9*(3*(B*a*b^2*e^3*x^6 + 2*B*a^2*b*e^3*x^3 + B*a^3*e^3)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*((4*B*a*b - A*b^2)*e^3*x^4 + 3*B*a^2*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)]

giac [A] time = 0.48, size = 102, normalized size = 0.89

$$\frac{2x^{\frac{3}{2}}\left(\frac{3Bae^8}{b^2} + \frac{(4Ba^5b^6e^{24} - Aa^4b^7e^{24})x^3e^{(-16)}}{a^5b^7}\right)e^{\frac{3}{2}}}{9\left(bx^3e^4 + ae^4\right)^{\frac{3}{2}}} - \frac{2Be^{\frac{7}{2}} \log\left(\left|-\sqrt{b}x^{\frac{3}{2}}e^2 + \sqrt{bx^3e^4 + ae^4}\right|\right)}{3b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2), x, algorithm="giac")

[Out] -2/9*x^(3/2)*(3*B*a*e^8/b^2 + (4*B*a^5*b^6*e^24 - A*a^4*b^7*e^24)*x^3*e^(-16)/(a^5*b^7))*e^(3/2)/(b*x^3*e^4 + a*e^4)^(3/2) - 2/3*B*e^(7/2)*log(abs(-sqrt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/b^(5/2)

maple [C] time = 1.05, size = 7081, normalized size = 62.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)(ex)^{\frac{7}{2}}}{(bx^3 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")`

[Out] `integrate((B*x^3 + A)*(e*x)^(7/2)/(b*x^3 + a)^(5/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^3 + A)(ex)^{7/2}}{(bx^3 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(5/2),x)`

[Out] `int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(5/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)`

[Out] Timed out

$$3.342 \quad \int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}} dx$$

Optimal. Leaf size=79

$$\frac{2(ex)^{3/2}(aB + 2Ab)}{9a^2be\sqrt{a + bx^3}} + \frac{2(ex)^{3/2}(Ab - aB)}{9abe(a + bx^3)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {457, 264}

$$\frac{2(ex)^{3/2}(aB + 2Ab)}{9a^2be\sqrt{a + bx^3}} + \frac{2(ex)^{3/2}(Ab - aB)}{9abe(a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2), x]

[Out] (2*(A*b - a*B)*(e*x)^(3/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(2*A*b + a*B)*(e*x)^(3/2))/(9*a^2*b*e*Sqrt[a + b*x^3])

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 457

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ((!IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))

Rubi steps

$$\int \frac{\sqrt{ex} (A + Bx^3)}{(a + bx^3)^{5/2}} dx = \frac{2(Ab - aB)(ex)^{3/2}}{9abe (a + bx^3)^{3/2}} + \frac{\left(2 \left(3Ab + \frac{3aB}{2}\right)\right) \int \frac{\sqrt{ex}}{(a+bx^3)^{3/2}} dx}{9ab}$$

$$= \frac{2(Ab - aB)(ex)^{3/2}}{9abe (a + bx^3)^{3/2}} + \frac{2(2Ab + aB)(ex)^{3/2}}{9a^2be\sqrt{a + bx^3}}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.56

$$\frac{2x\sqrt{ex} (3aA + aBx^3 + 2Abx^3)}{9a^2 (a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2), x]

[Out] (2*x*Sqrt[e*x]*(3*a*A + 2*A*b*x^3 + a*B*x^3))/(9*a^2*(a + b*x^3)^(3/2))

IntegrateAlgebraic [A] time = 1.26, size = 76, normalized size = 0.96

$$\frac{2\sqrt{a + bx^3} (3aAe^5(ex)^{3/2} + aBe^2(ex)^{9/2} + 2Abe^2(ex)^{9/2})}{9a^2 (ae^3 + be^3x^3)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2), x]

[Out] (2*Sqrt[a + b*x^3]*(3*a*A*e^5*(e*x)^(3/2) + 2*A*b*e^2*(e*x)^(9/2) + a*B*e^2*(e*x)^(9/2)))/(9*a^2*(a*e^3 + b*e^3*x^3)^2)

fricas [A] time = 0.61, size = 59, normalized size = 0.75

$$\frac{2 \left((Ba + 2Ab)x^4 + 3Aax \right) \sqrt{bx^3 + a} \sqrt{ex}}{9 (a^2b^2x^6 + 2a^3bx^3 + a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2), x, algorithm="fricas")

[Out] 2/9*((B*a + 2*A*b)*x^4 + 3*A*a*x)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)

giac [A] time = 0.32, size = 64, normalized size = 0.81

$$\frac{2x^{\frac{3}{2}} \left(\frac{3Ae^5}{a} + \frac{(Ba^5b^5e^{21} + 2Aa^4b^6e^{21})x^3e^{(-16)}}{a^6b^5} \right) e^{\frac{3}{2}}}{9(bx^3e^4 + ae^4)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="giac")

[Out] 2/9*x^(3/2)*(3*A*e^5/a + (B*a^5*b^5*e^21 + 2*A*a^4*b^6*e^21)*x^3*e^(-16)/(a^6*b^5))*e^(3/2)/(b*x^3*e^4 + a*e^4)^(3/2)

maple [A] time = 0.05, size = 39, normalized size = 0.49

$$\frac{2(2Ax^3b + Bax^3 + 3Aa)\sqrt{ex}x}{9(bx^3 + a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x)

[Out] 2/9*x*(2*A*b*x^3+B*a*x^3+3*A*a)*(e*x)^(1/2)/(b*x^3+a)^(3/2)/a^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)\sqrt{ex}}{(bx^3 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*sqrt(e*x)/(b*x^3 + a)^(5/2), x)

mupad [B] time = 4.63, size = 73, normalized size = 0.92

$$\frac{\left(\frac{2Ax\sqrt{ex}}{3ab^2} + \frac{x^4\sqrt{ex}(4Ab+2Ba)}{9a^2b^2} \right) \sqrt{bx^3+a}}{x^6 + \frac{a^2}{b^2} + \frac{2ax^3}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(5/2),x)

[Out]
$$\left(\frac{2Ax(e^x)^{1/2}}{3ab^2} + \frac{x^4(e^x)^{1/2}(4Ab + 2Ba)}{9a^2b^2} \right) \cdot (a + bx^3)^{1/2} / (x^6 + a^2/b^2 + (2ax^3)/b)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(5/2),x)`

[Out] Timed out

$$3.343 \quad \int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{5/2}} dx$$

Optimal. Leaf size=104

$$\frac{4(ex)^{3/2}(4Ab - aB)}{9a^3e^4\sqrt{a + bx^3}} - \frac{2(ex)^{3/2}(4Ab - aB)}{9a^2e^4(a + bx^3)^{3/2}} - \frac{2A}{3ae(ex)^{3/2}(a + bx^3)^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$\frac{4(ex)^{3/2}(4Ab - aB)}{9a^3e^4\sqrt{a + bx^3}} - \frac{2(ex)^{3/2}(4Ab - aB)}{9a^2e^4(a + bx^3)^{3/2}} - \frac{2A}{3ae(ex)^{3/2}(a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)), x]

[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^3)^(3/2)) - (2*(4*A*b - a*B)*(e*x)^(3/2))/(9*a^2*e^4*(a + b*x^3)^(3/2)) - (4*(4*A*b - a*B)*(e*x)^(3/2))/(9*a^3*e^4*Sqrt[a + b*x^3])

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 273

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (

LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^3}{(ex)^{5/2} (a + bx^3)^{5/2}} dx &= -\frac{2A}{3ae(ex)^{3/2} (a + bx^3)^{3/2}} - \frac{(4Ab - aB) \int \frac{\sqrt{ex}}{(a+bx^3)^{5/2}} dx}{ae^3} \\ &= -\frac{2A}{3ae(ex)^{3/2} (a + bx^3)^{3/2}} - \frac{2(4Ab - aB)(ex)^{3/2}}{9a^2e^4 (a + bx^3)^{3/2}} - \frac{(2(4Ab - aB)) \int \frac{\sqrt{ex}}{(a+bx^3)^{3/2}} dx}{3a^2e^3} \\ &= -\frac{2A}{3ae(ex)^{3/2} (a + bx^3)^{3/2}} - \frac{2(4Ab - aB)(ex)^{3/2}}{9a^2e^4 (a + bx^3)^{3/2}} - \frac{4(4Ab - aB)(ex)^{3/2}}{9a^3e^4\sqrt{a + bx^3}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.62

$$\frac{x(-6a^2(A - Bx^3) + 4abx^3(Bx^3 - 6A) - 16Ab^2x^6)}{9a^3(ex)^{5/2}(a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]

[Out] (x*(-16*A*b^2*x^6 - 6*a^2*(A - B*x^3) + 4*a*b*x^3*(-6*A + B*x^3)))/(9*a^3*(e*x)^(5/2)*(a + b*x^3)^(3/2))

IntegrateAlgebraic [A] time = 1.11, size = 100, normalized size = 0.96

$$\frac{2\sqrt{a + bx^3}(-3a^2Ae^6 + 3a^2Be^6x^3 - 12aAbe^6x^3 + 2abBe^6x^6 - 8Ab^2e^6x^6)}{9a^3e(ex)^{3/2}(ae^3 + be^3x^3)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]

[Out] (2*sqrt[a + b*x^3]*(-3*a^2*A*e^6 - 12*a*A*b*e^6*x^3 + 3*a^2*B*e^6*x^3 - 8*A*b^2*e^6*x^6 + 2*a*b*B*e^6*x^6))/(9*a^3*e*(e*x)^(3/2)*(a*e^3 + b*e^3*x^3)^2)

fricas [A] time = 0.63, size = 93, normalized size = 0.89

$$\frac{2 \left(2 (Bab - 4Ab^2)x^6 + 3 (Ba^2 - 4Aab)x^3 - 3Aa^2 \right) \sqrt{bx^3 + a} \sqrt{ex}}{9 (a^3b^2e^3x^8 + 2a^4be^3x^5 + a^5e^3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")

[Out] 2/9*(2*(B*a*b - 4*A*b^2)*x^6 + 3*(B*a^2 - 4*A*a*b)*x^3 - 3*A*a^2)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^3*b^2*e^3*x^8 + 2*a^4*b*e^3*x^5 + a^5*e^3*x^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{5}{2}} (ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="giac")

[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(5/2)), x)

maple [A] time = 0.05, size = 62, normalized size = 0.60

$$\frac{2 \left(8Ab^2x^6 - 2Babx^6 + 12Aabx^3 - 3Ba^2x^3 + 3Aa^2 \right) x}{9 (bx^3 + a)^{\frac{3}{2}} (ex)^{\frac{5}{2}} a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x)

[Out] -2/9*x*(8*A*b^2*x^6-2*B*a*b*x^6+12*A*a*b*x^3-3*B*a^2*x^3+3*A*a^2)/(b*x^3+a)^(3/2)/a^3/(e*x)^(5/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{5}{2}} (ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(5/2)), x)

mupad [B] time = 4.80, size = 115, normalized size = 1.11

$$\frac{\sqrt{bx^3 + a} \left(\frac{2A}{3ab^2e^2} - \frac{x^3(6Ba^2 - 24Aab)}{9a^3b^2e^2} + \frac{x^6(16Ab^2 - 4Bab)}{9a^3b^2e^2} \right)}{x^7 \sqrt{ex} + \frac{a^2x\sqrt{ex}}{b^2} + \frac{2ax^4\sqrt{ex}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)), x)

[Out] -((a + b*x^3)^(1/2)*((2*A)/(3*a*b^2*e^2) - (x^3*(6*B*a^2 - 24*A*a*b))/(9*a^3*b^2*e^2) + (x^6*(16*A*b^2 - 4*B*a*b))/(9*a^3*b^2*e^2)))/(x^7*(e*x)^(1/2) + (a^2*x*(e*x)^(1/2))/b^2 + (2*a*x^4*(e*x)^(1/2))/b)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(5/2), x)

[Out] Timed out

$$3.344 \quad \int \frac{x^{11} \sqrt[3]{a+bx^3}}{ad-bdx^3} dx$$

Optimal. Leaf size=220

$$\frac{a^{10/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^4 d} - \frac{a^{10/3} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2^{2/3} b^4 d} + \frac{\sqrt[3]{2} a^{10/3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^4 d} - \frac{a^3 \sqrt[3]{a+bx^3}}{b^4 d} - \frac{a^2 (a+bx^3)^4}{4b^4 d}$$

Rubi [A] time = 0.24, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 88, 50, 57, 617, 204, 31}

$$-\frac{a^3 \sqrt[3]{a+bx^3}}{b^4 d} - \frac{a^2 (a+bx^3)^{4/3}}{4b^4 d} + \frac{a^{10/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^4 d} - \frac{a^{10/3} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2^{2/3} b^4 d} + \frac{\sqrt[3]{2} a^{10/3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^4 d} + \frac{a (a+bx^3)^{7/3}}{7b^4 d} - \frac{(a+bx^3)^{10/3}}{10b^4 d}$$

Antiderivative was successfully verified.

[In] Int[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] -((a^3*(a + b*x^3)^(1/3))/(b^4*d)) - (a^2*(a + b*x^3)^(4/3))/(4*b^4*d) + (a*(a + b*x^3)^(7/3))/(7*b^4*d) - (a + b*x^3)^(10/3)/(10*b^4*d) + (2^(1/3)*a^(10/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^4*d) + (a^(10/3)*Log[a - b*x^3])/(3*2^(2/3)*b^4*d) - (a^(10/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^4*d)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x]

3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
]]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^{11} \sqrt[3]{a+bx^3}}{ad-bdx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3 \sqrt[3]{a+bx}}{ad-bdx} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{a^2 \sqrt[3]{a+bx}}{b^3 d} + \frac{a(a+bx)^{4/3}}{b^3 d} - \frac{(a+bx)^{7/3}}{b^3 d} + \frac{a^3 \sqrt[3]{a+bx}}{b^3(ad-bdx)} \right) dx, x, x^3 \right) \\
&= -\frac{a^2 (a+bx^3)^{4/3}}{4b^4 d} + \frac{a (a+bx^3)^{7/3}}{7b^4 d} - \frac{(a+bx^3)^{10/3}}{10b^4 d} + \frac{a^3 \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx} dx, x, x^3 \right)}{3b^3} \\
&= -\frac{a^3 \sqrt[3]{a+bx^3}}{b^4 d} - \frac{a^2 (a+bx^3)^{4/3}}{4b^4 d} + \frac{a (a+bx^3)^{7/3}}{7b^4 d} - \frac{(a+bx^3)^{10/3}}{10b^4 d} + \frac{(2a^4) \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}} dx, x, x^3 \right)}{3b^3} \\
&= -\frac{a^3 \sqrt[3]{a+bx^3}}{b^4 d} - \frac{a^2 (a+bx^3)^{4/3}}{4b^4 d} + \frac{a (a+bx^3)^{7/3}}{7b^4 d} - \frac{(a+bx^3)^{10/3}}{10b^4 d} + \frac{a^{10/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^4 d} + \frac{a^1}{\sqrt{3}} \\
&= -\frac{a^3 \sqrt[3]{a+bx^3}}{b^4 d} - \frac{a^2 (a+bx^3)^{4/3}}{4b^4 d} + \frac{a (a+bx^3)^{7/3}}{7b^4 d} - \frac{(a+bx^3)^{10/3}}{10b^4 d} + \frac{a^{10/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^4 d} - \frac{a^1}{\sqrt{3}} \\
&= -\frac{a^3 \sqrt[3]{a+bx^3}}{b^4 d} - \frac{a^2 (a+bx^3)^{4/3}}{4b^4 d} + \frac{a (a+bx^3)^{7/3}}{7b^4 d} - \frac{(a+bx^3)^{10/3}}{10b^4 d} + \frac{\sqrt[3]{2} a^{10/3} \tan^{-1} \left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3} b^4 d}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 230, normalized size = 1.05

$$\frac{140 \sqrt[3]{2} a^{10/3} \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}) - 70 \sqrt[3]{2} a^{10/3} \log(2^{2/3} a^{2/3} + \sqrt[3]{2} \sqrt[3]{a} \sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}) - 140 \sqrt[3]{2} \sqrt[3]{3} a^{10/3} \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + 1}{\sqrt[3]{3}} \right) + 507 a^3 \sqrt[3]{a+bx^3} + 111 a^2 b x^3 \sqrt[3]{a+bx^3} + 42 b^3 x^9 \sqrt[3]{a+bx^3} + 66 a^2 x^6 \sqrt[3]{a+bx^3}}{420 b^4 d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] -1/420*(507*a^3*(a + b*x^3)^(1/3) + 111*a^2*b*x^3*(a + b*x^3)^(1/3) + 66*a*b^2*x^6*(a + b*x^3)^(1/3) + 42*b^3*x^9*(a + b*x^3)^(1/3) - 140*2^(1/3)*Sqrt[3]*a^(10/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 140*2^(1/3)*a^(10/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] - 70*2^(1/3)*a^(10/3)*Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(b^4*d)

IntegrateAlgebraic [A] time = 0.25, size = 224, normalized size = 1.02

$$\frac{\sqrt[3]{2} a^{10/3} \log(2^{2/3} \sqrt[3]{a+bx^3} - 2 \sqrt[3]{a})}{3b^4 d} + \frac{a^{10/3} \log(2a^{2/3} + 2^{2/3} \sqrt[3]{a} \sqrt[3]{a+bx^3} + \sqrt[3]{2} (a+bx^3)^{2/3})}{3 \cdot 2^{2/3} b^4 d} + \frac{\sqrt[3]{2} a^{10/3} \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + 1}{\sqrt[3]{3}} \right)}{\sqrt{3} b^4 d} + \frac{\sqrt[3]{a+bx^3} (-169a^3 - 37a^2bx^3 - 22ab^2x^6 - 14b^3x^9)}{140b^4 d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-169*a^3 - 37*a^2*b*x^3 - 22*a*b^2*x^6 - 14*b^3*x^9))/(
140*b^4*d) + (2^(1/3)*a^(10/3)*ArcTan[1/Sqrt[3] + (2^(2/3)*(a + b*x^3)^(1/3)
) ]/(Sqrt[3]*a^(1/3)))/(Sqrt[3]*b^4*d) - (2^(1/3)*a^(10/3)*Log[-2*a^(1/3) +
2^(2/3)*(a + b*x^3)^(1/3)])/(3*b^4*d) + (a^(10/3)*Log[2*a^(2/3) + 2^(2/3)*
a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(2/3)*b^4*d)
```

fricas [A] time = 0.74, size = 186, normalized size = 0.85

$$\frac{140\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^3\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+70\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^3\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)-140\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^3\log\left(2^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right)+3(14b^3x^9+22ab^2x^6+37a^2bx^3+169a^3)(bx^3+a)^{\frac{1}{3}}}{420b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/420*(140*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^3*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x
^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 70*2^(1/3)*(-a)^(1/3)*a^3*log(2^
(2/3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)
) - 140*2^(1/3)*(-a)^(1/3)*a^3*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3))
+ 3*(14*b^3*x^9 + 22*a*b^2*x^6 + 37*a^2*b*x^3 + 169*a^3)*(b*x^3 + a)^(1/3))
/(b^4*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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```


$$\left(\frac{2}{3}\right) + 2^{(1/3)}*(b*x^3 + a)^{(1/3)}*a^{(1/3)} + (b*x^3 + a)^{(2/3)}/d - 140*2^{(1/3)}*a^{(10/3)}*\log(-2^{(1/3)}*a^{(1/3)} + (b*x^3 + a)^{(1/3)}/d - 3*(14*(b*x^3 + a)^{(10/3)} - 20*(b*x^3 + a)^{(7/3)}*a + 35*(b*x^3 + a)^{(4/3)}*a^2 + 140*(b*x^3 + a)^{(1/3)}*a^3)/d)/b^4$$

mupad [B] time = 4.81, size = 240, normalized size = 1.09

$$\frac{a(bx^3+a)^{7/3}}{7b^4d} - \frac{a^3(bx^3+a)^{1/3}}{b^4d} - \frac{a^2(bx^3+a)^{4/3}}{4b^4d} - \frac{(bx^3+a)^{10/3}}{10b^4d} - \frac{2^{1/3}a^{10/3}\ln\left(\frac{6a^4(bx^3+a)^{1/3}}{b^4d} - \frac{62^{1/3}a^{13/3}\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)}{b^4d}\right)}{3b^4d} - \frac{2^{1/3}a^{10/3}\ln\left(\frac{6a^4(bx^3+a)^{1/3}}{b^4d} + \frac{182^{1/3}a^{13/3}\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)}{b^4d}\right)}{3b^4d} + \frac{2^{1/3}a^{10/3}\ln\left(\frac{6a^4(bx^3+a)^{1/3}}{b^4d} + \frac{182^{1/3}a^{13/3}\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)}{b^4d}\right)}{b^4d} \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)

[Out] (a*(a + b*x^3)^(7/3))/(7*b^4*d) - (a^3*(a + b*x^3)^(1/3))/(b^4*d) - (a^2*(a + b*x^3)^(4/3))/(4*b^4*d) - (a + b*x^3)^(10/3)/(10*b^4*d) - (2^(1/3)*a^(10/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*a^(1/3)))/(3*b^4*d) - (2^(1/3)*a^(10/3)*log((6*a^4*(a + b*x^3)^(1/3))/(b^4*d) - (6*2^(1/3)*a^(13/3)*((3^(1/2)*1i)/2 - 1/2)))/(b^4*d))*((3^(1/2)*1i)/2 - 1/2))/(3*b^4*d) + (2^(1/3)*a^(10/3)*log((6*a^4*(a + b*x^3)^(1/3))/(b^4*d) + (18*2^(1/3)*a^(13/3)*((3^(1/2)*1i)/6 + 1/6)))/(b^4*d))*((3^(1/2)*1i)/6 + 1/6))/(b^4*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^{11} \sqrt[3]{a+bx^3}}{-a+bx^3} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d), x)

[Out] -Integral(x**11*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d

$$3.345 \quad \int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx$$

Optimal. Leaf size=174

$$\frac{a^{7/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^3 d} - \frac{a^{7/3} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2^{2/3} b^3 d} + \frac{\sqrt[3]{2} a^{7/3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^3 d} - \frac{a^2 \sqrt[3]{a+bx^3}}{b^3 d} - \frac{(a+bx^3)^{7/3}}{7b^3 d}$$

Rubi [A] time = 0.18, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 88, 50, 57, 617, 204, 31}

$$-\frac{a^2 \sqrt[3]{a+bx^3}}{b^3 d} + \frac{a^{7/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^3 d} - \frac{a^{7/3} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2^{2/3} b^3 d} + \frac{\sqrt[3]{2} a^{7/3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^3 d} - \frac{(a+bx^3)^{7/3}}{7b^3 d}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] -((a^2*(a + b*x^3)^(1/3))/(b^3*d)) - (a + b*x^3)^(7/3)/(7*b^3*d) + (2^(1/3)*a^(7/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^3*d) + (a^(7/3)*Log[a - b*x^3])/(3*2^(2/3)*b^3*d) - (a^(7/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^3*d)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]

]]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 \sqrt[3]{a+bx}}{ad-bdx} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{(a+bx)^{4/3}}{b^2 d} + \frac{a^2 \sqrt[3]{a+bx}}{b^2(ad-bdx)} \right) dx, x, x^3 \right) \\
&= -\frac{(a+bx^3)^{7/3}}{7b^3 d} + \frac{a^2 \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx} dx, x, x^3 \right)}{3b^2} \\
&= -\frac{a^2 \sqrt[3]{a+bx^3}}{b^3 d} - \frac{(a+bx^3)^{7/3}}{7b^3 d} + \frac{(2a^3) \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)} dx, x, x^3 \right)}{3b^2} \\
&= -\frac{a^2 \sqrt[3]{a+bx^3}}{b^3 d} - \frac{(a+bx^3)^{7/3}}{7b^3 d} + \frac{a^{7/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^3 d} + \frac{a^{7/3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{2} \sqrt[3]{a-x}} dx, x, \sqrt[3]{a+bx^3} \right)}{2^{2/3} b^3 d} \\
&= -\frac{a^2 \sqrt[3]{a+bx^3}}{b^3 d} - \frac{(a+bx^3)^{7/3}}{7b^3 d} + \frac{a^{7/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^3 d} - \frac{a^{7/3} \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3})}{2^{2/3} b^3 d} - \frac{(\sqrt[3]{2} a^{7/3})}{\sqrt{3} b^3 d} \tan^{-1} \left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right) + \frac{a^{7/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^3 d} - \frac{a^{7/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^3 d}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 207, normalized size = 1.19

$$\frac{1}{3} \left(-\frac{2 \sqrt[3]{2} a^{7/3} \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3})}{bd} - \frac{\sqrt[3]{2} a^{7/3} \left(\log(2^{2/3} a^{2/3} + \sqrt[3]{2} \sqrt[3]{a} \sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}) + 2\sqrt{3} \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}} \right) \right)}{2b^2} - \frac{3a^2 \sqrt[3]{a+bx^3}}{b^3 d} - \frac{3(a+bx^3)^{7/3}}{7b^3 d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] ((-3*a^2*(a + b*x^3)^(1/3))/(b^3*d) - (3*(a + b*x^3)^(7/3))/(7*b^3*d) - ((2*2^(1/3)*a^(7/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(b*d) - (2^(1/3)*a^(7/3)*(2*sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(sqrt[3]*a^(1/3)] + Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]))/(b*d))/(2*b^2))/3

[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] $\frac{1}{42} \cdot (14 \cdot \sqrt{3}) \cdot 2^{1/3} \cdot a^{7/3} \cdot \arctan\left(\frac{1}{6} \sqrt{3} \cdot 2^{2/3} \cdot (2^{1/3} \cdot a^{1/3}) + 2 \cdot (b \cdot x^3 + a)^{1/3}\right) / a^{1/3} / d + 7 \cdot 2^{1/3} \cdot a^{7/3} \cdot \log\left(\frac{2^{2/3} \cdot a^{2/3} + 2^{1/3} \cdot (b \cdot x^3 + a)^{1/3} \cdot a^{1/3} + (b \cdot x^3 + a)^{2/3}}{d}\right) - 14 \cdot 2^{1/3} \cdot a^{7/3} \cdot \log\left(\frac{-2^{1/3} \cdot a^{1/3} + (b \cdot x^3 + a)^{1/3}}{d}\right) - 6 \cdot ((b \cdot x^3 + a)^{7/3} + 7 \cdot (b \cdot x^3 + a)^{1/3} \cdot a^2) / d / b^3$

mupad [B] time = 4.65, size = 219, normalized size = 1.26

$$\frac{2^{1/3} (-a)^{7/3} \ln\left(\frac{6a^2(bx^3+a)^{1/3} - 6 \cdot 2^{1/3} (-a)^{10/3}}{3b^3d}\right) - a^2(bx^3+a)^{1/3} - \frac{(bx^3+a)^{7/3}}{7b^3d} - \frac{2^{1/3} (-a)^{7/3} \ln\left(\frac{6a^2(bx^3+a)^{1/3} + 6 \cdot 2^{1/3} (-a)^{10/3} \left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)}{3b^3d}\right) \left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)}{3b^3d} + \frac{2^{1/3} (-a)^{7/3} \ln\left(\frac{6a^2(bx^3+a)^{1/3} - 18 \cdot 2^{1/3} (-a)^{10/3} \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)}{b^3d}\right) \left(-\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)}{b^3d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)

[Out] $\frac{(2^{1/3} \cdot (-a)^{7/3} \cdot \log(6 \cdot a^3 \cdot (a + b \cdot x^3)^{1/3} - 6 \cdot 2^{1/3} \cdot (-a)^{10/3})) / (3 \cdot b^3 \cdot d) - (a^2 \cdot (a + b \cdot x^3)^{1/3}) / (b^3 \cdot d) - (a + b \cdot x^3)^{7/3} / (7 \cdot b^3 \cdot d) - (2^{1/3} \cdot (-a)^{7/3} \cdot \log((6 \cdot a^3 \cdot (a + b \cdot x^3)^{1/3}) / (b^3 \cdot d) + (6 \cdot 2^{1/3} \cdot (-a)^{10/3} \cdot ((3^{1/2} \cdot 1i) / 2 + 1/2)) / (b^3 \cdot d)) \cdot ((3^{1/2} \cdot 1i) / 2 + 1/2)) / (3 \cdot b^3 \cdot d) + (2^{1/3} \cdot (-a)^{7/3} \cdot \log((6 \cdot a^3 \cdot (a + b \cdot x^3)^{1/3}) / (b^3 \cdot d) - (18 \cdot 2^{1/3} \cdot (-a)^{10/3} \cdot ((3^{1/2} \cdot 1i) / 6 - 1/6)) / (b^3 \cdot d)) \cdot ((3^{1/2} \cdot 1i) / 6 - 1/6)) / (b^3 \cdot d)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^8 \sqrt[3]{a+bx^3}}{-a+bx^3} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d), x)

[Out] -Integral(x**8*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d

$$3.346 \quad \int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx$$

Optimal. Leaf size=172

$$\frac{a^{4/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^2 d} - \frac{a^{4/3} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2^{2/3} b^2 d} + \frac{\sqrt[3]{2} a^{4/3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^2 d} - \frac{a \sqrt[3]{a+bx^3}}{b^2 d} - \frac{(a+bx^3)^{4/3}}{4b^2 d}$$

Rubi [A] time = 0.15, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 80, 50, 57, 617, 204, 31}

$$\frac{a^{4/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^2 d} - \frac{a^{4/3} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2^{2/3} b^2 d} + \frac{\sqrt[3]{2} a^{4/3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^2 d} - \frac{a \sqrt[3]{a+bx^3}}{b^2 d} - \frac{(a+bx^3)^{4/3}}{4b^2 d}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] -((a*(a + b*x^3)^(1/3))/(b^2*d)) - (a + b*x^3)^(4/3)/(4*b^2*d) + (2^(1/3)*a^(4/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^2*d) + (a^(4/3)*Log[a - b*x^3])/(3*2^(2/3)*b^2*d) - (a^(4/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^2*d)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]

]]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x \sqrt[3]{a+bx}}{ad-bdx} dx, x, x^3 \right) \\
&= -\frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{a \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx} dx, x, x^3 \right)}{3b} \\
&= -\frac{a \sqrt[3]{a+bx^3}}{b^2d} - \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{(2a^2) \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)} dx, x, x^3 \right)}{3b} \\
&= -\frac{a \sqrt[3]{a+bx^3}}{b^2d} - \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{a^{4/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^2d} + \frac{a^{4/3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{2} \sqrt[3]{a-x}} dx, x, \sqrt[3]{a+bx^3} \right)}{2^{2/3} b^2d} \\
&= -\frac{a \sqrt[3]{a+bx^3}}{b^2d} - \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{a^{4/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^2d} - \frac{a^{4/3} \log \left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3} \right)}{2^{2/3} b^2d} - \frac{(\sqrt[3]{2} a^{4/3})}{2^{2/3} b^2d} \\
&= -\frac{a \sqrt[3]{a+bx^3}}{b^2d} - \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{\sqrt[3]{2} a^{4/3} \tan^{-1} \left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3} b^2d} + \frac{a^{4/3} \log(a-bx^3)}{3 \cdot 2^{2/3} b^2d} - \frac{a^{4/3} \log \left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3} \right)}{2^{2/3} b^2d}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 186, normalized size = 1.08

$$\frac{4\sqrt[3]{2} a^{4/3} \log \left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3} \right) - 2\sqrt[3]{2} a^{4/3} \log \left(2^{2/3} a^{2/3} + \sqrt[3]{2} \sqrt[3]{a} \sqrt[3]{a+bx^3} + (a+bx^3)^{2/3} \right) - 4\sqrt[3]{2} \sqrt{3} a^{4/3} \tan^{-1} \left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right) + 3bx^3 \sqrt[3]{a+bx^3} + 15a \sqrt[3]{a+bx^3}}{12b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] -1/12*(15*a*(a + b*x^3)^(1/3) + 3*b*x^3*(a + b*x^3)^(1/3) - 4*2^(1/3)*Sqrt[3]*a^(4/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 4*2^(1/3)*a^(4/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] - 2*2^(1/3)*a^(4/3)*Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/ (b^2*d)

IntegrateAlgebraic [A] time = 0.18, size = 202, normalized size = 1.17

$$-\frac{\sqrt[3]{2} a^{4/3} \log \left(2^{2/3} \sqrt[3]{a+bx^3} - 2\sqrt[3]{a} \right)}{3b^2d} + \frac{a^{4/3} \log \left(2a^{2/3} + 2^{2/3} \sqrt[3]{a} \sqrt[3]{a+bx^3} + \sqrt[3]{2} (a+bx^3)^{2/3} \right)}{3 \cdot 2^{2/3} b^2d} + \frac{\sqrt[3]{2} a^{4/3} \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{a+bx^3}}{\sqrt{3} \sqrt[3]{a}} + \frac{1}{\sqrt{3}} \right)}{\sqrt{3} b^2d} + \frac{(-5a - bx^3) \sqrt[3]{a+bx^3}}{4b^2d}$$

$/3) * \log(-2^{(1/3)} * a^{(1/3)} + (b * x^3 + a)^{(1/3)}) / d - 3 * ((b * x^3 + a)^{(4/3)} + 4 * (b * x^3 + a)^{(1/3)} * a) / d) / b^2$

mupad [B] time = 4.66, size = 200, normalized size = 1.16

$$\frac{(bx^3+a)^{4/3}}{4b^2d} - \frac{a(bx^3+a)^{1/3}}{b^2d} - \frac{2^{1/3}a^{4/3} \ln\left(\frac{(bx^3+a)^{1/3} - 2^{1/3}a^{1/3}}{3b^2d}\right)}{3b^2d} - \frac{2^{1/3}a^{4/3} \ln\left(\frac{6a^2(bx^3+a)^{1/3} - 6^{2/3}a^{7/3}\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)}{b^2d}\right)}{3b^2d} \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) + \frac{2^{1/3}a^{4/3} \ln\left(\frac{6a^2(bx^3+a)^{1/3} + 18^{2/3}a^{7/3}\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)}{b^2d}\right)}{b^2d} \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)`

[Out] $(2^{(1/3)} * a^{(4/3)} * \log((6 * a^2 * (a + b * x^3)^{(1/3)}) / (b^2 * d) + (18 * 2^{(1/3)} * a^{(7/3)}) * ((3^{(1/2)} * i) / 6 + 1/6)) / (b^2 * d)) * ((3^{(1/2)} * i) / 6 + 1/6)) / (b^2 * d) - (a * (a + b * x^3)^{(1/3)}) / (b^2 * d) - (2^{(1/3)} * a^{(4/3)} * \log((a + b * x^3)^{(1/3)} - 2^{(1/3)} * a^{(1/3)})) / (3 * b^2 * d) - (2^{(1/3)} * a^{(4/3)} * \log((6 * a^2 * (a + b * x^3)^{(1/3)}) / (b^2 * d) - (6 * 2^{(1/3)} * a^{(7/3)}) * ((3^{(1/2)} * i) / 2 - 1/2)) / (b^2 * d)) * ((3^{(1/2)} * i) / 2 - 1/2)) / (3 * b^2 * d) - (a + b * x^3)^{(4/3)} / (4 * b^2 * d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^5 \sqrt[3]{a+bx^3}}{-a+bx^3} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d), x)`

[Out] `-Integral(x**5*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d`

$$3.347 \quad \int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx$$

Optimal. Leaf size=150

$$-\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{a} \log(a-bx^3)}{3 \cdot 2^{2/3} bd} - \frac{\sqrt[3]{a} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2^{2/3} bd} + \frac{\sqrt[3]{2} \sqrt[3]{a} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} bd}$$

Rubi [A] time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {444, 50, 57, 617, 204, 31}

$$-\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{a} \log(a-bx^3)}{3 \cdot 2^{2/3} bd} - \frac{\sqrt[3]{a} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2^{2/3} bd} + \frac{\sqrt[3]{2} \sqrt[3]{a} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} bd}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] -((a + b*x^3)^(1/3)/(b*d)) + (2^(1/3)*a^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b*d) + (a^(1/3)*Log[a - b*x^3])/(3*2^(2/3)*b*d) - (a^(1/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]/(2^(2/3)*b*d)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x]

3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
]]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(-1)*((c_) + (d_.)*(x_)^(n_.))^(-1), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx} dx, x, x^3 \right) \\
 &= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{1}{3}(2a) \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)} dx, x, x^3 \right) \\
 &= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{a} \log(a-bx^3)}{3 \cdot 2^{2/3} bd} + \frac{\sqrt[3]{a} \text{Subst} \left(\int \frac{1}{\sqrt[3]{2} \sqrt[3]{a-x}} dx, x, \sqrt[3]{a+bx^3} \right)}{2^{2/3} bd} + \frac{a^{2/3} \text{Subst} \left(\int \frac{1}{-3} \right)}{bd} \\
 &= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{a} \log(a-bx^3)}{3 \cdot 2^{2/3} bd} - \frac{\sqrt[3]{a} \log \left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3} \right)}{2^{2/3} bd} - \frac{(\sqrt[3]{2} \sqrt[3]{a}) \text{Subst} \left(\int \frac{1}{-3} \right)}{bd} \\
 &= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{2} \sqrt[3]{a} \tan^{-1} \left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3} bd} + \frac{\sqrt[3]{a} \log(a-bx^3)}{3 \cdot 2^{2/3} bd} - \frac{\sqrt[3]{a} \log \left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3} \right)}{2^{2/3} bd}
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 167, normalized size = 1.11

$$\frac{\sqrt[3]{2} \sqrt[3]{a} \log\left(2^{2/3} a^{2/3} + \sqrt[3]{2} \sqrt[3]{a} \sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}\right) - 6\sqrt[3]{a+bx^3} - 2\sqrt[3]{2} \sqrt[3]{a} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right) + 2\sqrt[3]{2} \sqrt[3]{a} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + 1}{\sqrt[3]{a}}\right)}{6bd}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] (-6*(a + b*x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(1/3)*a^(1/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] + 2^(1/3)*a^(1/3)*Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*b*d)

IntegrateAlgebraic [A] time = 0.17, size = 190, normalized size = 1.27

$$\frac{\sqrt[3]{a} \log\left(2a^{2/3} + 2^{2/3} \sqrt[3]{a} \sqrt[3]{a+bx^3} + \sqrt[3]{2} (a+bx^3)^{2/3}\right)}{3 \cdot 2^{2/3} bd} - \frac{\sqrt[3]{a+bx^3}}{bd} - \frac{\sqrt[3]{2} \sqrt[3]{a} \log\left(2^{2/3} \sqrt[3]{a+bx^3} - 2\sqrt[3]{a}\right)}{3bd} + \frac{\sqrt[3]{2} \sqrt[3]{a} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3}}{\sqrt[3]{3} \sqrt[3]{a}} + \frac{1}{\sqrt[3]{3}}\right)}{\sqrt[3]{3} bd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] -((a + b*x^3)^(1/3)/(b*d)) + (2^(1/3)*a^(1/3)*ArcTan[1/Sqrt[3] + (2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b*d) - (2^(1/3)*a^(1/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)]/(3*b*d) + (a^(1/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]/(3*2^(2/3)*b*d))

fricas [A] time = 0.70, size = 144, normalized size = 0.96

$$\frac{2\sqrt[3]{2} \sqrt[3]{(-a)^{1/3}} \arctan\left(\frac{\sqrt[3]{3} \sqrt[3]{(bx^3+a)^{1/3} (-a)^{2/3} + \sqrt[3]{3} a}}{3a}\right) + 2^{1/3} (-a)^{1/3} \log\left(2^{2/3} (-a)^{2/3} - 2^{1/3} (bx^3+a)^{1/3} (-a)^{1/3} + (bx^3+a)^{2/3}\right) - 2 \cdot 2^{1/3} (-a)^{1/3} \log\left(2^{1/3} (-a)^{1/3} + (bx^3+a)^{1/3}\right) + 6(bx^3+a)^{1/3}}{6bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d), x, algorithm="fricas")

[Out] -1/6*(2*sqrt(3)*2^(1/3)*(-a)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2^(1/3)*(-a)^(1/3)*log(2^(2/3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 2*2^(1/3)*(-a)^(1/3)*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 6*(b*x^3 + a)^(1/3)/(b*d)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

[In] `int(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d), x)`

[Out] `int(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d), x)`

maxima [A] time = 1.27, size = 139, normalized size = 0.93

$$\frac{2\sqrt{3}2^{\frac{1}{3}}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(bx^3+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d} + \frac{2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{d} - \frac{2\cdot 2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right)}{d} - \frac{6(bx^3+a)^{\frac{1}{3}}}{d}$$

$6b$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d), x, algorithm="maxima")`

[Out] `1/6*(2*sqrt(3)*2^(1/3)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3))/d + 2^(1/3)*a^(1/3)*log(2^(2/3)*a^(2/3)+2^(1/3)*(b*x^3+a)^(1/3)*a^(1/3)+(b*x^3+a)^(2/3))/d - 2*2^(1/3)*a^(1/3)*log(-2^(1/3)*a^(1/3)+(b*x^3+a)^(1/3))/d - 6*(b*x^3+a)^(1/3)/d/b`

muPAD [B] time = 4.64, size = 194, normalized size = 1.29

$$\frac{2^{1/3}(-a)^{1/3}\ln\left(6a(bx^3+a)^{1/3}-6\cdot 2^{1/3}(-a)^{4/3}\right)}{3bd} - \frac{(bx^3+a)^{1/3}}{bd} + \frac{2^{1/3}(-a)^{1/3}\ln\left(\frac{6a(bx^3+a)^{1/3}}{bd}-\frac{6\cdot 2^{1/3}(-a)^{4/3}\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)}{bd}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)}{3bd} - \frac{2^{1/3}(-a)^{1/3}\ln\left(\frac{6a(bx^3+a)^{1/3}}{bd}+\frac{6\cdot 2^{1/3}(-a)^{4/3}\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)}{bd}\right)\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(a+b*x^3)^(1/3))/(a*d-b*d*x^3), x)`

[Out] `(2^(1/3)*(-a)^(1/3)*log(6*a*(a+b*x^3)^(1/3)-6*2^(1/3)*(-a)^(4/3)))/(3*b*d) - (a+b*x^3)^(1/3)/(b*d) + (2^(1/3)*(-a)^(1/3)*log((6*a*(a+b*x^3)^(1/3))/(b*d) - (6*2^(1/3)*(-a)^(4/3)*((3^(1/2)*1i)/2 - 1/2))/(b*d))*((3^(1/2)*1i)/2 - 1/2))/(3*b*d) - (2^(1/3)*(-a)^(1/3)*log((6*a*(a+b*x^3)^(1/3))/(b*d) + (6*2^(1/3)*(-a)^(4/3)*((3^(1/2)*1i)/2 + 1/2))/(b*d))*((3^(1/2)*1i)/2 + 1/2))/(3*b*d)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2 \sqrt[3]{a+bx^3}}{-a+bx^3} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d), x)`

[Out] `-Integral(x**2*(a+b*x**3)**(1/3)/(-a+b*x**3), x)/d`

$$3.348 \quad \int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)} dx$$

Optimal. Leaf size=214

$$\frac{\log(a-bx^3)}{3 \cdot 2^{2/3} a^{2/3} d} + \frac{\log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{2a^{2/3}d} - \frac{\log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3})}{2^{2/3} a^{2/3} d} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} d} + \frac{\sqrt[3]{2} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} d}$$

Rubi [A] time = 0.17, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {446, 83, 57, 617, 204, 31}

$$\frac{\log(a-bx^3)}{3 \cdot 2^{2/3} a^{2/3} d} + \frac{\log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{2a^{2/3}d} - \frac{\log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3})}{2^{2/3} a^{2/3} d} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} d} + \frac{\sqrt[3]{2} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} d} - \frac{\log(x)}{2a^{2/3}d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)), x]

[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*d) + (2^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*d) - Log[x]/(2*a^(2/3)*d) + Log[a - b*x^3]/(3*2^(2/3)*a^(2/3)*d) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(2/3)*d) - Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)*d)

Rule 31

Int[((a_) + (b_)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 83

Int[((e_) + (f_)*(x_))^(p_)/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p-1)/(a + b*x), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p-1)/(c + d*x), x]

], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{x(ad-bdx)} dx, x, x^3 \right) \\
 &= \frac{1}{3} (2b) \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)} dx, x, x^3 \right) + \frac{\text{Subst} \left(\int \frac{1}{x(a+bx)^{2/3}} dx, x, x^3 \right)}{3d} \\
 &= -\frac{\log(x)}{2a^{2/3}d} + \frac{\log(a-bx^3)}{3 \cdot 2^{2/3} a^{2/3} d} - \frac{\text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, \sqrt[3]{a+bx^3} \right)}{2a^{2/3}d} + \frac{\text{Subst} \left(\int \frac{1}{\sqrt[3]{2} \sqrt[3]{a-x}} dx, x, \sqrt[3]{a+bx^3} \right)}{2^{2/3} a^{2/3} d} \\
 &= -\frac{\log(x)}{2a^{2/3}d} + \frac{\log(a-bx^3)}{3 \cdot 2^{2/3} a^{2/3} d} + \frac{\log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{2a^{2/3}d} - \frac{\log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3})}{2^{2/3} a^{2/3} d} + \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, \sqrt[3]{a+bx^3} \right)}{3d} \\
 &= -\frac{\tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3} a^{2/3} d} + \frac{\sqrt[3]{2} \tan^{-1} \left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3} a^{2/3} d} - \frac{\log(x)}{2a^{2/3}d} + \frac{\log(a-bx^3)}{3 \cdot 2^{2/3} a^{2/3} d} + \frac{\log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{2a^{2/3}d}
 \end{aligned}$$

Mathematica [A] time = 0.11, size = 233, normalized size = 1.09

$$\frac{\log(a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}) - \sqrt[3]{2} \log(2^{2/3}a^{2/3} + \sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}) - 2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3}) + 2\sqrt[3]{2} \log(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+bx^3}) + 2\sqrt[3]{3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+1}{\sqrt[3]{a}}\right) - 2\sqrt[3]{2}\sqrt[3]{3} \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+1}{\sqrt[3]{a}}\right)}{6a^{2/3}d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)), x]

[Out] $-1/6 * (2 * \sqrt{3} * \text{ArcTan}[(1 + (2 * (a + b * x^3)^{1/3}) / a^{1/3}) / \sqrt{3}]) - 2 * 2^{1/3} * \sqrt{3} * \text{ArcTan}[(1 + (2^{2/3} * (a + b * x^3)^{1/3}) / a^{1/3}) / \sqrt{3}] - 2 * \text{Log}[a^{1/3} - (a + b * x^3)^{1/3}] + 2 * 2^{1/3} * \text{Log}[2^{1/3} * a^{1/3} - (a + b * x^3)^{1/3}] + \text{Log}[a^{2/3} + a^{1/3} * (a + b * x^3)^{1/3} + (a + b * x^3)^{2/3}] - 2^{1/3} * \text{Log}[2^{2/3} * a^{2/3} + 2^{1/3} * a^{1/3} * (a + b * x^3)^{1/3} + (a + b * x^3)^{2/3}]) / (a^{2/3} * d)$

IntegrateAlgebraic [A] time = 0.35, size = 286, normalized size = 1.34

$$\frac{\log(\sqrt[3]{a+bx^3} - \sqrt[3]{a})}{3a^{2/3}d} - \frac{\sqrt[3]{2} \log(2^{2/3}\sqrt[3]{a+bx^3} - 2\sqrt[3]{a})}{3a^{2/3}d} - \frac{\log(a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3})}{6a^{2/3}d} + \frac{\log(2a^{2/3} + 2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3} + \sqrt[3]{2}(a+bx^3)^{2/3})}{3 \cdot 2^{2/3}a^{2/3}d} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+1}{\sqrt[3]{a}}\right)}{\sqrt[3]{3}a^{2/3}d} + \frac{\sqrt[3]{2} \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+1}{\sqrt[3]{a}}\right)}{\sqrt[3]{3}a^{2/3}d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)), x]

[Out] $-(\text{ArcTan}[1/\sqrt{3}] + (2 * (a + b * x^3)^{1/3}) / (\sqrt{3} * a^{1/3})) / (\sqrt{3} * a^{2/3} * d) + (2^{1/3} * \text{ArcTan}[1/\sqrt{3}] + (2^{2/3} * (a + b * x^3)^{1/3}) / (\sqrt{3} * a^{1/3})) / (\sqrt{3} * a^{2/3} * d) + \text{Log}[-a^{1/3} + (a + b * x^3)^{1/3}] / (3 * a^{2/3} * d) - (2^{1/3} * \text{Log}[-2 * a^{1/3} + 2^{2/3} * (a + b * x^3)^{1/3}]) / (3 * a^{2/3} * d) - \text{Log}[a^{2/3} + a^{1/3} * (a + b * x^3)^{1/3} + (a + b * x^3)^{2/3}] / (6 * a^{2/3} * d) + \text{Log}[2 * a^{2/3} + 2^{2/3} * a^{1/3} * (a + b * x^3)^{1/3} + 2^{1/3} * (a + b * x^3)^{2/3}] / (3 * 2^{2/3} * a^{2/3} * d)$

fricas [B] time = 0.72, size = 1046, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out] $-2/3 * \text{sqrt}(3) * (1/2)^{1/3} * (-3 * a^2 * d^3 * \text{sqrt}(1/(a^4 * d^6)) + 1) / (a^2 * d^3)^{1/3} * \text{arctan}(1/12 * (1/2)^{2/3} * (\text{sqrt}(3) * a^3 * d^5 * \text{sqrt}(1/(a^4 * d^6)) - 3 * \text{sqrt}(3) * a * d^2 * \text{sqrt}(-4 * (1/2)^{1/3} * (b * x^3 + a)^{1/3} * a^3 * d^4 * (-3 * a^2 * d^3 * \text{sqrt}(1/(a^4 * d^6)) + 1) / (a^2 * d^3))^{1/3} * \text{sqrt}(1/(a^4 * d^6)) + 4 * (1/2)^{2/3} * a^2 * d^2 * (-3 * a^2 * d^3 * \text{sqrt}(1/(a^4 * d^6)) + 1) / (a^2 * d^3))^{2/3} + 4 * (b * x^3 + a)^{2/3} * (-3 * a^2 * d^3 * \text{sqrt}(1/(a^4 * d^6)) + 1) / (a^2 * d^3))^{2/3} - 1/6 * (1/2)^{2/3} * (\text{sqrt}(3) * a^3 * d^5 * \text{sqrt}(1/(a^4 * d^6)) - 3 * \text{sqrt}(3) * a * d^2 * (b * x^3 + a)^{1/3} * (-3 * a^2 * d^3 * \text{sqrt}(1/(a^4 * d^6)) + 1) / (a^2 * d^3))^{2/3} - 1/6 * (1/2)^{2/3} * (\text{sqrt}(3) * a^3 * d^5 * \text{sqrt}(1/(a^4 * d^6)) - 3 * \text{sqrt}(3) * a * d^2 * (b * x^3 + a)^{1/3} * (-3 * a^2 * d^3 * \text{sqrt}(1/(a^4 * d^6)) + 1) / (a^2 * d^3))^{2/3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x)`

[Out] $\log((a + b*x^3)^{(1/3)} - a*d*(1/(a^2*d^3))^{(1/3)})*(1/(27*a^2*d^3))^{(1/3)} + \log((a + b*x^3)^{(1/3)} + 2^{(1/3)*a*d*(-1/(a^2*d^3))^{(1/3)})*(-2/(27*a^2*d^3))^{(1/3)} - \log(2^{(1/3)*a*d*(-1/(a^2*d^3))^{(1/3)} - 2*(a + b*x^3)^{(1/3)} + 2^{(1/3)*3^{(1/2)*a*d*(-1/(a^2*d^3))^{(1/3)*1i}}*((3^{(1/2)*1i})/2 + 1/2)*(-2/(27*a^2*d^3))^{(1/3)} + \log(2*(a + b*x^3)^{(1/3)} - 2^{(1/3)*a*d*(-1/(a^2*d^3))^{(1/3)} + 2^{(1/3)*3^{(1/2)*a*d*(-1/(a^2*d^3))^{(1/3)*1i}}*((3^{(1/2)*1i})/2 - 1/2)*(-2/(27*a^2*d^3))^{(1/3)} + \log(2*(a + b*x^3)^{(1/3)} + a*d*(1/(a^2*d^3))^{(1/3)} - 3^{(1/2)*a*d*(1/(a^2*d^3))^{(1/3)*1i}}*((3^{(1/2)*1i})/2 - 1/2)*(1/(27*a^2*d^3))^{(1/3)}) - \log(2*(a + b*x^3)^{(1/3)} + a*d*(1/(a^2*d^3))^{(1/3)} + 3^{(1/2)*a*d*(1/(a^2*d^3))^{(1/3)*1i}}*((3^{(1/2)*1i})/2 + 1/2)*(1/(27*a^2*d^3))^{(1/3)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{-ax+bx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(1/3)/x/(-b*d*x**3+a*d),x)`

[Out] `-Integral((a + b*x**3)**(1/3)/(-a*x + b*x**4), x)/d`

$$3.349 \quad \int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)} dx$$

Optimal. Leaf size=268

$$\frac{b \log(a - bx^3)}{3 \cdot 2^{2/3} a^{5/3} d} + \frac{2b \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{3a^{5/3}d} - \frac{b \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3})}{2^{2/3} a^{5/3} d} - \frac{4b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} a^{5/3} d} + \frac{\sqrt[3]{2} b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.25, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {446, 103, 156, 50, 57, 617, 204, 31}

$$-\frac{(a+bx^3)^{4/3}}{3a^2dx^3} + \frac{b\sqrt[3]{a+bx^3}}{3a^2d} + \frac{b \log(a-bx^3)}{3 \cdot 2^{2/3} a^{5/3} d} + \frac{2b \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{3a^{5/3}d} - \frac{b \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3})}{2^{2/3} a^{5/3} d} - \frac{4b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} a^{5/3} d} + \frac{\sqrt[3]{2} b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{5/3} d} - \frac{2b \log(x)}{3a^{5/3}d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)), x]

[Out] (b*(a + b*x^3)^(1/3))/(3*a^2*d) - (a + b*x^3)^(4/3)/(3*a^2*d*x^3) - (4*b*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*d) + (2^(1/3)*b*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*d) - (2*b*Log[x])/(3*a^(5/3)*d) + (b*Log[a - b*x^3])/(3*2^(2/3)*a^(5/3)*d) + (2*b*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(3*a^(5/3)*d) - (b*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*a^(5/3)*d)

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m+1)*(c + d*x)^n)/(b*(m+n+1)), x] + Dist[(n*(b*c - a*d))/(b*(m+n+1)), Int[(a + b*x)^m*(c + d*x)^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 57

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),

$x] + (-\text{Dist}[3/(2*b*q), \text{Subst}[\text{Int}[1/(q^2 + q*x + x^2)], x], x, (c + d*x)^{(1/3)}], x] - \text{Dist}[3/(2*b*q^2), \text{Subst}[\text{Int}[1/(q - x)], x], x, (c + d*x)^{(1/3)}], x]) /;$ FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 103

$\text{Int}[\left((a_{.}) + (b_{.})*(x_{.})\right)^{(m_{.})}*\left((c_{.}) + (d_{.})*(x_{.})\right)^{(n_{.})}*\left((e_{.}) + (f_{.})*(x_{.})\right)^{(p_{.})}, x_Symbol] \rightarrow \text{Simp}[(b*(a + b*x)^{(m + 1)}*(c + d*x)^{(n + 1)}*(e + f*x)^{(p + 1)})/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + \text{Dist}[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n*(e + f*x)^p*\text{Simp}[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /;$ FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

$\text{Int}[\left(\left((e_{.}) + (f_{.})*(x_{.})\right)^{(p_{.})}*\left((g_{.}) + (h_{.})*(x_{.})\right)\right)/\left(\left((a_{.}) + (b_{.})*(x_{.})\right)*\left((c_{.}) + (d_{.})*(x_{.})\right)\right), x_Symbol] \rightarrow \text{Dist}[(b*g - a*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(a + b*x), x], x] - \text{Dist}[(d*g - c*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(c + d*x), x], x] /;$ FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 204

$\text{Int}[\left((a_{.}) + (b_{.})*(x_{.})^2\right)^{(-1)}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/\left(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]\right), x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

$\text{Int}[(x_{.})^{(m_{.})}*\left((a_{.}) + (b_{.})*(x_{.})^{(n_{.})}\right)^{(p_{.})}*\left((c_{.}) + (d_{.})*(x_{.})^{(n_{.})}\right)^{(q_{.})}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q}, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

$\text{Int}[\left((a_{.}) + (b_{.})*(x_{.}) + (c_{.})*(x_{.})^2\right)^{(-1)}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2)], x], x, 1 + (2*c*x)/b], x] /;$ RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /;

FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{x^2(ad-bdx)} dx, x, x^3 \right) \\
&= -\frac{(a+bx^3)^{4/3}}{3a^2 dx^3} - \frac{\text{Subst} \left(\int \frac{\sqrt[3]{a+bx} \left(-\frac{4}{3}abd + \frac{1}{3}b^2 dx \right)}{x(ad-bdx)} dx, x, x^3 \right)}{3a^2 d} \\
&= -\frac{(a+bx^3)^{4/3}}{3a^2 dx^3} + \frac{b^2 \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx} dx, x, x^3 \right)}{3a^2} + \frac{(4b) \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{x} dx, x, x^3 \right)}{9a^2 d} \\
&= \frac{b\sqrt[3]{a+bx^3}}{3a^2 d} - \frac{(a+bx^3)^{4/3}}{3a^2 dx^3} + \frac{(2b^2) \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)} dx, x, x^3 \right)}{3a} + \frac{(4b) \text{Subst} \left(\int \frac{1}{x} dx, x, x^3 \right)}{9a^2 d} \\
&= \frac{b\sqrt[3]{a+bx^3}}{3a^2 d} - \frac{(a+bx^3)^{4/3}}{3a^2 dx^3} - \frac{2b \log(x)}{3a^{5/3} d} + \frac{b \log(a-bx^3)}{3 \cdot 2^{2/3} a^{5/3} d} - \frac{(2b) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, \sqrt[3]{a+bx^3} \right)}{3a^{5/3} d} \\
&= \frac{b\sqrt[3]{a+bx^3}}{3a^2 d} - \frac{(a+bx^3)^{4/3}}{3a^2 dx^3} - \frac{2b \log(x)}{3a^{5/3} d} + \frac{b \log(a-bx^3)}{3 \cdot 2^{2/3} a^{5/3} d} + \frac{2b \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{3a^{5/3} d} \\
&= \frac{b\sqrt[3]{a+bx^3}}{3a^2 d} - \frac{(a+bx^3)^{4/3}}{3a^2 dx^3} - \frac{4b \tan^{-1} \left(\frac{1 + 2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}} \right)}{3\sqrt{3} a^{5/3} d} + \frac{\sqrt[3]{2} b \tan^{-1} \left(\frac{1 + 2^{2/3} \sqrt[3]{a+bx^3}}{\sqrt[3]{a}} \right)}{\sqrt{3} a^{5/3} d} - \frac{2b \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{3a^{5/3} d}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 280, normalized size = 1.04

$$\frac{6a^{2/3}\sqrt[3]{a+bx^3} + 4bx^3 \log(a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}) - 3\sqrt[3]{2}bx^3 \log(2^{2/3}a^{2/3} + \sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}) - 8bx^3 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3}) + 6\sqrt[3]{2}bx^3 \log(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+bx^3}) + 8\sqrt[3]{3}bx^3 \tan^{-1}\left(\frac{1 + 2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right) - 6\sqrt[3]{2}\sqrt[3]{3}bx^3 \tan^{-1}\left(\frac{1 + 2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{18a^{5/3}dx^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)), x]

[Out]
$$\begin{aligned}
& -1/18*(6*a^{(2/3)}*(a + b*x^3)^{(1/3)} + 8*sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*x^3)^{(1/3}))/a^{(1/3}))/sqrt[3]] - 6*2^{(1/3)}*sqrt[3]*b*x^3*ArcTan[(1 + (2^{(2/3)}*(a + b*x^3)^{(1/3}))/a^{(1/3}))/sqrt[3]] - 8*b*x^3*Log[a^{(1/3)} - (a + b*x^3)^{(1/3)}] + 6*2^{(1/3)}*b*x^3*Log[2^{(1/3)}*a^{(1/3)} - (a + b*x^3)^{(1/3)}] + 4*b*x^3*Log[a^{(2/3)} + a^{(1/3)}*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}] - 3*2^{(1/3)}*b*x^3*Log[2^{(2/3)}*a^{(2/3)} + 2^{(1/3)}*a^{(1/3)}*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}])/(a^{(5/3)}*d*x^3)
\end{aligned}$$

IntegrateAlgebraic [A] time = 0.57, size = 318, normalized size = 1.19

$$\frac{4b \log(\sqrt{a+bx^3} - \sqrt{a})}{9a^{5/3}d} - \frac{\sqrt{2}b \log(2^{2/3}\sqrt{a+bx^3} - 2\sqrt{a})}{3a^{5/3}d} - \frac{2b \log(a^{2/3} + \sqrt{a}\sqrt{a+bx^3} + (a+bx^3)^{2/3})}{9a^{5/3}d} + \frac{b \log(2a^{2/3} + 2^{2/3}\sqrt{a}\sqrt{a+bx^3} + \sqrt{2}(a+bx^3)^{2/3})}{3 \cdot 2^{2/3}a^{5/3}d} - \frac{4b \tan^{-1}\left(\frac{2\sqrt{a+bx^3}}{\sqrt{3}\sqrt{a}} + \frac{1}{\sqrt{3}}\right)}{3\sqrt{3}a^{5/3}d} + \frac{\sqrt{2}b \tan^{-1}\left(\frac{2^{2/3}\sqrt{a+bx^3}}{\sqrt{3}\sqrt{a}} + \frac{1}{\sqrt{3}}\right)}{\sqrt{3}a^{5/3}d} - \frac{\sqrt{a+bx^3}}{3adx^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]

[Out]
$$-1/3*(a + b*x^3)^{(1/3)}/(a*d*x^3) - (4*b*ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(3*Sqrt[3]*a^{(5/3)*d}) + (2^{(1/3)}*b*ArcTan[1/Sqrt[3] + (2^{(2/3)}*(a + b*x^3)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(5/3)*d}) + (4*b*Log[-a^{(1/3)} + (a + b*x^3)^{(1/3)}])/(9*a^{(5/3)*d}) - (2^{(1/3)}*b*Log[-2*a^{(1/3)} + 2^{(2/3)}*(a + b*x^3)^{(1/3)}])/(3*a^{(5/3)*d}) - (2*b*Log[a^{(2/3)} + a^{(1/3)}*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}])/(9*a^{(5/3)*d}) + (b*Log[2*a^{(2/3)} + 2^{(2/3)}*a^{(1/3)}*(a + b*x^3)^{(1/3)} + 2^{(1/3)}*(a + b*x^3)^{(2/3)}])/(3*2^{(2/3)}*a^{(5/3)*d})$$

fricas [A] time = 0.74, size = 321, normalized size = 1.20

$$\frac{6\sqrt{3}2^{1/3}a^{2/3}\log\left(\frac{2}{3}\sqrt{3}2^{1/3}(a^2+bx^3)^{1/3}x\left(-\frac{1}{3}\sqrt{3}+\frac{1}{3}\sqrt{3}\right)+3\cdot 2^{1/3}a^{2/3}\log\left(2^{2/3}a^{1/3}\sqrt{3}x\left(-\frac{1}{3}\sqrt{3}-2^{1/3}(a^2+bx^3)^{1/3}x\left(-\frac{1}{3}\sqrt{3}+(a^2+bx^3)^{1/3}\right)\right)-6\cdot 2^{1/3}a^{2/3}\log\left(2^{2/3}a^{1/3}\sqrt{3}x\left(-\frac{1}{3}\sqrt{3}+(a^2+bx^3)^{1/3}\right)+8\sqrt{3}(a^2)^{1/3}a^{2/3}\arctan\left(\frac{(a^2)^{1/3}\sqrt{3}(a^2)^{1/3}+2\sqrt{3}(a^2)^{1/3}(a^2)^{1/3}}{3a^2}\right)\right)+4(a^2)^{1/3}a^{2/3}\log\left((a^2+bx^3)^{1/3}x+(a^2+bx^3)^{1/3}x\right)-8(a^2)^{1/3}a^{2/3}\log\left((a^2+bx^3)^{1/3}x-(a^2+bx^3)^{1/3}x\right)+6(a^2+bx^3)^{1/3}x\right)}{18a^{5/3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out]
$$-1/18*(6*\sqrt{3})*2^{(1/3)}*a^2*b*x^3*(-1/a^2)^{(1/3)}*\arctan(1/3*\sqrt{3})*2^{(2/3)}*(b*x^3 + a)^{(1/3)}*a*(-1/a^2)^{(2/3)} + 1/3*\sqrt{3}) + 3*2^{(1/3)}*a^2*b*x^3*(-1/a^2)^{(1/3)}*\log(2^{(2/3)}*a^2*(-1/a^2)^{(2/3)} - 2^{(1/3)}*(b*x^3 + a)^{(1/3)}*a*(-1/a^2)^{(1/3)} + (b*x^3 + a)^{(2/3)}) - 6*2^{(1/3)}*a^2*b*x^3*(-1/a^2)^{(1/3)}*\log(2^{(1/3)}*a^2*(-1/a^2)^{(1/3)} + (b*x^3 + a)^{(1/3)}) + 8*\sqrt{3}*(a^2)^{(1/6)}*a*b*x^3*\arctan(1/3*(a^2)^{(1/6)}*(\sqrt{3}*(a^2)^{(1/3)}*a + 2*\sqrt{3}*(b*x^3 + a)^{(1/3)}*(a^2)^{(2/3)})/a^2) + 4*(a^2)^{(2/3)}*b*x^3*\log((b*x^3 + a)^{(2/3)}*a + (a^2)^{(1/3)}*a + (b*x^3 + a)^{(1/3)}*(a^2)^{(2/3)}) - 8*(a^2)^{(2/3)}*b*x^3*\log((b*x^3 + a)^{(1/3)}*a - (a^2)^{(2/3)}) + 6*(b*x^3 + a)^{(1/3)}*a^2/(a^3*d*x^3)$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algebraic extensions not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(bdx^3 - ad)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^4), x)

mupad [B] time = 5.35, size = 455, normalized size = 1.70

$$\frac{4 \sqrt[3]{b(a+bx^3)} \log\left(\frac{\sqrt[3]{b(a+bx^3)} - a^2 d \sqrt[3]{b/(a^5 d^3)}}{9}\right) + \log\left(\frac{\sqrt[3]{b(a+bx^3)} + 2^{1/3} a^2 d \sqrt[3]{-b/(a^5 d^3)}}{27 a^5 d^3}\right) - 3^{1/2} a^2 d \sqrt[3]{b/(a^5 d^3)} \operatorname{erfi}\left(\frac{3^{1/2} \sqrt[3]{b(a+bx^3)} - a^2 d \sqrt[3]{b/(a^5 d^3)}}{2}\right) + \log\left(\frac{2 \sqrt[3]{b(a+bx^3)} + a^2 d \sqrt[3]{b/(a^5 d^3)}}{27 a^5 d^3}\right) - 3^{1/2} a^2 d \sqrt[3]{b/(a^5 d^3)} \operatorname{erfi}\left(\frac{3^{1/2} \sqrt[3]{b(a+bx^3)} + a^2 d \sqrt[3]{b/(a^5 d^3)}}{2}\right) - 2 \sqrt[3]{b(a+bx^3)} + 2^{1/3} a^2 d \sqrt[3]{-b/(a^5 d^3)} \operatorname{erfi}\left(\frac{3^{1/2} \sqrt[3]{b(a+bx^3)} - a^2 d \sqrt[3]{b/(a^5 d^3)}}{2}\right) + \log\left(\frac{2 \sqrt[3]{b(a+bx^3)} - 2^{1/3} a^2 d \sqrt[3]{-b/(a^5 d^3)}}{27 a^5 d^3}\right) + 2^{1/3} a^2 d \sqrt[3]{-b/(a^5 d^3)} \operatorname{erfi}\left(\frac{3^{1/2} \sqrt[3]{b(a+bx^3)} - 2^{1/3} a^2 d \sqrt[3]{-b/(a^5 d^3)}}{2}\right) - (b(a+bx^3))^{1/3} / (3 a (d(a+bx^3) - a d))}{3 a (d(a+bx^3) - a d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x)

[Out] (4*log(b*(a + b*x^3)^(1/3) - a^2*d*(b^3/(a^5*d^3))^(1/3))*(b^3/(a^5*d^3))^(1/3))/9 + log(b*(a + b*x^3)^(1/3) + 2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3))*(-2*b^3)/(27*a^5*d^3)^(1/3) + log(2*b*(a + b*x^3)^(1/3) + a^2*d*(b^3/(a^5*d^3))^(1/3) - 3^(1/2)*a^2*d*(b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*((64*b^3)/(729*a^5*d^3))^(1/3) - log(2*b*(a + b*x^3)^(1/3) + a^2*d*(b^3/(a^5*d^3))^(1/3) + 3^(1/2)*a^2*d*(b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*((64*b^3)/(729*a^5*d^3))^(1/3) - log(2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3) - 2*b*(a + b*x^3)^(1/3) + 2^(1/3)*3^(1/2)*a^2*d*(-b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(-2*b^3)/(27*a^5*d^3)^(1/3) + log(2*b*(a + b*x^3)^(1/3) - 2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3) + 2^(1/3)*3^(1/2)*a^2*d*(-b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(-2*b^3)/(27*a^5*d^3)^(1/3) - (b*(a + b*x^3)^(1/3))/(3*a*(d*(a + b*x^3) - a*d))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^4+bx^7} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(1/3)/x**4/(-b*d*x**3+a*d),x)

[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**4 + b*x**7), x)/d

$$3.350 \quad \int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)} dx$$

Optimal. Leaf size=283

$$\frac{b^2 \log(a - bx^3)}{3 \cdot 2^{2/3} a^{8/3} d} + \frac{11b^2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{18a^{8/3} d} - \frac{b^2 \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3})}{2^{2/3} a^{8/3} d} - \frac{11b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{9\sqrt{3} a^{8/3} d} + \frac{\sqrt[3]{2} b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{9\sqrt{3} a^{8/3} d}$$

Rubi [A] time = 0.26, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28, number of rules / integrand size = 0.286, Rules used = {446, 103, 149, 156, 57, 617, 204, 31}

$$\frac{b^2 \log(a - bx^3)}{3 \cdot 2^{2/3} a^{8/3} d} + \frac{11b^2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{18a^{8/3} d} - \frac{b^2 \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3})}{2^{2/3} a^{8/3} d} - \frac{11b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{9\sqrt{3} a^{8/3} d} + \frac{\sqrt[3]{2} b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{8/3} d} - \frac{11b^2 \log(x)}{18a^{8/3} d} - \frac{2b\sqrt[3]{a+bx^3}}{9a^2 dx^3} - \frac{(a+bx^3)^{4/3}}{6a^2 dx^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)), x]

[Out] (-2*b*(a + b*x^3)^(1/3))/(9*a^2*d*x^3) - (a + b*x^3)^(4/3)/(6*a^2*d*x^6) - (11*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(8/3)*d) + (2^(1/3)*b^2*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(8/3)*d) - (11*b^2*Log[x])/(18*a^(8/3)*d) + (b^2*Log[a - b*x^3])/(3*2^(2/3)*a^(8/3)*d) + (11*b^2*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(8/3)*d) - (b^2*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*a^(8/3)*d)

Rule 31

Int[((a_) + (b_)*(x_))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 103

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m+1)*(c + d*x)^(n+1)*(e + f*x)^(p+1))/(b*(a + b*x)^(m+1)*(c + d*x)^(n+1)*(e + f*x)^(p+1)), x_Symbol]

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{x^3(ad-bdx)} dx, x, x^3 \right) \\
&= -\frac{(a+bx^3)^{4/3}}{6a^2 dx^6} - \frac{\text{Subst} \left(\int \frac{\sqrt[3]{a+bx} \left(-\frac{4}{3}abd - \frac{2}{3}b^2 dx \right)}{x^2(ad-bdx)} dx, x, x^3 \right)}{6a^2 d} \\
&= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2 dx^3} - \frac{(a+bx^3)^{4/3}}{6a^2 dx^6} - \frac{\text{Subst} \left(\int \frac{-\frac{22}{9}a^2 b^2 d^2 - \frac{14}{9}ab^3 d^2 x}{x(a+bx)^{2/3}(ad-bdx)} dx, x, x^3 \right)}{6a^3 d^2} \\
&= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2 dx^3} - \frac{(a+bx^3)^{4/3}}{6a^2 dx^6} + \frac{(2b^3) \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)} dx, x, x^3 \right)}{3a^2} + \frac{(11b^2) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, x^3 \right)}{18a^{8/3}d} \\
&= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2 dx^3} - \frac{(a+bx^3)^{4/3}}{6a^2 dx^6} - \frac{11b^2 \log(x)}{18a^{8/3}d} + \frac{b^2 \log(a-bx^3)}{3 \cdot 2^{2/3} a^{8/3}d} - \frac{(11b^2) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, x^3 \right)}{18a^{8/3}d} \\
&= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2 dx^3} - \frac{(a+bx^3)^{4/3}}{6a^2 dx^6} - \frac{11b^2 \log(x)}{18a^{8/3}d} + \frac{b^2 \log(a-bx^3)}{3 \cdot 2^{2/3} a^{8/3}d} + \frac{11b^2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{18a^{8/3}d} \\
&= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2 dx^3} - \frac{(a+bx^3)^{4/3}}{6a^2 dx^6} - \frac{11b^2 \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{9\sqrt{3} a^{8/3}d} + \frac{\sqrt[3]{2} b^2 \tan^{-1} \left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3} a^{8/3}d} - \frac{1}{18a^{8/3}d}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 314, normalized size = 1.11

$$\frac{11b^2 x^6 \log\left(x^{2/3} + \sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}\right) - 9\sqrt{2} b^2 x^6 \log\left(2^{2/3} x^{2/3} + \sqrt{2} \sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}\right) + 21a^{2/3} b x^3 \sqrt[3]{a+bx^3} + 9a^{5/3} \sqrt[3]{a+bx^3} - 22b^2 x^6 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right) + 18\sqrt{2} b^2 x^6 \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right) + 22\sqrt{3} b^2 x^6 \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 18\sqrt{3} b^2 x^6 \tan^{-1}\left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{54a^{8/3} dx^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)), x]

[Out] $-\frac{1}{54} \cdot (9a^{5/3})(a+bx^3)^{1/3} + 21a^{2/3} b x^3 (a+bx^3)^{1/3} + 2 \cdot 2 \cdot \text{Sqrt}[3] \cdot b^2 x^6 \cdot \text{ArcTan}\left[\frac{1 + (a+bx^3)^{1/3}}{a^{1/3}}\right] / \text{Sqrt}[3] - 18 \cdot 2^{1/3} \cdot \text{Sqrt}[3] \cdot b^2 x^6 \cdot \text{ArcTan}\left[\frac{1 + (2^{2/3})(a+bx^3)^{1/3}}{a^{1/3}}\right] / \text{Sqrt}[3] - 22 \cdot b^2 x^6 \cdot \text{Log}[a^{1/3} - (a+bx^3)^{1/3}] + 18 \cdot 2^{1/3} \cdot b^2 x^6 \cdot \text{Log}[2^{2/3} a^{1/3} - (a+bx^3)^{1/3}] + 11 \cdot b^2 x^6 \cdot \text{Log}[a^{2/3} + a^{1/3} (a+bx^3)^{1/3} + (a+bx^3)^{2/3}] - 9 \cdot 2^{1/3} \cdot b^2 x^6 \cdot \text{Log}[2^{2/3} a^{2/3} + 2^{1/3} a^{1/3} (a+bx^3)^{1/3} + (a+bx^3)^{2/3}] / (a^{8/3} d x^6)$

IntegrateAlgebraic [A] time = 0.72, size = 340, normalized size = 1.20

$$\frac{11b^2 \log(\sqrt{a+bx^3} - \sqrt{a})}{27a^{8/3}d} - \frac{\sqrt{2}b^2 \log(2^{2/3}\sqrt{a+bx^3} - 2\sqrt{a})}{3a^{8/3}d} - \frac{11b^2 \log(a^{2/3} + \sqrt{a}\sqrt{a+bx^3} + (a+bx^3)^{2/3})}{54a^{8/3}d} + \frac{b^2 \log(2a^{2/3} + 2^{2/3}\sqrt{a}\sqrt{a+bx^3} + \sqrt{2}(a+bx^3)^{2/3})}{3 \cdot 2^{2/3}a^{8/3}d} - \frac{11b^2 \tan^{-1}\left(\frac{2\sqrt{a+bx^3}}{\sqrt{3}\sqrt{a}} + \frac{1}{\sqrt{3}}\right)}{9\sqrt{3}a^{8/3}d} + \frac{\sqrt{2}b^2 \tan^{-1}\left(\frac{2^{2/3}\sqrt{a+bx^3}}{\sqrt{3}\sqrt{a}} + \frac{1}{\sqrt{3}}\right)}{\sqrt{3}a^{8/3}d} + \frac{(-3a-7bx^3)\sqrt{a+bx^3}}{18a^2dx^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)), x]

[Out]
$$\begin{aligned} &((-3*a - 7*b*x^3)*(a + b*x^3)^{(1/3)})/(18*a^2*d*x^6) - (11*b^2*ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(9*Sqrt[3]*a^{(8/3)*d}) + (2^{(1/3)}*b^2*ArcTan[1/Sqrt[3] + (2^{(2/3)}*(a + b*x^3)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(8/3)*d}) + (11*b^2*Log[-a^{(1/3)} + (a + b*x^3)^{(1/3)}])/(27*a^{(8/3)*d}) - (2^{(1/3)}*b^2*Log[-2*a^{(1/3)} + 2^{(2/3)}*(a + b*x^3)^{(1/3)}])/(3*a^{(8/3)*d}) - (11*b^2*Log[a^{(2/3)} + a^{(1/3)}*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}])/(54*a^{(8/3)*d}) + (b^2*Log[2*a^{(2/3)} + 2^{(2/3)}*a^{(1/3)}*(a + b*x^3)^{(1/3)} + 2^{(1/3)}*(a + b*x^3)^{(2/3)}])/(3*2^{(2/3)}*a^{(8/3)*d}) \end{aligned}$$

fricas [A] time = 0.64, size = 345, normalized size = 1.22

$$\frac{18\sqrt{3}b^2d^2x^6(-\frac{1}{3})^3 \arctan\left(\frac{1}{3}\sqrt{3}\frac{(bx^3+a)^{1/3}}{a^{1/3}} + \frac{1}{3}\sqrt{3}\right) + 9 \cdot 2^{1/3}b^2d^2x^6(-\frac{1}{3})^3 \log\left[2^{2/3}\frac{(bx^3+a)^{1/3}}{a^{1/3}} - 2^{1/3}\frac{(bx^3+a)^{1/3}}{a^{1/3}}\right] - 18 \cdot 2^{1/3}b^2d^2x^6(-\frac{1}{3})^3 \log\left[2^{2/3}\frac{(bx^3+a)^{1/3}}{a^{1/3}} + (bx^3+a)^{1/3}\right] + 22\sqrt{3}(a^{1/3})^2d^2x^6 \arctan\left(\frac{(a^{1/3})^2\sqrt{3}(bx^3+a)^{1/3}}{2a^{2/3}}\right) + 11(a^{1/3})^2d^2x^6 \log\left((bx^3+a)^{1/3} + (a^{1/3})^2 + (bx^3+a)^{1/3}(a^{1/3})^2\right) - 22(a^{1/3})^2d^2x^6 \log\left((bx^3+a)^{1/3} - (a^{1/3})^2\right) + 3(7a^2b^2x^3 + 3a^3)(bx^3+a)^{1/3}}{54a^6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d), x, algorithm="fricas")

[Out]
$$\begin{aligned} &-1/54*(18*\sqrt{3})^2*(1/3)*a^2*b^2*x^6*(-1/a^2)^{(1/3)}*\arctan(1/3*\sqrt{3})^2*(2/3)*(b*x^3 + a)^{(1/3)}*a*(-1/a^2)^{(2/3)} + 1/3*\sqrt{3}) + 9*2^{(1/3)}*a^2*b^2*x^6*(-1/a^2)^{(1/3)}*\log(2^{(2/3)}*a^2*(-1/a^2)^{(2/3)} - 2^{(1/3)}*(b*x^3 + a)^{(1/3)}*a*(-1/a^2)^{(1/3)} + (b*x^3 + a)^{(2/3)}) - 18*2^{(1/3)}*a^2*b^2*x^6*(-1/a^2)^{(1/3)}*\log(2^{(1/3)}*a*(-1/a^2)^{(1/3)} + (b*x^3 + a)^{(1/3)}) + 22*\sqrt{3}*(a^2)^{(1/6)}*a*b^2*x^6*\arctan(1/3*(a^2)^{(1/6)}*(\sqrt{3}*(a^2)^{(1/3)}*a + 2*\sqrt{3}*(b*x^3 + a)^{(1/3)}*(a^2)^{(2/3)})/a^2) + 11*(a^2)^{(2/3)}*b^2*x^6*\log((b*x^3 + a)^{(2/3)}*a + (a^2)^{(1/3)}*a + (b*x^3 + a)^{(1/3)}*(a^2)^{(2/3)}) - 22*(a^2)^{(2/3)}*b^2*x^6*\log((b*x^3 + a)^{(1/3)}*a - (a^2)^{(2/3)}) + 3*(7*a^2*b*x^3 + 3*a^3)*(b*x^3 + a)^{(1/3)}/(a^4*d*x^6) \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d), x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algebraic extensions not allowed in a rootofAlgebraic extensions not allowed in

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 a^(1/3)*1/27/a³/d*ln(abs((a+b*x³)^(1/3)-a^(1/3)))- (7*(a+b*x³)^(1/3)*(a+b
 *x³)*b²-4*(a+b*x³)^(1/3)*a*b²)/18/a²/d/(a+b*x³-a)²

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{(bx^3+a)^{\frac{1}{3}}}{(-bdx^3+ad)x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x³+a)^(1/3)/x⁷/(-b*d*x³+a*d), x)

[Out] $\text{int}((b*x^3+a)^{(1/3)}/x^7/(-b*d*x^3+a*d), x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(bdx^3 - ad)x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b*x^3+a)^{(1/3)}/x^7/(-b*d*x^3+a*d), x, \text{algorithm}="maxima")$

[Out] $-\text{integrate}((b*x^3 + a)^{(1/3)}/((b*d*x^3 - a*d)*x^7), x)$

mupad [B] time = 5.44, size = 490, normalized size = 1.73

$$\frac{\frac{\sqrt[3]{a+bx^3}}{(bdx^3-ad)x^7}}{\frac{\sqrt[3]{a+bx^3}}{(bdx^3-ad)x^7}} = \frac{\sqrt[3]{a+bx^3}}{(bdx^3-ad)x^7} \cdot \frac{(bdx^3-ad)x^7}{\sqrt[3]{a+bx^3}} = 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a + b*x^3)^{(1/3)}/(x^7*(a*d - b*d*x^3)), x)$

[Out] $((2*b^2*(a + b*x^3)^{(1/3)})/(9*a) - (7*b^2*(a + b*x^3)^{(4/3)})/(18*a^2))/(d*(a + b*x^3)^2 + a^2*d - 2*a*d*(a + b*x^3)) + (11*\log(b^2*(a + b*x^3)^{(1/3)} - a^3*d*(b^6/(a^8*d^3))^{(1/3)})*(b^6/(a^8*d^3))^{(1/3)})/27 + \log(b^2*(a + b*x^3)^{(1/3)} + 2^{(1/3)*a^3*d*(-b^6/(a^8*d^3))^{(1/3)})*(-(2*b^6)/(27*a^8*d^3))^{(1/3)} - \log(2^{(1/3)*a^3*d*(-b^6/(a^8*d^3))^{(1/3)} - 2*b^2*(a + b*x^3)^{(1/3)} + 2^{(1/3)*3^{(1/2)*a^3*d*(-b^6/(a^8*d^3))^{(1/3)*1i}}*((3^{(1/2)*1i})/2 + 1/2)*(-(2*b^6)/(27*a^8*d^3))^{(1/3)} + \log(2*b^2*(a + b*x^3)^{(1/3)} - 2^{(1/3)*a^3*d*(-b^6/(a^8*d^3))^{(1/3)} + 2^{(1/3)*3^{(1/2)*a^3*d*(-b^6/(a^8*d^3))^{(1/3)*1i}}*((3^{(1/2)*1i})/2 - 1/2)*(-(2*b^6)/(27*a^8*d^3))^{(1/3)} + (11*\log(2*b^2*(a + b*x^3)^{(1/3)} + a^3*d*(b^6/(a^8*d^3))^{(1/3)} - 3^{(1/2)*a^3*d*(b^6/(a^8*d^3))^{(1/3)})*1i)*(3^{(1/2)*1i} - 1)*(b^6/(a^8*d^3))^{(1/3)})/54 - (11*\log(2*b^2*(a + b*x^3)^{(1/3)} + a^3*d*(b^6/(a^8*d^3))^{(1/3)} + 3^{(1/2)*a^3*d*(b^6/(a^8*d^3))^{(1/3)})*1i)*(3^{(1/2)*1i} + 1)*(b^6/(a^8*d^3))^{(1/3)})/54$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^7+bx^{10}} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b*x**3+a)**(1/3)/x**7/(-b*d*x**3+a*d), x)$

[Out] $-\text{Integral}((a + b*x**3)**(1/3)/(-a*x**7 + b*x**10), x)/d$

$$3.351 \quad \int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx$$

Optimal. Leaf size=268

$$\frac{a^2 \log(ad - bdx^3)}{3 \cdot 2^{2/3} b^{8/3} d} + \frac{11a^2 \log(\sqrt[3]{b}x - \sqrt[3]{a+bx^3})}{18b^{8/3}d} - \frac{a^2 \log(\sqrt[3]{2} \sqrt[3]{b}x - \sqrt[3]{a+bx^3})}{2^{2/3} b^{8/3} d} + \frac{11a^2 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right)}{9\sqrt{3} b^{8/3} d} - \frac{\sqrt[3]{2} a^2}{\dots}$$

Rubi [C] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$\frac{x^8 \sqrt[3]{a+bx^3} F_1\left(\frac{8}{3}; -\frac{1}{3}, 1; \frac{11}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{8ad \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] (x^8*(a + b*x^3)^(1/3)*AppellF1[8/3, -1/3, 1, 11/3, -((b*x^3)/a), (b*x^3)/a])/ (8*a*d*(1 + (b*x^3)/a)^(1/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/ (e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^(m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx = \frac{\sqrt[3]{a+bx^3} \int \frac{x^7 \sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3} dx}{\sqrt[3]{1+\frac{bx^3}{a}}} = \frac{x^8 \sqrt[3]{a+bx^3} F_1\left(\frac{8}{3}; -\frac{1}{3}, 1; \frac{11}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{8ad \sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.21, size = 177, normalized size = 0.66

$$\frac{22abx^5 \left(1 - \frac{b^2x^6}{a^2}\right)^{2/3} F_1\left(\frac{5}{3}; \frac{2}{3}, 1; \frac{8}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) - 5x^2 \left(\left(1 - \frac{bx^3}{a}\right)^{2/3} (7a^2 + 10abx^3 + 3b^2x^6) - 7a^2 \left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{2bx^3}{a-bx^3}\right)\right)}{90b^2d(a+bx^3)^{2/3} \left(1 - \frac{bx^3}{a}\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] (22*a*b*x^5*(1 - (b^2*x^6)/a^2)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a] - 5*x^2*((1 - (b*x^3)/a)^(2/3)*(7*a^2 + 10*a*b*x^3 + 3*b^2*x^6) - 7*a^2*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^3)/(a - b*x^3)]))/(90*b^2*d*(a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))

IntegrateAlgebraic [A] time = 1.21, size = 364, normalized size = 1.36

$$\frac{11a^2 \log(\sqrt[3]{a+bx^3} - \sqrt[3]{bx^3})}{27b^3d} - \frac{\sqrt[3]{2} a^2 \log(2^{2/3} \sqrt[3]{a+bx^3} - 2\sqrt[3]{bx^3})}{36b^3d} + \frac{11a^2 \tan^{-1}\left(\frac{\sqrt[3]{5}\sqrt[3]{bx^3}}{2\sqrt[3]{a+bx^3} + \sqrt[3]{bx^3}}\right)}{9\sqrt[3]{5}b^3d} - \frac{\sqrt[3]{2} a^2 \tan^{-1}\left(\frac{\sqrt[3]{5}\sqrt[3]{bx^3}}{2\sqrt[3]{a+bx^3} + \sqrt[3]{bx^3}}\right)}{\sqrt[3]{5}b^3d} - \frac{11a^2 \log(\sqrt[3]{bx^3}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3} + b^{2/3}x^2)}{54b^3d} + \frac{a^2 \log(2^{2/3}\sqrt[3]{bx^3}\sqrt[3]{a+bx^3} + \sqrt[3]{2}(a+bx^3)^{2/3} + 2b^{2/3}x^2)}{3 \cdot 2^{2/3}b^3d} + \frac{\sqrt[3]{a+bx^3}(-7ax^2 - 3bx^5)}{18b^2d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] ((a + b*x^3)^(1/3)*(-7*a*x^2 - 3*b*x^5))/(18*b^2*d) + (11*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(9*Sqrt[3]*b^(8/3)*d) - (2^(1/3)*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))]/(Sqrt[3]*b^(8/3)*d) + (11*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(27*b^(8/3)*d) - (2^(1/3)*a^2*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*b^(8/3)*d) - (11*a^2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(8/3)*d) + (a^2*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(2/3)*b^(8/3)*d)

fricas [A] time = 0.64, size = 362, normalized size = 1.35

$$\frac{18\sqrt[3]{3}a^2b^2\left(-\frac{1}{b}\right) \arctan\left(\frac{\sqrt[3]{5}\sqrt[3]{bx^3}}{2\sqrt[3]{a+bx^3} + \sqrt[3]{bx^3}}\right) - 18 \cdot 2^{1/3}a^2b^2\left(-\frac{1}{b}\right) \log\left(\frac{2^{1/3}\sqrt[3]{bx^3}}{2\sqrt[3]{a+bx^3} + \sqrt[3]{bx^3}}\right) + 9 \cdot 2^{1/3}a^2b^2\left(-\frac{1}{b}\right) \log\left(\frac{2^{1/3}\sqrt[3]{bx^3}}{2\sqrt[3]{a+bx^3} + \sqrt[3]{bx^3}}\right) + 22\sqrt[3]{5}a^2b^2 \arctan\left(\frac{\sqrt[3]{5}\sqrt[3]{bx^3}}{2\sqrt[3]{a+bx^3} + \sqrt[3]{bx^3}}\right) - 22a^2b^2 \log\left(\frac{\sqrt[3]{5}\sqrt[3]{bx^3}}{2\sqrt[3]{a+bx^3} + \sqrt[3]{bx^3}}\right) + 11a^2b^2 \log\left(\frac{\sqrt[3]{5}\sqrt[3]{bx^3}}{2\sqrt[3]{a+bx^3} + \sqrt[3]{bx^3}}\right) + 3(3b^2x^6 + 7ab^2x^3)(bx^3 + a)^{2/3}}{54b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")`

[Out]
$$-1/54*(18*\sqrt{3})*2^{(1/3)}*a^2*b^2*(-1/b^2)^{(1/3)}*\arctan(1/3*(\sqrt{3})*2^{(2/3)}*(b*x^3 + a)^{(1/3)}*b*(-1/b^2)^{(2/3)} + \sqrt{3}*x)/x) - 18*2^{(1/3)}*a^2*b^2*(-1/b^2)^{(1/3)}*\log((2^{(1/3)}*b*x*(-1/b^2)^{(1/3)} + (b*x^3 + a)^{(1/3)})/x) + 9*2^{(1/3)}*a^2*b^2*(-1/b^2)^{(1/3)}*\log((2^{(2/3)}*b^2*x^2*(-1/b^2)^{(2/3)} - 2^{(1/3)}*(b*x^3 + a)^{(1/3)}*b*x*(-1/b^2)^{(1/3)} + (b*x^3 + a)^{(2/3)})/x^2) + 22*\sqrt{3})*a^2*(b^2)^{(1/6)}*b*\arctan(1/3*(\sqrt{3})*(b^2)^{(1/3)}*b*x + 2*\sqrt{3}*(b*x^3 + a)^{(1/3)}*(b^2)^{(2/3)})*(b^2)^{(1/6)}/(b^2*x)) - 22*a^2*(b^2)^{(2/3)}*\log(-((b^2)^{(2/3)}*x - (b*x^3 + a)^{(1/3)}*b)/x) + 11*a^2*(b^2)^{(2/3)}*\log(((b^2)^{(1/3)}*b*x^2 + (b*x^3 + a)^{(1/3)}*(b^2)^{(2/3)}*x + (b*x^3 + a)^{(2/3)}*b)/x^2) + 3*(3*b^3*x^5 + 7*a*b^2*x^2)*(b*x^3 + a)^{(1/3)})/(b^4*d)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{1}{3}}x^7}{bdx^3 - ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")`

[Out] `integrate(-(b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)`

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}x^7}{-bdx^3 + ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)`

[Out] `int(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{1}{3}}x^7}{bdx^3 - ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")`

[Out] -integrate((b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7 (bx^3 + a)^{1/3}}{ad - bdx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)

[Out] int((x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^7 \sqrt[3]{a+bx^3}}{-a+bx^3} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d), x)

[Out] -Integral(x**7*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d

$$3.352 \quad \int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx$$

Optimal. Leaf size=233

$$\frac{a \log(ad - bdx^3)}{3 \cdot 2^{2/3} b^{5/3} d} + \frac{2a \log(\sqrt[3]{b} x - \sqrt[3]{a + bx^3})}{3b^{5/3} d} - \frac{a \log(\sqrt[3]{2} \sqrt[3]{b} x - \sqrt[3]{a + bx^3})}{2^{2/3} b^{5/3} d} + \frac{4a \tan^{-1}\left(\frac{2\sqrt[3]{b} x + 1}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt{3} b^{5/3} d} - \frac{\sqrt[3]{2} a \tan^{-1}\left(\frac{2\sqrt[3]{b} x + 1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3} b^{5/3} d}$$

Rubi [C] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.28, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$\frac{x^5 \sqrt[3]{a + bx^3} F_1\left(\frac{5}{3}; -\frac{1}{3}, 1; \frac{8}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{5ad \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] (x^5*(a + b*x^3)^(1/3)*AppellF1[5/3, -1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a])/ (5*a*d*(1 + (b*x^3)/a)^(1/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/ (e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^(m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx = \frac{\sqrt[3]{a+bx^3} \int \frac{x^4 \sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3} dx}{\sqrt[3]{1+\frac{bx^3}{a}}} = \frac{x^5 \sqrt[3]{a+bx^3} F_1\left(\frac{5}{3}; -\frac{1}{3}, 1; \frac{8}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{5ad \sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.17, size = 160, normalized size = 0.69

$$\frac{4bx^5 \left(1 - \frac{b^2x^6}{a^2}\right)^{2/3} F_1\left(\frac{5}{3}; \frac{2}{3}, 1; \frac{8}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) - 5x^2 \left(\left(a+bx^3\right)\left(1 - \frac{bx^3}{a}\right)^{2/3} - a\left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{2bx^3}{a-bx^3}\right)\right)}{15bd \left(a+bx^3\right)^{2/3} \left(1 - \frac{bx^3}{a}\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] (4*b*x^5*(1 - (b^2*x^6)/a^2)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -(b*x^3)/a], (b*x^3)/a] - 5*x^2*((a + b*x^3)*(1 - (b*x^3)/a)^(2/3) - a*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^3)/(a - b*x^3)]))/(15*b*d*(a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))

IntegrateAlgebraic [A] time = 0.87, size = 342, normalized size = 1.47

$$\frac{4a \log(\sqrt[3]{a+bx^3} - \sqrt[3]{bx^3})}{9b^{5/3}d} - \frac{\sqrt[3]{2} a \log(2^{2/3}\sqrt[3]{a+bx^3} - 2\sqrt[3]{bx^3})}{3b^{5/3}d} + \frac{4a \tan^{-1}\left(\frac{\sqrt[3]{3}\sqrt[3]{bx^3}}{2\sqrt[3]{a+bx^3} + \sqrt[3]{bx^3}}\right)}{3\sqrt[3]{b^{5/3}d}} - \frac{\sqrt[3]{2} a \tan^{-1}\left(\frac{\sqrt[3]{3}\sqrt[3]{bx^3}}{2^{2/3}\sqrt[3]{a+bx^3} + \sqrt[3]{bx^3}}\right)}{\sqrt[3]{b^{5/3}d}} - \frac{2a \log(\sqrt[3]{bx^3}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3} + b^{2/3}x^2)}{9b^{5/3}d} + \frac{a \log(2^{2/3}\sqrt[3]{bx^3}\sqrt[3]{a+bx^3} + \sqrt[3]{2}(a+bx^3)^{2/3} + 2b^{2/3}x^2)}{3 \cdot 2^{2/3}b^{5/3}d} - \frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] -1/3*(x^2*(a + b*x^3)^(1/3))/(b*d) + (4*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(3*Sqrt[3]*b^(5/3)*d) - (2^(1/3)*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))]/(Sqrt[3]*b^(5/3)*d) + (4*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(9*b^(5/3)*d) - (2^(1/3)*a*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)]/(3*b^(5/3)*d) - (2*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(9*b^(5/3)*d) + (a*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]/(3*2^(2/3)*b^(5/3)*d)

fricas [A] time = 0.63, size = 338, normalized size = 1.45

$$\frac{6\sqrt{3}a^{\frac{1}{3}}\left(-\frac{1}{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{1}{2}\right)^{\frac{1}{3}}+\sqrt{3}}{3}\right)-6\cdot 2^{\frac{1}{3}}ab^2\left(-\frac{1}{2}\right)^{\frac{1}{3}}\log\left(\frac{\left(\frac{2}{3}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}}{3}\right)+3\cdot 2^{\frac{1}{3}}ab^2\left(-\frac{1}{2}\right)^{\frac{1}{3}}\log\left(\frac{\left(\frac{2}{3}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{1}{2}\right)^{\frac{1}{3}}+\sqrt{3}}{3}\right)+6\left(bx^3+a\right)^{\frac{1}{3}}b^2x^2+8\sqrt{3}a\left(b^2\right)^{\frac{1}{3}}b\arctan\left(\frac{\left(\sqrt{3}\left(b^2\right)^{\frac{1}{3}}bx+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(b^2\right)^{\frac{1}{3}}\right)^{\frac{1}{3}}}{3b^2}\right)-8a\left(b^2\right)^{\frac{1}{3}}\log\left(\frac{\left(b^2\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}}{3}\right)+4a\left(b^2\right)^{\frac{1}{3}}\log\left(\frac{\left(b^2\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\left(b^2\right)^{\frac{1}{3}}}{3}\right)}{18bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out]
$$-1/18*(6*\sqrt{3})*2^{(1/3)}*a*b^2*(-1/b^2)^{(1/3)}*\arctan(1/3*(\sqrt{3})*2^{(2/3)}*(b*x^3 + a)^{(1/3)}*b*(-1/b^2)^{(2/3)} + \sqrt{3}*x)/x - 6*2^{(1/3)}*a*b^2*(-1/b^2)^{(1/3)}*\log((2^{(1/3)}*b*x*(-1/b^2)^{(1/3)} + (b*x^3 + a)^{(1/3)})/x) + 3*2^{(1/3)}*a*b^2*(-1/b^2)^{(1/3)}*\log((2^{(2/3)}*b^2*x^2*(-1/b^2)^{(2/3)} - 2^{(1/3)}*(b*x^3 + a)^{(1/3)}*b*x*(-1/b^2)^{(1/3)} + (b*x^3 + a)^{(2/3)})/x^2) + 6*(b*x^3 + a)^{(1/3)}*b^2*x^2 + 8*\sqrt{3}*a*(b^2)^{(1/6)}*b*\arctan(1/3*(\sqrt{3}*(b^2)^{(1/3)}*b*x + 2*\sqrt{3}*(b*x^3 + a)^{(1/3)}*(b^2)^{(2/3)})*(b^2)^{(1/6)}/(b^2*x)) - 8*a*(b^2)^{(2/3)}*\log(-((b^2)^{(2/3)}*x - (b*x^3 + a)^{(1/3)}*b)/x) + 4*a*(b^2)^{(2/3)}*\log(((b^2)^{(1/3)}*b*x^2 + (b*x^3 + a)^{(1/3)}*(b^2)^{(2/3)}*x + (b*x^3 + a)^{(2/3)}*b/x^2))/(b^3*d)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{1}{3}}x^4}{bdx^3 - ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")

[Out] integrate(-(b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}x^4}{-bdx^3 + ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)

[Out] int(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{1}{3}}x^4}{bdx^3 - ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 (bx^3 + a)^{1/3}}{ad - bdx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)

[Out] int((x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^4 \sqrt[3]{a+bx^3}}{-a+bx^3} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)

[Out] -Integral(x**4*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d

$$3.353 \quad \int \frac{x \sqrt[3]{a+bx^3}}{ad-bdx^3} dx$$

Optimal. Leaf size=201

$$\frac{\log(ad-bdx^3)}{3 \cdot 2^{2/3} b^{2/3} d} + \frac{\log(\sqrt[3]{b}x - \sqrt[3]{a+bx^3})}{2b^{2/3}d} - \frac{\log(\sqrt[3]{2} \sqrt[3]{b}x - \sqrt[3]{a+bx^3})}{2^{2/3} b^{2/3} d} + \frac{\tan^{-1}\left(\frac{2 \sqrt[3]{b}x}{\sqrt[3]{a+bx^3}+1}\right)}{\sqrt{3} b^{2/3} d} - \frac{\sqrt[3]{2} \tan^{-1}\left(\frac{2 \sqrt[3]{2} \sqrt[3]{b}x}{\sqrt[3]{a+bx^3}+\sqrt{3}}\right)}{\sqrt{3} b^{2/3} d}$$

Rubi [C] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 0.33, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {511, 510}

$$\frac{x^2 \sqrt[3]{a+bx^3} F_1\left(\frac{2}{3}; -\frac{1}{3}, 1; \frac{5}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{2ad \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] (x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a])/ (2*a*d*(1 + (b*x^3)/a)^(1/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^(m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x \sqrt[3]{a + bx^3}}{ad - bdx^3} dx = \frac{\sqrt[3]{a + bx^3} \int \frac{x \sqrt[3]{1 + \frac{bx^3}{a}}}{ad - bdx^3} dx}{\sqrt[3]{1 + \frac{bx^3}{a}}}$$

$$= \frac{x^2 \sqrt[3]{a + bx^3} F_1\left(\frac{2}{3}; -\frac{1}{3}, 1; \frac{5}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{2ad \sqrt[3]{1 + \frac{bx^3}{a}}}$$

Mathematica [C] time = 0.04, size = 63, normalized size = 0.31

$$\frac{x^2 \left(\frac{bx^3}{a} + 1\right)^{2/3} F_1\left(\frac{2}{3}; -\frac{1}{3}, 1; \frac{5}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{2d \left(a + bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] (x^2*(1 + (b*x^3)/a)^(2/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a])/(2*d*(a + b*x^3)^(2/3))

IntegrateAlgebraic [A] time = 0.60, size = 309, normalized size = 1.54

$$\frac{\log\left(\sqrt[3]{a + bx^3} - \sqrt[3]{b}x\right)}{3b^{2/3}d} - \frac{\sqrt[3]{2} \log\left(2^{2/3}\sqrt[3]{a + bx^3} - 2\sqrt[3]{b}x\right)}{3b^{2/3}d} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}\sqrt[3]{bx^3}}{2\sqrt[3]{a + bx^3} + \sqrt[3]{b}x}\right)}{\sqrt[3]{3}b^{2/3}d} - \frac{\sqrt[3]{2} \tan^{-1}\left(\frac{\sqrt[3]{3}\sqrt[3]{bx^3}}{2^{2/3}\sqrt[3]{a + bx^3} + \sqrt[3]{b}x}\right)}{\sqrt[3]{3}b^{2/3}d} - \frac{\log\left(\sqrt[3]{b}x\sqrt[3]{a + bx^3} + (a + bx^3)^{2/3} + b^{2/3}x^2\right)}{6b^{2/3}d} + \frac{\log\left(2^{2/3}\sqrt[3]{b}x\sqrt[3]{a + bx^3} + \sqrt[3]{2}(a + bx^3)^{2/3} + 2b^{2/3}x^2\right)}{3 \cdot 2^{2/3}b^{2/3}d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x]

[Out] ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(Sqrt[3]*b^(2/3)*d) - (2^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))]/(Sqrt[3]*b^(2/3)*d) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(3*b^(2/3)*d) - (2^(1/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)]/(3*b^(2/3)*d) - Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(6*b^(2/3)*d) + Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]/(3*2^(2/3)*b^(2/3)*d)

fricas [B] time = 0.62, size = 313, normalized size = 1.56

$$\frac{2\sqrt[3]{2}b^{1/3} \arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{(a+bx^3)^{1/3}\left(\frac{1}{3}\right)^{1/3} + \sqrt[3]{b}x}}{3x}\right) - 2 \cdot 2^{1/3}b^{1/3} \arctan\left(\frac{2^{1/3}\sqrt[3]{(a+bx^3)^{1/3}\left(\frac{1}{3}\right)^{1/3}}}{x}\right) + 2^{1/3}b^{1/3} \log\left(\frac{2^{1/3}\sqrt[3]{(a+bx^3)^{1/3}\left(\frac{1}{3}\right)^{1/3}}}{x}\right) + 2\sqrt[3]{2}b^{1/3} \arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{(a+bx^3)^{1/3}\left(\frac{1}{3}\right)^{1/3}}}{3x}\right) - 2\left(\frac{1}{3}\right)^{1/3} \log\left(\frac{(a+bx^3)^{1/3}}{x}\right) + (a+bx^3)^{1/3} \log\left(\frac{(a+bx^3)^{1/3}}{x}\right) + (a+bx^3)^{1/3} \log\left(\frac{(a+bx^3)^{1/3}}{x}\right)}{6b^{2/3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out]
$$-1/6*(2*\sqrt{3})*2^{(1/3)}*b^2*(-1/b^2)^{(1/3)}*\arctan(1/3*(\sqrt{3})*2^{(2/3)}*(b*x^3 + a)^{(1/3)}*b*(-1/b^2)^{(2/3)} + \sqrt{3}*x)/x) - 2*2^{(1/3)}*b^2*(-1/b^2)^{(1/3)}* \log((2^{(1/3)}*b*x*(-1/b^2)^{(1/3)} + (b*x^3 + a)^{(1/3)})/x) + 2^{(1/3)}*b^2*(-1/b^2)^{(1/3)}* \log((2^{(2/3)}*b^2*x^2*(-1/b^2)^{(2/3)} - 2^{(1/3)}*(b*x^3 + a)^{(1/3)})*b*x*(-1/b^2)^{(1/3)} + (b*x^3 + a)^{(2/3)})/x^2) + 2*\sqrt{3}*(b^2)^{(1/6)}*b*\arctan(1/3*(\sqrt{3})*(b^2)^{(1/3)}*b*x + 2*\sqrt{3}*(b*x^3 + a)^{(1/3)}*(b^2)^{(2/3)})*(b^2)^{(1/6)}/(b^2*x)) - 2*(b^2)^{(2/3)}*\log(-((b^2)^{(2/3)}*x - (b*x^3 + a)^{(1/3)}*b)/x) + (b^2)^{(2/3)}*\log(((b^2)^{(1/3)}*b*x^2 + (b*x^3 + a)^{(1/3)}*(b^2)^{(2/3)}*x + (b*x^3 + a)^{(2/3)}*b)/x^2))/(b^2*d)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{1}{3}}x}{bdx^3 - ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")

[Out] integrate(-(b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}x}{-bdx^3 + ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)

[Out] int(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{1}{3}}x}{bdx^3 - ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x(bx^3 + a)^{1/3}}{ad - bdx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)

[Out] int((x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x\sqrt[3]{a+bx^3}}{-a+bx^3} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d), x)

[Out] -Integral(x*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d

$$3.354 \quad \int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)} dx$$

Optimal. Leaf size=156

$$\frac{\sqrt[3]{a+bx^3}}{adx} + \frac{\sqrt[3]{b} \log(ad-bdx^3)}{3 \cdot 2^{2/3} ad} - \frac{\sqrt[3]{b} \log\left(\sqrt[3]{2} \sqrt[3]{b} x - \sqrt[3]{a+bx^3}\right)}{2^{2/3} ad} - \frac{\sqrt[3]{2} \sqrt[3]{b} \tan^{-1}\left(\frac{2 \sqrt[3]{2} \sqrt[3]{b} x + 1}{\frac{\sqrt[3]{a+bx^3}}{\sqrt{3}}}\right)}{\sqrt{3} ad}$$

Rubi [C] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 0.49, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$\frac{\sqrt[3]{a+bx^3} \sqrt[3]{1-\frac{bx^3}{a}} {}_2F_1\left(-\frac{1}{3}, -\frac{1}{3}; \frac{2}{3}; -\frac{2bx^3}{a-bx^3}\right)}{adx \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)), x]

[Out] -(((a + b*x^3)^(1/3)*(1 - (b*x^3)/a)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, (-2*b*x^3)/(a - b*x^3)])/(a*d*x*(1 + (b*x^3)/a)^(1/3)))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -(b*x^n)/a, -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^(m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)} dx = \frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^2(ad-bdx^3)} dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{\sqrt[3]{a+bx^3} \sqrt[3]{1-\frac{bx^3}{a}} {}_2F_1\left(-\frac{1}{3}, -\frac{1}{3}; \frac{2}{3}; -\frac{2bx^3}{a-bx^3}\right)}{adx \sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.29

$$-\frac{\sqrt[3]{a+bx^3} {}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{2bx^3}{bx^3+a}\right)}{adx}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)), x]

[Out] -(((a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 1, 2/3, (2*b*x^3)/(a + b*x^3)])/(a*d*x))

IntegrateAlgebraic [A] time = 0.43, size = 205, normalized size = 1.31

$$\frac{\sqrt[3]{b} \log\left(2^{2/3} \sqrt[3]{b} x \sqrt[3]{a+bx^3} + \sqrt[3]{2} (a+bx^3)^{2/3} + 2b^{2/3}x^2\right)}{3 \cdot 2^{2/3} ad} - \frac{\sqrt[3]{a+bx^3}}{adx} - \frac{\sqrt[3]{2} \sqrt[3]{b} \log\left(2^{2/3} \sqrt[3]{a+bx^3} - 2\sqrt[3]{b}x\right)}{3ad} - \frac{\sqrt[3]{2} \sqrt[3]{b} \tan^{-1}\left(\frac{\sqrt[3]{3} \sqrt[3]{b} x}{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{b}x}\right)}{\sqrt[3]{3} ad}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)), x]

[Out] -((a + b*x^3)^(1/3)/(a*d*x)) - (2^(1/3)*b^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(Sqrt[3]*a*d) - (2^(1/3)*b^(1/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)]/(3*a*d) + (b^(1/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]/(3*2^(2/3)*a*d))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{1}{3}}}{(bdx^3 - ad)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="giac")

[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)

maple [F] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(-bdx^3 + ad)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x)

[Out] int((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(bdx^3 - ad)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^3 + a)^{1/3}}{x^2 (ad - bdx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)), x)
```

```
[Out] int((a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^2+bx^5} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**2/(-b*d*x**3+a*d), x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**2 + b*x**5), x)/d
```

$$3.355 \quad \int \frac{\sqrt[3]{a+bx^3}}{x^5(ad-bdx^3)} dx$$

Optimal. Leaf size=183

$$\frac{b^{4/3} \log(ad-bdx^3)}{3 \cdot 2^{2/3} a^2 d} - \frac{b^{4/3} \log\left(\sqrt[3]{2} \sqrt[3]{b} x - \sqrt[3]{a+bx^3}\right)}{2^{2/3} a^2 d} - \frac{\sqrt[3]{2} b^{4/3} \tan^{-1}\left(\frac{2 \sqrt[3]{2} \sqrt[3]{b} x + 1}{\frac{\sqrt[3]{a+bx^3}}{\sqrt{3}}}\right)}{\sqrt{3} a^2 d} - \frac{5b \sqrt[3]{a+bx^3}}{4a^2 dx} - \frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

Rubi [C] time = 0.42, antiderivative size = 117, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$\frac{a^2 - bx^3 \left(a + 3bx^3\right) {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{2bx^3}{bx^3+a}\right) + 3bx^3 \left(a - bx^3\right) {}_2F_1\left(\frac{2}{3}, 2; \frac{5}{3}; \frac{2bx^3}{bx^3+a}\right) + 4abx^3 + 3b^2x^6}{4a^2dx^4 \left(a + bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]

[Out] -(a^2 + 4*a*b*x^3 + 3*b^2*x^6 - b*x^3*(a + 3*b*x^3)*Hypergeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] + 3*b*x^3*(a - b*x^3)*Hypergeometric2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)])/(4*a^2*d*x^4*(a + b*x^3)^(2/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -(b*x^n)/a, -(d*x^n)/c]]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5(ad-bdx^3)} dx = \frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^5(ad-bdx^3)} dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{a^2 + 4abx^3 + 3b^2x^6 - bx^3(a+3bx^3) {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{2bx^3}{a+bx^3}\right) + 3bx^3(a-bx^3) {}_2F_1\left(\frac{2}{3}, 2; \frac{5}{3}; \frac{2bx^3}{a+bx^3}\right)}{4a^2dx^4(a+bx^3)^{2/3}}$$

Mathematica [C] time = 5.10, size = 125, normalized size = 0.68

$$\frac{b^2x^2 \left(\frac{a+bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{2bx^3}{a\left(1-\frac{bx^3}{a}\right)}\right)}{a^2d(a+bx^3)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}} - \frac{\left(\frac{5b}{4a^2x} + \frac{1}{4ax^4}\right) \sqrt[3]{a+bx^3}}{d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)), x]

[Out] -(((1/(4*a*x^4) + (5*b)/(4*a^2*x))*(a + b*x^3)^(1/3))/d) + (b^2*x^2*((a + b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^3)/(a*(1 - (b*x^3)/a))])/(a^2*d*(a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))

IntegrateAlgebraic [A] time = 0.48, size = 217, normalized size = 1.19

$$-\frac{\sqrt[3]{2}b^{4/3} \log\left(2^{2/3}\sqrt[3]{a+bx^3} - 2\sqrt[3]{bx}\right)}{3a^2d} - \frac{\sqrt[3]{2}b^{4/3} \tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{bx}}{2^{2/3}\sqrt[3]{a+bx^3} + \sqrt[3]{bx}}\right)}{\sqrt{3}a^2d} + \frac{b^{4/3} \log\left(2^{2/3}\sqrt[3]{bx}\sqrt[3]{a+bx^3} + \sqrt[3]{2}(a+bx^3)^{2/3} + 2b^{2/3}x^2\right)}{3 \cdot 2^{2/3}a^2d} + \frac{(-a-5bx^3)\sqrt[3]{a+bx^3}}{4a^2dx^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)), x]

[Out] ((-a - 5*b*x^3)*(a + b*x^3)^(1/3))/(4*a^2*d*x^4) - (2^(1/3)*b^(4/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(Sqrt[3]*a^2*d) - (2^(1/3)*b^(4/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a^2*d) + (b^(4/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(2/3)*a^2*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="fricas")`

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{1}{3}}}{(bdx^3 - ad)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="giac")`

[Out] `integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)`

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(-bdx^3 + ad)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x)`

[Out] `int((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(bdx^3 - ad)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="maxima")`

[Out] `-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{x^5 (ad - bdx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)), x)`

[Out] `int((a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^5+bx^8} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(1/3)/x**5/(-b*d*x**3+a*d), x)`

[Out] `-Integral((a + b*x**3)**(1/3)/(-a*x**5 + b*x**8), x)/d`

$$3.356 \quad \int \frac{\sqrt[3]{a+bx^3}}{x^8(ad-bdx^3)} dx$$

Optimal. Leaf size=210

$$\frac{b^{7/3} \log(ad - bdx^3)}{3 \cdot 2^{2/3} a^3 d} - \frac{b^{7/3} \log\left(\sqrt[3]{2} \sqrt[3]{b} x - \sqrt[3]{a + bx^3}\right)}{2^{2/3} a^3 d} - \frac{\sqrt[3]{2} b^{7/3} \tan^{-1}\left(\frac{2 \sqrt[3]{2} \sqrt[3]{b} x + 1}{\frac{\sqrt[3]{a+bx^3}}{\sqrt{3}}}\right)}{\sqrt{3} a^3 d} - \frac{8b^2 \sqrt[3]{a + bx^3}}{7a^3 dx} - \frac{2b \sqrt[3]{a + bx^3}}{7a^2 dx^4}$$

Rubi [C] time = 19.78, antiderivative size = 244, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$\frac{-9bx^3(a-bx^3)^2 {}_3F_2\left(\frac{2}{3}, 2, 2; 1, \frac{5}{3}; \frac{2bx^3}{bx^3+a}\right) - 2bx^3(2a^2 + 3abx^3 + 9b^2x^6) {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{2bx^3}{bx^3+a}\right) + 15a^2bx^3 {}_2F_1\left(\frac{2}{3}, 2; \frac{5}{3}; \frac{2bx^3}{bx^3+a}\right) + 10a^2bx^3 + 4a^3 - 27b^3x^9 {}_2F_1\left(\frac{2}{3}, 2; \frac{5}{3}; \frac{2bx^3}{bx^3+a}\right) + 12ab^2x^6 {}_2F_1\left(\frac{2}{3}, 2; \frac{5}{3}; \frac{2bx^3}{bx^3+a}\right) + 24ab^2x^6 + 18b^3x^9}{28a^3dx^7(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)), x]

[Out] $-(4a^3 + 10a^2bx^3 + 24a^2b^2x^6 + 18b^3x^9 - 2bx^3(2a^2 + 3a^2b^2x^3 + 9b^2x^6) \text{Hypergeometric2F1}[2/3, 1, 5/3, (2bx^3)/(a + bx^3)] + 15a^2b^2x^3 \text{Hypergeometric2F1}[2/3, 2, 5/3, (2bx^3)/(a + bx^3)] + 12a^2b^2x^6 \text{Hypergeometric2F1}[2/3, 2, 5/3, (2bx^3)/(a + bx^3)] - 27b^3x^9 \text{Hypergeometric2F1}[2/3, 2, 5/3, (2bx^3)/(a + bx^3)] - 9bx^3(a - bx^3)^2 \text{HypergeometricPFQ}[\{2/3, 2, 2\}, \{1, 5/3\}, (2bx^3)/(a + bx^3)]) / (28a^3d x^7(a + bx^3)^{2/3})$

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -(b*x^n)/a, -(d*x^n)/c])/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8(ad-bdx^3)} dx = \frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^8(ad-bdx^3)} dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{4a^3 + 10a^2bx^3 + 24ab^2x^6 + 18b^3x^9 - 2bx^3(2a^2 + 3abx^3 + 9b^2x^6) {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{2bx^3}{a+bx^3}\right) + 1}{7a^3dx^7(a+bx^3)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 5.12, size = 135, normalized size = 0.64

$$\frac{7b^3x^9\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{2bx^3}{a-bx^3}\right) - \left(1-\frac{bx^3}{a}\right)^{2/3} (a^3+3a^2bx^3+10ab^2x^6+8b^3x^9)}{7a^3dx^7(a+bx^3)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)), x]

[Out] (-((1 - (b*x^3)/a)^(2/3)*(a^3 + 3*a^2*b*x^3 + 10*a*b^2*x^6 + 8*b^3*x^9)) + 7*b^3*x^9*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^3)/(a - b*x^3)])/(7*a^3*d*x^7*(a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))

IntegrateAlgebraic [A] time = 0.52, size = 228, normalized size = 1.09

$$\frac{\sqrt[3]{2} b^{7/3} \log\left(2^{2/3} \sqrt[3]{a+bx^3} - 2\sqrt[3]{bx}\right)}{3a^3d} - \frac{\sqrt[3]{2} b^{7/3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[3]{bx}}{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{bx}}\right)}{\sqrt{3} a^3d} + \frac{b^{7/3} \log\left(2^{2/3} \sqrt[3]{bx} \sqrt[3]{a+bx^3} + \sqrt[3]{2} (a+bx^3)^{2/3} + 2b^{2/3}x^2\right)}{3 \cdot 2^{2/3} a^3d} + \frac{\sqrt[3]{a+bx^3} (-a^2 - 2abx^3 - 8b^2x^6)}{7a^3dx^7}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)), x]

[Out] ((a + b*x^3)^(1/3)*(-a^2 - 2*a*b*x^3 - 8*b^2*x^6))/(7*a^3*d*x^7) - (2^(1/3)*b^(7/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(Sqrt[3]*a^3*d) - (2^(1/3)*b^(7/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a^3*d) + (b^(7/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(2/3)*a^3*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="fricas")`

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{1}{3}}}{(bdx^3 - ad)x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="giac")`

[Out] `integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)`

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(-bdx^3 + ad)x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x)`

[Out] `int((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(bdx^3 - ad)x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="maxima")`

[Out] `-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{x^8 (ad - bdx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)), x)`

[Out] `int((a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^8+bx^{11}} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(1/3)/x**8/(-b*d*x**3+a*d), x)`

[Out] `-Integral((a + b*x**3)**(1/3)/(-a*x**8 + b*x**11), x)/d`

$$3.357 \quad \int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)} dx$$

Optimal. Leaf size=237

$$\frac{b^{10/3} \log(ad - bdx^3)}{3 \cdot 2^{2/3} a^4 d} - \frac{b^{10/3} \log\left(\sqrt[3]{2} \sqrt[3]{b} x - \sqrt[3]{a + bx^3}\right)}{2^{2/3} a^4 d} - \frac{\sqrt[3]{2} b^{10/3} \tan^{-1}\left(\frac{2\sqrt[3]{2} \sqrt[3]{b} x + 1}{\sqrt[3]{\frac{a+bx^3}{3}}}\right)}{\sqrt{3} a^4 d} - \frac{169b^3 \sqrt[3]{a + bx^3}}{140a^4 dx} - \frac{37b^2 \sqrt[3]{a - bdx^3}}{140a^3 d}$$

Rubi [C] time = 29.21, antiderivative size = 423, normalized size of antiderivative = 1.78, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$\frac{-54b^3(a-b)^2(2a+3bx)^2 F_2\left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right) + 27b^3(a-b)^2 F_2\left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right) - 36a^2b^2 F_1\left(\frac{2}{3}, \frac{2}{3}\right) + 99a^2b^2 F_1\left(\frac{2}{3}, \frac{2}{3}\right) + 99a^2b^2 F_1\left(\frac{2}{3}, \frac{2}{3}\right) - 28a^2b^2 F_1\left(\frac{2}{3}, \frac{2}{3}\right) + 117a^2b^2 F_1\left(\frac{2}{3}, \frac{2}{3}\right) + 64a^2b^2 + 28a^2 - 162a^{1/2} F_1\left(\frac{2}{3}, \frac{2}{3}\right) - 297a^{1/2} F_1\left(\frac{2}{3}, \frac{2}{3}\right) - 54a^{1/2} F_1\left(\frac{2}{3}, \frac{2}{3}\right) + 81a^{1/2} F_1\left(\frac{2}{3}, \frac{2}{3}\right) + 216a^{1/2} + 162a^{1/2}}{280a^4 \sqrt[3]{(a+bx^3)^2}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]

[Out] $-(28a^4 + 64a^3bx^3 + 90a^2b^2x^6 + 216a^2b^3x^9 + 162b^4x^{12} - 28a^3bx^3 \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(2bx^3)}{(a+bx^3)}\right] - 36a^2b^2x^6 \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(2bx^3)}{(a+bx^3)}\right] - 54a^2b^3x^9 \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(2bx^3)}{(a+bx^3)}\right] - 162b^4x^{12} \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(2bx^3)}{(a+bx^3)}\right] + 117a^3bx^3 \text{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(2bx^3)}{(a+bx^3)}\right] + 99a^2b^2x^6 \text{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(2bx^3)}{(a+bx^3)}\right] + 81a^2b^3x^9 \text{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(2bx^3)}{(a+bx^3)}\right] - 297b^4x^{12} \text{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(2bx^3)}{(a+bx^3)}\right] - 54b^2x^3(a-bx^3)^2(2a+3bx^3) \text{HypergeometricPFQ}\left[\left\{\frac{2}{3}, 2, 2\right\}, \{1, \frac{5}{3}\}, \frac{(2bx^3)}{(a+bx^3)}\right] + 27b^2x^3(a-bx^3)^3 \text{HypergeometricPFQ}\left[\left\{\frac{2}{3}, 2, 2, 2\right\}, \{1, 1, \frac{5}{3}\}, \frac{(2bx^3)}{(a+bx^3)}\right]) / (280a^4d^2x^{10}(a+bx^3)^{2/3})$

Rule 510

Int[((e_.)*(x_))^(m_)*((a_)+(b_)*(x_)^(n_))^(p_)*((c_)+(d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1+(m+1)/n, -(b*x^n)/a, -(d*x^n)/c])/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_)*((a_)+(b_)*(x_)^(n_))^(p_)*((c_)+(d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a+b*x^n)^FracPart[p])/(1+(b*x^n)^p), x]

$\int (e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x] /;$
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)} dx = \frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^{11}(ad-bdx^3)} dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{28a^4 + 64a^3bx^3 + 90a^2b^2x^6 + 216ab^3x^9 + 162b^4x^{12} - 28a^3bx^3 {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{2bx^3}{a+bx^3}\right) - 36a^4}{140a^4dx^{10}(a+bx^3)^{2/3}}$$

Mathematica [C] time = 5.20, size = 130, normalized size = 0.55

$$\frac{-14a^4 - 36a^3bx^3 - 59a^2b^2x^6 + \frac{140b^4x^{12}\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{2bx^3}{a-bx^3}\right)}{\left(1-\frac{bx^3}{a}\right)^{2/3}} - 206ab^3x^9 - 169b^4x^{12}}{140a^4dx^{10}(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)), x]

[Out] (-14*a^4 - 36*a^3*b*x^3 - 59*a^2*b^2*x^6 - 206*a*b^3*x^9 - 169*b^4*x^12 + (140*b^4*x^12*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^3)/(a - b*x^3)])/(1 - (b*x^3)/a)^(2/3))/(140*a^4*d*x^10*(a + b*x^3)^(2/3))

IntegrateAlgebraic [A] time = 0.57, size = 239, normalized size = 1.01

$$\frac{\sqrt[3]{2} b^{10/3} \log\left(2^{2/3} \sqrt[3]{a+bx^3} - 2\sqrt[3]{b} x\right)}{3a^4d} - \frac{\sqrt[3]{2} b^{10/3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[3]{b} x}{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{b} x}\right)}{\sqrt{3} a^4d} + \frac{b^{10/3} \log\left(2^{2/3} \sqrt[3]{b} x \sqrt[3]{a+bx^3} + \sqrt[3]{2} (a+bx^3)^{2/3} + 2b^{2/3}x^2\right)}{3 \cdot 2^{2/3} a^4d} + \frac{\sqrt[3]{a+bx^3} (-14a^3 - 22a^2bx^3 - 37ab^2x^6 - 169b^3x^9)}{140a^4dx^{10}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)), x]

[Out] ((a + b*x^3)^(1/3)*(-14*a^3 - 22*a^2*b*x^3 - 37*a*b^2*x^6 - 169*b^3*x^9))/(140*a^4*d*x^10) - (2^(1/3)*b^(10/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x +

$$\frac{2^{2/3} \cdot (a + b \cdot x^3)^{1/3}}{\sqrt[3]{a^4 \cdot d}} - \frac{2^{1/3} \cdot b^{10/3} \cdot \log[-2 \cdot b^{1/3} \cdot x + 2^{2/3} \cdot (a + b \cdot x^3)^{1/3}]}{3 \cdot a^4 \cdot d} + \frac{b^{10/3} \cdot \log[2 \cdot b^{2/3} \cdot x^2 + 2^{2/3} \cdot b^{1/3} \cdot x \cdot (a + b \cdot x^3)^{1/3} + 2^{1/3} \cdot (a + b \cdot x^3)^{2/3}]}{3 \cdot 2^{2/3} \cdot a^4 \cdot d}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{1}{3}}}{(bdx^3 - ad)x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="giac")

[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(-bdx^3 + ad)x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x)

[Out] int((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(bdx^3 - ad)x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{1/3}}{x^{11} (ad - bdx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x)

[Out] int((a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(1/3)/x**11/(-b*d*x**3+a*d),x)

[Out] Timed out

$$3.358 \quad \int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3} dx$$

Optimal. Leaf size=416

$$\frac{\log\left(2^{2/3} - \frac{\sqrt[3]{a} + \sqrt[3]{bx^3}}{\sqrt[3]{a+bx^3}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{a} \sqrt[3]{bd}} + \frac{\log\left(\frac{2^{2/3}(\sqrt[3]{a} + \sqrt[3]{bx^3})^2}{(a+bx^3)^{2/3}} - \frac{\sqrt[3]{2}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}} + 1\right)}{3 \cdot 2^{2/3} \sqrt[3]{a} \sqrt[3]{bd}} - \frac{\sqrt[3]{2} \log\left(\frac{\sqrt[3]{2}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}} + 1\right)}{3 \sqrt[3]{a} \sqrt[3]{bd}} + \frac{\log\left(\frac{(\sqrt[3]{a} + \sqrt[3]{bx^3})^2}{(a+bx^3)^{2/3}} + 1\right)}{6 \cdot 2^{2/3} \sqrt[3]{a} \sqrt[3]{bd}}$$

Rubi [C] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 0.15, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {430, 429}

$$\frac{x \sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{ad \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(1/3)/(a*d - b*d*x^3), x]

[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/ (a*d*(1 + (b*x^3)/a)^(1/3))

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3} dx = \frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3} dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{x\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{ad\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.15, size = 154, normalized size = 0.37

$$\frac{4ax\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{d(a-bx^3) \left(bx^3 \left(3F_1\left(\frac{4}{3}; -\frac{1}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) + F_1\left(\frac{4}{3}; \frac{2}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) \right) + 4aF_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(1/3)/(a*d - b*d*x^3), x]

[Out] (4*a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/(d*(a - b*x^3)*(4*a*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] + AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))

IntegrateAlgebraic [A] time = 3.09, size = 533, normalized size = 1.28

$$\frac{\log\left(\frac{2^{2/3}a^{3/2} - \sqrt{2}\sqrt{a+bx^3} + (a+bx^3)^{3/2} - \sqrt{2}\sqrt{a+bx^3} + 2^{2/3}\sqrt{a+bx^3} + 2^{2/3}a^{3/2}}{3^{2/3}\sqrt{a+bx^3}}\right)}{3^{2/3}\sqrt{a+bx^3}} + \frac{\log\left(\frac{2^{2/3}a^{3/2} + 2\sqrt{2}\sqrt{a+bx^3} + 4(a+bx^3)^{3/2} + 2\sqrt{2}\sqrt{a+bx^3} + 2^{2/3}\sqrt{a+bx^3} + 2^{2/3}a^{3/2}}{6^{2/3}\sqrt{a+bx^3}}\right)}{6^{2/3}\sqrt{a+bx^3}} - \frac{\sqrt{2}\log\left(\frac{\sqrt{a+bx^3} + \sqrt{2}\sqrt{a+bx^3} + \sqrt{2}\sqrt{a+bx^3}}{3\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt{a+bx^3}} + \frac{\log\left(\frac{2\sqrt{a+bx^3} - \sqrt{2}\sqrt{a+bx^3} - \sqrt{2}\sqrt{a+bx^3}}{3^{2/3}\sqrt{a+bx^3}}\right)}{3^{2/3}\sqrt{a+bx^3}} + \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}\sqrt{a+bx^3}} + \frac{\tan^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2^{2/3}\sqrt{2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(a*d - b*d*x^3), x]

[Out] (2^(1/3)*ArcTan[(Sqrt[3]*(a + b*x^3)^(1/3))/(-2*2^(1/3)*a^(1/3) - 2*2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))]/(Sqrt[3]*a^(1/3)*b^(1/3)*d) + ArcTan[(Sqrt[3]*(a + b*x^3)^(1/3))/(2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))]/(2^(2/3)*Sqrt[3]*a^(1/3)*b^(1/3)*d) - (2^(1/3)*Log[2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3)]/(3*a^(1/3)*b^(1/3)*d) - Log[-(2^(1/3)*a^(1/3) - 2^(1/3)*b^(1/3)*x + 2*(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)*b^(1/3)*d) + Log[2^(2/3)*a^(2/3) + 2*2^(2/3)*a^(1/3)*b^(1/3)*x + 2^(2/3)*b^(2/3)*x^2 - 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) - 2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(3*2^(2/3)*a^(1/3)*b^(1/3)*d) + Log[2^(2/3)*a^(2/3) + 2*2^(2/3)*a^(1/3)*b^(1/3)*x + 2^(2/3)*b^(2/3)*x^2 + 2*2^(1/3)*a^(1/3)*b^(1/3)*x + (a + b*x^3)^(2/3)]/(3*2^(2/3)*a^(1/3)*b^(1/3)*d) + Log[2^(2/3)*a^(2/3) + 2*2^(2/3)*a^(1/3)*b^(1/3)*x + 2^(2/3)*b^(2/3)*x^2 + 2*2^(1/3)*a^(1/3)*b^(1/3)*x + (a + b*x^3)^(2/3)]/(3*2^(2/3)*a^(1/3)*b^(1/3)*d)

$$\frac{1}{3}*(a + b*x^3)^{(1/3)} + 2*2^{(1/3)}*b^{(1/3)}*x*(a + b*x^3)^{(1/3)} + 4*(a + b*x^3)^{(2/3)}/(6*2^{(2/3)}*a^{(1/3)}*b^{(1/3)}*d)$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{1}{3}}}{bdx^3 - ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")

[Out] integrate(-(b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{-bdx^3 + ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)

[Out] int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{1}{3}}}{bdx^3 - ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{1/3}}{ad - bdx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(1/3)/(a*d - b*d*x^3), x)

[Out] int((a + b*x^3)^(1/3)/(a*d - b*d*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-a+bx^3} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(1/3)/(-b*d*x**3+a*d), x)

[Out] -Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)/d

$$3.359 \quad \int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3} dx$$

Optimal. Leaf size=223

$$\frac{a^{11/3} \log(a-bx^3)}{3\sqrt[3]{2}b^4d} - \frac{a^{11/3} \log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^4d} - \frac{2^{2/3}a^{11/3} \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^4d} - \frac{a^3(a+bx^3)^{2/3}}{2b^4d} - \frac{a^2(a+bx^3)^{1/3}}{5b^4d}$$

Rubi [A] time = 0.25, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 88, 50, 55, 617, 204, 31}

$$\frac{a^3(a+bx^3)^{2/3}}{2b^4d} - \frac{a^2(a+bx^3)^{5/3}}{5b^4d} + \frac{a^{11/3} \log(a-bx^3)}{3\sqrt[3]{2}b^4d} - \frac{a^{11/3} \log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^4d} - \frac{2^{2/3}a^{11/3} \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^4d} + \frac{a(a+bx^3)^{8/3}}{8b^4d} - \frac{(a+bx^3)^{11/3}}{11b^4d}$$

Antiderivative was successfully verified.

[In] Int[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

[Out] -(a^3*(a + b*x^3)^(2/3))/(2*b^4*d) - (a^2*(a + b*x^3)^(5/3))/(5*b^4*d) + (a*(a + b*x^3)^(8/3))/(8*b^4*d) - (a + b*x^3)^(11/3)/(11*b^4*d) - (2^(2/3)*a^(11/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^4*d) + (a^(11/3)*Log[a - b*x^3])/(3*2^(1/3)*b^4*d) - (a^(11/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^4*d)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m+1)*(c + d*x)^n)/(b*(m+n+1)), x] + Dist[(n*(b*c - a*d))/(b*(m+n+1)), Int[(a + b*x)^m*(c + d*x)^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],

$x] - \text{Dist}[3/(2*b*q), \text{Subst}[\text{Int}[1/(q - x), x], x, (c + d*x)^{(1/3)}, x]]] /;$
 $\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{PosQ}[(b*c - a*d)/b]$

Rule 88

$\text{Int}[(a_. + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}*((e_.) + (f_.)*(x_))^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e, f, p, x\} \ \&\& \ \text{IntegersQ}[m, n] \ \&\& \ (\text{IntegerQ}[p] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{GeQ}[n, -1]))$

Rule 204

$\text{Int}[(a_) + (b_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /;$
 $\text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 446

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}*((c_) + (d_.)*(x_)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$
 $\text{FreeQ}\{a, b, c, d, m, n, p, q, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 617

$\text{Int}[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{With}\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /;$
 $\text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ \text{!RationalQ}[b^2 - 4*a*c]) /;$
 $\text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x^{11} (a + bx^3)^{2/3}}{ad - bdx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3 (a + bx)^{2/3}}{ad - bdx} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{a^2 (a + bx)^{2/3}}{b^3 d} + \frac{a(a + bx)^{5/3}}{b^3 d} - \frac{(a + bx)^{8/3}}{b^3 d} + \frac{a^3 (a + bx)^{2/3}}{b^3 (ad - bdx)} \right) dx, x, x^3 \right) \\
&= -\frac{a^2 (a + bx^3)^{5/3}}{5b^4 d} + \frac{a (a + bx^3)^{8/3}}{8b^4 d} - \frac{(a + bx^3)^{11/3}}{11b^4 d} + \frac{a^3 \text{Subst} \left(\int \frac{(a+bx)^{2/3}}{ad-bdx} dx, x, x^3 \right)}{3b^3} \\
&= -\frac{a^3 (a + bx^3)^{2/3}}{2b^4 d} - \frac{a^2 (a + bx^3)^{5/3}}{5b^4 d} + \frac{a (a + bx^3)^{8/3}}{8b^4 d} - \frac{(a + bx^3)^{11/3}}{11b^4 d} + \frac{(2a^4) \text{Subst} \left(\int \frac{1}{ad-bdx} dx, x, x^3 \right)}{3b^3} \\
&= -\frac{a^3 (a + bx^3)^{2/3}}{2b^4 d} - \frac{a^2 (a + bx^3)^{5/3}}{5b^4 d} + \frac{a (a + bx^3)^{8/3}}{8b^4 d} - \frac{(a + bx^3)^{11/3}}{11b^4 d} + \frac{a^{11/3} \log(a - bx^3)}{3\sqrt[3]{2} b^4 d} \\
&= -\frac{a^3 (a + bx^3)^{2/3}}{2b^4 d} - \frac{a^2 (a + bx^3)^{5/3}}{5b^4 d} + \frac{a (a + bx^3)^{8/3}}{8b^4 d} - \frac{(a + bx^3)^{11/3}}{11b^4 d} + \frac{a^{11/3} \log(a - bx^3)}{3\sqrt[3]{2} b^4 d} \\
&= -\frac{a^3 (a + bx^3)^{2/3}}{2b^4 d} - \frac{a^2 (a + bx^3)^{5/3}}{5b^4 d} + \frac{a (a + bx^3)^{8/3}}{8b^4 d} - \frac{(a + bx^3)^{11/3}}{11b^4 d} - \frac{2^{2/3} a^{11/3} \tan^{-1} \left(\frac{1 + (2^{2/3} (a + bx^3)^{1/3})}{\sqrt{3}} \right)}{\sqrt{3} b^4 d}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 163, normalized size = 0.73

$$\frac{-220 \cdot 2^{2/3} a^{11/3} \log(a - bx^3) + 660 \cdot 2^{2/3} a^{11/3} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + bx^3}\right) + 440 \cdot 2^{2/3} \sqrt{3} a^{11/3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a + bx^3} + 1}{\sqrt{3} \sqrt[3]{a}}\right) + 3(a + bx^3)^{2/3} (293a^3 + 98a^2 bx^3 + 65ab^2 x^6 + 40b^3 x^9)}{1320b^4 d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

[Out] $-1/1320 \cdot (3 \cdot (a + b \cdot x^3)^{2/3} \cdot (293 \cdot a^3 + 98 \cdot a^2 \cdot b \cdot x^3 + 65 \cdot a \cdot b^2 \cdot x^6 + 40 \cdot b^3 \cdot x^9) + 440 \cdot 2^{2/3} \cdot \text{Sqrt}[3] \cdot a^{11/3} \cdot \text{ArcTan}[(1 + (2^{2/3} \cdot (a + b \cdot x^3)^{1/3})) / a^{1/3}] / \text{Sqrt}[3]] - 220 \cdot 2^{2/3} \cdot a^{11/3} \cdot \text{Log}[a - b \cdot x^3] + 660 \cdot 2^{2/3} \cdot a^{11/3} \cdot \text{Log}[2^{1/3} \cdot a^{1/3} - (a + b \cdot x^3)^{1/3}]) / (b^4 \cdot d)$

IntegrateAlgebraic [A] time = 0.27, size = 225, normalized size = 1.01

$$-\frac{2^{2/3} a^{11/3} \log\left(2^{2/3} \sqrt[3]{a + bx^3} - 2 \sqrt[3]{a}\right)}{3b^4 d} + \frac{a^{11/3} \log\left(2a^{2/3} + 2^{2/3} \sqrt[3]{a} \sqrt[3]{a + bx^3} + \sqrt[3]{2} (a + bx^3)^{2/3}\right)}{3\sqrt[3]{2} b^4 d} - \frac{2^{2/3} a^{11/3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a + bx^3} + 1}{\sqrt{3} \sqrt[3]{a}} + \frac{1}{\sqrt{3}}\right)}{\sqrt{3} b^4 d} + \frac{(a + bx^3)^{2/3} (-293a^3 - 98a^2 bx^3 - 65ab^2 x^6 - 40b^3 x^9)}{440b^4 d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-293*a^3 - 98*a^2*b*x^3 - 65*a*b^2*x^6 - 40*b^3*x^9))/(
440*b^4*d) - (2^(2/3)*a^(11/3)*ArcTan[1/Sqrt[3] + (2^(2/3)*(a + b*x^3)^(1/3
))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^4*d) - (2^(2/3)*a^(11/3)*Log[-2*a^(1/3) +
2^(2/3)*(a + b*x^3)^(1/3)]/(3*b^4*d) + (a^(11/3)*Log[2*a^(2/3) + 2^(2/3)*
a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]/(3*2^(1/3)*b^4*d)
```

fricas [A] time = 0.65, size = 209, normalized size = 0.94

$$\frac{440 \cdot 4^{\frac{1}{3}} \sqrt[3]{-a^2} \arctan\left(\frac{4^{\frac{1}{3}} \sqrt[3]{(bx^3+a)^{\frac{1}{3}} (-a^2)^{\frac{1}{3}} - \sqrt[3]{3a}}}{3a}\right) + 220 \cdot 4^{\frac{1}{3}} (-a^2)^{\frac{1}{3}} a^3 \log\left(4^{\frac{2}{3}} (bx^3+a)^{\frac{1}{3}} (-a^2)^{\frac{2}{3}} + 2(bx^3+a)^{\frac{2}{3}} a - 2 \cdot 4^{\frac{1}{3}} (-a^2)^{\frac{1}{3}} a\right) - 440 \cdot 4^{\frac{1}{3}} (-a^2)^{\frac{1}{3}} a^3 \log\left(-4^{\frac{1}{3}} (-a^2)^{\frac{2}{3}} + 2(bx^3+a)^{\frac{1}{3}} a\right) + 3(40 b^3 x^9 + 65 a b^2 x^6 + 98 a^2 b x^3 + 293 a^3)(bx^3+a)^{\frac{2}{3}}}{1320 b^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^3*arctan(1/3*(4^(1/3)*sqrt(3)*(
b*x^3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 220*4^(1/3)*(-a^2)^(1/3)*a^
3*log(4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^
(1/3)*(-a^2)^(1/3)*a) - 440*4^(1/3)*(-a^2)^(1/3)*a^3*log(-4^(2/3)*(-a^2)^(2
/3) + 2*(b*x^3 + a)^(1/3)*a) + 3*(40*b^3*x^9 + 65*a*b^2*x^6 + 98*a^2*b*x^3
+ 293*a^3)*(b*x^3 + a)^(2/3))/(b^4*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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 b*x³)^(1/3)+2*a)^(1/3)*(2*a)^(1/3))-a³((2*a)^(1/3))²/sqrt(3)/b⁴/d*ata
 n(((a+b*x³)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*a ⁴*b⁴⁴*d¹¹*(2*a)^(1/3)*1/6/a/b⁴⁸/d¹²*ln(abs((a+b*x³)^(1/3)-(2*a)^(1/3))
)-(1/11*((a+b*x³)^(1/3))²*(a+b*x³)³*b⁴⁰*d¹⁰-1/8*((a+b*x³)^(1/3))²*(
 a+b*x³)²*a*b⁴⁰*d¹⁰+1/5*((a+b*x³)^(1/3))²*(a+b*x³)*a²*b⁴⁰*d¹⁰+1/2*
 ((a+b*x³)^(1/3))²*a³*b⁴⁰*d¹⁰)/b⁴⁴/d¹¹

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}} x^{11}}{-bdx^3 + ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹*(b*x³+a)^(2/3)/(-b*d*x³+a*d), x)

[Out] int(x¹¹*(b*x³+a)^(2/3)/(-b*d*x³+a*d), x)

maxima [A] time = 1.25, size = 183, normalized size = 0.82

$$\frac{440 \sqrt{3} 2^{\frac{2}{3}} a^{\frac{11}{3}} \arctan\left(\frac{\sqrt{3} 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} a^{\frac{1}{3}} + 2 (bx^3+a)^{\frac{1}{3}}\right)}{6 a^{\frac{1}{3}}}\right)}{d} - \frac{220 \cdot 2^{\frac{2}{3}} a^{\frac{11}{3}} \log\left(2^{\frac{2}{3}} a^{\frac{2}{3}} + 2^{\frac{1}{3}} (bx^3+a)^{\frac{1}{3}} a^{\frac{1}{3}} + (bx^3+a)^{\frac{2}{3}}\right)}{d} + \frac{440 \cdot 2^{\frac{2}{3}} a^{\frac{11}{3}} \log\left(-2^{\frac{1}{3}} a^{\frac{1}{3}} + (bx^3+a)^{\frac{1}{3}}\right)}{d} + \frac{3 \left(40 (bx^3+a)^{\frac{11}{3}} - 55 (bx^3+a)^{\frac{8}{3}} a + 88 (bx^3+a)^{\frac{5}{3}} a^2 + 220 (bx^3+a)^{\frac{2}{3}} a^3\right)}{1320 b^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(b*x³+a)^(2/3)/(-b*d*x³+a*d),x, algorithm="maxima")

[Out] -1/1320*(440*sqrt(3)*2^(2/3)*a^(11/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*x³ + a)^(1/3))/a^(1/3))/d - 220*2^(2/3)*a^(11/3)*log(2^(2/3)

$$*a^{(2/3)} + 2^{(1/3)}*(b*x^3 + a)^{(1/3)}*a^{(1/3)} + (b*x^3 + a)^{(2/3)}/d + 440*2^{(2/3)}*a^{(11/3)}*\log(-2^{(1/3)}*a^{(1/3)} + (b*x^3 + a)^{(1/3)})/d + 3*(40*(b*x^3 + a)^{(11/3)} - 55*(b*x^3 + a)^{(8/3)}*a + 88*(b*x^3 + a)^{(5/3)}*a^2 + 220*(b*x^3 + a)^{(2/3)}*a^3)/d)/b^4$$

mupad [B] time = 4.85, size = 261, normalized size = 1.17

$$\frac{a(bx^3+a)^{8/3}}{8b^4d} - \frac{a^3(bx^3+a)^{2/3}}{2b^4d} - \frac{a^2(bx^3+a)^{5/3}}{5b^4d} - \frac{(bx^3+a)^{11/3}}{11b^4d} + \frac{4^{1/3}(-a)^{11/3} \ln\left(\frac{4d^8(bx^3+a)^{1/3}}{b^8d^2} + \frac{24^{2/3}(-a)^{25/3}\left(\frac{1+\sqrt{3}i}{2}\right)^2}{b^8d^2}\right)}{3b^4d} - \frac{4^{1/3}(-a)^{11/3} \ln\left(\frac{4d^8(bx^3+a)^{1/3}}{b^8d^2} + \frac{24^{2/3}(-a)^{25/3}\left(\frac{1+\sqrt{3}i}{2}\right)^2}{b^8d^2}\right)}{3b^4d} + \frac{4^{1/3}(-a)^{11/3} \ln\left(\frac{4d^8(bx^3+a)^{1/3}}{b^8d^2} + \frac{184^{2/3}(-a)^{25/3}\left(\frac{1+\sqrt{3}i}{2}\right)^2}{b^8d^2}\right)}{b^4d} + \frac{4^{1/3}(-a)^{11/3} \ln\left(\frac{4d^8(bx^3+a)^{1/3}}{b^8d^2} + \frac{184^{2/3}(-a)^{25/3}\left(\frac{1+\sqrt{3}i}{2}\right)^2}{b^8d^2}\right)}{b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)

[Out] (a*(a + b*x^3)^(8/3))/(8*b^4*d) - (a^3*(a + b*x^3)^(2/3))/(2*b^4*d) - (a^2*(a + b*x^3)^(5/3))/(5*b^4*d) - (a + b*x^3)^(11/3)/(11*b^4*d) + (4^(1/3)*(-a)^(11/3)*log(4*a^8*(a + b*x^3)^(1/3) + 4*2^(1/3)*(-a)^(25/3)))/(3*b^4*d) - (4^(1/3)*(-a)^(11/3)*log((4*a^8*(a + b*x^3)^(1/3))/(b^8*d^2) + (2*4^(2/3)*(-a)^(25/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(b^8*d^2)))*((3^(1/2)*1i)/2 + 1/2))/(3*b^4*d) + (4^(1/3)*(-a)^(11/3)*log((4*a^8*(a + b*x^3)^(1/3))/(b^8*d^2) + (18*4^(2/3)*(-a)^(25/3)*((3^(1/2)*1i)/6 - 1/6)^2)/(b^8*d^2)))*((3^(1/2)*1i)/6 - 1/6))/(b^4*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d), x)

[Out] Timed out

$$3.360 \quad \int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3} dx$$

Optimal. Leaf size=177

$$\frac{a^{8/3} \log(a-bx^3)}{3\sqrt[3]{2} b^3 d} - \frac{a^{8/3} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2} b^3 d} - \frac{2^{2/3} a^{8/3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^3 d} - \frac{a^2 (a+bx^3)^{2/3}}{2b^3 d} - \frac{(a+bx^3)^{8/3}}{8b^3 d}$$

Rubi [A] time = 0.20, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 88, 50, 55, 617, 204, 31}

$$-\frac{a^2 (a+bx^3)^{2/3}}{2b^3 d} + \frac{a^{8/3} \log(a-bx^3)}{3\sqrt[3]{2} b^3 d} - \frac{a^{8/3} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2} b^3 d} - \frac{2^{2/3} a^{8/3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^3 d} - \frac{(a+bx^3)^{8/3}}{8b^3 d}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

[Out] -(a^2*(a + b*x^3)^(2/3))/(2*b^3*d) - (a + b*x^3)^(8/3)/(8*b^3*d) - (2^(2/3)*a^(8/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^3*d) + (a^(8/3)*Log[a - b*x^3])/(3*2^(1/3)*b^3*d) - (a^(8/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^3*d)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /;

FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^8 (a + bx^3)^{2/3}}{ad - bdx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (a + bx)^{2/3}}{ad - bdx} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{(a + bx)^{5/3}}{b^2 d} + \frac{a^2 (a + bx)^{2/3}}{b^2 (ad - bdx)} \right) dx, x, x^3 \right) \\
&= -\frac{(a + bx^3)^{8/3}}{8b^3 d} + \frac{a^2 \text{Subst} \left(\int \frac{(a + bx)^{2/3}}{ad - bdx} dx, x, x^3 \right)}{3b^2} \\
&= -\frac{a^2 (a + bx^3)^{2/3}}{2b^3 d} - \frac{(a + bx^3)^{8/3}}{8b^3 d} + \frac{(2a^3) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a + bx} (ad - bdx)} dx, x, x^3 \right)}{3b^2} \\
&= -\frac{a^2 (a + bx^3)^{2/3}}{2b^3 d} - \frac{(a + bx^3)^{8/3}}{8b^3 d} + \frac{a^{8/3} \log(a - bx^3)}{3\sqrt[3]{2} b^3 d} + \frac{a^{8/3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{2} \sqrt[3]{a - x}} dx, x, \sqrt[3]{a - bx^3} \right)}{\sqrt[3]{2} b^3 d} \\
&= -\frac{a^2 (a + bx^3)^{2/3}}{2b^3 d} - \frac{(a + bx^3)^{8/3}}{8b^3 d} + \frac{a^{8/3} \log(a - bx^3)}{3\sqrt[3]{2} b^3 d} - \frac{a^{8/3} \log \left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + bx^3} \right)}{\sqrt[3]{2} b^3 d} + \\
&= -\frac{a^2 (a + bx^3)^{2/3}}{2b^3 d} - \frac{(a + bx^3)^{8/3}}{8b^3 d} - \frac{2^{2/3} a^{8/3} \tan^{-1} \left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a + bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3} b^3 d} + \frac{a^{8/3} \log(a - bx^3)}{3\sqrt[3]{2} b^3 d} - \frac{a^{8/3} \log \left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + bx^3} \right)}{\sqrt[3]{2} b^3 d}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 153, normalized size = 0.86

$$\frac{4 \cdot 2^{2/3} a^{8/3} \log(a - bx^3) - 8 \cdot 2^{2/3} \sqrt{3} a^{8/3} \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{a + bx^3} + 1}{\frac{\sqrt[3]{a}}{\sqrt{3}}} \right) - 3 \left(4 \cdot 2^{2/3} a^{8/3} \log \left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + bx^3} \right) + (a + bx^3)^{2/3} (5a^2 + 2abx^3 + b^2x^6) \right)}{24b^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

[Out] (-8*2^(2/3)*Sqrt[3]*a^(8/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 4*2^(2/3)*a^(8/3)*Log[a - b*x^3] - 3*((a + b*x^3)^(2/3)*(5*a^2 + 2*a*b*x^3 + b^2*x^6) + 4*2^(2/3)*a^(8/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]))/(24*b^3*d)

IntegrateAlgebraic [A] time = 0.22, size = 214, normalized size = 1.21

$$-\frac{2^{2/3} a^{8/3} \log \left(2^{2/3} \sqrt[3]{a + bx^3} - 2 \sqrt[3]{a} \right)}{3b^3 d} + \frac{a^{8/3} \log \left(2a^{2/3} + 2^{2/3} \sqrt[3]{a} \sqrt[3]{a + bx^3} + \sqrt[3]{2} (a + bx^3)^{2/3} \right)}{3\sqrt[3]{2} b^3 d} - \frac{2^{2/3} a^{8/3} \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{a + bx^3}}{\sqrt{3} \sqrt[3]{a}} + \frac{1}{\sqrt{3}} \right)}{\sqrt{3} b^3 d} + \frac{(a + bx^3)^{2/3} (-5a^2 - 2abx^3 - b^2x^6)}{8b^3 d}$$

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²¹*d⁷)/b²⁴/d⁸

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}} x^8}{-bdx^3 + ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x⁸*(b*x³+a)^(2/3)/(-b*d*x³+a*d), x)

[Out] int(x⁸*(b*x³+a)^(2/3)/(-b*d*x³+a*d), x)

maxima [A] time = 1.17, size = 155, normalized size = 0.88

$$\frac{8\sqrt{3}2^{\frac{2}{3}}a^{\frac{8}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(bx^3+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d} - \frac{4\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{d} + \frac{8\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right)}{d} + \frac{3\left((bx^3+a)^{\frac{8}{3}}+4(bx^3+a)^{\frac{2}{3}}a^2\right)}{d}$$

24b³

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁸*(b*x³+a)^(2/3)/(-b*d*x³+a*d), x, algorithm="maxima")

[Out] -1/24*(8*sqrt(3)*2^(2/3)*a^(8/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
) + 2*(b*x³ + a)^(1/3))/a^(1/3))/d - 4*2^(2/3)*a^(8/3)*log(2^(2/3)*a^(2/3))

+ $2^{(1/3)}*(b*x^3 + a)^{(1/3)}*a^{(1/3)} + (b*x^3 + a)^{(2/3)}/d + 8*2^{(2/3)}*a^{(8/3)}*\log(-2^{(1/3)}*a^{(1/3)} + (b*x^3 + a)^{(1/3)}/d + 3*((b*x^3 + a)^{(8/3)} + 4*(b*x^3 + a)^{(2/3)}*a^2)/d)/b^3$

mupad [B] time = 4.91, size = 206, normalized size = 1.16

$$\frac{\frac{(bx^3+a)^{8/3}}{8b^3d} - \frac{a^2(bx^3+a)^{2/3}}{2b^3d} - \frac{4^{1/3}a^{8/3}\ln\left(\frac{(bx^3+a)^{1/3} - 2^{1/3}a^{1/3}}{3b^3d}\right)}{3b^3d} - \frac{4^{1/3}a^{8/3}\ln\left(\frac{4a^6(bx^3+a)^{1/3}}{b^6d^2} - \frac{24^{2/3}a^{19/3}\left(\frac{1}{2} + \frac{\sqrt{3}11}{2}\right)^2}{b^6d^2}\right)}{3b^3d}\left(-\frac{1}{2} + \frac{\sqrt{3}11}{2}\right)}{b^3d} + \frac{4^{1/3}a^{8/3}\ln\left(\frac{4a^6(bx^3+a)^{1/3}}{b^6d^2} - \frac{184^{2/3}a^{19/3}\left(\frac{1}{6} + \frac{\sqrt{3}11}{6}\right)^2}{b^6d^2}\right)}{b^3d}\left(\frac{1}{6} + \frac{\sqrt{3}11}{6}\right)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)`

[Out] $(4^{(1/3)}*a^{(8/3)}*\log((4*a^6*(a + b*x^3)^{(1/3)})/(b^6*d^2) - (18*4^{(2/3)}*a^{(19/3)}*((3^{(1/2)}*1i)/6 + 1/6)^2)/(b^6*d^2))*(3^{(1/2)}*1i)/6 + 1/6)/(b^3*d) - (a^2*(a + b*x^3)^{(2/3)})/(2*b^3*d) - (4^{(1/3)}*a^{(8/3)}*\log((a + b*x^3)^{(1/3)} - 2^{(1/3)}*a^{(1/3)}))/(3*b^3*d) - (4^{(1/3)}*a^{(8/3)}*\log((4*a^6*(a + b*x^3)^{(1/3)})/(b^6*d^2) - (2*4^{(2/3)}*a^{(19/3)}*((3^{(1/2)}*1i)/2 - 1/2)^2)/(b^6*d^2))*(3^{(1/2)}*1i)/2 - 1/2)/(3*b^3*d) - (a + b*x^3)^{(8/3)}/(8*b^3*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^8(a+bx^3)^{\frac{2}{3}}}{-a+bx^3} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d), x)`

[Out] `-Integral(x**8*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d`

$$3.361 \quad \int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3} dx$$

Optimal. Leaf size=175

$$\frac{a^{5/3} \log(a-bx^3)}{3\sqrt[3]{2} b^2 d} - \frac{a^{5/3} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2} b^2 d} - \frac{2^{2/3} a^{5/3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^2 d} - \frac{a(a+bx^3)^{2/3}}{2b^2 d} - \frac{(a+bx^3)^{5/3}}{5b^2 d}$$

Rubi [A] time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 80, 50, 55, 617, 204, 31}

$$\frac{a^{5/3} \log(a-bx^3)}{3\sqrt[3]{2} b^2 d} - \frac{a^{5/3} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2} b^2 d} - \frac{2^{2/3} a^{5/3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^2 d} - \frac{a(a+bx^3)^{2/3}}{2b^2 d} - \frac{(a+bx^3)^{5/3}}{5b^2 d}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

[Out] -(a*(a + b*x^3)^(2/3))/(2*b^2*d) - (a + b*x^3)^(5/3)/(5*b^2*d) - (2^(2/3)*a^(5/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^2*d) + (a^(5/3)*Log[a - b*x^3])/(3*2^(1/3)*b^2*d) - (a^(5/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^2*d)

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 55

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],

$x] - \text{Dist}[3/(2*b*q), \text{Subst}[\text{Int}[1/(q - x), x], x, (c + d*x)^{(1/3)}], x]] /;$
 $\text{FreeQ}\{a, b, c, d\}, x] \&\& \text{PosQ}[(b*c - a*d)/b]$

Rule 80

$\text{Int}[(a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] := \text{Simp}[(b*(c + d*x)^{(n + 1)}*(e + f*x)^{(p + 1)})/(d*f*(n + p + 2)), x] + \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), \text{Int}[(c + d*x)^n*(e + f*x)^p, x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e, f, n, p\}, x] \&\& \text{NeQ}[n + p + 2, 0]$

Rule 204

$\text{Int}[(a_) + (b_.)*(x_)^2]^{-1}, x_Symbol] := -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /;$
 $\text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 446

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}*((c_) + (d_.)*(x_)^{(n_.)})^{(q_.)}, x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)}*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$
 $\text{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 617

$\text{Int}[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]^{-1}, x_Symbol] := \text{With}\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /;$
 $\text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /;$
 $\text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (a + bx^3)^{2/3}}{ad - bdx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(a + bx)^{2/3}}{ad - bdx} dx, x, x^3 \right) \\
&= -\frac{(a + bx^3)^{5/3}}{5b^2d} + \frac{a \text{Subst} \left(\int \frac{(a+bx)^{2/3}}{ad-bdx} dx, x, x^3 \right)}{3b} \\
&= -\frac{a(a + bx^3)^{2/3}}{2b^2d} - \frac{(a + bx^3)^{5/3}}{5b^2d} + \frac{(2a^2) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)} dx, x, x^3 \right)}{3b} \\
&= -\frac{a(a + bx^3)^{2/3}}{2b^2d} - \frac{(a + bx^3)^{5/3}}{5b^2d} + \frac{a^{5/3} \log(a - bx^3)}{3\sqrt[3]{2} b^2d} + \frac{a^{5/3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{2} \sqrt[3]{a-x}} dx, x, \sqrt[3]{a + bx^3} \right)}{\sqrt[3]{2} b^2d} \\
&= -\frac{a(a + bx^3)^{2/3}}{2b^2d} - \frac{(a + bx^3)^{5/3}}{5b^2d} + \frac{a^{5/3} \log(a - bx^3)}{3\sqrt[3]{2} b^2d} - \frac{a^{5/3} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + bx^3}\right)}{\sqrt[3]{2} b^2d} + \dots \\
&= -\frac{a(a + bx^3)^{2/3}}{2b^2d} - \frac{(a + bx^3)^{5/3}}{5b^2d} - \frac{2^{2/3} a^{5/3} \tan^{-1} \left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3} b^2d} + \frac{a^{5/3} \log(a - bx^3)}{3\sqrt[3]{2} b^2d} - \dots
\end{aligned}$$

Mathematica [A] time = 0.15, size = 143, normalized size = 0.82

$$\frac{5 \cdot 2^{2/3} a^{5/3} \log(a - bx^3) - 3 \left(5 \cdot 2^{2/3} a^{5/3} \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + bx^3}\right) + (a + bx^3)^{2/3} (7a + 2bx^3) \right) - 10 \cdot 2^{2/3} \sqrt{3} a^{5/3} \tan^{-1} \left(\frac{\frac{2^{2/3} \sqrt[3]{a+bx^3}}{\sqrt[3]{a}} + 1}{\sqrt{3}} \right)}{30b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

[Out] (-10*2^(2/3)*Sqrt[3]*a^(5/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3)))/a^(1/3)]/Sqrt[3] + 5*2^(2/3)*a^(5/3)*Log[a - b*x^3] - 3*((a + b*x^3)^(2/3)*(7*a + 2*b*x^3) + 5*2^(2/3)*a^(5/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]))/(30*b^2*d)

IntegrateAlgebraic [A] time = 0.21, size = 203, normalized size = 1.16

$$-\frac{2^{2/3} a^{5/3} \log\left(2^{2/3} \sqrt[3]{a + bx^3} - 2\sqrt[3]{a}\right)}{3b^2d} + \frac{a^{5/3} \log\left(2a^{2/3} + 2^{2/3} \sqrt[3]{a} \sqrt[3]{a + bx^3} + \sqrt[3]{2} (a + bx^3)^{2/3}\right)}{3\sqrt[3]{2} b^2d} - \frac{2^{2/3} a^{5/3} \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{a+bx^3}}{\sqrt{3} \sqrt[3]{a}} + \frac{1}{\sqrt{3}} \right)}{\sqrt{3} b^2d} + \frac{(-7a - 2bx^3)(a + bx^3)^{2/3}}{10b^2d}$$

) + $2^{(1/3)} * (b*x^3 + a)^{(1/3)} * a^{(1/3)} + (b*x^3 + a)^{(2/3)} / d + 10 * 2^{(2/3)} * a^{(5/3)} * \log(-2^{(1/3)} * a^{(1/3)} + (b*x^3 + a)^{(1/3)}) / d + 3 * (2 * (b*x^3 + a)^{(5/3)} + 5 * (b*x^3 + a)^{(2/3)} * a) / d / b^2$

mupad [B] time = 4.84, size = 221, normalized size = 1.26

$$\frac{4^{1/3}(-a)^{5/3} \ln\left(4a^4(bx^3+a)^{1/3} + 42^{1/3}(-a)^{13/3}\right)}{3b^2d} - \frac{a(bx^3+a)^{2/3}}{2b^2d} - \frac{(bx^3+a)^{5/3}}{5b^2d} - \frac{4^{1/3}(-a)^{5/3} \ln\left(\frac{4a^4(bx^3+a)^{1/3}}{b^4d^2} + \frac{24^{2/3}(-a)^{13/3}\left(\frac{1}{2} + \frac{\sqrt{3}11}{2}\right)^2}{b^4d^2}\right)}{3b^2d} \left(\frac{1}{2} + \frac{\sqrt{3}11}{2}\right) + \frac{4^{1/3}(-a)^{5/3} \ln\left(\frac{4a^4(bx^3+a)^{1/3}}{b^4d^2} + \frac{184^{2/3}(-a)^{13/3}\left(\frac{1}{6} + \frac{\sqrt{3}11}{6}\right)^2}{b^4d^2}\right)}{b^2d} \left(-\frac{1}{6} + \frac{\sqrt{3}11}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)`

[Out] $(4^{(1/3)} * (-a)^{(5/3)} * \log(4*a^4*(a + b*x^3)^{(1/3)} + 4*2^{(1/3)} * (-a)^{(13/3)})) / (3*b^2*d) - (a*(a + b*x^3)^{(2/3)}) / (2*b^2*d) - (a + b*x^3)^{(5/3)} / (5*b^2*d) - (4^{(1/3)} * (-a)^{(5/3)} * \log((4*a^4*(a + b*x^3)^{(1/3)}) / (b^4*d^2) + (2*4^{(2/3)} * (-a)^{(13/3)} * ((3^{(1/2)}*1i)/2 + 1/2)^2) / (b^4*d^2)) * ((3^{(1/2)}*1i)/2 + 1/2)) / (3*b^2*d) + (4^{(1/3)} * (-a)^{(5/3)} * \log((4*a^4*(a + b*x^3)^{(1/3)}) / (b^4*d^2) + (18*4^{(2/3)} * (-a)^{(13/3)} * ((3^{(1/2)}*1i)/6 - 1/6)^2) / (b^4*d^2)) * ((3^{(1/2)}*1i)/6 - 1/6)) / (b^2*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^5(a+bx^3)^{\frac{2}{3}}}{-a+bx^3} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d), x)`

[Out] `-Integral(x**5*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d`

$$3.362 \quad \int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3} dx$$

Optimal. Leaf size=153

$$\frac{a^{2/3} \log(a - bx^3)}{3\sqrt[3]{2}bd} - \frac{a^{2/3} \log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd} - \frac{2^{2/3}a^{2/3} \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}bd} - \frac{(a+bx^3)^{2/3}}{2bd}$$

Rubi [A] time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {444, 50, 55, 617, 204, 31}

$$\frac{a^{2/3} \log(a - bx^3)}{3\sqrt[3]{2}bd} - \frac{a^{2/3} \log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd} - \frac{2^{2/3}a^{2/3} \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}bd} - \frac{(a+bx^3)^{2/3}}{2bd}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

[Out] -(a + b*x^3)^(2/3)/(2*b*d) - (2^(2/3)*a^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b*d) + (a^(2/3)*Log[a - b*x^3])/(3*2^(1/3)*b*d) - (a^(2/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b*d)

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 55

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + bx^3)^{2/3}}{ad - bdx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^{2/3}}{ad - bdx} dx, x, x^3 \right) \\
&= -\frac{(a + bx^3)^{2/3}}{2bd} + \frac{1}{3} (2a) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a + bx} (ad - bdx)} dx, x, x^3 \right) \\
&= -\frac{(a + bx^3)^{2/3}}{2bd} + \frac{a^{2/3} \log(a - bx^3)}{3\sqrt[3]{2} bd} + \frac{a^{2/3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{2} \sqrt[3]{a - x}} dx, x, \sqrt[3]{a + bx^3} \right)}{\sqrt[3]{2} bd} - \frac{a \text{Subst} \left(\int \frac{1}{\sqrt[3]{2} \sqrt[3]{a - x}} dx, x, \sqrt[3]{a + bx^3} \right)}{\sqrt[3]{2} bd} \\
&= -\frac{(a + bx^3)^{2/3}}{2bd} + \frac{a^{2/3} \log(a - bx^3)}{3\sqrt[3]{2} bd} - \frac{a^{2/3} \log \left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + bx^3} \right)}{\sqrt[3]{2} bd} + \frac{(2^{2/3} a^{2/3}) \text{Subst} \left(\int \frac{1}{\sqrt[3]{2} \sqrt[3]{a - x}} dx, x, \sqrt[3]{a + bx^3} \right)}{\sqrt[3]{2} bd} \\
&= -\frac{(a + bx^3)^{2/3}}{2bd} - \frac{2^{2/3} a^{2/3} \tan^{-1} \left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a + bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3} bd} + \frac{a^{2/3} \log(a - bx^3)}{3\sqrt[3]{2} bd} - \frac{a^{2/3} \log \left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + bx^3} \right)}{\sqrt[3]{2} bd}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 130, normalized size = 0.85

$$\frac{2^{2/3} a^{2/3} \log(a - bx^3) - 3 \left(2^{2/3} a^{2/3} \log \left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + bx^3} \right) + (a + bx^3)^{2/3} \right) - 2 \cdot 2^{2/3} \sqrt{3} a^{2/3} \tan^{-1} \left(\frac{\frac{2^{2/3} \sqrt[3]{a + bx^3}}{\sqrt[3]{a}} + 1}{\sqrt{3}} \right)}{6bd}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

[Out] (-2*2^(2/3)*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3)))/a^(1/3)]/Sqrt[3]] + 2^(2/3)*a^(2/3)*Log[a - b*x^3] - 3*((a + b*x^3)^(2/3) + 2^(2/3)*a^(2/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(6*b*d)

IntegrateAlgebraic [A] time = 0.19, size = 193, normalized size = 1.26

$$-\frac{2^{2/3} a^{2/3} \log \left(2^{2/3} \sqrt[3]{a + bx^3} - 2 \sqrt[3]{a} \right)}{3bd} + \frac{a^{2/3} \log \left(2a^{2/3} + 2^{2/3} \sqrt[3]{a} \sqrt[3]{a + bx^3} + \sqrt[3]{2} (a + bx^3)^{2/3} \right)}{3\sqrt[3]{2} bd} - \frac{2^{2/3} a^{2/3} \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{a + bx^3}}{\sqrt{3} \sqrt[3]{a}} + \frac{1}{\sqrt{3}} \right)}{\sqrt{3} bd} - \frac{(a + bx^3)^{2/3}}{2bd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

```
[Out] -1/2*(a + b*x^3)^(2/3)/(b*d) - (2^(2/3)*a^(2/3)*ArcTan[1/Sqrt[3] + (2^(2/3)
*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b*d) - (2^(2/3)*a^(2/3)*Lo
g[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)]/(3*b*d) + (a^(2/3)*Log[2*a^(2/3)
+ 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]/(3*2^(1/
3)*b*d)
```

fricas [A] time = 1.17, size = 167, normalized size = 1.09

$$\frac{2 \cdot 4^{\frac{1}{3}} \sqrt{3} (-a^2)^{\frac{1}{3}} \arctan\left(\frac{4^{\frac{1}{3}} \sqrt{3} (bx^3+a)^{\frac{1}{3}} (-a^2)^{\frac{1}{3}} - \sqrt{3} a}{3a}\right) + 4^{\frac{1}{3}} (-a^2)^{\frac{1}{3}} \log\left(4^{\frac{2}{3}} (bx^3+a)^{\frac{1}{3}} (-a^2)^{\frac{2}{3}} + 2(bx^3+a)^{\frac{2}{3}} a - 2 \cdot 4^{\frac{1}{3}} (-a^2)^{\frac{1}{3}} a\right) - 2 \cdot 4^{\frac{1}{3}} (-a^2)^{\frac{1}{3}} \log\left(-4^{\frac{2}{3}} (-a^2)^{\frac{2}{3}} + 2(bx^3+a)^{\frac{1}{3}} a\right) + 3(bx^3+a)^{\frac{2}{3}}}{6bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4^(1/3)*(-a^2)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*(-a^2)^(1/3)*a) - 2*4^(1/3)*(-a^2)^(1/3)*log(-4^(2/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(1/3)*a) + 3*(b*x^3 + a)^(2/3))/(b*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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mupad [B] time = 4.83, size = 186, normalized size = 1.22

$$\frac{(bx^3+a)^{2/3}}{2bd} - \frac{4^{1/3}a^{2/3} \ln\left(\frac{(bx^3+a)^{1/3} - 2^{1/3}a^{1/3}}{3bd}\right)}{3bd} - \frac{4^{1/3}a^{2/3} \ln\left(\frac{4a^2(bx^3+a)^{1/3} - 24^{2/3}a^{7/3}\left(\frac{-1}{2} + \frac{\sqrt{3}11}{2}\right)^2}{b^2d^2}\right)}{3bd} \left(\frac{-1}{2} + \frac{\sqrt{3}11}{2}\right) + \frac{4^{1/3}a^{2/3} \ln\left(\frac{4a^2(bx^3+a)^{1/3} - 184^{2/3}a^{7/3}\left(\frac{1}{6} + \frac{\sqrt{3}11}{6}\right)^2}{b^2d^2}\right)}{bd} \left(\frac{1}{6} + \frac{\sqrt{3}11}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)

[Out] $4^{1/3}a^{2/3} \log\left(\frac{4a^2(a + bx^3)^{1/3}}{b^2d^2} - \frac{(3^{1/2}i + 1)/6}{b^2d^2}\right) - (18 \cdot 4^{2/3}a^{7/3}) \cdot \frac{(3^{1/2}i + 1)/6}{bd} - 4^{1/3}a^{2/3} \log\left(\frac{a + bx^3}{3bd} - \frac{2^{1/3}a^{1/3}}{3bd}\right) - 4^{1/3}a^{2/3} \log\left(\frac{4a^2(a + bx^3)^{1/3}}{b^2d^2} - \frac{(3^{1/2}i - 1)/2}{b^2d^2}\right) - (2 \cdot 4^{2/3}a^{7/3}) \cdot \frac{(3^{1/2}i - 1)/2}{3bd} - (a + bx^3)^{2/3} / (2bd)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^2(a+bx^3)^{\frac{2}{3}}}{-a+bx^3} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d), x)

[Out] -Integral(x**2*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d

$$3.363 \quad \int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)} dx$$

Optimal. Leaf size=214

$$\frac{\log(a-bx^3)}{3\sqrt[3]{2}\sqrt[3]{ad}} + \frac{\log(\sqrt[3]{a}-\sqrt[3]{a+bx^3})}{2\sqrt[3]{ad}} - \frac{\log(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3})}{\sqrt[3]{2}\sqrt[3]{ad}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ad}} - \frac{2^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ad}}$$

Rubi [A] time = 0.18, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {446, 83, 55, 617, 204, 31}

$$\frac{\log(a-bx^3)}{3\sqrt[3]{2}\sqrt[3]{ad}} + \frac{\log(\sqrt[3]{a}-\sqrt[3]{a+bx^3})}{2\sqrt[3]{ad}} - \frac{\log(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3})}{\sqrt[3]{2}\sqrt[3]{ad}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ad}} - \frac{2^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ad}} - \frac{\log(x)}{2\sqrt[3]{ad}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)), x]

[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*d) - (2^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/((Sqrt[3]*a^(1/3)*d) - Log[x]/(2*a^(1/3)*d) + Log[a - b*x^3]/(3*2^(1/3)*a^(1/3)*d) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(1/3)*d) - Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]/(2^(1/3)*a^(1/3)*d)

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 83

Int[((e_) + (f_)*(x_))^(p_)/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]

], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^{2/3}}{x(ad - bdx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^{2/3}}{x(ad - bdx)} dx, x, x^3 \right) \\
&= \frac{1}{3} (2b) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a + bx} (ad - bdx)} dx, x, x^3 \right) + \frac{\text{Subst} \left(\int \frac{1}{x \sqrt[3]{a + bx}} dx, x, x^3 \right)}{3d} \\
&= -\frac{\log(x)}{2\sqrt[3]{a}d} + \frac{\log(a - bx^3)}{3\sqrt[3]{2}\sqrt[3]{a}d} + \frac{\text{Subst} \left(\int \frac{1}{a^{2/3} + \sqrt[3]{a}x + x^2} dx, x, \sqrt[3]{a + bx^3} \right)}{2d} - \frac{\text{Subst} \left(\int \frac{1}{2^{2/3}a^{2/3} + \sqrt[3]{2}} \right)}{2d} \\
&= -\frac{\log(x)}{2\sqrt[3]{a}d} + \frac{\log(a - bx^3)}{3\sqrt[3]{2}\sqrt[3]{a}d} + \frac{\log(\sqrt[3]{a} - \sqrt[3]{a + bx^3})}{2\sqrt[3]{a}d} - \frac{\log(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a + bx^3})}{\sqrt[3]{2}\sqrt[3]{a}d} - \frac{\text{Subst} \left(\int \frac{1}{2^{2/3}a^{2/3} + \sqrt[3]{2}} \right)}{2d} \\
&= \frac{\tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a + bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3}\sqrt[3]{a}d} - \frac{2^{2/3} \tan^{-1} \left(\frac{1 + \frac{2^{2/3}\sqrt[3]{a + bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3}\sqrt[3]{a}d} - \frac{\log(x)}{2\sqrt[3]{a}d} + \frac{\log(a - bx^3)}{3\sqrt[3]{2}\sqrt[3]{a}d} + \frac{\log(\sqrt[3]{a} - \sqrt[3]{a + bx^3})}{2\sqrt[3]{a}d}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 164, normalized size = 0.77

$$\frac{2^{2/3} \log(a - bx^3) + 3 \log(\sqrt[3]{a} - \sqrt[3]{a + bx^3}) - 3 \cdot 2^{2/3} \log(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a + bx^3}) + 2\sqrt{3} \tan^{-1} \left(\frac{2\sqrt[3]{a + bx^3}}{\sqrt[3]{a}} + 1 \right) - 2 \cdot 2^{2/3} \sqrt{3} \tan^{-1} \left(\frac{2^{2/3}\sqrt[3]{a + bx^3}}{\sqrt[3]{a}} + 1 \right) - 3 \log(x)}{6\sqrt[3]{a}d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)), x]

[Out] (2*sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/sqrt[3]] - 2*2^(2/3)*sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/sqrt[3]] - 3*Log[x] + 2^(2/3)*Log[a - b*x^3] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)] - 3*2^(2/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(1/3)*d)

IntegrateAlgebraic [A] time = 0.35, size = 286, normalized size = 1.34

$$-\frac{\log(a^{2/3} + \sqrt[3]{a}\sqrt[3]{a + bx^3} + (a + bx^3)^{2/3})}{6\sqrt[3]{a}d} + \frac{\log(2a^{2/3} + 2^{2/3}\sqrt[3]{a}\sqrt[3]{a + bx^3} + \sqrt[3]{2}(a + bx^3)^{2/3})}{3\sqrt[3]{2}\sqrt[3]{a}d} + \frac{\log(\sqrt[3]{a + bx^3} - \sqrt[3]{a})}{3\sqrt[3]{a}d} - \frac{2^{2/3} \log(2^{2/3}\sqrt[3]{a + bx^3} - 2\sqrt[3]{a})}{3\sqrt[3]{a}d} + \frac{\tan^{-1} \left(\frac{2\sqrt[3]{a + bx^3}}{\sqrt{3}\sqrt[3]{a}} + \frac{1}{\sqrt{3}} \right)}{\sqrt{3}\sqrt[3]{a}d} - \frac{2^{2/3} \tan^{-1} \left(\frac{2^{2/3}\sqrt[3]{a + bx^3}}{\sqrt{3}\sqrt[3]{a}} + \frac{1}{\sqrt{3}} \right)}{\sqrt{3}\sqrt[3]{a}d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)), x]

```
[Out] ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*d) - (2^(2/3)*ArcTan[1/Sqrt[3] + (2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*d) + Log[-a^(1/3) + (a + b*x^3)^(1/3)]/(3*a^(1/3)*d) - (2^(2/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)]/(3*a^(1/3)*d) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(6*a^(1/3)*d) + Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]/(3*2^(1/3)*a^(1/3)*d)
```

fricas [A] time = 0.74, size = 530, normalized size = 2.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) + 4^(1/3)*a*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*d), -1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 4^(1/3)*a*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algebr
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 +(2*a)^(1/3)*(2*a)^(1/3))-((2*a)^(1/3))^2/sqrt(3)/a/d*atan(((a+b*x^3)^(1/3)
 +1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*(2*a)^(1/3)*1/6/a/d*
 ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-(a^(1/3))^2*1/6/a/d*ln(((a+b*x^3)^(1/3)
))^2+a^(1/3)*(a+b*x^3)^(1/3)+a^(1/3)*a^(1/3))+1/sqrt(3)/a^(1/3)/d*atan(((a+
 b*x^3)^(1/3)+1/2*a^(1/3))/sqrt(3)*2/a^(1/3))+a^(1/3)*a^(1/3)*1/3/a/d*ln(abs
 ((a+b*x^3)^(1/3)-a^(1/3)))

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(-bdx^3 + ad)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d), x)

[Out] int((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(bdx^3 - ad)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x), x)

mupad [B] time = 5.86, size = 369, normalized size = 1.72

$$\frac{b \left(\frac{1}{27} b^2 x^3 + \frac{1}{27} a^2 \right)^{\frac{1}{3}} \sqrt{3} \operatorname{arctan} \left(\frac{\sqrt{3} \left(\frac{1}{27} b^2 x^3 + \frac{1}{27} a^2 \right)^{\frac{1}{3}}}{\frac{1}{27} b^2 x^3 - \frac{1}{27} a^2} \right) - \sqrt{3} \operatorname{arctan} \left(\frac{\sqrt{3} \left(\frac{1}{27} b^2 x^3 + \frac{1}{27} a^2 \right)^{\frac{1}{3}}}{\frac{1}{27} b^2 x^3 - \frac{1}{27} a^2} \right) - \sqrt{3} \operatorname{arctan} \left(\frac{\sqrt{3} \left(\frac{1}{27} b^2 x^3 + \frac{1}{27} a^2 \right)^{\frac{1}{3}}}{\frac{1}{27} b^2 x^3 - \frac{1}{27} a^2} \right)}{b \left(\frac{1}{27} b^2 x^3 + \frac{1}{27} a^2 \right)^{\frac{1}{3}} \sqrt{3} \operatorname{arctan} \left(\frac{\sqrt{3} \left(\frac{1}{27} b^2 x^3 + \frac{1}{27} a^2 \right)^{\frac{1}{3}}}{\frac{1}{27} b^2 x^3 - \frac{1}{27} a^2} \right) - \sqrt{3} \operatorname{arctan} \left(\frac{\sqrt{3} \left(\frac{1}{27} b^2 x^3 + \frac{1}{27} a^2 \right)^{\frac{1}{3}}}{\frac{1}{27} b^2 x^3 - \frac{1}{27} a^2} \right) - \sqrt{3} \operatorname{arctan} \left(\frac{\sqrt{3} \left(\frac{1}{27} b^2 x^3 + \frac{1}{27} a^2 \right)^{\frac{1}{3}}}{\frac{1}{27} b^2 x^3 - \frac{1}{27} a^2} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x)

[Out] $\log(2*(a + b*x^3)^{(1/3)} - 2*2^{(1/3)}*a*d^2*(-1/(a*d^3))^{(2/3)}*(-4/(27*a*d^3))^{(1/3)} + \log((a + b*x^3)^{(1/3)} - a*d^2*(1/(a*d^3))^{(2/3)}*(1/(27*a*d^3))^{(1/3)} - \log(4*(a + b*x^3)^{(1/3)} + 2*2^{(1/3)}*a*d^2*(-1/(a*d^3))^{(2/3)} - 2^{(1/3)}*3^{(1/2)}*a*d^2*(-1/(a*d^3))^{(2/3)}*2i)*((3^{(1/2)}*1i)/2 + 1/2)*(-4/(27*a*d^3))^{(1/3)} + \log(4*(a + b*x^3)^{(1/3)} + 2*2^{(1/3)}*a*d^2*(-1/(a*d^3))^{(2/3)} + 2^{(1/3)}*3^{(1/2)}*a*d^2*(-1/(a*d^3))^{(2/3)}*2i)*((3^{(1/2)}*1i)/2 - 1/2)*(-4/(27*a*d^3))^{(1/3)} - \log(2*(a + b*x^3)^{(1/3)} + a*d^2*(1/(a*d^3))^{(2/3)} - 3^{(1/2)}*a*d^2*(1/(a*d^3))^{(2/3)}*1i)*((3^{(1/2)}*1i)/2 + 1/2)*(1/(27*a*d^3))^{(1/3)} + \log(2*(a + b*x^3)^{(1/3)} + a*d^2*(1/(a*d^3))^{(2/3)} + 3^{(1/2)}*a*d^2*(1/(a*d^3))^{(2/3)}*1i)*((3^{(1/2)}*1i)/2 - 1/2)*(1/(27*a*d^3))^{(1/3)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax+bx^4} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/x/(-b*d*x**3+a*d),x)

[Out] -Integral((a + b*x**3)**(2/3)/(-a*x + b*x**4), x)/d

$$3.364 \quad \int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)} dx$$

Optimal. Leaf size=269

$$\frac{b \log(a - bx^3)}{3\sqrt[3]{2} a^{4/3} d} + \frac{5b \log(\sqrt[3]{a} - \sqrt[3]{a + bx^3})}{6a^{4/3} d} - \frac{b \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + bx^3})}{\sqrt[3]{2} a^{4/3} d} + \frac{5b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} a^{4/3} d} - \frac{2^{2/3} b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.26, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {446, 103, 156, 50, 55, 617, 204, 31}

$$-\frac{(a+bx^3)^{5/3}}{3a^2 dx^3} + \frac{b(a+bx^3)^{2/3}}{3a^2 d} + \frac{b \log(a-bx^3)}{3\sqrt[3]{2} a^{4/3} d} + \frac{5b \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{6a^{4/3} d} - \frac{b \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3})}{\sqrt[3]{2} a^{4/3} d} + \frac{5b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{3\sqrt{3} a^{4/3} d} - \frac{2^{2/3} b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{4/3} d} - \frac{5b \log(x)}{6a^{4/3} d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)), x]

[Out] (b*(a + b*x^3)^(2/3))/(3*a^2*d) - (a + b*x^3)^(5/3)/(3*a^2*d*x^3) + (5*b*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*d) - (2^(2/3)*b*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*d) - (5*b*Log[x])/(6*a^(4/3)*d) + (b*Log[a - b*x^3])/(3*2^(1/3)*a^(4/3)*d) + (5*b*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(4/3)*d) - (b*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*a^(4/3)*d)

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m+1)*(c + d*x)^n)/(b*(m+n+1)), x] + Dist[(n*(b*c - a*d))/(b*(m+n+1)), Int[(a + b*x)^m*(c + d*x)^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 55

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x]

] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^{2/3}}{x^4(ad - bdx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^{2/3}}{x^2(ad - bdx)} dx, x, x^3 \right) \\
&= -\frac{(a + bx^3)^{5/3}}{3a^2 dx^3} - \frac{\text{Subst} \left(\int \frac{(a+bx)^{2/3} \left(-\frac{5}{3}abd + \frac{2}{3}b^2 dx\right)}{x(ad-bdx)} dx, x, x^3 \right)}{3a^2 d} \\
&= -\frac{(a + bx^3)^{5/3}}{3a^2 dx^3} + \frac{b^2 \text{Subst} \left(\int \frac{(a+bx)^{2/3}}{ad-bdx} dx, x, x^3 \right)}{3a^2} + \frac{(5b) \text{Subst} \left(\int \frac{(a+bx)^{2/3}}{x} dx, x, x^3 \right)}{9a^2 d} \\
&= \frac{b(a + bx^3)^{2/3}}{3a^2 d} - \frac{(a + bx^3)^{5/3}}{3a^2 dx^3} + \frac{(2b^2) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)} dx, x, x^3 \right)}{3a} + \frac{(5b) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, x^3 \right)}{6a^{4/3} d} \\
&= \frac{b(a + bx^3)^{2/3}}{3a^2 d} - \frac{(a + bx^3)^{5/3}}{3a^2 dx^3} - \frac{5b \log(x)}{6a^{4/3} d} + \frac{b \log(a - bx^3)}{3\sqrt[3]{2} a^{4/3} d} - \frac{(5b) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, x^3 \right)}{6a^{4/3} d} \\
&= \frac{b(a + bx^3)^{2/3}}{3a^2 d} - \frac{(a + bx^3)^{5/3}}{3a^2 dx^3} - \frac{5b \log(x)}{6a^{4/3} d} + \frac{b \log(a - bx^3)}{3\sqrt[3]{2} a^{4/3} d} + \frac{5b \log(\sqrt[3]{a} - \sqrt[3]{a + bx^3})}{6a^{4/3} d} \\
&= \frac{b(a + bx^3)^{2/3}}{3a^2 d} - \frac{(a + bx^3)^{5/3}}{3a^2 dx^3} + \frac{5b \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{3\sqrt{3} a^{4/3} d} - \frac{2^{2/3} b \tan^{-1} \left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3} a^{4/3} d} - \frac{5b \log(x)}{6a^{4/3} d}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 213, normalized size = 0.79

$$\frac{10\sqrt{3}bx^3 \tan^{-1} \left(\frac{2\sqrt[3]{a+bx^3} + 1}{\sqrt{3}} \right) - 3 \left(2\sqrt[3]{a} (a + bx^3)^{2/3} - 2^{2/3} bx^3 \log(a - bx^3) - 5bx^3 \log(\sqrt[3]{a} - \sqrt[3]{a + bx^3}) + 3 \cdot 2^{2/3} bx^3 \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + bx^3}) + 2 \cdot 2^{2/3} \sqrt{3} bx^3 \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + 1}{\sqrt{3}} \right) + 5bx^3 \log(x) \right)}{18a^{4/3} dx^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]

[Out] (10*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 3*(2*a^(1/3)*(a + b*x^3)^(2/3) + 2*2^(2/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 5*b*x^3*Log[x] - 2^(2/3)*b*x^3*Log[a - b*x^3] - 5*b*x^3*Log[a^(1/3) - (a + b*x^3)^(1/3)] + 3*2^(2/3)*b*x^3*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]))/(18*a^(4/3)*d*x^3)

IntegrateAlgebraic [A] time = 0.55, size = 319, normalized size = 1.19

$$\frac{5b \log(\sqrt[3]{a+bx^3} - \sqrt[3]{a})}{9a^{4/3}d} - \frac{2^{2/3}b \log(2^{2/3}\sqrt[3]{a+bx^3} - 2\sqrt[3]{a})}{3a^{4/3}d} - \frac{5b \log(a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3})}{18a^{4/3}d} + \frac{b \log(2a^{2/3} + 2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3} + \sqrt[3]{2}(a+bx^3)^{2/3})}{3\sqrt[3]{2}a^{4/3}d} + \frac{5b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{3}\sqrt[3]{a}} + \frac{1}{\sqrt[3]{3}}\right)}{3\sqrt[3]{3}a^{4/3}d} - \frac{2^{2/3}b \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{3}\sqrt[3]{a}} + \frac{1}{\sqrt[3]{3}}\right)}{\sqrt[3]{3}a^{4/3}d} - \frac{(a+bx^3)^{2/3}}{3adx^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)), x]

[Out]
$$-1/3*(a + b*x^3)^{(2/3)/(a*d*x^3)} + (5*b*ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(3*Sqrt[3]*a^{(4/3)*d}) - (2^{(2/3)*b*ArcTan[1/Sqrt[3] + (2^{(2/3)*(a + b*x^3)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])})/(Sqrt[3]*a^{(4/3)*d}) + (5*b*Log[-a^{(1/3)} + (a + b*x^3)^{(1/3)}])/(9*a^{(4/3)*d}) - (2^{(2/3)*b*Log[-2*a^{(1/3)} + 2^{(2/3)*(a + b*x^3)^{(1/3)}])})/(3*a^{(4/3)*d}) - (5*b*Log[a^{(2/3)} + a^{(1/3)*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}])/(18*a^{(4/3)*d}) + (b*Log[2*a^{(2/3)} + 2^{(2/3)*a^{(1/3)*(a + b*x^3)^{(1/3)} + 2^{(1/3)*(a + b*x^3)^{(2/3)}])})/(3*2^{(1/3)*a^{(4/3)*d})}$$

fricas [A] time = 0.51, size = 612, normalized size = 2.28

$$\frac{5b \log(\sqrt[3]{a+bx^3} - \sqrt[3]{a})}{9a^{4/3}d} - \frac{2^{2/3}b \log(2^{2/3}\sqrt[3]{a+bx^3} - 2\sqrt[3]{a})}{3a^{4/3}d} - \frac{5b \log(a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3})}{18a^{4/3}d} + \frac{b \log(2a^{2/3} + 2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3} + \sqrt[3]{2}(a+bx^3)^{2/3})}{3\sqrt[3]{2}a^{4/3}d} + \frac{5b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{3}\sqrt[3]{a}} + \frac{1}{\sqrt[3]{3}}\right)}{3\sqrt[3]{3}a^{4/3}d} - \frac{2^{2/3}b \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{3}\sqrt[3]{a}} + \frac{1}{\sqrt[3]{3}}\right)}{\sqrt[3]{3}a^{4/3}d} - \frac{(a+bx^3)^{2/3}}{3adx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d), x, algorithm="fricas")

[Out]
$$[-1/18*(6*4^{(1/3)*sqrt(3)*a*b*x^3*(-1/a)^{(1/3)*arctan(1/3*4^{(1/3)*sqrt(3)*b*x^3 + a)^{(1/3)*(-1/a)^{(1/3)} - 1/3*sqrt(3)} - 15*sqrt(1/3)*a*b*x^3*sqrt(-1/a^{(2/3)})*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^{(2/3)*a^{(2/3)} - (b*x^3 + a)^{(1/3)*a - a^{(4/3)})*sqrt(-1/a^{(2/3)}) - 3*(b*x^3 + a)^{(1/3)*a^{(2/3)} + 3*a)/x^3} + 3*4^{(1/3)*a*b*x^3*(-1/a)^{(1/3)*log(4^{(2/3)*(b*x^3 + a)^{(1/3)*a*(-1/a)^{(2/3)} - 2*4^{(1/3)*a*(-1/a)^{(1/3)} + 2*(b*x^3 + a)^{(2/3)}) - 6*4^{(1/3)*a*b*x^3*(-1/a)^{(1/3)*log(-4^{(2/3)*a*(-1/a)^{(2/3)} + 2*(b*x^3 + a)^{(1/3)}) + 5*a^{(2/3)*b*x^3*log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)*a^{(1/3)} + a^{(2/3)}) - 10*a^{(2/3)*b*x^3*log((b*x^3 + a)^{(1/3)} - a^{(1/3)}) + 6*(b*x^3 + a)^{(2/3)*a})/(a^2*d*x^3), -1/18*(6*4^{(1/3)*sqrt(3)*a*b*x^3*(-1/a)^{(1/3)*arctan(1/3*4^{(1/3)*sqrt(3)*b*x^3 + a)^{(1/3)*(-1/a)^{(1/3)} - 1/3*sqrt(3)} + 3*4^{(1/3)*a*b*x^3*(-1/a)^{(1/3)*log(4^{(2/3)*(b*x^3 + a)^{(1/3)*a*(-1/a)^{(2/3)} - 2*4^{(1/3)*a*(-1/a)^{(1/3)} + 2*(b*x^3 + a)^{(2/3)}) - 6*4^{(1/3)*a*b*x^3*(-1/a)^{(1/3)*log(-4^{(2/3)*a*(-1/a)^{(2/3)} + 2*(b*x^3 + a)^{(1/3)}) - 30*sqrt(1/3)*a^{(2/3)*b*x^3*arctan(sqrt(1/3)*(2*(b*x^3 + a)^{(1/3)} + a^{(1/3)})/a^{(1/3)}) + 5*a^{(2/3)*b*x^3*log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)*a^{(1/3)} + a^{(2/3)}) - 10*a^{(2/3)*b*x^3*log((b*x^3 + a)^{(1/3)} - a^{(1/3)}) + 6*(b*x^3 + a)^{(2/3)*a})/(a^2*d*x^3)]}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
```

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[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algebr
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3)+(2*a)^(1/3)*(2*a)^(1/3))-((2*a)^(1/3))^2*b/sqrt(3)/a^2/d*atan(((a+b*x^3
)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*b*(2*a)^(1/3)*
1/6/a^2/d*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-5*(a^(1/3))^2*b/1/18/a^2/d*1
n(((a+b*x^3)^(1/3))^2+a^(1/3)*(a+b*x^3)^(1/3)+a^(1/3)*a^(1/3))+5*b/3/sqrt(3
)/a/a^(1/3)/d*atan(((a+b*x^3)^(1/3)+1/2*a^(1/3))/sqrt(3)*2/a^(1/3))+5*a^(1/
3)*b*a^(1/3)*1/9/a^2/d*ln(abs((a+b*x^3)^(1/3)-a^(1/3)))-((a+b*x^3)^(1/3))^2
*b/3/a/d/(a+b*x^3-a)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(-bdx^3 + ad)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d), x)

[Out] int((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(bdx^3 - ad)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d), x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^4), x)

mupad [B] time = 5.56, size = 490, normalized size = 1.82

$$\frac{b^{\frac{2}{3}}(bx^3+a)^{\frac{2}{3}}(-bdx^3+ad)^{\frac{2}{3}}}{(bdx^3-ad)^{\frac{2}{3}}(bx^3+a)^{\frac{2}{3}}(-bdx^3+ad)^{\frac{2}{3}}} - \frac{b^{\frac{2}{3}}(bx^3+a)^{\frac{2}{3}}(-bdx^3+ad)^{\frac{2}{3}}}{(bdx^3-ad)^{\frac{2}{3}}(bx^3+a)^{\frac{2}{3}}(-bdx^3+ad)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)), x)

[Out] log(2*b^2*(a + b*x^3)^(1/3) - 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3))*(-(4*b^3)/(27*a^4*d^3))^(1/3) + (5*log(b^2*(a + b*x^3)^(1/3) - a^3*d^2*(b^3/(a^4*d^3))^(2/3))*(b^3/(a^4*d^3))^(1/3))/9 - log(4*b^2*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3) - 2^(1/3)*3^(1/2)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 + 1/2)*(-(4*b^3)/(27*a^4*d^3))^(1/3) + log(4*b^2*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3) + 2^(1/3)*3^(1/2)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-(4*b^3)/(27*a^4*d^3))^(1/3) - log(2*b^2*(a + b*x^3)^(1/3) + a^3*d^2*(b^3/(a^4*d^3))^(2/3) - 3^(1/2)*a^3*d^2*(b^3/(a^4*d^3))^(2/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*((125*b^3)/(729*a^4*d^3))^(1/3) + log(2*b^2*(a + b*x^3)^(1/3) + a^3*d^2*(b^3/(a^4*d^3))^(2/3) + 3^(1/2)*a^3*d^2*(b^3/(a^4*d^3))^(2/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*((125*b^3)/(729*a^4*d^3))^(1/3) - (b*(a + b*x^3)^(2/3))/(3*a*(d*(a + b*x^3) - a*d))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^4+bx^7} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/x**4/(-b*d*x**3+a*d), x)

[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**4 + b*x**7), x)/d

$$3.365 \quad \int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)} dx$$

Optimal. Leaf size=284

$$\frac{b^2 \log(a-bx^3)}{3\sqrt[3]{2} a^{7/3} d} + \frac{7b^2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{9a^{7/3} d} - \frac{b^2 \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3})}{\sqrt[3]{2} a^{7/3} d} + \frac{14b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{9\sqrt{3} a^{7/3} d} - \frac{2^{2/3} b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{9\sqrt{3} a^{7/3} d}$$

Rubi [A] time = 0.28, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {446, 103, 149, 156, 55, 617, 204, 31}

$$\frac{b^2 \log(a-bx^3)}{3\sqrt[3]{2} a^{7/3} d} + \frac{7b^2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{9a^{7/3} d} - \frac{b^2 \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3})}{\sqrt[3]{2} a^{7/3} d} + \frac{14b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{9\sqrt{3} a^{7/3} d} - \frac{2^{2/3} b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{7/3} d} - \frac{7b^2 \log(x)}{9a^{7/3} d} - \frac{5b(a+bx^3)^{2/3}}{18a^2 dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2 dx^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)), x]

[Out] (-5*b*(a + b*x^3)^(2/3)/(18*a^2*d*x^3) - (a + b*x^3)^(5/3)/(6*a^2*d*x^6) + (14*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*d) - (2^(2/3)*b^2*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*d) - (7*b^2*Log[x])/(9*a^(7/3)*d) + (b^2*Log[a - b*x^3])/(3*2^(1/3)*a^(7/3)*d) + (7*b^2*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(9*a^(7/3)*d) - (b^2*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*a^(7/3)*d)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x

$$\int \frac{(a + b*x)^{m+1} * (c + d*x)^n * (e + f*x)^p}{(m+1)*(b*c - a*d)*(b*e - a*f)} dx + \text{Dist}\left[\frac{1}{(m+1)*(b*c - a*d)*(b*e - a*f)}, \text{Int}[(a + b*x)^{m+1} * (c + d*x)^n * (e + f*x)^p * \text{Simp}[a*d*f*(m+1) - b*(d*e*(m+n+2) + c*f*(m+p+2)) - b*d*f*(m+n+p+3)*x, x], x\right] /;$$

$$\text{FreeQ}\{a, b, c, d, e, f, n, p\}, x \} \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ (\text{IntegerQ}[n] \ || \ \text{IntegersQ}[2*n, 2*p])$$

Rule 149

$$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p * \text{Simp}[(b*g - a*h)*(a + b*x)^{m+1} * (c + d*x)^n * (e + f*x)^{p+1} / (b*(b*e - a*f)*(m+1)), x] - \text{Dist}\left[\frac{1}{b*(b*e - a*f)*(m+1)}, \text{Int}[(a + b*x)^{m+1} * (c + d*x)^{n-1} * (e + f*x)^p * \text{Simp}[b*c*(f*g - e*h)*(m+1) + (b*g - a*h)*(d*e*n + c*f*(p+1)) + d*(b*(f*g - e*h)*(m+1) + f*(b*g - a*h)*(n+p+1))*x, x], x\right] /;$$

$$\text{FreeQ}\{a, b, c, d, e, f, g, h, p\}, x \} \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$$

Rule 156

$$\text{Int}[(e + f*x)^p * (g + h*x) / ((a + b*x) * (c + d*x)), x] \ \> \ \text{Dist}\left[\frac{b*g - a*h}{b*c - a*d}, \text{Int}[(e + f*x)^p / (a + b*x), x], x\right] - \text{Dist}\left[\frac{d*g - c*h}{b*c - a*d}, \text{Int}[(e + f*x)^p / (c + d*x), x], x\right] /;$$

$$\text{FreeQ}\{a, b, c, d, e, f, g, h\}, x \}$$

Rule 204

$$\text{Int}[(a + b*x)^{-1}, x] \ \> \ -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x] / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] * \text{Rt}[-b, 2]), x] /;$$

$$\text{FreeQ}\{a, b\}, x \} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

Rule 446

$$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p * \text{Simp}[(m+1)/n - 1 * (a + b*x)^p * (c + d*x)^q, x], x] /;$$

$$\text{FreeQ}\{a, b, c, d, m, n, p, q\}, x \} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$$

Rule 617

$$\text{Int}[(a + b*x + c*x^2)^{-1}, x] \ \> \ \text{With}\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /;$$

$$\text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c]) /;$$

$$\text{FreeQ}\{a, b, c\}, x \} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$$

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a+bx)^{2/3}}{x^3(ad-bdx)} dx, x, x^3 \right) \\
&= -\frac{(a+bx^3)^{5/3}}{6a^2 dx^6} - \frac{\text{Subst} \left(\int \frac{(a+bx)^{2/3} \left(-\frac{5}{3}abd - \frac{1}{3}b^2 dx \right)}{x^2(ad-bdx)} dx, x, x^3 \right)}{6a^2 d} \\
&= -\frac{5b(a+bx^3)^{2/3}}{18a^2 dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2 dx^6} - \frac{\text{Subst} \left(\int \frac{-\frac{28}{9}a^2 b^2 d^2 - \frac{8}{9}ab^3 d^2 x}{x^3 \sqrt[3]{a+bx}(ad-bdx)} dx, x, x^3 \right)}{6a^3 d^2} \\
&= -\frac{5b(a+bx^3)^{2/3}}{18a^2 dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2 dx^6} + \frac{(2b^3) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)} dx, x, x^3 \right)}{3a^2} + \frac{(14b^2) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, x^3 \right)}{3a^2} \\
&= -\frac{5b(a+bx^3)^{2/3}}{18a^2 dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2 dx^6} - \frac{7b^2 \log(x)}{9a^{7/3} d} + \frac{b^2 \log(a-bx^3)}{3\sqrt[3]{2} a^{7/3} d} - \frac{(7b^2) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, x^3 \right)}{9a^{7/3} d} \\
&= -\frac{5b(a+bx^3)^{2/3}}{18a^2 dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2 dx^6} - \frac{7b^2 \log(x)}{9a^{7/3} d} + \frac{b^2 \log(a-bx^3)}{3\sqrt[3]{2} a^{7/3} d} + \frac{7b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{9a^{7/3} d} \\
&= -\frac{5b(a+bx^3)^{2/3}}{18a^2 dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2 dx^6} + \frac{14b^2 \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{9\sqrt{3} a^{7/3} d} - \frac{2^{2/3} b^2 \tan^{-1} \left(\frac{1 + \frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3} a^{7/3} d}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 247, normalized size = 0.87

$$\frac{28\sqrt{3}b^2x^6 \tan^{-1} \left(\frac{2\sqrt[3]{a+bx^3} + 1}{\sqrt{3}} \right) - 3 \left(3a^{4/3}(a+bx^3)^{2/3} - 3 \cdot 2^{2/3} b^2 x^6 \log(a-bx^3) - 14b^2 x^6 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right) + 9 \cdot 2^{2/3} b^2 x^6 \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+bx^3}\right) + 6 \cdot 2^{2/3} \sqrt{3} b^2 x^6 \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{a+bx^3} + 1}{\sqrt{3}} \right) + 8 \sqrt[3]{a} b x^3 (a+bx^3)^{2/3} + 14b^2 x^6 \log(x) \right)}{54a^{7/3} d x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)), x]

[Out] (28*sqrt[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/sqrt[3]] - 3*(3*a^(4/3)*(a + b*x^3)^(2/3) + 8*a^(1/3)*b*x^3*(a + b*x^3)^(2/3) + 6*2^(2/3)*sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/sqrt[3]] + 14*b^2*x^6*Log[x] - 3*2^(2/3)*b^2*x^6*Log[a - b*x^3] - 14*b^2*x^6*Log[a^(1/3) - (a + b*x^3)^(1/3)] + 9*2^(2/3)*b^2*x^6*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(54*a^(7/3)*d*x^6)

IntegrateAlgebraic [A] time = 0.68, size = 341, normalized size = 1.20

$$\frac{14b^2 \log(\sqrt{a+bx^3} - \sqrt{a})}{27a^{7/3}d} - \frac{2^{2/3}b^2 \log(2^{2/3}\sqrt{a+bx^3} - 2\sqrt{a})}{3a^{7/3}d} - \frac{7b^2 \log(a^{2/3} + \sqrt{a}\sqrt{a+bx^3} + (a+bx^3)^{2/3})}{27a^{7/3}d} + \frac{b^2 \log(2a^{2/3} + 2^{2/3}\sqrt{a}\sqrt{a+bx^3} + \sqrt{2}(a+bx^3)^{2/3})}{3\sqrt{2}a^{7/3}d} + \frac{14b^2 \tan^{-1}\left(\frac{2\sqrt{a+bx^3}}{\sqrt{3}\sqrt{a}} + \frac{1}{\sqrt{3}}\right)}{9\sqrt{3}a^{7/3}d} - \frac{2^{2/3}b^2 \tan^{-1}\left(\frac{2^{2/3}\sqrt{a+bx^3}}{\sqrt{3}\sqrt{a}} + \frac{1}{\sqrt{3}}\right)}{\sqrt{3}a^{7/3}d} + \frac{(-3a-8bx^3)(a+bx^3)^{2/3}}{18a^2dx^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]

[Out]
$$\begin{aligned} &((-3*a - 8*b*x^3)*(a + b*x^3)^{(2/3)})/(18*a^2*d*x^6) + (14*b^2*ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(9*Sqrt[3]*a^{(7/3)*d}) - (2^{(2/3)}*b^2*ArcTan[1/Sqrt[3] + (2^{(2/3)}*(a + b*x^3)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(7/3)*d}) \\ &+ (14*b^2*Log[-a^{(1/3)} + (a + b*x^3)^{(1/3)}])/(27*a^{(7/3)*d}) - (2^{(2/3)}*b^2*Log[-2*a^{(1/3)} + 2^{(2/3)}*(a + b*x^3)^{(1/3)}])/(3*a^{(7/3)*d}) \\ &- (7*b^2*Log[a^{(2/3)} + a^{(1/3)}*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}])/(27*a^{(7/3)*d}) + (b^2*Log[2*a^{(2/3)} + 2^{(2/3)}*a^{(1/3)}*(a + b*x^3)^{(1/3)} + 2^{(1/3)}*(a + b*x^3)^{(2/3)}])/(3*2^{(1/3)}*a^{(7/3)*d}) \end{aligned}$$

fricas [A] time = 0.50, size = 660, normalized size = 2.32

$$\frac{14b^2 \log(\sqrt{a+bx^3} - \sqrt{a})}{27a^{7/3}d} - \frac{2^{2/3}b^2 \log(2^{2/3}\sqrt{a+bx^3} - 2\sqrt{a})}{3a^{7/3}d} - \frac{7b^2 \log(a^{2/3} + \sqrt{a}\sqrt{a+bx^3} + (a+bx^3)^{2/3})}{27a^{7/3}d} + \frac{b^2 \log(2a^{2/3} + 2^{2/3}\sqrt{a}\sqrt{a+bx^3} + \sqrt{2}(a+bx^3)^{2/3})}{3\sqrt{2}a^{7/3}d} + \frac{14b^2 \tan^{-1}\left(\frac{2\sqrt{a+bx^3}}{\sqrt{3}\sqrt{a}} + \frac{1}{\sqrt{3}}\right)}{9\sqrt{3}a^{7/3}d} - \frac{2^{2/3}b^2 \tan^{-1}\left(\frac{2^{2/3}\sqrt{a+bx^3}}{\sqrt{3}\sqrt{a}} + \frac{1}{\sqrt{3}}\right)}{\sqrt{3}a^{7/3}d} + \frac{(-3a-8bx^3)(a+bx^3)^{2/3}}{18a^2dx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out]
$$\begin{aligned} &[-1/54*(18*4^{(1/3)}*sqrt(3)*a*b^2*x^6*(-1/a)^{(1/3)}*arctan(1/3*4^{(1/3)}*sqrt(3)*(b*x^3 + a)^{(1/3)}*(-1/a)^{(1/3)} - 1/3*sqrt(3)) - 42*sqrt(1/3)*a*b^2*x^6*sqrt(-1/a^{(2/3)})*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^{(2/3)}*a^{(2/3)} - (b*x^3 + a)^{(1/3)}*a - a^{(4/3)})*sqrt(-1/a^{(2/3)}) - 3*(b*x^3 + a)^{(1/3)}*a^{(2/3)} + 3*a)/x^3) + 9*4^{(1/3)}*a*b^2*x^6*(-1/a)^{(1/3)}*log(4^{(2/3)}*(b*x^3 + a)^{(1/3)}*a*(-1/a)^{(2/3)} - 2*4^{(1/3)}*a*(-1/a)^{(1/3)} + 2*(b*x^3 + a)^{(2/3)}) - 18*4^{(1/3)}*a*b^2*x^6*(-1/a)^{(1/3)}*log(-4^{(2/3)}*a*(-1/a)^{(2/3)} + 2*(b*x^3 + a)^{(1/3)}) + 14*a^{(2/3)}*b^2*x^6*log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*a^{(1/3)} + a^{(2/3)}) - 28*a^{(2/3)}*b^2*x^6*log((b*x^3 + a)^{(1/3)} - a^{(1/3)}) + 3*(8*a*b*x^3 + 3*a^2)*(b*x^3 + a)^{(2/3)}/(a^3*d*x^6), -1/54*(18*4^{(1/3)}*sqrt(3)*a*b^2*x^6*(-1/a)^{(1/3)}*arctan(1/3*4^{(1/3)}*sqrt(3)*(b*x^3 + a)^{(1/3)}*(-1/a)^{(1/3)} - 1/3*sqrt(3)) + 9*4^{(1/3)}*a*b^2*x^6*(-1/a)^{(1/3)}*log(4^{(2/3)}*(b*x^3 + a)^{(1/3)}*a*(-1/a)^{(2/3)} - 2*4^{(1/3)}*a*(-1/a)^{(1/3)} + 2*(b*x^3 + a)^{(2/3)}) - 18*4^{(1/3)}*a*b^2*x^6*(-1/a)^{(1/3)}*log(-4^{(2/3)}*a*(-1/a)^{(2/3)} + 2*(b*x^3 + a)^{(1/3)}) - 84*sqrt(1/3)*a^{(2/3)}*b^2*x^6*arctan(sqrt(1/3)*(2*(b*x^3 + a)^{(1/3)} + a^{(1/3)})/a^{(1/3)}) + 14*a^{(2/3)}*b^2*x^6*log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*a^{(1/3)} + a^{(2/3)}) - 28*a^{(2/3)}*b^2*x^6*log((b*x^3 + a)^{(1/3)} - a^{(1/3)}) + 3*(8*a*b*x^3 + 3*a^2)*(b*x^3 + a)^{(2/3)}/(a^3*d*x^6)] \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(-bdx^3 + ad)x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x)

[Out] int((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(bdx^3 - ad)x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^7), x)

mupad [B] time = 5.45, size = 513, normalized size = 1.81

(Small, illegible mathematical expression)

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x)

[Out] ((5*b^2*(a + b*x^3)^(2/3))/(18*a) - (4*b^2*(a + b*x^3)^(5/3))/(9*a^2))/(d*(a + b*x^3)^2 + a^2*d - 2*a*d*(a + b*x^3)) + log(2*b^4*(a + b*x^3)^(1/3) - 2*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3))*(- (4*b^6)/(27*a^7*d^3))^(1/3) + (14*log(b^4*(a + b*x^3)^(1/3) - a^5*d^2*(b^6/(a^7*d^3))^(2/3))*(b^6/(a^7*d^3))^^(1/3))/27 - log(4*b^4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3) - 2^(1/3)*3^(1/2)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 + 1/2)*(- (4*b^6)/(27*a^7*d^3))^(1/3) + log(4*b^4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3) + 2^(1/3)*3^(1/2)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(- (4*b^6)/(27*a^7*d^3))^(1/3) - (7*log(2*b^4*(a + b*x^3)^(1/3) + a^5*d^2*(b^6/(a^7*d^3))^(2/3) - 3^(1/2)*a^5*d^2*(b^6/(a^7*d^3))^(2/3)*1i)*(3^(1/2)*1i + 1)*(b^6/(a^7*d^3))^(1/3))/27 + (7*log(2*b^4*(a + b*x^3)^(1/3) + a^5*d^2*(b^6/(a^7*d^3))^(2/3) + 3^(1/2)*a^5*d^2*(b^6/(a^7*d^3))^(2/3)*1i)*(3^(1/2)*1i - 1)*(b^6/(a^7*d^3))^(1/3))/27

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^7+bx^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/x**7/(-b*d*x**3+a*d), x)

[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**7 + b*x**10), x)/d

$$3.366 \quad \int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3} dx$$

Optimal. Leaf size=264

$$\frac{a^2 \log(ad - bdx^3)}{3\sqrt[3]{2} b^{7/3} d} - \frac{a^2 \log\left(\sqrt[3]{2} \sqrt[3]{b} x - \sqrt[3]{a + bx^3}\right)}{\sqrt[3]{2} b^{7/3} d} + \frac{7a^2 \log\left(\sqrt[3]{a + bx^3} - \sqrt[3]{b} x\right)}{9b^{7/3} d} - \frac{14a^2 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right)}{9\sqrt{3} b^{7/3} d} + \frac{2^{2/3} a^2}{9\sqrt{3} b^{7/3} d}$$

Rubi [C] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$\frac{x^7 (a + bx^3)^{2/3} F_1\left(\frac{7}{3}; -\frac{2}{3}, 1; \frac{10}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{7ad \left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

[Out] (x^7*(a + b*x^3)^(2/3)*AppellF1[7/3, -2/3, 1, 10/3, -((b*x^3)/a), (b*x^3)/a])/ (7*a*d*(1 + (b*x^3)/a)^(2/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/ (e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/ (1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^6 (a + bx^3)^{2/3}}{ad - bdx^3} dx = \frac{(a + bx^3)^{2/3} \int \frac{x^6 \left(1 + \frac{bx^3}{a}\right)^{2/3}}{ad - bdx^3} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x^7 (a + bx^3)^{2/3} F_1\left(\frac{7}{3}; -\frac{2}{3}, 1; \frac{10}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{7ad \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.30, size = 244, normalized size = 0.92

$$\frac{2 \cdot 2^{2/3} a^2 \sqrt[3]{a + bx^3} \left(\log\left(\frac{2^{2/3} b^{2/3} x^2}{(ax^3+b)^{2/3}} + \frac{\sqrt[3]{2} \sqrt[3]{bx^3}}{\sqrt[3]{ax^3+b}} + 1\right) - 2 \log\left(1 - \frac{\sqrt[3]{2} \sqrt[3]{bx^3}}{\sqrt[3]{ax^3+b}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{2 \sqrt[3]{2} \sqrt[3]{bx^3} + 1}{\sqrt[3]{ax^3+b}}\right) \right) + 21ab^{4/3} x^4 \sqrt[3]{\frac{bx^3}{a}} + 1 F_1\left(\frac{4}{3}; \frac{1}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) - 3\sqrt[3]{b} (a + bx^3) (8ax + 3bx^4)}{54b^{7/3} d \sqrt[3]{a + bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

[Out] (-3*b^(1/3)*(a + b*x^3)*(8*a*x + 3*b*x^4) + 21*a*b^(4/3)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -(b*x^3)/a, (b*x^3)/a] + 2*2^(2/3)*a^2*(a + b*x^3)^(1/3)*(2*sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3))/sqrt[3]] - 2*Log[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2/3)*x^2)/(b + a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)]))/(54*b^(7/3)*d*(a + b*x^3)^(1/3))

IntegrateAlgebraic [A] time = 1.14, size = 361, normalized size = 1.37

$$\frac{14a^2 \log(\sqrt[3]{a+bx^3} - \sqrt[3]{bx^3})}{27b^{7/3}d} - \frac{2^{2/3}a^2 \log(2^{2/3}\sqrt[3]{a+bx^3} - 2\sqrt[3]{bx^3})}{36b^{7/3}d} - \frac{14a^2 \tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{bx^3}}{2\sqrt[3]{a+bx^3} + \sqrt[3]{bx^3}}\right)}{9\sqrt[3]{b^{7/3}d}} + \frac{2^{2/3}a^2 \tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{bx^3}}{2\sqrt[3]{a+bx^3} + \sqrt[3]{bx^3}}\right)}{\sqrt[3]{b^{7/3}d}} - \frac{7a^2 \log(\sqrt[3]{bx^3}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3} + b^{2/3}x^2)}{27b^{7/3}d} + \frac{a^2 \log(2^{2/3}\sqrt[3]{bx^3}\sqrt[3]{a+bx^3} + \sqrt[3]{2}(a+bx^3)^{2/3} + 2b^{2/3}x^2)}{3\sqrt[3]{2}b^{7/3}d} + \frac{(a+bx^3)^{2/3}(-8ax - 3bx^4)}{18b^2d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

[Out] ((a + b*x^3)^(2/3)*(-8*a*x - 3*b*x^4))/(18*b^2*d) - (14*a^2*ArcTan[(sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(9*sqrt[3]*b^(7/3)*d) + (2^(2/3)*a^2*ArcTan[(sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))]/(sqrt[3]*b^(7/3)*d) + (14*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*7*b^(7/3)*d) - (2^(2/3)*a^2*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)]/(3*b^(7/3)*d) - (7*a^2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(27*b^(7/3)*d) + (a^2*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]/(3*2^(1/3)*b^(7/3)*d)

fricas [A] time = 0.52, size = 701, normalized size = 2.66



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/54*(18*4^{(1/3)}*\sqrt{3}*a^2*b*(-1/b)^{(1/3)}*\arctan(-1/3*(\sqrt{3}*x - 4^{(1/3)}*\sqrt{3}*(b*x^3 + a)^{(1/3)}*(-1/b)^{(1/3))}/x) - 42*\sqrt{1/3}*a^2*b*\sqrt{-1/b^{(2/3)}}*\log(3*b*x^3 - 3*(b*x^3 + a)^{(1/3)}*b^{(2/3)}*x^2 - 3*\sqrt{1/3}*(b^{(4/3)}*x^3 + (b*x^3 + a)^{(1/3)}*b*x^2 - 2*(b*x^3 + a)^{(2/3)}*b^{(2/3)}*x)*\sqrt{-1/b^{(2/3)}} + 2*a) - 18*4^{(1/3)}*a^2*b*(-1/b)^{(1/3)}*\log(-4^{(2/3)}*b*x*(-1/b)^{(2/3)} - 2*(b*x^3 + a)^{(1/3)})/x) + 9*4^{(1/3)}*a^2*b*(-1/b)^{(1/3)}*\log(-2*4^{(1/3)}*b*x^2*(-1/b)^{(1/3)} - 4^{(2/3)}*(b*x^3 + a)^{(1/3)}*b*x*(-1/b)^{(2/3)} - 2*(b*x^3 + a)^{(2/3)})/x^2) - 28*a^2*b^{(2/3)}*\log(-(b^{(1/3)}*x - (b*x^3 + a)^{(1/3)})/x) + 14*a^2*b^{(2/3)}*\log((b^{(2/3)}*x^2 + (b*x^3 + a)^{(1/3)}*b^{(1/3)}*x + (b*x^3 + a)^{(2/3)})/x^2) + 3*(3*b^2*x^4 + 8*a*b*x)*(b*x^3 + a)^{(2/3)}/(b^3*d), -1/54*(18*4^{(1/3)}*\sqrt{3}*a^2*b*(-1/b)^{(1/3)}*\arctan(-1/3*(\sqrt{3}*x - 4^{(1/3)}*\sqrt{3}*(b*x^3 + a)^{(1/3)}*(-1/b)^{(1/3))}/x) - 18*4^{(1/3)}*a^2*b*(-1/b)^{(1/3)}*\log(-4^{(2/3)}*b*x*(-1/b)^{(2/3)} - 2*(b*x^3 + a)^{(1/3)})/x) + 9*4^{(1/3)}*a^2*b*(-1/b)^{(1/3)}*\log(-2*4^{(1/3)}*b*x^2*(-1/b)^{(1/3)} - 4^{(2/3)}*(b*x^3 + a)^{(1/3)}*b*x*(-1/b)^{(2/3)} - 2*(b*x^3 + a)^{(2/3)})/x^2) - 84*\sqrt{1/3}*a^2*b^{(2/3)}*\arctan(\sqrt{1/3}*(b^{(1/3)}*x + 2*(b*x^3 + a)^{(1/3)})/(b^{(1/3)}*x)) - 28*a^2*b^{(2/3)}*\log(-(b^{(1/3)}*x - (b*x^3 + a)^{(1/3)})/x) + 14*a^2*b^{(2/3)}*\log((b^{(2/3)}*x^2 + (b*x^3 + a)^{(1/3)}*b^{(1/3)}*x + (b*x^3 + a)^{(2/3)})/x^2) + 3*(3*b^2*x^4 + 8*a*b*x)*(b*x^3 + a)^{(2/3)}/(b^3*d)] \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{2}{3}}x^6}{bdx^3 - ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")

[Out] integrate(-(b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}x^6}{-bdx^3 + ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)`

[Out] `int(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{2}{3}} x^6}{bdx^3 - ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")`

[Out] `-integrate((b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6 (bx^3 + a)^{\frac{2}{3}}}{ad - bdx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)`

[Out] `int((x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^6 (a+bx^3)^{\frac{2}{3}}}{-a+bx^3} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)`

[Out] `-Integral(x**6*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d`

$$3.367 \quad \int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3} dx$$

Optimal. Leaf size=229

$$\frac{a \log(ad - bdx^3)}{3\sqrt[3]{2}b^{4/3}d} - \frac{a \log\left(\sqrt[3]{2}\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^{4/3}d} + \frac{5a \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{6b^{4/3}d} - \frac{5a \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d} + \frac{2^{2/3}a \tan^{-1}\left(\frac{\sqrt[3]{a+bx^3}}{\sqrt[3]{b}x}\right)}{\sqrt{3}b^{4/3}d}$$

Rubi [C] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.29, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$\frac{x^4 (a + bx^3)^{2/3} F_1\left(\frac{4}{3}; -\frac{2}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{4ad \left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

[Out] (x^4*(a + b*x^3)^(2/3)*AppellF1[4/3, -2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])/ (4*a*d*(1 + (b*x^3)/a)^(2/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/ (e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/ (1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^3 (a + bx^3)^{2/3}}{ad - bdx^3} dx = \frac{(a + bx^3)^{2/3} \int \frac{x^3 \left(1 + \frac{bx^3}{a}\right)^{2/3}}{ad - bdx^3} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x^4 (a + bx^3)^{2/3} F_1\left(\frac{4}{3}; -\frac{2}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{4ad \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.22, size = 216, normalized size = 0.94

$$\frac{15x^4 \sqrt[3]{\frac{bx^3}{a}} + {}_1F_1\left(\frac{4}{3}; \frac{1}{3}; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} + \frac{2^{2/3} a \left(\log\left(\frac{2^{2/3} b^{2/3} x^2}{(ax^3+b)^{2/3}} + \frac{\sqrt[3]{2} \sqrt[3]{bx^3}}{\sqrt[3]{ax^3+b}} + 1\right) - 2 \log\left(1 - \frac{\sqrt[3]{2} \sqrt[3]{bx^3}}{\sqrt[3]{ax^3+b}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{2^{3/2} \sqrt[3]{bx^3} + 1}{\sqrt[3]{ax^3+b}}\right) \right)}{b^{4/3}} - \frac{12x(a+bx^3)^{2/3}}{b}$$

36d

Warning: Unable to verify antiderivative.

[In] Integrate[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

[Out] ((-12*x*(a + b*x^3)^(2/3))/b + (15*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -(b*x^3)/a, (b*x^3)/a])/(a + b*x^3)^(1/3) + (2^(2/3)*a*(2*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)]/Sqrt[3]] - 2*Log[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2/3)*x^2)/(b + a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)]))/b^(4/3))/(36*d)

IntegrateAlgebraic [A] time = 0.84, size = 339, normalized size = 1.48

$$\frac{5a \log(\sqrt{a+bx^3} - \sqrt{bx^3})}{9b^{4/3}d} - \frac{2^{2/3}a \log(2^{2/3}\sqrt[3]{a+bx^3} - 2\sqrt[3]{bx^3})}{3b^{4/3}d} - \frac{5a \tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{bx^3}}{2\sqrt[3]{a+bx^3} + \sqrt[3]{bx^3}}\right)}{3\sqrt{3}b^{4/3}d} + \frac{2^{2/3}a \tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{bx^3}}{2^{2/3}\sqrt[3]{a+bx^3} + \sqrt[3]{bx^3}}\right)}{\sqrt{3}b^{4/3}d} - \frac{5a \log(\sqrt[3]{bx^3}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3} + b^{2/3}x^2)}{18b^{4/3}d} + \frac{a \log(2^{2/3}\sqrt[3]{bx^3}\sqrt[3]{a+bx^3} + \sqrt[3]{2}(a+bx^3)^{2/3} + 2b^{2/3}x^2)}{3\sqrt{2}b^{4/3}d} - \frac{x(a+bx^3)^{2/3}}{3bd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x]

[Out] -1/3*(x*(a + b*x^3)^(2/3))/(b*d) - (5*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*b^(4/3)*d) + (2^(2/3)*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[3]*b^(4/3)*d) + (5*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(9*b^(4/3)*d) - (2^(2/3)*a*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*b^(4/3)*d) - (5*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*b^(4/3)*d)

+ (a*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(1/3)*b^(4/3)*d)

fricas [A] time = 0.51, size = 653, normalized size = 2.85

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out] [-1/18*(6*4^(1/3)*sqrt(3)*a*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 15*sqrt(1/3)*a*b*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-2*4^(1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d), -1/18*(6*4^(1/3)*sqrt(3)*a*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-2*4^(1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 30*sqrt(1/3)*a*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3)))/(b^(1/3)*x)) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{2}{3}}x^3}{bdx^3 - ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")

[Out] integrate(-(b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}x^3}{-bdx^3 + ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d), x)`

[Out] `int(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d), x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{2}{3}} x^3}{bdx^3 - ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d), x, algorithm="maxima")`

[Out] `-integrate((b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 (bx^3 + a)^{2/3}}{ad - bdx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)`

[Out] `int((x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^3(a+bx^3)^{\frac{2}{3}}}{-a+bx^3} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d), x)`

[Out] `-Integral(x**3*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d`

$$3.368 \quad \int \frac{(a+bx^3)^{2/3}}{ad-bdx^3} dx$$

Optimal. Leaf size=200

$$\frac{\log(ad-bdx^3)}{3\sqrt[3]{2}\sqrt[3]{bd}} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{bx} - \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{bd}} + \frac{\log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{bx}\right)}{2\sqrt[3]{bd}} - \frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{bx}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bd}} + \frac{2^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{bx}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bd}}$$

Rubi [C] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 0.30, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {430, 429}

$$\frac{x(a+bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{ad\left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(2/3)/(a*d - b*d*x^3), x]

[Out] (x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -(b*x^3)/a, (b*x^3)/a])/ (a*d*(1 + (b*x^3)/a)^(2/3))

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -(b*x^n)/a, -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a + bx^3)^{2/3}}{ad - bdx^3} dx = \frac{(a + bx^3)^{2/3} \int \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3}}{ad - bdx^3} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x(a + bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{ad \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.16, size = 156, normalized size = 0.78

$$\frac{4ax(a + bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{d(a - bx^3) \left(bx^3 \left(3F_1\left(\frac{4}{3}; -\frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) + 2F_1\left(\frac{4}{3}; \frac{1}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) \right) + 4aF_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(2/3)/(a*d - b*d*x^3), x]

[Out] (4*a*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/ (d*(a - b*x^3)*(4*a*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, -2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] + 2*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))

IntegrateAlgebraic [A] time = 0.60, size = 309, normalized size = 1.54

$$\frac{\log(\sqrt[3]{b}x\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3} + b^{2/3}x^2)}{6\sqrt[3]{bd}} + \frac{\log(2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3} + \sqrt[3]{2}(a+bx^3)^{2/3} + 2b^{2/3}x^2)}{3\sqrt[3]{2}\sqrt[3]{bd}} + \frac{\log(\sqrt[3]{a+bx^3} - \sqrt[3]{bx})}{3\sqrt[3]{bd}} - \frac{2^{2/3}\log(2^{2/3}\sqrt[3]{a+bx^3} - 2\sqrt[3]{bx})}{3\sqrt[3]{bd}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{bx}}{2\sqrt[3]{a+bx^3} + \sqrt[3]{bx}}\right)}{\sqrt{3}\sqrt[3]{bd}} + \frac{2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{bx}}{2^{2/3}\sqrt[3]{a+bx^3} + \sqrt[3]{bx}}\right)}{\sqrt{3}\sqrt[3]{bd}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(a*d - b*d*x^3), x]

[Out] -(ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(Sqrt[3]*b^(1/3)*d) + (2^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))]/(Sqrt[3]*b^(1/3)*d) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(3*b^(1/3)*d) - (2^(2/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)]/(3*b^(1/3)*d) - Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(6*b^(1/3)*d) + Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]/(3*2^(1/3)*b^(1/3)*d)

fricas [A] time = 0.54, size = 611, normalized size = 3.06

$$\frac{\log(\sqrt[3]{b}x\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3} + b^{2/3}x^2)}{6\sqrt[3]{bd}} + \frac{\log(2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3} + \sqrt[3]{2}(a+bx^3)^{2/3} + 2b^{2/3}x^2)}{3\sqrt[3]{2}\sqrt[3]{bd}} + \frac{\log(\sqrt[3]{a+bx^3} - \sqrt[3]{bx})}{3\sqrt[3]{bd}} - \frac{2^{2/3}\log(2^{2/3}\sqrt[3]{a+bx^3} - 2\sqrt[3]{bx})}{3\sqrt[3]{bd}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{bx}}{2\sqrt[3]{a+bx^3} + \sqrt[3]{bx}}\right)}{\sqrt{3}\sqrt[3]{bd}} + \frac{2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{bx}}{2^{2/3}\sqrt[3]{a+bx^3} + \sqrt[3]{bx}}\right)}{\sqrt{3}\sqrt[3]{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")`

[Out]
$$\begin{aligned} & [-1/6*(2*4^{(1/3)}*\sqrt{3}*b*(-1/b)^{(1/3)}*\arctan(-1/3*(\sqrt{3}*x - 4^{(1/3)}*\sqrt{3}*(b*x^3 + a)^{(1/3)}*(-1/b)^{(1/3))}/x) - 3*\sqrt{1/3}*b*\sqrt{-1/b^{(2/3)}}*1 \\ & \log(3*b*x^3 - 3*(b*x^3 + a)^{(1/3)}*b^{(2/3)}*x^2 - 3*\sqrt{1/3}*(b^{(4/3)}*x^3 + (\\ & b*x^3 + a)^{(1/3)}*b*x^2 - 2*(b*x^3 + a)^{(2/3)}*b^{(2/3)}*x)*\sqrt{-1/b^{(2/3)}} + \\ & 2*a) - 2*4^{(1/3)}*b*(-1/b)^{(1/3)}*\log(-(4^{(2/3)}*b*x*(-1/b)^{(2/3)} - 2*(b*x^3 + \\ & a)^{(1/3))}/x) + 4^{(1/3)}*b*(-1/b)^{(1/3)}*\log(-(2*4^{(1/3)}*b*x^2*(-1/b)^{(1/3)} - \\ & 4^{(2/3)}*(b*x^3 + a)^{(1/3)}*b*x*(-1/b)^{(2/3)} - 2*(b*x^3 + a)^{(2/3))}/x^2) - 2 \\ & *b^{(2/3)}*\log(-(b^{(1/3)}*x - (b*x^3 + a)^{(1/3))}/x) + b^{(2/3)}*\log((b^{(2/3)}*x^2 \\ & + (b*x^3 + a)^{(1/3)}*b^{(1/3)}*x + (b*x^3 + a)^{(2/3))}/x^2))/(b*d), -1/6*(2*4^{(1/3)} \\ & *\sqrt{3}*b*(-1/b)^{(1/3)}*\arctan(-1/3*(\sqrt{3}*x - 4^{(1/3)}*\sqrt{3}*(b*x^3 + a)^{(1/3)}*(-1/b)^{(1/3))}/x) - 2*4^{(1/3)}*b*(-1/b)^{(1/3)}*\log(-(4^{(2/3)}*b*x \\ & (-1/b)^{(2/3)} - 2*(b*x^3 + a)^{(1/3))}/x) + 4^{(1/3)}*b*(-1/b)^{(1/3)}*\log(-(2*4^{(1/3)} \\ & *b*x^2*(-1/b)^{(1/3)} - 4^{(2/3)}*(b*x^3 + a)^{(1/3)}*b*x*(-1/b)^{(2/3)} - 2*(b \\ & *x^3 + a)^{(2/3))}/x^2) - 6*\sqrt{1/3}*b^{(2/3)}*\arctan(\sqrt{1/3}*(b^{(1/3)}*x + 2 \\ & *(b*x^3 + a)^{(1/3))}/(b^{(1/3)}*x)) - 2*b^{(2/3)}*\log(-(b^{(1/3)}*x - (b*x^3 + a)^{(1/3))}/x) + b^{(2/3)} \\ & *\log((b^{(2/3)}*x^2 + (b*x^3 + a)^{(1/3)}*b^{(1/3)}*x + (b*x^3 + a)^{(2/3))}/x^2))/(b*d)] \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{bdx^3 - ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")`

[Out] `integrate(-(b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)`

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{-bdx^3 + ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)`

[Out] `int((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{2}{3}}}{bdx^3 - ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{ad - bdx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(a*d - b*d*x^3),x)

[Out] int((a + b*x^3)^(2/3)/(a*d - b*d*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-a+bx^3} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)

[Out] -Integral((a + b*x**3)**(2/3)/(-a + b*x**3), x)/d

$$3.369 \quad \int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)} dx$$

Optimal. Leaf size=157

$$\frac{b^{2/3} \log(ad - bdx^3)}{3\sqrt[3]{2} ad} - \frac{b^{2/3} \log\left(\sqrt[3]{2} \sqrt[3]{b} x - \sqrt[3]{a + bx^3}\right)}{\sqrt[3]{2} ad} + \frac{2^{2/3} b^{2/3} \tan^{-1}\left(\frac{2\sqrt[3]{2} \sqrt[3]{b} x + 1}{\sqrt[3]{a + bx^3} \sqrt{3}}\right)}{\sqrt{3} ad} - \frac{(a + bx^3)^{2/3}}{2adx^2}$$

Rubi [C] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 0.50, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$\frac{(a + bx^3)^{2/3} \left(1 - \frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(-\frac{2}{3}, -\frac{2}{3}; \frac{1}{3}; -\frac{2bx^3}{a - bx^3}\right)}{2adx^2 \left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)), x]

[Out] -((a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, (-2*b*x^3)/(a - b*x^3)]/(2*a*d*x^2*(1 + (b*x^3)/a)^(2/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -(b*x^n)/a, -(d*x^n)/c]]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^(m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out]
$$-1/18*(2*4^{(1/3)}*\sqrt{3})*(-b^2)^{(1/3)}*x^2*\arctan(1/3*(3*4^{(2/3)}*\sqrt{3})*(5*b^2*x^7 - 4*a*b*x^4 - a^2*x)*(b*x^3 + a)^{(2/3)}*(-b^2)^{(2/3)} + 6*4^{(1/3)}*\sqrt{3}*(19*b^3*x^8 + 16*a*b^2*x^5 + a^2*b*x^2)*(b*x^3 + a)^{(1/3)}*(-b^2)^{(1/3)} - \sqrt{3}*(71*b^4*x^9 + 111*a*b^3*x^6 + 33*a^2*b^2*x^3 + a^3*b))/(109*b^4*x^9 + 105*a*b^3*x^6 + 3*a^2*b^2*x^3 - a^3*b)) - 2*4^{(1/3)}*(-b^2)^{(1/3)}*x^2*\log((3*4^{(2/3)}*(b*x^3 + a)^{(1/3)}*(-b^2)^{(2/3)}*x^2 - 6*(b*x^3 + a)^{(2/3)}*b*x + 4^{(1/3)}*(b*x^3 - a)*(-b^2)^{(1/3)})/(b*x^3 - a)) + 4^{(1/3)}*(-b^2)^{(1/3)}*x^2*\log(-(6*4^{(1/3)}*(5*b^2*x^4 + a*b*x)*(b*x^3 + a)^{(2/3)}*(-b^2)^{(1/3)} - 4^{(2/3)}*(19*b^2*x^6 + 16*a*b*x^3 + a^2)*(-b^2)^{(2/3)} - 24*(2*b^3*x^5 + a*b^2*x^2)*(b*x^3 + a)^{(1/3)})/(b^2*x^6 - 2*a*b*x^3 + a^2)) + 9*(b*x^3 + a)^{(2/3)})/(a*d*x^2)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{2}{3}}}{(bdx^3 - ad)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="giac")

[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(-bdx^3 + ad)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d), x)

[Out] int((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(bdx^3 - ad)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^3 + a)^{2/3}}{x^3(ad - bdx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)), x)

[Out] int((a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(a+bx^3)^{2/3}}{-ax^3+bx^6} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/x**3/(-b*d*x**3+a*d), x)

[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**3 + b*x**6), x)/d

$$3.370 \quad \int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)} dx$$

Optimal. Leaf size=182

$$\frac{b^{5/3} \log(ad-bdx^3)}{3\sqrt[3]{2}a^2d} - \frac{b^{5/3} \log\left(\sqrt[3]{2}\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^2d} + \frac{2^{2/3}b^{5/3} \tan^{-1}\left(\frac{2\sqrt[3]{2}\sqrt[3]{bx^3}+1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}a^2d} - \frac{7b(a+bx^3)^{2/3}}{10a^2dx^2} - \frac{(a+bx^3)^{2/3}}{5adx^5}$$

Rubi [C] time = 0.43, antiderivative size = 121, normalized size of antiderivative = 0.66, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$\frac{2a^2 - 4bx^3(2a + 3bx^3) {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{2bx^3}{bx^3+a}\right) + 12bx^3(a - bx^3) {}_2F_1\left(\frac{1}{3}, 2; \frac{4}{3}; \frac{2bx^3}{bx^3+a}\right) + 5abx^3 + 3b^2x^6}{10a^2dx^5\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)), x]

[Out] -(2*a^2 + 5*a*b*x^3 + 3*b^2*x^6 - 4*b*x^3*(2*a + 3*b*x^3)*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] + 12*b*x^3*(a - b*x^3)*Hypergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)])/(10*a^2*d*x^5*(a + b*x^3)^(1/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1+(m+1)/n, -(b*x^n)/a, -(d*x^n)/c]]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n-1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n-1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{(a + bx^3)^{2/3}}{x^6(ad - bdx^3)} dx = \frac{(a + bx^3)^{2/3} \int \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3}}{x^6(ad - bdx^3)} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{2a^2 + 5abx^3 + 3b^2x^6 - 4bx^3(2a + 3bx^3) {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{2bx^3}{a+bx^3}\right) + 12bx^3(a - bx^3) {}_2F_1\left(\frac{1}{3}, 2;$$

$$10a^2dx^5\sqrt[3]{a + bx^3}}$$

Mathematica [A] time = 5.20, size = 170, normalized size = 0.93

$$\frac{5 \cdot 2^{2/3} b^{5/3} \left(\log\left(\frac{2^{2/3} b^{2/3} x^2}{(ax^3+b)^{2/3}} + \frac{\sqrt[3]{2} \sqrt[3]{b} x}{\sqrt[3]{ax^3+b}} + 1\right) - 2 \log\left(1 - \frac{\sqrt[3]{2} \sqrt[3]{b} x}{\sqrt[3]{ax^3+b}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{2 \sqrt[3]{2} \sqrt[3]{b} x}{\sqrt[3]{ax^3+b} + 1}\right) \right) - \frac{3(a+bx^3)^{2/3}(2a+7bx^3)}{x^5}}{30a^2d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)), x]

[Out] ((-3*(a + b*x^3)^(2/3)*(2*a + 7*b*x^3))/x^5 + 5*2^(2/3)*b^(5/3)*(2*sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3))/sqrt[3]] - 2*Log[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2/3)*x^2)/(b + a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)]))/(30*a^2*d)

IntegrateAlgebraic [A] time = 0.49, size = 216, normalized size = 1.19

$$-\frac{2^{2/3} b^{5/3} \log\left(2^{2/3} \sqrt[3]{a + bx^3} - 2\sqrt[3]{b} x\right)}{3a^2d} + \frac{2^{2/3} b^{5/3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[3]{b} x}{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{b} x}\right)}{\sqrt{3} a^2d} + \frac{b^{5/3} \log\left(2^{2/3} \sqrt[3]{b} x \sqrt[3]{a + bx^3} + \sqrt[3]{2} (a + bx^3)^{2/3} + 2b^{2/3} x^2\right)}{3\sqrt[3]{2} a^2d} + \frac{(-2a - 7bx^3)(a + bx^3)^{2/3}}{10a^2dx^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)), x]

[Out] ((-2*a - 7*b*x^3)*(a + b*x^3)^(2/3))/(10*a^2*d*x^5) + (2^(2/3)*b^(5/3)*ArcTan[(sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))]/(sqrt[3]*a^2*d) - (2^(2/3)*b^(5/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)]/(3*a^2*d) + (b^(5/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]/(3*2^(1/3)*a^2*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="fricas")`

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{2}{3}}}{(bdx^3 - ad)x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="giac")`

[Out] `integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)`

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(-bdx^3 + ad)x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x)`

[Out] `int((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(bdx^3 - ad)x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="maxima")`

[Out] `-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{x^6 (ad - bdx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)), x)`

[Out] `int((a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^6+bx^9} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(2/3)/x**6/(-b*d*x**3+a*d), x)`

[Out] `-Integral((a + b*x**3)**(2/3)/(-a*x**6 + b*x**9), x)/d`

$$3.371 \quad \int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)} dx$$

Optimal. Leaf size=209

$$\frac{b^{8/3} \log(ad-bdx^3)}{3\sqrt[3]{2}a^3d} - \frac{b^{8/3} \log\left(\sqrt[3]{2}\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^3d} + \frac{2^{2/3}b^{8/3} \tan^{-1}\left(\frac{2\sqrt[3]{2}\sqrt[3]{b}x+1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}a^3d} - \frac{5b^2(a+bx^3)^{2/3}}{8a^3dx^2} - \frac{b(a+bx^3)^{2/3}}{4a^2dx^5}$$

Rubi [C] time = 10.68, antiderivative size = 244, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$\frac{-18bx^3(a-bx^3)^2 {}_3F_2\left(\frac{1}{3}, 2, 2; 1, \frac{4}{3}; \frac{2bx^3}{bx^3+a}\right) - 4bx^3(5a^2+6abx^3+9b^2x^6) {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{2bx^3}{bx^3+a}\right) + 42a^2bx^3 {}_2F_1\left(\frac{1}{3}, 2; \frac{4}{3}; \frac{2bx^3}{bx^3+a}\right) + 11a^2bx^3 + 5a^3 - 54b^3x^9 {}_2F_1\left(\frac{1}{3}, 2; \frac{4}{3}; \frac{2bx^3}{bx^3+a}\right) + 12ab^2x^6 {}_2F_1\left(\frac{1}{3}, 2; \frac{4}{3}; \frac{2bx^3}{bx^3+a}\right) + 15ab^2x^6 + 9b^3x^9}{40a^3dx^8\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)), x]

[Out] -(5*a^3 + 11*a^2*b*x^3 + 15*a*b^2*x^6 + 9*b^3*x^9 - 4*b*x^3*(5*a^2 + 6*a*b*x^3 + 9*b^2*x^6)*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] + 42*a^2*b*x^3*Hypergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] + 12*a*b^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 54*b^3*x^9*Hypergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 18*b*x^3*(a - b*x^3)^2*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, (2*b*x^3)/(a + b*x^3)])/(40*a^3*d*x^8*(a + b*x^3)^(1/3))

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1+(m+1)/n, -(b*x^n)/a, -(d*x^n)/c]]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n-1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n-1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{(a + bx^3)^{2/3}}{x^9(ad - bdx^3)} dx = \frac{(a + bx^3)^{2/3} \int \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3}}{x^9(ad - bdx^3)} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{5a^3 + 11a^2bx^3 + 15ab^2x^6 + 9b^3x^9 - 4bx^3 \left(5a^2 + 6abx^3 + 9b^2x^6\right) {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{2bx^3}{a+bx^3}\right) + 42}{24a^3d}$$

Mathematica [A] time = 5.27, size = 179, normalized size = 0.86

$$\frac{4 \cdot 2^{2/3} b^{8/3} \left(\log\left(\frac{2^{2/3} b^{2/3} x^2}{(ax^3+b)^{2/3}} + \frac{\sqrt[3]{2} \sqrt[3]{bx}}{\sqrt[3]{ax^3+b}} + 1\right) - 2 \log\left(1 - \frac{\sqrt[3]{2} \sqrt[3]{bx}}{\sqrt[3]{ax^3+b}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{2} \sqrt[3]{bx}}{\sqrt[3]{ax^3+b}} + 1}{\sqrt{3}}\right) \right) - \frac{3(a+bx^3)^{2/3}(a^2+2abx^3+5b^2x^6)}{x^8}}{24a^3d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)), x]

[Out] ((-3*(a + b*x^3)^(2/3)*(a^2 + 2*a*b*x^3 + 5*b^2*x^6))/x^8 + 4*2^(2/3)*b^(8/3)*(2*sqrt[3]*ArcTan[(1 + (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3))/sqrt[3]] - 2*Log[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2/3)*x^2)/(b + a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)]))/(24*a^3*d)

IntegrateAlgebraic [A] time = 0.55, size = 227, normalized size = 1.09

$$-\frac{2^{2/3} b^{8/3} \log\left(2^{2/3} \sqrt[3]{a + bx^3} - 2\sqrt[3]{bx}\right)}{3a^3d} + \frac{2^{2/3} b^{8/3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[3]{bx}}{2^{2/3} \sqrt[3]{a+bx^3} + \sqrt[3]{bx}}\right)}{\sqrt{3} a^3d} + \frac{b^{8/3} \log\left(2^{2/3} \sqrt[3]{bx} \sqrt[3]{a + bx^3} + \sqrt[3]{2} (a + bx^3)^{2/3} + 2b^{2/3} x^2\right)}{3\sqrt[3]{2} a^3d} + \frac{(a + bx^3)^{2/3} (-a^2 - 2abx^3 - 5b^2x^6)}{8a^3dx^8}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)), x]

[Out] ((a + b*x^3)^(2/3)*(-a^2 - 2*a*b*x^3 - 5*b^2*x^6))/(8*a^3*d*x^8) + (2^(2/3)*b^(8/3)*ArcTan[(sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/((sqrt[3]*a^3*d) - (2^(2/3)*b^(8/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a^3*d) + (b^(8/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(1/3)*a^3*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{2}{3}}}{(bdx^3 - ad)x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="giac")

[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(-bdx^3 + ad)x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x)

[Out] int((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(bdx^3 - ad)x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{2/3}}{x^9 (ad - bdx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)), x)

[Out] int((a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/x**9/(-b*d*x**3+a*d), x)

[Out] Timed out

$$3.372 \quad \int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)} dx$$

Optimal. Leaf size=236

$$\frac{b^{11/3} \log(ad - bdx^3)}{3\sqrt[3]{2} a^4 d} - \frac{b^{11/3} \log\left(\sqrt[3]{2} \sqrt[3]{b} x - \sqrt[3]{a + bx^3}\right)}{\sqrt[3]{2} a^4 d} + \frac{2^{2/3} b^{11/3} \tan^{-1}\left(\frac{2\sqrt[3]{2} \sqrt[3]{b} x + 1}{\sqrt[3]{a + bx^3}}\right)}{\sqrt{3} a^4 d} - \frac{293b^3 (a + bx^3)^{2/3}}{440a^4 dx^2} - \frac{49b^2}{22}$$

Rubi [C] time = 17.60, antiderivative size = 391, normalized size of antiderivative = 1.66, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$\frac{-54b^3(a-bx^3)^2 F_2\left(\frac{1}{3}, 2, 2, 1, \frac{2bx^3}{a+bx^3}\right) + 54b^2(a-bx^3) F_2\left(\frac{1}{3}, 2, 2, 2, 1, \frac{2bx^3}{a+bx^3}\right) - 180a^2 b^2 F_1\left(\frac{1}{3}, 1, \frac{2bx^3}{a+bx^3}\right) + 198a^2 b^2 F_1\left(\frac{1}{3}, 2, \frac{2bx^3}{a+bx^3}\right) + 99a^2 b^2 F_1\left(\frac{1}{3}, 1, \frac{2bx^3}{a+bx^3}\right) - 160a^2 b^2 F_1\left(\frac{1}{3}, 1, \frac{2bx^3}{a+bx^3}\right) + 396a^2 b^2 F_1\left(\frac{1}{3}, 2, \frac{2bx^3}{a+bx^3}\right) + 85a^2 b^2 + 40a^4 - 324a^2 b^2 F_1\left(\frac{1}{3}, 1, \frac{2bx^3}{a+bx^3}\right) - 594a^2 b^2 F_1\left(\frac{1}{3}, 2, \frac{2bx^3}{a+bx^3}\right) - 216a^2 b^2 F_1\left(\frac{1}{3}, 1, \frac{2bx^3}{a+bx^3}\right) + 135a^2 b^2 + 81a^4}{440a^4 d \sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)), x]

[Out] $-(40*a^4 + 85*a^3*b*x^3 + 99*a^2*b^2*x^6 + 135*a*b^3*x^9 + 81*b^4*x^{12} - 160*a^3*b*x^3*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] - 180*a^2*b^2*x^6*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] - 216*a*b^3*x^9*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] - 324*b^4*x^{12}*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] + 396*a^3*b*x^3*Hypergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] + 198*a^2*b^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 594*b^4*x^{12}*Hypergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 54*b*x^3*(a - b*x^3)^2*(5*a + 6*b*x^3)*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, (2*b*x^3)/(a + b*x^3)] + 54*b*x^3*(a - b*x^3)^3*HypergeometricPFQ[{1/3, 2, 2, 2}, {1, 1, 4/3}, (2*b*x^3)/(a + b*x^3)))/(440*a^4*d*x^{11}*(a + b*x^3)^{(1/3)})$

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1+(m+1)/n, -(b*x^n)/a, -(d*x^n)/c])/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n-1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;

FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{(a + bx^3)^{2/3}}{x^{12}(ad - bdx^3)} dx = \frac{(a + bx^3)^{2/3} \int \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3}}{x^{12}(ad - bdx^3)} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{40a^4 + 85a^3bx^3 + 99a^2b^2x^6 + 135ab^3x^9 + 81b^4x^{12} - 160a^3bx^3 {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{2bx^3}{a+bx^3}\right) - 180}{220a^4x^{11}}$$

Mathematica [A] time = 5.35, size = 196, normalized size = 0.83

$$\frac{2^{2/3}b^{11/3} \left(\log\left(\frac{2^{2/3}b^{2/3}x^2 + \sqrt[3]{2}\sqrt[3]{bx}}{(ax^3+b)^{2/3} + \sqrt[3]{ax^3+b}} + 1\right) - 2\log\left(1 - \frac{\sqrt[3]{2}\sqrt[3]{bx}}{\sqrt[3]{ax^3+b}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{2}\sqrt[3]{bx} + 1}{\sqrt[3]{ax^3+b}}\right) \right)}{a^4} - \frac{3(a+bx^3)^{2/3}(40a^3 + 65a^2bx^3 + 98ab^2x^6 + 293b^3x^9)}{220a^4x^{11}}$$

6d

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)), x]

[Out] ((-3*(a + b*x^3)^(2/3)*(40*a^3 + 65*a^2*b*x^3 + 98*a*b^2*x^6 + 293*b^3*x^9)) / (220*a^4*d*x^11) + (2^(2/3)*b^(11/3)*(2*sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)]/sqrt[3]) - 2*Log[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2/3)*x^2)/(b + a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)])) / a^4) / (6*d)

IntegrateAlgebraic [A] time = 0.62, size = 238, normalized size = 1.01

$$-\frac{2^{2/3}b^{11/3} \log\left(2^{2/3}\sqrt[3]{a+bx^3} - 2\sqrt[3]{bx}\right)}{3a^4d} + \frac{2^{2/3}b^{11/3} \tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{bx}}{2^{2/3}\sqrt[3]{a+bx^3} + \sqrt[3]{bx}}\right)}{\sqrt{3}a^4d} + \frac{b^{11/3} \log\left(2^{2/3}\sqrt[3]{bx}\sqrt[3]{a+bx^3} + \sqrt[3]{2}\left(a+bx^3\right)^{2/3} + 2b^{2/3}x^2\right)}{3\sqrt[3]{2}a^4d} + \frac{(a+bx^3)^{2/3}(-40a^3 - 65a^2bx^3 - 98ab^2x^6 - 293b^3x^9)}{440a^4dx^{11}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)), x]

[Out] ((a + b*x^3)^(2/3)*(-40*a^3 - 65*a^2*b*x^3 - 98*a*b^2*x^6 - 293*b^3*x^9)) / (440*a^4*d*x^11) + (2^(2/3)*b^(11/3)*ArcTan[(sqrt[3]*b^(1/3)*x)/(b^(1/3)*x +

$$\frac{2^{2/3} \cdot (a + b \cdot x^3)^{1/3}}{\sqrt[3]{a^4 \cdot d}} - \frac{2^{2/3} \cdot b^{11/3} \cdot \log[-2 \cdot b^{1/3} \cdot x + 2^{2/3} \cdot (a + b \cdot x^3)^{1/3}]}{3 \cdot a^4 \cdot d} + \frac{b^{11/3} \cdot \log[2 \cdot b^{2/3} \cdot x^2 + 2^{2/3} \cdot b^{1/3} \cdot x \cdot (a + b \cdot x^3)^{1/3} + 2^{1/3} \cdot (a + b \cdot x^3)^{2/3}]}{3 \cdot 2^{1/3} \cdot a^4 \cdot d}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{2}{3}}}{(bdx^3 - ad)x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="giac")

[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(-bdx^3 + ad)x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x)

[Out] int((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(bdx^3 - ad)x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{2/3}}{x^{12} (ad - bdx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x)

[Out] int((a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/x**12/(-b*d*x**3+a*d),x)

[Out] Timed out

$$3.373 \quad \int \frac{x^{14}}{\sqrt[3]{1-x^3}(1+x^3)} dx$$

Optimal. Leaf size=127

$$\frac{1}{11}(1-x^3)^{11/3} - \frac{1}{4}(1-x^3)^{8/3} + \frac{2}{5}(1-x^3)^{5/3} - \frac{\log(x^3+1)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{2\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {446, 88, 55, 617, 204, 31}

$$\frac{1}{11}(1-x^3)^{11/3} - \frac{1}{4}(1-x^3)^{8/3} + \frac{2}{5}(1-x^3)^{5/3} - \frac{\log(x^3+1)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{2\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] (2*(1 - x^3)^(5/3))/5 - (1 - x^3)^(8/3)/4 + (1 - x^3)^(11/3)/11 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{14}}{\sqrt[3]{1-x^3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^4}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(-2(1-x)^{2/3} + 2(1-x)^{5/3} - (1-x)^{8/3} + \frac{1}{\sqrt[3]{1-x}(1+x)} \right) dx, x, x^3 \right) \\
 &= \frac{2}{5} (1-x^3)^{5/3} - \frac{1}{4} (1-x^3)^{8/3} + \frac{1}{11} (1-x^3)^{11/3} + \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\
 &= \frac{2}{5} (1-x^3)^{5/3} - \frac{1}{4} (1-x^3)^{8/3} + \frac{1}{11} (1-x^3)^{11/3} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2}x} dx, x, x^3 \right) \\
 &= \frac{2}{5} (1-x^3)^{5/3} - \frac{1}{4} (1-x^3)^{8/3} + \frac{1}{11} (1-x^3)^{11/3} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{2\sqrt[3]{2}} \\
 &= \frac{2}{5} (1-x^3)^{5/3} - \frac{1}{4} (1-x^3)^{8/3} + \frac{1}{11} (1-x^3)^{11/3} + \frac{\tan^{-1} \left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \dots
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 113, normalized size = 0.89

$$\frac{1}{660} \left(-55 \cdot 2^{2/3} \log(x^3 + 1) + 165 \cdot 2^{2/3} \log(\sqrt[3]{2} - \sqrt[3]{1-x^3}) + 110 \cdot 2^{2/3} \sqrt{3} \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{1-x^3} + 1}{\sqrt{3}} \right) + 3(1-x^3)^{2/3} (-20x^9 + 5x^6 - 38x^3 + 53) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^14/((1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] (3*(1 - x^3)^(2/3)*(53 - 38*x^3 + 5*x^6 - 20*x^9) + 110*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 55*2^(2/3)*Log[1 + x^3] + 165*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/660

IntegrateAlgebraic [A] time = 0.15, size = 148, normalized size = 1.17

$$\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}(1-x^3)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{1}{220}(1-x^3)^{2/3}(-20x^9+5x^6-38x^3+53)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^14/((1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] ((1 - x^3)^(2/3)*(53 - 38*x^3 + 5*x^6 - 20*x^9))/220 + ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))

fricas [A] time = 0.93, size = 118, normalized size = 0.93

$$-\frac{1}{220}(20x^9 - 5x^6 + 38x^3 - 53)(-x^3 + 1)^{2/3} + \frac{1}{6}\sqrt{6}2^{1/6}\arctan\left(\frac{1}{6}\cdot 2^{1/6}\left(\sqrt{6}2^{1/3} + 2\sqrt{6}(-x^3 + 1)^{1/3}\right)\right) - \frac{1}{12}\cdot 2^{2/3}\log\left(2^{2/3} + 2^{1/3}(-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}\right) + \frac{1}{6}\cdot 2^{2/3}\log\left(-2^{1/3} + (-x^3 + 1)^{1/3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1), x, algorithm="fricas")

[Out] -1/220*(20*x^9 - 5*x^6 + 38*x^3 - 53)*(-x^3 + 1)^(2/3) + 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))

giac [A] time = 0.17, size = 134, normalized size = 1.06

$$-\frac{1}{11}(x^3-1)^3(-x^3+1)^{2/3}-\frac{1}{4}(x^3-1)^2(-x^3+1)^{2/3}+\frac{1}{6}\sqrt{3}2^{2/3}\arctan\left(\frac{1}{6}\sqrt{3}2^{2/3}\left(2^{1/3}+2(-x^3+1)^{1/3}\right)\right)+\frac{2}{5}(-x^3+1)^{5/3}-\frac{1}{12}\cdot 2^{2/3}\log\left(2^{2/3}+2^{1/3}(-x^3+1)^{1/3}+(-x^3+1)^{2/3}\right)+\frac{1}{6}\cdot 2^{2/3}\log\left(-2^{1/3}+(-x^3+1)^{1/3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1), x, algorithm="giac")

[Out] $-1/11*(x^3 - 1)^3*(-x^3 + 1)^{(2/3)} - 1/4*(x^3 - 1)^2*(-x^3 + 1)^{(2/3)} + 1/6*\sqrt{3}*2^{(2/3)}*\arctan(1/6*\sqrt{3}*2^{(2/3)}*(2^{(1/3)} + 2*(-x^3 + 1)^{(1/3)})) + 2/5*(-x^3 + 1)^{(5/3)} - 1/12*2^{(2/3)}*\log(2^{(2/3)} + 2^{(1/3)}*(-x^3 + 1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) + 1/6*2^{(2/3)}*\log(\text{abs}(-2^{(1/3)} + (-x^3 + 1)^{(1/3)}))$

maple [C] time = 9.32, size = 792, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{14}/(-x^3+1)^{(1/3)}/(x^3+1), x)$

[Out] $1/220*(20*x^9-5*x^6+38*x^3-53)*(x^3-1)/(-x^3+1)^{(1/3)}+1/6*\text{RootOf}(_Z^3-4)*\ln((-3*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3-90*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3-3*\text{RootOf}(_Z^3-4)*x^3-90*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-21*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}-126*\text{RootOf}(_Z^3-4)*(-x^3+1)^{(1/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)+42*(-x^3+1)^{(2/3)}+7*\text{RootOf}(_Z^3-4)+210*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)))/(x+1)/(x^2-x+1))-1/6*\ln((-3*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3+72*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+\text{RootOf}(_Z^3-4)*x^3-24*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-21*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}-126*\text{RootOf}(_Z^3-4)*(-x^3+1)^{(1/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)+42*(-x^3+1)^{(2/3)}-7*\text{RootOf}(_Z^3-4)+168*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)))/(x+1)/(x^2-x+1))*\text{RootOf}(_Z^3-4)-\ln((-3*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3+72*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+\text{RootOf}(_Z^3-4)*x^3-24*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-21*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}-126*\text{RootOf}(_Z^3-4)*(-x^3+1)^{(1/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)+42*(-x^3+1)^{(2/3)}-7*\text{RootOf}(_Z^3-4)+168*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)))/(x+1)/(x^2-x+1))*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)$

maxima [A] time = 1.24, size = 119, normalized size = 0.94

$\frac{1}{11}(-x^3+1)^{\frac{11}{3}} - \frac{1}{4}(-x^3+1)^{\frac{8}{3}} + \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right) + \frac{2}{5}(-x^3+1)^{\frac{5}{3}} - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{14}/(-x^3+1)^{(1/3)}/(x^3+1), x, \text{algorithm}=\text{"maxima"})$

[Out] $1/11*(-x^3 + 1)^{(11/3)} - 1/4*(-x^3 + 1)^{(8/3)} + 1/6*\sqrt{3}*2^{(2/3)}*\arctan(1/6*\sqrt{3}*2^{(2/3)}*(2^{(1/3)} + 2*(-x^3 + 1)^{(1/3)})) + 2/5*(-x^3 + 1)^{(5/3)} - 1/12*2^{(2/3)}*\log(2^{(2/3)} + 2^{(1/3)}*(-x^3 + 1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) + 1/6*2^{(2/3)}*\log(-2^{(1/3)} + (-x^3 + 1)^{(1/3)})$

mupad [B] time = 5.03, size = 133, normalized size = 1.05

$$\frac{2^{2/3} \ln\left(\left(1-x^3\right)^{1/3} - 2^{1/3}\right)}{6} + \frac{2\left(1-x^3\right)^{5/3}}{5} - \frac{\left(1-x^3\right)^{8/3}}{4} + \frac{\left(1-x^3\right)^{11/3}}{11} + \frac{2^{2/3} \ln\left(\left(1-x^3\right)^{1/3} - \frac{2^{1/3}\left(-1+\sqrt{3}\right)}{4}\right)\left(-1+\sqrt{3}\right)}{12} - \frac{2^{2/3} \ln\left(\left(1-x^3\right)^{1/3} - \frac{2^{1/3}\left(1+\sqrt{3}\right)}{4}\right)\left(1+\sqrt{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^14/((1 - x^3)^(1/3)*(x^3 + 1)), x)`

[Out] $(2^{2/3} \log((1 - x^3)^{1/3} - 2^{1/3}))/6 + (2*(1 - x^3)^{5/3})/5 - (1 - x^3)^{8/3}/4 + (1 - x^3)^{11/3}/11 + (2^{2/3} \log((1 - x^3)^{1/3} - (2^{1/3} * (3^{1/2} * 1i - 1)^2)/4) * (3^{1/2} * 1i - 1))/12 - (2^{2/3} \log((1 - x^3)^{1/3} - (2^{1/3} * (3^{1/2} * 1i + 1)^2)/4) * (3^{1/2} * 1i + 1))/12$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**14/((-x**3+1)**(1/3)/(x**3+1)), x)`

[Out] `Integral(x**14/((-x - 1)*(x**2 + x + 1))** (1/3)*(x + 1)*(x**2 - x + 1)), x)`

$$3.374 \quad \int \frac{x^{11}}{\sqrt[3]{1-x^3}(1+x^3)} dx$$

Optimal. Leaf size=128

$$-\frac{1}{8}(1-x^3)^{8/3} + \frac{1}{5}(1-x^3)^{5/3} - \frac{1}{2}(1-x^3)^{2/3} + \frac{\log(x^3+1)}{6\sqrt[3]{2}} - \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {446, 88, 55, 617, 204, 31}

$$-\frac{1}{8}(1-x^3)^{8/3} + \frac{1}{5}(1-x^3)^{5/3} - \frac{1}{2}(1-x^3)^{2/3} + \frac{\log(x^3+1)}{6\sqrt[3]{2}} - \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(((1 - x^3)^(1/3)*(1 + x^3))),x]
```

```
[Out] -(1 - x^3)^(2/3)/2 + (1 - x^3)^(5/3)/5 - (1 - x^3)^(8/3)/8 - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11}}{\sqrt[3]{1-x^3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{\sqrt[3]{1-x}} - (1-x)^{2/3} + (1-x)^{5/3} - \frac{1}{\sqrt[3]{1-x}(1+x)} \right) dx, x, x^3 \right) \\
 &= -\frac{1}{2} (1-x^3)^{2/3} + \frac{1}{5} (1-x^3)^{5/3} - \frac{1}{8} (1-x^3)^{8/3} - \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\
 &= -\frac{1}{2} (1-x^3)^{2/3} + \frac{1}{5} (1-x^3)^{5/3} - \frac{1}{8} (1-x^3)^{8/3} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2}} dx, x, x^3 \right) \\
 &= -\frac{1}{2} (1-x^3)^{2/3} + \frac{1}{5} (1-x^3)^{5/3} - \frac{1}{8} (1-x^3)^{8/3} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{2\sqrt[3]{2}} \\
 &= -\frac{1}{2} (1-x^3)^{2/3} + \frac{1}{5} (1-x^3)^{5/3} - \frac{1}{8} (1-x^3)^{8/3} - \frac{\tan^{-1} \left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{6\sqrt[3]{2}}
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 128, normalized size = 1.00

$$\frac{1}{120} \left(6(1-x^3)^{2/3} x^3 - 51(1-x^3)^{2/3} + 10 \cdot 2^{2/3} \log(x^3+1) - 30 \cdot 2^{2/3} \log(\sqrt[3]{2} - \sqrt[3]{1-x^3}) - 20 \cdot 2^{2/3} \sqrt{3} \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{1-x^3} + 1}{\sqrt{3}} \right) - 15(1-x^3)^{2/3} x^6 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] (-51*(1 - x^3)^(2/3) + 6*x^3*(1 - x^3)^(2/3) - 15*x^6*(1 - x^3)^(2/3) - 20*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 10*2^(2/3)*Log[1 + x^3] - 30*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/120

IntegrateAlgebraic [A] time = 0.14, size = 144, normalized size = 1.12

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{1}{40}\left(1-x^3\right)^{2/3}\left(-5x^6+2x^3-17\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] ((1 - x^3)^(2/3)*(-17 + 2*x^3 - 5*x^6))/40 - ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))

fricas [A] time = 0.87, size = 137, normalized size = 1.07

$$-\frac{1}{6}\sqrt{6}2^{1/6}(-1)^{1/3}\arctan\left(\frac{1}{6}2^{1/6}\left(2\sqrt{6}(-1)^{1/3}(-x^3+1)^{1/3}-\sqrt{6}2^{1/6}\right)\right) - \frac{1}{12}2^{2/3}(-1)^{1/3}\log\left(2^{1/3}(-1)^{1/3}(-x^3+1)^{1/3}-2^{2/3}(-1)^{1/3}+(-x^3+1)^{2/3}\right) + \frac{1}{6}2^{2/3}(-1)^{1/3}\log\left(-2^{1/3}(-1)^{1/3}+(-x^3+1)^{1/3}\right) - \frac{1}{40}(5x^6-2x^3+17)(-x^3+1)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1), x, algorithm="fricas")

[Out] -1/6*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) - 1/12*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*(-1)^(1/3)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) - 1/40*(5*x^6 - 2*x^3 + 17)*(-x^3 + 1)^(2/3)

giac [A] time = 0.18, size = 127, normalized size = 0.99

$$-\frac{1}{8}(x^3-1)^2(-x^3+1)^{2/3} - \frac{1}{6}\sqrt{3}2^{2/3}\arctan\left(\frac{1}{6}\sqrt{3}2^{2/3}\left(2^{1/3}+2(-x^3+1)^{1/3}\right)\right) + \frac{1}{5}(-x^3+1)^{5/3} + \frac{1}{12}2^{2/3}\log\left(2^{2/3}+2^{1/3}(-x^3+1)^{1/3}+(-x^3+1)^{2/3}\right) - \frac{1}{6}2^{2/3}\log\left(\left|-2^{1/3}+(-x^3+1)^{1/3}\right|\right) - \frac{1}{2}(-x^3+1)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(-x³+1)^(1/3)/(x³+1),x, algorithm="giac")

[Out] $-\frac{1}{8}(x^3 - 1)^2(-x^3 + 1)^{2/3} - \frac{1}{6}\sqrt{3} \cdot 2^{2/3} \arctan\left(\frac{1}{6}\sqrt{3} \cdot 2^{2/3} \cdot (2^{1/3} + 2(-x^3 + 1)^{1/3})\right) + \frac{1}{5}(-x^3 + 1)^{5/3} + \frac{1}{12} \cdot 2^{2/3} \log(2^{2/3} + 2^{1/3}(-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}) - \frac{1}{6} \cdot 2^{2/3} \log(\text{abs}(-2^{1/3} + (-x^3 + 1)^{1/3})) - \frac{1}{2}(-x^3 + 1)^{2/3}$

maple [C] time = 8.41, size = 683, normalized size = 5.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/(-x³+1)^(1/3)/(x³+1),x)

[Out] $\frac{1}{40}(5x^6 - 2x^3 + 17)(x^3 - 1)/(-x^3 + 1)^{1/3} + \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot \ln\left(\frac{(15 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot \text{RootOf}(_Z^3 + 4)^3 \cdot x^3 + 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2)^2 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x^3 + 5 \cdot \text{RootOf}(_Z^3 + 4) \cdot x^3 + 6 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot x^3 + 21 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot (-x^3 + 1)^{1/3} + 42 \cdot (-x^3 + 1)^{2/3} - 35 \cdot \text{RootOf}(_Z^3 + 4) - 42 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2))}{(x+1)/(x^2-x+1)} - \frac{1}{6} \ln\left(\frac{(-12 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot \text{RootOf}(_Z^3 + 4)^3 \cdot x^3 + 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2)^2 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x^3 + 12 \cdot \text{RootOf}(_Z^3 + 4) \cdot x^3 - 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot x^3 + 21 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot (-x^3 + 1)^{1/3} + 42 \cdot (-x^3 + 1)^{2/3} - 28 \cdot \text{RootOf}(_Z^3 + 4) + 42 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2))}{(x+1)/(x^2-x+1)} \cdot \text{RootOf}(_Z^3 + 4) - \ln\left(\frac{(-12 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot \text{RootOf}(_Z^3 + 4)^3 \cdot x^3 + 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2)^2 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x^3 + 12 \cdot \text{RootOf}(_Z^3 + 4) \cdot x^3 - 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot x^3 + 21 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot (-x^3 + 1)^{1/3} + 42 \cdot (-x^3 + 1)^{2/3} - 28 \cdot \text{RootOf}(_Z^3 + 4) + 42 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2))}{(x+1)/(x^2-x+1)}\right) \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2)$

maxima [A] time = 1.25, size = 119, normalized size = 0.93

$-\frac{1}{8}(-x^3 + 1)^{\frac{8}{3}} - \frac{1}{6}\sqrt{3} \cdot 2^{\frac{2}{3}} \arctan\left(\frac{1}{6}\sqrt{3} \cdot 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} + 2(-x^3 + 1)^{\frac{1}{3}}\right)\right) + \frac{1}{5}(-x^3 + 1)^{\frac{5}{3}} + \frac{1}{12} \cdot 2^{\frac{2}{3}} \log\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}(-x^3 + 1)^{\frac{1}{3}} + (-x^3 + 1)^{\frac{2}{3}}\right) - \frac{1}{6} \cdot 2^{\frac{2}{3}} \log\left(-2^{\frac{1}{3}} + (-x^3 + 1)^{\frac{1}{3}}\right) - \frac{1}{2}(-x^3 + 1)^{\frac{2}{3}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(-x³+1)^(1/3)/(x³+1),x, algorithm="maxima")

[Out] $-\frac{1}{8}(-x^3 + 1)^{8/3} - \frac{1}{6}\sqrt{3} \cdot 2^{2/3} \arctan\left(\frac{1}{6}\sqrt{3} \cdot 2^{2/3} \cdot (2^{1/3} + 2(-x^3 + 1)^{1/3})\right) + \frac{1}{5}(-x^3 + 1)^{5/3} + \frac{1}{12} \cdot 2^{2/3} \log(2^{2/3} + 2^{1/3}(-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}) - \frac{1}{6} \cdot 2^{2/3} \log(-2^{1/3} + (-x^3 + 1)^{1/3}) - \frac{1}{2}(-x^3 + 1)^{2/3}$

mupad [B] time = 4.73, size = 133, normalized size = 1.04

$$\frac{(1-x^3)^{5/3}}{5} - \frac{(1-x^3)^{2/3}}{2} - \frac{2^{2/3} \ln\left(\left(1-x^3\right)^{1/3} - 2^{1/3}\right)}{6} - \frac{(1-x^3)^{8/3}}{8} - \frac{2^{2/3} \ln\left(\left(1-x^3\right)^{1/3} - \frac{2^{1/3}(-1+\sqrt{3}i)^2}{4}\right)(-1+\sqrt{3}i)}{12} + \frac{2^{2/3} \ln\left(\left(1-x^3\right)^{1/3} - \frac{2^{1/3}(1+\sqrt{3}i)^2}{4}\right)(1+\sqrt{3}i)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/((1 - x^3)^(1/3)*(x^3 + 1)),x)`

[Out] $(1 - x^3)^{5/3}/5 - (1 - x^3)^{2/3}/2 - (2^{2/3} \log((1 - x^3)^{1/3} - 2^{1/3}))/6 - (1 - x^3)^{8/3}/8 - (2^{2/3} \log((1 - x^3)^{1/3} - (2^{1/3} * (3^{1/2} * 1i - 1)^2)/4) * (3^{1/2} * 1i - 1))/12 + (2^{2/3} \log((1 - x^3)^{1/3} - (2^{1/3} * (3^{1/2} * 1i + 1)^2)/4) * (3^{1/2} * 1i + 1))/12$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11/(-x**3+1)**(1/3)/(x**3+1),x)`

[Out] `Integral(x**11/((-x - 1)*(x**2 + x + 1))**1/3*(x + 1)*(x**2 - x + 1)), x)`

$$3.375 \quad \int \frac{x^8}{\sqrt[3]{1-x^3}(1+x^3)} dx$$

Optimal. Leaf size=97

$$\frac{1}{5}(1-x^3)^{5/3} - \frac{\log(x^3+1)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{2\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {446, 88, 55, 617, 204, 31}

$$\frac{1}{5}(1-x^3)^{5/3} - \frac{\log(x^3+1)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{2\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] (1 - x^3)^(5/3)/5 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^8}{\sqrt[3]{1-x^3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(-(1-x)^{2/3} + \frac{1}{\sqrt[3]{1-x}(1+x)} \right) dx, x, x^3 \right) \\
 &= \frac{1}{5} (1-x^3)^{5/3} + \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\
 &= \frac{1}{5} (1-x^3)^{5/3} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2}x + x^2} dx, x, \sqrt[3]{1-x^3} \right) - \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 + 2^{2/3}\sqrt[3]{1-x^3} \right)}{\sqrt[3]{2}} \\
 &= \frac{1}{5} (1-x^3)^{5/3} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{2\sqrt[3]{2}} - \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 + 2^{2/3}\sqrt[3]{1-x^3} \right)}{\sqrt[3]{2}} \\
 &= \frac{1}{5} (1-x^3)^{5/3} + \frac{\tan^{-1} \left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{2\sqrt[3]{2}}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 96, normalized size = 0.99

$$\frac{1}{60} \left(12(1-x^3)^{5/3} - 5 \cdot 2^{2/3} \log(x^3+1) + 15 \cdot 2^{2/3} \log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right) + 10 \cdot 2^{2/3} \sqrt{3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{1-x^3} + 1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] (12*(1 - x^3)^(5/3) + 10*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 5*2^(2/3)*Log[1 + x^3] + 15*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/60

IntegrateAlgebraic [A] time = 0.11, size = 131, normalized size = 1.35

$$\frac{1}{5} (1-x^3)^{5/3} + \frac{\log\left(2^{2/3} \sqrt[3]{1-x^3} - 2\right)}{3\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2} (1-x^3)^{2/3} + 2^{2/3} \sqrt[3]{1-x^3} + 2\right)}{6\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{\sqrt[3]{2} \sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] (1 - x^3)^(5/3)/5 + ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))

fricas [A] time = 1.16, size = 106, normalized size = 1.09

$$-\frac{1}{5}(x^3-1)(-x^3+1)^{\frac{2}{3}} + \frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/5*(x^3 - 1)*(-x^3 + 1)^(2/3) + 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))

giac [A] time = 0.20, size = 98, normalized size = 1.01

$$\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right) + \frac{1}{5}(-x^3+1)^{\frac{5}{3}} - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] $\frac{1}{6}\sqrt{3} \cdot 2^{2/3} \arctan\left(\frac{1}{6}\sqrt{3} \cdot 2^{2/3} \left(2^{1/3} + 2(-x^3 + 1)^{1/3}\right)\right) + \frac{1}{5}(-x^3 + 1)^{5/3} - \frac{1}{12} \cdot 2^{2/3} \log\left(2^{2/3} + 2^{1/3}(-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}\right) + \frac{1}{6} \cdot 2^{2/3} \log\left(\sqrt{-2^{1/3} + (-x^3 + 1)^{1/3}}\right)$

maple [C] time = 6.73, size = 673, normalized size = 6.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^8/(-x^3+1)^{1/3}/(x^3+1), x)$

[Out] $\frac{1}{5}(x^3-1)^2/(-x^3+1)^{1/3} + \text{RootOf}\left(\text{RootOf}(_Z^3-4)^2+6_Z \cdot \text{RootOf}(_Z^3-4)+36_Z^2\right) \cdot \ln\left(\frac{(18 \cdot \text{RootOf}(\text{RootOf}(_Z^3-4)^2+6_Z \cdot \text{RootOf}(_Z^3-4)+36_Z^2)^2 \cdot \text{RootOf}(_Z^3-4)^2 \cdot x^3 + 15 \cdot \text{RootOf}(\text{RootOf}(_Z^3-4)^2+6_Z \cdot \text{RootOf}(_Z^3-4)+36_Z^2) \cdot \text{RootOf}(_Z^3-4)^3 \cdot x^3 - 6 \cdot \text{RootOf}(\text{RootOf}(_Z^3-4)^2+6_Z \cdot \text{RootOf}(_Z^3-4)+36_Z^2) \cdot x^3 - 5 \cdot \text{RootOf}(_Z^3-4) \cdot x^3 + 21 \cdot \text{RootOf}(_Z^3-4)^2 \cdot (-x^3+1)^{1/3} + 42 \cdot (-x^3+1)^{2/3} + 42 \cdot \text{RootOf}(\text{RootOf}(_Z^3-4)^2+6_Z \cdot \text{RootOf}(_Z^3-4)+36_Z^2) + 35 \cdot \text{RootOf}(_Z^3-4))}{(x+1)/(x^2-x+1)} - \frac{1}{6} \ln\left(\frac{(18 \cdot \text{RootOf}(\text{RootOf}(_Z^3-4)^2+6_Z \cdot \text{RootOf}(_Z^3-4)+36_Z^2)^2 \cdot \text{RootOf}(_Z^3-4)^2 \cdot x^3 - 12 \cdot \text{RootOf}(\text{RootOf}(_Z^3-4)^2+6_Z \cdot \text{RootOf}(_Z^3-4)+36_Z^2) \cdot \text{RootOf}(_Z^3-4)^3 \cdot x^3 + 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3-4)^2+6_Z \cdot \text{RootOf}(_Z^3-4)+36_Z^2) \cdot x^3 - 12 \cdot \text{RootOf}(_Z^3-4) \cdot x^3 + 21 \cdot \text{RootOf}(_Z^3-4)^2 \cdot (-x^3+1)^{1/3} + 42 \cdot (-x^3+1)^{2/3} - 42 \cdot \text{RootOf}(\text{RootOf}(_Z^3-4)^2+6_Z \cdot \text{RootOf}(_Z^3-4)+36_Z^2) + 28 \cdot \text{RootOf}(_Z^3-4))}{(x+1)/(x^2-x+1)} \cdot \text{RootOf}(_Z^3-4) - \ln\left(\frac{(18 \cdot \text{RootOf}(\text{RootOf}(_Z^3-4)^2+6_Z \cdot \text{RootOf}(_Z^3-4)+36_Z^2)^2 \cdot \text{RootOf}(_Z^3-4)^2 \cdot x^3 - 12 \cdot \text{RootOf}(\text{RootOf}(_Z^3-4)^2+6_Z \cdot \text{RootOf}(_Z^3-4)+36_Z^2) \cdot \text{RootOf}(_Z^3-4)^3 \cdot x^3 + 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3-4)^2+6_Z \cdot \text{RootOf}(_Z^3-4)+36_Z^2) \cdot x^3 - 12 \cdot \text{RootOf}(_Z^3-4) \cdot x^3 + 21 \cdot \text{RootOf}(_Z^3-4)^2 \cdot (-x^3+1)^{1/3} + 42 \cdot (-x^3+1)^{2/3} - 42 \cdot \text{RootOf}(\text{RootOf}(_Z^3-4)^2+6_Z \cdot \text{RootOf}(_Z^3-4)+36_Z^2) + 28 \cdot \text{RootOf}(_Z^3-4))}{(x+1)/(x^2-x+1)} \cdot \text{RootOf}(\text{RootOf}(_Z^3-4)^2+6_Z \cdot \text{RootOf}(_Z^3-4)+36_Z^2)\right)$

maxima [A] time = 1.19, size = 97, normalized size = 1.00

$\frac{1}{6} \sqrt{3} \cdot 2^{2/3} \arctan\left(\frac{1}{6} \sqrt{3} \cdot 2^{2/3} \left(2^{1/3} + 2(-x^3 + 1)^{1/3}\right)\right) + \frac{1}{5}(-x^3 + 1)^{5/3} - \frac{1}{12} \cdot 2^{2/3} \log\left(2^{2/3} + 2^{1/3}(-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}\right) + \frac{1}{6} \cdot 2^{2/3} \log\left(\sqrt{-2^{1/3} + (-x^3 + 1)^{1/3}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^8/(-x^3+1)^{1/3}/(x^3+1), x, \text{algorithm}=\text{"maxima"})$

[Out] $\frac{1}{6}\sqrt{3} \cdot 2^{2/3} \arctan\left(\frac{1}{6}\sqrt{3} \cdot 2^{2/3} \left(2^{1/3} + 2(-x^3 + 1)^{1/3}\right)\right) + \frac{1}{5}(-x^3 + 1)^{5/3} - \frac{1}{12} \cdot 2^{2/3} \log\left(2^{2/3} + 2^{1/3}(-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}\right) + \frac{1}{6} \cdot 2^{2/3} \log\left(\sqrt{-2^{1/3} + (-x^3 + 1)^{1/3}}\right)$

mupad [B] time = 4.65, size = 111, normalized size = 1.14

$\frac{2^{2/3} \ln\left(\left(1-x^3\right)^{1/3} - 2^{1/3}\right) + \left(1-x^3\right)^{5/3}}{6} + \frac{2^{2/3} \ln\left(\left(1-x^3\right)^{1/3} - \frac{2^{1/3}(-1+\sqrt{3}i)^2}{4}\right) (-1+\sqrt{3}i)}{12} - \frac{2^{2/3} \ln\left(\left(1-x^3\right)^{1/3} - \frac{2^{1/3}(1+\sqrt{3}i)^2}{4}\right) (1+\sqrt{3}i)}{12}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/((1 - x^3)^(1/3)*(x^3 + 1)),x)`

[Out] $(2^{2/3} \log((1 - x^3)^{1/3} - 2^{1/3}))/6 + (1 - x^3)^{5/3}/5 + (2^{2/3} \log((1 - x^3)^{1/3} - (2^{1/3} * (3^{1/2} * 1i - 1)^2)/4) * (3^{1/2} * 1i - 1))/12 - (2^{2/3} \log((1 - x^3)^{1/3} - (2^{1/3} * (3^{1/2} * 1i + 1)^2)/4) * (3^{1/2} * 1i + 1))/12$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8/(-x**3+1)**(1/3)/(x**3+1),x)`

[Out] `Integral(x**8/((-x - 1)*(x**2 + x + 1))** (1/3)*(x + 1)*(x**2 - x + 1)), x)`

$$3.376 \quad \int \frac{x^5}{\sqrt[3]{1-x^3}(1+x^3)} dx$$

Optimal. Leaf size=98

$$-\frac{1}{2}(1-x^3)^{2/3} + \frac{\log(x^3+1)}{6\sqrt[3]{2}} - \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {446, 80, 55, 617, 204, 31}

$$-\frac{1}{2}(1-x^3)^{2/3} + \frac{\log(x^3+1)}{6\sqrt[3]{2}} - \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^5/((1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] -(1 - x^3)^(2/3)/2 - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f}

, n, p}, x] && NeQ[n + p + 2, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{\sqrt[3]{1-x^3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\
 &= -\frac{1}{2} (1-x^3)^{2/3} - \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\
 &= -\frac{1}{2} (1-x^3)^{2/3} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2}x + x^2} dx, x, \sqrt[3]{1-x^3} \right) + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 + 2^{2/3} \sqrt[3]{1-x^3} \right)}{2\sqrt[3]{2}} \\
 &= -\frac{1}{2} (1-x^3)^{2/3} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 + 2^{2/3} \sqrt[3]{1-x^3} \right)}{2\sqrt[3]{2}} \\
 &= -\frac{1}{2} (1-x^3)^{2/3} - \frac{\tan^{-1} \left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 95, normalized size = 0.97

$$\frac{1}{12} \left(-6(1-x^3)^{2/3} + 2^{2/3} \log(x^3+1) - 3 \cdot 2^{2/3} \log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right) - 2 \cdot 2^{2/3} \sqrt{3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{1-x^3} + 1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] (-6*(1 - x^3)^(2/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 2^(2/3)*Log[1 + x^3] - 3*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/12

IntegrateAlgebraic [A] time = 0.12, size = 132, normalized size = 1.35

$$-\frac{1}{2}(1-x^3)^{2/3} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} - 2\right)}{3\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}(1-x^3)^{2/3} + 2^{2/3}\sqrt[3]{1-x^3} + 2\right)}{6\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] -1/2*(1 - x^3)^(2/3) - ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))

fricas [A] time = 1.01, size = 125, normalized size = 1.28

$$-\frac{1}{6}\sqrt{6}2^{1/3}(-1)^{1/3}\arctan\left(\frac{1}{6}\cdot 2^{1/3}\left(2\sqrt{6}(-1)^{1/3}(-x^3+1)^{1/3}-\sqrt{6}2^{1/3}\right)\right)-\frac{1}{12}\cdot 2^{2/3}(-1)^{1/3}\log\left(2^{1/3}(-1)^{2/3}(-x^3+1)^{1/3}-2^{2/3}(-1)^{1/3}+(-x^3+1)^{2/3}\right)+\frac{1}{6}\cdot 2^{2/3}(-1)^{1/3}\log\left(-2^{1/3}(-1)^{2/3}+(-x^3+1)^{1/3}\right)-\frac{1}{2}(-x^3+1)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1), x, algorithm="fricas")

[Out] -1/6*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*(-1)^(1/3)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)

giac [A] time = 0.18, size = 98, normalized size = 1.00

$$-\frac{1}{6}\sqrt{3}2^{2/3}\arctan\left(\frac{1}{6}\sqrt{3}2^{2/3}\left(2^{1/3}+2(-x^3+1)^{1/3}\right)\right)+\frac{1}{12}\cdot 2^{2/3}\log\left(2^{2/3}+2^{1/3}(-x^3+1)^{1/3}+(-x^3+1)^{2/3}\right)-\frac{1}{6}\cdot 2^{2/3}\log\left(\left|-2^{1/3}+(-x^3+1)^{1/3}\right|\right)-\frac{1}{2}(-x^3+1)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] $-1/6\sqrt{3} \cdot 2^{2/3} \cdot \arctan(1/6\sqrt{3} \cdot 2^{2/3} \cdot (2^{1/3} + 2 \cdot (-x^3 + 1)^{1/3})) + 1/12 \cdot 2^{2/3} \cdot \log(2^{2/3} + 2^{1/3} \cdot (-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}) - 1/6 \cdot 2^{2/3} \cdot \log(\text{abs}(-2^{1/3} + (-x^3 + 1)^{1/3})) - 1/2 \cdot (-x^3 + 1)^{2/3}$

maple [C] time = 3.82, size = 671, normalized size = 6.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(-x^3+1)^(1/3)/(x^3+1),x)

[Out] $1/2 \cdot (x^3 - 1) / (-x^3 + 1)^{1/3} + \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 + 4) + 36 \cdot _Z^2) \cdot \ln((15 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 + 4) + 36 \cdot _Z^2) \cdot \text{RootOf}(_Z^3 + 4)^3 \cdot x^3 + 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 + 4) + 36 \cdot _Z^2)^2 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x^3 + 5 \cdot \text{RootOf}(_Z^3 + 4) \cdot x^3 + 6 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 + 4) + 36 \cdot _Z^2) \cdot x^3 + 21 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot (-x^3 + 1)^{1/3} + 42 \cdot (-x^3 + 1)^{2/3} - 35 \cdot \text{RootOf}(_Z^3 + 4) - 42 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 + 4) + 36 \cdot _Z^2)) / (x + 1) / (x^2 - x + 1)) - 1/6 \cdot \ln((-12 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 + 4) + 36 \cdot _Z^2) \cdot \text{RootOf}(_Z^3 + 4)^3 \cdot x^3 + 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 + 4) + 36 \cdot _Z^2)^2 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x^3 + 12 \cdot \text{RootOf}(_Z^3 + 4) \cdot x^3 - 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 + 4) + 36 \cdot _Z^2) \cdot x^3 + 21 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot (-x^3 + 1)^{1/3} + 42 \cdot (-x^3 + 1)^{2/3} - 28 \cdot \text{RootOf}(_Z^3 + 4) + 42 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 + 4) + 36 \cdot _Z^2)) / (x + 1) / (x^2 - x + 1)) \cdot \text{RootOf}(_Z^3 + 4) - \ln((-12 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 + 4) + 36 \cdot _Z^2) \cdot \text{RootOf}(_Z^3 + 4)^3 \cdot x^3 + 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 + 4) + 36 \cdot _Z^2)^2 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x^3 + 12 \cdot \text{RootOf}(_Z^3 + 4) \cdot x^3 - 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 + 4) + 36 \cdot _Z^2) \cdot x^3 + 21 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot (-x^3 + 1)^{1/3} + 42 \cdot (-x^3 + 1)^{2/3} - 28 \cdot \text{RootOf}(_Z^3 + 4) + 42 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 + 4) + 36 \cdot _Z^2)) / (x + 1) / (x^2 - x + 1)) \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 + 4) + 36 \cdot _Z^2)$

maxima [A] time = 1.21, size = 97, normalized size = 0.99

$-\frac{1}{6} \sqrt{3} \cdot 2^{2/3} \arctan\left(\frac{1}{6} \sqrt{3} \cdot 2^{2/3} \left(2^{1/3} + 2(-x^3 + 1)^{1/3}\right)\right) + \frac{1}{12} \cdot 2^{2/3} \log\left(2^{2/3} + 2^{1/3}(-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}\right) - \frac{1}{6} \cdot 2^{2/3} \log\left(-2^{1/3} + (-x^3 + 1)^{1/3}\right) - \frac{1}{2}(-x^3 + 1)^{2/3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")

[Out] $-1/6\sqrt{3} \cdot 2^{2/3} \cdot \arctan(1/6\sqrt{3} \cdot 2^{2/3} \cdot (2^{1/3} + 2 \cdot (-x^3 + 1)^{1/3})) + 1/12 \cdot 2^{2/3} \cdot \log(2^{2/3} + 2^{1/3} \cdot (-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}) - 1/6 \cdot 2^{2/3} \cdot \log(-2^{1/3} + (-x^3 + 1)^{1/3}) - 1/2 \cdot (-x^3 + 1)^{2/3}$

mupad [B] time = 4.69, size = 111, normalized size = 1.13

$$\frac{2^{2/3} \ln\left((1-x^3)^{1/3} - 2^{1/3}\right)}{6} - \frac{(1-x^3)^{2/3}}{2} - \frac{2^{2/3} \ln\left((1-x^3)^{1/3} - \frac{2^{1/3}(-1+\sqrt{3}1i)^2}{4}\right)(-1+\sqrt{3}1i)}{12} + \frac{2^{2/3} \ln\left((1-x^3)^{1/3} - \frac{2^{1/3}(1+\sqrt{3}1i)^2}{4}\right)(1+\sqrt{3}1i)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/((1 - x^3)^(1/3)*(x^3 + 1)),x)`

[Out] $(2^{2/3} \log((1-x^3)^{1/3} - (2^{1/3} * (3^{1/2} * 1i + 1)^2) / 4) * (3^{1/2} * 1i + 1)) / 12 - (1-x^3)^{2/3} / 2 - (2^{2/3} \log((1-x^3)^{1/3} - (2^{1/3} * (3^{1/2} * 1i - 1)^2) / 4) * (3^{1/2} * 1i - 1)) / 12 - (2^{2/3} \log((1-x^3)^{1/3} - 2^{1/3})) / 6$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(-x**3+1)**(1/3)/(x**3+1),x)`

[Out] `Integral(x**5/((-x - 1)*(x**2 + x + 1))** (1/3)*(x + 1)*(x**2 - x + 1)), x)`

$$3.377 \quad \int \frac{x^2}{\sqrt[3]{1-x^3}(1+x^3)} dx$$

Optimal. Leaf size=82

$$-\frac{\log(x^3+1)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{2\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {444, 55, 617, 204, 31}

$$-\frac{\log(x^3+1)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{2\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 444

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt[3]{1-x^3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\ &= -\frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2}x + x^2} dx, x, \sqrt[3]{1-x^3} \right) - \frac{\text{Subst} \left(\int \frac{1}{\sqrt[3]{2}-x} dx, x, \sqrt[3]{1-x^3} \right)}{2\sqrt[3]{2}} \\ &= -\frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{2\sqrt[3]{2}} - \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 + 2^{2/3}\sqrt[3]{1-x^3} \right)}{\sqrt[3]{2}} \\ &= \frac{\tan^{-1} \left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{2\sqrt[3]{2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 0.89

$$\frac{-\log(x^3 + 1) + 3 \log(\sqrt[3]{2} - \sqrt[3]{1-x^3}) + 2\sqrt{3} \tan^{-1} \left(\frac{2^{2/3}\sqrt[3]{1-x^3} + 1}{\sqrt{3}} \right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)), x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + x^3] + 3
*Log[2^(1/3) - (1 - x^3)^(1/3)])/(6*2^(1/3))
```

IntegrateAlgebraic [A] time = 0.09, size = 116, normalized size = 1.41

$$\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))

fricas [A] time = 0.58, size = 90, normalized size = 1.10

$$\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")

[Out] 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))

giac [A] time = 0.18, size = 87, normalized size = 1.06

$$\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))

maple [C] time = 5.59, size = 476, normalized size = 5.80

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-x^3+1)^(1/3)/(x^3+1),x)

```
[Out] RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+72*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+15*RootOf(_Z^3-4)*x^3+72*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+126*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+42*(-x^3+1)^(2/3)-35*RootOf(_Z^3-4)-168*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))+1/6*RootOf(_Z^3-4)*ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+45*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-2*RootOf(_Z^3-4)*x^3-15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-63*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)-21*(-x^3+1)^(2/3)+14*RootOf(_Z^3-4)+105*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))
```

maxima [A] time = 1.30, size = 86, normalized size = 1.05

$$\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

mupad [B] time = 4.89, size = 100, normalized size = 1.22

$$\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}1i\right)^2}{4}\right)\left(-1+\sqrt{3}1i\right)}{12}-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}1i\right)^2}{4}\right)\left(1+\sqrt{3}1i\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{-(x-1)(x^2+x+1)(x+1)(x^2-x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

$$3.378 \quad \int \frac{1}{x \sqrt[3]{1-x^3} (1+x^3)} dx$$

Optimal. Leaf size=137

$$\frac{\log(x^3 + 1)}{6\sqrt[3]{2}} + \frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2}$$

Rubi [A] time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {446, 86, 55, 618, 204, 31, 617}

$$\frac{\log(x^3 + 1)}{6\sqrt[3]{2}} + \frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(6*2^(1/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 86

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x] - Dist[d/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && !IntegerQ[p]

Rule 204

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 446

$\text{Int}[(x_)^{(m_.)}*((a_.) + (b_.)*(x_)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_)^{(n_.)})^{(q_.)}], x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*x)^p * (c + d*x)^q, x}], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rule 617

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2)], x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 618

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x\sqrt[3]{1-x^3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}x(1+x)} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}x} dx, x, x^3 \right) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\
 &= -\frac{\log(x)}{2} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x} dx, x, \sqrt[3]{1-x^3} \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x} dx, x, \sqrt[3]{1-x^3} \right) \\
 &= -\frac{\log(x)}{2} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{2} \log \left(1 - \sqrt[3]{1-x^3} \right) - \frac{\log \left(\sqrt[3]{2} - \sqrt[3]{1-x^3} \right)}{2\sqrt[3]{2}} + \frac{\text{Subst} \left(\int \frac{1}{1+x} dx, x, \sqrt[3]{1-x^3} \right)}{2\sqrt[3]{2}} \\
 &= \frac{\tan^{-1} \left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{\tan^{-1} \left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{2} \log \left(1 - \sqrt[3]{1-x^3} \right) - \frac{\log \left(\sqrt[3]{2} - \sqrt[3]{1-x^3} \right)}{2\sqrt[3]{2}} + \frac{\text{Subst} \left(\int \frac{1}{1+x} dx, x, \sqrt[3]{1-x^3} \right)}{2\sqrt[3]{2}}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 133, normalized size = 0.97

$$\frac{1}{12} \left(2^{2/3} \log(x^3 + 1) + 6 \log(1 - \sqrt[3]{1-x^3}) - 3 \cdot 2^{2/3} \log(\sqrt[3]{2} - \sqrt[3]{1-x^3}) + 4\sqrt{3} \tan^{-1} \left(\frac{2\sqrt[3]{1-x^3} + 1}{\sqrt{3}} \right) - 2 \cdot 2^{2/3} \sqrt{3} \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{1-x^3} + 1}{\sqrt{3}} \right) - 6 \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] (4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 6*Log[x] + 2^(2/3)*Log[1 + x^3] + 6*Log[1 - (1 - x^3)^(1/3)] - 3*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/12

IntegrateAlgebraic [A] time = 0.20, size = 195, normalized size = 1.42

$$\frac{1}{3} \log(\sqrt[3]{1-x^3} - 1) - \frac{\log(2^{2/3} \sqrt[3]{1-x^3} - 2)}{3\sqrt[3]{2}} - \frac{1}{6} \log((1-x^3)^{2/3} + \sqrt[3]{1-x^3} + 1) + \frac{\log(\sqrt[3]{2}(1-x^3)^{2/3} + 2^{2/3} \sqrt[3]{1-x^3} + 2)}{6\sqrt[3]{2}} + \frac{\tan^{-1} \left(\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{\tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}} \right)}{\sqrt[3]{2} \sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] ArcTan[1/Sqrt[3] + (2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[-1 + (1 - x^3)^(1/3)]/3 - Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6 + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))

fricas [C] time = 1.48, size = 410, normalized size = 2.99

⋯

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")

[Out] 1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(1/8*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^3 - 3/4*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 3*(-x^3 + 1)^(1/3) + 1) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) - 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2))*log(3/8*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2))*log(-3/8*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 3*(-x^3 + 1)^(1/3))

$\frac{1}{3} - (-1)^{(1/3)}^2 + 3*(-x^3 + 1)^{(1/3)} + 1/3*\sqrt{3}*\arctan(2/3*\sqrt{3}*(-x^3 + 1)^{(1/3)} + 1/3*\sqrt{3}) + 1/3*\log(-1/24*(1*\sqrt{3}*(-1)^{(1/3)} - (-1)^{(1/3)}^3 + (-x^3 + 1)^{(1/3)} - 4/3) - 1/6*\log((-x^3 + 1)^{(2/3)} + (-x^3 + 1)^{(1/3)} + 1)$

giac [A] time = 0.20, size = 149, normalized size = 1.09

$-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{2}{3}}+2(-x^3+1)^{\frac{2}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{2}{3}}(-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{2}{3}}+(-x^3+1)^{\frac{2}{3}}\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(-x^3+1)^{\frac{2}{3}}+1\right)\right)-\frac{1}{6}\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] $-1/6*\sqrt{3}*2^{(2/3)}*\arctan(1/6*\sqrt{3}*2^{(2/3)}*(2^{(1/3)} + 2*(-x^3 + 1)^{(1/3)})) + 1/12*2^{(2/3)}*\log(2^{(2/3)} + 2^{(1/3)}*(-x^3 + 1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) - 1/6*2^{(2/3)}*\log(\text{abs}(-2^{(1/3)} + (-x^3 + 1)^{(1/3)})) + 1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*(-x^3 + 1)^{(1/3)} + 1)) - 1/6*\log((-x^3 + 1)^{(2/3)} + (-x^3 + 1)^{(1/3)} + 1) + 1/3*\log(\text{abs}((-x^3 + 1)^{(1/3)} - 1))$

maple [F] time = 2.44, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^3 + 1)^{\frac{1}{3}} (x^3 + 1) x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-x^3+1)^(1/3)/(x^3+1),x)

[Out] int(1/x/(-x^3+1)^(1/3)/(x^3+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x), x)

mupad [B] time = 4.80, size = 256, normalized size = 1.87

$\frac{\ln(6-6(-x^3)^{1/3})}{3} + \ln\left(\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)^3 \left(1458\left(-\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)^2 - 135(-x^3)^{1/3}\right) - (-x^3)^{1/3}\right) \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) - \ln\left(\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)^3 \left(1458\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)^2 - 135(-x^3)^{1/3}\right) - (-x^3)^{1/3}\right) \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) - \frac{2^{2/3} \ln\left(\frac{2(1-x^3)^{1/3} - 3^{2/3}}{3}\right)}{6} + \frac{(-1)^{1/3} 2^{2/3} \ln\left(\frac{3(1-x^3)^{1/3} - 3i \cdot 2^{2/3}}{3}\right)}{6} - \frac{(-1)^{1/3} 2^{2/3} \ln\left(\frac{(1+\sqrt{3}i)\left(\ln\left(\frac{1-x^3}{3}\right) - \frac{11x^{2/3} \sqrt{1-x^3}}{4}\right)}{4x} - (1-x^3)^{1/3}\right)}{12} (1+\sqrt{3}i)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(1 - x^3)^(1/3)*(x^3 + 1)),x)`

[Out] $\log(6 - 6*(1 - x^3)^{1/3})/3 + \log((3^{1/2}*1i)/6 - 1/6)^3*(1458*(3^{1/2}*1i)/6 - 1/6)^2 - 135*(1 - x^3)^{1/3}) - (1 - x^3)^{1/3})*((3^{1/2}*1i)/6 - 1/6) - \log(- ((3^{1/2}*1i)/6 + 1/6)^3*(1458*(3^{1/2}*1i)/6 + 1/6)^2 - 135*(1 - x^3)^{1/3}) - (1 - x^3)^{1/3})*((3^{1/2}*1i)/6 + 1/6) - (2^{2/3}*\log((3*(1 - x^3)^{1/3})/2 - (3*2^{1/3})/2))/6 + ((-1)^{1/3}*2^{2/3}*\log((3*(1 - x^3)^{1/3})/2 - (3*(-1)^{2/3}*2^{1/3})/2))/6 - ((-1)^{1/3}*2^{2/3}*\log(- (3^{1/2}*1i + 1)^3*(135*(1 - x^3)^{1/3} - (81*(-1)^{2/3}*2^{1/3}*(3^{1/2}*1i + 1)^2)/4))/432 - (1 - x^3)^{1/3})*(3^{1/2}*1i + 1))/12$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-x**3+1)**(1/3)/(x**3+1),x)`

[Out] `Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)`

$$3.379 \quad \int \frac{1}{x^4 \sqrt[3]{1-x^3} (1+x^3)} dx$$

Optimal. Leaf size=157

$$\frac{(1-x^3)^{2/3}}{3x^3} - \frac{\log(x^3+1)}{6\sqrt[3]{2}} - \frac{1}{3} \log\left(1 - \sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{2 \tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {446, 103, 156, 55, 618, 204, 31, 617}

$$\frac{(1-x^3)^{2/3}}{3x^3} - \frac{\log(x^3+1)}{6\sqrt[3]{2}} - \frac{1}{3} \log\left(1 - \sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{2 \tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] $-(1-x^3)^{2/3}/(3*x^3) - (2*\text{ArcTan}[(1 + 2*(1-x^3)^{1/3})/\text{Sqrt}[3]])/(3*\text{Sqrt}[3]) + \text{ArcTan}[(1 + 2^{2/3}*(1-x^3)^{1/3})/\text{Sqrt}[3]]/(2^{1/3}*\text{Sqrt}[3]) + \text{Log}[x]/3 - \text{Log}[1 + x^3]/(6*2^{1/3}) - \text{Log}[1 - (1-x^3)^{1/3}]/3 + \text{Log}[2^{2/3} - (1-x^3)^{1/3}]/(2*2^{1/3})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_)*((c_.) + (d_.)*(x_))^(1/3), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m+1)*(c + d*x)^(n+1)*(e + f*x)^(p+1))/((m+1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m+1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m+1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m+1) - b*(d*e*(m+n+2) + c*f*(m+p+2)) - b*d*f*(m+n+p+3)*x,

$x], x], x] /;$ FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \sqrt[3]{1-x^3} (1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x} x^2 (1+x)} dx, x, x^3 \right) \\
&= -\frac{(1-x^3)^{2/3}}{3x^3} - \frac{1}{3} \text{Subst} \left(\int \frac{\frac{2}{3} - \frac{x}{3}}{\sqrt[3]{1-x} x (1+x)} dx, x, x^3 \right) \\
&= -\frac{(1-x^3)^{2/3}}{3x^3} - \frac{2}{9} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x} x} dx, x, x^3 \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x} (1+x)} dx, x, x^3 \right) \\
&= -\frac{(1-x^3)^{2/3}}{3x^3} + \frac{\log(x)}{3} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{3} \text{Subst} \left(\int \frac{1}{1-x} dx, x, \sqrt[3]{1-x^3} \right) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{1+x} dx, x, \sqrt[3]{1-x^3} \right) \\
&= -\frac{(1-x^3)^{2/3}}{3x^3} + \frac{\log(x)}{3} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{1}{3} \log\left(1 - \sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} \\
&= -\frac{(1-x^3)^{2/3}}{3x^3} - \frac{2 \tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(x)}{3} - \frac{\log(1+x^3)}{6\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 153, normalized size = 0.97

$$\frac{1}{36} \left(3 \left(-\frac{4(1-x^3)^{2/3}}{x^3} - 2^{2/3} \log(x^3+1) - 4 \log\left(1 - \sqrt[3]{1-x^3}\right) + 3 \cdot 2^{2/3} \log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right) + 2 \cdot 2^{2/3} \sqrt{3} \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right) + 4 \log(x) \right) - 8\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] (-8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 3*((-4*(1 - x^3)^(2/3))/x^3 + 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 4*Log[x] - 2^(2/3)*Log[1 + x^3] - 4*Log[1 - (1 - x^3)^(1/3)] + 3*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)]))/36

IntegrateAlgebraic [A] time = 0.27, size = 215, normalized size = 1.37

$$-\frac{(1-x^3)^{2/3}}{3x^3} - \frac{2}{9} \log\left(\sqrt[3]{1-x^3} - 1\right) + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} - 2\right)}{3\sqrt[3]{2}} + \frac{1}{9} \log\left((1-x^3)^{2/3} + \sqrt[3]{1-x^3} + 1\right) - \frac{\log\left(\sqrt[3]{2}(1-x^3)^{2/3} + 2^{2/3}\sqrt[3]{1-x^3} + 2\right)}{6\sqrt[3]{2}} - \frac{2 \tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] $-1/3*(1 - x^3)^{(2/3)}/x^3 - (2*\text{ArcTan}[1/\text{Sqrt}[3] + (2*(1 - x^3)^{(1/3)})/\text{Sqrt}[3]])/(3*\text{Sqrt}[3]) + \text{ArcTan}[1/\text{Sqrt}[3] + (2^{(2/3)}*(1 - x^3)^{(1/3)})/\text{Sqrt}[3]]/(2^{(1/3)}*\text{Sqrt}[3]) - (2*\text{Log}[-1 + (1 - x^3)^{(1/3)})]/9 + \text{Log}[-2 + 2^{(2/3)}*(1 - x^3)^{(1/3)})/(3*2^{(1/3)}) + \text{Log}[1 + (1 - x^3)^{(1/3)} + (1 - x^3)^{(2/3)}]/9 - \text{Log}[2 + 2^{(2/3)}*(1 - x^3)^{(1/3)} + 2^{(1/3)}*(1 - x^3)^{(2/3)}]/(6*2^{(1/3)})$

fricas [A] time = 0.48, size = 187, normalized size = 1.19

$$\frac{6\sqrt{6}2^{\frac{2}{3}}x^3\arctan\left(\frac{1}{6}\cdot 2^{\frac{2}{3}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right)-3\cdot 2^{\frac{2}{3}}x^3\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)+6\cdot 2^{\frac{2}{3}}x^3\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)-8\sqrt{3}x^3\arctan\left(\frac{2}{3}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+4x^3\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)-8x^3\log\left((-x^3+1)^{\frac{1}{3}}-1\right)-12(-x^3+1)^{\frac{2}{3}}}{36x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")

[Out] $1/36*(6*\text{sqrt}(6)*2^{(1/6)}*x^3*\arctan(1/6*2^{(1/6)}*(\text{sqrt}(6)*2^{(1/3)} + 2*\text{sqrt}(6)*(-x^3 + 1)^{(1/3)})) - 3*2^{(2/3)}*x^3*\log(2^{(2/3)} + 2^{(1/3)}*(-x^3 + 1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) + 6*2^{(2/3)}*x^3*\log(-2^{(1/3)} + (-x^3 + 1)^{(1/3)}) - 8*\text{sqrt}(3)*x^3*\arctan(2/3*\text{sqrt}(3)*(-x^3 + 1)^{(1/3)} + 1/3*\text{sqrt}(3)) + 4*x^3*\log((-x^3 + 1)^{(2/3)} + (-x^3 + 1)^{(1/3)} + 1) - 8*x^3*\log((-x^3 + 1)^{(1/3)} - 1) - 12*(-x^3 + 1)^{(2/3)}/x^3$

giac [A] time = 0.19, size = 163, normalized size = 1.04

$$\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(-x^3+1)^{\frac{1}{3}}+1\right)\right)-\frac{(-x^3+1)^{\frac{2}{3}}}{3x^3}+\frac{1}{9}\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left((-x^3+1)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] $1/6*\text{sqrt}(3)*2^{(2/3)}*\arctan(1/6*\text{sqrt}(3)*2^{(2/3)}*(2^{(1/3)} + 2*(-x^3 + 1)^{(1/3)})) - 1/12*2^{(2/3)}*\log(2^{(2/3)} + 2^{(1/3)}*(-x^3 + 1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) + 1/6*2^{(2/3)}*\log(\text{abs}(-2^{(1/3)} + (-x^3 + 1)^{(1/3)})) - 2/9*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*(-x^3 + 1)^{(1/3)} + 1)) - 1/3*(-x^3 + 1)^{(2/3)}/x^3 + 1/9*\log((-x^3 + 1)^{(2/3)} + (-x^3 + 1)^{(1/3)} + 1) - 2/9*\log(\text{abs}((-x^3 + 1)^{(1/3)} - 1))$

maple [F] time = 2.82, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^3 + 1)^{\frac{1}{3}} (x^3 + 1) x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x)

[Out] int(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^4), x)

mupad [B] time = 4.86, size = 382, normalized size = 2.43

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(1 - x^3)^(1/3)*(x^3 + 1)),x)

[Out] $(2^{2/3} \log((2^{1/3} * ((2^{2/3} * (81 * 2^{1/3}) - 75 * (1 - x^3)^{1/3}))/6 - 38/3)) / 18 + (16 * (1 - x^3)^{1/3}) / 27) / 6 - (1 - x^3)^{2/3} / (3 * x^3) - (2 * \log((344 * (1 - x^3)^{1/3}) / 243 - 344 / 243)) / 9 + \log(((3^{1/2} * 1i) / 9 + 1/9)^2 * ((3^{1/2} * 1i) / 9 + 1/9) * (1458 * ((3^{1/2} * 1i) / 9 + 1/9)^2 - 75 * (1 - x^3)^{1/3})) - 38/3) + (16 * (1 - x^3)^{1/3}) / 27 * ((3^{1/2} * 1i) / 9 + 1/9) - \log((16 * (1 - x^3)^{1/3}) / 27 - ((3^{1/2} * 1i) / 9 - 1/9)^2 * ((3^{1/2} * 1i) / 9 - 1/9) * (1458 * ((3^{1/2} * 1i) / 9 - 1/9)^2 - 75 * (1 - x^3)^{1/3})) + 38/3) * ((3^{1/2} * 1i) / 9 - 1/9) + (2^{2/3} * \log((16 * (1 - x^3)^{1/3}) / 27 + (2^{1/3} * (3^{1/2} * 1i - 1))^2 * ((2^{2/3} * (3^{1/2} * 1i - 1) * ((81 * 2^{1/3} * (3^{1/2} * 1i - 1))^2 / 4 - 75 * (1 - x^3)^{1/3}))) / 12 - 38/3)) / 72 * (3^{1/2} * 1i - 1)) / 12 - (2^{2/3} * \log((16 * (1 - x^3)^{1/3}) / 27 - (2^{1/3} * (3^{1/2} * 1i + 1))^2 * ((2^{2/3} * (3^{1/2} * 1i + 1) * ((81 * 2^{1/3} * (3^{1/2} * 1i + 1))^2 / 4 - 75 * (1 - x^3)^{1/3}))) / 12 + 38/3)) / 72 * (3^{1/2} * 1i + 1)) / 12$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(-x**3+1)**(1/3)/(x**3+1),x)

[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)

$$3.380 \quad \int \frac{x^6}{\sqrt[3]{1-x^3}(1+x^3)} dx$$

Optimal. Leaf size=154

$$-\frac{1}{3}(1-x^3)^{2/3} x - \frac{\log(x^3+1)}{6\sqrt[3]{2}} + \frac{\log(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x)}{2\sqrt[3]{2}} - \frac{1}{3} \log(\sqrt[3]{1-x^3} + x) + \frac{2 \tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.15, antiderivative size = 226, normalized size of antiderivative = 1.47, number of steps used = 15, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {494, 470, 522, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{3}(1-x^3)^{2/3} x + \frac{1}{9} \log\left(\frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} + 1\right) - \frac{2}{9} \log\left(\frac{x}{\sqrt[3]{1-x^3}} + 1\right) - \frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} + \frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} + \frac{2 \tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^6/((1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] $-(x*(1-x^3)^{2/3})/3 + (2*\text{ArcTan}[(1-(2*x)/(1-x^3)^{1/3})/\text{Sqrt}[3]])/(3*\text{Sqrt}[3]) - \text{ArcTan}[(1-(2*2^{1/3}*x)/(1-x^3)^{1/3})/\text{Sqrt}[3]]/(2^{1/3}*\text{Sqrt}[3]) + \text{Log}[1+x^2/(1-x^3)^{2/3} - x/(1-x^3)^{1/3}]/9 - (2*\text{Log}[1+x/(1-x^3)^{1/3}])/9 - \text{Log}[1+(2^{2/3}*x^2)/(1-x^3)^{2/3} - (2^{1/3}*x)/(1-x^3)^{1/3}]/(6*2^{1/3}) + \text{Log}[1+(2^{1/3}*x)/(1-x^3)^{1/3}]/(3*2^{1/3})$

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 494

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && LtQ[-1, p, 0]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\sqrt[3]{1-x^3}(1+x^3)} dx &= \text{Subst} \left(\int \frac{x^6}{(1+x^3)^2(1+2x^3)} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{1}{3}x(1-x^3)^{2/3} + \frac{1}{3} \text{Subst} \left(\int \frac{1-x^3}{(1+x^3)(1+2x^3)} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{1}{3}x(1-x^3)^{2/3} - \frac{2}{3} \text{Subst} \left(\int \frac{1}{1+x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) + \text{Subst} \left(\int \frac{1}{1+2x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{1}{3}x(1-x^3)^{2/3} - \frac{2}{9} \text{Subst} \left(\int \frac{1}{1+x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{2}{9} \text{Subst} \left(\int \frac{2-x}{1-x+x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{1}{3}x(1-x^3)^{2/3} - \frac{2}{9} \log \left(1 + \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} + \frac{1}{9} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{1}{3}x(1-x^3)^{2/3} + \frac{1}{9} \log \left(1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{2}{9} \log \left(1 + \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} \\
&= -\frac{1}{3}x(1-x^3)^{2/3} + \frac{2 \tan^{-1} \left(\frac{1 - \frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{3\sqrt{3}} - \frac{\tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{\sqrt[3]{2}\sqrt{3}} + \frac{1}{9} \log \left(1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} \right)
\end{aligned}$$

Mathematica [C] time = 0.16, size = 144, normalized size = 0.94

$$\frac{1}{36} \left(-6x^4 F_1 \left(\frac{4}{3}; \frac{1}{3}, \frac{7}{3}; x^3, -x^3 \right) - 12(1-x^3)^{2/3} x + 2^{2/3} \left(2 \log \left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + 1 \right) + 2\sqrt{3} \tan^{-1} \left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} - 1}{\sqrt{3}} \right) - \log \left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + \frac{2^{2/3}x^2}{(x^3-1)^{2/3}} + 1 \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] $(-12*x*(1 - x^3)^{(2/3)} - 6*x^4*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3] + 2^{(2/3)}*(2*sqrt[3]*ArcTan[(-1 + (2*2^{(1/3)}*x)/(-1 + x^3)^{(1/3)})/sqrt[3]] - Log[1 + (2^{(2/3)}*x^2)/(-1 + x^3)^{(2/3)} - (2^{(1/3)}*x)/(-1 + x^3)^{(1/3)}] + 2*Log[1 + (2^{(1/3)}*x)/(-1 + x^3)^{(1/3)}]))/36$

IntegrateAlgebraic [A] time = 0.54, size = 230, normalized size = 1.49

$$-\frac{1}{3}(1-x^3)^{2/3}x - \frac{2}{9}\log(\sqrt[3]{1-x^3}+x) + \frac{\log(2^{2/3}\sqrt[3]{1-x^3}+2x)}{3\sqrt{2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}-x}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{\sqrt{2}\sqrt{3}} + \frac{1}{9}\log(-\sqrt[3]{1-x^3}x+(1-x^3)^{2/3}+x^2) - \frac{\log(2^{2/3}\sqrt[3]{1-x^3}x - \sqrt[3]{2}(1-x^3)^{2/3} - 2x^2)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] $-1/3*(x*(1 - x^3)^{(2/3)}) - (2*ArcTan[(sqrt[3]*x)/(-x + 2*(1 - x^3)^{(1/3)})])/(3*sqrt[3]) + ArcTan[(sqrt[3]*x)/(-x + 2^{(2/3)}*(1 - x^3)^{(1/3)})]/(2^{(1/3)}*sqrt[3]) - (2*Log[x + (1 - x^3)^{(1/3)}])/9 + Log[2*x + 2^{(2/3)}*(1 - x^3)^{(1/3)}]/(3*2^{(1/3)}) + Log[x^2 - x*(1 - x^3)^{(1/3)} + (1 - x^3)^{(2/3)}]/9 - Log[-2*x^2 + 2^{(2/3)}*x*(1 - x^3)^{(1/3)} - 2^{(1/3)}*(1 - x^3)^{(2/3)}]/(6*2^{(1/3)})$

fricas [A] time = 0.47, size = 201, normalized size = 1.31

$$-\frac{1}{3}(-x^3+1)^{2/3}x - \frac{1}{6}\sqrt{6}^{1/2}\arctan\left(-\frac{2^{1/3}(\sqrt{6}^{1/2}x - 2\sqrt{6}(-x^3+1)^{1/3})}{6x}\right) + \frac{1}{6}2^{1/3}\log\left(\frac{2^{1/3}x + (-x^3+1)^{1/3}}{x}\right) - \frac{1}{12}2^{1/3}\log\left(\frac{2^{1/3}x^2 - 2^{1/3}(-x^3+1)^{1/3}x + (-x^3+1)^{2/3}}{x^2}\right) + \frac{2}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x - 2\sqrt{3}(-x^3+1)^{1/3}}{3x}\right) - \frac{2}{9}\log\left(\frac{x + (-x^3+1)^{1/3}}{x}\right) + \frac{1}{9}\log\left(\frac{x^2 - (-x^3+1)^{1/3}x + (-x^3+1)^{2/3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")

[Out] $-1/3*(-x^3 + 1)^{(2/3)}*x - 1/6*sqrt(6)*2^{(1/6)}*arctan(-1/6*2^{(1/6)}*(sqrt(6)*2^{(1/3)}*x - 2*sqrt(6)*(-x^3 + 1)^{(1/3)})/x) + 1/6*2^{(2/3)}*log((2^{(1/3)}*x + (-x^3 + 1)^{(1/3)})/x) - 1/12*2^{(2/3)}*log((2^{(2/3)}*x^2 - 2^{(1/3)}*(-x^3 + 1)^{(1/3)}*x + (-x^3 + 1)^{(2/3)})/x^2) + 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^{(1/3)})/x) - 2/9*log((x + (-x^3 + 1)^{(1/3)})/x) + 1/9*log((x^2 - (-x^3 + 1)^{(1/3)}*x + (-x^3 + 1)^{(2/3)})/x^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(x^3 + 1)(-x^3 + 1)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)

maple [F] time = 1.92, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(-x^3 + 1)^{\frac{1}{3}} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(-x^3+1)^(1/3)/(x^3+1),x)

[Out] int(x^6/(-x^3+1)^(1/3)/(x^3+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{(1 - x^3)^{\frac{1}{3}} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/((1 - x^3)^(1/3)*(x^3 + 1)),x)

[Out] int(x^6/((1 - x^3)^(1/3)*(x^3 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(-x**3+1)**(1/3)/(x**3+1),x)

[Out] Integral(x**6/((-x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)

$$3.381 \quad \int \frac{x^3}{\sqrt[3]{1-x^3}(1+x^3)} dx$$

Optimal. Leaf size=135

$$\frac{\log(x^3 + 1)}{6\sqrt[3]{2}} - \frac{\log(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x)}{2\sqrt[3]{2}} + \frac{1}{2} \log(\sqrt[3]{1-x^3} + x) - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.10, antiderivative size = 207, normalized size of antiderivative = 1.53, number of steps used = 14, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {494, 481, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{6} \log\left(\frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} + 1\right) + \frac{1}{3} \log\left(\frac{x}{\sqrt[3]{1-x^3}} + 1\right) + \frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/6 + Log[1 + x/(1 - x^3)^(1/3)]/3 + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.))),
  x_Symbol] := -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
  x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)
, x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q]/(1 - b*x^k)^(p + q +
(m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```


Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt[3]{1-x^3}(1+x^3)} dx &= \text{Subst} \left(\int \frac{x^3}{(1+x^3)(1+2x^3)} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= \text{Subst} \left(\int \frac{1}{1+x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) - \text{Subst} \left(\int \frac{1}{1+2x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \frac{1}{1+x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{1+\sqrt[3]{2}x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{1-x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= \frac{1}{3} \log \left(1 + \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} - \frac{1}{6} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{1-x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{1}{6} \log \left(1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{1}{3} \log \left(1 + \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}} \\
&= \frac{\tan^{-1} \left(\frac{-1 + \frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{\tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{\sqrt[3]{2}\sqrt{3}} - \frac{1}{6} \log \left(1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{1}{3} \log \left(1 + \frac{x}{\sqrt[3]{1-x^3}} \right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 26, normalized size = 0.19

$$\frac{1}{4}x^4F_1\left(\frac{4}{3}; \frac{1}{3}, 1; \frac{7}{3}; x^3, -x^3\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/((1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] (x^4*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3])/4

IntegrateAlgebraic [A] time = 0.37, size = 212, normalized size = 1.57

$$\frac{1}{3} \log(\sqrt[3]{1-x^3} + x) - \frac{\log(2^{2/3}\sqrt[3]{1-x^3} + 2x)}{3\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}-x}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt[3]{2}\sqrt[3]{1-x^3}-x}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{1}{6} \log(-\sqrt[3]{1-x^3}x + (1-x^3)^{2/3} + x^2) + \frac{\log(2^{2/3}\sqrt[3]{1-x^3}x - \sqrt[3]{2}(1-x^3)^{2/3} - 2x^2)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/((1 - x^3)^(1/3)*(1 + x^3)), x]

```
[Out] ArcTan[(Sqrt[3]*x)/(-x + 2*(1 - x^3)^(1/3))]/Sqrt[3] - ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2^(1/3)*Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/3 - Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6 + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))
```

fricas [C] time = 1.43, size = 452, normalized size = 3.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(-1/8*(x*(I*sqrt(3))*(-1)^(1/3) - (-1)^(1/3))^3 - 6*2^(1/3)*x*(I*sqrt(3))*(-1)^(1/3) - (-1)^(1/3))^2 + 8*x - 24*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sqrt(3))*(-1)^(1/3) - (-1)^(1/3)) - 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3))*(-1)^(1/3) - (-1)^(1/3))^2)*log(-3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3))*(-1)^(1/3) - (-1)^(1/3))^2)*x*(I*sqrt(3))*(-1)^(1/3) - (-1)^(1/3)) + 2^(1/3)*x*(I*sqrt(3))*(-1)^(1/3) - (-1)^(1/3))^2 - 8*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sqrt(3))*(-1)^(1/3) - (-1)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3))*(-1)^(1/3) - (-1)^(1/3))^2)*log(3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3))*(-1)^(1/3) - (-1)^(1/3))^2)*x*(I*sqrt(3))*(-1)^(1/3) - (-1)^(1/3)) - 2^(1/3)*x*(I*sqrt(3))*(-1)^(1/3) - (-1)^(1/3))^2 + 8*(-x^3 + 1)^(1/3))/x) - 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3*log(1/24*(x*(I*sqrt(3))*(-1)^(1/3) - (-1)^(1/3))^3 + 32*x + 24*(-x^3 + 1)^(1/3))/x) - 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

maple [F] time = 1.74, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(-x^3 + 1)^{\frac{1}{3}}(x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(-x^3+1)^(1/3)/(x^3+1),x)`

[Out] `int(x^3/(-x^3+1)^(1/3)/(x^3+1),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")`

[Out] `integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(1 - x^3)^{1/3} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/((1 - x^3)^(1/3)*(x^3 + 1)),x)`

[Out] `int(x^3/((1 - x^3)^(1/3)*(x^3 + 1)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(-x**3+1)**(1/3)/(x**3+1),x)`

[Out] `Integral(x**3/((-x - 1)*(x**2 + x + 1))**1/3*(x + 1)*(x**2 - x + 1)), x)`

$$3.382 \quad \int \frac{1}{\sqrt[3]{1-x^3}(1+x^3)} dx$$

Optimal. Leaf size=88

$$-\frac{\log(x^3+1)}{6\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x\right)}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} + \frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{1-x^3}(1+x^3)} dx &= \text{Subst} \left(\int \frac{1}{1+2x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \frac{1}{1+\sqrt[3]{2}x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{1}{3} \text{Subst} \left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{\text{Subst} \left(\int \frac{-\sqrt[3]{2}+2}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}} \\
&= -\frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}} + \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{\sqrt[3]{2}} \\
&= -\frac{\tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}} + \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 112, normalized size = 1.27

$$\frac{2 \log \left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1 \right) + 2\sqrt{3} \tan^{-1} \left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} - 1}{\sqrt{3}} \right) - \log \left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} + 1 \right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*x^(1/3))/sqrt[3]{1-x^3})/sqrt[3]{3}] - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)] + 2*Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)])/(6*2^(1/3))

IntegrateAlgebraic [A] time = 0.27, size = 126, normalized size = 1.43

$$\frac{\log \left(2^{2/3} \sqrt[3]{1-x^3} + 2x \right)}{3\sqrt[3]{2}} + \frac{\tan^{-1} \left(\frac{\sqrt{3}x}{2^{2/3} \sqrt[3]{1-x^3} - x} \right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log \left(2^{2/3} \sqrt[3]{1-x^3} x - \sqrt[3]{2} (1-x^3)^{2/3} - 2x^2 \right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((1 - x^3)^(1/3)*(1 + x^3)), x]

[Out] ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2^(1/3)*Sqrt[3]) + Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))

fricas [B] time = 2.60, size = 253, normalized size = 2.88

$$\frac{1}{18} \sqrt{6} \arctan \left(\frac{2^{\frac{2}{3}} (6 \sqrt{6} 2^{\frac{2}{3}} (5x^2 + 4x^4 - 3)(-x^3 + 1)^{\frac{1}{3}} - \sqrt{6} 2^{\frac{2}{3}} (71x^9 - 111x^6 + 33x^3 - 1) + 12 \sqrt{6} (19x^6 - 16x^5 + x^2)(-x^3 + 1)^{\frac{1}{3}})}{6(109x^9 - 105x^6 + 3x^3 + 1)} \right) + \frac{1}{18} \cdot 2^{\frac{2}{3}} \log \left(\frac{6 \cdot 2^{\frac{2}{3}} (-x^3 + 1)^{\frac{1}{3}} x^2 + 2^{\frac{2}{3}} (x^3 + 1) + 6(-x^3 + 1)^{\frac{2}{3}}}{x^3 + 1} \right) - \frac{1}{36} \cdot 2^{\frac{2}{3}} \log \left(\frac{3 \cdot 2^{\frac{2}{3}} (5x^4 - 3)(-x^3 + 1)^{\frac{1}{3}} + 2^{\frac{2}{3}} (19x^6 - 16x^5 + 1) - 12(2x^5 - x^2)(-x^3 + 1)^{\frac{1}{3}}}{x^6 + 2x^3 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) + 1/18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)

maple [C] time = 4.94, size = 931, normalized size = 10.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^3+1)^(1/3)/(x^3+1),x)

[Out] RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x-5*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2+9*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+10*x*(-x^3+1)^(2/3)-3*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2

2)))/(x+1)/(x^2-x+1))-1/6*ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)+2*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)+2*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(1 - x^3)^{1/3} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((1 - x^3)^(1/3)*(x^3 + 1)),x)

[Out] int(1/((1 - x^3)^(1/3)*(x^3 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/((-x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

$$3.383 \quad \int \frac{1}{x^3 \sqrt[3]{1-x^3} (1+x^3)} dx$$

Optimal. Leaf size=105

$$\frac{\log(x^3 + 1)}{6\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x\right)}{2\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{(1-x^3)^{2/3}}{2x^2}$$

Rubi [A] time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {494, 453, 200, 31, 634, 617, 204, 628}

$$-\frac{(1-x^3)^{2/3}}{2x^2} + \frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] -(1 - x^3)^(2/3)/(2*x^2) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q]/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && LtQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \sqrt[3]{1-x^3} (1+x^3)} dx &= \text{Subst} \left(\int \frac{1+x^3}{x^3 (1+2x^3)} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{(1-x^3)^{2/3}}{2x^2} - \text{Subst} \left(\int \frac{1}{1+2x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{(1-x^3)^{2/3}}{2x^2} - \frac{1}{3} \text{Subst} \left(\int \frac{1}{1+\sqrt[3]{2}x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{1}{3} \text{Subst} \left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{(1-x^3)^{2/3}}{2x^2} - \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) + \dots \\
&= -\frac{(1-x^3)^{2/3}}{2x^2} + \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}} - \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} - \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{\sqrt[3]{2}} \\
&= -\frac{(1-x^3)^{2/3}}{2x^2} + \frac{\tan^{-1} \left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}} - \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}}
\end{aligned}$$

Mathematica [C] time = 0.60, size = 82, normalized size = 0.78

$$\frac{(-6x^6 + 4x^3 + 2) {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{2x^3}{x^3-1}\right) - 3x^3(x^3+1) {}_2F_1\left(\frac{4}{3}, 2; \frac{7}{3}; \frac{2x^3}{x^3-1}\right)}{4x^2(1-x^3)^{4/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^3*(1-x^3)^(1/3)*(1+x^3)),x]

[Out] -1/4*((2+4*x^3-6*x^6)*Hypergeometric2F1[1/3,1,4/3,(2*x^3)/(-1+x^3)] - 3*x^3*(1+x^3)*Hypergeometric2F1[4/3,2,7/3,(2*x^3)/(-1+x^3)])/(x^2*(1-x^3)^(4/3))

IntegrateAlgebraic [A] time = 0.29, size = 145, normalized size = 1.38

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{(1-x^3)^{2/3}}{2x^2} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}(1-x^3)^{2/3}-2x^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] $-1/2*(1 - x^3)^{(2/3)}/x^2 - \text{ArcTan}[\text{Sqrt}[3]*x]/(-x + 2^{(2/3)}*(1 - x^3)^{(1/3)})]/(2^{(1/3)}*\text{Sqrt}[3]) - \text{Log}[2*x + 2^{(2/3)}*(1 - x^3)^{(1/3)}]/(3*2^{(1/3)}) + \text{Log}[-2*x^2 + 2^{(2/3)}*x*(1 - x^3)^{(1/3)} - 2^{(1/3)}*(1 - x^3)^{(2/3)}]/(6*2^{(1/3)})$

fricas [B] time = 2.58, size = 307, normalized size = 2.92

$$\frac{2\sqrt{6}(-1)^{\frac{1}{3}}x^2 \arctan\left(\frac{z^{\frac{1}{3}}\left(6\sqrt{6}z^{\frac{1}{3}}(-1)^{\frac{1}{3}}(5z^2+4z-1)^{\frac{1}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}(9z^4-16z^2+z^2)(-z^3+1)^{\frac{1}{3}}-\sqrt{6}z^{\frac{1}{3}}(71z^6-111z^4+33z^2-1)\right)}{6(109z^9-105z^6+3z^3+1)}\right)}{36z^2} - 2(-1)^{\frac{1}{3}}x^2 \log\left(\frac{6z^{\frac{1}{3}}(-1)^{\frac{1}{3}}(-z^3+1)^{\frac{1}{3}}z^{\frac{1}{3}}-2z^{\frac{1}{3}}(-1)^{\frac{1}{3}}(z^3+1)+6(-z^3+1)^{\frac{1}{3}}}{z^3+1}\right) + 2(-1)^{\frac{1}{3}}x^2 \log\left(\frac{3z^{\frac{1}{3}}(-1)^{\frac{1}{3}}(5z^2+4z-1)^{\frac{1}{3}}-2z^{\frac{1}{3}}(-1)^{\frac{1}{3}}(9z^4-16z^2+1)+12(2z^6-z^2)(-z^3+1)^{\frac{1}{3}}}{z^2+2z+1}\right) + 18(-z^3+1)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")

[Out] $-1/36*(2*\text{sqrt}(6)*2^{(1/6)}*(-1)^{(1/3)}*x^2*\arctan(1/6*2^{(1/6)}*(6*\text{sqrt}(6)*2^{(2/3)}*(-1)^{(2/3)}*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^{(2/3)} - 12*\text{sqrt}(6)*(-1)^{(1/3)}*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^{(1/3)} - \text{sqrt}(6)*2^{(1/3)}*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^{(2/3)}*(-1)^{(1/3)}*x^2*\log((6*2^{(1/3)}*(-1)^{(2/3)}*(-x^3 + 1)^{(1/3)}*x^2 - 2^{(2/3)}*(-1)^{(1/3)}*(x^3 + 1) + 6*(-x^3 + 1)^{(2/3)}*x)/(x^3 + 1)) + 2^{(2/3)}*(-1)^{(1/3)}*x^2*\log(-(3*2^{(2/3)}*(-1)^{(1/3)}*(5*x^4 - x)*(-x^3 + 1)^{(2/3)} - 2^{(1/3)}*(-1)^{(2/3)}*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^{(1/3)))/(x^6 + 2*x^3 + 1)) + 18*(-x^3 + 1)^{(2/3)}/x^2$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)

maple [C] time = 4.49, size = 929, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x)

[Out] $1/2*(x^3-1)/x^2/(-x^3+1)^{(1/3)}-1/6*\ln(-(-36*\text{RootOf}(\text{RootOf}(_Z^3+4)^2+6*_Z*\text{RootOf}(_Z^3+4)+36*_Z^2)^2*\text{RootOf}(_Z^3+4)^2*x^3+3*\text{RootOf}(\text{RootOf}(_Z^3+4)^2+6*_Z*\text{RootOf}(_Z^3+4)+36*_Z^2)*\text{RootOf}(_Z^3+4)^3*x^3+12*(-x^3+1)^{(2/3)}*\text{RootOf}(\text{RootOf}(_Z^3+4)^2+6*_Z*\text{RootOf}(_Z^3+4)+36*_Z^2)*\text{RootOf}(_Z^3+4)^2*x-30*(-x^3+1)^{(1/3)}))$

$$\begin{aligned} & /3) * \text{RootOf}(_Z^3+4) * \text{RootOf}(\text{RootOf}(_Z^3+4)^2+6*_Z * \text{RootOf}(_Z^3+4)+36*_Z^2) * x^2 \\ & - (-x^3+1)^{(1/3)} * \text{RootOf}(_Z^3+4)^2 * x^2 + 36 * \text{RootOf}(\text{RootOf}(_Z^3+4)^2+6*_Z * \text{RootOf} \\ & (_Z^3+4)+36*_Z^2) * x^3 - 3 * \text{RootOf}(_Z^3+4) * x^3 - 10 * (-x^3+1)^{(2/3)} * x - 12 * \text{RootOf}(\text{Ro} \\ & \text{otOf}(_Z^3+4)^2+6*_Z * \text{RootOf}(_Z^3+4)+36*_Z^2) + \text{RootOf}(_Z^3+4)) / (x+1) / (x^2-x+1) \\ &) * \text{RootOf}(_Z^3+4) - \ln(-(-36 * \text{RootOf}(\text{RootOf}(_Z^3+4)^2+6*_Z * \text{RootOf}(_Z^3+4)+36*_Z \\ & ^2)^2 * \text{RootOf}(_Z^3+4)^2 * x^3 + 3 * \text{RootOf}(\text{RootOf}(_Z^3+4)^2+6*_Z * \text{RootOf}(_Z^3+4)+36 \\ & *_Z^2) * \text{RootOf}(_Z^3+4)^3 * x^3 + 12 * (-x^3+1)^{(2/3)} * \text{RootOf}(\text{RootOf}(_Z^3+4)^2+6*_Z * \\ & \text{RootOf}(_Z^3+4)+36*_Z^2) * \text{RootOf}(_Z^3+4)^2 * x - 30 * (-x^3+1)^{(1/3)} * \text{RootOf}(_Z^3+4) \\ & * \text{RootOf}(\text{RootOf}(_Z^3+4)^2+6*_Z * \text{RootOf}(_Z^3+4)+36*_Z^2) * x^2 - (-x^3+1)^{(1/3)} * \text{Ro} \\ & \text{otOf}(_Z^3+4)^2 * x^2 + 36 * \text{RootOf}(\text{RootOf}(_Z^3+4)^2+6*_Z * \text{RootOf}(_Z^3+4)+36*_Z^2) * \\ & x^3 - 3 * \text{RootOf}(_Z^3+4) * x^3 - 10 * (-x^3+1)^{(2/3)} * x - 12 * \text{RootOf}(\text{RootOf}(_Z^3+4)^2+6*_ \\ & _Z * \text{RootOf}(_Z^3+4)+36*_Z^2) + \text{RootOf}(_Z^3+4)) / (x+1) / (x^2-x+1)) * \text{RootOf}(\text{RootOf}(_Z \\ & ^3+4)^2+6*_Z * \text{RootOf}(_Z^3+4)+36*_Z^2) + 1/6 * \text{RootOf}(_Z^3+4) * \ln((-54 * \text{RootOf}(\text{Root} \\ & \text{Of}(_Z^3+4)^2+6*_Z * \text{RootOf}(_Z^3+4)+36*_Z^2)^2 * \text{RootOf}(_Z^3+4)^2 * x^3 - 3 * \text{RootOf}(\text{R} \\ & \text{oof}(_Z^3+4)^2+6*_Z * \text{RootOf}(_Z^3+4)+36*_Z^2) * \text{RootOf}(_Z^3+4)^3 * x^3 + 12 * (-x^3+ \\ & 1)^{(2/3)} * \text{RootOf}(\text{RootOf}(_Z^3+4)^2+6*_Z * \text{RootOf}(_Z^3+4)+36*_Z^2) * \text{RootOf}(_Z^3+4) \\ &)^2 * x + 6 * (-x^3+1)^{(1/3)} * \text{RootOf}(_Z^3+4) * \text{RootOf}(\text{RootOf}(_Z^3+4)^2+6*_Z * \text{RootOf}(_ \\ & _Z^3+4)+36*_Z^2) * x^2 + 5 * (-x^3+1)^{(1/3)} * \text{RootOf}(_Z^3+4)^2 * x^2 - 18 * \text{RootOf}(\text{RootOf}(\ \\ & _Z^3+4)^2+6*_Z * \text{RootOf}(_Z^3+4)+36*_Z^2) * x^3 - \text{RootOf}(_Z^3+4) * x^3 + 2 * (-x^3+1)^{(2 \\ & /3)} * x + 18 * \text{RootOf}(\text{RootOf}(_Z^3+4)^2+6*_Z * \text{RootOf}(_Z^3+4)+36*_Z^2) + \text{RootOf}(_Z^3+4 \\ &)) / (x+1) / (x^2-x+1)) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3+1)*(-x^3+1)^(1/3)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3(1-x^3)^{1/3}(x^3+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(1-x^3)^(1/3)*(x^3+1)),x)

[Out] int(1/(x^3*(1-x^3)^(1/3)*(x^3+1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(-x**3+1)**(1/3)/(x**3+1), x)

[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)

$$3.384 \quad \int \frac{1}{x^6 \sqrt[3]{1-x^3} (1+x^3)} dx$$

Optimal. Leaf size=124

$$-\frac{\log(x^3+1)}{6\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x\right)}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{(1-x^3)^{2/3}}{5x^5} + \frac{(1-x^3)^{2/3}}{5x^2}$$

Rubi [A] time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$\frac{(1-x^3)^{5/3}}{5x^5} - \frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} + \frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] -(1 - x^3)^(5/3)/(5*x^5) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 461

Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 494

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && LtQ[-1, p, 0]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6 \sqrt[3]{1-x^3} (1+x^3)} dx &= \text{Subst} \left(\int \frac{(1+x^3)^2}{x^6 (1+2x^3)} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= \text{Subst} \left(\int \left(\frac{1}{x^6} + \frac{1}{1+2x^3} \right) dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{(1-x^3)^{5/3}}{5x^5} + \text{Subst} \left(\int \frac{1}{1+2x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{(1-x^3)^{5/3}}{5x^5} + \frac{1}{3} \text{Subst} \left(\int \frac{1}{1+\sqrt[3]{2}x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{1}{3} \text{Subst} \left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{(1-x^3)^{5/3}}{5x^5} + \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{(1-x^3)^{5/3}}{5x^5} - \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}} + \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{\sqrt[3]{2}} \\
&= -\frac{(1-x^3)^{5/3}}{5x^5} - \frac{\tan^{-1} \left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}} + \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A] time = 5.25, size = 123, normalized size = 0.99

$$\frac{2 \log \left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + 1 \right) + 2\sqrt{3} \tan^{-1} \left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} - 1}{\sqrt{3}} \right) - \log \left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + \frac{2^{2/3}x^2}{(x^3-1)^{2/3}} + 1 \right)}{6\sqrt[3]{2}} - \frac{(1-x^3)^{5/3}}{5x^5}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^6*(1-x^3)^(1/3)*(1+x^3)),x]

[Out] -1/5*(1-x^3)^(5/3)/x^5 + (2*Sqrt[3]*ArcTan[(-1+(2*2^(1/3)*x)/(-1+x^3)^(1/3))/Sqrt[3]] - Log[1+(2^(2/3)*x^2)/(-1+x^3)^(2/3)-(2^(1/3)*x)/(-1+x^3)^(1/3)] + 2*Log[1+(2^(1/3)*x)/(-1+x^3)^(1/3)])/(6*2^(1/3))

IntegrateAlgebraic [A] time = 0.31, size = 149, normalized size = 1.20

$$\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{(1-x^3)^{2/3}(x^3-1)}{5x^5} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}(1-x^3)^{2/3}-2x^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^6*(1-x^3)^(1/3)*(1+x^3)),x]

[Out] ((1-x^3)^(2/3)*(-1+x^3))/(5*x^5) + ArcTan[(Sqrt[3]*x)/(-x+2^(2/3)*(1-x^3)^(1/3))]/(2^(1/3)*Sqrt[3]) + Log[2*x+2^(2/3)*(1-x^3)^(1/3)]/(3*2^(1/3)) - Log[-2*x^2+2^(2/3)*x*(1-x^3)^(1/3)-2^(1/3)*(1-x^3)^(2/3)]/(6*2^(1/3))

fricas [B] time = 2.55, size = 283, normalized size = 2.28

$$\frac{10\sqrt{6}2^{1/6}x^5 \arctan\left(\frac{2^{1/6}\sqrt{6}\sqrt[3]{5x^4-x^3}(-x^3+1)^{1/3}-\sqrt{6}2^{1/6}(71x^9-111x^6+33x^3-1)+12\sqrt{6}(19x^8-16x^5+x^2)(-x^3+1)^{1/3}}{6(109x^9-105x^6+3x^3+1)}\right)-10\cdot 2^{1/6}x^5 \log\left(\frac{62^{1/6}(-x^3+1)^{1/3}x^2+2^{1/6}(x^3+1)+6(-x^3+1)^{1/3}x}{x^3+1}\right)+5\cdot 2^{1/6}x^5 \log\left(\frac{32^{1/6}(5x^4-x^3)(-x^3+1)^{1/3}+2^{1/6}(19x^6-16x^3+1)-12(2x^5-x^2)(-x^3+1)^{1/3}}{x^6+2x^3+1}\right)-36(x^3-1)(-x^3+1)^{2/3}}{180x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/180*(10*sqrt(6)*2^(1/6)*x^5*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7+4*x^4-x)*(-x^3+1)^(2/3)-sqrt(6)*2^(1/3)*(71*x^9-111*x^6+33*x^3-1)+12*sqrt(6)*(19*x^8-16*x^5+x^2)*(-x^3+1)^(1/3))/(109*x^9-105*x^6+3*x^3+1))-10*2^(1/6)*x^5*log((6*2^(1/3)*(-x^3+1)^(1/3)*x^2+2^(2/3)*(x^3+1)+6*(-x^3+1)^(2/3)*x)/(x^3+1))+5*2^(2/3)*x^5*log((3*2^(2/3)*(5*x^4-x)*(-x^3+1)^(2/3)+2^(1/3)*(19*x^6-16*x^3+1)-12*(2*x^5-x^2)*(-x^3+1)^(1/3))/(x^6+2*x^3+1))-36*(x^3-1)*(-x^3+1)^(2/3))/x^5

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(1/((x^3+1)*(-x^3+1)^(1/3)*x^6),x)

maple [C] time = 2.75, size = 955, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x)`

[Out]
$$-1/5*(x^6-2*x^3+1)/x^5/(-x^3+1)^{(1/3)}+\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\ln((9*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3+18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+12*(-x^3+1)^{(2/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^2*x-5*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}*x^2-24*(-x^3+1)^{(1/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)*x^2+9*\text{RootOf}(_Z^3-4)*x^3+18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3+10*(-x^3+1)^{(2/3)}*x-3*\text{RootOf}(_Z^3-4)-6*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/6*\ln(-(6*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3-18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+12*(-x^3+1)^{(2/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^2*x+\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}*x^2-24*(-x^3+1)^{(1/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)*x^2-2*\text{RootOf}(_Z^3-4)*x^3+6*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^{(2/3)}*x+2*\text{RootOf}(_Z^3-4)-6*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)))/(x+1)/(x^2-x+1))*\text{RootOf}(_Z^3-4)-\ln(-(6*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3-18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+12*(-x^3+1)^{(2/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^2*x+\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}*x^2-24*(-x^3+1)^{(1/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)*x^2-2*\text{RootOf}(_Z^3-4)*x^3+6*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^{(2/3)}*x+2*\text{RootOf}(_Z^3-4)-6*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)))/(x+1)/(x^2-x+1))*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}}x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")`

[Out] `integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6(1-x^3)^{1/3}(x^3+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^6*(1 - x^3)^(1/3)*(x^3 + 1)), x)`

[Out] `int(1/(x^6*(1 - x^3)^(1/3)*(x^3 + 1)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(-x**3+1)**(1/3)/(x**3+1), x)`

[Out] `Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)`

$$3.385 \quad \int \frac{1}{x^9 \sqrt[3]{1-x^3} (1+x^3)} dx$$

Optimal. Leaf size=141

$$\frac{\log(x^3+1)}{6\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x\right)}{2\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{(1-x^3)^{2/3}}{8x^8} + \frac{(1-x^3)^{2/3}}{20x^5} - \frac{17(1-x^3)^{2/3}}{40x^2}$$

Rubi [A] time = 0.09, antiderivative size = 175, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$-\frac{(1-x^3)^{8/3}}{8x^8} - \frac{(1-x^3)^{5/3}}{5x^5} - \frac{(1-x^3)^{2/3}}{2x^2} + \frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] -(1 - x^3)^(2/3)/(2*x^2) - (1 - x^3)^(5/3)/(5*x^5) - (1 - x^3)^(8/3)/(8*x^8) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 461

Int[(((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_))/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 494

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_) , x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && LtQ[-1, p, 0]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^9 \sqrt[3]{1-x^3} (1+x^3)} dx &= \text{Subst} \left(\int \frac{(1+x^3)^3}{x^9 (1+2x^3)} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= \text{Subst} \left(\int \left(\frac{1}{x^9} + \frac{1}{x^6} + \frac{1}{x^3} + \frac{1}{-1-2x^3} \right) dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{(1-x^3)^{2/3}}{2x^2} - \frac{(1-x^3)^{5/3}}{5x^5} - \frac{(1-x^3)^{8/3}}{8x^8} + \text{Subst} \left(\int \frac{1}{-1-2x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{(1-x^3)^{2/3}}{2x^2} - \frac{(1-x^3)^{5/3}}{5x^5} - \frac{(1-x^3)^{8/3}}{8x^8} + \frac{1}{3} \text{Subst} \left(\int \frac{1}{-1-\sqrt[3]{2}x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) + \\
&= -\frac{(1-x^3)^{2/3}}{2x^2} - \frac{(1-x^3)^{5/3}}{5x^5} - \frac{(1-x^3)^{8/3}}{8x^8} - \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-\sqrt[3]{2}x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{(1-x^3)^{2/3}}{2x^2} - \frac{(1-x^3)^{5/3}}{5x^5} - \frac{(1-x^3)^{8/3}}{8x^8} + \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}} - \frac{\log \left(1 + \frac{\sqrt[3]{2}}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} \\
&= -\frac{(1-x^3)^{2/3}}{2x^2} - \frac{(1-x^3)^{5/3}}{5x^5} - \frac{(1-x^3)^{8/3}}{8x^8} + \frac{\tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A] time = 5.11, size = 133, normalized size = 0.94

$$\frac{-2 \log \left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + 1 \right) - 2\sqrt{3} \tan^{-1} \left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} - 1}{\sqrt{3}} \right) + \log \left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + \frac{2^{2/3}x^2}{(x^3-1)^{2/3}} + 1 \right)}{6\sqrt[3]{2}} - \frac{(1-x^3)^{2/3} (17x^6 - 2x^3 + 5)}{40x^8}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^9*(1-x^3)^(1/3)*(1+x^3)),x]

[Out] -1/40*((1-x^3)^(2/3)*(5-2*x^3+17*x^6))/x^8 + (-2*sqrt[3]*ArcTan[(-1+(2*2^(1/3)*x)/(-1+x^3)^(1/3))/sqrt[3]] + Log[1+(2^(2/3)*x^2)/(-1+x^3)^(2/3) - (2^(1/3)*x)/(-1+x^3)^(1/3)] - 2*Log[1+(2^(1/3)*x)/(-1+x^3)^(1/3)])/(6*2^(1/3))

IntegrateAlgebraic [A] time = 0.30, size = 157, normalized size = 1.11

$$\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x - \sqrt[3]{2}(1-x^3)^{2/3} - 2x^2\right)}{6\sqrt[3]{2}} + \frac{(1-x^3)^{2/3}(-17x^6+2x^3-5)}{40x^8}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] $((1 - x^3)^{(2/3)}*(-5 + 2*x^3 - 17*x^6))/(40*x^8) - \text{ArcTan}[(\text{Sqrt}[3]*x)/(-x + 2^{(2/3)}*(1 - x^3)^{(1/3)})]/(2^{(1/3)}*\text{Sqrt}[3]) - \text{Log}[2*x + 2^{(2/3)}*(1 - x^3)^{(1/3)}]/(3*2^{(1/3)}) + \text{Log}[-2*x^2 + 2^{(2/3)}*x*(1 - x^3)^{(1/3)} - 2^{(1/3)}*(1 - x^3)^{(2/3)}]/(6*2^{(1/3)})$

fricas [B] time = 2.61, size = 320, normalized size = 2.27

$$\frac{20\sqrt{6}2^{2/3}(-1)^{1/3}x^8\arctan\left(\frac{2^{1/3}\sqrt{6}\sqrt{2}(-1)^{1/3}(6x^2+4x-1)^{1/3}-12\sqrt{6}(-1)^{1/3}(9x^6-16x^5+x^2)^{1/3}-\sqrt{6}2^{1/3}(71x^9-111x^6+33x^3-1)}{6(109x^9-105x^6+3x^3+1)}\right)}{360x^8} - 20\cdot 2^{1/3}(-1)^{1/3}x^8\log\left(\frac{2^{2/3}(-1)^{1/3}(6x^2+4x-1)^{1/3}-2^{1/3}(-1)^{1/3}(9x^6-16x^5+x^2)^{1/3}+2^{1/3}(-1)^{1/3}(71x^9-111x^6+33x^3-1)}{3x^3+1}\right) + 10\cdot 2^{2/3}(-1)^{1/3}x^8\log\left(\frac{32^{1/3}(-1)^{1/3}(6x^2+4x-1)^{1/3}-2^{1/3}(-1)^{1/3}(9x^6-16x^5+x^2)^{1/3}+12(2x^3+1)^{1/3}}{x^2+1}\right) + 9(17x^6-2x^3+5)(-x^3+1)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")

[Out] $-1/360*(20*\text{sqrt}(6)*2^{(1/6)}*(-1)^{(1/3)}*x^8*\arctan(1/6*2^{(1/6)}*(6*\text{sqrt}(6)*2^{(2/3)}*(-1)^{(2/3)}*(5*x^7+4*x^4-x)*(-x^3+1)^{(2/3)} - 12*\text{sqrt}(6)*(-1)^{(1/3)}*(19*x^8-16*x^5+x^2)*(-x^3+1)^{(1/3)} - \text{sqrt}(6)*2^{(1/3)}*(71*x^9-111*x^6+33*x^3-1))/(109*x^9-105*x^6+3*x^3+1)) - 20*2^{(2/3)}*(-1)^{(1/3)}*x^8*\log((6*2^{(1/3)}*(-1)^{(2/3)}*(-x^3+1)^{(1/3)}*x^2 - 2^{(2/3)}*(-1)^{(1/3)}*(x^3+1) + 6*(-x^3+1)^{(2/3)}*x)/(x^3+1)) + 10*2^{(2/3)}*(-1)^{(1/3)}*x^8*\log(-3*2^{(2/3)}*(-1)^{(1/3)}*(5*x^4-x)*(-x^3+1)^{(2/3)} - 2^{(1/3)}*(-1)^{(2/3)}*(19*x^6-16*x^3+1) + 12*(2*x^5-x^2)*(-x^3+1)^{(1/3)})/(x^6+2*x^3+1)) + 9*(17*x^6-2*x^3+5)*(-x^3+1)^{(2/3)}/x^8$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(1/((x^3+1)*(-x^3+1)^(1/3)*x^9), x)

maple [C] time = 4.07, size = 963, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x)`

[Out] $\frac{1}{40} \cdot (17x^9 - 19x^6 + 7x^3 - 5) / x^8 / (-x^3 + 1)^{1/3} + \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot \ln(-(-18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2)^2 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x^3 - 9 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot \text{RootOf}(_Z^3 + 4)^3 \cdot x^3 + 12 \cdot (-x^3 + 1)^{2/3} \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x + 24 \cdot (-x^3 + 1)^{1/3} \cdot \text{RootOf}(_Z^3 + 4) \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot x^2 + 5 \cdot (-x^3 + 1)^{1/3} \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x^2 + 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot x^3 + 9 \cdot \text{RootOf}(_Z^3 + 4) \cdot x^3 - 10 \cdot (-x^3 + 1)^{2/3} \cdot x - 6 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) - 3 \cdot \text{RootOf}(_Z^3 + 4)) / (x + 1) / (x^2 - x + 1)) - 1/6 \cdot \ln((18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2)^2 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x^3 - 6 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot \text{RootOf}(_Z^3 + 4)^3 \cdot x^3 + 12 \cdot (-x^3 + 1)^{2/3} \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x + 24 \cdot (-x^3 + 1)^{1/3} \cdot \text{RootOf}(_Z^3 + 4) \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot x^2 - (-x^3 + 1)^{1/3} \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x^2 + 6 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot x^3 - 2 \cdot \text{RootOf}(_Z^3 + 4) \cdot x^3 + 2 \cdot (-x^3 + 1)^{2/3} \cdot x - 6 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot \text{RootOf}(_Z^3 + 4) + 2 \cdot \text{RootOf}(_Z^3 + 4)) / (x + 1) / (x^2 - x + 1)) \cdot \text{RootOf}(_Z^3 + 4) - \ln((18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2)^2 \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x^3 - 6 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot \text{RootOf}(_Z^3 + 4)^3 \cdot x^3 + 12 \cdot (-x^3 + 1)^{2/3} \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x + 24 \cdot (-x^3 + 1)^{1/3} \cdot \text{RootOf}(_Z^3 + 4) \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot x^2 - (-x^3 + 1)^{1/3} \cdot \text{RootOf}(_Z^3 + 4)^2 \cdot x^2 + 6 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot x^3 - 2 \cdot \text{RootOf}(_Z^3 + 4) \cdot x^3 + 2 \cdot (-x^3 + 1)^{2/3} \cdot x - 6 \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2) \cdot \text{RootOf}(_Z^3 + 4) + 2 \cdot \text{RootOf}(_Z^3 + 4)) / (x + 1) / (x^2 - x + 1)) \cdot \text{RootOf}(\text{RootOf}(_Z^3 + 4)^2 + 6_Z \cdot \text{RootOf}(_Z^3 + 4) + 36_Z^2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}} x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")`

[Out] `integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^9 (1 - x^3)^{1/3} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^9*(1 - x^3)^(1/3)*(x^3 + 1)), x)`

[Out] `int(1/(x^9*(1 - x^3)^(1/3)*(x^3 + 1)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9 \sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**9/(-x**3+1)**(1/3)/(x**3+1), x)`

[Out] `Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)`

$$3.386 \quad \int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)} dx$$

Optimal. Leaf size=233

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{1 - 2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Rubi [C] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 0.11, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {510}

$$\frac{1}{2}x^2F_1\left(\frac{2}{3}; \frac{1}{3}, 1; \frac{5}{3}; x^3, -x^3\right)$$

Warning: Unable to verify antiderivative.

[In] Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -(b*x^n)/a, -(d*x^n)/c]]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)} dx = \frac{1}{2}x^2F_1\left(\frac{2}{3}; \frac{1}{3}, 1; \frac{5}{3}; x^3, -x^3\right)$$

Mathematica [C] time = 0.02, size = 26, normalized size = 0.11

$$\frac{1}{2}x^2F_1\left(\frac{2}{3}; \frac{1}{3}, 1; \frac{5}{3}; x^3, -x^3\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]

[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2

IntegrateAlgebraic [A] time = 1.08, size = 340, normalized size = 1.46

$$\frac{\log\left(\frac{-\sqrt{1-x^3}+\sqrt{2x-\sqrt{2}}}{3\sqrt{2}}\right)-\log\left(\frac{2\sqrt{1-x^3}+\sqrt{2x-\sqrt{2}}}{6\sqrt{2}}\right)-\tan^{-1}\left(\frac{\sqrt{3}\sqrt{1-x^3}}{\sqrt{1-x^3}-\sqrt{2x+\sqrt{2}}}\right)-\tan^{-1}\left(\frac{\sqrt{3}\sqrt{1-x^3}}{\sqrt{1-x^3}+\sqrt{2x+\sqrt{2}}}\right)}{2\sqrt{2}\sqrt{3}}+\frac{\log\left((1-x^3)^{2/3}+(\sqrt{2x-\sqrt{2}})\sqrt{1-x^3}+2^{2/3}x^2-2\cdot 2^{2/3}x+2^{2/3}\right)}{6\sqrt{2}}+\frac{\log\left(4(1-x^3)^{2/3}+(2\sqrt{2}-2\sqrt{2}x)\sqrt{1-x^3}+2^{2/3}x^2-2\cdot 2^{2/3}x+2^{2/3}\right)}{12\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/((1 - x^3)^(1/3)*(1 + x^3)),x]

[Out]
$$\begin{aligned} & -1/2*\text{ArcTan}[(\text{Sqrt}[3]*(1 - x^3)^{(1/3)})/(2^{(1/3)} - 2^{(1/3)}*x + (1 - x^3)^{(1/3)})] / (2^{(1/3)}*\text{Sqrt}[3]) - \text{ArcTan}[(\text{Sqrt}[3]*(1 - x^3)^{(1/3)})/(-2*2^{(1/3)} + 2*2^{(1/3)}*x + (1 - x^3)^{(1/3)})] / (2^{(1/3)}*\text{Sqrt}[3]) - \text{Log}[-2^{(1/3)} + 2^{(1/3)}*x - (1 - x^3)^{(1/3)}] / (3*2^{(1/3)}) - \text{Log}[-2^{(1/3)} + 2^{(1/3)}*x + 2*(1 - x^3)^{(1/3)}] / (6*2^{(1/3)}) + \text{Log}[2^{(2/3)} - 2*2^{(2/3)}*x + 2^{(2/3)}*x^2 + (-2^{(1/3)} + 2^{(1/3)}*x)*(1 - x^3)^{(1/3)} + (1 - x^3)^{(2/3)}] / (6*2^{(1/3)}) + \text{Log}[2^{(2/3)} - 2*2^{(2/3)}*x + 2^{(2/3)}*x^2 + (2*2^{(1/3)} - 2*2^{(1/3)}*x)*(1 - x^3)^{(1/3)} + 4*(1 - x^3)^{(2/3)}] / (12*2^{(1/3)}) \end{aligned}$$

fricas [B] time = 2.32, size = 373, normalized size = 1.60

$$\frac{1}{3\sqrt{2}} \arctan\left(\frac{2\sqrt{2}\sqrt{1-x^3}(x^2-2x+1)^2 + 2\sqrt{2}\sqrt{1-x^3}(x^2-2x+1) + 2\sqrt{2}\sqrt{1-x^3}(x^2-2x+1)^2 + 2\sqrt{2}\sqrt{1-x^3}(x^2-2x+1)}{4(x^2-2x+1)^2 + 4\sqrt{2}\sqrt{1-x^3}(x^2-2x+1)}\right) - \frac{1}{2} \arctan\left(\frac{2\sqrt{2}\sqrt{1-x^3}(x^2-2x+1)^2 + 2\sqrt{2}\sqrt{1-x^3}(x^2-2x+1) + 2\sqrt{2}\sqrt{1-x^3}(x^2-2x+1)^2 + 2\sqrt{2}\sqrt{1-x^3}(x^2-2x+1)}{4(x^2-2x+1)^2 + 4\sqrt{2}\sqrt{1-x^3}(x^2-2x+1)}\right) + \frac{1}{3\sqrt{2}} \arctan\left(\frac{2\sqrt{2}\sqrt{1-x^3}(x^2-2x+1)^2 + 2\sqrt{2}\sqrt{1-x^3}(x^2-2x+1) + 2\sqrt{2}\sqrt{1-x^3}(x^2-2x+1)^2 + 2\sqrt{2}\sqrt{1-x^3}(x^2-2x+1)}{4(x^2-2x+1)^2 + 4\sqrt{2}\sqrt{1-x^3}(x^2-2x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/36*\text{sqrt}(6)*2^{(1/6)}*(-1)^{(1/3)}*\text{arctan}(1/6*2^{(1/6)}*(24*\text{sqrt}(6)*2^{(2/3)}*(-1)^{(2/3)}*(x^{14} - 2*x^{11} - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^{(2/3)} + 12*\text{sqrt}(6)*(-1)^{(1/3)}*(x^{16} - 33*x^{13} + 110*x^{10} - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^{(1/3)} + \text{sqrt}(6)*2^{(1/3)}*(x^{18} + 42*x^{15} - 417*x^{12} + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^{18} - 102*x^{15} + 447*x^{12} - 628*x^9 + 447*x^6 - 102*x^3 + 1)) - 1/72*2^{(2/3)}*(-1)^{(1/3)}*\log(-(12*2^{(2/3)}*(-1)^{(1/3)}*(x^8 - 4*x^5 + x^2)*(-x^3 + 1)^{(2/3)} - 2^{(1/3)}*(-1)^{(2/3)}*(x^{12} - 32*x^9 + 78*x^6 - 32*x^3 + 1) - 6*(x^{10} - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^{(1/3)))/(x^{12} + 4*x^9 + 6*x^6 + 4*x^3 + 1)) + 1/36*2^{(2/3)}*(-1)^{(1/3)}*\log(-(12*(-x^3 + 1)^{(2/3)}*x^2 - 6*2^{(1/3)}*(-1)^{(2/3)}*(x^4 - x)*(-x^3 + 1)^{(1/3)} - 2^{(2/3)}*(-1)^{(1/3)}*(x^6 + 2*x^3 + 1))/(x^6 + 2*x^3 + 1)) \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)

maple [F] time = 3.08, size = 0, normalized size = 0.00

$$\int \frac{x}{(-x^3 + 1)^{\frac{1}{3}} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^3+1)^(1/3)/(x^3+1),x)

[Out] int(x/(-x^3+1)^(1/3)/(x^3+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(1 - x^3)^{\frac{1}{3}} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((1 - x^3)^(1/3)*(x^3 + 1)),x)

[Out] int(x/((1 - x^3)^(1/3)*(x^3 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)

[Out] Integral(x/((-x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)

$$3.387 \quad \int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)} dx$$

Optimal. Leaf size=125

$$-\frac{1}{7}(1-x^3)^{7/3} + \frac{1}{4}(1-x^3)^{4/3} - \sqrt[3]{1-x^3} + \frac{\log(x^3+1)}{6 \cdot 2^{2/3}} - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Rubi [A] time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {446, 88, 57, 617, 204, 31}

$$-\frac{1}{7}(1-x^3)^{7/3} + \frac{1}{4}(1-x^3)^{4/3} - \sqrt[3]{1-x^3} + \frac{\log(x^3+1)}{6 \cdot 2^{2/3}} - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] -(1 - x^3)^(1/3) + (1 - x^3)^(4/3)/4 - (1 - x^3)^(7/3)/7 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(2/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3}{(1-x)^{2/3}(1+x)} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{(1-x)^{2/3}} - \sqrt[3]{1-x} + (1-x)^{4/3} - \frac{1}{(1-x)^{2/3}(1+x)} \right) dx, x, x^3 \right) \\
 &= -\sqrt[3]{1-x^3} + \frac{1}{4}(1-x^3)^{4/3} - \frac{1}{7}(1-x^3)^{7/3} - \frac{1}{3} \text{Subst} \left(\int \frac{1}{(1-x)^{2/3}(1+x)} dx, x, x^3 \right) \\
 &= -\sqrt[3]{1-x^3} + \frac{1}{4}(1-x^3)^{4/3} - \frac{1}{7}(1-x^3)^{7/3} + \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{\sqrt[3]{2-x}} dx, x, \sqrt[3]{1-x^3} \right)}{2 \cdot 2^{2/3}} \\
 &= -\sqrt[3]{1-x^3} + \frac{1}{4}(1-x^3)^{4/3} - \frac{1}{7}(1-x^3)^{7/3} + \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} - \frac{\log \left(\sqrt[3]{2} - \sqrt[3]{1-x^3} \right)}{2 \cdot 2^{2/3}} - \frac{\log \left(\sqrt[3]{1-x^3} \right)}{2 \cdot 2^{2/3}} \\
 &= -\sqrt[3]{1-x^3} + \frac{1}{4}(1-x^3)^{4/3} - \frac{1}{7}(1-x^3)^{7/3} + \frac{\tan^{-1} \left(\frac{1+2^{2/3} \sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{2^{2/3} \sqrt{3}} + \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} - \frac{\log \left(\sqrt[3]{1-x^3} \right)}{2 \cdot 2^{2/3}}
 \end{aligned}$$

Mathematica [A] time = 0.11, size = 151, normalized size = 1.21

$$\frac{1}{84} \left(3\sqrt[3]{1-x^3}x^3 - 75\sqrt[3]{1-x^3} - 14\sqrt[3]{2} \log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right) + 7\sqrt[3]{2} \log\left((1-x^3)^{2/3} + \sqrt[3]{2-2x^3} + 2^{2/3}\right) + 14\sqrt[3]{2}\sqrt{3} \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3} + 1}{\sqrt{3}}\right) - 12\sqrt[3]{1-x^3}x^6 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((1 - x^3)^(2/3)*(1 + x^3)), x]

[Out] $(-75*(1 - x^3)^{(1/3)} + 3*x^3*(1 - x^3)^{(1/3)} - 12*x^6*(1 - x^3)^{(1/3)} + 14*2^{(1/3)}*\text{Sqrt}[3]*\text{ArcTan}[(1 + 2^{(2/3)}*(1 - x^3)^{(1/3)})/\text{Sqrt}[3]] - 14*2^{(1/3)}*\text{Log}[2^{(1/3)} - (1 - x^3)^{(1/3)}] + 7*2^{(1/3)}*\text{Log}[2^{(2/3)} + (2 - 2*x^3)^{(1/3)} + (1 - x^3)^{(2/3)}])/84$

IntegrateAlgebraic [A] time = 0.14, size = 141, normalized size = 1.13

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} - 2\right)}{3 \cdot 2^{2/3}} + \frac{\log\left(\sqrt[3]{2} (1-x^3)^{2/3} + 2^{2/3}\sqrt[3]{1-x^3} + 2\right)}{6 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{1}{28}\sqrt[3]{1-x^3}(-4x^6 + x^3 - 25)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((1 - x^3)^(2/3)*(1 + x^3)), x]

[Out] $((1 - x^3)^{(1/3)}*(-25 + x^3 - 4*x^6))/28 + \text{ArcTan}[1/\text{Sqrt}[3] + (2^{(2/3)}*(1 - x^3)^{(1/3)})/\text{Sqrt}[3]]/(2^{(2/3)}*\text{Sqrt}[3]) - \text{Log}[-2 + 2^{(2/3)}*(1 - x^3)^{(1/3)}]/(3*2^{(2/3)}) + \text{Log}[2 + 2^{(2/3)}*(1 - x^3)^{(1/3)} + 2^{(1/3)}*(1 - x^3)^{(2/3)}]/(6*2^{(2/3)})$

fricas [A] time = 0.45, size = 142, normalized size = 1.14

$$-\frac{1}{6} \cdot 4^{1/3} \sqrt{3} (-1)^{1/3} \arctan\left(\frac{1}{6} \cdot 4^{1/3} \left(4^{2/3} \sqrt{3} (-1)^{2/3} (-x^3 + 1)^{1/3} + 4^{1/3} \sqrt{3}\right)\right) - \frac{1}{24} \cdot 4^{2/3} (-1)^{1/3} \log\left(-4^{2/3} (-1)^{1/3} (-x^3 + 1)^{1/3} + 2 \cdot 4^{1/3} (-1)^{2/3} + 2(-x^3 + 1)^{2/3}\right) + \frac{1}{12} \cdot 4^{2/3} (-1)^{1/3} \log\left(4^{2/3} (-1)^{1/3} + 2(-x^3 + 1)^{1/3}\right) - \frac{1}{28} (4x^6 - x^3 + 25)(-x^3 + 1)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1), x, algorithm="fricas")

[Out] $-1/6*4^{(1/6)}*\text{sqrt}(3)*(-1)^{(1/3)}*\text{arctan}(1/6*4^{(1/6)}*(4^{(2/3)}*\text{sqrt}(3)*(-1)^{(2/3)}*(-x^3 + 1)^{(1/3)} + 4^{(1/3)}*\text{sqrt}(3))) - 1/24*4^{(2/3)}*(-1)^{(1/3)}*\text{log}(-4^{(2/3)}*(-1)^{(1/3)}*(-x^3 + 1)^{(1/3)} + 2*4^{(1/3)}*(-1)^{(2/3)} + 2*(-x^3 + 1)^{(2/3})) + 1/12*4^{(2/3)}*(-1)^{(1/3)}*\text{log}(4^{(2/3)}*(-1)^{(1/3)} + 2*(-x^3 + 1)^{(1/3})) - 1/28*(4*x^6 - x^3 + 25)*(-x^3 + 1)^{(1/3)}$

giac [A] time = 0.18, size = 127, normalized size = 1.02

$$-\frac{1}{7} (x^3 - 1)^2 (-x^3 + 1)^{1/3} + \frac{1}{6} \sqrt{3} 2^{1/3} \arctan\left(\frac{1}{6} \sqrt{3} 2^{2/3} (2^{1/3} + 2(-x^3 + 1)^{1/3})\right) + \frac{1}{4} (-x^3 + 1)^{4/3} + \frac{1}{12} \cdot 2^{1/3} \log\left(2^{2/3} + 2^{1/3}(-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}\right) - \frac{1}{6} \cdot 2^{1/3} \log\left(-2^{1/3} + (-x^3 + 1)^{1/3}\right) - (-x^3 + 1)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(-x³+1)^(2/3)/(x³+1),x, algorithm="giac")

[Out] $-1/7*(x^3 - 1)^2*(-x^3 + 1)^{1/3} + 1/6*\sqrt{3}*2^{1/3}*\arctan(1/6*\sqrt{3})*2^{2/3}*(2^{1/3} + 2*(-x^3 + 1)^{1/3})) + 1/4*(-x^3 + 1)^{4/3} + 1/12*2^{1/3}*\log(2^{2/3} + 2^{1/3}*(-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}) - 1/6*2^{1/3}*\log(\text{abs}(-2^{1/3} + (-x^3 + 1)^{1/3})) - (-x^3 + 1)^{1/3}$

maple [C] time = 7.45, size = 1579, normalized size = 12.63

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/(-x³+1)^(2/3)/(x³+1),x)

[Out] $1/28*(4*x^6-x^3+25)*(x^3-1)/(-x^3+1)^{2/3}+(1/6*\text{RootOf}(_Z^3+2)*\ln((36*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)^2*\text{RootOf}(_Z^3+2)^2*x^6-24*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)^3*x^6-36*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)^2*\text{RootOf}(_Z^3+2)^2*x^3+24*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)^3*x^3+6*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^6-4*\text{RootOf}(_Z^3+2)*x^6-144*(x^6-2*x^3+1)^{1/3}*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)*x^3-9*\text{RootOf}(_Z^3+2)^2*(x^6-2*x^3+1)^{1/3}*x^3+90*(x^6-2*x^3+1)^{2/3}*\text{RootOf}(_Z^3+2)^2*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)-48*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^3+32*\text{RootOf}(_Z^3+2)*x^3+144*(x^6-2*x^3+1)^{1/3}*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)+9*\text{RootOf}(_Z^3+2)^2*(x^6-2*x^3+1)^{1/3}-48*(x^6-2*x^3+1)^{2/3}+42*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)-28*\text{RootOf}(_Z^3+2)))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1))-1/6*\ln(-(90*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)^2*\text{RootOf}(_Z^3+2)^2*x^6+12*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)^3*x^6-90*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)^2*\text{RootOf}(_Z^3+2)^2*x^3-12*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)^3*x^3-45*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^6-6*\text{RootOf}(_Z^3+2)*x^6+27*(x^6-2*x^3+1)^{1/3}*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)*x^3+12*\text{RootOf}(_Z^3+2)^2*(x^6-2*x^3+1)^{1/3}*x^3+45*(x^6-2*x^3+1)^{2/3}*\text{RootOf}(_Z^3+2)^2*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)+150*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^3+20*\text{RootOf}(_Z^3+2)*x^3-27*(x^6-2*x^3+1)^{1/3}*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)-12*\text{RootOf}(_Z^3+2)^2*(x^6-2*x^3+1)^{1/3}+9*(x^6-2*x^3+1)^{2/3}-105*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)-14*\text{RootOf}(_Z^3+2)))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1))*\text{RootOf}(_Z^3+2)-\ln(-(90*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)^2*\text{RootOf}(_Z^3+2)^2*x^6+12*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)^3*x^6-90*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)^2*\text{RootOf}(_Z^3+2)^2*x^3-12*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*$

$$\text{RootOf}(_Z^3+2)^3*x^3-45*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^6-6*\text{RootOf}(_Z^3+2)*x^6+27*(x^6-2*x^3+1)^{(1/3)}*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)*x^3+12*\text{RootOf}(_Z^3+2)^2*(x^6-2*x^3+1)^{(1/3)}*x^3+45*(x^6-2*x^3+1)^{(2/3)}*\text{RootOf}(_Z^3+2)^2*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)+150*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^3+20*\text{RootOf}(_Z^3+2)*x^3-27*(x^6-2*x^3+1)^{(1/3)}*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)-12*\text{RootOf}(_Z^3+2)^2*(x^6-2*x^3+1)^{(1/3)}+9*(x^6-2*x^3+1)^{(2/3)}-105*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)-14*\text{RootOf}(_Z^3+2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1))*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2))/(-x^3+1)^{(2/3)}*((x^3-1)^2)^{(1/3)}$$

maxima [A] time = 1.52, size = 119, normalized size = 0.95

$$-\frac{1}{7}(-x^3+1)^{\frac{7}{3}}+\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)+\frac{1}{4}(-x^3+1)^{\frac{4}{3}}+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)-(-x^3+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(-x³+1)^(2/3)/(x³+1),x, algorithm="maxima")

[Out] -1/7*(-x³+1)^(7/3)+1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)+2*(-x³+1)^(1/3)))+1/4*(-x³+1)^(4/3)+1/12*2^(1/3)*log(2^(2/3)+2^(1/3)*(-x³+1)^(1/3)+(-x³+1)^(2/3))-1/6*2^(1/3)*log(-2^(1/3)+(-x³+1)^(1/3))-(-x³+1)^(1/3)

mupad [B] time = 5.18, size = 135, normalized size = 1.08

$$\frac{(1-x^3)^{4/3}}{4}-(1-x^3)^{1/3}-\frac{2^{1/3}\ln\left(32^{2/3}-3(1-x^3)^{1/3}\right)}{6}-\frac{(1-x^3)^{7/3}}{7}-\frac{2^{1/3}\ln\left(3(1-x^3)^{1/3}-\frac{32^{2/3}(-1+\sqrt{3}1i)}{2}\right)(-1+\sqrt{3}1i)}{12}+\frac{2^{1/3}\ln\left(\frac{32^{2/3}(1+\sqrt{3}1i)}{2}+3(1-x^3)^{1/3}\right)(1+\sqrt{3}1i)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/((1-x³)^(2/3)*(x³+1)),x)

[Out] (1-x³)^(4/3)/4-(1-x³)^(1/3)-(2^(1/3)*log(3*2^(1/3)-3*(1-x³)^(1/3)))/6-(1-x³)^(7/3)/7-(2^(1/3)*log(3*(1-x³)^(1/3)-(3*2^(1/3)*(3^(1/2)*1i-1))/2*(3^(1/2)*1i-1))/12+(2^(1/3)*log((3*2^(1/3)*(3^(1/2)*1i+1))/2+3*(1-x³)^(1/3))*(3^(1/2)*1i+1))/12

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x  
)
```

$$3.388 \quad \int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)} dx$$

Optimal. Leaf size=98

$$\frac{1}{4}(1-x^3)^{4/3} - \frac{\log(x^3+1)}{6 \cdot 2^{2/3}} + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {446, 88, 57, 617, 204, 31}

$$\frac{1}{4}(1-x^3)^{4/3} - \frac{\log(x^3+1)}{6 \cdot 2^{2/3}} + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] (1 - x^3)^(4/3)/4 - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(1-x)^{2/3}(1+x)} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(-\sqrt[3]{1-x} + \frac{1}{(1-x)^{2/3}(1+x)} \right) dx, x, x^3 \right) \\
 &= \frac{1}{4} (1-x^3)^{4/3} + \frac{1}{3} \text{Subst} \left(\int \frac{1}{(1-x)^{2/3}(1+x)} dx, x, x^3 \right) \\
 &= \frac{1}{4} (1-x^3)^{4/3} - \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} - \frac{\text{Subst} \left(\int \frac{1}{\sqrt[3]{2-x}} dx, x, \sqrt[3]{1-x^3} \right)}{2 \cdot 2^{2/3}} - \frac{\text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2}x + x^3} dx, x, \sqrt[3]{1-x^3} \right)}{2 \cdot \sqrt[3]{2}} \\
 &= \frac{1}{4} (1-x^3)^{4/3} - \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2 \cdot 2^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 + 2^{2/3} \sqrt[3]{1-x^3} \right)}{2^{2/3}} \\
 &= \frac{1}{4} (1-x^3)^{4/3} - \frac{\tan^{-1} \left(\frac{1+2^{2/3} \sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{2^{2/3} \sqrt{3}} - \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2 \cdot 2^{2/3}}
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 135, normalized size = 1.38

$$\frac{1}{12} \left(-3\sqrt[3]{1-x^3}x^3 + 3\sqrt[3]{1-x^3} + 2\sqrt[3]{2} \log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right) - \sqrt[3]{2} \log\left((1-x^3)^{2/3} + \sqrt[3]{2-2x^3} + 2^{2/3}\right) - 2\sqrt[3]{2}\sqrt{3} \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3} + 1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((1 - x^3)^(2/3)*(1 + x^3)), x]

[Out] (3*(1 - x^3)^(1/3) - 3*x^3*(1 - x^3)^(1/3) - 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)] - 2^(1/3)*Log[2^(2/3) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)])/12

IntegrateAlgebraic [A] time = 0.12, size = 132, normalized size = 1.35

$$\frac{1}{4} (1-x^3)^{4/3} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} - 2\right)}{3 \cdot 2^{2/3}} - \frac{\log\left(\sqrt[3]{2} (1-x^3)^{2/3} + 2^{2/3}\sqrt[3]{1-x^3} + 2\right)}{6 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((1 - x^3)^(2/3)*(1 + x^3)), x]

[Out] (1 - x^3)^(4/3)/4 - ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))

fricas [A] time = 0.48, size = 114, normalized size = 1.16

$$-\frac{1}{6} \cdot 4^{\frac{1}{6}} \sqrt{3} \arctan\left(\frac{1}{6} \cdot 4^{\frac{1}{6}} \left(4^{\frac{2}{3}} \sqrt{3} (-x^3 + 1)^{\frac{1}{3}} + 4^{\frac{1}{3}} \sqrt{3}\right)\right) - \frac{1}{24} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{2}{3}} (-x^3 + 1)^{\frac{1}{3}} + 2(-x^3 + 1)^{\frac{2}{3}} + 2 \cdot 4^{\frac{1}{3}}\right) + \frac{1}{12} \cdot 4^{\frac{2}{3}} \log\left(-4^{\frac{2}{3}} + 2(-x^3 + 1)^{\frac{1}{3}}\right) - \frac{1}{4} (x^3 - 1)(-x^3 + 1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1), x, algorithm="fricas")

[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 + 1)^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3)) - 1/4*(x^3 - 1)*(-x^3 + 1)^(1/3)

giac [A] time = 0.18, size = 98, normalized size = 1.00

$$-\frac{1}{6} \sqrt{3} 2^{\frac{1}{3}} \arctan\left(\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} + 2(-x^3 + 1)^{\frac{1}{3}}\right)\right) + \frac{1}{4} (-x^3 + 1)^{\frac{4}{3}} - \frac{1}{12} \cdot 2^{\frac{1}{3}} \log\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}(-x^3 + 1)^{\frac{1}{3}} + (-x^3 + 1)^{\frac{2}{3}}\right) + \frac{1}{6} \cdot 2^{\frac{1}{3}} \log\left(\left|-2^{\frac{1}{3}} + (-x^3 + 1)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1), x, algorithm="giac")

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/4*(-x^3 + 1)^(4/3) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

maple [C] time = 7.46, size = 1584, normalized size = 16.16

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] 1/4*(x^3-1)^2/(-x^3+1)^(2/3)+(RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*ln((144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^6-6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^6-144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3+6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3+72*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^6-3*RootOf(_Z^3-2)*x^6-90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3+9*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)*x^3+90*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-240*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+10*RootOf(_Z^3-2)*x^3+90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)-9*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)-18*(x^6-2*x^3+1)^(2/3)+168*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-7*RootOf(_Z^3-2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1))-1/6*ln((144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^6+30*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^6-144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-30*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3-24*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^6-5*RootOf(_Z^3-2)*x^6+90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3+24*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)*x^3-90*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+192*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+40*RootOf(_Z^3-2)*x^3-90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)-24*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)-48*(x^6-2*x^3+1)^(2/3)-168*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-35*RootOf(_Z^3-2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1))*RootOf(_Z^3-2)-ln((144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^6+30*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^6-144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-30*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3-24*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^6-5*RootOf(_Z^3-2)*x^6+90*(x^6-2*x^3+1)^(1/3)*Ro
```


otOf (RootOf (_Z^3-2)^2+6*_Z*RootOf (_Z^3-2)+36*_Z^2)*RootOf (_Z^3-2)*x^3+24*Ro
 otOf (_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)*x^3-90*(x^6-2*x^3+1)^(2/3)*RootOf (_Z^3-2
)^2*RootOf (RootOf (_Z^3-2)^2+6*_Z*RootOf (_Z^3-2)+36*_Z^2)+192*RootOf (RootOf (
 _Z^3-2)^2+6*_Z*RootOf (_Z^3-2)+36*_Z^2)*x^3+40*RootOf (_Z^3-2)*x^3-90*(x^6-2*
 x^3+1)^(1/3)*RootOf (RootOf (_Z^3-2)^2+6*_Z*RootOf (_Z^3-2)+36*_Z^2)*RootOf (_Z
 ^3-2)-24*RootOf (_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)-48*(x^6-2*x^3+1)^(2/3)-168*Ro
 otOf (RootOf (_Z^3-2)^2+6*_Z*RootOf (_Z^3-2)+36*_Z^2)-35*RootOf (_Z^3-2))/(x-1)
 /(x+1)/(x^2+x+1)/(x^2-x+1)*RootOf (RootOf (_Z^3-2)^2+6*_Z*RootOf (_Z^3-2)+36*
 _Z^2))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)

maxima [A] time = 1.19, size = 97, normalized size = 0.99

$$-\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)+\frac{1}{4}(-x^3+1)^{\frac{4}{3}}-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1), x, algorithm="maxima")

[Out] $-1/6*\sqrt{3}*2^{(1/3)}*\arctan(1/6*\sqrt{3}*2^{(2/3)}*(2^{(1/3)} + 2*(-x^3 + 1)^{(1/3}))) + 1/4*(-x^3 + 1)^{(4/3)} - 1/12*2^{(1/3)}*\log(2^{(2/3)} + 2^{(1/3)}*(-x^3 + 1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) + 1/6*2^{(1/3)}*\log(-2^{(1/3)} + (-x^3 + 1)^{(1/3)})$

mupad [B] time = 4.98, size = 113, normalized size = 1.15

$$\frac{2^{1/3}\ln\left(\frac{(1-x^3)^{1/3}}{2}-\frac{2^{1/3}}{2}\right)}{6}+\frac{(1-x^3)^{4/3}}{4}+\frac{2^{1/3}\ln\left(3(1-x^3)^{1/3}-\frac{32^{1/3}(-1+\sqrt{3}1i)}{2}\right)(-1+\sqrt{3}1i)}{12}-\frac{2^{1/3}\ln\left(\frac{32^{1/3}(1+\sqrt{3}1i)}{2}+3(1-x^3)^{1/3}\right)(1+\sqrt{3}1i)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((1 - x^3)^(2/3)*(x^3 + 1)), x)

[Out] $(2^{(1/3)}*\log((1 - x^3)^{(1/3)}/2 - 2^{(1/3)}/2))/6 + (1 - x^3)^{(4/3)}/4 + (2^{(1/3)}*\log(3*(1 - x^3)^{(1/3)} - (3*2^{(1/3)}*(3^{(1/2)}*1i - 1))/2)*(3^{(1/2)}*1i - 1))/12 - (2^{(1/3)}*\log((3*2^{(1/3)}*(3^{(1/2)}*1i + 1))/2 + 3*(1 - x^3)^{(1/3)}*(3^{(1/2)}*1i + 1))/12$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(- (x - 1) (x^2 + x + 1)\right)^{\frac{2}{3}} (x + 1) (x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(-x**3+1)**(2/3)/(x**3+1), x)

[Out] Integral(x**8/((- (x - 1) (x**2 + x + 1))** (2/3) * (x + 1) (x**2 - x + 1)), x)

$$3.389 \quad \int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)} dx$$

Optimal. Leaf size=95

$$-\sqrt[3]{1-x^3} + \frac{\log(x^3+1)}{6 \cdot 2^{2/3}} - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {446, 80, 57, 617, 204, 31}

$$-\sqrt[3]{1-x^3} + \frac{\log(x^3+1)}{6 \cdot 2^{2/3}} - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] -(1 - x^3)^(1/3) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(2/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f}

, n, p}, x] && NeQ[n + p + 2, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{(1-x)^{2/3}(1+x)} dx, x, x^3 \right) \\
 &= -\sqrt[3]{1-x^3} - \frac{1}{3} \text{Subst} \left(\int \frac{1}{(1-x)^{2/3}(1+x)} dx, x, x^3 \right) \\
 &= -\sqrt[3]{1-x^3} + \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{\sqrt[3]{2-x}} dx, x, \sqrt[3]{1-x^3} \right)}{2 \cdot 2^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2} \cdot x + x^2} dx, x, 1 + 2^{2/3} \sqrt[3]{1-x^3} \right)}{2 \sqrt[3]{2}} \\
 &= -\sqrt[3]{1-x^3} + \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} - \frac{\log \left(\sqrt[3]{2} - \sqrt[3]{1-x^3} \right)}{2 \cdot 2^{2/3}} - \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 + 2^{2/3} \sqrt[3]{1-x^3} \right)}{2^{2/3}} \\
 &= -\sqrt[3]{1-x^3} + \frac{\tan^{-1} \left(\frac{1+2^{2/3} \sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{2^{2/3} \sqrt{3}} + \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} - \frac{\log \left(\sqrt[3]{2} - \sqrt[3]{1-x^3} \right)}{2 \cdot 2^{2/3}}
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 118, normalized size = 1.24

$$\frac{1}{12} \left(-12\sqrt[3]{1-x^3} - 2\sqrt[3]{2} \log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right) + \sqrt[3]{2} \log\left((1-x^3)^{2/3} + \sqrt[3]{2-2x^3} + 2^{2/3}\right) + 2\sqrt[3]{2} \sqrt{3} \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3} + 1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] (-12*(1 - x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 2*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)] + 2^(1/3)*Log[2^(2/3) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)])/12

IntegrateAlgebraic [A] time = 0.10, size = 129, normalized size = 1.36

$$-\sqrt[3]{1-x^3} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} - 2\right)}{3 \cdot 2^{2/3}} + \frac{\log\left(\sqrt[3]{2} (1-x^3)^{2/3} + 2^{2/3}\sqrt[3]{1-x^3} + 2\right)}{6 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] -(1 - x^3)^(1/3) + ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))

fricas [A] time = 0.46, size = 130, normalized size = 1.37

$$-\frac{1}{6} \cdot 4^{\frac{1}{6}} \sqrt{3} (-1)^{\frac{1}{3}} \arctan\left(\frac{1}{6} \cdot 4^{\frac{2}{3}} \sqrt{3} (-1)^{\frac{2}{3}} (-x^3 + 1)^{\frac{1}{3}} + 4^{\frac{1}{3}} \sqrt{3}\right) - \frac{1}{24} \cdot 4^{\frac{2}{3}} (-1)^{\frac{1}{3}} \log\left(-4^{\frac{2}{3}} (-1)^{\frac{1}{3}} (-x^3 + 1)^{\frac{1}{3}} + 2 \cdot 4^{\frac{1}{3}} (-1)^{\frac{2}{3}} + 2(-x^3 + 1)^{\frac{2}{3}}\right) + \frac{1}{12} \cdot 4^{\frac{2}{3}} (-1)^{\frac{1}{3}} \log\left(4^{\frac{2}{3}} (-1)^{\frac{1}{3}} + 2(-x^3 + 1)^{\frac{1}{3}}\right) - (-x^3 + 1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)

giac [A] time = 0.23, size = 98, normalized size = 1.03

$$\frac{1}{6} \sqrt{3} 2^{\frac{1}{3}} \arctan\left(\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} + 2(-x^3 + 1)^{\frac{1}{3}}\right)\right) + \frac{1}{12} \cdot 2^{\frac{1}{3}} \log\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}(-x^3 + 1)^{\frac{1}{3}} + (-x^3 + 1)^{\frac{2}{3}}\right) - \frac{1}{6} \cdot 2^{\frac{1}{3}} \log\left(-2^{\frac{1}{3}} + (-x^3 + 1)^{\frac{1}{3}}\right) - (-x^3 + 1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")

[Out] $\frac{1}{6}\sqrt{3}2^{1/3}\arctan\left(\frac{1}{6}\sqrt{3}2^{2/3}\left(2^{1/3}+2(-x^3+1)^{1/3}\right)\right)+\frac{1}{12}2^{1/3}\log\left(2^{2/3}+2^{1/3}(-x^3+1)^{1/3}+(-x^3+1)^{2/3}\right)-\frac{1}{6}2^{1/3}\log\left(\operatorname{abs}\left(-2^{1/3}+(-x^3+1)^{1/3}\right)\right)-(-x^3+1)^{1/3}$

maple [C] time = 5.10, size = 1582, normalized size = 16.65

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(-x^3+1)^(2/3)/(x^3+1),x)

[Out] $\frac{(x^3-1)/(-x^3+1)^{2/3}+(-1/6\ln((30\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)\operatorname{RootOf}(_Z^3+2)^3x^6+144\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)^2\operatorname{RootOf}(_Z^3+2)^2x^6-30\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)\operatorname{RootOf}(_Z^3+2)^3x^3-144\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)^2\operatorname{RootOf}(_Z^3+2)^2x^3+5\operatorname{RootOf}(_Z^3+2)x^6+24\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)x^6+24\operatorname{RootOf}(_Z^3+2)^2(x^6-2x^3+1)^{1/3}x^3+90(x^6-2x^3+1)^{1/3}\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)\operatorname{RootOf}(_Z^3+2)x^3+90(x^6-2x^3+1)^{2/3}\operatorname{RootOf}(_Z^3+2)^2\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)-40\operatorname{RootOf}(_Z^3+2)x^3-192\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)x^3-24\operatorname{RootOf}(_Z^3+2)^2(x^6-2x^3+1)^{1/3}-90(x^6-2x^3+1)^{1/3}\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)\operatorname{RootOf}(_Z^3+2)-48(x^6-2x^3+1)^{2/3}+35\operatorname{RootOf}(_Z^3+2)+168\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1))\operatorname{RootOf}(_Z^3+2)-\ln((30\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)\operatorname{RootOf}(_Z^3+2)^3x^6+144\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)^2\operatorname{RootOf}(_Z^3+2)^2x^6-30\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)\operatorname{RootOf}(_Z^3+2)^3x^3-144\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)^2\operatorname{RootOf}(_Z^3+2)^2x^3+5\operatorname{RootOf}(_Z^3+2)x^6+24\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)x^6+24\operatorname{RootOf}(_Z^3+2)^2(x^6-2x^3+1)^{1/3}x^3+90(x^6-2x^3+1)^{1/3}\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)\operatorname{RootOf}(_Z^3+2)x^3+90(x^6-2x^3+1)^{2/3}\operatorname{RootOf}(_Z^3+2)^2\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)-40\operatorname{RootOf}(_Z^3+2)x^3-192\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)x^3-24\operatorname{RootOf}(_Z^3+2)^2(x^6-2x^3+1)^{1/3}-90(x^6-2x^3+1)^{1/3}\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)\operatorname{RootOf}(_Z^3+2)-48(x^6-2x^3+1)^{2/3}+35\operatorname{RootOf}(_Z^3+2)+168\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1))\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)+\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)\ln(-(6\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)\operatorname{RootOf}(_Z^3+2)^3x^6-144\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)^2\operatorname{RootOf}(_Z^3+2)^2x^6-6\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)\operatorname{RootOf}(_Z^3+2)^3x^3+144\operatorname{RootOf}(\operatorname{RootOf}(_Z^3+2)^2+6_Z\operatorname{RootOf}(_Z^3+2)+36_Z^2)^2$

RootOf(_Z^3+2)^2*x^3-3*RootOf(_Z^3+2)*x^6+72*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^6-9*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)*x^3+90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3+90*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)+10*RootOf(_Z^3+2)*x^3-240*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^3+9*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)-90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)+18*(x^6-2*x^3+1)^(2/3)-7*RootOf(_Z^3+2)+168*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)

maxima [A] time = 1.38, size = 97, normalized size = 1.02

$$\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)-(-x^3+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")

[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)+2*(-x^3+1)^(1/3))) + 1/12*2^(1/3)*log(2^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+(-x^3+1)^(2/3)) - 1/6*2^(1/3)*log(-2^(1/3)+(-x^3+1)^(1/3)) - (-x^3+1)^(1/3)

mupad [B] time = 4.89, size = 113, normalized size = 1.19

$$-\frac{2^{1/3}\ln\left(\frac{(1-x^3)^{1/3}}{2}-\frac{2^{1/3}}{2}\right)}{6}-\frac{2^{1/3}\ln\left(3(1-x^3)^{1/3}-\frac{32^{1/3}(-1+\sqrt{3}1i)}{2}\right)(-1+\sqrt{3}1i)}{12}+\frac{2^{1/3}\ln\left(\frac{32^{1/3}(1+\sqrt{3}1i)}{2}+3(1-x^3)^{1/3}\right)(1+\sqrt{3}1i)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((1-x^3)^(2/3)*(x^3+1)),x)

[Out] (2^(1/3)*log((3*2^(1/3)*(3^(1/2)*1i+1))/2+3*(1-x^3)^(1/3))*(3^(1/2)*1i+1))/12 - (1-x^3)^(1/3) - (2^(1/3)*log(3*(1-x^3)^(1/3)-(3*2^(1/3)*(3^(1/2)*1i-1))/2)*(3^(1/2)*1i-1))/12 - (2^(1/3)*log((1-x^3)^(1/3)/2-2^(1/3)/2))/6

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(-x-1)(x^2+x+1)^{\frac{2}{3}}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(-x**3+1)**(2/3)/(x**3+1),x)

[Out] Integral(x**5/((-x-1)*(x**2+x+1)**(2/3)*(x+1)*(x**2-x+1)),x)

$$3.390 \quad \int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)} dx$$

Optimal. Leaf size=83

$$-\frac{\log(x^3+1)}{6 \cdot 2^{2/3}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{2 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {444, 57, 617, 204, 31}

$$-\frac{\log(x^3+1)}{6 \cdot 2^{2/3}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{2 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((1 - x^3)^(2/3)*(1 + x^3)), x]

[Out] -(ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) - Log[1 + x^3]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))

Rule 31

Int[((a_) + (b_)*(x_)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_)^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 444

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(1-x)^{2/3}(1+x)} dx, x, x^3 \right) \\ &= -\frac{\log(1+x^3)}{6 \cdot 2^{2/3}} - \frac{\text{Subst} \left(\int \frac{1}{\sqrt[3]{2-x}} dx, x, \sqrt[3]{1-x^3} \right)}{2 \cdot 2^{2/3}} - \frac{\text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2} x + x^2} dx, x, \sqrt[3]{1-x^3} \right)}{2 \sqrt[3]{2}} \\ &= -\frac{\log(1+x^3)}{6 \cdot 2^{2/3}} + \frac{\log \left(\sqrt[3]{2} - \sqrt[3]{1-x^3} \right)}{2 \cdot 2^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 + 2^{2/3} \sqrt[3]{1-x^3} \right)}{2^{2/3}} \\ &= -\frac{\tan^{-1} \left(\frac{1+2^{2/3} \sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{2^{2/3} \sqrt{3}} - \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} + \frac{\log \left(\sqrt[3]{2} - \sqrt[3]{1-x^3} \right)}{2 \cdot 2^{2/3}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 94, normalized size = 1.13

$$\frac{-2 \log \left(\sqrt[3]{2} - \sqrt[3]{1-x^3} \right) + \log \left((1-x^3)^{2/3} + \sqrt[3]{2-2x^3} + 2^{2/3} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{1-x^3} + 1}{\sqrt{3}} \right)}{6 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(2/3)*(1 + x^3)), x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 2*Log[2^(1/
3) - (1 - x^3)^(1/3)] + Log[2^(2/3) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)])
/2^(2/3)
```


IntegrateAlgebraic [A] time = 0.10, size = 117, normalized size = 1.41

$$\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3 \cdot 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))

fricas [A] time = 0.49, size = 98, normalized size = 1.18

$$-\frac{1}{6} \cdot 4^{1/6} \sqrt{3} \arctan\left(\frac{1}{6} \cdot 4^{1/6} \left(4^{2/3} \sqrt{3} (-x^3 + 1)^{1/3} + 4^{1/3} \sqrt{3}\right)\right) - \frac{1}{24} \cdot 4^{2/3} \log\left(4^{2/3} (-x^3 + 1)^{1/3} + 2(-x^3 + 1)^{2/3} + 2 \cdot 4^{1/3}\right) + \frac{1}{12} \cdot 4^{2/3} \log\left(-4^{2/3} + 2(-x^3 + 1)^{1/3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 + 1)^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3))

giac [A] time = 0.20, size = 87, normalized size = 1.05

$$-\frac{1}{6} \sqrt{3} 2^{1/3} \arctan\left(\frac{1}{6} \sqrt{3} 2^{2/3} \left(2^{1/3} + 2(-x^3 + 1)^{1/3}\right)\right) - \frac{1}{12} \cdot 2^{1/3} \log\left(2^{2/3} + 2^{1/3}(-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}\right) + \frac{1}{6} \cdot 2^{1/3} \log\left(\left|-2^{1/3} + (-x^3 + 1)^{1/3}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")

[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))

maple [C] time = 6.28, size = 529, normalized size = 6.37

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-x^3+1)^(2/3)/(x^3+1),x)

```
[Out] RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*ln((144*RootOf(RootOf(
_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^3*x^3-6*RootOf(Root
Of(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^4*x^3-24*RootOf(Ro
otOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3+RootOf(_Z^3-
2)^2*x^3+168*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z
^3-2)-252*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*(-x^3+1)^(1/
3)-7*RootOf(_Z^3-2)^2-42*RootOf(_Z^3-2)*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3))/(
x+1)/(x^2-x+1))+1/6*RootOf(_Z^3-2)*ln(-(180*RootOf(RootOf(_Z^3-2)^2+6*_Z*Ro
otOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^3*x^3+6*RootOf(RootOf(_Z^3-2)^2+6*_Z
*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^4*x^3+90*RootOf(RootOf(_Z^3-2)^2+6*_
_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3+3*RootOf(_Z^3-2)^2*x^3-210*Ro
otOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)+252*RootO
f(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*(-x^3+1)^(1/3)-7*RootOf(_Z
^3-2)^2+42*RootOf(_Z^3-2)*(-x^3+1)^(1/3)-42*(-x^3+1)^(2/3))/(x+1)/(x^2-x+1))
```

maxima [A] time = 1.18, size = 86, normalized size = 1.04

$$-\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)+2*(-x^3+1)^(1/
3))) - 1/12*2^(1/3)*log(2^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+(-x^3+1)^(2/
3)) + 1/6*2^(1/3)*log(-2^(1/3)+(-x^3+1)^(1/3))
```

mupad [B] time = 5.05, size = 102, normalized size = 1.23

$$\frac{2^{1/3}\ln\left(32^{1/3}-3(1-x^3)^{1/3}\right)}{6} + \frac{2^{1/3}\ln\left(3(1-x^3)^{1/3}-\frac{32^{1/3}(-1+\sqrt{3}i)}{2}\right)(-1+\sqrt{3}i)}{12} - \frac{2^{1/3}\ln\left(\frac{32^{1/3}(1+\sqrt{3}i)}{2}+3(1-x^3)^{1/3}\right)(1+\sqrt{3}i)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1-x^3)^(2/3)*(x^3+1)),x)
```

```
[Out] (2^(1/3)*log(3*2^(1/3)-3*(1-x^3)^(1/3)))/6 + (2^(1/3)*log(3*(1-x^3)^(
1/3)-(3*2^(1/3)*(3^(1/2)*1i-1))/2)*(3^(1/2)*1i-1))/12 - (2^(1/3)*log(
(3*2^(1/3)*(3^(1/2)*1i+1))/2+3*(1-x^3)^(1/3))*(3^(1/2)*1i+1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-x-1)(x^2+x+1)^{\frac{2}{3}}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

$$3.391 \quad \int \frac{1}{x(1-x^3)^{2/3}(1+x^3)} dx$$

Optimal. Leaf size=137

$$\frac{\log(x^3+1)}{6 \cdot 2^{2/3}} + \frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(x)}{2}$$

Rubi [A] time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {446, 86, 57, 618, 204, 31, 617}

$$\frac{\log(x^3+1)}{6 \cdot 2^{2/3}} + \frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] -(ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(6*2^(2/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])]/; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 86

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && !IntegerQ[p]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(1-x^3)^{2/3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(1-x)^{2/3}x(1+x)} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(1-x)^{2/3}x} dx, x, x^3 \right) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{(1-x)^{2/3}(1+x)} dx, x, x^3 \right) \\
 &= -\frac{\log(x)}{2} + \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x} dx, x, \sqrt[3]{1-x^3} \right) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x} dx, x, \sqrt[3]{1+x^3} \right) \\
 &= -\frac{\log(x)}{2} + \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} + \frac{1}{2} \log \left(1 - \sqrt[3]{1-x^3} \right) - \frac{\log \left(\sqrt[3]{2} - \sqrt[3]{1-x^3} \right)}{2 \cdot 2^{2/3}} - \frac{\text{Subst} \left(\int \frac{1}{1+x} dx, x, \sqrt[3]{1+x^3} \right)}{2} \\
 &= -\frac{\tan^{-1} \left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{\tan^{-1} \left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{2^{2/3}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} + \frac{1}{2} \log \left(1 - \sqrt[3]{1-x^3} \right) - \frac{\log \left(\sqrt[3]{2} - \sqrt[3]{1-x^3} \right)}{2 \cdot 2^{2/3}} - \frac{\text{Subst} \left(\int \frac{1}{1+x} dx, x, \sqrt[3]{1+x^3} \right)}{2}
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 179, normalized size = 1.31

$$\frac{1}{12} \left(4 \log(1 - \sqrt[3]{1-x^3}) - 2\sqrt[3]{2} \log(\sqrt[3]{2} - \sqrt[3]{1-x^3}) + \sqrt[3]{2} \log((1-x^3)^{2/3} + \sqrt[3]{2-2x^3} + 2^{2/3}) - 2 \log((1-x^3)^{2/3} + \sqrt[3]{1-x^3} + 1) - 4\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{1-x^3} + 1}{\sqrt{3}}\right) + 2\sqrt[3]{2} \sqrt{3} \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3} + 1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 - x^3)^(2/3)*(1 + x^3)), x]

[Out] (-4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 4*Log[1 - (1 - x^3)^(1/3)] - 2*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)] + 2^(1/3)*Log[2^(2/3) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] - 2*Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)])/12

IntegrateAlgebraic [A] time = 0.19, size = 195, normalized size = 1.42

$$\frac{1}{3} \log(\sqrt[3]{1-x^3} - 1) - \frac{\log(2^{2/3}\sqrt[3]{1-x^3} - 2)}{3 \cdot 2^{2/3}} - \frac{1}{6} \log((1-x^3)^{2/3} + \sqrt[3]{1-x^3} + 1) + \frac{\log(\sqrt[3]{2}(1-x^3)^{2/3} + 2^{2/3}\sqrt[3]{1-x^3} + 2)}{6 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3} + 1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3} + 1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(1 - x^3)^(2/3)*(1 + x^3)), x]

[Out] -(ArcTan[1/Sqrt[3] + (2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[-1 + (1 - x^3)^(1/3)]/3 - Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6 + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))

fricas [A] time = 0.48, size = 182, normalized size = 1.33

$$-\frac{1}{6} \cdot 4^{1/3} \sqrt{3} (-1)^{1/3} \arctan\left(\frac{1}{6} \cdot 4^{1/3} \sqrt{3} (-1)^{1/3} (-x^3 + 1)^{1/3} + 4^{1/3} \sqrt{3}\right) - \frac{1}{24} \cdot 4^{1/3} (-1)^{1/3} \log\left(-4^{1/3} (-1)^{1/3} (-x^3 + 1)^{1/3} + 2 \cdot 4^{1/3} (-1)^{1/3} + 2(-x^3 + 1)^{1/3}\right) + \frac{1}{12} \cdot 4^{1/3} (-1)^{1/3} \log\left(4^{1/3} (-1)^{1/3} + 2(-x^3 + 1)^{1/3}\right) - \frac{1}{3} \sqrt{3} \arctan\left(\frac{2}{3} \sqrt{3} (-x^3 + 1)^{1/3} + \frac{1}{3} \sqrt{3}\right) - \frac{1}{6} \log\left((-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3} + 1\right) + \frac{1}{3} \log\left((-x^3 + 1)^{1/3} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1), x, algorithm="fricas")

[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3)) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)

giac [A] time = 0.24, size = 149, normalized size = 1.09

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{6} \sqrt{3} 2^{1/3} (2^{1/3} + 2(-x^3 + 1)^{1/3})\right) - \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2(-x^3 + 1)^{1/3} + 1)\right) + \frac{1}{12} \cdot 2^{1/3} \log\left(2^{1/3} + 2^{1/3}(-x^3 + 1)^{1/3} + (-x^3 + 1)^{1/3}\right) - \frac{1}{6} \cdot 2^{1/3} \log\left(-2^{1/3} + (-x^3 + 1)^{1/3}\right) - \frac{1}{6} \log\left((-x^3 + 1)^{2/3} + (-x^3 + 1)^{1/3} + 1\right) + \frac{1}{3} \log\left((-x^3 + 1)^{1/3} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")

[Out] $\frac{1}{6}\sqrt{3}2^{1/3}\arctan\left(\frac{1}{6}\sqrt{3}2^{2/3}(2^{1/3} + 2(-x^3 + 1)^{1/3})\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2(-x^3 + 1)^{1/3} + 1)\right) + \frac{1}{12}2^{1/3}\log(2^{2/3} + 2^{1/3}(-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}) - \frac{1}{6}2^{1/3}\log(\text{abs}(-2^{1/3} + (-x^3 + 1)^{1/3})) - \frac{1}{6}\log((-x^3 + 1)^{2/3} + (-x^3 + 1)^{1/3} + 1) + \frac{1}{3}\log(\text{abs}((-x^3 + 1)^{1/3} - 1))$

maple [F] time = 2.38, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^3 + 1)^{\frac{2}{3}}(x^3 + 1)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-x^3+1)^(2/3)/(x^3+1),x)

[Out] int(1/x/(-x^3+1)^(2/3)/(x^3+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{2}{3}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x), x)

mupad [B] time = 4.92, size = 344, normalized size = 2.51

$$\frac{\ln\left(\frac{5 - 46(-x^3)^{1/3}}{3}\right)}{3} - \frac{\ln\left(\frac{5 - 46(-x^3)^{1/3}}{3}\right)}{3} + \frac{\ln\left(\frac{5 - 46(-x^3)^{1/3}}{3}\right)}{3} + \frac{\ln\left(\frac{5 - 46(-x^3)^{1/3}}{3}\right)}{3} + \frac{\ln\left(\frac{5 - 46(-x^3)^{1/3}}{3}\right)}{3} + \frac{\ln\left(\frac{5 - 46(-x^3)^{1/3}}{3}\right)}{3} + \frac{\ln\left(\frac{5 - 46(-x^3)^{1/3}}{3}\right)}{3} + \frac{\ln\left(\frac{5 - 46(-x^3)^{1/3}}{3}\right)}{3} + \frac{\ln\left(\frac{5 - 46(-x^3)^{1/3}}{3}\right)}{3} + \frac{\ln\left(\frac{5 - 46(-x^3)^{1/3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(1 - x^3)^(2/3)*(x^3 + 1)),x)

[Out] $\log(5 - 5(1 - x^3)^{1/3})/3 - (2^{1/3}\log(6(1 - x^3)^{1/3} - (2^{1/3})((2^{2/3}(243*2^{1/3} + 243(1 - x^3)^{1/3}))/36 + 9))/6) + \log(((3^{1/2}) * 1i)/6 - 1/6) * (((3^{1/2}) * 1i)/6 - 1/6)^2 * (243(1 - x^3)^{1/3} - 3^{1/2} * 243i + 243) + 9) + 6(1 - x^3)^{1/3} * (((3^{1/2}) * 1i)/6 - 1/6) - \log(6(1 - x^3)^{1/3} - ((3^{1/2}) * 1i)/6 + 1/6) * (((3^{1/2}) * 1i)/6 + 1/6)^2 * (3^{1/2} * 243i + 24$

```

3*(1 - x^3)^(1/3) + 243) + 9))*((3^(1/2)*1i)/6 + 1/6) + ((-1)^(1/3)*2^(1/3)
*log(6*(1 - x^3)^(1/3) - ((-1)^(1/3)*2^(1/3)*((-1)^(2/3)*2^(2/3)*(243*(-1)
^(1/3)*2^(1/3) - 243*(1 - x^3)^(1/3)))/36 - 9))/6))/6 - ((-1)^(1/3)*2^(1/3)
*log(6*(1 - x^3)^(1/3) - ((-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1)*((-1)^(2/3)*
2^(2/3)*(3^(1/2)*1i + 1)^2*(243*(1 - x^3)^(1/3) + (243*(-1)^(1/3)*2^(1/3)*(
3^(1/2)*1i + 1))/2))/144 + 9))/12)*(3^(1/2)*1i + 1))/12

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left(-(x-1) \left(x^2 + x + 1 \right) \right)^{\frac{2}{3}} (x+1) (x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x**3+1)**(2/3)/(x**3+1), x)

[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)

$$3.392 \quad \int \frac{1}{x^4(1-x^3)^{2/3}(1+x^3)} dx$$

Optimal. Leaf size=158

$$-\frac{\sqrt[3]{1-x^3}}{3x^3} - \frac{\log(x^3+1)}{6 \cdot 2^{2/3}} - \frac{1}{6} \log\left(1 - \sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} +$$

Rubi [A] time = 0.10, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {446, 103, 156, 57, 618, 204, 31, 617}

$$-\frac{\sqrt[3]{1-x^3}}{3x^3} - \frac{\log(x^3+1)}{6 \cdot 2^{2/3}} - \frac{1}{6} \log\left(1 - \sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(x)}{6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] $-(1 - x^3)^{(1/3)}/(3*x^3) + \text{ArcTan}[(1 + 2*(1 - x^3)^{(1/3)})/\text{Sqrt}[3]]/(3*\text{Sqrt}[3]) - \text{ArcTan}[(1 + 2^{(2/3)}*(1 - x^3)^{(1/3)})/\text{Sqrt}[3]]/(2^{(2/3)}*\text{Sqrt}[3]) + \text{Log}[x]/6 - \text{Log}[1 + x^3]/(6*2^{(2/3)}) - \text{Log}[1 - (1 - x^3)^{(1/3)}]/6 + \text{Log}[2^{(1/3)} - (1 - x^3)^{(1/3)}]/(2*2^{(2/3)})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,

$x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n, p\}, x] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ (\text{IntegerQ}[n] \ || \ \text{IntegersQ}[2*n, 2*p])$

Rule 156

$\text{Int}[(((e_.) + (f_.)*(x_))^{(p_.)}*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] \ :> \ \text{Dist}[(b*g - a*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(a + b*x), x], x] - \text{Dist}[(d*g - c*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(c + d*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x]$

Rule 204

$\text{Int}[((a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \ :> \ -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 446

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}*((c_) + (d_.)*(x_)^{(n_.)})^{(q_.)}, x_Symbol] \ :> \ \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 617

$\text{Int}[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x_Symbol] \ :> \ \text{With}[\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 618

$\text{Int}[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x_Symbol] \ :> \ \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(1-x)^{2/3} x^2 (1+x)} dx, x, x^3 \right) \\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3} - \frac{1}{3} \text{Subst} \left(\int \frac{\frac{1}{3} - \frac{2x}{3}}{(1-x)^{2/3} x (1+x)} dx, x, x^3 \right) \\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3} - \frac{1}{9} \text{Subst} \left(\int \frac{1}{(1-x)^{2/3} x} dx, x, x^3 \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{(1-x)^{2/3} (1+x)} dx, x, x^3 \right) \\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3} + \frac{\log(x)}{6} - \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} + \frac{1}{6} \text{Subst} \left(\int \frac{1}{1-x} dx, x, \sqrt[3]{1-x^3} \right) + \frac{1}{6} \text{Subst} \left(\int \frac{1}{1+x} dx, x, \sqrt[3]{1-x^3} \right) \\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3} + \frac{\log(x)}{6} - \frac{\log(1+x^3)}{6 \cdot 2^{2/3}} - \frac{1}{6} \log \left(1 - \sqrt[3]{1-x^3} \right) + \frac{\log \left(\sqrt[3]{2} - \sqrt[3]{1-x^3} \right)}{2 \cdot 2^{2/3}} \\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3} + \frac{\tan^{-1} \left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{3\sqrt{3}} - \frac{\tan^{-1} \left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{2^{2/3}\sqrt{3}} + \frac{\log(x)}{6} - \frac{\log(1+x^3)}{6 \cdot 2^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 196, normalized size = 1.24

$$\frac{1}{36} \left(-\frac{12\sqrt[3]{1-x^3}}{x^3} - 4\log(1-\sqrt[3]{1-x^3}) + 6\sqrt[3]{2} \log(\sqrt[3]{2} - \sqrt[3]{1-x^3}) - 3\sqrt[3]{2} \log((1-x^3)^{2/3} + \sqrt[3]{2-2x^3} + 2^{2/3}) + 2\log((1-x^3)^{2/3} + \sqrt[3]{1-x^3} + 1) + 4\sqrt{3} \tan^{-1} \left(\frac{2\sqrt[3]{1-x^3} + 1}{\sqrt{3}} \right) - 6\sqrt[3]{2} \sqrt{3} \tan^{-1} \left(\frac{2^{2/3}\sqrt[3]{1-x^3} + 1}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(1-x^3)^(2/3)*(1+x^3)),x]

[Out] ((-12*(1-x^3)^(1/3))/x^3 + 4*Sqrt[3]*ArcTan[(1+2*(1-x^3)^(1/3))/Sqrt[3]] - 6*2^(1/3)*Sqrt[3]*ArcTan[(1+2^(2/3)*(1-x^3)^(1/3))/Sqrt[3]] - 4*Log[1-(1-x^3)^(1/3)] + 6*2^(1/3)*Log[2^(1/3)-(1-x^3)^(1/3)] - 3*2^(1/3)*Log[2^(2/3)+(2-2*x^3)^(1/3)+(1-x^3)^(2/3)] + 2*Log[1+(1-x^3)^(1/3)+(1-x^3)^(2/3)])/36

IntegrateAlgebraic [A] time = 0.25, size = 216, normalized size = 1.37

$$-\frac{\sqrt[3]{1-x^3}}{3x^3} - \frac{1}{9} \log(\sqrt[3]{1-x^3} - 1) + \frac{\log(2^{2/3}\sqrt[3]{1-x^3} - 2)}{3 \cdot 2^{2/3}} + \frac{1}{18} \log((1-x^3)^{2/3} + \sqrt[3]{1-x^3} + 1) - \frac{\log(\sqrt[3]{2}(1-x^3)^{2/3} + 2^{2/3}\sqrt[3]{1-x^3} + 2)}{6 \cdot 2^{2/3}} + \frac{\tan^{-1} \left(\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}} \right)}{3\sqrt{3}} - \frac{\tan^{-1} \left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}} \right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(1-x^3)^(2/3)*(1+x^3)),x]

```
[Out] -1/3*(1 - x^3)^(1/3)/x^3 + ArcTan[1/Sqrt[3] + (2*(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[3]) - ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[-1 + (1 - x^3)^(1/3)]/9 + Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) + Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/18 - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

fricas [A] time = 0.49, size = 195, normalized size = 1.23

$$\frac{12 \cdot 4^{\frac{2}{3}} \sqrt{3} x^3 \arctan\left(\frac{1}{6} \cdot 4^{\frac{2}{3}} \sqrt{3} (-x^3 + 1)^{\frac{1}{3}} + 4^{\frac{1}{3}} \sqrt{3}\right) + 3 \cdot 4^{\frac{2}{3}} x^3 \log\left(4^{\frac{2}{3}} (-x^3 + 1)^{\frac{1}{3}} + 2(-x^3 + 1)^{\frac{2}{3}} + 2 \cdot 4^{\frac{1}{3}}\right) - 6 \cdot 4^{\frac{2}{3}} x^3 \log\left(-4^{\frac{2}{3}} + 2(-x^3 + 1)^{\frac{1}{3}}\right) - 8 \sqrt{3} x^3 \arctan\left(\frac{2}{3} \sqrt{3} (-x^3 + 1)^{\frac{1}{3}} + \frac{1}{3} \sqrt{3}\right) - 4 x^3 \log\left((-x^3 + 1)^{\frac{2}{3}} + (-x^3 + 1)^{\frac{1}{3}} + 1\right) + 8 x^3 \log\left((-x^3 + 1)^{\frac{2}{3}} - 1\right) + 24(-x^3 + 1)^{\frac{2}{3}}}{72 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(12*4^(1/6)*sqrt(3)*x^3*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) + 3*4^(2/3)*x^3*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 + 1)^(2/3) + 2*4^(1/3)) - 6*4^(2/3)*x^3*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3)) - 8*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 4*x^3*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 8*x^3*log((-x^3 + 1)^(1/3) - 1) + 24*(-x^3 + 1)^(1/3)/x^3
```

giac [A] time = 0.22, size = 163, normalized size = 1.03

$$-\frac{1}{6} \sqrt{3} x^3 \arctan\left(\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} (2^{\frac{2}{3}} + 2(-x^3 + 1)^{\frac{1}{3}})\right) + \frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2(-x^3 + 1)^{\frac{1}{3}} + 1)\right) - \frac{1}{12} 2^{\frac{2}{3}} \log\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}(-x^3 + 1)^{\frac{1}{3}} + (-x^3 + 1)^{\frac{2}{3}}\right) + \frac{1}{6} 2^{\frac{2}{3}} \log\left(-2^{\frac{2}{3}} + (-x^3 + 1)^{\frac{1}{3}}\right) - \frac{(-x^3 + 1)^{\frac{1}{3}}}{3 x^3} + \frac{1}{18} \log\left((-x^3 + 1)^{\frac{2}{3}} + (-x^3 + 1)^{\frac{1}{3}} + 1\right) - \frac{1}{9} \log\left((-x^3 + 1)^{\frac{2}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/3*(-x^3 + 1)^(1/3)/x^3 + 1/18*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 1/9*log(abs((-x^3 + 1)^(1/3) - 1))
```

maple [F] time = 3.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^3 + 1)^{\frac{2}{3}} (x^3 + 1) x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{2}{3}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^4), x)

mupad [B] time = 5.07, size = 368, normalized size = 2.33

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(1 - x^3)^(2/3)*(x^3 + 1)),x)

[Out] $(2^{(1/3)} \log((10*(1 - x^3)^{(1/3)})/9 - (2^{(1/3)}*((2^{(2/3)}*(243*2^{(1/3)} + 27*(1 - x^3)^{(1/3}))/36 - 25/3))/6))/6 - (1 - x^3)^{(1/3)}/(3*x^3) - \log((31*(1 - x^3)^{(1/3)}/243 - 31/243)/9 - \log(((3^{(1/2)}*1i)/18 - 1/18)*((3^{(1/2)}*1i)/18 - 1/18)^2*(27*(1 - x^3)^{(1/3)} - 3^{(1/2)}*81i + 81) - 25/3) + (10*(1 - x^3)^{(1/3)}/9)*((3^{(1/2)}*1i)/18 - 1/18) + \log((10*(1 - x^3)^{(1/3)}/9 - ((3^{(1/2)}*1i)/18 + 1/18)*((3^{(1/2)}*1i)/18 + 1/18)^2*(3^{(1/2)}*81i + 27*(1 - x^3)^{(1/3)} + 81) - 25/3))*((3^{(1/2)}*1i)/18 + 1/18) + (2^{(1/3)} \log((10*(1 - x^3)^{(1/3)}/9 - (2^{(1/3)}*(3^{(1/2)}*1i - 1))*((2^{(2/3)}*(3^{(1/2)}*1i - 1))^2*((243*2^{(1/3)}*(3^{(1/2)}*1i - 1))/2 + 27*(1 - x^3)^{(1/3}))/144 - 25/3))/12)*(3^{(1/2)}*1i - 1))/12 - (2^{(1/3)} \log((10*(1 - x^3)^{(1/3)}/9 - (2^{(1/3)}*(3^{(1/2)}*1i + 1))*((2^{(2/3)}*(3^{(1/2)}*1i + 1))^2*((243*2^{(1/3)}*(3^{(1/2)}*1i + 1))/2 - 27*(1 - x^3)^{(1/3}))/144 + 25/3))/12)*(3^{(1/2)}*1i + 1))/12$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (-(x-1)(x^2+x+1))^{\frac{2}{3}} (x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(-x**3+1)**(2/3)/(x**3+1),x)

[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)

$$3.393 \quad \int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)} dx$$

Optimal. Leaf size=160

$$\frac{\log(x^3+1)}{6 \cdot 2^{2/3}} + \frac{1}{6} \log\left(-\sqrt[3]{1-x^3} - x\right) - \frac{\log\left(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x\right)}{2 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{1}{3}\sqrt[3]{1-x^3}x^2$$

Rubi [A] time = 0.16, antiderivative size = 228, normalized size of antiderivative = 1.42, number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {494, 470, 584, 634, 618, 204, 628, 292, 31, 617}

$$-\frac{1}{3}\sqrt[3]{1-x^3}x^2 - \frac{1}{18} \log\left(\frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} + 1\right) + \frac{1}{9} \log\left(\frac{x}{\sqrt[3]{1-x^3}} + 1\right) + \frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{6 \cdot 2^{2/3}} - \frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{3 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] -(x^2*(1 - x^3)^(1/3))/3 + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/18 + Log[1 + x/(1 - x^3)^(1/3)]/9 + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(n-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x

$\wedge 2), x], x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 470

$\text{Int}[(e_.)*(x_)]^{(m_)}*((a_)+(b_)*(x_)^{(n_)})^{(p_)}*((c_)+(d_)*(x_)^{(n_)})^{(q_)}, x_Symbol] \rightarrow -\text{Simp}[(a*e^{(2*n-1)}*(e*x)^{(m-2*n+1)}*(a+b*x^n)^{(p+1)}*(c+d*x^n)^{(q+1)})/(b*n*(b*c-a*d)*(p+1)), x] + \text{Dist}[e^{(2*n)}/(b*n*(b*c-a*d)*(p+1)), \text{Int}[(e*x)^{(m-2*n)}*(a+b*x^n)^{(p+1)}*(c+d*x^n)^q * \text{Simp}[a*c*(m-2*n+1) + (a*d*(m-n+n*q+1) + b*c*n*(p+1))*x^n, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, q\}, x] \&\& \text{NeQ}[b*c-a*d, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[m-n+1, n] \&\& \text{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 494

$\text{Int}[(x_)]^{(m_)}*((a_)+(b_)*(x_)^{(n_)})^{(p_)}*((c_)+(d_)*(x_)^{(n_)})^{(q_)}, x_Symbol] \rightarrow \text{With}[\{k = \text{Denominator}[p]\}, \text{Dist}[(k*a^{(p+(m+1)/n)})/n, \text{Subst}[\text{Int}[(x^{((k*(m+1))/n-1)}*(c-(b*c-a*d)*x^k)^q)/(1-b*x^k)^{(p+q+(m+1)/n+1)}], x], x, x^{(n/k)}/(a+b*x^n)^{(1/k)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{RationalQ}[m, p] \&\& \text{IntegersQ}[p+(m+1)/n, q] \&\& \text{LtQ}[-1, p, 0]$

Rule 584

$\text{Int}[(g_.)*(x_)]^{(m_)}*((a_)+(b_)*(x_)^{(n_)})^{(p_)}*((e_)+(f_)*(x_)^{(n_)}))/((c_)+(d_)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(g*x)^m*(a+b*x^n)^p*(e+f*x^n)/(c+d*x^n), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, m, p\}, x] \&\& \text{IGtQ}[n, 0]$

Rule 617

$\text{Int}[(a_)+(b_)*(x_)+(c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q-x^2)], x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \|\ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 618

$\text{Int}[(a_)+(b_)*(x_)+(c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)} dx &= \text{Subst} \left(\int \frac{x^7}{(1+x^3)^2(1+2x^3)} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3} \text{Subst} \left(\int \frac{x(2+x^3)}{(1+x^3)(1+2x^3)} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{3(1+x)} + \frac{-1-x}{3(1-x+x^2)} + \frac{3x}{1+2x^3} \right) dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{9} \log \left(1 + \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{1}{9} \text{Subst} \left(\int \frac{-1-x}{1-x+x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) + \text{Subst} \left(\int \frac{3x}{1+2x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{9} \log \left(1 + \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{1}{18} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{1}{6} \text{Subst} \left(\int \frac{3x}{1+2x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{1}{18} \log \left(1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{1}{9} \log \left(1 + \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{\log \left(1 + \frac{x}{\sqrt[3]{1-x^3}} \right)}{3} \\
&= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{\tan^{-1} \left(\frac{1 - \frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{3\sqrt{3}} - \frac{1}{18} \log \left(1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{1}{9} \log \left(1 + \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{\tan^{-1} \left(\frac{1 - \frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{3\sqrt{3}} - \frac{\tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{2^{2/3}\sqrt{3}} - \frac{1}{18} \log \left(1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{1}{9} \log \left(1 + \frac{x}{\sqrt[3]{1-x^3}} \right)
\end{aligned}$$

Mathematica [C] time = 0.09, size = 78, normalized size = 0.49

$$\frac{1}{15}x^2 \left(x^3 \left(-F_1 \left(\frac{5}{3}; \frac{2}{3}, 1; \frac{8}{3}; x^3, -x^3 \right) \right) + \frac{{}_5F_1 \left(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{2x^3}{x^3+1} \right)}{(x^3+1)^{2/3}} - 5\sqrt[3]{1-x^3} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] (x^2*(-5*(1 - x^3)^(1/3) - x^3*AppellF1[5/3, 2/3, 1, 8/3, x^3, -x^3] + (5*Hypergeometric2F1[2/3, 2/3, 5/3, (2*x^3)/(1 + x^3)]))/(1 + x^3)^(2/3))/15

IntegrateAlgebraic [A] time = 0.54, size = 232, normalized size = 1.45

$$\frac{1}{9} \log(\sqrt[3]{1-x^3}+x) - \frac{\log(2^{2/3}\sqrt[3]{1-x^3}+2x)}{3 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}-x}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2^{2/3}\sqrt{3}} - \frac{1}{3}\sqrt[3]{1-x^3}x^2 - \frac{1}{18} \log(-\sqrt[3]{1-x^3}x + (1-x^3)^{2/3} + x^2) + \frac{\log(2^{2/3}\sqrt[3]{1-x^3}x - \sqrt[3]{2}(1-x^3)^{2/3} - 2x^2)}{6 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] -1/3*(x^2*(1 - x^3)^(1/3)) - ArcTan[(Sqrt[3]*x)/(-x + 2*(1 - x^3)^(1/3))]/(3*Sqrt[3]) + ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2^(2/3)*Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/9 - Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/18 + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))

fricas [A] time = 0.46, size = 232, normalized size = 1.45

$$\frac{1}{9}(-x^3+1)^{1/3}x^2 + \frac{1}{9}4^{1/3}\sqrt{3}(-1)^{1/3}\arctan\left(\frac{4^{1/3}\sqrt{3}(-1)^{1/3}(-x^3+1)^{1/3}-4^{1/3}\sqrt{3}x}{6x}\right) + \frac{1}{12}4^{1/3}(-1)^{1/3}\log\left(\frac{4^{1/3}(-1)^{1/3}x-2(-x^3+1)^{1/3}}{x}\right) - \frac{1}{24}4^{1/3}(-1)^{1/3}\log\left(\frac{2\cdot 4^{1/3}(-1)^{1/3}x^2+4^{1/3}(-1)^{1/3}(-x^3+1)^{1/3}x+2(-x^3+1)^{1/3}}{x^2}\right) + \frac{1}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{1/3}}{3x}\right) + \frac{1}{9}\log\left(\frac{x+(-x^3+1)^{1/3}}{x}\right) - \frac{1}{18}\log\left(\frac{x^2-(-x^3+1)^{1/3}x+(-x^3+1)^{1/3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/3*(-x^3 + 1)^(1/3)*x^2 + 1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*4^(2/3)*sqrt(3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 4^(1/3)*sqrt(3)*x/x) + 1/12*4^(2/3)*(-1)^(1/3)*log((-4^(2/3)*(-1)^(1/3)*x - 2*(-x^3 + 1)^(1/3))/x) - 1/24*4^(2/3)*(-1)^(1/3)*log((2*4^(1/3)*(-1)^(2/3)*x^2 + 4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) + 1/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/9*log((x + (-x^3 + 1)^(1/3))/x) - 1/18*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)

maple [F] time = 2.04, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(-x^3 + 1)^{\frac{2}{3}} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(-x^3+1)^(2/3)/(x^3+1),x)

[Out] int(x^7/(-x^3+1)^(2/3)/(x^3+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(x^3 + 1)(-x^3 + 1)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{(1 - x^3)^{\frac{2}{3}} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/((1 - x^3)^(2/3)*(x^3 + 1)),x)

[Out] int(x^7/((1 - x^3)^(2/3)*(x^3 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(-(x - 1)(x^2 + x + 1))^{\frac{2}{3}} (x + 1)(x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**7/((-x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

$$3.394 \quad \int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)} dx$$

Optimal. Leaf size=139

$$-\frac{\log(x^3+1)}{6 \cdot 2^{2/3}} - \frac{1}{2} \log\left(-\sqrt[3]{1-x^3} - x\right) + \frac{\log\left(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x\right)}{2 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Rubi [A] time = 0.10, antiderivative size = 207, normalized size of antiderivative = 1.49, number of steps used = 14, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {494, 481, 292, 31, 634, 618, 204, 628, 617}

$$\frac{1}{6} \log\left(\frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} + 1\right) - \frac{1}{3} \log\left(\frac{x}{\sqrt[3]{1-x^3}} + 1\right) - \frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{6 \cdot 2^{2/3}} + \frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{3 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/6 - Log[1 + x/(1 - x^3)^(1/3)]/3 - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(n-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x

$\wedge 2), x], x] /; \text{FreeQ}\{a, b\}, x]$

Rule 481

$\text{Int}[(e_{.})*(x_{.})^{(m_{.})}/((a_{.}) + (b_{.})*(x_{.})^{(n_{.})})*((c_{.}) + (d_{.})*(x_{.})^{(n_{.})})], x_Symbol] :> -\text{Dist}[(a*e^n)/(b*c - a*d), \text{Int}[(e*x)^{(m-n)}/(a + b*x^n), x], x] + \text{Dist}[(c*e^n)/(b*c - a*d), \text{Int}[(e*x)^{(m-n)}/(c + d*x^n), x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{LeQ}[n, m, 2*n - 1]$

Rule 494

$\text{Int}[(x_{.})^{(m_{.})}*((a_{.}) + (b_{.})*(x_{.})^{(n_{.})})^{(p_{.})}*((c_{.}) + (d_{.})*(x_{.})^{(n_{.})})^{(q_{.})}, x_Symbol] :> \text{With}\{k = \text{Denominator}[p]\}, \text{Dist}[(k*a^{(p + (m + 1)/n)})/n, \text{Subst}[\text{Int}[(x^{((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^{(p + q + (m + 1)/n + 1)}], x], x, x^{(n/k)}/(a + b*x^n)^{(1/k)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{RationalQ}[m, p] \&\& \text{IntegersQ}[p + (m + 1)/n, q] \&\& \text{LtQ}[-1, p, 0]$

Rule 617

$\text{Int}[(a_{.}) + (b_{.})*(x_{.}) + (c_{.})*(x_{.})^2]^{(-1)}, x_Symbol] :> \text{With}\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \|\ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 618

$\text{Int}[(a_{.}) + (b_{.})*(x_{.}) + (c_{.})*(x_{.})^2]^{(-1)}, x_Symbol] :> \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_{.}) + (e_{.})*(x_{.})]/((a_{.}) + (b_{.})*(x_{.}) + (c_{.})*(x_{.})^2), x_Symbol] :> \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[(d_{.}) + (e_{.})*(x_{.})]/((a_{.}) + (b_{.})*(x_{.}) + (c_{.})*(x_{.})^2), x_Symbol] :> \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)} dx &= \text{Subst} \left(\int \frac{x^4}{(1+x^3)(1+2x^3)} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= \text{Subst} \left(\int \frac{x}{1+x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) - \text{Subst} \left(\int \frac{x}{1+2x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\left(\frac{1}{3} \text{Subst} \left(\int \frac{1}{1+x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1+x}{1-x+x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) + \dots \\
&= -\frac{1}{3} \log \left(1 + \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3 \cdot 2^{2/3}} + \frac{1}{6} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= \frac{1}{6} \log \left(1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{1}{3} \log \left(1 + \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}}{\sqrt[3]{1-x^3}} \right)}{6 \cdot 2^{2/3}} \\
&= \frac{\tan^{-1} \left(\frac{-1 + \frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{\tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{2^{2/3}\sqrt{3}} + \frac{1}{6} \log \left(1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{1}{3} \log \left(1 + \frac{x}{\sqrt[3]{1-x^3}} \right)
\end{aligned}$$

Mathematica [C] time = 0.05, size = 26, normalized size = 0.19

$$\frac{1}{5}x^5F_1\left(\frac{5}{3}; \frac{2}{3}, 1; \frac{8}{3}; x^3, -x^3\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4/((1 - x^3)^(2/3)*(1 + x^3)), x]

[Out] (x^5*AppellF1[5/3, 2/3, 1, 8/3, x^3, -x^3])/5

IntegrateAlgebraic [A] time = 0.38, size = 212, normalized size = 1.53

$$-\frac{1}{3} \log(\sqrt[3]{1-x^3} + x) + \frac{\log(2^{2/3}\sqrt[3]{1-x^3} + 2x)}{3 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}-x}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2^{2/3}\sqrt{3}} + \frac{1}{6} \log(-\sqrt[3]{1-x^3}x + (1-x^3)^{2/3} + x^2) - \frac{\log(2^{2/3}\sqrt[3]{1-x^3}x - \sqrt[3]{2}(1-x^3)^{2/3} - 2x^2)}{6 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] ArcTan[(Sqrt[3]*x)/(-x + 2*(1 - x^3)^(1/3))]/Sqrt[3] - ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2^(2/3)*Sqrt[3]) - Log[x + (1 - x^3)^(1/3)]/3 + Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) + Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6 - Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))

fricas [A] time = 0.46, size = 197, normalized size = 1.42

$$\frac{1}{6} \cdot 4^{\frac{1}{6}} \sqrt{3} \arctan\left(\frac{4^{\frac{1}{6}}(4^{\frac{1}{6}}\sqrt{3}x - 4^{\frac{1}{6}}\sqrt{3}(-x^3+1)^{\frac{1}{3}})}{6x}\right) + \frac{1}{12} \cdot 4^{\frac{2}{3}} \log\left(\frac{4^{\frac{2}{3}}x + 2(-x^3+1)^{\frac{1}{3}}}{x}\right) - \frac{1}{24} \cdot 4^{\frac{2}{3}} \log\left(\frac{2 \cdot 4^{\frac{1}{3}}x^2 - 4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}x + 2(-x^3+1)^{\frac{2}{3}}}{x^2}\right) - \frac{1}{3} \sqrt{3} \arctan\left(\frac{-\sqrt{3}x - 2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3x}\right) - \frac{1}{3} \log\left(\frac{x + (-x^3+1)^{\frac{1}{3}}}{x}\right) + \frac{1}{6} \log\left(\frac{x^2 - (-x^3+1)^{\frac{1}{3}}x + (-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")

[Out] 1/6*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*(4^(1/3)*sqrt(3)*x - 4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/12*4^(2/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3))/x) - 1/24*4^(2/3)*log((2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) - 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) - 1/3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(x^3 + 1)(-x^3 + 1)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)

maple [F] time = 1.76, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-x^3 + 1)^{\frac{2}{3}}(x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(-x^3+1)^(2/3)/(x^3+1),x)

[Out] int(x^4/(-x^3+1)^(2/3)/(x^3+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(x^3 + 1)(-x^3 + 1)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{(1 - x^3)^{\frac{2}{3}} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((1 - x^3)^(2/3)*(x^3 + 1)),x)

[Out] int(x^4/((1 - x^3)^(2/3)*(x^3 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-(x - 1)(x^2 + x + 1))^{\frac{2}{3}} (x + 1)(x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(-x**3+1)**(2/3)/(x**3+1),x)

[Out] Integral(x**4/((-x - 1)*(x**2 + x + 1))**2/3*(x + 1)*(x**2 - x + 1)), x)

$$3.395 \quad \int \frac{x}{(1-x^3)^{2/3}(1+x^3)} dx$$

Optimal. Leaf size=88

$$\frac{\log(x^3 + 1)}{6 \cdot 2^{2/3}} - \frac{\log(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x)}{2 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {494, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{6 \cdot 2^{2/3}} - \frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{3 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x/((1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(n-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
(m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1-x^3)^{2/3}(1+x^3)} dx &= \text{Subst} \left(\int \frac{x}{1+2x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{\text{Subst} \left(\int \frac{1}{1+\sqrt[3]{2}x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} + \frac{\text{Subst} \left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} \\
&= -\frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3 \cdot 2^{2/3}} + \frac{\text{Subst} \left(\int \frac{-\sqrt[3]{2}+2 \cdot 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{6 \cdot 2^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{2\sqrt[3]{2}} \\
&= \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6 \cdot 2^{2/3}} - \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3 \cdot 2^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{2^{2/3}} \\
&= -\frac{\tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{2^{2/3}\sqrt{3}} + \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6 \cdot 2^{2/3}} - \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3 \cdot 2^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.43

$$\frac{x^2 {}_2F_1 \left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; \frac{2x^3}{x^3+1} \right)}{2(x^3+1)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/((1 - x^3)^(2/3)*(1 + x^3)), x]

[Out] (x^2*Hypergeometric2F1[2/3, 2/3, 5/3, (2*x^3)/(1 + x^3)])/(2*(1 + x^3)^(2/3))

IntegrateAlgebraic [A] time = 0.29, size = 126, normalized size = 1.43

$$-\frac{\log \left(2^{2/3} \sqrt[3]{1-x^3} + 2x \right)}{3 \cdot 2^{2/3}} + \frac{\tan^{-1} \left(\frac{\sqrt{3}x}{2^{2/3} \sqrt[3]{1-x^3} - x} \right)}{2^{2/3} \sqrt{3}} + \frac{\log \left(2^{2/3} \sqrt[3]{1-x^3} x - \sqrt{2} (1-x^3)^{2/3} - 2x^2 \right)}{6 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/((1 - x^3)^(2/3)*(1 + x^3)), x]

[Out] ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2^(2/3)*Sqrt[3]) - Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))

fricas [B] time = 1.92, size = 283, normalized size = 3.22

$$\frac{1}{18} \cdot 4^{\frac{1}{3}} \sqrt{3} (-1)^{\frac{1}{3}} \arctan \left(\frac{4^{\frac{1}{3}} (6 - 4^{\frac{1}{3}} \sqrt{3} (-1)^{\frac{1}{3}} (19x^3 - 16x^2 + x^2)(-x^3 + 1)^{\frac{1}{3}} - 12 \sqrt{3} (-1)^{\frac{1}{3}} (5x^2 + 4x^2 - x)(-x^3 + 1)^{\frac{1}{3}} - 4^{\frac{1}{3}} \sqrt{3} (71x^9 - 111x^6 + 33x^3 - 1))}{6(109x^9 - 105x^6 + 3x^3 + 1)}} \right) + \frac{1}{36} \cdot 4^{\frac{1}{3}} (-1)^{\frac{1}{3}} \log \left(\frac{3 \cdot 4^{\frac{1}{3}} (-1)^{\frac{1}{3}} (-x^3 + 1)^{\frac{1}{3}} x^2 - 4^{\frac{1}{3}} (-1)^{\frac{1}{3}} (x^3 + 1) - 6(-x^3 + 1)^{\frac{1}{3}}}{x^3 + 1}} \right) - \frac{1}{72} \cdot 4^{\frac{1}{3}} (-1)^{\frac{1}{3}} \log \left(\frac{6 \cdot 4^{\frac{1}{3}} (-1)^{\frac{1}{3}} (5x^4 - x)(-x^3 + 1)^{\frac{1}{3}} - 4^{\frac{1}{3}} (-1)^{\frac{1}{3}} (19x^6 - 16x^3 + 1) - 24(2x^5 - x^2)(-x^3 + 1)^{\frac{1}{3}}}{x^6 + 2x^3 + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/18*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(-1)^(2/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - 12*sqrt(3)*(-1)^(1/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1) + 1/36*4^(2/3)*(-1)^(1/3)*log(-3*4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(1/3)*(-1)^(2/3)*(x^3 + 1) - 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1) - 1/72*4^(2/3)*(-1)^(1/3)*log((6*4^(1/3)*(-1)^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(19*x^6 - 16*x^3 + 1) - 24*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x^3 + 1)(-x^3 + 1)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)

maple [C] time = 4.24, size = 938, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^3+1)^(2/3)/(x^3+1),x)

[Out] 1/6*RootOf(_Z^3+2)*ln((54*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^3*x^3+6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^4*x^3-12*(-x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x-27*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3-3*x^3*RootOf(_Z^3+2)^2+6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*(-x^3+1)^(1/3)*x^2-4*RootOf(_Z^3+2)*(-x^3+1)^(1/3)*x^2+5*(-x^3+1)^(2/3)*x+9*RootOf(RootOf(_Z^3+2)^2+6*_Z

*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)+RootOf(_Z^3+2)^2)/(x+1)/(x^2-x+1))-
 1/6*ln(-(72*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_
 Z^3+2)^3*x^3+18*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(
 (_Z^3+2)^4*x^3-24*(-x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6
 *_Z*RootOf(_Z^3+2)+36*_Z^2)*x+12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2
)+36*_Z^2)*RootOf(_Z^3+2)*x^3+3*x^3*RootOf(_Z^3+2)^2-60*RootOf(RootOf(_Z^3+
 2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*(-x^3+1)^(1/3)*x^2-8*RootOf(_Z^3+2)*(-x^3
 +1)^(1/3)*x^2-2*(-x^3+1)^(2/3)*x-12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^
 3+2)+36*_Z^2)*RootOf(_Z^3+2)-3*RootOf(_Z^3+2)^2)/(x+1)/(x^2-x+1))*RootOf(_Z
 ^3+2)-ln(-(72*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(
 (_Z^3+2)^3*x^3+18*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*Root
 Of(_Z^3+2)^4*x^3-24*(-x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2
 +6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x+12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3
 +2)+36*_Z^2)*RootOf(_Z^3+2)*x^3+3*x^3*RootOf(_Z^3+2)^2-60*RootOf(RootOf(_Z^
 3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*(-x^3+1)^(1/3)*x^2-8*RootOf(_Z^3+2)*(-x
 ^3+1)^(1/3)*x^2-2*(-x^3+1)^(2/3)*x-12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_
 Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)-3*RootOf(_Z^3+2)^2)/(x+1)/(x^2-x+1))*RootOf(
 RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x^3 + 1)(-x^3 + 1)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(1 - x^3)^{\frac{2}{3}}(x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((1 - x^3)^(2/3)*(x^3 + 1)),x)

[Out] int(x/((1 - x^3)^(2/3)*(x^3 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(-(x - 1)(x^2 + x + 1))^{\frac{2}{3}}(x + 1)(x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

$$3.396 \quad \int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)} dx$$

Optimal. Leaf size=103

$$-\frac{\sqrt[3]{1-x^3}}{x} - \frac{\log(x^3+1)}{6 \cdot 2^{2/3}} + \frac{\log(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x)}{2 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 137, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {494, 453, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{1-x^3}}{x} - \frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{6 \cdot 2^{2/3}} + \frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{3 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] -((1 - x^3)^(1/3)/x) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(n-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x

$^2), x], x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 453

$\text{Int}[(e_.)*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})], x_Symbol] \rightarrow \text{Simp}[(c*(e*x)^{(m+1)}*(a + b*x^n)^{(p+1)})/(a*e^{(m+1)}), x] + \text{Dist}[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), \text{Int}[(e*x)^{(m+n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& (\text{IntegerQ}[n] \parallel \text{GtQ}[e, 0]) \&\& ((\text{GtQ}[n, 0] \&\& \text{LtQ}[m, -1]) \parallel (\text{LtQ}[n, 0] \&\& \text{GtQ}[m+n, -1])) \&\& !\text{ILtQ}[p, -1]$

Rule 494

$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}], x_Symbol] \rightarrow \text{With}[\{k = \text{Denominator}[p]\}, \text{Dist}[(k*a^{(p+(m+1)/n)})/n, \text{Subst}[\text{Int}[(x^{((k*(m+1)/n-1)*(c - (b*c - a*d)*x^k)})^{(p+q+(m+1)/n+1)}], x], x, x^{(n/k)}/(a + b*x^n)^{(1/k)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{RationalQ}[m, p] \&\& \text{IntegersQ}[p + (m+1)/n, q] \&\& \text{LtQ}[-1, p, 0]$

Rule 617

$\text{Int}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2]^{-1}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel !\text{RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_.) + (e_.)*(x_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[(d_.) + (e_.)*(x_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)} dx &= \text{Subst} \left(\int \frac{1+x^3}{x^2(1+2x^3)} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{\sqrt[3]{1-x^3}}{x} - \text{Subst} \left(\int \frac{x}{1+2x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\text{Subst} \left(\int \frac{1}{1+\sqrt[3]{2}x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} - \frac{\text{Subst} \left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} \\
&= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3 \cdot 2^{2/3}} - \frac{\text{Subst} \left(\int \frac{-\sqrt[3]{2}+2 \cdot 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{6 \cdot 2^{2/3}} - \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{2^{2/3}} \\
&= -\frac{\sqrt[3]{1-x^3}}{x} - \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6 \cdot 2^{2/3}} + \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3 \cdot 2^{2/3}} - \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{2^{2/3}} \\
&= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{2^{2/3}\sqrt{3}} - \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6 \cdot 2^{2/3}} + \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3 \cdot 2^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.55, size = 81, normalized size = 0.79

$$\frac{5(-3x^6 + x^3 + 2) {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{2x^3}{x^3-1}\right) - 12x^3(x^3 + 1) {}_2F_1\left(\frac{5}{3}, 2; \frac{8}{3}; \frac{2x^3}{x^3-1}\right)}{10x(1-x^3)^{5/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)), x]

[Out] -1/10*(5*(2 + x^3 - 3*x^6)*Hypergeometric2F1[2/3, 1, 5/3, (2*x^3)/(-1 + x^3)]) - 12*x^3*(1 + x^3)*Hypergeometric2F1[5/3, 2, 8/3, (2*x^3)/(-1 + x^3)]/(x*(1 - x^3)^(5/3))

IntegrateAlgebraic [A] time = 0.30, size = 143, normalized size = 1.39

$$-\frac{\sqrt[3]{1-x^3}}{x} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} + 2x\right)}{3 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2^{2/3}\sqrt{3}} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x - \sqrt[3]{2}(1-x^3)^{2/3} - 2x^2\right)}{6 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] $-\left(\frac{(1 - x^3)^{1/3}}{x} - \text{ArcTan}\left[\frac{\sqrt{3}x}{-x + 2^{2/3}(1 - x^3)^{1/3}}\right]\right) / \left(2^{2/3}\sqrt{3}\right) + \text{Log}\left[\frac{2x + 2^{2/3}(1 - x^3)^{1/3}}{3 \cdot 2^{2/3}}\right] - \text{Log}\left[\frac{-2x^2 + 2^{2/3}x(1 - x^3)^{1/3} - 2^{1/3}(1 - x^3)^{2/3}}{6 \cdot 2^{2/3}}\right]$

fricas [B] time = 1.94, size = 272, normalized size = 2.64

$$4 \cdot 4^{\frac{1}{2}} \sqrt{3} x \arctan\left(\frac{4^{\frac{1}{2}} \left(6 \cdot 4^{\frac{2}{3}} \sqrt{3} (19x^5 - 16x^4 + x^2)(-x^3 + 1)^{\frac{1}{3}} + 12 \sqrt{3} (5x^2 + 4x^4 - x)(-x^3 + 1)^{\frac{2}{3}} - 4^{\frac{1}{2}} \sqrt{3} (71x^9 - 111x^6 + 33x^3 - 1)\right)}{6(109x^9 - 105x^6 + 3x^3 + 1)}}\right) + 2 \cdot 4^{\frac{2}{3}} x \log\left(\frac{3 \cdot 4^{\frac{1}{2}} (-x^3 + 1)^{\frac{1}{3}} (2x^2 + 6(-x^3 + 1)^{\frac{2}{3}} x + 4^{\frac{1}{2}} (x^3 + 1))}{x^3 + 1}\right) - 4^{\frac{2}{3}} x \log\left(\frac{6 \cdot 4^{\frac{1}{2}} (5x^4 - x)(-x^3 + 1)^{\frac{2}{3}} + 4^{\frac{1}{2}} (19x^6 - 16x^3 + 1) - 24(2x^5 - x^2)(-x^3 + 1)^{\frac{1}{3}}}{x^6 + 2x^3 + 1}\right) - 72(-x^3 + 1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")

[Out] $\frac{1}{72} \cdot (4 \cdot 4^{1/6} \cdot \sqrt{3} \cdot x \cdot \arctan(1/6 \cdot 4^{1/6} \cdot (6 \cdot 4^{2/3} \cdot \sqrt{3} \cdot (19x^8 - 16x^5 + x^2) \cdot (-x^3 + 1)^{1/3} + 12 \cdot \sqrt{3} \cdot (5x^7 + 4x^4 - x) \cdot (-x^3 + 1)^{2/3} - 4^{1/3} \cdot \sqrt{3} \cdot (71x^9 - 111x^6 + 33x^3 - 1)) / (109x^9 - 105x^6 + 3x^3 + 1)) + 2 \cdot 4^{2/3} \cdot x \cdot \log((3 \cdot 4^{2/3} \cdot (-x^3 + 1)^{1/3} \cdot x^2 + 6 \cdot (-x^3 + 1)^{2/3} \cdot x + 4^{1/3} \cdot (x^3 + 1)) / (x^3 + 1)) - 4^{2/3} \cdot x \cdot \log((6 \cdot 4^{1/3} \cdot (5x^4 - x) \cdot (-x^3 + 1)^{2/3} + 4^{2/3} \cdot (19x^6 - 16x^3 + 1) - 24 \cdot (2x^5 - x^2) \cdot (-x^3 + 1)^{1/3}) / (x^6 + 2x^3 + 1)) - 72 \cdot (-x^3 + 1)^{1/3}) / x$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{2}{3}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)

maple [C] time = 4.06, size = 1387, normalized size = 13.47

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x)

[Out] $(x^3 - 1) / x / (-x^3 + 1)^{2/3} + (-1/6 \cdot \ln((18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 2)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 2) + 36 \cdot _Z^2)^2 \cdot \text{RootOf}(_Z^3 - 2)^2 \cdot x^6 + 6 \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 2)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 2) + 36 \cdot _Z^2) \cdot \text{RootOf}(_Z^3 - 2)^3 \cdot x^6 + 9 \cdot (x^6 - 2 \cdot x^3 + 1)^{2/3} \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 2)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 2) + 36 \cdot _Z^2) \cdot \text{RootOf}(_Z^3 - 2)^2 \cdot x^2 - 9 \cdot (x^6 - 2 \cdot x^3 + 1)^{1/3} \cdot \text{RootOf}(_Z^3 - 2) \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 2)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 2) + 36 \cdot _Z^2))$

```

*x^4-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-
2)^2*x^3-6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3
-2)^3*x^3-3*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^6-RootOf
(_Z^3-2)*x^6+3*(x^6-2*x^3+1)^(2/3)*x^2+9*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)
*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x+6*RootOf(RootOf(_Z^
3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+2*RootOf(_Z^3-2)*x^3-3*RootOf(RootO
f(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-RootOf(_Z^3-2))/(x-1)/(x^2+x+1)/(x
+1)/(x^2-x+1))*RootOf(_Z^3-2)-ln((18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z
^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^6+6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf
(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^6+9*(x^6-2*x^3+1)^(2/3)*RootOf(RootOf(
_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x^2-9*(x^6-2*x^3+1)
^(1/3)*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*
x^4-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)
)^2*x^3-6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-
2)^3*x^3-3*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^6-RootOf(
_Z^3-2)*x^6+3*(x^6-2*x^3+1)^(2/3)*x^2+9*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)*
RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x+6*RootOf(RootOf(_Z^3
-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+2*RootOf(_Z^3-2)*x^3-3*RootOf(RootO
f(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-RootOf(_Z^3-2))/(x-1)/(x^2+x+1)/(x+
1)/(x^2-x+1))*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+1/6*Root
Of(_Z^3-2)*ln(-(-36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*
RootOf(_Z^3-2)^2*x^6-12*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)
)*RootOf(_Z^3-2)^3*x^6+18*(x^6-2*x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*
RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x^2+3*(x^6-2*x^3+1)^(1/3)*RootOf(_
Z^3-2)^2*x^4+36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*Root
Of(_Z^3-2)^2*x^3+12*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*Ro
otOf(_Z^3-2)^3*x^3-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*
x^6-6*RootOf(_Z^3-2)*x^6-3*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x+24*RootOf
(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+8*RootOf(_Z^3-2)*x^3-6*R
ootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-2*RootOf(_Z^3-2))/(x-1)
/(x^2+x+1)/(x+1)/(x^2-x+1)))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{2}{3}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 (1 - x^3)^{2/3} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(1 - x^3)^(2/3)*(x^3 + 1)),x)`

[Out] `int(1/(x^2*(1 - x^3)^(2/3)*(x^3 + 1)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(-(x-1)(x^2+x+1) \right)^{2/3} (x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(-x**3+1)**(2/3)/(x**3+1),x)`

[Out] `Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)`

$$3.397 \quad \int \frac{1}{x^5(1-x^3)^{2/3}(1+x^3)} dx$$

Optimal. Leaf size=124

$$\frac{\sqrt[3]{1-x^3}}{4x} + \frac{\log(x^3+1)}{6 \cdot 2^{2/3}} - \frac{\log(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x)}{2 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\sqrt[3]{1-x^3}}{4x^4}$$

Rubi [A] time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {494, 461, 292, 31, 634, 617, 204, 628}

$$-\frac{(1-x^3)^{4/3}}{4x^4} + \frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{6 \cdot 2^{2/3}} - \frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{3 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] $-(1-x^3)^{4/3}/(4x^4) - \text{ArcTan}[(1 - (2 \cdot 2^{1/3}x)/(1-x^3)^{1/3})/\text{Sqrt}[3]]/(2^{2/3} \cdot \text{Sqrt}[3]) + \text{Log}[1 + (2^{2/3}x^2)/(1-x^3)^{2/3} - (2^{1/3}x)/(1-x^3)^{1/3}]/(6 \cdot 2^{2/3}) - \text{Log}[1 + (2^{1/3}x)/(1-x^3)^{1/3}]/(3 \cdot 2^{2/3})$

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^-1, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x

$\wedge 2), x], x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 461

$\text{Int}[\frac{(e \cdot x)^m \cdot (a + b \cdot x^n)^p}{(c + d \cdot x^n)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[\frac{(e \cdot x)^m \cdot (a + b \cdot x^n)^p}{(c + d \cdot x^n)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{IGtQ}[2 \cdot (m + 1), 0] \ || \ !\text{RationalQ}[m])$

Rule 494

$\text{Int}[(x^m \cdot (a + b \cdot x^n)^p \cdot (c + d \cdot x^n)^q), x_Symbol] := \text{With}[\{k = \text{Denominator}[p]\}, \text{Dist}[\frac{k \cdot a^{(p + (m + 1)/n)}}{n}, \text{Subst}[\text{Int}[(x^{((k \cdot (m + 1))/n - 1) \cdot (c - (b \cdot c - a \cdot d) \cdot x^k)^q) / (1 - b \cdot x^k)^{(p + q + (m + 1)/n + 1)}], x], x, x^{(n/k)} / (a + b \cdot x^n)^{(1/k)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{RationalQ}[m, p] \ \&\& \ \text{IntegersQ}[p + (m + 1)/n, q] \ \&\& \ \text{LtQ}[-1, p, 0]$

Rule 617

$\text{Int}[(a + b \cdot x + c \cdot x^2)^{-1}, x_Symbol] := \text{With}[\{q = 1 - 4 \cdot \text{Simplify}[(a \cdot c) / b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1 / (q - x^2)], x], x, 1 + (2 \cdot c \cdot x) / b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4 \cdot a \cdot c]) /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 628

$\text{Int}[(d + e \cdot x) / (a + b \cdot x + c \cdot x^2), x_Symbol] := \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rule 634

$\text{Int}[(d + e \cdot x) / (a + b \cdot x + c \cdot x^2), x_Symbol] := \text{Dist}[(2 \cdot c \cdot d - b \cdot e) / (2 \cdot c), \text{Int}[1 / (a + b \cdot x + c \cdot x^2), x], x] + \text{Dist}[e / (2 \cdot c), \text{Int}[(b + 2 \cdot c \cdot x) / (a + b \cdot x + c \cdot x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2 \cdot c \cdot d - b \cdot e, 0] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4 \cdot a \cdot c]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5 (1-x^3)^{2/3} (1+x^3)} dx &= \text{Subst} \left(\int \frac{(1+x^3)^2}{x^5 (1+2x^3)} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= \text{Subst} \left(\int \left(\frac{1}{x^5} + \frac{x}{1+2x^3} \right) dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{(1-x^3)^{4/3}}{4x^4} + \text{Subst} \left(\int \frac{x}{1+2x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= -\frac{(1-x^3)^{4/3}}{4x^4} - \frac{\text{Subst} \left(\int \frac{1}{1+\sqrt[3]{2}x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} + \frac{\text{Subst} \left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} \\
&= -\frac{(1-x^3)^{4/3}}{4x^4} - \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3 \cdot 2^{2/3}} + \frac{\text{Subst} \left(\int \frac{-\sqrt[3]{2}+2 \cdot 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{6 \cdot 2^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{2} \\
&= -\frac{(1-x^3)^{4/3}}{4x^4} + \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6 \cdot 2^{2/3}} - \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3 \cdot 2^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{2} \\
&= -\frac{(1-x^3)^{4/3}}{4x^4} - \frac{\tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{2^{2/3}\sqrt{3}} + \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6 \cdot 2^{2/3}} - \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3 \cdot 2^{2/3}}
\end{aligned}$$

Mathematica [C] time = 13.90, size = 145, normalized size = 1.17

$$\frac{81(x^3+1)^2 x^3 {}_4F_3\left(\frac{2}{3}, 2, 2, 2; 1, 1, \frac{8}{3}; \frac{2x^3}{x^3-1}\right) + 216(x^9+x^6) {}_3F_2\left(\frac{2}{3}, 2, 2; 1, \frac{8}{3}; \frac{2x^3}{x^3-1}\right) + 5\left((9x^9-20x^6-13x^3+4) {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{2x^3}{x^3-1}\right) + 9x^9+x^6-9x^3-1\right)}{60x^4(1-x^3)^{5/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)), x]

[Out] -1/60*(5*(-1 - 9*x^3 + x^6 + 9*x^9 + (4 - 13*x^3 - 20*x^6 + 9*x^9)*Hypergeometric2F1[2/3, 1, 5/3, (2*x^3)/(-1 + x^3)]) + 216*(x^6 + x^9)*HypergeometricPFQ[{2/3, 2, 2}, {1, 8/3}, (2*x^3)/(-1 + x^3)] + 81*x^3*(1 + x^3)^2*HypergeometricPFQ[{2/3, 2, 2, 2}, {1, 1, 8/3}, (2*x^3)/(-1 + x^3)])/(x^4*(1 - x^3)^(5/3))

IntegrateAlgebraic [A] time = 0.33, size = 149, normalized size = 1.20

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\cdot 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2^{2/3}\sqrt{3}}+\frac{\sqrt[3]{1-x^3}(x^3-1)}{4x^4}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}(1-x^3)^{2/3}-2x^2\right)}{6\cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] ((1 - x^3)^(1/3)*(-1 + x^3))/(4*x^4) + ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2^(2/3)*Sqrt[3]) - Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))

fricas [B] time = 1.86, size = 312, normalized size = 2.52

$$4\cdot 4^{1/3}\sqrt{3}\cdot (-1)^{1/3}x^4\arctan\left(\frac{4^{1/3}\sqrt{3}\sqrt{(19x^8-16x^5+x^2)(-x^3+1)^{1/3}-12\sqrt{3}(5x^7+4x^4-x)(-x^3+1)^{1/3}+4^3\sqrt{3}(71x^9-111x^6+33x^3-1)}}{4(109x^9-105x^6+3x^3+1)}\right)-2\cdot 4^{2/3}(-1)^{1/3}x^4\log\left(\frac{32^{1/3}(-1)^{1/3}(x^3+1)^{1/3}+2\cdot 4^{1/3}(-1)^{1/3}(x^3+1)^{1/3}-6(-x^3+1)^{1/3}}{x^3+1}\right)+4^{1/3}(-1)^{1/3}x^4\log\left(\frac{64^{1/3}(-1)^{1/3}(5x^4-x)(-x^3+1)^{1/3}-4^{1/3}(-1)^{1/3}(19x^8-16x^5+x^2)-24(2x^5-x^2)(-x^3+1)^{1/3}}{x^6+2x^3+1}\right)-18(x^3-1)(-x^3+1)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/72*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^4*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(-1)^(2/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - 12*sqrt(3)*(-1)^(1/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*4^(2/3)*(-1)^(1/3)*x^4*log((-3*4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(1/3)*(-1)^(2/3)*(x^3 + 1) - 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 4^(2/3)*(-1)^(1/3)*x^4*log((6*4^(1/3)*(-1)^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(19*x^6 - 16*x^3 + 1) - 24*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 18*(x^3 - 1)*(-x^3 + 1)^(1/3)/x^4

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{2}{3}}x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)

maple [C] time = 3.51, size = 1386, normalized size = 11.18

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x)`

[Out]
$$-1/4*(x^6-2*x^3+1)/x^4/(-x^3+1)^{(2/3)}+(-1/6*\ln((18*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)^2*\text{RootOf}(_Z^3+2)^2*x^6+6*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)^3*x^6-18*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)^2*x^3-6*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)^3*x^3-9*(x^6-2*x^3+1)^{(2/3)}*\text{RootOf}(_Z^3+2)^2*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^2-9*(x^6-2*x^3+1)^{(1/3)}*\text{RootOf}(_Z^3+2)*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^4+3*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^6+\text{RootOf}(_Z^3+2)*x^6+9*(x^6-2*x^3+1)^{(1/3)}*\text{RootOf}(_Z^3+2)*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x-6*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^3-2*\text{RootOf}(_Z^3+2)*x^3+3*(x^6-2*x^3+1)^{(2/3)}*x^2+3*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)+\text{RootOf}(_Z^3+2)))/(x-1)/(x^2+x+1)/(x+1)/(x^2-x+1))*\text{RootOf}(_Z^3+2)-\ln((18*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)^2*\text{RootOf}(_Z^3+2)^2*x^6+6*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)^3*x^6-18*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)^2*x^3-6*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)^3*x^3-9*(x^6-2*x^3+1)^{(2/3)}*\text{RootOf}(_Z^3+2)^2*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^2-9*(x^6-2*x^3+1)^{(1/3)}*\text{RootOf}(_Z^3+2)*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^4+3*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^6+\text{RootOf}(_Z^3+2)*x^6+9*(x^6-2*x^3+1)^{(1/3)}*\text{RootOf}(_Z^3+2)*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x-6*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^3-2*\text{RootOf}(_Z^3+2)*x^3+3*(x^6-2*x^3+1)^{(2/3)}*x^2+3*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)+\text{RootOf}(_Z^3+2)))/(x-1)/(x^2+x+1)/(x+1)/(x^2-x+1))*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)+1/6*\text{RootOf}(_Z^3+2)*\ln((36*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)^2*\text{RootOf}(_Z^3+2)^2*x^6+12*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)^3*x^6-36*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)^2*\text{RootOf}(_Z^3+2)^2*x^3-12*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*\text{RootOf}(_Z^3+2)^3*x^3+18*(x^6-2*x^3+1)^{(2/3)}*\text{RootOf}(_Z^3+2)^2*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^2-18*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^6-3*(x^6-2*x^3+1)^{(1/3)}*\text{RootOf}(_Z^3+2)^2*x^4-6*\text{RootOf}(_Z^3+2)*x^6+24*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)*x^3+3*(x^6-2*x^3+1)^{(1/3)}*\text{RootOf}(_Z^3+2)^2*x+8*\text{RootOf}(_Z^3+2)*x^3-6*\text{RootOf}(\text{RootOf}(_Z^3+2)^2+6*_Z*\text{RootOf}(_Z^3+2)+36*_Z^2)-2*\text{RootOf}(_Z^3+2)))/(x-1)/(x^2+x+1)/(x+1)/(x^2-x+1)))/(-x^3+1)^{(2/3)}*((x^3-1)^2)^{(1/3)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5 (1 - x^3)^{2/3} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(1 - x^3)^(2/3)*(x^3 + 1)),x)

[Out] int(1/(x^5*(1 - x^3)^(2/3)*(x^3 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(-(x-1)(x^2+x+1) \right)^{2/3} (x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(-x**3+1)**(2/3)/(x**3+1),x)

[Out] Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)

$$3.398 \quad \int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)} dx$$

Optimal. Leaf size=291

$$-\frac{1}{2} \sqrt[3]{1-x^3} x + \frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6 \cdot 2^{2/3}} - \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{6 \cdot 2^{2/3}} + \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{3 \cdot 2^{2/3}} - \frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{12 \cdot 2^{2/3}}$$

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {510}

$$\frac{1}{7} x^7 F_1\left(\frac{7}{3}; \frac{2}{3}, 1; \frac{10}{3}; x^3, -x^3\right)$$

Warning: Unable to verify antiderivative.

[In] Int[x^6/((1 - x^3)^(2/3)*(1 + x^3)), x]

[Out] (x^7*AppellF1[7/3, 2/3, 1, 10/3, x^3, -x^3])/7

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -(b*x^n)/a, -(d*x^n)/c]]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)} dx = \frac{1}{7} x^7 F_1\left(\frac{7}{3}; \frac{2}{3}, 1; \frac{10}{3}; x^3, -x^3\right)$$

Mathematica [C] time = 0.16, size = 115, normalized size = 0.40

$$\frac{1}{2} x \sqrt[3]{1-x^3} \left(-\frac{4F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; x^3, -x^3\right)}{(x^3+1)\left(x^3\left(3F_1\left(\frac{4}{3}; -\frac{1}{3}, 2; \frac{7}{3}; x^3, -x^3\right) + F_1\left(\frac{4}{3}; \frac{2}{3}, 1; \frac{7}{3}; x^3, -x^3\right)\right) - 4F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; x^3, -x^3\right)} - 1 \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] (x*(1 - x^3)^(1/3)*(-1 - (4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/3, 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3]))))/2

IntegrateAlgebraic [F] time = 11.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] Defer[IntegrateAlgebraic][x^6/((1 - x^3)^(2/3)*(1 + x^3)), x]

fricas [A] time = 2.52, size = 356, normalized size = 1.22

$$\frac{1}{30} \sqrt[3]{5} \arctan\left(\frac{4\sqrt[3]{6}\sqrt[3]{(x^6-33x^3+110x^0-110x^3+33x^6-x^9)^2-48\sqrt[3]{(x^6-2x^3+1)^2-4\sqrt[3]{(x^6+42x^3+417x^0+812x^3-417x^6+42x^9)}}}{6(x^6-102x^3+447x^0-628x^3+447x^6-102x^9)}}\right) - \frac{1}{22} \sqrt[3]{10} \log\left(\frac{12(x^3+1)^2x^3-3\sqrt[3]{(x^3+1)^2+4\sqrt[3]{(x^3+2x^6+1)}}}{x^6+2x^3+1}\right) - \frac{1}{144} \sqrt[3]{10} \log\left(\frac{24\sqrt[3]{(x^6-4x^3+1)^2+4\sqrt[3]{(x^6-32x^3+78x^0-32x^3+1)+12(x^6-11x^3-x^9)(x^3+1)}}}{x^6+4x^3+6x^0+4x^3+1}\right) - \frac{1}{2}(x^3+1)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")

[Out] 1/36*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 48*sqrt(3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/72*4^(2/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2 - 3*4^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(x^6 + 2*x^3 + 1)))/(x^6 + 2*x^3 + 1)) - 1/144*4^(2/3)*log((24*4^(1/3)*(x^8 - 4*x^5 + x^2)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 12*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3)))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1)) - 1/2*(-x^3 + 1)^(1/3)*x

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)

maple [C] time = 14.51, size = 696, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(-x^3+1)^(2/3)/(x^3+1),x)

[Out] $\frac{1}{2}x(x^3-1)/(-x^3+1)^{2/3} + (1/12\sqrt[3]{Z^3-2}\ln((6\sqrt[3]{Z^3-2})^2+3\sqrt[3]{Z^3-2}+9\sqrt[3]{Z^3-2})\sqrt[3]{Z^3-2}x^3+36\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2x^3-6\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2x^3-6\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2+3\sqrt[3]{Z^3-2}+9\sqrt[3]{Z^3-2})x^6-9(x^6-2x^3+1)^{2/3}\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2+3\sqrt[3]{Z^3-2}+9\sqrt[3]{Z^3-2})x^6-9(x^6-2x^3+1)^{2/3}\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2x-6(x^6-2x^3+1)^{1/3}\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2x^2+6\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}x^3+36\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2+3\sqrt[3]{Z^3-2}+9\sqrt[3]{Z^3-2})x^3-6(x^6-2x^3+1)^{2/3}\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}-6\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2+3\sqrt[3]{Z^3-2}+9\sqrt[3]{Z^3-2})/(x+1)^2/(x^2-x+1)^2+1/4\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2+3\sqrt[3]{Z^3-2}+9\sqrt[3]{Z^3-2})\ln(-(12\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2+3\sqrt[3]{Z^3-2}+9\sqrt[3]{Z^3-2})\sqrt[3]{Z^3-2}^3x^3+18\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2+3\sqrt[3]{Z^3-2}+9\sqrt[3]{Z^3-2})^2\sqrt[3]{Z^3-2}^2x^3+2\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}x^6+3\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2+3\sqrt[3]{Z^3-2}+9\sqrt[3]{Z^3-2})x^6-9(x^6-2x^3+1)^{2/3}\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2+3\sqrt[3]{Z^3-2}+9\sqrt[3]{Z^3-2})\sqrt[3]{Z^3-2}^2x-6(x^6-2x^3+1)^{1/3}\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2x^2-18\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}x^6-2x^3+1)^{1/3}\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2+3\sqrt[3]{Z^3-2}+9\sqrt[3]{Z^3-2})x^2-4\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}x^3-6\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2+3\sqrt[3]{Z^3-2}+9\sqrt[3]{Z^3-2})x^3+2\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}+3\sqrt[3]{Z^3-2}\sqrt[3]{Z^3-2}^2+3\sqrt[3]{Z^3-2}+9\sqrt[3]{Z^3-2})/(x+1)^2/(x^2-x+1)^2)/(-x^3+1)^{2/3}*((x^3-1)^2)^{1/3}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(x^3 + 1)(-x^3 + 1)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{(1-x^3)^{2/3}(x^3+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/((1 - x^3)^(2/3)*(x^3 + 1)),x)`

[Out] `int(x^6/((1 - x^3)^(2/3)*(x^3 + 1)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(- (x - 1) (x^2 + x + 1))^{\frac{2}{3}} (x + 1) (x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(-x**3+1)**(2/3)/(x**3+1),x)`

[Out] `Integral(x**6/((- (x - 1) * (x**2 + x + 1))** (2/3) * (x + 1) * (x**2 - x + 1)), x)`

$$3.399 \quad \int \frac{1}{x^3(1-x^3)^{2/3}(1+x^3)} dx$$

Optimal. Leaf size=294

$$\frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6 \cdot 2^{2/3}} + \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{6 \cdot 2^{2/3}} - \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{3 \cdot 2^{2/3}} + \frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}} + 2\sqrt[3]{2}\right)}{12 \cdot 2^{2/3}} \tan^{-1}$$

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {510}

$$-\frac{F_1\left(-\frac{2}{3}; \frac{2}{3}, 1; \frac{1}{3}; x^3, -x^3\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

[In] Int[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)), x]

[Out] -AppellF1[-2/3, 2/3, 1, 1/3, x^3, -x^3]/(2*x^2)

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -(b*x^n)/a, -((d*x^n)/c)]]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\int \frac{1}{x^3(1-x^3)^{2/3}(1+x^3)} dx = -\frac{F_1\left(-\frac{2}{3}; \frac{2}{3}, 1; \frac{1}{3}; x^3, -x^3\right)}{2x^2}$$

Mathematica [C] time = 0.10, size = 120, normalized size = 0.41

$$\frac{\sqrt[3]{1-x^3} \left(\frac{4x^3 F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; x^3, -x^3\right)}{\left(x^3+1\right)\left(x^3\left(3F_1\left(\frac{4}{3}; -\frac{1}{3}, 2; \frac{7}{3}; x^3, -x^3\right)+F_1\left(\frac{4}{3}; \frac{2}{3}, 1; \frac{7}{3}; x^3, -x^3\right)\right)-4F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; x^3, -x^3\right)}\right) - 1}{2x^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] ((1 - x^3)^(1/3)*(-1 + (4*x^3*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3]))/((1 + x^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/3, 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))

IntegrateAlgebraic [F] time = 34.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (1 - x^3)^{2/3} (1 + x^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] Defer[IntegrateAlgebraic][1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)), x]

fricas [A] time = 2.42, size = 396, normalized size = 1.35

$$4 \cdot 4^{1/3} \sqrt{3} (-1)^{1/3} \arctan\left(\frac{4^{1/3} \sqrt{3} (-1)^{1/3} (x^{18} - 102x^{15} + 447x^{12} - 628x^9 + 447x^6 - 102x^3 + 1)}{4^{1/3} \sqrt{3} (-1)^{1/3} (x^{18} - 102x^{15} + 447x^{12} - 628x^9 + 447x^6 - 102x^3 + 1)}\right) + 4^{1/3} (-1)^{1/3} \log\left(\frac{24 \cdot 4^{1/3} (-1)^{1/3} (x^8 - 4x^5 + x^2) (x^3 + 1)^{2/3} - 4^{2/3} (-1)^{1/3} (x^{12} - 32x^9 + 78x^6 - 32x^3 + 1) + 12(x^{10} - 11x^7 + 11x^4 - x)(-x^3 + 1)^{1/3}}{(x^{12} + 4x^9 + 6x^6 + 4x^3 + 1)}\right) - 2 \cdot 4^{1/3} (-1)^{1/3} \log\left(\frac{12(x^3 + 1)^{2/3} (x^6 + 2x^3 + 1) + 72(-x^3 + 1)^{1/3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/144*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^2*arctan(1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(-1)^(2/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) + 48*sqrt(3)*(-1)^(1/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1) + 4^(2/3)*(-1)^(1/3)*x^2*log((24*4^(1/3)*(-1)^(2/3)*(x^8 - 4*x^5 + x^2)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 12*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1) - 2*4^(2/3)*(-1)^(1/3)*x^2*log(-(12*(-x^3 + 1)^(2/3)*x^2 + 3*4^(2/3)*(-1)^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(-1)^(2/3)*(x^6 + 2*x^3 + 1))/(x^6 + 2*x^3 + 1) + 72*(-x^3 + 1)^(1/3))/x^2

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{2/3} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)

maple [C] time = 14.43, size = 991, normalized size = 3.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x)

[Out] $\frac{1}{2} \frac{(x^3-1)}{x^2} (-x^3+1)^{-2/3} + (-1/12 \ln((6 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2}} + 9 \sqrt[3]{Z^2}) \sqrt[3]{Z^3+2}^3 x^3 - 18 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2})^2 \sqrt[3]{Z^3+2}^2 x^3 + \sqrt[3]{Z^3+2} x^6 - 3 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) x^6 + 9(x^6 - 2x^3 + 1)^{2/3} \sqrt[3]{\sqrt[3]{Z^3+2}}^2 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) x - 18 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) \sqrt[3]{\sqrt[3]{Z^3+2}}^2 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) x^6 - 2x^3 + 1)^{1/3} x^2 - 6 \sqrt[3]{\sqrt[3]{Z^3+2}} x^3 + 18 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) x^3 - 6(x^6 - 2x^3 + 1)^{2/3} x + \sqrt[3]{Z^3+2} - 3 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) / (x+1)^2 / (x^2 - x + 1)^2 \sqrt[3]{Z^3+2} - 1/4 \ln((6 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2}} + 9 \sqrt[3]{Z^2}) \sqrt[3]{Z^3+2}^3 x^3 - 18 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2})^2 \sqrt[3]{Z^3+2}^2 x^3 + \sqrt[3]{Z^3+2} x^6 - 3 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) x^6 + 9(x^6 - 2x^3 + 1)^{2/3} \sqrt[3]{\sqrt[3]{Z^3+2}}^2 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) x - 18 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) \sqrt[3]{\sqrt[3]{Z^3+2}} (x^6 - 2x^3 + 1)^{1/3} x^2 - 6 \sqrt[3]{\sqrt[3]{Z^3+2}} x^3 + 18 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) x^3 - 6(x^6 - 2x^3 + 1)^{2/3} x + \sqrt[3]{Z^3+2} - 3 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) / (x+1)^2 / (x^2 - x + 1)^2 \sqrt[3]{\sqrt[3]{Z^3+2}}^2 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) + 1/12 \sqrt[3]{\sqrt[3]{Z^3+2}} \ln((6 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2}} + 9 \sqrt[3]{Z^2}) \sqrt[3]{Z^3+2}^3 x^3 + 36 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2})^2 \sqrt[3]{\sqrt[3]{Z^3+2}}^2 x^3 - \sqrt[3]{\sqrt[3]{Z^3+2}} x^6 - 6 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) x^6 - 9(x^6 - 2x^3 + 1)^{2/3} \sqrt[3]{\sqrt[3]{Z^3+2}}^2 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) x - 6 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) x^3 - \sqrt[3]{\sqrt[3]{Z^3+2}} - 6 \sqrt[3]{\sqrt[3]{Z^3+2}} + 3 \sqrt[3]{Z} \sqrt[3]{Z^3+2} + 9 \sqrt[3]{Z^2}) / (x+1)^2 / (x^2 - x + 1)^2) / (-x^3+1)^{2/3} * ((x^3-1)^2)^{1/3}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{2}{3}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 (1 - x^3)^{2/3} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(1 - x^3)^(2/3)*(x^3 + 1)), x)

[Out] int(1/(x^3*(1 - x^3)^(2/3)*(x^3 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(-(x-1)(x^2+x+1) \right)^{2/3} (x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(-x**3+1)**(2/3)/(x**3+1), x)

[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)

$$3.400 \quad \int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)} dx$$

Optimal. Leaf size=141

$$\frac{1}{8}(1-x^3)^{8/3} - \frac{2}{5}(1-x^3)^{5/3} + (1-x^3)^{2/3} + \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {446, 87, 43, 627, 51, 55, 617, 204, 31}

$$\frac{1}{8}(1-x^3)^{8/3} - \frac{2}{5}(1-x^3)^{5/3} + (1-x^3)^{2/3} + \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3) - (2*(1 - x^3)^(5/3))/5 + (1 - x^3)^(8/3)/8 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^(m)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 627

```
Int[(((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^4}{(1-x)^{4/3}(1+x)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{1}{\sqrt[3]{1-x}} - \frac{x^2}{\sqrt[3]{1-x}} + \frac{1}{\sqrt[3]{1-x}(1-x^2)} \right) dx, x, x^3 \right) \\
&= \frac{1}{2} (1-x^3)^{2/3} - \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{\sqrt[3]{1-x}} dx, x, x^3 \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1-x^2)} dx, x, x^3 \right) \\
&= \frac{1}{2} (1-x^3)^{2/3} - \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{\sqrt[3]{1-x}} - 2(1-x)^{2/3} + (1-x)^{5/3} \right) dx, x, x^3 \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1-x^2)} dx, x, x^3 \right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + (1-x^3)^{2/3} - \frac{2}{5} (1-x^3)^{5/3} + \frac{1}{8} (1-x^3)^{8/3} + \frac{1}{6} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + (1-x^3)^{2/3} - \frac{2}{5} (1-x^3)^{5/3} + \frac{1}{8} (1-x^3)^{8/3} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + (1-x^3)^{2/3} - \frac{2}{5} (1-x^3)^{5/3} + \frac{1}{8} (1-x^3)^{8/3} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{4\sqrt[3]{2}} \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + (1-x^3)^{2/3} - \frac{2}{5} (1-x^3)^{5/3} + \frac{1}{8} (1-x^3)^{8/3} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 53, normalized size = 0.38

$$\frac{20 {}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{1}{2}(1-x^3)\right) - 5x^9 - x^6 - 23x^3 + 29}{40\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^14/((1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] (29 - 23*x^3 - x^6 - 5*x^9 + 20*Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2])/(40*(1 - x^3)^(1/3))

IntegrateAlgebraic [A] time = 0.18, size = 156, normalized size = 1.11

$$\frac{\log\left(\frac{2^{2/3}\sqrt[3]{1-x^3}-2}{6\sqrt[3]{2}}\right) - \log\left(\frac{\sqrt[3]{2}(1-x^3)^{2/3} + 2^{2/3}\sqrt[3]{1-x^3} + 2}{12\sqrt[3]{2}}\right) + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{(1-x^3)^{2/3}(5x^9 + x^6 + 23x^3 - 49)}{40(x^3 - 1)}}{1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] $((1 - x^3)^{2/3}*(-49 + 23*x^3 + x^6 + 5*x^9))/(40*(-1 + x^3)) + \text{ArcTan}[1/\text{Sqrt}[3] + (2^{2/3}*(1 - x^3)^{1/3})/\text{Sqrt}[3]]/(2*2^{1/3}*\text{Sqrt}[3]) + \text{Log}[-2 + 2^{2/3}*(1 - x^3)^{1/3}]/(6*2^{1/3}) - \text{Log}[2 + 2^{2/3}*(1 - x^3)^{1/3} + 2^{1/3}*(1 - x^3)^{2/3}]/(12*2^{1/3})$

fricas [A] time = 0.47, size = 140, normalized size = 0.99

$$\frac{10\sqrt{6}2^{1/6}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{1/6}\left(\sqrt{6}2^{1/6}+2\sqrt{6}(-x^3+1)^{1/6}\right)\right)-5\cdot 2^{2/3}(x^3-1)\log\left(2^{2/3}+2^{1/3}(-x^3+1)^{1/3}+(-x^3+1)^{2/3}\right)+10\cdot 2^{2/3}(x^3-1)\log\left(-2^{1/3}+(-x^3+1)^{1/6}\right)+3(5x^9+x^6+23x^3-49)(-x^3+1)^{2/3}}{120(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")

[Out] $1/120*(10*\text{sqrt}(6)*2^{1/6}*(x^3 - 1)*\arctan(1/6*2^{1/6}*(\text{sqrt}(6)*2^{1/3} + 2*\text{sqrt}(6)*(-x^3 + 1)^{1/3})) - 5*2^{2/3}*(x^3 - 1)*\log(2^{2/3} + 2^{1/3}*(-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}) + 10*2^{2/3}*(x^3 - 1)*\log(-2^{1/3} + (-x^3 + 1)^{1/3}) + 3*(5*x^9 + x^6 + 23*x^3 - 49)*(-x^3 + 1)^{2/3})/(x^3 - 1)$

giac [A] time = 0.19, size = 136, normalized size = 0.96

$$\frac{1}{8}(x^3-1)^2(-x^3+1)^{2/3}+\frac{1}{12}\sqrt{3}2^{2/3}\arctan\left(\frac{1}{6}\sqrt{3}2^{2/3}\left(2^{1/3}+2(-x^3+1)^{1/3}\right)\right)-\frac{2}{5}(-x^3+1)^{5/3}-\frac{1}{24}\cdot 2^{2/3}\log\left(2^{2/3}+2^{1/3}(-x^3+1)^{1/3}+(-x^3+1)^{2/3}\right)+\frac{1}{12}\cdot 2^{2/3}\log\left(-2^{1/3}+(-x^3+1)^{1/3}\right)+(-x^3+1)^{2/3}+\frac{1}{2(-x^3+1)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")

[Out] $1/8*(x^3 - 1)^2*(-x^3 + 1)^{2/3} + 1/12*\text{sqrt}(3)*2^{2/3}*\arctan(1/6*\text{sqrt}(3)*2^{2/3}*(2^{1/3} + 2*(-x^3 + 1)^{1/3})) - 2/5*(-x^3 + 1)^{5/3} - 1/24*2^{2/3}*\log(2^{2/3} + 2^{1/3}*(-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}) + 1/12*2^{2/3}*\log(\text{abs}(-2^{1/3} + (-x^3 + 1)^{1/3})) + (-x^3 + 1)^{2/3} + 1/2/(-x^3 + 1)^{1/3}$

maple [C] time = 3.78, size = 682, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^14/(-x^3+1)^(4/3)/(x^3+1),x)

[Out] $-1/40*(5*x^9+x^6+23*x^3-49)/(-x^3+1)^{1/3}-1/12*\ln\left(\left(18*\text{RootOf}\left(\text{RootOf}\left(_Z^3-4\right)\right)^2+6*_Z*\text{RootOf}\left(_Z^3-4\right)+36*_Z^2\right)^2*\text{RootOf}\left(_Z^3-4\right)^2*x^3-12*\text{RootOf}\left(\text{RootOf}\left(_Z^3-4\right)\right)^2+6*_Z*\text{RootOf}\left(_Z^3-4\right)+36*_Z^2\right)*\text{RootOf}\left(_Z^3-4\right)^3*x^3+18*\text{RootOf}\left(\text{RootOf}\left(_Z^3-4\right)\right)^2+6*_Z*\text{RootOf}\left(_Z^3-4\right)+36*_Z^2\right)*x^3-12*\text{RootOf}\left(_Z^3-4\right)*x^3+21*\text{RootOf}\left(\text{RootOf}\left(_Z^3-4\right)\right)^2+6*_Z*\text{RootOf}\left(_Z^3-4\right)+36*_Z^2\right)$

$$\begin{aligned} & _Z^3-4)^2*(-x^3+1)^{(1/3)}+42*(-x^3+1)^{(2/3)}-42*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z* \\ & \text{RootOf}(_Z^3-4)+36*_Z^2)+28*\text{RootOf}(_Z^3-4))/(x+1)/(x^2-x+1))*\text{RootOf}(_Z^3-4)- \\ & 1/2*\ln((18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z \\ & ^3-4)^2*x^3-12*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z \\ & ^3-4)^3*x^3+18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-1 \\ & 2*\text{RootOf}(_Z^3-4)*x^3+21*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}+42*(-x^3+1)^{(2/3)}-4 \\ & 2*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)+28*\text{RootOf}(_Z^3-4))/(\\ & x+1)/(x^2-x+1))*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)+1/2*\text{Ro} \\ & \text{otOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\ln((18*\text{RootOf}(\text{RootOf}(_Z^ \\ & 3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+15*\text{RootOf}(\text{RootOf} \\ & (_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3-6*\text{RootOf}(\text{RootO} \\ & f(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-5*\text{RootOf}(_Z^3-4)*x^3+21*\text{RootOf} \\ & (_Z^3-4)^2*(-x^3+1)^{(1/3)}+42*(-x^3+1)^{(2/3)}+42*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z \\ & *\text{RootOf}(_Z^3-4)+36*_Z^2)+35*\text{RootOf}(_Z^3-4))/(x+1)/(x^2-x+1)) \end{aligned}$$

maxima [A] time = 1.07, size = 128, normalized size = 0.91

$$\frac{1}{8}(-x^3+1)^{\frac{8}{3}} + \frac{1}{12}\sqrt{3} \arctan\left(\frac{1}{6}\sqrt{3} \left(2^{\frac{2}{3}} + 2(-x^3+1)^{\frac{1}{3}}\right)\right) - \frac{2}{5}(-x^3+1)^{\frac{5}{3}} - \frac{1}{24} \cdot 2^{\frac{2}{3}} \log\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}} + (-x^3+1)^{\frac{2}{3}}\right) + \frac{1}{12} \cdot 2^{\frac{2}{3}} \log\left(-2^{\frac{1}{3}} + (-x^3+1)^{\frac{1}{3}}\right) + (-x^3+1)^{\frac{2}{3}} + \frac{1}{2(-x^3+1)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")

[Out] $1/8*(-x^3 + 1)^{(8/3)} + 1/12*\text{sqrt}(3)*2^{(2/3)}*\text{arctan}(1/6*\text{sqrt}(3)*2^{(2/3)}*(2^{(1/3)} + 2*(-x^3 + 1)^{(1/3)})) - 2/5*(-x^3 + 1)^{(5/3)} - 1/24*2^{(2/3)}*\log(2^{(2/3)} + 2^{(1/3)}*(-x^3 + 1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) + 1/12*2^{(2/3)}*\log(-2^{(1/3)} + (-x^3 + 1)^{(1/3)}) + (-x^3 + 1)^{(2/3)} + 1/2/(-x^3 + 1)^{(1/3)}$

mupad [B] time = 4.86, size = 148, normalized size = 1.05

$$\frac{2^{2/3} \ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}}{4}\right)}{12} + \frac{1}{2(1-x^3)^{1/3}} + (1-x^3)^{2/3} - \frac{2(1-x^3)^{5/3}}{5} + \frac{(1-x^3)^{8/3}}{8} + \frac{2^{2/3} \ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}(-1+\sqrt{3}1i)^2}{16}\right)(-1+\sqrt{3}1i)}{24} - \frac{2^{2/3} \ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}(1+\sqrt{3}1i)^2}{16}\right)(1+\sqrt{3}1i)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^14/((1 - x^3)^(4/3)*(x^3 + 1)),x)

[Out] $(2^{(2/3)}*\log((1 - x^3)^{(1/3)}/4 - 2^{(1/3)}/4))/12 + 1/(2*(1 - x^3)^{(1/3)}) + (1 - x^3)^{(2/3)} - (2*(1 - x^3)^{(5/3)})/5 + (1 - x^3)^{(8/3)}/8 + (2^{(2/3)}*\log((1 - x^3)^{(1/3)}/4 - (2^{(1/3)}*(3^{(1/2)}*1i - 1)^2)/16)*(3^{(1/2)}*1i - 1))/24 - (2^{(2/3)}*\log((1 - x^3)^{(1/3)}/4 - (2^{(1/3)}*(3^{(1/2)}*1i + 1)^2)/16)*(3^{(1/2)}*1i + 1))/24$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(- (x - 1) (x^2 + x + 1)\right)^{\frac{4}{3}} (x + 1) (x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**14/((-x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x  
)
```


$$3.401 \quad \int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)} dx$$

Optimal. Leaf size=130

$$-\frac{1}{5}(1-x^3)^{5/3} + \frac{1}{2}(1-x^3)^{2/3} + \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log(x^3+1)}{12\sqrt[3]{2}} - \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{4\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {446, 87, 43, 783, 78, 55, 617, 204, 31}

$$-\frac{1}{5}(1-x^3)^{5/3} + \frac{1}{2}(1-x^3)^{2/3} + \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log(x^3+1)}{12\sqrt[3]{2}} - \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{4\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 - (1 - x^3)^(5/3)/5 - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])]/; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x_)), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 783

```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3}{(1-x)^{4/3}(1+x)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{x}{\sqrt[3]{1-x}} - \frac{x}{\sqrt[3]{1-x}(-1+x^2)} \right) dx, x, x^3 \right) \\
&= -\left(\frac{1}{3} \text{Subst} \left(\int \frac{x}{\sqrt[3]{1-x}} dx, x, x^3 \right) \right) - \frac{1}{3} \text{Subst} \left(\int \frac{x}{\sqrt[3]{1-x}(-1+x^2)} dx, x, x^3 \right) \\
&= -\left(\frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{\sqrt[3]{1-x}} - (1-x)^{2/3} \right) dx, x, x^3 \right) \right) - \frac{1}{3} \text{Subst} \left(\int \frac{x}{(-1-x)(1-x)^{4/3}} dx, x, x^3 \right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}(1-x^3)^{2/3} - \frac{1}{5}(1-x^3)^{5/3} + \frac{1}{6} \text{Subst} \left(\int \frac{1}{(-1-x)\sqrt[3]{1-x}} dx, x, x^3 \right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}(1-x^3)^{2/3} - \frac{1}{5}(1-x^3)^{5/3} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2}x} dx, x, x^3 \right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}(1-x^3)^{2/3} - \frac{1}{5}(1-x^3)^{5/3} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{4\sqrt[3]{2}} + \frac{1}{6} \text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2}x} dx, x, x^3 \right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}(1-x^3)^{2/3} - \frac{1}{5}(1-x^3)^{5/3} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{4\sqrt[3]{2}} + \frac{1}{6} \text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2}x} dx, x, x^3 \right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.37

$$\frac{-5 {}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{1}{2}(1-x^3)\right) - 2x^6 - x^3 + 13}{10\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((1-x^3)^(4/3)*(1+x^3)),x]

[Out] (13 - x^3 - 2*x^6 - 5*Hypergeometric2F1[-1/3, 1, 2/3, (1-x^3)/2])/(10*(1-x^3)^(1/3))

IntegrateAlgebraic [A] time = 0.16, size = 151, normalized size = 1.16

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{12\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\left(1-x^3\right)^{2/3}\left(2x^6+x^3-8\right)}{10\left(x^3-1\right)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] $((1 - x^3)^{(2/3)}*(-8 + x^3 + 2*x^6))/(10*(-1 + x^3)) - \text{ArcTan}[1/\text{Sqrt}[3] + (2^{(2/3)}*(1 - x^3)^{(1/3)})/\text{Sqrt}[3]]/(2*2^{(1/3)}*\text{Sqrt}[3]) - \text{Log}[-2 + 2^{(2/3)}*(1 - x^3)^{(1/3)}]/(6*2^{(1/3)}) + \text{Log}[2 + 2^{(2/3)}*(1 - x^3)^{(1/3)} + 2^{(1/3)}*(1 - x^3)^{(2/3)}]/(12*2^{(1/3)})$

fricas [A] time = 0.45, size = 159, normalized size = 1.22

$$\frac{10\sqrt{6}2^{\frac{1}{3}}(-1)^{\frac{1}{3}}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{\frac{2}{3}}(2\sqrt{6}(-1)^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}})\right)+5\cdot 2^{\frac{5}{3}}(-1)^{\frac{1}{3}}(x^3-1)\log\left(2^{\frac{1}{3}}(-1)^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)-10\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^3-1)\log\left(-2^{\frac{1}{3}}(-1)^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)-12(2x^6+x^3-8)(-x^3+1)^{\frac{2}{3}}}{120(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")

[Out] $-1/120*(10*\text{sqrt}(6)*2^{(1/6)}*(-1)^{(1/3)}*(x^3 - 1)*\arctan(1/6*2^{(1/6)}*(2*\text{sqrt}(6)*(-1)^{(1/3)}*(-x^3 + 1)^{(1/3)} - \text{sqrt}(6)*2^{(1/3)})) + 5*2^{(2/3)}*(-1)^{(1/3)}*(x^3 - 1)*\log(2^{(1/3)}*(-1)^{(2/3)}*(-x^3 + 1)^{(1/3)} - 2^{(2/3)}*(-1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) - 10*2^{(2/3)}*(-1)^{(1/3)}*(x^3 - 1)*\log(-2^{(1/3)}*(-1)^{(2/3)} + (-x^3 + 1)^{(1/3)}) - 12*(2*x^6 + x^3 - 8)*(-x^3 + 1)^{(2/3)}/(x^3 - 1)$

giac [A] time = 0.18, size = 120, normalized size = 0.92

$$-\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)-\frac{1}{5}(-x^3+1)^{\frac{5}{3}}+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)+\frac{1}{2}(-x^3+1)^{\frac{2}{3}}+\frac{1}{2(-x^3+1)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")

[Out] $-1/12*\text{sqrt}(3)*2^{(2/3)}*\arctan(1/6*\text{sqrt}(3)*2^{(2/3)}*(2^{(1/3)} + 2*(-x^3 + 1)^{(1/3)})) - 1/5*(-x^3 + 1)^{(5/3)} + 1/24*2^{(2/3)}*\log(2^{(2/3)} + 2^{(1/3)}*(-x^3 + 1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) - 1/12*2^{(2/3)}*\log(\text{abs}(-2^{(1/3)} + (-x^3 + 1)^{(1/3)})) + 1/2*(-x^3 + 1)^{(2/3)} + 1/2/(-x^3 + 1)^{(1/3)}$

maple [C] time = 3.79, size = 497, normalized size = 3.82

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/(-x^3+1)^(4/3)/(x^3+1),x)

[Out] $-1/10*(2*x^6+x^3-8)/(-x^3+1)^{(1/3)}-1/12*\text{RootOf}(_Z^3-4)*\ln((-6*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3-45*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+2*\text{RootOf}(_Z^3-4)^2*x^3-8)/(-x^3+1)^{(1/3)})$

$Z^3-4)*x^3+15*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3+63*\text{RootOf}(_Z^3-4)*(-x^3+1)^{(1/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)+21*(-x^3+1)^{(2/3)}-14*\text{RootOf}(_Z^3-4)-105*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/2*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\ln((15*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3+72*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+15*\text{RootOf}(_Z^3-4)*x^3+72*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3+126*\text{RootOf}(_Z^3-4)*(-x^3+1)^{(1/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)+42*(-x^3+1)^{(2/3)}-35*\text{RootOf}(_Z^3-4)-168*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))$

maxima [A] time = 1.27, size = 119, normalized size = 0.92

$$-\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)-\frac{1}{5}(-x^3+1)^{\frac{5}{3}}+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)+\frac{1}{2}(-x^3+1)^{\frac{2}{3}}+\frac{1}{2(-x^3+1)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(-x³+1)^(4/3)/(x³+1), x, algorithm="maxima")

[Out] $-1/12*\text{sqrt}(3)*2^{(2/3)}*\arctan(1/6*\text{sqrt}(3)*2^{(2/3)}*(2^{(1/3)}+2*(-x^3+1)^{(1/3)}))-1/5*(-x^3+1)^{(5/3)}+1/24*2^{(2/3)}*\log(2^{(2/3)}+2^{(1/3)}*(-x^3+1)^{(1/3)}+(-x^3+1)^{(2/3)})-1/12*2^{(2/3)}*\log(-2^{(1/3)}+(-x^3+1)^{(1/3)})+1/2*(-x^3+1)^{(2/3)}+1/2/(-x^3+1)^{(1/3)}$

mupad [B] time = 5.15, size = 139, normalized size = 1.07

$$\frac{1}{2(1-x^3)^{1/3}}-\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{(1-x^3)^{2/3}}{2}-\frac{(1-x^3)^{5/3}}{5}-\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}(-1+\sqrt{3}1i)^2}{16}\right)(-1+\sqrt{3}1i)}{24}+\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}(1+\sqrt{3}1i)^2}{16}\right)(1+\sqrt{3}1i)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/((1-x³)^(4/3)*(x³+1)), x)

[Out] $1/(2*(1-x^3)^{(1/3)})-(2^{(2/3)}*\log((1-x^3)^{(1/3)}/4-2^{(1/3)}/4))/12+(1-x^3)^{(2/3)}/2-(1-x^3)^{(5/3)}/5-(2^{(2/3)}*\log((1-x^3)^{(1/3)}/4-(2^{(1/3)}*(3^{(1/2)}*1i-1)^2)/16)*(3^{(1/2)}*1i-1))/24+(2^{(2/3)}*\log((1-x^3)^{(1/3)}/4-(2^{(1/3)}*(3^{(1/2)}*1i+1)^2)/16)*(3^{(1/2)}*1i+1))/24$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(- (x-1) (x^2+x+1)\right)^{\frac{4}{3}} (x+1) (x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

$$3.402 \quad \int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)} dx$$

Optimal. Leaf size=115

$$\frac{1}{2}(1-x^3)^{2/3} + \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {446, 87, 627, 51, 55, 617, 204, 31}

$$\frac{1}{2}(1-x^3)^{2/3} + \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^8/((1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m+1)*(c + d*x)^(n+1))/((b*c - a*d)*(m+1)), x] - Dist[(d*(m+n+2))/((b*c - a*d)*(m+1)), Int[(a + b*x)^(m+1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntegerQ[n]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /;

FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 87

Int[((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 627

Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && IntegerQ[m + p]))

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(1-x)^{4/3}(1+x)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{1}{\sqrt[3]{1-x}} + \frac{1}{\sqrt[3]{1-x}(1-x^2)} \right) dx, x, x^3 \right) \\
&= \frac{1}{2} (1-x^3)^{2/3} + \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1-x^2)} dx, x, x^3 \right) \\
&= \frac{1}{2} (1-x^3)^{2/3} + \frac{1}{3} \text{Subst} \left(\int \frac{1}{(1-x)^{4/3}(1+x)} dx, x, x^3 \right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2} (1-x^3)^{2/3} + \frac{1}{6} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2} (1-x^3)^{2/3} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2}x + x^2} dx, x, \sqrt[3]{1-x^3} \right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2} (1-x^3)^{2/3} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{4\sqrt[3]{2}} - \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \sqrt[3]{1-x^3} \right)}{4\sqrt[3]{2}} \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2} (1-x^3)^{2/3} + \frac{\tan^{-1} \left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{4\sqrt[3]{2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.36

$$\frac{{}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{1}{2}(1-x^3)\right) - x^3 + 1}{2\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] (1 - x^3 + Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2])/(2*(1 - x^3)^(1/3))

IntegrateAlgebraic [A] time = 0.15, size = 146, normalized size = 1.27

$$\frac{(1-x^3)^{2/3}(x^3-2)}{2(x^3-1)} + \frac{\log(2^{2/3}\sqrt[3]{1-x^3}-2)}{6\sqrt[3]{2}} - \frac{\log(\sqrt[3]{2}(1-x^3)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2)}{12\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] $((1 - x^3)^{(2/3)}*(-2 + x^3))/(2*(-1 + x^3)) + \text{ArcTan}[1/\text{Sqrt}[3] + (2^{(2/3)}*(1 - x^3)^{(1/3)})/\text{Sqrt}[3]]/(2*2^{(1/3)}*\text{Sqrt}[3]) + \text{Log}[-2 + 2^{(2/3)}*(1 - x^3)^{(1/3)}]/(6*2^{(1/3)}) - \text{Log}[2 + 2^{(2/3)}*(1 - x^3)^{(1/3)} + 2^{(1/3)}*(1 - x^3)^{(2/3)}]/(12*2^{(1/3)})$

fricas [A] time = 0.47, size = 130, normalized size = 1.13

$$\frac{2\sqrt{6}^{\frac{1}{2}}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{3}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}(x^3-1)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)+2\cdot 2^{\frac{2}{3}}(x^3-1)\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)+12(x^3-2)(-x^3+1)^{\frac{2}{3}}}{24(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")

[Out] $1/24*(2*\text{sqrt}(6)*2^{(1/6)}*(x^3 - 1)*\arctan(1/6*2^{(1/6)}*(\text{sqrt}(6)*2^{(1/3)} + 2*\text{sqrt}(6)*(-x^3 + 1)^{(1/3)})) - 2^{(2/3)}*(x^3 - 1)*\log(2^{(2/3)} + 2^{(1/3)}*(-x^3 + 1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) + 2*2^{(2/3)}*(x^3 - 1)*\log(-2^{(1/3)} + (-x^3 + 1)^{(1/3)}) + 12*(x^3 - 2)*(-x^3 + 1)^{(2/3)})/(x^3 - 1)$

giac [A] time = 0.18, size = 109, normalized size = 0.95

$$\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)+\frac{1}{2}(-x^3+1)^{\frac{2}{3}}+\frac{1}{2(-x^3+1)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")

[Out] $1/12*\text{sqrt}(3)*2^{(2/3)}*\arctan(1/6*\text{sqrt}(3)*2^{(2/3)}*(2^{(1/3)} + 2*(-x^3 + 1)^{(1/3)})) - 1/24*2^{(2/3)}*\log(2^{(2/3)} + 2^{(1/3)}*(-x^3 + 1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) + 1/12*2^{(2/3)}*\log(\text{abs}(-2^{(1/3)} + (-x^3 + 1)^{(1/3)})) + 1/2*(-x^3 + 1)^{(2/3)} + 1/2/(-x^3 + 1)^{(1/3)}$

maple [C] time = 3.78, size = 672, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(-x^3+1)^(4/3)/(x^3+1),x)

[Out] $-1/2*(x^3-2)/(-x^3+1)^{(1/3)}+1/2*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\ln((18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+15*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3-6*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*$

$x^3-5*\text{RootOf}(_Z^3-4)*x^3+21*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}+42*(-x^3+1)^{(2/3)}+42*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)+35*\text{RootOf}(_Z^3-4)))/(x+1)/(x^2-x+1))-1/12*\ln((18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3-12*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3+18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-12*\text{RootOf}(_Z^3-4)*x^3+21*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}+42*(-x^3+1)^{(2/3)}-42*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)+28*\text{RootOf}(_Z^3-4)))/(x+1)/(x^2-x+1))*\text{RootOf}(_Z^3-4)-1/2*\ln((18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3-12*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3+18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-12*\text{RootOf}(_Z^3-4)*x^3+21*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}+42*(-x^3+1)^{(2/3)}-42*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)+28*\text{RootOf}(_Z^3-4)))/(x+1)/(x^2-x+1))*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)$

maxima [A] time = 1.27, size = 108, normalized size = 0.94

$$\frac{1}{12} \sqrt{3} 2^{\frac{2}{3}} \arctan\left(\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} + 2(-x^3+1)^{\frac{1}{3}}\right)\right) - \frac{1}{24} \cdot 2^{\frac{2}{3}} \log\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}} + (-x^3+1)^{\frac{2}{3}}\right) + \frac{1}{12} \cdot 2^{\frac{2}{3}} \log\left(-2^{\frac{1}{3}} + (-x^3+1)^{\frac{1}{3}}\right) + \frac{1}{2}(-x^3+1)^{\frac{2}{3}} + \frac{1}{2(-x^3+1)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1), x, algorithm="maxima")

[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2*(-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)

mupad [B] time = 4.89, size = 128, normalized size = 1.11

$$\frac{2^{2/3} \ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}}{4}\right)}{12} + \frac{1}{2(1-x^3)^{1/3}} + \frac{(1-x^3)^{2/3}}{2} + \frac{2^{2/3} \ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}(-1+\sqrt{3}1i)^2}{16}\right)(-1+\sqrt{3}1i)}{24} - \frac{2^{2/3} \ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}(1+\sqrt{3}1i)^2}{16}\right)(1+\sqrt{3}1i)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((1 - x^3)^(4/3)*(x^3 + 1)), x)

[Out] (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 + (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1i + 1))/24

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(- (x-1) (x^2+x+1)\right)^{\frac{4}{3}} (x+1) (x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

$$3.403 \quad \int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)} dx$$

Optimal. Leaf size=100

$$\frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log(x^3+1)}{12\sqrt[3]{2}} - \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{4\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {446, 78, 55, 617, 204, 31}

$$\frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log(x^3+1)}{12\sqrt[3]{2}} - \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{4\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^5/((1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] 1/(2*(1 - x^3)^(1/3)) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int

egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{(1-x)^{4/3}(1+x)} dx, x, x^3 \right) \\
 &= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{1}{6} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\
 &= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2}x + x^2} dx, x, \sqrt[3]{1-x^3} \right) + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 + 2^{2/3}\sqrt[3]{1-x^3} \right)}{2\sqrt[3]{2}} \\
 &= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 + 2^{2/3}\sqrt[3]{1-x^3} \right)}{2\sqrt[3]{2}} \\
 &= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 95, normalized size = 0.95

$$\frac{1}{24} \left(\frac{12}{\sqrt[3]{1-x^3}} + 2^{2/3} \log(x^3 + 1) - 3 \cdot 2^{2/3} \log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right) - 2 \cdot 2^{2/3} \sqrt{3} \tan^{-1}\left(\frac{2^{2/3} \sqrt[3]{1-x^3} + 1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] (12/(1 - x^3)^(1/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 2^(2/3)*Log[1 + x^3] - 3*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/24

IntegrateAlgebraic [A] time = 0.14, size = 141, normalized size = 1.41

$$\frac{(1-x^3)^{2/3}}{2(x^3-1)} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}(1-x^3)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{12\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] -1/2*(1 - x^3)^(2/3)/(-1 + x^3) - ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(12*2^(1/3))

fricas [B] time = 0.46, size = 148, normalized size = 1.48

$$\frac{2\sqrt{6}^{1/2}(-1)^{1/3}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{2/3}\left(2\sqrt{6}(-1)^{1/3}(-x^3+1)^{1/3}-\sqrt{6}2^{2/3}\right)\right)+2^{2/3}(-1)^{1/3}(x^3-1)\log\left(2^{1/3}(-1)^{2/3}(-x^3+1)^{1/3}-2^{2/3}(-1)^{1/3}+(-x^3+1)^{2/3}\right)-2\cdot 2^{2/3}(-1)^{1/3}(x^3-1)\log\left(-2^{1/3}(-1)^{2/3}+(-x^3+1)^{1/3}\right)+12(-x^3+1)^{2/3}}{24(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1), x, algorithm="fricas")

[Out] -1/24*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) - 2*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) + 12*(-x^3 + 1)^(2/3)/(x^3 - 1)

giac [A] time = 0.18, size = 98, normalized size = 0.98

$$-\frac{1}{12}\sqrt{3}2^{2/3}\arctan\left(\frac{1}{6}\sqrt{3}2^{2/3}\left(2^{1/3}+2(-x^3+1)^{1/3}\right)\right)+\frac{1}{24}\cdot 2^{2/3}\log\left(2^{2/3}+2^{1/3}(-x^3+1)^{1/3}+(-x^3+1)^{2/3}\right)-\frac{1}{12}\cdot 2^{2/3}\log\left(\left(-2^{1/3}+(-x^3+1)^{1/3}\right)\right)+\frac{1}{2(-x^3+1)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")

[Out] $-1/12*\sqrt{3}*2^{(2/3)}*\arctan(1/6*\sqrt{3}*2^{(2/3)}*(2^{(1/3)} + 2*(-x^3 + 1)^{(1/3)})) + 1/24*2^{(2/3)}*\log(2^{(2/3)} + 2^{(1/3)}*(-x^3 + 1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) - 1/12*2^{(2/3)}*\log(\text{abs}(-2^{(1/3)} + (-x^3 + 1)^{(1/3)})) + 1/2/(-x^3 + 1)^{(1/3)}$

maple [C] time = 3.76, size = 667, normalized size = 6.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(-x^3+1)^(4/3)/(x^3+1),x)

[Out] $1/2/(-x^3+1)^{(1/3)} - 1/2*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2) * \ln((18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+15*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3-6*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-5*\text{RootOf}(_Z^3-4)*x^3+21*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}+42*(-x^3+1)^{(2/3)}+42*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)+35*\text{RootOf}(_Z^3-4)))/(x+1)/(x^2-x+1)) + 1/12*\ln((18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3-12*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3+18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-12*\text{RootOf}(_Z^3-4)*x^3+21*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}+42*(-x^3+1)^{(2/3)}-42*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)+28*\text{RootOf}(_Z^3-4)))/(x+1)/(x^2-x+1))*\text{RootOf}(_Z^3-4) + 1/2*\ln((18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3-12*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3+18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-12*\text{RootOf}(_Z^3-4)*x^3+21*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}+42*(-x^3+1)^{(2/3)}-42*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)+28*\text{RootOf}(_Z^3-4)))/(x+1)/(x^2-x+1))*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)$

maxima [A] time = 1.53, size = 97, normalized size = 0.97

$$-\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)+\frac{1}{2(-x^3+1)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")

[Out] $-1/12*\sqrt{3}*2^{(2/3)}*\arctan(1/6*\sqrt{3}*2^{(2/3)}*(2^{(1/3)} + 2*(-x^3 + 1)^{(1/3)})) + 1/24*2^{(2/3)}*\log(2^{(2/3)} + 2^{(1/3)}*(-x^3 + 1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) - 1/12*2^{(2/3)}*\log(-2^{(1/3)} + (-x^3 + 1)^{(1/3)}) + 1/2/(-x^3 + 1)^{(1/3)}$

mupad [B] time = 4.85, size = 117, normalized size = 1.17

$$\frac{1}{2(1-x^3)^{1/3}} - \frac{2^{2/3} \ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}}{4}\right)}{12} - \frac{2^{2/3} \ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}(-1+\sqrt{3}1i)^2}{16}\right)(-1+\sqrt{3}1i)}{24} + \frac{2^{2/3} \ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}(1+\sqrt{3}1i)^2}{16}\right)(1+\sqrt{3}1i)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/((1 - x^3)^(4/3)*(x^3 + 1)), x)`

[Out] $\frac{1}{2(1-x^3)^{1/3}} - \frac{2^{2/3} \log((1-x^3)^{1/3}/4 - 2^{1/3}/4)}{12} - \left(\frac{2^{2/3} \log((1-x^3)^{1/3}/4 - (2^{1/3}(3^{1/2}1i - 1)^2)/16) \cdot (3^{1/2}1i - 1)}{24} + \frac{2^{2/3} \log((1-x^3)^{1/3}/4 - (2^{1/3}(3^{1/2}1i + 1)^2)/16) \cdot (3^{1/2}1i + 1)}{24} \right)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(- (x - 1) (x^2 + x + 1))^{4/3} (x + 1) (x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(-x**3+1)**(4/3)/(x**3+1), x)`

[Out] `Integral(x**5/((- (x - 1) (x**2 + x + 1))** (4/3) (x + 1) (x**2 - x + 1)), x)`

$$3.404 \quad \int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)} dx$$

Optimal. Leaf size=100

$$\frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {444, 51, 55, 617, 204, 31}

$$\frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] 1/(2*(1 - x^3)^(1/3)) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;

FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 444

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(1-x)^{4/3}(1+x)} dx, x, x^3 \right) \\
 &= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{6} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\
 &= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2}x + x^2} dx, x, \sqrt[3]{1-x^3} \right) - \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 + 2^{2/3}\sqrt[3]{1-x^3} \right)}{2\sqrt[3]{2}} \\
 &= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 + 2^{2/3}\sqrt[3]{1-x^3} \right)}{2\sqrt[3]{2}} \\
 &= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.34

$$\frac{{}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{1}{2}(1-x^3)\right)}{2\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2]/(2*(1 - x^3)^(1/3))

IntegrateAlgebraic [A] time = 0.12, size = 141, normalized size = 1.41

$$-\frac{(1-x^3)^{2/3}}{2(x^3-1)} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}(1-x^3)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{12\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] -1/2*(1 - x^3)^(2/3)/(-1 + x^3) + ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(12*2^(1/3))

fricas [A] time = 0.47, size = 125, normalized size = 1.25

$$\frac{2\sqrt{6}^{1/6}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{1/6}\left(\sqrt{6}2^{1/3}+2\sqrt{6}(-x^3+1)^{1/3}\right)\right)-2^{2/3}(x^3-1)\log\left(2^{2/3}+2^{1/3}(-x^3+1)^{1/3}+(-x^3+1)^{2/3}\right)+2\cdot 2^{2/3}(x^3-1)\log\left(-2^{1/3}+(-x^3+1)^{1/3}\right)-12(-x^3+1)^{2/3}}{24(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1), x, algorithm="fricas")

[Out] 1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 12*(-x^3 + 1)^(2/3))/(x^3 - 1)

giac [A] time = 0.19, size = 98, normalized size = 0.98

$$\frac{1}{12}\sqrt{3}2^{2/3}\arctan\left(\frac{1}{6}\sqrt{3}2^{2/3}\left(2^{1/3}+2(-x^3+1)^{1/3}\right)\right)-\frac{1}{24}\cdot 2^{2/3}\log\left(2^{2/3}+2^{1/3}(-x^3+1)^{1/3}+(-x^3+1)^{2/3}\right)+\frac{1}{12}\cdot 2^{2/3}\log\left(\left|-2^{1/3}+(-x^3+1)^{1/3}\right|\right)+\frac{1}{2(-x^3+1)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")

[Out] $\frac{1}{12}\sqrt{3} \cdot 2^{2/3} \cdot \arctan\left(\frac{1}{6}\sqrt{3} \cdot 2^{2/3} \cdot (2^{1/3} + 2 \cdot (-x^3 + 1)^{1/3})\right) - \frac{1}{24} \cdot 2^{2/3} \cdot \log(2^{2/3} + 2^{1/3} \cdot (-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}) + \frac{1}{12} \cdot 2^{2/3} \cdot \log(\text{abs}(-2^{1/3} + (-x^3 + 1)^{1/3})) + \frac{1}{2} \cdot (-x^3 + 1)^{-1/3}$

maple [C] time = 3.87, size = 667, normalized size = 6.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-x^3+1)^(4/3)/(x^3+1),x)

[Out] $\frac{1}{2} \cdot (-x^3 + 1)^{-1/3} + \frac{1}{2} \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 4) + 36 \cdot _Z^2) \cdot \ln\left(\frac{(18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 4) + 36 \cdot _Z^2)^2 \cdot \text{RootOf}(_Z^3 - 4)^2 \cdot x^3 + 15 \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 4) + 36 \cdot _Z^2) \cdot \text{RootOf}(_Z^3 - 4)^3 \cdot x^3 - 6 \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 4) + 36 \cdot _Z^2) \cdot x^3 - 5 \cdot \text{RootOf}(_Z^3 - 4) \cdot x^3 + 21 \cdot \text{RootOf}(_Z^3 - 4)^2 \cdot (-x^3 + 1)^{1/3} + 42 \cdot (-x^3 + 1)^{2/3} + 42 \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 4) + 36 \cdot _Z^2) + 35 \cdot \text{RootOf}(_Z^3 - 4)}{(x + 1) \cdot (x^2 - x + 1)} - \frac{1}{12} \cdot \ln\left(\frac{(18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 4) + 36 \cdot _Z^2)^2 \cdot \text{RootOf}(_Z^3 - 4)^2 \cdot x^3 - 12 \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 4) + 36 \cdot _Z^2) \cdot \text{RootOf}(_Z^3 - 4)^3 \cdot x^3 + 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 4) + 36 \cdot _Z^2) \cdot x^3 - 12 \cdot \text{RootOf}(_Z^3 - 4) \cdot x^3 + 21 \cdot \text{RootOf}(_Z^3 - 4)^2 \cdot (-x^3 + 1)^{1/3} + 42 \cdot (-x^3 + 1)^{2/3} - 42 \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 4) + 36 \cdot _Z^2) + 28 \cdot \text{RootOf}(_Z^3 - 4)}{(x + 1) \cdot (x^2 - x + 1)} \cdot \text{RootOf}(_Z^3 - 4) - \frac{1}{2} \cdot \ln\left(\frac{(18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 4) + 36 \cdot _Z^2)^2 \cdot \text{RootOf}(_Z^3 - 4)^2 \cdot x^3 - 12 \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 4) + 36 \cdot _Z^2) \cdot \text{RootOf}(_Z^3 - 4)^3 \cdot x^3 + 18 \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 4) + 36 \cdot _Z^2) \cdot x^3 - 12 \cdot \text{RootOf}(_Z^3 - 4) \cdot x^3 + 21 \cdot \text{RootOf}(_Z^3 - 4)^2 \cdot (-x^3 + 1)^{1/3} + 42 \cdot (-x^3 + 1)^{2/3} - 42 \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 4) + 36 \cdot _Z^2) + 28 \cdot \text{RootOf}(_Z^3 - 4)}{(x + 1) \cdot (x^2 - x + 1)} \cdot \text{RootOf}(\text{RootOf}(_Z^3 - 4)^2 + 6 \cdot _Z \cdot \text{RootOf}(_Z^3 - 4) + 36 \cdot _Z^2)\right)$

maxima [A] time = 1.34, size = 97, normalized size = 0.97

$$\frac{1}{12} \sqrt{3} 2^{2/3} \arctan\left(\frac{1}{6} \sqrt{3} 2^{2/3} \left(2^{1/3} + 2(-x^3 + 1)^{1/3}\right)\right) - \frac{1}{24} \cdot 2^{2/3} \log\left(2^{2/3} + 2^{1/3}(-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}\right) + \frac{1}{12} \cdot 2^{2/3} \log\left(-2^{1/3} + (-x^3 + 1)^{1/3}\right) + \frac{1}{2(-x^3 + 1)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")

[Out] $\frac{1}{12}\sqrt{3} \cdot 2^{2/3} \cdot \arctan\left(\frac{1}{6}\sqrt{3} \cdot 2^{2/3} \cdot (2^{1/3} + 2 \cdot (-x^3 + 1)^{1/3})\right) - \frac{1}{24} \cdot 2^{2/3} \cdot \log(2^{2/3} + 2^{1/3} \cdot (-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}) + \frac{1}{12} \cdot 2^{2/3} \cdot \log(-2^{1/3} + (-x^3 + 1)^{1/3}) + \frac{1}{2} \cdot (-x^3 + 1)^{-1/3}$

mupad [B] time = 4.90, size = 117, normalized size = 1.17

$$\frac{2^{2/3} \ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}}{4}\right)}{12} + \frac{1}{2(1-x^3)^{1/3}} + \frac{2^{2/3} \ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}(-1+\sqrt{3}i)^2}{16}\right)(-1+\sqrt{3}i)}{24} - \frac{2^{2/3} \ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}(1+\sqrt{3}i)^2}{16}\right)(1+\sqrt{3}i)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((1 - x^3)^(4/3)*(x^3 + 1)),x)`

[Out] $(2^{2/3} \log((1 - x^3)^{1/3}/4 - 2^{1/3}/4))/12 + 1/(2(1 - x^3)^{1/3}) + (2^{2/3} \log((1 - x^3)^{1/3}/4 - (2^{1/3} \cdot (3^{1/2} \cdot 1i - 1)^2)/16) \cdot (3^{1/2} \cdot 1i - 1))/24 - (2^{2/3} \log((1 - x^3)^{1/3}/4 - (2^{1/3} \cdot (3^{1/2} \cdot 1i + 1)^2)/16) \cdot (3^{1/2} \cdot 1i + 1))/24$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-x-1)(x^2+x+1)^{4/3}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-x**3+1)**(4/3)/(x**3+1),x)`

[Out] `Integral(x**2/((-x - 1)*(x**2 + x + 1)**(4/3)*(x + 1)*(x**2 - x + 1)), x)`

$$3.405 \quad \int \frac{1}{x(1-x^3)^{4/3}(1+x^3)} dx$$

Optimal. Leaf size=154

$$\frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {446, 85, 156, 55, 618, 204, 31, 617}

$$\frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] 1/(2*(1 - x^3)^(1/3)) + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(12*2^(1/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 85

Int[((e_.) + (f_.)*(x_))^(p)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)), x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e}

, f}, x] && LtQ[p, -1]

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(1-x)^{4/3}x(1+x)} dx, x, x^3 \right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{6} \text{Subst} \left(\int \frac{2+x}{\sqrt[3]{1-x}x(1+x)} dx, x, x^3 \right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{1}{6} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x}x} dx, x, x^3 \right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x)}{2} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{2^{2/3} + \sqrt[3]{2}x + x^2} dx, x, \sqrt[3]{1-x^3} \right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x)}{2} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} +
\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.35

$$\frac{{}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; 1-x^3\right) - {}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{1}{2}(1-x^3)\right)}{2\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] (-Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2] + 2*Hypergeometric2F1[-1/3, 1, 2/3, 1 - x^3])/(2*(1 - x^3)^(1/3))

IntegrateAlgebraic [A] time = 0.25, size = 219, normalized size = 1.42

$$-\frac{(1-x^3)^{2/3}}{2(x^3-1)} + \frac{1}{3} \log(\sqrt[3]{1-x^3}-1) - \frac{\log(2^{2/3}\sqrt[3]{1-x^3}-2)}{6\sqrt[3]{2}} - \frac{1}{6} \log((1-x^3)^{2/3} + \sqrt[3]{1-x^3} + 1) + \frac{\log(\sqrt[3]{2}(1-x^3)^{2/3} + 2^{2/3}\sqrt[3]{1-x^3} + 2)}{12\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] -1/2*(1 - x^3)^(2/3)/(-1 + x^3) + ArcTan[1/Sqrt[3] + (2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2

$$\begin{aligned} & \sqrt[3]{-1} \sqrt[3]{3} + \frac{\log[-1 + (1 - x^3)^{1/3}]}{3} - \frac{\log[-2 + 2^{2/3}(1 - x^3)^{1/3}]}{6} \\ & - \frac{\log[1 + (1 - x^3)^{1/3} + (1 - x^3)^{2/3}]}{6} + \frac{\log[2 + 2^{2/3}(1 - x^3)^{1/3} + 2^{1/3}(1 - x^3)^{2/3}]}{12} \end{aligned}$$

fricas [A] time = 0.49, size = 226, normalized size = 1.47

$$\frac{2\sqrt{2}(-1)^{\frac{1}{3}}(x^3-1)\arctan\left(\frac{1}{3}\sqrt[3]{2}\sqrt[3]{(-x^3+1)^2-\sqrt{6}x}\right)+2^{\frac{1}{3}}(-1)^{\frac{1}{3}}(x^3-1)\log\left(\frac{1}{3}\sqrt[3]{(-x^3+1)^2-2^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}}\right)-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^3-1)\log\left(-2^{\frac{1}{3}}(-1)^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)-8\sqrt{3}(x^3-1)\arctan\left(\frac{1}{3}\sqrt{3}\sqrt[3]{(-x^3+1)^2+\frac{1}{3}\sqrt{3}}\right)+4(x^3-1)\log\left((-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}+1\right)-8(x^3-1)\log\left((-x^3+1)^{\frac{1}{3}}-1\right)+12(-x^3+1)^{\frac{1}{3}}}{24(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -\frac{1}{24} \cdot (2\sqrt{6}) \cdot 2^{1/6} \cdot (-1)^{1/3} \cdot (x^3 - 1) \cdot \arctan\left(\frac{1}{6} \cdot 2^{1/6} \cdot (2\sqrt{6}) \cdot (-1)^{1/3} \cdot (-x^3 + 1)^{1/3} - \sqrt{6} \cdot 2^{1/3}\right) \\ & + 2^{2/3} \cdot (-1)^{1/3} \cdot (x^3 - 1) \cdot \log\left(2^{1/3} \cdot (-1)^{2/3} \cdot (-x^3 + 1)^{1/3} - 2^{2/3} \cdot (-1)^{1/3} + (-x^3 + 1)^{2/3}\right) \\ & - 2 \cdot 2^{2/3} \cdot (-1)^{1/3} \cdot (x^3 - 1) \cdot \log\left(-2^{1/3} \cdot (-1)^{2/3} + (-x^3 + 1)^{1/3}\right) \\ & - 8 \cdot \sqrt{3} \cdot (x^3 - 1) \cdot \arctan\left(\frac{2}{3} \cdot \sqrt{3} \cdot (-x^3 + 1)^{1/3} + \frac{1}{3} \cdot \sqrt{3}\right) \\ & + 4 \cdot (x^3 - 1) \cdot \log\left((-x^3 + 1)^{2/3} + (-x^3 + 1)^{1/3} + 1\right) - 8 \cdot (x^3 - 1) \cdot \log\left((-x^3 + 1)^{1/3} - 1\right) \\ & + 12 \cdot (-x^3 + 1)^{2/3} / (x^3 - 1) \end{aligned}$$

giac [A] time = 0.19, size = 160, normalized size = 1.04

$$-\frac{1}{12}\sqrt{3}^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{1}{3}}+2^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(-x^3+1)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2(-x^3+1)^{\frac{2}{3}}}-\frac{1}{6}\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|(-x^3+1)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")

[Out]
$$\begin{aligned} & -\frac{1}{12} \cdot \sqrt{3} \cdot 2^{2/3} \cdot \arctan\left(\frac{1}{6} \cdot \sqrt{3} \cdot 2^{2/3} \cdot (2^{1/3} + 2 \cdot (-x^3 + 1)^{1/3})\right) \\ & + \frac{1}{24} \cdot 2^{2/3} \cdot \log\left(2^{2/3} + 2^{1/3} \cdot (-x^3 + 1)^{1/3} + (-x^3 + 1)^{2/3}\right) \\ & - \frac{1}{12} \cdot 2^{2/3} \cdot \log\left(\left| -2^{1/3} + (-x^3 + 1)^{1/3} \right|\right) + \frac{1}{3} \cdot \sqrt{3} \cdot \arctan\left(\frac{1}{3} \cdot \sqrt{3} \cdot (2 \cdot (-x^3 + 1)^{1/3} + 1)\right) \\ & + \frac{1}{2 \cdot (-x^3 + 1)^{2/3}} - \frac{1}{6} \cdot \log\left((-x^3 + 1)^{2/3} + (-x^3 + 1)^{1/3} + 1\right) + \frac{1}{3} \cdot \log\left(\left| (-x^3 + 1)^{1/3} - 1 \right|\right) \end{aligned}$$

maple [F] time = 2.72, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^3 + 1)^{\frac{4}{3}} (x^3 + 1) x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-x^3+1)^(4/3)/(x^3+1),x)

[Out] int(1/x/(-x^3+1)^(4/3)/(x^3+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1), x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x), x)

mupad [B] time = 5.40, size = 253, normalized size = 1.64

$$\frac{\ln\left(\frac{2}{3} - \frac{2(1-x^3)^{1/3}}{3}\right) + \ln\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) \left(1458\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)^2 - \frac{459(1-x^3)^{1/3}}{4}\right) + \frac{63}{4} \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) - \ln\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) \left(1458\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)^2 - \frac{459(1-x^3)^{1/3}}{4}\right) + \frac{63}{4} \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) - \frac{2^{2/3} \ln\left(\frac{2^{1/3} \left(\frac{2(1-x^3)^{1/3}}{3} + \frac{2}{3}\right)}{12}\right) + \frac{2^{2/3}}{2(1-x^3)^{1/3}} + \frac{(-1)^{1/3} 2^{2/3} \ln\left(\frac{(-1)^{1/3} 2^{1/3} \left(\frac{2(1-x^3)^{1/3}}{3} + \frac{2}{3}\right)}{12}\right) - \frac{2^{2/3}}{4}}{12} - \frac{(-1)^{1/3} 2^{2/3} \ln\left(\frac{(-1)^{1/3} 2^{1/3} \left(\frac{2(1-x^3)^{1/3}}{3} + \frac{2}{3}\right)}{24}\right) - \frac{2^{2/3}}{4}}{24}}{(1 + \sqrt{3}i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(1 - x^3)^(4/3)*(x^3 + 1)), x)

[Out] log(17/4 - (17*(1 - x^3)^(1/3))/4)/3 + log(((3^(1/2)*1i)/6 - 1/6)*(1458*((3^(1/2)*1i)/6 - 1/6)^2 - (459*(1 - x^3)^(1/3))/4) - 63/4)*((3^(1/2)*1i)/6 - 1/6) - log(((3^(1/2)*1i)/6 + 1/6)*(1458*((3^(1/2)*1i)/6 + 1/6)^2 - (459*(1 - x^3)^(1/3))/4) + 63/4)*((3^(1/2)*1i)/6 + 1/6) - (2^(2/3)*log((2^(2/3)*((81*2^(1/3))/4 - (459*(1 - x^3)^(1/3))/4)))/12 + 63/4))/12 + 1/(2*(1 - x^3)^(1/3)) + ((-1)^(1/3)*2^(2/3)*log(((1)^(1/3)*2^(2/3)*((81*(-1)^(2/3)*2^(1/3))/4 - (459*(1 - x^3)^(1/3))/4)))/12 - 63/4))/12 - ((-1)^(1/3)*2^(2/3)*log(((1)^(1/3)*2^(2/3)*((3^(1/2)*1i + 1)*((459*(1 - x^3)^(1/3))/4 - (81*(-1)^(2/3)*2^(1/3)*((3^(1/2)*1i + 1)^2)/16)))/24 - 63/4)*(3^(1/2)*1i + 1))/24

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(-x-1)(x^2+x+1)^{\frac{4}{3}}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x**3+1)**(4/3)/(x**3+1), x)

[Out] Integral(1/(x*(-x - 1)*(x**2 + x + 1)**(4/3)*(x + 1)*(x**2 - x + 1)), x)

$$3.406 \quad \int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)} dx$$

Optimal. Leaf size=175

$$-\frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{5}{6\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{1}{6} \log\left(1 - \sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {446, 103, 156, 51, 55, 618, 204, 31, 617}

$$-\frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{5}{6\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{1}{6} \log\left(1 - \sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}-1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] 5/(6*(1 - x^3)^(1/3)) - 1/(3*x^3*(1 - x^3)^(1/3)) + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[x]/6 - Log[1 + x^3]/(12*2^(1/3)) + Log[1 - (1 - x^3)^(1/3)]/6 + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;

FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

Int[((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (1-x^3)^{4/3} (1+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(1-x)^{4/3} x^2 (1+x)} dx, x, x^3 \right) \\
&= -\frac{1}{3x^3 \sqrt[3]{1-x^3}} - \frac{1}{3} \text{Subst} \left(\int \frac{-\frac{1}{3} - \frac{4x}{3}}{(1-x)^{4/3} x (1+x)} dx, x, x^3 \right) \\
&= -\frac{1}{3x^3 \sqrt[3]{1-x^3}} + \frac{1}{9} \text{Subst} \left(\int \frac{1}{(1-x)^{4/3} x} dx, x, x^3 \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{(1-x)^{4/3} (1+x)} dx, x, x^3 \right) \\
&= \frac{5}{6 \sqrt[3]{1-x^3}} - \frac{1}{3x^3 \sqrt[3]{1-x^3}} + \frac{1}{9} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x} x} dx, x, x^3 \right) + \frac{1}{6} \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-x} (1+x)} dx, x, x^3 \right) \\
&= \frac{5}{6 \sqrt[3]{1-x^3}} - \frac{1}{3x^3 \sqrt[3]{1-x^3}} - \frac{\log(x)}{6} - \frac{\log(1+x^3)}{12 \sqrt[3]{2}} - \frac{1}{6} \text{Subst} \left(\int \frac{1}{1-x} dx, x, \sqrt[3]{1-x^3} \right) \\
&= \frac{5}{6 \sqrt[3]{1-x^3}} - \frac{1}{3x^3 \sqrt[3]{1-x^3}} - \frac{\log(x)}{6} - \frac{\log(1+x^3)}{12 \sqrt[3]{2}} + \frac{1}{6} \log \left(1 - \sqrt[3]{1-x^3} \right) + \frac{\log \left(\sqrt[3]{1-x^3} + 1 \right)}{6} \\
&= \frac{5}{6 \sqrt[3]{1-x^3}} - \frac{1}{3x^3 \sqrt[3]{1-x^3}} + \frac{\tan^{-1} \left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{\tan^{-1} \left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{6} - \frac{\log(1+x^3)}{12\sqrt[3]{2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 64, normalized size = 0.37

$$\frac{3x^3 {}_2F_1 \left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{1}{2} (1-x^3) \right) + 2x^3 {}_2F_1 \left(-\frac{1}{3}, 1; \frac{2}{3}; 1-x^3 \right) - 2}{6x^3 \sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(1-x^3)^(4/3)*(1+x^3)),x]

[Out] (-2 + 3*x^3*Hypergeometric2F1[-1/3, 1, 2/3, (1-x^3)/2] + 2*x^3*Hypergeometric2F1[-1/3, 1, 2/3, 1-x^3])/(6*x^3*(1-x^3)^(1/3))

IntegrateAlgebraic [A] time = 0.30, size = 232, normalized size = 1.33

$$\frac{(1-x^3)^{2/3} (2-5x^3)}{6x^3 (x^3-1)} + \frac{1}{9} \log(\sqrt[3]{1-x^3}-1) + \frac{\log(2^{2/3}\sqrt[3]{1-x^3}-2)}{6\sqrt[3]{2}} - \frac{1}{18} \log((1-x^3)^{2/3} + \sqrt[3]{1-x^3} + 1) - \frac{\log(\sqrt[3]{2}(1-x^3)^{2/3} + 2^{2/3}\sqrt[3]{1-x^3} + 2)}{12\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] $\frac{((2 - 5x^3)(1 - x^3)^{(2/3)})/(6x^3(-1 + x^3)) + \text{ArcTan}[1/\text{Sqrt}[3] + (2*(1 - x^3)^{(1/3)})/\text{Sqrt}[3]]/(3*\text{Sqrt}[3]) + \text{ArcTan}[1/\text{Sqrt}[3] + (2^{(2/3)}*(1 - x^3)^{(1/3)})/\text{Sqrt}[3]]/(2*2^{(1/3)}*\text{Sqrt}[3]) + \text{Log}[-1 + (1 - x^3)^{(1/3)}]/9 + \text{Log}[-2 + 2^{(2/3)}*(1 - x^3)^{(1/3)}]/(6*2^{(1/3)}) - \text{Log}[1 + (1 - x^3)^{(1/3)} + (1 - x^3)^{(2/3)}]/18 - \text{Log}[2 + 2^{(2/3)}*(1 - x^3)^{(1/3)} + 2^{(1/3)}*(1 - x^3)^{(2/3)}]/(12*2^{(1/3)})$

fricas [A] time = 0.46, size = 238, normalized size = 1.36

$$\frac{6\sqrt{6}2^{\frac{1}{2}}(x^6-x^3)\arctan\left(\frac{2}{3}\sqrt{6}2^{\frac{1}{2}}+2\sqrt{6}(-x^3+1)^{\frac{1}{2}}\right)-3\cdot 2^{\frac{1}{2}}(x^6-x^3)\log\left(2^{\frac{1}{2}}+2^{\frac{1}{2}}(-x^3+1)^{\frac{1}{2}}+(-x^3+1)^{\frac{1}{2}}\right)+6\cdot 2^{\frac{1}{2}}(x^6-x^3)\log\left(-2^{\frac{1}{2}}+(-x^3+1)^{\frac{1}{2}}\right)+8\sqrt{6}(x^6-x^3)\arctan\left(\frac{2}{3}\sqrt{6}(-x^3+1)^{\frac{1}{2}}+\frac{1}{3}\sqrt{6}\right)-4(x^6-x^3)\log\left((-x^3+1)^{\frac{1}{2}}+(-x^3+1)^{\frac{1}{2}}+1\right)+8(x^6-x^3)\log\left((-x^3+1)^{\frac{1}{2}}-1\right)-12(5x^3-2)(-x^3+1)^{\frac{1}{2}}}{72(x^6-x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")

[Out] $\frac{1}{72}*(6*\text{sqrt}(6)*2^{(1/6)}*(x^6 - x^3)*\arctan(1/6*2^{(1/6)}*(\text{sqrt}(6)*2^{(1/3)} + 2*\text{sqrt}(6)*(-x^3 + 1)^{(1/3)})) - 3*2^{(2/3)}*(x^6 - x^3)*\log(2^{(2/3)} + 2^{(1/3)}*(-x^3 + 1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) + 6*2^{(2/3)}*(x^6 - x^3)*\log(-2^{(1/3)} + (-x^3 + 1)^{(1/3)}) + 8*\text{sqrt}(3)*(x^6 - x^3)*\arctan(2/3*\text{sqrt}(3)*(-x^3 + 1)^{(1/3)} + 1/3*\text{sqrt}(3)) - 4*(x^6 - x^3)*\log((-x^3 + 1)^{(2/3)} + (-x^3 + 1)^{(1/3)} + 1) + 8*(x^6 - x^3)*\log((-x^3 + 1)^{(1/3)} - 1) - 12*(5*x^3 - 2)*(-x^3 + 1)^{(2/3)})/(x^6 - x^3)$

giac [A] time = 0.21, size = 181, normalized size = 1.03

$$\frac{1}{12}\sqrt{2}^{\frac{1}{2}}\arctan\left(\frac{1}{6}\sqrt{2}^{\frac{1}{2}}(2^{\frac{1}{2}}+2(-x^3+1)^{\frac{1}{2}})\right)-\frac{1}{24}\cdot 2^{\frac{1}{2}}\log\left(2^{\frac{1}{2}}+2^{\frac{1}{2}}(-x^3+1)^{\frac{1}{2}}+(-x^3+1)^{\frac{1}{2}}\right)+\frac{1}{12}\cdot 2^{\frac{1}{2}}\log\left(-2^{\frac{1}{2}}+(-x^3+1)^{\frac{1}{2}}\right)+\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2(-x^3+1)^{\frac{1}{2}}+1)\right)-\frac{5x^3-2}{6((-x^3+1)^{\frac{1}{2}}-(-x^3+1)^{\frac{1}{2}})}-\frac{1}{18}\log\left((-x^3+1)^{\frac{1}{2}}+(-x^3+1)^{\frac{1}{2}}+1\right)+\frac{1}{9}\log\left((-x^3+1)^{\frac{1}{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")

[Out] $\frac{1}{12}*\text{sqrt}(3)*2^{(2/3)}*\arctan(1/6*\text{sqrt}(3)*2^{(2/3)}*(2^{(1/3)} + 2*(-x^3 + 1)^{(1/3)})) - 1/24*2^{(2/3)}*\log(2^{(2/3)} + 2^{(1/3)}*(-x^3 + 1)^{(1/3)} + (-x^3 + 1)^{(2/3)}) + 1/12*2^{(2/3)}*\log(\text{abs}(-2^{(1/3)} + (-x^3 + 1)^{(1/3)})) + 1/9*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*(-x^3 + 1)^{(1/3)} + 1)) - 1/6*(5*x^3 - 2)/((-x^3 + 1)^{(4/3)} - (-x^3 + 1)^{(1/3)}) - 1/18*\log((-x^3 + 1)^{(2/3)} + (-x^3 + 1)^{(1/3)} + 1) + 1/9*\log(\text{abs}((-x^3 + 1)^{(1/3)} - 1))$

maple [F] time = 2.29, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^3 + 1)^{\frac{4}{3}} (x^3 + 1) x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x)`

[Out] `int(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")`

[Out] `integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^4), x)`

mupad [B] time = 5.02, size = 399, normalized size = 2.28

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(1 - x^3)^(4/3)*(x^3 + 1)),x)`

[Out] `log((11*(1 - x^3)^(1/3))/972 - 11/972)/9 + (2^(2/3)*log((2^(1/3)*((2^(2/3)*((81*2^(1/3))/4 - (75*(1 - x^3)^(1/3))/4))/12 - 35/12))/72 + (1 - x^3)^(1/3)/27))/12 + log(((3^(1/2)*1i)/18 - 1/18)^2*((3^(1/2)*1i)/18 - 1/18)*(1458*((3^(1/2)*1i)/18 - 1/18)^2 - (75*(1 - x^3)^(1/3))/4) - 35/12) + (1 - x^3)^(1/3)/27)*((3^(1/2)*1i)/18 - 1/18) - log((1 - x^3)^(1/3)/27 - ((3^(1/2)*1i)/18 + 1/18)^2*((3^(1/2)*1i)/18 + 1/18)*(1458*((3^(1/2)*1i)/18 + 1/18)^2 - (75*(1 - x^3)^(1/3))/4) + 35/12))*((3^(1/2)*1i)/18 + 1/18) + ((5*x^3)/6 - 1/3)/((1 - x^3)^(1/3) - (1 - x^3)^(4/3)) + (2^(2/3)*log((1 - x^3)^(1/3)/27 + (2^(1/3)*(3^(1/2)*1i - 1)^2*((2^(2/3)*(3^(1/2)*1i - 1)*((81*2^(1/3)*(3^(1/2)*1i - 1)^2)/16 - (75*(1 - x^3)^(1/3))/4))/24 - 35/12))/288)*(3^(1/2)*1i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/27 - (2^(1/3)*(3^(1/2)*1i + 1)^2*((2^(2/3)*(3^(1/2)*1i + 1)*((81*2^(1/3)*(3^(1/2)*1i + 1)^2)/16 - (75*(1 - x^3)^(1/3))/4))/24 + 35/12))/288)*(3^(1/2)*1i + 1))/24`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(-(x-1)(x^2+x+1) \right)^{\frac{4}{3}} (x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(-x**3+1)**(4/3)/(x**3+1),x)`

[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)

$$3.407 \quad \int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)} dx$$

Optimal. Leaf size=174

$$\frac{5}{6}(1-x^3)^{2/3} x + \frac{\log(x^3+1)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x\right)}{4\sqrt[3]{2}} - \frac{1}{6} \log\left(\sqrt[3]{1-x^3} + x\right) + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.15, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {510}

$$\frac{1}{10}x^{10}F_1\left(\frac{10}{3}; \frac{4}{3}, 1; \frac{13}{3}; x^3, -x^3\right)$$

Warning: Unable to verify antiderivative.

[In] Int[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] (x^10*AppellF1[10/3, 4/3, 1, 13/3, x^3, -x^3])/10

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1+(m+1)/n, -(b*x^n)/a, -(d*x^n)/c])/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n-1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)} dx = \frac{1}{10}x^{10}F_1\left(\frac{10}{3}; \frac{4}{3}, 1; \frac{13}{3}; x^3, -x^3\right)$$

Mathematica [C] time = 0.25, size = 152, normalized size = 0.87

$$\frac{1}{72} \left(-6x^4 F_1\left(\frac{4}{3}; \frac{1}{3}, 1; \frac{7}{3}; x^3, -x^3\right) - \frac{12(2x^3-5)x}{\sqrt[3]{1-x^3}} - 5 \cdot 2^{2/3} \left(2 \log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + 1\right) + 2\sqrt{3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} - 1}{\sqrt{3}}\right) - \log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + \frac{2^{2/3}x^2}{(x^3-1)^{2/3}} + 1\right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] $((-12*x*(-5 + 2*x^3))/(1 - x^3)^{(1/3)} - 6*x^4*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3] - 5*2^{(2/3)}*(2*sqrt[3]*ArcTan[(-1 + (2*2^{(1/3)}*x)/(-1 + x^3)^{(1/3)})/sqrt[3]] - Log[1 + (2^{(2/3)}*x^2)/(-1 + x^3)^{(2/3)} - (2^{(1/3)}*x)/(-1 + x^3)^{(1/3)}] + 2*Log[1 + (2^{(1/3)}*x)/(-1 + x^3)^{(1/3)}]))/72$

IntegrateAlgebraic [A] time = 0.57, size = 248, normalized size = 1.43

$$-\frac{1}{9} \log(\sqrt[3]{1-x^3} + x) - \frac{\log(2^{2/3}\sqrt[3]{1-x^3} + 2x)}{6\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}-x}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}-x}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{(1-x^3)^{2/3}(2x^4-5x)}{6(x^3-1)} + \frac{1}{18} \log(-\sqrt[3]{1-x^3}x + (1-x^3)^{2/3} + x^2) + \frac{\log(2^{2/3}\sqrt[3]{1-x^3}x - \sqrt[3]{2}(1-x^3)^{2/3} - 2x^2)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] $((1 - x^3)^{(2/3)}*(-5*x + 2*x^4))/(6*(-1 + x^3)) - ArcTan[(sqrt[3]*x)/(-x + 2*(1 - x^3)^{(1/3)})]/(3*sqrt[3]) - ArcTan[(sqrt[3]*x)/(-x + 2^{(2/3)}*(1 - x^3)^{(1/3)})]/(2*2^{(1/3)}*sqrt[3]) - Log[x + (1 - x^3)^{(1/3)}]/9 - Log[2*x + 2^{(2/3)}*(1 - x^3)^{(1/3)}]/(6*2^{(1/3)}) + Log[x^2 - x*(1 - x^3)^{(1/3)} + (1 - x^3)^{(2/3)}]/18 + Log[-2*x^2 + 2^{(2/3)}*x*(1 - x^3)^{(1/3)} - 2^{(1/3)}*(1 - x^3)^{(2/3)}]/(12*2^{(1/3)})$

fricas [B] time = 0.49, size = 271, normalized size = 1.56

$$\frac{6\sqrt{6}2^{\frac{1}{2}}(-1)^{\frac{1}{2}}(x^3-1)\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{2+2\sqrt[3]{1-x^3}}}{x}\right) + 6\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{2}}(x^3-1)\log\left(\frac{\sqrt[3]{1-x^3}\sqrt[3]{1-x^3}}{x}\right) - 3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{2}}(x^3-1)\log\left(\frac{\sqrt[3]{1-x^3}\sqrt[3]{2+2\sqrt[3]{1-x^3}}}{x}\right) + 8\sqrt{5}(x^3-1)\arctan\left(\frac{\sqrt{5+2\sqrt[3]{1-x^3}}}{3x}\right) - 8(x^3-1)\log\left(\frac{x+\sqrt[3]{1-x^3}}{x}\right) + 4(x^3-1)\log\left(\frac{x+\sqrt[3]{1-x^3}}{x}\right) + 12(2x^4-5x)(-x^3+1)^{\frac{2}{3}}}{72(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")

[Out] $1/72*(6*sqrt(6)*2^{(1/6)}*(-1)^{(1/3)}*(x^3 - 1)*arctan(1/6*2^{(1/6)}*(sqrt(6)*2^{(1/3)}*x + 2*sqrt(6)*(-1)^{(1/3)}*(-x^3 + 1)^{(1/3)})/x) + 6*2^{(2/3)}*(-1)^{(1/3)}*(x^3 - 1)*log((2^{(1/3)}*(-1)^{(2/3)}*x + (-x^3 + 1)^{(1/3)})/x) - 3*2^{(2/3)}*(-1)^{(1/3)}*(x^3 - 1)*log((-2^{(2/3)}*(-1)^{(1/3)}*x^2 + 2^{(1/3)}*(-1)^{(2/3)}*(-x^3 + 1)^{(1/3)}*x - (-x^3 + 1)^{(2/3)})/x^2) + 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^{(1/3)})/x) - 8*(x^3 - 1)*log((x + (-x^3 + 1)^{(1/3)})/x) + 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^{(1/3)}*x + (-x^3 + 1)^{(2/3)})/x^2) + 12*(2*x^4 - 5*x)*(-x^3 + 1)^{(2/3)}/(x^3 - 1)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)

maple [F] time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(-x^3 + 1)^{\frac{4}{3}} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(-x^3+1)^(4/3)/(x^3+1),x)

[Out] int(x^9/(-x^3+1)^(4/3)/(x^3+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{(1 - x^3)^{\frac{4}{3}} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/((1 - x^3)^(4/3)*(x^3 + 1)),x)

[Out] int(x^9/((1 - x^3)^(4/3)*(x^3 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(-(x - 1)(x^2 + x + 1))^{\frac{4}{3}} (x + 1)(x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**9/((-x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

$$3.408 \quad \int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)} dx$$

Optimal. Leaf size=153

$$\frac{x}{2\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{\log(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x)}{4\sqrt[3]{2}} - \frac{1}{2} \log(\sqrt[3]{1-x^3} + x) + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.17, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {510}

$$\frac{1}{7}x^7F_1\left(\frac{7}{3}; \frac{4}{3}, 1; \frac{10}{3}; x^3, -x^3\right)$$

Warning: Unable to verify antiderivative.

[In] Int[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] (x^7*AppellF1[7/3, 4/3, 1, 10/3, x^3, -x^3])/7

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -(b*x^n)/a, -(d*x^n)/c]]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)} dx = \frac{1}{7}x^7F_1\left(\frac{7}{3}; \frac{4}{3}, 1; \frac{10}{3}; x^3, -x^3\right)$$

Mathematica [C] time = 0.16, size = 142, normalized size = 0.93

$$\frac{1}{24} \left(-6x^4F_1\left(\frac{4}{3}; \frac{1}{3}, 1; \frac{7}{3}; x^3, -x^3\right) + \frac{12x}{\sqrt[3]{1-x^3}} + 2^{2/3} \left(-2 \log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + 1\right) - 2\sqrt{3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} - 1}{\sqrt{3}}\right) + \log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + \frac{2^{2/3}x^2}{(x^3-1)^{2/3}} + 1\right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] ((12*x)/(1 - x^3)^(1/3) - 6*x^4*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3] + 2^(2/3)*(-2*sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/sqrt[3]] + Log[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] - 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)]))/24

IntegrateAlgebraic [A] time = 0.46, size = 238, normalized size = 1.56

$$-\frac{(1-x^3)^{2/3}x}{2(x^3-1)} - \frac{1}{3} \log(\sqrt[3]{1-x^3}+x) + \frac{\log(2^{2/3}\sqrt[3]{1-x^3}+2x)}{6\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}-x}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{1}{6} \log(-\sqrt[3]{1-x^3}x+(1-x^3)^{2/3}+x^2) - \frac{\log(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}(1-x^3)^{2/3}-2x^2)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] -1/2*(x*(1 - x^3)^(2/3))/(-1 + x^3) - ArcTan[(sqrt[3]*x)/(-x + 2*(1 - x^3)^(1/3))]/sqrt[3] + ArcTan[(sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2*2^(1/3)*sqrt[3]) - Log[x + (1 - x^3)^(1/3)]/3 + Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6 - Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(12*2^(1/3))

fricas [B] time = 0.46, size = 239, normalized size = 1.56

$$\frac{2\sqrt{6}2^{2/3}(x^3-1)\arctan\left(\frac{2^{2/3}\sqrt{6}2^{1/6}\sqrt{-(x^3+1)^{1/2}}}{6x}\right) - 2\cdot 2^{2/3}(x^3-1)\log\left(\frac{2^{2/3}x+(-x^3+1)^{1/2}}{x}\right) + 2^{2/3}(x^3-1)\log\left(\frac{2^{2/3}x^2-2^{1/3}(-x^3+1)^{1/2}x+(-x^3+1)^{3/2}}{x^2}\right) - 8\sqrt{5}(x^3-1)\arctan\left(\frac{\sqrt{5}x-2\sqrt{-(x^3+1)^{1/2}}}{3x}\right) + 8(x^3-1)\log\left(\frac{x+(-x^3+1)^{1/2}}{x}\right) - 4(x^3-1)\log\left(\frac{x^2-(-x^3+1)^{1/2}x+(-x^3+1)^{3/2}}{x^2}\right) + 12(-x^3+1)^{3/2}x}{24(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(-1/6*2^(1/6)*(sqrt(6)*2^(1/3)*x - 2*sqrt(6)*(-x^3 + 1)^(1/3))/x) - 2*2^(2/3)*(x^3 - 1)*log((2^(1/3)*x + (-x^3 + 1)^(1/3))/x) + 2^(2/3)*(x^3 - 1)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2) - 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 8*(x^3 - 1)*log((x + (-x^3 + 1)^(1/3))/x) - 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2) + 12*(-x^3 + 1)^(2/3)*x/(x^3 - 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)

maple [F] time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(-x^3 + 1)^{\frac{4}{3}} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(-x^3+1)^(4/3)/(x^3+1),x)

[Out] int(x^6/(-x^3+1)^(4/3)/(x^3+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{(1 - x^3)^{\frac{4}{3}} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/((1 - x^3)^(4/3)*(x^3 + 1)),x)

[Out] int(x^6/((1 - x^3)^(4/3)*(x^3 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(-(x - 1)(x^2 + x + 1))^{\frac{4}{3}} (x + 1)(x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(-x**3+1)**(4/3)/(x**3+1),x)

[Out] Integral(x**6/((-x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)

$$3.409 \quad \int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)} dx$$

Optimal. Leaf size=106

$$\frac{x}{2\sqrt[3]{1-x^3}} + \frac{\log(x^3+1)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x\right)}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Rubi [C] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 0.36, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {510}

$$\frac{x^4 {}_2F_1\left(\frac{4}{3}, \frac{4}{3}; \frac{7}{3}; \frac{2x^3}{x^3+1}\right)}{4(x^3+1)^{4/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] (x^4*Hypergeometric2F1[4/3, 4/3, 7/3, (2*x^3)/(1 + x^3)]/(4*(1 + x^3)^(4/3)))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1+(m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)} dx = \frac{x^4 {}_2F_1\left(\frac{4}{3}, \frac{4}{3}; \frac{7}{3}; \frac{2x^3}{1+x^3}\right)}{4(1+x^3)^{4/3}}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.36

$$\frac{x^4 {}_2F_1\left(\frac{4}{3}, \frac{4}{3}; \frac{7}{3}; \frac{2x^3}{x^3+1}\right)}{4(x^3+1)^{4/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] (x^4*Hypergeometric2F1[4/3, 4/3, 7/3, (2*x^3)/(1 + x^3)]/(4*(1 + x^3)^(4/3)))

IntegrateAlgebraic [A] time = 0.36, size = 152, normalized size = 1.43

$$\frac{(1-x^3)^{2/3} x}{2(x^3-1)} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} + 2x\right)}{6\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2\sqrt[3]{2}\sqrt[3]{3}} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x - \sqrt[3]{2}(1-x^3)^{2/3} - 2x^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] -1/2*(x*(1 - x^3)^(2/3))/(-1 + x^3) - ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2*2^(1/3)*Sqrt[3]) - Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(12*2^(1/3))

fricas [B] time = 2.58, size = 318, normalized size = 3.00

$$\frac{2\sqrt[3]{2}(-1)^{\frac{1}{3}}(x^3-1)\arctan\left(\frac{2^{\frac{2}{3}}\sqrt[3]{2}\sqrt[3]{(9x^2+4x+1)^{\frac{2}{3}}-12\sqrt[3]{-1}}\sqrt[3]{(9x^2-16x^2+1)^{\frac{2}{3}}-12\sqrt[3]{-1}}\sqrt[3]{(71x^2-111x^2+33x^2-1)}}{6(109x^2-105x^2+3x^2+1)}}\right) - 2\sqrt[3]{-1}(x^3-1)\log\left(\frac{62^{\frac{1}{3}}(-1)^{\frac{1}{3}}(x^3+1)^{\frac{2}{3}}2^{\frac{1}{3}}(-1)^{\frac{1}{3}}(x^3+1)^{\frac{2}{3}}+6(-x^3+1)^{\frac{2}{3}}}{x^3+1}\right) + 2^{\frac{1}{3}}(-1)^{\frac{1}{3}}(x^3-1)\log\left(\frac{32^{\frac{1}{3}}(-1)^{\frac{1}{3}}(9x^2+4x+1)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{1}{3}}\sqrt[3]{(9x^2-16x^2+1)-12(2x^2+1)^{\frac{2}{3}}}}{x^2+1}\right) + 36(-x^3+1)^{\frac{1}{3}}x}{72(x^3-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/72*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log((-3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 36*(-x^3 + 1)^(2/3)*x)/(x^3 - 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)

maple [C] time = 4.23, size = 627, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(-x^3+1)^(4/3)/(x^3+1),x)

[Out] $\frac{1}{2}x/(-x^3+1)^{(1/3)} - \frac{1}{2}\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z\text{RootOf}(_Z^3-4)+36*_Z^2)*\ln(-(-9*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3-36*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+12*(-x^3+1)^{(2/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^2*x+4*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}*x^2+30*(-x^3+1)^{(1/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)*x^2+3*\text{RootOf}(_Z^3-4)*x^3+12*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^{(2/3)}*x-3*\text{RootOf}(_Z^3-4)-12*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z\text{RootOf}(_Z^3-4)+36*_Z^2))/((x+1)/(x^2-x+1)) - \frac{1}{12}\text{RootOf}(_Z^3-4)*\ln((-3*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3-27*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+6*(-x^3+1)^{(2/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^2*x+2*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}*x^2-3*(-x^3+1)^{(1/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)*x^2-3*\text{RootOf}(_Z^3-4)*x^3-27*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3+5*(-x^3+1)^{(2/3)}*x+\text{RootOf}(_Z^3-4)+9*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z\text{RootOf}(_Z^3-4)+36*_Z^2))/((x+1)/(x^2-x+1))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(1-x^3)^{4/3} (x^3+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((1 - x^3)^(4/3)*(x^3 + 1)),x)

[Out] int(x^3/((1 - x^3)^(4/3)*(x^3 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(- (x-1) (x^2+x+1))^{4/3} (x+1) (x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(-x**3+1)**(4/3)/(x**3+1),x)

[Out] Integral(x**3/((- (x - 1) * (x**2 + x + 1))** (4/3) * (x + 1) * (x**2 - x + 1)), x)

$$3.410 \quad \int \frac{1}{(1-x^3)^{4/3}(1+x^3)} dx$$

Optimal. Leaf size=106

$$\frac{x}{2\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{\log(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x)}{4\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Rubi [A] time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$\frac{x}{2\sqrt[3]{1-x^3}} - \frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{12\sqrt[3]{2}} + \frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] x/(2*(1 - x^3)^(1/3)) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(12*2^(1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^-1, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 382

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-x^3)^{4/3}(1+x^3)} dx &= \frac{x}{2\sqrt[3]{1-x^3}} + \frac{1}{2} \int \frac{1}{\sqrt[3]{1-x^3}(1+x^3)} dx \\
&= \frac{x}{2\sqrt[3]{1-x^3}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+2x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= \frac{x}{2\sqrt[3]{1-x^3}} + \frac{1}{6} \text{Subst} \left(\int \frac{1}{1+\sqrt[3]{2}x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{1}{6} \text{Subst} \left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= \frac{x}{2\sqrt[3]{1-x^3}} + \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\
&= \frac{x}{2\sqrt[3]{1-x^3}} - \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{12\sqrt[3]{2}} + \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{2\sqrt[3]{2}} \\
&= \frac{x}{2\sqrt[3]{1-x^3}} - \frac{\tan^{-1} \left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log \left(1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{12\sqrt[3]{2}} + \frac{\log \left(1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 100, normalized size = 0.94

$$\frac{-7(3x^3+4)(x^3-1)^2 {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{2x^3}{x^3-1}\right) - 12(x^9+x^6) {}_2F_1\left(2, \frac{7}{3}; \frac{10}{3}; \frac{2x^3}{x^3-1}\right) + 7(3x^3+4)(x^3-1)^2}{14x^2(1-x^3)^{7/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((1-x^3)^(4/3)*(1+x^3)),x]

[Out] (7*(-1+x^3)^2*(4+3*x^3) - 7*(-1+x^3)^2*(4+3*x^3)*Hypergeometric2F1[1/3, 1, 4/3, (2*x^3)/(-1+x^3)] - 12*(x^6+x^9)*Hypergeometric2F1[2, 7/3, 10/3, (2*x^3)/(-1+x^3)])/(14*x^2*(1-x^3)^(7/3))

IntegrateAlgebraic [A] time = 0.34, size = 152, normalized size = 1.43

$$\frac{(1-x^3)^{2/3}x}{2(x^3-1)} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{6\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{2}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}(1-x^3)^{2/3}-2x^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] $-1/2*(x*(1 - x^3)^{(2/3)})/(-1 + x^3) + \text{ArcTan}[(\text{Sqrt}[3]*x)/(-x + 2^{(2/3)}*(1 - x^3)^{(1/3)})]/(2*2^{(1/3)}*\text{Sqrt}[3]) + \text{Log}[2*x + 2^{(2/3)}*(1 - x^3)^{(1/3)}]/(6*2^{(1/3)}) - \text{Log}[-2*x^2 + 2^{(2/3)}*x*(1 - x^3)^{(1/3)} - 2^{(1/3)}*(1 - x^3)^{(2/3)}]/(12*2^{(1/3)})$

fricas [B] time = 2.60, size = 288, normalized size = 2.72

$$\frac{2\sqrt{2}^{\frac{1}{2}}(x^3-1)\arctan\left(\frac{2^{\frac{2}{3}}(6\sqrt{6}^{\frac{2}{3}}(5x^2+4x^4-x)(-x^2+1)^{\frac{1}{2}}-\sqrt{6}^{\frac{2}{3}}(71x^6-111x^4+33x^2-1)+12\sqrt{6}(19x^6-16x^4+x^2)(-x^2+1)^{\frac{1}{2}})}{6(109x^6-105x^4+3x^2+1)}\right)-2\cdot 2^{\frac{2}{3}}(x^3-1)\log\left(\frac{6\cdot 2^{\frac{2}{3}}(-x^3+1)^{\frac{1}{2}}x^2+2^{\frac{2}{3}}(x^3+1)+6(-x^3+1)^{\frac{3}{2}}x}{x^3+1}\right)+2^{\frac{2}{3}}(x^3-1)\log\left(\frac{3\cdot 2^{\frac{2}{3}}(5x^4-x)(-x^3+1)^{\frac{1}{2}}+2^{\frac{2}{3}}(19x^6-16x^4+x^2)-12(2x^5-x^2)(-x^3+1)^{\frac{1}{2}}}{x^6+2x^3+1}\right)+36(-x^3+1)^{\frac{1}{2}}x}{72(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")

[Out] $-1/72*(2*\text{sqrt}(6)*2^{(1/6)}*(x^3 - 1)*\arctan(1/6*2^{(1/6)}*(6*\text{sqrt}(6)*2^{(2/3)}*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^{(2/3)} - \text{sqrt}(6)*2^{(1/3)}*(71*x^9 - 111*x^6 + 33*x^3 - 1) + 12*\text{sqrt}(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^{(1/3)}))/(109*x^9 - 105*x^6 + 3*x^3 + 1) - 2*2^{(2/3)}*(x^3 - 1)*\log((6*2^{(1/3)}*(-x^3 + 1)^{(1/3)})*x^2 + 2^{(2/3)}*(x^3 + 1) + 6*(-x^3 + 1)^{(2/3)}*x)/(x^3 + 1) + 2^{(2/3)}*(x^3 - 1)*\log((3*2^{(2/3)}*(5*x^4 - x)*(-x^3 + 1)^{(2/3)} + 2^{(1/3)}*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^{(1/3)}))/(x^6 + 2*x^3 + 1) + 36*(-x^3 + 1)^{(2/3)}*x)/(x^3 - 1)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)

maple [C] time = 3.01, size = 944, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^3+1)^(4/3)/(x^3+1),x)

[Out] $1/2/(-x^3+1)^{(1/3)}*x+1/2*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4))+36*_Z^2)*\ln((9*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4))+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3+18*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4))+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+12*(-x^3+1)^{(2/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4))+36*_Z^2)*\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4))+36*_Z^2)$


```

6*_Z^2)*RootOf(_Z^3-4)^2*x-5*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)
)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)
*x^2+9*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*x^3+10*(-x^3+1)^(2/3)*x-3*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/12*ln(-(6*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2
/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x
+RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)
)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*
RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^(2/3)*x
+2*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(
x+1)/(x^2-x+1))*RootOf(_Z^3-4)-1/2*ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*Root
Of(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*Ro
otOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+RootOf(_Z^3-4)^
2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(
_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^(2/3)*x+2*RootOf(_Z^3-4)
)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*
RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(1 - x^3)^{4/3} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((1 - x^3)^(4/3)*(x^3 + 1)),x)

[Out] int(1/((1 - x^3)^(4/3)*(x^3 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x-1)(x^2+x+1)^{\frac{4}{3}}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**3+1)**(4/3)/(x**3+1), x)

[Out] Integral(1/((-x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)

$$3.411 \quad \int \frac{1}{x^3(1-x^3)^{4/3}(1+x^3)} dx$$

Optimal. Leaf size=124

$$\frac{\log(x^3+1)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x\right)}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{(1-x^3)^{2/3}}{x^2} + \frac{1}{2x^2\sqrt[3]{1-x^3}}$$

Rubi [C] time = 8.12, antiderivative size = 204, normalized size of antiderivative = 1.65, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {510}

$$\frac{-18(x^3+1)^2 x^6 {}_3F_2\left(2, 2, \frac{7}{3}; 1, \frac{10}{3}; -\frac{2x^3}{1-x^3}\right) - 54x^{12} {}_2F_1\left(2, \frac{7}{3}; \frac{10}{3}; -\frac{2x^3}{1-x^3}\right) - 84x^9 {}_2F_1\left(2, \frac{7}{3}; \frac{10}{3}; -\frac{2x^3}{1-x^3}\right) - 30x^6 {}_2F_1\left(2, \frac{7}{3}; \frac{10}{3}; -\frac{2x^3}{1-x^3}\right) - 7(1-x^3)^2 (9x^6 + 12x^3 + 2) {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; -\frac{2x^3}{1-x^3}\right) + 63x^{12} - 42x^9 - 91x^6 + 56x^3 + 14}{14x^5(1-x^3)^{7/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] -(14 + 56*x^3 - 91*x^6 - 42*x^9 + 63*x^12 - 7*(1 - x^3)^2*(2 + 12*x^3 + 9*x^6)*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 30*x^6*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] - 84*x^9*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] - 54*x^12*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] - 18*x^6*(1 + x^3)^2*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, (-2*x^3)/(1 - x^3)])/(14*x^5*(1 - x^3)^(7/3))

Rule 510

Int[((e._)*(x._))^(m._)*((a._) + (b._)*(x._)^(n._))^(p._)*((c._) + (d._)*(x._)^(n._))^(q._), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -(b*x^n)/a, -(d*x^n)/c]]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\int \frac{1}{x^3(1-x^3)^{4/3}(1+x^3)} dx = -\frac{14 + 56x^3 - 91x^6 - 42x^9 + 63x^{12} - 7(1-x^3)^2(2 + 12x^3 + 9x^6) {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; -\frac{2x^3}{1-x^3}\right)}{14x^5(1-x^3)^{7/3}}$$

Mathematica [C] time = 3.11, size = 192, normalized size = 1.55

$$\frac{18(x^3+1)^2 x^6 {}_3F_2\left(2, 2, \frac{7}{3}; 1, \frac{10}{3}; \frac{2x^3}{x^3-1}\right) + 54x^{12} {}_2F_1\left(2, \frac{7}{3}; \frac{10}{3}; \frac{2x^3}{x^3-1}\right) + 84x^9 {}_2F_1\left(2, \frac{7}{3}; \frac{10}{3}; \frac{2x^3}{x^3-1}\right) + 30x^6 {}_2F_1\left(2, \frac{7}{3}; \frac{10}{3}; \frac{2x^3}{x^3-1}\right) + 7(x^3-1)^2 (9x^6 + 12x^3 + 2) {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{2x^3}{x^3-1}\right) - 63x^{12} + 42x^9 + 91x^6 - 56x^3 - 14}{14x^5(1-x^3)^{7/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] (-14 - 56*x^3 + 91*x^6 + 42*x^9 - 63*x^12 + 7*(-1 + x^3)^2*(2 + 12*x^3 + 9*x^6)*Hypergeometric2F1[1/3, 1, 4/3, (2*x^3)/(-1 + x^3)] + 30*x^6*Hypergeometric2F1[2, 7/3, 10/3, (2*x^3)/(-1 + x^3)] + 84*x^9*Hypergeometric2F1[2, 7/3, 10/3, (2*x^3)/(-1 + x^3)] + 54*x^12*Hypergeometric2F1[2, 7/3, 10/3, (2*x^3)/(-1 + x^3)] + 18*x^6*(1 + x^3)^2*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, (2*x^3)/(-1 + x^3)]/(14*x^5*(1 - x^3)^(7/3))

IntegrateAlgebraic [A] time = 0.37, size = 161, normalized size = 1.30

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{6\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{(1-x^3)^{2/3}(1-2x^3)}{2x^2(x^3-1)} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}(1-x^3)^{2/3}-2x^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] ((1 - 2*x^3)*(1 - x^3)^(2/3))/(2*x^2*(-1 + x^3)) - ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2*2^(1/3)*Sqrt[3]) - Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(12*2^(1/3))

fricas [B] time = 2.60, size = 340, normalized size = 2.74

$$\frac{2\sqrt{2}^{\frac{1}{2}}(-1)^{\frac{1}{2}}(x^2-x^2)\arctan\left(\frac{\sqrt[3]{6}\sqrt[3]{-1}\sqrt[3]{5x^2+4x^2}\sqrt[3]{(-x^3)^{\frac{1}{3}}}-12\sqrt[3]{-1}\sqrt[3]{109x^9-16x^6x^2}\sqrt[3]{(-x^3)^{\frac{1}{3}}}-\sqrt[3]{2}\sqrt[3]{71x^9-111x^6x^2-1}}{\sqrt[3]{109x^9-105x^6x^2+1}}}\right)-2\sqrt[3]{-1}\sqrt[3]{-1}\sqrt[3]{x^2-x^2}\log\left(\frac{6\sqrt[3]{-1}\sqrt[3]{(-x^3)^{\frac{1}{3}}}\sqrt[3]{2}\sqrt[3]{(-x^3)^{\frac{1}{3}}}\sqrt[3]{(-x^3)^{\frac{1}{3}}}\sqrt[3]{(-x^3)^{\frac{1}{3}}}}{x^2}\right)+\sqrt[3]{-1}\sqrt[3]{-1}\sqrt[3]{x^2-x^2}\log\left(\frac{-3\sqrt[3]{-1}\sqrt[3]{6x^2+4x^2}\sqrt[3]{(-x^3)^{\frac{1}{3}}}\sqrt[3]{(-x^3)^{\frac{1}{3}}}\sqrt[3]{(-x^3)^{\frac{1}{3}}}\sqrt[3]{(-x^3)^{\frac{1}{3}}}}{x^2+2x^2+1}\right)+36(2x^3-1)(-x^2+1)^{\frac{1}{2}}}{72(x^3-x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/72*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^5 - x^2)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1) - 2*2^(2/3)*(-1)^(1/3)*(x^5 - x^2)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3)*(x^5 - x^2)*log(-3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(

$$\frac{1}{3}*(-1)^{(2/3)}*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^{(1/3)}/(x^6 + 2*x^3 + 1) + 36*(2*x^3 - 1)*(-x^3 + 1)^{(2/3)}/(x^5 - x^2)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)

maple [C] time = 2.92, size = 570, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x)

[Out] $\frac{1}{2}*(2*x^3-1)/x^2/(-x^3+1)^{(1/3)}-1/12*\text{RootOf}(_Z^3-4)*\ln((-3*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3-9*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)*x^2-3*\text{RootOf}(_Z^3-4)*x^3-9*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3+3*(-x^3+1)^{(2/3)}*x+\text{RootOf}(_Z^3-4)+3*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)))/(x+1)/(x^2-x+1))-1/2*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\ln(-(-9*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3-36*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+12*(-x^3+1)^{(2/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^2*x+4*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}*x^2+30*(-x^3+1)^{(1/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)*x^2+3*\text{RootOf}(_Z^3-4)*x^3+12*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^{(2/3)}*x-3*\text{RootOf}(_Z^3-4)-12*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)))/(x+1)/(x^2-x+1))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 (1 - x^3)^{4/3} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(1 - x^3)^(4/3)*(x^3 + 1)), x)

[Out] int(1/(x^3*(1 - x^3)^(4/3)*(x^3 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(-(x-1)(x^2+x+1) \right)^{4/3} (x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(-x**3+1)**(4/3)/(x**3+1), x)

[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (1+x^3)} dx = -\frac{28 - 182x^3 - 476x^6 + 819x^9 + 378x^{12} - 567x^{15} - 28 {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; -\frac{2x^3}{1-x^3}\right) + 182x^3}{12\sqrt[3]{2}}$$

Mathematica [A] time = 5.16, size = 133, normalized size = 0.92

$$\frac{2 \log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + 1\right) + 2\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{2}x-1}{\sqrt{3}\sqrt[3]{x^3-1}}\right) - \log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + \frac{2^{2/3}x^2}{(x^3-1)^{2/3}} + 1\right)}{12\sqrt[3]{2}} - \frac{-8x^6 + x^3 + 2}{10x^5\sqrt[3]{1-x^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] $-\frac{1}{10} \frac{(2 + x^3 - 8x^6)}{(x^5(1 - x^3)^{1/3})} + \frac{(2\sqrt{3} \operatorname{ArcTan}[-1 + (2^{1/3}x)/(-1 + x^3)^{1/3}])}{\sqrt{3}} - \frac{\operatorname{Log}[1 + (2^{2/3}x^2)/(-1 + x^3)^{2/3} - (2^{1/3}x)/(-1 + x^3)^{1/3}]}{(12 \cdot 2^{1/3})} + 2 \frac{\operatorname{Log}[1 + (2^{1/3}x)/(-1 + x^3)^{1/3}]}{(12 \cdot 2^{1/3})}$

IntegrateAlgebraic [A] time = 0.39, size = 164, normalized size = 1.14

$$\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} + 2x\right)}{6\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x - \sqrt{2}(1-x^3)^{2/3} - 2x^2\right)}{12\sqrt[3]{2}} + \frac{(1-x^3)^{2/3}(-8x^6 + x^3 + 2)}{10x^5(x^3-1)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] $\frac{(1-x^3)^{2/3}(2+x^3-8x^6)}{(10x^5(-1+x^3))} + \frac{\operatorname{ArcTan}[(\sqrt{3}x)/(-x + 2^{2/3}(1-x^3)^{1/3})]}{(2 \cdot 2^{1/3} \sqrt{3})} + \frac{\operatorname{Log}[2x + 2^{2/3}(1-x^3)^{1/3}]}{(6 \cdot 2^{1/3})} - \frac{\operatorname{Log}[-2x^2 + 2^{2/3}x(1-x^3)^{1/3} - 2^{1/3}(1-x^3)^{2/3}]}{(12 \cdot 2^{1/3})}$

fricas [B] time = 2.71, size = 316, normalized size = 2.19

$$\frac{10\sqrt{6}2^{1/3}(x^6-x^3)\arctan\left(\frac{2^{2/3}\sqrt{3}(5x^2+4x^3-1)(-x^3+1)^{1/3}-\sqrt{6}2^{1/3}(71x^9-111x^6+33x^3-1)+12\sqrt{6}(19x^6-16x^3+x^2)(-x^3+1)^{1/3}}{6(109x^9-105x^6+3x^3+1)}\right)}{360(x^6-x^3)} - \frac{10 \cdot 2^{1/3}(x^6-x^3)\log\left(\frac{6x^2(-x^3+1)^{1/3}x^2+2^{2/3}(x^3+1)^{1/3}}{x^3+1}\right)+5 \cdot 2^{1/3}(x^6-x^3)\log\left(\frac{3x^2(5x^2-1)(-x^3+1)^{1/3}+2^{1/3}(19x^6-16x^3+1)-12(2x^2-x^2)(-x^3+1)^{1/3}}{x^2+2x^3+1}\right)}{360(x^6-x^3)} + \frac{36(5x^6-x^3-2)(-x^3+1)^{2/3}}{360(x^6-x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1), x, algorithm="fricas")


```
[Out] -1/360*(10*sqrt(6)*2^(1/6)*(x^8 - x^5)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)
)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6
+ 33*x^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x
^9 - 105*x^6 + 3*x^3 + 1) - 10*2^(2/3)*(x^8 - x^5)*log((6*2^(1/3)*(-x^3 +
1)^(1/3)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(
2/3)*(x^8 - x^5)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*
x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) +
36*(8*x^6 - x^3 - 2)*(-x^3 + 1)^(2/3))/(x^8 - x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

maple [C] time = 4.02, size = 851, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/10*(8*x^6-x^3-2)/x^5/(-x^3+1)^(1/3)+1/12*RootOf(_Z^3-4)*ln(-(3*RootOf(Roo
tOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+54*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^
3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3
-4)^2*x-5*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-6*(-x^3+1)^(1/3)*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-RootOf(_Z^3-4)*
x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^(
2/3)*x+RootOf(_Z^3-4)+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z
^2))/(x+1)/(x^2-x+1)-1/12*ln((3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3
-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+3*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2+1
8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootO
f(_Z^3-4)*x^2+3*RootOf(_Z^3-4)*x^3+36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_
Z^3-4)+36*_Z^2)*x^3+6*(-x^3+1)^(2/3)*x-RootOf(_Z^3-4)-12*RootOf(RootOf(_Z^3
-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-1/2*ln(
(3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^
3+36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^
2*x^3+3*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2+18*(-x^3+1)^(1/3)*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2+3*RootOf(_Z^3-4)
```

```
*x^3+36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+6*(-x^3+1)
^(2/3)*x-RootOf(_Z^3-4)-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6 (1 - x^3)^{\frac{4}{3}} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^6*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \left(-(x-1)(x^2+x+1) \right)^{\frac{4}{3}} (x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

$$3.413 \quad \int \frac{1}{x^9(1-x^3)^{4/3}(1+x^3)} dx$$

Optimal. Leaf size=162

$$\frac{\log(x^3+1)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x\right)}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{5(1-x^3)^{2/3}}{8x^8} + \frac{1}{2x^8\sqrt[3]{1-x^3}} - \frac{13(1-x^3)^{2/3}}{20x^5} - \frac{49(1-x^3)}{40x^2}$$

Rubi [C] time = 10.59, antiderivative size = 643, normalized size of antiderivative = 3.97, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {510}

Warning: Unable to verify antiderivative.

[In] Int[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] -(70 - 308*x^3 + 1162*x^6 + 2856*x^9 - 4914*x^12 - 2268*x^15 + 3402*x^18 - 70*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] + 308*x^3*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 1162*x^6*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 2856*x^9*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] + 4914*x^12*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] + 2268*x^15*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 3402*x^18*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 66*x^6*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] + 312*x^9*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] - 2268*x^12*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] - 6696*x^15*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] - 4050*x^18*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] + 27*x^6*(1 + x^3)^2*(7 - 18*x^3 - 105*x^6)*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, (-2*x^3)/(1 - x^3)] + 54*x^6*(1 - 15*x^3)*(1 + x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 81*x^6*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 324*x^9*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 486*x^12*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 324*x^15*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 81*x^18*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3)]/(280*x^11*(1 - x^3)^(7/3))

Rule 510

Int[((e._)*(x._))^(m._)*((a._) + (b._)*(x._)^(n._))^(p._)*((c._) + (d._)*(x._)^(n._))^(q._), x_Symbol] := Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -

q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} dx = -\frac{70 - 308x^3 + 1162x^6 + 2856x^9 - 4914x^{12} - 2268x^{15} + 3402x^{18} - 70 {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{x^3}{1-x^3}\right)}{(1-x^3)^{4/3} (1+x^3)}$$

Mathematica [A] time = 5.21, size = 136, normalized size = 0.84

$$\frac{1}{120} \left(5 \cdot 2^{2/3} \left(-2 \log \left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + 1 \right) - 2\sqrt{3} \tan^{-1} \left(\frac{2\sqrt[3]{2}x-1}{\sqrt{3}} \right) + \log \left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + \frac{2^{2/3}x^2}{(x^3-1)^{2/3}} + 1 \right) \right) - \frac{3(-49x^9 + 23x^6 + x^3 + 5)}{x^8 \sqrt[3]{1-x^3}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] ((-3*(5 + x^3 + 23*x^6 - 49*x^9))/(x^8*(1 - x^3)^(1/3)) + 5*2^(2/3)*(-2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] + Log[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] - 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)]))/120

IntegrateAlgebraic [A] time = 0.39, size = 169, normalized size = 1.04

$$\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{6\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{12\sqrt[3]{2}} + \frac{\left(1-x^3\right)^{2/3}\left(-49x^9+23x^6+x^3+5\right)}{40x^8\left(x^3-1\right)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)), x]

[Out] ((1 - x^3)^(2/3)*(5 + x^3 + 23*x^6 - 49*x^9))/(40*x^8*(-1 + x^3)) - ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2*2^(1/3)*Sqrt[3]) - Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(12*2^(1/3))

fricas [B] time = 2.59, size = 351, normalized size = 2.17

$$\frac{10\sqrt{2}(-1)^{\frac{1}{3}}(x^3-x^6)\arctan\left(\frac{\sqrt[3]{6}\sqrt[3]{1-x^3}\sqrt[3]{x^3-x^6}-\sqrt[3]{2}\sqrt[3]{1-x^3}\sqrt[3]{19x^6-16x^3+7}\sqrt[3]{x^3-x^6}}{6(10x^6-10x^3-x^6)}\right)}{360(x^3-x^6)} - 10\sqrt[3]{2}(-1)^{\frac{1}{3}}(x^3-x^6)\log\left(\frac{6x^3-x^6\sqrt[3]{1-x^3}\sqrt[3]{2-x^3-x^6}\sqrt[3]{1-x^3}+5\sqrt[3]{2}(-1)^{\frac{1}{3}}(x^3-x^6)\log\left(\frac{3x^3-x^6\sqrt[3]{1-x^3}\sqrt[3]{1-x^3}\sqrt[3]{19x^6-16x^3+7}\sqrt[3]{1-x^3}\sqrt[3]{1-x^3}}{2x^3-x^6}\right)+9(49x^9-23x^6-x^3-5)(-x^3+1)^{\frac{2}{3}}}{2x^3-x^6}\right)}{360(x^3-x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")

[Out]
$$-1/360*(10*\sqrt{6}*2^{(1/6)}*(-1)^{(1/3)}*(x^{11} - x^8)*\arctan(1/6*2^{(1/6)}*(6*\sqrt{6}*2^{(2/3)}*(-1)^{(2/3)}*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^{(2/3)} - 12*\sqrt{6})*(-1)^{(1/3)}*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^{(1/3)} - \sqrt{6}*2^{(1/3)}*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1) - 10*2^{(2/3)}*(-1)^{(1/3)}*(x^{11} - x^8)*\log((6*2^{(1/3)}*(-1)^{(2/3)}*(-x^3 + 1)^{(1/3)}*x^2 - 2^{(2/3)}*(-1)^{(1/3)}*(x^3 + 1) + 6*(-x^3 + 1)^{(2/3)}*x)/(x^3 + 1)) + 5*2^{(2/3)}*(-1)^{(1/3)}*(x^{11} - x^8)*\log(-3*2^{(2/3)}*(-1)^{(1/3)}*(5*x^4 - x)*(-x^3 + 1)^{(2/3)} - 2^{(1/3)}*(-1)^{(2/3)}*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^{(1/3)})/(x^6 + 2*x^3 + 1) + 9*(49*x^9 - 23*x^6 - x^3 - 5)*(-x^3 + 1)^{(2/3)}/(x^{11} - x^8)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}}x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)

maple [C] time = 2.68, size = 649, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x)

[Out]
$$1/40*(49*x^9-23*x^6-x^3-5)/x^8/(-x^3+1)^{(1/3)}-1/2*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\ln(-(-9*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3-36*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3+12*(-x^3+1)^{(2/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^2*x+4*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)}*x^2+30*(-x^3+1)^{(1/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)*x^2+3*\text{RootOf}(_Z^3-4)*x^3+12*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^{(2/3)}*x-3*\text{RootOf}(_Z^3-4)-12*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)))/(x+1)/(x^2-x+1))-1/12*\text{RootOf}(_Z^3-4)*\ln(-(3*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^3*x^3+27*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)^2*\text{RootOf}(_Z^3-4)^2*x^3-6*(-x^3+1)^{(2/3)}*\text{RootOf}(\text{RootOf}(_Z^3-4)^2+6*_Z*\text{RootOf}(_Z^3-4)+36*_Z^2)*\text{RootOf}(_Z^3-4)^2*x-2*\text{RootOf}(_Z^3-4)^2*(-x^3+1)^{(1/3)})$$

```
*x^2+3*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*
RootOf(_Z^3-4)*x^2+3*RootOf(_Z^3-4)*x^3+27*RootOf(RootOf(_Z^3-4)^2+6*_Z*Ro
otOf(_Z^3-4)+36*_Z^2)*x^3-5*(-x^3+1)^(2/3)*x-RootOf(_Z^3-4)-9*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{4}{3}} x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^9 (1 - x^3)^{4/3} (x^3 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^9*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9 \left(-(x-1)(x^2+x+1) \right)^{\frac{4}{3}} (x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

$$3.414 \quad \int \frac{x^{11} \sqrt[3]{a+bx^3}}{c+dx^3} dx$$

Optimal. Leaf size=264

$$\frac{(a+bx^3)^{4/3} (a^2d^2 + abcd + b^2c^2)}{4b^3d^3} - \frac{(a+bx^3)^{7/3} (2ad+bc)}{7b^3d^2} + \frac{(a+bx^3)^{10/3}}{10b^3d} - \frac{c^3 \sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{13/3}} + \frac{c^3 \sqrt[3]{bc-ad}}{\sqrt{3}d^{13/3}}$$

Rubi [A] time = 0.39, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 88, 50, 58, 617, 204, 31}

$$\frac{(a+bx^3)^{4/3} (a^2d^2 + abcd + b^2c^2)}{4b^3d^3} - \frac{(a+bx^3)^{7/3} (2ad+bc)}{7b^3d^2} + \frac{(a+bx^3)^{10/3}}{10b^3d} - \frac{c^3 \sqrt[3]{a+bx^3}}{d^4} - \frac{c^3 \sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{13/3}} + \frac{c^3 \sqrt[3]{bc-ad} \log(\sqrt[3]{bc-ad} + \sqrt[3]{a+bx^3})}{2d^{13/3}} - \frac{c^3 \sqrt[3]{bc-ad} \tan^{-1}\left(\frac{1-2\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{13/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] -((c^3*(a + b*x^3)^(1/3))/d^4) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(4/3))/(4*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(7/3))/(7*b^3*d^2) + (a + b*x^3)^(10/3)/(10*b^3*d) - (c^3*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d))/Sqrt[3]]/(Sqrt[3]*d^(13/3)) - (c^3*(b*c - a*d)^(1/3)*Log[c + d*x^3]/(6*d^(13/3)) + (c^3*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(13/3))

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m+1)*(c + d*x)^n)/(b*(m+n+1)), x] + Dist[(n*(b*c - a*d))/(b*(m+n+1)), Int[(a + b*x)^m*(c + d*x)^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 58

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^

2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^{11} \sqrt[3]{a+bx^3}}{c+dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{(b^2c^2 + abcd + a^2d^2) \sqrt[3]{a+bx^3}}{b^2d^3} + \frac{(-bc - 2ad)(a+bx^3)^{4/3}}{b^2d^2} + \frac{(a+bx^3)^{7/3}}{b^2d} - \frac{c^3 \sqrt[3]{a+bx^3}}{d^3} \right) dx \right) \\
&= \frac{(b^2c^2 + abcd + a^2d^2)(a+bx^3)^{4/3}}{4b^3d^3} - \frac{(bc + 2ad)(a+bx^3)^{7/3}}{7b^3d^2} + \frac{(a+bx^3)^{10/3}}{10b^3d} - \frac{c^3 \text{Subst} \left(\int \frac{\sqrt[3]{a+bx^3}}{c+dx} dx \right)}{3} \\
&= -\frac{c^3 \sqrt[3]{a+bx^3}}{d^4} + \frac{(b^2c^2 + abcd + a^2d^2)(a+bx^3)^{4/3}}{4b^3d^3} - \frac{(bc + 2ad)(a+bx^3)^{7/3}}{7b^3d^2} + \frac{(a+bx^3)^{10/3}}{10b^3d} \\
&= -\frac{c^3 \sqrt[3]{a+bx^3}}{d^4} + \frac{(b^2c^2 + abcd + a^2d^2)(a+bx^3)^{4/3}}{4b^3d^3} - \frac{(bc + 2ad)(a+bx^3)^{7/3}}{7b^3d^2} + \frac{(a+bx^3)^{10/3}}{10b^3d} \\
&= -\frac{c^3 \sqrt[3]{a+bx^3}}{d^4} + \frac{(b^2c^2 + abcd + a^2d^2)(a+bx^3)^{4/3}}{4b^3d^3} - \frac{(bc + 2ad)(a+bx^3)^{7/3}}{7b^3d^2} + \frac{(a+bx^3)^{10/3}}{10b^3d}
\end{aligned}$$

Mathematica [A] time = 0.68, size = 270, normalized size = 1.02

$$\frac{105d(a+bx^3)^{4/3}(a^2d^2+abcd+b^2c^2)}{b^3} - \frac{60d^2(a+bx^3)^{7/3}(2ad+bc)}{b^3} + \frac{42d^3(a+bx^3)^{10/3}}{b^3} - \frac{70c^3 \sqrt[3]{bc-ad} \left(\log \left(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3} \right) - 2 \log \left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{1 - 2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt{3}} \right) \right)}{420d^4 \sqrt[3]{d}} - 420c^3 \sqrt[3]{a+bx^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] (-420*c^3*(a + b*x^3)^(1/3) + (105*d*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(4/3))/b^3 - (60*d^2*(b*c + 2*a*d)*(a + b*x^3)^(7/3))/b^3 + (42*d^3*(a + b*x^3)^(10/3))/b^3 - (70*c^3*(b*c - a*d)^(1/3)*(2*sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)]/sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]])/d^(1/3))/(420*d^4)

IntegrateAlgebraic [A] time = 0.50, size = 340, normalized size = 1.29

$$\frac{\sqrt{a+bx^3} (9a^2d^3 + 15a^2bcd^2 - 3a^2bd^3x^3 + 35ab^2c^2d - 5ab^2cd^2x^3 + 2ab^2d^3x^6 - 140b^3c^3 + 35b^3c^2dx^3 - 20b^3cd^2x^6 + 14b^3d^3x^9)}{140b^3d^4} + \frac{c^3\sqrt{bc-ad} \log(\sqrt{bc-ad} + \sqrt{d}\sqrt{a+bx^3})}{3d^{13/3}} - \frac{c^3\sqrt{bc-ad} \log(-\sqrt{d}\sqrt{a+bx^3}\sqrt{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3})}{6d^{13/3}} - \frac{c^3\sqrt{bc-ad} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt{d}\sqrt{a+bx^3}}{\sqrt{3}\sqrt{bc-ad}}\right)}{\sqrt{3}d^{13/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]

[Out] ((a + b*x^3)^(1/3)*(-140*b^3*c^3 + 35*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 9*a^3*d^3 + 35*b^3*c^2*d*x^3 - 5*a*b^2*c*d^2*x^6 - 3*a^2*b*d^3*x^9 - 20*b^3*c*d^2*x^6 + 2*a*b^2*d^3*x^9) / (140*b^3*d^4) - (c^3*(b*c - a*d)^(1/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]) / (Sqrt[3]*d^(13/3)) + (c^3*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]) / (3*d^(13/3)) - (c^3*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]) / (6*d^(13/3)))

fricas [A] time = 0.45, size = 325, normalized size = 1.23

$$\frac{140\sqrt{3}b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(b^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+70b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((b^3+a)^{\frac{1}{3}}-(bc^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)-140b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((b^3+a)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)-3(14b^3d^3x^9-2(10b^3cd^2-ab^2d^3)x^6-140b^3c^3+35a^2b^2cd^2+15a^2bd^3+9a^3d^3+(35b^3c^2d-5ab^2cd^2-3a^2bd^3)x^3)(b^3+a)^{\frac{1}{3}}}{420b^3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] -1/420*(140*sqrt(3)*b^3*c^3*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 70*b^3*c^3*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 140*b^3*c^3*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(14*b^3*d^3*x^9 - 2*(10*b^3*c*d^2 - a*b^2*d^3)*x^6 - 140*b^3*c^3 + 35*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 9*a^3*d^3 + (35*b^3*c^2*d - 5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^3)*(b*x^3 + a)^(1/3))/(b^3*d^4)

giac [A] time = 0.29, size = 379, normalized size = 1.44

$$\frac{(b^3+a)^{\frac{1}{3}}-ab^2cd^3}{3(b^3cd^2-ab^2d^3)}\log\left[\frac{(b^3+a)^{\frac{1}{3}}-\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}{\sqrt{3}\left(-bc^3+ad\right)^{\frac{1}{3}}}\arctan\left(\frac{\sqrt{3}\left((b^3+a)^{\frac{1}{3}}-\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)\right]}{3d^6}+\frac{\sqrt{3}\left(-bc^3+ad\right)^{\frac{1}{3}}c^3\arctan\left(\frac{\sqrt{3}\left((b^3+a)^{\frac{1}{3}}-\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{6d^6}+\frac{(-bc^3+ad)^{\frac{1}{3}}c^3\log\left[\frac{(b^3+a)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}{3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right]}{6d^6}+\frac{140(b^3+a)^{\frac{1}{3}}b^3c^2d^2-35(b^3+a)^{\frac{1}{3}}b^2cd^2+20(b^3+a)^{\frac{1}{3}}bd^2cd^2-35(b^3+a)^{\frac{1}{3}}ad^2cd^2-14(b^3+a)^{\frac{1}{3}}d^2cd^2+40(b^3+a)^{\frac{1}{3}}ad^2d^2-35(b^3+a)^{\frac{1}{3}}d^2d^2d^2}{140b^3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out] -1/3*(b^34*c^4*d^6 - a*b^33*c^3*d^7)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - ((b*c - a*d)/d)^(1/3)))/(b^34*c*d^10 - a*b^33*d^11) + 1/3*sqrt(3)*(-(b*c*d^2 + a*d^3)^(1/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +

$$\begin{aligned} & \frac{(-b*c - a*d)/d)^{1/3}}{(-b*c - a*d)/d)^{1/3}}/d^5 + 1/6*(-b*c*d^2 + a*d^3)^{1/3}*c^3*\log((b*x^3 + a)^{2/3} + (b*x^3 + a)^{1/3}*(-b*c - a*d)/d)^{1/3} \\ & + (-b*c - a*d)/d)^{2/3}}/d^5 - 1/140*(140*(b*x^3 + a)^{1/3}*b^30*c^3*d^6 - 35*(b*x^3 + a)^{4/3}*b^29*c^2*d^7 + 20*(b*x^3 + a)^{7/3}*b^28*c*d^8 - 3 \\ & 5*(b*x^3 + a)^{4/3}*a*b^28*c*d^8 - 14*(b*x^3 + a)^{10/3}*b^27*d^9 + 40*(b*x^3 + a)^{7/3}*a*b^27*d^9 - 35*(b*x^3 + a)^{4/3}*a^2*b^27*d^9)/(b^30*d^10) \end{aligned}$$

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}} x^{11}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11*(b*x^3+a)^(1/3)/(d*x^3+c), x)

[Out] int(x^11*(b*x^3+a)^(1/3)/(d*x^3+c), x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.98, size = 442, normalized size = 1.67

$$\left(\frac{3a^2}{4b^2} + \frac{3a^2 - a^2 d^2}{4b^2 d} \right) (bx^3 + a)^{1/3} + \frac{3a}{7b^2} + \frac{3a - a^2 d}{7b^2 d} (bx^3 + a)^{2/3} + \frac{a^2}{b^2 d} \left(\frac{3a}{2b^2} + \frac{3a^2 - a^2 d^2}{4b^2 d} \right) (bx^3 + a)^{1/3} + \frac{3a^2}{10b^2 d} + \frac{3a^2 - a^2 d^2}{10b^2 d} \ln \left(\frac{bx^3 + a}{d} \right) + \frac{3a^2}{3a^{1/3} b^2 d} \ln \left(\frac{10bx^3 + 10bx^2 d + 10bx d^2 + 10b^2 d^3 + 3a}{d} \right) \left(\frac{1}{2} + \frac{a^2}{3a} \right) \ln \left(\frac{10bx^3 + 10bx^2 d + 10bx d^2 + 10b^2 d^3 + 3a}{d} \right) \left(\frac{1}{2} + \frac{a^2}{3a} \right) \ln \left(\frac{10bx^3 + 10bx^2 d + 10bx d^2 + 10b^2 d^3 + 3a}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^11*(a + b*x^3)^(1/3))/(c + d*x^3), x)

[Out] ((3*a^2)/(4*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c - a*b^3*d))/(4*b^3*d))*(a + b*x^3)^(4/3) - ((3*a)/(7*b^3*d) + (b^4*c - a*b^3*d)/(7*b^6*d^2))*(a + b*x^3)^(7/3) - (a + b*x^3)^(1/3)*(a^3/(b^3*d) + (((3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c - a*b^3*d))/(b^3*d))*(b^4*c - a*b^3*d))/(b^3*d)) + (a + b*x^3)^(10/3)/(10*b^3*d) - (c^3*log((a*d - b*c)^(1/3) - d^(1/3)*(a + b*x^3)^(1/3))*(a*d - b*c)^(1/3))/(3*d^(13/3)) - (c^3*log((3*(a + b*x^3)^(1/3)*(b*c^4 - a*c^3*d))/d^2 + (3

```
*c^3*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(4/3))/d^(7/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(1/3))/(3*d^(13/3)) + (c^3*log((3*(a + b*x^3)^(1/3)*(b*c^4 - a*c^3*d))/d^2 - (9*c^3*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(4/3))/d^(7/3)))*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(1/3))/d^(13/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11} \sqrt[3]{a + bx^3}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11*(b*x**3+a)**(1/3)/(d*x**3+c), x)

[Out] Integral(x**11*(a + b*x**3)**(1/3)/(c + d*x**3), x)

$$3.415 \quad \int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3} dx$$

Optimal. Leaf size=220

$$\frac{(a+bx^3)^{4/3}(ad+bc)}{4b^2d^2} + \frac{(a+bx^3)^{7/3}}{7b^2d} + \frac{c^2\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{10/3}} - \frac{c^2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}$$

Rubi [A] time = 0.26, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 88, 50, 58, 617, 204, 31}

$$-\frac{(a+bx^3)^{4/3}(ad+bc)}{4b^2d^2} + \frac{(a+bx^3)^{7/3}}{7b^2d} + \frac{c^2\sqrt[3]{a+bx^3}}{d^3} + \frac{c^2\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{10/3}} - \frac{c^2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}} + \frac{c^2\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{3}d^{10/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] (c^2*(a + b*x^3)^(1/3))/d^3 - ((b*c + a*d)*(a + b*x^3)^(4/3))/(4*b^2*d^2) + (a + b*x^3)^(7/3)/(7*b^2*d) + (c^2*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d))/Sqrt[3]])/(Sqrt[3]*d^(10/3)) + (c^2*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(10/3)) - (c^2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m+1)*(c + d*x)^n)/(b*(m+1)), x] + Dist[(n*(b*c - a*d))/(b*(m+n+1)), Int[(a + b*x)^m*(c + d*x)^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 58

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^

2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 \sqrt[3]{a+bx}}{c+dx} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{(-bc-ad) \sqrt[3]{a+bx}}{bd^2} + \frac{(a+bx)^{4/3}}{bd} + \frac{c^2 \sqrt[3]{a+bx}}{d^2(c+dx)} \right) dx, x, x^3 \right) \\
&= -\frac{(bc+ad)(a+bx^3)^{4/3}}{4b^2d^2} + \frac{(a+bx^3)^{7/3}}{7b^2d} + \frac{c^2 \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{c+dx} dx, x, x^3 \right)}{3d^2} \\
&= \frac{c^2 \sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)(a+bx^3)^{4/3}}{4b^2d^2} + \frac{(a+bx^3)^{7/3}}{7b^2d} - \frac{(c^2(bc-ad)) \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} \right)}{3d^3} \\
&= \frac{c^2 \sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)(a+bx^3)^{4/3}}{4b^2d^2} + \frac{(a+bx^3)^{7/3}}{7b^2d} + \frac{c^2 \sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{10/3}} - \frac{(c^2 \sqrt[3]{bc-ad})}{\sqrt{3} d^{10/3}} \\
&= \frac{c^2 \sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)(a+bx^3)^{4/3}}{4b^2d^2} + \frac{(a+bx^3)^{7/3}}{7b^2d} + \frac{c^2 \sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{10/3}} - \frac{c^2 \sqrt[3]{bc-ad}}{\sqrt{3} d^{10/3}} \\
&= \frac{c^2 \sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)(a+bx^3)^{4/3}}{4b^2d^2} + \frac{(a+bx^3)^{7/3}}{7b^2d} + \frac{c^2 \sqrt[3]{bc-ad} \tan^{-1} \left(\frac{1 - \frac{2 \sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}} \right)}{\sqrt{3} d^{10/3}} +
\end{aligned}$$

Mathematica [A] time = 0.41, size = 230, normalized size = 1.05

$$\frac{-\frac{21d(a+bx^3)^{4/3}(ad+bc)}{b^2} + \frac{12d^2(a+bx^3)^{7/3}}{b^2} + \frac{14c^2 \sqrt[3]{bc-ad} \left(\log \left(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3} (a+bx^3)^{2/3} \right) - 2 \log \left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{1 - \frac{2 \sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}} \right) \right)}{84d^3} + 84c^2 \sqrt[3]{a+bx^3}}{84d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]

[Out] (84*c^2*(a + b*x^3)^(1/3) - (21*d*(b*c + a*d)*(a + b*x^3)^(4/3))/b^2 + (12*d^2*(a + b*x^3)^(7/3))/b^2 + (14*c^2*(b*c - a*d)^(1/3)*(2*sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)]/sqrt[3]) - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/d^(1/3))/(84*d^3)

IntegrateAlgebraic [A] time = 0.37, size = 283, normalized size = 1.29

$$\frac{\sqrt{a+bx^3}(-3a^2d^2-7abcd+abd^2x^3+28b^2c^2-7b^2cdx^3+4b^2d^2x^6)}{28b^2d^3} - \frac{c^2\sqrt{bc-ad}\log(\sqrt{bc-ad}+\sqrt{d}\sqrt{a+bx^3})}{3d^{10/3}} + \frac{c^2\sqrt{bc-ad}\log(-\sqrt{d}\sqrt{a+bx^3}\sqrt{bc-ad}+(bc-ad)^{2/3}+d^{2/3}(a+bx^3)^{2/3})}{6d^{10/3}} + \frac{c^2\sqrt{bc-ad}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt{d}\sqrt{a+bx^3}}{\sqrt{3}\sqrt{bc-ad}}\right)}{\sqrt{3}d^{10/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]

[Out] ((a + b*x^3)^(1/3)*(28*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2 - 7*b^2*c*d*x^3 + a*b*d^2*x^3 + 4*b^2*d^2*x^6))/(28*b^2*d^3) + (c^2*(b*c - a*d)^(1/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*d^(10/3)) - (c^2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(10/3)) + (c^2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(10/3))

fricas [A] time = 0.46, size = 282, normalized size = 1.28

$$\frac{28\sqrt{3}b^2c^2\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bc+ad)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+14b^2c^2\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-28b^2c^2\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)-3(4b^2d^2x^6+28b^2c^2-7abcd-3a^2d^2-(7b^2cd-abd^2)x^3)(bx^3+a)^{\frac{1}{3}}}{84b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] -1/84*(28*sqrt(3)*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d) + 14*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 28*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 3*(4*b^2*d^2*x^6 + 28*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2 - (7*b^2*c*d - a*b*d^2)*x^3)*(b*x^3 + a)^(1/3)/(b^2*d^3)

giac [A] time = 0.25, size = 320, normalized size = 1.45

$$\frac{(b^{17}c^4d^4-ab^{16}c^2d^5)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left((bx^3+a)^{\frac{1}{3}}-\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\right)}{3(b^{17}cd^7-ab^{16}d^8)} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}-\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^4} - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^4} + \frac{28(bx^3+a)^{\frac{1}{3}}b^{14}c^2d^4-7(bx^3+a)^{\frac{1}{3}}b^{13}cd^5+4(bx^3+a)^{\frac{1}{3}}b^{12}d^6-7(bx^3+a)^{\frac{1}{3}}ab^{12}d^6}{28b^4d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out] 1/3*(b^17*c^3*d^4 - a*b^16*c^2*d^5)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^17*c*d^7 - a*b^16*d^8) - 1/3*sqrt(3)*(-(b*c*d^2 + a*d^3)^(1/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3)))/(-(b*c - a*d)/d)^(1/3))/d^4 - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3))

+ $(-(b*c - a*d)/d)^{(2/3)}/d^4 + 1/28*(28*(b*x^3 + a)^{(1/3)}*b^{14}*c^2*d^4 - 7*(b*x^3 + a)^{(4/3)}*b^{13}*c*d^5 + 4*(b*x^3 + a)^{(7/3)}*b^{12}*d^6 - 7*(b*x^3 + a)^{(4/3)}*a*b^{12}*d^6)/(b^{14}*d^7)$

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}} x^8}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x)

[Out] int(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.93, size = 336, normalized size = 1.53

$$\left(\frac{a^2}{b^2 d} + \frac{\left(\frac{2a}{b^2} + \frac{b^2 c - a^2 d}{b^2 d}\right) (b^3 c - a^2 d)}{b^2 d}\right) (b^3 + a)^{1/3} - \left(\frac{a}{2 b^2 d} + \frac{b^3 c - a^2 d}{4 b^2 d}\right) (b^3 + a)^{4/3} + \frac{(b^3 + a)^{7/3}}{7 b^2 d} + \frac{c^2 \ln((a d - b c)^{1/3} - d^{1/3} (b^3 + a)^{1/3}) (a d - b c)^{1/3}}{3 d^{10/3}} - \frac{c^2 \ln\left(\frac{3(b^3 + a)^{1/3} (b^3 c - a^2 d) - \frac{3d}{4} \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) a d + d^{1/3}}{d}\right) \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) (a d - b c)^{1/3}}{3 d^{10/3}} + \frac{c^2 \ln\left(\frac{3(b^3 + a)^{1/3} (b^3 c - a^2 d) + \frac{3d}{4} \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) a d + d^{1/3}}{d}\right) \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) (a d - b c)^{1/3}}{d^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x)

[Out] $(a^2/(b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*b^2*d))/(b^2*d))*(a + b*x^3)^{(1/3)} - (a/(2*b^2*d) + (b^3*c - a*b^2*d)/(4*b^4*d^2))*(a + b*x^3)^{(4/3)} + (a + b*x^3)^{(7/3)}/(7*b^2*d) + (c^2*\log((a*d - b*c)^{(1/3)} - d^{(1/3)}*(a + b*x^3)^{(1/3)})*(a*d - b*c)^{(1/3)})/(3*d^{(10/3)}) - (c^2*\log(((3*(a + b*x^3)^{(1/3)}*(b*c^3 - a*c^2*d))/d - (3*c^2*((3^{(1/2)}*1i)/2 + 1/2)*(a*d - b*c)^{(4/3)}))/d^{(4/3)})*((3^{(1/2)}*1i)/2 + 1/2)*(a*d - b*c)^{(1/3)})/(3*d^{(10/3)}) + (c^2*\log((3*(a + b*x^3)^{(1/3)}*(b*c^3 - a*c^2*d))/d + (9*c^2*((3^{(1/2)}*1i)/6 - 1/6)*(a*d - b*c)^{(4/3)}))/d^{(4/3)})*((3^{(1/2)}*1i)/6 - 1/6)*(a*d - b*c)^{(1/3)}/d^{(10/3)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 \sqrt[3]{a + bx^3}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(b*x**3+a)**(1/3)/(d*x**3+c), x)

[Out] Integral(x**8*(a + b*x**3)**(1/3)/(c + d*x**3), x)

$$3.416 \quad \int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} dx$$

Optimal. Leaf size=186

$$\frac{c\sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{7/3}} + \frac{c\sqrt[3]{bc-ad} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}} - \frac{c\sqrt[3]{bc-ad} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{7/3}} - \frac{c\sqrt[3]{a+bx^3}}{d}$$

Rubi [A] time = 0.20, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 80, 50, 58, 617, 204, 31}

$$\frac{c\sqrt[3]{a+bx^3}}{d^2} - \frac{c\sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{7/3}} + \frac{c\sqrt[3]{bc-ad} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}} - \frac{c\sqrt[3]{bc-ad} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{7/3}} + \frac{(a+bx^3)^{4/3}}{4bd}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] -((c*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*b*d) - (c*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(7/3)) - (c*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(7/3)) + (c*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(7/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x \sqrt[3]{a+bx}}{c+dx} dx, x, x^3 \right) \\
&= \frac{(a+bx^3)^{4/3}}{4bd} - \frac{c \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{c+dx} dx, x, x^3 \right)}{3d} \\
&= -\frac{c \sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4bd} + \frac{(c(bc-ad)) \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3d^2} \\
&= -\frac{c \sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4bd} - \frac{c \sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{7/3}} + \frac{(c \sqrt[3]{bc-ad}) \text{Subst} \left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{a}} + x} dx, x, x^3 \right)}{2d^{7/3}} \\
&= -\frac{c \sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4bd} - \frac{c \sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{7/3}} + \frac{c \sqrt[3]{bc-ad} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d})}{2d^{7/3}} \\
&= -\frac{c \sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4bd} - \frac{c \sqrt[3]{bc-ad} \tan^{-1} \left(\frac{1 - \frac{2 \sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}} \right)}{\sqrt{3} d^{7/3}} - \frac{c \sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{7/3}}
\end{aligned}$$

Mathematica [A] time = 0.47, size = 204, normalized size = 1.10

$$\frac{c \sqrt[3]{bc-ad} \left(-\log \left(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3} (a+bx^3)^{2/3} \right) + 2 \log \left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{2 \sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}} - 1 \right) \right)}{6d^{7/3}} - \frac{c \sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4bd}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] -((c*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*b*d) + (c*(b*c - a*d)^(1/3)*(2*sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)])/sqrt[3]) + 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(7/3))

IntegrateAlgebraic [A] time = 0.22, size = 239, normalized size = 1.28

$$\frac{c \sqrt[3]{bc-ad} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{3d^{7/3}} - \frac{c \sqrt[3]{bc-ad} \log(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3} (a+bx^3)^{2/3})}{6d^{7/3}} - \frac{c \sqrt[3]{bc-ad} \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2 \sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt{3} \sqrt[3]{bc-ad}} \right)}{\sqrt{3} d^{7/3}} + \frac{\sqrt[3]{a+bx^3} (ad - 4bc + bdx^3)}{4bd^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]

[Out] ((a + b*x^3)^(1/3)*(-4*b*c + a*d + b*d*x^3))/(4*b*d^2) - (c*(b*c - a*d)^(1/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*d^(7/3)) + (c*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(3*d^(7/3)) - (c*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(7/3))

fricas [A] time = 0.45, size = 222, normalized size = 1.19

$$\frac{4\sqrt{3}bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+2bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-4bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)-3(bdx^3-4bc+ad)(bx^3+a)^{\frac{1}{3}}}{12bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] -1/12*(4*sqrt(3)*b*c*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*b*c*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 4*b*c*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(b*d*x^3 - 4*b*c + a*d)*(b*x^3 + a)^(1/3))/(b*d^2)

giac [A] time = 0.25, size = 276, normalized size = 1.48

$$\frac{(b^6c^2d^2 - ab^5cd^3)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3(b^6cd^4 - ab^5d^5)} + \frac{\sqrt{3}(-bcd^2 + ad^3)^{\frac{1}{3}}c\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^3} + \frac{(-bcd^2 + ad^3)^{\frac{1}{3}}c\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^3} - \frac{4(bx^3+a)^{\frac{1}{3}}b^4cd^2 - (bx^3+a)^{\frac{4}{3}}b^3d^3}{4b^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out] -1/3*(b^6*c^2*d^2 - a*b^5*c*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - ((b*c - a*d)/d)^(1/3)))/(b^6*c*d^4 - a*b^5*d^5) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)))/(-((b*c - a*d)/d)^(1/3))/d^3 + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*c*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/3)*b^4*c*d^2 - (b*x^3 + a)^(4/3)*b^3*d^3)/(b^4*d^4)

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}} x^5}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(b*x^3+a)^(1/3)/(d*x^3+c), x)`

[Out] `int(x^5*(b*x^3+a)^(1/3)/(d*x^3+c), x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c), x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.62, size = 298, normalized size = 1.60

$$\frac{\frac{(bx^3+a)^{4/3}}{4bd} - (bx^3+a)^{1/3} \left(\frac{a}{bd} + \frac{b^2c-abd}{b^2d^2} \right) - \frac{c \ln\left((bx^3+a)^{1/3} (3bc^2-3acd) + \frac{c(ad-bc)^{1/3}(9a^2-9bd^2)}{3d^3}\right) (ad-bc)^{1/3}}{3d^{7/3}}}{3d^{7/3}} - \frac{c \ln\left((bx^3+a)^{1/3} (3bc^2-3acd) + \frac{\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) (ad-bc)^{1/3} (9a^2-9bd^2)}{3d^3}\right) \left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) (ad-bc)^{1/3}}{3d^{7/3}}}{3d^{7/3}} + \frac{c \ln\left((bx^3+a)^{1/3} (3bc^2-3acd) - \frac{\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) (ad-bc)^{1/3} (9a^2-9bd^2)}{3d^3}\right) \left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) (ad-bc)^{1/3}}{3d^{7/3}}}{3d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(a + b*x^3)^(1/3))/(c + d*x^3), x)`

[Out] $(a + bx^3)^{4/3}/(4bd) - (a + bx^3)^{1/3} * (a/(bd) + (b^2c - abd)/(b^2d^2)) - (c * \log((a + bx^3)^{1/3} * (3bc^2 - 3acd) + (c * (a*d - b*c)^{1/3} * (9ad^3 - 9b*c*d^2))/(3*d^{7/3}))) * (a*d - b*c)^{1/3} / (3*d^{7/3}) - (c * \log((a + bx^3)^{1/3} * (3bc^2 - 3acd) + (c * ((3^{1/2} * 1i) / 2 - 1/2) * (a*d - b*c)^{1/3} * (9ad^3 - 9b*c*d^2))/(3*d^{7/3}))) * ((3^{1/2} * 1i) / 2 - 1/2) * (a*d - b*c)^{1/3} / (3*d^{7/3}) + (c * \log((a + bx^3)^{1/3} * (3bc^2 - 3acd) - (c * ((3^{1/2} * 1i) / 2 + 1/2) * (a*d - b*c)^{1/3} * (9ad^3 - 9b*c*d^2))/(3*d^{7/3}))) * ((3^{1/2} * 1i) / 2 + 1/2) * (a*d - b*c)^{1/3} / (3*d^{7/3})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \sqrt[3]{a + bx^3}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b*x**3+a)**(1/3)/(d*x**3+c), x)`

[Out] `Integral(x**5*(a + b*x**3)**(1/3)/(c + d*x**3), x)`

$$3.417 \quad \int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} dx$$

Optimal. Leaf size=159

$$\frac{\sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{4/3}} + \frac{\sqrt[3]{bc-ad} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{4/3}} + \frac{\sqrt[3]{a+bx^3}}{d}$$

Rubi [A] time = 0.15, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {444, 50, 58, 617, 204, 31}

$$\frac{\sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{4/3}} + \frac{\sqrt[3]{bc-ad} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{4/3}} + \frac{\sqrt[3]{a+bx^3}}{d}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] (a + b*x^3)^(1/3)/d + ((b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d))/Sqrt[3]]/(Sqrt[3]*d^(4/3)) + ((b*c - a*d)^(1/3)*Log[c + d*x^3]/(6*d^(4/3)) - ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(4/3)))

Rule 31

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 58

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^


```
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{c+dx} dx, x, x^3 \right) \\
&= \frac{\sqrt[3]{a+bx^3}}{d} - \frac{(bc-ad) \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3d} \\
&= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad} \text{Subst} \left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}} + x} dx, x, \sqrt[3]{a+bx^3} \right)}{2d^{4/3}} \quad (bc \\
&= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{2d^{4/3}} - \frac{\sqrt[3]{bc-ad}}{d} \\
&= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad} \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}} \right)}{\sqrt{3} d^{4/3}} + \frac{\sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{2d^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 205, normalized size = 1.29

$$\frac{\sqrt[3]{bc-ad} \log(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3} (a+bx^3)^{2/3}) - 2\sqrt[3]{bc-ad} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}) - 2\sqrt{3} \sqrt[3]{bc-ad} \tan^{-1} \left(\frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3} - 1}{\sqrt{3} \sqrt[3]{bc-ad}} \right) + 6\sqrt[3]{d} \sqrt[3]{a+bx^3}}{6d^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] (6*d^(1/3)*(a + b*x^3)^(1/3) - 2*sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d))/sqrt[3]] - 2*(b*c - a*d)^(1/3) *Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(4/3))

IntegrateAlgebraic [A] time = 0.23, size = 215, normalized size = 1.35

$$-\frac{\sqrt[3]{bc-ad} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{3d^{4/3}} + \frac{\sqrt[3]{bc-ad} \log(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3} (a+bx^3)^{2/3})}{6d^{4/3}} + \frac{\sqrt[3]{bc-ad} \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt{3} \sqrt[3]{bc-ad}}}{\sqrt{3}} \right)}{\sqrt{3} d^{4/3}} + \frac{\sqrt[3]{a+bx^3}}{d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] $(a + b*x^3)^{(1/3)}/d + ((b*c - a*d)^{(1/3)}*ArcTan[1/Sqrt[3] - (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})/(Sqrt[3]*(b*c - a*d)^{(1/3)})]/(Sqrt[3]*d^{(4/3)}) - ((b*c - a*d)^{(1/3)}*Log[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)}]/(3*d^{(4/3)}) + ((b*c - a*d)^{(1/3)}*Log[(b*c - a*d)^{(2/3)} - d^{(1/3)}*(b*c - a*d)^{(1/3)}*(a + b*x^3)^{(1/3)} + d^{(2/3)}*(a + b*x^3)^{(2/3)}]/(6*d^{(4/3)})$

fricas [A] time = 0.44, size = 206, normalized size = 1.30

$$\frac{2\sqrt{3}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)-6(bx^3+a)^{\frac{1}{3}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")`

[Out] $-1/6*(2*\sqrt{3}*(-(b*c - a*d)/d)^{(1/3)}*\arctan(-1/3*(2*\sqrt{3}*(b*x^3 + a)^{(1/3)}*d*(-(b*c - a*d)/d)^{(2/3)} - \sqrt{3}*(b*c - a*d))/(b*c - a*d)) + (-(b*c - a*d)/d)^{(1/3)}*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(-(b*c - a*d)/d)^{(1/3)} + (-(b*c - a*d)/d)^{(2/3)}) - 2*(-(b*c - a*d)/d)^{(1/3)}*\log((b*x^3 + a)^{(1/3)} - (-(b*c - a*d)/d)^{(1/3)}) - 6*(b*x^3 + a)^{(1/3)}/d$

giac [A] time = 0.27, size = 223, normalized size = 1.40

$$\frac{(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3(bcd-ad^2)}-\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^2}+\frac{(bx^3+a)^{\frac{1}{3}}}{d}-\frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")`

[Out] $1/3*(b*c - a*d)*(-(b*c - a*d)/d)^{(1/3)}*\log(\text{abs}((b*x^3 + a)^{(1/3)} - (-(b*c - a*d)/d)^{(1/3)}))/(b*c*d - a*d^2) - 1/3*\sqrt{3}*(-b*c*d^2 + a*d^3)^{(1/3)}*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{(1/3)} + (-(b*c - a*d)/d)^{(1/3)})/(-(b*c - a*d)/d)^{(1/3)})/d^2 + (b*x^3 + a)^{(1/3)}/d - 1/6*(-b*c*d^2 + a*d^3)^{(1/3)}*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(-(b*c - a*d)/d)^{(1/3)} + (-(b*c - a*d)/d)^{(2/3)})/d^2$

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}} x^2}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x)`

[Out] $\int (x^2(bx^3+a)^{1/3}/(dx^3+c), x)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c), x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.62, size = 249, normalized size = 1.57

$$\frac{(bx^3+a)^{1/3}}{d} + \frac{\ln\left(\frac{(bx^3+a)^{1/3}(3ad^2-3bcd) - \frac{(ad-bc)^{1/3}(9a^2b-9bc^2)}{3a^2}}{(ad-bc)^{1/3}}\right)}{3d^{4/3}} - \frac{\ln\left(\frac{(bx^3+a)^{1/3}(3ad^2-3bcd) + \frac{(\frac{1}{2} + \frac{\sqrt{3}i}{2})(ad-bc)^{1/3}(9a^2b-9bc^2)}{3a^2}}{(ad-bc)^{1/3}}\right)}{3d^{4/3}} + \frac{\ln\left(\frac{(bx^3+a)^{1/3}(3ad^2-3bcd) - \frac{(-\frac{1}{2} + \frac{\sqrt{3}i}{2})(ad-bc)^{1/3}(9a^2b-9bc^2)}{3a^2}}{(ad-bc)^{1/3}}\right)}{d^{4/3}} + \frac{\ln\left(\frac{(bx^3+a)^{1/3}(3ad^2-3bcd) + \frac{(-\frac{1}{2} + \frac{\sqrt{3}i}{2})(ad-bc)^{1/3}(9a^2b-9bc^2)}{3a^2}}{(ad-bc)^{1/3}}\right)}{d^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(a + b*x^3)^(1/3))/(c + d*x^3), x)`

[Out] $(a + bx^3)^{1/3}/d + (\log((a + bx^3)^{1/3}*(3ad^2 - 3b^2cd) - ((ad - bc)^{1/3}*(9ad^3 - 9b^2cd^2))/(3d^{4/3}))) * (ad - bc)^{1/3} / (3d^{4/3}) - (\log((a + bx^3)^{1/3}*(3ad^2 - 3b^2cd) + ((3^{1/2}*1i)/2 + 1/2)*(ad - bc)^{1/3}*(9ad^3 - 9b^2cd^2))/(3d^{4/3})) * ((3^{1/2}*1i)/2 + 1/2) * (ad - bc)^{1/3} / (3d^{4/3}) + (\log((a + bx^3)^{1/3}*(3ad^2 - 3b^2cd) - ((3^{1/2}*1i)/6 - 1/6)*(ad - bc)^{1/3}*(9ad^3 - 9b^2cd^2))/d^{4/3}) * ((3^{1/2}*1i)/6 - 1/6) * (ad - bc)^{1/3} / d^{4/3}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt[3]{a + bx^3}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**3+a)**(1/3)/(d*x**3+c), x)`

[Out] `Integral(x**2*(a + b*x**3)**(1/3)/(c + d*x**3), x)`

$$3.418 \quad \int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)} dx$$

Optimal. Leaf size=246

$$\frac{\sqrt[3]{bc-ad} \log(c+dx^3)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{bc-ad} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{2c\sqrt[3]{d}} - \frac{\sqrt[3]{bc-ad} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{d}} + \frac{\sqrt[3]{a} \log(x)}{2c}$$

Rubi [A] time = 0.23, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, number of rules / integrand size = 0.292, Rules used = {446, 83, 57, 617, 204, 31, 58}

$$\frac{\sqrt[3]{bc-ad} \log(c+dx^3)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{bc-ad} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{2c\sqrt[3]{d}} - \frac{\sqrt[3]{bc-ad} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{d}} + \frac{\sqrt[3]{a} \log(x)}{2c}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(1/3)/(x*(c + d*x^3)), x]

[Out] -((a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*c)) - ((b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sqrt[3]*c*d^(1/3)) - (a^(1/3)*Log[x])/(2*c) - ((b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c*d^(1/3)) + (a^(1/3)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*c) + ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*d^(1/3))

Rule 31

Int[((a_) + (b_)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 58

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

```
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x]]) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{x(c+dx)} dx, x, x^3 \right) \\
&= \frac{a \text{Subst} \left(\int \frac{1}{x(a+bx)^{2/3}} dx, x, x^3 \right)}{3c} + \frac{(bc-ad) \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3c} \\
&= -\frac{\sqrt[3]{a} \log(x)}{2c} - \frac{\sqrt[3]{bc-ad} \log(c+dx^3)}{6c\sqrt[3]{d}} - \frac{\sqrt[3]{a} \text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, \sqrt[3]{a+bx^3} \right)}{2c} - \frac{a^{2/3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, \sqrt[3]{a+bx^3} \right)}{2c} \\
&= -\frac{\sqrt[3]{a} \log(x)}{2c} - \frac{\sqrt[3]{bc-ad} \log(c+dx^3)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{a} \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2c} + \frac{\sqrt[3]{bc-ad} \log\left(\sqrt[3]{bc-ad} - \sqrt[3]{bc-ad} \sqrt[3]{a+bx^3}\right)}{2c} \\
&= -\frac{\sqrt[3]{a} \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3}c} - \frac{\sqrt[3]{bc-ad} \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}} \right)}{\sqrt{3}c\sqrt[3]{d}} - \frac{\sqrt[3]{a} \log(x)}{2c} - \frac{\sqrt[3]{bc-ad} \log(c+dx^3)}{6c\sqrt[3]{d}}
\end{aligned}$$

Mathematica [A] time = 0.47, size = 268, normalized size = 1.09

$$\frac{\sqrt[3]{bc-ad} \left(-\log\left(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + a^{2/3}(a+bx^3)^{2/3}\right) + 2 \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}-1}{\sqrt[3]{bc-ad}\sqrt{3}}\right) \right)}{\sqrt[3]{d}} - \sqrt[3]{a} \left(\log\left(a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}\right) - 2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(1/3)/(x*(c + d*x^3)), x]

[Out] $-(a^{1/3}*(2*\text{Sqrt}[3]*\text{ArcTan}[(1 + (2*(a + b*x^3)^{1/3}))/a^{1/3}])/\text{Sqrt}[3]) - 2*\text{Log}[a^{1/3} - (a + b*x^3)^{1/3}] + \text{Log}[a^{2/3} + a^{1/3}*(a + b*x^3)^{1/3} + (a + b*x^3)^{2/3}] + ((b*c - a*d)^{1/3}*(2*\text{Sqrt}[3]*\text{ArcTan}[-1 + (2*d^{1/3}*(a + b*x^3)^{1/3}]/(b*c - a*d)^{1/3}))/\text{Sqrt}[3] + 2*\text{Log}[(b*c - a*d)^{1/3} + d^{1/3}*(a + b*x^3)^{1/3}] - \text{Log}[(b*c - a*d)^{2/3} - d^{1/3}*(b*c - a*d)^{1/3}*(a + b*x^3)^{1/3} + d^{2/3}*(a + b*x^3)^{2/3}])/d^{1/3}/(6*c)$

IntegrateAlgebraic [A] time = 0.44, size = 334, normalized size = 1.36

$$\frac{\sqrt[3]{a} \log\left(a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}\right)}{6c} - \frac{\sqrt[3]{bc-ad} \log\left(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + a^{2/3}(a+bx^3)^{2/3}\right)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{bc-ad} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{3c\sqrt[3]{d}} - \frac{\sqrt[3]{bc-ad} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}c\sqrt[3]{d}} + \frac{\sqrt[3]{a} \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{a}\right)}{3c} - \frac{\sqrt[3]{a} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+1}{\sqrt{3}}\right)}{\sqrt{3}c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]

[Out] $-\left(\frac{a^{1/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(a + b x^3)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} c}\right) - \left(\frac{(b c - a d)^{1/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/3} (a + b x^3)^{1/3}}{\sqrt{3} (b c - a d)^{1/3}}\right]}{\sqrt{3} c d^{1/3}}\right) + \frac{a^{1/3} \operatorname{Log}\left[-a^{1/3} + (a + b x^3)^{1/3}\right]}{3 c} + \frac{(b c - a d)^{1/3} \operatorname{Log}\left[(b c - a d)^{1/3} + d^{1/3} (a + b x^3)^{1/3}\right]}{3 c d^{1/3}} - \frac{a^{1/3} \operatorname{Log}\left[a^{2/3} + a^{1/3} (a + b x^3)^{1/3} + (a + b x^3)^{2/3}\right]}{6 c} - \frac{(b c - a d)^{1/3} \operatorname{Log}\left[(b c - a d)^{2/3} - d^{1/3} (b c - a d)^{1/3} (a + b x^3)^{1/3} + d^{2/3} (a + b x^3)^{2/3}\right]}{6 c d^{1/3}}$

fricas [A] time = 0.45, size = 276, normalized size = 1.12

$$\frac{2\sqrt{3}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} - \sqrt{3}(bc-ad)}{3(bc-ad)}\right) + 2\sqrt{3}a^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + \sqrt{3}a}{3a}\right) + a^{\frac{1}{3}} \log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right) + \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left((bx^3+a)^{\frac{2}{3}} - (bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right) - 2a^{\frac{1}{3}} \log\left((bx^3+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right) - 2\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left((bx^3+a)^{\frac{1}{3}} + \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="fricas")

[Out] $-\frac{1}{6} * (2 * \sqrt{3}) * \left(\frac{b c - a d}{d}\right)^{\frac{1}{3}} * \arctan\left(-\frac{1}{3} * (2 * \sqrt{3}) * \frac{(b x^3 + a)^{\frac{1}{3}} * d * \left(\frac{b c - a d}{d}\right)^{\frac{2}{3}} - \sqrt{3} * (b c - a d)}{(b c - a d)} + 2 * \sqrt{3} * \frac{a^{\frac{1}{3}} * \arctan\left(\frac{1}{3} * (2 * \sqrt{3}) * \frac{(b x^3 + a)^{\frac{1}{3}} * a^{\frac{2}{3}} + \sqrt{3} * a}{a} + a^{\frac{1}{3}} * \log\left(\frac{(b x^3 + a)^{\frac{2}{3}} + (b x^3 + a)^{\frac{1}{3}} * a^{\frac{1}{3}} + a^{\frac{2}{3}}\right) + \left(\frac{b c - a d}{d}\right)^{\frac{1}{3}} * \log\left(\frac{(b x^3 + a)^{\frac{2}{3}} - (b x^3 + a)^{\frac{1}{3}} * \left(\frac{b c - a d}{d}\right)^{\frac{1}{3}} + \left(\frac{b c - a d}{d}\right)^{\frac{2}{3}}\right) - 2 * a^{\frac{1}{3}} * \log\left(\frac{(b x^3 + a)^{\frac{1}{3}} - a^{\frac{1}{3}}}{(b c - a d)^{\frac{1}{3}}}\right) - 2 * \left(\frac{b c - a d}{d}\right)^{\frac{1}{3}} * \log\left(\frac{(b x^3 + a)^{\frac{1}{3}} + \left(\frac{b c - a d}{d}\right)^{\frac{1}{3}}}{(b c - a d)^{\frac{1}{3}}}\right)}{c}\right)$

giac [A] time = 0.80, size = 311, normalized size = 1.26

$$\frac{(bc-ad)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left(\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right) + \sqrt{3} a^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3a}\right) + a^{\frac{1}{3}} \log\left(\frac{(bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}}{6c}\right) + a^{\frac{1}{3}} \log\left(\frac{(bx^3+a)^{\frac{1}{3}} - a^{\frac{1}{3}}}{3c}\right) + \frac{\sqrt{3}(-bc d^2 + a d^3)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}} + \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right) + (-bc d^2 + a d^3)^{\frac{1}{3}} \log\left(\frac{(bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}}{6cd}\right)}{3(bc^2 - acd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="giac")

[Out] $-\frac{1}{3} * (b c - a d) * \left(-\frac{(b c - a d)}{d}\right)^{\frac{1}{3}} * \log\left(\frac{\operatorname{abs}\left(\frac{(b x^3 + a)^{\frac{1}{3}}}{(b c - a d)^{\frac{1}{3}}}\right) - \left(-\frac{(b c - a d)}{d}\right)^{\frac{1}{3}}}{(b c^2 - a c d)} - \frac{1}{3} * \sqrt{3} * \frac{a^{\frac{1}{3}} * \arctan\left(\frac{1}{3} * \sqrt{3} * \frac{2 * (b x^3 + a)^{\frac{1}{3}} + a^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right)}{c} - \frac{1}{6} * a^{\frac{1}{3}} * \log\left(\frac{(b x^3 + a)^{\frac{2}{3}} + (b x^3 + a)^{\frac{1}{3}} * a^{\frac{1}{3}} + a^{\frac{2}{3}}}{c} + \frac{1}{3} * a^{\frac{1}{3}} * \log\left(\frac{\operatorname{abs}\left(\frac{(b x^3 + a)^{\frac{1}{3}}}{(b c - a d)^{\frac{1}{3}}}\right) - a^{\frac{1}{3}}}{c} + \frac{1}{3} * \sqrt{3} * \frac{(-b c d^2 + a d^3)^{\frac{1}{3}} * \arctan\left(\frac{1}{3} * \sqrt{3} * \frac{2 * (b x^3 + a)^{\frac{1}{3}} + \left(-\frac{(b c - a d)}{d}\right)^{\frac{1}{3}}}{\left(-\frac{(b c - a d)}{d}\right)^{\frac{1}{3}}}\right)}{(c * d)} + \frac{1}{6} * \frac{(-b c d^2 + a d^3)^{\frac{1}{3}} * \log\left(\frac{(b x^3 + a)^{\frac{2}{3}} + (b x^3 + a)^{\frac{1}{3}} * \left(-\frac{(b c - a d)}{d}\right)^{\frac{1}{3}} + \left(-\frac{(b c - a d)}{d}\right)^{\frac{2}{3}}}{(c * d)}\right)}{c * d}\right)$

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(1/3)/x/(d*x^3+c),x)

[Out] int((b*x^3+a)^(1/3)/x/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x), x)

mupad [B] time = 4.74, size = 1607, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(1/3)/(x*(c + d*x^3)),x)

[Out] log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) - (a/(27*c^3))^(1/3)*(((243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a/(27*c^3))^(1/3) - (a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4))*(a/(27*c^3))^(2/3) - 9*a*b^7*c^4*d^2 + 27*a^2*b^6*c^3*d^3 - 18*a^3*b^5*c^2*d^4))*(a/(27*c^3))^(1/3) + log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) - (((243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a*d - b*c)/(27*c^3*d))^(1/3) - (a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4)*(-(a*d - b*c)/(27*c^3*d))^(2/3) - 9*a*b^7*c^4*d^2 + 27*a^2*b^6*c^3*d^3 - 18*a^3*b^5*c^2*d^4)*(-(a*d - b*c)/(27*c^3*d))^(1/3))*(-(a*d - b*c)/(27*c^3*d))^(1/3) + log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) + ((3^(1/2)*1i)/2 - 1/2)*(-(a*d - b*c)/(27*c^3*d))^(1/3)*(((3^(1/2)*1i)/2 - 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4) - ((3^(1/2)*1i)/2 - 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a*d - b*c)/(27*c

```

^3*d))^(1/3))*(-(a*d - b*c)/(27*c^3*d))^(2/3) + 9*a*b^7*c^4*d^2 - 27*a^2*b^
6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 - 1/2)*(-(a*d - b*c)/(27*c
^3*d))^(1/3) - log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*
a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) - ((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b*c)/(
27*c^3*d))^(1/3)*(((3^(1/2)*1i)/2 + 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5
*d^3 - 81*a^2*b^5*c^4*d^4) + ((3^(1/2)*1i)/2 + 1/2)*(243*a*b^6*c^6*d^3 - 72
9*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5))*(-(a*d - b*c)/(27*c^3*d))^(1/3))*(-
(a*d - b*c)/(27*c^3*d))^(2/3) + 9*a*b^7*c^4*d^2 - 27*a^2*b^6*c^3*d^3 + 18*
a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b*c)/(27*c^3*d))^(1/3) +
log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 +
9*a^2*b^6*c^2*d^3) + ((3^(1/2)*1i)/2 - 1/2)*(a/(27*c^3))^(1/3)*(((3^(1/2)*
1i)/2 - 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4) -
((3^(1/2)*1i)/2 - 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*
b^4*c^4*d^5)*(a/(27*c^3))^(1/3))*(a/(27*c^3))^(2/3) + 9*a*b^7*c^4*d^2 - 27*
a^2*b^6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 - 1/2)*(a/(27*c^3))^(
1/3) - log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5
*c*d^4 + 9*a^2*b^6*c^2*d^3) - ((3^(1/2)*1i)/2 + 1/2)*(a/(27*c^3))^(1/3)*(((
3^(1/2)*1i)/2 + 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^
4*d^4) + ((3^(1/2)*1i)/2 + 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 +
486*a^3*b^4*c^4*d^5)*(a/(27*c^3))^(1/3))*(a/(27*c^3))^(2/3) + 9*a*b^7*c^4*d
^2 - 27*a^2*b^6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 + 1/2)*(a/(2
7*c^3))^(1/3)

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{x(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(1/3)/x/(d*x**3+c),x)

[Out] Integral((a + b*x**3)**(1/3)/(x*(c + d*x**3)), x)

$$3.419 \quad \int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)} dx$$

Optimal. Leaf size=340

$$\frac{(bc-3ad) \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2} - \frac{(bc-3ad) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}c^2} - \frac{\log(x)(bc-3ad)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad} \log(c+dx^3)}{6c^2}$$

Rubi [A] time = 0.38, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {446, 103, 156, 50, 57, 617, 204, 31, 58}

$$\frac{(bc-3ad) \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2} - \frac{(bc-3ad) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}c^2} - \frac{\log(x)(bc-3ad)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad} \log(c+dx^3)}{6c^2} - \frac{d^{2/3}\sqrt[3]{bc-ad} \log\left(\frac{\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}}{2c^2}\right)}{2c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad} \tan^{-1}\left(\frac{1-2\frac{\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2} + \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{\sqrt[3]{a+bx^3}(bc-3ad)}{3ac^2} - \frac{(a+bx^3)^{4/3}}{3acx^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)), x]

[Out] (d*(a + b*x^3)^(1/3))/c^2 + ((b*c - 3*a*d)*(a + b*x^3)^(1/3))/(3*a*c^2) - (a + b*x^3)^(4/3)/(3*a*c*x^3) - ((b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(2/3)*c^2) + (d^(2/3)*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sqrt[3]*c^2) - ((b*c - 3*a*d)*Log[x])/(6*a^(2/3)*c^2) + (d^(2/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c^2) + ((b*c - 3*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(2/3)*c^2) - (d^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m+1)*(c + d*x)^n)/(b*(m+n+1)), x] + Dist[(n*(b*c - a*d))/(b*(m+n+1)), Int[(a + b*x)^m*(c + d*x)^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x]]) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{x^2(c+dx)} dx, x, x^3 \right) \\
 &= -\frac{(a+bx^3)^{4/3}}{3acx^3} - \frac{\text{Subst} \left(\int \frac{\sqrt[3]{a+bx} \left(\frac{1}{3}(-bc+3ad) - \frac{bdx}{3} \right)}{x(c+dx)} dx, x, x^3 \right)}{3ac} \\
 &= -\frac{(a+bx^3)^{4/3}}{3acx^3} + \frac{d^2 \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{c+dx} dx, x, x^3 \right)}{3c^2} + \frac{(bc-3ad) \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{x} dx, x, x^3 \right)}{9ac^2} \\
 &= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{(a+bx^3)^{4/3}}{3acx^3} + \frac{(bc-3ad) \text{Subst} \left(\int \frac{1}{x(a+bx)^{2/3}} dx, x, x^3 \right)}{9c^2} \\
 &= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{(a+bx^3)^{4/3}}{3acx^3} - \frac{(bc-3ad) \log(x)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad} \log}{6c^2} \\
 &= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{(a+bx^3)^{4/3}}{3acx^3} - \frac{(bc-3ad) \log(x)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad} \log}{6c^2} \\
 &= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{(a+bx^3)^{4/3}}{3acx^3} - \frac{(bc-3ad) \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{3\sqrt{3} a^{2/3} c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad} \log}{6c^2}
 \end{aligned}$$

Mathematica [A] time = 1.24, size = 366, normalized size = 1.08

$$\frac{(bc-3ad) \left(3\sqrt[3]{a+bx^3} - \frac{1}{2}\sqrt[3]{a} \left(\log(a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}) - 2\log(\sqrt[3]{a} - \sqrt[3]{a+bx^3}) + 2\sqrt{3} \tan^{-1} \left(\frac{2\sqrt[3]{a+bx^3} + 1}{\sqrt{3}} \right) \right) \right)}{3c} + \frac{ad^{2/3} \left(\sqrt[3]{bc-ad} \log \left(-\sqrt[3]{d}\sqrt[3]{a+bx^3} - \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3} \right) - 2\sqrt[3]{bc-ad} \log \left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3} \right) - 2\sqrt{3}\sqrt[3]{bc-ad} \tan^{-1} \left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3} - 1}{\sqrt{3}} \right) + 6\sqrt[3]{d}\sqrt[3]{a+bx^3} \right)}{3ac} - \frac{(a+bx^3)^{4/3}}{3c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]

[Out]
$$\begin{aligned} & \left(-\frac{(a + b x^3)^{4/3}}{x^3} + \frac{((b c - 3 a d) (3 (a + b x^3)^{1/3} - a^{1/3}) (2 \sqrt{3} \operatorname{ArcTan}\left[\frac{1 + (2 (a + b x^3)^{1/3})}{a^{1/3}}\right]) - 2 \operatorname{Log}\left[\frac{a^{1/3} - (a + b x^3)^{1/3}}{a^{1/3} + (a + b x^3)^{1/3}}\right] + \operatorname{Log}\left[\frac{a^{2/3} + a^{1/3} (a + b x^3)^{1/3} + (a + b x^3)^{2/3}}{a^{2/3} - a^{1/3} (a + b x^3)^{1/3} + (a + b x^3)^{2/3}}\right])}{3 c} \right. \\ & \left. + \frac{(a d^{2/3} (6 d^{1/3} (a + b x^3)^{1/3} - 2 \sqrt{3} \operatorname{ArcTan}\left[\frac{b c - a d}{d^{1/3} + (2 d^{1/3} (a + b x^3)^{1/3})}\right]) - 2 (b c - a d)^{1/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} + d^{1/3} (a + b x^3)^{1/3}}{(b c - a d)^{1/3} - d^{1/3} (a + b x^3)^{1/3}}\right] + (b c - a d)^{1/3} \operatorname{Log}\left[\frac{(b c - a d)^{2/3} - d^{1/3} (b c - a d)^{1/3} (a + b x^3)^{1/3} + d^{2/3} (a + b x^3)^{2/3}}{(b c - a d)^{2/3} + d^{1/3} (b c - a d)^{1/3} (a + b x^3)^{1/3} + d^{2/3} (a + b x^3)^{2/3}}\right])}{2 c} \right) / (3 a c) \end{aligned}$$

IntegrateAlgebraic [A] time = 0.67, size = 383, normalized size = 1.13

$$\frac{(bc - 3ad) \log\left(\frac{\sqrt{a + bx^3} - \sqrt{a}}{\sqrt{a + bx^3} + \sqrt{a}}\right) + (3ad - bc) \log\left(\frac{a^{2/3} + \sqrt{a} \sqrt{a + bx^3} + (a + bx^3)^{2/3}}{18a^{2/3}c^2}\right) + (3ad - bc) \tan^{-1}\left(\frac{2\sqrt{a + bx^3} + \sqrt{a}}{\sqrt{a + bx^3}}\right)}{3\sqrt{3}a^{2/3}c^2} - \frac{d^{2/3}\sqrt{bc - ad} \log\left(\frac{\sqrt{bc - ad} + \sqrt{d} \sqrt{a + bx^3}}{3c^2}\right) + d^{2/3}\sqrt{bc - ad} \log\left(\frac{-\sqrt{d} \sqrt{a + bx^3} \sqrt{bc - ad} + (bc - ad)^{2/3} + d^{2/3}(a + bx^3)^{2/3}}{6c^2}\right) + d^{2/3}\sqrt{bc - ad} \tan^{-1}\left(\frac{1}{\sqrt{3}} \frac{2\sqrt{a + bx^3}}{\sqrt{a + bx^3} - \sqrt{bc - ad}}\right)}{3c^2} - \frac{\sqrt{a + bx^3}}{3ax^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]

[Out]
$$\begin{aligned} & -\frac{1}{3} \frac{(a + b x^3)^{1/3}}{c x^3} + \frac{((-b c) + 3 a d) \operatorname{ArcTan}\left[\frac{a^{1/3} + 2(a + b x^3)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{(3 \sqrt{3} a^{1/3} c^2)} + \frac{d^{2/3} (b c - a d)^{1/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/3} (a + b x^3)^{1/3}}{\sqrt{3} (b c - a d)^{1/3}}\right]}{\sqrt{3} c^2} + \frac{((b c - 3 a d) \operatorname{Log}\left[\frac{-a^{1/3} + (a + b x^3)^{1/3}}{9 a^{2/3} c^2}\right] - (d^{2/3} (b c - a d)^{1/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3}}{d^{1/3} (a + b x^3)^{1/3}}\right])}{(3 c^2)} \right. \\ & \left. + \frac{((-b c) + 3 a d) \operatorname{Log}\left[\frac{a^{2/3} + a^{1/3} (a + b x^3)^{1/3} + (a + b x^3)^{2/3}}{18 a^{2/3} c^2}\right] + (d^{2/3} (b c - a d)^{1/3} \operatorname{Log}\left[\frac{(b c - a d)^{2/3} - d^{1/3} (b c - a d)^{1/3} (a + b x^3)^{1/3} + d^{2/3} (a + b x^3)^{2/3}}{6 c^2}\right])}{(6 c^2)} \end{aligned}$$

fricas [A] time = 0.56, size = 429, normalized size = 1.26

$$\frac{c \sqrt{3} (bc - ad)^{1/3} x^2 \operatorname{arctan}\left(\frac{2 \sqrt{a + bx^3} + \sqrt{a}}{\sqrt{a + bx^3}}\right) - 3 (bc - ad)^{1/3} x^2 \operatorname{Log}\left(\frac{(bc - ad)^{1/3} + d^{1/3} (a + bx^3)^{1/3}}{(bc - ad)^{1/3} - d^{1/3} (a + bx^3)^{1/3}}\right) - 6 (bc - ad)^{1/3} x^2 \operatorname{Log}\left(\frac{(bc - ad)^{2/3} - d^{1/3} (bc - ad)^{1/3} (a + bx^3)^{1/3} + d^{2/3} (a + bx^3)^{2/3}}{(bc - ad)^{2/3} + d^{1/3} (bc - ad)^{1/3} (a + bx^3)^{1/3} + d^{2/3} (a + bx^3)^{2/3}}\right) + 2 \sqrt{3} (bc - 3ad)^{1/3} \sqrt{c - d} \operatorname{arctan}\left(\frac{1}{\sqrt{3}} \frac{2 \sqrt{a + bx^3} + \sqrt{a}}{\sqrt{a + bx^3} - \sqrt{bc - ad}}\right) + (d^{2/3} (bc - 3ad)^{1/3} \operatorname{Log}\left(\frac{(bc - ad)^{1/3} + d^{1/3} (a + bx^3)^{1/3}}{(bc - ad)^{1/3} - d^{1/3} (a + bx^3)^{1/3}}\right) - 2 (d^{2/3} (bc - 3ad)^{1/3} \operatorname{Log}\left(\frac{(bc - ad)^{2/3} - d^{1/3} (bc - ad)^{1/3} (a + bx^3)^{1/3} + d^{2/3} (a + bx^3)^{2/3}}{(bc - ad)^{2/3} + d^{1/3} (bc - ad)^{1/3} (a + bx^3)^{1/3} + d^{2/3} (a + bx^3)^{2/3}}\right))}{18 a^{2/3} c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -\frac{1}{18} \frac{(6 \sqrt{3} (-b c d^2 + a d^3)^{1/3} a^2 x^3 \operatorname{arctan}\left(\frac{-1/3 (2 \sqrt{3} (-b c d^2 + a d^3)^{1/3} (b x^3 + a)^{1/3} - \sqrt{3} (b c d - a d^2))}{(b c d - a d^2)}\right) + 3 (-b c d^2 + a d^3)^{1/3} a^2 x^3 \operatorname{Log}\left(\frac{(b x^3 + a)^{2/3} d^2 + (-b c d^2 + a d^3)^{1/3} (b x^3 + a)^{1/3} d + (-b c d^2 + a d^3)^{2/3}}{(b c d - a d^2) d - (-b c d^2 + a d^3)^{1/3}}\right) - 6 (-b c d^2 + a d^3)^{1/3} a^2 x^3 \operatorname{Log}\left(\frac{(b x^3 + a)^{1/3} d - (-b c d^2 + a d^3)^{1/3}}{(b c d - a d^2) d - (-b c d^2 + a d^3)^{1/3}}\right) + 2 \sqrt{3} (a b c - 3 a^2 d) x^3 \sqrt{-(-a^2)^{1/3}} \operatorname{arctan}\left(\frac{-1/3 (\sqrt{3} (-a^2)^{1/3} a - 2 \sqrt{3} (b x^3 + a)^{1/3} (-a^2)^{2/3})}{(-(-a^2)^{1/3})/a^2} + (-a^2)^{2/3} (b c - 3 a d) x^3 \operatorname{Log}\left(\frac{(b x^3 + a)^{2/3}}{a} - (-a^2)^{1/3} a + (b x^3 + a)^{1/3} (-a^2)^{2/3}\right) - 2 (-a^2)^{2/3} (b c} \right. \end{aligned}$$

$- 3*a*d)*x^3*\log((b*x^3 + a)^{(1/3)}*a - (-a^2)^{(2/3)}) + 6*(b*x^3 + a)^{(1/3)}*a^2*c)/(a^2*c^2*x^3)$

giac [A] time = 0.78, size = 351, normalized size = 1.03

$$\frac{(bc-d) \left(\frac{bc-d}{a} \right)^{\frac{1}{3}} \log \left(\left((bx^3+a)^{\frac{1}{3}} - \left(\frac{bc-d}{a} \right)^{\frac{1}{3}} \right) \right)}{3(bc^3-a^2d)} - \frac{\sqrt{3}(bc-3ad) \arctan \left(\frac{\sqrt{3} \left((bx^3+a)^{\frac{1}{3}} + a^{\frac{1}{3}} \right)}{3a^{\frac{1}{3}}} \right)}{9a^{\frac{1}{3}}c^2} - \frac{\sqrt{3}(-bcd+ad)^{\frac{1}{3}} \arctan \left(\frac{\sqrt{3} \left((bx^3+a)^{\frac{1}{3}} - \left(\frac{bc-d}{a} \right)^{\frac{1}{3}} \right)}{3 \left(\frac{bc-d}{a} \right)^{\frac{1}{3}}} \right)}{3c^2} - \frac{(bc-3ad) \log \left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}} \right)}{18a^{\frac{1}{3}}c^2} - \frac{(-bcd+ad)^{\frac{1}{3}} \log \left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}} \left(\frac{bc-d}{a} \right)^{\frac{1}{3}} + \left(\frac{bc-d}{a} \right)^{\frac{2}{3}} \right)}{6c^2} + \frac{(bc-3ad) \log \left((bx^3+a)^{\frac{1}{3}} - a^{\frac{1}{3}} \right)}{9a^{\frac{1}{3}}c^2} + \frac{(bx^3+a)^{\frac{1}{3}}}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="giac")

[Out] $\frac{1}{3}*(b*c*d - a*d^2)*(-b*c - a*d)/d)^{(1/3)}*\log(\text{abs}((b*x^3 + a)^{(1/3)} - (b*c - a*d)/d)^{(1/3)))/(b*c^3 - a*c^2*d) - 1/9*\text{sqrt}(3)*(b*c - 3*a*d)*\arctan(1/3*\text{sqrt}(3)*(2*(b*x^3 + a)^{(1/3)} + a^{(1/3)})/a^{(1/3)})/(a^{(2/3)}*c^2) - 1/3*\text{sqrt}(3)*(-b*c*d^2 + a*d^3)^{(1/3)}*\arctan(1/3*\text{sqrt}(3)*(2*(b*x^3 + a)^{(1/3)} + (-b*c - a*d)/d)^{(1/3)})/(-b*c - a*d)/d)^{(1/3))/c^2 - 1/18*(b*c - 3*a*d)*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*a^{(1/3)} + a^{(2/3)})/(a^{(2/3)}*c^2) - 1/6*(-b*c*d^2 + a*d^3)^{(1/3)}*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(-b*c - a*d)/d)^{(1/3)} + (-b*c - a*d)/d)^{(2/3))/c^2 + 1/9*(b*c - 3*a*d)*\log(\text{abs}((b*x^3 + a)^{(1/3)} - a^{(1/3)})/(a^{(2/3)}*c^2) - 1/3*(b*x^3 + a)^{(1/3)}/(c*x^3)$

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x)

[Out] int((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^4), x)

mupad [B] time = 9.99, size = 1917, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a + b*x^3)^{(1/3)}/(x^4*(c + d*x^3)),x)$

[Out] $\log(-(((27*b^5*c^3*d^3*(a + b*x^3)^{(1/3)}*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-3*a*d - b*c)^3/(a^2*c^6))^{(1/3)}*(-(3*a*d - b*c)^3/(a^2*c^6))^{(2/3)})/81 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*(-3*a*d - b*c)^3/(a^2*c^6))^{(1/3)})/9 - (2*b^4*d^5*(a + b*x^3)^{(1/3)}*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*(-27*a^3*d^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^{(1/3)} + \log(-(((27*b^5*c^3*d^3*(a + b*x^3)^{(1/3)}*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) - 81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^6))^{(1/3)})*((d^2*(a*d - b*c))/c^6)^{(2/3)})/9 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^{(1/3)})/3 - (2*b^4*d^5*(a + b*x^3)^{(1/3)}*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((a*d^3 - b*c*d^2)/(27*c^6))^{(1/3)} + \log(((3^{(1/2)*1i})/2 - 1/2)*(((3^{(1/2)*1i})/2 + 1/2)*(27*b^5*c^3*d^3*(a + b*x^3)^{(1/3)}*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) - 81*a*b^4*c^4*d^3*((3^{(1/2)*1i})/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^6))^{(1/3)})*((d^2*(a*d - b*c))/c^6)^{(2/3)})/9 + (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^{(1/3)})/3 - (2*b^4*d^5*(a + b*x^3)^{(1/3)}*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((3^{(1/2)*1i})/2 - 1/2)*((a*d^3 - b*c*d^2)/(27*c^6))^{(1/3)} - \log((2*b^4*d^5*(a + b*x^3)^{(1/3)}*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4) - (((3^{(1/2)*1i})/2 + 1/2)*(((3^{(1/2)*1i})/2 - 1/2)*(27*b^5*c^3*d^3*(a + b*x^3)^{(1/3)}*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) + 81*a*b^4*c^4*d^3*((3^{(1/2)*1i})/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^6))^{(1/3)})*((d^2*(a*d - b*c))/c^6)^{(2/3)})/9 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^{(1/3)})/3)*((3^{(1/2)*1i})/2 + 1/2)*((a*d^3 - b*c*d^2)/(27*c^6))^{(1/3)} + \log(((3^{(1/2)*1i})/2 - 1/2)*(((3^{(1/2)*1i})/2 + 1/2)*(27*b^5*c^3*d^3*(a + b*x^3)^{(1/3)}*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) - 27*a*b^4*c^4*d^3*((3^{(1/2)*1i})/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-3*a*d - b*c)^3/(a^2*c^6))^{(1/3)})*(-3*a*d - b*c)^3/(a^2*c^6))^{(2/3)})/81 + (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*(-3*a*d - b*c)^3/(a^2*c^6))^{(1/3)})/9 - (2*b^4*d^5*(a + b*x^3)^{(1/3)}*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((3^{(1/2)*1i})/2 - 1/2)*(-27*a^3*d^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^{(1/3)} - \log((2*b^4*d^5*(a + b*x^3)^{(1/3)}*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4) - (((3^{(1/2)*1i})/2 + 1/2)*(((3^{(1/2)*1i})/2 - 1/2)*(27*b^5*c^3*d^3*(a + b*x^3)^{(1/3)}*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) + 27*a*b^4*c^4*d^3*((3^{(1/2)*1i})/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 -$

$$\frac{3* a * b * c * d * (- (3 * a * d - b * c)^3 / (a^2 * c^6))^{1/3} * (- (3 * a * d - b * c)^3 / (a^2 * c^6))^{2/3}}{81} - \frac{(b^5 * d^4 * (27 * a^3 * d^3 + b^3 * c^3 + 17 * a * b^2 * c^2 * d - 45 * a^2 * b * c * d^2)) / (3 * c) * (- (3 * a * d - b * c)^3 / (a^2 * c^6))^{1/3}}{9} * ((3^{1/2} * 1i) / 2 + 1/2) * \frac{(- (27 * a^3 * d^3 - b^3 * c^3 + 9 * a * b^2 * c^2 * d - 27 * a^2 * b * c * d^2)) / (729 * a^2 * c^6))^{1/3}}{3} - \frac{(a + b * x^3)^{1/3}}{3 * c * x^3}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{x^4 (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(1/3)/x**4/(d*x**3+c), x)

[Out] Integral((a + b*x**3)**(1/3)/(x**4*(c + d*x**3)), x)

$$3.420 \quad \int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)} dx$$

Optimal. Leaf size=370

$$\frac{(-9a^2d^2 + 3abcd + b^2c^2) \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3} + \frac{(-9a^2d^2 + 3abcd + b^2c^2) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}c^3} + \frac{\log(x)(-9a^2d^2 + 3abcd + b^2c^2)}{18a^{5/3}c^3}$$

Rubi [A] time = 0.49, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {446, 103, 149, 156, 57, 617, 204, 31, 58}

$$\frac{(-9a^2d^2 + 3abcd + b^2c^2) \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3} + \frac{(-9a^2d^2 + 3abcd + b^2c^2) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}c^3} + \frac{\log(x)(-9a^2d^2 + 3abcd + b^2c^2)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad} \log(c+dx^3)}{6c^3} + \frac{d^{5/3}\sqrt[3]{bc-ad} \log\left(\frac{\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}}{2c^3}\right)}{2c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c^3} + \frac{\sqrt[3]{a+bx^3}(3ad+bc)}{9ac^2x^3} - \frac{(a+bx^3)^{4/3}}{6acx^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)), x]

[Out] ((b*c + 3*a*d)*(a + b*x^3)^(1/3))/(9*a*c^2*x^3) - (a + b*x^3)^(4/3)/(6*a*c*x^6) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(5/3)*c^3) - (d^(5/3)*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sqrt[3]*c^3) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*Log[x])/(18*a^(5/3)*c^3) - (d^(5/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c^3) - ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(5/3)*c^3) + (d^(5/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^3)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])]/; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
x]]) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_)*((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)} dx = \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{x^3(c+dx)} dx, x, x^3 \right)$$

$$= -\frac{(a+bx^3)^{4/3}}{6acx^6} - \frac{\text{Subst} \left(\int \frac{\sqrt[3]{a+bx} \left(\frac{2}{3}(bc+3ad) + \frac{2bdx}{3} \right)}{x^2(c+dx)} dx, x, x^3 \right)}{6ac}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{(a+bx^3)^{4/3}}{6acx^6} - \frac{\text{Subst} \left(\int \frac{\frac{2}{9}(b^2c^2+3abcd-9a^2d^2) + \frac{2}{9}bd(bc-6ad)x}{x(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{6ac^2}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{(a+bx^3)^{4/3}}{6acx^6} + \frac{(d^2(bc-ad)) \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3c^3} - \frac{(b^2c^2+3abcd-9a^2d^2) \log(x)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad} \log(c)}{6c^3}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{(a+bx^3)^{4/3}}{6acx^6} + \frac{(b^2c^2+3abcd-9a^2d^2) \log(x)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad} \log(c)}{6c^3}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{(a+bx^3)^{4/3}}{6acx^6} + \frac{(b^2c^2+3abcd-9a^2d^2) \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{9\sqrt{3}a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad} \log(c)}{6c^3}$$

Mathematica [A] time = 1.84, size = 411, normalized size = 1.11

$$\frac{2(-9a^2d^2+3abcd+d^2)^2 \left(3\sqrt[3]{a+bx^3} - \frac{1}{2} \sqrt[3]{c} \left(\log(a^{2/3} + \sqrt[3]{c} \sqrt[3]{a+bx^3} + (a+bx^3)^{2/3} \right) - 2 \log(\sqrt[3]{c} - \sqrt[3]{a+bx^3}) + 2\sqrt{3} \tan^{-1} \left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{c}}{\sqrt{3}} \right) \right)}{9ac^2} + \frac{a^{5/3} \left(\sqrt[3]{bc-ad} \log(-\sqrt[3]{c} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} (a+bx^3)^{2/3} \right) - 2 \sqrt[3]{bc-ad} \log(\sqrt[3]{bc-ad} + \sqrt[3]{c} \sqrt[3]{a+bx^3}) - 2\sqrt{3} \sqrt[3]{bc-ad} \tan^{-1} \left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{c}}{\sqrt{3}} \right) + 6 \sqrt[3]{c} \sqrt[3]{a+bx^3} \right)}{6ac} - \frac{2(a+bx^3)^{4/3} (3ad+bc)}{3ac^3} + \frac{(a+bx^3)^{4/3}}{6c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]

[Out]
$$-1/6*((a + b*x^3)^{(4/3)}/x^6 - (2*(b*c + 3*a*d)*(a + b*x^3)^{(4/3)})/(3*a*c*x^3) + (2*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(3*(a + b*x^3)^{(1/3)} - (a^{(1/3)}*(2*\sqrt{3}*\text{ArcTan}[(1 + (2*(a + b*x^3)^{(1/3)})/a^{(1/3)})/\sqrt{3}]) - 2*\text{Log}[a^{(1/3)} - (a + b*x^3)^{(1/3)}] + \text{Log}[a^{(2/3)} + a^{(1/3)}*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}]))/2)/(9*a*c^2) + (a*d^{(5/3)}*(6*d^{(1/3)}*(a + b*x^3)^{(1/3)} - 2*\sqrt{3}*(b*c - a*d)^{(1/3)}*\text{ArcTan}[-1 + (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})/(b*c - a*d)^{(1/3)})/\sqrt{3}] - 2*(b*c - a*d)^{(1/3)}*\text{Log}[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)}] + (b*c - a*d)^{(1/3)}*\text{Log}[(b*c - a*d)^{(2/3)} - d^{(1/3)}*(b*c - a*d)^{(1/3)}*(a + b*x^3)^{(1/3)} + d^{(2/3)}*(a + b*x^3)^{(2/3)}]))/c^2)/(a*c)$$

IntegrateAlgebraic [A] time = 0.81, size = 448, normalized size = 1.21

$$\frac{(9a^2d^2 - 3abcd - b^2c^2)\log(\sqrt{a+bx^3} - \sqrt{c})}{27a^2c^2} + \frac{(-9a^2d^2 + 3abcd + b^2c^2)\log(\sqrt{a+bx^3} + \sqrt{c})}{54a^2c^2} + \frac{(9a^2d^2 - 3abcd - b^2c^2)\tan^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{c}}\right)}{9\sqrt{a}d^{5/3}c^2} + \frac{d^{5/3}\sqrt{c-ad}\log\left(\frac{\sqrt{c-ad} + \sqrt{d}\sqrt{a+bx^3}}{3c^2}\right)}{3c^2} + \frac{d^{5/3}\sqrt{c-ad}\log\left(\frac{-\sqrt{d}\sqrt{a+bx^3} + \sqrt{c-ad}}{6c^2}\right)}{6c^2} + \frac{d^{5/3}\sqrt{c-ad}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}\frac{\sqrt{a+bx^3}}{\sqrt{c}}\right)}{\sqrt{3}c^2} + \frac{\sqrt{a+bx^3}(-3ac + 6ads^2 - bc^2)}{18a^2c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]

[Out]
$$\left(\frac{(a + b*x^3)^{(1/3)}*(-3*a*c - b*c*x^3 + 6*a*d*x^3)}{(18*a*c^2*x^6) - ((-b^2*c^2 - 3*a*b*c*d + 9*a^2*d^2)*\text{ArcTan}[(a^{(1/3)} + 2*(a + b*x^3)^{(1/3)})/\sqrt{3}]*a^{(1/3)})}{(9*\sqrt{3}*a^{(5/3)}*c^3) - (d^{(5/3)}*(b*c - a*d)^{(1/3)}*\text{ArcTan}[1/\sqrt{3} - (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})/(\sqrt{3}*(b*c - a*d)^{(1/3)})]})(\sqrt{3}*c^3) + ((-b^2*c^2 - 3*a*b*c*d + 9*a^2*d^2)*\text{Log}[-a^{(1/3)} + (a + b*x^3)^{(1/3)})]})(27*a^{(5/3)}*c^3) + (d^{(5/3)}*(b*c - a*d)^{(1/3)}*\text{Log}[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)}]})(3*c^3) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*\text{Log}[a^{(2/3)} + a^{(1/3)}*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}]})(54*a^{(5/3)}*c^3) - (d^{(5/3)}*(b*c - a*d)^{(1/3)}*\text{Log}[(b*c - a*d)^{(2/3)} - d^{(1/3)}*(b*c - a*d)^{(1/3)}*(a + b*x^3)^{(1/3)} + d^{(2/3)}*(a + b*x^3)^{(2/3)}]})(6*c^3)$$

fricas [A] time = 2.00, size = 472, normalized size = 1.28

$$\frac{18\sqrt{3}(9a^2d^2 - 3abcd - b^2c^2)\log(\sqrt{a+bx^3} - \sqrt{c})}{27a^2c^2} + \frac{(-18a^2d^2 + 3abcd + b^2c^2)\log(\sqrt{a+bx^3} + \sqrt{c})}{54a^2c^2} + \frac{(9a^2d^2 - 3abcd - b^2c^2)\tan^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{c}}\right)}{9\sqrt{a}d^{5/3}c^2} + \frac{d^{5/3}\sqrt{c-ad}\log\left(\frac{\sqrt{c-ad} + \sqrt{d}\sqrt{a+bx^3}}{3c^2}\right)}{3c^2} + \frac{d^{5/3}\sqrt{c-ad}\log\left(\frac{-\sqrt{d}\sqrt{a+bx^3} + \sqrt{c-ad}}{6c^2}\right)}{6c^2} + \frac{d^{5/3}\sqrt{c-ad}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}\frac{\sqrt{a+bx^3}}{\sqrt{c}}\right)}{\sqrt{3}c^2} + \frac{\sqrt{a+bx^3}(-3ac + 6ads^2 - bc^2)}{18a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="fricas")

[Out]
$$-1/54*(18*\sqrt{3}*(b*c*d^2 - a*d^3)^{(1/3)}*a^3*d*x^6*\arctan(-1/3*(2*\sqrt{3}*(b*c*d^2 - a*d^3)^{(2/3)}*(b*x^3 + a)^{(1/3)} - \sqrt{3}*(b*c*d - a*d^2))/(b*c*d - a*d^2)) + 9*(b*c*d^2 - a*d^3)^{(1/3)}*a^3*d*x^6*\log((b*x^3 + a)^{(2/3)}*d^2 - (b*c*d^2 - a*d^3)^{(1/3)}*(b*x^3 + a)^{(1/3)}*d + (b*c*d^2 - a*d^3)^{(2/3)}) - 18*(b*c*d^2 - a*d^3)^{(1/3)}*a^3*d*x^6*\log((b*x^3 + a)^{(1/3)}*d + (b*c*d^2 - a*d^3)^{(1/3)}) - 2*\sqrt{3}*(a*b^2*c^2 + 3*a^2*b*c*d - 9*a^3*d^2)*(a^2)^{(1/6)}*$$

$$x^6 \arctan\left(\frac{1}{3} (a^2)^{1/6} \sqrt{3} (a^2)^{1/3} a + 2\sqrt{3} (bx^3 + a)^{1/3} (a^2)^{2/3}\right) / a^2 - (b^2c^2 + 3ab^2cd - 9a^2d^2) (a^2)^{2/3} x^6 \log\left(\frac{(bx^3 + a)^{2/3} a + (a^2)^{1/3} a + (bx^3 + a)^{1/3} (a^2)^{2/3}}{(bx^3 + a)^{1/3} a - (a^2)^{2/3}}\right) + 3(3a^3c^2 + (a^2b^2c^2 - 6a^3cd)x^3) (bx^3 + a)^{1/3} / (a^3c^3x^6)$$

giac [A] time = 0.81, size = 465, normalized size = 1.26

$$\frac{(b^2c^2 - a^2d^2) \sqrt{3} \arctan\left(\frac{1}{3} (a^2)^{1/6} \sqrt{3} (a^2)^{1/3} a + 2\sqrt{3} (bx^3 + a)^{1/3} (a^2)^{2/3}\right)}{3(a^3c^3x^6)} - \frac{\sqrt{3} (b^2c^2 + a^2d^2) \log\left(\frac{(bx^3 + a)^{2/3} a + (a^2)^{1/3} a + (bx^3 + a)^{1/3} (a^2)^{2/3}}{(bx^3 + a)^{1/3} a - (a^2)^{2/3}}\right)}{3a^2} + \frac{3(3a^3c^2 + (a^2b^2c^2 - 6a^3cd)x^3) (bx^3 + a)^{1/3}}{a^3c^3x^6} - \frac{(b^2c^2 + 3ab^2cd - 9a^2d^2) (a^2)^{2/3} x^6 \log\left(\frac{(bx^3 + a)^{2/3} a + (a^2)^{1/3} a + (bx^3 + a)^{1/3} (a^2)^{2/3}}{(bx^3 + a)^{1/3} a - (a^2)^{2/3}}\right)}{27a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="giac")

[Out]
$$-1/3*(b*c*d^2 - a*d^3)*(-(b*c - a*d)/d)^{1/3}*\log(\text{abs}((b*x^3 + a)^{1/3} - (b*c - a*d)/d)^{1/3})/(b*c^4 - a*c^3*d) + 1/3*\sqrt{3}*(-b*c*d^2 + a*d^3)^{1/3}*d*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{1/3} + (b*c - a*d)/d)^{1/3}/(-(b*c - a*d)/d)^{1/3})/c^3 + 1/6*(-b*c*d^2 + a*d^3)^{1/3}*d*\log((b*x^3 + a)^{2/3} + (b*x^3 + a)^{1/3}*(-(b*c - a*d)/d)^{1/3} + (b*c - a*d)/d)^{2/3})/c^3 + 1/27*\sqrt{3}*(a^{1/3}*b^2*c^2 + 3*a^{4/3}*b*c*d - 9*a^{7/3}*d^2)*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{1/3} + a^{1/3})/a^{1/3})/(a^2*c^3) - 1/27*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*\log(\text{abs}((b*x^3 + a)^{1/3} - a^{1/3}))/a^{5/3}*c^3 + 1/54*(a^{1/3}*b^2*c^2 + 3*a^{4/3}*b*c*d - 9*a^{7/3}*d^2)*\log((b*x^3 + a)^{2/3} + (b*x^3 + a)^{1/3}*a^{1/3} + a^{2/3})/(a^2*c^3) - 1/18*((b*x^3 + a)^{4/3}*b^2*c + 2*(b*x^3 + a)^{1/3}*a*b^2*c - 6*(b*x^3 + a)^{4/3}*a*b*d + 6*(b*x^3 + a)^{1/3}*a^2*b*d)/(a*b^2*c^2*x^6)$$

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{1/3}}{(dx^3 + c)x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x)

[Out] int((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{1/3}}{(dx^3 + c)x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^7), x)

mupad [B] time = 12.55, size = 2767, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x)

[Out] $\log\left(\frac{\left(\frac{81ab^4c^4d^3(2a^2d^2 + b^2c^2 - 3abc*d)(-d^5(ad - bc))}{c^9}\right)^{1/3} + (9b^5c^2d^3(a + bx^3)^{1/3}(12a^3d^3 + b^3c^3 + ab^2c^2d - 14a^2b*c*d^2))/a \cdot (-d^5(ad - bc))/c^9)^{2/3}}{9} - (b^5d^4(729a^6d^6 + b^6c^6 - 9a^2b^4c^4d^2 - 135a^3b^3c^3d^3 + 864a^4b^2c^2d^4 + 8ab^5c^5d - 1458a^5b*c*d^5))/(81a^3c^4) \cdot (-d^5(ad - bc))/c^9)^{1/3}}{3} - (b^4d^6(a + bx^3)^{1/3}(1458a^7d^7 + b^7c^7 + 72a^2b^5c^5d^2 - 135a^3b^4c^4d^3 - 1080a^4b^3c^3d^4 + 3564a^5b^2c^2d^5 + 8ab^6c^6d - 3888a^6b*c*d^6))/(243a^3c^8) \cdot (-ad^6 - b*c*d^5)/(27c^9)\right)^{1/3} + \log\left(\frac{\left(\frac{9b^5c^2d^3(a + bx^3)^{1/3}(12a^3d^3 + b^3c^3 + ab^2c^2d - 14a^2b*c*d^2)}{a} + 9ab^4c^4d^3(2a^2d^2 + b^2c^2 - 3abc*d)(-b^2c^2 - 9a^2d^2 + 3abc*d)^3/(a^5c^9)\right)^{1/3} \cdot (-b^2c^2 - 9a^2d^2 + 3abc*d)^3/(a^5c^9)^{2/3}}{729} - (b^5d^4(729a^6d^6 + b^6c^6 - 9a^2b^4c^4d^2 - 135a^3b^3c^3d^3 + 864a^4b^2c^2d^4 + 8ab^5c^5d - 1458a^5b*c*d^5))/(81a^3c^4) \cdot (-b^2c^2 - 9a^2d^2 + 3abc*d)^3/(a^5c^9)^{1/3}}{27} - (b^4d^6(a + bx^3)^{1/3}(1458a^7d^7 + b^7c^7 + 72a^2b^5c^5d^2 - 135a^3b^4c^4d^3 - 1080a^4b^3c^3d^4 + 3564a^5b^2c^2d^5 + 8ab^6c^6d - 3888a^6b*c*d^6))/(243a^3c^8) \cdot (-b^6c^6 - 729a^6d^6 - 135a^3b^3c^3d^3 + 9ab^5c^5d + 729a^5b*c*d^5)/(19683a^5c^9)\right)^{1/3} - \left(\frac{(a + bx^3)^{1/3}(b^2c + 3abd)}{(9c^2)} - \frac{b(a + bx^3)^{4/3}(6ad - bc)}{(18ac^2)}\right) / \left(\frac{(a + bx^3)^2 - 2a(a + bx^3) + a^2}{(3^{1/2}i)/2 - 1/2}\right) + \log\left(-\frac{(3^{1/2}i)/2 - 1/2}{((9b^5c^2d^3(a + bx^3)^{1/3}(12a^3d^3 + b^3c^3 + ab^2c^2d - 14a^2b*c*d^2))/a + 81ab^4c^4d^3((3^{1/2}i)/2 - 1/2)(2a^2d^2 + b^2c^2 - 3abc*d)(-d^5(ad - bc))/c^9)^{1/3}} \cdot \frac{(3^{1/2}i)/2 + 1/2}{(-d^5(ad - bc))/c^9)^{2/3}}{9} + (b^5d^4(729a^6d^6 + b^6c^6 - 9a^2b^4c^4d^2 - 135a^3b^3c^3d^3 + 864a^4b^2c^2d^4 + 8ab^5c^5d - 1458a^5b*c*d^5))/(81a^3c^4) \cdot (-d^5(ad - bc))/c^9)^{1/3}}{3} - (b^4d^6(a + bx^3)^{1/3}(1458a^7d^7 + b^7c^7 + 72a^2b^5c^5d^2 - 135a^3b^4c^4d^3 - 1080a^4b^3c^3d^4 + 3564a^5b^2c^2d^5 + 8ab^6c^6d - 3888a^6b*c*d^6))/(243a^3c^8) \cdot \frac{(3^{1/2}i)/2 - 1/2}{(-ad^6 - b*c*d^5)/(27c^9)^{1/3}} - \log\left(\frac{(3^{1/2}i)/2 + 1/2}{((9b^5c^2d^3(a + bx^3)^{1/3}(12a^3d^3 + b^3c^3 + ab^2c^2d - 14a^2b*c*d^2))/a - 81ab^4c^4d^3((3^{1/2}i)/2 + 1/2)(2a^2d^2 + b^2c^2 - 3abc*d)(-d^5(ad - bc))/c^9)^{1/3}} \cdot \frac{(3^{1/2}i)/2 - 1/2}{(-ad^6 - b*c*d^5)/(27c^9)^{1/3}}\right)$

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*(a*d - b*c))/c^9)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(-(d^5*(a*d - b*c))/c^9)^(
2/3))/9 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3
*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3
*c^4))*(-(d^5*(a*d - b*c))/c^9)^(1/3))/3 + (b^4*d^6*(a + b*x^3)^(1/3)*(1458
*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^
3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*
a^3*c^8))*((3^(1/2)*1i)/2 + 1/2)*(-(a*d^6 - b*c*d^5)/(27*c^9))^(1/3) + log(
- (((3^(1/2)*1i)/2 - 1/2)*(((3^(1/2)*1i)/2 + 1/2)*((9*b^5*c^2*d^3*(a + b*x
^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a + 9*a*b^
4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c
^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*
a*b*c*d)^3/(a^5*c^9))^(2/3))/729 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*
b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1
458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*
c^9))^(1/3))/27 - (b^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a
^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*
c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2
- 1/2)*(-(b^6*c^6 - 729*a^6*d^6 - 135*a^3*b^3*c^3*d^3 + 9*a*b^5*c^5*d + 72
9*a^5*b*c*d^5)/(19683*a^5*c^9))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2)*(((3^(
1/2)*1i)/2 - 1/2)*((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 +
a*b^2*c^2*d - 14*a^2*b*c*d^2))/a - 9*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*
(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^
5*c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(2/3))/729
- (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3
+ 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-
(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))/27 + (b^4*d^6*(a + b
*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*
d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^
6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2 + 1/2)*(-(b^6*c^6 - 729*a^6*d^6
- 135*a^3*b^3*c^3*d^3 + 9*a*b^5*c^5*d + 729*a^5*b*c*d^5)/(19683*a^5*c^9))^(
1/3)

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{x^7(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(1/3)/x**7/(d*x**3+c), x)

[Out] Integral((a + b*x**3)**(1/3)/(x**7*(c + d*x**3)), x)

$$3.421 \quad \int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3} dx$$

Optimal. Leaf size=336

$$\frac{(-a^2d^2 - 3abcd + 9b^2c^2) \log\left(\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right)}{18b^{5/3}d^3} - \frac{(-a^2d^2 - 3abcd + 9b^2c^2) \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}d^3} - \frac{c^{5/3}\sqrt[3]{bc-ad} \log\left(\frac{c\sqrt[3]{a+bx^3} + \sqrt[3]{bc-ad}}{c}\right)}{6d^3}$$

Rubi [C] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{x^8 \sqrt[3]{a+bx^3} F_1\left(\frac{8}{3}; -\frac{1}{3}, 1; \frac{11}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{8c \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] (x^8*(a + b*x^3)^(1/3)*AppellF1[8/3, -1/3, 1, 11/3, -((b*x^3)/a), -((d*x^3)/c)]/(8*c*(1 + (b*x^3)/a)^(1/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^(m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^7 \sqrt[3]{a + bx^3}}{c + dx^3} dx = \frac{\sqrt[3]{a + bx^3} \int \frac{x^7 \sqrt[3]{1 + \frac{bx^3}{a}}}{c + dx^3} dx}{\sqrt[3]{1 + \frac{bx^3}{a}}}$$

$$= \frac{x^8 \sqrt[3]{a + bx^3} F_1\left(\frac{8}{3}; -\frac{1}{3}, 1; \frac{11}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{8c \sqrt[3]{1 + \frac{bx^3}{a}}}$$

Mathematica [C] time = 0.37, size = 226, normalized size = 0.67

$$\frac{5cx^2 \left(a \left(\frac{bx^3}{a} + 1\right)^{2/3} (6bc - ad) {}_2F_1\left(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(ad-bc)x^3}{a(dx^3+c)}\right) + (a + bx^3) \left(\frac{dx^3}{c} + 1\right)^{2/3} (ad - 6bc + 3bdx^3)\right) - 2x^5 \left(\frac{bx^3}{a} + 1\right)^{2/3} \left(\frac{dx^3}{c} + 1\right)^{2/3} (a^2d^2 + 3abcd - 9b^2c^2) F_1\left(\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{90bcd^2 (a + bx^3)^{2/3} \left(\frac{dx^3}{c} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] (-2*(-9*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*c*x^2*((a + b*x^3)*(-6*b*c + a*d + 3*b*d*x^3)*(1 + (d*x^3)/c)^(2/3) + a*(6*b*c - a*d)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-b*c) + a*d)*x^3/(a*(c + d*x^3))])/ (90*b*c*d^2*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3))

IntegrateAlgebraic [C] time = 5.50, size = 613, normalized size = 1.82

$$\frac{(a^2d^2 - 3abcd - 9b^2c^2) \operatorname{arctan}\left(\frac{\sqrt[3]{a+bx^3}}{\sqrt[3]{c+dx^3}}\right) + (a+bx^3) \sqrt[3]{\frac{dx^3}{c}+1} (ad-6bc+3bdx^3) - 2x^5 \sqrt[3]{\frac{bx^3}{a}+1} \sqrt[3]{\frac{dx^3}{c}+1} (a^2d^2+3abcd-9b^2c^2) F_1\left(\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{90bcd^2 (a+bx^3)^{2/3} \left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] ((a + b*x^3)^(1/3)*(-6*b*c*x^2 + a*d*x^2 + 3*b*d*x^5))/(18*b*d^2) - ((9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(9*Sqrt[3]*b^(5/3)*d^3) - (Sqrt[-1 - I*Sqrt[3]]*c^(5/3)*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]/(Sqrt[6]*d^3) + ((-9*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(27*b^(5/3)*d^3) + ((I/6)*(I*c^(5/3)*(b*c - a*d)^(1/3) + Sqrt[3]*c^(5/3)*(b*c - a*d)^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]/d^3 + ((9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*Log[b^

$$\frac{(2/3)*x^2 + b^{(1/3)}*x*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}}{(54*b^{(5/3)}*d^3) + ((c^{(5/3)}*(b*c - a*d)^{(1/3)} - I*\text{Sqrt}[3]*c^{(5/3)}*(b*c - a*d)^{(1/3)})*\text{Log}[(-2*I)*(b*c - a*d)^{(2/3)}*x^2 + c^{(1/3)}*(b*c - a*d)^{(1/3)}*(I*x - \text{Sqrt}[3]*x)]*(a + b*x^3)^{(1/3)} + (I + \text{Sqrt}[3])*c^{(2/3)}*(a + b*x^3)^{(2/3)}}{(12*d^3)}$$

fricas [A] time = 1.95, size = 494, normalized size = 1.47

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] $\frac{1}{54} * (18 * \text{sqrt}(3) * (b * c^3 - a * c^2 * d)^{(1/3)} * b^3 * c * \arctan(-1/3 * (\text{sqrt}(3) * (b * c^2 - a * c * d) * x + 2 * \text{sqrt}(3) * (b * c^3 - a * c^2 * d)^{(2/3)} * (b * x^3 + a)^{(1/3)})) / ((b * c^2 - a * c * d) * x)) + 18 * (b * c^3 - a * c^2 * d)^{(1/3)} * b^3 * c * \log(((b * x^3 + a)^{(1/3)} * c - (b * c^3 - a * c^2 * d)^{(1/3)} * x) / x) - 9 * (b * c^3 - a * c^2 * d)^{(1/3)} * b^3 * c * \log(((b * x^3 + a)^{(2/3)} * c^2 + (b * c^3 - a * c^2 * d)^{(1/3)} * (b * x^3 + a)^{(1/3)} * c * x + (b * c^3 - a * c^2 * d)^{(2/3)} * x^2) / x^2) + 2 * \text{sqrt}(3) * (9 * b^3 * c^2 - 3 * a * b^2 * c * d - a^2 * b * d^2) * (b^2)^{(1/6)} * \arctan(1/3 * (\text{sqrt}(3) * (b^2)^{(1/3)} * b * x + 2 * \text{sqrt}(3) * (b * x^3 + a)^{(1/3)}) * (b^2)^{(2/3)}) * (b^2)^{(1/6)} / (b^2 * x)) - 2 * (9 * b^2 * c^2 - 3 * a * b * c * d - a^2 * d^2) * (b^2)^{(2/3)} * \log(-((b^2)^{(2/3)} * x - (b * x^3 + a)^{(1/3)} * b) / x) + (9 * b^2 * c^2 - 3 * a * b * c * d - a^2 * d^2) * (b^2)^{(2/3)} * \log(((b^2)^{(1/3)} * b * x^2 + (b * x^3 + a)^{(1/3)} * (b^2)^{(2/3)} * x + (b * x^3 + a)^{(2/3)} * b) / x^2) + 3 * (3 * b^3 * d^2 * x^5 - (6 * b^3 * c * d - a * b^2 * d^2) * x^2) * (b * x^3 + a)^{(1/3)} / (b^3 * d^3)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}} x^7}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}} x^7}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x)

[Out] `int(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}} x^7}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7 (bx^3 + a)^{1/3}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x)`

[Out] `int((x^7*(a + b*x^3)^(1/3))/(c + d*x^3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 \sqrt[3]{a + bx^3}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(b*x**3+a)**(1/3)/(d*x**3+c),x)`

[Out] `Integral(x**7*(a + b*x**3)**(1/3)/(c + d*x**3), x)`

$$3.422 \quad \int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3} dx$$

Optimal. Leaf size=276

$$\frac{(3bc - ad) \log\left(\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right)}{6b^{2/3}d^2} + \frac{(3bc - ad) \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}d^2} + \frac{c^{2/3}\sqrt[3]{bc-ad} \log(c+dx^3)}{6d^2} - \frac{c^{2/3}\sqrt[3]{bc-ad} \log\left(\frac{c+dx^3}{\sqrt[3]{a+bx^3}}\right)}{6d^2}$$

Rubi [C] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 0.23, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{x^5 \sqrt[3]{a+bx^3} F_1\left(\frac{5}{3}; -\frac{1}{3}, 1; \frac{8}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{5c \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] (x^5*(a + b*x^3)^(1/3)*AppellF1[5/3, -1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]/(5*c*(1 + (b*x^3)/a)^(1/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3} dx = \frac{\sqrt[3]{a+bx^3} \int \frac{x^4 \sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3} dx}{\sqrt[3]{1+\frac{bx^3}{a}}} = \frac{x^5 \sqrt[3]{a+bx^3} F_1\left(\frac{5}{3}; -\frac{1}{3}, 1; \frac{8}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{5c \sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.19, size = 185, normalized size = 0.67

$$\frac{x^5 \left(\frac{bx^3}{a} + 1\right)^{2/3} \left(\frac{dx^3}{c} + 1\right)^{2/3} (ad - 3bc) F_1\left(\frac{5}{3}; \frac{2}{3}, 1; \frac{8}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) + 5cx^2 \left((a + bx^3) \left(\frac{dx^3}{c} + 1\right)^{2/3} - a \left(\frac{bx^3}{a} + 1\right)^{2/3}\right) {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{15cd (a + bx^3)^{2/3} \left(\frac{dx^3}{c} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] ((-3*b*c + a*d)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*c*x^2*((a + b*x^3)*(1 + (d*x^3)/c)^(2/3) - a*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-b*c) + a*d)*x^3/(a*(c + d*x^3))]))/(15*c*d*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3))

IntegrateAlgebraic [C] time = 4.32, size = 549, normalized size = 1.99

$\frac{d(c-a)\log(\sqrt{a+bx^3}-\sqrt{c})}{3\sqrt{3}d^2} - \frac{d(c-a)\log\left(\frac{\sqrt{3}\sqrt{c}}{\sqrt{a+bx^3}+\sqrt{c}}\right)}{3\sqrt{3}d^2} - \frac{(d-3c)\log\left(\sqrt{c}\sqrt{a+bx^3}+(a+bx^3)^{3/2}+c^{3/2}\right)}{18d^2} - \frac{(c^2\sqrt{c-a}-c\sqrt{c}\sqrt{c-a})\log\left(\frac{2\sqrt{c-a}+(1+\sqrt{3})\sqrt{c}\sqrt{a+bx^3}}{\sqrt{3}\sqrt{c-a}}\right)}{6d^2} - \frac{\sqrt{1-\sqrt{3}}c^2\sqrt{c-a}\arctan\left(\frac{\sqrt{a+bx^3}}{\sqrt{3}\sqrt{c-a}}\right)}{3\sqrt{3}d^2} - \frac{((\sqrt{3}c^2\sqrt{c-a}+c^2\sqrt{c}\sqrt{c-a})\log((\sqrt{3}+1)c^2(a+bx^3)^{3/2}+\sqrt{c}(-\sqrt{3}+1)\sqrt{a+bx^3}\sqrt{c-a}-2c^2(c-a)^{3/2}))}{12d^2} - \frac{c^2\sqrt{c-a}}{3d^2}$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] (x^2*(a + b*x^3)^(1/3))/(3*d) + ((3*b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(3*Sqrt[3]*b^(2/3)*d^2) + (Sqrt[-1 - I*Sqrt[3]]*c^(2/3)*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]/(Sqrt[6]*d^2) + ((3*b*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(9*b^(2/3)*d^2) + ((c^(2/3)*(b*c - a*d)^(1/3) - I*Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]/(6*d^2) + ((-3*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(18*b^(2/3)*d^2) + ((I/12)*(I*c^(2/3)*(b*c - a*d)^(1/3) + Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3))*Log[(-2*I)*(b*c - a

$*d)^{(2/3)} * x^2 + c^{(1/3)} * (b*c - a*d)^{(1/3)} * (I*x - \text{Sqrt}[3]*x) * (a + b*x^3)^{(1/3)} + (I + \text{Sqrt}[3]) * c^{(2/3)} * (a + b*x^3)^{(2/3)}] / d^2$

fricas [B] time = 0.55, size = 452, normalized size = 1.64

$$\frac{6(b^2+d)^2 b d^2 + 6\sqrt{3}(bc^2+ad^2)^2 \arctan\left(\frac{\sqrt{3}(bc^2+ad^2)\sqrt{(bc^2+ad^2)^2+d^3}}{3(d^2+bd)}\right) + 6(bc^2+ad^2)^2 \log\left(\frac{(bc^2+ad^2)\sqrt{(bc^2+ad^2)^2+d^3}}{3(d^2+bd)}\right) - 3(bc^2+ad^2)^2 \log\left(\frac{(bc^2+ad^2)\sqrt{(bc^2+ad^2)^2+d^3}}{3(d^2+bd)}\right) - 2\sqrt{3}(b^2c-ad)\sqrt{-d^3} \arctan\left(\frac{\sqrt{3}(bc^2+ad^2)\sqrt{(bc^2+ad^2)^2+d^3}}{3(d^2+bd)}\right) + 2(-d)^2(bc-ad)\log\left(\frac{(bc^2+ad^2)\sqrt{(bc^2+ad^2)^2+d^3}}{3(d^2+bd)}\right) - (-d)^2(bc-ad)\log\left(\frac{(bc^2+ad^2)\sqrt{(bc^2+ad^2)^2+d^3}}{3(d^2+bd)}\right)}{18b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] $1/18*(6*(b*x^3 + a)^{(1/3)}*b^2*d*x^2 + 6*\text{sqrt}(3)*(-b*c^3 + a*c^2*d)^{(1/3)}*b^2*2*\arctan(-1/3*(\text{sqrt}(3)*(b*c^2 - a*c*d))*x + 2*\text{sqrt}(3)*(-b*c^3 + a*c^2*d)^{(2/3)}*(b*x^3 + a)^{(1/3)})/((b*c^2 - a*c*d)*x)) + 6*(-b*c^3 + a*c^2*d)^{(1/3)}*b^2*\log(((b*x^3 + a)^{(1/3)}*c + (-b*c^3 + a*c^2*d)^{(1/3)}*x)/x) - 3*(-b*c^3 + a*c^2*d)^{(1/3)}*b^2*\log(((b*x^3 + a)^{(2/3)}*c^2 - (-b*c^3 + a*c^2*d)^{(1/3)}*(b*x^3 + a)^{(1/3)}*c*x + (-b*c^3 + a*c^2*d)^{(2/3)}*x^2)/x^2) - 2*\text{sqrt}(3)*(3*b^2*c - a*b*d)*\text{sqrt}(-(-b^2)^{(1/3)})*\arctan(-1/3*(\text{sqrt}(3)*(-b^2)^{(1/3)}*b*x - 2*\text{sqrt}(3)*(b*x^3 + a)^{(1/3)}*(-b^2)^{(2/3)}))*\text{sqrt}(-(-b^2)^{(1/3)})/(b^2*x)) + 2*(-b^2)^{(2/3)}*(3*b*c - a*d)*\log(-((-b^2)^{(2/3)}*x - (b*x^3 + a)^{(1/3)}*b)/x) - (-b^2)^{(2/3)}*(3*b*c - a*d)*\log(-((-b^2)^{(1/3)}*b*x^2 - (b*x^3 + a)^{(1/3)}*(-b^2)^{(2/3)}*x - (b*x^3 + a)^{(2/3)}*b)/x^2))/ (b^2*d^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}} x^4}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}} x^4}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x)

[Out] int(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}} x^4}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 (bx^3 + a)^{1/3}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x)

[Out] int((x^4*(a + b*x^3)^(1/3))/(c + d*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sqrt[3]{a + bx^3}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(b*x**3+a)**(1/3)/(d*x**3+c),x)

[Out] Integral(x**4*(a + b*x**3)**(1/3)/(c + d*x**3), x)

$$3.423 \quad \int \frac{x \sqrt[3]{a+bx^3}}{c+dx^3} dx$$

Optimal. Leaf size=234

$$\frac{\sqrt[3]{bc-ad} \log(c+dx^3)}{6\sqrt[3]{cd}} + \frac{\sqrt[3]{bc-ad} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{cd}} + \frac{\sqrt[3]{bc-ad} \tan^{-1}\left(\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}} + 1\right)}{\sqrt{3}\sqrt[3]{cd}} - \frac{\sqrt[3]{b} \log(\sqrt[3]{b}x)}{2a}$$

Rubi [C] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 0.27, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {511, 510}

$$\frac{x^2 \sqrt[3]{a+bx^3} F_1\left(\frac{2}{3}; -\frac{1}{3}, 1; \frac{5}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{2c \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(x*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] (x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)]/(2*c*(1 + (b*x^3)/a)^(1/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^(m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^3 \sqrt[3]{a + bx^3}}{c + dx^3} dx = \frac{\sqrt[3]{a + bx^3} \int \frac{x^3 \sqrt[3]{1 + \frac{bx^3}{a}}}{c + dx^3} dx}{\sqrt[3]{1 + \frac{bx^3}{a}}}$$

$$= \frac{x^2 \sqrt[3]{a + bx^3} F_1\left(\frac{2}{3}; -\frac{1}{3}, 1; \frac{5}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{2c \sqrt[3]{1 + \frac{bx^3}{a}}}$$

Mathematica [C] time = 0.04, size = 65, normalized size = 0.28

$$\frac{x^2 \sqrt[3]{a + bx^3} F_1\left(\frac{2}{3}; -\frac{1}{3}, 1; \frac{5}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{2c \sqrt[3]{\frac{a + bx^3}{a}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] (x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)])/(2*c*((a + b*x^3)/a)^(1/3))

IntegrateAlgebraic [C] time = 3.79, size = 490, normalized size = 2.09

$$\frac{\sqrt[3]{d} \log\left(\frac{\sqrt[3]{a + bx^3} + (a + bx^3)^{2/3}}{a}\right) + (\sqrt[3]{bc - ad} - i\sqrt[3]{\sqrt[3]{bc - ad}}) \log\left(\frac{\sqrt[3]{a + bx^3} + (a + bx^3)^{2/3}}{\sqrt[3]{a + bx^3} + i\sqrt[3]{bc - ad}}\right) + \sqrt[3]{c}(-\sqrt[3]{a + i}) \sqrt[3]{a + bx^3} \sqrt[3]{bc - ad} - 2i\sqrt[3]{bc - ad}^2}{12\sqrt[3]{cd}} + i(\sqrt[3]{\sqrt[3]{bc - ad} + i\sqrt[3]{bc - ad}}) \log\left(\frac{2i\sqrt[3]{bc - ad} + (1 + i\sqrt[3]{c}) \sqrt[3]{c} \sqrt[3]{a + bx^3}}{6\sqrt[3]{d}}\right) + \sqrt[3]{-1 - i\sqrt[3]{c}} \sqrt[3]{bc - ad} \tan^{-1}\left(\frac{\sqrt[3]{a + bx^3}}{\sqrt[3]{a + i\sqrt[3]{bc - ad}} + \sqrt[3]{a + i\sqrt[3]{bc - ad}}}\right) + \sqrt[3]{d} \log\left(\frac{\sqrt[3]{a + bx^3} - \sqrt[3]{a}}{\sqrt[3]{a + bx^3} + \sqrt[3]{a}}\right) + \frac{\sqrt[3]{d} \tan^{-1}\left(\frac{\sqrt[3]{a + bx^3}}{\sqrt[3]{a + i\sqrt[3]{bc - ad}} + \sqrt[3]{a + i\sqrt[3]{bc - ad}}}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(a + b*x^3)^(1/3))/(c + d*x^3), x]

[Out] -((b^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(Sqrt[3]*d)) - (Sqrt[-1 - I*Sqrt[3]]*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])/((Sqrt[6]*c^(1/3)*d) - (b^(1/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*d) + ((I/6)*(I*(b*c - a*d)^(1/3) + Sqrt[3]*(b*c - a*d)^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]/(c^(1/3)*d) + (b^(1/3)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*d) + (((b*c - a*d)^(1/3) - I*Sqrt[3]*(b*c - a*d)^(1/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(1/3)*d)

fricas [A] time = 0.48, size = 330, normalized size = 1.41

$$\frac{2\sqrt{3}\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}(bc-ad)+2\sqrt{3}(b^3+a^3)^{\frac{1}{3}}\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3(bc-ad)x}\right)-2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}bx+2\sqrt{3}(b^3+a^3)^{\frac{1}{3}}(c-b)^{\frac{2}{3}}}{3bx}\right)+2(-b)^{\frac{1}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(b^3+a^3)^{\frac{1}{3}}}{x}\right)+2\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\log\left(\frac{-\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}-(b^3+a^3)^{\frac{1}{3}}}{x}\right)-(-b)^{\frac{1}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x^2-(b^3+a^3)^{\frac{1}{3}}(c-b)^{\frac{1}{3}}x+(b^3+a^3)^{\frac{2}{3}}}{x^2}\right)-\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\log\left(\frac{x^2\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}+(b^3+a^3)^{\frac{1}{3}}x\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}+(b^3+a^3)^{\frac{2}{3}}}{x^2}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] $\frac{1}{6}*(2*\sqrt{3})*((b*c - a*d)/c)^{(1/3)}*\arctan(-1/3*(\sqrt{3}*(b*c - a*d)*x + 2*\sqrt{3}*(b*x^3 + a)^{(1/3)}*c*((b*c - a*d)/c)^{(2/3)})/((b*c - a*d)*x) - 2*\sqrt{3}*(-b)^{(1/3)}*\arctan(1/3*(\sqrt{3}*b*x + 2*\sqrt{3}*(b*x^3 + a)^{(1/3)}*(-b)^{(2/3)})/(b*x)) + 2*(-b)^{(1/3)}*\log(((b)^{(1/3)}*x + (b*x^3 + a)^{(1/3)})/x) + 2*((b*c - a*d)/c)^{(1/3)}*\log(-(x*((b*c - a*d)/c)^{(1/3)} - (b*x^3 + a)^{(1/3)})/x) - (-b)^{(1/3)}*\log(((b)^{(2/3)}*x^2 - (b*x^3 + a)^{(1/3)}*(-b)^{(1/3)}*x + (b*x^3 + a)^{(2/3)})/x^2) - ((b*c - a*d)/c)^{(1/3)}*\log((x^2*((b*c - a*d)/c)^{(2/3)} + (b*x^3 + a)^{(1/3)}*x*((b*c - a*d)/c)^{(1/3)} + (b*x^3 + a)^{(2/3)})/x^2))/d$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}} x}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}} x}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a)^(1/3)/(d*x^3+c),x)

[Out] int(x*(b*x^3+a)^(1/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}} x}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x(bx^3 + a)^{1/3}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*x^3)^(1/3))/(c + d*x^3),x)

[Out] int((x*(a + b*x^3)^(1/3))/(c + d*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{a + bx^3}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**3+a)**(1/3)/(d*x**3+c),x)

[Out] Integral(x*(a + b*x**3)**(1/3)/(c + d*x**3), x)

$$3.424 \quad \int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)} dx$$

Optimal. Leaf size=168

$$\frac{\sqrt[3]{bc-ad} \log(c+dx^3)}{6c^{4/3}} - \frac{\sqrt[3]{bc-ad} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{4/3}} - \frac{\sqrt[3]{bc-ad} \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}c^{4/3}} - \frac{\sqrt[3]{a+bx^3}}{cx}$$

Rubi [C] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 0.52, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{\sqrt[3]{a+bx^3} \sqrt[3]{\frac{dx^3}{c} + 1} {}_2F_1\left(-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}; -\frac{c\left(\frac{bx^3}{a} - \frac{dx^3}{c}\right)}{dx^3+c}\right)}{cx\sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)), x]

[Out] -(((a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, -((c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3))]/(c*x*(1 + (b*x^3)/a)^(1/3)))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^(m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)} dx = \frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^2(c+dx^3)} dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{\sqrt[3]{a+bx^3} \sqrt[3]{1+\frac{dx^3}{c}} {}_2F_1\left(-\frac{1}{3}, -\frac{1}{3}; \frac{2}{3}; -\frac{c\left(\frac{bx^3}{a} - \frac{dx^3}{c}\right)}{c+dx^3}\right)}{cx \sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.04, size = 81, normalized size = 0.48

$$\frac{\sqrt[3]{a+bx^3} \sqrt[3]{\frac{dx^3}{c} + 1} {}_2F_1\left(-\frac{1}{3}, -\frac{1}{3}; \frac{2}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{cx \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)), x]

[Out] -(((a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, ((-b*c) + a*d)*x^3/(a*(c + d*x^3))])/(c*x*(1 + (b*x^3)/a)^(1/3)))

IntegrateAlgebraic [C] time = 2.27, size = 363, normalized size = 2.16

$$\frac{(\sqrt[3]{bc-ad} - i\sqrt[3]{\sqrt[3]{bc-ad}}) \log(2x\sqrt[3]{bc-ad} + (1+i\sqrt[3]{\sqrt[3]{bc-ad}}) \sqrt[3]{c\sqrt[3]{a+bx^3}})}{6c^{4/3}} + \frac{\sqrt{-1-i\sqrt[3]{\sqrt[3]{bc-ad}} \tan^{-1}\left(\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{3x\sqrt[3]{bc-ad}} - \sqrt[3]{3}\sqrt[3]{\sqrt[3]{bc-ad}} - 3\sqrt[3]{\sqrt[3]{bc-ad}}}\right)}}{\sqrt[3]{6}c^{4/3}} + \frac{i(\sqrt[3]{\sqrt[3]{bc-ad}} + i\sqrt[3]{\sqrt[3]{bc-ad}}) \log((\sqrt[3]{3} + i)^{2/3}(a+bx^3)^{2/3} + \sqrt[3]{c}(-\sqrt[3]{3}x + i)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} - 2ix^2(bc-ad)^{2/3})}{12c^{4/3}} - \frac{\sqrt[3]{a+bx^3}}{cx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)), x]

[Out] -((a + b*x^3)^(1/3)/(c*x)) + (Sqrt[-1 - I*Sqrt[3]]*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])/(Sqrt[6]*c^(4/3)) + (((b*c - a*d)^(1/3) - I*Sqrt[3]*(b*c - a*d)^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*c^(4/3)) + ((I/12)*(I*(b*c - a*d)^(1/3) + Sqrt[3]*(b*c - a*d)^(1/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(4/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x)

[Out] int((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^3 + a)^{1/3}}{x^2(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(1/3)/(x^2*(c + d*x^3)), x)`

[Out] `int((a + b*x^3)^(1/3)/(x^2*(c + d*x^3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{x^2(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(1/3)/x**2/(d*x**3+c), x)`

[Out] `Integral((a + b*x**3)**(1/3)/(x**2*(c + d*x**3)), x)`

$$3.425 \quad \int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)} dx$$

Optimal. Leaf size=204

$$\frac{d\sqrt[3]{bc-ad} \log(c+dx^3)}{6c^{7/3}} + \frac{d\sqrt[3]{bc-ad} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{7/3}} + \frac{d\sqrt[3]{bc-ad} \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}\sqrt[3]{a+bx^3}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}} - \frac{\sqrt[3]{a+bx^3}}{4ac}$$

Rubi [C] time = 0.13, antiderivative size = 145, normalized size of antiderivative = 0.71, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{x^3(c-3dx^3)(-bc-ad) {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right) + 3x^3(c+dx^3)(bc-ad) {}_2F_1\left(\frac{2}{3}, 2; \frac{5}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right) + 2c(a+bx^3)(c-3dx^3)}{8c^3x^4(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)), x]

[Out] $-(2*c*(a + b*x^3)*(c - 3*d*x^3) - (b*c - a*d)*x^3*(c - 3*d*x^3)*\text{Hypergeometric2F1}[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]] + 3*(b*c - a*d)*x^3*(c + d*x^3)*\text{Hypergeometric2F1}[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]]/(8*c^3*x^4*(a + b*x^3)^(2/3))$

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^(m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)} dx = \frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^5(c+dx^3)} dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{2c(a+bx^3)(c-3dx^3) - (bc-ad)x^3(c-3dx^3) {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{(bc-ad)x^3}{c(a+bx^3)}\right) + 3(bc-ad)x^3(c+bx^3) {}_2F_1\left(\frac{2}{3}, 2; \frac{5}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{8c^3x^4(a+bx^3)^{2/3}}$$

Mathematica [C] time = 0.78, size = 146, normalized size = 0.72

$$\frac{x^3(3dx^3-c)(bc-ad) {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right) + 3x^3(c+dx^3)(bc-ad) {}_2F_1\left(\frac{2}{3}, 2; \frac{5}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right) + 2c(a+bx^3)(c-3dx^3)}{8c^3x^4(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)), x]

[Out] -1/8*(2*c*(a + b*x^3)*(c - 3*d*x^3) + (b*c - a*d)*x^3*(-c + 3*d*x^3)*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*(b*c - a*d)*x^3*(c + d*x^3)*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(c^3*x^4*(a + b*x^3)^(2/3))

IntegrateAlgebraic [C] time = 2.30, size = 394, normalized size = 1.93

$$\frac{i(\sqrt{3}d\sqrt{bc-ad} + id\sqrt{bc-ad}) \log(2x\sqrt{bc-ad} + (1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3})}{6c^{2/3}} - \frac{\sqrt{-1-i\sqrt{3}}d\sqrt{bc-ad} \tan^{-1}\left(\frac{2x\sqrt{bc-ad}}{\sqrt{3}\sqrt{bc-ad}-\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{6}c^{2/3}} + \frac{(d\sqrt{bc-ad} - i\sqrt{3}d\sqrt{bc-ad}) \log((\sqrt{3}+i)^{2/3}(a+bx^3)^{1/3} + \sqrt[3]{c}(-\sqrt{3}x+ix)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} - 2ix^2(bc-ad)^{2/3})}{12c^{2/3}} + \frac{\sqrt{a+bx^3}(-bc+4adx^3-bcx^3)}{4cx^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)), x]

[Out] ((a + b*x^3)^(1/3)*(-(a*c) - b*c*x^3 + 4*a*d*x^3))/(4*a*c^2*x^4) - (Sqrt[-1 - I*Sqrt[3]]*d*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])/((Sqrt[6]*c^(7/3)) + ((I/6)*(I*d*(b*c - a*d)^(1/3) + Sqrt[3]*d*(b*c - a*d)^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/c^(7/3) + ((d*(b*c - a*d)^(1/3) - I*Sqrt[3]*d*(b*c - a*d)^(1/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/((12*c^(7/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x)

[Out] int((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{1/3}}{x^5(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(1/3)/(x^5*(c + d*x^3)), x)`

[Out] `int((a + b*x^3)^(1/3)/(x^5*(c + d*x^3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{x^5(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(1/3)/x**5/(d*x**3+c), x)`

[Out] `Integral((a + b*x**3)**(1/3)/(x**5*(c + d*x**3)), x)`

$$3.426 \quad \int \frac{\sqrt[3]{a+bx^3}}{x^8(c+dx^3)} dx$$

Optimal. Leaf size=258

$$\frac{\sqrt[3]{a+bx^3} (-28a^2d^2 + 7abcd + 3b^2c^2)}{28a^2c^3x} + \frac{d^2\sqrt[3]{bc-ad} \log(c+dx^3)}{6c^{10/3}} - \frac{d^2\sqrt[3]{bc-ad} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{10/3}} - d^2\sqrt[3]{bc-ad}$$

Rubi [C] time = 0.94, antiderivative size = 451, normalized size of antiderivative = 1.75, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{-9d^2(c+dx^3)^2(bc-ad)^2 \left(\frac{2}{3}, 2, 2, \frac{2}{3}, \frac{bc-ad}{(c+dx^3)}\right) - 2d^2(2c^2-3cd^2+9d^3d^2)(bc-ad)^2 \left(\frac{2}{3}, 1, \frac{bc-ad}{(c+dx^3)}\right) - 12b^2d^4 \left(\frac{2}{3}, 2, \frac{bc-ad}{(c+dx^3)}\right) + 15bc^2d^4 \left(\frac{2}{3}, 2, \frac{bc-ad}{(c+dx^3)}\right) - 15bc^2d^4 \left(\frac{2}{3}, 2, \frac{bc-ad}{(c+dx^3)}\right) + 27ad^4 \left(\frac{2}{3}, 2, \frac{bc-ad}{(c+dx^3)}\right) - 27bc^2d^4 \left(\frac{2}{3}, 2, \frac{bc-ad}{(c+dx^3)}\right) + 12ad^4 \left(\frac{2}{3}, 2, \frac{bc-ad}{(c+dx^3)}\right) - 12a^2d^4 + 8bc^2 + 36acd^2 - 12b^2d^4 + 8bc^2 + 36acd^2}{56c^4(a+bx^3)^{25}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)), x]

[Out] $-(8*a*c^3 + 8*b*c^3*x^3 - 12*a*c^2*d*x^3 - 12*b*c^2*d*x^6 + 36*a*c*d^2*x^6 + 36*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(2*c^2 - 3*c*d*x^3 + 9*d^2*x^6)*\text{Hypergeometric2F1}[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 15*b*c^3*x^3*\text{Hypergeometric2F1}[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 15*a*c^2*d*x^3*\text{Hypergeometric2F1}[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 12*b*c^2*d*x^6*\text{Hypergeometric2F1}[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 12*a*c*d^2*x^6*\text{Hypergeometric2F1}[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 27*b*c*d^2*x^9*\text{Hypergeometric2F1}[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*a*d^3*x^9*\text{Hypergeometric2F1}[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*\text{HypergeometricPFQ}[\{2/3, 2, 2\}, \{1, 5/3\}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(56*c^4*x^7*(a + b*x^3)^(2/3))$

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1+(m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x

```

~n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

```

Rubi steps

$$\int \frac{\sqrt[3]{a + bx^3}}{x^8(c + dx^3)} dx = \frac{\sqrt[3]{a + bx^3} \int \frac{\sqrt[3]{1 + \frac{bx^3}{a}}}{x^8(c + dx^3)} dx}{\sqrt[3]{1 + \frac{bx^3}{a}}}$$

$$= -\frac{8ac^3 + 8bc^3x^3 - 12ac^2dx^3 - 12bc^2dx^6 + 36acd^2x^6 + 36bcd^2x^9 - 2(bc - ad)x^3(2c^2 - 3cdx^3)}{56c^2x^7(a + bx^3)^2}$$

Mathematica [C] time = 2.75, size = 451, normalized size = 1.75

$$\frac{-9a^2(c + dx^3)^2(bc - ad)^2F_1\left(\frac{5}{2}, 2, \frac{5}{2}, \frac{(bc - ad)x^3}{(c + dx^3)}\right) + 15bc^3x^3F_1\left(\frac{5}{2}, 2, \frac{5}{2}, \frac{(bc - ad)x^3}{(c + dx^3)}\right) - 2a^2(2c^2 - 3cdx^3 + 9d^2x^6)(bc - ad)F_1\left(\frac{5}{2}, 1, \frac{(bc - ad)x^3}{(c + dx^3)}\right) - 15bc^2dx^3F_1\left(\frac{5}{2}, 2, \frac{(bc - ad)x^3}{(c + dx^3)}\right) - 12bc^2dx^6F_1\left(\frac{5}{2}, 2, \frac{(bc - ad)x^3}{(c + dx^3)}\right) + 27ad^2x^6F_1\left(\frac{5}{2}, 2, \frac{(bc - ad)x^3}{(c + dx^3)}\right) - 27cd^2x^9F_1\left(\frac{5}{2}, 2, \frac{(bc - ad)x^3}{(c + dx^3)}\right) + 12acd^2x^6F_1\left(\frac{5}{2}, 2, \frac{(bc - ad)x^3}{(c + dx^3)}\right) + 6ac^2 - 12ac^2dx^3 + 36acd^2x^6 + 6bc^2x^3 - 12bc^2dx^6 + 36bcd^2x^9}{56c^2x^7(a + bx^3)^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)), x]

[Out] -1/56*(8*a*c^3 + 8*b*c^3*x^3 - 12*a*c^2*d*x^3 - 12*b*c^2*d*x^6 + 36*a*c*d^2*x^6 + 36*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(2*c^2 - 3*c*d*x^3 + 9*d^2*x^6)*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 15*b*c^3*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 15*a*c^2*d*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 12*b*c^2*d*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 12*a*c*d^2*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*HypergeometricPFQ[{2/3, 2, 2}, {1, 5/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(c^4*x^7*(a + b*x^3)^(2/3))

IntegrateAlgebraic [C] time = 2.62, size = 444, normalized size = 1.72

$$\frac{\sqrt[3]{a + bx^3}(-4a^2c^2 + 7a^2cdx^3 - 28a^2d^2x^6 - abc^2x^3 + 7abcdx^6 + 3b^2c^2x^6)}{28a^2c^2x^7} \sqrt[3]{\frac{d^2\sqrt[3]{bc - ad} - \sqrt[3]{d^2\sqrt[3]{bc - ad}}}{6c^{103}}} \log\left(\frac{2x\sqrt[3]{bc - ad} + (1 + \sqrt[3]{3})\sqrt[3]{c}\sqrt[3]{a + bx^3}}{\sqrt[3]{1 - \sqrt[3]{3}}d^2\sqrt[3]{bc - ad} \tan^{-1}\left(\frac{3c\sqrt[3]{bc - ad}}{\sqrt[3]{3}\sqrt[3]{bc - ad} - \sqrt[3]{3}\sqrt[3]{c}\sqrt[3]{a + bx^3}}\right)}\right) + \frac{\sqrt[3]{1 - \sqrt[3]{3}}d^2\sqrt[3]{bc - ad}}{\sqrt[3]{6c^{103}}} \log\left(\frac{\sqrt[3]{3}d^2\sqrt[3]{bc - ad} + d^2\sqrt[3]{bc - ad}}{\sqrt[3]{3}d^2\sqrt[3]{bc - ad} - d^2\sqrt[3]{bc - ad}}\right) \log\left(\frac{\sqrt[3]{3} + i}{\sqrt[3]{3} - i}\right)^{2/3} + \sqrt[3]{c}(-\sqrt[3]{3} + i)\sqrt[3]{a + bx^3}\sqrt[3]{bc - ad} - 2a^2(bc - ad)^{2/3}}{12c^{103}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]

[Out] ((a + b*x^3)^(1/3)*(-4*a^2*c^2 - a*b*c^2*x^3 + 7*a^2*c*d*x^3 + 3*b^2*c^2*x^6 + 7*a*b*c*d*x^6 - 28*a^2*d^2*x^6))/(28*a^2*c^3*x^7) + (Sqrt[-1 - I*Sqrt[3]]*d^2*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]/(Sqrt[6]*c^(10/3)) + ((d^2*(b*c - a*d)^(1/3) - I*Sqrt[3]*d^2*(b*c - a*d)^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]/(6*c^(10/3)) + ((I/12)*(I*d^2*(b*c - a*d)^(1/3) + Sqrt[3]*d^2*(b*c - a*d)^(1/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)]*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]/c^(10/3)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x)

[Out] int((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{1/3}}{x^8 (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x)

[Out] int((a + b*x^3)^(1/3)/(x^8*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{x^8 (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(1/3)/x**8/(d*x**3+c),x)

[Out] Integral((a + b*x**3)**(1/3)/(x**8*(c + d*x**3)), x)

$$3.427 \quad \int \frac{\sqrt[3]{a+bx^3}}{x^{11}(c+dx^3)} dx$$

Optimal. Leaf size=318

$$\frac{\sqrt[3]{a+bx^3} (-35a^2d^2 + 5abcd + 3b^2c^2)}{140a^2c^3x^4} - \frac{\sqrt[3]{a+bx^3} (-140a^3d^3 + 35a^2bcd^2 + 15ab^2c^2d + 9b^3c^3)}{140a^3c^4x} - \frac{d^3\sqrt[3]{bc-ad} \log}{6c^{13/3}}$$

Rubi [C] time = 2.64, antiderivative size = 905, normalized size of antiderivative = 2.85, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]

[Out] $-(56*a*c^4 + 56*b*c^4*x^3 - 72*a*c^3*d*x^3 - 72*b*c^3*d*x^6 + 108*a*c^2*d^2*x^6 + 108*b*c^2*d^2*x^9 - 324*a*c*d^3*x^9 - 324*b*c*d^3*x^{12} - 28*b*c^4*x^3*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 28*a*c^3*d*x^3*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 36*b*c^3*d*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 36*a*c^2*d^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 54*b*c^2*d^2*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 54*a*c*d^3*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b*c*d^3*x^{12}*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a*d^4*x^{12}*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 117*b*c^4*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 117*a*c^3*d*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 99*b*c^3*d*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 99*a*c^2*d^2*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 81*b*c^2*d^2*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 81*a*c*d^3*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*b*c*d^3*x^{12}*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^{12}*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 54*(b*c - a*d)*x^3*(2*c - 3*d*x^3)*(c + d*x^3)^2*HypergeometricPFQ[{2/3, 2, 2}, {1, 5/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*HypergeometricPFQ[{2/3, 2, 2, 2}, {1, 1, 5/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(560*c^5*x^{10}*(a + b*x^3)^(2/3))$

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a + bx^3}}{x^{11}(c + dx^3)} dx = \frac{\sqrt[3]{a + bx^3} \int \frac{\sqrt[3]{1 + \frac{bx^3}{a}}}{x^{11}(c + dx^3)} dx}{\sqrt[3]{1 + \frac{bx^3}{a}}}$$

$$= \frac{56ac^4 + 56bc^4x^3 - 72ac^3dx^3 - 72bc^3dx^6 + 108ac^2d^2x^6 + 108bc^2d^2x^9 - 324acd^3x^9 - 324b^2d^3x^{12}}{70a^3c^4x^{10} + d^3x^2\left(\frac{bx^3}{a} + 1\right)^{2/3} (ad-bc) {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)}\right) + c^5\left(\frac{dx^3}{c} + 1\right)^{2/3}}$$

Mathematica [C] time = 5.24, size = 214, normalized size = 0.67

$$\frac{(a+bx^3)(a^3(-14c^3+20c^2dx^3-35cd^2x^6+140d^3x^9)+a^2bcx^3(-2c^2+5cdx^3-35d^2x^6)+3ab^2c^2x^6(c-5dx^3)-9b^3c^3x^9)}{70a^3c^4x^{10}} + \frac{d^3x^2\left(\frac{bx^3}{a}+1\right)^{2/3} (ad-bc) {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)}\right) + c^5\left(\frac{dx^3}{c}+1\right)^{2/3}}{2(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)), x]

[Out] (((a + b*x^3)*(-9*b^3*c^3*x^9 + 3*a*b^2*c^2*x^6*(c - 5*d*x^3) + a^2*b*c*x^3*(-2*c^2 + 5*c*d*x^3 - 35*d^2*x^6) + a^3*(-14*c^3 + 20*c^2*d*x^3 - 35*c*d^2*x^6 + 140*d^3*x^9)))/(70*a^3*c^4*x^10) + (d^3*(-(b*c) + a*d)*x^2*(1 + (b*x

$\sqrt{3}/a)^{2/3} \cdot \text{Hypergeometric2F1}[2/3, 2/3, 5/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))]/(c^5*(1 + (d*x^3)/c)^{(2/3)})/(2*(a + b*x^3)^{(2/3)})$

IntegrateAlgebraic [C] time = 2.99, size = 503, normalized size = 1.58

$\frac{\sqrt{3} \sqrt{3} (-14a^2c^2 + 20a^2cd^2 - 35a^2d^2c^2 + 140a^2d^2c^2 - 2a^2b^2c^2 + 5a^2b^2cd^2 - 35a^2b^2cd^2 + 3a^2b^2c^2d^2 - 15a^2b^2c^2d^2 - 9a^2b^2c^2d^2)}{140c^{13/3}}$, $\frac{(\sqrt{3}d^2\sqrt{c-ad} + a^2\sqrt{c-ad})\log(2\sqrt{c-ad} + (1+\sqrt{3})\sqrt{c-ad})}{6c^{13/3}}$, $\frac{\sqrt{3-\sqrt{3}}d^2\sqrt{c-ad}\arctan\left(\frac{2\sqrt{c-ad}}{d\sqrt{c-ad} + \sqrt{3}d\sqrt{c-ad}}\right)}{\sqrt{6}c^{13/3}}$, $\frac{(d^2\sqrt{c-ad} - \sqrt{3}d^2\sqrt{c-ad})\log(\sqrt{3} + 1)c^{13/3} + \sqrt{3}(-\sqrt{3}x + a)\sqrt{c-ad} - 2a^2(c-ad)^{3/2}}{12c^{13/3}}$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]

[Out] $((a + b*x^3)^{1/3}*(-14*a^3*c^3 - 2*a^2*b*c^3*x^3 + 20*a^3*c^2*d*x^3 + 3*a*b^2*c^3*x^6 + 5*a^2*b*c^2*d*x^6 - 35*a^3*c*d^2*x^6 - 9*b^3*c^3*x^9 - 15*a*b^2*c^2*d*x^9 - 35*a^2*b*c*d^2*x^9 + 140*a^3*d^3*x^9))/(140*a^3*c^4*x^{10} - (\text{Sqrt}[-1 - I*\text{Sqrt}[3]]*d^3*(b*c - a*d)^{1/3}*\text{ArcTan}[(3*(b*c - a*d)^{1/3}*x)/(\text{Sqrt}[3]*(b*c - a*d)^{1/3}*x - (3*I)*c^{1/3}*(a + b*x^3)^{1/3} - \text{Sqrt}[3]*c^{1/3}*(a + b*x^3)^{1/3})])/(\text{Sqrt}[6]*c^{13/3}) + ((I/6)*(I*d^3*(b*c - a*d)^{1/3} + \text{Sqrt}[3]*d^3*(b*c - a*d)^{1/3})*\text{Log}[2*(b*c - a*d)^{1/3}*x + (1 + I*\text{Sqrt}[3])*c^{1/3}*(a + b*x^3)^{1/3}])/c^{13/3} + ((d^3*(b*c - a*d)^{1/3} - I*\text{Sqrt}[3]*d^3*(b*c - a*d)^{1/3})*\text{Log}[(-2*I)*(b*c - a*d)^{2/3}*x^2 + c^{1/3}*(b*c - a*d)^{1/3}*(I*x - \text{Sqrt}[3]*x)*(a + b*x^3)^{1/3} + (I + \text{Sqrt}[3])*c^{2/3}*(a + b*x^3)^{2/3}])/ (12*c^{13/3})$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x)`

[Out] `int((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{1/3}}{x^{11} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x)`

[Out] `int((a + b*x^3)^(1/3)/(x^11*(c + d*x^3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{x^{11} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(1/3)/x**11/(d*x**3+c),x)`

[Out] `Integral((a + b*x**3)**(1/3)/(x**11*(c + d*x**3)), x)`

$$3.428 \quad \int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3} dx$$

Optimal. Leaf size=266

$$\frac{(a+bx^3)^{5/3}(a^2d^2+abcd+b^2c^2)}{5b^3d^3} - \frac{(a+bx^3)^{8/3}(2ad+bc)}{8b^3d^2} + \frac{(a+bx^3)^{11/3}}{11b^3d} + \frac{c^3(bc-ad)^{2/3} \log(c+dx^3)}{6d^{14/3}} - \frac{c^3(bc-ad)^{2/3} \log(c+dx^3)}{6d^{14/3}}$$

Rubi [A] time = 0.32, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 88, 50, 56, 617, 204, 31}

$$\frac{(a+bx^3)^{5/3}(a^2d^2+abcd+b^2c^2)}{5b^3d^3} - \frac{(a+bx^3)^{8/3}(2ad+bc)}{8b^3d^2} + \frac{(a+bx^3)^{11/3}}{11b^3d} - \frac{c^3(a+bx^3)^{2/3}}{2d^4} + \frac{c^3(bc-ad)^{2/3} \log(c+dx^3)}{6d^{14/3}} - \frac{c^3(bc-ad)^{2/3} \log\left(\frac{\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{2d^{14/3}} - \frac{c^3(bc-ad)^{2/3} \tan^{-1}\left(\frac{1-2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d} d^{14/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] $-(c^3(a + b*x^3)^{2/3})/(2*d^4) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^{5/3})/(5*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^{8/3})/(8*b^3*d^2) + (a + b*x^3)^{11/3}/(11*b^3*d) - (c^3*(b*c - a*d)^{2/3}*ArcTan[(1 - (2*d^{1/3})*(a + b*x^3)^{1/3})/(b*c - a*d)^{1/3}]/(sqrt[3]))/(sqrt[3]*d^{14/3}) + (c^3*(b*c - a*d)^{2/3}*Log[c + d*x^3])/(6*d^{14/3}) - (c^3*(b*c - a*d)^{2/3}*Log[(b*c - a*d)^{1/3} + d^{1/3}*(a + b*x^3)^{1/3}])/(2*d^{14/3})$

Rule 31

Int[((a_) + (b_)*(x_))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11} (a + bx^3)^{2/3}}{c + dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3 (a + bx)^{2/3}}{c + dx} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{(b^2c^2 + abcd + a^2d^2)(a + bx)^{2/3}}{b^2d^3} + \frac{(-bc - 2ad)(a + bx)^{5/3}}{b^2d^2} + \frac{(a + bx)^{8/3}}{b^2d} \right) dx, x, x^3 \right) \\
&= \frac{(b^2c^2 + abcd + a^2d^2)(a + bx^3)^{5/3}}{5b^3d^3} - \frac{(bc + 2ad)(a + bx^3)^{8/3}}{8b^3d^2} + \frac{(a + bx^3)^{11/3}}{11b^3d} - \frac{c^3 \text{Subst} \left(\int \frac{x^3 (a + bx)^{2/3}}{c + dx} dx, x, x^3 \right)}{11b^3d} \\
&= -\frac{c^3 (a + bx^3)^{2/3}}{2d^4} + \frac{(b^2c^2 + abcd + a^2d^2)(a + bx^3)^{5/3}}{5b^3d^3} - \frac{(bc + 2ad)(a + bx^3)^{8/3}}{8b^3d^2} + \frac{(a + bx^3)^{11/3}}{11b^3d} \\
&= -\frac{c^3 (a + bx^3)^{2/3}}{2d^4} + \frac{(b^2c^2 + abcd + a^2d^2)(a + bx^3)^{5/3}}{5b^3d^3} - \frac{(bc + 2ad)(a + bx^3)^{8/3}}{8b^3d^2} + \frac{(a + bx^3)^{11/3}}{11b^3d} \\
&= -\frac{c^3 (a + bx^3)^{2/3}}{2d^4} + \frac{(b^2c^2 + abcd + a^2d^2)(a + bx^3)^{5/3}}{5b^3d^3} - \frac{(bc + 2ad)(a + bx^3)^{8/3}}{8b^3d^2} + \frac{(a + bx^3)^{11/3}}{11b^3d} \\
&= -\frac{c^3 (a + bx^3)^{2/3}}{2d^4} + \frac{(b^2c^2 + abcd + a^2d^2)(a + bx^3)^{5/3}}{5b^3d^3} - \frac{(bc + 2ad)(a + bx^3)^{8/3}}{8b^3d^2} + \frac{(a + bx^3)^{11/3}}{11b^3d}
\end{aligned}$$

Mathematica [C] time = 0.12, size = 148, normalized size = 0.56

$$\frac{(a + bx^3)^{2/3} \left(18a^3d^3 + 3a^2bd^2(11c - 4dx^3) + 220b^3c^3 {}_2F_1 \left(\frac{2}{3}, 1; \frac{5}{3}; \frac{d(bx^3+a)}{ad-bc} \right) + 2ab^2d(44c^2 - 11cdx^3 + 5d^2x^6) + b^3(-220c^3 + 88c^2dx^3 - 55cd^2x^6 + 40d^3x^9) \right)}{440b^3d^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] ((a + b*x^3)^(2/3)*(18*a^3*d^3 + 3*a^2*b*d^2*(11*c - 4*d*x^3) + 2*a*b^2*d*(44*c^2 - 11*c*d*x^3 + 5*d^2*x^6) + b^3*(-220*c^3 + 88*c^2*d*x^3 - 55*c*d^2*x^6 + 40*d^3*x^9) + 220*b^3*c^3*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-b*c + a*d)]))/(440*b^3*d^4)

IntegrateAlgebraic [A] time = 0.65, size = 340, normalized size = 1.28

$$\frac{(a + bx^3)^{2/3} (18a^3d^3 + 33a^2bd^2 - 12a^2cd^2x^3 + 88ab^2c^2d - 22ab^2cd^2x^3 + 10ab^2d^2x^6 - 220b^3c^3 + 88b^3c^2dx^3 - 55b^3cd^2x^6 + 40b^3d^3x^9)}{440b^3d^4} - \frac{c^2(bc - ad)^{2/3} \log \left(\frac{\sqrt{bc - ad} + \sqrt{d} \sqrt{a + bx^3}}{3d^{4/3}} \right)}{3d^{4/3}} + \frac{c^2(bc - ad)^{2/3} \log \left(\frac{-\sqrt{d} \sqrt{a + bx^3} \sqrt{bc - ad} + (bc - ad)^{2/3} + d^{2/3}(a + bx^3)^{2/3}}{6d^{4/3}} \right)}{6d^{4/3}} - \frac{c^2(bc - ad)^{2/3} \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2\sqrt{d} \sqrt{a + bx^3}}{\sqrt{3} \sqrt{bc - ad}} \right)}{\sqrt{3}d^{4/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]

[Out] ((a + b*x^3)^(2/3)*(-220*b^3*c^3 + 88*a*b^2*c^2*d + 33*a^2*b*c*d^2 + 18*a^3*d^3 + 88*b^3*c^2*d*x^3 - 22*a*b^2*c*d^2*x^3 - 12*a^2*b*d^3*x^3 - 55*b^3*c*d^2*x^6 + 10*a*b^2*d^3*x^6 + 40*b^3*d^3*x^9))/(440*b^3*d^4) - (c^3*(b*c - a*d)^(2/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*d^(14/3)) - (c^3*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(3*d^(14/3)) + (c^3*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/(6*d^(14/3))

fricas [B] time = 0.69, size = 455, normalized size = 1.71

$$\frac{440 \sqrt{3} b^3 c^3 \left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} \arctan\left(\frac{1}{\sqrt{3}} \frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{(b^2 c^2 d^2 - 2 a b c d + a^2 d^2)^{1/3}} \right) + 220 b^3 c^3 \log\left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} \log\left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} + (b^2 c^2 d^2 - 2 a b c d + a^2 d^2)^{1/3} \log\left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} - (b^2 c^2 d^2 - 2 a b c d + a^2 d^2)^{1/3} \log\left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} - 440 b^3 c^3 \left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} \log\left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} - (b^2 c^2 d^2 - 2 a b c d + a^2 d^2)^{1/3} \log\left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} - 3(40 b^3 d^3 x^9 - 5(11 b^3 c d^2 - 2 a b^2 d^3) x^6 - 220 b^3 c^3 + 88 a b^2 c^2 d + 33 a^2 b c d^2 + 18 a^3 d^3 + 2(44 b^3 c^2 d - 11 a b^2 c d^2 - 6 a^2 b d^3) x^3) (b^2 c^2 d^2 - 2 a b c d + a^2 d^2)^{1/3}}{1320 b^3 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] -1/1320*(440*sqrt(3)*b^3*c^3*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 220*b^3*c^3*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) - 440*b^3*c^3*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) - 3*(40*b^3*d^3*x^9 - 5*(11*b^3*c*d^2 - 2*a*b^2*d^3)*x^6 - 220*b^3*c^3 + 88*a*b^2*c^2*d + 33*a^2*b*c*d^2 + 18*a^3*d^3 + 2*(44*b^3*c^2*d - 11*a*b^2*c*d^2 - 6*a^2*b*d^3)*x^3)*(b*x^3 + a)^(2/3))/(b^3*d^4)

giac [A] time = 0.29, size = 409, normalized size = 1.54

$$\frac{\left(b^2 c^2 d^2 - 2 a b c d + a^2 d^2 \right)^{1/3} \arctan\left(\frac{1}{\sqrt{3}} \frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{(b^2 c^2 d^2 - 2 a b c d + a^2 d^2)^{1/3}} \right) + 220 b^3 c^3 \log\left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} \log\left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} + (b^2 c^2 d^2 - 2 a b c d + a^2 d^2)^{1/3} \log\left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} - (b^2 c^2 d^2 - 2 a b c d + a^2 d^2)^{1/3} \log\left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} - 440 b^3 c^3 \left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} \log\left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} - (b^2 c^2 d^2 - 2 a b c d + a^2 d^2)^{1/3} \log\left(\frac{b^2 c^2 d^2 - 2 a b c d + a^2 d^2}{d^2} \right)^{1/3} - 3(40 b^3 d^3 x^9 - 5(11 b^3 c d^2 - 2 a b^2 d^3) x^6 - 220 b^3 c^3 + 88 a b^2 c^2 d + 33 a^2 b c d^2 + 18 a^3 d^3 + 2(44 b^3 c^2 d - 11 a b^2 c d^2 - 6 a^2 b d^3) x^3) (b^2 c^2 d^2 - 2 a b c d + a^2 d^2)^{1/3}}{1320 b^3 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out] -1/3*(b^37*c^4*d^7*(-(b*c - a*d)/d)^(1/3) - a*b^36*c^3*d^8*(-(b*c - a*d)/d)^(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^37*c*d^11 - a*b^36*d^12) - 1/3*sqrt(3)*(-(b*c*d^2 + a*d^3)^(2/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3)))/(-(b*c - a*d)/d)^(1/3))/d^6 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3))

$$\frac{(2/3) + (b*x^3 + a)^{(1/3)}*(-(b*c - a*d)/d)^{(1/3)} + (-(b*c - a*d)/d)^{(2/3)}}{d^6} - \frac{1}{440} * (220*(b*x^3 + a)^{(2/3)}*b^{33}*c^3*d^7 - 88*(b*x^3 + a)^{(5/3)}*b^{32}*c^2*d^8 + 55*(b*x^3 + a)^{(8/3)}*b^{31}*c*d^9 - 88*(b*x^3 + a)^{(5/3)}*a*b^{31}*c*d^9 - 40*(b*x^3 + a)^{(11/3)}*b^{30}*d^{10} + 110*(b*x^3 + a)^{(8/3)}*a*b^{30}*d^{10} - 88*(b*x^3 + a)^{(5/3)}*a^2*b^{30}*d^{10}) / (b^{33}*d^{11})$$

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}} x^{11}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 5.13, size = 490, normalized size = 1.84

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x)

[Out] ((3*a^2)/(5*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c - a*b^3*d))/(5*b^3*d))*(a + b*x^3)^(5/3) - ((3*a)/(8*b^3*d) + (b^4*c - a*b^3*d)/(8*b^6*d^2))*(a + b*x^3)^(8/3) - (a + b*x^3)^(2/3)*(a^3/(2*b^3*d) + ((3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c - a*b^3*d))/(b^3*d))*(b^4*c - a*b^3*d)/(2*b^3*d)) + (a + b*x^3)^(11/3)/(11*b^3*d) - (c^3*log(((a + b*x^3)^(1/3)*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))/d^7 - (c^6*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(28/3))))*(a*d - b*c)^(2/3)/(3*d^(14/3)) - (c^3*log((c^6*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7

$$\begin{aligned} & /3)) / d^{22/3} + (c^6 * (a + b * x^3)^{1/3} * (a * d - b * c)^2 / d^7 * ((3^{1/2} * 1i) / 2 \\ & - 1/2) * (a * d - b * c)^{2/3}) / (3 * d^{14/3}) + (c^3 * \log((c^6 * (a + b * x^3)^{1/3} * (a \\ & * d - b * c)^2 / d^7 - (c^6 * (3^{1/2} * 1i + 1)^2 * (a * d - b * c)^{7/3}) / (4 * d^{22/3}))) \\ & * ((3^{1/2} * 1i) / 6 + 1/6) * (a * d - b * c)^{2/3}) / d^{14/3} \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11} (a + bx^3)^{\frac{2}{3}}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11*(b*x**3+a)**(2/3)/(d*x**3+c), x)

[Out] Integral(x**11*(a + b*x**3)**(2/3)/(c + d*x**3), x)

$$3.429 \quad \int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3} dx$$

Optimal. Leaf size=223

$$\frac{(a+bx^3)^{5/3}(ad+bc)}{5b^2d^2} + \frac{(a+bx^3)^{8/3}}{8b^2d} - \frac{c^2(bc-ad)^{2/3} \log(c+dx^3)}{6d^{11/3}} + \frac{c^2(bc-ad)^{2/3} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{11/3}}$$

Rubi [A] time = 0.26, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 88, 50, 56, 617, 204, 31}

$$\frac{(a+bx^3)^{5/3}(ad+bc)}{5b^2d^2} + \frac{(a+bx^3)^{8/3}}{8b^2d} + \frac{c^2(a+bx^3)^{2/3}}{2d^3} - \frac{c^2(bc-ad)^{2/3} \log(c+dx^3)}{6d^{11/3}} + \frac{c^2(bc-ad)^{2/3} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{11/3}} + \frac{c^2(bc-ad)^{2/3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] (c^2*(a + b*x^3)^(2/3))/(2*d^3) - ((b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^2*d^2) + (a + b*x^3)^(8/3)/(8*b^2*d) + (c^2*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sqrt[3]*d^(11/3)) - (c^2*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(11/3)) + (c^2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(11/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m+1)*(c + d*x)^n)/(b*(m+n+1)), x] + Dist[(n*(b*c - a*d))/(b*(m+n+1)), Int[(a + b*x)^m*(c + d*x)^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 56

Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),

```
x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x]]) /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8 (a + bx^3)^{2/3}}{c + dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (a + bx)^{2/3}}{c + dx} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{(-bc - ad)(a + bx)^{2/3}}{bd^2} + \frac{(a + bx)^{5/3}}{bd} + \frac{c^2 (a + bx)^{2/3}}{d^2 (c + dx)} \right) dx, x, x^3 \right) \\
&= -\frac{(bc + ad)(a + bx^3)^{5/3}}{5b^2 d^2} + \frac{(a + bx^3)^{8/3}}{8b^2 d} + \frac{c^2 \text{Subst} \left(\int \frac{(a + bx)^{2/3}}{c + dx} dx, x, x^3 \right)}{3d^2} \\
&= \frac{c^2 (a + bx^3)^{2/3}}{2d^3} - \frac{(bc + ad)(a + bx^3)^{5/3}}{5b^2 d^2} + \frac{(a + bx^3)^{8/3}}{8b^2 d} - \frac{(c^2(bc - ad)) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a + bx}} dx, x, x^3 \right)}{3d^3} \\
&= \frac{c^2 (a + bx^3)^{2/3}}{2d^3} - \frac{(bc + ad)(a + bx^3)^{5/3}}{5b^2 d^2} + \frac{(a + bx^3)^{8/3}}{8b^2 d} - \frac{c^2(bc - ad)^{2/3} \log(c + dx^3)}{6d^{11/3}} + \frac{c^2(bc - ad)^{2/3} \tan^{-1} \left(\frac{1 - 2\sqrt[3]{a} \sqrt[3]{a + bx}}{\sqrt[3]{bc - ad}} \right)}{\sqrt{3} d^{11/3}} \\
&= \frac{c^2 (a + bx^3)^{2/3}}{2d^3} - \frac{(bc + ad)(a + bx^3)^{5/3}}{5b^2 d^2} + \frac{(a + bx^3)^{8/3}}{8b^2 d} - \frac{c^2(bc - ad)^{2/3} \log(c + dx^3)}{6d^{11/3}} + \frac{c^2(bc - ad)^{2/3} \tan^{-1} \left(\frac{1 - 2\sqrt[3]{a} \sqrt[3]{a + bx}}{\sqrt[3]{bc - ad}} \right)}{\sqrt{3} d^{11/3}}
\end{aligned}$$

Mathematica [C] time = 0.10, size = 104, normalized size = 0.47

$$\frac{(a + bx^3)^{2/3} \left(-3a^2 d^2 - 20b^2 c^2 {}_2F_1 \left(\frac{2}{3}, 1; \frac{5}{3}; \frac{d(bx^3 + a)}{ad - bc} \right) + 2abd(dx^3 - 4c) + b^2(20c^2 - 8cdx^3 + 5d^2 x^6) \right)}{40b^2 d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] ((a + b*x^3)^(2/3)*(-3*a^2*d^2 + 2*a*b*d*(-4*c + d*x^3) + b^2*(20*c^2 - 8*c*d*x^3 + 5*d^2*x^6) - 20*b^2*c^2*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-b*c + a*d)]))/(40*b^2*d^3)

IntegrateAlgebraic [A] time = 0.40, size = 284, normalized size = 1.27

$$\frac{(a + bx^3)^{2/3} (-3a^2 d^2 - 8abcd + 2abd^2 x^3 + 20b^2 c^2 - 8b^2 cdx^3 + 5b^2 d^2 x^6)}{40b^2 d^3} + \frac{c^2 (bc - ad)^{2/3} \log(\sqrt[3]{bc - ad} + \sqrt[3]{d} \sqrt[3]{a + bx^3})}{3d^{11/3}} - \frac{c^2 (bc - ad)^{2/3} \log(-\sqrt[3]{d} \sqrt[3]{a + bx^3} \sqrt[3]{bc - ad} + (bc - ad)^{2/3} + d^{2/3} (a + bx^3)^{2/3})}{6d^{11/3}} + \frac{c^2 (bc - ad)^{2/3} \tan^{-1} \left(\frac{1 - 2\sqrt[3]{a} \sqrt[3]{a + bx^3}}{\sqrt[3]{bc - ad}} \right)}{\sqrt{3} d^{11/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]

[Out] ((a + b*x^3)^(2/3)*(20*b^2*c^2 - 8*a*b*c*d - 3*a^2*d^2 - 8*b^2*c*d*x^3 + 2*a*b*d^2*x^3 + 5*b^2*d^2*x^6))/(40*b^2*d^3) + (c^2*(b*c - a*d)^(2/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*d^(11/3)) + (c^2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(3*d^(11/3)) - (c^2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(11/3))

fricas [B] time = 0.61, size = 398, normalized size = 1.78

$$\frac{40\sqrt{3}b^2c^2\left(\frac{b^2-2abd+ad^2}{d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(\frac{b^2-2abd+ad^2}{d^3}\right)^{\frac{1}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)-20b^2c^2\left(\frac{b^2-2abd+ad^2}{d^3}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}d\left(\frac{b^2-2abd+ad^2}{d^3}\right)^{\frac{1}{3}}-(bx^3+a)^{\frac{1}{3}}(bc-ad)-(bc-ad)\left(\frac{b^2-2abd+ad^2}{d^3}\right)^{\frac{1}{3}}\right)+40b^2c^2\left(\frac{b^2-2abd+ad^2}{d^3}\right)^{\frac{1}{3}}\log\left(-d\left(\frac{b^2-2abd+ad^2}{d^3}\right)^{\frac{1}{3}}-(bx^3+a)^{\frac{1}{3}}(bc-ad)\right)+3(5b^2d^3+20b^2c^2-8abd-3a^2d^2-2(4b^2cd-abd^2)d^2)(bx^3+a)^{\frac{2}{3}}}{120b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] 1/120*(40*sqrt(3)*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 20*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + 40*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) + 3*(5*b^2*d^2*x^6 + 20*b^2*c^2 - 8*a*b*c*d - 3*a^2*d^2 - 2*(4*b^2*c*d - a*b*d^2)*x^3)*(b*x^3 + a)^(2/3)/(b^2*d^3)

giac [A] time = 0.27, size = 350, normalized size = 1.57

$$\frac{\left(\frac{b^2c^2d^2\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-ab^2c^2d^2\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\right)}{3(b^2cd^2-ab^2d^2)}+\frac{\sqrt{3}(-bcd^2+ad^2)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left((bx^3+a)^{\frac{1}{3}}\sqrt{\frac{bc-ad}{d}}\right)^{\frac{2}{3}}}{3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^3}-\frac{(-bcd^2+ad^2)^{\frac{2}{3}}c^2\log\left(\left(bx^3+a\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{6d^3}+\frac{20\left(bx^3+a\right)^{\frac{2}{3}}b^2c^2d^2-8\left(bx^3+a\right)^{\frac{1}{3}}b^2cd^2+5\left(bx^3+a\right)^{\frac{1}{3}}b^2d^2-8\left(bx^3+a\right)^{\frac{1}{3}}abd^2d^2}{40b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out] 1/3*(b^19*c^3*d^5*(-(b*c - a*d)/d)^(1/3) - a*b^18*c^2*d^6*(-(b*c - a*d)/d)^(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^19*c*d^8 - a*b^18*d^9) + 1/3*sqrt(3)*(-(b*c*d^2 + a*d^3)^(2/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3)))/(-(b*c - a*d)/d)^(1/3))/d^5 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^5 + 1/40*(20*(b*x^3 + a)^(2/3)*b^16*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^15*c*d^6

+ 5*(b*x^3 + a)^(8/3)*b^14*d^7 - 8*(b*x^3 + a)^(5/3)*a*b^14*d^7)/(b^16*d^8)

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}} x^8}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 5.11, size = 385, normalized size = 1.73

$$\left(\frac{x^8}{2d^2} + \frac{(2a + \frac{b^2 x^3 + a^2}{2d^2}) (b^3 c - a^2 d)}{2d^2} \right) (bx^3 + a)^{2/3} - \left(\frac{2a}{5d^2} + \frac{b^3 c - a^2 d}{5d^2} \right) (bx^3 + a)^{5/3} + \frac{(bx^3 + a)^{8/3}}{8d^2} + \frac{c^2 \ln \left(\frac{(bx^3 + a)^{1/3} (bx^3 + a^2) - a^2 (bx^3 + a)^{2/3}}{3d^{1/3}} \right)}{3d^{1/3}} - \frac{c^2 \ln \left(\frac{a^2 (bx^3 + a)^{1/3} (bx^3 + a^2) - a^2 (bx^3 + a)^{2/3}}{3d^{1/3}} \right)}{3d^{1/3}} \left(\frac{1}{2} + \frac{\sqrt{3}i}{2} \right) (ad - bc)^{2/3} + \frac{c^2 \ln \left(\frac{a^2 (bx^3 + a)^{1/3} (bx^3 + a^2) - a^2 (bx^3 + a)^{2/3}}{3d^{1/3}} \right)}{3d^{1/3}} \left(\frac{1}{2} + \frac{\sqrt{3}i}{2} \right) (ad - bc)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x)

[Out] (a^2/(2*b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*b^2*d))/(2*b^2*d))*(a + b*x^3)^(2/3) - ((2*a)/(5*b^2*d) + (b^3*c - a*b^2*d)/(5*b^4*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^2*d) + (c^2*log(((a + b*x^3)^(1/3)*(b^2*c^6 + a^2*c^4*d^2 - 2*a*b*c^5*d))/d^5 - (c^4*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(22/3)))*(a*d - b*c)^(2/3))/(3*d^(11/3)) - (c^2*log((c^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^5 - (c^4*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(16/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(2/3))/(3*d^(11/3)) + (c^2*log((c^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^5 - (c^4*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(7/3))/(4*d^(16/3)))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(2/3))/d^(11/3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 (a + bx^3)^{\frac{2}{3}}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(b*x**3+a)**(2/3)/(d*x**3+c), x)

[Out] Integral(x**8*(a + b*x**3)**(2/3)/(c + d*x**3), x)

$$3.430 \quad \int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3} dx$$

Optimal. Leaf size=188

$$\frac{c(bc-ad)^{2/3} \log(c+dx^3)}{6d^{8/3}} - \frac{c(bc-ad)^{2/3} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{8/3}} - \frac{c(bc-ad)^{2/3} \tan^{-1}\left(\frac{1-2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{8/3}} - c$$

Rubi [A] time = 0.20, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 80, 50, 56, 617, 204, 31}

$$-\frac{c(a+bx^3)^{2/3}}{2d^2} + \frac{c(bc-ad)^{2/3} \log(c+dx^3)}{6d^{8/3}} - \frac{c(bc-ad)^{2/3} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{8/3}} - \frac{c(bc-ad)^{2/3} \tan^{-1}\left(\frac{1-2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{8/3}} + \frac{(a+bx^3)^{5/3}}{5bd}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] -(c*(a + b*x^3)^(2/3))/(2*d^2) + (a + b*x^3)^(5/3)/(5*b*d) - (c*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(8/3)) + (c*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(8/3)) - (c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(8/3))

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m+1)*(c + d*x)^n)/(b*(m+n+1)), x] + Dist[(n*(b*c - a*d))/(b*(m+n+1)), Int[(a + b*x)^m*(c + d*x)^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (a + bx^3)^{2/3}}{c + dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(a + bx)^{2/3}}{c + dx} dx, x, x^3 \right) \\
&= \frac{(a + bx^3)^{5/3}}{5bd} - \frac{c \text{Subst} \left(\int \frac{(a+bx)^{2/3}}{c+dx} dx, x, x^3 \right)}{3d} \\
&= -\frac{c(a + bx^3)^{2/3}}{2d^2} + \frac{(a + bx^3)^{5/3}}{5bd} + \frac{(c(bc - ad)) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right)}{3d^2} \\
&= -\frac{c(a + bx^3)^{2/3}}{2d^2} + \frac{(a + bx^3)^{5/3}}{5bd} + \frac{c(bc - ad)^{2/3} \log(c + dx^3)}{6d^{8/3}} - \frac{(c(bc - ad)^{2/3}) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right)}{2d^2} \\
&= -\frac{c(a + bx^3)^{2/3}}{2d^2} + \frac{(a + bx^3)^{5/3}}{5bd} + \frac{c(bc - ad)^{2/3} \log(c + dx^3)}{6d^{8/3}} - \frac{c(bc - ad)^{2/3} \log(\sqrt[3]{bc - ad})}{2d^{8/3}} \\
&= -\frac{c(a + bx^3)^{2/3}}{2d^2} + \frac{(a + bx^3)^{5/3}}{5bd} - \frac{c(bc - ad)^{2/3} \tan^{-1} \left(\frac{1 - 2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}} \right)}{\sqrt{3} d^{8/3}} + \frac{c(bc - ad)^{2/3} \log(\sqrt[3]{bc - ad})}{6d^{8/3}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 68, normalized size = 0.36

$$\frac{(a + bx^3)^{2/3} \left(5bc {}_2F_1 \left(\frac{2}{3}, 1; \frac{5}{3}; \frac{d(bx^3+a)}{ad-bc} \right) + 2ad - 5bc + 2bdx^3 \right)}{10bd^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] ((a + b*x^3)^(2/3)*(-5*b*c + 2*a*d + 2*b*d*x^3 + 5*b*c*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-b*c) + a*d]))/(10*b*d^2)

IntegrateAlgebraic [A] time = 0.31, size = 241, normalized size = 1.28

$$-\frac{c(bc - ad)^{2/3} \log(\sqrt[3]{bc - ad} + \sqrt[3]{d}\sqrt[3]{a + bx^3})}{3d^{8/3}} + \frac{c(bc - ad)^{2/3} \log(-\sqrt[3]{d}\sqrt[3]{a + bx^3}\sqrt[3]{bc - ad} + (bc - ad)^{2/3} + d^{2/3}(a + bx^3)^{2/3})}{6d^{8/3}} - \frac{c(bc - ad)^{2/3} \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}} \right)}{\sqrt{3}d^{8/3}} + \frac{(a + bx^3)^{2/3}(2ad - 5bc + 2bdx^3)}{10bd^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]

[Out] ((a + b*x^3)^(2/3)*(-5*b*c + 2*a*d + 2*b*d*x^3))/(10*b*d^2) - (c*(b*c - a*d)^(2/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*d^(8/3)) - (c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(3*d^(8/3)) + (c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/(6*d^(8/3))

fricas [B] time = 0.41, size = 353, normalized size = 1.88

$$\frac{10\sqrt{3}c\left(\frac{\sqrt{2}\sqrt{b^2+ad}}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(b^2+ad)^{\frac{1}{3}}d\left(\frac{b^2-2abd+ad^2}{d^3}\right)^{\frac{1}{3}}+\sqrt{3}(b^2+ad)}{3(b^2+ad)}\right)+5bc\left(\frac{\sqrt{2}\sqrt{b^2+ad}}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}d\left(\frac{b^2-2abd+ad^2}{d^3}\right)^{\frac{1}{3}}-(b^2+ad)^{\frac{1}{3}}(bc-ad)+(bc-ad)\left(\frac{b^2-2abd+ad^2}{d^3}\right)^{\frac{1}{3}}\right)-10bc\left(\frac{\sqrt{2}\sqrt{b^2+ad}}{d}\right)^{\frac{1}{3}}\log\left(-d\left(\frac{\sqrt{2}\sqrt{b^2+ad}}{d}\right)^{\frac{1}{3}}-(b^2+ad)^{\frac{1}{3}}(bc-ad)\right)-3(2bdx^3-5bc+2ad)(bx^3+a)^{\frac{1}{3}}}{30bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] -1/30*(10*sqrt(3)*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 5*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) - 10*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) - 3*(2*b*d*x^3 - 5*b*c + 2*a*d)*(b*x^3 + a)^(2/3)/(b*d^2)

giac [B] time = 0.55, size = 306, normalized size = 1.63

$$\frac{\left(\frac{b^7c^2d^3}{3(b^7cd^3-ab^6d^6)}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^6cd^4\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left((bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\right)}{\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}c\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^4}}+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c\log\left(\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)\right)}{6d^4}}-\frac{5(bx^3+a)^{\frac{2}{3}}b^5cd^3-2(bx^3+a)^{\frac{1}{3}}b^4d^4}{10b^5d^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out] -1/3*(b^7*c^2*d^3*(-(b*c - a*d)/d)^(1/3) - a*b^6*c*d^4*(-(b*c - a*d)/d)^(1/3)))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (- (b*c - a*d)/d)^(1/3)))/(b^7*c*d^5 - a*b^6*d^6) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (- (b*c - a*d)/d)^(1/3)))/(- (b*c - a*d)/d)^(1/3))/d^4 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c*log(((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(- (b*c - a*d)/d)^(1/3) + (- (b*c - a*d)/d)^(2/3))/d^4 - 1/10*(5*(b*x^3 + a)^(2/3)*b^5*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^4*d^4)/(b^5*d^5)

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}} x^5}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 5.06, size = 302, normalized size = 1.61

$$\frac{(bx^3+a)^{5/3}}{5bd} - (bx^3+a)^{2/3} \left(\frac{a}{2bd} + \frac{b^2c-abd}{2b^2d^2} \right) - \frac{c \ln\left(\frac{(bx^3+a)^{10}(a^2d^2-2ab^2d+bd^2) - c^2(ad-bc)^4(bd^2-abc^2)}{9d^{10}}\right)}{3d^{10}} - \frac{c \ln\left(\frac{c^2\left(\frac{1}{2}+\frac{\sqrt{11}}{2}\right)(ad-bc)^{10} + c^2(bx^3+a)^{10}(ad-bc)^2}{9d^{10}}\right)}{3d^{10}} \left(\frac{1}{2}+\frac{\sqrt{11}}{2}\right)(ad-bc)^{2/3} + \frac{c \ln\left(\frac{c^2(bx^3+a)^{10}(ad-bc)^2 - c^2\left(\frac{1}{2}+\frac{\sqrt{11}}{2}\right)(ad-bc)^{10}}{9d^{10}}\right)}{3d^{10}} \left(\frac{1}{2}+\frac{\sqrt{11}}{2}\right)(ad-bc)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x)

[Out] (a + b*x^3)^(5/3)/(5*b*d) - (a + b*x^3)^(2/3)*(a/(2*b*d) + (b^2*c - a*b*d)/(2*b^2*d^2)) - (c*log(((a + b*x^3)^(1/3)*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))/d^3 - (c^2*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(16/3))))*(a*d - b*c)^(2/3)/(3*d^(8/3)) - (c*log(((c^2*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7/3))/d^(10/3) + (c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^3*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(2/3))/(3*d^(8/3)) + (c*log(((c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^3 - (c^2*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(10/3)))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(2/3))/(3*d^(8/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + bx^3)^{\frac{2}{3}}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

$$3.431 \quad \int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3} dx$$

Optimal. Leaf size=162

$$-\frac{(bc-ad)^{2/3} \log(c+dx^3)}{6d^{5/3}} + \frac{(bc-ad)^{2/3} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{5/3}} + \frac{(bc-ad)^{2/3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3} d^{5/3}} + (a$$

Rubi [A] time = 0.17, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {444, 50, 56, 617, 204, 31}

$$-\frac{(bc-ad)^{2/3} \log(c+dx^3)}{6d^{5/3}} + \frac{(bc-ad)^{2/3} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{5/3}} + \frac{(bc-ad)^{2/3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3} d^{5/3}} + \frac{(a+bx^3)^{2/3}}{2d}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] (a + b*x^3)^(2/3)/(2*d) + ((b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sqrt[3]*d^(5/3)) - ((b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(5/3)) + ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(5/3))

Rule 31

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m+1)*(c + d*x)^n)/(b*(m+n+1)), x] + Dist[(n*(b*c - a*d))/(b*(m+n+1)), Int[(a + b*x)^m*(c + d*x)^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 56

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),

```
x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x]]) /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + bx^3)^{2/3}}{c + dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^{2/3}}{c + dx} dx, x, x^3 \right) \\
&= \frac{(a + bx^3)^{2/3}}{2d} - \frac{(bc - ad) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right)}{3d} \\
&= \frac{(a + bx^3)^{2/3}}{2d} - \frac{(bc - ad)^{2/3} \log(c + dx^3)}{6d^{5/3}} + \frac{(bc - ad)^{2/3} \text{Subst} \left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}} + x} dx, x, \sqrt[3]{a + bx^3} \right)}{2d^{5/3}} \\
&= \frac{(a + bx^3)^{2/3}}{2d} - \frac{(bc - ad)^{2/3} \log(c + dx^3)}{6d^{5/3}} + \frac{(bc - ad)^{2/3} \log \left(\sqrt[3]{bc - ad} + \sqrt[3]{d} \sqrt[3]{a + bx^3} \right)}{2d^{5/3}} \\
&= \frac{(a + bx^3)^{2/3}}{2d} + \frac{(bc - ad)^{2/3} \tan^{-1} \left(\frac{1 - 2 \sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}} \right)}{\sqrt{3} d^{5/3}} - \frac{(bc - ad)^{2/3} \log(c + dx^3)}{6d^{5/3}} + \frac{(bc - ad)^{2/3}}{2d}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 47, normalized size = 0.29

$$\frac{(a + bx^3)^{2/3} \left({}_2F_1 \left(\frac{2}{3}, 1; \frac{5}{3}; \frac{d(bx^3+a)}{ad-bc} \right) - 1 \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] -1/2*((a + b*x^3)^(2/3)*(-1 + Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-b*c) + a*d]))/d

IntegrateAlgebraic [A] time = 0.25, size = 218, normalized size = 1.35

$$\frac{(bc - ad)^{2/3} \log \left(\sqrt[3]{bc - ad} + \sqrt[3]{d} \sqrt[3]{a + bx^3} \right)}{3d^{5/3}} - \frac{(bc - ad)^{2/3} \log \left(-\sqrt[3]{d} \sqrt[3]{a + bx^3} \sqrt[3]{bc - ad} + (bc - ad)^{2/3} + d^{2/3} (a + bx^3)^{2/3} \right)}{6d^{5/3}} + \frac{(bc - ad)^{2/3} \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2 \sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt{3} \sqrt[3]{bc-ad}} \right)}{\sqrt{3} d^{5/3}} + \frac{(a + bx^3)^{2/3}}{2d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] (a + b*x^3)^(2/3)/(2*d) + ((b*c - a*d)^(2/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*d^(5/3)) + ((b*c

$$- a*d)^{(2/3)} * \text{Log}[(b*c - a*d)^{(1/3)} + d^{(1/3)} * (a + b*x^3)^{(1/3)}] / (3*d^{(5/3)}) - ((b*c - a*d)^{(2/3)} * \text{Log}[(b*c - a*d)^{(2/3)} - d^{(1/3)} * (b*c - a*d)^{(1/3)} * (a + b*x^3)^{(1/3)} + d^{(2/3)} * (a + b*x^3)^{(2/3)}]) / (6*d^{(5/3)})$$

fricas [B] time = 0.44, size = 323, normalized size = 1.99

$$2\sqrt{3} \left(\frac{b^2c^2 - 2abcd + a^2d^2}{d^2} \right)^{\frac{1}{2}} \arctan \left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{2}} \left(\frac{b^2c^2 - 2abcd + a^2d^2}{d^2} \right)^{\frac{1}{2}} - \sqrt{3}(bc-ad)}{3(bc-ad)} \right) - \left(\frac{b^2c^2 - 2abcd + a^2d^2}{d^2} \right)^{\frac{1}{2}} \log \left((bx^3+a)^{\frac{1}{2}} d \left(\frac{b^2c^2 - 2abcd + a^2d^2}{d^2} \right)^{\frac{1}{2}} - (bx^3+a)^{\frac{3}{2}}(bc-ad) - (bc-ad) \left(\frac{b^2c^2 - 2abcd + a^2d^2}{d^2} \right)^{\frac{1}{2}} \right) + 2 \left(\frac{b^2c^2 - 2abcd + a^2d^2}{d^2} \right)^{\frac{1}{2}} \log \left(-d \left(\frac{b^2c^2 - 2abcd + a^2d^2}{d^2} \right)^{\frac{1}{2}} - (bx^3+a)^{\frac{1}{2}}(bc-ad) \right) + 3(bx^3+a)^{\frac{3}{2}}$$

6d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] 1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + 2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) + 3*(b*x^3 + a)^(2/3)/d

giac [B] time = 0.27, size = 259, normalized size = 1.60

$$\frac{\left(bcd \left(\frac{bc-ad}{d} \right)^{\frac{1}{2}} - ad^2 \left(\frac{bc-ad}{d} \right)^{\frac{1}{2}} \right) \left(\frac{bc-ad}{d} \right)^{\frac{1}{2}} \log \left(\left(bx^3+a \right)^{\frac{1}{2}} - \left(\frac{bc-ad}{d} \right)^{\frac{1}{2}} \right)}{3(bc^2 - ad^2)} + \frac{(bx^3+a)^{\frac{3}{2}}}{2d} + \frac{\sqrt{3}(-bcd^2 + ad^3)^{\frac{3}{2}} \arctan \left(\frac{\sqrt{3} \left(2 \left(bx^3+a \right)^{\frac{1}{2}} + \left(\frac{bc-ad}{d} \right)^{\frac{1}{2}} \right)}{3 \left(\frac{bc-ad}{d} \right)^{\frac{1}{2}}} \right)}{3d^3} - \frac{(-bcd^2 + ad^3)^{\frac{2}{3}} \log \left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}} \left(\frac{bc-ad}{d} \right)^{\frac{1}{2}} + \left(\frac{bc-ad}{d} \right)^{\frac{3}{2}} \right)}{6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out] 1/3*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^2 - a*d^3) + 1/2*(b*x^3 + a)^(2/3)/d + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3)))/(-(b*c - a*d)/d)^(1/3)/d^3 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}} x^2}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] $\int (x^2(bx^3+a)^{2/3}/(dx^3+c), x)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c), x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation may help (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more details) Is a*d-b*c positive or negative?

mupad [B] time = 5.05, size = 238, normalized size = 1.47

$$\frac{(bx^3+d)^{2/3}}{2d} + \frac{\ln\left(\frac{(bx^3+a)^{10}}{d} \frac{(d^2d^2-2abcd+d^2c^2)}{9d^{10/3}} - \frac{(ad-bc)^{10}}{9d^{10/3}}\right)(ad-bc)^{2/3}}{3d^{8/3}} - \frac{\ln\left(\frac{(bx^3+a)^{10}}{d} \frac{(ad-bc)^2}{d^{4/3}} - \frac{\left(\frac{1}{2} + \frac{\sqrt{3}11}{2}\right)(ad-bc)^{7/3}}{d^{4/3}}\right)\left(\frac{1}{2} + \frac{\sqrt{3}11}{2}\right)(ad-bc)^{2/3}}{3d^{8/3}} + \frac{\ln\left(\frac{(bx^3+a)^{10}}{d} \frac{(ad-bc)^2}{4d^{4/3}} - \frac{(-1+\sqrt{3}11)^2(ad-bc)^{7/3}}{4d^{4/3}}\right)\left(-\frac{1}{6} + \frac{\sqrt{3}11}{6}\right)(ad-bc)^{2/3}}{d^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(a + b*x^3)^(2/3))/(c + d*x^3), x)`

[Out] $(a + bx^3)^{2/3}/(2d) + (\log(((a + bx^3)^{1/3}*(a^2d^2 + b^2c^2 - 2*ab*c*d))/d - ((ad - bc)^{4/3}*(9*a*d^3 - 9*b*c*d^2))/(9*d^{10/3}))*((a*d - b*c)^{2/3})/(3*d^{5/3}) - (\log(((a + bx^3)^{1/3}*(a*d - b*c)^2)/d - ((3^{1/2}*1i)/2 - 1/2)*(a*d - b*c)^{7/3})/d^{4/3})*((3^{1/2}*1i)/2 + 1/2)*(a*d - b*c)^{2/3})/(3*d^{5/3}) + (\log(((a + bx^3)^{1/3}*(a*d - b*c)^2)/d - ((3^{1/2}*1i - 1)^2*(a*d - b*c)^{7/3})/(4*d^{4/3}))*((3^{1/2}*1i)/6 - 1/6)*(a*d - b*c)^{2/3})/d^{5/3}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + bx^3)^{\frac{2}{3}}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**3+a)**(2/3)/(d*x**3+c), x)`

[Out] `Integral(x**2*(a + b*x**3)**(2/3)/(c + d*x**3), x)`

$$3.432 \quad \int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)} dx$$

Optimal. Leaf size=245

$$\frac{a^{2/3} \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2c} + \frac{a^{2/3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} - \frac{a^{2/3} \log(x)}{2c} + \frac{(bc-ad)^{2/3} \log(c+dx^3)}{6cd^{2/3}} - \frac{(bc-ad)^{2/3} \log\left(\sqrt[3]{\frac{bc-ad}{a+bx^3}}\right)}{\sqrt{3}cd^{2/3}}$$

Rubi [A] time = 0.23, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 83, 55, 617, 204, 31, 56}

$$\frac{a^{2/3} \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2c} + \frac{a^{2/3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} - \frac{a^{2/3} \log(x)}{2c} + \frac{(bc-ad)^{2/3} \log(c+dx^3)}{6cd^{2/3}} - \frac{(bc-ad)^{2/3} \log\left(\sqrt[3]{\frac{bc-ad}{a+bx^3}}\right)}{2cd^{2/3}} - \frac{(bc-ad)^{2/3} \tan^{-1}\left(\frac{1-2\sqrt[3]{\frac{bc-ad}{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(2/3)/(x*(c + d*x^3)), x]

[Out] (a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*c) - ((b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sqrt[3]*c*d^(2/3)) - (a^(2/3)*Log[x])/(2*c) + ((b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c*d^(2/3)) + (a^(2/3)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*c) - ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*d^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(−1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^{2/3}}{x(c + dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^{2/3}}{x(c + dx)} dx, x, x^3 \right) \\
&= \frac{a \text{Subst} \left(\int \frac{1}{x \sqrt[3]{a+bx}} dx, x, x^3 \right)}{3c} + \frac{(bc - ad) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right)}{3c} \\
&= -\frac{a^{2/3} \log(x)}{2c} + \frac{(bc - ad)^{2/3} \log(c + dx^3)}{6cd^{2/3}} - \frac{a^{2/3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, \sqrt[3]{a + bx^3} \right)}{2c} + \frac{a \text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, \sqrt[3]{a + bx^3} \right)}{2c} \\
&= -\frac{a^{2/3} \log(x)}{2c} + \frac{(bc - ad)^{2/3} \log(c + dx^3)}{6cd^{2/3}} + \frac{a^{2/3} \log \left(\sqrt[3]{a} - \sqrt[3]{a + bx^3} \right)}{2c} - \frac{(bc - ad)^{2/3} \log \left(\sqrt[3]{a} - \sqrt[3]{a + bx^3} \right)}{2c} \\
&= \frac{a^{2/3} \tan^{-1} \left(\frac{1 + 2 \sqrt[3]{a+bx^3}}{\sqrt[3]{a}} \right)}{\sqrt{3} c} - \frac{(bc - ad)^{2/3} \tan^{-1} \left(\frac{1 - 2 \sqrt[3]{a} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}} \right)}{\sqrt{3} cd^{2/3}} - \frac{a^{2/3} \log(x)}{2c} + \frac{(bc - ad)^{2/3} \log(c)}{6cd^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.12, size = 115, normalized size = 0.47

$$\frac{a^{2/3} \left(3 \log \left(\sqrt[3]{a} - \sqrt[3]{a + bx^3} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{2 \sqrt[3]{a+bx^3} + 1}{\sqrt[3]{a}} \right) - 3 \log(x) \right) + 3(a + bx^3)^{2/3} {}_2F_1 \left(\frac{2}{3}, 1; \frac{5}{3}; \frac{d(bx^3+a)}{ad-bc} \right)}{6c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(2/3)/(x*(c + d*x^3)), x]

[Out] (3*(a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)] + a^(2/3)*(2*sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/sqrt[3]] - 3*Log[x] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*c)

IntegrateAlgebraic [A] time = 0.47, size = 333, normalized size = 1.36

$$\frac{a^{2/3} \log \left(\sqrt[3]{a + bx^3} - \sqrt[3]{a} \right)}{3c} - \frac{a^{2/3} \log \left(a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^3} + (a + bx^3)^{2/3} \right)}{6c} + \frac{a^{2/3} \tan^{-1} \left(\frac{2 \sqrt[3]{a+bx^3} + 1}{\sqrt[3]{a}} \right)}{\sqrt{3} c} - \frac{(bc - ad)^{2/3} \log \left(\sqrt[3]{bc-ad} + \sqrt[3]{a} \sqrt[3]{a + bx^3} \right)}{3cd^{2/3}} + \frac{(bc - ad)^{2/3} \log \left(-\sqrt[3]{a} \sqrt[3]{a + bx^3} \sqrt[3]{bc-ad} + (bc - ad)^{2/3} + d^{2/3} (a + bx^3)^{2/3} \right)}{6cd^{2/3}} - \frac{(bc - ad)^{2/3} \tan^{-1} \left(\frac{1 - 2 \sqrt[3]{a} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}} \right)}{\sqrt{3} cd^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x*(c + d*x^3)), x]

[Out] $(a^{2/3} \operatorname{ArcTan}[1/\sqrt{3}] + (2(a + bx^3)^{1/3})/(\sqrt{3}a^{1/3})) / (\sqrt{3}c) - ((bc - ad)^{2/3} \operatorname{ArcTan}[1/\sqrt{3}] - (2d^{1/3}(a + bx^3)^{1/3}) / (\sqrt{3}(bc - ad)^{1/3})) / (\sqrt{3}cd^{2/3}) + (a^{2/3} \operatorname{Log}[-a^{1/3} + (a + bx^3)^{1/3}]) / (3c) - ((bc - ad)^{2/3} \operatorname{Log}[(bc - ad)^{1/3} + d^{1/3}(a + bx^3)^{1/3}]) / (3cd^{2/3}) - (a^{2/3} \operatorname{Log}[a^{2/3} + a^{1/3}(a + bx^3)^{1/3} + (a + bx^3)^{2/3}]) / (6c) + ((bc - ad)^{2/3} \operatorname{Log}[(bc - ad)^{2/3} - d^{1/3}(bc - ad)^{1/3}(a + bx^3)^{1/3} + d^{2/3}(a + bx^3)^{2/3}]) / (6cd^{2/3})$

fricas [B] time = 0.46, size = 425, normalized size = 1.73

$$2\sqrt{3}\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\sqrt{bc-ad}\left(\frac{bc-ad}{3c}\right)^{\frac{1}{3}}+\sqrt{bc-ad}}{3c}\right)-2\sqrt{3}(d)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\sqrt{bc-ad}\left(\frac{bc-ad}{3c}\right)^{\frac{1}{3}}}{3c}\right)+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\frac{bc-ad}{3c}\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}-(bc-a)^{\frac{1}{3}}(bc-ad)+bc-ad\left(\frac{bc-ad}{3c}\right)^{\frac{1}{3}}+(d)^{\frac{1}{3}}\log\left((bc-a)^{\frac{1}{3}}+d^{1/3}\right)+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(-2\left(\frac{bc-ad}{3c}\right)^{\frac{1}{3}}\log\left(-2\left(\frac{bc-ad}{3c}\right)^{\frac{1}{3}}\log\left(\frac{bc-ad}{3c}\right)^{\frac{1}{3}}-(bc-a)^{\frac{1}{3}}(bc-ad)\right)-2(d)^{\frac{1}{3}}\log\left((bc-a)^{\frac{1}{3}}+d^{1/3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="fricas")`

[Out] $-1/6*(2*\sqrt{3})*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^{1/3}*\arctan(-1/3*(2*\sqrt{3})*(b*x^3 + a)^{1/3}*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^{1/3} + \sqrt{3}*(bc - ad))/(bc - ad) - 2*\sqrt{3}*(a^2)^{1/3}*\arctan(1/3*(\sqrt{3}*a + 2*\sqrt{3}*(b*x^3 + a)^{1/3}*(a^2)^{1/3})/a) + (-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^{1/3}*\log((b*x^3 + a)^{1/3}*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^{2/3} - (b*x^3 + a)^{2/3}*(bc - ad) + (bc - ad)*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^{1/3}) + (a^2)^{1/3}*\log((b*x^3 + a)^{2/3}*a + (a^2)^{1/3}*a + (b*x^3 + a)^{1/3}*(a^2)^{2/3}) - 2*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^{1/3}*\log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^{2/3} - (b*x^3 + a)^{1/3}*(bc - ad)) - 2*(a^2)^{1/3}*\log((b*x^3 + a)^{1/3}*a - (a^2)^{2/3}))/c$

giac [A] time = 0.80, size = 341, normalized size = 1.39

$$\left(\frac{bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}{3(bc^2-adc)}\right)^{\frac{1}{3}}\log\left(\frac{bc-ad}{3c}\right)^{\frac{1}{3}}\log\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}-\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)+\frac{\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+a^{\frac{1}{3}}}{3a^{\frac{1}{3}}}\right)}{3c}-\frac{a^{\frac{1}{3}}\log\left(\frac{bc^2+ad}{6c}\right)+\left(\frac{bc^2+ad}{6c}\right)^{\frac{1}{3}}}{6c}+\frac{a^{\frac{1}{3}}\log\left(\frac{bc^2+ad}{3c}\right)}{3c}-\frac{\sqrt{3}(-bc^2+ad)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}-\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}{3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3cd^2}+\frac{(-bc^2+ad)^{\frac{1}{3}}\log\left(\frac{bc^2+ad}{6cd^2}\right)+\left(\frac{bc^2+ad}{6cd^2}\right)^{\frac{1}{3}}}{6cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="giac")`

[Out] $-1/3*(bc*(-(bc - ad)/d)^{1/3} - ad*(-(bc - ad)/d)^{1/3})*(-(bc - ad)/d)^{1/3}*\log(\operatorname{abs}((b*x^3 + a)^{1/3} - (-(bc - ad)/d)^{1/3}))/((bc^2 - ac*d) + 1/3*\sqrt{3}*a^{2/3}*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{1/3} + a^{1/3}))/a^{1/3})/c - 1/6*a^{2/3}*\log((b*x^3 + a)^{2/3} + (b*x^3 + a)^{1/3}*a^{1/3} + a^{2/3})/c + 1/3*a^{2/3}*\log(\operatorname{abs}((b*x^3 + a)^{1/3} - a^{1/3}))/c - 1/3*\sqrt{3}*(-bc*d^2 + a*d^3)^{2/3}*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{1/3} + (-(bc - ad)/d)^{1/3}))/(-(bc - ad)/d)^{1/3} + 1/6*(-bc*d^2 + a*d^3)^{2/3}*\log((b*x^3 + a)^{2/3} + (b*x^3 + a)^{1/3}*(-(bc - ad)/d)^{1/3} + (-(bc - ad)/d)^{2/3}))/((c*d^2)$

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(2/3)/x/(d*x^3+c),x)

[Out] int((b*x^3+a)^(2/3)/x/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x), x)

mupad [B] time = 4.77, size = 1963, normalized size = 8.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x*(c + d*x^3)),x)

[Out] log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - (a^2/(27*c^3))^(2/3)*((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - (243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3))^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*(a^2/(27*c^3))^(1/3) + log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - ((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)*((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - (243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) - log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) + ((3^(1/2)*i)/2 + 1/2)^2*(a^2/(27*c^3))^(2/3)*((3^(1/2)*i)/2 + 1/2)*((a +

$$\begin{aligned}
& b*x^3)^{(1/3)}*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^{(1/2)}*1i)/2 + 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^{(2/3)}*(a^2/(27*c^3))^{(1/3)} - 36*a^2*b^7*c^4*d^2 + 54*a^3*b^6*c^3*d^3 - 27*a^4*b^5*c^2*d^4 + 9*a*b^8*c^5*d))*((3^{(1/2)}*1i)/2 + 1/2)*(a^2/(27*c^3))^{(1/3)} + \log((a + b*x^3)^{(1/3)}*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - ((3^{(1/2)}*1i)/2 - 1/2)^2*(a^2/(27*c^3))^{(2/3)}*((3^{(1/2)}*1i)/2 - 1/2)*((a + b*x^3)^{(1/3)}*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^{(1/2)}*1i)/2 - 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5))*(a^2/(27*c^3))^{(2/3)}*(a^2/(27*c^3))^{(1/3)} + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*((3^{(1/2)}*1i)/2 - 1/2)*(a^2/(27*c^3))^{(1/3)} - \log((a + b*x^3)^{(1/3)}*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) + ((3^{(1/2)}*1i)/2 + 1/2)^2*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^{(2/3)}*((3^{(1/2)}*1i)/2 + 1/2)*((a + b*x^3)^{(1/3)}*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^{(1/2)}*1i)/2 + 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^{(2/3)}*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^{(1/3)} - 36*a^2*b^7*c^4*d^2 + 54*a^3*b^6*c^3*d^3 - 27*a^4*b^5*c^2*d^4 + 9*a*b^8*c^5*d))*((3^{(1/2)}*1i)/2 + 1/2)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^{(1/3)} + \log((a + b*x^3)^{(1/3)}*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - ((3^{(1/2)}*1i)/2 - 1/2)^2*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^{(2/3)}*((3^{(1/2)}*1i)/2 - 1/2)*((a + b*x^3)^{(1/3)}*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^{(1/2)}*1i)/2 - 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^{(2/3)}*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^{(1/3)} + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*((3^{(1/2)}*1i)/2 - 1/2)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^{(1/3)}
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{\frac{2}{3}}}{x(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/x/(d*x**3+c), x)

[Out] Integral((a + b*x**3)**(2/3)/(x*(c + d*x**3)), x)

$$3.433 \quad \int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)} dx$$

Optimal. Leaf size=347

$$\frac{d(a+bx^3)^{2/3}}{2c^2} + \frac{(a+bx^3)^{2/3}(2bc-3ad)}{6ac^2} - \frac{\sqrt[3]{d}(bc-ad)^{2/3} \log(c+dx^3)}{6c^2} + \frac{(2bc-3ad) \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{6\sqrt[3]{a}c^2} + \frac{\sqrt[3]{d}(bc-ad)^{2/3} \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{6\sqrt[3]{a}c^2} + \dots$$

Rubi [A] time = 0.39, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {446, 103, 156, 50, 55, 617, 204, 31, 56}

$$\frac{d(a+bx^3)^{2/3}}{2c^2} + \frac{(a+bx^3)^{2/3}(2bc-3ad)}{6ac^2} - \frac{\sqrt[3]{d}(bc-ad)^{2/3} \log(c+dx^3)}{6c^2} + \frac{(2bc-3ad) \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{6\sqrt[3]{a}c^2} + \frac{\sqrt[3]{d}(bc-ad)^{2/3} \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{6\sqrt[3]{a}c^2} + \frac{(2bc-3ad) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt[3]{3}\sqrt[3]{d}}\right)}{3\sqrt[3]{3}\sqrt[3]{a}c^2} + \frac{\sqrt[3]{d}(bc-ad)^{2/3} \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\frac{\sqrt[3]{d}}{\sqrt[3]{a}}}\right)}{\sqrt[3]{3}c^2} - \frac{\log(x)(2bc-3ad)}{6\sqrt[3]{a}c^2} - \frac{(a+bx^3)^{5/3}}{3ac^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)), x]

[Out] (d*(a + b*x^3)^(2/3))/(2*c^2) + ((2*b*c - 3*a*d)*(a + b*x^3)^(2/3))/(6*a*c^2) - (a + b*x^3)^(5/3)/(3*a*c*x^3) + ((2*b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*c^2) + (d^(1/3)*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c^2) - ((2*b*c - 3*a*d)*Log[x])/(6*a^(1/3)*c^2) - (d^(1/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^2) + ((2*b*c - 3*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(1/3)*c^2) + (d^(1/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[(((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

$b*c - a*d, 0]$ && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^3)^{2/3}}{x^4(c + dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^{2/3}}{x^2(c + dx)} dx, x, x^3 \right) \\
 &= -\frac{(a + bx^3)^{5/3}}{3acx^3} - \frac{\text{Subst} \left(\int \frac{(a+bx)^{2/3} \left(\frac{1}{3}(-2bc+3ad) - \frac{2bdx}{3} \right)}{x(c+dx)} dx, x, x^3 \right)}{3ac} \\
 &= -\frac{(a + bx^3)^{5/3}}{3acx^3} + \frac{d^2 \text{Subst} \left(\int \frac{(a+bx)^{2/3}}{c+dx} dx, x, x^3 \right)}{3c^2} + \frac{(2bc - 3ad) \text{Subst} \left(\int \frac{(a+bx)^{2/3}}{x} dx, x, x^3 \right)}{9ac^2} \\
 &= \frac{d(a + bx^3)^{2/3}}{2c^2} + \frac{(2bc - 3ad)(a + bx^3)^{2/3}}{6ac^2} - \frac{(a + bx^3)^{5/3}}{3acx^3} + \frac{(2bc - 3ad) \text{Subst} \left(\int \frac{1}{x\sqrt[3]{a+bx}} dx, x, x^3 \right)}{9c^2} \\
 &= \frac{d(a + bx^3)^{2/3}}{2c^2} + \frac{(2bc - 3ad)(a + bx^3)^{2/3}}{6ac^2} - \frac{(a + bx^3)^{5/3}}{3acx^3} - \frac{(2bc - 3ad) \log(x)}{6\sqrt[3]{a}c^2} - \frac{\sqrt[3]{d}(bc - ad)}{\sqrt[3]{a}c^2} \\
 &= \frac{d(a + bx^3)^{2/3}}{2c^2} + \frac{(2bc - 3ad)(a + bx^3)^{2/3}}{6ac^2} - \frac{(a + bx^3)^{5/3}}{3acx^3} - \frac{(2bc - 3ad) \log(x)}{6\sqrt[3]{a}c^2} - \frac{\sqrt[3]{d}(bc - ad)}{\sqrt[3]{a}c^2} \\
 &= \frac{d(a + bx^3)^{2/3}}{2c^2} + \frac{(2bc - 3ad)(a + bx^3)^{2/3}}{6ac^2} - \frac{(a + bx^3)^{5/3}}{3acx^3} + \frac{(2bc - 3ad) \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{3\sqrt{3}\sqrt[3]{a}c^2} +
 \end{aligned}$$

Mathematica [C] time = 0.16, size = 202, normalized size = 0.58

$$\frac{-9\sqrt[3]{a}dx^3(a+bx^3)^{2/3} {}_2F_1\left(\frac{2}{3}; 1; \frac{5}{3}; \frac{d(bx^3+a)}{ad-bc}\right) + 2\sqrt{3}x^3(2bc-3ad)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+1}{\sqrt[3]{a}}\right) - 6\sqrt[3]{a}c(a+bx^3)^{2/3} + 6bcx^3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right) - 9adx^3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right) + 9adx^3\log(x) - 6bcx^3\log(x)}{18\sqrt[3]{a}c^2x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)), x]

[Out] $(-6*a^{1/3}*c*(a + b*x^3)^{2/3} + 2*\text{Sqrt}[3]*(2*b*c - 3*a*d)*x^3*\text{ArcTan}[(1 + (2*(a + b*x^3)^{1/3})/a^{1/3})/\text{Sqrt}[3]] - 9*a^{1/3}*d*x^3*(a + b*x^3)^{2/3})*\text{Hypergeometric2F1}[2/3, 1, 5/3, (d*(a + b*x^3))/(-b*c) + a*d] - 6*b*c*x^3*\text{Log}[x] + 9*a*d*x^3*\text{Log}[x] + 6*b*c*x^3*\text{Log}[a^{1/3} - (a + b*x^3)^{1/3}] - 9*a*d*x^3*\text{Log}[a^{1/3} - (a + b*x^3)^{1/3}]/(18*a^{1/3}*c^2*x^3)$

IntegrateAlgebraic [A] time = 0.71, size = 384, normalized size = 1.11

$$\frac{(3ad-2bc)\log\left(\frac{a^{2/3}+\sqrt[3]{a}\sqrt{a+bx^3}+(a+bx^3)^{2/3}}{18\sqrt[3]{a}c^2}\right) - \sqrt[3]{d}(bc-ad)^{2/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}(a+bx^3)^{2/3}\right)}{6c^2} + \frac{(2bc-3ad)\log\left(\frac{\sqrt[3]{a+bx^3}-\sqrt[3]{a}}{9\sqrt[3]{a}c^2}\right) + \sqrt[3]{d}(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}}{3c^2}\right)}{9\sqrt[3]{a}c^2} - \frac{(3ad-2bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+1}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}c^2} + \frac{\sqrt[3]{d}(bc-ad)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt[3]{a}}-\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{3}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{3}c^2} - \frac{(a+bx^3)^{2/3}}{3c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)), x]

[Out] $-1/3*(a + b*x^3)^{2/3}/(c*x^3) - ((-2*b*c + 3*a*d)*\text{ArcTan}[(a^{1/3} + 2*(a + b*x^3)^{1/3})/(\text{Sqrt}[3]*a^{1/3})])/(3*\text{Sqrt}[3]*a^{1/3}*c^2) + (d^{1/3}*(b*c - a*d)^{2/3}*\text{ArcTan}[1/\text{Sqrt}[3] - (2*d^{1/3}*(a + b*x^3)^{1/3})/(\text{Sqrt}[3]*(b*c - a*d)^{1/3})])/(3*\text{Sqrt}[3]*c^2) + ((2*b*c - 3*a*d)*\text{Log}[-a^{1/3} + (a + b*x^3)^{1/3}])/(9*a^{1/3}*c^2) + (d^{1/3}*(b*c - a*d)^{2/3}*\text{Log}[(b*c - a*d)^{1/3} + d^{1/3}*(a + b*x^3)^{1/3}])/(3*c^2) + ((-2*b*c + 3*a*d)*\text{Log}[a^{2/3} + a^{1/3}*(a + b*x^3)^{1/3} + (a + b*x^3)^{2/3}])/(18*a^{1/3}*c^2) - (d^{1/3}*(b*c - a*d)^{2/3}*\text{Log}[(b*c - a*d)^{2/3} - d^{1/3}*(b*c - a*d)^{1/3}*(a + b*x^3)^{1/3} + d^{2/3}*(a + b*x^3)^{2/3}])/(6*c^2)$

fricas [A] time = 0.55, size = 1030, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c), x, algorithm="fricas")

[Out] $[-1/18*(3*\text{sqrt}(1/3)*(2*a*b*c - 3*a^2*d)*x^3*\text{sqrt}((-a)^{1/3}/a)*\log((2*b*x^3 - 3*\text{sqrt}(1/3)*(2*(b*x^3 + a)^{2/3}*(-a)^{2/3} - (b*x^3 + a)^{1/3}*a + (-a)^{1/3}*a)*\text{sqrt}((-a)^{1/3}/a) - 3*(b*x^3 + a)^{1/3}*(-a)^{2/3} + 3*a)/x^3) - 6*\text{sqrt}(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{1/3}*a*x^3*\text{arctan}(1/3*(\text{sqrt}(3)*(b*c - a*d) - 2*\text{sqrt}(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{1/3}*(b*x^3 + a)^{1/3})/(b*c - a*d)) + (2*b*c - 3*a*d)*(-a)^{2/3}*x^3*\log((b*x^3 + a)^{2/3} - (b*x^3 + a)^{1/3}*(-a)^{1/3} + (-a)^{2/3}) - 2*(2*b*c - 3*a*d)*(-a)$

$$\begin{aligned} &)^{2/3} * x^3 * \log((b*x^3 + a)^{1/3} + (-a)^{1/3}) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{1/3} * a*x^3 * \log(-(b*x^3 + a)^{2/3} * (b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{1/3} * (b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{2/3} * (b*x^3 + a)^{1/3}) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{1/3} * a*x^3 * \log(-(b*x^3 + a)^{1/3} * (b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{2/3}) + 6*(b*x^3 + a)^{2/3} * a*c / (a*c^2*x^3), \\ & 1/18*(6*\sqrt{1/3})*(2*a*b*c - 3*a^2*d)*x^3*\sqrt{-(-a)^{1/3}/a}*\arctan(\sqrt{1/3}*(2*(b*x^3 + a)^{1/3} - (-a)^{1/3}))*\sqrt{-(-a)^{1/3}/a}) + 6*\sqrt{3}*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{1/3} * a*x^3 * \arctan(1/3*(\sqrt{3}*(b*c - a*d) - 2*\sqrt{3}*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{1/3} * (b*x^3 + a)^{1/3}) / (b*c - a*d)) - (2*b*c - 3*a*d)*(-a)^{2/3} * x^3 * \log((b*x^3 + a)^{2/3} - (b*x^3 + a)^{1/3} * (-a)^{1/3} + (-a)^{2/3}) + 2*(2*b*c - 3*a*d)*(-a)^{2/3} * x^3 * \log((b*x^3 + a)^{1/3} + (-a)^{1/3}) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{1/3} * a*x^3 * \log(-(b*x^3 + a)^{2/3} * (b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{1/3} * (b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{2/3} * (b*x^3 + a)^{1/3}) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{1/3} * a*x^3 * \log(-(b*x^3 + a)^{1/3} * (b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{2/3}) - 6*(b*x^3 + a)^{2/3} * a*c / (a*c^2*x^3) \end{aligned}$$

giac [A] time = 0.81, size = 400, normalized size = 1.15

$$\frac{\left(\frac{bx^3+a}{3}\right)^{\frac{2}{3}} - ad^{\frac{2}{3}} \left(\frac{bx^3+a}{3}\right)^{\frac{1}{3}} \log\left(\left|\frac{bx^3+a}{3} - \left(\frac{bx^3+a}{3}\right)^{\frac{1}{3}}\right|\right)}{3(b^2c^2d - 2abd^2)} \cdot \frac{(2bc - 3ad) \log\left(\frac{(bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}d^{\frac{2}{3}} + d^{\frac{2}{3}}}{3d}\right)}{18a^{\frac{2}{3}}d^2} \cdot \frac{\sqrt{3}(2a^{\frac{2}{3}}bc - 3a^{\frac{1}{3}}d^2) \arctan\left(\frac{\sqrt{3}\left(\frac{(bx^3+a)^{\frac{2}{3}} + \left(\frac{bx^3+a}{3}\right)^{\frac{1}{3}}d^{\frac{2}{3}} + d^{\frac{2}{3}}\right)}{3d}\right)}{3d}\right)}{9a^{\frac{2}{3}}d^2} \cdot \frac{(2a^{\frac{2}{3}}bc - 3ad^2) \log\left(\left|\frac{bx^3+a}{3} - d\right|\right)}{9a^{\frac{2}{3}}d^2} \cdot \frac{\sqrt{3}(-3ad^2 + ad^{\frac{2}{3}}) \arctan\left(\frac{\sqrt{3}\left(\frac{(bx^3+a)^{\frac{2}{3}} + \left(\frac{bx^3+a}{3}\right)^{\frac{1}{3}}d^{\frac{2}{3}} + d^{\frac{2}{3}}\right)}{3d}\right)}{3\left(\frac{bx^3+a}{3}\right)^{\frac{1}{3}}}\right)}{3c^2d} \cdot \frac{(-3ad^2 + ad^{\frac{2}{3}}) \log\left(\frac{(bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}d^{\frac{2}{3}} + \left(\frac{bx^3+a}{3}\right)^{\frac{2}{3}}}{6c^2d}\right)}{6c^2d} \cdot \frac{(bx^3+a)^{\frac{2}{3}}}{3c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="giac")

[Out] $\frac{1}{3}*(b*c*d*(-(b*c - a*d)/d)^{1/3} - a*d^2*(-(b*c - a*d)/d)^{1/3})*(-(b*c - a*d)/d)^{1/3}*\log(\text{abs}((b*x^3 + a)^{1/3} - (-(b*c - a*d)/d)^{1/3}))/((b*c^3 - a*c^2*d) - 1/18*(2*b*c - 3*a*d)*\log((b*x^3 + a)^{2/3} + (b*x^3 + a)^{1/3}*a^{1/3} + a^{2/3}))/((a^{1/3}*c^2) + 1/9*\sqrt{3}*(2*a^{2/3}*b*c - 3*a^{5/3}*d)*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{1/3} + a^{1/3}))/a^{1/3}))/((a*c^2) + 1/9*(2*a^{1/3}*b*c - 3*a^{4/3}*d)*\log(\text{abs}((b*x^3 + a)^{1/3} - a^{1/3}))/((a^{2/3}*c^2) + 1/3*\sqrt{3}*(-b*c*d^2 + a*d^3)^{2/3}*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{1/3} + (-b*c - a*d)/d)^{1/3}))/((-b*c - a*d)/d)^{1/3}))/((c^2*d) - 1/6*(-b*c*d^2 + a*d^3)^{2/3}*\log((b*x^3 + a)^{2/3} + (b*x^3 + a)^{1/3}*(-b*c - a*d)/d)^{1/3} + (-b*c - a*d)/d)^{2/3}))/((c^2*d) - 1/3*(b*x^3 + a)^{2/3}/(c*x^3))$

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x)

[Out] int((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^4), x)

mupad [B] time = 10.57, size = 1908, normalized size = 5.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x)

[Out] log(- (((6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))*((d*(a*d - b*c)^2)/c^6)^(2/3))*((d*(a*d - b*c)^2)/c^6)^(1/3))/3 - (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*((d*(a*d - b*c)^2)/c^6)^(2/3))/9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)/(27*c^6))^(1/3) + log(- (((6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) - 3*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1/3))/9 - (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*(-(27*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^(1/3) - log(((3^(1/2)*1i)/2 - 1/2)*(((3^(1/2)*1i)/2 + 1/2)*(6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) - 3*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1/3))/9 + (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^(1/3) + log(((3^(1/2)*1i)/2 + 1/2)*(((3^(1/2)*1i)/2 - 1/2)*(6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) + 3*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))*(-(3*a*d - 2*b*

$$\begin{aligned}
& c)^3/(a*c^6))^{(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^{(1/3)}/9 - (a*b^5*d^4*(27 \\
& *a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*(-(3*a*d - \\
& 2*b*c)^3/(a*c^6))^{(2/3)}/81 - (b^5*d^4*(a + b*x^3)^{(1/3)}*(4*a*d - 3*b*c)*(\\
& 3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^{(1/2)*1i)/2 - 1/2)*(-2 \\
& 7*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^{(1/3)} \\
& - (a + b*x^3)^{(2/3)}/(3*c*x^3) - \log((((3^{(1/2)*1i)/2 - 1/2)*(((3^{(1/2)*1i} \\
&)/2 + 1/2)*(6*b^4*d^3*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^ \\
& 2 - 6*a*b*c*d) - 27*a*b^4*c^4*d^3*((3^{(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c \\
& ^2 - 3*a*b*c*d))*((d*(a*d - b*c)^2)/c^6)^{(2/3)))*((d*(a*d - b*c)^2)/c^6)^{(1/3} \\
&))/3 + (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^ \\
& 2))/(3*c))*((d*(a*d - b*c)^2)/c^6)^{(2/3)}/9 - (b^5*d^4*(a + b*x^3)^{(1/3)}*(4 \\
& *a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^{(1/2)*1i} \\
&)/2 + 1/2)*((a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)/(27*c^6))^{(1/3)} + \log((((3^{(1/2)*1i} \\
&)/2 + 1/2)*(((3^{(1/2)*1i)/2 - 1/2)*(6*b^4*d^3*(a + b*x^3)^{(1/3)}*(a \\
& *d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) + 27*a*b^4*c^4*d^3*((3^{(1/2} \\
&)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))*((d*(a*d - b*c)^2)/c^6)^{(2 \\
& /3))*((d*(a*d - b*c)^2)/c^6)^{(1/3)}/3 - (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 \\
& + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*((d*(a*d - b*c)^2)/c^6)^{(2/3)}/ \\
& 9 - (b^5*d^4*(a + b*x^3)^{(1/3)}*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a \\
& *b*c*d)^2)/(27*c^5))*((3^{(1/2)*1i)/2 - 1/2)*((a^2*d^3 + b^2*c^2*d - 2*a*b*c \\
& *d^2)/(27*c^6))^{(1/3)}
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{\frac{2}{3}}}{x^4(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/x**4/(d*x**3+c), x)

[Out] Integral((a + b*x**3)**(2/3)/(x**4*(c + d*x**3)), x)

$$3.434 \quad \int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)} dx$$

Optimal. Leaf size=370

$$\frac{(-9a^2d^2 + 6abcd + b^2c^2) \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3} - \frac{(-9a^2d^2 + 6abcd + b^2c^2) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}c^3} + \frac{\log(x)(-9a^2d^2 + 6abcd + b^2c^2)}{18a^{4/3}c^3}$$

Rubi [A] time = 0.50, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {446, 103, 149, 156, 55, 617, 204, 31, 56}

$$\frac{(-9a^2d^2 + 6abcd + b^2c^2) \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3} - \frac{(-9a^2d^2 + 6abcd + b^2c^2) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}c^3} + \frac{\log(x)(-9a^2d^2 + 6abcd + b^2c^2)}{18a^{4/3}c^3} + \frac{d^{4/3}(bc - ad)^{2/3} \log(c + dx^3)}{6c^3} - \frac{d^{4/3}(bc - ad)^{2/3} \log\left(\sqrt[3]{c-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3} - \frac{d^{4/3}(bc - ad)^{2/3} \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}c^3} + \frac{(a+bx^3)^{2/3}(ad+bc)}{18a^{2/3}c^3} - \frac{(a+bx^3)^{5/3}}{6acx^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)), x]

[Out] ((b*c + 6*a*d)*(a + b*x^3)^(2/3))/(18*a*c^2*x^3) - (a + b*x^3)^(5/3)/(6*a*c*x^6) - ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(4/3)*c^3) - (d^(4/3)*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sqrt[3]*c^3) + ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*Log[x])/(18*a^(4/3)*c^3) + (d^(4/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^3) - ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(4/3)*c^3) - (d^(4/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^3)

Rule 31

Int[((a_) + (b_)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^p_], x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

$*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 617

$\text{Int}[\{(a_)+(b_)*(x_)+(c_)*(x_)^2\}^{-1}, x_Symbol] :> \text{With}[\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2)], x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \|\ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^3)^{2/3}}{x^7(c + dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^{2/3}}{x^3(c + dx)} dx, x, x^3 \right) \\
 &= \frac{(a + bx^3)^{5/3}}{6acx^6} - \frac{\text{Subst} \left(\int \frac{(a+bx)^{2/3} \left(\frac{1}{3}(bc+6ad) + \frac{bdx}{3} \right)}{x^2(c+dx)} dx, x, x^3 \right)}{6ac} \\
 &= \frac{(bc + 6ad)(a + bx^3)^{2/3}}{18ac^2x^3} - \frac{(a + bx^3)^{5/3}}{6acx^6} - \frac{\text{Subst} \left(\int \frac{\frac{2}{9}(b^2c^2 + 6abcd - 9a^2d^2) + \frac{2}{9}bd(bc - 3ad)x}{x^3\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right)}{6ac^2} \\
 &= \frac{(bc + 6ad)(a + bx^3)^{2/3}}{18ac^2x^3} - \frac{(a + bx^3)^{5/3}}{6acx^6} + \frac{(d^2(bc - ad)) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right)}{3c^3} - \frac{(b^2c^2 + 6abcd - 9a^2d^2) \log(x)}{18a^{4/3}c^3} + \frac{d^{4/3}(bc - ad)^{2/3}}{6c^3} \\
 &= \frac{(bc + 6ad)(a + bx^3)^{2/3}}{18ac^2x^3} - \frac{(a + bx^3)^{5/3}}{6acx^6} + \frac{(b^2c^2 + 6abcd - 9a^2d^2) \log(x)}{18a^{4/3}c^3} + \frac{d^{4/3}(bc - ad)^{2/3}}{6c^3} \\
 &= \frac{(bc + 6ad)(a + bx^3)^{2/3}}{18ac^2x^3} - \frac{(a + bx^3)^{5/3}}{6acx^6} - \frac{(b^2c^2 + 6abcd - 9a^2d^2) \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{9\sqrt{3}a^{4/3}c^3} + \frac{d^{4/3}(bc - ad)^{2/3}}{6c^3}
 \end{aligned}$$

Mathematica [C] time = 0.38, size = 240, normalized size = 0.65

$$\frac{27a^{4/3}d^2x^6(a+bx^3)^{2/3} {}_2F_1\left(\frac{5}{3}, 1; \frac{5}{3}; \frac{d(bx^3+a)}{ad-bc}\right) + 3x^6 \log(x) (-9a^2d^2 + 6abcd + b^2c^2) - 3x^6 (-9a^2d^2 + 6abcd + b^2c^2) \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right) - 2\sqrt[3]{3}x^6 (-9a^2d^2 + 6abcd + b^2c^2) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{3}} + 1\right) + 3\sqrt[3]{a}c(a+bx^3)^{2/3} (-3ac + 6ad^2 - 2bcx^3)}{54a^{4/3}c^3x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)), x]

[Out] (3*a^(1/3)*c*(a + b*x^3)^(2/3)*(-3*a*c - 2*b*c*x^3 + 6*a*d*x^3) - 2*sqrt[3]*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/sqrt[3]] + 27*a^(4/3)*d^2*x^6*(a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-b*c) + a*d]] + 3*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*x^6*Log[x] - 3*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*x^6*Log[a^(1/3) - (a + b*x^3)^(1/3)]/(54*a^(4/3)*c^3*x^6)

IntegrateAlgebraic [A] time = 0.88, size = 448, normalized size = 1.21

$$\frac{(9a^2d^2 - 6abcd - b^2c^2) \log\left(\frac{\sqrt{a+bx^3} - \sqrt{c}}{\sqrt{a+bx^3} + \sqrt{c}}\right) + (-9a^2d^2 + 6abcd + b^2c^2) \log\left(\frac{a^{2/3} + \sqrt{c}\sqrt{a+bx^3} + (a+bx^3)^{2/3}}{a^{2/3}}\right) + (9a^2d^2 - 6abcd - b^2c^2) \tan^{-1}\left(\frac{2\sqrt{a+bx^3}}{\sqrt{3}} + 1\right) + d^{4/3}(bc - ad^2) \log\left(\frac{\sqrt{bc-ad} + \sqrt{3}\sqrt{a+bx^3}}{3c}\right) + d^{4/3}(bc - ad^2) \log\left(\frac{-\sqrt{3}\sqrt{a+bx^3}\sqrt{bc-ad} + (bc - ad^2) + d^{2/3}(a+bx^3)^{2/3}}{6c}\right) + d^{4/3}(bc - ad^2) \tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt{a+bx^3}}{\sqrt{3}\sqrt{bc-ad}}\right) + (a+bx^3)^{2/3} \frac{(-3ac + 6ad^2 - 2bcx^3)}{18a^2d^2}}{27a^{4/3}c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)), x]

[Out] ((a + b*x^3)^(2/3)*(-3*a*c - 2*b*c*x^3 + 6*a*d*x^3))/(18*a*c^2*x^6) + ((-b^2*c^2 - 6*a*b*c*d + 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(sqrt[3]*a^(1/3))])/(9*sqrt[3]*a^(4/3)*c^3) - (d^(4/3)*(b*c - a*d)^(2/3)*ArcTan[1/sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(sqrt[3]*(b*c - a*d)^(1/3))])/(sqrt[3]*c^3) + (((-b^2*c^2) - 6*a*b*c*d + 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/(27*a^(4/3)*c^3) - (d^(4/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*c^3) + ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*a^(4/3)*c^3) + (d^(4/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*c^3)

fricas [A] time = 1.62, size = 1151, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c), x, algorithm="fricas")

[Out] [-1/54*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*arc tan(-1/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 -

$$\begin{aligned}
& a^2 d^3)^{(2/3)} (b x^3 + a)^{(1/3)} - 18 (-b^2 c^2 d + 2 a b c d^2 - a^2 d^3)^{(1/3)} a^2 d x^6 \log(- (b x^3 + a)^{(1/3)} (b c d - a d^2) - (-b^2 c^2 d + 2 a b c d^2 - a^2 d^3)^{(2/3)}) + 3 \sqrt{1/3} (a b^2 c^2 + 6 a^2 b c d - 9 a^3 d^2) x^6 \sqrt{-1/a^{(2/3)}} \log((2 b x^3 + 3 \sqrt{1/3} (2 (b x^3 + a)^{(2/3)} a^{(2/3)} - (b x^3 + a)^{(1/3)} a - a^{(4/3)})) \sqrt{-1/a^{(2/3)}} - 3 (b x^3 + a)^{(1/3)} a^{(2/3)} + 3 a) / x^3) - (b^2 c^2 + 6 a b c d - 9 a^2 d^2) a^{(2/3)} x^6 \log((b x^3 + a)^{(2/3)} + (b x^3 + a)^{(1/3)} a^{(1/3)} + a^{(2/3)}) + 2 (b^2 c^2 + 6 a b c d - 9 a^2 d^2) a^{(2/3)} x^6 \log((b x^3 + a)^{(1/3)} - a^{(1/3)}) + 3 (3 a^2 c^2 + 2 (a b c^2 - 3 a^2 c d) x^3) (b x^3 + a)^{(2/3)} / (a^2 c^3 x^6), -1/54 * (18 \sqrt{3} (-b^2 c^2 d + 2 a b c d^2 - a^2 d^3)^{(1/3)} a^2 d x^6 \arctan(-1/3 * (\sqrt{3} (b c - a d) + 2 \sqrt{3} (-b^2 c^2 d + 2 a b c d^2 - a^2 d^3)^{(1/3)} (b x^3 + a)^{(1/3)}) / (b c - a d)) + 9 (-b^2 c^2 d + 2 a b c d^2 - a^2 d^3)^{(1/3)} a^2 d x^6 \log(- (b x^3 + a)^{(2/3)} (b c d - a d^2) + (-b^2 c^2 d + 2 a b c d^2 - a^2 d^3)^{(1/3)} (b c - a d) + (-b^2 c^2 d + 2 a b c d^2 - a^2 d^3)^{(2/3)} (b x^3 + a)^{(1/3)}) - 18 (-b^2 c^2 d + 2 a b c d^2 - a^2 d^3)^{(1/3)} a^2 d x^6 \log(- (b x^3 + a)^{(1/3)} (b c d - a d^2) - (-b^2 c^2 d + 2 a b c d^2 - a^2 d^3)^{(2/3)}) - (b^2 c^2 + 6 a b c d - 9 a^2 d^2) a^{(2/3)} x^6 \log((b x^3 + a)^{(2/3)} + (b x^3 + a)^{(1/3)} a^{(1/3)} + a^{(2/3)}) + 2 (b^2 c^2 + 6 a b c d - 9 a^2 d^2) a^{(2/3)} x^6 \log((b x^3 + a)^{(1/3)} - a^{(1/3)}) + 6 \sqrt{1/3} (a b^2 c^2 + 6 a^2 b c d - 9 a^3 d^2) x^6 \arctan(\sqrt{1/3} (2 (b x^3 + a)^{(1/3)} + a^{(1/3)}) / a^{(1/3)}) / a^{(1/3)} + 3 (3 a^2 c^2 + 2 (a b c^2 - 3 a^2 c d) x^3) (b x^3 + a)^{(2/3)} / (a^2 c^3 x^6)]
\end{aligned}$$

giac [A] time = 0.78, size = 493, normalized size = 1.33

$$\frac{\left(\frac{b c d^2 - a d^3}{3 (b^2 c^2 - a d^2)} \right) \left(\frac{b c d^2 - a d^3}{3} \right) \log \left(\left| \frac{b c d^2 - a d^3}{3} \right| \right) \sqrt[3]{\frac{b c d^2 - a d^3}{3}} \arctan \left(\frac{\sqrt[3]{\frac{b c d^2 - a d^3}{3}}}{\frac{b c d^2 - a d^3}{3}} \right) + \frac{\sqrt[3]{\frac{b c d^2 - a d^3}{3}} \log \left(\left| \frac{b c d^2 - a d^3}{3} \right| \right)}{3 c^2} + \frac{\sqrt[3]{\frac{b c d^2 - a d^3}{3}} \log \left(\left| \frac{b c d^2 - a d^3}{3} \right| \right)}{3 c^2} + \frac{\sqrt[3]{\frac{b c d^2 - a d^3}{3}} \log \left(\left| \frac{b c d^2 - a d^3}{3} \right| \right)}{27 c^3} + \frac{\sqrt[3]{\frac{b c d^2 - a d^3}{3}} \log \left(\left| \frac{b c d^2 - a d^3}{3} \right| \right)}{27 c^3} + \frac{\sqrt[3]{\frac{b c d^2 - a d^3}{3}} \log \left(\left| \frac{b c d^2 - a d^3}{3} \right| \right)}{27 c^3} + \frac{\sqrt[3]{\frac{b c d^2 - a d^3}{3}} \log \left(\left| \frac{b c d^2 - a d^3}{3} \right| \right)}{27 c^3} + \frac{\sqrt[3]{\frac{b c d^2 - a d^3}{3}} \log \left(\left| \frac{b c d^2 - a d^3}{3} \right| \right)}{27 c^3} + \frac{\sqrt[3]{\frac{b c d^2 - a d^3}{3}} \log \left(\left| \frac{b c d^2 - a d^3}{3} \right| \right)}{27 c^3} + \frac{\sqrt[3]{\frac{b c d^2 - a d^3}{3}} \log \left(\left| \frac{b c d^2 - a d^3}{3} \right| \right)}{27 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/3 (b c d^2 (- (b c - a d) / d)^{(1/3)} - a d^3 (- (b c - a d) / d)^{(1/3)}) (- (b c - a d) / d)^{(1/3)} \log(\text{abs}((b x^3 + a)^{(1/3)} - (- (b c - a d) / d)^{(1/3)})) / (b c^4 - a c^3 d) - 1/3 \sqrt{3} (-b c d^2 + a d^3)^{(2/3)} \arctan(1/3 \sqrt{3} (2 (b x^3 + a)^{(1/3)} + (- (b c - a d) / d)^{(1/3)}) / (- (b c - a d) / d)^{(1/3)}) / c^3 + 1/6 (-b c d^2 + a d^3)^{(2/3)} \log((b x^3 + a)^{(2/3)} + (b x^3 + a)^{(1/3)} (- (b c - a d) / d)^{(1/3)} + (- (b c - a d) / d)^{(2/3)}) / c^3 - 1/27 \sqrt{3} (b^2 c^2 + 6 a b c d - 9 a^2 d^2) \arctan(1/3 \sqrt{3} (2 (b x^3 + a)^{(1/3)} + a^{(1/3)}) / a^{(1/3)}) / (a^{(4/3)} c^3) - 1/27 (a^{(1/3)} b^2 c^2 + 6 a^{(4/3)} b c d - 9 a^{(7/3)} d^2) \log(\text{abs}((b x^3 + a)^{(1/3)} - a^{(1/3)})) / (a^{(5/3)} c^3) + 1/54 (a^{(2/3)} b^2 c^2 + 6 a^{(5/3)} b c d - 9 a^{(8/3)} d^2) \log((b x^3 + a)^{(2/3)} + (b x^3 + a)^{(1/3)} a^{(1/3)} + a^{(2/3)}) / (a^2 c^3) - 1/18 (2 (b x^3 + a)^{(5/3)} b^2 c + (b x^3 + a)^{(2/3)} a b^2 c - 6 (b x^3 + a)^{(5/3)} a b d + 6 (b x^3 + a)^{(2/3)} a^2 b d) / (a b^2 c^2 x^6)
\end{aligned}$$

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x)

[Out] int((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^7), x)

mupad [B] time = 15.19, size = 2788, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x)

[Out] log((((((27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3) - (b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2)))*(-(d^4*(a*d - b*c)^2)/c^9)^(1/3))/3 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3)) /9 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*(-(a^2*d^6 + b^2*c^2*d^4 - 2*a*b*c*d^5)/(27*c^9))^(1/3) + log((((((a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/3 - (b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2)))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(1/3))/27 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(1/3))

$$\begin{aligned}
& *c*d)^3/(a^4*c^9))^{(2/3)}/729 + (2*b^5*d^7*(a + b*x^3)^{(1/3)}*(6*a*d - 5*b*c) \\
&)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2/(729*a^2*c^{10})) \\
& *(-b^6*c^6 - 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*b^3*c^3*d^3 - 729* \\
& a^4*b^2*c^2*d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19683*a^4*c^9))^{(1/3)} \\
& - (((a + b*x^3)^{(2/3)}*(b^2*c + 6*a*b*d))/(18*c^2) - (b*(a + b*x^3)^{(5/3)}*(\\
& 3*a*d - b*c))/(9*a*c^2)))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) - \log((((3 \\
& ^{(1/2)}*1i)/2 - 1/2)*(((3^{(1/2)}*1i)/2 + 1/2)*((b^4*d^3*(a + b*x^3)^{(1/3)}*(a \\
& *d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - \\
& 108*a^3*b*c*d^3))/(3*a^2*c^2) - 27*a*b^4*c^4*d^3*((3^{(1/2)}*1i)/2 - 1/2)*(2* \\
& a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^4*(a*d - b*c)^2)/c^9)^{(2/3)}*(-(d^4*(a \\
& d - b*c)^2)/c^9)^{(1/3)})/3 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4 \\
& *d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187* \\
& a^5*b*c*d^5))/(81*a^2*c^4))*(-(d^4*(a*d - b*c)^2)/c^9)^{(2/3)}/9 + (2*b^5*d^7 \\
& *(a + b*x^3)^{(1/3)}*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - \\
& 15*a^2*b*c*d^2)^2/(729*a^2*c^{10}))*((3^{(1/2)}*1i)/2 + 1/2)*(-(a^2*d^6 + b^2* \\
& c^2*d^4 - 2*a*b*c*d^5)/(27*c^9))^{(1/3)} + \log((((3^{(1/2)}*1i)/2 + 1/2)*(((3 \\
& ^{(1/2)}*1i)/2 - 1/2)*((b^4*d^3*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(162*a^4*d^4 + \\
& b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c \\
& ^2) + 27*a*b^4*c^4*d^3*((3^{(1/2)}*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b* \\
& c*d)*(-(d^4*(a*d - b*c)^2)/c^9)^{(2/3)}*(-(d^4*(a*d - b*c)^2)/c^9)^{(1/3)})/3 \\
& + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 \\
& + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4) \\
&)*(-(d^4*(a*d - b*c)^2)/c^9)^{(2/3)}/9 + (2*b^5*d^7*(a + b*x^3)^{(1/3)}*(6*a*d \\
& - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2/(729*a^2 \\
& *c^{10}))*((3^{(1/2)}*1i)/2 - 1/2)*(-(a^2*d^6 + b^2*c^2*d^4 - 2*a*b*c*d^5)/(27 \\
& *c^9))^{(1/3)} - \log((((3^{(1/2)}*1i)/2 - 1/2)*(((3^{(1/2)}*1i)/2 + 1/2)*((b^4*d \\
& ^3*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2* \\
& d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2) - (a*b^4*c^4*d^3*((3^{(\\
& 1/2)}*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 \\
& + 6*a*b*c*d)^3/(a^4*c^9))^{(2/3)}/3)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(\\
& a^4*c^9))^{(1/3)})/27 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 \\
& - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b* \\
& c*d^5))/(81*a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^{(2/3) \\
&)}/729 + (2*b^5*d^7*(a + b*x^3)^{(1/3)}*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 \\
& + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2/(729*a^2*c^{10}))*((3^{(1/2)}*1i)/2 + 1/2) \\
& *(-b^6*c^6 - 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*b^3*c^3*d^3 - 729* \\
& a^4*b^2*c^2*d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19683*a^4*c^9))^{(1/3)} \\
& + \log((((3^{(1/2)}*1i)/2 + 1/2)*(((3^{(1/2)}*1i)/2 - 1/2)*((b^4*d^3*(a + b*x^ \\
& 3)^{(1/3)}*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b \\
& ^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2) + (a*b^4*c^4*d^3*((3^{(1/2)}*1i)/2 + \\
& 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d) \\
& ^3/(a^4*c^9))^{(2/3)}/3)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^{(1 \\
& /3)})/27 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^ \\
& 3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81* \\
& a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^{(2/3)}/729 + (2*
\end{aligned}$$

$$b^5 d^7 (a + b x^3)^{1/3} (6 a d - 5 b c) (9 a^3 d^3 + b^3 c^3 + 5 a b^2 c^2 d - 15 a^2 b c d^2)^2 / (729 a^2 c^{10}) \left(\frac{\sqrt{3}}{2} i - \frac{1}{2} \right) \left(- (b^6 c^6 - 729 a^6 d^6 + 81 a^2 b^4 c^4 d^2 - 108 a^3 b^3 c^3 d^3 - 729 a^4 b^2 c^2 d^4 + 18 a b^5 c^5 d + 1458 a^5 b c d^5) / (19683 a^4 c^9) \right)^{1/3}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{\frac{2}{3}}}{x^7 (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/x**7/(d*x**3+c),x)

[Out] Integral((a + b*x**3)**(2/3)/(x**7*(c + d*x**3)), x)

$$3.435 \quad \int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3} dx$$

Optimal. Leaf size=334

$$\frac{(-a^2d^2 - 6abcd + 9b^2c^2) \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{18b^{4/3}d^3} + \frac{(-a^2d^2 - 6abcd + 9b^2c^2) \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}d^3} - \frac{c^{4/3}(bc - ad)^{2/3}}{6d}$$

Rubi [C] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{x^7(a+bx^3)^{2/3} F_1\left(\frac{7}{3}; -\frac{2}{3}, 1; \frac{10}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{7c\left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] (x^7*(a + b*x^3)^(2/3)*AppellF1[7/3, -2/3, 1, 10/3, -((b*x^3)/a), -((d*x^3)/c)]/(7*c*(1 + (b*x^3)/a)^(2/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^6 (a + bx^3)^{2/3}}{c + dx^3} dx = \frac{(a + bx^3)^{2/3} \int \frac{x^6 \left(1 + \frac{bx^3}{a}\right)^{2/3}}{c + dx^3} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x^7 (a + bx^3)^{2/3} F_1\left(\frac{7}{3}; -\frac{2}{3}, 1; \frac{10}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{7c \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.76, size = 525, normalized size = 1.57

$$\frac{3x^4 \sqrt{\frac{bx^3}{a} + 1} \sqrt{bx^3 - ad} (-c^2 d^2 - 6abcd + 9d^2) F_1\left(\frac{7}{3}; 1, \frac{1}{3}, \frac{10}{3}, -\frac{bx^3}{a}\right) + 2 \left(-c^2 \sqrt{c^2 d^2 \sqrt{bx^3 - ad} + 9d^2} \log\left(\frac{\sqrt{c^2 d^2 \sqrt{bx^3 - ad} + 9d^2} + \sqrt{bx^3 - ad}}{(c^2 d^2)^{3/2}} + c^2\right) + 6c^2 d \sqrt{bx^3 - ad} + 9d^2 c^2 \sqrt{bx^3 - ad} - 18d^2 c^2 \sqrt{bx^3 - ad} + 3abx^{10} \sqrt{bx^3 - ad} \log\left(\frac{\sqrt{c^2 d^2 \sqrt{bx^3 - ad} + 9d^2}}{(c^2 d^2)^{3/2}} + c^2\right) + 15abcd \sqrt{bx^3 - ad} + 2a \sqrt{c^2 d^2 \sqrt{bx^3 - ad} + 9d^2} \log\left(\sqrt{c^2 d^2 \sqrt{bx^3 - ad} + 9d^2} - \frac{15abcd}{(c^2 d^2)^{3/2}}\right) - 2 \sqrt{c^2 d^2 \sqrt{bx^3 - ad} + 9d^2} \tan^{-1}\left(\frac{\sqrt{c^2 d^2 \sqrt{bx^3 - ad} + 9d^2}}{c}\right) - 18abcd \sqrt{bx^3 - ad} \right)}{108bd^2 \sqrt{a + bx^3} \sqrt{bx^3 - ad}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] (3*(b*c - a*d)^(1/3)*(9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c*(-18*a*b*c*(b*c - a*d)^(1/3)*x + 6*a^2*d*(b*c - a*d)^(1/3)*x - 18*b^2*c*(b*c - a*d)^(1/3)*x^4 + 15*a*b*d*(b*c - a*d)^(1/3)*x^4 + 9*b^2*d*(b*c - a*d)^(1/3)*x^7 - 2*Sqrt[3]*a*c^(1/3)*(-3*b*c + a*d)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*c^(1/3)*(-3*b*c + a*d)*(a + b*x^3)^(1/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + 3*a*b*c^(4/3)*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - a^2*c^(1/3)*d*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)])/(108*b*c*d^2*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3))

IntegrateAlgebraic [C] time = 6.45, size = 608, normalized size = 1.82

$$\frac{(c^2 d^2 - 9d^2) \log\left(\sqrt{c^2 d^2 - 9d^2} - \sqrt{c^2 d^2}\right)}{225d^2} - \frac{(c^2 d^2 - 9d^2) \tan^{-1}\left(\frac{\sqrt{c^2 d^2}}{3d}\right)}{225d^2} - \frac{(c^2 d^2 - 9d^2) \log\left(\sqrt{c^2 d^2 + 9d^2} + (c + b x^3)^{1/3}\right)}{225d^2} - \frac{(c^2 d^2 - 9d^2) \log\left(\sqrt{c^2 d^2 + 9d^2} - (c + b x^3)^{1/3}\right)}{225d^2} - \frac{\sqrt{c^2 d^2 - 9d^2} \tan^{-1}\left(\frac{\sqrt{c^2 d^2 - 9d^2}}{3d}\right)}{225d^2} - \frac{\sqrt{c^2 d^2 - 9d^2} \tan^{-1}\left(\frac{\sqrt{c^2 d^2 - 9d^2}}{3d}\right)}{225d^2} - \frac{(c^2 d^2 - 9d^2) \log\left(\sqrt{c^2 d^2 - 9d^2} + (c + b x^3)^{1/3}\right)}{225d^2} - \frac{(c^2 d^2 - 9d^2) \log\left(\sqrt{c^2 d^2 - 9d^2} - (c + b x^3)^{1/3}\right)}{225d^2} - \frac{(c + b x^3)^{1/3} \log\left(\frac{c^2 d^2 - 9d^2}{(c + b x^3)^{1/3}}\right)}{18bd^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] ((a + b*x^3)^(2/3)*(-6*b*c*x + 2*a*d*x + 3*b*d*x^4))/(18*b*d^2) + ((9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(9*Sqrt[3]*b^(4/3)*d^3) + (Sqrt[(-1 + I*Sqrt[3])/6]*c^(4/3)*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*

$$x - (3*I)*c^{(1/3)}*(a + b*x^3)^{(1/3)} - \text{Sqrt}[3]*c^{(1/3)}*(a + b*x^3)^{(1/3)})/d^3 + ((-9*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*\text{Log}[-(b^{(1/3)}*x) + (a + b*x^3)^{(1/3)}])/(27*b^{(4/3)}*d^3) - ((I/6)*((-I)*c^{(4/3)}*(b*c - a*d)^{(2/3)} + \text{Sqrt}[3]*c^{(4/3)}*(b*c - a*d)^{(2/3)})*\text{Log}[2*(b*c - a*d)^{(1/3)}*x + (1 + I*\text{Sqrt}[3])*c^{(1/3)}*(a + b*x^3)^{(1/3)}])/d^3 + ((9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*\text{Log}[b^{(2/3)}*x^2 + b^{(1/3)}*x*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}])/(54*b^{(4/3)}*d^3) + ((c^{(4/3)}*(b*c - a*d)^{(2/3)} + I*\text{Sqrt}[3]*c^{(4/3)}*(b*c - a*d)^{(2/3)})*\text{Log}[(-2*I)*(b*c - a*d)^{(2/3)}*x^2 + c^{(1/3)}*(b*c - a*d)^{(1/3)}*(I*x - \text{Sqrt}[3]*x)*(a + b*x^3)^{(1/3)} + (I + \text{Sqrt}[3])*c^{(2/3)}*(a + b*x^3)^{(2/3)}])/(12*d^3)$$

fricas [B] time = 1.60, size = 1164, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/54*(18*\text{sqrt}(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^{(1/3)}*b^2*c*\arctan(- \\ & 1/3*(\text{sqrt}(3)*(b*c - a*d)*x + 2*\text{sqrt}(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^{(1/3)}*(b*x^3 + a)^{(1/3)}))/((b*c - a*d)*x)) - 18*(b^2*c^3 - 2*a*b*c^2*d + a^2 \\ & *c*d^2)^{(1/3)}*b^2*c*\log(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^{(2/3)}*x - (b*x \\ & ^3 + a)^{(1/3)}*(b*c^2 - a*c*d))/x) + 9*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^{(1/3)}*b^2*c*\log(-((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^{(1/3)}*(b*c - a*d)*x^2 \\ & + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^{(2/3)}*(b*x^3 + a)^{(1/3)}*x + (b*x^3 + \\ & a)^{(2/3)}*(b*c^2 - a*c*d))/x^2) + 3*\text{sqrt}(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d - a^2 \\ & *b*d^2)*\text{sqrt}(-1/b^{(2/3)})*\log(3*b*x^3 - 3*(b*x^3 + a)^{(1/3)}*b^{(2/3)}*x^2 - 3* \\ & \text{sqrt}(1/3)*(b^{(4/3)}*x^3 + (b*x^3 + a)^{(1/3)}*b*x^2 - 2*(b*x^3 + a)^{(2/3)}*b^{(2/3)}*x) \\ & *\text{sqrt}(-1/b^{(2/3)}) + 2*a) + 2*(9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*b^{(2/3)} \\ & *\log(-(b^{(1/3)}*x - (b*x^3 + a)^{(1/3)})/x) - (9*b^2*c^2 - 6*a*b*c*d - a^2*d^2) \\ & *b^{(2/3)}*\log((b^{(2/3)}*x^2 + (b*x^3 + a)^{(1/3)}*b^{(1/3)}*x + (b*x^3 + a)^{(2/3)})/x^2) \\ & - 3*(3*b^2*d^2*x^4 - 2*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 + a)^{(2/3)})/(b^2*d^3), \\ & -1/54*(18*\text{sqrt}(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^{(1/3)}*b^2 \\ & *c*\arctan(-1/3*(\text{sqrt}(3)*(b*c - a*d)*x + 2*\text{sqrt}(3)*(b^2*c^3 - 2*a*b*c^2*d + \\ & a^2*c*d^2)^{(1/3)}*(b*x^3 + a)^{(1/3)}))/((b*c - a*d)*x)) - 18*(b^2*c^3 - 2*a*b \\ & *c^2*d + a^2*c*d^2)^{(1/3)}*b^2*c*\log(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^{(2/3)}*x \\ & - (b*x^3 + a)^{(1/3)}*(b*c^2 - a*c*d))/x) + 9*(b^2*c^3 - 2*a*b*c^2*d + a^2 \\ & *c*d^2)^{(1/3)}*b^2*c*\log(-((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^{(1/3)}*(b*c \\ & - a*d)*x^2 + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^{(2/3)}*(b*x^3 + a)^{(1/3)}*x \\ & + (b*x^3 + a)^{(2/3)}*(b*c^2 - a*c*d))/x^2) + 2*(9*b^2*c^2 - 6*a*b*c*d - a^2 \\ & *d^2)*b^{(2/3)}*\log(-(b^{(1/3)}*x - (b*x^3 + a)^{(1/3)})/x) - (9*b^2*c^2 - 6*a*b*c \\ & *d - a^2*d^2)*b^{(2/3)}*\log((b^{(2/3)}*x^2 + (b*x^3 + a)^{(1/3)}*b^{(1/3)}*x + (b*x \\ & ^3 + a)^{(2/3)})/x^2) + 6*\text{sqrt}(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d - a^2*b*d^2)*\text{arc} \\ & \text{tan}(\text{sqrt}(1/3)*(b^{(1/3)}*x + 2*(b*x^3 + a)^{(1/3)})/(b^{(1/3)}*x))/b^{(1/3)} - 3*(3 \\ & *b^2*d^2*x^4 - 2*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 + a)^{(2/3)})/(b^2*d^3)] \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}} x^6}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}} x^6}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}} x^6}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6 (bx^3 + a)^{\frac{2}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x)

[Out] int((x^6*(a + b*x^3)^(2/3))/(c + d*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 (a + bx^3)^{\frac{2}{3}}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(b*x**3+a)**(2/3)/(d*x**3+c), x)

[Out] Integral(x**6*(a + b*x**3)**(2/3)/(c + d*x**3), x)

$$3.436 \quad \int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3} dx$$

Optimal. Leaf size=272

$$\frac{\sqrt[3]{c}(bc-ad)^{2/3} \log(c+dx^3)}{6d^2} - \frac{\sqrt[3]{c}(bc-ad)^{2/3} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d^2} + \frac{(3bc-2ad) \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{6\sqrt[3]{b}d^2} \quad (3)$$

Rubi [C] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 0.24, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{x^4(a+bx^3)^{2/3} F_1\left(\frac{4}{3}; -\frac{2}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{4c\left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] (x^4*(a + b*x^3)^(2/3)*AppellF1[4/3, -2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]/(4*c*(1 + (b*x^3)/a)^(2/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^3 (a + bx^3)^{2/3}}{c + dx^3} dx = \frac{(a + bx^3)^{2/3} \int \frac{x^3 \left(1 + \frac{bx^3}{a}\right)^{2/3}}{c + dx^3} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x^4 (a + bx^3)^{2/3} F_1\left(\frac{4}{3}; -\frac{2}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{4c \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.59, size = 286, normalized size = 1.05

$$\frac{3x^4 \sqrt[3]{\frac{bx^3}{a} + 1} (2ad - 3bc) F_1\left(\frac{4}{3}; \frac{1}{3}, \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c \sqrt[3]{a + bx^3}} + \frac{2 \left(-a \sqrt[3]{c} \log\left(\frac{\sqrt[3]{cx} \sqrt[3]{bc-ad} + x^2 (bc-ad)^{2/3}}{\sqrt[3]{ax^3+b}} + c^{2/3}\right) + 6x(a+bx^3)^{2/3} \sqrt[3]{bc-ad} + 2a \sqrt[3]{c} \log\left(\sqrt[3]{c} - \frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right) - 2\sqrt{3} a \sqrt[3]{c} \tan^{-1}\left(\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{c} \sqrt[3]{ax^3+b}} + 1\right) \right)}{\sqrt[3]{bc-ad}}$$

36d

Warning: Unable to verify antiderivative.

[In] Integrate[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] ((3*(-3*b*c + 2*a*d)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -(b*x^3)/a, -(d*x^3)/c])/(c*(a + b*x^3)^(1/3)) + (2*(6*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(2/3) - 2*Sqrt[3]*a*c^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))]/Sqrt[3]] + 2*a*c^(1/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - a*c^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b*c - a*d)^(1/3))/(36*d)

IntegrateAlgebraic [C] time = 4.96, size = 549, normalized size = 2.02

$$\frac{(2ad - 3bc) \log\left(\sqrt[3]{a + bx^3} + (a + bx^3)^{2/3}\right)}{3\sqrt[3]{c}} + \frac{(\sqrt[3]{c} \sqrt[3]{bc - ad} - \sqrt[3]{c} \sqrt[3]{bc - ad}) \log\left(\sqrt[3]{a + bx^3} + (a + bx^3)^{2/3}\right)}{3\sqrt[3]{c}} + \frac{(\sqrt[3]{c} \sqrt[3]{bc - ad} - \sqrt[3]{c} \sqrt[3]{bc - ad}) \sqrt[3]{a + bx^3} \sqrt[3]{bc - ad} - 2a \sqrt[3]{c} \sqrt[3]{bc - ad}}{3\sqrt[3]{c}} + \frac{(3bc - 2ad) \log\left(\sqrt[3]{a + bx^3} + (a + bx^3)^{2/3}\right)}{3\sqrt[3]{c}} + \frac{(\sqrt[3]{c} \sqrt[3]{bc - ad} + \sqrt[3]{c} \sqrt[3]{bc - ad}) \log\left(\sqrt[3]{a + bx^3} + (a + bx^3)^{2/3}\right)}{3\sqrt[3]{c}} + \frac{(3bc - 2ad) \tan^{-1}\left(\frac{\sqrt[3]{c} \sqrt[3]{bc - ad}}{\sqrt[3]{a + bx^3} + (a + bx^3)^{2/3}}\right)}{3\sqrt[3]{c}} + \frac{\sqrt[3]{c} \sqrt[3]{bc - ad} \tan^{-1}\left(\frac{\sqrt[3]{c} \sqrt[3]{bc - ad}}{\sqrt[3]{a + bx^3} + (a + bx^3)^{2/3}}\right)}{3\sqrt[3]{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3), x]

[Out] (x*(a + b*x^3)^(2/3))/(3*d) - ((3*b*c - 2*a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(3*Sqrt[3]*b^(1/3)*d^2) - (Sqrt[-1 + I*Sqrt[3]]*c^(1/3)*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]/(Sqrt[6]*d^2) + ((3*b*c - 2*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(9*b^(1/3)*d^2) + ((c^(1/3)*(b*c - a*d)^(2/3) + I*Sqrt[3]*c^(1/3)*(b*c - a*d)^(1/3)*x)/(36*d)

$$\frac{1}{3}(bc - ad)^{2/3} \text{Log}[2(bc - ad)^{1/3}x + (1 + I\sqrt{3})c^{1/3}(a + bx^3)^{1/3}] / (6d^2) + ((-3bc + 2ad) \text{Log}[b^{2/3}x^2 + b^{1/3}x(a + bx^3)^{1/3} + (a + bx^3)^{2/3}]) / (18b^{1/3}d^2) - ((I/12)((-I)c^{1/3}(bc - ad)^{2/3} + \sqrt{3}c^{1/3}(bc - ad)^{2/3}) \text{Log}[-2I(bc - ad)^{2/3}x^2 + c^{1/3}(bc - ad)^{1/3}(Ix - \sqrt{3}x)(a + bx^3)^{1/3} + (I + \sqrt{3})c^{2/3}(a + bx^3)^{2/3}]) / d^2$$

fricas [B] time = 0.57, size = 1091, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c), x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/18(6(bx^3 + a)^{2/3}b^2dx - 3\sqrt{1/3}(3b^2c - 2ab^2d)\sqrt{(-b)^{1/3}/b})\log(3bx^3 - 3(bx^3 + a)^{1/3}(-b)^{2/3}x^2 - 3\sqrt{1/3}((-b)^{1/3}b^2x^3 - (bx^3 + a)^{1/3}b^2x^2 + 2(bx^3 + a)^{2/3}(-b)^{2/3}x)\sqrt{(-b)^{1/3}/b}) + 2a) + 6\sqrt{3}(-b^2c^3 + 2ab^2cd^2 - a^2c^2d^2)^{1/3}b^2\arctan(1/3(\sqrt{3}(bc - ad)x - 2\sqrt{3}(-b^2c^3 + 2ab^2cd^2 - a^2c^2d^2)^{1/3}(bx^3 + a)^{1/3})/((bc - ad)x)) + 2(3b^2c - 2ab^2d)(-b)^{2/3}\log((-b)^{1/3}x + (bx^3 + a)^{1/3})/x - (3b^2c - 2ab^2d)(-b)^{2/3}\log((-b)^{2/3}x^2 - (bx^3 + a)^{1/3}(-b)^{1/3}x + (bx^3 + a)^{2/3})/x^2) + 6(-b^2c^3 + 2ab^2cd^2 - a^2c^2d^2)^{1/3}b^2\log((-b^2c^3 + 2ab^2cd^2 - a^2c^2d^2)^{2/3}x - (bx^3 + a)^{1/3}(b^2c^2 - a^2cd)) / x - 3(-b^2c^3 + 2ab^2cd^2 - a^2c^2d^2)^{1/3}b^2\log((-b^2c^3 + 2ab^2cd^2 - a^2c^2d^2)^{1/3}(bc - ad)x^2 - (-b^2c^3 + 2ab^2cd^2 - a^2c^2d^2)^{2/3}(bx^3 + a)^{1/3}x - (bx^3 + a)^{2/3}(b^2c^2 - a^2cd)) / x^2) / (bd^2), 1/18(6(bx^3 + a)^{2/3}b^2dx + 6\sqrt{1/3}(3b^2c - 2ab^2d)\sqrt{(-b)^{1/3}/b})\arctan(-\sqrt{1/3}((-b)^{1/3}x - 2(bx^3 + a)^{1/3}))\sqrt{(-b)^{1/3}/b}) / x + 6\sqrt{3}(-b^2c^3 + 2ab^2cd^2 - a^2c^2d^2)^{1/3}b^2\arctan(1/3(\sqrt{3}(bc - ad)x - 2\sqrt{3}(-b^2c^3 + 2ab^2cd^2 - a^2c^2d^2)^{1/3}(bx^3 + a)^{1/3})/((bc - ad)x)) + 2(3b^2c - 2ab^2d)(-b)^{2/3}\log((-b)^{1/3}x + (bx^3 + a)^{1/3})/x - (3b^2c - 2ab^2d)(-b)^{2/3}\log((-b)^{2/3}x^2 - (bx^3 + a)^{1/3}(-b)^{1/3}x + (bx^3 + a)^{2/3})/x^2) + 6(-b^2c^3 + 2ab^2cd^2 - a^2c^2d^2)^{1/3}b^2\log((-b^2c^3 + 2ab^2cd^2 - a^2c^2d^2)^{2/3}x - (bx^3 + a)^{1/3}(b^2c^2 - a^2cd)) / x - 3(-b^2c^3 + 2ab^2cd^2 - a^2c^2d^2)^{1/3}b^2\log((-b^2c^3 + 2ab^2cd^2 - a^2c^2d^2)^{1/3}(bc - ad)x^2 - (-b^2c^3 + 2ab^2cd^2 - a^2c^2d^2)^{2/3}(bx^3 + a)^{1/3}x - (bx^3 + a)^{2/3}(b^2c^2 - a^2cd)) / x^2) / (bd^2)] \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{2/3}x^3}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}} x^3}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}} x^3}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 (bx^3 + a)^{\frac{2}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x)

[Out] int((x^3*(a + b*x^3)^(2/3))/(c + d*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + bx^3)^{\frac{2}{3}}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(b*x**3+a)**(2/3)/(d*x**3+c),x)

[Out] Integral(x**3*(a + b*x**3)**(2/3)/(c + d*x**3), x)

$$3.437 \quad \int \frac{(a+bx^3)^{2/3}}{c+dx^3} dx$$

Optimal. Leaf size=233

$$\frac{b^{2/3} \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{2d} + \frac{b^{2/3} \tan^{-1}\left(\frac{2\sqrt[3]{b}x + \sqrt[3]{a+bx^3}}{\sqrt{3}}\right)}{\sqrt{3}d} - \frac{(bc-ad)^{2/3} \log(c+dx^3)}{6c^{2/3}d} + \frac{(bc-ad)^{2/3} \log\left(\frac{x\sqrt[3]{bc-ad} - \sqrt[3]{a}}{\sqrt[3]{c}}\right)}{2c^{2/3}d}$$

Rubi [C] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {430, 429}

$$\frac{x(a+bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3), x]

[Out] (x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c*(1 + (b*x^3)/a)^(2/3))

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a + bx^3)^{2/3}}{c + dx^3} dx = \frac{(a + bx^3)^{2/3} \int \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3}}{c + dx^3} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x (a + bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.06, size = 161, normalized size = 0.69

$$\frac{4acx (a + bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{(c + dx^3) \left(x^3 \left(2bcF_1\left(\frac{4}{3}; \frac{1}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) - 3adF_1\left(\frac{4}{3}; -\frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) \right) + 4acF_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3), x]

[Out] (4*a*c*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)]/((c + d*x^3)*(4*a*c*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))

IntegrateAlgebraic [C] time = 0.00, size = 487, normalized size = 2.09

$$\frac{\frac{b^{2/3} \log(\sqrt{a+bx^3} - \sqrt{c})}{3d} + \frac{b^{2/3} \tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}}{\sqrt{a+bx^3} - \sqrt{c}}\right)}{\sqrt{3}d} + \frac{b^{2/3} \log(\sqrt{b}\sqrt{a+bx^3} + (a+bx^3)^{2/3} + b^{2/3})}{6d} - \frac{i(\sqrt{3}(bc-ad)^3 - (bc-ad)^2) \log(2i\sqrt{c-ad} + (1+i\sqrt{3})\sqrt[3]{c^2(a+bx^3)}})}{6c^{3/2}d} + \frac{\sqrt{2(-1+i\sqrt{3})}(bc-ad)^2 \tan^{-1}\left(\frac{2i\sqrt{c-ad}}{\sqrt{1+3i}\sqrt{c^2(a+bx^3)} + \sqrt{3}\sqrt{c-ad}}\right)}{c^{3/2}d} - \frac{(bc-ad)^2 + i\sqrt{3}(bc-ad)^2) \log(\sqrt{3} + i) a^{2/3} (a+bx^3)^{2/3} + \sqrt{3}(-\sqrt{3} + i) \sqrt{a+bx^3} \sqrt{c-ad} - 2i^2(bc-ad)^2)}{12c^{3/2}d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(c + d*x^3), x]

[Out] (b^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(Sqrt[3]*d) + (Sqrt[(-1 + I*Sqrt[3])/6]*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])/((c^(2/3)*d) - (b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/((3*d) - ((I/6)*((-I)*(b*c - a*d)^(2/3) + Sqrt[3]*(b*c - a*d)^(2/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/((c^(2/3)*d) + (b^(2/3)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/((6*d) + (((b*c - a*d)^(2/3) + I*Sqrt[3]*(b*c - a*d)^(2/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)]))

$(I*x - \text{Sqrt}[3]*x)*(a + b*x^3)^{(1/3)} + (I + \text{Sqrt}[3])*c^{(2/3)}*(a + b*x^3)^{(2/3)}]/(12*c^{(2/3)}*d)$

fricas [B] time = 0.45, size = 469, normalized size = 2.01

$$2\sqrt{3}\left(\frac{(b^2c^2+2\sqrt{3}bd^2)^{\frac{1}{2}}}{3(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}\right)^{\frac{1}{2}}\arctan\left(\frac{\sqrt{3}(b^2c^2+2\sqrt{3}bd^2)^{\frac{1}{2}}}{3(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}\right) + 2\sqrt{3}(-b)^{\frac{1}{2}}\arctan\left(\frac{\sqrt{3}(b^2c^2+2\sqrt{3}bd^2)^{\frac{1}{2}}}{3(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}\right) - 2\left(\frac{(b^2c^2+2\sqrt{3}bd^2)^{\frac{1}{2}}}{3(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}\right)^{\frac{1}{2}}\log\left(\frac{(b^2c^2+2\sqrt{3}bd^2)^{\frac{1}{2}}(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}{(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}\right) - 2(-b)^{\frac{1}{2}}\log\left(\frac{(b^2c^2+2\sqrt{3}bd^2)^{\frac{1}{2}}(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}{(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}\right) + (-b)^{\frac{1}{2}}\log\left(\frac{(b^2c^2+2\sqrt{3}bd^2)^{\frac{1}{2}}(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}{(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}\right) + \left(\frac{(b^2c^2+2\sqrt{3}bd^2)^{\frac{1}{2}}}{3(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}\right)^{\frac{1}{2}}\log\left(\frac{(b^2c^2+2\sqrt{3}bd^2)^{\frac{1}{2}}(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}{(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}\right) - (b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}\log\left(\frac{(b^2c^2+2\sqrt{3}bd^2)^{\frac{1}{2}}(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}{(b^2c^2-2\sqrt{3}bd^2)^{\frac{1}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] $-1/6*(2*\text{sqrt}(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^{(1/3)}*\text{arctan}(-1/3*(\text{sqrt}(3)*(b*c - a*d)*x + 2*\text{sqrt}(3)*(b*x^3 + a)^{(1/3)}*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^{(1/3)})/((b*c - a*d)*x)) + 2*\text{sqrt}(3)*(-b^2)^{(1/3)}*\text{arctan}(-1/3*(\text{sqrt}(3)*b*x - 2*\text{sqrt}(3)*(b*x^3 + a)^{(1/3)}*(-b^2)^{(1/3)})/(b*x)) - 2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^{(1/3)}*\log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^{(2/3)} - (b*x^3 + a)^{(1/3)}*(b*c - a*d))/x) - 2*(-b^2)^{(1/3)}*\log(-((-b^2)^{(2/3)}*x - (b*x^3 + a)^{(1/3)}*b)/x) + (-b^2)^{(1/3)}*\log(-((-b^2)^{(1/3)}*b*x^2 - (b*x^3 + a)^{(1/3)}*(-b^2)^{(2/3)}*x - (b*x^3 + a)^{(2/3)}*b)/x^2) + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^{(1/3)}*\log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^{(1/3)} + (b*x^3 + a)^{(1/3)}*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^{(2/3)} + (b*x^3 + a)^{(2/3)}*(b*c - a*d))/x^2))/d$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int((b*x^3+a)^(2/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(c + d*x^3),x)

[Out] int((a + b*x^3)^(2/3)/(c + d*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{\frac{2}{3}}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)

[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)

$$3.438 \quad \int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)} dx$$

Optimal. Leaf size=169

$$\frac{(bc-ad)^{2/3} \log(c+dx^3)}{6c^{5/3}} - \frac{(bc-ad)^{2/3} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{5/3}} + \frac{(bc-ad)^{2/3} \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}c^{5/3}} - \frac{(a+bx^3)^{2/3}}{2cx^2}$$

Rubi [C] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 0.53, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{(a+bx^3)^{2/3} \left(\frac{dx^3}{c} + 1\right)^{2/3} {}_2F_1\left(-\frac{2}{3}, -\frac{2}{3}; \frac{1}{3}; -\frac{c\left(\frac{bx^3-dx^3}{a-c}\right)}{dx^3+c}\right)}{2cx^2 \left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]

[Out] -((a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, -(c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3)])/(2*c*x^2*(1 + (b*x^3)/a)^(2/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -(b*x^n)/a, -(d*x^n)/c]]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{(a + bx^3)^{2/3}}{x^3(c + dx^3)} dx = \frac{(a + bx^3)^{2/3} \int \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3}}{x^3(c + dx^3)} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{(a + bx^3)^{2/3} \left(1 + \frac{dx^3}{c}\right)^{2/3} {}_2F_1\left(-\frac{2}{3}, -\frac{2}{3}; \frac{1}{3}; -\frac{c\left(\frac{bx^3}{a} - \frac{dx^3}{c}\right)}{c + dx^3}\right)}{2cx^2 \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.05, size = 83, normalized size = 0.49

$$\frac{(a + bx^3)^{2/3} \left(\frac{dx^3}{c} + 1\right)^{2/3} {}_2F_1\left(-\frac{2}{3}, -\frac{2}{3}; \frac{1}{3}; \frac{(ad - bc)x^3}{a(dx^3 + c)}\right)}{2cx^2 \left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)), x]

[Out] -1/2*((a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*x^2*(1 + (b*x^3)/a)^(2/3))

IntegrateAlgebraic [C] time = 2.38, size = 365, normalized size = 2.16

$$\frac{(bc - ad)^{2/3} + i\sqrt{3}(bc - ad)^{2/3} \log\left(\frac{2x\sqrt{bc - ad} + (1 + i\sqrt{3})\sqrt{c}\sqrt{a + bx^3}}{c^{3/3}}\right) - \sqrt{\frac{(-1 + i\sqrt{3})(bc - ad)^{2/3} \tan^{-1}\left(\frac{\sqrt{3}\sqrt{bc - ad}}{\sqrt{3}\sqrt{bc - ad} - \sqrt{3}\sqrt{c}\sqrt{a + bx^3}}\right) - \frac{2\sqrt{bc - ad}}{c^{3/3}}}{c^{3/3}}}}{12c^{5/3}} + i\sqrt{3}(bc - ad)^{2/3} - i(bc - ad)^{2/3} \log\left(\frac{(\sqrt{3} + i)c^{2/3}(a + bx^3)^{2/3} + \sqrt{3}c(-\sqrt{3}x + ix)\sqrt{a + bx^3}\sqrt{bc - ad} - 2ix^2(bc - ad)^{2/3}}{12c^{5/3}}\right) - \frac{(a + bx^3)^{2/3}}{2cx^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)), x]

[Out] -1/2*(a + b*x^3)^(2/3)/(c*x^2) - (Sqrt[(-1 + I*Sqrt[3])/6]*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])]/c^(5/3) + (((b*c - a*d)^(2/3) + I*Sqrt[3]*(b*c - a*d)^(2/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*c^(5/3)) - ((I/12)*((-I)*(b*c - a*d)^(2/3) + Sqrt[3]*(b*c - a*d)^(2/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/c^(5/3)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x)

[Out] int((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^3 + a)^{2/3}}{x^3(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x^3*(c + d*x^3)), x)

[Out] int((a + b*x^3)^(2/3)/(x^3*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{2/3}}{x^3(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/x**3/(d*x**3+c), x)

[Out] Integral((a + b*x**3)**(2/3)/(x**3*(c + d*x**3)), x)

$$3.439 \quad \int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)} dx$$

Optimal. Leaf size=206

$$\frac{d(bc-ad)^{2/3} \log(c+dx^3)}{6c^{8/3}} + \frac{d(bc-ad)^{2/3} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{8/3}} - \frac{d(bc-ad)^{2/3} \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\frac{\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt{3}}}\right)}{\sqrt{3}c^{8/3}} (a+bx^3)$$

Rubi [C] time = 0.13, antiderivative size = 148, normalized size of antiderivative = 0.72, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{-2x^3(2c-3dx^3)(bc-ad) {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right) + 6x^3(c+dx^3)(bc-ad) {}_2F_1\left(\frac{1}{3}, 2; \frac{4}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right) + c(a+bx^3)(2c-3dx^3)}{10c^3x^5\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)), x]

[Out] -(c*(a + b*x^3)*(2*c - 3*d*x^3) - 2*(b*c - a*d)*x^3*(2*c - 3*d*x^3)*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6*(b*c - a*d)*x^3*(c + d*x^3)*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(10*c^3*x^5*(a + b*x^3)^(1/3))

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{(a + bx^3)^{2/3}}{x^6(c + dx^3)} dx = \frac{(a + bx^3)^{2/3} \int \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3}}{x^6(c + dx^3)} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{c(a + bx^3)(2c - 3dx^3) - 2(bc - ad)x^3(2c - 3dx^3) {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{(bc - ad)x^3}{c(a + bx^3)}\right) + 6(bc - ad)x^3}{10c^3x^5\sqrt[3]{a + bx^3}}$$

Mathematica [C] time = 0.64, size = 148, normalized size = 0.72

$$\frac{2x^3(3dx^3 - 2c)(bc - ad) {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{(bc - ad)x^3}{c(bx^3 + a)}\right) + 6x^3(c + dx^3)(bc - ad) {}_2F_1\left(\frac{1}{3}, 2; \frac{4}{3}; \frac{(bc - ad)x^3}{c(bx^3 + a)}\right) + c(a + bx^3)(2c - 3dx^3)}{10c^3x^5\sqrt[3]{a + bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)), x]

[Out] -1/10*(c*(a + b*x^3)*(2*c - 3*d*x^3) + 2*(b*c - a*d)*x^3*(-2*c + 3*d*x^3))*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6*(b*c - a*d)*x^3*(c + d*x^3)*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(c^3*x^5*(a + b*x^3)^(1/3))

IntegrateAlgebraic [C] time = 2.50, size = 392, normalized size = 1.90

$$\frac{i(\sqrt{3}d(bc - ad)^{2/3} - id(bc - ad)^{2/3}) \log(2x\sqrt{bc - ad} + (1 + i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a + bx^3})}{6c^{5/3}} + \frac{\sqrt[3]{\frac{-1 + i\sqrt{3}}{2}} d(bc - ad)^{2/3} \tan^{-1}\left(\frac{2x\sqrt{bc - ad}}{\sqrt{3}\sqrt{bc - ad} - \sqrt[3]{c}\sqrt[3]{a + bx^3}}\right)}{c^{5/3}} + \frac{(d(bc - ad)^{2/3} + i\sqrt{3}d(bc - ad)^{2/3}) \log\left(\sqrt{3} + i\right) 2^{2/3} (a + bx^3)^{2/3} + \sqrt[3]{c}(-\sqrt{3} + i)\sqrt[3]{a + bx^3}\sqrt[3]{bc - ad} - 2ix^2(bc - ad)^{2/3}}{12c^{5/3}} + \frac{(a + bx^3)^{2/3}(-2ac + 5ad^3 - 2bc^3)}{10c^2x^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)), x]

[Out] ((a + b*x^3)^(2/3)*(-2*a*c - 2*b*c*x^3 + 5*a*d*x^3))/(10*a*c^2*x^5) + (Sqrt[(-1 + I*Sqrt[3])/6]*d*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])/c^(8/3) - ((I/6)*((-I)*d*(b*c - a*d)^(2/3) + Sqrt[3]*d*(b*c - a*d)^(2/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]/c^(8/3) + ((d*(b*c - a*d)^(2/3) + I*Sqrt[3]*d*(b*c - a*d)^(2/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]/(12*c^(8/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x)

[Out] int((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{2/3}}{x^6(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x^6*(c + d*x^3)), x)

[Out] int((a + b*x^3)^(2/3)/(x^6*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{2/3}}{x^6(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/x**6/(d*x**3+c), x)

[Out] Integral((a + b*x**3)**(2/3)/(x**6*(c + d*x**3)), x)

$$3.440 \quad \int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)} dx$$

Optimal. Leaf size=257

$$\frac{(a+bx^3)^{2/3}(-20a^2d^2+8abcd+3b^2c^2)}{40a^2c^3x^2} + \frac{d^2(bc-ad)^{2/3}\log(c+dx^3)}{6c^{11/3}} - \frac{d^2(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{11/3}}$$

Rubi [C] time = 0.97, antiderivative size = 451, normalized size of antiderivative = 1.75, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{-9x^2(c+dx)^2(bc-ad)^2F_1\left(\frac{1}{2}, 2, 2, 1; \frac{1}{2}, \frac{1}{2}, \frac{1}{2}; \frac{bc-ad}{c}\right) - 2x^2(bc^2-6acd^2+9a^2d^3)(bc-ad)^2F_1\left(\frac{3}{2}, 1; \frac{3}{2}; \frac{bc-ad}{c}\right) - 6b^2cd^2F_1\left(\frac{1}{2}, 2; \frac{1}{2}, \frac{1}{2}; \frac{bc-ad}{c}\right) + 21bc^2d^2F_1\left(\frac{3}{2}, 2; \frac{3}{2}; \frac{bc-ad}{c}\right) - 21ac^2d^3F_1\left(\frac{1}{2}, 2; \frac{1}{2}; \frac{bc-ad}{c}\right) + 27ad^3F_1\left(\frac{1}{2}, 2; \frac{1}{2}; \frac{bc-ad}{c}\right) - 27bc^2d^3F_1\left(\frac{1}{2}, 2; \frac{1}{2}; \frac{bc-ad}{c}\right) + 6acd^2F_1\left(\frac{1}{2}, 2; \frac{1}{2}; \frac{bc-ad}{c}\right) - 6ac^2d^2 + 5ac^3 + 9acd^2d^2 - 6b^2cd^2 + 5bc^2d^2 + 9acd^2d^2}{40c^4\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]

[Out] -(5*a*c^3 + 5*b*c^3*x^3 - 6*a*c^2*d*x^3 - 6*b*c^2*d*x^6 + 9*a*c*d^2*x^6 + 9*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(5*c^2 - 6*c*d*x^3 + 9*d^2*x^6)*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21*b*c^3*x^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 21*a*c^2*d*x^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 6*b*c^2*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6*a*c*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(40*c^4*x^8*(a + b*x^3)^(1/3))

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x

```

(n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

```

Rubi steps

$$\int \frac{(a + bx^3)^{2/3}}{x^9(c + dx^3)} dx = \frac{(a + bx^3)^{2/3} \int \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3}}{x^9(c + dx^3)} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= - \frac{5ac^3 + 5bc^3x^3 - 6ac^2dx^3 - 6bc^2dx^6 + 9acd^2x^6 + 9bcd^2x^9 - 2(bc - ad)x^3(5c^2 - 6cdx^3 + 9d^2x^6)}{40c^2\sqrt{a + bx^3}}$$

Mathematica [C] time = 2.20, size = 451, normalized size = 1.75

$$\frac{-9c^2(c + dx^3)^2(bc - ad)^2 \operatorname{F}_1\left(\frac{1}{3}, 2, 2, \frac{1}{3}, \frac{(bc - ad)^2}{(c + dx^3)^2}\right) + 21bc^2x^3 \operatorname{F}_1\left(\frac{1}{3}, 2, \frac{1}{3}, \frac{(bc - ad)^2}{(c + dx^3)^2}\right) - 2a^2(5c^2 - 6ad^2 + 9d^2x^3)(bc - ad) \operatorname{F}_1\left(\frac{1}{3}, 1, \frac{1}{3}, \frac{(bc - ad)^2}{(c + dx^3)^2}\right) - 21ac^2dx^3 \operatorname{F}_1\left(\frac{1}{3}, 2, \frac{1}{3}, \frac{(bc - ad)^2}{(c + dx^3)^2}\right) - 6bc^2dx^6 \operatorname{F}_1\left(\frac{1}{3}, 2, \frac{1}{3}, \frac{(bc - ad)^2}{(c + dx^3)^2}\right) + 27ad^2x^6 \operatorname{F}_1\left(\frac{1}{3}, 2, \frac{1}{3}, \frac{(bc - ad)^2}{(c + dx^3)^2}\right) + 27bcd^2x^9 \operatorname{F}_1\left(\frac{1}{3}, 2, \frac{1}{3}, \frac{(bc - ad)^2}{(c + dx^3)^2}\right) - 27bc^2dx^3 \operatorname{F}_1\left(\frac{1}{3}, 2, \frac{1}{3}, \frac{(bc - ad)^2}{(c + dx^3)^2}\right) + 6acd^2x^6 \operatorname{F}_1\left(\frac{1}{3}, 2, \frac{1}{3}, \frac{(bc - ad)^2}{(c + dx^3)^2}\right) + 5ac^3 - 6ac^2dx^3 + 9acd^2x^6 + 5bc^3x^3 - 6bc^2dx^6 + 9bcd^2x^9}{40c^2\sqrt{a + bx^3}}$$

Warning: Unable to verify antiderivative.

```

[In] Integrate[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
[Out] -1/40*(5*a*c^3 + 5*b*c^3*x^3 - 6*a*c^2*d*x^3 - 6*b*c^2*d*x^6 + 9*a*c*d^2*x^
6 + 9*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(5*c^2 - 6*c*d*x^3 + 9*d^2*x^6)*Hyper
geometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21*b*c^3*x^3
*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 21*a*c
^2*d*x^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
- 6*b*c^2*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b
x^3))] + 6*a*c*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c
(a + b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*
x^3)/(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c -
a*d)*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*Hypergeometri
cPFQ[{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(c^4*x^8*(a
+ b*x^3)^(1/3))

```

IntegrateAlgebraic [C] time = 2.91, size = 444, normalized size = 1.73

$$\frac{(a + bx^3)^{2/3}(-5a^2c^2 + 8a^2cd^2 - 20a^2d^2x^3 - 2ab^2c^2 + 8abcdx^3 + 3a^2d^2x^6)}{40c^2\sqrt{a + bx^3}} - \frac{(a^2(bc - ad)^2 + \sqrt{3}d^2(bc - ad)^2) \log\left(2\sqrt{bc - ad} + (1 + \sqrt{3})\sqrt{c + dx^3}\right)}{6c^{1/3}} - \frac{\sqrt{\frac{1}{3}(1 + \sqrt{3})}d^2(bc - ad)^2 \tan^{-1}\left(\frac{bc - ad}{\sqrt{3}\sqrt{bc - ad} - \sqrt{3}\sqrt{c + dx^3}}\right)}{2c^{1/3}} - \frac{i(\sqrt{3}d^2(bc - ad)^2 - ad^2(bc - ad)^2) \log\left(\sqrt{3} + i\right)^{2/3}(a + bx^3)^{2/3} + \sqrt{c}(-\sqrt{3}x + i)\sqrt{a + bx^3}\sqrt{bc - ad} - 2a^2(bc - ad)^2}{12c^{1/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]

[Out] ((a + b*x^3)^(2/3)*(-5*a^2*c^2 - 2*a*b*c^2*x^3 + 8*a^2*c*d*x^3 + 3*b^2*c^2*x^6 + 8*a*b*c*d*x^6 - 20*a^2*d^2*x^6))/(40*a^2*c^3*x^8) - (Sqrt[(-1 + I*Sqrt[3])/6]*d^2*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]/c^(11/3) + ((d^2*(b*c - a*d)^(2/3) + I*Sqrt[3]*d^2*(b*c - a*d)^(2/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]/(6*c^(11/3)) - ((I/12)*((-I)*d^2*(b*c - a*d)^(2/3) + Sqrt[3]*d^2*(b*c - a*d)^(2/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)]*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]/c^(11/3)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x)

[Out] int((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{x^9(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x)

[Out] int((a + b*x^3)^(2/3)/(x^9*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{\frac{2}{3}}}{x^9(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/x**9/(d*x**3+c),x)

[Out] Integral((a + b*x**3)**(2/3)/(x**9*(c + d*x**3)), x)

$$3.441 \quad \int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)} dx$$

Optimal. Leaf size=320

$$\frac{(a+bx^3)^{2/3}(-44a^2d^2+11abcd+6b^2c^2)}{220a^2c^3x^5} - \frac{(a+bx^3)^{2/3}(-220a^3d^3+88a^2bcd^2+33ab^2c^2d+18b^3c^3)}{440a^3c^4x^2} - \frac{d^3(bc-ad)}{c^2}$$

Rubi [C] time = 2.63, antiderivative size = 819, normalized size of antiderivative = 2.56, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]

[Out] $-(40*a*c^4 + 40*b*c^4*x^3 - 45*a*c^3*d*x^3 - 45*b*c^3*d*x^6 + 54*a*c^2*d^2*x^6 + 54*b*c^2*d^2*x^9 - 81*a*c*d^3*x^9 - 81*b*c*d^3*x^{12} - 80*b*c^4*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 80*a*c^3*d*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 90*b*c^3*d*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 90*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 108*b*c^2*d^2*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 108*a*c*d^3*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b*c*d^3*x^{12}*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a*d^4*x^{12}*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 198*b*c^4*x^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 198*a*c^3*d*x^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 99*b*c^3*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 99*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*b*c*d^3*x^{12}*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^{12}*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 27*(b*c - a*d)*x^3*(5*c - 6*d*x^3)*(c + d*x^3)^2*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*HypergeometricPFQ[{1/3, 2, 2, 2}, {1, 1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(440*c^5*x^{11}*(a + b*x^3)^(1/3))$

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -(b*x^n)/a, -(d*x^n)/c]]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a + bx^3)^{2/3}}{x^{12}(c + dx^3)} dx = \frac{(a + bx^3)^{2/3} \int \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3}}{x^{12}(c + dx^3)} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{40ac^4 + 40bc^4x^3 - 45ac^3dx^3 - 45bc^3dx^6 + 54ac^2d^2x^6 + 54bc^2d^2x^9 - 81acd^3x^9 - 81bcd^3x^{12}}{\dots}$$

Mathematica [C] time = 4.99, size = 819, normalized size = 2.56

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

```
[Out] -1/440*(40*a*c^4 + 40*b*c^4*x^3 - 45*a*c^3*d*x^3 - 45*b*c^3*d*x^6 + 54*a*c^2*d^2*x^6 + 54*b*c^2*d^2*x^9 - 81*a*c*d^3*x^9 - 81*b*c*d^3*x^12 - 80*b*c^4*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 80*a*c^3*d*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 90*b*c^3*d*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 90*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 108*b*c^2*d^2*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 108*a*c*d^3*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b*c*d^3*x^12*Hypergeometric2
```

$$\begin{aligned}
& F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a*d^4*x^12*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 198*b*c^4*x^3* \\
& Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 198*a*c^3*d*x^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] \\
& - 99*b*c^3*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 99*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3) \\
& / (c*(a + b*x^3))] + 297*b*c*d^3*x^12*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hypergeometric2F1[1/3, 2, 4/3, \\
& ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^2*(-5*c + 6*d*x^3)*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*HypergeometricPFQ[{1/3, 2, \\
& 2, 2}, {1, 1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(c^5*x^11*(a + b*x^3)^(1/3))
\end{aligned}$$

IntegrateAlgebraic [C] time = 3.35, size = 501, normalized size = 1.57

$$\frac{(a+bx^3)^{11}(-40a^3c^3-10a^2b^2c^3x^3+55a^3c^2d^2x^6+12a^2b^2c^2d^2x^6+22a^2b^2c^2d^2x^6-88a^3c^2d^2x^6-18b^3c^3x^9-33a^2b^2c^2d^2x^9-88a^2b^2c^2d^2x^9+220a^3d^3x^9)}{440c^{14}} \frac{(\sqrt{d^3bc-ad^3}) \log\left(\frac{2\sqrt{d^3bc-ad^3} + (1+\sqrt{3})\sqrt{d^3bc-ad^3}}{2\sqrt{d^3bc-ad^3}}\right)}{d^3bc-ad^3} \frac{\sqrt{2(1+\sqrt{3})d^3bc-ad^3} \tan^{-1}\left(\frac{2\sqrt{d^3bc-ad^3}}{\sqrt{2(1+\sqrt{3})d^3bc-ad^3} + \sqrt{d^3bc-ad^3}}\right)}{d^3bc-ad^3} \frac{(d^3bc-ad^3 + \sqrt{d^3bc-ad^3}) \log\left(\frac{\sqrt{d^3bc-ad^3} + \sqrt{d^3bc-ad^3}}{\sqrt{d^3bc-ad^3} + \sqrt{d^3bc-ad^3}}\right) + \sqrt{d^3bc-ad^3} \log\left(\frac{\sqrt{d^3bc-ad^3} + \sqrt{d^3bc-ad^3}}{\sqrt{d^3bc-ad^3} + \sqrt{d^3bc-ad^3}}\right)}{12c^{14}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]

[Out] ((a + b*x^3)^(2/3)*(-40*a^3*c^3 - 10*a^2*b*c^3*x^3 + 55*a^3*c^2*d*x^3 + 12*a*b^2*c^3*x^6 + 22*a^2*b*c^2*d*x^6 - 88*a^3*c*d^2*x^6 - 18*b^3*c^3*x^9 - 33*a*b^2*c^2*d*x^9 - 88*a^2*b*c*d^2*x^9 + 220*a^3*d^3*x^9))/(440*a^3*c^4*x^11) + (Sqrt[(-1 + I*Sqrt[3])/6]*d^3*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])]/c^(14/3) - ((I/6)*((-I)*d^3*(b*c - a*d)^(2/3) + Sqrt[3]*d^3*(b*c - a*d)^(2/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]/c^(14/3) + ((d^3*(b*c - a*d)^(2/3) + I*Sqrt[3]*d^3*(b*c - a*d)^(2/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]/(12*c^(14/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x)

[Out] int((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{x^{12}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x)

[Out] int((a + b*x^3)^(2/3)/(x^12*(c + d*x^3)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(2/3)/x**12/(d*x**3+c),x)

[Out] Timed out

$$3.442 \quad \int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3} dx$$

Optimal. Leaf size=251

$$\frac{(a+bx^3)^{7/3}(ad+bc)}{7b^2d^2} + \frac{(a+bx^3)^{10/3}}{10b^2d} - \frac{c^2(bc-ad)^{4/3} \log(c+dx^3)}{6d^{13/3}} + \frac{c^2(bc-ad)^{4/3} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{13/3}}$$

Rubi [A] time = 0.36, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 88, 50, 58, 617, 204, 31}

$$-\frac{(a+bx^3)^{7/3}(ad+bc)}{7b^2d^2} + \frac{(a+bx^3)^{10/3}}{10b^2d} + \frac{c^2(a+bx^3)^{4/3}}{4d^3} - \frac{c^2\sqrt[3]{a+bx^3}(bc-ad)}{d^4} - \frac{c^2(bc-ad)^{4/3} \log(c+dx^3)}{6d^{13/3}} + \frac{c^2(bc-ad)^{4/3} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{13/3}} - \frac{c^2(bc-ad)^{4/3} \tan^{-1}\left(\frac{1-2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{3}d^{13/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3), x]

[Out] -((c^2*(b*c - a*d)*(a + b*x^3)^(1/3))/d^4) + (c^2*(a + b*x^3)^(4/3))/(4*d^3) - ((b*c + a*d)*(a + b*x^3)^(7/3))/(7*b^2*d^2) + (a + b*x^3)^(10/3)/(10*b^2*d) - (c^2*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(13/3)) - (c^2*(b*c - a*d)^(4/3)*Log[c + d*x^3]/(6*d^(13/3)) + (c^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(13/3)))

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8 (a + bx^3)^{4/3}}{c + dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (a + bx)^{4/3}}{c + dx} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{(-bc - ad)(a + bx)^{4/3}}{bd^2} + \frac{(a + bx)^{7/3}}{bd} + \frac{c^2 (a + bx)^{4/3}}{d^2 (c + dx)} \right) dx, x, x^3 \right) \\
&= -\frac{(bc + ad)(a + bx^3)^{7/3}}{7b^2 d^2} + \frac{(a + bx^3)^{10/3}}{10b^2 d} + \frac{c^2 \text{Subst} \left(\int \frac{(a + bx)^{4/3}}{c + dx} dx, x, x^3 \right)}{3d^2} \\
&= \frac{c^2 (a + bx^3)^{4/3}}{4d^3} - \frac{(bc + ad)(a + bx^3)^{7/3}}{7b^2 d^2} + \frac{(a + bx^3)^{10/3}}{10b^2 d} - \frac{(c^2(bc - ad)) \text{Subst} \left(\int \frac{\sqrt[3]{a + bx}}{c + dx} \right)}{3d^3} \\
&= -\frac{c^2(bc - ad)\sqrt[3]{a + bx^3}}{d^4} + \frac{c^2 (a + bx^3)^{4/3}}{4d^3} - \frac{(bc + ad)(a + bx^3)^{7/3}}{7b^2 d^2} + \frac{(a + bx^3)^{10/3}}{10b^2 d} + \frac{c^2(bc - ad)\sqrt[3]{a + bx^3}}{d^4} \\
&= -\frac{c^2(bc - ad)\sqrt[3]{a + bx^3}}{d^4} + \frac{c^2 (a + bx^3)^{4/3}}{4d^3} - \frac{(bc + ad)(a + bx^3)^{7/3}}{7b^2 d^2} + \frac{(a + bx^3)^{10/3}}{10b^2 d} - \frac{c^2(bc - ad)\sqrt[3]{a + bx^3}}{d^4} \\
&= -\frac{c^2(bc - ad)\sqrt[3]{a + bx^3}}{d^4} + \frac{c^2 (a + bx^3)^{4/3}}{4d^3} - \frac{(bc + ad)(a + bx^3)^{7/3}}{7b^2 d^2} + \frac{(a + bx^3)^{10/3}}{10b^2 d} - \frac{c^2(bc - ad)\sqrt[3]{a + bx^3}}{d^4} \\
&= -\frac{c^2(bc - ad)\sqrt[3]{a + bx^3}}{d^4} + \frac{c^2 (a + bx^3)^{4/3}}{4d^3} - \frac{(bc + ad)(a + bx^3)^{7/3}}{7b^2 d^2} + \frac{(a + bx^3)^{10/3}}{10b^2 d} - \frac{c^2(bc - ad)\sqrt[3]{a + bx^3}}{d^4}
\end{aligned}$$

Mathematica [A] time = 0.54, size = 258, normalized size = 1.03

$$\frac{-\frac{60d(a+bx^3)^{7/3}}{b^2} + \frac{42d^2(a+bx^3)^{10/3}}{b^2} - \frac{70c^2(bc-ad)\sqrt[3]{bc-ad} \left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3}\right) - 2\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 6\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{d^{4/3}}}{420d^3} + 105c^2(a+bx^3)^{4/3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3), x]

[Out] (105*c^2*(a + b*x^3)^(4/3) - (60*d*(b*c + a*d)*(a + b*x^3)^(7/3))/b^2 + (42*d^2*(a + b*x^3)^(10/3))/b^2 - (70*c^2*(b*c - a*d)*(6*d^(1/3)*(a + b*x^3)^(1/3) + (b*c - a*d)^(1/3)*(2*sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)]/sqrt[3]) - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x

$$\sqrt[3]{a+bx^3} + \text{Log}[(b*c - a*d)^{2/3} - d^{1/3}*(b*c - a*d)^{1/3}*(a + b*x^3)^{1/3} + d^{2/3}*(a + b*x^3)^{2/3}]/d^{4/3})/(420*d^3)$$

IntegrateAlgebraic [A] time = 0.46, size = 340, normalized size = 1.35

$$\frac{\sqrt{a+bx^3}(-6a^3d^3-20a^2bd^2+2a^2bd^3+175a^2cd^3-40a^2cd^3+22a^2cd^3-140b^3c^3+35b^3cd^3-20b^3cd^3+14b^3cd^3)}{140b^2d^4} + \frac{c^2(bc-ad)^{4/3}\log\left(\frac{\sqrt{bc-ad}+\sqrt{d}\sqrt{a+bx^3}}{3d^{1/3}}\right)}{3d^{1/3}} - \frac{c^2(bc-ad)^{4/3}\log\left(-\sqrt{d}\sqrt{a+bx^3}+\sqrt{bc-ad}+(bc-ad)^{2/3}+d^{2/3}(a+bx^3)^{2/3}\right)}{6d^{1/3}} - \frac{c^2(bc-ad)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt{3}\sqrt{a+bx^3}}{\sqrt{3}\sqrt{bc-ad}}\right)}{\sqrt{3}d^{1/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-140*b^3*c^3 + 175*a*b^2*c^2*d - 20*a^2*b*c*d^2 - 6*a^3*d^3 + 35*b^3*c^2*d*x^3 - 40*a*b^2*c*d^2*x^3 + 2*a^2*b*d^3*x^3 - 20*b^3*c*d^2*x^6 + 22*a*b^2*d^3*x^6 + 14*b^3*d^3*x^9))/(140*b^2*d^4) - (c^2*(b*c - a*d)^(4/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*d^(13/3)) + (c^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(3*d^(13/3)) - (c^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/(6*d^(13/3)))
```

fricas [A] time = 0.56, size = 369, normalized size = 1.47

$$\frac{140\sqrt{3}(b^3c^3-ad^2cd)\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(b^3c^3-ad^2cd)\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}-\sqrt{3}(bc-a)}{3(b^3c^3-ad^2cd)\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}}\right)+70(b^3c^3-ad^2cd)\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}\log\left((b^3+a)^{\frac{1}{3}}+(b^3+a)^{\frac{1}{3}}\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}+\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}\right)-140(b^3c^3-ad^2cd)\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}\log\left((b^3+a)^{\frac{1}{3}}-\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}\right)+3(14b^3d^3x^9-2(10b^3cd^2-11ab^2cd-6a^2d^2+35b^3cd-40ab^2cd+2a^2bd^3)x^6-140b^3c^3+175a*b^2*c^2*d-20a^2*b*c*d^2-6a^3*d^3+(35b^3c^2*d-40a*b^2*c*d^2+2a^2*b*d^3)x^3)(b^3+a)^{\frac{1}{3}}}{420b^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/420*(140*sqrt(3)*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 70*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*log(((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 140*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) + 3*(14*b^3*d^3*x^9 - 2*(10*b^3*c*d^2 - 11*a*b^2*d^3)*x^6 - 140*b^3*c^3 + 175*a*b^2*c^2*d - 20*a^2*b*c*d^2 - 6*a^3*d^3 + (35*b^3*c^2*d - 40*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^3)*(b*x^3 + a)^(1/3))/(b^2*d^4)
```

giac [A] time = 0.30, size = 394, normalized size = 1.57

$$\frac{(b^3c^3-2ab^2cd^2+ab^2cd^3)\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}\log\left(\frac{\sqrt{3}(b^3c^3-ad^2cd)\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}-\sqrt{3}(bc-a)}{3(b^3c^3-ad^2cd)\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}}\right)+70(b^3c^3-ad^2cd)\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}\log\left((b^3+a)^{\frac{1}{3}}+(b^3+a)^{\frac{1}{3}}\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}+\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}\right)-140(b^3c^3-ad^2cd)\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}\log\left((b^3+a)^{\frac{1}{3}}-\left(\frac{b^2c^2}{20}\right)^{\frac{1}{3}}\right)+3(14b^3d^3x^9-2(10b^3cd^2-11ab^2cd-6a^2d^2+35b^3cd-40ab^2cd+2a^2bd^3)x^6-140b^3c^3+175a*b^2*c^2*d-20a^2*b*c*d^2-6a^3*d^3+(35b^3c^2*d-40a*b^2*c*d^2+2a^2*b*d^3)x^3)(b^3+a)^{\frac{1}{3}}}{140b^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^24*c^4*d^6 - 2*a*b^23*c^3*d^7 + a^2*b^22*c^2*d^8)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - ((b*c - a*d)/d)^(1/3)))/(b^23*c*d^10 - a*b^22*d^11) + 1/3*sqrt(3)*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^5 + 1/6*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^21*c^3*d^6 - 35*(b*x^3 + a)^(4/3)*b^20*c^2*d^7 - 140*(b*x^3 + a)^(1/3)*a*b^20*c^2*d^7 + 20*(b*x^3 + a)^(7/3)*b^19*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^18*d^9 + 20*(b*x^3 + a)^(7/3)*a*b^18*d^9)/(b^20*d^10)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}} x^8}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.12, size = 477, normalized size = 1.90

$$\left(\frac{x^2}{4b^2d} + \frac{(2a + \frac{3c}{4b^2d})\sqrt{c - a^2/d}}{4b^2d}\right) (bx^3 + a)^{1/3} - \left(\frac{2a}{7b^2d} + \frac{b^3c - a^2/d}{7b^2d}\right) (bx^3 + a)^{7/3} + \frac{(bx^3 + a)^{10/3}}{10b^2d} + \frac{c^2 \ln\left(\frac{(bx^3 + a)^{1/3} \sqrt{c - a^2/d} - (bx^3 + a)^{1/3} \sqrt{c - a^2/d}}{2}\right)}{3b^2d} - \frac{c^2 \ln\left(\frac{(bx^3 + a)^{1/3} \sqrt{c - a^2/d} + (bx^3 + a)^{1/3} \sqrt{c - a^2/d}}{2}\right)}{3b^2d} + \frac{c^2 \ln\left(\frac{(bx^3 + a)^{1/3} \sqrt{c - a^2/d} - (bx^3 + a)^{1/3} \sqrt{c - a^2/d}}{2}\right)}{3b^2d} - \frac{c^2 \ln\left(\frac{(bx^3 + a)^{1/3} \sqrt{c - a^2/d} + (bx^3 + a)^{1/3} \sqrt{c - a^2/d}}{2}\right)}{3b^2d} + \frac{c^2 \ln\left(\frac{(bx^3 + a)^{1/3} \sqrt{c - a^2/d} - (bx^3 + a)^{1/3} \sqrt{c - a^2/d}}{2}\right)}{3b^2d} - \frac{c^2 \ln\left(\frac{(bx^3 + a)^{1/3} \sqrt{c - a^2/d} + (bx^3 + a)^{1/3} \sqrt{c - a^2/d}}{2}\right)}{3b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] (a^2/(4*b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*b^2*d))/(4*b^2*d))*(a + b*x^3)^(4/3) - ((2*a)/(7*b^2*d) + (b^3*c - a*b^2*d)/(7*b^4*d^2))*(a + b*x^3)^(7/3) + (a + b*x^3)^(10/3)/(10*b^2*d) + (c^2*log(
```

$$\begin{aligned} & (3*(a + b*x^3)^{(1/3)}*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))/d^2 - (c^2*(a*d \\ & - b*c)^{(4/3)}*(9*a*d^3 - 9*b*c*d^2))/(3*d^{(13/3)})*(a*d - b*c)^{(4/3)}/(3*d^{(13/3)}) \\ & - ((a^2/(b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*b^2*d))/(b^2*d)) \\ & *(a + b*x^3)^{(1/3)}*(b^3*c - a*b^2*d))/(b^2*d) - (c^2*\log((3*c^2*((3^{(1/2)}*1i)/2 + 1/2)*(a*d - b*c)^{(7/3)})/d^{(7/3)} + (3*c^2*(a \\ & + b*x^3)^{(1/3)}*(a*d - b*c)^2)/d^2)*((3^{(1/2)}*1i)/2 + 1/2)*(a*d - b*c)^{(4/3)})/(3*d^{(13/3)}) \\ & + (c^2*\log((3*c^2*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2)/d^2 - (9*c^2*((3^{(1/2)}*1i)/6 - 1/6)*(a*d - b*c)^{(7/3)})/d^{(7/3)})*((3^{(1/2)}*1i)/6 - 1/6)*(a*d - b*c)^{(4/3)})/d^{(13/3)} \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(b*x**3+a)**(4/3)/(d*x**3+c),x)

[Out] Timed out

$$3.443 \quad \int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3} dx$$

Optimal. Leaf size=211

$$\frac{c(bc-ad)^{4/3} \log(c+dx^3)}{6d^{10/3}} - \frac{c(bc-ad)^{4/3} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{10/3}} + \frac{c(bc-ad)^{4/3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}} + \dots$$

Rubi [A] time = 0.24, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 80, 50, 58, 617, 204, 31}

$$-\frac{c(a+bx^3)^{4/3}}{4d^2} + \frac{c\sqrt[3]{a+bx^3}(bc-ad)}{d^3} + \frac{c(bc-ad)^{4/3} \log(c+dx^3)}{6d^{10/3}} - \frac{c(bc-ad)^{4/3} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{10/3}} + \frac{c(bc-ad)^{4/3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}} + \frac{(a+bx^3)^{7/3}}{7bd}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3), x]

[Out] (c*(b*c - a*d)*(a + b*x^3)^(1/3))/d^3 - (c*(a + b*x^3)^(4/3))/(4*d^2) + (a + b*x^3)^(7/3)/(7*b*d) + (c*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d))/Sqrt[3]]/(Sqrt[3]*d^(10/3)) + (c*(b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*d^(10/3)) - (c*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m+1)*(c + d*x)^n)/(b*(m+n+1)), x] + Dist[(n*(b*c - a*d))/(b*(m+n+1)), Int[(a + b*x)^m*(c + d*x)^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 58

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^

2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (a + bx^3)^{4/3}}{c + dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(a + bx)^{4/3}}{c + dx} dx, x, x^3 \right) \\
&= \frac{(a + bx^3)^{7/3}}{7bd} - \frac{c \text{Subst} \left(\int \frac{(a+bx)^{4/3}}{c+dx} dx, x, x^3 \right)}{3d} \\
&= -\frac{c(a + bx^3)^{4/3}}{4d^2} + \frac{(a + bx^3)^{7/3}}{7bd} + \frac{(c(bc - ad)) \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{c+dx} dx, x, x^3 \right)}{3d^2} \\
&= \frac{c(bc - ad) \sqrt[3]{a + bx^3}}{d^3} - \frac{c(a + bx^3)^{4/3}}{4d^2} + \frac{(a + bx^3)^{7/3}}{7bd} - \frac{(c(bc - ad)^2) \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3d^3} \\
&= \frac{c(bc - ad) \sqrt[3]{a + bx^3}}{d^3} - \frac{c(a + bx^3)^{4/3}}{4d^2} + \frac{(a + bx^3)^{7/3}}{7bd} + \frac{c(bc - ad)^{4/3} \log(c + dx^3)}{6d^{10/3}} - \frac{c(bc - ad)^2 \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3d^3} \\
&= \frac{c(bc - ad) \sqrt[3]{a + bx^3}}{d^3} - \frac{c(a + bx^3)^{4/3}}{4d^2} + \frac{(a + bx^3)^{7/3}}{7bd} + \frac{c(bc - ad)^{4/3} \log(c + dx^3)}{6d^{10/3}} - \frac{c(bc - ad)^2 \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3d^3} \\
&= \frac{c(bc - ad) \sqrt[3]{a + bx^3}}{d^3} - \frac{c(a + bx^3)^{4/3}}{4d^2} + \frac{(a + bx^3)^{7/3}}{7bd} + \frac{c(bc - ad)^{4/3} \log(c + dx^3)}{6d^{10/3}} - \frac{c(bc - ad)^2 \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3d^3} \\
&= \frac{c(bc - ad) \sqrt[3]{a + bx^3}}{d^3} - \frac{c(a + bx^3)^{4/3}}{4d^2} + \frac{(a + bx^3)^{7/3}}{7bd} + \frac{c(bc - ad)^{4/3} \tan^{-1} \left(\frac{1 - 2 \sqrt[3]{a} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}} \right)}{\sqrt{3} d^{10/3}}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 255, normalized size = 1.21

$$\frac{c(bc - ad) \left(\sqrt[3]{bc - ad} \log \left(-\sqrt[3]{a} \sqrt[3]{a + bx^3} \sqrt[3]{bc - ad} + (bc - ad)^{2/3} + d^{2/3} (a + bx^3)^{2/3} \right) - 2\sqrt[3]{bc - ad} \log \left(\sqrt[3]{bc - ad} + \sqrt[3]{a} \sqrt[3]{a + bx^3} \right) - 2\sqrt{3} \sqrt[3]{bc - ad} \tan^{-1} \left(\frac{2\sqrt[3]{a} \sqrt[3]{a+bx^3} - 1}{\sqrt[3]{bc-ad}} \right) + 6\sqrt[3]{a} \sqrt[3]{a + bx^3} \right)}{6d^{10/3}} - \frac{c(a + bx^3)^{4/3}}{4d^2} + \frac{(a + bx^3)^{7/3}}{7bd}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3), x]

[Out] -1/4*(c*(a + b*x^3)^(4/3))/d^2 + (a + b*x^3)^(7/3)/(7*b*d) + (c*(b*c - a*d) * (6*d^(1/3)*(a + b*x^3)^(1/3) - 2*sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)]/sqrt[3]] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/(6*d^(10/3))

IntegrateAlgebraic [A] time = 0.27, size = 278, normalized size = 1.32

$$\frac{\sqrt{a+bx^3} (4a^2d^2 - 35abcd + 8abd^2x^3 + 28b^2c^2 - 7b^2cdx^3 + 4b^2d^2x^6)}{28bd^3} - \frac{c(bc-ad)^{4/3} \log\left(\frac{\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}}{3d^{1/3}}\right)}{3d^{1/3}} + \frac{c(bc-ad)^{4/3} \log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3} - \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3}\right)}{6d^{1/3}} + \frac{c(bc-ad)^{4/3} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{1/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]

[Out] ((a + b*x^3)^(1/3)*(28*b^2*c^2 - 35*a*b*c*d + 4*a^2*d^2 - 7*b^2*c*d*x^3 + 8*a*b*d^2*x^3 + 4*b^2*d^2*x^6))/(28*b*d^3) + (c*(b*c - a*d)^(4/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*d^(10/3)) - (c*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(10/3)) + (c*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(10/3))

fricas [A] time = 0.43, size = 298, normalized size = 1.41

$$\frac{28\sqrt{3}(b^2c^2 - abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}(b^2c^2 - abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} - \sqrt{3}(bc-ad)}{3(bc-ad)}\right) + 14(b^2c^2 - abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left((b^2c^2 - abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} - (b^2c^2 - abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right) - 28(b^2c^2 - abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left((b^2c^2 - abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right) + 3(4b^2d^2x^6 + 28b^2c^2 - 35abcd + 4a^2d^2 - (7b^2cd - 8abd^2)x^3)(b^2c^2 - abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}{84bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] 1/84*(28*sqrt(3)*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 14*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 28*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) + 3*(4*b^2*d^2*x^6 + 28*b^2*c^2 - 35*a*b*c*d + 4*a^2*d^2 - (7*b^2*c*d - 8*a*b*d^2)*x^3)*(b*x^3 + a)^(1/3)/(b*d^3)

giac [B] time = 0.29, size = 348, normalized size = 1.65

$$\frac{(b^2c^2d^4 - 2abd^2c^2d^4 + a^2b^2cd^4)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left(\left|(b^2c^2 - abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right) - \sqrt{3}(-bcd + ad^2)^{\frac{1}{3}}(bc^2 - acd) \arctan\left(\frac{\sqrt{3}\left(2(b^2c^2 - abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right) - (-bcd + ad^2)^{\frac{1}{3}}(bc^2 - acd) \log\left((b^2c^2 - abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} + (b^2c^2 - abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right) + \frac{(-bcd + ad^2)^{\frac{1}{3}}(bc^2 - acd) \log\left((b^2c^2 - abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^4} + \frac{28(b^2c^2 - abcd)^{\frac{1}{3}}b^2c^2d^4 - 7(b^2c^2 - abcd)^{\frac{1}{3}}b^2cd^4 - 28(b^2c^2 - abcd)^{\frac{1}{3}}abd^2cd^4 + 4(b^2c^2 - abcd)^{\frac{1}{3}}b^2d^4}{28b^2d^4}}{3(b^2c^2 - abcd)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] 1/3*(b^10*c^3*d^4 - 2*a*b^9*c^2*d^5 + a^2*b^8*c*d^6)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-b*c - a*d)/d)^(1/3)))/(b^9*c*d^7 - a*b^8*d^8) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-b*c - a*d)/d)^(1/3))/(-b*c - a*d)/d^4 - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*log((b*x^3 + a)^(2/3) + (b

$x^3 + a)^{1/3} * (-b*c - a*d)/d)^{1/3} + (-b*c - a*d)/d)^{2/3})/d^4 + 1/28$
 $* (28*(b*x^3 + a)^{1/3} * b^8*c^2*d^4 - 7*(b*x^3 + a)^{4/3} * b^7*c*d^5 - 28*(b*$
 $x^3 + a)^{1/3} * a*b^7*c*d^5 + 4*(b*x^3 + a)^{7/3} * b^6*d^6)/(b^7*d^7)$

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}} x^5}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] int(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
 dditional constraints; using the 'assume' command before evaluation *may* h
 elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?

mupad [B] time = 5.06, size = 348, normalized size = 1.65

$$\frac{(bx^3+a)^{7/3}}{7bd} - (bx^3+a)^{4/3} \left(\frac{a}{4bd} + \frac{b^2c-abd}{4b^2d^2} \right) - \frac{c \ln \left(\frac{3(b^2x^3+a)^3 (b^2c-2ab^2d+bd^2) - (ad-bc)^3 (b+d-bc)}{3d^{10/3}} \right) (ad-bc)^{4/3}}{3d^{10/3}} - \frac{c \ln \left(\frac{3(b^2x^3+a)^3 (ad-bc)^2 - 3 \left(\frac{1+\sqrt{3}i}{2} \right) (ad-bc)^{7/3}}{3d^{10/3}} \right) \left(\frac{1+\sqrt{3}i}{2} \right) (ad-bc)^{4/3}}{3d^{10/3}} + \frac{c \ln \left(\frac{3(b^2x^3+a)^3 (ad-bc)^2 + 3 \left(\frac{1+\sqrt{3}i}{2} \right) (ad-bc)^{7/3}}{3d^{10/3}} \right) \left(\frac{1+\sqrt{3}i}{2} \right) (ad-bc)^{4/3}}{3d^{10/3}} + \frac{(bx^3+a)^{1/3} (b^2c-abd) \left(\frac{1+\sqrt{3}i}{2} \right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x)

[Out] $(a + b*x^3)^{7/3}/(7*b*d) - (a + b*x^3)^{4/3}*(a/(4*b*d) + (b^2*c - a*b*d)/$
 $(4*b^2*d^2)) - (c*\log((3*(a + b*x^3)^{1/3}*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2$
 $*d))/d - (c*(a*d - b*c)^{4/3}*(9*a*d^3 - 9*b*c*d^2))/(3*d^{10/3}))* (a*d - b$
 $*c)^{4/3})/(3*d^{10/3}) - (c*\log((3*c*(a + b*x^3)^{1/3}*(a*d - b*c)^2)/d -$
 $(3*c*((3^{1/2}*1i)/2 - 1/2)*(a*d - b*c)^{7/3})/d^{4/3}))* ((3^{1/2}*1i)/2 - 1$
 $/2)*(a*d - b*c)^{4/3})/(3*d^{10/3}) + (c*\log((3*c*(a + b*x^3)^{1/3}*(a*d -$
 $b*c)^2)/d + (3*c*((3^{1/2}*1i)/2 + 1/2)*(a*d - b*c)^{7/3})/d^{4/3}))* ((3^{1/2}$
 $*1i)/2 + 1/2)*(a*d - b*c)^{4/3})/(3*d^{10/3}) + ((a + b*x^3)^{1/3}*(b^2*c$
 $- a*b*d)*(a/(b*d) + (b^2*c - a*b*d)/(b^2*d^2)))/(b*d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b*x**3+a)**(4/3)/(d*x**3+c),x)`

[Out] Timed out

$$3.444 \quad \int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3} dx$$

Optimal. Leaf size=187

$$\frac{(bc-ad)^{4/3} \log(c+dx^3)}{6d^{7/3}} + \frac{(bc-ad)^{4/3} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{7/3}} - \frac{(bc-ad)^{4/3} \tan^{-1}\left(\frac{1-2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{7/3}} - \sqrt[3]{a}$$

Rubi [A] time = 0.21, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {444, 50, 58, 617, 204, 31}

$$-\frac{\sqrt[3]{a+bx^3}(bc-ad)}{d^2} - \frac{(bc-ad)^{4/3} \log(c+dx^3)}{6d^{7/3}} + \frac{(bc-ad)^{4/3} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{7/3}} - \frac{(bc-ad)^{4/3} \tan^{-1}\left(\frac{1-2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{7/3}} + \frac{(a+bx^3)^{4/3}}{4d}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3), x]

[Out] -(((b*c - a*d)*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*d) - ((b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(7/3)) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*d^(7/3)) + ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(7/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m+1)*(c + d*x)^n/(b*(m+n+1)), x] + Dist[(n*(b*c - a*d))/(b*(m+n+1)), Int[(a + b*x)^m*(c + d*x)^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + bx^3)^{4/3}}{c + dx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^{4/3}}{c + dx} dx, x, x^3 \right) \\
&= \frac{(a + bx^3)^{4/3}}{4d} - \frac{(bc - ad) \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{c+dx} dx, x, x^3 \right)}{3d} \\
&= -\frac{(bc - ad) \sqrt[3]{a + bx^3}}{d^2} + \frac{(a + bx^3)^{4/3}}{4d} + \frac{(bc - ad)^2 \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3d^2} \\
&= -\frac{(bc - ad) \sqrt[3]{a + bx^3}}{d^2} + \frac{(a + bx^3)^{4/3}}{4d} - \frac{(bc - ad)^{4/3} \log(c + dx^3)}{6d^{7/3}} + \frac{(bc - ad)^{4/3} \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3d^2} \\
&= -\frac{(bc - ad) \sqrt[3]{a + bx^3}}{d^2} + \frac{(a + bx^3)^{4/3}}{4d} - \frac{(bc - ad)^{4/3} \log(c + dx^3)}{6d^{7/3}} + \frac{(bc - ad)^{4/3} \log \left(\sqrt[3]{bc - ad} \right)}{2d^2} \\
&= -\frac{(bc - ad) \sqrt[3]{a + bx^3}}{d^2} + \frac{(a + bx^3)^{4/3}}{4d} - \frac{(bc - ad)^{4/3} \tan^{-1} \left(\frac{1 - 2 \sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}} \right)}{\sqrt{3} d^{7/3}} - \frac{(bc - ad)^{4/3} \log \left(\sqrt[3]{bc - ad} \right)}{6d^{7/3}}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 232, normalized size = 1.24

$$\frac{(ad - bc) \left(\sqrt[3]{bc - ad} \log \left(-\sqrt[3]{d} \sqrt[3]{a + bx^3} \sqrt[3]{bc - ad} + (bc - ad)^{2/3} + d^{2/3} (a + bx^3)^{2/3} \right) - 2 \sqrt[3]{bc - ad} \log \left(\sqrt[3]{bc - ad} + \sqrt[3]{d} \sqrt[3]{a + bx^3} \right) - 2 \sqrt{3} \sqrt[3]{bc - ad} \tan^{-1} \left(\frac{2 \sqrt[3]{d} \sqrt[3]{a + bx^3} - 1}{\sqrt[3]{bc - ad}} \right) + 6 \sqrt[3]{d} \sqrt[3]{a + bx^3} \right)}{6d^{7/3}} + \frac{(a + bx^3)^{4/3}}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3), x]

[Out] (a + b*x^3)^(4/3)/(4*d) + ((- (b*c) + a*d)*(6*d^(1/3)*(a + b*x^3)^(1/3) - 2* Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3)))/(b*c - a*d)^(1/3)]/Sqrt[3]) - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/(6*d^(7/3))

IntegrateAlgebraic [A] time = 0.24, size = 234, normalized size = 1.25

$$\frac{(bc - ad)^{4/3} \log \left(\sqrt[3]{bc - ad} + \sqrt[3]{d} \sqrt[3]{a + bx^3} \right)}{3d^{7/3}} - \frac{(bc - ad)^{4/3} \log \left(-\sqrt[3]{d} \sqrt[3]{a + bx^3} \sqrt[3]{bc - ad} + (bc - ad)^{2/3} + d^{2/3} (a + bx^3)^{2/3} \right)}{6d^{7/3}} - \frac{(bc - ad)^{4/3} \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2 \sqrt[3]{d} \sqrt[3]{a + bx^3}}{\sqrt{3} \sqrt[3]{bc - ad}} \right)}{\sqrt{3} d^{7/3}} + \frac{\sqrt[3]{a + bx^3} (5ad - 4bc + bdx^3)}{4d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]

[Out] ((a + b*x^3)^(1/3)*(-4*b*c + 5*a*d + b*d*x^3))/(4*d^2) - ((b*c - a*d)^(4/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*d^(7/3)) + ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(7/3)) - ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(7/3))

fricas [A] time = 0.44, size = 246, normalized size = 1.32

$$\frac{4\sqrt{3}(bc-ad)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+2(bc-ad)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-4(bc-ad)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)+3(bdx^3-4bc+5ad)(bx^3+a)^{\frac{1}{3}}}{12d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] 1/12*(4*sqrt(3)*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 4*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) + 3*(b*d*x^3 - 4*b*c + 5*a*d)*(b*x^3 + a)^(1/3))/d^2

giac [A] time = 0.30, size = 297, normalized size = 1.59

$$\frac{\left(\frac{b^2c^2d^2-2abcd^2+a^2d^4}{3(bc-d^2)}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3(bc-d^2)}+\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{3\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}}\right)}{3d^3}+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^3}-\frac{4\left(bx^3+a\right)^{\frac{1}{3}}bcd^2-\left(bx^3+a\right)^{\frac{4}{3}}d^3-4\left(bx^3+a\right)^{\frac{1}{3}}ad^3}{4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] -1/3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^4 - a*d^5) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3)))/(-(b*c - a*d)/d)^(1/3)/d^3 + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/3)*b*c*d^2 - (b*x^3 + a)^(4/3)*d^3 - 4*(b*x^3 + a)^(1/3)*a*d^3)/d^4

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}} x^2}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x)`

[Out] `int(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.72, size = 304, normalized size = 1.63

$$\frac{(b^3+a)^{4/3}}{4d} + \frac{\ln\left((b^3+a)^{1/3}(3a^2d-6abcd+3b^2c^2) - \frac{6d-bc^2(ba^2+3b^2c^2)}{3d^2}\right)(ad-bc)^{4/3}}{3d^{1/3}} + \frac{(b^3+a)^{1/3}(ad-bc)^{4/3}}{d^2} + \frac{\ln\left((b^3+a)^{1/3}(3a^2d-6abcd+3b^2c^2) + \frac{(1-\sqrt{3})\sqrt{ad-bc^2}(ba^2+3b^2c^2)}{3d^2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)(ad-bc)^{4/3}}{3d^{1/3}} + \frac{\ln\left((b^3+a)^{1/3}(3a^2d-6abcd+3b^2c^2) - \frac{(1-\sqrt{3})\sqrt{ad-bc^2}(ba^2+3b^2c^2)}{3d^2}\right)\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)(ad-bc)^{4/3}}{d^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x)`

[Out] $(a + b*x^3)^{4/3}/(4*d) + (\log((a + b*x^3)^{1/3}*(3*a^2*d^2 + 3*b^2*c^2 - 6*a*b*c*d) - ((a*d - b*c)^{4/3}*(9*a*d^3 - 9*b*c*d^2))/(3*d^{7/3}))*((a*d - b*c)^{4/3})/(3*d^{7/3}) + ((a + b*x^3)^{1/3}*(a*d - b*c))/d^2 - (\log((a + b*x^3)^{1/3}*(3*a^2*d^2 + 3*b^2*c^2 - 6*a*b*c*d) + (((3^{1/2}*1i)/2 + 1/2)*(a*d - b*c)^{4/3}*(9*a*d^3 - 9*b*c*d^2))/(3*d^{7/3}))*((3^{1/2}*1i)/2 + 1/2)*(a*d - b*c)^{4/3})/(3*d^{7/3}) + (\log((a + b*x^3)^{1/3}*(3*a^2*d^2 + 3*b^2*c^2 - 6*a*b*c*d) - (((3^{1/2}*1i)/6 - 1/6)*(a*d - b*c)^{4/3}*(9*a*d^3 - 9*b*c*d^2))/d^{7/3}))*((3^{1/2}*1i)/6 - 1/6)*(a*d - b*c)^{4/3})/d^{7/3}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + bx^3)^{\frac{4}{3}}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**3+a)**(4/3)/(d*x**3+c),x)`

[Out] `Integral(x**2*(a + b*x**3)**(4/3)/(c + d*x**3), x)`

$$3.445 \quad \int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)} dx$$

Optimal. Leaf size=261

$$\frac{a^{4/3} \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2c} - \frac{a^{4/3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} - \frac{a^{4/3} \log(x)}{2c} + \frac{(bc-ad)^{4/3} \log(c+dx^3)}{6cd^{4/3}} - \frac{(bc-ad)^{4/3} \log\left(\sqrt[3]{\frac{bc-ad}{a+bx^3}}\right)}{6cd^{4/3}}$$

Rubi [A] time = 0.30, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {446, 84, 156, 57, 617, 204, 31, 58}

$$\frac{a^{4/3} \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2c} - \frac{a^{4/3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} - \frac{a^{4/3} \log(x)}{2c} + \frac{(bc-ad)^{4/3} \log(c+dx^3)}{6cd^{4/3}} - \frac{(bc-ad)^{4/3} \log\left(\sqrt[3]{\frac{bc-ad}{a+bx^3}}\right)}{2cd^{4/3}} + \frac{(bc-ad)^{4/3} \tan^{-1}\left(\frac{1-2\sqrt[3]{\frac{bc-ad}{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{4/3}} + \frac{b\sqrt[3]{a+bx^3}}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(4/3)/(x*(c + d*x^3)), x]

[Out] (b*(a + b*x^3)^(1/3))/d - (a^(4/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*c) + ((b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d))/Sqrt[3]]/(Sqrt[3]*c*d^(4/3)) - (a^(4/3)*Log[x])/(2*c) + ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c*d^(4/3)) + (a^(4/3)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*c) - ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*d^(4/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 58

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 84

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), Int[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x]*(e + f*x)^(p - 2))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]

Rule 156

Int((((e_.) + (f_.)*(x_))^(p_))*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 204

Int(((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^{4/3}}{x(c + dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^{4/3}}{x(c + dx)} dx, x, x^3 \right) \\
&= \frac{b\sqrt[3]{a + bx^3}}{d} + \frac{\text{Subst} \left(\int \frac{a^2 d + b(-bc + 2ad)x}{x(a + bx)^{2/3}(c + dx)} dx, x, x^3 \right)}{3d} \\
&= \frac{b\sqrt[3]{a + bx^3}}{d} + \frac{a^2 \text{Subst} \left(\int \frac{1}{x(a + bx)^{2/3}} dx, x, x^3 \right)}{3c} - \frac{(bc - ad)^2 \text{Subst} \left(\int \frac{1}{(a + bx)^{2/3}(c + dx)} dx, x, x^3 \right)}{3cd} \\
&= \frac{b\sqrt[3]{a + bx^3}}{d} - \frac{a^{4/3} \log(x)}{2c} + \frac{(bc - ad)^{4/3} \log(c + dx^3)}{6cd^{4/3}} - \frac{a^{4/3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{a - x}} dx, x, \sqrt[3]{a + bx^3} \right)}{2c} \\
&= \frac{b\sqrt[3]{a + bx^3}}{d} - \frac{a^{4/3} \log(x)}{2c} + \frac{(bc - ad)^{4/3} \log(c + dx^3)}{6cd^{4/3}} + \frac{a^{4/3} \log \left(\sqrt[3]{a} - \sqrt[3]{a + bx^3} \right)}{2c} - \frac{(bc - ad)^{4/3} \log \left(\sqrt[3]{a} - \sqrt[3]{a + bx^3} \right)}{2c} \\
&= \frac{b\sqrt[3]{a + bx^3}}{d} - \frac{a^{4/3} \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a + bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3}c} + \frac{(bc - ad)^{4/3} \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{d} \sqrt[3]{a + bx^3}}{\sqrt[3]{bc - ad}}}{\sqrt{3}} \right)}{\sqrt{3}cd^{4/3}} - \frac{a^{4/3} \log(x)}{2c} + \frac{(bc - ad)^{4/3} \log(c + dx^3)}{6cd^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.67, size = 331, normalized size = 1.27

$$\frac{-\left(a^{4/3} \left(\log \left(a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^3} + (a + bx^3)^{2/3} \right) - 2 \log \left(\sqrt[3]{a} - \sqrt[3]{a + bx^3} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{\frac{2\sqrt[3]{a + bx^3}}{\sqrt[3]{a}} + 1}{\sqrt{3}} \right) \right) + \frac{(bc - ad)^{4/3} \log \left(\sqrt[3]{bc - ad} \log \left(-\sqrt[3]{d} \sqrt[3]{a + bx^3} \sqrt[3]{bc - ad} + (bc - ad)^{2/3} + a^{2/3} (a + bx^3)^{2/3} \right) - 2 \sqrt[3]{bc - ad} \log \left(\sqrt[3]{bc - ad} + \sqrt[3]{d} \sqrt[3]{a + bx^3} \right) - 2\sqrt{3} \sqrt[3]{bc - ad} \tan^{-1} \left(\frac{\frac{2\sqrt[3]{d} \sqrt[3]{a + bx^3}}{\sqrt[3]{bc - ad}} - 1}{\sqrt{3}} \right) + 6 \sqrt[3]{d} \sqrt[3]{a + bx^3}}{\sqrt{3}cd^{4/3}}}{6c} + 6a\sqrt[3]{a + bx^3}}{6c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(4/3)/(x*(c + d*x^3)), x]

[Out] (6*a*(a + b*x^3)^(1/3) - a^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3)))/a^(1/3)]/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]) + ((b*c - a*d)*(6*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3)))/(b*c - a*d)]/Sqrt[3]] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/d^(4/3)/(6*c)

IntegrateAlgebraic [A] time = 0.65, size = 349, normalized size = 1.34

$$\frac{a^{4/3} \log \left(\sqrt[3]{a + bx^3} - \sqrt[3]{a} \right) - a^{4/3} \log \left(a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^3} + (a + bx^3)^{2/3} \right) - a^{4/3} \tan^{-1} \left(\frac{2\sqrt[3]{a + bx^3}}{\sqrt{3}\sqrt[3]{a}} + \frac{1}{\sqrt{3}} \right) - (bc - ad)^{4/3} \log \left(\sqrt[3]{bc - ad} + \sqrt[3]{d} \sqrt[3]{a + bx^3} \right) - (bc - ad)^{4/3} \log \left(-\sqrt[3]{d} \sqrt[3]{a + bx^3} \sqrt[3]{bc - ad} + (bc - ad)^{2/3} + a^{2/3} (a + bx^3)^{2/3} \right) + (bc - ad)^{4/3} \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{d} \sqrt[3]{a + bx^3}}{\sqrt{3}\sqrt[3]{bc - ad}}}{\sqrt{3}} \right) + b\sqrt[3]{a + bx^3}}{\sqrt{3}cd^{4/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]

[Out] (b*(a + b*x^3)^(1/3))/d - (a^(4/3)*ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*c) + ((b*c - a*d)^(4/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*c*d^(4/3)) + (a^(4/3)*Log[-a^(1/3) + (a + b*x^3)^(1/3)]/(3*c) - ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(3*c*d^(4/3)) - (a^(4/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(6*c) + ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/(6*c*d^(4/3))

fricas [A] time = 0.45, size = 320, normalized size = 1.23

$$\frac{2\sqrt{3}a^{\frac{1}{3}}d\arctan\left(\frac{2\sqrt{3}(b^2+a^2)^{\frac{1}{3}}+a^{\frac{1}{3}}\sqrt{3}}{3a}\right) + a^{\frac{1}{3}}d\log\left((bx^3+a)^{\frac{1}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{1}{3}}\right) - 2a^{\frac{1}{3}}d\log\left((bx^3+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right) - 2\sqrt{3}(bc-ad)\left(\frac{bc-ad}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(b^2+a^2)^{\frac{1}{3}}(\frac{bc-ad}{a^2}) - \sqrt{3}(bc-ad)}{3(bc-ad)}\right) - 6(bx^3+a)^{\frac{1}{3}}bc - (bc-ad)\left(\frac{bc-ad}{a^2}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}} - (bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{a^2}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{a^2}\right)^{\frac{1}{3}}\right) + 2(bc-ad)\left(\frac{bc-ad}{a^2}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}} + \left(\frac{bc-ad}{a^2}\right)^{\frac{1}{3}}\right)}{6cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="fricas")

[Out] -1/6*(2*sqrt(3)*a^(4/3)*d*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) + sqrt(3)*a)/a) + a^(4/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(4/3)*d*log((b*x^3 + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*(b*c - a*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d) - 6*(b*x^3 + a)^(1/3)*b*c - (b*c - a*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) + 2*(b*c - a*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3))/(c*d)

giac [A] time = 0.84, size = 357, normalized size = 1.37

$$\frac{\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}(b^2+a^2)^{\frac{1}{3}}+a^{\frac{1}{3}}}{3a^{\frac{1}{3}}}\right) - a^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{1}{3}}\right) + a^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right) + \frac{(b^2c^2-2abcd+a^2d^2)\left(\frac{bc-ad}{a^2}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}} - \left(\frac{bc-ad}{a^2}\right)^{\frac{1}{3}}\right)}{3(bc^2-ad^2)} + \frac{(b^2c^2-2abcd+a^2d^2)\left(\frac{bc-ad}{a^2}\right)^{\frac{1}{3}}}{d} + \frac{\sqrt{3}(-bc^2+ad^2)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}(b^2+a^2)^{\frac{1}{3}}(\frac{bc-ad}{a^2})}{3(\frac{bc-ad}{a^2})^{\frac{1}{3}}}\right) - (-bc^2+ad^2)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{1}{3}} + (bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{a^2}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{a^2}\right)^{\frac{1}{3}}\right)}{3ad^2}}{6cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="giac")

[Out] -1/3*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/c - 1/6*a^(4/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/c + 1/3*a^(4/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/c + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - ((b*c - a*d)/d)^(1/3)))/(b*c^2*d - a*c*d^2) + (b*x^3 + a)^(1/3)*b/d - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-b*c - a*d)/d)^(1/3))/(-b*c - a*d)/d^(1/3)/(c*d^2) - 1/

$6*(-b*c*d^2 + a*d^3)^{(1/3)}*(b*c - a*d)*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(-(b*c - a*d)/d)^{(1/3)} + (-(b*c - a*d)/d)^{(2/3)})/(c*d^2)$

maple [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(4/3)/x/(d*x^3+c),x)

[Out] int((b*x^3+a)^(4/3)/x/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x), x)

mupad [B] time = 6.08, size = 796, normalized size = 3.05

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(4/3)/(x*(c + d*x^3)),x)

[Out] $\log(c*d*(-(a*d - b*c)^4/(c^3*d^4))^{(1/3)} + a*d*(a + b*x^3)^{(1/3)} - b*c*(a + b*x^3)^{(1/3)})*(-(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^3*d^4))^{(1/3)} + \log(c*(a^4/c^3)^{(1/3)} - a*(a + b*x^3)^{(1/3)})*(a^4/(27*c^3))^{(1/3)} + (b*(a + b*x^3)^{(1/3)})/d - \log(c*(a^4/c^3)^{(1/3)} + 2*a*(a + b*x^3)^{(1/3)} + 3^{(1/2)}*c*(a^4/c^3)^{(1/3)*1i})*((3^{(1/2)*1i})/2 + 1/2)*(a^4/(27*c^3))^{(1/3)} + \log(c*(a^4/c^3)^{(1/3)*1i} + a*(a + b*x^3)^{(1/3)})*2i + 3^{(1/2)}*c*(a^4/c^3)^{(1/3)})*((3^{(1/2)*1i})/2 - 1/2)*(a^4/(27*c^3))^{(1/3)} + \log((3*a^2*b^4*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(2*a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + 3*a^2*b^4*c*((3^{(1/2)*1i})/2 - 1/2)*(-(a*d - b*c)^4/(c^3*d^4))^{(1/3)}*(2*a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 6*a^4*b*c*d^4))*((3^{(1/2)*1i})/2 - 1/2)*(-(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d$

$$- 4a^3bc^3d^3/(27c^3d^4)^{1/3} - \log((3a^2b^4(a + bx^3)^{1/3}(ad - bc)^2(2a^4d^4 + b^4c^4 + 6a^2b^2c^2d^2 - 4ab^3c^3d - 4a^3b^2c^2d^3))/d - 3a^2b^4c((3^{1/2}i)/2 + 1/2)*(-(ad - bc)^4/(c^3d^4))^{1/3}(2a^5d^5 - b^5c^5 - 10a^2b^3c^3d^2 + 10a^3b^2c^2d^3 + 5a^4b^2c^4d - 6a^4b^2c^4d^2 - 4a^4b^3c^3d - 4a^3b^2c^3d^2)/(27c^3d^4))^{1/3})$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{\frac{4}{3}}}{x(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(4/3)/x/(d*x**3+c), x)

[Out] Integral((a + b*x**3)**(4/3)/(x*(c + d*x**3)), x)

$$3.446 \quad \int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)} dx$$

Optimal. Leaf size=399

$$\frac{d(a+bx^3)^{4/3}}{4c^2} + \frac{(a+bx^3)^{4/3}(4bc-3ad)}{12ac^2} + \frac{\sqrt[3]{a+bx^3}(4bc-3ad)}{3c^2} - \frac{\sqrt[3]{a+bx^3}(bc-ad)}{c^2} - \frac{(bc-ad)^{4/3} \log(c+dx^3)}{6c^2 \sqrt[3]{d}}$$

Rubi [A] time = 0.48, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {446, 103, 156, 50, 57, 617, 204, 31, 58}

$$\frac{d(a+bx^3)^{4/3}}{4c^2} + \frac{(a+bx^3)^{4/3}(4bc-3ad)}{12ac^2} + \frac{\sqrt[3]{a+bx^3}(4bc-3ad)}{3c^2} - \frac{\sqrt[3]{a+bx^3}(bc-ad)}{c^2} - \frac{(bc-ad)^{4/3} \log(c+dx^3)}{6c^2 \sqrt[3]{d}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]

[Out] ((4*b*c - 3*a*d)*(a + b*x^3)^(1/3))/(3*c^2) - ((b*c - a*d)*(a + b*x^3)^(1/3))/c^2 + (d*(a + b*x^3)^(4/3))/(4*c^2) + ((4*b*c - 3*a*d)*(a + b*x^3)^(4/3))/(12*a*c^2) - (a + b*x^3)^(7/3)/(3*a*c*x^3) - (a^(1/3)*(4*b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*c^2) - ((b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sqrt[3]*c^2*d^(1/3)) - (a^(1/3)*(4*b*c - 3*a*d)*Log[x])/(6*c^2) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^2*d^(1/3)) + (a^(1/3)*(4*b*c - 3*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*c^2) + ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*d^(1/3))

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[
{q = Rt[-(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x]]) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[(((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

$b*c - a*d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 617

$\text{Int}[(a_ + (b_)*(x_ + (c_)*(x_)^2)^{-1}), x_Symbol] \text{ :> With}[\{q = 1 - 4*S$
 $\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b$
 $], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \mid\mid \text{!RationalQ}[b^2 - 4*a*c]) /; \text{Free}$
 $\text{Q}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^{4/3}}{x^4(c + dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^{4/3}}{x^2(c + dx)} dx, x, x^3 \right) \\ &= -\frac{(a + bx^3)^{7/3}}{3acx^3} - \frac{\text{Subst} \left(\int \frac{(a+bx)^{4/3} \left(\frac{1}{3}(-4bc+3ad) - \frac{4bdx}{3} \right)}{x(c+dx)} dx, x, x^3 \right)}{3ac} \\ &= -\frac{(a + bx^3)^{7/3}}{3acx^3} + \frac{d^2 \text{Subst} \left(\int \frac{(a+bx)^{4/3}}{c+dx} dx, x, x^3 \right)}{3c^2} + \frac{(4bc - 3ad) \text{Subst} \left(\int \frac{(a+bx)^{4/3}}{x} dx, x, x^3 \right)}{9ac^2} \\ &= \frac{d(a + bx^3)^{4/3}}{4c^2} + \frac{(4bc - 3ad)(a + bx^3)^{4/3}}{12ac^2} - \frac{(a + bx^3)^{7/3}}{3acx^3} + \frac{(4bc - 3ad) \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{x} dx, x, x^3 \right)}{9c^2} \\ &= \frac{(4bc - 3ad)\sqrt[3]{a + bx^3}}{3c^2} - \frac{(bc - ad)\sqrt[3]{a + bx^3}}{c^2} + \frac{d(a + bx^3)^{4/3}}{4c^2} + \frac{(4bc - 3ad)(a + bx^3)^{4/3}}{12ac^2} - \frac{(a + bx^3)^{7/3}}{3acx^3} \\ &= \frac{(4bc - 3ad)\sqrt[3]{a + bx^3}}{3c^2} - \frac{(bc - ad)\sqrt[3]{a + bx^3}}{c^2} + \frac{d(a + bx^3)^{4/3}}{4c^2} + \frac{(4bc - 3ad)(a + bx^3)^{4/3}}{12ac^2} - \frac{(a + bx^3)^{7/3}}{3acx^3} \\ &= \frac{(4bc - 3ad)\sqrt[3]{a + bx^3}}{3c^2} - \frac{(bc - ad)\sqrt[3]{a + bx^3}}{c^2} + \frac{d(a + bx^3)^{4/3}}{4c^2} + \frac{(4bc - 3ad)(a + bx^3)^{4/3}}{12ac^2} - \frac{(a + bx^3)^{7/3}}{3acx^3} \end{aligned}$$

Mathematica [A] time = 1.55, size = 389, normalized size = 0.97

$$\frac{(4bc-3ad)\left(-\frac{1}{2}a^3\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}-1}{\sqrt{3}}\right)\right)+3a\sqrt[3]{a+bx^3}+\frac{2}{3}(a+bx^3)^{4/3}\right)}{3c} + \frac{a\left(3a^{4/3}(a+bx^3)^{4/3}-2(bc-ad)\left(\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{a}-\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+(a+bx^3)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{a+bx^3}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}-1}{\sqrt{3}}\right)\right)+6\sqrt[3]{a+bx^3}\right)}{4c\sqrt[3]{d}} - \frac{(a+bx^3)^{7/3}}{3a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)), x]

[Out] $-\left(\frac{(a + b*x^3)^{7/3}}{x^3}\right) + \left(\frac{(4*b*c - 3*a*d)*(3*a*(a + b*x^3)^{1/3} + (3*(a + b*x^3)^{4/3}))/4 - (a^{4/3}*(2*\sqrt{3}*ArcTan[(1 + (2*(a + b*x^3)^{1/3}))/a^{1/3}])/sqrt{3} - 2*Log[a^{1/3} - (a + b*x^3)^{1/3}] + Log[a^{2/3} + a^{1/3}*(a + b*x^3)^{1/3} + (a + b*x^3)^{2/3}]}{2}\right)/(3*c) + \left(\frac{a*(3*d^{4/3}*(a + b*x^3)^{4/3} - 2*(b*c - a*d)*(6*d^{1/3}*(a + b*x^3)^{1/3} + (b*c - a*d)^{1/3}*(-2*\sqrt{3}*ArcTan[(-1 + (2*d^{1/3}*(a + b*x^3)^{1/3}))/b*c - a*d]^{1/3}))/sqrt{3} - 2*Log[(b*c - a*d)^{1/3} + d^{1/3}*(a + b*x^3)^{1/3}] + Log[(b*c - a*d)^{2/3} - d^{1/3}*(b*c - a*d)^{1/3}*(a + b*x^3)^{1/3} + d^{2/3}*(a + b*x^3)^{2/3}]}{4*c*d^{1/3}}\right)/(3*a*c)$

IntegrateAlgebraic [A] time = 0.77, size = 398, normalized size = 1.00

$$\frac{(4\sqrt[3]{bc}-3a^3d)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{9c^2} + \frac{(3a^3d-4\sqrt[3]{abc})\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{18c^2} + \frac{(3a^3d-4\sqrt[3]{abc})\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}-1}{\sqrt{3}}\right)}{3\sqrt{3}c^2} - \frac{(bc-ad)^3\log\left(-\sqrt[3]{a}-\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+(a+bx^3)^{2/3}\right)}{6c^2\sqrt[3]{d}} + \frac{(bc-ad)^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}\right)}{3c^2\sqrt[3]{d}} - \frac{(bc-ad)^3\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx^3}-1}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{d}} - \frac{a\sqrt[3]{a+bx^3}}{3c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)), x]

[Out] $-1/3*(a*(a + b*x^3)^{1/3})/(c*x^3) + ((-4*a^{1/3}*b*c + 3*a^{4/3}*d)*ArcTan[(a^{1/3} + 2*(a + b*x^3)^{1/3})/(sqrt{3}*a^{1/3})])/(3*sqrt{3}*c^2) - ((b*c - a*d)^{4/3}*ArcTan[1/sqrt{3} - (2*d^{1/3}*(a + b*x^3)^{1/3})/(sqrt{3}*(b*c - a*d)^{1/3})])/(sqrt{3}*c^2*d^{1/3}) + ((4*a^{1/3}*b*c - 3*a^{4/3}*d)*Log[-a^{1/3} + (a + b*x^3)^{1/3}])/(9*c^2) + ((b*c - a*d)^{4/3}*Log[(b*c - a*d)^{1/3} + d^{1/3}*(a + b*x^3)^{1/3}])/(3*c^2*d^{1/3}) + ((-4*a^{1/3}*b*c + 3*a^{4/3}*d)*Log[a^{2/3} + a^{1/3}*(a + b*x^3)^{1/3} + (a + b*x^3)^{2/3}])/(18*c^2) - ((b*c - a*d)^{4/3}*Log[(b*c - a*d)^{2/3} - d^{1/3}*(b*c - a*d)^{1/3}*(a + b*x^3)^{1/3} + d^{2/3}*(a + b*x^3)^{2/3}])/(6*c^2*d^{1/3})$

fricas [A] time = 0.70, size = 383, normalized size = 0.96

$$\frac{6\sqrt{3}(bc-ad)^3\arctan\left(\frac{2\sqrt[3]{a+bx^3}-1}{\sqrt{3}}\right)+2\sqrt{3}(4bc-3ad)(c-a)^3\arctan\left(\frac{2\sqrt[3]{a+bx^3}-1}{\sqrt{3}}\right)+(4bc-3ad)(c-a)^3\log\left((b^2+a)^3-(b^2+a)^2(c-a)^3+(c-a)^3\right)+3(bc-ad)^3\log\left((b^2+a)^3+(b^2+a)^2\left(\frac{2a}{c}\right)+\left(\frac{2a}{c}\right)^3\right)-2(4bc-3ad)(c-a)^3\log\left((b^2+a)^3+(b^2+a)^2\left(\frac{2a}{c}\right)+\left(\frac{2a}{c}\right)^3\right)-2(4bc-3ad)(c-a)^3\log\left((b^2+a)^3+(b^2+a)^2\left(\frac{2a}{c}\right)+\left(\frac{2a}{c}\right)^3\right)-6(bc-ad)^3\log\left((b^2+a)^3-\left(\frac{2a}{c}\right)^3\right)-6(b^2+a)^3c}{18c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c), x, algorithm="fricas")

[Out] $1/18*(6*sqrt{3}*(b*c - a*d)*x^3*(-(b*c - a*d)/d)^{1/3}*arctan(-1/3*(2*sqrt{3}*(b*x^3 + a)^{1/3}*d*(-(b*c - a*d)/d)^{2/3} - sqrt{3}*(b*c - a*d))/(b*c -$

$$a*d)) + 2*\sqrt{3}*(4*b*c - 3*a*d)*(-a)^{(1/3)}*x^3*\arctan(1/3*(2*\sqrt{3})*(b*x^3 + a)^{(1/3)}*(-a)^{(2/3)} + \sqrt{3}*a)/a) + (4*b*c - 3*a*d)*(-a)^{(1/3)}*x^3*\log((b*x^3 + a)^{(2/3)} - (b*x^3 + a)^{(1/3)}*(-a)^{(1/3)} + (-a)^{(2/3)}) + 3*(b*c - a*d)*x^3*(-(b*c - a*d)/d)^{(1/3)}*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)})*(-(b*c - a*d)/d)^{(1/3)} + ((-b*c - a*d)/d)^{(2/3)}) - 2*(4*b*c - 3*a*d)*(-a)^{(1/3)}*x^3*\log((b*x^3 + a)^{(1/3)} + (-a)^{(1/3)}) - 6*(b*c - a*d)*x^3*(-(b*c - a*d)/d)^{(1/3)}*\log((b*x^3 + a)^{(1/3)} - (-b*c - a*d)/d)^{(1/3)}) - 6*(b*x^3 + a)^{(1/3)}*a*c)/(c^2*x^3)$$

giac [A] time = 0.86, size = 394, normalized size = 0.99

$$\frac{(b^2c - 2abd + a^2d^2)\left(\frac{b^2c - 2abd + a^2d^2}{3(b^2 - a^2d)}\right)^{\frac{1}{3}} \log\left(\left|\frac{(b^2 + a)^2 - \left(\frac{b^2c - 2abd + a^2d^2}{3(b^2 - a^2d)}\right)^{\frac{1}{3}}}{(b^2 + a)^2 - a^2}\right|\right)}{3(b^2 - a^2d)} + \frac{\sqrt{3}(4a^2bc - 3a^2d^2)\arctan\left(\frac{\sqrt{3}\left(\frac{(b^2 + a)^2 - \left(\frac{b^2c - 2abd + a^2d^2}{3(b^2 - a^2d)}\right)^{\frac{1}{3}}}{(b^2 + a)^2 - a^2}\right)}{\frac{b^2c - 2abd + a^2d^2}{3(b^2 - a^2d)}}\right)}{9c^2} + \frac{(4a^2bc - 3a^2d^2)\log\left(\frac{(b^2 + a)^2 + (b^2 + a)^2\left(\frac{b^2c - 2abd + a^2d^2}{3(b^2 - a^2d)}\right)^{\frac{1}{3}}}{(b^2 + a)^2 - a^2}\right)}{18c^2} + \frac{\sqrt{3}(-bc^2 + ad^2)(bc - ad)\arctan\left(\frac{\sqrt{3}\left(\frac{(b^2 + a)^2 - \left(\frac{b^2c - 2abd + a^2d^2}{3(b^2 - a^2d)}\right)^{\frac{1}{3}}}{(b^2 + a)^2 - a^2}\right)}{\frac{b^2c - 2abd + a^2d^2}{3(b^2 - a^2d)}}\right)}{3cd} + \frac{(-bc^2 + ad^2)(bc - ad)\log\left(\frac{(b^2 + a)^2 + (b^2 + a)^2\left(\frac{b^2c - 2abd + a^2d^2}{3(b^2 - a^2d)}\right)^{\frac{1}{3}}}{(b^2 + a)^2 - a^2}\right)}{6cd} + \frac{(4abc - 3a^2d)\log\left(\left|\frac{(b^2 + a)^2 - a^2}{(b^2 + a)^2}\right|\right)}{9a^2c^2} + \frac{(b^2 + a)^2/a}{3ca^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="giac")

[Out]
$$-1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^{(1/3)}*\log(\text{abs}((b*x^3 + a)^{(1/3)} - (-b*c - a*d)/d)^{(1/3)})/(b*c^3 - a*c^2*d) - 1/9*\sqrt{3}*(4*a^{(1/3)}*b*c - 3*a^{(4/3)}*d)*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{(1/3)} + a^{(1/3)})/a^{(1/3)})/c^2 - 1/18*(4*a^{(1/3)}*b*c - 3*a^{(4/3)}*d)*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*a^{(1/3)} + a^{(2/3)})/c^2 + 1/3*\sqrt{3}*(-b*c*d^2 + a*d^3)^{(1/3)}*(b*c - a*d)*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{(1/3)} + (-b*c - a*d)/d)^{(1/3)})/((-b*c - a*d)/d)^{(1/3)}/(c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^{(1/3)}*(b*c - a*d)*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(-b*c - a*d)/d)^{(1/3)} + (-b*c - a*d)/d)^{(2/3)}/(c^2*d) + 1/9*(4*a*b*c - 3*a^2*d)*\log(\text{abs}((b*x^3 + a)^{(1/3)} - a^{(1/3)}))/((a^{(2/3)}*c^2) - 1/3*(b*x^3 + a)^{(1/3)}*a/(c*x^3))$$

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x)

[Out] int((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^4), x)
```

mupad [B] time = 10.88, size = 2047, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x)
```

```
[Out] log(c^2*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3) + 3*a*d*(a + b*x^3)^(1/3) - 4*b*c*(a + b*x^3)^(1/3))*(-(27*a^4*d^3 - 64*a*b^3*c^3 + 144*a^2*b^2*c^2*d - 108*a^3*b*c*d^2)/(729*c^6))^(1/3) + log((((81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))*((a*d - b*c)^4/(c^6*d))^(1/3) - 108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*((a*d - b*c)^4/(c^6*d))^(2/3))/9 + (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3 - (a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4))*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^6*d))^(1/3) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 - 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))*((a*d - b*c)^4/(c^6*d))^(1/3))*((a*d - b*c)^4/(c^6*d))^(2/3))/9 + (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3 - (a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4))*((3^(1/2)*1i)/2 - 1/2)*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^6*d))^(1/3) - log((a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4) - (((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 + 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))*((a*d - b*c)^4/(c^6*d))^(1/3))*((a*d - b*c)^4/(c^6*d))^(2/3))/9 - (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3*((3^(1/2)*1i)/2 + 1/2)*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^6*d))^(1/3) + log((a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4) - (((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 - 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))*
```

$$\begin{aligned}
& -\frac{(a(3ad - 4bc)^3/c^6)^{1/3} \cdot (-\frac{(a(3ad - 4bc)^3/c^6)^{2/3}}{81} + \frac{(ab^5d^2(27a^5d^5 - 27b^5c^5 - 341a^2b^3c^3d^2 + 332a^3b^2c^2d^3 + 162ab^4c^4d - 153a^4b^2c^2d^4))/(3c)) \cdot (-\frac{(a(3ad - 4bc)^3/c^6)^{1/3}}{9})}{(3^{1/2}i)/2 - 1/2} \cdot (-\frac{(27a^4d^3 - 64ab^3c^3 + 144a^2b^2c^2d - 108a^3b^2c^2d - 108a^3b^2c^2d)/(729c^6))^{1/3} - \log((ab^4d^2(a + bx^3))^{1/3} \cdot (ad - bc)^2(54a^5d^5 - 36b^5c^5 - 388a^2b^3c^3d^2 + 450a^3b^2c^2d^3 + 171ab^4c^4d - 252a^4b^2c^2d^4))/(9c^4) - (((3^{1/2}i)/2 + 1/2) \cdot (((3^{1/2}i)/2 - 1/2) \cdot (108ab^5c^3d^3(a + bx^3)^{1/3} \cdot (ad - bc)^2 + 27ab^4c^4d^3 \cdot ((3^{1/2}i)/2 + 1/2) \cdot (2a^2d^2 + b^2c^2 - 3ab^2cd)) \cdot (-\frac{(a(3ad - 4bc)^3/c^6)^{1/3}}{81} - \frac{(ab^5d^2(27a^5d^5 - 27b^5c^5 - 341a^2b^3c^3d^2 + 332a^3b^2c^2d^3 + 162ab^4c^4d - 153a^4b^2c^2d^4))/(3c)) \cdot (-\frac{(a(3ad - 4bc)^3/c^6)^{1/3}}{9})}{(3^{1/2}i)/2 + 1/2} \cdot (-\frac{(27a^4d^3 - 64ab^3c^3 + 144a^2b^2c^2d - 108a^3b^2c^2d)/(729c^6))^{1/3} - (a(a + bx^3)^{1/3})/(3cx^3))}
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{\frac{4}{3}}}{x^4(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(4/3)/x**4/(d*x**3+c), x)

[Out] Integral((a + b*x**3)**(4/3)/(x**4*(c + d*x**3)), x)

$$3.447 \quad \int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)} dx$$

Optimal. Leaf size=440

$$\frac{\sqrt[3]{a+bx^3} (9a^2d^2 - 12abcd + 2b^2c^2)}{9ac^3} + \frac{(9a^2d^2 - 12abcd + 2b^2c^2) \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3} - \frac{(9a^2d^2 - 12abcd + 2b^2c^2)}{9\sqrt[3]{a}}$$

Rubi [A] time = 0.62, antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {446, 103, 149, 156, 50, 57, 617, 204, 31, 58}

$$\frac{\sqrt{a+bx^3} (9a^2d^2 - 12abcd + 2b^2c^2)}{9ac^3} + \frac{(9a^2d^2 - 12abcd + 2b^2c^2) \log\left(\sqrt{a} - \sqrt{a+bx^3}\right)}{18a^{2/3}c^3} - \frac{(9a^2d^2 - 12abcd + 2b^2c^2) \tan^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{9\sqrt[3]{a^3c^3}} + \frac{\log(x) (9a^2d^2 - 12abcd + 2b^2c^2)}{18a^{2/3}c^3} + \frac{d^{2/3}(bc-ad) \log(c+dx^3)}{6c^3} - \frac{d^{2/3}(bc-ad) \log\left(\frac{\sqrt{bc-ad} + \sqrt{d}\sqrt{a+bx^3}}{2c}\right)}{2c^3} + \frac{d^{2/3}(bc-ad) \tan^{-1}\left(\frac{1+\frac{d^2}{3}\frac{a+bx^3}{a}}{\sqrt{3}}\right)}{\sqrt{3}c^3} - \frac{(a+bx^3)^{5/3} (bc-6ad)}{18a^{2/3}c^3} + \frac{d\sqrt{a+bx^3} (bc-ad)}{c^3} - \frac{(a+bx^3)^{5/3}}{6ac^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)), x]

[Out] (d*(b*c - a*d)*(a + b*x^3)^(1/3))/c^3 + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(a + b*x^3)^(1/3))/(9*a*c^3) - ((b*c - 6*a*d)*(a + b*x^3)^(4/3))/(18*a*c^2*x^3) - (a + b*x^3)^(7/3)/(6*a*c*x^6) - ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sqrt[3]*c^3) - ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*Log[x])/(18*a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^3) + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(2/3)*c^3) - (d^(2/3)*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^3)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m+1)*(c + d*x)^n)/(b*(m+n+1)), x] + Dist[(n*(b*c - a*d))/(b*(m+n+1)), Int[(a + b*x)^m*(c + d*x)^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n

+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x]]) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_.*((c_) + (d_.)*(x_)^(n_.))^q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^{4/3}}{x^7(c + dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx)^{4/3}}{x^3(c + dx)} dx, x, x^3 \right) \\
&= \frac{(a + bx^3)^{7/3}}{6acx^6} - \frac{\text{Subst} \left(\int \frac{(a+bx)^{4/3} \left(\frac{1}{3}(-bc+6ad) - \frac{bdx}{3} \right)}{x^2(c+dx)} dx, x, x^3 \right)}{6ac} \\
&= -\frac{(bc - 6ad)(a + bx^3)^{4/3}}{18ac^2x^3} - \frac{(a + bx^3)^{7/3}}{6acx^6} - \frac{\text{Subst} \left(\int \frac{\sqrt[3]{a+bx} \left(-\frac{2}{9}(2b^2c^2 - 12abcd + 9a^2d^2) - \frac{2}{9}bd(2bc - 3ad)x \right)}{x(c+dx)} dx, x, x^3 \right)}{6ac^2} \\
&= -\frac{(bc - 6ad)(a + bx^3)^{4/3}}{18ac^2x^3} - \frac{(a + bx^3)^{7/3}}{6acx^6} + \frac{(d^2(bc - ad)) \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{c+dx} dx, x, x^3 \right)}{3c^3} + \frac{(2b^2c^2 - 12abcd + 9a^2d^2) \sqrt[3]{a + bx^3}}{9ac^3} - \frac{(bc - 6ad)(a + bx^3)^{4/3}}{18ac^2x^3} - \frac{(a + bx^3)^{7/3}}{6acx^6} \\
&= \frac{d(bc - ad) \sqrt[3]{a + bx^3}}{c^3} + \frac{(2b^2c^2 - 12abcd + 9a^2d^2) \sqrt[3]{a + bx^3}}{9ac^3} - \frac{(bc - 6ad)(a + bx^3)^{4/3}}{18ac^2x^3} - \frac{(a + bx^3)^{7/3}}{6acx^6} \\
&= \frac{d(bc - ad) \sqrt[3]{a + bx^3}}{c^3} + \frac{(2b^2c^2 - 12abcd + 9a^2d^2) \sqrt[3]{a + bx^3}}{9ac^3} - \frac{(bc - 6ad)(a + bx^3)^{4/3}}{18ac^2x^3} - \frac{(a + bx^3)^{7/3}}{6acx^6} \\
&= \frac{d(bc - ad) \sqrt[3]{a + bx^3}}{c^3} + \frac{(2b^2c^2 - 12abcd + 9a^2d^2) \sqrt[3]{a + bx^3}}{9ac^3} - \frac{(bc - 6ad)(a + bx^3)^{4/3}}{18ac^2x^3} - \frac{(a + bx^3)^{7/3}}{6acx^6} \\
&= \frac{d(bc - ad) \sqrt[3]{a + bx^3}}{c^3} + \frac{(2b^2c^2 - 12abcd + 9a^2d^2) \sqrt[3]{a + bx^3}}{9ac^3} - \frac{(bc - 6ad)(a + bx^3)^{4/3}}{18ac^2x^3} - \frac{(a + bx^3)^{7/3}}{6acx^6}
\end{aligned}$$

Mathematica [A] time = 1.25, size = 429, normalized size = 0.98

$$\frac{x^6 \left(4(9d^2d^2 - 12abcd + 9a^2d^2) \left(\frac{1}{3} \sqrt[3]{a + bx^3} \left(\log(a^2 + \sqrt{3}\sqrt{a + bx^3} + (a + bx^3)^{2/3}) - 2 \log(\sqrt{3} - \sqrt{a + bx^3}) + 2\sqrt{3} \tan^{-1} \left(\frac{\sqrt{3}\sqrt{a + bx^3}}{3 - \sqrt{a + bx^3}} \right) \right) + 3\sqrt{3}\sqrt{a + bx^3} + \frac{1}{3}(a + bx^3)^{2/3} \right) - 6d^2d^2 \left(3d^2(a + bx^3)^{2/3} - 20(bc - ad) \left(\sqrt{3c - ad} \left(\log(-\sqrt{3}\sqrt{a + bx^3}\sqrt{3c - ad} + (bc - ad)^{3/2} + d^2(a + bx^3)^{2/3}) - 2 \log(\sqrt{3c - ad} + \sqrt{3}\sqrt{a + bx^3}) - 2\sqrt{3} \tan^{-1} \left(\frac{\sqrt{3}\sqrt{a + bx^3}}{3 - \sqrt{a + bx^3}} \right) \right) + 3\sqrt{3}\sqrt{a + bx^3} \right) - 18ac^2(a + bx^3)^{2/3} - ac^2(a + bx^3)^{2/3}(bc - 6ad) \right)}{108c^6x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)), x]

[Out] (-18*a*c^2*(a + b*x^3)^(7/3) - 6*c*(b*c - 6*a*d)*x^3*(a + b*x^3)^(7/3) + x^6*4*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(3*a*(a + b*x^3)^(1/3) + (3*(a + b*x^3)^(4/3)))/4 - (a^(4/3)*(2*sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3)))/a^(1/3)]))

$$\frac{1}{3})/\text{Sqrt}[3]] - 2*\text{Log}[a^{(1/3)} - (a + b*x^3)^{(1/3)}] + \text{Log}[a^{(2/3)} + a^{(1/3)} * (a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}])]/2 - 9*a^2*d^{(2/3)}*(3*d^{(4/3)}*(a + b*x^3)^{(4/3)} - 2*(b*c - a*d)*(6*d^{(1/3)}*(a + b*x^3)^{(1/3)} + (b*c - a*d)^{(1/3)}*(-2*\text{Sqrt}[3]*\text{ArcTan}[(-1 + (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})/(b*c - a*d)^{(1/3)})]/\text{Sqrt}[3]] - 2*\text{Log}[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)}] + \text{Log}[(b*c - a*d)^{(2/3)} - d^{(1/3)}*(b*c - a*d)^{(1/3)}*(a + b*x^3)^{(1/3)} + d^{(2/3)}*(a + b*x^3)^{(2/3)}])))/(108*a^2*c^3*x^6)$$

IntegrateAlgebraic [A] time = 1.07, size = 445, normalized size = 1.01

$$\frac{(9a^2d^2 - 12abcd + 2d^2c^2)\log(\sqrt{a+bx^3} - \sqrt{d})}{27a^2c^2} - \frac{(-9a^2d^2 + 12abcd - 2d^2c^2)\log(\sqrt{a+bx^3} + \sqrt{d})}{54a^2c^2} + \frac{(9a^2d^2 - 12abcd + 2d^2c^2)\tan^{-1}\left(\frac{\sqrt{a+bx^3} - \sqrt{d}}{\sqrt{3}}\right)}{9\sqrt{3}a^2c^2} - \frac{d^{2/3}(bc - ad)^{4/3}\log\left(\frac{\sqrt{bc-ad} + \sqrt{d}\sqrt{a+bx^3}}{3c}\right)}{3c^2} - \frac{d^{2/3}(bc - ad)^{4/3}\log\left(\frac{\sqrt{d}\sqrt{a+bx^3} - \sqrt{bc-ad} + (bc - ad)^{3/2} + d^{3/2}(a+bx^3)^{3/2}}{6c}\right)}{6c^2} + \frac{d^{2/3}(bc - ad)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{3}c^2} + \frac{\sqrt{d+bx^3}(-3bc+6abd^2-7bd^2c^2)}{18c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]

$$[Out] ((a + b*x^3)^{(1/3)}*(-3*a*c - 7*b*c*x^3 + 6*a*d*x^3))/(18*c^2*x^6) - ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*\text{ArcTan}[(a^{(1/3)} + 2*(a + b*x^3)^{(1/3)})/\text{Sqrt}[3]*a^{(1/3)}])/(9*\text{Sqrt}[3]*a^{(2/3)}*c^3) + (d^{(2/3)}*(b*c - a*d)^{(4/3)}*\text{ArcTan}[1/\text{Sqrt}[3] - (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})/\text{Sqrt}[3]*(b*c - a*d)^{(1/3)}])/(9*\text{Sqrt}[3]*c^3) + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*\text{Log}[-a^{(1/3)} + (a + b*x^3)^{(1/3)}])/(27*a^{(2/3)}*c^3) - (d^{(2/3)}*(b*c - a*d)^{(4/3)}*\text{Log}[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)}])/(3*c^3) + ((-2*b^2*c^2 + 12*a*b*c*d - 9*a^2*d^2)*\text{Log}[a^{(2/3)} + a^{(1/3)}*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}])/(54*a^{(2/3)}*c^3) + (d^{(2/3)}*(b*c - a*d)^{(4/3)}*\text{Log}[(b*c - a*d)^{(2/3)} - d^{(1/3)}*(b*c - a*d)^{(1/3)}*(a + b*x^3)^{(1/3)} + d^{(2/3)}*(a + b*x^3)^{(2/3)}])/(6*c^3)$$

fricas [A] time = 2.02, size = 503, normalized size = 1.14

$$\frac{18\sqrt{3}(bc - ad)^{4/3} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{bc-ad}}\right)}{9\sqrt{3}c^2} - \frac{d^{2/3}(bc - ad)^{4/3} \log\left(\frac{\sqrt{bc-ad} + \sqrt{d}\sqrt{a+bx^3}}{3c}\right)}{3c^2} - \frac{d^{2/3}(bc - ad)^{4/3} \log\left(\frac{\sqrt{d}\sqrt{a+bx^3} - \sqrt{bc-ad} + (bc - ad)^{3/2} + d^{3/2}(a+bx^3)^{3/2}}{6c}\right)}{6c^2} + \frac{d^{2/3}(bc - ad)^{4/3} \tan^{-1}\left(\frac{1}{\sqrt{3}}\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{3}c^2} + \frac{\sqrt{d+bx^3}(-3bc+6abd^2-7bd^2c^2)}{18c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="fricas")

$$[Out] \frac{1}{54}*(18*\text{sqrt}(3)*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^{(1/3)}*x^6*\text{arctan}(-1/3*(2*\text{sqrt}(3)*(b*c*d^2 - a*d^3)^{(2/3)}*(b*x^3 + a)^{(1/3)} - \text{sqrt}(3)*(b*c*d - a*d^2))/(b*c*d - a*d^2)) - 2*\text{sqrt}(3)*(2*a*b^2*c^2 - 12*a^2*b*c*d + 9*a^3*d^2)*(a^2)^{(1/6)}*x^6*\text{arctan}(1/3*(a^2)^{(1/6)}*(\text{sqrt}(3)*(a^2)^{(1/3)}*a + 2*\text{sqrt}(3)*(b*x^3 + a)^{(1/3)}*(a^2)^{(2/3)})/a^2) - (2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(a^2)^{(2/3)}*x^6*\log((b*x^3 + a)^{(2/3)}*a + (a^2)^{(1/3)}*a + (b*x^3 + a)^{(1/3)}*(a^2)^{(2/3})) + 2*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(a^2)^{(2/3)}*x^6*\log((b*x^3 + a)^{(1/3)}*a - (a^2)^{(2/3})) + 9*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^{(1/3)}*x^6*\log((b*x^3 + a)^{(2/3)}*d^2 - (b*c*d^2 - a*d^3)^{(1/3)}*(b*x^3 + a)^{(1/3)}*d + (b*c*d^2 - a*d^3)^{(2/3})) - 18*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^{(1/3)}*x^6*\log((b*x^3 + a)^{(1/3)}*d + (b*c*d^2 - a*d^3)^{(1/3})) - 3*(3*a^3*c^2 + (7*a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b*x^3 + a)^{(1/3)})/(a^2*c^3*x^6)$$

giac [A] time = 0.86, size = 481, normalized size = 1.09

$$\frac{(b^2 d - 2 a b c + a^2) \sqrt{\frac{d x^3 + a}{3(b^2 - a^2)}} \log\left(\left|\frac{d x^3 + a}{3(b^2 - a^2)}\right|\right) - \frac{\sqrt{(b^2 - a^2)(b^2 c - a d)} \arctan\left(\frac{\sqrt{\frac{d x^3 + a}{3(b^2 - a^2)}}}{\frac{d x^3 + a}{3(b^2 - a^2)}}\right)}{3 a^2} + \frac{(b^2 + a^2) \log\left(\frac{b^2 + a^2}{b^2 - a^2}\right) \sqrt{\frac{d x^3 + a}{3(b^2 - a^2)}} + (b^2 + a^2) \sqrt{\frac{d x^3 + a}{3(b^2 - a^2)}}}{3 a^2} - \frac{\sqrt{(b^2 - 12 a b c + 9 a^2 d)} \arctan\left(\frac{\sqrt{\frac{d x^3 + a}{3(b^2 - a^2)}}}{\frac{d x^3 + a}{3(b^2 - a^2)}}\right)}{27 a^2} + \frac{(b^2 - 12 a b c + 9 a^2 d) \log\left(\left|\frac{d x^3 + a}{3(b^2 - a^2)}\right|\right)}{27 a^2} - \frac{(b^2 + a^2) \sqrt{\frac{d x^3 + a}{3(b^2 - a^2)}} + (b^2 + a^2) \sqrt{\frac{d x^3 + a}{3(b^2 - a^2)}}}{27 a^2} + \frac{(b^2 - 12 a b c + 9 a^2 d) \log\left(\left|\frac{d x^3 + a}{3(b^2 - a^2)}\right|\right)}{27 a^2} - \frac{(b^2 + a^2) \sqrt{\frac{d x^3 + a}{3(b^2 - a^2)}} + (b^2 + a^2) \sqrt{\frac{d x^3 + a}{3(b^2 - a^2)}}}{27 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="giac")

[Out] $\frac{1}{3} * (b^2 * c^2 * d - 2 * a * b * c * d^2 + a^2 * d^3) * (- (b * c - a * d) / d)^{(1/3)} * \log(\text{abs}((b * x^3 + a)^{(1/3)} - (- (b * c - a * d) / d)^{(1/3)})) / (b * c^4 - a * c^3 * d) - \frac{1}{3} * \sqrt{3} * (- b * c * d^2 + a * d^3)^{(1/3)} * (b * c - a * d) * \arctan(\frac{1}{3} * \sqrt{3} * (2 * (b * x^3 + a)^{(1/3)} + (- (b * c - a * d) / d)^{(1/3)}) / (- (b * c - a * d) / d)^{(1/3)}) / c^3 - \frac{1}{6} * (- b * c * d^2 + a * d^3)^{(1/3)} * (b * c - a * d) * \log((b * x^3 + a)^{(2/3)} + (b * x^3 + a)^{(1/3)} * (- (b * c - a * d) / d)^{(1/3)} + (- (b * c - a * d) / d)^{(2/3)}) / c^3 - \frac{1}{27} * \sqrt{3} * (2 * b^2 * c^2 - 12 * a * b * c * d + 9 * a^2 * d^2) * \arctan(\frac{1}{3} * \sqrt{3} * (2 * (b * x^3 + a)^{(1/3)} + a^{(1/3)}) / a^{(1/3)}) / (a^{(2/3)} * c^3) - \frac{1}{54} * (2 * b^2 * c^2 - 12 * a * b * c * d + 9 * a^2 * d^2) * \log((b * x^3 + a)^{(2/3)} + (b * x^3 + a)^{(1/3)} * a^{(1/3)} + a^{(2/3)}) / (a^{(2/3)} * c^3) + \frac{1}{27} * (2 * b^2 * c^2 - 12 * a * b * c * d + 9 * a^2 * d^2) * \log(\text{abs}((b * x^3 + a)^{(1/3)} - a^{(1/3)})) / (a^{(2/3)} * c^3) - \frac{1}{18} * (7 * (b * x^3 + a)^{(4/3)} * b^2 * c - 4 * (b * x^3 + a)^{(1/3)} * a * b^2 * c - 6 * (b * x^3 + a)^{(4/3)} * a * b * d + 6 * (b * x^3 + a)^{(1/3)} * a^2 * b * d) / (b^2 * c^2 * x^6)$

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{(b x^3 + a)^{\frac{4}{3}}}{(d x^3 + c) x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x)

[Out] int((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b x^3 + a)^{\frac{4}{3}}}{(d x^3 + c) x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^7), x)

mupad [B] time = 13.25, size = 2841, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a + b*x^3)^{(4/3)}/(x^7*(c + d*x^3)), x)$

[Out] $\log(\frac{((18*b^5*c^2*d^3*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(6*a*d - b*c) + 9*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^{(1/3)}*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^{(2/3)})/729 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4)*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^{(1/3)})/27 - (b^4*d^5*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8)*((729*a^6*d^6 + 8*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^4*b^2*c^2*d^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5))/(19683*a^2*c^9))^{(1/3)} + \log(\frac{((18*b^5*c^2*d^3*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(6*a*d - b*c) + 81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^{(1/3)}*(-(d^2*(a*d - b*c)^4)/c^9)^{(2/3)})/9 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4)*(-(d^2*(a*d - b*c)^4)/c^9)^{(1/3)}/3 - (b^4*d^5*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8)*(-(a^4*d^6 + b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5))/(27*c^9))^{(1/3)} + (((2*a*b^2*c - 3*a^2*b*d)*(a + b*x^3)^{(1/3)})/(9*c^2) + (b*(a + b*x^3)^{(4/3)}*(6*a*d - 7*b*c))/(18*c^2)))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) + \log(\frac{((3^{(1/2)}*1i)/2 - 1/2)*(((3^{(1/2)}*1i)/2 + 1/2)*(18*b^5*c^2*d^3*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(6*a*d - b*c) + 81*a*b^4*c^4*d^3*((3^{(1/2)}*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^{(1/3)}*(-(d^2*(a*d - b*c)^4)/c^9)^{(2/3)})/9 - (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4)*(-(d^2*(a*d - b*c)^4)/c^9)^{(1/3)}/3 + (b^4*d^5*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8)*((3^{(1/2)}*1i)/2 - 1/2)*(-(a^4*d^6 + b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5))/(27*c^9))^{(1/3)} - \log(\frac{((3^{(1/2)}*1i)/2 + 1/2)*(((3^{(1/2)}*1i)/2 - 1/2)*(18*b^5*c^2*d^3*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(6*a*d - b*c) - 81*a*b^4*c^4*d^3*((3^{(1/2)}*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^{(1/3)}*(-(d^2*(a*d - b*c)^4)/c^9)^{(2/3)})/9 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4)*(-(d^2*(a*d - b*c)^4)/c^9)^{(1/3)}/3 + (b^4*d^5*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d$

$$\begin{aligned}
& - 6804*a^5*b*c*d^5)/(243*c^8))*((3^{(1/2)*1i})/2 + 1/2)*(-(a^4*d^6 + b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^{(1/3)} \\
&) + \log((((3^{(1/2)*1i})/2 - 1/2)*(((3^{(1/2)*1i})/2 + 1/2)*(18*b^5*c^2*d^3*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(6*a*d - b*c) + 9*a*b^4*c^4*d^3*((3^{(1/2)*1i})/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^{(1/3)}*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^{(2/3)}))/729 - (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^{(1/3)})/27 + (b^4*d^5*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((3^{(1/2)*1i})/2 - 1/2)*((729*a^6*d^6 + 8*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^4*b^2*c^2*d^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^{(1/3)} \\
& - \log((((3^{(1/2)*1i})/2 + 1/2)*(((3^{(1/2)*1i})/2 - 1/2)*(18*b^5*c^2*d^3*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(6*a*d - b*c) - 9*a*b^4*c^4*d^3*((3^{(1/2)*1i})/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^{(1/3)}*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^{(2/3)}))/729 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^{(1/3)})/27 + (b^4*d^5*(a + b*x^3)^{(1/3)}*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((3^{(1/2)*1i})/2 + 1/2)*((729*a^6*d^6 + 8*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^4*b^2*c^2*d^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^{(1/3)}
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(4/3)/x**7/(d*x**3+c),x)

[Out] Timed out

$$3.448 \quad \int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3} dx$$

Optimal. Leaf size=334

$$\frac{(2a^2d^2 - 12abcd + 9b^2c^2) \log\left(\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right) (2a^2d^2 - 12abcd + 9b^2c^2) \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right) c^{2/3}(bc - ad)^{4/3}}{18b^{2/3}d^3 \cdot 9\sqrt{3}b^{2/3}d^3} \quad 6$$

Rubi [C] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{ax^5\sqrt[3]{a+bx^3} F_1\left(\frac{5}{3}; -\frac{4}{3}, 1; \frac{8}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{5c\sqrt[3]{\frac{bx^3}{a}} + 1}$$

Warning: Unable to verify antiderivative.

[In] Int[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3), x]

[Out] (a*x^5*(a + b*x^3)^(1/3)*AppellF1[5/3, -4/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]/(5*c*(1 + (b*x^3)/a)^(1/3))

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^4 (a + bx^3)^{4/3}}{c + dx^3} dx = \frac{\left(a \sqrt[3]{a + bx^3}\right) \int \frac{x^4 \left(1 + \frac{bx^3}{a}\right)^{4/3}}{c + dx^3} dx}{\sqrt[3]{1 + \frac{bx^3}{a}}}$$

$$= \frac{ax^5 \sqrt[3]{a + bx^3} F_1\left(\frac{5}{3}; -\frac{4}{3}, 1; \frac{8}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{5c \sqrt[3]{1 + \frac{bx^3}{a}}}$$

Mathematica [C] time = 0.31, size = 225, normalized size = 0.67

$$\frac{2x^5 \left(\frac{bx^3}{a} + 1\right)^{2/3} \left(\frac{dx^3}{c} + 1\right)^{2/3} (2a^2d^2 - 12abcd + 9b^2c^2) F_1\left(\frac{5}{3}; \frac{2}{3}, 1; \frac{8}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) + 5cx^2 \left(a \left(\frac{bx^3}{a} + 1\right)^{2/3} (6bc - 7ad) {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)}\right) + (a + bx^3) \left(\frac{dx^3}{c} + 1\right)^{2/3} (7ad - 6bc + 3bdx^3)\right)}{90cd^2 (a + bx^3)^{2/3} \left(\frac{dx^3}{c} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3), x]

[Out] (2*(9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*c*x^2*((a + b*x^3)*(-6*b*c + 7*a*d + 3*b*d*x^3)*(1 + (d*x^3)/c)^(2/3) + a*(6*b*c - 7*a*d)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-b*c) + a*d)*x^3/(a*(c + d*x^3))]))/(90*c*d^2*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3))

IntegrateAlgebraic [C] time = 9.96, size = 611, normalized size = 1.83

$$\frac{(2a^2d^2 - 12abcd + 9b^2c^2) \sqrt[3]{a + bx^3} \sqrt[3]{c + dx^3} \operatorname{AppellF1}\left(\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) + 5cx^2 \left(a \left(\frac{bx^3}{a} + 1\right)^{2/3} (6bc - 7ad) {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)}\right) + (a + bx^3) \left(\frac{dx^3}{c} + 1\right)^{2/3} (7ad - 6bc + 3bdx^3)\right)}{90cd^2 (a + bx^3)^{2/3} \left(\frac{dx^3}{c} + 1\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3), x]

[Out] ((a + b*x^3)^(1/3)*(-6*b*c*x^2 + 7*a*d*x^2 + 3*b*d*x^5))/(18*d^2) - ((9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(9*Sqrt[3]*b^(2/3)*d^3) - (Sqrt[(-1 - I*Sqrt[3])/6]*c^(2/3)*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/d^3 + ((-9*b^2*c^2 + 12*a*b*c*d - 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(27*b^(2/3)*d^3) + ((I/6)*(I*c^(2/3)*(b*c - a*d)^(4/3) + Sqrt[3]*c^(2/3)*(b*c - a*d)^(4/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]/d^3 + ((9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*Log

$$\frac{[b^{(2/3)}x^2 + b^{(1/3)}x(a + bx^3)^{(1/3)} + (a + bx^3)^{(2/3)}]}{(54b^{(2/3)}d^3) + ((c^{(2/3)}(bc - ad)^{(4/3)} - I\sqrt{3}c^{(2/3)}(bc - ad)^{(4/3)})} \\ * \text{Log}[(-2I)(bc - ad)^{(2/3)}x^2 + c^{(1/3)}(bc - ad)^{(1/3)}(Ix - \sqrt{3})] * x * (a + bx^3)^{(1/3)} + (I + \sqrt{3})c^{(2/3)}(a + bx^3)^{(2/3)}] / (12d^3)$$

fricas [A] time = 2.05, size = 550, normalized size = 1.65

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] $\frac{1}{54} * (2 * \sqrt{3}) * (9 * b^3 * c^2 - 12 * a * b^2 * c * d + 2 * a^2 * b * d^2) * \sqrt{-(-b^2)^{(1/3)}} * \arctan\left(\frac{-1/3 * (\sqrt{3}) * (-b^2)^{(1/3)} * b * x - 2 * \sqrt{3} * (b * x^3 + a)^{(1/3)} * (-b^2)^{(2/3)}}{(-b^2)^{(1/3)} * (b^2 * x)}\right) - 18 * \sqrt{3} * (b^3 * c - a * b^2 * d) * (-b * c^3 + a * c^2 * d)^{(1/3)} * \arctan\left(\frac{-1/3 * (\sqrt{3}) * (b * c^2 - a * c * d) * x + 2 * \sqrt{3} * (-b * c^3 + a * c^2 * d)^{(2/3)} * (b * x^3 + a)^{(1/3)}}{(b * c^2 - a * c * d) * x}\right) - 2 * (9 * b^2 * c^2 - 12 * a * b * c * d + 2 * a^2 * d^2) * (-b^2)^{(2/3)} * \log\left(\frac{-((-b^2)^{(2/3)} * x - (b * x^3 + a)^{(1/3)} * b)}{x}\right) + (9 * b^2 * c^2 - 12 * a * b * c * d + 2 * a^2 * d^2) * (-b^2)^{(2/3)} * \log\left(\frac{-((-b^2)^{(1/3)} * b * x^2 - (b * x^3 + a)^{(1/3)} * (-b^2)^{(2/3)} * x - (b * x^3 + a)^{(2/3)} * b)}{x^2}\right) - 18 * (b^3 * c - a * b^2 * d) * (-b * c^3 + a * c^2 * d)^{(1/3)} * \log\left(\frac{((b * x^3 + a)^{(1/3)} * c + (-b * c^3 + a * c^2 * d)^{(1/3)} * x)}{x}\right) + 9 * (b^3 * c - a * b^2 * d) * (-b * c^3 + a * c^2 * d)^{(1/3)} * \log\left(\frac{((b * x^3 + a)^{(2/3)} * c^2 - (-b * c^3 + a * c^2 * d)^{(1/3)} * (b * x^3 + a)^{(1/3)} * c * x + (-b * c^3 + a * c^2 * d)^{(2/3)} * x^2)}{x^2}\right) + 3 * (3 * b^3 * d^2 * x^5 - (6 * b^3 * c * d - 7 * a * b^2 * d^2) * x^2) * (b * x^3 + a)^{(1/3)} / (b^2 * d^3)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}} x^4}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}} x^4}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] `int(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}} x^4}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 (bx^3 + a)^{\frac{4}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x)`

[Out] `int((x^4*(a + b*x^3)^(4/3))/(c + d*x^3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + bx^3)^{\frac{4}{3}}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(b*x**3+a)**(4/3)/(d*x**3+c),x)`

[Out] `Integral(x**4*(a + b*x**3)**(4/3)/(c + d*x**3), x)`

$$3.449 \quad \int \frac{x(a+bx^3)^{4/3}}{c+dx^3} dx$$

Optimal. Leaf size=277

$$\frac{(bc-ad)^{4/3} \log(c+dx^3)}{6\sqrt[3]{c}d^2} + \frac{\sqrt[3]{b}(3bc-4ad) \log\left(\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right)}{6d^2} - \frac{(bc-ad)^{4/3} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}d^2} + \sqrt[3]{b}$$

Rubi [C] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 0.23, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {511, 510}

$$\frac{ax^2\sqrt[3]{a+bx^3} F_1\left(\frac{2}{3}; -\frac{4}{3}, 1; \frac{5}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{2c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(x*(a + b*x^3)^(4/3))/(c + d*x^3), x]

[Out] (a*x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -4/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)]/(2*c*(1 + (b*x^3)/a)^(1/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^(m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3} dx = \frac{\left(a\sqrt[3]{a+bx^3}\right) \int \frac{x\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3} dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{ax^2\sqrt[3]{a+bx^3} F_1\left(\frac{2}{3}; -\frac{4}{3}, 1; \frac{5}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{2c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.32, size = 198, normalized size = 0.71

$$\frac{2bx^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}(4ad-3bc)F_1\left(\frac{5}{3}; \frac{2}{3}, 1; \frac{8}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)+5x^2\left(a\left(\frac{bx^3}{a}+1\right)\right)^{2/3}(3ad-2bc)F_1\left(\frac{2}{3}; \frac{2}{3}; \frac{5}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)}\right)+2bc(a+bx^3)\left(\frac{dx^3}{c}+1\right)^{2/3}}{30cd(a+bx^3)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x*(a + b*x^3)^(4/3))/(c + d*x^3), x]

[Out] (2*b*(-3*b*c + 4*a*d)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(2/3)*Appell1F1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*x^2*(2*b*c*(a + b*x^3)*(1 + (d*x^3)/c)^(2/3) + a*(-2*b*c + 3*a*d)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-b*c) + a*d)*x^3/(a*(c + d*x^3))])/(30*c*d*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3))

IntegrateAlgebraic [C] time = 7.07, size = 552, normalized size = 1.99

$$\frac{(b^2c - 4\sqrt[3]{d})\log(\sqrt[3]{d^2x^3 - d^2})}{3d^2} - \frac{(3b^2c - 4\sqrt[3]{d})\log\left(\frac{d^2x^3}{12d^2x^3 - d^2}\right)}{3\sqrt[3]{d}} - \frac{(4\sqrt[3]{d} - 3b^2)\log(\sqrt[3]{d^2x^3 - d^2} - (a + bx^3)^{1/3})}{18d} - \frac{(b^2c - ad^2 + d^2c - ad^2)\log(\sqrt[3]{d^2x^3 - d^2} - (a + bx^3)^{1/3})}{12\sqrt[3]{d}} - \frac{(b^2c - ad^2 + d^2c - ad^2)\log(\sqrt[3]{d^2x^3 - d^2} - 2a\sqrt[3]{bc - ad^2})}{12\sqrt[3]{d}} - \frac{(bc - ad^2 - \sqrt[3]{d}bc - ad^2)\log\left(\frac{2\sqrt[3]{d^2x^3 - d^2} + (1 + \sqrt[3]{d})\sqrt[3]{d^2x^3 - d^2}}{2\sqrt[3]{d}}\right)}{2\sqrt[3]{d}} - \frac{\sqrt{1 - \sqrt[3]{d}}(bc - ad^2)\log\left(\frac{2\sqrt[3]{d^2x^3 - d^2} + (1 + \sqrt[3]{d})\sqrt[3]{d^2x^3 - d^2}}{2\sqrt[3]{d}}\right)}{2\sqrt[3]{d}} - \frac{2\sqrt[3]{d}bc}{3d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(a + b*x^3)^(4/3))/(c + d*x^3), x]

[Out] (b*x^2*(a + b*x^3)^(1/3))/(3*d) + ((3*b^(4/3)*c - 4*a*b^(1/3)*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(3*Sqrt[3]*d^2) + (Sqrt[-1 - I*Sqrt[3]]*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]/(Sqrt[6]*c^(1/3)*d^2) + ((3*b^(4/3)*c - 4*a*b^(1/3)*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(9*d^2) + (((b*c - a*d)^(4/3) - I*Sqrt[3]*(b*c - a*d)^(4/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]/(6*c^(1/3)*d^2) + ((-3*b^(4/3)*c + 4*a*b^(1/3)*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(18*d^2) + ((I/12)*(I*(b*c - a*d)^(4/3) + Sqrt[3]*(b*c - a*d)^(4/3))*Log[(-2*I)*(b*c -

$a*d)^{(2/3)}*x^2 + c^{(1/3)}*(b*c - a*d)^{(1/3)}*(I*x - \text{Sqrt}[3]*x)*(a + b*x^3)^{(1/3)} + (I + \text{Sqrt}[3])*c^{(2/3)}*(a + b*x^3)^{(2/3)}/(c^{(1/3)}*d^2)$

fricas [A] time = 0.71, size = 396, normalized size = 1.43

$$\frac{6(bx^3+a)^{\frac{1}{3}}bdx^2 - 6\sqrt{3}(bc-ad)\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}(bc-ad)+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}}{3(bc-ad)}\right) + 2\sqrt{3}(3bc-4ad)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}(bc-ad)\sqrt{3}(bx^3+a)^{\frac{1}{3}}+3b^{\frac{1}{3}}}{3bc}\right) - 2(3bc-4ad)^{\frac{1}{3}}\log\left(\frac{c(b^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}})}{c}\right) - 6(bc-ad)\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\log\left(\frac{-(\frac{bc-ad}{c})^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}}{c}\right) + (3bc-4ad)^{\frac{1}{3}}\log\left(\frac{(bx^3+a)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}}{c}\right) + 3(bc-ad)\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\log\left(\frac{-(\frac{bc-ad}{c})^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}}{c}\right)}{18d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] $\frac{1}{18}*(6*(b*x^3 + a)^{(1/3)}*b*d*x^2 - 6*\text{sqrt}(3)*(b*c - a*d)*((b*c - a*d)/c)^{(1/3)}*\text{arctan}(-1/3*(\text{sqrt}(3)*(b*c - a*d)*x + 2*\text{sqrt}(3)*(b*x^3 + a)^{(1/3)}*c*((b*c - a*d)/c)^{(2/3)})/((b*c - a*d)*x)) + 2*\text{sqrt}(3)*(3*b*c - 4*a*d)*(-b)^{(1/3)}*\text{arctan}(1/3*(\text{sqrt}(3)*b*x + 2*\text{sqrt}(3)*(b*x^3 + a)^{(1/3)}*(-b)^{(2/3)})/(b*x)) - 2*(3*b*c - 4*a*d)*(-b)^{(1/3)}*\log(((b)^{(1/3)}*x + (b*x^3 + a)^{(1/3)})/x) - 6*(b*c - a*d)*((b*c - a*d)/c)^{(1/3)}*\log(-(x*((b*c - a*d)/c)^{(1/3)} - (b*x^3 + a)^{(1/3)})/x) + (3*b*c - 4*a*d)*(-b)^{(1/3)}*\log(((b)^{(2/3)}*x^2 - (b*x^3 + a)^{(1/3)}*(-b)^{(1/3)}*x + (b*x^3 + a)^{(2/3)})/x^2) + 3*(b*c - a*d)*((b*c - a*d)/c)^{(1/3)}*\log((x^2*((b*c - a*d)/c)^{(2/3)} + (b*x^3 + a)^{(1/3)}*x*((b*c - a*d)/c)^{(1/3)} + (b*x^3 + a)^{(2/3)})/x^2))/d^2$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}x}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}x}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] int(x*(b*x^3+a)^(4/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}} x}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x (bx^3 + a)^{4/3}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*x^3)^(4/3))/(c + d*x^3),x)

[Out] int((x*(a + b*x^3)^(4/3))/(c + d*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x (a + bx^3)^{\frac{4}{3}}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**3+a)**(4/3)/(d*x**3+c),x)

[Out] Integral(x*(a + b*x**3)**(4/3)/(c + d*x**3), x)

$$3.450 \quad \int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)} dx$$

Optimal. Leaf size=254

$$\frac{b^{4/3} \log\left(\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right)}{2d} - \frac{b^{4/3} \tan^{-1}\left(\frac{2\sqrt[3]{b}x+1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}d} - \frac{(bc-ad)^{4/3} \log(c+dx^3)}{6c^{4/3}d} + \frac{(bc-ad)^{4/3} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a}\right)}{2c^{4/3}d}$$

Rubi [C] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{a\sqrt[3]{a+bx^3} F_1\left(-\frac{1}{3}; -\frac{4}{3}, 1; \frac{2}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{cx\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)), x]

[Out] -((a*(a + b*x^3)^(1/3)*AppellF1[-1/3, -4/3, 1, 2/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*x*(1 + (b*x^3)/a)^(1/3)))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^(m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{(a + bx^3)^{4/3}}{x^2(c + dx^3)} dx = \frac{\left(a\sqrt[3]{a + bx^3}\right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^{4/3}}{x^2(c + dx^3)} dx}{\sqrt[3]{1 + \frac{bx^3}{a}}}$$

$$= -\frac{a\sqrt[3]{a + bx^3} F_1\left(-\frac{1}{3}; -\frac{4}{3}, 1; \frac{2}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{cx\sqrt[3]{1 + \frac{bx^3}{a}}}$$

Mathematica [C] time = 0.55, size = 161, normalized size = 0.63

$$\frac{2b^2cx^6 \left(\frac{bx^3}{a} + 1\right)^{2/3} F_1\left(\frac{5}{3}; \frac{2}{3}, 1; \frac{8}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) - \frac{5ax^3 \left(\frac{bx^3}{a} + 1\right)^{2/3} (ad - 2bc) {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; \frac{(ad - bc)x^3}{a(dx^3 + c)}\right)}{\left(\frac{dx^3}{c} + 1\right)^{2/3}} - 10ac(a + bx^3)}{10c^2x(a + bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)), x]

[Out] (-10*a*c*(a + b*x^3) + 2*b^2*c*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] - (5*a*(-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-b*c) + a*d)*x^3/(a*(c + d*x^3))])/(1 + (d*x^3)/c)^(2/3))/(10*c^2*x*(a + b*x^3)^(2/3))

IntegrateAlgebraic [C] time = 5.54, size = 509, normalized size = 2.00

$$\frac{b^{4/3} \log(\sqrt{a + bx^3} - \sqrt{c})}{3d} - \frac{b^{4/3} \tan^{-1}\left(\frac{\sqrt{a + bx^3}}{\sqrt{c}}\right)}{\sqrt{3}d} + \frac{b^{4/3} \log(\sqrt{a + bx^3} + \sqrt{a + bx^3})}{6d} + \frac{b^{4/3} \log(\sqrt{a + bx^3} + \sqrt{a + bx^3})}{6d} + \frac{b^{4/3} \log(\sqrt{a + bx^3} + \sqrt{a + bx^3})}{6d} + \frac{b^{4/3} \log(\sqrt{a + bx^3} + \sqrt{a + bx^3})}{6d} + \frac{b^{4/3} \log(\sqrt{a + bx^3} + \sqrt{a + bx^3})}{6d} + \frac{b^{4/3} \log(\sqrt{a + bx^3} + \sqrt{a + bx^3})}{6d} + \frac{b^{4/3} \log(\sqrt{a + bx^3} + \sqrt{a + bx^3})}{6d} + \frac{b^{4/3} \log(\sqrt{a + bx^3} + \sqrt{a + bx^3})}{6d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)), x]

[Out] -((a*(a + b*x^3)^(1/3))/(c*x)) - (b^(4/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(Sqrt[3]*d) - (Sqrt[(-1 - I*Sqrt[3])/6]*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(4/3)*d) - (b^(4/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(3*d) + ((I/6)*(I*(b*c - a*d)^(4/3) + Sqrt[3]*(b*c - a*d)^(4/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]/(c^(4/3)*d) + (b^(4/3)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(6*d) + ((b*c

$$- a*d)^{(4/3)} - I*\text{Sqrt}[3]*(b*c - a*d)^{(4/3)}*\text{Log}[(-2*I)*(b*c - a*d)^{(2/3)}*x^2 + c^{(1/3)}*(b*c - a*d)^{(1/3)}*(I*x - \text{Sqrt}[3]*x)*(a + b*x^3)^{(1/3)} + (I + \text{Sqrt}[3])*c^{(2/3)}*(a + b*x^3)^{(2/3)}]]/(12*c^{(4/3)}*d)$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x)

[Out] int((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{4/3}}{x^2(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(4/3)/(x^2*(c + d*x^3)), x)

[Out] int((a + b*x^3)^(4/3)/(x^2*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{4/3}}{x^2(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(4/3)/x**2/(d*x**3+c), x)

[Out] Integral((a + b*x**3)**(4/3)/(x**2*(c + d*x**3)), x)

$$3.451 \quad \int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)} dx$$

Optimal. Leaf size=201

$$\frac{(bc-ad)^{4/3} \log(c+dx^3)}{6c^{7/3}} - \frac{(bc-ad)^{4/3} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{7/3}} - \frac{(bc-ad)^{4/3} \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}c^{7/3}} - \frac{\sqrt[3]{a+bx^3}}{4c^2}$$

Rubi [C] time = 0.06, antiderivative size = 90, normalized size of antiderivative = 0.45, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{a\sqrt[3]{a+bx^3} \left(\frac{dx^3}{c} + 1\right)^{4/3} {}_2F_1\left(-\frac{4}{3}, -\frac{4}{3}; -\frac{1}{3}; -\frac{c\left(\frac{bx^3}{a} - \frac{dx^3}{c}\right)}{dx^3+c}\right)}{4cx^4 \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)), x]

[Out] -(a*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3)*Hypergeometric2F1[-4/3, -4/3, -1/3, -(c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3)))/(4*c*x^4*(1 + (b*x^3)/a)^(1/3))

Rule 510

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{(a + bx^3)^{4/3}}{x^5 (c + dx^3)} dx = \frac{\left(a \sqrt[3]{a + bx^3}\right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^{4/3}}{x^5 (c + dx^3)} dx}{\sqrt[3]{1 + \frac{bx^3}{a}}}$$

$$= \frac{a \sqrt[3]{a + bx^3} \left(1 + \frac{dx^3}{c}\right)^{4/3} {}_2F_1\left(-\frac{4}{3}, -\frac{4}{3}; -\frac{1}{3}; -\frac{c\left(\frac{bx^3}{a} - \frac{dx^3}{c}\right)}{c + dx^3}\right)}{4cx^4 \sqrt[3]{1 + \frac{bx^3}{a}}}$$

Mathematica [C] time = 0.04, size = 84, normalized size = 0.42

$$\frac{a \sqrt[3]{a + bx^3} \left(\frac{dx^3}{c} + 1\right)^{4/3} {}_2F_1\left(-\frac{4}{3}, -\frac{4}{3}; -\frac{1}{3}; \frac{(ad - bc)x^3}{a(dx^3 + c)}\right)}{4cx^4 \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)), x]

[Out] -1/4*(a*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3)*Hypergeometric2F1[-4/3, -4/3, -1/3, ((-b*c) + a*d)*x^3/(a*(c + d*x^3))]/(c*x^4*(1 + (b*x^3)/a)^(1/3))

IntegrateAlgebraic [C] time = 2.90, size = 383, normalized size = 1.91

$$\frac{((bc - ad)^{4/3} - i\sqrt{3}(bc - ad)^{2/3}) \log\left(\frac{2x\sqrt{bc - ad} + (1 + i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a + bx^3}}{c^{2/3}}\right) + \sqrt[3]{(-1 - i\sqrt{3})(bc - ad)^{4/3}} \tan^{-1}\left(\frac{2i\sqrt[3]{bc - ad}}{\sqrt[3]{(-1 - i\sqrt{3})(bc - ad)^{4/3}} - \sqrt[3]{3}\sqrt[3]{a + bx^3}}\right) + i(\sqrt{3}(bc - ad)^{4/3} + i(bc - ad)^{2/3}) \log\left(\frac{(\sqrt{3} + i)c^{2/3}(a + bx^3)^{2/3} + \sqrt[3]{c}(-\sqrt{3}x + i)\sqrt[3]{a + bx^3}\sqrt[3]{bc - ad} - 2ix^2(bc - ad)^{2/3}}{12c^{2/3}}\right) + \frac{\sqrt[3]{a + bx^3}(-ic + 4ad^3 - 5bc^2)}{4c^{2/3}}}{c^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)), x]

[Out] ((a + b*x^3)^(1/3)*(-(a*c) - 5*b*c*x^3 + 4*a*d*x^3))/(4*c^2*x^4) + (Sqrt[(-1 - I*Sqrt[3])/6]*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])/c^(7/3) + (((b*c - a*d)^(4/3) - I*Sqrt[3]*(b*c - a*d)^(4/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*c^(7/3)) + ((I/12)*(I*(b*c - a*d)^(4/3) + Sqrt[3]*(b*c - a*d)^(4/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/c^(7/3)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x)

[Out] int((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{4/3}}{x^5(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(4/3)/(x^5*(c + d*x^3)), x)`

[Out] `int((a + b*x^3)^(4/3)/(x^5*(c + d*x^3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{4/3}}{x^5(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(4/3)/x**5/(d*x**3+c), x)`

[Out] `Integral((a + b*x**3)**(4/3)/(x**5*(c + d*x**3)), x)`

$$3.452 \quad \int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)} dx$$

Optimal. Leaf size=250

$$\frac{\sqrt[3]{a+bx^3} (28a^2d^2 - 35abcd + 4b^2c^2)}{28ac^3x} - \frac{d(bc-ad)^{4/3} \log(c+dx^3)}{6c^{10/3}} + \frac{d(bc-ad)^{4/3} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{10/3}} + \dots$$

Rubi [C] time = 0.51, antiderivative size = 169, normalized size of antiderivative = 0.68, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{12cx^3(a+bx^3)(c+dx^3)(bc-ad) {}_2F_1\left(-\frac{1}{3}, 2; \frac{2}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right) - (4c-3dx^3)\left(c(a+bx^3)(a(c-4dx^3)+5bcx^3) - 2x^6(bc-ad)^2 {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)\right)}{28c^4x^7(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)), x]

[Out] (12*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - (4*c - 3*d*x^3)*(c*(a + b*x^3)*(5*b*c*x^3 + a*(c - 4*d*x^3)) - 2*(b*c - a*d)^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(28*c^4*x^7*(a + b*x^3)^(2/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -((b*x^n)/a), -((d*x^n)/c)])/((e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{(a + bx^3)^{4/3}}{x^8(c + dx^3)} dx = \frac{\left(a\sqrt[3]{a + bx^3}\right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^{4/3}}{x^8(c + dx^3)} dx}{\sqrt[3]{1 + \frac{bx^3}{a}}}$$

$$= \frac{12c(bc - ad)x^3(a + bx^3)(c + dx^3) {}_2F_1\left(-\frac{1}{3}, 2; \frac{2}{3}; \frac{(bc - ad)x^3}{c(a + bx^3)}\right) - (4c - 3dx^3)\left(c(a + bx^3)(5bcx^3)\right)}{28c^4x^7(a + bx^3)^{2/3}}$$

Mathematica [C] time = 0.60, size = 179, normalized size = 0.72

$$\frac{a\left(\frac{bx^3}{a} + 1\right)\left(12cx^3(a + bx^3)(c + dx^3)(ad - bc) {}_2F_1\left(-\frac{1}{3}, 2; \frac{2}{3}; \frac{(bc - ad)x^3}{c(bx^3 + a)}\right) + (4c - 3dx^3)\left(c(a + bx^3)(c - 4dx^3) + 5bcx^3\right) - 2x^6(bc - ad)^2 {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{(bc - ad)x^3}{c(bx^3 + a)}\right)\right)}{28c^4x^7(a + bx^3)^{5/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)), x]

[Out] -1/28*(a*(1 + (b*x^3)/a)*(12*c*(-(b*c) + a*d)*x^3*(a + b*x^3)*(c + d*x^3)*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) + (4*c - 3*d*x^3)*(c*(a + b*x^3)*(5*b*c*x^3 + a*(c - 4*d*x^3)) - 2*(b*c - a*d)^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/(c^4*x^7*(a + b*x^3)^(5/3))

IntegrateAlgebraic [C] time = 3.20, size = 434, normalized size = 1.74

$$\frac{\sqrt{a + bx^3} \left(-4a^2c^2 + 7a^2cd^3 - 28a^2d^4e - 8ab^2c^3 + 35ab^2cd^2e - 4b^2c^2e^2\right) + i\left(\sqrt{3}d(bc - ad)^{1/3} + id(bc - ad)^{2/3}\right) \log\left(\frac{2x\sqrt{bc - ad} + (1 + i\sqrt{3})\sqrt{c}\sqrt{a + bx^3}}{6c^{1/3}}\right) + \sqrt[3]{\frac{-1 - i\sqrt{3}}{2}} d(bc - ad)^{1/3} \tan^{-1}\left(\frac{b\sqrt{3}\sqrt{a + bx^3}}{\sqrt{3}(bc - ad) - \sqrt{3}\sqrt{c}\sqrt{a + bx^3} - \sqrt{3}\sqrt{c}\sqrt{a + bx^3}}\right) + \frac{d(bc - ad)^{1/3} - i\sqrt{3}d(bc - ad)^{2/3} \log\left(\frac{(\sqrt{3} + i)c^{2/3}(a + bx^3)^{2/3} + \sqrt{c}(-\sqrt{3}x + i)\sqrt{a + bx^3}\sqrt{bc - ad} - 2ix^2(bc - ad)^{2/3}}{12c^{1/3}}\right)}{12c^{1/3}}}{28c^4x^7(a + bx^3)^{5/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)), x]

[Out] ((a + b*x^3)^(1/3)*(-4*a^2*c^2 - 8*a*b*c^2*x^3 + 7*a^2*c*d*x^3 - 4*b^2*c^2*x^6 + 35*a*b*c*d*x^6 - 28*a^2*d^2*x^6))/(28*a*c^3*x^7) - (Sqrt[(-1 - I*Sqrt[3])/6]*d*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]/c^(10/3) + ((I/6)*(I*d*(b*c - a*d)^(4/3) + Sqrt[3]*d*(b*c - a*d)^(4/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/c^(10/3) + ((d*(b*c - a*d)^(4/3) - I*Sqrt[3]*d*(b*c - a*d)^(4/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/((12*c^(10/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x)

[Out] int((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{4/3}}{x^8 (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x)

[Out] int((a + b*x^3)^(4/3)/(x^8*(c + d*x^3)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(4/3)/x**8/(d*x**3+c),x)

[Out] Timed out

$$3.453 \quad \int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)} dx$$

Optimal. Leaf size=318

$$\frac{\sqrt[3]{a+bx^3} (35a^2d^2 - 40abcd + 2b^2c^2)}{140ac^3x^4} + \frac{\sqrt[3]{a+bx^3} (140a^3d^3 - 175a^2bcd^2 + 20ab^2c^2d + 6b^3c^3)}{140a^2c^4x} + \frac{d^2(bc-ad)^{4/3}}{6c^{13}}$$

Rubi [C] time = 1.80, antiderivative size = 260, normalized size of antiderivative = 0.82, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{-18cx^3(a+bx^3)(c+dx^3)^2(bc-ad)_3F_2\left(-\frac{1}{3}, 2, 2; \frac{2}{3}, 1; \frac{(bc-ad)x^3}{(bx^3+a)}\right) + 6cx^3(a+bx^3)(11c^2+2cdx^3-9d^2x^6)(bc-ad)_2F_1\left(-\frac{1}{3}, 2; \frac{2}{3}; \frac{(bc-ad)x^3}{(bx^3+a)}\right) - (14c^2-12cdx^3+9d^2x^6)\left(c(a+bx^3)(a(c-4dx^3)+5bcx^3)-2x^6(bc-ad)\right)_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{(bc-ad)x^3}{(bx^3+a)}\right)}{140c^5x^{10}(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)), x]

[Out] (6*c*(b*c - a*d)*x^3*(a + b*x^3)*(11*c^2 + 2*c*d*x^3 - 9*d^2*x^6)*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - (14*c^2 - 12*c*d*x^3 + 9*d^2*x^6)*(c*(a + b*x^3)*(5*b*c*x^3 + a*(c - 4*d*x^3)) - 2*(b*c - a*d)^2*x^6)*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^2*HypergeometricPFQ[{-1/3, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(140*c^5*x^10*(a + b*x^3)^(2/3))

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1+(m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n-1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^(m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n-1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{(a + bx^3)^{4/3}}{x^{11}(c + dx^3)} dx = \frac{\left(a \sqrt[3]{a + bx^3}\right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^{4/3}}{x^{11}(c + dx^3)} dx}{\sqrt[3]{1 + \frac{bx^3}{a}}} = \frac{6c(bc - ad)x^3 (a + bx^3) (11c^2 + 2cdx^3 - 9d^2x^6) {}_2F_1\left(-\frac{1}{3}, 2; \frac{2}{3}; \frac{(bc-ad)x^3}{c(a+bx^3)}\right) - (14c^2 - 12cdx^3 + 9d^2x^6) \left(\frac{bc-ad}{c}\right)^{1/3}}{140c^5 x^{10} (a + bx^3)^{5/3}}$$

Mathematica [C] time = 2.70, size = 270, normalized size = 0.85

$$\frac{a\left(\frac{bc}{a}+1\right)\left(18cx^3(a+bx^3)(c+dx^3)^2(bc-ad){}_2F_2\left(-\frac{1}{3}, 2, 2; \frac{2}{3}, 1; \frac{(bc-ad)x^3}{c(a+bx^3)}\right) + 6cx^3(a+bx^3)(-11c^2-2cdx^3+9d^2x^6)(bc-ad){}_2F_1\left(-\frac{1}{3}, 2; \frac{2}{3}; \frac{(bc-ad)x^3}{c(a+bx^3)}\right) + (14c^2-12cdx^3+9d^2x^6)\left(c(a+bx^3)(a(c-4dx^3)+5bcx^3)-2x^6(bc-ad)^2{}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{(bc-ad)x^3}{c(a+bx^3)}\right)\right)}{140c^5 x^{10} (a + bx^3)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] -1/140*(a*(1 + (b*x^3)/a)*(6*c*(b*c - a*d)*x^3*(a + b*x^3)*(-11*c^2 - 2*c*d*x^3 + 9*d^2*x^6)*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) + (14*c^2 - 12*c*d*x^3 + 9*d^2*x^6)*(c*(a + b*x^3)*(5*b*c*x^3 + a*(c - 4*d*x^3)) - 2*(b*c - a*d)^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) + 18*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^2*HypergeometricPFQ[{-1/3, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3)))]/(c^5*x^10*(a + b*x^3)^(5/3))
```

IntegrateAlgebraic [C] time = 3.69, size = 501, normalized size = 1.58

$$\frac{\sqrt{x^3+1}(-14d^2x^2+20d^2x^2-35d^2x^2+140d^2x^2-22d^2x^2+40d^2x^2-175d^2x^2-20d^2x^2+60d^2x^2)}{140d^5x^{10}} \sqrt[3]{\frac{bc-ad}{c}} \sqrt[3]{\frac{bc-ad}{c}} \sqrt[3]{\frac{bc-ad}{c}} \log\left(\frac{2\sqrt[3]{bc-ad} + (1+\sqrt{3})\sqrt[3]{d^3x^3+1}}{2\sqrt[3]{bc-ad}}\right) + \frac{\sqrt[3]{(1-\sqrt{3})d^3x^3+1}}{2\sqrt[3]{bc-ad}} \log\left(\frac{2\sqrt[3]{bc-ad} + (1+\sqrt{3})\sqrt[3]{d^3x^3+1}}{2\sqrt[3]{bc-ad}}\right) + \frac{(1-\sqrt{3})d^3x^3+1}{2\sqrt[3]{bc-ad}} \log\left(\frac{2\sqrt[3]{bc-ad} + (1+\sqrt{3})\sqrt[3]{d^3x^3+1}}{2\sqrt[3]{bc-ad}}\right) + \frac{d^3x^3+1}{2\sqrt[3]{bc-ad}} \log\left(\frac{2\sqrt[3]{bc-ad} + (1+\sqrt{3})\sqrt[3]{d^3x^3+1}}{2\sqrt[3]{bc-ad}}\right) + \frac{d^3x^3+1}{2\sqrt[3]{bc-ad}} \log\left(\frac{2\sqrt[3]{bc-ad} + (1+\sqrt{3})\sqrt[3]{d^3x^3+1}}{2\sqrt[3]{bc-ad}}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-14*a^3*c^3 - 22*a^2*b*c^3*x^3 + 20*a^3*c^2*d*x^3 - 2*a*b^2*c^3*x^6 + 40*a^2*b*c^2*d*x^6 - 35*a^3*c*d^2*x^6 + 6*b^3*c^3*x^9 + 20*a*b^2*c^2*d*x^9 - 175*a^2*b*c*d^2*x^9 + 140*a^3*d^3*x^9))/(140*a^2*c^4*x^10) + (Sqrt[(-1 - I*Sqrt[3])/6]*d^2*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]/c^(13/3) + ((d^2*(b*c - a*d)^(4/3) - I*Sqrt[3]*d^2*(b*c - a*d)^(4/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/c^(13/3)
```

$$\frac{(1/3)*(a + b*x^3)^{(1/3)}}{(6*c^{(13/3)})} + ((I/12)*(I*d^2*(b*c - a*d)^{(4/3)} + \text{Sqrt}[3]*d^2*(b*c - a*d)^{(4/3)}*\text{Log}[(-2*I)*(b*c - a*d)^{(2/3)}*x^2 + c^{(1/3)}*(b*c - a*d)^{(1/3)}*(I*x - \text{Sqrt}[3]*x)*(a + b*x^3)^{(1/3)} + (I + \text{Sqrt}[3])*c^{(2/3)}*(a + b*x^3)^{(2/3)}])/c^{(13/3)}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x)

[Out] int((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{4/3}}{x^{11} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x)

[Out] int((a + b*x^3)^(4/3)/(x^11*(c + d*x^3)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(4/3)/x**11/(d*x**3+c),x)

[Out] Timed out

$$3.454 \quad \int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)} dx$$

Optimal. Leaf size=392

$$\frac{\sqrt[3]{a+bx^3} (130a^2d^2 - 143abcd + 4b^2c^2)}{910ac^3x^7} + \frac{\sqrt[3]{a+bx^3} (455a^3d^3 - 520a^2bcd^2 + 26ab^2c^2d + 12b^3c^3)}{1820a^2c^4x^4} \sqrt[3]{a+bx^3} ($$

Rubi [C] time = 4.39, antiderivative size = 1446, normalized size of antiderivative = 3.69, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

result too large to display

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]

[Out] $-(140*a^2*c^5 + 840*a*b*c^5*x^3 - 686*a^2*c^4*d*x^3 + 700*b^2*c^5*x^6 - 1316*a*b*c^4*d*x^6 + 612*a^2*c^3*d^2*x^6 - 630*b^2*c^4*d*x^9 + 1152*a*b*c^3*d^2*x^9 - 513*a^2*c^2*d^3*x^9 + 540*b^2*c^3*d^2*x^12 - 918*a*b*c^2*d^3*x^12 + 324*a^2*c*d^4*x^12 - 405*b^2*c^2*d^3*x^15 + 324*a*b*c*d^4*x^15 - 828*a*b*c^5*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 828*a^2*c^4*d*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 828*b^2*c^5*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 918*a*b*c^4*d*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 90*a^2*c^3*d^2*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 90*b^2*c^4*d*x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 234*a*b*c^3*d^2*x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 324*a^2*c^2*d^3*x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 324*b^2*c^3*d^2*x^12*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 918*a*b*c^2*d^3*x^12*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 594*a^2*c*d^4*x^12*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 594*b^2*c^2*d^3*x^15*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 594*a*b*c*d^4*x^15*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 280*b^2*c^5*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 560*a*b*c^4*d*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 280*a^2*c^3*d^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*b^2*c^4*d*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 504*a*b*c^3*d^2*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]$

```

ergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*a^2*c^
2*d^3*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
- 216*b^2*c^3*d^2*x^12*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] + 432*a*b*c^2*d^3*x^12*Hypergeometric2F1[2/3, 1, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] - 216*a^2*c*d^4*x^12*Hypergeometric2F1[2/3, 1,
5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b^2*c^2*d^3*x^15*Hypergeomet
ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 324*a*b*c*d^4*x^15
*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*a^
2*d^5*x^15*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3)
] + 54*c*(b*c - a*d)*x^3*(a + b*x^3)*(7*c - 6*d*x^3)*(c + d*x^3)^2*Hypergeo
metricPFQ[{-1/3, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 54*c
*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^3*HypergeometricPFQ[{-1/3, 2, 2
}, {2/3, 1, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(1820*c^6*x^13*(a + b*x
^3)^(2/3))

```

Rule 510

```

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^q, x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a,
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

```

Rule 511

```

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^q, x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

```

Rubi steps

$$\int \frac{(a + bx^3)^{4/3}}{x^{14}(c + dx^3)} dx = \frac{\left(a\sqrt[3]{a + bx^3}\right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^{4/3}}{x^{14}(c + dx^3)} dx}{\sqrt[3]{1 + \frac{bx^3}{a}}}$$

$$= -\frac{140a^2c^5 + 840abc^5x^3 - 686a^2c^4dx^3 + 700b^2c^5x^6 - 1316abc^4dx^6 + 612a^2c^3d^2x^6 - 630b^2c^4}{\dots}$$

Mathematica [C] time = 5.44, size = 1446, normalized size = 3.69

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]

[Out]
$$\begin{aligned} & (-140*a^2*c^5 - 840*a*b*c^5*x^3 + 686*a^2*c^4*d*x^3 - 700*b^2*c^5*x^6 + 131 \\ & 6*a*b*c^4*d*x^6 - 612*a^2*c^3*d^2*x^6 + 630*b^2*c^4*d*x^9 - 1152*a*b*c^3*d^2 \\ & 2*x^9 + 513*a^2*c^2*d^3*x^9 - 540*b^2*c^3*d^2*x^{12} + 918*a*b*c^2*d^3*x^{12} - \\ & 324*a^2*c*d^4*x^{12} + 405*b^2*c^2*d^3*x^{15} - 324*a*b*c*d^4*x^{15} + 828*a*b*c \\ & ^5*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - \\ & 828*a^2*c^4*d*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a \\ & + b*x^3))] + 828*b^2*c^5*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x \\ & ^3)/(c*(a + b*x^3))] - 918*a*b*c^4*d*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((\\ & b*c - a*d)*x^3)/(c*(a + b*x^3))] + 90*a^2*c^3*d^2*x^6*Hypergeometric2F1[-1/ \\ & 3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 90*b^2*c^4*d*x^9*Hypergeome \\ & tric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 234*a*b*c^3*d^2* \\ & x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 32 \\ & 4*a^2*c^2*d^3*x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + \\ & b*x^3))] - 324*b^2*c^3*d^2*x^{12}*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a* \\ & d)*x^3)/(c*(a + b*x^3))] + 918*a*b*c^2*d^3*x^{12}*Hypergeometric2F1[-1/3, 2, \\ & 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 594*a^2*c*d^4*x^{12}*Hypergeometric \\ & 2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 594*b^2*c^2*d^3*x^{15} \\ & *Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 594*a \\ & *b*c*d^4*x^{15}*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x \\ & ^3))] + 280*b^2*c^5*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c \\ & *(a + b*x^3))] - 560*a*b*c^4*d*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a \\ & *d)*x^3)/(c*(a + b*x^3))] + 280*a^2*c^3*d^2*x^6*Hypergeometric2F1[2/3, 1, 5 \\ & /3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 252*b^2*c^4*d*x^9*Hypergeometric2F \\ & 1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 504*a*b*c^3*d^2*x^9*Hyp \\ & ergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 252*a^2*c^ \\ & 2*d^3*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] \\ & + 216*b^2*c^3*d^2*x^{12}*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c \\ & *(a + b*x^3))] - 432*a*b*c^2*d^3*x^{12}*Hypergeometric2F1[2/3, 1, 5/3, ((b*c \\ & - a*d)*x^3)/(c*(a + b*x^3))] + 216*a^2*c*d^4*x^{12}*Hypergeometric2F1[2/3, 1, \\ & 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*b^2*c^2*d^3*x^{15}*Hypergeomet \\ & ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 324*a*b*c*d^4*x^{15} \\ & *Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a^ \\ & 2*d^5*x^{15}*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3)) \\ &] + 54*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^2*(-7*c + 6*d*x^3)*Hyperge \\ & ometricPFQ[{-1/3, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 54* \\ & c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^3*HypergeometricPFQ[{-1/3, 2, 2, \\ & 2}, {2/3, 1, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(1820*c^6*x^{13}*(a + b* \\ & x^3)^(2/3)) \end{aligned}$$

IntegrateAlgebraic [C] time = 4.75, size = 577, normalized size = 1.47

$\frac{d^2 \sqrt{bx^3+a}}{dx^2} = \frac{3bdx^2}{2\sqrt{bx^3+a}} + \frac{3a^2}{4(bx^3+a)^{3/2}}$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]

[Out]
$$\begin{aligned} & ((a + b*x^3)^{(1/3)} * (-140*a^4*c^4 - 196*a^3*b*c^4*x^3 + 182*a^4*c^3*d*x^3 - \\ & 8*a^2*b^2*c^4*x^6 + 286*a^3*b*c^3*d*x^6 - 260*a^4*c^2*d^2*x^6 + 12*a*b^3*c^4*x^9 + 26*a^2*b^2*c^3*d*x^9 - \\ & 520*a^3*b*c^2*d^2*x^9 + 455*a^4*c*d^3*x^9 - 36*b^4*c^4*x^{12} - 78*a*b^3*c^3*d*x^{12} - \\ & 260*a^2*b^2*c^2*d^2*x^{12} + 2275*a^3*b*c*d^3*x^{12} - 1820*a^4*d^4*x^{12})) / (1820*a^3*c^5*x^{13}) - \\ & (\text{Sqrt}[-1 - I*\text{Sqrt}[3])/6] * d^3 * (b*c - a*d)^{(4/3)} * \text{ArcTan}[(3*(b*c - a*d)^{(1/3)}*x) / (\text{Sqrt}[3] * (b*c - \\ & a*d)^{(1/3)}*x - (3*I)*c^{(1/3)}*(a + b*x^3)^{(1/3)} - \text{Sqrt}[3]*c^{(1/3)}*(a + b*x^3)^{(1/3)})] / c^{(16/3)} + \\ & ((I/6) * (I*d^3*(b*c - a*d)^{(4/3)} + \text{Sqrt}[3]*d^3*(b*c - a*d)^{(4/3)}) * \text{Log}[2*(b*c - a*d)^{(1/3)}*x + \\ & (1 + I*\text{Sqrt}[3])*c^{(1/3)}*(a + b*x^3)^{(1/3)}] / c^{(16/3)} + ((d^3*(b*c - a*d)^{(4/3)} - I*\text{Sqrt}[3]*d^3*(b*c - a*d)^{(4/3)}) * \\ & \text{Log}[(-2*I)*(b*c - a*d)^{(2/3)}*x^2 + c^{(1/3)}*(b*c - a*d)^{(1/3)}*(I*x - \text{Sqrt}[3]*x) * (a + b*x^3)^{(1/3)} + \\ & (I + \text{Sqrt}[3])*c^{(2/3)}*(a + b*x^3)^{(2/3)}] / (12*c^{(16/3)}) \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^{14}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^{14}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x)`

[Out] `int((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^{14}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{x^{14}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x)`

[Out] `int((a + b*x^3)^(4/3)/(x^14*(c + d*x^3)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(4/3)/x**14/(d*x**3+c),x)`

[Out] Timed out

$$3.455 \quad \int \frac{x^{14}}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$$

Optimal. Leaf size=290

$$-\frac{(a+bx^3)^{2/3}(ad+bc)(a^2d^2+b^2c^2)}{2b^4d^4} + \frac{(a+bx^3)^{5/3}(3a^2d^2+2abcd+b^2c^2)}{5b^4d^3} - \frac{(a+bx^3)^{8/3}(3ad+bc)}{8b^4d^2} + \frac{(a+bx^3)^{11/3}}{11b^4d}$$

Rubi [A] time = 0.32, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 88, 56, 617, 204, 31}

$$\frac{(a+bx^3)^{5/3}(3a^2d^2+2abcd+b^2c^2)}{5b^4d^3} - \frac{(a+bx^3)^{2/3}(ad+bc)(a^2d^2+b^2c^2)}{2b^4d^4} - \frac{(a+bx^3)^{8/3}(3ad+bc)}{8b^4d^2} + \frac{(a+bx^3)^{11/3}}{11b^4d} + \frac{c^4 \log(c+dx^3)}{6d^{14/3}\sqrt[3]{bc-ad}} - \frac{c^4 \log(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3})}{2d^{14/3}\sqrt[3]{bc-ad}} - \frac{c^4 \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{14/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] -((b*c + a*d)*(b^2*c^2 + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^4) + ((b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*(a + b*x^3)^(5/3))/(5*b^4*d^3) - ((b*c + 3*a*d)*(a + b*x^3)^(8/3))/(8*b^4*d^2) + (a + b*x^3)^(11/3)/(11*b^4*d) - (c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)]/Sqrt[3])/Sqrt[3]*d^(14/3)*(b*c - a*d)^(1/3)) + (c^4*Log[c + d*x^3])/(6*d^(14/3)*(b*c - a*d)^(1/3)) - (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(14/3)*(b*c - a*d)^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 56

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])]/; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out]
$$\frac{((a + b*x^3)^{(2/3)}*(-220*b^3*c^3 - 132*a*b^2*c^2*d - 99*a^2*b*c*d^2 - 81*a^3*d^3 + 88*b^3*c^2*d*x^3 + 66*a*b^2*c*d^2*x^3 + 54*a^2*b*d^3*x^3 - 55*b^3*c*d^2*x^6 - 45*a*b^2*d^3*x^6 + 40*b^3*d^3*x^9))/(440*b^4*d^4) - (c^4*ArcTan[1/Sqrt[3] - (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})/(Sqrt[3]*(b*c - a*d)^{(1/3)})])/(Sqrt[3]*d^{(14/3)}*(b*c - a*d)^{(1/3)}) - (c^4*Log[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)})/(3*d^{(14/3)}*(b*c - a*d)^{(1/3)}) + (c^4*Log[(b*c - a*d)^{(2/3)} - d^{(1/3)}*(b*c - a*d)^{(1/3)}*(a + b*x^3)^{(1/3)} + d^{(2/3)}*(a + b*x^3)^{(2/3)})/(6*d^{(14/3)}*(b*c - a*d)^{(1/3)})$$

fricas [A] time = 0.76, size = 1004, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out]
$$\frac{1}{1320} * (220 * (-b*c*d^2 + a*d^3)^{(2/3)} * b^4*c^4 * \log((b*x^3 + a)^{(2/3)} * d^2 + (-b*c*d^2 + a*d^3)^{(1/3)} * (b*x^3 + a)^{(1/3)} * d + (-b*c*d^2 + a*d^3)^{(2/3)}) - 440 * (-b*c*d^2 + a*d^3)^{(2/3)} * b^4*c^4 * \log((b*x^3 + a)^{(1/3)} * d - (-b*c*d^2 + a*d^3)^{(1/3)}) + 660 * \sqrt{1/3} * (b^5*c^5*d - a*b^4*c^4*d^2) * \sqrt{(-b*c*d^2 + a*d^3)^{(1/3)} / (b*c - a*d)} * \log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*\sqrt{1/3} * (2*(-b*c*d^2 + a*d^3)^{(2/3)} * (b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)} * (b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^{(1/3)} * (b*c - a*d)) * \sqrt{(-b*c*d^2 + a*d^3)^{(1/3)} / (b*c - a*d)} - 3 * (-b*c*d^2 + a*d^3)^{(2/3)} * (b*x^3 + a)^{(1/3)}) / (d*x^3 + c)) - 3 * (220*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 - 18*a^3*b*c*d^5 - 81*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b^4*c^2*d^4 - 2*a*b^3*c*d^5 - 9*a^2*b^2*d^6)*x^6 - 2*(44*b^4*c^3*d^3 - 11*a*b^3*c^2*d^4 - 6*a^2*b^2*c*d^5 - 27*a^3*b*d^6)*x^3) * (b*x^3 + a)^{(2/3)} / (b^5*c*d^6 - a*b^4*d^7), \frac{1}{1320} * (220 * (-b*c*d^2 + a*d^3)^{(2/3)} * b^4*c^4 * \log((b*x^3 + a)^{(2/3)} * d^2 + (-b*c*d^2 + a*d^3)^{(1/3)} * (b*x^3 + a)^{(1/3)} * d + (-b*c*d^2 + a*d^3)^{(2/3)}) - 440 * (-b*c*d^2 + a*d^3)^{(2/3)} * b^4*c^4 * \log((b*x^3 + a)^{(1/3)} * d - (-b*c*d^2 + a*d^3)^{(1/3)}) + 1320 * \sqrt{1/3} * (b^5*c^5*d - a*b^4*c^4*d^2) * \sqrt{(-b*c*d^2 + a*d^3)^{(1/3)} / (b*c - a*d)} * \arctan(\sqrt{1/3} * (2*(b*x^3 + a)^{(1/3)} * d + (-b*c*d^2 + a*d^3)^{(1/3)}) * \sqrt{(-b*c*d^2 + a*d^3)^{(1/3)} / (b*c - a*d)}) / d - 3 * (220*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 - 18*a^3*b*c*d^5 - 81*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b^4*c^2*d^4 - 2*a*b^3*c*d^5 - 9*a^2*b^2*d^6)*x^6 - 2*(44*b^4*c^3*d^3 - 11*a*b^3*c^2*d^4 - 6*a^2*b^2*c*d^5 - 27*a^3*b*d^6)*x^3) * (b*x^3 + a)^{(2/3)} / (b^5*c*d^6 - a*b^4*d^7)]$$

giac [A] time = 0.33, size = 454, normalized size = 1.57

$$\frac{b^4 c^4 d^2 \sqrt{\frac{b^2 + a d}{3(b^2 - a d)}} \operatorname{arctan}\left(\frac{\sqrt{3} \sqrt{b^2 + a d} \sqrt{\frac{b^2 + a d}{3(b^2 - a d)}}}{\sqrt{3} \sqrt{b^2 - a d}}\right) + (-b^4 c^4 d^2 + a^2 b^3 c^3 d^3) \sqrt{\frac{b^2 + a d}{3(b^2 - a d)}} \operatorname{arctan}\left(\frac{\sqrt{3} \sqrt{b^2 + a d} \sqrt{\frac{b^2 + a d}{3(b^2 - a d)}}}{\sqrt{3} \sqrt{b^2 - a d}}\right) + \frac{220(b^2 + a d)^2 d^2 \sqrt{1/3} \sqrt{b^5 c^5 d - a b^4 c^4 d^2} \sqrt{(-b^2 c^2 d^2 + a^2 d^3)^{(1/3)} / (b^2 c - a^2 d)}}{440 \sqrt{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁴/(b*x³+a)^(1/3)/(d*x³+c),x, algorithm="giac")

[Out]
$$-1/3*b^{48}*c^4*d^7*(-(b*c - a*d)/d)^{(2/3)}*\log(\text{abs}((b*x^3 + a)^{(1/3)} - (-(b*c - a*d)/d)^{(1/3)}))/((b^{49}*c*d^{11} - a*b^{48}*d^{12}) - (-(b*c*d^2 + a*d^3)^{(2/3)}*c^4*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{(1/3)} + (-(b*c - a*d)/d)^{(1/3)}))/(-(b*c - a*d)/d)^{(1/3)})/(\sqrt{3}*b*c*d^6 - \sqrt{3}*a*d^7) + 1/6*(-b*c*d^2 + a*d^3)^{(2/3)}*c^4*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(-(b*c - a*d)/d)^{(1/3)} + (-(b*c - a*d)/d)^{(2/3)})/(b*c*d^6 - a*d^7) - 1/440*(220*(b*x^3 + a)^{(2/3)}*b^{43}*c^3*d^7 - 88*(b*x^3 + a)^{(5/3)}*b^{42}*c^2*d^8 + 220*(b*x^3 + a)^{(2/3)}*a*b^{42}*c^2*d^8 + 55*(b*x^3 + a)^{(8/3)}*b^{41}*c*d^9 - 176*(b*x^3 + a)^{(5/3)}*a*b^{41}*c*d^9 + 220*(b*x^3 + a)^{(2/3)}*a^2*b^{41}*c*d^9 - 40*(b*x^3 + a)^{(11/3)}*b^{40}*d^{10} + 165*(b*x^3 + a)^{(8/3)}*a*b^{40}*d^{10} - 264*(b*x^3 + a)^{(5/3)}*a^2*b^{40}*d^{10} + 220*(b*x^3 + a)^{(2/3)}*a^3*b^{40}*d^{10})/(b^{44}*d^{11})$$

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁴/(b*x³+a)^(1/3)/(d*x³+c),x)

[Out] int(x¹⁴/(b*x³+a)^(1/3)/(d*x³+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁴/(b*x³+a)^(1/3)/(d*x³+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 5.11, size = 438, normalized size = 1.51

$$\left(\frac{6a^2}{5b^4d} + \frac{\left(\frac{a}{b^2} + \frac{d^2c^2}{5b^4d}\right)(b^3c - ab^4d)}{5b^4d} \right) (bx^3 + a)^{5/3} - \left(\frac{a}{2b^4d} + \frac{b^3c - ab^4d}{8b^4d^2} \right) (bx^3 + a)^{5/3} - (bx^3 + a)^{2/3} \left(\frac{2a^3}{b^4d} + \frac{\left(\frac{6a^2}{5b^4} + \frac{\left(\frac{a}{b^2} + \frac{d^2c^2}{5b^4d}\right)(b^3c - ab^4d)}{5b^4d}\right)(b^3c - ab^4d)}{2b^4d} \right) + \frac{(bx^3 + a)^{1/3}}{11b^4d} + \frac{c^4 \ln\left(\frac{d(b^2x^3)^{1/3} - d^2c^2}{d^2b^2}\right)}{3d^{6/3}(bd - bc)^{1/3}} - \frac{\ln\left(\frac{d(b^2x^3)^{1/3} - d^2c^2}{d^2b^2}\right)}{6d^{6/3}(bd - bc)^{1/3}} \left(c^4 + \sqrt{3}c^3i \right) + \frac{c^4 \ln\left(\frac{d(b^2x^3)^{1/3} - d^2c^2}{d^2b^2}\right)}{d^{6/3}(bd - bc)^{1/3}} \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] ((6*a^2)/(5*b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c - a*b^4*d))/(5*b^4*d))*(a + b*x^3)^(5/3) - (a/(2*b^4*d) + (b^5*c - a*b^4*d)/(8*b^8*d^2))*(a + b*x^3)^(8/3) - (a + b*x^3)^(2/3)*((2*a^3)/(b^4*d) + ((6*a^2)/(b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c - a*b^4*d))/(b^4*d))*(b^5*c - a*b^4*d))/(2*b^4*d) + (a + b*x^3)^(11/3)/(11*b^4*d) + (c^4*log((c^8*(a + b*x^3)^(1/3))/d^7 - (c^8*(a*d - b*c)^(1/3))/d^(22/3)))/(3*d^(14/3)*(a*d - b*c)^(1/3)) - (log((c^8*(a + b*x^3)^(1/3))/d^7 - (c^8*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(22/3)))*(3^(1/2)*c^4*1i + c^4))/(6*d^(14/3)*(a*d - b*c)^(1/3)) + (c^4*log((c^8*(a + b*x^3)^(1/3))/d^7 - (c^8*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(22/3)))*((3^(1/2)*1i)/6 - 1/6))/(d^(14/3)*(a*d - b*c)^(1/3))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^{14}}{\sqrt[3]{a + bx^3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**14/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

$$3.456 \quad \int \frac{x^{11}}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$$

Optimal. Leaf size=244

$$\frac{(a+bx^3)^{2/3}(a^2d^2+abcd+b^2c^2)}{2b^3d^3} - \frac{(a+bx^3)^{5/3}(2ad+bc)}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} - \frac{c^3 \log(c+dx^3)}{6d^{11/3}\sqrt[3]{bc-ad}} + \frac{c^3 \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{a+bx^3}\right)}{2d^{11/3}\sqrt[3]{bc-ad}}$$

Rubi [A] time = 0.24, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 88, 56, 617, 204, 31}

$$\frac{(a+bx^3)^{2/3}(a^2d^2+abcd+b^2c^2)}{2b^3d^3} - \frac{(a+bx^3)^{5/3}(2ad+bc)}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} - \frac{c^3 \log(c+dx^3)}{6d^{11/3}\sqrt[3]{bc-ad}} + \frac{c^3 \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{a+bx^3}\right)}{2d^{11/3}\sqrt[3]{bc-ad}} + \frac{c^3 \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(5/3))/(5*b^3*d^2) + (a + b*x^3)^(8/3)/(8*b^3*d) + (c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/Sqrt[3]*d^(11/3)*(b*c - a*d)^(1/3) - (c^3*Log[c + d*x^3])/(6*d^(11/3)*(b*c - a*d)^(1/3)) + (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(11/3)*(b*c - a*d)^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(−1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 56

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 88


```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{\sqrt[3]{a+bx^3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{b^2c^2 + abcd + a^2d^2}{b^2d^3\sqrt[3]{a+bx}} + \frac{(-bc-2ad)(a+bx)^{2/3}}{b^2d^2} + \frac{(a+bx)^{5/3}}{b^2d} - \frac{c^3}{d^3\sqrt[3]{a+bx}} \right) dx, x, x^3 \right) \\
&= \frac{(b^2c^2 + abcd + a^2d^2)(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} - \frac{c^3 \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}} dx, x, x^3 \right)}{d^3} \\
&= \frac{(b^2c^2 + abcd + a^2d^2)(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} - \frac{c^3 \log \left(\frac{\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt[3]{a+bx^3} - \sqrt[3]{a}} \right)}{6d^{11/3}} \\
&= \frac{(b^2c^2 + abcd + a^2d^2)(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} - \frac{c^3 \log \left(\frac{\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt[3]{a+bx^3} - \sqrt[3]{a}} \right)}{6d^{11/3}} \\
&= \frac{(b^2c^2 + abcd + a^2d^2)(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} + \frac{c^3 \tan^{-1} \left(\frac{\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt[3]{a+bx^3} - \sqrt[3]{a}} \right)}{\sqrt[3]{3}d^{11/3}}
\end{aligned}$$

Mathematica [C] time = 0.14, size = 145, normalized size = 0.59

$$\frac{(a+bx^3)^{2/3} \left(9a^3d^3 + 3a^2bd^2(c-2dx^3) + 20b^3c^3 {}_2F_1 \left(\frac{2}{3}, 1; \frac{5}{3}; \frac{d(bx^3+a)}{ad-bc} \right) + ab^2d(8c^2-2cdx^3+5d^2x^6) + b^3c(-20c^2+8cdx^3-5d^2x^6) \right)}{40b^3d^3(bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)), x]

[Out] -1/40*((a + b*x^3)^(2/3)*(9*a^3*d^3 + 3*a^2*b*d^2*(c - 2*d*x^3) + b^3*c*(-20*c^2 + 8*c*d*x^3 - 5*d^2*x^6) + a*b^2*d*(8*c^2 - 2*c*d*x^3 + 5*d^2*x^6) + 20*b^3*c^3*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-b*c) + a*d]))/(b^3*d^3*(b*c - a*d))

IntegrateAlgebraic [A] time = 0.42, size = 284, normalized size = 1.16

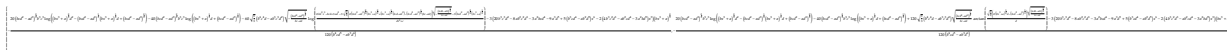
$$\frac{(a+bx^3)^{2/3} (9a^2d^2 + 12abcd - 6abd^2x^3 + 20b^2c^2 - 8b^2cdx^3 + 5b^2d^2x^6)}{40b^3d^3} + \frac{c^3 \log \left(\frac{\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}}{3d^{1/3}\sqrt[3]{bc-ad}} \right)}{3d^{1/3}\sqrt[3]{bc-ad}} - \frac{c^3 \log \left(-\sqrt[3]{d}\sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3} \right)}{6d^{11/3}\sqrt[3]{bc-ad}} + \frac{c^3 \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}} \right)}{\sqrt{3}d^{11/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out]
$$\frac{(a + b x^3)^{2/3} (20 b^2 c^2 + 12 a b c d + 9 a^2 d^2 - 8 b^2 c d x^3 - 6 a b d^2 x^3 + 5 b^2 d^2 x^6)}{(40 b^3 d^3) + (c^3 \operatorname{ArcTan}[1/\sqrt{3} - (2 d^{1/3} (a + b x^3)^{1/3}) / (\sqrt{3} (b c - a d)^{1/3})]) / (\sqrt{3} d^{11/3} (b c - a d)^{1/3})} + \frac{(c^3 \operatorname{Log}[(b c - a d)^{1/3} + d^{1/3} (a + b x^3)^{1/3}]) / (3 d^{11/3} (b c - a d)^{1/3}) - (c^3 \operatorname{Log}[(b c - a d)^{2/3} - d^{1/3} (b c - a d)^{1/3} (a + b x^3)^{1/3} + d^{2/3} (a + b x^3)^{2/3}]) / (6 d^{11/3} (b c - a d)^{1/3})}{(40 b^3 d^3) + (c^3 \operatorname{ArcTan}[1/\sqrt{3} - (2 d^{1/3} (a + b x^3)^{1/3}) / (\sqrt{3} (b c - a d)^{1/3})]) / (\sqrt{3} d^{11/3} (b c - a d)^{1/3})}$$

fricas [A] time = 0.74, size = 873, normalized size = 3.58



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/120 * (20 * (b * c * d^2 - a * d^3)^{2/3} * b^3 * c^3 * \log((b * x^3 + a)^{2/3} * d^2 - (b * c * d^2 - a * d^3)^{1/3} * (b * x^3 + a)^{1/3} * d + (b * c * d^2 - a * d^3)^{2/3}) - 40 * (b * c * d^2 - a * d^3)^{2/3} * b^3 * c^3 * \log((b * x^3 + a)^{1/3} * d + (b * c * d^2 - a * d^3)^{1/3}) - 60 * \sqrt{1/3} * (b^4 * c^4 * d - a * b^3 * c^3 * d^2) * \sqrt{- (b * c * d^2 - a * d^3)^{1/3} / (b * c - a * d)} * \log((2 * b * d^2 * x^3 - b * c * d + 3 * a * d^2 - 3 * \sqrt{1/3} * (2 * (b * c * d^2 - a * d^3)^{2/3} * (b * x^3 + a)^{2/3} + (b * x^3 + a)^{1/3} * (b * c * d - a * d^2) - (b * c * d^2 - a * d^3)^{1/3} * (b * c - a * d)) * \sqrt{- (b * c * d^2 - a * d^3)^{1/3} / (b * c - a * d)} - 3 * (b * c * d^2 - a * d^3)^{2/3} * (b * x^3 + a)^{1/3}) / (d * x^3 + c) - 3 * (20 * b^3 * c^3 * d^2 - 8 * a * b^2 * c^2 * d^3 - 3 * a^2 * b * c * d^4 - 9 * a^3 * d^5 + 5 * (b^3 * c * d^4 - a * b^2 * d^5) * x^6 - 2 * (4 * b^3 * c^2 * d^3 - a * b^2 * c * d^4 - 3 * a^2 * b * d^5) * x^3) * (b * x^3 + a)^{2/3} / (b^4 * c * d^5 - a * b^3 * d^6), \\ & -1/120 * (20 * (b * c * d^2 - a * d^3)^{2/3} * b^3 * c^3 * \log((b * x^3 + a)^{2/3} * d^2 - (b * c * d^2 - a * d^3)^{1/3} * (b * x^3 + a)^{1/3} * d + (b * c * d^2 - a * d^3)^{2/3}) - 40 * (b * c * d^2 - a * d^3)^{2/3} * b^3 * c^3 * \log((b * x^3 + a)^{1/3} * d + (b * c * d^2 - a * d^3)^{1/3}) + 120 * \sqrt{1/3} * (b^4 * c^4 * d - a * b^3 * c^3 * d^2) * \sqrt{(b * c * d^2 - a * d^3)^{1/3} / (b * c - a * d)} * \arctan(\sqrt{1/3} * (2 * (b * x^3 + a)^{1/3} * d - (b * c * d^2 - a * d^3)^{1/3}) * \sqrt{(b * c * d^2 - a * d^3)^{1/3} / (b * c - a * d)}) / d - 3 * (20 * b^3 * c^3 * d^2 - 8 * a * b^2 * c^2 * d^3 - 3 * a^2 * b * c * d^4 - 9 * a^3 * d^5 + 5 * (b^3 * c * d^4 - a * b^2 * d^5) * x^6 - 2 * (4 * b^3 * c^2 * d^3 - a * b^2 * c * d^4 - 3 * a^2 * b * d^5) * x^3) * (b * x^3 + a)^{2/3} / (b^4 * c * d^5 - a * b^3 * d^6)] \end{aligned}$$

giac [A] time = 0.28, size = 371, normalized size = 1.52

$$\frac{b^2 c^3 d^6 \left(-\frac{b c d^2}{3(b^2 c d^2 - a b^2 d^2)} \log\left(\frac{(b x^3 + a)^{2/3} - \left(-\frac{b c d^2}{3} \right)^{1/3}}{(b x^3 + a)^{1/3}} \right) + \frac{(-b c d^2 + a d^3)^{2/3} \arctan\left(\frac{\sqrt{3} \left(2(b x^3 + a)^{1/3} d - (b c d^2 - a d^3)^{1/3} \right)}{3 \left(-\frac{b c d^2}{3} \right)^{1/3}} \right)}{\sqrt{3} b c d^2 - \sqrt{3} a d^3} - \frac{(-b c d^2 + a d^3)^{2/3} \log\left((b x^3 + a)^{2/3} + (b x^3 + a)^{1/3} \left(-\frac{b c d^2}{3} \right)^{1/3} + \left(-\frac{b c d^2}{3} \right)^{2/3} \right)}{6 (b c d^2 - a d^3)} + \frac{20 (b x^3 + a)^{1/3} b^2 c^2 d^6 - 8 (b x^3 + a)^{2/3} b^2 c^2 d^6 + 20 (b x^3 + a)^{1/3} a b^2 c^2 d^6 + 5 (b x^3 + a)^{2/3} b^2 c^2 d^6 - 16 (b x^3 + a)^{1/3} a b^2 c^2 d^6 + 20 (b x^3 + a)^{2/3} a b^2 c^2 d^6}{40 b^2 c^2 d^6}}{3(b^2 c d^2 - a b^2 d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

```
[Out] 1/3*b^27*c^3*d^5*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^28*c*d^8 - a*b^27*d^9) + (-b*c*d^2 + a*d^3)^(2/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3)))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^5 - sqrt(3)*a*d^6) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^5 - a*d^6) + 1/40*(20*(b*x^3 + a)^(2/3)*b^23*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^22*c*d^6 + 20*(b*x^3 + a)^(2/3)*a*b^22*c*d^6 + 5*(b*x^3 + a)^(8/3)*b^21*d^7 - 16*(b*x^3 + a)^(5/3)*a*b^21*d^7 + 20*(b*x^3 + a)^(2/3)*a^2*b^21*d^7)/(b^24*d^8)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.09, size = 339, normalized size = 1.39

$$\left(\frac{3a^2}{2b^3d} + \frac{\left(\frac{3a}{b^2d} + \frac{b^4c - a^2d}{b^2d}\right) (b^4c - a^2d)}{2b^3d} \right) (bx^3 + a)^{2/3} - \left(\frac{3a}{5b^3d} + \frac{b^4c - a^2d}{5b^2d^2} \right) (bx^3 + a)^{5/3} + \frac{(bx^3 + a)^{8/3}}{8b^3d} - \frac{c^3 \ln\left(\frac{c^2(bx^3 + a)^{1/3} + b^2c - a^2d}{d^2} + \frac{b^2c - a^2d}{d^2(bx^3 + a)^{1/3}}\right)}{3d^{1/3}(ad - bc)^{1/3}} + \frac{\ln\left(\frac{c^2(bx^3 + a)^{1/3} - c^2(1 + \sqrt{3})^2(ad - bc)^{1/3}}{4d^{4/3}}\right)}{6d^{1/3}(ad - bc)^{1/3}} (c^3 + \sqrt{3}c^3i) - \frac{c^3 \ln\left(\frac{c^2(bx^3 + a)^{1/3} + c^2\left(\frac{1 + \sqrt{3}}{2}\right)(ad - bc)^{1/3}}{3d^{4/3}}\right)}{3d^{1/3}(ad - bc)^{1/3}} \left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] ((3*a^2)/(2*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))* (b^4*c - a*b^3*d))/(2*b^3*d)) * (a + b*x^3)^(2/3) - ((3*a)/(5*b^3*d) + (b^4*c - a*b^3*d)/(5*b^6*d^2)) * (a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^3*d) - (c^3*lo
```

$$g\left(\frac{c^6(a + bx^3)^{1/3}}{d^5} + \frac{(bc^7 - ac^6d)}{d^{16/3}(ad - bc)^{2/3}}\right) / (3d^{11/3}(ad - bc)^{1/3}) + \left(\log\left(\frac{c^6(a + bx^3)^{1/3}}{d^5} - \frac{c^6(3^{1/2}i + 1)^2(ad - bc)^{1/3}}{4d^{16/3}}\right)\right) \cdot \frac{3^{1/2}c^3i + c^3}{6d^{11/3}(ad - bc)^{1/3}} - \left(\frac{c^3 \log\left(\frac{c^6(a + bx^3)^{1/3}}{d^5} + \frac{c^6(3^{1/2}i)/2 + 1/2}{d^{16/3}}(ad - bc)^{1/3}\right)}{3d^{11/3}(ad - bc)^{1/3}}\right)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt[3]{a + bx^3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11/(b*x**3+a)**(1/3)/(d*x**3+c), x)

[Out] Integral(x**11/((a + b*x**3)**(1/3)*(c + d*x**3)), x)

$$3.457 \quad \int \frac{x^8}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$$

Optimal. Leaf size=203

$$-\frac{(a+bx^3)^{2/3}(ad+bc)}{2b^2d^2} + \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{c^2 \log(c+dx^3)}{6d^{8/3}\sqrt[3]{bc-ad}} - \frac{c^2 \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}} - \frac{c^2 \tan^{-1}\left(\frac{1-2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{8/3}\sqrt[3]{bc-ad}}$$

Rubi [A] time = 0.21, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 88, 56, 617, 204, 31}

$$-\frac{(a+bx^3)^{2/3}(ad+bc)}{2b^2d^2} + \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{c^2 \log(c+dx^3)}{6d^{8/3}\sqrt[3]{bc-ad}} - \frac{c^2 \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}} - \frac{c^2 \tan^{-1}\left(\frac{1-2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{8/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)), x]

[Out] -((b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^2*d^2) + (a + b*x^3)^(5/3)/(5*b^2*d) - (c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d))/Sqrt[3]])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(1/3)) + (c^2*Log[c + d*x^3])/(6*d^(8/3)*(b*c - a*d)^(1/3)) - (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(8/3)*(b*c - a*d)^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 56

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x]

$x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{IntegersQ}[m, n] \ \&\& \ (\text{IntegerQ}[p] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{GeQ}[n, -1]))$

Rule 204

$\text{Int}[(a_ + (b_ \cdot x)^2)^{-1}, x_Symbol] \ :> \ -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2] \cdot x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 446

$\text{Int}[(x_)^{(m_)} \cdot ((a_ + (b_ \cdot x)^{n_})^{(p_)} \cdot ((c_ + (d_ \cdot x)^{n_})^{(q_)}), x_Symbol] \ :> \ \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) \cdot (a + b \cdot x)^p \cdot (c + d \cdot x)^q}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 617

$\text{Int}[(a_ + (b_ \cdot x) + (c_ \cdot x)^2)^{-1}, x_Symbol] \ :> \ \text{With}[\{q = 1 - 4 \cdot \text{Simplify}[(a \cdot c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2)], x], x, 1 + (2 \cdot c \cdot x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4 \cdot a \cdot c]) /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\sqrt[3]{a+bx^3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{-bc-ad}{bd^2 \sqrt[3]{a+bx}} + \frac{(a+bx)^{2/3}}{bd} + \frac{c^2}{d^2 \sqrt[3]{a+bx}(c+dx)} \right) dx, x, x^3 \right) \\
&= -\frac{(bc+ad)(a+bx^3)^{2/3}}{2b^2d^2} + \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{c^2 \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right)}{3d^2} \\
&= -\frac{(bc+ad)(a+bx^3)^{2/3}}{2b^2d^2} + \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{c^2 \log(c+dx^3)}{6d^{8/3} \sqrt[3]{bc-ad}} + \frac{c^2 \text{Subst} \left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc}}{\sqrt[3]{a+bx^3}}} dx, x, x^3 \right)}{2d^{8/3} \sqrt[3]{bc-ad}} \\
&= -\frac{(bc+ad)(a+bx^3)^{2/3}}{2b^2d^2} + \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{c^2 \log(c+dx^3)}{6d^{8/3} \sqrt[3]{bc-ad}} - \frac{c^2 \log(\sqrt[3]{bc-ad} + \sqrt[3]{d(a+bx^3)})}{2d^{8/3} \sqrt[3]{bc-ad}} \\
&= -\frac{(bc+ad)(a+bx^3)^{2/3}}{2b^2d^2} + \frac{(a+bx^3)^{5/3}}{5b^2d} - \frac{c^2 \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}} \right)}{\sqrt{3} d^{8/3} \sqrt[3]{bc-ad}} + \frac{c^2 \log(c+dx^3)}{6d^{8/3} \sqrt[3]{bc-ad}}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 103, normalized size = 0.51

$$\frac{(a+bx^3)^{2/3} \left(3a^2d^2 + 5b^2c^2 {}_2F_1 \left(\frac{2}{3}, 1; \frac{5}{3}; \frac{d(bx^3+a)}{ad-bc} \right) + 2abd(c-dx^3) + b^2c(2dx^3-5c) \right)}{10b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)), x]

[Out] ((a + b*x^3)^(2/3)*(3*a^2*d^2 + 2*a*b*d*(c - d*x^3) + b^2*c*(-5*c + 2*d*x^3) + 5*b^2*c^2*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-b*c) + a*d]))/(10*b^2*d^2*(b*c - a*d))

IntegrateAlgebraic [A] time = 0.40, size = 247, normalized size = 1.22

$$\frac{(a+bx^3)^{2/3}(-3ad-5bc+2bdx^3)}{10b^2d^2} - \frac{c^2 \log(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3})}{3d^{8/3} \sqrt[3]{bc-ad}} + \frac{c^2 \log(-\sqrt[3]{d}\sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3})}{6d^{8/3} \sqrt[3]{bc-ad}} - \frac{c^2 \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}} \right)}{\sqrt{3} d^{8/3} \sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] ((a + b*x^3)^(2/3)*(-5*b*c - 3*a*d + 2*b*d*x^3)/(10*b^2*d^2) - (c^2*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(1/3)) - (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(8/3)*(b*c - a*d)^(1/3)) + (c^2*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(8/3)*(b*c - a*d)^(1/3))

fricas [B] time = 0.79, size = 768, normalized size = 3.78

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] [1/30*(5*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) + 15*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(5*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3*a^2*d^4 - 2*(b^2*c*d^3 - a*b*d^4)*x^3)*(b*x^3 + a)^(2/3)/(b^3*c*d^4 - a*b^2*d^5), 1/30*(5*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) + 30*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)))/d - 3*(5*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3*a^2*d^4 - 2*(b^2*c*d^3 - a*b*d^4)*x^3)*(b*x^3 + a)^(2/3)/(b^3*c*d^4 - a*b^2*d^5)]

giac [A] time = 0.26, size = 313, normalized size = 1.54

$$\frac{b^2 c^2 d^3 \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}} \log\left(\left(bx^3+a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3(b^3 cd^3 - ab^2 d^4)} - \frac{(-bcd^2 + ad^3)^{\frac{2}{3}} c^2 \arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^4 - \sqrt{3}ad^5} + \frac{(-bcd^2 + ad^3)^{\frac{2}{3}} c^2 \log\left(\left(bx^3+a\right)^{\frac{1}{3}} + \left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6(bcd^4 - ad^5)} - \frac{5(bx^3+a)^{\frac{2}{3}} b^3 cd^3 - 2(bx^3+a)^{\frac{5}{3}} b^2 d^4 + 5(bx^3+a)^{\frac{2}{3}} ab^2 d^4}{10b^3 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out] -1/3*b^12*c^2*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-b*c - a*d)/d)^(1/3))/(b^13*c*d^5 - a*b^12*d^6) - (-b*c*d^2 + a*d^3)^(2/3)*c^2

*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-b*c - a*d)/d)^(1/3))/(-b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-b*c - a*d)/d)^(1/3) + (-b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) - 1/10*(5*(b*x^3 + a)^(2/3)*b^9*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^8*d^4 + 5*(b*x^3 + a)^(2/3)*a*b^8*d^4)/(b^10*d^5)

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x)

[Out] int(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 5.11, size = 267, normalized size = 1.32

$$\frac{(bx^3 + a)^{5/3}}{5b^2d} - \left(\frac{a}{b^2d} + \frac{b^3c - ab^2d}{2b^4d^2}\right)(bx^3 + a)^{2/3} + \frac{c^2 \ln\left(\frac{c^4(bx^3+a)^{1/3}}{d^3} + \frac{bc^5 - ac^4d}{d^{10/3}(ad-bc)^{2/3}}\right)}{3d^{8/3}(ad-bc)^{1/3}} - \frac{\ln\left(\frac{c^4(bx^3+a)^{1/3}}{d^3} - \frac{c^4(1+\sqrt{3}i)^2(ad-bc)^{1/3}}{4d^{10/3}}\right)(c^2 + \sqrt{3}c^2i)}{6d^{8/3}(ad-bc)^{1/3}} + \frac{c^2 \ln\left(\frac{c^4(bx^3+a)^{1/3}}{d^3} - \frac{c^4(-1+\sqrt{3}i)^2(ad-bc)^{1/3}}{4d^{10/3}}\right)\left(-\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)}{d^{8/3}(ad-bc)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x)

[Out] (a + b*x^3)^(5/3)/(5*b^2*d) - (a/(b^2*d) + (b^3*c - a*b^2*d)/(2*b^4*d^2))* (a + b*x^3)^(2/3) + (c^2*log((c^4*(a + b*x^3)^(1/3))/d^3 + (b*c^5 - a*c^4*d)/(d^(10/3)*(a*d - b*c)^(2/3))))/(3*d^(8/3)*(a*d - b*c)^(1/3)) - (log((c^4*(a + b*x^3)^(1/3))/d^3 - (c^4*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(10/3))))*(3^(1/2)*c^2*1i + c^2))/(6*d^(8/3)*(a*d - b*c)^(1/3)) + (c^2*log((c^4*(a + b*x^3)^(1/3))/d^3 - (c^4*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(10/3))))*(3^(1/2)*1i)/6 - 1/6))/(d^(8/3)*(a*d - b*c)^(1/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt[3]{a + bx^3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(b*x**3+a)**(1/3)/(d*x**3+c), x)

[Out] Integral(x**8/((a + b*x**3)**(1/3)*(c + d*x**3)), x)

$$3.458 \quad \int \frac{x^5}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$$

Optimal. Leaf size=168

$$\frac{c \log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}} + \frac{c \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}\sqrt[3]{bc-ad}} + \frac{(a+bx^3)^{2/3}}{2bd}$$

Rubi [A] time = 0.16, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 80, 56, 617, 204, 31}

$$\frac{c \log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}} + \frac{c \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}\sqrt[3]{bc-ad}} + \frac{(a+bx^3)^{2/3}}{2bd}$$

Antiderivative was successfully verified.

[In] Int[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] (a + b*x^3)^(2/3)/(2*b*d) + (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3)))/(b*c - a*d)^(1/3)]/Sqrt[3])/Sqrt[3]*d^(5/3)*(b*c - a*d)^(1/3) - (c*Log[c + d*x^3])/(6*d^(5/3)*(b*c - a*d)^(1/3)) + (c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(5/3)*(b*c - a*d)^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 56

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p

+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt[3]{a+bx^3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right) \\
&= \frac{(a+bx^3)^{2/3}}{2bd} - \frac{c \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right)}{3d} \\
&= \frac{(a+bx^3)^{2/3}}{2bd} - \frac{c \log(c+dx^3)}{6d^{5/3} \sqrt[3]{bc-ad}} - \frac{c \text{Subst} \left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}} + x^2} dx, x, \sqrt[3]{a+bx^3} \right)}{2d^2} + \dots \\
&= \frac{(a+bx^3)^{2/3}}{2bd} - \frac{c \log(c+dx^3)}{6d^{5/3} \sqrt[3]{bc-ad}} + \frac{c \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{2d^{5/3} \sqrt[3]{bc-ad}} - \frac{c \text{Subst} \left(\int \frac{1}{-3-} \right)}{d^{5/3}} \\
&= \frac{(a+bx^3)^{2/3}}{2bd} + \frac{c \tan^{-1} \left(\frac{1 - 2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad} \sqrt{3}} \right)}{\sqrt{3} d^{5/3} \sqrt[3]{bc-ad}} - \frac{c \log(c+dx^3)}{6d^{5/3} \sqrt[3]{bc-ad}} + \frac{c \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{2d^{5/3} \sqrt[3]{bc-ad}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 69, normalized size = 0.41

$$\frac{(a+bx^3)^{2/3} \left(bc {}_2F_1 \left(\frac{2}{3}, 1; \frac{5}{3}; \frac{d(bx^3+a)}{ad-bc} \right) + ad - bc \right)}{2bd(bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] -1/2*((a + b*x^3)^(2/3)*(-(b*c) + a*d + b*c*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)])))/(b*d*(b*c - a*d))

IntegrateAlgebraic [A] time = 0.21, size = 224, normalized size = 1.33

$$\frac{c \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{3d^{5/3} \sqrt[3]{bc-ad}} - \frac{c \log(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3})}{6d^{5/3} \sqrt[3]{bc-ad}} + \frac{c \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt{3} \sqrt[3]{bc-ad}} \right)}{\sqrt{3} d^{5/3} \sqrt[3]{bc-ad}} + \frac{(a+bx^3)^{2/3}}{2bd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

```
[Out] (a + b*x^3)^(2/3)/(2*b*d) + (c*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(1/3)) + (c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(3*d^(5/3)*(b*c - a*d)^(1/3)) - (c*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/(6*d^(5/3)*(b*c - a*d)^(1/3))
```

fricas [B] time = 0.73, size = 667, normalized size = 3.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c), x, algorithm="fricas")
```

```
[Out] [-1/6*((b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*sqrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(b*c*d^2 - a*d^3)*(b*x^3 + a)^(2/3)/(b^2*c*d^3 - a*b*d^4), -1/6*((b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 6*sqrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))/d) - 3*(b*c*d^2 - a*d^3)*(b*x^3 + a)^(2/3)/(b^2*c*d^3 - a*b*d^4)]
```

giac [A] time = 0.28, size = 257, normalized size = 1.53

$$\frac{2bcd\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}} \log\left(\left(bx^3+a\right)^{\frac{1}{3}} - \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{bcd^2-ad^3} + \frac{6(-bcd^2+ad^3)^{\frac{2}{3}} bc \arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}} bc \log\left(\left(bx^3+a\right)^{\frac{2}{3}} + \left(bx^3+a\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{bcd^3-ad^4} + \frac{3\left(bx^3+a\right)^{\frac{2}{3}}}{d}$$

6 b

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c), x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^2 - a*d^3) + 6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*log((
```

$b*x^3 + a)^{2/3} + (b*x^3 + a)^{1/3}*(-(b*c - a*d)/d)^{1/3} + (-(b*c - a*d)/d)^{2/3})/(b*c*d^3 - a*d^4) + 3*(b*x^3 + a)^{2/3}/d)/b$

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)`

[Out] `int(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 5.10, size = 219, normalized size = 1.30

$$\frac{(bx^3 + a)^{2/3}}{2bd} + \frac{\ln\left(\frac{c^2(bx^3+a)^{1/3}}{d} - \frac{c^2(-1+\sqrt{3}i)^2(ad-bc)^{1/3}}{4d^{4/3}}\right)(c-\sqrt{3}ci)}{6d^{5/3}(ad-bc)^{1/3}} + \frac{\ln\left(\frac{c^2(bx^3+a)^{1/3}}{d} - \frac{c^2(1+\sqrt{3}i)^2(ad-bc)^{1/3}}{4d^{4/3}}\right)(c+\sqrt{3}ci)}{6d^{5/3}(ad-bc)^{1/3}} - \frac{c \ln\left(\frac{c^2(bx^3+a)^{1/3}}{d} + \frac{bc^3-ad^2d}{d^{4/3}(ad-bc)^{2/3}}\right)}{3d^{5/3}(ad-bc)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x)`

[Out] $(a + b*x^3)^{2/3}/(2*b*d) + (\log((c^2*(a + b*x^3)^{1/3})/d - (c^2*(3^{1/2}*1i - 1)^2*(a*d - b*c)^{1/3})/(4*d^{4/3}))* (c - 3^{1/2}*c*1i))/(6*d^{5/3}*(a*d - b*c)^{1/3}) + (\log((c^2*(a + b*x^3)^{1/3})/d - (c^2*(3^{1/2}*1i + 1)^2*(a*d - b*c)^{1/3})/(4*d^{4/3}))* (c + 3^{1/2}*c*1i))/(6*d^{5/3}*(a*d - b*c)^{1/3}) - (c*\log((c^2*(a + b*x^3)^{1/3})/d + (b*c^3 - a*c^2*d)/(d^{4/3})*(a*d - b*c)^{2/3}))/ (3*d^{5/3}*(a*d - b*c)^{1/3})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt[3]{a + bx^3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

$$3.459 \quad \int \frac{x^2}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$$

Optimal. Leaf size=145

$$\frac{\log(c+dx^3)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}} - \frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}}$$

Rubi [A] time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {444, 56, 617, 204, 31}

$$\frac{\log(c+dx^3)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}} - \frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] -(ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sqrt[3]*d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(6*d^(2/3)*(b*c - a*d)^(1/3)) - Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(2/3)*(b*c - a*d)^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 56

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 444

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{\sqrt[3]{a+bx^3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right) \\
 &= \frac{\log(c+dx^3)}{6d^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst} \left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}} + x^2} dx, x, \sqrt[3]{a+bx^3} \right)}{2d} - \frac{\text{Subst} \left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}} dx, x, \sqrt[3]{a+bx^3} \right)}{2d^{2/3}\sqrt[3]{bc-ad}} \\
 &= \frac{\log(c+dx^3)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2\sqrt[3]{d}}{\sqrt[3]{b}} \sqrt[3]{a+bx^3} \right)}{d^{2/3}\sqrt[3]{bc-ad}} \\
 &= -\frac{\tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}} \right)}{\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}} + \frac{\log(c+dx^3)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}
 \end{aligned}$$

Mathematica [C] time = 0.02, size = 50, normalized size = 0.34

$$\frac{(a+bx^3)^{2/3} {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{d(bx^3+a)}{ad-bc}\right)}{2bc-2ad}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] ((a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)]/(2*b*c - 2*a*d))

IntegrateAlgebraic [A] time = 0.17, size = 201, normalized size = 1.39

$$-\frac{\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3}\right)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] -(ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*d^(2/3)*(b*c - a*d)^(1/3))) - Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(3*d^(2/3)*(b*c - a*d)^(1/3)) + Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/(6*d^(2/3)*(b*c - a*d)^(1/3))

fricas [B] time = 0.82, size = 592, normalized size = 4.08

$$\frac{\sqrt{3}\sqrt[3]{bc-ad}\sqrt{\frac{1}{3}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}}}{3d^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3}\right)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] [1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) + (-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)))/(b*c*d^2 - a*d^3), 1/6*(6*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d) + (-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)))/(b*c*d^2 - a*d^3)]

giac [A] time = 0.29, size = 226, normalized size = 1.56

$$\frac{(-bcd^2 + ad^3)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2 - \sqrt{3}ad^3} + \frac{(-bcd^2 + ad^3)^{\frac{2}{3}} \log\left(\left(bx^3+a\right)^{\frac{2}{3}} + \left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6(bcd^2 - ad^3)} - \frac{\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}} \log\left(\left(bx^3+a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out]
$$\begin{aligned} & -(-b*c*d^2 + a*d^3)^{(2/3)}*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{(1/3)} + (-b*c \\ & - a*d)/d)^{(1/3)})/(-b*c - a*d)/d)^{(1/3)})/(\sqrt{3}*b*c*d^2 - \sqrt{3}*a*d^3) \\ & + 1/6*(-b*c*d^2 + a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(- \\ & (b*c - a*d)/d)^{(1/3)} + (-b*c - a*d)/d)^{(2/3)})/(b*c*d^2 - a*d^3) - 1/3*(-(b \\ & *c - a*d)/d)^{(2/3)}*\log(\text{abs}((b*x^3 + a)^{(1/3)} - (-b*c - a*d)/d)^{(1/3)})/(b* \\ & c - a*d) \end{aligned}$$

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)

[Out] int(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.93, size = 208, normalized size = 1.43

$$\frac{\ln\left(d(bx^3+a)^{1/3} - \frac{9ad^3-9bcd^2}{9d^{4/3}(ad-bc)^{2/3}}\right)}{3d^{2/3}(ad-bc)^{1/3}} + \frac{\ln\left(d(bx^3+a)^{1/3} - \frac{(-1+\sqrt{3}1i)^2(9ad^3-9bcd^2)}{36d^{4/3}(ad-bc)^{2/3}}\right)(-1+\sqrt{3}1i)}{6d^{2/3}(ad-bc)^{1/3}} - \frac{\ln\left(d(bx^3+a)^{1/3} - \frac{(1+\sqrt{3}1i)^2(9ad^3-9bcd^2)}{36d^{4/3}(ad-bc)^{2/3}}\right)(1+\sqrt{3}1i)}{6d^{2/3}(ad-bc)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x)`

[Out] $\log(d*(a + b*x^3)^{(1/3)} - (9*a*d^3 - 9*b*c*d^2)/(9*d^{(4/3)}*(a*d - b*c)^{(2/3)})) / (3*d^{(2/3)}*(a*d - b*c)^{(1/3)}) + (\log(d*(a + b*x^3)^{(1/3)} - ((3^{(1/2)}*1i - 1)^2*(9*a*d^3 - 9*b*c*d^2))/(36*d^{(4/3)}*(a*d - b*c)^{(2/3)})) * (3^{(1/2)}*1i - 1)) / (6*d^{(2/3)}*(a*d - b*c)^{(1/3)}) - (\log(d*(a + b*x^3)^{(1/3)} - ((3^{(1/2)}*1i + 1)^2*(9*a*d^3 - 9*b*c*d^2))/(36*d^{(4/3)}*(a*d - b*c)^{(2/3)})) * (3^{(1/2)}*1i + 1)) / (6*d^{(2/3)}*(a*d - b*c)^{(1/3)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{a + bx^3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**3+a)**(1/3)/(d*x**3+c),x)`

[Out] `Integral(x**2/((a + b*x**3)**(1/3)*(c + d*x**3)), x)`

$$3.460 \quad \int \frac{1}{x \sqrt[3]{a+bx^3} (c+dx^3)} dx$$

Optimal. Leaf size=244

$$-\frac{\sqrt[3]{d} \log(c+dx^3)}{6c\sqrt[3]{bc-ad}} + \frac{\sqrt[3]{d} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{2c\sqrt[3]{bc-ad}} + \frac{\sqrt[3]{d} \tan^{-1}\left(\frac{1-2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}c\sqrt[3]{bc-ad}} + \frac{\log(\sqrt[3]{a}-\sqrt[3]{a+bx^3})}{2\sqrt[3]{a}c} + \dots$$

Rubi [A] time = 0.21, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 86, 55, 617, 204, 31, 56}

$$-\frac{\sqrt[3]{d} \log(c+dx^3)}{6c\sqrt[3]{bc-ad}} + \frac{\sqrt[3]{d} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{2c\sqrt[3]{bc-ad}} + \frac{\sqrt[3]{d} \tan^{-1}\left(\frac{1-2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}c\sqrt[3]{bc-ad}} + \frac{\log(\sqrt[3]{a}-\sqrt[3]{a+bx^3})}{2\sqrt[3]{a}c} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}c} - \frac{\log(x)}{2\sqrt[3]{a}c}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)), x]

[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*c) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(1/3)) - Log[x]/(2*a^(1/3)*c) - (d^(1/3)*Log[c + d*x^3])/(6*c*(b*c - a*d)^(1/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(1/3)*c) + (d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*(b*c - a*d)^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])]/; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] := Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
p}, x] && !IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[3]{a+bx^3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{x\sqrt[3]{a+bx}} dx, x, x^3 \right)}{3c} - \frac{d \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right)}{3c} \\
&= \frac{\log(x)}{2\sqrt[3]{ac}} - \frac{\sqrt[3]{d} \log(c+dx^3)}{6c\sqrt[3]{bc-ad}} + \frac{\text{Subst} \left(\int \frac{1}{a^{2/3} + \sqrt[3]{a}x+x^2} dx, x, \sqrt[3]{a+bx^3} \right)}{2c} - \frac{\text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx^3}} dx, x, \sqrt[3]{a+bx^3} \right)}{2c} \\
&= \frac{\log(x)}{2\sqrt[3]{ac}} - \frac{\sqrt[3]{d} \log(c+dx^3)}{6c\sqrt[3]{bc-ad}} + \frac{\log(\sqrt[3]{a} - \sqrt[3]{a+bx^3})}{2\sqrt[3]{ac}} + \frac{\sqrt[3]{d} \log(\sqrt[3]{bc-ad} + \sqrt[3]{a+bx^3})}{2c\sqrt[3]{bc-ad}} \\
&= \frac{\tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3}\sqrt[3]{ac}} + \frac{\sqrt[3]{d} \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}} \right)}{\sqrt{3}c\sqrt[3]{bc-ad}} - \frac{\log(x)}{2\sqrt[3]{ac}} - \frac{\sqrt[3]{d} \log(c+dx^3)}{6c\sqrt[3]{bc-ad}} + \frac{\log(\sqrt[3]{a+bx^3})}{2c\sqrt[3]{bc-ad}}
\end{aligned}$$

Mathematica [C] time = 0.18, size = 140, normalized size = 0.57

$$\frac{3\sqrt[3]{ad}(a+bx^3)^{2/3} {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{d(bx^3+a)}{ad-bc}\right) - (bc-ad) \left(3 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^3}) + 2\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + 1}{\sqrt{3}}\right) - 3 \log(x) \right)}{6\sqrt[3]{ac}(ad-bc)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)), x]

[Out] (3*a^(1/3)*d*(a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-b*c) + a*d] - (b*c - a*d)*(2*sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3)))/a^(1/3)]/sqrt[3]] - 3*Log[x] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(1/3)*c*(-b*c) + a*d)

IntegrateAlgebraic [A] time = 0.49, size = 332, normalized size = 1.36

$$-\frac{\log(a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3})}{6\sqrt[3]{ac}} - \frac{\sqrt[3]{d} \log(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3})}{6c\sqrt[3]{bc-ad}} + \frac{\sqrt[3]{d} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3})}{3c\sqrt[3]{bc-ad}} + \frac{\sqrt[3]{d} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}c\sqrt[3]{bc-ad}} + \frac{\log(\sqrt[3]{a+bx^3} - \sqrt[3]{a})}{3\sqrt[3]{ac}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}} + \frac{1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{ac}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*c) + (d^(1/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*c*(b*c - a*d)^(1/3)) + Log[-a^(1/3) + (a + b*x^3)^(1/3)]/(3*a^(1/3)*c) + (d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(3*c*(b*c - a*d)^(1/3)) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(6*a^(1/3)*c) - (d^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/(6*c*(b*c - a*d)^(1/3))

fricas [A] time = 0.45, size = 628, normalized size = 2.57

$$\frac{\sqrt{3} \arctan\left(\frac{1 + \frac{2(a + bx^3)^{1/3}}{\sqrt{3}a^{1/3}}}{\sqrt{3} + \frac{2d^{1/3}(a + bx^3)^{1/3}}{\sqrt{3}(bc - ad)^{1/3}}}\right)}{\sqrt{3}a^{1/3}c} + \frac{d^{1/3} \arctan\left(\frac{1 - \frac{2d^{1/3}(a + bx^3)^{1/3}}{\sqrt{3}(bc - ad)^{1/3}}}{\sqrt{3} + \frac{2(a + bx^3)^{1/3}}{\sqrt{3}a^{1/3}}}\right)}{\sqrt{3}c(bc - ad)^{1/3}} + \frac{\log\left(-a^{1/3} + (a + bx^3)^{1/3}\right)}{3a^{1/3}c} + \frac{d^{1/3} \log\left(\frac{(bc - ad)^{1/3} + d^{1/3}(a + bx^3)^{1/3}}{3c(bc - ad)^{1/3}}\right)}{3c(bc - ad)^{1/3}} - \frac{\log\left(a^{2/3} + a^{1/3}(a + bx^3)^{1/3} + (a + bx^3)^{2/3}\right)}{6a^{1/3}c} - \frac{d^{1/3} \log\left(\frac{(bc - ad)^{2/3} - d^{1/3}(bc - ad)^{1/3}(a + bx^3)^{1/3} + d^{2/3}(a + bx^3)^{2/3}}{6c(bc - ad)^{1/3}}\right)}{6c(bc - ad)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] [1/6*(3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - 2*sqrt(3)*a*(d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3*sqrt(3)) - a*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) + 2*a*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) - a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*c), -1/6*(2*sqrt(3)*a*(d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3*sqrt(3)) + a*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) - 2*a*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*c)]

giac [A] time = 0.74, size = 326, normalized size = 1.34

$$\frac{d \left(-\frac{bc-ad}{d} \right)^{\frac{1}{3}} \log \left(\left(bx^3 + a \right)^{\frac{1}{3}} - \left(\frac{bc-ad}{d} \right)^{\frac{1}{3}} \right)}{3(bc^2 - acd)} + \frac{(-bc d^2 + ad^3)^{\frac{1}{3}} \arctan \left(\frac{\sqrt{3} \left(2(bx^3 + a)^{\frac{1}{3}} + \left(\frac{bc-ad}{d} \right)^{\frac{1}{3}} \right)}{3 \left(\frac{bc-ad}{d} \right)^{\frac{1}{3}}} \right)}{\sqrt{3}bc^2d - \sqrt{3}acd^2} - \frac{(-bc d^2 + ad^3)^{\frac{1}{3}} \log \left((bx^3 + a)^{\frac{2}{3}} + (bx^3 + a)^{\frac{1}{3}} \left(\frac{bc-ad}{d} \right)^{\frac{1}{3}} + \left(\frac{bc-ad}{d} \right)^{\frac{2}{3}} \right)}{6(bc^2d - acd^2)} + \frac{\sqrt{3} \arctan \left(\frac{\sqrt{3} \left(2(bx^3 + a)^{\frac{1}{3}} + a^{\frac{1}{3}} \right)}{3a^{\frac{1}{3}}} \right)}{3a^{\frac{1}{3}}c} - \frac{\log \left((bx^3 + a)^{\frac{2}{3}} + (bx^3 + a)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}} \right)}{6a^{\frac{1}{3}}c} + \frac{\log \left((bx^3 + a)^{\frac{1}{3}} - a^{\frac{1}{3}} \right)}{3a^{\frac{1}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out] 1/3*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - ((b*c - a*d)/d)^(1/3)))/(b*c^2 - a*c*d) + (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*

$$x^3 + a)^{1/3} + (-b*c - a*d)/d)^{1/3})/(-b*c - a*d)/d)^{1/3})/(\sqrt{3}*b*c^2*d - \sqrt{3}*a*c*d^2) - 1/6*(-b*c*d^2 + a*d^3)^{2/3}*\log((b*x^3 + a)^{2/3} + (b*x^3 + a)^{1/3})*(-b*c - a*d)/d)^{1/3} + (-b*c - a*d)/d)^{2/3})/(b*c^2*d - a*c*d^2) + 1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{1/3} + a^{1/3})/a^{1/3})/(a^{1/3}*c) - 1/6*\log((b*x^3 + a)^{2/3} + (b*x^3 + a)^{1/3})*a^{1/3} + a^{2/3})/(a^{1/3}*c) + 1/3*\log(\text{abs}((b*x^3 + a)^{1/3} - a^{1/3}))/a^{1/3}*c)$$

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^3+a)^(1/3)/(d*x^3+c), x)

[Out] int(1/x/(b*x^3+a)^(1/3)/(d*x^3+c), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c), x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x), x)

mupad [B] time = 6.44, size = 702, normalized size = 2.88

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)), x)

[Out] $\log(b^5*d^4*(a + b*x^3)^{1/3} - (d*(27*b^4*c^2*d^3*(a + b*x^3)^{1/3}*(2*a^2*d^2 + b^2*c^2 - 2*a*b*c*d) - 243*a*b^4*c^4*d^3*(d/(27*b*c^4 - 27*a*c^3*d))^{2/3}*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)))/(27*b*c^4 - 27*a*c^3*d))^{1/3} + \log((a + b*x^3)^{1/3} - a*c^2*(1/(a*c^3))^{2/3})*(1/(27*a*c^3))^{1/3} + (\log(b^5*d^4*(a + b*x^3)^{1/3} - (d*(3^{1/2}*1i - 1)^3*(27*b^4*c^2*d^3*(a + b*x^3)^{1/3}*(2*a^2*d^2 + b^2*c^2 - 2*a*b*c*d) - (243*a*b^4*c^4*d^3*(3^{1/2}*1i - 1)^2*(d/(27*b*c^4 - 27*a*c^3*d))^{2/3}*(2$

```

*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/4)/(8*(27*b*c^4 - 27*a*c^3*d)))*(3^(1/2)*
1i - 1)*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3))/2 - (log(b^5*d^4*(a + b*x^3)^(1/
3) + (d*(3^(1/2)*1i + 1)^3*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2*d^2 + b
^2*c^2 - 2*a*b*c*d) - (243*a*b^4*c^4*d^3*(3^(1/2)*1i + 1)^2*(d/(27*b*c^4 -
27*a*c^3*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/(8*(27*b*c^4 - 27
*a*c^3*d)))*(3^(1/2)*1i + 1)*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3))/2 - log((a
+ b*x^3)^(1/3)*2i + a*c^2*(1/(a*c^3))^(2/3)*1i + 3^(1/2)*a*c^2*(1/(a*c^3))^(
2/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a*c^3))^(1/3) + log((a + b*x^3)^(1/3)*
2i + a*c^2*(1/(a*c^3))^(2/3)*1i - 3^(1/2)*a*c^2*(1/(a*c^3))^(2/3))*((3^(1/2
)*1i)/2 - 1/2)*(1/(27*a*c^3))^(1/3)

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{a+bx^3}(c+dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**3+a)**(1/3)/(d*x**3+c),x)

[Out] Integral(1/(x*(a + b*x**3)**(1/3)*(c + d*x**3)), x)

$$3.461 \quad \int \frac{1}{x^4 \sqrt[3]{a+bx^3} (c+dx^3)} dx$$

Optimal. Leaf size=296

$$\frac{(3ad + bc) \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2} - \frac{(3ad + bc) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}c^2} + \frac{\log(x)(3ad + bc)}{6a^{4/3}c^2} + \frac{d^{4/3} \log(c + dx^3)}{6c^2\sqrt[3]{bc - ad}} - \frac{d^{4/3}}{3acx^3}$$

Rubi [A] time = 0.31, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {446, 103, 156, 55, 617, 204, 31, 56}

$$\frac{(3ad + bc) \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2} - \frac{(3ad + bc) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}c^2} + \frac{\log(x)(3ad + bc)}{6a^{4/3}c^2} + \frac{d^{4/3} \log(c + dx^3)}{6c^2\sqrt[3]{bc - ad}} - \frac{d^{4/3} \log\left(\sqrt[3]{bc - ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{bc - ad}} - \frac{d^{4/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc - ad}}\right)}{\sqrt{3}c^2\sqrt[3]{bc - ad}} - \frac{(a + bx^3)^{2/3}}{3acx^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)), x]

[Out] $-(a + b*x^3)^{(2/3)}/(3*a*c*x^3) - ((b*c + 3*a*d)*\text{ArcTan}[(a^{(1/3)} + 2*(a + b*x^3)^{(1/3)})/(\text{Sqrt}[3]*a^{(1/3)})])/(3*\text{Sqrt}[3]*a^{(4/3)}*c^2) - (d^{(4/3)}*\text{ArcTan}[(1 - (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})/(b*c - a*d)^{(1/3)})/\text{Sqrt}[3]])/(\text{Sqrt}[3]*c^2*(b*c - a*d)^{(1/3)}) + ((b*c + 3*a*d)*\text{Log}[x])/(6*a^{(4/3)}*c^2) + (d^{(4/3)}*\text{Log}[c + d*x^3])/(6*c^2*(b*c - a*d)^{(1/3)}) - ((b*c + 3*a*d)*\text{Log}[a^{(1/3)} - (a + b*x^3)^{(1/3)})/(6*a^{(4/3)}*c^2) - (d^{(4/3)}*\text{Log}[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)})/(2*c^2*(b*c - a*d)^{(1/3)})$

Rule 31

Int[((a_) + (b_)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^p_., x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \sqrt[3]{a+bx^3} (c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2 \sqrt[3]{a+bx} (c+dx)} dx, x, x^3 \right) \\
&= -\frac{(a+bx^3)^{2/3}}{3acx^3} - \frac{\text{Subst} \left(\int \frac{\frac{1}{3}(bc+3ad) + \frac{bdx}{3}}{x \sqrt[3]{a+bx} (c+dx)} dx, x, x^3 \right)}{3ac} \\
&= -\frac{(a+bx^3)^{2/3}}{3acx^3} + \frac{d^2 \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx} (c+dx)} dx, x, x^3 \right)}{3c^2} - \frac{(bc+3ad) \text{Subst} \left(\int \frac{1}{x \sqrt[3]{a+bx}} dx, x, x^3 \right)}{9ac^2} \\
&= -\frac{(a+bx^3)^{2/3}}{3acx^3} + \frac{(bc+3ad) \log(x)}{6a^{4/3}c^2} + \frac{d^{4/3} \log(c+dx^3)}{6c^2 \sqrt[3]{bc-ad}} + \frac{d \text{Subst} \left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc}}{d}} dx, x, x^3 \right)}{6a^{4/3}c^2} \\
&= -\frac{(a+bx^3)^{2/3}}{3acx^3} + \frac{(bc+3ad) \log(x)}{6a^{4/3}c^2} + \frac{d^{4/3} \log(c+dx^3)}{6c^2 \sqrt[3]{bc-ad}} - \frac{(bc+3ad) \log(\sqrt[3]{a+bx^3})}{6a^{4/3}c^2} \\
&= -\frac{(a+bx^3)^{2/3}}{3acx^3} - \frac{(bc+3ad) \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{3\sqrt{3} a^{4/3} c^2} - \frac{d^{4/3} \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3} c^2 \sqrt[3]{bc-ad}} + \frac{(bc+3ad) \log(\sqrt[3]{a+bx^3})}{6a^{4/3}c^2}
\end{aligned}$$

Mathematica [C] time = 0.61, size = 156, normalized size = 0.53

$$\frac{(3ad+bc) \left(3 \log \left(\sqrt[3]{a} - \sqrt[3]{a+bx^3} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{2\sqrt[3]{a+bx^3} + 1}{\frac{\sqrt[3]{a}}{\sqrt{3}}} \right) - 3 \log(x) \right)}{a^{4/3}} - \frac{9d^2(a+bx^3)^{2/3} {}_2F_1 \left(\frac{2}{3}, 1; \frac{5}{3}; \frac{d(bx^3+a)}{ad-bc} \right)}{bc-ad} + \frac{6c(a+bx^3)^{2/3}}{ax^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)), x]

[Out] -1/18*((6*c*(a + b*x^3)^(2/3))/(a*x^3) - (9*d^2*(a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)]/(b*c - a*d) + ((b*c + 3*a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3)]/Sqrt[3]) - 3*Log[x] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)]))/a^(4/3))/c^2

IntegrateAlgebraic [A] time = 0.71, size = 386, normalized size = 1.30

$$\frac{(-3ad-bc) \log(\sqrt[3]{a+bx^3} - \sqrt[3]{a})}{9a^{4/3}c^2} + \frac{(3ad+bc) \log(a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^3} + (a+bx^3)^{2/3})}{18a^{4/3}c^2} - \frac{(3ad+bc) \tan^{-1} \left(\frac{2\sqrt[3]{a+bx^3} + 1}{\sqrt{3}\sqrt[3]{a}} \right)}{3\sqrt{3}a^{4/3}c^2} - \frac{d^{4/3} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{3c^2 \sqrt[3]{bc-ad}} + \frac{d^{4/3} \log(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3})}{6c^2 \sqrt[3]{bc-ad}} - \frac{d^{4/3} \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3}c^2 \sqrt[3]{bc-ad}} + \frac{(a+bx^3)^{2/3}}{3acx^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out]
$$-1/3*(a + b*x^3)^{(2/3)}/(a*c*x^3) - ((b*c + 3*a*d)*\text{ArcTan}[(a^{(1/3)} + 2*(a + b*x^3)^{(1/3)})/(\text{Sqrt}[3]*a^{(1/3)})])/(3*\text{Sqrt}[3]*a^{(4/3)}*c^2) - (d^{(4/3)}*\text{ArcTan}[1/\text{Sqrt}[3] - (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})/(\text{Sqrt}[3]*(b*c - a*d)^{(1/3)})])/(3*\text{Sqrt}[3]*c^2*(b*c - a*d)^{(1/3)}) + ((-b*c) - 3*a*d)*\text{Log}[-a^{(1/3)} + (a + b*x^3)^{(1/3})]/(9*a^{(4/3)}*c^2) - (d^{(4/3)}*\text{Log}[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)})]/(3*c^2*(b*c - a*d)^{(1/3)}) + ((b*c + 3*a*d)*\text{Log}[a^{(2/3)} + a^{(1/3)}*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3})])/(18*a^{(4/3)}*c^2) + (d^{(4/3)}*\text{Log}[(b*c - a*d)^{(2/3)} - d^{(1/3)}*(b*c - a*d)^{(1/3)}*(a + b*x^3)^{(1/3)} + d^{(2/3)}*(a + b*x^3)^{(2/3})])/(6*c^2*(b*c - a*d)^{(1/3)})$$

fricas [A] time = 0.62, size = 837, normalized size = 2.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/18*(6*\text{sqrt}(3)*a^2*d*x^3*(-d/(b*c - a*d))^{(1/3)}*\text{arctan}(2/3*\text{sqrt}(3)*(b*x^3 + a)^{(1/3)}*(-d/(b*c - a*d))^{(1/3)} + 1/3*\text{sqrt}(3)) - 3*a^2*d*x^3*(-d/(b*c - a*d))^{(1/3)}*\log(-(b*x^3 + a)^{(1/3)}*(b*c - a*d)*(-d/(b*c - a*d))^{(2/3)} + (b*x^3 + a)^{(2/3)}*d - (b*c - a*d)*(-d/(b*c - a*d))^{(1/3)}) + 6*a^2*d*x^3*(-d/(b*c - a*d))^{(1/3)}*\log((b*c - a*d)*(-d/(b*c - a*d))^{(2/3)} + (b*x^3 + a)^{(1/3)}*d) + 3*\text{sqrt}(1/3)*(a*b*c + 3*a^2*d)*x^3*\text{sqrt}((-a)^{(1/3)}/a)*\log((2*b*x^3 - 3*\text{sqrt}(1/3)*(2*(b*x^3 + a)^{(2/3)}*(-a)^{(2/3)} - (b*x^3 + a)^{(1/3)}*a + (-a)^{(1/3)})*a)*\text{sqrt}((-a)^{(1/3)}/a) - 3*(b*x^3 + a)^{(1/3)}*(-a)^{(2/3)} + 3*a)/x^3) + (b*c + 3*a*d)*(-a)^{(2/3)}*x^3*\log((b*x^3 + a)^{(2/3)} - (b*x^3 + a)^{(1/3)}*(-a)^{(1/3)} + (-a)^{(2/3)}) - 2*(b*c + 3*a*d)*(-a)^{(2/3)}*x^3*\log((b*x^3 + a)^{(1/3)} + (-a)^{(1/3)}) - 6*(b*x^3 + a)^{(2/3)}*a*c)/(a^2*c^2*x^3), 1/18*(6*\text{sqrt}(3)*a^2*d*x^3*(-d/(b*c - a*d))^{(1/3)}*\text{arctan}(2/3*\text{sqrt}(3)*(b*x^3 + a)^{(1/3)}*(-d/(b*c - a*d))^{(1/3)} + 1/3*\text{sqrt}(3)) - 3*a^2*d*x^3*(-d/(b*c - a*d))^{(1/3)}*\log(-(b*x^3 + a)^{(1/3)}*(b*c - a*d)*(-d/(b*c - a*d))^{(2/3)} + (b*x^3 + a)^{(2/3)}*d - (b*c - a*d)*(-d/(b*c - a*d))^{(1/3)}) + 6*a^2*d*x^3*(-d/(b*c - a*d))^{(1/3)}*\log((b*c - a*d)*(-d/(b*c - a*d))^{(2/3)} + (b*x^3 + a)^{(1/3)}*d) - 6*\text{sqrt}(1/3)*(a*b*c + 3*a^2*d)*x^3*\text{sqrt}((-a)^{(1/3)}/a)*\text{arctan}(\text{sqrt}(1/3)*(2*(b*x^3 + a)^{(1/3)} - (-a)^{(1/3)})*\text{sqrt}(-(-a)^{(1/3)}/a)) + (b*c + 3*a*d)*(-a)^{(2/3)}*x^3*\log((b*x^3 + a)^{(2/3)} - (b*x^3 + a)^{(1/3)}*(-a)^{(1/3)} + (-a)^{(2/3)}) - 2*(b*c + 3*a*d)*(-a)^{(2/3)}*x^3*\log((b*x^3 + a)^{(1/3)} + (-a)^{(1/3)}) - 6*(b*x^3 + a)^{(2/3)}*a*c)/(a^2*c^2*x^3)] \end{aligned}$$

giac [A] time = 0.88, size = 383, normalized size = 1.29

$$\frac{d^2 \left(\frac{b^2 - ad}{a^2} \right)^{\frac{1}{2}} \log \left(\frac{(bx^3 + a)^{\frac{1}{2}} - \left(\frac{b^2 - ad}{a^2} \right)^{\frac{1}{2}}}{\sqrt{3}} \right)}{3(b^3 - ad^2)} \cdot \frac{(-bca^2 + ad^3)^{\frac{1}{2}} \arctan \left(\frac{\sqrt{3} \left((bx^3 + a)^{\frac{1}{2}} - \left(\frac{b^2 - ad}{a^2} \right)^{\frac{1}{2}} \right)}{3 \left(\frac{b^2 - ad}{a^2} \right)^{\frac{1}{2}} \right)}{\sqrt{3}b^3 - \sqrt{3}ad^2}}{\sqrt{3}b^3 - \sqrt{3}ad^2}} + \frac{(-bca^2 + ad^3)^{\frac{1}{2}} \log \left(\frac{(bx^3 + a)^{\frac{1}{2}} + (bx^3 + a)^{\frac{1}{2}} \left(\frac{b^2 - ad}{a^2} \right)^{\frac{1}{2}} + \left(\frac{b^2 - ad}{a^2} \right)^{\frac{1}{2}}}{6(b^3 - ad^2)} \right)}{6(b^3 - ad^2)} + \frac{(bc + 3ad) \log \left(\frac{(bx^3 + a)^{\frac{1}{2}} + (bx^3 + a)^{\frac{1}{2}} a^{\frac{1}{2}}}{18a^{\frac{1}{2}}c^2} \right)}{18a^{\frac{1}{2}}c^2}} + \frac{\sqrt{3} \left(a^{\frac{1}{2}}bc + 3a^{\frac{1}{2}}d \right) \arctan \left(\frac{\sqrt{3} \left((bx^3 + a)^{\frac{1}{2}} - a^{\frac{1}{2}} \right)}{3a^{\frac{1}{2}}} \right)}{9a^{\frac{1}{2}}c^2}} + \frac{(a^{\frac{1}{2}}bc + 3a^{\frac{1}{2}}d) \log \left(\frac{(bx^3 + a)^{\frac{1}{2}} - a^{\frac{1}{2}}}{9a^{\frac{1}{2}}c^2} \right)}{9a^{\frac{1}{2}}c^2}} - \frac{(bx^3 + a)^{\frac{1}{2}}}{3ac^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out]
$$-1/3*d^2*(-(b*c - a*d)/d)^{(2/3)}*\log(\text{abs}((b*x^3 + a)^{(1/3)} - (-(b*c - a*d)/d)^{(1/3)}))/((b*c^3 - a*c^2*d) - (-b*c*d^2 + a*d^3)^{(2/3)}*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{(1/3)} + (-(b*c - a*d)/d)^{(1/3)})/(-(b*c - a*d)/d)^{(1/3)})/(\sqrt{3}*b*c^3 - \sqrt{3}*a*c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(-(b*c - a*d)/d)^{(1/3)} + (-(b*c - a*d)/d)^{(2/3)})/(b*c^3 - a*c^2*d) + 1/18*(b*c + 3*a*d)*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*a^{(1/3)} + a^{(2/3)})/(a^{(4/3)}*c^2) - 1/9*\sqrt{3}*(a^{(2/3)}*b*c + 3*a^{(5/3)}*d)*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{(1/3)} + a^{(1/3)})/a^{(1/3)})/(a^2*c^2) - 1/9*(a^{(1/3)}*b*c + 3*a^{(4/3)}*d)*\log(\text{abs}((b*x^3 + a)^{(1/3)} - a^{(1/3)}))/((a^{(5/3)}*c^2) - 1/3*(b*x^3 + a)^{(2/3)})/(a*c*x^3)$$

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)

[Out] int(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^4), x)

mupad [B] time = 11.36, size = 1929, normalized size = 6.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x)

[Out]
$$\log(-(((3*b^4*d^3*(a + b*x^3)^{(1/3)}*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 3*a*b^4*c^4*d^3*(2*a^2*d^2 +$$

$$\begin{aligned}
& b^2c^2 - 3ab^2cd) * (-3ad + bc)^3 / (a^4c^6)^{2/3} * (-3ad + bc)^3 / (a^4c^6)^{1/3} / 9 + (b^5d^4(b^3c^3 - 27a^3d^3 + 8ab^2c^2d + 18a^2b^2cd^2)) / (3a^2c) * (-3ad + bc)^3 / (a^4c^6)^{2/3} / 81 - (4b^5d^7 * (a + bx^3)^{1/3} * (3ad + bc)^2) / (27a^2c^5) * (-27a^3d^3 + b^3c^3 + 9ab^2c^2d + 27a^2b^2cd^2) / (729a^4c^6)^{1/3} + \log(-(-d^4 / (27b^3c^7 - 27a^3c^6d))^{2/3} * ((-d^4 / (27b^3c^7 - 27a^3c^6d))^{1/3} * ((3b^4d^3 * (a + bx^3)^{1/3} * (18a^4d^4 + b^4c^4 - 2a^2b^2c^2d^2 + 4ab^3c^3d - 12a^3b^2cd^3)) / a^2 - 243ab^4c^4d^3 * (-d^4 / (27b^3c^7 - 27a^3c^6d))^{2/3} * (2a^2d^2 + b^2c^2 - 3ab^2cd)) + (b^5d^4 * (b^3c^3 - 27a^3d^3 + 8ab^2c^2d + 18a^2b^2cd^2)) / (3a^2c)) - (4b^5d^7 * (a + bx^3)^{1/3} * (3ad + bc)^2) / (27a^2c^5) * (-d^4 / (27b^3c^7 - 27a^3c^6d))^{1/3} - \log((((3b^4d^3 * (a + bx^3)^{1/3} * (18a^4d^4 + b^4c^4 - 2a^2b^2c^2d^2 + 4ab^3c^3d - 12a^3b^2cd^3)) / a^2 - 3ab^4c^4d^3 * ((3^{1/2} * 1i) / 2 - 1/2) * (2a^2d^2 + b^2c^2 - 3ab^2cd) * (-3ad + bc)^3 / (a^4c^6)^{2/3}) * ((3^{1/2} * 1i) / 2 + 1/2) * (-3ad + bc)^3 / (a^4c^6)^{1/3}) / 9 - (b^5d^4 * (b^3c^3 - 27a^3d^3 + 8ab^2c^2d + 18a^2b^2cd^2)) / (3a^2c) * ((3^{1/2} * 1i) / 2 - 1/2) * (-3ad + bc)^3 / (a^4c^6)^{2/3} / 81 - (4b^5d^7 * (a + bx^3)^{1/3} * (3ad + bc)^2) / (27a^2c^5) * ((3^{1/2} * 1i) / 2 + 1/2) * (-27a^3d^3 + b^3c^3 + 9ab^2c^2d + 27a^2b^2cd^2) / (729a^4c^6)^{1/3} + \log((((3b^4d^3 * (a + bx^3)^{1/3} * (18a^4d^4 + b^4c^4 - 2a^2b^2c^2d^2 + 4ab^3c^3d - 12a^3b^2cd^3)) / a^2 + 3ab^4c^4d^3 * ((3^{1/2} * 1i) / 2 + 1/2) * (2a^2d^2 + b^2c^2 - 3ab^2cd) * (-3ad + bc)^3 / (a^4c^6)^{2/3}) * ((3^{1/2} * 1i) / 2 - 1/2) * (-3ad + bc)^3 / (a^4c^6)^{1/3}) / 9 + (b^5d^4 * (b^3c^3 - 27a^3d^3 + 8ab^2c^2d + 18a^2b^2cd^2)) / (3a^2c) * ((3^{1/2} * 1i) / 2 + 1/2) * (-3ad + bc)^3 / (a^4c^6)^{2/3} / 81 - (4b^5d^7 * (a + bx^3)^{1/3} * (3ad + bc)^2) / (27a^2c^5) * ((3^{1/2} * 1i) / 2 - 1/2) * (-27a^3d^3 + b^3c^3 + 9ab^2c^2d + 27a^2b^2cd^2) / (729a^4c^6)^{1/3} + (\log(-((3^{1/2} * 1i - 1)^2 * (((3^{1/2} * 1i - 1) * (-d^4 / (27b^3c^7 - 27a^3c^6d))^{1/3} * ((3b^4d^3 * (a + bx^3)^{1/3} * (18a^4d^4 + b^4c^4 - 2a^2b^2c^2d^2 + 4ab^3c^3d - 12a^3b^2cd^3)) / a^2 - (243ab^4c^4d^3 * (3^{1/2} * 1i - 1)^2 * (-d^4 / (27b^3c^7 - 27a^3c^6d))^{2/3} * (2a^2d^2 + b^2c^2 - 3ab^2cd)) / 4)) / 2 + (b^5d^4 * (b^3c^3 - 27a^3d^3 + 8ab^2c^2d + 18a^2b^2cd^2)) / (3a^2c) * (-d^4 / (27b^3c^7 - 27a^3c^6d))^{2/3}) / 4 - (4b^5d^7 * (a + bx^3)^{1/3} * (3ad + bc)^2) / (27a^2c^5) * (3^{1/2} * 1i - 1) * (-d^4 / (27b^3c^7 - 27a^3c^6d))^{1/3}) / 2 - (\log((((3^{1/2} * 1i + 1)^2 * (((3^{1/2} * 1i + 1) * (-d^4 / (27b^3c^7 - 27a^3c^6d))^{1/3} * ((3b^4d^3 * (a + bx^3)^{1/3} * (18a^4d^4 + b^4c^4 - 2a^2b^2c^2d^2 + 4ab^3c^3d - 12a^3b^2cd^3)) / a^2 - (243ab^4c^4d^3 * (3^{1/2} * 1i + 1)^2 * (-d^4 / (27b^3c^7 - 27a^3c^6d))^{2/3} * (2a^2d^2 + b^2c^2 - 3ab^2cd)) / 4)) / 2 - (b^5d^4 * (b^3c^3 - 27a^3d^3 + 8ab^2c^2d + 18a^2b^2cd^2)) / (3a^2c) * (-d^4 / (27b^3c^7 - 27a^3c^6d))^{2/3}) / 4 - (4b^5d^7 * (a + bx^3)^{1/3} * (3ad + bc)^2) / (27a^2c^5) * (3^{1/2} * 1i + 1) * (-d^4 / (27b^3c^7 - 27a^3c^6d))^{1/3}) / 2 - (a + bx^3)^{2/3} / (3a^3c^3)
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt[3]{a + bx^3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**3+a)**(1/3)/(d*x**3+c), x)

[Out] Integral(1/(x**4*(a + b*x**3)**(1/3)*(c + d*x**3)), x)

$$3.462 \quad \int \frac{x^6}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$$

Optimal. Leaf size=273

$$\frac{(ad + 3bc) \log\left(\sqrt[3]{a + bx^3} - \sqrt[3]{b}x\right)}{6b^{4/3}d^2} - \frac{(ad + 3bc) \tan^{-1}\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1\right)}{3\sqrt{3}b^{4/3}d^2} + \frac{c^{4/3} \log(c + dx^3)}{6d^2\sqrt[3]{bc - ad}} - \frac{c^{4/3} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a + bx^3}\right)}{2d^2\sqrt[3]{bc - ad}}$$

Rubi [A] time = 0.51, antiderivative size = 394, normalized size of antiderivative = 1.44, number of steps used = 15, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {494, 470, 522, 200, 31, 634, 617, 204, 628}

$$\frac{(ad + 3bc) \log\left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{4/3}d^2} - \frac{(ad + 3bc) \log\left(\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1\right)}{18b^{4/3}d^2} - \frac{(ad + 3bc) \tan^{-1}\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1\right)}{3\sqrt{3}b^{4/3}d^2} - \frac{c^{4/3} \log\left(\sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3d^2\sqrt[3]{bc-ad}} + \frac{c^{4/3} \log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6d^2\sqrt[3]{bc-ad}} + \frac{c^{4/3} \tan^{-1}\left(\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \sqrt[3]{c}\right)}{\sqrt{3}d^2\sqrt[3]{bc-ad}} + \frac{x(a+bx^3)^{2/3}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)), x]

[Out] (x*(a + b*x^3)^(2/3))/(3*b*d) - ((3*b*c + a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)*d^2) + (c^(4/3)*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))]/(Sqrt[3]*d^2*(b*c - a*d)^(1/3)) + ((3*b*c + a*d)*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*b^(4/3)*d^2) - ((3*b*c + a*d)*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(18*b^(4/3)*d^2) - (c^(4/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*d^2*(b*c - a*d)^(1/3)) + (c^(4/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*d^2*(b*c - a*d)^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 494

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q]/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && LtQ[-1, p, 0]
```

Rule 522

```
Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\sqrt[3]{a+bx^3} (c+dx^3)} dx &= a^2 \operatorname{Subst} \left(\int \frac{x^6}{(1-bx^3)^2 (c-(bc-ad)x^3)} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right) \\
&= \frac{x(a+bx^3)^{2/3}}{3bd} - \frac{a \operatorname{Subst} \left(\int \frac{c+(2bc+ad)x^3}{(1-bx^3)(c+(-bc+ad)x^3)} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3bd} \\
&= \frac{x(a+bx^3)^{2/3}}{3bd} + \frac{c^2 \operatorname{Subst} \left(\int \frac{1}{c+(-bc+ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{d^2} - \frac{(3bc+ad) \operatorname{Subst} \left(\int \frac{1}{1-bx^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3bd^2} \\
&= \frac{x(a+bx^3)^{2/3}}{3bd} + \frac{c^{4/3} \operatorname{Subst} \left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3d^2} + \frac{c^{4/3} \operatorname{Subst} \left(\int \frac{2\sqrt[3]{c}}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3d^2} \\
&= \frac{x(a+bx^3)^{2/3}}{3bd} + \frac{(3bc+ad) \log \left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right)}{9b^{4/3}d^2} - \frac{c^{4/3} \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3d^2 \sqrt[3]{bc-ad}} + \frac{c^{5/3} \operatorname{Subst} \left(\int \frac{1}{1-bx^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3bd^2} \\
&= \frac{x(a+bx^3)^{2/3}}{3bd} + \frac{(3bc+ad) \log \left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right)}{9b^{4/3}d^2} - \frac{(3bc+ad) \log \left(1 + \frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right)}{18b^{4/3}d^2} \\
&= \frac{x(a+bx^3)^{2/3}}{3bd} - \frac{(3bc+ad) \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{3\sqrt{3}b^{4/3}d^2} + \frac{c^{4/3} \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{\sqrt{3}d^2 \sqrt[3]{bc-ad}} + \frac{(3bc+ad) \operatorname{Subst} \left(\int \frac{1}{1-bx^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3bd^2}
\end{aligned}$$

Mathematica [C] time = 0.82, size = 288, normalized size = 1.05

$$\frac{2 \left(-a \sqrt[3]{c} \log \left(\frac{\sqrt[3]{c} \sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}} + \frac{x^2(bc-ad)^{2/3}}{(ax^3+b)^{2/3}} + c^{2/3} \right) + 6x(a+bx^3)^{2/3} \sqrt[3]{bc-ad} + 2a \sqrt[3]{c} \log \left(\sqrt[3]{c} - \frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}} \right) - 2\sqrt{3}a \sqrt[3]{c} \tan^{-1} \left(\frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}} + 1}{\sqrt{3}} \right) \right)}{\sqrt[3]{bc-ad}} - \frac{3x^4 \sqrt[3]{\frac{bx^3}{a} + 1} (ad+3bc) F_1 \left(\frac{4}{3}, \frac{1}{3}, \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{c \sqrt[3]{a+bx^3}}$$

36bd

Warning: Unable to verify antiderivative.

[In] Integrate[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out]
$$\left((-3*(3*b*c + a*d)*x^4*(1 + (b*x^3)/a)^{(1/3)}*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]/(c*(a + b*x^3)^{(1/3)}) + (2*(6*(b*c - a*d)^{(1/3)}*x*(a + b*x^3)^{(2/3)} - 2*Sqrt[3]*a*c^{(1/3)}*ArcTan[(1 + (2*(b*c - a*d)^{(1/3)}*x)/(c^{(1/3)}*(b + a*x^3)^{(1/3)})])/Sqrt[3]] + 2*a*c^{(1/3)}*Log[c^{(1/3)} - ((b*c - a*d)^{(1/3)}*x)/(b + a*x^3)^{(1/3)}] - a*c^{(1/3)}*Log[c^{(2/3)} + ((b*c - a*d)^{(2/3)}*x^2)/(b + a*x^3)^{(2/3)} + (c^{(1/3)}*(b*c - a*d)^{(1/3)}*x)/(b + a*x^3)^{(1/3)}]) \right) / (b*c - a*d)^{(1/3)} / (36*b*d)$$

IntegrateAlgebraic [C] time = 4.25, size = 525, normalized size = 1.92

$$\frac{(af+3bc)\log(\sqrt{a+bx^3}-\sqrt{c})}{9b^2d^2} - \frac{(af+3bc)\tan^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a+bx^3}+\sqrt{c}}\right)}{3\sqrt{b}d^2} - \frac{(af-3bc)\log(\sqrt{a+bx^3}+\sqrt{c})}{18b^2d^2} + \frac{(a^2+4\sqrt{a}b^2)\log(2\sqrt{a+bx^3}+(1+i\sqrt{3})\sqrt{c}\sqrt{a+bx^3})}{6a^2\sqrt{c}d^2} + \frac{\sqrt{-1+i\sqrt{3}}e^{i\pi/3}\tan^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a+bx^3}+\sqrt{c}}\right)}{\sqrt{6}d^2\sqrt{c-ad}} - \frac{i(\sqrt{3}e^{i\pi/3}-ie^{i\pi/6})\log((\sqrt{3}+i)^{2/3}(a+bx^3)^{1/3}+\sqrt{c}(\sqrt{3}+i)\sqrt{a+bx^3}\sqrt{c-ad}-2a^2(c-ad)^{1/3})}{12d^2\sqrt{c-ad}} + \frac{i(a+bx^3)^{2/3}}{36d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out]
$$\left(\frac{x*(a + b*x^3)^{(2/3)}}{3*b*d} - \left((3*b*c + a*d)*ArcTan[(Sqrt[3]*b^{(1/3)}*x)/(b^{(1/3)}*x + 2*(a + b*x^3)^{(1/3)})] \right) / (3*Sqrt[3]*b^{(4/3)}*d^2) - \left(Sqrt[-1 + I*Sqrt[3]]*c^{(4/3)}*ArcTan[(3*(b*c - a*d)^{(1/3)}*x)/(Sqrt[3]*(b*c - a*d)^{(1/3)}*x - (3*I)*c^{(1/3)}*(a + b*x^3)^{(1/3)} - Sqrt[3]*c^{(1/3)}*(a + b*x^3)^{(1/3)})] \right) / (Sqrt[6]*d^2*(b*c - a*d)^{(1/3)}) + \left((3*b*c + a*d)*Log[-(b^{(1/3)}*x) + (a + b*x^3)^{(1/3)}] \right) / (9*b^{(4/3)}*d^2) + \left((c^{(4/3)} + I*Sqrt[3]*c^{(4/3)})*Log[2*(b*c - a*d)^{(1/3)}*x + (1 + I*Sqrt[3])*c^{(1/3)}*(a + b*x^3)^{(1/3)}] \right) / (6*d^2*(b*c - a*d)^{(1/3)}) + \left((-3*b*c - a*d)*Log[b^{(2/3)}*x^2 + b^{(1/3)}*x*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}] \right) / (18*b^{(4/3)}*d^2) - \left((I/12)*((-I)*c^{(4/3)} + Sqrt[3]*c^{(4/3)})*Log[(-2*I)*(b*c - a*d)^{(2/3)}*x^2 + c^{(1/3)}*(b*c - a*d)^{(1/3)}*(I*x - Sqrt[3]*x)*(a + b*x^3)^{(1/3)} + (I + Sqrt[3])*c^{(2/3)}*(a + b*x^3)^{(2/3)}] \right) / (d^2*(b*c - a*d)^{(1/3)}) \right)$$

fricas [A] time = 0.63, size = 826, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out]
$$\left(-1/18*(6*\sqrt{3}*b^2*c*(-c/(b*c - a*d))^{(1/3)}*\arctan(-1/3*(\sqrt{3}*x - 2*\sqrt{3}*b*x^3 + a)^{(1/3)}*(-c/(b*c - a*d))^{(1/3)})/x - 6*b^2*c*(-c/(b*c - a*d))^{(1/3)}*\log(-((b*c - a*d)*x*(-c/(b*c - a*d))^{(2/3)} - (b*x^3 + a)^{(1/3)}*c)/x) + 3*b^2*c*(-c/(b*c - a*d))^{(1/3)}*\log(-((b*c - a*d)*x^2*(-c/(b*c - a*d))^{(1/3)} - (b*x^3 + a)^{(1/3)}*(b*c - a*d)*x*(-c/(b*c - a*d))^{(2/3)} - (b*x^3 + a)^{(2/3)}*c)/x^2) - 6*(b*x^3 + a)^{(2/3)}*b*d*x - 3*\sqrt{1/3}*(3*b^2*c + a*b*d)*\sqrt{-1/b^{(2/3)}}*\log(3*b*x^3 - 3*(b*x^3 + a)^{(1/3)}*b^{(2/3)}*x^2 - 3*\sqrt{1/3}*(b^{(4/3)}*x^3 + (b*x^3 + a)^{(1/3)}*b*x^2 - 2*(b*x^3 + a)^{(2/3)}*b^{(2/3)}*x)*\sqrt{-1/b^{(2/3)}} + 2*a) - 2*(3*b*c + a*d)*b^{(2/3)}*\log(-b^{(1/3)}*x - (b*x^3$$

+ a)^(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)/(b^2*d^2), -1/18*(6*sqrt(3)*b^2*c*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) + 3*b^2*c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2) - 6*(b*x^3 + a)^(2/3)*b*d*x - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*(3*b^2*c + a*b*d)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3))/(b^2*d^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)

[Out] int(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{(bx^3 + a)^{1/3} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/((a + b*x^3)^(1/3)*(c + d*x^3)), x)

[Out] int(x^6/((a + b*x^3)^(1/3)*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt[3]{a + bx^3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(b*x**3+a)**(1/3)/(d*x**3+c), x)

[Out] Integral(x**6/((a + b*x**3)**(1/3)*(c + d*x**3)), x)

$$3.463 \quad \int \frac{x^3}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$$

Optimal. Leaf size=233

$$-\frac{\sqrt[3]{c} \log(c+dx^3)}{6d\sqrt[3]{bc-ad}} + \frac{\sqrt[3]{c} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d\sqrt[3]{bc-ad}} - \frac{\sqrt[3]{c} \tan^{-1}\left(\frac{2x\sqrt[3]{bc-ad} + \sqrt[3]{c}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}d\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{2\sqrt[3]{b}d} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1\right)}{\sqrt{3}\sqrt[3]{b}d}$$

Rubi [A] time = 0.29, antiderivative size = 346, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {494, 481, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1\right)}{6\sqrt[3]{b}d} - \frac{\sqrt[3]{c} \log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6d\sqrt[3]{bc-ad}} + \frac{\sqrt[3]{c} \log\left(\sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3d\sqrt[3]{bc-ad}} - \frac{\sqrt[3]{c} \tan^{-1}\left(\frac{2x\sqrt[3]{bc-ad} + \sqrt[3]{c}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}d\sqrt[3]{bc-ad}} - \frac{\log\left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}d} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1\right)}{\sqrt{3}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

[In] Int[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)*d) - (c^(1/3)*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))]/(Sqrt[3]*d*(b*c - a*d)^(1/3)) - Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*b^(1/3)*d) + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]/(6*b^(1/3)*d) + (c^(1/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*d*(b*c - a*d)^(1/3)) - (c^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(6*d*(b*c - a*d)^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_)^(m_.))/(((a_) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.))), x_Symbol] := -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x], x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q]/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && LtQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt[3]{a+bx^3}(c+dx^3)} dx &= a \operatorname{Subst} \left(\int \frac{x^3}{(1-bx^3)(c-(bc-ad)x^3)} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right) \\
&= \frac{\operatorname{Subst} \left(\int \frac{1}{1-bx^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{d} - \frac{c \operatorname{Subst} \left(\int \frac{1}{c+(-bc+ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{d} \\
&= \frac{\operatorname{Subst} \left(\int \frac{1}{1-\sqrt[3]{b}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3d} + \frac{\operatorname{Subst} \left(\int \frac{2+\sqrt[3]{b}x}{1+\sqrt[3]{b}x+b^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3d} - \frac{\sqrt[3]{c} \operatorname{Subst} \left(\int \frac{1}{1+\sqrt[3]{b}x+b^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{2d} \\
&= -\frac{\log \left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right)}{3\sqrt[3]{b}d} + \frac{\sqrt[3]{c} \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3d\sqrt[3]{bc-ad}} + \frac{\operatorname{Subst} \left(\int \frac{1}{1+\sqrt[3]{b}x+b^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{2d} \\
&= -\frac{\log \left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right)}{3\sqrt[3]{b}d} + \frac{\log \left(1 + \frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right)}{6\sqrt[3]{b}d} + \frac{\sqrt[3]{c} \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3d\sqrt[3]{bc-ad}} - \frac{\sqrt[3]{c}}{6\sqrt[3]{b}d} \\
&= \frac{\tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{\sqrt{3}\sqrt[3]{b}d} - \frac{\sqrt[3]{c} \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{\sqrt{3}d\sqrt[3]{bc-ad}} - \frac{\log \left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right)}{3\sqrt[3]{b}d} + \frac{\log \left(1 + \frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} \right)}{6\sqrt[3]{b}d}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 65, normalized size = 0.28

$$\frac{x^4 \sqrt[3]{\frac{a+bx^3}{a}} F_1 \left(\frac{4}{3}; \frac{1}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{4c \sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)), x]

[Out] (x^4*((a + b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -(b*x^3)/a, -((d*x^3)/c)])/(4*c*(a + b*x^3)^(1/3))

IntegrateAlgebraic [C] time = 3.13, size = 474, normalized size = 2.03

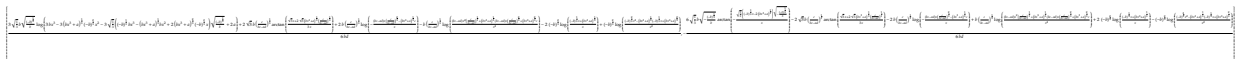
$$\frac{\log \left(\sqrt[3]{b}x\sqrt{a+bx^3} + (a+bx^3)^{3/3} + b^{2/3}x^2 \right)}{6\sqrt[3]{b}d} + \frac{(\sqrt[3]{c} + i\sqrt{3}\sqrt[3]{c}) \log \left((\sqrt{3} + i)c^{2/3}(a+bx^3)^{3/3} + \sqrt[3]{c}(-\sqrt{3}x + i)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} - 2i\sqrt[3]{bc-ad} \right)}{12i\sqrt[3]{bc-ad}} + \frac{i(\sqrt{3}\sqrt[3]{c} - i\sqrt[3]{c}) \log \left(2i\sqrt[3]{bc-ad} + (1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3} \right)}{6i\sqrt[3]{bc-ad}} + \frac{\sqrt{-1+i\sqrt{3}}\sqrt[3]{c} \tan^{-1} \left(\frac{3\sqrt[3]{bc-ad}}{\sqrt{3}\sqrt[3]{bc-ad} - \sqrt[3]{c}\sqrt[3]{a+bx^3}} \right)}{\sqrt[3]{6}d\sqrt[3]{bc-ad}} + \frac{\log \left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x \right)}{3\sqrt[3]{b}d} + \frac{\tan^{-1} \left(\frac{\sqrt{3}\sqrt[3]{c}}{\sqrt[3]{a+bx^3} + \sqrt[3]{c}} \right)}{\sqrt{3}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)), x]

```
[Out] ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(Sqrt[3]*b^(1/3)*d) + (Sqrt[-1 + I*Sqrt[3]]*c^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]/(Sqrt[6]*d*(b*c - a*d)^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(3*b^(1/3)*d) - ((I/6)*((-I)*c^(1/3) + Sqrt[3]*c^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]/(d*(b*c - a*d)^(1/3)) + Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(6*b^(1/3)*d) + ((c^(1/3) + I*Sqrt[3]*c^(1/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]/(12*d*(b*c - a*d)^(1/3))
```

fricas [A] time = 0.47, size = 761, normalized size = 3.27



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*sqrt(3)*b*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) + 2*b*(c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) - b*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) - 2*(-b)^(2/3)*log(((b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (-b)^(2/3)*log(((b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)]/(b*d), -1/6*(6*sqrt(1/3)*b*sqrt((-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt((-b)^(1/3)/b)/x) - 2*sqrt(3)*b*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) - 2*b*(c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) + b*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) + 2*(-b)^(2/3)*log(((b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (-b)^(2/3)*log(((b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)]/(b*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)

[Out] int(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(bx^3 + a)^{1/3} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x)

[Out] int(x^3/((a + b*x^3)^(1/3)*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[3]{a + bx^3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)

[Out] Integral(x**3/((a + b*x**3)**(1/3)*(c + d*x**3)), x)

$$3.464 \quad \int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$$

Optimal. Leaf size=148

$$\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}} + \frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}$$

Rubi [A] time = 0.13, antiderivative size = 207, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6c^{2/3}\sqrt[3]{bc-ad}} + \frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)), x]

[Out] ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) - Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(6*c^(2/3)*(b*c - a*d)^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{a+bx^3} (c+dx^3)} dx &= \text{Subst} \left(\int \frac{1}{c - (bc-ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{\sqrt[3]{c} - \sqrt[3]{bc-ad} x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3c^{2/3}} + \frac{\text{Subst} \left(\int \frac{2\sqrt[3]{c} + \sqrt[3]{bc-ad} x}{c^{2/3} + \sqrt[3]{c} \sqrt[3]{bc-ad} x + (bc-ad)^{2/3} x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3c^{2/3}} \\
&= -\frac{\log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad} x}{\sqrt[3]{a+bx^3}} \right)}{3c^{2/3} \sqrt[3]{bc-ad}} + \frac{\text{Subst} \left(\int \frac{1}{c^{2/3} + \sqrt[3]{c} \sqrt[3]{bc-ad} x + (bc-ad)^{2/3} x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{2\sqrt[3]{c}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{c^{2/3} \sqrt[3]{bc-ad}} \\
&= -\frac{\log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad} x}{\sqrt[3]{a+bx^3}} \right)}{3c^{2/3} \sqrt[3]{bc-ad}} + \frac{\log \left(c^{2/3} + \frac{(bc-ad)^{2/3} x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c} \sqrt[3]{bc-ad} x}{\sqrt[3]{a+bx^3}} \right)}{6c^{2/3} \sqrt[3]{bc-ad}} - \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{c^{2/3} \sqrt[3]{bc-ad}} \\
&= \frac{\tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{bc-ad} x}{\sqrt[3]{c} \sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{\sqrt{3} c^{2/3} \sqrt[3]{bc-ad}} - \frac{\log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad} x}{\sqrt[3]{a+bx^3}} \right)}{3c^{2/3} \sqrt[3]{bc-ad}} + \frac{\log \left(c^{2/3} + \frac{(bc-ad)^{2/3} x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c} \sqrt[3]{bc-ad} x}{\sqrt[3]{a+bx^3}} \right)}{6c^{2/3} \sqrt[3]{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 168, normalized size = 1.14

$$\frac{\log \left(\frac{\sqrt[3]{c} x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \frac{x^2 (bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + c^{2/3} \right) - 2 \log \left(\sqrt[3]{c} - \frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{c} \sqrt[3]{a+bx^3}} + 1}{\sqrt{3}} \right)}{6c^{2/3} \sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)), x]

[Out] (2*sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(2/3)*(b*c - a*d)^(1/3))

IntegrateAlgebraic [C] time = 0.00, size = 320, normalized size = 2.16

$$\frac{(1+i\sqrt{3}) \log \left(2x\sqrt[3]{bc-ad} + (1+i\sqrt{3}) \sqrt[3]{c} \sqrt[3]{a+bx^3} \right)}{6c^{2/3} \sqrt[3]{bc-ad}} - \frac{\sqrt{-1+i\sqrt{3}} \tan^{-1} \left(\frac{3x \sqrt[3]{bc-ad}}{\sqrt{3} x \sqrt[3]{bc-ad} - \sqrt{3} \sqrt[3]{c} \sqrt[3]{a+bx^3} - 3i \sqrt[3]{c} \sqrt[3]{a+bx^3}} \right)}{\sqrt{6} c^{2/3} \sqrt[3]{bc-ad}} - \frac{i(\sqrt{3}-i) \log \left((\sqrt{3}+i) c^{2/3} (a+bx^3)^{2/3} + \sqrt[3]{c} (-\sqrt{3}x+ix) \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} - 2ix^2 (bc-ad)^{2/3} \right)}{12c^{2/3} \sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] -((Sqrt[-1 + I*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])/((Sqrt[6]*c^(2/3)*(b*c - a*d)^(1/3))) + ((1 + I*Sqrt[3])*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/((6*c^(2/3)*(b*c - a*d)^(1/3))) - ((I/12)*(-I + Sqrt[3])*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/((c^(2/3)*(b*c - a*d)^(1/3)))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)

[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(bx^3 + a)^{1/3} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^3)^(1/3)*(c + d*x^3)),x)`

[Out] `int(1/((a + b*x^3)^(1/3)*(c + d*x^3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a + bx^3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)`

[Out] `Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)`

$$3.465 \quad \int \frac{1}{x^3 \sqrt[3]{a+bx^3} (c+dx^3)} dx$$

Optimal. Leaf size=176

$$-\frac{d \log(c+dx^3)}{6c^{5/3} \sqrt[3]{bc-ad}} + \frac{d \log\left(\frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{5/3} \sqrt[3]{bc-ad}} - \frac{d \tan^{-1}\left(\frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt{3}}\right)}{\sqrt{3} c^{5/3} \sqrt[3]{bc-ad}} - \frac{(a+bx^3)^{2/3}}{2acx^2}$$

Rubi [A] time = 0.23, antiderivative size = 235, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {494, 453, 200, 31, 634, 617, 204, 628}

$$\frac{d \log\left(\sqrt[3]{c} - \frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3} \sqrt[3]{bc-ad}} - \frac{d \log\left(\frac{x^2 (bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c} x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6c^{5/3} \sqrt[3]{bc-ad}} - \frac{d \tan^{-1}\left(\frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \sqrt[3]{c}}{\sqrt{3} \sqrt[3]{c}}\right)}{\sqrt{3} c^{5/3} \sqrt[3]{bc-ad}} - \frac{(a+bx^3)^{2/3}}{2acx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] $-(a + b*x^3)^{(2/3)}/(2*a*c*x^2) - (d*ArcTan[(c^{(1/3)} + (2*(b*c - a*d)^{(1/3)}*x)/(a + b*x^3)^{(1/3)})]/(Sqrt[3]*c^{(1/3)})]/(Sqrt[3]*c^{(5/3)}*(b*c - a*d)^{(1/3)}) + (d*Log[c^{(1/3)} - ((b*c - a*d)^{(1/3)}*x)/(a + b*x^3)^{(1/3)})]/(3*c^{(5/3)}*(b*c - a*d)^{(1/3)}) - (d*Log[c^{(2/3)} + ((b*c - a*d)^{(2/3)}*x^2)/(a + b*x^3)^{(2/3)} + (c^{(1/3)}*(b*c - a*d)^{(1/3)}*x)/(a + b*x^3)^{(1/3)})]/(6*c^{(5/3)}*(b*c - a*d)^{(1/3)})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 453

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 494

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && LtQ[-1, p, 0]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \sqrt[3]{a+bx^3} (c+dx^3)} dx &= \frac{\text{Subst}\left(\int \frac{1-bx^3}{x^3(c-(bc-ad)x^3)} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a} \\
&= -\frac{(a+bx^3)^{2/3}}{2acx^2} - \frac{d \text{Subst}\left(\int \frac{1}{c+(-bc+ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c} \\
&= -\frac{(a+bx^3)^{2/3}}{2acx^2} - \frac{d \text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}} - \frac{d \text{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{bc-ad}}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}} \\
&= -\frac{(a+bx^3)^{2/3}}{2acx^2} + \frac{d \log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} - \frac{d \text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{2c^{4/3}} \\
&= -\frac{(a+bx^3)^{2/3}}{2acx^2} + \frac{d \log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} - \frac{d \log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{6c^{5/3}\sqrt[3]{bc-ad}} + \\
&= -\frac{(a+bx^3)^{2/3}}{2acx^2} - \frac{d \tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}} + \frac{d \log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} - \frac{d \log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{6c^{5/3}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 124, normalized size = 0.70

$$\frac{-3x^3(c+dx^3)(bc-ad) {}_2F_1\left(\frac{4}{3}, 2; \frac{7}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right) - 4c(a+bx^3)(c+3dx^3) {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{8c^3x^2(a+bx^3)^{4/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)), x]

[Out] (-4*c*(a + b*x^3)*(c + 3*d*x^3)*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3*(b*c - a*d)*x^3*(c + d*x^3)*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(8*c^3*x^2*(a + b*x^3)^(4/3))

IntegrateAlgebraic [C] time = 1.96, size = 349, normalized size = 1.98

$$\frac{i(\sqrt{3}d-id)\log(2x\sqrt[3]{bc-ad}+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3})}{6c^{5/3}\sqrt[3]{bc-ad}} + \frac{\sqrt{-1+i\sqrt{3}}d \tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}} + \frac{(d+i\sqrt{3}d)\log((\sqrt{3}+i)c^{2/3}(a+bx^3)^{2/3}+\sqrt[3]{c}(-\sqrt{3}x+ix)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3})}{12c^{5/3}\sqrt[3]{bc-ad}} - \frac{(a+bx^3)^{2/3}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/2*(a + b*x^3)^(2/3)/(a*c*x^2) + (Sqrt[-1 + I*Sqrt[3]]*d*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])/(Sqrt[6]*c^(5/3)*(b*c - a*d)^(1/3)) - ((I/6)*((-I)*d + Sqrt[3]*d)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]/(c^(5/3)*(b*c - a*d)^(1/3)) + ((d + I*Sqrt[3]*d)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(5/3)*(b*c - a*d)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 (bx^3 + a)^{1/3} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x)

[Out] int(1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt[3]{a + bx^3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)

[Out] Integral(1/(x**3*(a + b*x**3)**(1/3)*(c + d*x**3)), x)

$$3.466 \quad \int \frac{1}{x^6 \sqrt[3]{a+bx^3} (c+dx^3)} dx$$

Optimal. Leaf size=214

$$\frac{(a+bx^3)^{2/3} (5ad+3bc)}{10a^2c^2x^2} + \frac{d^2 \log(c+dx^3)}{6c^{8/3} \sqrt[3]{bc-ad}} - \frac{d^2 \log\left(\frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{8/3} \sqrt[3]{bc-ad}} + \frac{d^2 \tan^{-1}\left(\frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right)}{\sqrt{3} c^{8/3} \sqrt[3]{bc-ad}} - \frac{(a+bx^3)^{2/3}}{5acx^5}$$

Rubi [A] time = 0.29, antiderivative size = 271, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$\frac{(a+bx^3)^{2/3} (ad+bc)}{2a^2c^2x^2} - \frac{(a+bx^3)^{5/3}}{5a^2cx^5} - \frac{d^2 \log\left(\sqrt[3]{c} - \frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3} \sqrt[3]{bc-ad}} + \frac{d^2 \log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c} x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6c^{8/3} \sqrt[3]{bc-ad}} + \frac{d^2 \tan^{-1}\left(\frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \sqrt[3]{c}}{\sqrt{3} \sqrt[3]{c}}\right)}{\sqrt{3} c^{8/3} \sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] ((b*c + a*d)*(a + b*x^3)^(2/3))/(2*a^2*c^2*x^2) - (a + b*x^3)^(5/3)/(5*a^2*c*x^5) + (d^2*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(Sqrt[3]*c^(1/3)))/(Sqrt[3]*c^(8/3)*(b*c - a*d)^(1/3)) - (d^2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(8/3)*(b*c - a*d)^(1/3)) + (d^2*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(8/3)*(b*c - a*d)^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q]/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && LtQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6 \sqrt[3]{a+bx^3} (c+dx^3)} dx &= \frac{\text{Subst} \left(\int \frac{(1-bx^3)^2}{x^6 (c-(bc-ad)x^3)} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{a^2} \\
&= \frac{\text{Subst} \left(\int \left(\frac{1}{cx^6} + \frac{-bc-ad}{c^2 x^3} + \frac{a^2 d^2}{c^2 (c-(bc-ad)x^3)} \right) dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{a^2} \\
&= \frac{(bc+ad)(a+bx^3)^{2/3}}{2a^2 c^2 x^2} - \frac{(a+bx^3)^{5/3}}{5a^2 cx^5} + \frac{d^2 \text{Subst} \left(\int \frac{1}{c-(bc-ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{c^2} \\
&= \frac{(bc+ad)(a+bx^3)^{2/3}}{2a^2 c^2 x^2} - \frac{(a+bx^3)^{5/3}}{5a^2 cx^5} + \frac{d^2 \text{Subst} \left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad} x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3c^{8/3}} + \dots \\
&= \frac{(bc+ad)(a+bx^3)^{2/3}}{2a^2 c^2 x^2} - \frac{(a+bx^3)^{5/3}}{5a^2 cx^5} - \frac{d^2 \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad} x}{\sqrt[3]{a+bx^3}} \right)}{3c^{8/3} \sqrt[3]{bc-ad}} + \frac{d^2 \text{Subst} \left(\int \frac{1}{c^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{6c^8} \\
&= \frac{(bc+ad)(a+bx^3)^{2/3}}{2a^2 c^2 x^2} - \frac{(a+bx^3)^{5/3}}{5a^2 cx^5} - \frac{d^2 \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad} x}{\sqrt[3]{a+bx^3}} \right)}{3c^{8/3} \sqrt[3]{bc-ad}} + \frac{d^2 \log \left(c^{2/3} + \frac{d}{\sqrt[3]{a+bx^3}} \right)}{6c^8} \\
&= \frac{(bc+ad)(a+bx^3)^{2/3}}{2a^2 c^2 x^2} - \frac{(a+bx^3)^{5/3}}{5a^2 cx^5} + \frac{d^2 \tan^{-1} \left(\frac{1 + \frac{2 \sqrt[3]{bc-ad} x}{\sqrt[3]{c} \sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{\sqrt{3} c^{8/3} \sqrt[3]{bc-ad}} - \frac{d^2 \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad} x}{\sqrt[3]{a+bx^3}} \right)}{3c^{8/3} \sqrt[3]{bc-ad}}
\end{aligned}$$

Mathematica [C] time = 1.03, size = 207, normalized size = 0.97

$$\frac{-9x^3 (c+dx^3)^2 (bc-ad) {}_3F_2 \left(\frac{4}{3}, 2, 2; 1, \frac{7}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)} \right) - 3x^3 (-c^2 + 8cdx^3 + 9d^2x^6) (bc-ad) {}_2F_1 \left(\frac{4}{3}, 2; \frac{7}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)} \right) + 8c(a+bx^3) (c^2 - 3cdx^3 - 9d^2x^6) {}_2F_1 \left(\frac{1}{3}, 1; \frac{4}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)} \right)}{40c^4 x^5 (a+bx^3)^{4/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)), x]

[Out] $-1/40*(8*c*(a + b*x^3)*(c^2 - 3*c*d*x^3 - 9*d^2*x^6)*\text{Hypergeometric2F1}[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3*(b*c - a*d)*x^3*(-c^2 + 8*c*d*x^3 + 9*d^2*x^6)*\text{Hypergeometric2F1}[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*\text{HypergeometricPFQ}[\{4/3, 2, 2\}, \{1, 7/3\}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(c^4*x^5*(a + b*x^3)^(4/3))$

IntegrateAlgebraic [C] time = 2.31, size = 379, normalized size = 1.77

$$\frac{(a+bx^3)^{2/3}(-2ac+5adt^3+3bcx^3)}{10a^2c^2x^5} + \frac{(d+i\sqrt{3}d^2)\log(2x\sqrt{bc-ad}+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3})}{6c^{8/3}\sqrt[3]{bc-ad}} - \frac{\sqrt{-1+i\sqrt{3}}d^2\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}c^{8/3}\sqrt[3]{bc-ad}} - \frac{i(\sqrt{3}d^2-i d^2)\log((\sqrt{3}+i)^{2/3}(a+bx^3)^{2/3}+\sqrt[3]{c}(-\sqrt{3}x+ix)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3})}{12c^{8/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] $((a + b*x^3)^{(2/3)}*(-2*a*c + 3*b*c*x^3 + 5*a*d*x^3))/(10*a^2*c^2*x^5) - (\text{Sqrt}[-1 + I*\text{Sqrt}[3]]*d^2*\text{ArcTan}[(3*(b*c - a*d)^{(1/3)}*x)/(\text{Sqrt}[3]*(b*c - a*d)^{(1/3)}*x - (3*I)*c^{(1/3)}*(a + b*x^3)^{(1/3)} - \text{Sqrt}[3]*c^{(1/3)}*(a + b*x^3)^{(1/3)})]/(\text{Sqrt}[6]*c^{(8/3)}*(b*c - a*d)^{(1/3)}) + ((d^2 + I*\text{Sqrt}[3]*d^2)*\text{Log}[2*(b*c - a*d)^{(1/3)}*x + (1 + I*\text{Sqrt}[3])*c^{(1/3)}*(a + b*x^3)^{(1/3)}]/(6*c^{(8/3)}*(b*c - a*d)^{(1/3)}) - ((I/12)*(-I)*d^2 + \text{Sqrt}[3]*d^2)*\text{Log}[(-2*I)*(b*c - a*d)^{(2/3)}*x^2 + c^{(1/3)}*(b*c - a*d)^{(1/3)}*(I*x - \text{Sqrt}[3]*x)*(a + b*x^3)^{(1/3)} + (I + \text{Sqrt}[3])*c^{(2/3)}*(a + b*x^3)^{(2/3)}]/(c^{(8/3)}*(b*c - a*d)^{(1/3)})$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)

[Out] `int(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6 (bx^3 + a)^{1/3} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x)`

[Out] `int(1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \sqrt[3]{a + bx^3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)`

[Out] `Integral(1/(x**6*(a + b*x**3)**(1/3)*(c + d*x**3)), x)`

$$3.467 \quad \int \frac{1}{x^9 \sqrt[3]{a+bx^3} (c+dx^3)} dx$$

Optimal. Leaf size=262

$$\frac{(a+bx^3)^{2/3} (4ad+3bc)}{20a^2c^2x^5} - \frac{(a+bx^3)^{2/3} (20a^2d^2+12abcd+9b^2c^2)}{40a^3c^3x^2} - \frac{d^3 \log(c+dx^3)}{6c^{11/3} \sqrt[3]{bc-ad}} + \frac{d^3 \log\left(\frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{11/3} \sqrt[3]{bc-ad}}$$

Rubi [A] time = 0.32, antiderivative size = 317, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$-\frac{(a+bx^3)^{2/3} (a^2d^2+abcd+b^2c^2)}{2a^3c^3x^2} + \frac{(a+bx^3)^{5/3} (ad+2bc)}{5a^3c^2x^5} - \frac{(a+bx^3)^{8/3}}{8a^3cx^8} + \frac{d^3 \log\left(\sqrt[3]{c} - \frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{11/3} \sqrt[3]{bc-ad}} - \frac{d^3 \log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c} x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6c^{11/3} \sqrt[3]{bc-ad}} - \frac{d^3 \tan^{-1}\left(\frac{2x \sqrt[3]{bc-ad} + \sqrt[3]{c}}{\sqrt{3} \sqrt[3]{c}}\right)}{\sqrt{3} c^{11/3} \sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] -((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*a^3*c^3*x^2) + ((2*b*c + a*d)*(a + b*x^3)^(5/3))/(5*a^3*c^2*x^5) - (a + b*x^3)^(8/3)/(8*a^3*c*x^8) - (d^3*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(Sqrt[3]*c^(1/3)))/(Sqrt[3]*c^(11/3)*(b*c - a*d)^(1/3)) + (d^3*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(11/3)*(b*c - a*d)^(1/3)) - (d^3*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(11/3)*(b*c - a*d)^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(−1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(−1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(−1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 461

Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 494

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && LtQ[-1, p, 0]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^9 \sqrt[3]{a + bx^3} (c + dx^3)} dx &= \frac{\text{Subst} \left(\int \frac{(1-bx^3)^3}{x^9 (c-(bc-ad)x^3)} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{a^3} \\
&= \frac{\text{Subst} \left(\int \left(\frac{1}{cx^9} + \frac{-2bc-ad}{c^2x^6} + \frac{b^2c^2+abcd+a^2d^2}{c^3x^3} + \frac{a^3d^3}{c^3(-c+(bc-ad)x^3)} \right) dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{a^3} \\
&= -\frac{(b^2c^2 + abcd + a^2d^2)(a + bx^3)^{2/3}}{2a^3c^3x^2} + \frac{(2bc + ad)(a + bx^3)^{5/3}}{5a^3c^2x^5} - \frac{(a + bx^3)^{8/3}}{8a^3cx^8} + \frac{d^3}{\dots} \\
&= -\frac{(b^2c^2 + abcd + a^2d^2)(a + bx^3)^{2/3}}{2a^3c^3x^2} + \frac{(2bc + ad)(a + bx^3)^{5/3}}{5a^3c^2x^5} - \frac{(a + bx^3)^{8/3}}{8a^3cx^8} + \frac{d^3}{\dots} \\
&= -\frac{(b^2c^2 + abcd + a^2d^2)(a + bx^3)^{2/3}}{2a^3c^3x^2} + \frac{(2bc + ad)(a + bx^3)^{5/3}}{5a^3c^2x^5} - \frac{(a + bx^3)^{8/3}}{8a^3cx^8} + \frac{d^3}{\dots} \\
&= -\frac{(b^2c^2 + abcd + a^2d^2)(a + bx^3)^{2/3}}{2a^3c^3x^2} + \frac{(2bc + ad)(a + bx^3)^{5/3}}{5a^3c^2x^5} - \frac{(a + bx^3)^{8/3}}{8a^3cx^8} + \frac{d^3}{\dots} \\
&= -\frac{(b^2c^2 + abcd + a^2d^2)(a + bx^3)^{2/3}}{2a^3c^3x^2} + \frac{(2bc + ad)(a + bx^3)^{5/3}}{5a^3c^2x^5} - \frac{(a + bx^3)^{8/3}}{8a^3cx^8} + \frac{d^3}{\dots}
\end{aligned}$$

Mathematica [C] time = 2.55, size = 821, normalized size = 3.13

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] $-1/320*(40*a*c^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) + 40*b*c^4*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 72*a*c^3*d*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 72*b*c^3*d*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 216*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 216*b*c^2*d^2*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 648*a*c*d^3*x^9*Hyper$

geometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 648*b*c*d^3*x^12*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 9*b*c^4*x^3*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*a*c^3*d*x^3*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 45*b*c^3*d*x^6*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 45*a*c^2*d^2*x^6*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 243*b*c^2*d^2*x^9*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 243*a*c*d^3*x^9*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*b*c*d^3*x^12*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 54*(b*c - a*d)*x^3*(c - 3*d*x^3)*(c + d*x^3)^2*HypergeometricPFQ[{4/3, 2, 2}, {1, 7/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*HypergeometricPFQ[{4/3, 2, 2, 2}, {1, 1, 7/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(c^5*x^8*(a + b*x^3)^(4/3))

IntegrateAlgebraic [C] time = 3.11, size = 419, normalized size = 1.60

$$\frac{(a+bx^3)^{2/3}(-5a^2c^2+8a^2cd^2-20a^2d^2e^2+6abc^2x^3-12abcdx^6-9b^2c^2x^6)}{40a^3c^3x^8} - \frac{i(\sqrt{3}d^3-ad)\log\left(\frac{2x\sqrt{bc-ad}+(1+i\sqrt{3})\sqrt{c}\sqrt{a+bx^3}}{6c^{1/3}\sqrt{bc-ad}}\right)}{6c^{1/3}\sqrt{bc-ad}} + \frac{\sqrt{-1+i\sqrt{3}}d^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{3}\sqrt{bc-ad}-\sqrt{3}\sqrt{a+bx^3}}\right)}{\sqrt{6}c^{1/3}\sqrt{bc-ad}} + \frac{(d^3+i\sqrt{3}d^3)\log\left(\frac{(\sqrt{3}+i)^{2/3}(a+bx^3)^{2/3}+\sqrt{c}(-\sqrt{3}x+ix)\sqrt{a+bx^3}\sqrt{bc-ad}-2ix^2(bc-ad)^{2/3}}{12c^{1/3}\sqrt{bc-ad}}\right)}{12c^{1/3}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] ((a + b*x^3)^(2/3)*(-5*a^2*c^2 + 6*a*b*c^2*x^3 + 8*a^2*c*d*x^3 - 9*b^2*c^2*x^6 - 12*a*b*c*d*x^6 - 20*a^2*d^2*x^6))/(40*a^3*c^3*x^8) + (Sqrt[-1 + I*Sqrt[3]]*d^3*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])/(Sqrt[6]*c^(11/3)*(b*c - a*d)^(1/3)) - ((I/6)*((-I)*d^3 + Sqrt[3]*d^3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]/(c^(11/3)*(b*c - a*d)^(1/3)) + ((d^3 + I*Sqrt[3]*d^3)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]/(12*c^(11/3)*(b*c - a*d)^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x)

[Out] int(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^9 (bx^3 + a)^{1/3} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x)

[Out] int(1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9 \sqrt[3]{a + bx^3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**9/(b*x**3+a)**(1/3)/(d*x**3+c),x)

[Out] Integral(1/(x**9*(a + b*x**3)**(1/3)*(c + d*x**3)), x)

$$3.468 \quad \int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)} dx$$

Optimal. Leaf size=241

$$\frac{\sqrt[3]{a+bx^3} (a^2d^2 + abcd + b^2c^2)}{b^3d^3} - \frac{(a+bx^3)^{4/3} (2ad+bc)}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} + \frac{c^3 \log(c+dx^3)}{6d^{10/3}(bc-ad)^{2/3}} - \frac{c^3 \log(\sqrt[3]{bc-ad} + \sqrt[3]{a+bx^3})}{2d^{10/3}(bc-ad)^{2/3}}$$

Rubi [A] time = 0.26, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 88, 58, 617, 204, 31}

$$\frac{\sqrt[3]{a+bx^3} (a^2d^2 + abcd + b^2c^2)}{b^3d^3} - \frac{(a+bx^3)^{4/3} (2ad+bc)}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} + \frac{c^3 \log(c+dx^3)}{6d^{10/3}(bc-ad)^{2/3}} - \frac{c^3 \log(\sqrt[3]{bc-ad} + \sqrt[3]{a+bx^3})}{2d^{10/3}(bc-ad)^{2/3}} + \frac{c^3 \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{10/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out] ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(1/3))/(b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(4/3))/(4*b^3*d^2) + (a + b*x^3)^(7/3)/(7*b^3*d) + (c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)]^(1/3))/Sqrt[3])/(Sqrt[3]*d^(10/3)*(b*c - a*d)^(2/3)) + (c^3*Log[c + d*x^3])/(6*d^(10/3)*(b*c - a*d)^(2/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3)*(b*c - a*d)^(2/3))

Rule 31

Int[((a_) + (b_)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 58

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{b^2c^2 + abcd + a^2d^2}{b^2d^3(a+bx)^{2/3}} + \frac{(-bc - 2ad)\sqrt[3]{a+bx}}{b^2d^2} + \frac{(a+bx)^{4/3}}{b^2d} - \frac{(a+bx)^{7/3}}{d^3(a+bx)} \right) dx, x, x^3 \right) \\
&= \frac{(b^2c^2 + abcd + a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc + 2ad)(a+bx^3)^{4/3}}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} - \frac{c^3 \text{Subst} \left(\int \frac{1}{a+bx} dx, x, x^3 \right)}{6d^{10/3}} \\
&= \frac{(b^2c^2 + abcd + a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc + 2ad)(a+bx^3)^{4/3}}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} + \frac{c^3 \log(a+bx^3)}{6d^{10/3}} \\
&= \frac{(b^2c^2 + abcd + a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc + 2ad)(a+bx^3)^{4/3}}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} + \frac{c^3 \log(a+bx^3)}{6d^{10/3}} \\
&= \frac{(b^2c^2 + abcd + a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc + 2ad)(a+bx^3)^{4/3}}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} + \frac{c^3 \tan^{-1} \left(\frac{1 - 2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}} \right)}{\sqrt{3}d^{10/3}}
\end{aligned}$$

Mathematica [A] time = 0.66, size = 251, normalized size = 1.04

$$\frac{84\sqrt[3]{a+bx^3}(a^2d^2+abcd+b^2c^2)}{b^3} - \frac{21d(a+bx^3)^{4/3}(2ad+bc)}{b^3} + \frac{12d^2(a+bx^3)^{7/3}}{b^3} + \frac{14c^3 \left(\log \left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3} \right) - 2 \log \left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{1 - 2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}} \right) \right)}{\sqrt[3]{d}(bc-ad)^{2/3}}}{84d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out] ((84*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(1/3))/b^3 - (21*d*(b*c + 2*a*d)*(a + b*x^3)^(4/3))/b^3 + (12*d^2*(a + b*x^3)^(7/3))/b^3 + (14*c^3*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)]/Sqrt[3] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])))/(d^(1/3)*(b*c - a*d)^(2/3)))/(84*d^3)

IntegrateAlgebraic [A] time = 0.50, size = 284, normalized size = 1.18

$$\frac{\sqrt[3]{a+bx^3}(18a^2d^2+21abcd-6abd^2x^3+28b^2c^2-7b^2cdx^3+4b^2d^2x^6)}{28b^3d^3} - \frac{c^3 \log(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3})}{3d^{10/3}(bc-ad)^{2/3}} + \frac{c^3 \log(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3})}{6d^{10/3}(bc-ad)^{2/3}} + \frac{c^3 \tan^{-1} \left(\frac{1 - 2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}} \right)}{\sqrt{3}d^{10/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out]
$$\begin{aligned} & ((a + b*x^3)^{(1/3)}*(28*b^2*c^2 + 21*a*b*c*d + 18*a^2*d^2 - 7*b^2*c*d*x^3 - \\ & 6*a*b*d^2*x^3 + 4*b^2*d^2*x^6))/(28*b^3*d^3) + (c^3*ArcTan[1/Sqrt[3] - (2*d \\ & ^{(1/3)}*(a + b*x^3)^{(1/3})]/(Sqrt[3]*(b*c - a*d)^{(1/3}))]/(Sqrt[3]*d^{(10/3)}*(\\ & b*c - a*d)^{(2/3)}) - (c^3*Log[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)}] \\ &)/(3*d^{(10/3)}*(b*c - a*d)^{(2/3)}) + (c^3*Log[(b*c - a*d)^{(2/3)} - d^{(1/3)}*(b* \\ & c - a*d)^{(1/3)}*(a + b*x^3)^{(1/3)} + d^{(2/3)}*(a + b*x^3)^{(2/3)}]/(6*d^{(10/3)}* \\ & (b*c - a*d)^{(2/3)}) \end{aligned}$$

fricas [B] time = 0.83, size = 1322, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/84*(14*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(2/3)}*b^3*c^3*\log(-(b*x^3 + \\ & a)^{(2/3)}*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(1/3)}*(b*c \\ & - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(2/3)}*(b*x^3 + a)^{(1/3})) - 28 \\ & *(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(2/3)}*b^3*c^3*\log(-(b*x^3 + a)^{(1/3)}* \\ & (b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(2/3)}) - 42*\sqrt{1/3} \\ & *(b^4*c^4*d - a*b^3*c^3*d^2)*\sqrt{(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(1/3)} \\ & /d}*\log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 - 3* \\ & \sqrt{1/3}*(2*(b*x^3 + a)^{(2/3)}*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 \\ & - a^2*d^3)^{(1/3)}*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(2/3)}*(\\ & b*x^3 + a)^{(1/3}))*\sqrt{(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(1/3)}/d} - 3*(- \\ & b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(1/3)}*(b*x^3 + a)^{(1/3)}*(b*c - a*d))/(d* \\ & x^3 + c)) + 3*(28*b^4*c^4*d - 35*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3 - 15*a^3 \\ & *b*c*d^4 + 18*a^4*d^5 + 4*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 - \\ & (7*b^4*c^3*d^2 - 8*a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b* \\ & x^3 + a)^{(1/3)}/(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6), 1/84*(14*(-b^2 \\ & *c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(2/3)}*b^3*c^3*\log(-(b*x^3 + a)^{(2/3)}*(b*c*d \\ & - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(1/3)}*(b*c - a*d) + (-b^2* \\ & c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(2/3)}*(b*x^3 + a)^{(1/3})) - 28*(-b^2*c^2*d + \\ & 2*a*b*c*d^2 - a^2*d^3)^{(2/3)}*b^3*c^3*\log(-(b*x^3 + a)^{(1/3)}*(b*c*d - a*d^2) \\ & - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(2/3)}) - 84*\sqrt{1/3}*(b^4*c^4*d - \\ & a*b^3*c^3*d^2)*\sqrt{(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(1/3)}/d}*\arctan(\sqrt{1/3}*(\\ & (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^{(1/3)}*(b*c - a*d) + 2*(-b^2*c^2*d \\ & + 2*a*b*c*d^2 - a^2*d^3)^{(2/3)}*(b*x^3 + a)^{(1/3}))*\sqrt{(-b^2*c^2*d + \\ & 2*a*b*c*d^2 - a^2*d^3)^{(1/3)}/d}/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + 3*(28*b \\ & ^4*c^4*d - 35*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3 - 15*a^3*b*c*d^4 + 18*a^4*d \\ & ^5 + 4*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 - (7*b^4*c^3*d^2 - 8 \\ & *a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^{(1/3)}/(b^ \end{aligned}$$

$$5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)]$$

giac [A] time = 0.26, size = 372, normalized size = 1.54

$$\frac{b^{24}c^4d^4 \left(-\frac{bc}{d} \right)^{\frac{1}{3}} \log \left(\left(bx^3 + a \right)^{\frac{1}{3}} - \left(-\frac{bc}{d} \right)^{\frac{1}{3}} \right)}{3(b^{24}c^4d^4 - ab^{24}d^4)} - \frac{(-bc^2 + ad)^{\frac{1}{3}} c^3 \arctan \left(\frac{\sqrt{3} \left((bx^3 + a)^{\frac{1}{3}} - \left(-\frac{bc}{d} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{bc}{d} \right)^{\frac{1}{3}} \right)}{\sqrt{3} bc^2 - \sqrt{3} ad^2}}{\sqrt{3} bc^2 - \sqrt{3} ad^2}} - \frac{(-bc^2 + ad)^{\frac{1}{3}} c^3 \log \left(\left(bx^3 + a \right)^{\frac{1}{3}} + \left(bx^3 + a \right)^{\frac{1}{3}} \left(-\frac{bc}{d} \right)^{\frac{1}{3}} + \left(-\frac{bc}{d} \right)^{\frac{1}{3}} \right)}{6(bc^2 - ad^2)} + \frac{28(bx^3 + a)^{\frac{1}{3}} b^{20} c^2 d^4 - 7(bx^3 + a)^{\frac{1}{3}} b^{19} c d^5 + 28(bx^3 + a)^{\frac{1}{3}} a b^{18} c d^6 + 4(bx^3 + a)^{\frac{1}{3}} b^{18} d^6 - 14(bx^3 + a)^{\frac{1}{3}} a b^{17} d^6 + 28(bx^3 + a)^{\frac{1}{3}} a^2 b^{16} d^6}{28 b^{21} d^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out] $\frac{1}{3} b^{24} c^3 d^4 \left(-\frac{bc}{d} \right)^{\frac{1}{3}} \log \left(\text{abs} \left(\left(bx^3 + a \right)^{\frac{1}{3}} - \left(-\frac{bc}{d} \right)^{\frac{1}{3}} \right) \right) / \left(b^{25} c^3 d^7 - a b^{24} d^8 \right) - \left(-bc^2 d^2 + a d^3 \right)^{\frac{1}{3}} c^3 \arctan \left(\frac{1}{3} \sqrt{3} \left(2 \left(bx^3 + a \right)^{\frac{1}{3}} + \left(-\frac{bc}{d} \right)^{\frac{1}{3}} \right) / \left(-\frac{bc}{d} \right)^{\frac{1}{3}} \right) / \left(-\frac{bc}{d} \right)^{\frac{1}{3}} / \left(\sqrt{3} b^2 c^3 d^4 - \sqrt{3} a d^5 \right) - \frac{1}{6} \left(-bc^2 d^2 + a d^3 \right)^{\frac{1}{3}} c^3 \log \left(\left(bx^3 + a \right)^{\frac{2}{3}} + \left(bx^3 + a \right)^{\frac{1}{3}} \left(-\frac{bc}{d} \right)^{\frac{1}{3}} + \left(-\frac{bc}{d} \right)^{\frac{2}{3}} \right) / \left(b^2 c^3 d^4 - a d^5 \right) + \frac{1}{28} \left(28 \left(bx^3 + a \right)^{\frac{1}{3}} b^2 0 c^2 d^4 - 7 \left(bx^3 + a \right)^{\frac{4}{3}} b^{19} c^2 d^5 + 28 \left(bx^3 + a \right)^{\frac{1}{3}} a b^{19} c d^5 + 4 \left(bx^3 + a \right)^{\frac{7}{3}} b^{18} d^6 - 14 \left(bx^3 + a \right)^{\frac{4}{3}} a b^{18} d^6 + 28 \left(bx^3 + a \right)^{\frac{1}{3}} a^2 b^{18} d^6 \right) / \left(b^{21} d^7 \right)$

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b x^3 + a \right)^{\frac{2}{3}} \left(d x^3 + c \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details) Is a*d-b*c positive or negative?

mupad [B] time = 5.01, size = 331, normalized size = 1.37

$$\left(\frac{3a^2}{b^3 d} + \frac{\left(\frac{3a}{b^2 d} + \frac{b^2 c - a b^2 d}{b^2 d} \right) (b^3 c - a b^2 d)}{b^3 d} \right) (b x^3 + a)^{1/3} - \left(\frac{3a}{4 b^3 d} + \frac{b^4 c - a b^2 d}{4 b^2 d^2} \right) (b x^3 + a)^{4/3} + \frac{(b x^3 + a)^{7/3}}{7 b^3 d} + \frac{\ln \left(\frac{3 c^2 (b x^3 + a)^{1/3} + 3 c^2 (1 + \sqrt{3} i) (a d - b c)^{1/3}}{d} \right)}{6 d^{1/3} (a d - b c)^{2/3}} - \frac{c^2 + \sqrt{3} c^2 i}{3 d^{1/3} (a d - b c)^{2/3}} - c^2 \ln \left(\frac{3 c^2 (b x^3 + a)^{1/3} - 3 c^2 (a d - b c)^{1/3}}{d} \right) - \frac{c^2 \ln \left(\frac{3 c^2 (b x^3 + a)^{1/3} - 3 c^2 \left(-\frac{1}{2} + \frac{\sqrt{3} i}{2} \right) (a d - b c)^{1/3}}{d} \right)}{3 d^{1/3} (a d - b c)^{2/3}} \left(\frac{1}{2} + \frac{\sqrt{3} i}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x)`

[Out]
$$\begin{aligned} & \left(\frac{3a^2}{b^3d} + \left(\frac{3a}{b^3d} + \frac{b^4c - ab^3d}{b^6d^2} \right) (b^4c - ab^3d) \right) / (b^3d) * (a + b*x^3)^{1/3} - \left(\frac{3a}{4*b^3*d} + \frac{b^4*c - a*b^3*d}{4*b^6*d^2} \right) * (a + b*x^3)^{4/3} + (a + b*x^3)^{7/3} / (7*b^3*d) + \log((3*c^3 * (a + b*x^3)^{1/3}) / d + (3*c^3*(3^{1/2}*i + 1)*(a*d - b*c)^{1/3}) / (2*d^{4/3})) * (3^{1/2}*c^3*i + c^3) / (6*d^{10/3}*(a*d - b*c)^{2/3}) - (c^3*\log((3*c^3*(a + b*x^3)^{1/3}) / d - (3*c^3*(a*d - b*c)^{1/3}) / d^{4/3})) / (3*d^{10/3}*(a*d - b*c)^{2/3}) - (c^3*\log((3*c^3*(a + b*x^3)^{1/3}) / d - (3*c^3*((3^{1/2}*i)/2 - 1/2)*(a*d - b*c)^{1/3}) / d^{4/3})) * ((3^{1/2}*i)/2 - 1/2)) / (3*d^{10/3}*(a*d - b*c)^{2/3}) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a + bx^3)^{\frac{2}{3}}(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11/(b*x**3+a)**(2/3)/(d*x**3+c),x)`

[Out] `Integral(x**11/((a + b*x**3)**(2/3)*(c + d*x**3)), x)`

$$3.469 \quad \int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)} dx$$

Optimal. Leaf size=201

$$-\frac{\sqrt[3]{a+bx^3}(ad+bc)}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} - \frac{c^2 \log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2 \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}} - \frac{c^2 \tan^{-1}\left(\frac{1-2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{7/3}(bc-ad)^{2/3}}$$

Rubi [A] time = 0.21, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 88, 58, 617, 204, 31}

$$-\frac{\sqrt[3]{a+bx^3}(ad+bc)}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} - \frac{c^2 \log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2 \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}} - \frac{c^2 \tan^{-1}\left(\frac{1-2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{7/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out] -(((b*c + a*d)*(a + b*x^3)^(1/3))/(b^2*d^2)) + (a + b*x^3)^(4/3)/(4*b^2*d) - (c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(7/3)*(b*c - a*d)^(2/3)) - (c^2*Log[c + d*x^3])/(6*d^(7/3)*(b*c - a*d)^(2/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(7/3)*(b*c - a*d)^(2/3))

Rule 31

Int[((a_) + (b_)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 58

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{-bc-ad}{bd^2(a+bx)^{2/3}} + \frac{\sqrt[3]{a+bx}}{bd} + \frac{c^2}{d^2(a+bx)^{2/3}(c+dx)} \right) dx, x, x^3 \right) \\
&= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{c^2 \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3d^2} \\
&= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} - \frac{c^2 \log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2 \text{Subst} \left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x} dx, x, x^3 \right)}{2d^{7/3}(bc-ad)^{2/3}} \\
&= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} - \frac{c^2 \log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2 \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}} \\
&= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} - \frac{c^2 \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}(bc-ad)^{2/3}} - \frac{c^2 \log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 211, normalized size = 1.05

$$\frac{-\frac{12\sqrt[3]{a+bx^3}(ad+bc)}{b^2} + \frac{3d(a+bx^3)^{4/3}}{b^2} - \frac{2c^2 \left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3}\right) - 2\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) \right)}{\sqrt[3]{d}(bc-ad)^{2/3}}}{12d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out] ((-12*(b*c + a*d)*(a + b*x^3)^(1/3))/b^2 + (3*d*(a + b*x^3)^(4/3))/b^2 - (2*c^2*(2*sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)]/sqrt[3] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/(d^(1/3)*(b*c - a*d)^(2/3)))/(12*d^2)

IntegrateAlgebraic [A] time = 0.42, size = 246, normalized size = 1.22

$$\frac{\sqrt[3]{a+bx^3}(-3ad-4bc+bdx^3)}{4b^2d^2} + \frac{c^2 \log\left(\frac{\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}}{3d^{7/3}(bc-ad)^{2/3}}\right)}{3d^{7/3}(bc-ad)^{2/3}} - \frac{c^2 \log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3}\right)}{6d^{7/3}(bc-ad)^{2/3}} - \frac{c^2 \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{7/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-4*b*c - 3*a*d + b*d*x^3))/(4*b^2*d^2) - (c^2*ArcTan[1/
Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt
[3]*d^(7/3)*(b*c - a*d)^(2/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a +
b*x^3)^(1/3)])/(3*d^(7/3)*(b*c - a*d)^(2/3)) - (c^2*Log[(b*c - a*d)^(2/3) -
d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/
(6*d^(7/3)*(b*c - a*d)^(2/3))
```

fricas [B] time = 0.47, size = 1156, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a
)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c -
a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 4*(b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a)^(1/3)*(b*c*
d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)) + 6*sqrt(1/3)*(b^3*
c^3*d - a*b^2*c^2*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)*l
og((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 + 3*sqrt(1/
3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a
)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d) + 3*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c))
+ 3*(4*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + 3*a^3*d^4 - (b^3*c^2*d
^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(1/3))/(b^4*c^2*d^3 - 2*a*
b^3*c*d^4 + a^2*b^2*d^5), -1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)
)*b^2*c^2*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2
+ a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(
b*x^3 + a)^(1/3)) - 4*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log
(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(
2/3)) - 12*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d
^3)^(1/3)*(b*c - a*d) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3
+ a)^(1/3))*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d))/(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)) + 3*(4*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + 3*
a^3*d^4 - (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(1/3))
/(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)]
```

giac [A] time = 0.30, size = 312, normalized size = 1.55

$$\frac{b^{10}c^2d^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3(b^{11}cd^4-ab^{10}d^5)}+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4}+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6(bcd^3-ad^4)}-\frac{4\left(bx^3+a\right)^{\frac{1}{3}}b^7cd^2-\left(bx^3+a\right)^{\frac{4}{3}}b^6d^3+4\left(bx^3+a\right)^{\frac{5}{3}}ab^6d^3}{4b^6d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out]
$$-1/3*b^{10}*c^2*d^2*(-(b*c - a*d)/d)^{(1/3)}*\log(\text{abs}((b*x^3 + a)^{(1/3)} - ((b*c - a*d)/d)^{(1/3)}))/(b^{11}*c*d^4 - a*b^{10}*d^5) + (-b*c*d^2 + a*d^3)^{(1/3)}*c^2*\arctan(1/3*\text{sqrt}(3)*(2*(b*x^3 + a)^{(1/3)} + ((b*c - a*d)/d)^{(1/3)})/((-b*c - a*d)/d)^{(1/3)})/(\text{sqrt}(3)*b*c*d^3 - \text{sqrt}(3)*a*d^4) + 1/6*(-b*c*d^2 + a*d^3)^{(1/3)}*c^2*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*((b*c - a*d)/d)^{(1/3)} + ((b*c - a*d)/d)^{(2/3)})/(b*c*d^3 - a*d^4) - 1/4*(4*(b*x^3 + a)^{(1/3)}*b^7*c*d^2 - (b*x^3 + a)^{(4/3)}*b^6*d^3 + 4*(b*x^3 + a)^{(1/3)}*a*b^6*d^3)/(b^8*d^4)$$

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details) Is a*d-b*c positive or negative?

mupad [B] time = 4.69, size = 292, normalized size = 1.45

$$\frac{(bx^3+a)^{4/3}}{4b^2d} - \left(\frac{2a}{b^2d} + \frac{b^3c-ab^2d}{b^4d^2}\right)(bx^3+a)^{1/3} - \frac{\ln\left(3c^2(bx^3+a)^{1/3} + \frac{(c^2+\sqrt{3}c^2i)(9ad^3-9bc d^2)}{6d^{7/3}(ad-bc)^{2/3}}\right)(c^2+\sqrt{3}c^2i)}{6d^{7/3}(ad-bc)^{2/3}} + \frac{c^2\ln\left(3c^2(bx^3+a)^{1/3} - \frac{c^2(9ad^3-9bc d^2)}{3d^{7/3}(ad-bc)^{2/3}}\right)}{3d^{7/3}(ad-bc)^{2/3}} + \frac{c^2\ln\left(3c^2(bx^3+a)^{1/3} - \frac{c^2\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)(9ad^3-9bc d^2)}{d^{7/3}(ad-bc)^{2/3}}\right)}{d^{7/3}(ad-bc)^{2/3}}\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x)`

[Out] $(a + b*x^3)^{4/3}/(4*b^2*d) - ((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))$
 $* (a + b*x^3)^{1/3} - (\log(3*c^2*(a + b*x^3)^{1/3} + ((3^{1/2}*c^2*1i + c^2)$
 $* (9*a*d^3 - 9*b*c*d^2))/(6*d^{7/3}*(a*d - b*c)^{2/3}))* (3^{1/2}*c^2*1i + c^2$
 $2))/(6*d^{7/3}*(a*d - b*c)^{2/3}) + (c^2*\log(3*c^2*(a + b*x^3)^{1/3} - (c^2$
 $* (9*a*d^3 - 9*b*c*d^2))/(3*d^{7/3}*(a*d - b*c)^{2/3}))/ (3*d^{7/3}*(a*d - b$
 $*c)^{2/3}) + (c^2*\log(3*c^2*(a + b*x^3)^{1/3} - (c^2*((3^{1/2}*1i)/6 - 1/6)$
 $* (9*a*d^3 - 9*b*c*d^2))/(d^{7/3}*(a*d - b*c)^{2/3}))* ((3^{1/2}*1i)/6 - 1/6)$
 $)/(d^{7/3}*(a*d - b*c)^{2/3})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a + bx^3)^{\frac{2}{3}}(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8/(b*x**3+a)**(2/3)/(d*x**3+c),x)`

[Out] `Integral(x**8/((a + b*x**3)**(2/3)*(c + d*x**3)), x)`

$$3.470 \quad \int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)} dx$$

Optimal. Leaf size=165

$$\frac{c \log(c+dx^3)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}} + \frac{c \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}(bc-ad)^{2/3}} + \frac{\sqrt[3]{a+bx^3}}{bd}$$

Rubi [A] time = 0.16, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 80, 58, 617, 204, 31}

$$\frac{c \log(c+dx^3)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}} + \frac{c \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}(bc-ad)^{2/3}} + \frac{\sqrt[3]{a+bx^3}}{bd}$$

Antiderivative was successfully verified.

[In] Int[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out] (a + b*x^3)^(1/3)/(b*d) + (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sqrt[3]*d^(4/3)*(b*c - a*d)^(2/3)) + (c*Log[c + d*x^3])/(6*d^(4/3)*(b*c - a*d)^(2/3)) - (c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(4/3)*(b*c - a*d)^(2/3))

Rule 31

Int[((a_) + (b_)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 58

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right) \\
&= \frac{\sqrt[3]{a+bx^3}}{bd} - \frac{c \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3d} \\
&= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c \log(c+dx^3)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c \text{Subst} \left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}+x}{\sqrt[3]{a}}}} dx, x, \sqrt[3]{a+bx^3} \right)}{2d^{4/3}(bc-ad)^{2/3}} - \frac{c \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx^3}} dx, x, \sqrt[3]{a+bx^3} \right)}{3d} \\
&= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c \log(c+dx^3)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c \log \left(\sqrt[3]{bc-ad} + \sqrt[3]{a+bx^3} \right)}{2d^{4/3}(bc-ad)^{2/3}} - \frac{c \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx^3}} dx, x, \sqrt[3]{a+bx^3} \right)}{3d} \\
&= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}} \right)}{\sqrt{3} d^{4/3}(bc-ad)^{2/3}} + \frac{c \log(c+dx^3)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c \log \left(\sqrt[3]{bc-ad} + \sqrt[3]{a+bx^3} \right)}{2d^{4/3}(bc-ad)^{2/3}} - \frac{c \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx^3}} dx, x, \sqrt[3]{a+bx^3} \right)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 202, normalized size = 1.22

$$\frac{bc \log \left(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3} (a+bx^3)^{2/3} \right) + 6\sqrt[3]{d} \sqrt[3]{a+bx^3} (bc-ad)^{2/3} - 2bc \log \left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3} \right) - 2\sqrt{3} bc \tan^{-1} \left(\frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3} - 1}{\sqrt[3]{bc-ad}} \right)}{6bd^{4/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out] (6*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*b*c*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d))/Sqrt[3]] - 2*b*c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + b*c*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*b*d^(4/3)*(b*c - a*d)^(2/3))

IntegrateAlgebraic [A] time = 0.21, size = 221, normalized size = 1.34

$$-\frac{c \log \left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3} \right)}{3d^{4/3}(bc-ad)^{2/3}} + \frac{c \log \left(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3} (a+bx^3)^{2/3} \right)}{6d^{4/3}(bc-ad)^{2/3}} + \frac{c \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}} \right)}{\sqrt{3} d^{4/3}(bc-ad)^{2/3}} + \frac{\sqrt[3]{a+bx^3}}{bd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] (a + b*x^3)^(1/3)/(b*d) + (c*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*d^(4/3)*(b*c - a*d)^(2/3)) - (c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(3*d^(4/3)*(b*c - a*d)^(2/3)) + (c*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/(6*d^(4/3)*(b*c - a*d)^(2/3))

fricas [B] time = 0.61, size = 1060, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] [1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 3*sqrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)*log((b^2*c^2*d - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d) - 3*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x^3 + a)^(1/3))/(b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4), 1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 6*sqrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)*arctan(sqrt(1/3)*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x^3 + a)^(1/3))/(b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)]

giac [A] time = 0.23, size = 253, normalized size = 1.53

$$\frac{6(-bcd^2+ad^3)^{\frac{1}{3}}bc \arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}}bc \log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{bcd^2-ad^3} - \frac{2bc\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left(\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{bcd-ad^2} - \frac{6(bx^3+a)^{\frac{1}{3}}}{d}$$

6b

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out]
$$-1/6*(6*(-b*c*d^2 + a*d^3)^{1/3}*b*c*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{1/3} + (-b*c - a*d)/d)^{1/3})/(-b*c - a*d)/d)^{1/3})/(\sqrt{3}*b*c*d^2 - \sqrt{3}*a*d^3) + (-b*c*d^2 + a*d^3)^{1/3}*b*c*\log((b*x^3 + a)^{2/3} + (b*x^3 + a)^{1/3})*(-b*c - a*d)/d)^{1/3} + (-b*c - a*d)/d)^{2/3})/(b*c*d^2 - a*d^3) - 2*b*c*(-b*c - a*d)/d)^{1/3}*\log(\text{abs}((b*x^3 + a)^{1/3} - (-b*c - a*d)/d)^{1/3}))/b$$

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.68, size = 232, normalized size = 1.41

$$\frac{(bx^3 + a)^{1/3}}{bd} - \frac{c \ln\left(3cd(bx^3 + a)^{1/3} - \frac{c(9ad^3 - 9bcd^2)}{3d^{4/3}(ad - bc)^{2/3}}\right)}{3d^{4/3}(ad - bc)^{2/3}} + \frac{\ln\left(3cd(bx^3 + a)^{1/3} + \frac{(9ad^3 - 9bcd^2)(c - \sqrt{3}ci)}{6d^{4/3}(ad - bc)^{2/3}}\right)(c - \sqrt{3}ci)}{6d^{4/3}(ad - bc)^{2/3}} + \frac{\ln\left(3cd(bx^3 + a)^{1/3} + \frac{(9ad^3 - 9bcd^2)(c + \sqrt{3}ci)}{6d^{4/3}(ad - bc)^{2/3}}\right)(c + \sqrt{3}ci)}{6d^{4/3}(ad - bc)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x)

[Out]
$$(a + b*x^3)^{1/3}/(b*d) - (c*\log(3*c*d*(a + b*x^3)^{1/3} - (c*(9*a*d^3 - 9*b*c*d^2))/(3*d^{4/3)*(a*d - b*c)^{2/3}))/ (3*d^{4/3)*(a*d - b*c)^{2/3}) + (\log(3*c*d*(a + b*x^3)^{1/3} + ((9*a*d^3 - 9*b*c*d^2)*(c - 3^{1/2}*c*1i)))/(6*d^{4/3)*(a*d - b*c)^{2/3}))* (c - 3^{1/2}*c*1i))/(6*d^{4/3)*(a*d - b*c)^{2/3}) + (\log(3*c*d*(a + b*x^3)^{1/3} + ((9*a*d^3 - 9*b*c*d^2)*(c + 3^{1/2}*c*1i)))/(6*d^{4/3)*(a*d - b*c)^{2/3}))* (c + 3^{1/2}*c*1i))/(6*d^{4/3)*(a*d - b*c)^{2/3})$$

1i)))/(6*d^(4/3)*(a*d - b*c)^(2/3)))*(c + 3^(1/2)*c*1i))/(6*d^(4/3)*(a*d - b*c)^(2/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + bx^3)^{\frac{2}{3}}(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x**3+a)**(2/3)/(d*x**3+c), x)

[Out] Integral(x**5/((a + b*x**3)**(2/3)*(c + d*x**3)), x)

$$3.471 \quad \int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)} dx$$

Optimal. Leaf size=145

$$-\frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}} - \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}}$$

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {444, 58, 617, 204, 31}

$$-\frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}} - \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] -(ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sqrt[3]*d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(6*d^(1/3)*(b*c - a*d)^(2/3)) + Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(1/3)*(b*c - a*d)^(2/3))

Rule 31

Int[((a_) + (b_)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 58

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 444

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{(a + bx^3)^{2/3} (c + dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(a + bx)^{2/3} (c + dx)} dx, x, x^3 \right) \\
 &= -\frac{\log(c + dx^3)}{6\sqrt[3]{d} (bc - ad)^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}} + x} dx, x, \sqrt[3]{a + bx^3} \right)}{2\sqrt[3]{d} (bc - ad)^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}} dx, x, \sqrt[3]{a + bx^3} \right)}{2d^{2/3} \sqrt[3]{d}} \\
 &= -\frac{\log(c + dx^3)}{6\sqrt[3]{d} (bc - ad)^{2/3}} + \frac{\log \left(\sqrt[3]{bc - ad} + \sqrt[3]{d} \sqrt[3]{a + bx^3} \right)}{2\sqrt[3]{d} (bc - ad)^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2\sqrt[3]{d} \sqrt[3]{a + bx^3}}{\sqrt[3]{bc - ad}} \right)}{\sqrt[3]{d} (bc - ad)^{2/3}} \\
 &= -\frac{\tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{d} \sqrt[3]{a + bx^3}}{\sqrt[3]{bc - ad}}}{\sqrt{3}} \right)}{\sqrt{3} \sqrt[3]{d} (bc - ad)^{2/3}} - \frac{\log(c + dx^3)}{6\sqrt[3]{d} (bc - ad)^{2/3}} + \frac{\log \left(\sqrt[3]{bc - ad} + \sqrt[3]{d} \sqrt[3]{a + bx^3} \right)}{2\sqrt[3]{d} (bc - ad)^{2/3}}
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 164, normalized size = 1.13

$$\frac{-\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}(a+bx^3)^{2/3}\right)+2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}-1}{\frac{\sqrt[3]{bc-ad}}{\sqrt{3}}}\right)}{6\sqrt[3]{d}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(1/3)*(b*c - a*d)^(2/3))

IntegrateAlgebraic [A] time = 0.16, size = 201, normalized size = 1.39

$$\frac{\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}(a+bx^3)^{2/3}\right)}{6\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{d}(bc-ad)^{2/3}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] -(ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*d^(1/3)*(b*c - a*d)^(2/3))) + Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(3*d^(1/3)*(b*c - a*d)^(2/3)) - Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/(6*d^(1/3)*(b*c - a*d)^(2/3))

fricas [B] time = 0.56, size = 927, normalized size = 6.39



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] [-1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(2

$$\begin{aligned} & /3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{(1/3)}*(b*c - a*d) \\ & + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{(2/3)}*(b*x^3 + a)^{(1/3)} - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{(2/3)}*log(-(b*x^3 + a)^{(1/3)}*(b*c*d - a*d^2) - \\ & (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{(2/3)})) / (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3), 1/6*(6*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{(1/3)}/d) * \\ & arctan(-sqrt(1/3)*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{(1/3)}*(b*c - a*d) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{(2/3)}*(b*x^3 + a)^{(1/3)})) * \\ & sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{(1/3)}/d) / (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^2)) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{(2/3)}*log(-(b*x^3 + a)^{(2/3)} * \\ & (b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{(1/3)}*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{(2/3)}*(b*x^3 + a)^{(1/3)}) + 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{(2/3)}*log(-(b*x^3 + a)^{(1/3)}*(b*c*d - a*d^2) - \\ & (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^{(2/3)})) / (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)] \end{aligned}$$

giac [A] time = 0.29, size = 221, normalized size = 1.52

$$\frac{(-bcd^2 + ad^3)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd - \sqrt{3}ad^2} + \frac{(-bcd^2 + ad^3)^{\frac{1}{3}} \log\left(\left(bx^3+a\right)^{\frac{2}{3}} + \left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6(bcd - ad^2)} - \frac{\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left(\left(bx^3+a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out]
$$\begin{aligned} & (-b*c*d^2 + a*d^3)^{(1/3)}*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^{(1/3)} + (-b*c - a*d)/d)^{(1/3)}) / (-b*c - a*d)/d)^{(1/3)} / (sqrt(3)*b*c*d - sqrt(3)*a*d^2) + 1 \\ & /6*(-b*c*d^2 + a*d^3)^{(1/3)}*log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(-b*c - a*d)/d)^{(1/3)} + (-b*c - a*d)/d)^{(2/3)}) / (b*c*d - a*d^2) - 1/3*(-b*c - a*d)/d)^{(1/3)}*log(abs((b*x^3 + a)^{(1/3)} - (-b*c - a*d)/d)^{(1/3)}) / (b*c - a*d) \end{aligned}$$

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.85, size = 213, normalized size = 1.47

$$\frac{\ln\left(3d^2(bx^3+a)^{1/3} - \frac{9ad^3-9bcd^2}{3d^{1/3}(ad-bc)^{2/3}}\right)}{3d^{1/3}(ad-bc)^{2/3}} + \frac{\ln\left(3d^2(bx^3+a)^{1/3} - \frac{(-1+\sqrt{3}i)(9ad^3-9bcd^2)}{6d^{1/3}(ad-bc)^{2/3}}\right)(-1+\sqrt{3}i)}{6d^{1/3}(ad-bc)^{2/3}} - \frac{\ln\left(3d^2(bx^3+a)^{1/3} + \frac{(1+\sqrt{3}i)(9ad^3-9bcd^2)}{6d^{1/3}(ad-bc)^{2/3}}\right)(1+\sqrt{3}i)}{6d^{1/3}(ad-bc)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x)`

[Out] $\log(3d^2(a + bx^3)^{1/3} - (9ad^3 - 9b^2cd^2)/(3d^{1/3}(ad - b^2c)^{2/3}))/ (3d^{1/3}(ad - b^2c)^{2/3}) + (\log(3d^2(a + bx^3)^{1/3} - ((3^{1/2}i - 1)(9ad^3 - 9b^2cd^2))/(6d^{1/3}(ad - b^2c)^{2/3}))) * (3^{1/2}i - 1) / (6d^{1/3}(ad - b^2c)^{2/3}) - (\log(3d^2(a + bx^3)^{1/3} + ((3^{1/2}i + 1)(9ad^3 - 9b^2cd^2))/(6d^{1/3}(ad - b^2c)^{2/3}))) * (3^{1/2}i + 1) / (6d^{1/3}(ad - b^2c)^{2/3})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + bx^3)^{\frac{2}{3}}(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)`

[Out] `Integral(x**2/((a + b*x**3)**(2/3)*(c + d*x**3)), x)`

$$3.472 \quad \int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)} dx$$

Optimal. Leaf size=245

$$\frac{\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}c} - \frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log(c+dx^3)}{6c(bc-ad)^{2/3}} - \frac{d^{2/3}\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}} + \dots$$

Rubi [A] time = 0.21, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 86, 57, 617, 204, 31, 58}

$$\frac{\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}c} - \frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log(c+dx^3)}{6c(bc-ad)^{2/3}} - \frac{d^{2/3}\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}} + \frac{d^{2/3}\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*c)) + (d^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sqrt[3]*c*(b*c - a*d)^(2/3)) - Log[x]/(2*a^(2/3)*c) + (d^(2/3)*Log[c + d*x^3]/(6*c*(b*c - a*d)^(2/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(2/3)*c) - (d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/((2*c*(b*c - a*d)^(2/3)))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])]/; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] := Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
p}, x] && !IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{x(a+bx)^{2/3}} dx, x, x^3 \right)}{3c} - \frac{d \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3c} \\
&= -\frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3} \log(c+dx^3)}{6c(bc-ad)^{2/3}} - \frac{\text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, \sqrt[3]{a+bx^3} \right)}{2a^{2/3}c} - \frac{\text{Subst} \left(\int \frac{1}{a^{2/3}} dx, x, \sqrt[3]{a+bx^3} \right)}{2a^{2/3}c} \\
&= -\frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3} \log(c+dx^3)}{6c(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{d^{2/3} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}} \\
&= -\frac{\tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}c} + \frac{d^{2/3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{2/3}} - \frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3} \log(c+dx^3)}{6c(bc-ad)^{2/3}} +
\end{aligned}$$

Mathematica [A] time = 0.45, size = 308, normalized size = 1.26

$$-\frac{2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{a^{2/3}} + \frac{\log\left(a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}\right)}{a^{2/3}} + \frac{2\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + 1}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2a^{2/3} \log\left(\frac{\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}}{(bc-ad)^{2/3}}\right)}{(bc-ad)^{2/3}} - \frac{d^{2/3} \log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3} - \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{2/3}} + \frac{2\sqrt{3}d^{2/3} \tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3} - 1}{\sqrt{3}}\right)}{(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out] $-\frac{1}{6} \left(\frac{2\sqrt{3} \text{ArcTan}\left[\frac{1 + (2(a + b x^3)^{1/3})}{a^{1/3}}\right]}{\sqrt{3}} \right) / a^{2/3} + \frac{2\sqrt{3} d^{2/3} \text{ArcTan}\left[\frac{-1 + (2d^{1/3}(a + b x^3)^{1/3})}{b c - a d}\right]}{(b c - a d)^{2/3}} - \frac{2 \text{Log}\left[a^{1/3} - (a + b x^3)^{1/3}\right]}{a^{2/3}} + \frac{2 d^{2/3} \text{Log}\left[(b c - a d)^{1/3} + d^{1/3}(a + b x^3)^{1/3}\right]}{(b c - a d)^{2/3}} + \frac{\text{Log}\left[a^{2/3} + a^{1/3}(a + b x^3)^{1/3} + (a + b x^3)^{2/3}\right]}{a^{2/3}} - \frac{d^{2/3} \text{Log}\left[(b c - a d)^{2/3} - d^{1/3}(b c - a d)^{1/3}(a + b x^3)^{1/3} + d^{2/3}(a + b x^3)^{2/3}\right]}{(b c - a d)^{2/3}} / c$

IntegrateAlgebraic [A] time = 0.59, size = 333, normalized size = 1.36

$$\frac{\log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{a}\right)}{3a^{2/3}c} - \frac{\log\left(a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}\right)}{6a^{2/3}c} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + 1}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}c} - \frac{d^{2/3} \log\left(\frac{\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}}{3c(bc-ad)^{2/3}}\right)}{3c(bc-ad)^{2/3}} + \frac{d^{2/3} \log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3} - \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3}\right)}{6c(bc-ad)^{2/3}} + \frac{d^{2/3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] $-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(a + b x^3)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} c} + \frac{d^{2/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/3} (a + b x^3)^{1/3}}{\sqrt{3} (b c - a d)^{1/3}}\right]}{\sqrt{3} c (b c - a d)^{2/3}} + \frac{\text{Log}\left[-a^{1/3} + (a + b x^3)^{1/3}\right]}{3 a^{2/3} c} - \frac{d^{2/3} \text{Log}\left[(b c - a d)^{1/3} + d^{1/3} (a + b x^3)^{1/3}\right]}{3 c (b c - a d)^{2/3}} - \frac{\text{Log}\left[a^{2/3} + a^{1/3} (a + b x^3)^{1/3} + (a + b x^3)^{2/3}\right]}{6 a^{2/3} c} + \frac{d^{2/3} \text{Log}\left[(b c - a d)^{2/3} - d^{1/3} (b c - a d)^{1/3} (a + b x^3)^{1/3} + d^{2/3} (a + b x^3)^{2/3}\right]}{6 c (b c - a d)^{2/3}}$

fricas [B] time = 0.50, size = 472, normalized size = 1.93

$$\frac{2\sqrt{3}\left(\frac{d}{3(bc-ad)}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(a+b x^3)^{\frac{1}{3}}-\frac{bc-ad}{d}}{\frac{d}{3(bc-ad)}\right)^{\frac{1}{3}}}{3(bc-ad)} + \frac{d^{\frac{2}{3}}\left(\frac{d}{3(bc-ad)}\right)^{\frac{1}{3}}\log\left(\frac{(bc-a)^{\frac{1}{3}}d + (bc+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} + (d^2-2ad+d^2)\left(\frac{d}{3(bc-ad)}\right)^{\frac{1}{3}}}{6d}\right)}{6d} - \frac{2d^{\frac{2}{3}}\left(\frac{d}{3(bc-ad)}\right)^{\frac{1}{3}}\log\left(\frac{(bc+a)^{\frac{1}{3}}d - (bc-a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} + 2\sqrt{3}(a+b x^3)^{\frac{1}{3}}\left(\frac{d}{3(bc-ad)}\right)^{\frac{1}{3}}}{6d}\right)}{6d} + \frac{2\sqrt{3}\left(\frac{d}{3(bc-ad)}\right)^{\frac{1}{3}}\arctan\left(\frac{(a+b x^3)^{\frac{1}{3}}-\frac{bc-ad}{d}}{\frac{d}{3(bc-ad)}\right)^{\frac{1}{3}}}{3a^{\frac{2}{3}}c} - \frac{d^{\frac{2}{3}}\log\left(\frac{(bc-a)^{\frac{1}{3}}d + (bc+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} - 2(d^2-2ad+d^2)\left(\frac{d}{3(bc-ad)}\right)^{\frac{1}{3}}}{6d}\right)}{6c(bc-ad)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] $-\frac{1}{6} * (2 * \sqrt{3}) * a^2 * (-d^2 / (b^2 * c^2 - 2 * a * b * c * d + a^2 * d^2))^{1/3} * \arctan(-1 / (3 * (2 * \sqrt{3}) * (b * x^3 + a)^{1/3} * (b * c - a * d) * (-d^2 / (b^2 * c^2 - 2 * a * b * c * d + a^2 * d^2))^{1/3} - \sqrt{3} * d) / d) + a^2 * (-d^2 / (b^2 * c^2 - 2 * a * b * c * d + a^2 * d^2))^{1/3} * \log((b * x^3 + a)^{2/3} * d^2 + (b * x^3 + a)^{1/3} * (b * c * d - a * d^2) * (-d^2 / (b^2 * c^2 - 2 * a * b * c * d + a^2 * d^2))^{1/3} + (b^2 * c^2 - 2 * a * b * c * d + a^2 * d^2) * (-d^2 / (b^2 * c^2 - 2 * a * b * c * d + a^2 * d^2))^{2/3}) - 2 * a^2 * (-d^2 / (b^2 * c^2 - 2 * a * b * c * d + a^2 * d^2))^{1/3} * \log((b * x^3 + a)^{1/3} * d - (b * c - a * d) * (-d^2 / (b^2 * c^2 - 2 * a * b * c * d + a^2 * d^2))^{1/3}) + 2 * \sqrt{3} * (a^2)^{1/6} * a * \arctan(1/3 * (a^2)^{1/6} * (\sqrt{3} * (a^2)^{1/3} * a + 2 * \sqrt{3} * (b * x^3 + a)^{1/3} * (a^2)^{2/3}) / a^2) + (a^2)^{2/3} * \log((b * x^3 + a)^{2/3} * a + (a^2)^{1/3} * a + (b * x^3 + a)^{1/3} * (a^2)^{2/3}) - 2 * (a^2)^{2/3} * \log((b * x^3 + a)^{1/3} * a - (a^2)^{2/3}) / (a^2 * c)$

giac [A] time = 0.76, size = 321, normalized size = 1.31

$$\frac{d \left(\frac{bc-ad}{d} \right)^{\frac{1}{3}} \log \left(\left(bx^3 + a \right)^{\frac{1}{3}} - \left(\frac{bc-ad}{d} \right)^{\frac{1}{3}} \right)}{3(bc^2 - acd)} - \frac{(-bcd^2 + ad^3)^{\frac{1}{3}} \arctan \left(\frac{\sqrt{3} \left(2(bx^3 + a)^{\frac{1}{3}} - \frac{bc-ad}{d} \right)^{\frac{1}{3}}}{3 \left(\frac{bc-ad}{d} \right)^{\frac{1}{3}}} \right)}{\sqrt{3}bc^2 - \sqrt{3}acd} - \frac{(-bcd^2 + ad^3)^{\frac{1}{3}} \log \left((bx^3 + a)^{\frac{2}{3}} + (bx^3 + a)^{\frac{1}{3}} \left(\frac{bc-ad}{d} \right)^{\frac{1}{3}} + \left(\frac{bc-ad}{d} \right)^{\frac{1}{3}} \right)}{6(bc^2 - acd)} - \frac{\sqrt{3} \arctan \left(\frac{\sqrt{3} \left(2(bx^3 + a)^{\frac{1}{3}} - \frac{bc-ad}{d} \right)^{\frac{1}{3}}}{3a^{\frac{1}{3}}c} \right)}{3a^{\frac{1}{3}}c} - \frac{\log \left((bx^3 + a)^{\frac{2}{3}} + (bx^3 + a)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}} \right)}{6a^{\frac{1}{3}}c} + \frac{\log \left((bx^3 + a)^{\frac{1}{3}} - a^{\frac{1}{3}} \right)}{3a^{\frac{1}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out] $\frac{1}{3} * d * (- (b * c - a * d) / d)^{1/3} * \log(\text{abs}((b * x^3 + a)^{1/3} - (- (b * c - a * d) / d)^{1/3})) / (b * c^2 - a * c * d) - (- b * c * d^2 + a * d^3)^{1/3} * \arctan(1/3 * \sqrt{3} * (2 * (b * x^3 + a)^{1/3} + (- (b * c - a * d) / d)^{1/3})) / (- (b * c - a * d) / d)^{1/3} / (\sqrt{3} * b * c^2 - \sqrt{3} * a * c * d) - 1/6 * (- b * c * d^2 + a * d^3)^{1/3} * \log((b * x^3 + a)^{2/3} + (b * x^3 + a)^{1/3} * (- (b * c - a * d) / d)^{1/3} + (- (b * c - a * d) / d)^{2/3}) / (b * c^2 - a * c * d) - 1/3 * \sqrt{3} * \arctan(1/3 * \sqrt{3} * (2 * (b * x^3 + a)^{1/3} + a^{1/3})) /$

$a^{(1/3)}/(a^{(2/3)*c}) - 1/6*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)*a^{(1/3)}} + a^{(2/3)})/(a^{(2/3)*c}) + 1/3*\log(\text{abs}((b*x^3 + a)^{(1/3)} - a^{(1/3)}))/(a^{(2/3)*c})$

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x)`

[Out] `int(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x), x)`

mupad [B] time = 4.94, size = 1413, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x)`

[Out] `log((((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - (243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^(1/3))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^(1/3) + 6*b^4*d^5*(a + b*x^3)^(1/3))*(1/(27*a^2*c^3))^(1/3) + log(- (((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - (-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5))*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(2/3) - 9*b^5*c^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3) - 6*b^4*d^5*(a + b*x^3)^(1/3))*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3) + log(((3^(1/2)*1i)/2 - 1/2)*(((3^(1/2)*1i)/2 - 1/2)^2*((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - ((3^(1/2)*1i)/2 - 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^(1/3))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^(1/3) + 6*b^4*d^5*(a + b*x^3)^(1/3))`

$$d^5*(a + b*x^3)^{(1/3)}*((3^{(1/2)*1i})/2 - 1/2)*(1/(27*a^2*c^3))^{(1/3)} - \log(6*b^4*d^5*(a + b*x^3)^{(1/3)} - ((3^{(1/2)*1i})/2 + 1/2)*(((3^{(1/2)*1i})/2 + 1/2)^2*((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^{(1/3)} + ((3^{(1/2)*1i})/2 + 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^{(1/3)}*(1/(27*a^2*c^3))^{(2/3)} - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^{(1/3)}*((3^{(1/2)*1i})/2 + 1/2)*(1/(27*a^2*c^3))^{(1/3)} + (\log(6*b^4*d^5*(a + b*x^3)^{(1/3)} + ((3^{(1/2)*1i} - 1)*(((3^{(1/2)*1i} - 1)^2*((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^{(1/3)} - ((3^{(1/2)*1i} - 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d)))^{(1/3)}*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5))/2)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d)))^{(2/3)}/4 - 9*b^5*c^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^{(1/3)}/2*(3^{(1/2)*1i} - 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^{(1/3)}/2 - (\log(6*b^4*d^5*(a + b*x^3)^{(1/3)} - ((3^{(1/2)*1i} + 1)*(((3^{(1/2)*1i} + 1)^2*((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^{(1/3)} + ((3^{(1/2)*1i} + 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d)))^{(1/3)}*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5))/2)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d)))^{(2/3)}/4 - 9*b^5*c^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^{(1/3)}/2*(3^{(1/2)*1i} + 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^{(1/3)}/2$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + bx^3)^{\frac{2}{3}}(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**3+a)**(2/3)/(d*x**3+c), x)

[Out] Integral(1/(x*(a + b*x**3)**(2/3)*(c + d*x**3)), x)

$$3.473 \quad \int \frac{1}{x^4(a+bx^3)^{2/3}(c+dx^3)} dx$$

Optimal. Leaf size=299

$$\frac{(3ad + 2bc) \log\left(\sqrt[3]{a} - \sqrt[3]{a + bx^3}\right)}{6a^{5/3}c^2} + \frac{(3ad + 2bc) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}c^2} + \frac{\log(x)(3ad + 2bc)}{6a^{5/3}c^2} - \frac{d^{5/3} \log(c + dx^3)}{6c^2(bc - ad)^{2/3}} + \dots$$

Rubi [A] time = 0.32, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {446, 103, 156, 57, 617, 204, 31, 58}

$$\frac{(3ad + 2bc) \log\left(\sqrt[3]{a} - \sqrt[3]{a + bx^3}\right)}{6a^{5/3}c^2} + \frac{(3ad + 2bc) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}c^2} + \frac{\log(x)(3ad + 2bc)}{6a^{5/3}c^2} - \frac{d^{5/3} \log(c + dx^3)}{6c^2(bc - ad)^{2/3}} + \frac{d^{5/3} \log\left(\sqrt[3]{bc - ad} + \sqrt[3]{d}\sqrt[3]{a + bx^3}\right)}{2c^2(bc - ad)^{2/3}} - \frac{d^{5/3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{bc - ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2(bc - ad)^{2/3}} - \frac{\sqrt[3]{a + bx^3}}{3acx^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] $-(a + b*x^3)^{(1/3)}/(3*a*c*x^3) + ((2*b*c + 3*a*d)*\text{ArcTan}[(a^{(1/3)} + 2*(a + b*x^3)^{(1/3)})/(\text{Sqrt}[3]*a^{(1/3)})])/(3*\text{Sqrt}[3]*a^{(5/3)}*c^2) - (d^{(5/3)}*\text{ArcTan}[(1 - (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})/(b*c - a*d)^{(1/3)})]/\text{Sqrt}[3])]/(\text{Sqrt}[3]*c^2*(b*c - a*d)^{(2/3)}) + ((2*b*c + 3*a*d)*\text{Log}[x])/(6*a^{(5/3)}*c^2) - (d^{(5/3)}*\text{Log}[c + d*x^3])/(6*c^2*(b*c - a*d)^{(2/3)}) - ((2*b*c + 3*a*d)*\text{Log}[a^{(1/3)} - (a + b*x^3)^{(1/3)})]/(6*a^{(5/3)}*c^2) + (d^{(5/3)}*\text{Log}[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)}])/(2*c^2*(b*c - a*d)^{(2/3)})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])]/; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 58


```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (a + bx^3)^{2/3} (c + dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2 (a + bx)^{2/3} (c + dx)} dx, x, x^3 \right) \\
&= -\frac{\sqrt[3]{a + bx^3}}{3acx^3} - \frac{\text{Subst} \left(\int \frac{\frac{1}{3}(2bc+3ad) + \frac{2bdx}{3}}{x(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3ac} \\
&= -\frac{\sqrt[3]{a + bx^3}}{3acx^3} + \frac{d^2 \text{Subst} \left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} dx, x, x^3 \right)}{3c^2} - \frac{(2bc + 3ad) \text{Subst} \left(\int \frac{1}{x(a+bx)} dx, x, x^3 \right)}{9ac^2} \\
&= -\frac{\sqrt[3]{a + bx^3}}{3acx^3} + \frac{(2bc + 3ad) \log(x)}{6a^{5/3}c^2} - \frac{d^{5/3} \log(c + dx^3)}{6c^2(bc - ad)^{2/3}} + \frac{d^{5/3} \text{Subst} \left(\int \frac{1}{\frac{\sqrt[3]{bc-ad} + x}{\sqrt[3]{a}}} dx, x, x^3 \right)}{2c^2(bc - ad)^{2/3}} \\
&= -\frac{\sqrt[3]{a + bx^3}}{3acx^3} + \frac{(2bc + 3ad) \log(x)}{6a^{5/3}c^2} - \frac{d^{5/3} \log(c + dx^3)}{6c^2(bc - ad)^{2/3}} - \frac{(2bc + 3ad) \log(\sqrt[3]{a} - \sqrt[3]{a + bx^3})}{6a^{5/3}c^2} \\
&= -\frac{\sqrt[3]{a + bx^3}}{3acx^3} + \frac{(2bc + 3ad) \tan^{-1} \left(\frac{1 + 2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}} \right)}{3\sqrt{3}a^{5/3}c^2} - \frac{d^{5/3} \tan^{-1} \left(\frac{1 - 2\sqrt[3]{a}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}} \right)}{\sqrt{3}c^2(bc - ad)^{2/3}} + \frac{(2bc + 3ad) \log(\sqrt[3]{a} - \sqrt[3]{a + bx^3})}{6a^{5/3}c^2}
\end{aligned}$$

Mathematica [A] time = 0.59, size = 303, normalized size = 1.01

$$\frac{(3ad+2bc) \log \left(a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^3} + (a+bx^3)^{2/3} \right) - 2 \log \left(\sqrt[3]{a} - \sqrt[3]{a+bx^3} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{2\sqrt[3]{a+bx^3} + 1}{\sqrt[3]{a}} \right)}{a^{2/3}c} + \frac{3ad^{5/3} \left(-\log \left(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + a^{2/3}(a+bx^3)^{2/3} \right) + 2 \log \left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3} - 1}{\sqrt[3]{bc-ad}} \right) \right)}{c(bc-ad)^{2/3}} - \frac{6\sqrt[3]{a+bx^3}}{x^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out] ((-6*(a + b*x^3)^(1/3))/x^3 + ((2*b*c + 3*a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3)]/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]))/(a^(2/3)*c) + (3*a*d^(5/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)]/Sqrt[3]] + 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/(c*(b*c - a*d)^(2/3)))/(18*a*c)

IntegrateAlgebraic [A] time = 0.73, size = 388, normalized size = 1.30

$$\frac{(-3ad-2bc) \log(\sqrt[3]{a+bx^3} - \sqrt[3]{a})}{9a^{5/3}c^2} + \frac{(3ad+2bc) \log(a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^3} + (a+bx^3)^{2/3})}{18a^{5/3}c^2} + \frac{(3ad+2bc) \tan^{-1} \left(\frac{2\sqrt[3]{a+bx^3} + 1}{\sqrt[3]{a}} \right)}{3\sqrt{3}a^{5/3}c^2} + \frac{d^{5/3} \log \left(\sqrt[3]{bc-ad} - \sqrt[3]{d} \sqrt[3]{a+bx^3} \right)}{3c^2(bc-ad)^{2/3}} - \frac{d^{5/3} \log \left(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + a^{2/3}(a+bx^3)^{2/3} \right)}{6c^2(bc-ad)^{2/3}} - \frac{d^{5/3} \tan^{-1} \left(\frac{1}{\sqrt[3]{a}} - \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}} \right)}{\sqrt{3}c^2(bc-ad)^{2/3}} - \frac{\sqrt[3]{a+bx^3}}{3acx^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out]
$$-1/3*(a + b*x^3)^{(1/3)}/(a*c*x^3) + ((2*b*c + 3*a*d)*\text{ArcTan}[(a^{(1/3)} + 2*(a + b*x^3)^{(1/3)})/(\text{Sqrt}[3]*a^{(1/3)})])/(3*\text{Sqrt}[3]*a^{(5/3)*c^2} - (d^{(5/3)*\text{ArcTan}[1/\text{Sqrt}[3] - (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})/(\text{Sqrt}[3]*(b*c - a*d)^{(1/3)})])]/(\text{Sqrt}[3]*c^2*(b*c - a*d)^{(2/3)}) + ((-2*b*c - 3*a*d)*\text{Log}[-a^{(1/3)} + (a + b*x^3)^{(1/3)}])/(9*a^{(5/3)*c^2} + (d^{(5/3)*\text{Log}[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)}])/(3*c^2*(b*c - a*d)^{(2/3)}) + ((2*b*c + 3*a*d)*\text{Log}[a^{(2/3)} + a^{(1/3)}*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}])/(18*a^{(5/3)*c^2} - (d^{(5/3)*\text{Log}[(b*c - a*d)^{(2/3)} - d^{(1/3)}*(b*c - a*d)^{(1/3)}*(a + b*x^3)^{(1/3)} + d^{(2/3)}*(a + b*x^3)^{(2/3)}])/(6*c^2*(b*c - a*d)^{(2/3)})$$

fricas [B] time = 1.25, size = 562, normalized size = 1.88

$$\frac{d^2 \sqrt{3} \log\left(\frac{(b^2 x^3 + a)^{1/3} (b^2 c^2 - 2 a b c d + a^2 d^2)^{1/3}}{(b^2 c^2 - 2 a b c d + a^2 d^2)^{1/3}}\right) + 3 a^2 d \sqrt{3} \log\left(\frac{(b^2 x^3 + a)^{2/3} d^2 - (b^2 x^3 + a)^{1/3} (b^2 c d - a d^2) (d^2 / (b^2 c^2 - 2 a b c d + a^2 d^2))^{1/3}}{(b^2 x^3 + a)^{2/3} d^2 - (b^2 x^3 + a)^{1/3} (b^2 c d - a d^2) (d^2 / (b^2 c^2 - 2 a b c d + a^2 d^2))^{1/3}}\right) - 6 a^2 d \sqrt{3} \log\left(\frac{(b^2 x^3 + a)^{1/3} d + (b^2 c - a d) (d^2 / (b^2 c^2 - 2 a b c d + a^2 d^2))^{1/3}}{(b^2 x^3 + a)^{1/3} d + (b^2 c - a d) (d^2 / (b^2 c^2 - 2 a b c d + a^2 d^2))^{1/3}}\right) - 2 \sqrt{3} (2 a b c + 3 a^2 d) x^3 \sqrt{-(-a^2)^{1/3}} \arctan\left(\frac{-1/3 \sqrt{3} (-a^2)^{1/3} a - 2 \sqrt{3} (b^2 x^3 + a)^{1/3} (-a^2)^{2/3}}{-1/3 \sqrt{3} (-a^2)^{1/3} a - 2 \sqrt{3} (b^2 x^3 + a)^{1/3} (-a^2)^{2/3}}\right) \sqrt{-(-a^2)^{1/3}} / a^2 - (-a^2)^{2/3} (2 b^2 c + 3 a^2 d) x^3 \log\left(\frac{(b^2 x^3 + a)^{2/3} a - (-a^2)^{1/3} a + (b^2 x^3 + a)^{1/3} (-a^2)^{2/3}}{(b^2 x^3 + a)^{2/3} a - (-a^2)^{1/3} a + (b^2 x^3 + a)^{1/3} (-a^2)^{2/3}}\right) + 2 (-a^2)^{2/3} (2 b^2 c + 3 a^2 d) x^3 \log\left(\frac{(b^2 x^3 + a)^{1/3} a - (-a^2)^{2/3}}{(b^2 x^3 + a)^{1/3} a - (-a^2)^{2/3}}\right) + 6 (b^2 x^3 + a)^{1/3} a^2 c / (a^3 c^2 x^3)}{3 (b^3 - a c^2 d) \sqrt{3} b c^3 - \sqrt{3} a c^2 d} + \frac{(-b c^2 + a d)^2 d \arctan\left(\frac{\sqrt{3} \left(\frac{(b^2 x^3 + a)^{1/3} \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}{3 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}\right)}{\sqrt{3} \left(\frac{(b^2 x^3 + a)^{1/3} \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}{3 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}\right)}\right)}{\sqrt{3} b c^3 - \sqrt{3} a c^2 d} + \frac{(-b c^2 + a d)^2 d \log\left(\frac{(b^2 x^3 + a)^2 + (b^2 x^3 + a)^2 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3} + \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^2}{(b^2 x^3 + a)^2 + (b^2 x^3 + a)^2 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3} + \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^2}\right)}{6 (b^3 - a c^2 d)} + \frac{\sqrt{3} (2 b c + 3 a d) \arctan\left(\frac{\sqrt{3} \left(\frac{(b^2 x^3 + a)^{1/3} \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}{3 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}\right)}{\sqrt{3} \left(\frac{(b^2 x^3 + a)^{1/3} \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}{3 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}\right)}\right)}{9 a^2 c^2} + \frac{(2 b c + 3 a d) \log\left(\frac{(b^2 x^3 + a)^2 + (b^2 x^3 + a)^2 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3} + \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^2}{(b^2 x^3 + a)^2 + (b^2 x^3 + a)^2 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3} + \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^2}\right)}{18 a^2 c^2} - \frac{(2 b c + 3 a d) \log\left(\frac{(b^2 x^3 + a)^2 - a^2}{(b^2 x^3 + a)^2 - a^2}\right)}{9 a^2 c^2} + \frac{(b^2 x^3 + a)^2}{3 a c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out]
$$-1/18*(6*\text{sqrt}(3)*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^{1/3}*x^3*\text{arctan}(-1/3*(2*\text{sqrt}(3)*(b*x^3 + a)^{(1/3)}*(b*c - a*d)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^{1/3} - \text{sqrt}(3)*d)/d) + 3*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^{1/3}*x^3*\log((b*x^3 + a)^{(2/3)}*d^2 - (b*x^3 + a)^{(1/3)}*(b*c*d - a*d^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^{1/3} + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^{2/3}) - 6*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^{1/3}*x^3*\log((b*x^3 + a)^{(1/3)}*d + (b*c - a*d)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^{1/3}) - 2*\text{sqrt}(3)*(2*a*b*c + 3*a^2*d)*x^3*\text{sqrt}(-(-a^2)^{(1/3)})*\text{arctan}(-1/3*(\text{sqrt}(3)*(-a^2)^{(1/3)}*a - 2*\text{sqrt}(3)*(b*x^3 + a)^{(1/3)}*(-a^2)^{(2/3)})*\text{sqrt}(-(-a^2)^{(1/3)})/a^2) - (-a^2)^{(2/3)}*(2*b*c + 3*a*d)*x^3*\log((b*x^3 + a)^{(2/3)}*a - (-a^2)^{(1/3)}*a + (b*x^3 + a)^{(1/3)}*(-a^2)^{(2/3})) + 2*(-a^2)^{(2/3)}*(2*b*c + 3*a*d)*x^3*\log((b*x^3 + a)^{(1/3)}*a - (-a^2)^{(2/3})) + 6*(b*x^3 + a)^{(1/3)}*a^2*c/(a^3*c^2*x^3)$$

giac [A] time = 0.76, size = 377, normalized size = 1.26

$$\frac{d^2 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3} \log\left(\frac{(b^2 x^3 + a)^2 - \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)}{(b^2 x^3 + a)^2 - \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)}\right)}{3 (b^3 - a c^2 d) \sqrt{3} b c^3 - \sqrt{3} a c^2 d} + \frac{(-b c^2 + a d)^2 d \arctan\left(\frac{\sqrt{3} \left(\frac{(b^2 x^3 + a)^{1/3} \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}{3 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}\right)}{\sqrt{3} \left(\frac{(b^2 x^3 + a)^{1/3} \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}{3 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}\right)}\right)}{\sqrt{3} b c^3 - \sqrt{3} a c^2 d} + \frac{(-b c^2 + a d)^2 d \log\left(\frac{(b^2 x^3 + a)^2 + (b^2 x^3 + a)^2 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3} + \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^2}{(b^2 x^3 + a)^2 + (b^2 x^3 + a)^2 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3} + \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^2}\right)}{6 (b^3 - a c^2 d)} + \frac{\sqrt{3} (2 b c + 3 a d) \arctan\left(\frac{\sqrt{3} \left(\frac{(b^2 x^3 + a)^{1/3} \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}{3 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}\right)}{\sqrt{3} \left(\frac{(b^2 x^3 + a)^{1/3} \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}{3 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3}}\right)}\right)}{9 a^2 c^2} + \frac{(2 b c + 3 a d) \log\left(\frac{(b^2 x^3 + a)^2 + (b^2 x^3 + a)^2 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3} + \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^2}{(b^2 x^3 + a)^2 + (b^2 x^3 + a)^2 \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^{1/3} + \left(\frac{b^2 c^2 - 2 a b c d + a^2 d^2}{3}\right)^2}\right)}{18 a^2 c^2} - \frac{(2 b c + 3 a d) \log\left(\frac{(b^2 x^3 + a)^2 - a^2}{(b^2 x^3 + a)^2 - a^2}\right)}{9 a^2 c^2} + \frac{(b^2 x^3 + a)^2}{3 a c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out]
$$-1/3*d^2*(-(b*c - a*d)/d)^{(1/3)}*\log(\text{abs}((b*x^3 + a)^{(1/3)} - (-(b*c - a*d)/d)^{(1/3)}))/(b*c^3 - a*c^2*d) + (-b*c*d^2 + a*d^3)^{(1/3)}*d*\text{arctan}(1/3*\text{sqrt}(3))$$

```

*(2*(b*x^3 + a)^(1/3) + (-b*c - a*d)/d)^(1/3))/(-b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-b*c - a*d)/d)^(1/3) + (-b*c - a*d)/d)^(2/3))/b*c^3 - a*c^2*d) + 1/9*sqrt(3)*(2*b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(5/3)*c^2) + 1/18*(2*b*c + 3*a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(5/3)*c^2) - 1/9*(2*b*c + 3*a*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*c^2) - 1/3*(b*x^3 + a)^(1/3)/(a*c*x^3)

```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c), x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c), x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c), x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^4), x)
```

mupad [B] time = 11.14, size = 1959, normalized size = 6.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

```
[Out] log(- ((((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - 81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(a*d - b*c)^2))^(1/3))*(d^5/(c^6*(a*d - b*c)^2))^(2/3))/9 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5/(c^6*(a*d - b*c)^2))^(1/3))/3 - (2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4))*(d^5/(27*b^2*c^8 + 27*a^2*c^6*d^2 - 54*a*b*c^7*d))^(1/3) + log(- ((((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c
```

$$\begin{aligned}
&^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^{(1/3)}*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^{(2/3)}/81 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c^3)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^{(1/3)}/9 - (2*b^4*d^6*(a + b*x^3)^{(1/3)}*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4)*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^2*c^2*d + 54*a^2*b*c*d^2)/(729*a^5*c^6))^{(1/3)} + (\log(((3^{(1/2)}*i - 1)*(((27*b^5*c^3*d^3*(a + b*x^3)^{(1/3)}*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - (81*a*b^4*c^4*d^3*(3^{(1/2)}*i - 1)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(a*d - b*c)^2))^{(1/3)}))/2)*(3^{(1/2)}*i - 1)^2*(d^5/(c^6*(a*d - b*c)^2))^{(2/3)})/36 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c^3)*((d^5/(c^6*(a*d - b*c)^2))^{(1/3)})/6 + (2*b^4*d^6*(a + b*x^3)^{(1/3)}*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4)*(3^{(1/2)}*i - 1)*(d^5/(27*b^2*c^8 + 27*a^2*c^6*d^2 - 54*a*b*c^7*d))^{(1/3)}/2 - (\log(((3^{(1/2)}*i + 1)*(((27*b^5*c^3*d^3*(a + b*x^3)^{(1/3)}*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a + (81*a*b^4*c^4*d^3*(3^{(1/2)}*i + 1)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(a*d - b*c)^2))^{(1/3)}))/2)*(3^{(1/2)}*i + 1)^2*(d^5/(c^6*(a*d - b*c)^2))^{(2/3)})/36 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c^3)*((d^5/(c^6*(a*d - b*c)^2))^{(1/3)})/6 - (2*b^4*d^6*(a + b*x^3)^{(1/3)}*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4)*(3^{(1/2)}*i + 1)*(d^5/(27*b^2*c^8 + 27*a^2*c^6*d^2 - 54*a*b*c^7*d))^{(1/3)}/2 + \log((2*b^4*d^6*(a + b*x^3)^{(1/3)}*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4) - (((((27*b^5*c^3*d^3*(a + b*x^3)^{(1/3)}*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - 27*a*b^4*c^4*d^3*(3^{(1/2)}*i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^{(1/3)})*((3^{(1/2)}*i)/2 + 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^{(2/3)})/81 - (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c^3)*((3^{(1/2)}*i)/2 - 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^{(1/3)}/9)*((3^{(1/2)}*i)/2 - 1/2)*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^2*c^2*d + 54*a^2*b*c*d^2)/(729*a^5*c^6))^{(1/3)} - \log((((((27*b^5*c^3*d^3*(a + b*x^3)^{(1/3)}*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a + 27*a*b^4*c^4*d^3*((3^{(1/2)}*i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^{(1/3)})*((3^{(1/2)}*i)/2 - 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^{(2/3)})/81 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c^3)*(((3^{(1/2)}*i)/2 + 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^{(1/3)}/9 - (2*b^4*d^6*(a + b*x^3)^{(1/3)}*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4))*((3^{(1/2)}*i)/2 + 1/2)*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^2*c^2*d + 54*a^2*b*c*d^2)/(729*a^5*c^6))^{(1/3)} - (a + b*x^3)^{(1/3)}/(3*a*c*x^3)
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + bx^3)^{\frac{2}{3}} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

$$3.474 \quad \int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)} dx$$

Optimal. Leaf size=279

$$\frac{(2ad + 3bc) \log\left(\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right)}{6b^{5/3}d^2} + \frac{(2ad + 3bc) \tan^{-1}\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1\right)}{3\sqrt{3}b^{5/3}d^2} + \frac{c^{5/3} \log(c + dx^3)}{6d^2(bc - ad)^{2/3}} - \frac{c^{5/3} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d^2(bc - ad)^{2/3}}$$

Rubi [A] time = 0.50, antiderivative size = 400, normalized size of antiderivative = 1.43, number of steps used = 16, number of rules used = 9, integrand size = 24, number of rules / integrand size = 0.375, Rules used = {494, 470, 584, 292, 31, 634, 617, 204, 628}

$$\frac{(2ad + 3bc) \log\left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}d^2} - \frac{(2ad + 3bc) \log\left(\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1\right)}{18b^{5/3}d^2} + \frac{(2ad + 3bc) \tan^{-1}\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1\right)}{3\sqrt{3}b^{5/3}d^2} - \frac{c^{5/3} \log\left(\sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3d^2(bc - ad)^{2/3}} + \frac{c^{5/3} \log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6d^2(bc - ad)^{2/3}} - \frac{c^{5/3} \tan^{-1}\left(\frac{2x\sqrt[3]{bc-ad}}{\sqrt{3}\sqrt[3]{c}} + \sqrt[3]{c}\right)}{\sqrt{3}d^2(bc - ad)^{2/3}} + \frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out] (x^2*(a + b*x^3)^(1/3))/(3*b*d) + ((3*b*c + 2*a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(5/3)*d^2) - (c^(5/3)*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3)))/(Sqrt[3]*d^2*(b*c - a*d)^(2/3)) + ((3*b*c + 2*a*d)*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*b^(5/3)*d^2) - ((3*b*c + 2*a*d)*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(18*b^(5/3)*d^2) - (c^(5/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*d^2*(b*c - a*d)^(2/3)) + (c^(5/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*d^2*(b*c - a*d)^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 470

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 494

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q]/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && LtQ[-1, p, 0]
```

Rule 584

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((e_) + (f_)*(x_)^(n_)))/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((g*x)^m*(a + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```


Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
 \int \frac{x^7}{(a + bx^3)^{2/3} (c + dx^3)} dx &= a^2 \operatorname{Subst} \left(\int \frac{x^7}{(1 - bx^3)^2 (c - (bc - ad)x^3)} dx, x, \frac{x}{\sqrt[3]{a + bx^3}} \right) \\
 &= \frac{x^2 \sqrt[3]{a + bx^3}}{3bd} - \frac{a \operatorname{Subst} \left(\int \frac{x(2c + (bc + 2ad)x^3)}{(1 - bx^3)(c + (-bc + ad)x^3)} dx, x, \frac{x}{\sqrt[3]{a + bx^3}} \right)}{3bd} \\
 &= \frac{x^2 \sqrt[3]{a + bx^3}}{3bd} - \frac{a \operatorname{Subst} \left(\int \left(\frac{(3bc + 2ad)x}{ad(1 - bx^3)} + \frac{3bc^2x}{ad(-c + (bc - ad)x^3)} \right) dx, x, \frac{x}{\sqrt[3]{a + bx^3}} \right)}{3bd} \\
 &= \frac{x^2 \sqrt[3]{a + bx^3}}{3bd} - \frac{c^2 \operatorname{Subst} \left(\int \frac{x}{-c + (bc - ad)x^3} dx, x, \frac{x}{\sqrt[3]{a + bx^3}} \right)}{d^2} - \frac{(3bc + 2ad) \operatorname{Subst} \left(\int \frac{x}{1 - bx^3} dx, x, \frac{x}{\sqrt[3]{a + bx^3}} \right)}{3bd^2} \\
 &= \frac{x^2 \sqrt[3]{a + bx^3}}{3bd} - \frac{c^{5/3} \operatorname{Subst} \left(\int \frac{1}{-\sqrt[3]{c} + \sqrt[3]{bc - ad}x} dx, x, \frac{x}{\sqrt[3]{a + bx^3}} \right)}{3d^2 \sqrt[3]{bc - ad}} + \frac{c^{5/3} \operatorname{Subst} \left(\int \frac{1}{c^{2/3} + \sqrt[3]{c}x} dx, x, \frac{x}{\sqrt[3]{a + bx^3}} \right)}{3d^2 \sqrt[3]{c}} \\
 &= \frac{x^2 \sqrt[3]{a + bx^3}}{3bd} + \frac{(3bc + 2ad) \log \left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a + bx^3}} \right)}{9b^{5/3}d^2} - \frac{c^{5/3} \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc - ad}x}{\sqrt[3]{a + bx^3}} \right)}{3d^2(bc - ad)^{2/3}} + \frac{c^{5/3} \log \left(\sqrt[3]{c} + \frac{\sqrt[3]{bc - ad}x}{\sqrt[3]{a + bx^3}} \right)}{3d^2(bc - ad)^{2/3}} \\
 &= \frac{x^2 \sqrt[3]{a + bx^3}}{3bd} + \frac{(3bc + 2ad) \log \left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a + bx^3}} \right)}{9b^{5/3}d^2} - \frac{(3bc + 2ad) \log \left(1 + \frac{b^{2/3}x^2}{(a + bx^3)^{2/3}} + \frac{b^{1/3}x}{\sqrt[3]{a + bx^3}} \right)}{18b^{5/3}d^2} + \frac{c^{5/3} \log \left(1 + \frac{b^{2/3}x^2}{(a + bx^3)^{2/3}} + \frac{b^{1/3}x}{\sqrt[3]{a + bx^3}} \right)}{18b^{5/3}d^2} \\
 &= \frac{x^2 \sqrt[3]{a + bx^3}}{3bd} + \frac{(3bc + 2ad) \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{b}x}{\sqrt[3]{a + bx^3}}}{\sqrt{3}} \right)}{3\sqrt{3}b^{5/3}d^2} - \frac{c^{5/3} \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{bc - ad}x}{\sqrt[3]{c}\sqrt[3]{a + bx^3}}}{\sqrt{3}} \right)}{\sqrt{3}d^2(bc - ad)^{2/3}} + \frac{c^{5/3} \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{bc - ad}x}{\sqrt[3]{c}\sqrt[3]{a + bx^3}}}{\sqrt{3}} \right)}{\sqrt{3}d^2(bc - ad)^{2/3}}
 \end{aligned}$$

Mathematica [C] time = 0.29, size = 190, normalized size = 0.68

$$\frac{5cx^2 \left((a + bx^3) \left(\frac{dx^3}{c} + 1 \right)^{2/3} - a \left(\frac{bx^3}{a} + 1 \right)^{2/3} {}_2F_1 \left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)} \right) \right) - x^5 \left(\frac{bx^3}{a} + 1 \right)^{2/3} \left(\frac{dx^3}{c} + 1 \right)^{2/3} (2ad + 3bc) {}_2F_1 \left(\frac{5}{3}; \frac{2}{3}, 1; \frac{8}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{15bcd (a + bx^3)^{2/3} \left(\frac{dx^3}{c} + 1 \right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] (-((3*b*c + 2*a*d)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -(b*x^3)/a, -(d*x^3)/c]) + 5*c*x^2*((a + b*x^3)*(1 + (d*x^3)/c)^(2/3) - a*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-b*c) + a*d)*x^3/(a*(c + d*x^3))])/(15*b*c*d*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3))

IntegrateAlgebraic [C] time = 4.57, size = 527, normalized size = 1.89

$$\frac{(2ad + 3bc) \log(\sqrt{a + bx^3} - \sqrt{c})}{9b^2d^2} - \frac{(2ad + 3bc) \tan^{-1}\left(\frac{\sqrt{d} \sqrt{c}}{\sqrt{a + bx^3} \sqrt{c}}\right)}{3\sqrt{3}b^2d^2} - \frac{(c - 2ad - 3bc) \log(\sqrt{a + bx^3} + (a + bx^3)^{2/3} + b^2d^2)}{18b^2d^2} - \frac{(c^{5/3} - \sqrt{3}c^{2/3}) \log(2\sqrt{c} - ad + (1 + \sqrt{3})\sqrt{c}\sqrt{a + bx^3})}{6b^2d^2} - \frac{\sqrt{d}(1 - \sqrt{3}) \tan^{-1}\left(\frac{2\sqrt{3}\sqrt{a + bx^3}}{\sqrt{d}\sqrt{c} - \sqrt{3}\sqrt{a + bx^3} - \sqrt{c}}\right)}{2b^2d^2} - \frac{i(\sqrt{3}c^{5/3} + c^{2/3}) \log((\sqrt{3} + i)c^{2/3}(a + bx^3)^{2/3} + \sqrt{c}(-\sqrt{3} + i)\sqrt{a + bx^3}\sqrt{c} - ad)}{12b^2d^2} - \frac{c^2\sqrt{a + bx^3}}{36d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] (x^2*(a + b*x^3)^(1/3))/(3*b*d) + ((3*b*c + 2*a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(3*Sqrt[3]*b^(5/3)*d^2) + (Sqrt[(-1 - I*Sqrt[3])/6]*c^(5/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]/(d^2*(b*c - a*d)^(2/3)) + ((3*b*c + 2*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(9*b^(5/3)*d^2) + ((c^(5/3) - I*Sqrt[3]*c^(5/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]/(6*d^2*(b*c - a*d)^(2/3)) + ((-3*b*c - 2*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(18*b^(5/3)*d^2) + ((I/12)*(I*c^(5/3) + Sqrt[3]*c^(5/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]/(d^2*(b*c - a*d)^(2/3))

fricas [B] time = 1.28, size = 558, normalized size = 2.00

$$\frac{c\sqrt{3}d\sqrt{\frac{c}{a+bx^3}} \arctan\left(\frac{2\sqrt{3}d\sqrt{a+bx^3} - \sqrt{c}}{2\sqrt{3}d\sqrt{a+bx^3} + \sqrt{c}}\right) + c\sqrt{3}d\sqrt{\frac{c}{a+bx^3}} \arctan\left(\frac{2\sqrt{3}d\sqrt{a+bx^3} - \sqrt{c}}{2\sqrt{3}d\sqrt{a+bx^3} + \sqrt{c}}\right) - 3d\sqrt{\frac{c}{a+bx^3}} \log\left(\frac{2\sqrt{3}d\sqrt{a+bx^3} - \sqrt{c}}{2\sqrt{3}d\sqrt{a+bx^3} + \sqrt{c}}\right) - 2\sqrt{3}(3d + 2ad)\sqrt{c} \arctan\left(\frac{2\sqrt{3}d\sqrt{a+bx^3} - \sqrt{c}}{2\sqrt{3}d\sqrt{a+bx^3} + \sqrt{c}}\right) + 2(3d^2 + 2ad)\log\left(\frac{2\sqrt{3}d\sqrt{a+bx^3} - \sqrt{c}}{2\sqrt{3}d\sqrt{a+bx^3} + \sqrt{c}}\right) - (3d^2 + 2ad)\log\left(\frac{2\sqrt{3}d\sqrt{a+bx^3} - \sqrt{c}}{2\sqrt{3}d\sqrt{a+bx^3} + \sqrt{c}}\right)}{18b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] 1/18*(6*sqrt(3)*b^3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a

$$\begin{aligned} & \sqrt[3]{-2*d^2})^{2/3} + \sqrt{3}*c*x)/(c*x)) + 6*(b*x^3 + a)^{1/3}*b^2*d*x^2 + 6*b^3*c* \\ & (-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^{1/3}*log(((b*c - a*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^{1/3}*x + (b*x^3 + a)^{1/3}*c)/x) - 3*b^3*c* \\ & (-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^{1/3}*log(((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^{2/3}*x^2 + (b*x^3 + a)^{2/3}* \\ & c^2 - (b*x^3 + a)^{1/3}*(b*c^2 - a*c*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^{1/3}*x)/x^2) - 2*\sqrt{3}*(3*b^2*c + 2*a*b*d)*(b^2)^{1/6}*arctan(1/3*(\sqrt{3}*(b^2)^{1/3}*b*x + 2*\sqrt{3}*(b*x^3 + a)^{1/3}*(b^2)^{2/3}))* \\ & (b^2)^{1/6}/(b^2*x)) + 2*(b^2)^{2/3}*(3*b*c + 2*a*d)*log(-((b^2)^{2/3}*x - (b*x^3 + a)^{1/3}*b)/x) - (b^2)^{2/3}*(3*b*c + 2*a*d)*log(((b^2)^{1/3}*b*x^2 + (b*x^3 + a)^{1/3}*(b^2)^{2/3}*x + (b*x^3 + a)^{2/3}*b)/x^2))/(b^3*d^2) \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{(bx^3 + a)^{2/3} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/((a + b*x^3)^(2/3)*(c + d*x^3)), x)`

[Out] `int(x^7/((a + b*x^3)^(2/3)*(c + d*x^3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a + bx^3)^{\frac{2}{3}} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(b*x**3+a)**(2/3)/(d*x**3+c), x)`

[Out] `Integral(x**7/((a + b*x**3)**(2/3)*(c + d*x**3)), x)`

$$3.475 \quad \int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)} dx$$

Optimal. Leaf size=234

$$\frac{\log\left(\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right)}{2b^{2/3}d} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{bx} + 1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}b^{2/3}d} - \frac{c^{2/3}\log(c+dx^3)}{6d(bc-ad)^{2/3}} + \frac{c^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{2/3}} + \frac{c^{2/3}\tan^{-1}\left(\frac{2x}{\sqrt[3]{c}}\right)}{\sqrt{3}d(bc-ad)^{2/3}}$$

Rubi [A] time = 0.31, antiderivative size = 346, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {494, 481, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(1 - \frac{\sqrt[3]{bx}}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} + \frac{\log\left(\frac{1^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{bx}}{\sqrt[3]{a+bx^3}} + 1\right)}{6b^{2/3}d} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{bx} + 1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}b^{2/3}d} + \frac{c^{2/3}\log\left(\sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3d(bc-ad)^{2/3}} - \frac{c^{2/3}\log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{cx}\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6d(bc-ad)^{2/3}} + \frac{c^{2/3}\tan^{-1}\left(\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \sqrt[3]{c}\right)}{\sqrt{3}d(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out] -(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*d) + (c^(2/3)*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))]/(Sqrt[3]*d*(b*c - a*d)^(2/3)) - Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*b^(2/3)*d) + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]/(6*b^(2/3)*d) + (c^(2/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*d*(b*c - a*d)^(2/3)) - (c^(2/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(6*d*(b*c - a*d)^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 481

```
Int[((e_)*(x_)^(m_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x], x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]
```

Rule 494

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q]/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && LtQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)} dx &= a \operatorname{Subst} \left(\int \frac{x^4}{(1-bx^3)(c-(bc-ad)x^3)} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right) \\
&= \frac{\operatorname{Subst} \left(\int \frac{x}{1-bx^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{d} - \frac{c \operatorname{Subst} \left(\int \frac{x}{c+(-bc+ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{d} \\
&= \frac{\operatorname{Subst} \left(\int \frac{1}{1-\sqrt[3]{b}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3\sqrt[3]{b}d} - \frac{\operatorname{Subst} \left(\int \frac{1-\sqrt[3]{b}x}{1+\sqrt[3]{b}x+b^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3\sqrt[3]{b}d} - \frac{c^{2/3} \operatorname{Subst} \left(\int \frac{1}{1+\sqrt[3]{b}x+b^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3\sqrt[3]{b}d} \\
&= -\frac{\log \left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right)}{3b^{2/3}d} + \frac{c^{2/3} \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3d(bc-ad)^{2/3}} + \frac{\operatorname{Subst} \left(\int \frac{\sqrt[3]{b}+2b^{2/3}x}{1+\sqrt[3]{b}x+b^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{6b^{2/3}d} \\
&= -\frac{\log \left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right)}{3b^{2/3}d} + \frac{\log \left(1 + \frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right)}{6b^{2/3}d} + \frac{c^{2/3} \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3d(bc-ad)^{2/3}} - \frac{c^{2/3} \operatorname{Subst} \left(\int \frac{1}{1+\sqrt[3]{b}x+b^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3d(bc-ad)^{2/3}} \\
&= -\frac{\tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{\sqrt{3}b^{2/3}d} + \frac{c^{2/3} \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{\sqrt{3}d(bc-ad)^{2/3}} - \frac{\log \left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right)}{3b^{2/3}d} + \frac{\log \left(1 + \frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right)}{6b^{2/3}d}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 65, normalized size = 0.28

$$\frac{x^5 \left(\frac{a+bx^3}{a} \right)^{2/3} F_1 \left(\frac{5}{3}; \frac{2}{3}, 1; \frac{8}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{5c (a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] (x^5*((a + b*x^3)/a)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]/(5*c*(a + b*x^3)^(2/3))

IntegrateAlgebraic [C] time = 3.42, size = 475, normalized size = 2.03

$$\frac{\log \left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x \right)}{3b^{2/3}d} - \frac{\tan^{-1} \left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right)}{\sqrt{3}b^{2/3}d} + \frac{\log \left(\sqrt[3]{b}x\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3} + b^{2/3}x^2 \right)}{6b^{2/3}d} + \frac{((\sqrt{5}+i)^{2/3} \log(2\sqrt[3]{bc-ad} + (1+i\sqrt{5})\sqrt[3]{a+bx^3}) - \sqrt[3]{2(-1-i\sqrt{5})})^{2/3} \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{6d(bc-ad)^{2/3}} + \frac{\sqrt[3]{2(-1-i\sqrt{5})}^{2/3} \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{d(bc-ad)^{2/3}} + \frac{((2/3 - i\sqrt{5}^{2/3}) \log((\sqrt{5}+i)^{2/3}(a+bx^3)^{2/3} + \sqrt[3]{c}(-\sqrt{5}x+ia)\sqrt[3]{a+bx^3} - 2ix^2(bc-ad)^{2/3}))}{12d(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] $-(\text{ArcTan}[\frac{\sqrt{3}b^{1/3}x}{b^{1/3}x + 2(a + b x^3)^{1/3}}]/(\sqrt{3}b^{2/3}d)) - (\sqrt{(-1 - I\sqrt{3})/6}c^{2/3}\text{ArcTan}[\frac{3(b c - a d)^{1/3}x}{\sqrt{3}(b c - a d)^{1/3}x - (3I)c^{1/3}(a + b x^3)^{1/3} - \sqrt{3}c^{1/3}(a + b x^3)^{1/3}}])/(\sqrt{3}(b c - a d)^{1/3}x - (3I)c^{1/3}(a + b x^3)^{1/3} - \sqrt{3}c^{1/3}(a + b x^3)^{1/3}))/(\sqrt{3}(b c - a d)^{2/3}) - \text{Log}[-(b^{1/3}x + (a + b x^3)^{1/3})/(3b^{2/3}d) + ((I/6)(Ic^{2/3} + \sqrt{3}c^{2/3})\text{Log}[2(b c - a d)^{1/3}x + (1 + I\sqrt{3})c^{1/3}(a + b x^3)^{1/3}])/(\sqrt{3}(b c - a d)^{2/3}) + \text{Log}[b^{2/3}x^2 + b^{1/3}x(a + b x^3)^{1/3} + (a + b x^3)^{2/3}]/(6b^{2/3}d) + ((c^{2/3} - I\sqrt{3}c^{2/3})\text{Log}[(-2I)(b c - a d)^{1/3}x^2 + c^{1/3}(b c - a d)^{1/3}(Ix - \sqrt{3}x)(a + b x^3)^{1/3} + (I + \sqrt{3})c^{2/3}(a + b x^3)^{2/3}])/(12d(b c - a d)^{2/3}))$

fricas [B] time = 0.55, size = 530, normalized size = 2.26

$$2\sqrt{3}\sqrt{\frac{d}{(2d^2+3bd+3a^2)^2}}\arctan\left(\frac{2\sqrt{3}b^{1/3}x - \sqrt{3}(a + bx^3)^{1/3}}{3a + 2bx^3}\right) + 2\sqrt{3}\sqrt{\frac{d}{(2d^2+3bd+3a^2)^2}}\log\left(\frac{3(b c - a d)^{1/3}x - \sqrt{3}(a + bx^3)^{1/3} - \sqrt{3}c^{1/3}(a + bx^3)^{1/3}}{3(b c - a d)^{1/3}x - (3I)c^{1/3}(a + bx^3)^{1/3} - \sqrt{3}c^{1/3}(a + bx^3)^{1/3}}\right) + 2\sqrt{3}\sqrt{\frac{d}{(2d^2+3bd+3a^2)^2}}\log\left(\frac{(b c - a d)^{1/3}x + (1 + I\sqrt{3})c^{1/3}(a + bx^3)^{1/3}}{2(b c - a d)^{1/3}x + (1 + I\sqrt{3})c^{1/3}(a + bx^3)^{1/3}}\right) + 2\sqrt{3}\sqrt{\frac{d}{(2d^2+3bd+3a^2)^2}}\arctan\left(\frac{\sqrt{3}(a + bx^3)^{1/3} - 2\sqrt{3}c^{1/3}(a + bx^3)^{1/3}}{3a + 2bx^3}\right) - 2(-d)^{1/3}\log\left(\frac{(b c - a d)^{1/3}x^2 + c^{1/3}(b c - a d)^{1/3}(Ix - \sqrt{3}x)(a + bx^3)^{1/3} + (I + \sqrt{3})c^{2/3}(a + bx^3)^{2/3}}{(b c - a d)^{2/3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] $\frac{1}{6}(2\sqrt{3}b^2(c^2/(b^2c^2 - 2ab^2cd + a^2d^2))^{1/3}\arctan(-\frac{1}{3}(2\sqrt{3}(b^2c^2 - 2ab^2cd + a^2d^2)^{1/3} + \sqrt{3}cx)/(cx)) + 2b^2(c^2/(b^2c^2 - 2ab^2cd + a^2d^2))^{1/3}\log(-((b c - a d)(c^2/(b^2c^2 - 2ab^2cd + a^2d^2))^{1/3}x - (b x^3 + a)^{1/3}c)/x) - b^2(c^2/(b^2c^2 - 2ab^2cd + a^2d^2))^{1/3}\log(((b^2c^2 - 2ab^2cd + a^2d^2)(c^2/(b^2c^2 - 2ab^2cd + a^2d^2))^{2/3}x^2 + (b x^3 + a)^{2/3}c^2 + (b x^3 + a)^{1/3}(b c^2 - a c d)(c^2/(b^2c^2 - 2ab^2cd + a^2d^2))^{1/3}x)/x^2) + 2\sqrt{3}b\sqrt{3}(-b^2)^{1/3}\arctan(-\frac{1}{3}(\sqrt{3}(-b^2)^{1/3}bx - 2\sqrt{3}(b x^3 + a)^{1/3}(-b^2)^{2/3})\sqrt{3}/(b^2x)) - 2(-b^2)^{2/3}\log(-((-b^2)^{2/3}x - (b x^3 + a)^{1/3}b)/x) + (-b^2)^{2/3}\log(-((-b^2)^{1/3}bx^2 - (b x^3 + a)^{1/3}(-b^2)^{2/3}x - (b x^3 + a)^{2/3}b)/x^2))/(b^2d)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^3+a)^(2/3)/(d*x^3+c), x)

[Out] int(x^4/(b*x^3+a)^(2/3)/(d*x^3+c), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c), x, algorithm="maxima")

[Out] integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((a + b*x^3)^(2/3)*(c + d*x^3)), x)

[Out] int(x^4/((a + b*x^3)^(2/3)*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a + bx^3)^{\frac{2}{3}}(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**3+a)**(2/3)/(d*x**3+c), x)

[Out] Integral(x**4/((a + b*x**3)**(2/3)*(c + d*x**3)), x)

$$3.476 \quad \int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)} dx$$

Optimal. Leaf size=149

$$\frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}} - \frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}} - \frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}$$

Rubi [A] time = 0.14, antiderivative size = 208, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {494, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}} - \frac{\log\left(\sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}(bc-ad)^{2/3}} - \frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] -(ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))]/(Sqrt[3]*c^(1/3))]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3)) - Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*c^(1/3)*(b*c - a*d)^(2/3)) + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(6*c^(1/3)*(b*c - a*d)^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I

nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 494

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_) , x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && LtQ[-1, p, 0]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)} dx &= \text{Subst} \left(\int \frac{x}{c-(bc-ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3\sqrt[3]{c}\sqrt[3]{bc-ad}} - \frac{\text{Subst} \left(\int \frac{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3\sqrt[3]{c}\sqrt[3]{bc-ad}} \\
&= -\frac{\log \left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\text{Subst} \left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{6\sqrt[3]{c}(bc-ad)^{2/3}} - \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{\sqrt[3]{c}(bc-ad)^{2/3}} \\
&= -\frac{\log \left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\log \left(c^{2/3} + \frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{6\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{\sqrt[3]{c}(bc-ad)^{2/3}} \\
&= -\frac{\tan^{-1} \left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}} - \frac{\log \left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\log \left(c^{2/3} + \frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 83, normalized size = 0.56

$$\frac{x^2 \left(\frac{bx^3}{a} + 1 \right)^{2/3} {}_2F_1 \left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)} \right)}{2c(a+bx^3)^{2/3} \left(\frac{dx^3}{c} + 1 \right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/((a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out] (x^2*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(2*c*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3))

IntegrateAlgebraic [C] time = 1.96, size = 319, normalized size = 2.14

$$\frac{i(\sqrt{3}+i)\log\left((\sqrt{3}+i)c^{2/3}(a+bx^3)^{2/3}+\sqrt[3]{c}(-\sqrt{3}x+ix)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{(1-i\sqrt{3})\log\left(2x\sqrt[3]{bc-ad}+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\sqrt{-1-i\sqrt{3}}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}\sqrt[3]{c}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/((a + b*x^3)^(2/3)*(c + d*x^3)), x]

```
[Out] (Sqrt[-1 - I*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])/((Sqrt[6]*c^(1/3)*(b*c - a*d)^(2/3)) + ((1 - I*Sqrt[3])*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*c^(1/3)*(b*c - a*d)^(2/3)) + ((I/12)*(I + Sqrt[3])*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(1/3)*(b*c - a*d)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

[Out] integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

[Out] int(x/(b*x^3+a)^(2/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(bx^3 + a)^{2/3} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + b*x^3)^(2/3)*(c + d*x^3)),x)

[Out] int(x/((a + b*x^3)^(2/3)*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + bx^3)^{\frac{2}{3}} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**3+a)**(2/3)/(d*x**3+c),x)

[Out] Integral(x/((a + b*x**3)**(2/3)*(c + d*x**3)), x)

$$3.477 \quad \int \frac{1}{x^2(a+bx^3)^{2/3}(c+dx^3)} dx$$

Optimal. Leaf size=173

$$-\frac{d \log(c+dx^3)}{6c^{4/3}(bc-ad)^{2/3}} + \frac{d \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{4/3}(bc-ad)^{2/3}} + \frac{d \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}(bc-ad)^{2/3}} - \frac{\sqrt[3]{a+bx^3}}{acx}$$

Rubi [A] time = 0.22, antiderivative size = 232, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {494, 453, 292, 31, 634, 617, 204, 628}

$$\frac{d \log\left(\sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d \log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6c^{4/3}(bc-ad)^{2/3}} + \frac{d \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{4/3}(bc-ad)^{2/3}} - \frac{\sqrt[3]{a+bx^3}}{acx}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out] -((a + b*x^3)^(1/3)/(a*c*x)) + (d*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))]/(Sqrt[3]*c^(4/3)*(b*c - a*d)^(2/3)) + (d*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*c^(4/3)*(b*c - a*d)^(2/3)) - (d*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3)] + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(6*c^(4/3)*(b*c - a*d)^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^-1, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I

Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 494

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q]/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && LtQ[-1, p, 0]

Rule 617

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a + bx^3)^{2/3} (c + dx^3)} dx &= \frac{\text{Subst} \left(\int \frac{1-bx^3}{x^2(c-(bc-ad)x^3)} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{a} \\
&= -\frac{\sqrt[3]{a+bx^3}}{acx} - \frac{d \text{Subst} \left(\int \frac{x}{c+(-bc+ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{c} \\
&= -\frac{\sqrt[3]{a+bx^3}}{acx} - \frac{d \text{Subst} \left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3c^{4/3}\sqrt[3]{bc-ad}} + \frac{d \text{Subst} \left(\int \frac{\sqrt[3]{c}-\sqrt[3]{bc-ad}}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3c^{4/3}} \\
&= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d \text{Subst} \left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{6c^{4/3}(bc-ad)^{2/3}} \\
&= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d \log \left(c^{2/3} + \frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{6c^{4/3}(bc-ad)^{2/3}} \\
&= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{\sqrt{3}c^{4/3}(bc-ad)^{2/3}} + \frac{d \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d \log \left(c^{2/3} + \frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{6c^{4/3}(bc-ad)^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 128, normalized size = 0.74

$$\frac{6x^3(c+dx^3)(bc-ad) {}_2F_1\left(\frac{5}{3}, 2; \frac{8}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{a+bx^3} + 5c(2c+3dx^3) {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)$$

$$10c^3x(a+bx^3)^{2/3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out] -1/10*(5*c*(2*c + 3*d*x^3)*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) + (6*(b*c - a*d)*x^3*(c + d*x^3)*Hypergeometric2F1[5/3, 2, 8/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(a + b*x^3)/(c^3*x*(a + b*x^3)^(2/3))

IntegrateAlgebraic [C] time = 2.02, size = 347, normalized size = 2.01

$$\frac{i(\sqrt{3}d+id)\log\left(2x\sqrt[3]{bc-ad}+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{4/3}(bc-ad)^{2/3}} - \frac{\sqrt[3]{6}(-1-i\sqrt{3})d\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{4/3}(bc-ad)^{2/3}} + \frac{(d-i\sqrt{3}d)\log\left((\sqrt{3}+i)c^{2/3}(a+bx^3)^{2/3}+\sqrt[3]{c}(-\sqrt{3}x+ix)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{4/3}(bc-ad)^{2/3}} - \frac{\sqrt[3]{a+bx^3}}{acx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] $-\frac{(a + b x^3)^{1/3}}{a c x} - \frac{(\sqrt{3}(-1 - I \sqrt{3})/6) d \operatorname{ArcTan}\left[\frac{3(b c - a d)^{1/3} x}{\sqrt{3}(b c - a d)^{1/3} x - (3 I) c^{1/3} (a + b x^3)^{1/3}} - \sqrt{3} c^{1/3} (a + b x^3)^{1/3}\right]}{c^{4/3} (b c - a d)^{2/3}} + \frac{(I/6)(I d + \sqrt{3} d) \operatorname{Log}\left[2(b c - a d)^{1/3} x + (1 + I \sqrt{3}) c^{1/3} (a + b x^3)^{1/3}\right]}{c^{4/3} (b c - a d)^{2/3}} + \frac{(d - I \sqrt{3} d) \operatorname{Log}\left[(-2 I)(b c - a d)^{2/3} x^2 + c^{1/3} (b c - a d)^{1/3} (I x - \sqrt{3} x)(a + b x^3)^{1/3} + (I + \sqrt{3}) c^{2/3} (a + b x^3)^{2/3}\right]}{12 c^{4/3} (b c - a d)^{2/3}}$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b x^3 + a)^{\frac{2}{3}} (d x^3 + c) x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{1}{(b x^3 + a)^{\frac{2}{3}} (d x^3 + c) x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 (bx^3 + a)^{\frac{2}{3}} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x)

[Out] int(1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + bx^3)^{\frac{2}{3}} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)

[Out] Integral(1/(x**2*(a + b*x**3)**(2/3)*(c + d*x**3)), x)

$$3.478 \quad \int \frac{1}{x^5(a+bx^3)^{2/3}(c+dx^3)} dx$$

Optimal. Leaf size=215

$$\frac{\sqrt[3]{a+bx^3}(4ad+3bc)}{4a^2c^2x} + \frac{d^2 \log(c+dx^3)}{6c^{7/3}(bc-ad)^{2/3}} - \frac{d^2 \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{7/3}(bc-ad)^{2/3}} - \frac{d^2 \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}(bc-ad)^{2/3}} - \frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

Rubi [A] time = 0.30, antiderivative size = 269, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {494, 461, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a+bx^3}(ad+bc)}{a^2c^2x} - \frac{(a+bx^3)^{4/3}}{4a^2cx^4} - \frac{d^2 \log\left(\sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}(bc-ad)^{2/3}} + \frac{d^2 \log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6c^{7/3}(bc-ad)^{2/3}} - \frac{d^2 \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{7/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out] ((b*c + a*d)*(a + b*x^3)^(1/3))/(a^2*c^2*x) - (a + b*x^3)^(4/3)/(4*a^2*c*x^4) - (d^2*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(Sqrt[3]*c^(1/3)))/(Sqrt[3]*c^(7/3)*(b*c - a*d)^(2/3)) - (d^2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*c^(7/3)*(b*c - a*d)^(2/3)) + (d^2*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(6*c^(7/3)*(b*c - a*d)^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(n-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I

nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 461

Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 494

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && LtQ[-1, p, 0]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5 (a + bx^3)^{2/3} (c + dx^3)} dx &= \frac{\text{Subst} \left(\int \frac{(1-bx^3)^2}{x^5 (c-(bc-ad)x^3)} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{a^2} \\
&= \frac{\text{Subst} \left(\int \left(\frac{1}{cx^5} + \frac{-bc-ad}{c^2 x^2} + \frac{a^2 d^2 x}{c^2 (c-(bc-ad)x^3)} \right) dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{a^2} \\
&= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2 c^2 x} - \frac{(a+bx^3)^{4/3}}{4a^2 cx^4} + \frac{d^2 \text{Subst} \left(\int \frac{x}{c-(bc-ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{c^2} \\
&= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2 c^2 x} - \frac{(a+bx^3)^{4/3}}{4a^2 cx^4} + \frac{d^2 \text{Subst} \left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3c^{7/3}\sqrt[3]{bc-ad}} \\
&= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2 c^2 x} - \frac{(a+bx^3)^{4/3}}{4a^2 cx^4} - \frac{d^2 \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3c^{7/3}(bc-ad)^{2/3}} + \frac{d^2 \text{Subst} \left(\int \frac{\sqrt[3]{bc-ad}x}{c^{2/3} + \frac{bc-ad}{a}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{6c^{7/3}(bc-ad)^{2/3}} \\
&= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2 c^2 x} - \frac{(a+bx^3)^{4/3}}{4a^2 cx^4} - \frac{d^2 \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3c^{7/3}(bc-ad)^{2/3}} + \frac{d^2 \log \left(c^{2/3} + \frac{bc-ad}{a}x \right)}{6c^{7/3}(bc-ad)^{2/3}} \\
&= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2 c^2 x} - \frac{(a+bx^3)^{4/3}}{4a^2 cx^4} - \frac{d^2 \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{\sqrt{3}c^{7/3}(bc-ad)^{2/3}} - \frac{d^2 \log \left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3c^{7/3}(bc-ad)^{2/3}}
\end{aligned}$$

Mathematica [C] time = 1.89, size = 267, normalized size = 1.24

$$\frac{-81x^3(c+dx)^2(bc-ad) {}_4F_3\left(\frac{2}{3}, 2, 2, 2, 1, \frac{8}{3}; \frac{(bc-ad)x^3}{c(a+bx^3)}\right) + 216dx^6(c+dx^3)(ad-bc) {}_3F_2\left(\frac{2}{3}, 2, 2, 1, \frac{8}{3}; \frac{(bc-ad)x^3}{c(a+bx^3)}\right) - 5\left(2c(a+bx^3)(c^2+10cdx^3+9d^2x^6) + (a(-8c^3+17c^2dx^3+46cd^2x^6+9d^3x^9) + 3bcx^3(-3c^2+2cdx^3+9d^2x^6))\right) {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{(bc-ad)x^3}{c(a+bx^3)}\right)}{120c^4x^4(a+bx^3)^{5/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)), x]

[Out]
$$\begin{aligned}
& -1/120*(-5*(2*c*(a + b*x^3)*(c^2 + 10*c*d*x^3 + 9*d^2*x^6) + (3*b*c*x^3*(-3 \\
& *c^2 + 2*c*d*x^3 + 9*d^2*x^6) + a*(-8*c^3 + 17*c^2*d*x^3 + 46*c*d^2*x^6 + 9 \\
& *d^3*x^9))*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3)) \\
&]) + 216*d*(-(b*c) + a*d)*x^6*(c + d*x^3)*HypergeometricPFQ[{2/3, 2, 2}, {1 \\
& , 8/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 81*(b*c - a*d)*x^3*(c + d*x^3) \\
& ^2*HypergeometricPFQ[{2/3, 2, 2, 2}, {1, 1, 8/3}, ((b*c - a*d)*x^3)/(c*(a + \\
& b*x^3))]/(c^4*x^4*(a + b*x^3)^(5/3))
\end{aligned}$$

IntegrateAlgebraic [C] time = 2.41, size = 377, normalized size = 1.75

$$\frac{\sqrt{a+bx^3}(-ac+4ad^2x^3+3bcx^3)}{4d^2c^2x^4} + \frac{(d^2-i\sqrt{3}d^2)\log(2x\sqrt{bc-ad}+(1+i\sqrt{3})\sqrt[3]{c}\sqrt{a+bx^3})}{6c^{2/3}(bc-ad)^{2/3}} + \frac{\sqrt[3]{2}(-1-i\sqrt{3})d^2\tan^{-1}\left(\frac{3x\sqrt{bc-ad}}{\sqrt{3x\sqrt{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt{a+bx^3}-3i\sqrt[3]{c}\sqrt{a+bx^3}}}\right)}{c^{2/3}(bc-ad)^{2/3}} + \frac{i(\sqrt{3}d^2+id^2)\log\left(\left(\sqrt{3}+i\right)c^{2/3}(a+bx^3)^{2/3}+\sqrt[3]{c}(-\sqrt{3}x+ix)\sqrt{a+bx^3}\sqrt{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{2/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out]
$$\frac{((a + b*x^3)^{1/3}*(-(a*c) + 3*b*c*x^3 + 4*a*d*x^3))/(4*a^2*c^2*x^4) + (\text{Sqrt}[-1 - I*\text{Sqrt}[3]])/6*d^2*\text{ArcTan}[(3*(b*c - a*d)^{1/3}*x)/(\text{Sqrt}[3]*(b*c - a*d)^{1/3}*x - (3*I)*c^{1/3}*(a + b*x^3)^{1/3} - \text{Sqrt}[3]*c^{1/3}*(a + b*x^3)^{1/3})]/(c^{7/3}*(b*c - a*d)^{2/3}) + ((d^2 - I*\text{Sqrt}[3]*d^2)*\text{Log}[2*(b*c - a*d)^{1/3}*x + (1 + I*\text{Sqrt}[3])*c^{1/3}*(a + b*x^3)^{1/3}]/(6*c^{7/3}*(b*c - a*d)^{2/3}) + ((I/12)*(I*d^2 + \text{Sqrt}[3]*d^2)*\text{Log}[(-2*I)*(b*c - a*d)^{2/3}*x^2 + c^{1/3}*(b*c - a*d)^{1/3}*(I*x - \text{Sqrt}[3]*x)*(a + b*x^3)^{1/3} + (I + \text{Sqrt}[3])*c^{2/3}*(a + b*x^3)^{2/3}]/(c^{7/3}*(b*c - a*d)^{2/3})$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] `int(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5 (bx^3 + a)^{\frac{2}{3}} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x)`

[Out] `int(1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 (a + bx^3)^{\frac{2}{3}} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)`

[Out] `Integral(1/(x**5*(a + b*x**3)**(2/3)*(c + d*x**3)), x)`

$$3.479 \quad \int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal. Leaf size=347

$$\frac{a^4}{b^4 \sqrt[3]{a+bx^3} (bc-ad)} + \frac{a^2 (a+bx^3)^{2/3}}{2b^4 d} + \frac{(a+bx^3)^{2/3} (a^2 d^2 + abcd + b^2 c^2)}{2b^4 d^3} + \frac{a (a+bx^3)^{2/3} (ad+bc)}{2b^4 d^2} - \frac{(a+bx^3)^{5/3}}{5b^4 d^2}$$

Rubi [A] time = 0.44, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 87, 43, 56, 617, 204, 31}

$$\frac{(a+bx^3)^{2/3} (a^2 d^2 + abcd + b^2 c^2)}{2b^4 d^3} - \frac{a^4}{b^4 \sqrt[3]{a+bx^3} (bc-ad)} + \frac{a^2 (a+bx^3)^{2/3}}{2b^4 d} + \frac{a (a+bx^3)^{2/3} (ad+bc)}{2b^4 d^2} - \frac{(a+bx^3)^{5/3} (ad+bc)}{5b^4 d^2} - \frac{2a (a+bx^3)^{5/3}}{5b^4 d} + \frac{(a+bx^3)^{8/3}}{8b^4 d} - \frac{c^4 \log(c+dx^3)}{6d^{11/3} (bc-ad)^{4/3}} + \frac{c^4 \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{2d^{11/3} (bc-ad)^{4/3}} + \frac{c^4 \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3} d^{11/3} (bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] $-(a^4/(b^4*(b*c - a*d)*(a + b*x^3)^{(1/3)})) + (a^2*(a + b*x^3)^{(2/3)})/(2*b^4*d) + (a*(b*c + a*d)*(a + b*x^3)^{(2/3)})/(2*b^4*d^2) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^{(2/3)})/(2*b^4*d^3) - (2*a*(a + b*x^3)^{(5/3)})/(5*b^4*d) - ((b*c + a*d)*(a + b*x^3)^{(5/3)})/(5*b^4*d^2) + (a + b*x^3)^{(8/3)}/(8*b^4*d) + (c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)]/Sqrt[3])]/(Sqrt[3]*d^(11/3)*(b*c - a*d)^(4/3)) - (c^4*Log[c + d*x^3])/(6*d^(11/3)*(b*c - a*d)^(4/3)) + (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(11/3)*(b*c - a*d)^(4/3))$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^(n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^4}{(a+bx)^{4/3}(c+dx)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{a^4}{b^3(bc-ad)(a+bx)^{4/3}} + \frac{b^2c^2+abcd+a^2d^2}{b^3d^3\sqrt[3]{a+bx}} - \frac{(bc+ad)x}{b^2d^2\sqrt[3]{a+bx}} + \frac{bd^2}{bd^3} \right) dx \right) \\
&= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{2/3}}{2b^4d^3} + \frac{\text{Subst} \left(\int \frac{x^2}{\sqrt[3]{a+bx}} dx \right)}{3bd} \\
&= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{2/3}}{2b^4d^3} - \frac{c^4 \log(c+dx^3)}{6d^{11/3}(bc-ad)^{4/3}} \\
&= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{a^2(a+bx^3)^{2/3}}{2b^4d} + \frac{a(bc+ad)(a+bx^3)^{2/3}}{2b^4d^2} + \frac{(b^2c^2+abcd)}{2b^4d^2} \\
&= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{a^2(a+bx^3)^{2/3}}{2b^4d} + \frac{a(bc+ad)(a+bx^3)^{2/3}}{2b^4d^2} + \frac{(b^2c^2+abcd)}{2b^4d^2}
\end{aligned}$$

Mathematica [C] time = 0.27, size = 157, normalized size = 0.45

$$\frac{81a^3d^3+9a^2bd^2(8c+3dx^3)+3ab^2d(20c^2+8cdx^3-3d^2x^6)+b^3(40c^3+20c^2dx^3-8cd^2x^6+5d^3x^9)}{b^4} - \frac{40c^4 {}_2F_1\left(-\frac{1}{3}, 1, \frac{2}{3}; \frac{d(bx^3+a)}{ad-bc}\right)}{bc-ad}$$

$$\frac{40d^4\sqrt[3]{a+bx^3}}{40d^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] ((81*a^3*d^3 + 9*a^2*b*d^2*(8*c + 3*d*x^3) + 3*a*b^2*d*(20*c^2 + 8*c*d*x^3 - 3*d^2*x^6) + b^3*(40*c^3 + 20*c^2*d*x^3 - 8*c*d^2*x^6 + 5*d^3*x^9))/b^4 - (40*c^4*Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)]/(b*c - a*d))/(40*d^4*(a + b*x^3)^(1/3))

IntegrateAlgebraic [A] time = 0.71, size = 393, normalized size = 1.13

$$\frac{81a^3d^3 - 9a^2bcd^2 + 27a^2bd^3 - 12a^2b^2c^2d - 3a^2b^2cd^2 - 9a^2b^2d^3x^6 - 20ab^3c^3 - 4ab^3c^2dx^3 + ab^3cd^2x^6 + 5ab^3d^3x^9 - 20b^4c^3x^3 + 8b^4c^2dx^6 - 5b^4cd^2x^9}{40b^4d^3\sqrt[3]{a+bx^3}(bc-ad)} + \frac{c^4 \log\left(\frac{\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}}{3a^{1/3}(bc-ad)^{1/3}}\right) - c^4 \log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3}\right)}{6a^{1/3}(bc-ad)^{4/3}} + \frac{c^4 \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{1/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out]
$$-1/40*(-20*a*b^3*c^3 - 12*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 81*a^4*d^3 - 20*b^4*c^3*x^3 - 4*a*b^3*c^2*d*x^3 - 3*a^2*b^2*c*d^2*x^3 + 27*a^3*b*d^3*x^3 + 8*b^4*c^2*d*x^6 + a*b^3*c*d^2*x^6 - 9*a^2*b^2*d^3*x^6 - 5*b^4*c*d^2*x^9 + 5*a*b^3*d^3*x^9)/(b^4*d^3*(b*c - a*d)*(a + b*x^3)^{(1/3)}) + (c^4*ArcTan[1/Sqrt[3] - (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})/(Sqrt[3]*(b*c - a*d)^{(1/3)})])/(Sqrt[3]*d^{(11/3)}*(b*c - a*d)^{(4/3)}) + (c^4*Log[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)}])/(3*d^{(11/3)}*(b*c - a*d)^{(4/3)}) - (c^4*Log[(b*c - a*d)^{(2/3)} - d^{(1/3)}*(b*c - a*d)^{(1/3)}*(a + b*x^3)^{(1/3)} + d^{(2/3)}*(a + b*x^3)^{(2/3)}])/(6*d^{(11/3)}*(b*c - a*d)^{(4/3)})$$

fricas [B] time = 0.84, size = 1300, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/120*(60*\sqrt{1/3}*(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*c^4*d^2)*x^3)*\sqrt{(-b*c*d^2 + a*d^3)^{(1/3)}/(b*c - a*d)}*\log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*\sqrt{1/3}*(2*(-b*c*d^2 + a*d^3)^{(2/3)}*(b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^{(1/3)}*(b*c - a*d)))*\sqrt{(-b*c*d^2 + a*d^3)^{(1/3)}/(b*c - a*d)} - 3*(-b*c*d^2 + a*d^3)^{(2/3)}*(b*x^3 + a)^{(1/3)})/(d*x^3 + c) + 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(2/3)}*d^2 + (-b*c*d^2 + a*d^3)^{(1/3)}*(b*x^3 + a)^{(1/3)}*d + (-b*c*d^2 + a*d^3)^{(2/3)}) - 40*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(1/3)}*d - (-b*c*d^2 + a*d^3)^{(1/3)}) - 3*(20*a*b^4*c^4*d^2 - 8*a^2*b^3*c^3*d^3 - 3*a^3*b^2*c^2*d^4 - 90*a^4*b*c*d^5 + 81*a^5*d^6 + 5*(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x^9 - (8*b^5*c^3*d^3 - 7*a*b^4*c^2*d^4 - 10*a^2*b^3*c*d^5 + 9*a^3*b^2*d^6)*x^6 + (20*b^5*c^4*d^2 - 16*a*b^4*c^3*d^3 - a^2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^5 + 27*a^4*b*d^6)*x^3)*(b*x^3 + a)^{(2/3)}/(a*b^6*c^2*d^5 - 2*a^2*b^5*c*d^6 + a^3*b^4*d^7 + (b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^3), -1/120*(120*\sqrt{1/3}*(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*c^4*d^2)*x^3)*\sqrt{(-b*c*d^2 + a*d^3)^{(1/3)}/(b*c - a*d)}*\arctan(\sqrt{1/3}*(2*(b*x^3 + a)^{(1/3)}*d + (-b*c*d^2 + a*d^3)^{(1/3)})*\sqrt{(-b*c*d^2 + a*d^3)^{(1/3)}/(b*c - a*d)})/d + 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(2/3)}*d^2 + (-b*c*d^2 + a*d^3)^{(1/3)}*(b*x^3 + a)^{(1/3)}*d + (-b*c*d^2 + a*d^3)^{(2/3)}) - 40*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(1/3)}*d - (-b*c*d^2 + a*d^3)^{(1/3)}) - 3*(20*a*b^4*c^4*d^2 - 8*a^2*b^3*c^3*d^3 - 3*a^3*b^2*c^2*d^4 - 90*a^4*b*c*d^5 + 81*a^5*d^6 + 5*(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x^9 - (8*b^5*c^3*d^3 - 7*a*b^4*c^2*d^4 - 10*a^2*b^3*c*d^5 + 9*a^3*b^2*d^6)*x^6 + (20*b^5*c^4*d^2 - 16*a*b^4*c^3*d^3 - a^2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^5 - a^4*b*d^6)*x^3)] \end{aligned}$$

$$2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^5 + 27*a^4*b*d^6)*x^3)*(b*x^3 + a)^{(2/3)} / (a*b^6*c^2*d^5 - 2*a^2*b^5*c*d^6 + a^3*b^4*d^7 + (b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^3]$$

giac [A] time = 0.26, size = 431, normalized size = 1.24

$$\frac{(-bc^2 + ad)^{\frac{1}{2}} c^4 \arctan\left(\frac{\sqrt{3}\sqrt{(bx^3+a)^{\frac{1}{3}} - \left(\frac{bc-d}{a}\right)^{\frac{1}{3}}}}{\sqrt{3}\sqrt{3d^2d^2 - 2\sqrt{3}abc^2d + \sqrt{3}d^2d^2}}\right) - (-bc^2 + ad)^{\frac{1}{2}} c^4 \log\left(\frac{(bx^3+a)^{\frac{1}{3}} + (bx^3+a)^{\frac{1}{3}}\left(\frac{bc-d}{a}\right)^{\frac{1}{3}} - \left(\frac{bc-d}{a}\right)^{\frac{1}{3}}}{3\sqrt{3}\sqrt{3d^2d^2 - 2\sqrt{3}abc^2d + \sqrt{3}d^2d^2}}\right) + c^4\left(\frac{bc-d}{a}\right)^{\frac{1}{2}} \log\left(\frac{(bx^3+a)^{\frac{1}{3}} - \left(\frac{bc-d}{a}\right)^{\frac{1}{3}}}{(bc-ad)\sqrt{(bx^3+a)^{\frac{1}{3}}}}\right)}{\sqrt{3}\sqrt{3d^2d^2 - 2\sqrt{3}abc^2d + \sqrt{3}d^2d^2} - \frac{20(bx^3+a)^{\frac{1}{3}}b^3c^2d^5 - 8(bx^3+a)^{\frac{1}{3}}b^2c^2d^6 + 40(bx^3+a)^{\frac{1}{3}}ab^2c^2d^6 + 5(bx^3+a)^{\frac{1}{3}}b^2c^2d^7 - 24(bx^3+a)^{\frac{1}{3}}ab^2c^2d^7 + 60(bx^3+a)^{\frac{1}{3}}b^2c^2d^8}{40b^2c^2d^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] $(-b*c*d^2 + a*d^3)^{(2/3)}*c^4*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{(1/3)} + (-b*c - a*d)/d)^{(1/3)})/(-b*c - a*d)/d)^{(1/3)})/(\sqrt{3}*b^2*c^2*d^5 - 2*\sqrt{3}*(a*b*c*d^6 + \sqrt{3}*a^2*d^7) - 1/6*(-b*c*d^2 + a*d^3)^{(2/3)}*c^4*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(-b*c - a*d)/d)^{(1/3)} + (-b*c - a*d)/d)^{(2/3)})/(b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7) + 1/3*c^4*(-b*c - a*d)/d)^{(2/3)}*\log(\text{abs}((b*x^3 + a)^{(1/3)} - (-b*c - a*d)/d)^{(1/3)})/(b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5) - a^4/((b^5*c - a*b^4*d)*(b*x^3 + a)^{(1/3)}) + 1/40*(20*(b*x^3 + a)^{(2/3)}*b^30*c^2*d^5 - 8*(b*x^3 + a)^{(5/3)}*b^29*c*d^6 + 40*(b*x^3 + a)^{(2/3)}*a*b^29*c*d^6 + 5*(b*x^3 + a)^{(8/3)}*b^28*d^7 - 24*(b*x^3 + a)^{(5/3)}*a*b^28*d^7 + 60*(b*x^3 + a)^{(2/3)}*a^2*b^28*d^7)/(b^32*d^8)$

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] int(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 5.19, size = 564, normalized size = 1.63

$$\left[\frac{\frac{3d^2}{2b^2} \left(\frac{bx^3 + a}{b^2} \right)^{\frac{b^2 - 3d^2}{2b^2}}}{(bx^3 + a)^{\frac{b^2 - 3d^2}{2b^2}}} - \frac{\frac{4d}{5b^2} \left(\frac{bx^3 + a}{5b^2} \right)^{\frac{b^2 - 4d}{5b^2}}}{(bx^3 + a)^{\frac{b^2 - 4d}{5b^2}}} + \frac{\frac{b^2 - 3d^2}{8b^2} \left(\frac{bx^3 + a}{8b^2} \right)^{\frac{b^2 - 3d^2}{8b^2}}}{8(b^2 - 3d^2)^{\frac{b^2 - 3d^2}{8b^2}}} - \frac{a^{\frac{1}{3}} \ln\left(\frac{bx^3 + a}{(bx^3 + a)^{\frac{1}{3}} (bx^3 + a)^{\frac{1}{3}}}\right)}{3a^{\frac{1}{3}}(bx^3 + a)^{\frac{1}{3}}} - \frac{\ln\left(\frac{bx^3 + a}{(bx^3 + a)^{\frac{1}{3}} (bx^3 + a)^{\frac{1}{3}}}\right)}{3a^{\frac{1}{3}}(bx^3 + a)^{\frac{1}{3}}} - \frac{\frac{c^4 \ln\left(\frac{bx^3 + a}{(bx^3 + a)^{\frac{1}{3}} (bx^3 + a)^{\frac{1}{3}}}\right)}{6d^{\frac{11}{3}}(bx^3 + a)^{\frac{1}{3}}}}{6d^{\frac{11}{3}}(bx^3 + a)^{\frac{1}{3}}} - \frac{c^4 \ln\left(\frac{bx^3 + a}{(bx^3 + a)^{\frac{1}{3}} (bx^3 + a)^{\frac{1}{3}}}\right)}{6d^{\frac{11}{3}}(bx^3 + a)^{\frac{1}{3}}} \right] \left(\frac{1}{3} + \frac{2d^2}{3b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x)`

[Out]
$$\begin{aligned} & \left(\frac{3a^2}{b^4d} + \left(\frac{4a}{b^4d} + \frac{b^5c - ab^4d}{b^8d^2} \right) (b^5c - ab^4d) / (2b^4d) \right) (a + bx^3)^{2/3} - \left(\frac{4a}{5b^4d} + \frac{b^5c - ab^4d}{5b^8d^2} \right) (a + bx^3)^{5/3} + (a + bx^3)^{8/3} / (8b^4d) + a^4 / (b^4d) \\ & (a + bx^3)^{1/3} (ad - bc) + (c^4 \log((a + bx^3)^{1/3} (ac^8d^5 - bc^9d^4) - (c^8(9a^4d^{15} + 9b^4c^4d^{11} - 36ab^3c^3d^{12} + 54a^2b^2c^2d^{13} - 36a^3b^2c^2d^{14})) / (9d^{22/3} (ad - bc)^{8/3}))) / (3d^{11/3} (ad - bc)^{4/3}) \\ & - (\log((a + bx^3)^{1/3} (ac^8d^5 - bc^9d^4) - ((3^{1/2})c^4i + c^4)^2 (9a^4d^{15} + 9b^4c^4d^{11} - 36ab^3c^3d^{12} + 54a^2b^2c^2d^{13} - 36a^3b^2c^2d^{14})) / (36d^{22/3} (ad - bc)^{8/3})) * (3^{1/2}c^4i + c^4) / (6d^{11/3} (ad - bc)^{4/3}) + (c^4 \log((a + bx^3)^{1/3} (ac^8d^5 - bc^9d^4) - (c^8((3^{1/2})i)/6 - 1/6)^2 (9a^4d^{15} + 9b^4c^4d^{11} - 36ab^3c^3d^{12} + 54a^2b^2c^2d^{13} - 36a^3b^2c^2d^{14})) / (d^{22/3} (ad - bc)^{8/3})) * ((3^{1/2})i)/6 - 1/6) / (d^{11/3} (ad - bc)^{4/3}) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(a + bx^3)^{\frac{4}{3}} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**14/(b*x**3+a)**(4/3)/(d*x**3+c),x)`

[Out] `Integral(x**14/((a + b*x**3)**(4/3)*(c + d*x**3)), x)`

$$3.480 \quad \int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal. Leaf size=253

$$\frac{a^3}{b^3 \sqrt[3]{a+bx^3} (bc-ad)} - \frac{(a+bx^3)^{2/3} (ad+bc)}{2b^3 d^2} - \frac{a(a+bx^3)^{2/3}}{2b^3 d} + \frac{(a+bx^3)^{5/3}}{5b^3 d} + \frac{c^3 \log(c+dx^3)}{6d^{8/3} (bc-ad)^{4/3}} - \frac{c^3 \log(\sqrt[3]{bc-ad})}{2d^{8/3} (bc-ad)^{4/3}}$$

Rubi [A] time = 0.34, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 87, 43, 56, 617, 204, 31}

$$\frac{a^3}{b^3 \sqrt[3]{a+bx^3} (bc-ad)} - \frac{(a+bx^3)^{2/3} (ad+bc)}{2b^3 d^2} - \frac{a(a+bx^3)^{2/3}}{2b^3 d} + \frac{(a+bx^3)^{5/3}}{5b^3 d} + \frac{c^3 \log(c+dx^3)}{6d^{8/3} (bc-ad)^{4/3}} - \frac{c^3 \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{2d^{8/3} (bc-ad)^{4/3}} - \frac{c^3 \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3} d^{8/3} (bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] a^3/(b^3*(b*c - a*d)*(a + b*x^3)^(1/3)) - (a*(a + b*x^3)^(2/3))/(2*b^3*d) - ((b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^3*d^2) + (a + b*x^3)^(5/3)/(5*b^3*d) - (c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(4/3)) + (c^3*Log[c + d*x^3])/(6*d^(8/3)*(b*c - a*d)^(4/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(8/3)*(b*c - a*d)^(4/3))

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 43

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 56

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),

```
x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x]) /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3}{(a+bx)^{4/3}(c+dx)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{a^3}{b^2(bc-ad)(a+bx)^{4/3}} + \frac{-bc-ad}{b^2d^2\sqrt[3]{a+bx}} + \frac{x}{bd\sqrt[3]{a+bx}} - \frac{1}{d^2(-bc+ad)} \right) dx, x, x^3 \right) \\
&= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{(bc+ad)(a+bx^3)^{2/3}}{2b^3d^2} + \frac{\text{Subst} \left(\int \frac{x}{\sqrt[3]{a+bx}} dx, x, x^3 \right)}{3bd} + \frac{c^3 \log(c+dx^3)}{6d^{8/3}(bc-ad)^{4/3}} \\
&= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{(bc+ad)(a+bx^3)^{2/3}}{2b^3d^2} + \frac{c^3 \log(c+dx^3)}{6d^{8/3}(bc-ad)^{4/3}} + \frac{\text{Subst} \left(\int \left(-\frac{1}{b^2(bc-ad)(a+bx)^{4/3}} + \frac{-bc-ad}{b^2d^2\sqrt[3]{a+bx}} + \frac{x}{bd\sqrt[3]{a+bx}} - \frac{1}{d^2(-bc+ad)} \right) dx, x, x^3 \right)}{3} \\
&= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{a(a+bx^3)^{2/3}}{2b^3d} - \frac{(bc+ad)(a+bx^3)^{2/3}}{2b^3d^2} + \frac{(a+bx^3)^{5/3}}{5b^3d} \\
&= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{a(a+bx^3)^{2/3}}{2b^3d} - \frac{(bc+ad)(a+bx^3)^{2/3}}{2b^3d^2} + \frac{(a+bx^3)^{5/3}}{5b^3d} - \frac{c^3 \log(c+dx^3)}{6d^{8/3}(bc-ad)^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.11, size = 147, normalized size = 0.58

$$\frac{18a^3d^3 + 3a^2bd^2(2dx^3 - c) + 10b^3c^3 {}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{d(bx^3+a)}{ad-bc}\right) - ab^2d(5c^2 + cdx^3 + 2d^2x^6) + b^3c(-10c^2 - 5cdx^3 + 2d^2x^6)}{10b^3d^3\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] (18*a^3*d^3 + 3*a^2*b*d^2*(-c + 2*d*x^3) + b^3*c*(-10*c^2 - 5*c*d*x^3 + 2*d^2*x^6) - a*b^2*d*(5*c^2 + c*d*x^3 + 2*d^2*x^6) + 10*b^3*c^3*Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-b*c) + a*d])/(10*b^3*d^3*(b*c - a*d)*(a + b*x^3)^(1/3))

IntegrateAlgebraic [A] time = 0.50, size = 322, normalized size = 1.27

$$\frac{-18a^3d^2 + 3a^2bcd - 6a^2bd^2x^3 + 5ab^2c^2 + ab^2cdx^3 + 2ab^2d^2x^6 + 5b^3c^2x^3 - 2b^3cdx^6}{10b^3d^2\sqrt[3]{a+bx^3}(bc-ad)} - \frac{c^3 \log\left(\frac{\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}}{3d^{8/3}(bc-ad)^{4/3}}\right)}{3d^{8/3}(bc-ad)^{4/3}} + \frac{c^3 \log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3}\right)}{6d^{8/3}(bc-ad)^{4/3}} - \frac{c^3 \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{8/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out]
$$-1/10*(5*a*b^2*c^2 + 3*a^2*b*c*d - 18*a^3*d^2 + 5*b^3*c^2*x^3 + a*b^2*c*d*x^3 - 6*a^2*b*d^2*x^3 - 2*b^3*c*d*x^6 + 2*a*b^2*d^2*x^6)/(b^3*d^2*(b*c - a*d)*(a + b*x^3)^{(1/3)}) - (c^3*ArcTan[1/Sqrt[3] - (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})]/(Sqrt[3]*(b*c - a*d)^{(1/3)})]/(Sqrt[3]*d^{(8/3)}*(b*c - a*d)^{(4/3)}) - (c^3*Log[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)}]/(3*d^{(8/3)}*(b*c - a*d)^{(4/3)}) + (c^3*Log[(b*c - a*d)^{(2/3)} - d^{(1/3)}*(b*c - a*d)^{(1/3)}*(a + b*x^3)^{(1/3)} + d^{(2/3)}*(a + b*x^3)^{(2/3)}]/(6*d^{(8/3)}*(b*c - a*d)^{(4/3)}))$$

fricas [B] time = 0.73, size = 1141, normalized size = 4.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/30*(15*\sqrt{1/3}*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*d - a*b^4*c^3*d^2)*x^3)*\sqrt{-(b*c*d^2 - a*d^3)^{(1/3)}/(b*c - a*d)}*\log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*\sqrt{1/3}*(2*(b*c*d^2 - a*d^3)^{(2/3)}*(b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^{(1/3)}*(b*c - a*d))*\sqrt{-(b*c*d^2 - a*d^3)^{(1/3)}/(b*c - a*d)} - 3*(b*c*d^2 - a*d^3)^{(2/3)}*(b*x^3 + a)^{(1/3)})/(d*x^3 + c) - 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(2/3)}*d^2 - (b*c*d^2 - a*d^3)^{(1/3)}*(b*x^3 + a)^{(1/3)}*d + (b*c*d^2 - a*d^3)^{(2/3)}) + 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(1/3)}*d + (b*c*d^2 - a*d^3)^{(1/3)}) + 3*(5*a*b^3*c^3*d^2 - 2*a^2*b^2*c^2*d^3 - 21*a^3*b*c*d^4 + 18*a^4*d^5 - 2*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 + (5*b^4*c^3*d^2 - 4*a*b^3*c^2*d^3 - 7*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^{(2/3)}/(a*b^5*c^2*d^4 - 2*a^2*b^4*c*d^5 + a^3*b^3*d^6 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^4*d^6)*x^3), \\ & 1/30*(30*\sqrt{1/3}*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*d - a*b^4*c^3*d^2)*x^3)*\sqrt{((b*c*d^2 - a*d^3)^{(1/3)}/(b*c - a*d))*\arctan(\sqrt{1/3}*(2*(b*x^3 + a)^{(1/3)}*d - (b*c*d^2 - a*d^3)^{(1/3)})*\sqrt{((b*c*d^2 - a*d^3)^{(1/3)}/(b*c - a*d))/d} + 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(2/3)}*d^2 - (b*c*d^2 - a*d^3)^{(1/3)}*(b*x^3 + a)^{(1/3)}*d + (b*c*d^2 - a*d^3)^{(2/3)}) - 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(1/3)}*d + (b*c*d^2 - a*d^3)^{(1/3)}) - 3*(5*a*b^3*c^3*d^2 - 2*a^2*b^2*c^2*d^3 - 21*a^3*b*c*d^4 + 18*a^4*d^5 - 2*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 + (5*b^4*c^3*d^2 - 4*a*b^3*c^2*d^3 - 7*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^{(2/3)}/(a*b^5*c^2*d^4 - 2*a^2*b^4*c*d^5 + a^3*b^3*d^6 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^4*d^6)*x^3)] \end{aligned}$$

giac [A] time = 0.26, size = 372, normalized size = 1.47

$$\frac{(-bc^2 + ad^2)^{\frac{2}{3}} c^2 \arctan\left(\frac{\sqrt{3}\left(2(bx^3 + a)^{\frac{1}{3}} - \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^4 - 2\sqrt{3}abcd^2 + \sqrt{3}a^2d^6} + \frac{(-bc^2 + ad^2)^{\frac{2}{3}} c^3 \log\left(\left(bx^3 + a\right)^{\frac{2}{3}} + \left(bx^3 + a\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6(b^2c^2d^4 - 2abcd^2 + a^2d^6)} - \frac{c^3\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}} \log\left(\left(bx^3 + a\right)^{\frac{1}{3}} - \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3(b^2c^2d^4 - 2abcd^2 + a^2d^6)} + \frac{a^3}{(b^4c - ab^3d)(bx^3 + a)^{\frac{5}{3}}} - \frac{5(bx^3 + a)^{\frac{2}{3}} b^{13}cd^3 - 2(bx^3 + a)^{\frac{5}{3}} b^{12}d^4 + 10(bx^3 + a)^{\frac{2}{3}} ab^{12}d^4}{10b^{15}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c), x, algorithm="giac")

[Out] $-(b^2cd^2 + ad^3)^{\frac{2}{3}} c^3 \arctan\left(\frac{1}{3} \sqrt{3} (2(bx^3 + a)^{\frac{1}{3}} + (bc - ad)/d)^{\frac{1}{3}}\right) / ((bc - ad)/d)^{\frac{1}{3}} / (\sqrt{3} b^2 c^2 d^4 - 2 \sqrt{3} (3) a b c d^2 + 3 a^2 d^6) + 1/6 (-b^2 c d^2 + a d^3)^{\frac{2}{3}} c^3 \log((bx^3 + a)^{\frac{2}{3}} + (bx^3 + a)^{\frac{1}{3}} (bc - ad)/d)^{\frac{1}{3}} + (-b^2 c d^2 + a d^3)^{\frac{2}{3}} c^3 \log((bx^3 + a)^{\frac{1}{3}} - (bc - ad)/d)^{\frac{1}{3}} / (b^2 c^2 d^4 - 2 a b c d^2 + a^2 d^6) - 1/3 c^3 ((bc - ad)/d)^{\frac{2}{3}} \log(\text{abs}((bx^3 + a)^{\frac{1}{3}} - (bc - ad)/d)^{\frac{1}{3}}) / (b^2 c^2 d^4 - 2 a b c d^2 + a^2 d^6) + a^3 / ((b^4 c - a b^3 d) (bx^3 + a)^{\frac{5}{3}}) - 1/10 (5 (bx^3 + a)^{\frac{2}{3}} b^{13} c d^3 - 2 (bx^3 + a)^{\frac{5}{3}} b^{12} d^4 + 10 (bx^3 + a)^{\frac{2}{3}} a b^{12} d^4) / (b^{15} d^5)$

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(bx^3 + a)^{\frac{4}{3}} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/(b*x^3+a)^(4/3)/(d*x^3+c), x)

[Out] int(x^11/(b*x^3+a)^(4/3)/(d*x^3+c), x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details) Is a*d-b*c positive or negative?

mupad [B] time = 5.16, size = 493, normalized size = 1.95

$$\frac{(b^2 + a)^{\frac{5}{3}}}{5b^3d} \left(\frac{3a}{2b^2} + \frac{b^2c - a^2d}{2b^2d} \right) (bx^3 + a)^{\frac{5}{3}} - \frac{a^3}{b^3(bx^3 + a)^{\frac{5}{3}}(ad - bc)} \ln\left(\frac{(bx^3 + a)^{\frac{5}{3}}(a^2d^2 - b^2c^2d)}{b^2d^2(bx^3 + a)^{\frac{5}{3}}(a^2d^2 - b^2c^2d)}\right) - \frac{a^3 \ln\left(\frac{(bx^3 + a)^{\frac{5}{3}}(a^2d^2 - b^2c^2d)}{b^2d^2(bx^3 + a)^{\frac{5}{3}}(a^2d^2 - b^2c^2d)}\right)}{6b^3d(ad - bc)^2} + \frac{c^2 \ln\left(\frac{(bx^3 + a)^{\frac{5}{3}}(a^2d^2 - b^2c^2d)}{b^2d^2(bx^3 + a)^{\frac{5}{3}}(a^2d^2 - b^2c^2d)}\right)}{3d^3(ad - bc)^2} - \frac{c^2 \ln\left(\frac{(bx^3 + a)^{\frac{5}{3}}(a^2d^2 - b^2c^2d)}{b^2d^2(bx^3 + a)^{\frac{5}{3}}(a^2d^2 - b^2c^2d)}\right)}{3d^3(ad - bc)^2} \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x)`

[Out] $(a + b*x^3)^{5/3}/(5*b^3*d) - ((3*a)/(2*b^3*d) + (b^4*c - a*b^3*d)/(2*b^6*d^2))*(a + b*x^3)^{2/3} - a^3/(b^3*(a + b*x^3)^{1/3}*(a*d - b*c)) - (c^3*\log((a + b*x^3)^{1/3}*(a*c^6*d^4 - b*c^7*d^3) - (c^6*(9*a^4*d^12 + 9*b^4*c^4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(9*d^{16/3}*(a*d - b*c)^{8/3}))) / (3*d^{8/3}*(a*d - b*c)^{4/3}) + (\log((a + b*x^3)^{1/3}*(a*c^6*d^4 - b*c^7*d^3) - ((3^{1/2}*c^3*1i + c^3)^2*(9*a^4*d^12 + 9*b^4*c^4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(36*d^{16/3}*(a*d - b*c)^{8/3}))* (3^{1/2}*c^3*1i + c^3) / (6*d^{8/3}*(a*d - b*c)^{4/3}) - (c^3*\log((a + b*x^3)^{1/3}*(a*c^6*d^4 - b*c^7*d^3) - (c^6*((3^{1/2}*1i)/2 - 1/2)^2*(9*a^4*d^12 + 9*b^4*c^4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(9*d^{16/3}*(a*d - b*c)^{8/3}))) * ((3^{1/2}*1i)/2 - 1/2) / (3*d^{8/3}*(a*d - b*c)^{4/3})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a + bx^3)^{\frac{4}{3}}(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11/(b*x**3+a)**(4/3)/(d*x**3+c),x)`

[Out] `Integral(x**11/((a + b*x**3)**(4/3)*(c + d*x**3)), x)`

$$3.481 \quad \int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal. Leaf size=203

$$\frac{a^2}{b^2 \sqrt[3]{a+bx^3} (bc-ad)} + \frac{(a+bx^3)^{2/3}}{2b^2d} - \frac{c^2 \log(c+dx^3)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2 \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}} + \frac{c^2 \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3} d^{5/3}(bc-ad)}$$

Rubi [A] time = 0.24, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 87, 56, 617, 204, 31}

$$-\frac{a^2}{b^2 \sqrt[3]{a+bx^3} (bc-ad)} + \frac{(a+bx^3)^{2/3}}{2b^2d} - \frac{c^2 \log(c+dx^3)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2 \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}} + \frac{c^2 \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3} d^{5/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] $-(a^2/(b^2*(b*c - a*d)*(a + b*x^3)^{(1/3)})) + (a + b*x^3)^{(2/3)/(2*b^2*d)} + (c^2*ArcTan[(1 - (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})/(b*c - a*d)^{(1/3)})/Sqrt[3]])/(Sqrt[3]*d^{(5/3)}*(b*c - a*d)^{(4/3)}) - (c^2*Log[c + d*x^3])/(6*d^{(5/3)}*(b*c - a*d)^{(4/3)}) + (c^2*Log[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)})/(2*d^{(5/3)}*(b*c - a*d)^{(4/3)})$

Rule 31

Int[((a_) + (b_)*(x_))^(−1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 56

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]], x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x]]) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x_)), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(a+bx)^{4/3}(c+dx)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^{4/3}} + \frac{1}{bd\sqrt[3]{a+bx}} + \frac{c^2}{d(-bc+ad)\sqrt[3]{a+bx}(c+dx)} \right) dx, x, x^3 \right) \\
&= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(a+bx^3)^{2/3}}{2b^2d} - \frac{c^2 \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right)}{3d(bc-ad)} \\
&= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(a+bx^3)^{2/3}}{2b^2d} - \frac{c^2 \log(c+dx^3)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2 \text{Subst} \left(\int \frac{1}{\sqrt[3]{bc-ad} + \sqrt[3]{a}} dx, x, x^3 \right)}{2d^{5/3}(bc-ad)} \\
&= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(a+bx^3)^{2/3}}{2b^2d} - \frac{c^2 \log(c+dx^3)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2 \log(\sqrt[3]{bc-ad} + \sqrt[3]{a})}{2d^{5/3}(bc-ad)} \\
&= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(a+bx^3)^{2/3}}{2b^2d} + \frac{c^2 \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}} \right)}{\sqrt{3}d^{5/3}(bc-ad)^{4/3}} - \frac{c^2 \log(c+dx^3)}{6d^{5/3}(bc-ad)}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 101, normalized size = 0.50

$$\frac{-3a^2d^2 - 2b^2c^2 {}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{d(bx^3+a)}{ad-bc}\right) + abd(c-dx^3) + b^2c(2c+dx^3)}{2b^2d^2\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] (-3*a^2*d^2 + a*b*d*(c - d*x^3) + b^2*c*(2*c + d*x^3) - 2*b^2*c^2*Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-b*c + a*d)]/(2*b^2*d^2*(b*c - a*d)*(a + b*x^3)^(1/3))

IntegrateAlgebraic [A] time = 0.46, size = 268, normalized size = 1.32

$$-\frac{3a^2d - abc + abdx^3 - b^2cx^3}{2b^2d\sqrt[3]{a+bx^3}(bc-ad)} + \frac{c^2 \log(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3})}{3d^{5/3}(bc-ad)^{4/3}} - \frac{c^2 \log(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3})}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2 \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{5/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out]
$$-1/2*(-(a*b*c) + 3*a^2*d - b^2*c*x^3 + a*b*d*x^3)/(b^2*d*(b*c - a*d)*(a + b*x^3)^{(1/3)} + (c^2*ArcTan[1/Sqrt[3] - (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})]/(Sqrt[3]*(b*c - a*d)^{(1/3)})))/(Sqrt[3]*d^{(5/3)}*(b*c - a*d)^{(4/3)} + (c^2*Log[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)}])/(3*d^{(5/3)}*(b*c - a*d)^{(4/3)} - (c^2*Log[(b*c - a*d)^{(2/3)} - d^{(1/3)}*(b*c - a*d)^{(1/3)}*(a + b*x^3)^{(1/3)} + d^{(2/3)}*(a + b*x^3)^{(2/3)}])/(6*d^{(5/3)}*(b*c - a*d)^{(4/3)})$$

fricas [B] time = 0.71, size = 1004, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out]
$$[-1/6*(3*\sqrt{1/3}*(a*b^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2*d^2)*x^3)*\sqrt{(-b*c*d^2 + a*d^3)^{(1/3)}/(b*c - a*d)}*\log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*\sqrt{1/3}*(2*(-b*c*d^2 + a*d^3)^{(2/3)}*(b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^{(1/3)}*(b*c - a*d))*\sqrt{(-b*c*d^2 + a*d^3)^{(1/3)}/(b*c - a*d)} - 3*(-b*c*d^2 + a*d^3)^{(2/3)}*(b*x^3 + a)^{(1/3)})/(d*x^3 + c)) + (b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(2/3)}*d^2 + (-b*c*d^2 + a*d^3)^{(1/3)}*(b*x^3 + a)^{(1/3)})*d + (-b*c*d^2 + a*d^3)^{(2/3)} - 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(1/3)}*d - (-b*c*d^2 + a*d^3)^{(1/3)}) - 3*(a*b^2*c^2*d^2 - 4*a^2*b*c*d^3 + 3*a^3*d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^{(2/3)})/(a*b^4*c^2*d^3 - 2*a^2*b^3*c*d^4 + a^3*b^2*d^5 + (b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*x^3), -1/6*(6*\sqrt{1/3}*(a*b^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2*d^2)*x^3)*\sqrt{(-b*c*d^2 + a*d^3)^{(1/3)}/(b*c - a*d)}*\arctan(\sqrt{1/3}*(2*(b*x^3 + a)^{(1/3)}*d + (-b*c*d^2 + a*d^3)^{(1/3)})*\sqrt{(-b*c*d^2 + a*d^3)^{(1/3)}/(b*c - a*d)})/d + (b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(2/3)}*d^2 + (-b*c*d^2 + a*d^3)^{(1/3)}*(b*x^3 + a)^{(1/3)}*d + (-b*c*d^2 + a*d^3)^{(2/3)}) - 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(1/3)}*d - (-b*c*d^2 + a*d^3)^{(1/3)}) - 3*(a*b^2*c^2*d^2 - 4*a^2*b*c*d^3 + 3*a^3*d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^{(2/3)})/(a*b^4*c^2*d^3 - 2*a^2*b^3*c*d^4 + a^3*b^2*d^5 + (b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*x^3)]$$

giac [A] time = 0.25, size = 325, normalized size = 1.60

$$\frac{(-bcd^2 + ad^3)^{\frac{2}{3}} c^2 \arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}} + \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^3 - 2\sqrt{3}abcd^4 + \sqrt{3}a^2d^5} - \frac{(-bcd^2 + ad^3)^{\frac{2}{3}} c^2 \log\left(\left(bx^3 + a\right)^{\frac{2}{3}} + \left(bx^3 + a\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6(b^2c^2d^3 - 2abcd^4 + a^2d^5)} + \frac{c^2\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}} \log\left(\left(bx^3 + a\right)^{\frac{1}{3}} - \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3(b^2c^2d - 2abcd^2 + a^2d^3)} - \frac{a^2}{(b^3c - ab^2d)(bx^3 + a)^{\frac{1}{3}}} + \frac{(bx^3 + a)^{\frac{2}{3}}}{2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] $(-b*c*d^2 + a*d^3)^{2/3} * c^2 * \arctan\left(\frac{1}{3}\sqrt{3} * (2*(b*x^3 + a)^{1/3} + (-b*c - a*d)/d)^{1/3}\right) / (-b*c - a*d)/d^{1/3} / (\sqrt{3} * b^2 * c^2 * d^3 - 2 * \sqrt{3} * a * b * c * d^4 + \sqrt{3} * a^2 * d^5) - 1/6 * (-b*c*d^2 + a*d^3)^{2/3} * c^2 * \log\left(\frac{(b*x^3 + a)^{2/3} + (b*x^3 + a)^{1/3} * (-b*c - a*d)/d^{1/3} + (-b*c - a*d)/d^{2/3}}{b^2 * c^2 * d^3 - 2 * a * b * c * d^4 + a^2 * d^5}\right) + 1/3 * c^2 * (-b*c - a*d)/d^{2/3} * \log\left(\frac{\text{abs}\left((b*x^3 + a)^{1/3} - (-b*c - a*d)/d^{1/3}\right)}{b^2 * c^2 * d - 2 * a * b * c * d^2 + a^2 * d^3} - a^2 / ((b^3 * c - a * b^2 * d) * (b*x^3 + a)^{1/3}) + 1/2 * (b*x^3 + a)^{2/3} / (b^2 * d)\right)$

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(bx^3 + a)^{\frac{4}{3}} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] int(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.99, size = 449, normalized size = 2.21

$$\frac{(bx^3+a)^{2/3}}{2b^2d^2} + \frac{c^2}{b^2(b^3+a)^{1/3}(ad-bc)} + \frac{c^2 \ln\left(\frac{(bx^3+a)^{1/3}(ad-bc) - \sqrt{3} \sqrt{bx^3+a} \sqrt{ad-bc}}{3d^{1/3}(ad-bc)^{2/3}}\right)}{3d^{1/3}(ad-bc)^{2/3}} + \frac{\ln\left(\frac{(bx^3+a)^{1/3}(ad-bc) - \sqrt{3} \sqrt{bx^3+a} \sqrt{ad-bc}}{6d^{1/3}(ad-bc)^{2/3}}\right)}{6d^{1/3}(ad-bc)^{2/3}} + \frac{c^2 \ln\left(\frac{(bx^3+a)^{1/3}(ad-bc) - \sqrt{3} \sqrt{bx^3+a} \sqrt{ad-bc}}{d^{1/3}(ad-bc)^{2/3}}\right)}{d^{1/3}(ad-bc)^{2/3}} + \frac{c^2 \ln\left(\frac{(bx^3+a)^{1/3}(ad-bc) - \sqrt{3} \sqrt{bx^3+a} \sqrt{ad-bc}}{d^{1/3}(ad-bc)^{2/3}}\right)}{d^{1/3}(ad-bc)^{2/3}} + \frac{c^2 \ln\left(\frac{(bx^3+a)^{1/3}(ad-bc) - \sqrt{3} \sqrt{bx^3+a} \sqrt{ad-bc}}{d^{1/3}(ad-bc)^{2/3}}\right)}{d^{1/3}(ad-bc)^{2/3}} + \frac{c^2 \ln\left(\frac{(bx^3+a)^{1/3}(ad-bc) - \sqrt{3} \sqrt{bx^3+a} \sqrt{ad-bc}}{d^{1/3}(ad-bc)^{2/3}}\right)}{d^{1/3}(ad-bc)^{2/3}} + \frac{c^2 \ln\left(\frac{(bx^3+a)^{1/3}(ad-bc) - \sqrt{3} \sqrt{bx^3+a} \sqrt{ad-bc}}{d^{1/3}(ad-bc)^{2/3}}\right)}{d^{1/3}(ad-bc)^{2/3}} + \frac{c^2 \ln\left(\frac{(bx^3+a)^{1/3}(ad-bc) - \sqrt{3} \sqrt{bx^3+a} \sqrt{ad-bc}}{d^{1/3}(ad-bc)^{2/3}}\right)}{d^{1/3}(ad-bc)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x)

[Out] $(a + b*x^3)^{2/3} / (2*b^2*d) + a^2 / (b^2*(a + b*x^3)^{1/3}*(a*d - b*c)) + (c^2 * \log((a + b*x^3)^{1/3}*(a*c^4*d^3 - b*c^5*d^2) - (c^4*(9*a^4*d^9 + 9*b^4*c$

$$\frac{(d^4 - 36ab^3c^3d^6 + 54a^2b^2c^2d^7 - 36a^3b^2c^2d^8)/(9d^{10/3}(ad - bc)^{8/3})}{(3d^{5/3}(ad - bc)^{4/3})} - (\log((a + bx^3)^{1/3}(ac^4d^3 - bc^5d^2) - ((3^{1/2})c^2i + c^2)^2(9a^4d^9 + 9b^4c^4d^5 - 36a^3b^3c^3d^6 + 54a^2b^2c^2d^7 - 36a^3b^2c^2d^8)))/(36d^{10/3}(ad - bc)^{8/3})) * (3^{1/2}c^2i + c^2) / (6d^{5/3}(ad - bc)^{4/3}) + (c^2 \log((a + bx^3)^{1/3}(ac^4d^3 - bc^5d^2) - (c^4((3^{1/2})i)/6 - 1/6)^2(9a^4d^9 + 9b^4c^4d^5 - 36a^3b^3c^3d^6 + 54a^2b^2c^2d^7 - 36a^3b^2c^2d^8)))/(d^{10/3}(ad - bc)^{8/3})) * ((3^{1/2})i)/6 - 1/6) / (d^{5/3}(ad - bc)^{4/3})$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a + bx^3)^{\frac{4}{3}}(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(b*x**3+a)**(4/3)/(d*x**3+c), x)

[Out] Integral(x**8/((a + b*x**3)**(4/3)*(c + d*x**3)), x)

$$3.482 \quad \int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal. Leaf size=174

$$\frac{c \log(c+dx^3)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}} - \frac{c \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}(bc-ad)^{4/3}} + \frac{a}{b\sqrt[3]{a+bx^3}(bc-ad)}$$

Rubi [A] time = 0.17, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 78, 56, 617, 204, 31}

$$\frac{c \log(c+dx^3)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}} - \frac{c \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}(bc-ad)^{4/3}} + \frac{a}{b\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] a/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) - (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d))/Sqrt[3]]/Sqrt[3])/((Sqrt[3]*d^(2/3)*(b*c - a*d)^(4/3)) + (c*Log[c + d*x^3])/(6*d^(2/3)*(b*c - a*d)^(4/3)) - (c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(2/3)*(b*c - a*d)^(4/3)))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 56

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])]/; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{(a+bx)^{4/3}(c+dx)} dx, x, x^3 \right) \\
&= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right)}{3(bc-ad)} \\
&= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c \log(c+dx^3)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c \text{Subst} \left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}+x}{\sqrt[3]{a}}}} dx, x, \sqrt[3]{a+bx^3} \right)}{2d^{2/3}(bc-ad)^{4/3}} \\
&= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c \log(c+dx^3)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c \log \left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3} \right)}{2d^{2/3}(bc-ad)^{4/3}} + \dots \\
&= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} - \frac{c \tan^{-1} \left(\frac{1 - 2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}} \right)}{\sqrt{3} d^{2/3} (bc-ad)^{4/3}} + \frac{c \log(c+dx^3)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c \log \left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3} \right)}{2d^{2/3}(bc-ad)^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 77, normalized size = 0.44

$$\frac{bc(a+bx^3) {}_2F_1 \left(\frac{2}{3}, 1; \frac{5}{3}; \frac{d(bx^3+a)}{ad-bc} \right) + 2a(bc-ad)}{2b\sqrt[3]{a+bx^3}(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] (2*a*(b*c - a*d) + b*c*(a + b*x^3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)]/(2*b*(b*c - a*d)^2*(a + b*x^3)^(1/3))

IntegrateAlgebraic [A] time = 0.26, size = 230, normalized size = 1.32

$$-\frac{c \log \left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3} \right)}{3d^{2/3}(bc-ad)^{4/3}} + \frac{c \log \left(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3} (a+bx^3)^{2/3} \right)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt{3} \sqrt[3]{bc-ad}} \right)}{\sqrt{3} d^{2/3} (bc-ad)^{4/3}} + \frac{a}{b\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

```
[Out] a/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) - (c*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a +
b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*d^(2/3)*(b*c - a*d)^(
4/3)) - (c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(3*d^(2/3)*(
b*c - a*d)^(4/3)) + (c*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
+ b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/(6*d^(2/3)*(b*c - a*d)^(4/3))
```

fricas [B] time = 0.82, size = 872, normalized size = 5.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(a*b^2*c^2*d - a^2*b*c*d^2 + (b^3*c^2*d - a*b^2*c*d^2)*x
^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3
*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3
+ a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(-(b*
c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(
1/3))/(d*x^3 + c) - (b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log((b*x^3
+ a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 -
a*d^3)^(2/3)) + 2*(b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log((b*x^3 +
a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 6*(a*b*c*d^2 - a^2*d^3)*(b*x^3 + a)
^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + a^3*b*d^4 + (b^4*c^2*d^2 - 2*a*b
^3*c*d^3 + a^2*b^2*d^4)*x^3), 1/6*(6*sqrt(1/3)*(a*b^2*c^2*d - a^2*b*c*d^2 +
(b^3*c^2*d - a*b^2*c*d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*a
rctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c
*d^2 - a*d^3)^(1/3)/(b*c - a*d))/d) + (b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)
^(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)
)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2
/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 6*(a*b*c*d^2 - a^2
*d^3)*(b*x^3 + a)^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + a^3*b*d^4 + (b^
4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^3)]
```

giac [B] time = 0.25, size = 301, normalized size = 1.73

$$\frac{6(-bcd^2+ad^3)^{\frac{2}{3}}bc \arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^2-2\sqrt{3}abcd^3+\sqrt{3}a^2d^4} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}bc \log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{b^2c^2d^2-2abcd^3+a^2d^4} + \frac{2bc\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}} \log\left(\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{b^2c^2-2abcd+a^2d^2} - \frac{6a}{(bx^3+a)^{\frac{1}{3}}(bc-ad)}$$

6b

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/6*(6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3)
) + (-b*c - a*d)/d)^(1/3))/(-b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^2 -
```

$2\sqrt{3}abc^2d^3 + \sqrt{3}a^2d^4 - (-b^2cd^2 + a^2d^3)^{2/3}bc \log\left(\frac{bx^3 + a}{b^2c^2d^2 - 2abc^2d^3 + a^2d^4} + \frac{(bx^3 + a)^{1/3}(-b^2cd^2 + a^2d^3)^{1/3} + (-b^2cd^2 + a^2d^3)^{2/3}}{b^2c^2d^2 - 2abc^2d^3 + a^2d^4} + 2bc \frac{(-b^2cd^2 + a^2d^3)^{1/3}}{b^2c^2d^2 - 2abc^2d^3 + a^2d^4} \log\left(\frac{(bx^3 + a)^{1/3} - (-b^2cd^2 + a^2d^3)^{1/3}}{b^2c^2d^2 - 2abc^2d^3 + a^2d^4} - 6a \frac{(-b^2cd^2 + a^2d^3)^{1/3}}{(bx^3 + a)^{1/3}bc}\right)\right)$

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] int(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.98, size = 412, normalized size = 2.37

$$\frac{a \ln\left(\frac{(bx^3 + a)^{1/3} (a^2 d^2 - b^2 c^3 d)}{3 d^{2/3} (a d - b c)^{2/3}}\right) + \ln\left(\frac{(bx^3 + a)^{1/3} (a^2 d^2 - b^2 c^3 d)}{3 d^{2/3} (a d - b c)^{2/3}}\right) (c - \sqrt{3} i)}{6 d^{2/3} (a d - b c)^{2/3}} + \frac{a \ln\left(\frac{(bx^3 + a)^{1/3} (a^2 d^2 - b^2 c^3 d)}{3 d^{2/3} (a d - b c)^{2/3}}\right) + \ln\left(\frac{(bx^3 + a)^{1/3} (a^2 d^2 - b^2 c^3 d)}{3 d^{2/3} (a d - b c)^{2/3}}\right) (c + \sqrt{3} i)}{6 d^{2/3} (a d - b c)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x)

[Out] $\frac{\log\left(\frac{(a + bx^3)^{1/3} (a^2 d^2 - b^2 c^3 d)}{3 d^{2/3} (a d - b c)^{2/3}}\right) - \left((c - 3^{1/2} c i)^2 (9 a^4 d^6 + 9 b^4 c^4 d^2 - 36 a b^3 c^3 d^3 + 54 a^2 b^2 c^2 d^4 - 36 a^3 b c d^5)\right) / (36 d^{4/3} (a d - b c)^{8/3})}{(6 d^{2/3} (a d - b c)^{2/3})} + \frac{\log\left(\frac{(a + bx^3)^{1/3} (a^2 d^2 - b^2 c^3 d)}{3 d^{2/3} (a d - b c)^{2/3}}\right) + \left((c + 3^{1/2} c i)^2 (9 a^4 d^6 + 9 b^4 c^4 d^2 - 36 a b^3 c^3 d^3 + 54 a^2 b^2 c^2 d^4 - 36 a^3 b c d^5)\right) / (9 d^{4/3} (a d - b c)^{8/3})}{(3 d^{2/3} (a d - b c)^{2/3})} - \frac{a}{b (a + bx^3)^{1/3} (a d - b c)} + \frac{\log\left(\frac{(a + bx^3)^{1/3} (a^2 d^2 - b^2 c^3 d)}{3 d^{2/3} (a d - b c)^{2/3}}\right) - \left((c - 3^{1/2} c i)^2 (9 a^4 d^6 + 9 b^4 c^4 d^2 - 36 a b^3 c^3 d^3 + 54 a^2 b^2 c^2 d^4 - 36 a^3 b c d^5)\right) / (36 d^{4/3} (a d - b c)^{8/3})}{(6 d^{2/3} (a d - b c)^{2/3})} + \frac{\log\left(\frac{(a + bx^3)^{1/3} (a^2 d^2 - b^2 c^3 d)}{3 d^{2/3} (a d - b c)^{2/3}}\right) + \left((c + 3^{1/2} c i)^2 (9 a^4 d^6 + 9 b^4 c^4 d^2 - 36 a b^3 c^3 d^3 + 54 a^2 b^2 c^2 d^4 - 36 a^3 b c d^5)\right) / (9 d^{4/3} (a d - b c)^{8/3})}{(3 d^{2/3} (a d - b c)^{2/3})} - \frac{a}{b (a + bx^3)^{1/3} (a d - b c)}$

$(a^2 b^2 c^2 d^4 - 36 a^3 b c d^5) / (36 d^{4/3} (a d - b c)^{8/3}) * (c + 3^{1/2} c^{1/2}) / (6 d^{2/3} (a d - b c)^{4/3})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + bx^3)^{4/3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x**3+a)**(4/3)/(d*x**3+c), x)

[Out] Integral(x**5/((a + b*x**3)**(4/3)*(c + d*x**3)), x)

$$3.483 \quad \int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal. Leaf size=167

$$\frac{1}{\sqrt[3]{a+bx^3}(bc-ad)} - \frac{\sqrt[3]{d} \log(c+dx^3)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{2(bc-ad)^{4/3}} + \frac{\sqrt[3]{d} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}(bc-ad)^{4/3}}$$

Rubi [A] time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {444, 51, 56, 617, 204, 31}

$$\frac{1}{\sqrt[3]{a+bx^3}(bc-ad)} - \frac{\sqrt[3]{d} \log(c+dx^3)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{2(bc-ad)^{4/3}} + \frac{\sqrt[3]{d} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt{3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] -(1/((b*c - a*d)*(a + b*x^3)^(1/3))) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3))*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)]/Sqrt[3])/(Sqrt[3]*(b*c - a*d)^(4/3)) - (d^(1/3)*Log[c + d*x^3]/(6*(b*c - a*d)^(4/3)) + (d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*(b*c - a*d)^(4/3)))

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 51

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(a+bx)^{4/3}(c+dx)} dx, x, x^3 \right) \\
&= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right)}{3(bc-ad)} \\
&= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\sqrt[3]{d} \log(c+dx^3)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d} \text{Subst} \left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}} + x} dx, x, \sqrt[3]{a+bx^3} \right)}{2(bc-ad)^{4/3}} \\
&= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\sqrt[3]{d} \log(c+dx^3)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{2(bc-ad)^{4/3}} \\
&= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{d} \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}} \right)}{\sqrt{3}(bc-ad)^{4/3}} - \frac{\sqrt[3]{d} \log(c+dx^3)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d} \log(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3})}{2(bc-ad)^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 50, normalized size = 0.30

$$-\frac{{}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{d(bx^3+a)}{ad-bc}\right)}{\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] -(Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-b*c) + a*d])/((b*c - a*d)*(a + b*x^3)^(1/3))

IntegrateAlgebraic [A] time = 0.24, size = 223, normalized size = 1.34

$$-\frac{\sqrt[3]{d} \log\left(-\sqrt[3]{d} \sqrt[3]{a+bx^3} \sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3} (a+bx^3)^{2/3}\right)}{6(bc-ad)^{4/3}} - \frac{1}{\sqrt[3]{a+bx^3}(bc-ad)} + \frac{\sqrt[3]{d} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d} \sqrt[3]{a+bx^3}\right)}{3(bc-ad)^{4/3}} + \frac{\sqrt[3]{d} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{d} \sqrt[3]{a+bx^3}}{\sqrt{3} \sqrt[3]{bc-ad}}\right)}{\sqrt{3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] $-(1/((b*c - a*d)*(a + b*x^3)^{(1/3)})) + (d^{(1/3)}*ArcTan[1/Sqrt[3] - (2*d^{(1/3)}*(a + b*x^3)^{(1/3)})/(Sqrt[3]*(b*c - a*d)^{(1/3)})])/(Sqrt[3]*(b*c - a*d)^{(4/3)}) + (d^{(1/3)}*Log[(b*c - a*d)^{(1/3)} + d^{(1/3)}*(a + b*x^3)^{(1/3)}])/(3*(b*c - a*d)^{(4/3)}) - (d^{(1/3)}*Log[(b*c - a*d)^{(2/3)} - d^{(1/3)}*(b*c - a*d)^{(1/3)}*(a + b*x^3)^{(1/3)} + d^{(2/3)}*(a + b*x^3)^{(2/3)}])/(6*(b*c - a*d)^{(4/3)})$

fricas [A] time = 0.67, size = 262, normalized size = 1.57

$$\frac{2\sqrt{3}(bx^3+a)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-(bx^3+a)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left(-\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\left(-\frac{d}{bc-ad}\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}d-(bc-ad)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\right)+2(bx^3+a)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left((bc-ad)\left(-\frac{d}{bc-ad}\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}d\right)+6(bx^3+a)^{\frac{2}{3}}}{6\left((b^2c-abd)x^3+abc-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")`

[Out] $-1/6*(2*\sqrt{3}*(b*x^3 + a)*(-d/(b*c - a*d))^{(1/3)}*\arctan(2/3*\sqrt{3}*(b*x^3 + a)^{(1/3)}*(-d/(b*c - a*d))^{(1/3)} + 1/3*\sqrt{3})) - (b*x^3 + a)*(-d/(b*c - a*d))^{(1/3)}*\log(-(b*x^3 + a)^{(1/3)}*(b*c - a*d)*(-d/(b*c - a*d))^{(2/3)} + (b*x^3 + a)^{(2/3)}*d - (b*c - a*d)*(-d/(b*c - a*d))^{(1/3)}) + 2*(b*x^3 + a)*(-d/(b*c - a*d))^{(1/3)}*\log((b*c - a*d)*(-d/(b*c - a*d))^{(2/3)} + (b*x^3 + a)^{(1/3)}*d) + 6*(b*x^3 + a)^{(2/3)})/((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)$

giac [B] time = 0.26, size = 285, normalized size = 1.71

$$\frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3(b^2c^2-2abcd+a^2d^2)}+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d-2\sqrt{3}abcd^2+\sqrt{3}a^2d^3}-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6(b^2c^2d-2abcd^2+a^2d^3)}-\frac{1}{(bx^3+a)^{\frac{1}{3}}(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")`

[Out] $1/3*d*(-(b*c - a*d)/d)^{(2/3)}*\log(ABS((b*x^3 + a)^{(1/3)} - (-b*c - a*d)/d)^{(1/3)})/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + (-b*c*d^2 + a*d^3)^{(2/3)}*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{(1/3)} + (-b*c - a*d)/d)^{(1/3)})/((-b*c - a*d)/d)^{(1/3)}/(\sqrt{3}*b^2*c^2*d - 2*\sqrt{3}*a*b*c*d^2 + \sqrt{3}*a^2*d^3) - 1/6*(-b*c*d^2 + a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(-b*c - a*d)/d)^{(1/3)} + (-b*c - a*d)/d)^{(2/3)})/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/((b*x^3 + a)^{(1/3)}*(b*c - a*d))$

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)`

[Out] `int(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.92, size = 389, normalized size = 2.33

$$\frac{1}{(bx^3+a)^{1/3}(ad-bc)} + \frac{d^{1/3} \ln\left(\frac{(bx^3+a)^{1/3}(ad-bc)^{1/3} - \frac{d^{1/3}(bx^3+a)^{1/3}(ad-bc)^{1/3}}{3(ad-bc)^{1/3}}}{\frac{d^{1/3}(bx^3+a)^{1/3}(ad-bc)^{1/3}}{3(ad-bc)^{1/3}}}\right)}{3(ad-bc)^{1/3}} + \frac{d^{1/3} \ln\left(\frac{(bx^3+a)^{1/3}(ad-bc)^{1/3} - \frac{d^{1/3}(bx^3+a)^{1/3}(ad-bc)^{1/3}}{3(ad-bc)^{1/3}}}{\frac{d^{1/3}(bx^3+a)^{1/3}(ad-bc)^{1/3}}{3(ad-bc)^{1/3}}}\right)}{3(ad-bc)^{1/3}} + \frac{d^{1/3} \ln\left(\frac{(bx^3+a)^{1/3}(ad-bc)^{1/3} - \frac{d^{1/3}(bx^3+a)^{1/3}(ad-bc)^{1/3}}{3(ad-bc)^{1/3}}}{\frac{d^{1/3}(bx^3+a)^{1/3}(ad-bc)^{1/3}}{3(ad-bc)^{1/3}}}\right)}{3(ad-bc)^{1/3}} + \frac{d^{1/3} \ln\left(\frac{(bx^3+a)^{1/3}(ad-bc)^{1/3} - \frac{d^{1/3}(bx^3+a)^{1/3}(ad-bc)^{1/3}}{3(ad-bc)^{1/3}}}{\frac{d^{1/3}(bx^3+a)^{1/3}(ad-bc)^{1/3}}{3(ad-bc)^{1/3}}}\right)}{3(ad-bc)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x)`

[Out] $\frac{1}{(a + bx^3)^{1/3}(ad - bc)} + \frac{d^{1/3} \log((a + bx^3)^{1/3}(ad^4 - bc^4 - b^2c^2d^4 - 36a^3b^2cd^5))}{9(a + bx^3)^{8/3}} + \frac{d^{1/3} \log((a + bx^3)^{1/3}(ad^4 - bc^4 - b^2c^2d^4 - 36a^3b^2cd^5))}{9(a + bx^3)^{8/3}} + \frac{d^{1/3} \log((a + bx^3)^{1/3}(ad^4 - bc^4 - b^2c^2d^4 - 36a^3b^2cd^5))}{9(a + bx^3)^{8/3}} + \frac{d^{1/3} \log((a + bx^3)^{1/3}(ad^4 - bc^4 - b^2c^2d^4 - 36a^3b^2cd^5))}{9(a + bx^3)^{8/3}} + \frac{d^{1/3} \log((a + bx^3)^{1/3}(ad^4 - bc^4 - b^2c^2d^4 - 36a^3b^2cd^5))}{9(a + bx^3)^{8/3}}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + bx^3)^{4/3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**3+a)**(4/3)/(d*x**3+c),x)`

[Out] `Integral(x**2/((a + b*x**3)**(4/3)*(c + d*x**3)), x)`

$$3.484 \quad \int \frac{1}{x(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal. Leaf size=271

$$\frac{\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2a^{4/3}c} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}c} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log(c+dx^3)}{6c(bc-ad)^{4/3}} - \frac{d^{4/3}\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}}$$

Rubi [A] time = 0.32, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {446, 85, 156, 55, 617, 204, 31, 56}

$$\frac{\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2a^{4/3}c} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}c} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log(c+dx^3)}{6c(bc-ad)^{4/3}} - \frac{d^{4/3}\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}} - \frac{d^{4/3}\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{4/3}} + \frac{b}{a\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] b/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) + ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(4/3)*c) - (d^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(4/3)) - Log[x]/(2*a^(4/3)*c) + (d^(4/3)*Log[c + d*x^3])/(6*c*(b*c - a*d)^(4/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(4/3)*c) - (d^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*(b*c - a*d)^(4/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 55

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 56

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),

```
x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])) /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_))*((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx^3)^{4/3}(c+dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(a+bx)^{4/3}(c+dx)} dx, x, x^3 \right) \\
&= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\text{Subst} \left(\int \frac{-bc+ad-bdx}{x\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right)}{3a(bc-ad)} \\
&= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\text{Subst} \left(\int \frac{1}{x\sqrt[3]{a+bx}} dx, x, x^3 \right)}{3ac} + \frac{d^2 \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)} dx, x, x^3 \right)}{3c(bc-ad)} \\
&= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3} \log(c+dx^3)}{6c(bc-ad)^{4/3}} - \frac{\text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, \sqrt[3]{a+bx^3} \right)}{2a^{4/3}c} \\
&= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3} \log(c+dx^3)}{6c(bc-ad)^{4/3}} + \frac{\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2a^{4/3}c} - \frac{d^2 \text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, \sqrt[3]{a+bx^3} \right)}{2a^{4/3}c} \\
&= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3}a^{4/3}c} - \frac{d^{4/3} \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{a}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}} \right)}{\sqrt{3}c(bc-ad)^{4/3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^2 \text{Subst} \left(\int \frac{1}{\sqrt[3]{a-x}} dx, x, \sqrt[3]{a+bx^3} \right)}{2a^{4/3}c}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 86, normalized size = 0.32

$$\frac{ad {}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{d(bx^3+a)}{ad-bc}\right) + (bc-ad) {}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{bx^3}{a} + 1\right)}{ac\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] (a*d*Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-b*c) + a*d]) + (b*c - a*d)*Hypergeometric2F1[-1/3, 1, 2/3, 1 + (b*x^3)/a])/(a*c*(b*c - a*d)*(a + b*x^3)^(1/3))

IntegrateAlgebraic [A] time = 0.85, size = 360, normalized size = 1.33

$$\frac{\log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{a}\right)}{3a^{4/3}c} - \frac{\log\left(a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^3} + (a+bx^3)^{2/3}\right)}{6a^{4/3}c} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}} + \frac{1}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}c} - \frac{d^{4/3} \log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3c(bc-ad)^{4/3}} + \frac{d^{4/3} \log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad} + (bc-ad)^{2/3} + d^{2/3}(a+bx^3)^{2/3}\right)}{6c(bc-ad)^{4/3}} - \frac{d^{4/3} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}c(bc-ad)^{4/3}} - \frac{b}{a\sqrt[3]{a+bx^3}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(b/(a*(-(b*c) + a*d)*(a + b*x^3)^(1/3))) + ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(4/3)*c) - (d^(4/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*c*(b*c - a*d)^(4/3)) + Log[-a^(1/3) + (a + b*x^3)^(1/3)]/(3*a^(4/3)*c) - (d^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(3*c*(b*c - a*d)^(4/3)) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(6*a^(4/3)*c) + (d^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/(6*c*(b*c - a*d)^(4/3))
```

fricas [B] time = 0.80, size = 975, normalized size = 3.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 3*sqrt(1/3)*(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^3)*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) + 2*sqrt(3)*(a^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (a^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) - 2*(a^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d)/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 2*sqrt(3)*(a^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (a^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) - 2*(a^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) + 6*sqrt(1/3)*(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3)]/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3)]
```

giac [A] time = 0.79, size = 389, normalized size = 1.44

$$\frac{a^2 \left(\frac{bc-ad}{a}\right)^{\frac{2}{3}} \log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(\frac{bc-ad}{a}\right)^{\frac{1}{3}}\right|\right)}{3(b^2c^3 - 2abc^2d + a^2cd^2)} - \frac{(-bcd + ad^2)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(\frac{bc-ad}{a}\right)^{\frac{1}{3}} - \left(\frac{bc-ad}{a}\right)^{\frac{1}{3}}}{3\left(\frac{bc-ad}{a}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^3 - 2\sqrt{3}abc^2d + \sqrt{3}a^2cd^2} + \frac{(-bcd + ad^2)^{\frac{2}{3}} \log\left(\left|(bx^3+a)^{\frac{1}{3}} + \left(\frac{bc-ad}{a}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{a}\right)^{\frac{1}{3}}\right|\right)}{6(b^2c^3 - 2abc^2d + a^2cd^2)} + \frac{b}{(bx^3+a)^{\frac{1}{3}}(abc - a^2d)} + \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(\frac{bc-ad}{a}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{a}\right)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{1}{3}}c} - \frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{1}{3}}\right|\right)}{6a^{\frac{1}{3}}c} + \frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{3a^{\frac{1}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out]
$$-1/3*d^2*(-(b*c - a*d)/d)^{(2/3)}*\log(\text{abs}((b*x^3 + a)^{(1/3)} - (- (b*c - a*d)/d)^{(1/3)}))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2) - (-b*c*d^2 + a*d^3)^{(2/3)}*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{(1/3)} + (- (b*c - a*d)/d)^{(1/3)})/(- (b*c - a*d)/d)^{(1/3)})/(\sqrt{3}*b^2*c^3 - 2*\sqrt{3}*a*b*c^2*d + \sqrt{3}*a^2*c*d^2) + 1/6*(-b*c*d^2 + a*d^3)^{(2/3)}*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*(- (b*c - a*d)/d)^{(1/3)} + (- (b*c - a*d)/d)^{(2/3)})/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2) + b/((b*x^3 + a)^{(1/3)}*(a*b*c - a^2*d)) + 1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*(b*x^3 + a)^{(1/3)} + a^{(1/3)})/a^{(1/3)})/a^{(4/3)}*c - 1/6*\log((b*x^3 + a)^{(2/3)} + (b*x^3 + a)^{(1/3)}*a^{(1/3)} + a^{(2/3)})/a^{(4/3)}*c + 1/3*\log(\text{abs}((b*x^3 + a)^{(1/3)} - a^{(1/3)}))/a^{(4/3)}*c$$

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] int(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x), x)

mupad [B] time = 5.34, size = 3804, normalized size = 14.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x*(a + b*x^3)^{(4/3)}*(c + d*x^3)), x)$

[Out] $\log(9*a^7*b^{14}*c^{11}*d^4 - ((a + b*x^3)^{(1/3)}*(27*a^7*b^{15}*c^{13}*d^3 - 297*a^8*b^{14}*c^{12}*d^4 + 1485*a^9*b^{13}*c^{11}*d^5 - 4455*a^{10}*b^{12}*c^{10}*d^6 + 8937*a^{11}*b^{11}*c^9*d^7 - 12663*a^{12}*b^{10}*c^8*d^8 + 13041*a^{13}*b^9*c^7*d^9 - 9855*a^{14}*b^8*c^6*d^{10} + 5400*a^{15}*b^7*c^5*d^{11} - 2052*a^{16}*b^6*c^4*d^{12} + 486*a^{17}*b^5*c^3*d^{13} - 54*a^{18}*b^4*c^2*d^{14}) - (-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^{(2/3)}*(243*a^{10}*b^{15}*c^{15}*d^3 - 2916*a^{11}*b^{14}*c^{14}*d^4 + 15795*a^{12}*b^{13}*c^{13}*d^5 - 51030*a^{13}*b^{12}*c^{12}*d^6 + 109350*a^{14}*b^{11}*c^{11}*d^7 - 163296*a^{15}*b^{10}*c^{10}*d^8 + 173502*a^{16}*b^9*c^9*d^9 - 131220*a^{17}*b^8*c^8*d^{10} + 69255*a^{18}*b^7*c^7*d^{11} - 24300*a^{19}*b^6*c^6*d^{12} + 5103*a^{20}*b^5*c^5*d^{13} - 486*a^{21}*b^4*c^4*d^{14}))*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^{(1/3)} - 90*a^8*b^{13}*c^{10}*d^5 + 405*a^9*b^{12}*c^9*d^6 - 1071*a^{10}*b^{11}*c^8*d^7 + 1827*a^{11}*b^{10}*c^7*d^8 - 2079*a^{12}*b^9*c^6*d^9 + 1575*a^{13}*b^8*c^5*d^{10} - 765*a^{14}*b^7*c^4*d^{11} + 216*a^{15}*b^6*c^3*d^{12} - 27*a^{16}*b^5*c^2*d^{13})*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^{(1/3)} + \log(9*a^7*b^{14}*c^{11}*d^4 - ((a + b*x^3)^{(1/3)}*(27*a^7*b^{15}*c^{13}*d^3 - 297*a^8*b^{14}*c^{12}*d^4 + 1485*a^9*b^{13}*c^{11}*d^5 - 4455*a^{10}*b^{12}*c^{10}*d^6 + 8937*a^{11}*b^{11}*c^9*d^7 - 12663*a^{12}*b^{10}*c^8*d^8 + 13041*a^{13}*b^9*c^7*d^9 - 9855*a^{14}*b^8*c^6*d^{10} + 5400*a^{15}*b^7*c^5*d^{11} - 2052*a^{16}*b^6*c^4*d^{12} + 486*a^{17}*b^5*c^3*d^{13} - 54*a^{18}*b^4*c^2*d^{14}) - (1/(27*a^4*c^3))^{(2/3)}*(243*a^{10}*b^{15}*c^{15}*d^3 - 2916*a^{11}*b^{14}*c^{14}*d^4 + 15795*a^{12}*b^{13}*c^{13}*d^5 - 51030*a^{13}*b^{12}*c^{12}*d^6 + 109350*a^{14}*b^{11}*c^{11}*d^7 - 163296*a^{15}*b^{10}*c^{10}*d^8 + 173502*a^{16}*b^9*c^9*d^9 - 131220*a^{17}*b^8*c^8*d^{10} + 69255*a^{18}*b^7*c^7*d^{11} - 24300*a^{19}*b^6*c^6*d^{12} + 5103*a^{20}*b^5*c^5*d^{13} - 486*a^{21}*b^4*c^4*d^{14}))* (1/(27*a^4*c^3))^{(1/3)} - 90*a^8*b^{13}*c^{10}*d^5 + 405*a^9*b^{12}*c^9*d^6 - 1071*a^{10}*b^{11}*c^8*d^7 + 1827*a^{11}*b^{10}*c^7*d^8 - 2079*a^{12}*b^9*c^6*d^9 + 1575*a^{13}*b^8*c^5*d^{10} - 765*a^{14}*b^7*c^4*d^{11} + 216*a^{15}*b^6*c^3*d^{12} - 27*a^{16}*b^5*c^2*d^{13})*(1/(27*a^4*c^3))^{(1/3)} + (\log(((3^{(1/2)}*i - 1)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^{(1/3)}*((a + b*x^3)^{(1/3)}*(27*a^7*b^{15}*c^{13}*d^3 - 297*a^8*b^{14}*c^{12}*d^4 + 1485*a^9*b^{13}*c^{11}*d^5 - 4455*a^{10}*b^{12}*c^{10}*d^6 + 8937*a^{11}*b^{11}*c^9*d^7 - 12663*a^{12}*b^{10}*c^8*d^8 + 13041*a^{13}*b^9*c^7*d^9 - 9855*a^{14}*b^8*c^6*d^{10} + 5400*a^{15}*b^7*c^5*d^{11} - 2052*a^{16}*b^6*c^4*d^{12} + 486*a^{17}*b^5*c^3*d^{13} - 54*a^{18}*b^4*c^2*d^{14}) - ((3^{(1/2)}*i - 1)^2*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^{(2/3)}*(243*a^{10}*b^{15}*c^{15}*d^3 - 2916*a^{11}*b^{14}*c^{14}*d^4 + 15795*a^{12}*b^{13}*c^{13}*d^5 - 51030*a^{13}*b^{12}*c^{12}*d^6 + 109350*a^{14}*b^{11}*c^{11}*d^7 - 163296*a^{15}*b^{10}*c^{10}*d^8 + 173502*a^{16}*b^9*c^9*d^9 - 131220*a^{17}*b^8*c^8*d^{10} + 69255*a^{18}*b^7*c^7*d^{11} - 24300*a^{19}*b^6*c^6*d^{12} + 5103*a^{20}*b^5*c^5*d^{13} - 486*a^{21}*b^4*c^4*d^{14}))/4))/2 - 9*a^7*b^{14}*c^{11}*d^4 + 90*a^8*b^{13}*c^{10}*d^5 - 405*a^$

$$\begin{aligned}
& 9*b^{12}*c^9*d^6 + 1071*a^{10}*b^{11}*c^8*d^7 - 1827*a^{11}*b^{10}*c^7*d^8 + 2079*a^{12}*b^9*c^6*d^9 - 1575*a^{13}*b^8*c^5*d^{10} + 765*a^{14}*b^7*c^4*d^{11} - 216*a^{15}*b^6*c^3*d^{12} + 27*a^{16}*b^5*c^2*d^{13} \\
& * (3^{(1/2)}*i - 1) * (-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^{(1/3)} \\
&) / 2 - (\log(((3^{(1/2)}*i + 1) * (-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^{(1/3)} * ((a + b*x^3)^{(1/3)} * (27*a^7*b^{15}*c^{13}*d^3 - 297*a^8*b^{14}*c^{12}*d^4 + 1485*a^9*b^{13}*c^{11}*d^5 - 4455*a^{10}*b^{12}*c^{10}*d^6 + 8937*a^{11}*b^{11}*c^9*d^7 - 12663*a^{12}*b^{10}*c^8*d^8 + 13041*a^{13}*b^9*c^7*d^9 - 9855*a^{14}*b^8*c^6*d^{10} + 5400*a^{15}*b^7*c^5*d^{11} - 2052*a^{16}*b^6*c^4*d^{12} + 486*a^{17}*b^5*c^3*d^{13} - 54*a^{18}*b^4*c^2*d^{14}) - ((3^{(1/2)}*i + 1)^2 * (-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^{(2/3)} * (243*a^{10}*b^{15}*c^{15}*d^3 - 2916*a^{11}*b^{14}*c^{14}*d^4 + 15795*a^{12}*b^{13}*c^{13}*d^5 - 51030*a^{13}*b^{12}*c^{12}*d^6 + 109350*a^{14}*b^{11}*c^{11}*d^7 - 163296*a^{15}*b^{10}*c^{10}*d^8 + 173502*a^{16}*b^9*c^9*d^9 - 131220*a^{17}*b^8*c^8*d^{10} + 69255*a^{18}*b^7*c^7*d^{11} - 24300*a^{19}*b^6*c^6*d^{12} + 5103*a^{20}*b^5*c^5*d^{13} - 486*a^{21}*b^4*c^4*d^{14}))/4)) / 2 + 9*a^7*b^{14}*c^{11}*d^4 - 90*a^8*b^{13}*c^{10}*d^5 + 405*a^9*b^{12}*c^9*d^6 - 1071*a^{10}*b^{11}*c^8*d^7 + 1827*a^{11}*b^{10}*c^7*d^8 - 2079*a^{12}*b^9*c^6*d^9 + 1575*a^{13}*b^8*c^5*d^{10} - 765*a^{14}*b^7*c^4*d^{11} + 216*a^{15}*b^6*c^3*d^{12} - 27*a^{16}*b^5*c^2*d^{13} \\
& * (3^{(1/2)}*i + 1) * (-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^{(1/3)} \\
&) / 2 - b / ((a + b*x^3)^{(1/3)} * (a^2*d - a*b*c)) + \log(((a + b*x^3)^{(1/3)} * (27*a^7*b^{15}*c^{13}*d^3 - 297*a^8*b^{14}*c^{12}*d^4 + 1485*a^9*b^{13}*c^{11}*d^5 - 4455*a^{10}*b^{12}*c^{10}*d^6 + 8937*a^{11}*b^{11}*c^9*d^7 - 12663*a^{12}*b^{10}*c^8*d^8 + 13041*a^{13}*b^9*c^7*d^9 - 9855*a^{14}*b^8*c^6*d^{10} + 5400*a^{15}*b^7*c^5*d^{11} - 2052*a^{16}*b^6*c^4*d^{12} + 486*a^{17}*b^5*c^3*d^{13} - 54*a^{18}*b^4*c^2*d^{14}) - ((3^{(1/2)}*i)/2 - 1/2)^2 * (1/(27*a^4*c^3))^{(2/3)} * (243*a^{10}*b^{15}*c^{15}*d^3 - 2916*a^{11}*b^{14}*c^{14}*d^4 + 15795*a^{12}*b^{13}*c^{13}*d^5 - 51030*a^{13}*b^{12}*c^{12}*d^6 + 109350*a^{14}*b^{11}*c^{11}*d^7 - 163296*a^{15}*b^{10}*c^{10}*d^8 + 173502*a^{16}*b^9*c^9*d^9 - 131220*a^{17}*b^8*c^8*d^{10} + 69255*a^{18}*b^7*c^7*d^{11} - 24300*a^{19}*b^6*c^6*d^{12} + 5103*a^{20}*b^5*c^5*d^{13} - 486*a^{21}*b^4*c^4*d^{14}) * ((3^{(1/2)}*i)/2 - 1/2) * (1/(27*a^4*c^3))^{(1/3)} - 9*a^7*b^{14}*c^{11}*d^4 + 90*a^8*b^{13}*c^{10}*d^5 - 405*a^9*b^{12}*c^9*d^6 + 1071*a^{10}*b^{11}*c^8*d^7 - 1827*a^{11}*b^{10}*c^7*d^8 + 2079*a^{12}*b^9*c^6*d^9 - 1575*a^{13}*b^8*c^5*d^{10} + 765*a^{14}*b^7*c^4*d^{11} - 216*a^{15}*b^6*c^3*d^{12} + 27*a^{16}*b^5*c^2*d^{13} \\
& * ((3^{(1/2)}*i)/2 - 1/2) * (1/(27*a^4*c^3))^{(1/3)} - \log(((a + b*x^3)^{(1/3)} * (27*a^7*b^{15}*c^{13}*d^3 - 297*a^8*b^{14}*c^{12}*d^4 + 1485*a^9*b^{13}*c^{11}*d^5 - 4455*a^{10}*b^{12}*c^{10}*d^6 + 8937*a^{11}*b^{11}*c^9*d^7 - 12663*a^{12}*b^{10}*c^8*d^8 + 13041*a^{13}*b^9*c^7*d^9 - 9855*a^{14}*b^8*c^6*d^{10} + 5400*a^{15}*b^7*c^5*d^{11} - 2052*a^{16}*b^6*c^4*d^{12} + 486*a^{17}*b^5*c^3*d^{13} - 54*a^{18}*b^4*c^2*d^{14}) - ((3^{(1/2)}*i)/2 + 1/2)^2 * (1/(27*a^4*c^3))^{(2/3)} * (243*a^{10}*b^{15}*c^{15}*d^3 - 2916*a^{11}*b^{14}*c^{14}*d^4 + 15795*a^{12}*b^{13}*c^{13}*d^5 - 51030*a^{13}*b^{12}*c^{12}*d^6 + 109350*a^{14}*b^{11}*c^{11}*d^7 - 163296*a^{15}*b^{10}*c^{10}*d^8 + 173502*a^{16}*b^9*c^9*d^9 - 131220*a^{17}*b^8*c^8*d^{10} + 69255*a^{18}*b^7*c^7*d^{11} - 24300*a^{19}*b^6*c^6*d^{12} + 5103*a^{20}*b^5*c^5*d^{13} - 486*a^{21}*b^4*c^4*d^{14}) * ((3^{(1/2)}*i)/2 + 1/2) * (1/(27*a^4*c^3))^{(1/3)} + 9*a^7*b^{14}*c^{11}*d^4 -
\end{aligned}$$

$90*a^8*b^{13}*c^{10}*d^5 + 405*a^9*b^{12}*c^9*d^6 - 1071*a^{10}*b^{11}*c^8*d^7 + 1827*a^{11}*b^{10}*c^7*d^8 - 2079*a^{12}*b^9*c^6*d^9 + 1575*a^{13}*b^8*c^5*d^{10} - 765*a^{14}*b^7*c^4*d^{11} + 216*a^{15}*b^6*c^3*d^{12} - 27*a^{16}*b^5*c^2*d^{13})*((3^{(1/2)}*1i)/2 + 1/2)*(1/(27*a^4*c^3))^{(1/3)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+bx^3)^{\frac{4}{3}}(c+dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**3+a)**(4/3)/(d*x**3+c),x)

[Out] Integral(1/(x*(a + b*x**3)**(4/3)*(c + d*x**3)), x)

$$3.485 \quad \int \frac{1}{x^4(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal. Leaf size=357

$$\frac{(3ad + 4bc) \log\left(\sqrt[3]{a} - \sqrt[3]{a + bx^3}\right)}{6a^{7/3}c^2} - \frac{(3ad + 4bc) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}c^2} + \frac{\log(x)(3ad + 4bc)}{6a^{7/3}c^2} - \frac{3ad + 4bc}{3a^2c^2\sqrt[3]{a + bx^3}} - \frac{d^{7/3}}{6c}$$

Rubi [A] time = 0.39, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {446, 103, 156, 51, 55, 617, 204, 31, 56}

$$\frac{3ad + 4bc}{3a^2c^2\sqrt[3]{a + bx^3}} - \frac{(3ad + 4bc) \log\left(\sqrt[3]{a} - \sqrt[3]{a + bx^3}\right)}{6a^{7/3}c^2} - \frac{(3ad + 4bc) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}c^2} + \frac{\log(x)(3ad + 4bc)}{6a^{7/3}c^2} - \frac{d^2}{c^2\sqrt[3]{a + bx^3}(bc - ad)} - \frac{d^{7/3} \log(c + dx^3)}{6c^2(bc - ad)^{4/3}} + \frac{d^{7/3} \log\left(\sqrt[3]{bc - ad} + \sqrt[3]{d}\sqrt[3]{a + bx^3}\right)}{2c^2(bc - ad)^{4/3}} + \frac{d^{7/3} \tan^{-1}\left(\frac{1 + \sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}c^2(bc - ad)^{4/3}} - \frac{1}{3acx^3\sqrt[3]{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] $-(d^2/(c^2*(b*c - a*d)*(a + b*x^3)^{1/3})) - (4*b*c + 3*a*d)/(3*a^2*c^2*(a + b*x^3)^{1/3}) - 1/(3*a*c*x^3*(a + b*x^3)^{1/3}) - ((4*b*c + 3*a*d)*\text{ArcTan}[(a^{1/3} + 2*(a + b*x^3)^{1/3})/(\text{Sqrt}[3]*a^{1/3})])/(3*\text{Sqrt}[3]*a^{7/3}*c^2) + (d^{7/3}*\text{ArcTan}[(1 - (2*d^{1/3}*(a + b*x^3)^{1/3})/(b*c - a*d)^{1/3})/\text{Sqrt}[3]])/(\text{Sqrt}[3]*c^2*(b*c - a*d)^{4/3}) + ((4*b*c + 3*a*d)*\text{Log}[x])/(6*a^{7/3}*c^2) - (d^{7/3}*\text{Log}[c + d*x^3])/(6*c^2*(b*c - a*d)^{4/3}) - ((4*b*c + 3*a*d)*\text{Log}[a^{1/3} - (a + b*x^3)^{1/3}])/(6*a^{7/3}*c^2) + (d^{7/3}*\text{Log}[(b*c - a*d)^{1/3} + d^{1/3}*(a + b*x^3)^{1/3}])/(2*c^2*(b*c - a*d)^{4/3})$

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 51

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])) /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])) /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[Rt[-b, 2]*x]/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_)*((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^4 (a + bx^3)^{4/3} (c + dx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2 (a + bx)^{4/3} (c + dx)} dx, x, x^3 \right) \\
 &= -\frac{1}{3acx^3 \sqrt[3]{a + bx^3}} - \frac{\text{Subst} \left(\int \frac{\frac{1}{3}(4bc+3ad) + \frac{4bdx}{3}}{x(a+bx)^{4/3}(c+dx)} dx, x, x^3 \right)}{3ac} \\
 &= -\frac{1}{3acx^3 \sqrt[3]{a + bx^3}} + \frac{d^2 \text{Subst} \left(\int \frac{1}{(a+bx)^{4/3}(c+dx)} dx, x, x^3 \right)}{3c^2} - \frac{(4bc + 3ad) \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx^3}} dx, x, x^3 \right)}{9ac^2} \\
 &= -\frac{d^2}{c^2(bc - ad) \sqrt[3]{a + bx^3}} - \frac{4bc + 3ad}{3a^2c^2 \sqrt[3]{a + bx^3}} - \frac{1}{3acx^3 \sqrt[3]{a + bx^3}} - \frac{d^3 \text{Subst} \left(\int \frac{1}{\sqrt[3]{a+bx^3}} dx, x, x^3 \right)}{3c^2(bc - ad)} \\
 &= -\frac{d^2}{c^2(bc - ad) \sqrt[3]{a + bx^3}} - \frac{4bc + 3ad}{3a^2c^2 \sqrt[3]{a + bx^3}} - \frac{1}{3acx^3 \sqrt[3]{a + bx^3}} + \frac{(4bc + 3ad) \log\left(\frac{c + dx^3}{\sqrt[3]{a + bx^3}}\right)}{6a^{7/3}c^2} \\
 &= -\frac{d^2}{c^2(bc - ad) \sqrt[3]{a + bx^3}} - \frac{4bc + 3ad}{3a^2c^2 \sqrt[3]{a + bx^3}} - \frac{1}{3acx^3 \sqrt[3]{a + bx^3}} + \frac{(4bc + 3ad) \log\left(\frac{c + dx^3}{\sqrt[3]{a + bx^3}}\right)}{6a^{7/3}c^2} \\
 &= -\frac{d^2}{c^2(bc - ad) \sqrt[3]{a + bx^3}} - \frac{4bc + 3ad}{3a^2c^2 \sqrt[3]{a + bx^3}} - \frac{1}{3acx^3 \sqrt[3]{a + bx^3}} - \frac{(4bc + 3ad) \tan^{-1}\left(\frac{c + dx^3}{\sqrt[3]{a + bx^3}}\right)}{3\sqrt[3]{3}a^{7/3}c^2}
 \end{aligned}$$

Mathematica [C] time = 0.05, size = 117, normalized size = 0.33

$$\frac{3a^2d^2x^3 {}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{d(bx^3+a)}{ad-bc}\right) + (bc - ad) \left(x^3(3ad + 4bc) {}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{bx^3}{a} + 1\right) + ac\right)}{3a^2c^2x^3 \sqrt[3]{a + bx^3} (ad - bc)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] (3*a^2*d^2*x^3*Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)] + (b*c - a*d)*(a*c + (4*b*c + 3*a*d)*x^3*Hypergeometric2F1[-1/3, 1, 2/3, 1 + (b*x^3)/a]))/(3*a^2*c^2*(-(b*c) + a*d)*x^3*(a + b*x^3)^(1/3))

IntegrateAlgebraic [A] time = 0.99, size = 425, normalized size = 1.19

$$\frac{(-3ad - 4bc) \log(\sqrt[3]{a + bx^3} - \sqrt[3]{a})}{9a^{2/3}c^2} + \frac{(3ad + 4bc) \log(a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^3} + (a + bx^3)^{2/3})}{18a^{2/3}c^2} - \frac{(3ad + 4bc) \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3} + \sqrt[3]{a}}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}a^{2/3}c^2} + \frac{-a^2d + abc - abdx^3 + 4b^2cx^3}{3a^2c^3\sqrt[3]{a + bx^3}(ad - bc)} + \frac{a^{2/3} \log(\sqrt[3]{bc - ad} + \sqrt[3]{d} \sqrt[3]{a + bx^3})}{3c^2(bc - ad)^{4/3}} - \frac{a^{2/3} \log(-\sqrt[3]{d} \sqrt[3]{a + bx^3} \sqrt[3]{bc - ad} + (bc - ad)^{2/3} + a^{2/3}(a + bx^3)^{2/3})}{6c^2(bc - ad)^{4/3}} + \frac{a^{2/3} \tan^{-1}\left(\frac{1}{\sqrt[3]{a}} - \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a} \sqrt[3]{bc - ad}}\right)}{\sqrt[3]{a}c^2(bc - ad)^{4/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] (a*b*c - a^2*d + 4*b^2*c*x^3 - a*b*d*x^3)/(3*a^2*c*(-(b*c) + a*d)*x^3*(a + b*x^3)^(1/3)) - ((4*b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3)])/(3*Sqrt[3]*a^(7/3)*c^2) + (d^(7/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]/(Sqrt[3]*c^2*(b*c - a*d)^(4/3)) + ((-4*b*c - 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)]/(9*a^(7/3)*c^2) + (d^(7/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(3*c^2*(b*c - a*d)^(4/3)) + ((4*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(18*a^(7/3)*c^2) - (d^(7/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/(6*c^2*(b*c - a*d)^(4/3))

fricas [B] time = 1.39, size = 1386, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] [1/18*(3*sqrt(1/3)*((4*a*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2*b^2*c^2 - a^3*b*c*d - 3*a^4*d^2)*x^3)*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3 - 6*sqrt(3)*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) + ((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 3*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) - 6*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log

$$\begin{aligned} & ((b*c - a*d)*(-d/(b*c - a*d))^{(2/3)} + (b*x^3 + a)^{(1/3)*d} - 6*(a^2*b*c^2 - \\ & a^3*c*d + (4*a*b^2*c^2 - a^2*b*c*d)*x^3)*(b*x^3 + a)^{(2/3)})/((a^3*b^2*c^3 \\ & - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/18*(6*sqrt(1/3)*((4*a \\ & *b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2*b^2*c^2 - a^3*b*c*d - 3* \\ & a^4*d^2)*x^3)*sqrt(-(-a)^{(1/3)}/a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^{(1/3)} - (\\ & -a)^{(1/3}))*sqrt(-(-a)^{(1/3)}/a)) + 6*sqrt(3)*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(\\ & -d/(b*c - a*d))^{(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^{(1/3)*(-d/(b*c - a*d))} \\ & ^{(1/3)} + 1/3*sqrt(3)) - ((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b \\ & ^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^{(2/3)*log((b*x^3 + a)^{(2/3)} - (b* \\ & x^3 + a)^{(1/3)*(-a)^{(1/3)} + (-a)^{(2/3)})} + 2*((4*b^3*c^2 - a*b^2*c*d - 3*a^2 \\ & *b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^{(2/3)*log((b* \\ & x^3 + a)^{(1/3)} + (-a)^{(1/3)})} - 3*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a \\ & *d))^{(1/3)*log(-(b*x^3 + a)^{(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^{(2/3)} + (b*x \\ & ^3 + a)^{(2/3)*d} - (b*c - a*d)*(-d/(b*c - a*d))^{(1/3)})} + 6*(a^3*b*d^2*x^6 + \\ & a^4*d^2*x^3)*(-d/(b*c - a*d))^{(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^{(2/3)} \\ & + (b*x^3 + a)^{(1/3)*d} + 6*(a^2*b*c^2 - a^3*c*d + (4*a*b^2*c^2 - a^2*b*c*d) \\ & *x^3)*(b*x^3 + a)^{(2/3)})/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^ \\ & 5*c^2*d)*x^3)] \end{aligned}$$

giac [A] time = 0.79, size = 486, normalized size = 1.36

$$\frac{d^2 \left(\frac{b*c - a*d}{d} \right)^{\frac{2}{3}} \log \left(\left| \frac{(b*x^3 + a)^{\frac{1}{3}} - (-a)^{\frac{1}{3}}}{d} \right| \right)}{3 \left(b^2*c^4 - 2*abc^2*d + a^2*d^2 \right)} + \frac{(-b*c*d^2 + a*d^3)^{\frac{2}{3}} \arctan \left(\frac{\sqrt{3} \left((b*x^3 + a)^{\frac{1}{3}} - (-a)^{\frac{1}{3}} \right)}{3 \left(b^2*c^4 - 2*abc^2*d + a^2*d^2 \right)} \right)}{\sqrt{3} \sqrt{b^2*c^4 - 2*abc^2*d + a^2*d^2}} + \frac{(-b*c*d^2 + a*d^3)^{\frac{2}{3}} \log \left(\left| \frac{(b*x^3 + a)^{\frac{2}{3}} + (b*x^3 + a)^{\frac{1}{3}} \left(\frac{b*c - a*d}{d} \right)^{\frac{2}{3}} + \left(\frac{b*c - a*d}{d} \right)^{\frac{4}{3}}}{3 \left(b^2*c^4 - 2*abc^2*d + a^2*d^2 \right)} \right| \right)}{6 \left(b^2*c^4 - 2*abc^2*d + a^2*d^2 \right)} - \frac{4 \left(b^2*c^4 - 2*abc^2*d - (b*x^3 + a) \left(\frac{b*c - a*d}{d} \right)^{\frac{2}{3}} - (b*x^3 + a) \left(\frac{b*c - a*d}{d} \right)^{\frac{4}{3}} \right)}{3 \left(b^2*c^4 - 2*abc^2*d - (b*x^3 + a) \left(\frac{b*c - a*d}{d} \right)^{\frac{2}{3}} - (b*x^3 + a) \left(\frac{b*c - a*d}{d} \right)^{\frac{4}{3}} \right)} + \frac{\sqrt{3} \left(4*b*c + 3*a*d \right) \arctan \left(\frac{\sqrt{3} \left((b*x^3 + a)^{\frac{1}{3}} - (-a)^{\frac{1}{3}} \right)}{3 \left(b^2*c^4 - 2*abc^2*d + a^2*d^2 \right)} \right)}{9 \sqrt{b^2*c^4 - 2*abc^2*d + a^2*d^2}} + \frac{\left(4*b^3*c^2 + 3*a^2*d \right) \log \left(\left| \frac{(b*x^3 + a)^{\frac{2}{3}} - (-a)^{\frac{2}{3}}}{d} \right| \right)}{9 \sqrt{b^2*c^4 - 2*abc^2*d + a^2*d^2}} + \frac{\left(4*b^3*c^2 + 3*a^2*d \right) \log \left(\left| \frac{(b*x^3 + a)^{\frac{2}{3}} + (b*x^3 + a)^{\frac{1}{3}} \left(\frac{b*c - a*d}{d} \right)^{\frac{2}{3}} + \left(\frac{b*c - a*d}{d} \right)^{\frac{4}{3}}}{d} \right| \right)}{18 \sqrt{b^2*c^4 - 2*abc^2*d + a^2*d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] 1/3*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-a)^(1/3)/d)^(1/3)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) + (-b*c*d^2 + a*d^3)^(2/3)*d*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-a)^(1/3)/d)^(1/3))/(-b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^2*d^2) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-a)^(1/3)/d)^(1/3) + (-a)^(2/3)/d)^(2/3))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*(4*(b*x^3 + a)*b^2*c - 3*a*b^2*c - (b*x^3 + a)*a*b*d)/((a^2*b*c^2 - a^3*c*d)*((b*x^3 + a)^(4/3) - (b*x^3 + a)^(1/3)*a) - 1/9*sqrt(3)*(4*b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)))/(a^(7/3)*c^2) - 1/9*(4*a^(1/3)*b*c + 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(8/3)*c^2) + 1/18*(4*a^(2/3)*b*c + 3*a^(5/3)*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^3*c^2)

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}} (dx^3 + c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)`

[Out] `int(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^4), x)`

mupad [B] time = 6.39, size = 5875, normalized size = 16.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x)`

[Out] `log((d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((a + b*x^3)^(1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + (d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3) - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^15*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 15529887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 7801029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*d^16) - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^13*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 - 1889568*a^18*b^9*c^8*d^14 + 10156428*a^19*b^8*c^7*d^15 - 3254256*a^20*b^7*c^6*d^16 + 10156428*a^21*b^6*c^5*d^17 - 14781933*a^22*b^5*c^4*d^18 + 4920750*a^23*b^4*c^3*d^19 - 15713595*a^24*b^3*c^2*d^20 + 13404123*a^25*b^2*c*d^21 - 10156428*a^26*b*c*d^22 + 10156428*a^27*b*d^23 - 10156428*a^28*d^24))`

$$\begin{aligned}
& 17*b^{10}*c^9*d^{13} - 551124*a^{18}*b^9*c^8*d^{14} + 224532*a^{19}*b^8*c^7*d^{15} + 10 \\
& 7892*a^{20}*b^7*c^6*d^{16} - 113724*a^{21}*b^6*c^5*d^{17} + 26244*a^{22}*b^5*c^4*d^{18} \\
&))*(d^7/(27*b^4*c^{10} + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8 \\
& *d^2 - 108*a*b^3*c^9*d))^{(1/3)} + (b^2/(a^2*d - a*b*c) + (b*(a + b*x^3)*(a*d \\
& - 4*b*c))/(3*a^2*c*(a*d - b*c)))/(a*(a + b*x^3)^{(1/3)} - (a + b*x^3)^{(4/3)}) \\
& + \log((-27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729 \\
& *a^7*c^6))^{(2/3)}*(419904*a^{13}*b^{17}*c^{20}*d^4 - ((a + b*x^3)^{(1/3)}*(8975448*a \\
& ^{15}*b^{16}*c^{21}*d^4 - 944784*a^{14}*b^{17}*c^{22}*d^3 - 36905625*a^{16}*b^{15}*c^{20}*d^5 \\
& + 83790531*a^{17}*b^{14}*c^{19}*d^6 - 107173935*a^{18}*b^{13}*c^{18}*d^7 + 56509893*a^{19} \\
& *b^{12}*c^{17}*d^8 + 42338133*a^{20}*b^{11}*c^{16}*d^9 - 93710763*a^{21}*b^{10}*c^{15}*d^{10} \\
& + 55092717*a^{22}*b^9*c^{14}*d^{11} + 12105045*a^{23}*b^8*c^{13}*d^{12} - 38736144*a \\
& ^{24}*b^7*c^{12}*d^{13} + 25745364*a^{25}*b^6*c^{11}*d^{14} - 8148762*a^{26}*b^5*c^{10}*d^{15} \\
& + 1062882*a^{27}*b^4*c^9*d^{16}) + (-27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2 \\
& *d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^{(2/3)}*(4782969*a^{19}*b^{15}*c^{24}*d^3 - 57 \\
& 395628*a^{20}*b^{14}*c^{23}*d^4 + 310892985*a^{21}*b^{13}*c^{22}*d^5 - 1004423490*a^{22} \\
& *b^{12}*c^{21}*d^6 + 2152336050*a^{23}*b^{11}*c^{20}*d^7 - 3214155168*a^{24}*b^{10}*c^{19}*d \\
& ^8 + 3415039866*a^{25}*b^9*c^{18}*d^9 - 2582803260*a^{26}*b^8*c^{17}*d^{10} + 1363146 \\
& 165*a^{27}*b^7*c^{16}*d^{11} - 478296900*a^{28}*b^6*c^{15}*d^{12} + 100442349*a^{29}*b^5* \\
& c^{14}*d^{13} - 9565938*a^{30}*b^4*c^{13}*d^{14}))*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a \\
& *b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^{(1/3)} - 3254256*a^{14}*b^{16}*c^{19} \\
& *d^5 + 10156428*a^{15}*b^{15}*c^{18}*d^6 - 14781933*a^{16}*b^{14}*c^{17}*d^7 + 4920750* \\
& a^{17}*b^{13}*c^{16}*d^8 + 15529887*a^{18}*b^{12}*c^{15}*d^9 - 22182741*a^{19}*b^{11}*c^{14} \\
& *d^{10} + 5412825*a^{20}*b^{10}*c^{13}*d^{11} + 13404123*a^{21}*b^9*c^{12}*d^{12} - 15713595 \\
& *a^{22}*b^8*c^{11}*d^{13} + 7801029*a^{23}*b^7*c^{10}*d^{14} - 1889568*a^{24}*b^6*c^9*d^{15} \\
& + 177147*a^{25}*b^5*c^8*d^{16}) - (a + b*x^3)^{(1/3)}*(256608*a^{14}*b^{13}*c^{12}*d^{10} \\
& - 46656*a^{13}*b^{14}*c^{13}*d^9 - 516132*a^{15}*b^{12}*c^{11}*d^{11} + 347004*a^{16}*b^{11} \\
& *c^{10}*d^{12} + 265356*a^{17}*b^{10}*c^9*d^{13} - 551124*a^{18}*b^9*c^8*d^{14} + 22453 \\
& 2*a^{19}*b^8*c^7*d^{15} + 107892*a^{20}*b^7*c^6*d^{16} - 113724*a^{21}*b^6*c^5*d^{17} + \\
& 26244*a^{22}*b^5*c^4*d^{18}))*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 1 \\
& 08*a^2*b*c*d^2)/(729*a^7*c^6))^{(1/3)} + (\log(((3^{(1/2)}*i - 1)^2*(d^7/(27*b^ \\
& 4*c^{10} + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b \\
& ^3*c^9*d))^{(2/3)}*(419904*a^{13}*b^{17}*c^{20}*d^4 - ((3^{(1/2)}*i - 1)*(d^7/(27*b^ \\
& 4*c^{10} + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b \\
& ^3*c^9*d))^{(1/3)}*((a + b*x^3)^{(1/3)}*(8975448*a^{15}*b^{16}*c^{21}*d^4 - 944784*a^{14} \\
& *b^{17}*c^{22}*d^3 - 36905625*a^{16}*b^{15}*c^{20}*d^5 + 83790531*a^{17}*b^{14}*c^{19}*d^6 \\
& - 107173935*a^{18}*b^{13}*c^{18}*d^7 + 56509893*a^{19}*b^{12}*c^{17}*d^8 + 42338133*a \\
& ^{20}*b^{11}*c^{16}*d^9 - 93710763*a^{21}*b^{10}*c^{15}*d^{10} + 55092717*a^{22}*b^9*c^{14} \\
& *d^{11} + 12105045*a^{23}*b^8*c^{13}*d^{12} - 38736144*a^{24}*b^7*c^{12}*d^{13} + 25745364* \\
& a^{25}*b^6*c^{11}*d^{14} - 8148762*a^{26}*b^5*c^{10}*d^{15} + 1062882*a^{27}*b^4*c^9*d^{16} \\
&) + ((3^{(1/2)}*i - 1)^2*(d^7/(27*b^4*c^{10} + 27*a^4*c^6*d^4 - 108*a^3*b*c^7* \\
& d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^{(2/3)}*(4782969*a^{19}*b^{15}*c^{24} \\
& *d^3 - 57395628*a^{20}*b^{14}*c^{23}*d^4 + 310892985*a^{21}*b^{13}*c^{22}*d^5 - 1004423 \\
& 490*a^{22}*b^{12}*c^{21}*d^6 + 2152336050*a^{23}*b^{11}*c^{20}*d^7 - 3214155168*a^{24}*b^{10} \\
& *c^{19}*d^8 + 3415039866*a^{25}*b^9*c^{18}*d^9 - 2582803260*a^{26}*b^8*c^{17}*d^{10} \\
& + 1363146165*a^{27}*b^7*c^{16}*d^{11} - 478296900*a^{28}*b^6*c^{15}*d^{12} + 100442349*
\end{aligned}$$

$$\begin{aligned}
& a^{29}b^5c^{14}d^{13} - 9565938a^{30}b^4c^{13}d^{14})/4)/2 - 3254256a^{14}b^{16} \\
& *c^{19}d^5 + 10156428a^{15}b^{15}c^{18}d^6 - 14781933a^{16}b^{14}c^{17}d^7 + 492 \\
& 0750a^{17}b^{13}c^{16}d^8 + 15529887a^{18}b^{12}c^{15}d^9 - 22182741a^{19}b^{11}c^{14}d^{10} + 5412825a^{20}b^{10}c^{13}d^{11} + 13404123a^{21}b^9c^{12}d^{12} - 157 \\
& 13595a^{22}b^8c^{11}d^{13} + 7801029a^{23}b^7c^{10}d^{14} - 1889568a^{24}b^6c^9d^{15} + 177147a^{25}b^5c^8d^{16})/4 - (a + b*x^3)^{(1/3)}*(256608a^{14}b^{13} \\
& *c^{12}d^{10} - 46656a^{13}b^{14}c^{13}d^9 - 516132a^{15}b^{12}c^{11}d^{11} + 347004 \\
& *a^{16}b^{11}c^{10}d^{12} + 265356a^{17}b^{10}c^9d^{13} - 551124a^{18}b^9c^8d^{14} \\
& + 224532a^{19}b^8c^7d^{15} + 107892a^{20}b^7c^6d^{16} - 113724a^{21}b^6c^5d^{17} + 26244a^{22}b^5c^4d^{18}))*(3^{(1/2)}*i - 1)*(d^7/(27*b^4*c^{10} + 27* \\
& a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^{(1/3)})/2 - (\log(((3^{(1/2)}*i + 1)^2*(d^7/(27*b^4*c^{10} + 27*a^4*c^6*d^4 - 108 \\
& *a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^{(2/3)}*((3^{(1/2)}*i \\
& + 1)*(d^7/(27*b^4*c^{10} + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2 \\
& *c^8*d^2 - 108*a*b^3*c^9*d))^{(1/3)}*((a + b*x^3)^{(1/3)}*(8975448a^{15}b^{16}c^{21}d^4 - 944784a^{14}b^{17}c^{22}d^3 - 36905625a^{16}b^{15}c^{20}d^5 + 83790531 \\
& *a^{17}b^{14}c^{19}d^6 - 107173935a^{18}b^{13}c^{18}d^7 + 56509893a^{19}b^{12}c^{17}d^8 + 42338133a^{20}b^{11}c^{16}d^9 - 93710763a^{21}b^{10}c^{15}d^{10} + 550927 \\
& 17*a^{22}b^9c^{14}d^{11} + 12105045a^{23}b^8c^{13}d^{12} - 38736144a^{24}b^7c^{12}d^{13} + 25745364a^{25}b^6c^{11}d^{14} - 8148762a^{26}b^5c^{10}d^{15} + 1062882 \\
& *a^{27}b^4c^9d^{16}) + ((3^{(1/2)}*i + 1)^2*(d^7/(27*b^4*c^{10} + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^{(2/3)}*(4782 \\
& 969*a^{19}b^{15}c^{24}d^3 - 57395628a^{20}b^{14}c^{23}d^4 + 310892985a^{21}b^{13}c^{22}d^5 - 1004423490a^{22}b^{12}c^{21}d^6 + 2152336050a^{23}b^{11}c^{20}d^7 - \\
& 3214155168a^{24}b^{10}c^{19}d^8 + 3415039866a^{25}b^9c^{18}d^9 - 2582803260a^{26}b^8c^{17}d^{10} + 1363146165a^{27}b^7c^{16}d^{11} - 478296900a^{28}b^6c^{15} \\
& *d^{12} + 100442349*a^{29}b^5c^{14}d^{13} - 9565938a^{30}b^4c^{13}d^{14}))/4)/2 + \\
& 419904a^{13}b^{17}c^{20}d^4 - 3254256a^{14}b^{16}c^{19}d^5 + 10156428a^{15}b^{15} \\
& *c^{18}d^6 - 14781933a^{16}b^{14}c^{17}d^7 + 4920750a^{17}b^{13}c^{16}d^8 + 155 \\
& 29887a^{18}b^{12}c^{15}d^9 - 22182741a^{19}b^{11}c^{14}d^{10} + 5412825a^{20}b^{10} \\
& *c^{13}d^{11} + 13404123a^{21}b^9c^{12}d^{12} - 15713595a^{22}b^8c^{11}d^{13} + 78 \\
& 01029a^{23}b^7c^{10}d^{14} - 1889568a^{24}b^6c^9d^{15} + 177147a^{25}b^5c^8d^{16})/4 - (a + b*x^3)^{(1/3)}*(256608a^{14}b^{13}c^{12}d^{10} - 46656a^{13}b^{14}c^{13}d^9 - 516132a^{15}b^{12}c^{11}d^{11} + 347004a^{16}b^{11}c^{10}d^{12} + 265356 \\
& *a^{17}b^{10}c^9d^{13} - 551124a^{18}b^9c^8d^{14} + 224532a^{19}b^8c^7d^{15} + \\
& 107892a^{20}b^7c^6d^{16} - 113724a^{21}b^6c^5d^{17} + 26244a^{22}b^5c^4d^{18}))*(3^{(1/2)}*i + 1)*(d^7/(27*b^4*c^{10} + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^{(1/3)})/2 - \log(((3^{(1/2)}*i)/2 \\
& + 1/2)^2*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(\\
& 729*a^7*c^6))^{(2/3)}*((3^{(1/2)}*i)/2 + 1/2)*((a + b*x^3)^{(1/3)}*(8975448a^{15}b^{16}c^{21}d^4 - 944784a^{14}b^{17}c^{22}d^3 - 36905625a^{16}b^{15}c^{20}d^5 + \\
& 83790531a^{17}b^{14}c^{19}d^6 - 107173935a^{18}b^{13}c^{18}d^7 + 56509893a^{19} \\
& *b^{12}c^{17}d^8 + 42338133a^{20}b^{11}c^{16}d^9 - 93710763a^{21}b^{10}c^{15}d^{10} \\
& + 55092717a^{22}b^9c^{14}d^{11} + 12105045a^{23}b^8c^{13}d^{12} - 38736144a^{24} \\
& *b^7c^{12}d^{13} + 25745364a^{25}b^6c^{11}d^{14} - 8148762a^{26}b^5c^{10}d^{15}
\end{aligned}$$

$$\begin{aligned}
& + 1062882*a^{27}*b^4*c^9*d^{16}) + ((3^{(1/2)}*i)/2 + 1/2)^2*(-(27*a^3*d^3 + 64* \\
& b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^{(2/3)}*(4782969* \\
& a^{19}*b^{15}*c^{24}*d^3 - 57395628*a^{20}*b^{14}*c^{23}*d^4 + 310892985*a^{21}*b^{13}*c^{22} \\
& *d^5 - 1004423490*a^{22}*b^{12}*c^{21}*d^6 + 2152336050*a^{23}*b^{11}*c^{20}*d^7 - 3214 \\
& 155168*a^{24}*b^{10}*c^{19}*d^8 + 3415039866*a^{25}*b^9*c^{18}*d^9 - 2582803260*a^{26}* \\
& b^8*c^{17}*d^{10} + 1363146165*a^{27}*b^7*c^{16}*d^{11} - 478296900*a^{28}*b^6*c^{15}*d^{12} \\
& + 100442349*a^{29}*b^5*c^{14}*d^{13} - 9565938*a^{30}*b^4*c^{13}*d^{14}))*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^{(1/3)} + \\
& 419904*a^{13}*b^{17}*c^{20}*d^4 - 3254256*a^{14}*b^{16}*c^{19}*d^5 + 10156428*a^{15}*b^{15}*c^{18}*d^6 - 14781933*a^{16}*b^{14}*c^{17}*d^7 + 4920750*a^{17}*b^{13}*c^{16}*d^8 + 155 \\
& 29887*a^{18}*b^{12}*c^{15}*d^9 - 22182741*a^{19}*b^{11}*c^{14}*d^{10} + 5412825*a^{20}*b^{10} \\
& *c^{13}*d^{11} + 13404123*a^{21}*b^9*c^{12}*d^{12} - 15713595*a^{22}*b^8*c^{11}*d^{13} + 78 \\
& 01029*a^{23}*b^7*c^{10}*d^{14} - 1889568*a^{24}*b^6*c^9*d^{15} + 177147*a^{25}*b^5*c^8* \\
& d^{16}) - (a + b*x^3)^{(1/3)}*(256608*a^{14}*b^{13}*c^{12}*d^{10} - 46656*a^{13}*b^{14}*c^{11} \\
& *d^9 - 516132*a^{15}*b^{12}*c^{11}*d^{11} + 347004*a^{16}*b^{11}*c^{10}*d^{12} + 265356*a^{17} \\
& *b^{10}*c^9*d^{13} - 551124*a^{18}*b^9*c^8*d^{14} + 224532*a^{19}*b^8*c^7*d^{15} + 10 \\
& 7892*a^{20}*b^7*c^6*d^{16} - 113724*a^{21}*b^6*c^5*d^{17} + 26244*a^{22}*b^5*c^4*d^{18} \\
&))*((3^{(1/2)}*i)/2 + 1/2)*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 10 \\
& 8*a^2*b*c*d^2)/(729*a^7*c^6))^{(1/3)} + \log(((3^{(1/2)}*i)/2 - 1/2)^2*(-(27*a^3 \\
& *d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^{(2/3)} \\
&)*(419904*a^{13}*b^{17}*c^{20}*d^4 - ((3^{(1/2)}*i)/2 - 1/2)*((a + b*x^3)^{(1/3)}*(8 \\
& 975448*a^{15}*b^{16}*c^{21}*d^4 - 944784*a^{14}*b^{17}*c^{22}*d^3 - 36905625*a^{16}*b^{15}* \\
& c^{20}*d^5 + 83790531*a^{17}*b^{14}*c^{19}*d^6 - 107173935*a^{18}*b^{13}*c^{18}*d^7 + 565 \\
& 09893*a^{19}*b^{12}*c^{17}*d^8 + 42338133*a^{20}*b^{11}*c^{16}*d^9 - 93710763*a^{21}*b^{10} \\
& *c^{15}*d^{10} + 55092717*a^{22}*b^9*c^{14}*d^{11} + 12105045*a^{23}*b^8*c^{13}*d^{12} - 38 \\
& 736144*a^{24}*b^7*c^{12}*d^{13} + 25745364*a^{25}*b^6*c^{11}*d^{14} - 8148762*a^{26}*b^5* \\
& c^{10}*d^{15} + 1062882*a^{27}*b^4*c^9*d^{16}) + ((3^{(1/2)}*i)/2 - 1/2)^2*(-(27*a^3 \\
& *d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^{(2/3)} \\
& *(4782969*a^{19}*b^{15}*c^{24}*d^3 - 57395628*a^{20}*b^{14}*c^{23}*d^4 + 310892985*a^{21} \\
& *b^{13}*c^{22}*d^5 - 1004423490*a^{22}*b^{12}*c^{21}*d^6 + 2152336050*a^{23}*b^{11}*c^{20}* \\
& d^7 - 3214155168*a^{24}*b^{10}*c^{19}*d^8 + 3415039866*a^{25}*b^9*c^{18}*d^9 - 258280 \\
& 3260*a^{26}*b^8*c^{17}*d^{10} + 1363146165*a^{27}*b^7*c^{16}*d^{11} - 478296900*a^{28}*b^6* \\
& c^{15}*d^{12} + 100442349*a^{29}*b^5*c^{14}*d^{13} - 9565938*a^{30}*b^4*c^{13}*d^{14}))*(- \\
& (27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6) \\
&))^{(1/3)} - 3254256*a^{14}*b^{16}*c^{19}*d^5 + 10156428*a^{15}*b^{15}*c^{18}*d^6 - 14781 \\
& 933*a^{16}*b^{14}*c^{17}*d^7 + 4920750*a^{17}*b^{13}*c^{16}*d^8 + 15529887*a^{18}*b^{12}*c^{15} \\
& *d^9 - 22182741*a^{19}*b^{11}*c^{14}*d^{10} + 5412825*a^{20}*b^{10}*c^{13}*d^{11} + 13404 \\
& 123*a^{21}*b^9*c^{12}*d^{12} - 15713595*a^{22}*b^8*c^{11}*d^{13} + 7801029*a^{23}*b^7*c^{10} \\
& *d^{14} - 1889568*a^{24}*b^6*c^9*d^{15} + 177147*a^{25}*b^5*c^8*d^{16}) - (a + b*x^3) \\
&)^{(1/3)}*(256608*a^{14}*b^{13}*c^{12}*d^{10} - 46656*a^{13}*b^{14}*c^{13}*d^9 - 516132*a^{15} \\
& *b^{12}*c^{11}*d^{11} + 347004*a^{16}*b^{11}*c^{10}*d^{12} + 265356*a^{17}*b^{10}*c^9*d^{13} - \\
& 551124*a^{18}*b^9*c^8*d^{14} + 224532*a^{19}*b^8*c^7*d^{15} + 107892*a^{20}*b^7*c^6* \\
& d^{16} - 113724*a^{21}*b^6*c^5*d^{17} + 26244*a^{22}*b^5*c^4*d^{18}))*((3^{(1/2)}*i)/2 \\
& - 1/2)*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(72 \\
& 9*a^7*c^6))^{(1/3)}
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + bx^3)^{\frac{4}{3}} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**3+a)**(4/3)/(d*x**3+c),x)

[Out] Integral(1/(x**4*(a + b*x**3)**(4/3)*(c + d*x**3)), x)

$$3.486 \quad \int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal. Leaf size=322

$$\frac{(4ad + 3bc) \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{bx^3}\right)}{6b^{7/3}d^2} - \frac{(4ad + 3bc) \tan^{-1}\left(\frac{\frac{2\sqrt[3]{bx^3}}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}d^2} + \frac{x(a+bx^3)^{2/3}(bc-4ad)}{3b^2d(bc-ad)} + \frac{c^{7/3} \log(c+dx^3)}{6d^2(bc-ad)^{4/3}}$$

Rubi [C] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 0.21, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{x^{10} \sqrt[3]{\frac{bx^3}{a}} + 1 F_1\left(\frac{10}{3}; \frac{4}{3}, 1; \frac{13}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{10ac \sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Int[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] (x^10*(1 + (b*x^3)/a)^(1/3)*AppellF1[10/3, 4/3, 1, 13/3, -((b*x^3)/a), -((d*x^3)/c)]/(10*a*c*(a + b*x^3)^(1/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^9}{(a + bx^3)^{4/3} (c + dx^3)} dx = \frac{\sqrt[3]{1 + \frac{bx^3}{a}} \int \frac{x^9}{\left(1 + \frac{bx^3}{a}\right)^{4/3} (c + dx^3)} dx}{a \sqrt[3]{a + bx^3}}$$

$$= \frac{x^{10} \sqrt[3]{1 + \frac{bx^3}{a}} F_1\left(\frac{10}{3}; \frac{4}{3}, 1; \frac{13}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{10ac \sqrt[3]{a + bx^3}}$$

Mathematica [C] time = 1.08, size = 504, normalized size = 1.57

$$\frac{3x^4 \sqrt{\frac{c}{a}} + 1 \sqrt{c-a} (-4d^2 c^2 + abd + 3b^2 d^2) F_1\left(\frac{10}{3}; \frac{4}{3}, 1; \frac{13}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) + 2 \left(-4d^2 \sqrt{c} \sqrt{a+bx^3} \log\left(\frac{\sqrt{c} \sqrt{a+bx^3}}{\sqrt{a+bx^3}} + \frac{d \sqrt{c-a} x^3}{(a+bx^3)^2} + c^2\right) + 24d^2 dx \sqrt{c-a} - 6d^2 c x^4 \sqrt{c-a} + ab d^2 x^4 \sqrt{c-a} + a b^2 x^4 \sqrt{c-a} \log\left(\frac{\sqrt{c} \sqrt{a+bx^3}}{\sqrt{a+bx^3}} + \frac{d \sqrt{c-a} x^3}{(a+bx^3)^2} + c^2\right) + 6ab d^2 x^4 \sqrt{c-a} + 2a \sqrt{c} \sqrt{a+bx^3} (4ad-bc) \log\left(\sqrt{c} - \frac{\sqrt{a+bx^3}}{\sqrt{c}}\right) - 2\sqrt{3} a \sqrt{c} \sqrt{a+bx^3} (4ad-bc) \tan^{-1}\left(\frac{\sqrt{3} \sqrt{a+bx^3}}{\sqrt{c}}\right) - 6ab c x \sqrt{c-a} \right)}{36b^2 c^2 \sqrt{a+bx^3} (c-a)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] $-1/36*(3*(b*c - a*d)^{(1/3)}*(3*b^2*c^2 + a*b*c*d - 4*a^2*d^2)*x^4*(1 + (b*x^3)/a)^{(1/3)}*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c*(-6*a*b*c*(b*c - a*d)^{(1/3)}*x + 24*a^2*d*(b*c - a*d)^{(1/3)}*x - 6*b^2*c*(b*c - a*d)^{(1/3)}*x^4 + 6*a*b*d*(b*c - a*d)^{(1/3)}*x^4 - 2*sqrt[3]*a*c^{(1/3)}*(-(b*c) + 4*a*d)*(a + b*x^3)^{(1/3)}*ArcTan[(1 + (2*(b*c - a*d)^{(1/3)}*x)/(c^{(1/3)}*(b + a*x^3)^{(1/3)})]/sqrt[3]] + 2*a*c^{(1/3)}*(-(b*c) + 4*a*d)*(a + b*x^3)^{(1/3)}*Log[c^{(1/3)} - ((b*c - a*d)^{(1/3)}*x)/(b + a*x^3)^{(1/3)}] + a*b*c^{(4/3)}*(a + b*x^3)^{(1/3)}*Log[c^{(2/3)} + ((b*c - a*d)^{(2/3)}*x^2)/(b + a*x^3)^{(2/3)} + (c^{(1/3)}*(b*c - a*d)^{(1/3)}*x)/(b + a*x^3)^{(1/3)}] - 4*a^2*c^{(1/3)}*d*(a + b*x^3)^{(1/3)}*Log[c^{(2/3)} + ((b*c - a*d)^{(2/3)}*x^2)/(b + a*x^3)^{(2/3)} + (c^{(1/3)}*(b*c - a*d)^{(1/3)}*x)/(b + a*x^3)^{(1/3)}]]/(b^2*c*d*(b*c - a*d)^{(4/3)}*(a + b*x^3)^{(1/3)})$

IntegrateAlgebraic [C] time = 9.27, size = 565, normalized size = 1.75

$$\frac{4d^2 dx - abdx + abdx^4 - d^2 c x^4}{36b^2 c^2 \sqrt{c-a}} + \frac{(4ad + 3bc) \log(\sqrt{c-a} - \sqrt{c})}{9b^2 d^2} + \frac{(4ad + 3bc) \tan^{-1}\left(\frac{\sqrt{3} \sqrt{c}}{\sqrt{c-a} - \sqrt{c}}\right)}{3\sqrt{3} b^2 d^2} + \frac{-4ad - 3bc \log(\sqrt{c} \sqrt{a+bx^3} + (a+bx^3)^{2/3})}{18b^2 d^2} + \frac{(c^2 + \sqrt{3} c^2) \log(2\sqrt{c-a} + (1 + \sqrt{3}) \sqrt{c} \sqrt{a+bx^3})}{6d^2 (c-a)^{3/2}} + \frac{\sqrt{[-(1 + \sqrt{3})] c^2 \tan^{-1}\left(\frac{\sqrt{3} \sqrt{c}}{\sqrt{c-a} - \sqrt{c}}\right)}}{d^2 (c-a)^{3/2}} + \frac{d \sqrt{c-a}}{d^2 (c-a)^{3/2}} + \frac{(\sqrt{c} c^2 - c^2) \log((\sqrt{c} + i)^{2/3} (a + bx^3)^{1/3} + \sqrt{c} (-\sqrt{3} + i) \sqrt{a+bx^3} \sqrt{c-a} - 2i^2 (c-a)^{2/3})}{12b^2 (c-a)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] $-1/3*(-(a*b*c*x) + 4*a^2*d*x - b^2*c*x^4 + a*b*d*x^4)/(b^2*d*(b*c - a*d)*(a + b*x^3)^{(1/3)}) - ((3*b*c + 4*a*d)*ArcTan[(sqrt[3]*b^{(1/3)}*x)/(b^{(1/3)}*x + 2*(a + b*x^3)^{(1/3)})]/(3*sqrt[3]*b^{(7/3)}*d^2) - (sqrt[3]*(-1 + I*sqrt[3])/6)*c^{(7/3)}*ArcTan[(3*(b*c - a*d)^{(1/3)}*x)/(sqrt[3]*(b*c - a*d)^{(1/3)}*x - (3*I)*c^{(1/3)}*(a + b*x^3)^{(1/3)} - sqrt[3]*c^{(1/3)}*(a + b*x^3)^{(1/3)})]/(d^2*(b$

$$c - a*d)^{(4/3)} + ((3*b*c + 4*a*d)*\text{Log}[-(b^{(1/3)}*x) + (a + b*x^3)^{(1/3)}])/(9*b^{(7/3)}*d^2) + ((c^{(7/3)} + I*\text{Sqrt}[3]*c^{(7/3)})*\text{Log}[2*(b*c - a*d)^{(1/3)}*x + (1 + I*\text{Sqrt}[3])*c^{(1/3)}*(a + b*x^3)^{(1/3)}])/(6*d^2*(b*c - a*d)^{(4/3)}) + ((-3*b*c - 4*a*d)*\text{Log}[b^{(2/3)}*x^2 + b^{(1/3)}*x*(a + b*x^3)^{(1/3)} + (a + b*x^3)^{(2/3)}])/(18*b^{(7/3)}*d^2) - ((I/12)*((-I)*c^{(7/3)} + \text{Sqrt}[3]*c^{(7/3)})*\text{Log}[(-2*I)*(b*c - a*d)^{(2/3)}*x^2 + c^{(1/3)}*(b*c - a*d)^{(1/3)}*(I*x - \text{Sqrt}[3]*x)*(a + b*x^3)^{(1/3)} + (I + \text{Sqrt}[3])*c^{(2/3)}*(a + b*x^3)^{(2/3)}])/(d^2*(b*c - a*d)^{(4/3)})$$

fricas [B] time = 1.36, size = 1329, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] [1/18*(3*sqrt(1/3)*(3*a*b^3*c^2 + a^2*b^2*c*d - 4*a^3*b*d^2 + (3*b^4*c^2 + a*b^3*c*d - 4*a^2*b^2*d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) - 6*sqrt(3)*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) + 2*(3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*log(-(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) + 3*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) + 6*((b^3*c*d - a*b^2*d^2)*x^4 + (a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^3), -1/18*(6*sqrt(3)*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) - 2*(3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 6*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*log(-(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) - 3*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) - 6*sqrt(1/3)*(3*a*b^3*c^2 + a^2*b^2*c*d - 4*a^3*b*d^2 + (3*b^4*c^2 + a*b^3*c*d - 4*a^2*b^2*d^2)*x^3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 6*((b^3*c*d - a*b^2*d^2)*x^4 + (a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] int(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^9}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x)

[Out] int(x^9/((a + b*x^3)^(4/3)*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(a + bx^3)^{\frac{4}{3}} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9/(b*x**3+a)**(4/3)/(d*x**3+c), x)

[Out] Integral(x**9/((a + b*x**3)**(4/3)*(c + d*x**3)), x)

$$3.487 \quad \int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal. Leaf size=260

$$\frac{\log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{bx}\right)}{2b^{4/3}d} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{bx}}{\sqrt[3]{a+bx^3}+1}\right)}{\sqrt{3}b^{4/3}d} - \frac{c^{4/3}\log(c+dx^3)}{6d(bc-ad)^{4/3}} + \frac{c^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{4/3}} - \frac{c^{4/3}\tan^{-1}\left(\frac{2x}{\sqrt[3]{c}}\right)}{\sqrt{3}d(bc-ad)^{4/3}}$$

Rubi [C] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 0.26, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{x^7 \sqrt[3]{\frac{bx^3}{a} + 1} {}_1F_1\left(\frac{7}{3}; \frac{4}{3}, 1; \frac{10}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{7ac\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Int[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] (x^7*(1 + (b*x^3)/a)^(1/3)*AppellF1[7/3, 4/3, 1, 10/3, -((b*x^3)/a), -((d*x^3)/c)]/(7*a*c*(a + b*x^3)^(1/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^(m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^6}{(a + bx^3)^{4/3} (c + dx^3)} dx = \frac{\sqrt[3]{1 + \frac{bx^3}{a}} \int \frac{x^6}{\left(1 + \frac{bx^3}{a}\right)^{4/3} (c + dx^3)} dx}{a \sqrt[3]{a + bx^3}}$$

$$= \frac{x^7 \sqrt[3]{1 + \frac{bx^3}{a}} F_1\left(\frac{7}{3}; \frac{4}{3}, 1; \frac{10}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{7ac \sqrt[3]{a + bx^3}}$$

Mathematica [C] time = 0.45, size = 309, normalized size = 1.19

$$\frac{3x^4 \sqrt[3]{\frac{bx^3}{a}} + 1 (bc - ad)^{4/3} F_1\left(\frac{4}{3}; \frac{1}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) - 2ac \left(\sqrt[3]{c} \sqrt[3]{a + bx^3} \log\left(\frac{\sqrt[3]{c} \sqrt[3]{bc - ad}}{\sqrt[3]{ax^3 + b}} + \frac{x^2 (bc - ad)^{2/3}}{(ax^3 + b)^{2/3}} + c^{2/3}\right) - 2 \sqrt[3]{c} \sqrt[3]{a + bx^3} \log\left(\sqrt[3]{c} - \frac{x \sqrt[3]{bc - ad}}{\sqrt[3]{ax^3 + b}}\right) + 2 \sqrt{3} \sqrt[3]{c} \sqrt[3]{a + bx^3} \tan^{-1}\left(\frac{2x \sqrt[3]{bc - ad}}{\sqrt[3]{c} \sqrt[3]{ax^3 + b}} + 1\right) - 6x \sqrt[3]{bc - ad} \right)}{12bc \sqrt[3]{a + bx^3} (bc - ad)^{4/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] (3*(b*c - a*d)^(4/3)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] - 2*a*c*(-6*(b*c - a*d)^(1/3)*x + 2*sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/sqrt[3]] - 2*c^(1/3)*(a + b*x^3)^(1/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + c^(1/3)*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)])/(12*b*c*(b*c - a*d)^(4/3)*(a + b*x^3)^(1/3))

IntegrateAlgebraic [C] time = 5.72, size = 500, normalized size = 1.92

$$\frac{\log(\sqrt{a + bx^3} - \sqrt{b})}{3b^{4/3}d} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{3}\sqrt{a+d} + \sqrt{3}}\right)}{\sqrt{3}b^{4/3}d} + \frac{\log(\sqrt{b}x\sqrt{a + bx^3} + (a + bx^3)^{2/3} + b^{2/3}x^2)}{6b^{4/3}d} + \frac{i(\sqrt{3}c^{4/3} - ic^{4/3})\log(2\sqrt{3}c - ad + (1 + i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a + bx^3})}{6i(bc - ad)^{4/3}} + \frac{\sqrt{2}(-1 + i\sqrt{3})c^{4/3}\tan^{-1}\left(\frac{2\sqrt{3}c - ad}{\sqrt{3}\sqrt{bc - ad} - \sqrt{3}\sqrt[3]{c}\sqrt[3]{a + bx^3}}\right)}{d(bc - ad)^{4/3}} + \frac{(c^{4/3} + i\sqrt{3}c^{4/3})\log((\sqrt{3} + i)c^{2/3}(a + bx^3)^{2/3} + \sqrt[3]{c}(-\sqrt{3}x + a)\sqrt[3]{a + bx^3}\sqrt[3]{bc - ad} - 2x^2(bc - ad)^{2/3})}{12i(bc - ad)^{4/3}} + \frac{ax}{b\sqrt{a + bx^3}(bc - ad)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] (a*x)/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) + ArcTan[(sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3)]/(sqrt[3]*b^(4/3)*d) + (sqrt[3]*(-1 + I*sqrt[3])/6]*c^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])/((d*(b*c - a*d)^(4/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(3*b^(4/3)*d) - ((I/6)*((-I)*c^(4/3) + sqrt[3]*c^(4/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*sqrt[3]*sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/((d*(b*c - a*d)^(4/3)) + Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(6*b^(4/3)*d) + ((c^(4/3) + I*sqrt[3]*c^(4/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)]

$$\frac{(I\sqrt[3]{x} - \sqrt[3]{3}x)(a + bx^3)^{1/3} + (I + \sqrt[3]{3})c^{2/3}(a + bx^3)^{2/3}}{(12d(bc - ad)^{4/3})}$$

fricas [B] time = 0.48, size = 1127, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] [1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x + 3*sqrt(1/3)*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*(b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 2*(b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) + (b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-(b*c - a*d)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d)))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2)]/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x - 6*sqrt(1/3)*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*sqrt(3)*(b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 2*(b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) + (b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-(b*c - a*d)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d)))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2)]/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b*x^3+a)^(4/3)/(d*x^3+c), x)

[Out] int(x^6/(b*x^3+a)^(4/3)/(d*x^3+c), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c), x, algorithm="maxima")

[Out] integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/((a + b*x^3)^(4/3)*(c + d*x^3)), x)

[Out] int(x^6/((a + b*x^3)^(4/3)*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a + bx^3)^{\frac{4}{3}}(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(b*x**3+a)**(4/3)/(d*x**3+c), x)

[Out] Integral(x**6/((a + b*x**3)**(4/3)*(c + d*x**3)), x)

$$3.488 \quad \int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal. Leaf size=172

$$\frac{x}{\sqrt[3]{a+bx^3}(bc-ad)} + \frac{\sqrt[3]{c} \log(c+dx^3)}{6(bc-ad)^{4/3}} - \frac{\sqrt[3]{c} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}} + \frac{\sqrt[3]{c} \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}(bc-ad)^{4/3}}$$

Rubi [C] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 0.53, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{x^4 \sqrt[3]{\frac{bx^3}{a} + 1} {}_2F_1\left(\frac{4}{3}, \frac{4}{3}, \frac{7}{3}, -\frac{c\left(\frac{bx^3}{a} - \frac{dx^3}{c}\right)}{dx^3+c}\right)}{4ac\sqrt[3]{a+bx^3} \left(\frac{dx^3}{c} + 1\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] (x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -((c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3))]/(4*a*c*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^3}{(a + bx^3)^{4/3} (c + dx^3)} dx = \frac{\sqrt[3]{1 + \frac{bx^3}{a}} \int \frac{x^3}{\left(1 + \frac{bx^3}{a}\right)^{4/3} (c + dx^3)} dx}{a \sqrt[3]{a + bx^3}}$$

$$= \frac{x^4 \sqrt[3]{1 + \frac{bx^3}{a}} {}_2F_1\left(\frac{4}{3}, \frac{4}{3}; \frac{7}{3}; -\frac{c\left(\frac{bx^3}{a} - \frac{dx^3}{c}\right)}{c + dx^3}\right)}{4ac \sqrt[3]{a + bx^3} \left(1 + \frac{dx^3}{c}\right)^{4/3}}$$

Mathematica [C] time = 0.06, size = 86, normalized size = 0.50

$$\frac{x^4 \sqrt[3]{\frac{bx^3}{a} + 1} {}_2F_1\left(\frac{4}{3}, \frac{4}{3}; \frac{7}{3}; \frac{(ad - bc)x^3}{a(dx^3 + c)}\right)}{4ac \sqrt[3]{a + bx^3} \left(\frac{dx^3}{c} + 1\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] (x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))]/(4*a*c*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3))

IntegrateAlgebraic [C] time = 2.56, size = 355, normalized size = 2.06

$$\frac{i(\sqrt{3}\sqrt{c} - i\sqrt{c})\log\left((\sqrt{3} + i)c^{2/3}(a + bx^3)^{2/3} + \sqrt{c}(-\sqrt{3}x + ix)\sqrt[3]{a + bx^3}\sqrt[3]{bc - ad} - 2ix^2(bc - ad)^{2/3}\right)}{12(bc - ad)^{4/3}} - \frac{x}{\sqrt[3]{a + bx^3}(bc - ad)} + \frac{(\sqrt{c} + i\sqrt{3}\sqrt{c})\log\left(2x\sqrt[3]{bc - ad} + (1 + i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a + bx^3}\right)}{6(bc - ad)^{4/3}} - \frac{\sqrt{-1 + i\sqrt{3}}\sqrt{c}\tan^{-1}\left(\frac{3x\sqrt[3]{bc - ad}}{\sqrt{3}\sqrt[3]{bc - ad} - \sqrt{3}\sqrt[3]{c}\sqrt[3]{a + bx^3} - 3i\sqrt[3]{c}\sqrt[3]{a + bx^3}}\right)}{\sqrt{6}(bc - ad)^{4/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] -(x/((b*c - a*d)*(a + b*x^3)^(1/3))) - (Sqrt[-1 + I*Sqrt[3]]*c^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)])/((Sqrt[6]*(b*c - a*d)^(4/3)) + ((c^(1/3) + I*Sqrt[3]*c^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*(b*c - a*d)^(4/3)) - ((I/12)*((-I)*c^(1/3) + Sqrt[3]*c^(1/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]/(b*c - a*d)^(4/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] int(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/((a + b*x^3)^(4/3)*(c + d*x^3)), x)`

[Out] `int(x^3/((a + b*x^3)^(4/3)*(c + d*x^3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + bx^3)^{\frac{4}{3}}(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b*x**3+a)**(4/3)/(d*x**3+c), x)`

[Out] `Integral(x**3/((a + b*x**3)**(4/3)*(c + d*x**3)), x)`

$$3.489 \quad \int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal. Leaf size=179

$$\frac{d \log(c+dx^3)}{6c^{2/3}(bc-ad)^{4/3}} + \frac{d \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}} - \frac{d \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} + \frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}$$

Rubi [A] time = 0.20, antiderivative size = 238, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$\frac{d \log\left(\sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d \log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6c^{2/3}(bc-ad)^{4/3}} - \frac{d \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} + \frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(4/3)) + (d*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*c^(2/3)*(b*c - a*d)^(4/3)) - (d*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]/(6*c^(2/3)*(b*c - a*d)^(4/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*a*c}, Simplify[(a*c)/b^2], Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)} dx &= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d \int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)} dx}{bc-ad} \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d \operatorname{Subst}\left(\int \frac{1}{c-(bc-ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{bc-ad} \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d \operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)} - \frac{d \operatorname{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}}\right)}{c^{2/3}+\sqrt[3]{c}} \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{d \log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d \operatorname{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)}\right)}{6c^{2/3}(bc-ad)^{4/3}} \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{d \log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d \log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}(bc-ad)^{4/3}} \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d \tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} + \frac{d \log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d \log\left(c^2\right)}{c^2}
\end{aligned}$$

Mathematica [C] time = 0.12, size = 256, normalized size = 1.43

$$\frac{28c^3(a+bx^3)^2 {}_2F_1\left(\frac{1}{3}, 1, \frac{4}{3}, \frac{(bc-ad)x^3}{c(bx^3+a)}\right) - 28c^3(a+bx^3)^2 + 21c^2dx^3(a+bx^3)^2 {}_2F_1\left(\frac{1}{3}, 1, \frac{4}{3}, \frac{(bc-ad)x^3}{c(bx^3+a)}\right) - 21c^2dx^3(a+bx^3)^2 + 3dx^9(bc-ad)^2 {}_2F_1\left(2, \frac{7}{3}, \frac{10}{3}, \frac{(bc-ad)x^3}{c(bx^3+a)}\right) + 3cx^6(bc-ad)^2 {}_2F_1\left(2, \frac{7}{3}, \frac{10}{3}, \frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{7c^3x^2(a+bx^3)^{7/3}(ad-bc)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] $-1/7*(-28*c^3*(a + b*x^3)^2 - 21*c^2*d*x^3*(a + b*x^3)^2 + 28*c^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21*c^2*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*c*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(c^3*(-(b*c) + a*d)*x^2*(a + b*x^3)^(7/3))$

IntegrateAlgebraic [C] time = 0.00, size = 352, normalized size = 1.97

$$\frac{i(\sqrt{3}d-id)\log\left(2x\sqrt{bc-ad}+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{2/3}(bc-ad)^{4/3}} + \frac{\sqrt[3]{-1+i\sqrt{3}}d\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{4/3}} + \frac{(d+i\sqrt{3}d)\log\left((\sqrt{3}+i)^{2/3}(a+bx^3)^{2/3}+\sqrt[3]{c}(-\sqrt{3}x+ix)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{2/3}(bc-ad)^{4/3}} - \frac{bx}{a\sqrt[3]{a+bx^3}(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] $-\left(\frac{b*x}{a*(-b*c) + a*d}\right) * (a + b*x^3)^{(1/3)} + \frac{\text{Sqrt}[-1 + I*\text{Sqrt}[3]]/6 * d * \text{ArcTan}[(3*(b*c - a*d)^{(1/3)}*x) / (\text{Sqrt}[3]*(b*c - a*d)^{(1/3)}*x - (3*I)*c^{(1/3)}*(a + b*x^3)^{(1/3)} - \text{Sqrt}[3]*c^{(1/3)}*(a + b*x^3)^{(1/3)})]}{c^{(2/3)}*(b*c - a*d)^{(4/3)}} - \left(\frac{I/6 * (-I)*d + \text{Sqrt}[3]*d}{c^{(2/3)}*(b*c - a*d)^{(4/3)}}\right) * \text{Log}[2*(b*c - a*d)^{(1/3)}*x + (1 + I*\text{Sqrt}[3])*c^{(1/3)}*(a + b*x^3)^{(1/3)}] + \left(\frac{d + I*\text{Sqrt}[3]*d}{c^{(2/3)}*(b*c - a*d)^{(4/3)}}\right) * \text{Log}[(-2*I)*(b*c - a*d)^{(2/3)}*x^2 + c^{(1/3)}*(b*c - a*d)^{(1/3)}*(I*x - \text{Sqrt}[3]*x)*(a + b*x^3)^{(1/3)} + (I + \text{Sqrt}[3])*c^{(2/3)}*(a + b*x^3)^{(2/3)}] / (12*c^{(2/3)}*(b*c - a*d)^{(4/3)})$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] `int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^3)^(4/3)*(c + d*x^3)),x)`

[Out] `int(1/((a + b*x^3)^(4/3)*(c + d*x^3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{\frac{4}{3}}(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)`

[Out] `Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)`

$$3.490 \quad \int \frac{1}{x^3(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal. Leaf size=229

$$-\frac{(a+bx^3)^{2/3}(3bc-ad)}{2a^2cx^2(bc-ad)} + \frac{d^2 \log(c+dx^3)}{6c^{5/3}(bc-ad)^{4/3}} - \frac{d^2 \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt{a+bx^3}\right)}{2c^{5/3}(bc-ad)^{4/3}} + \frac{d^2 \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt{a+bx^3}}\right)}{\sqrt{3}c^{5/3}(bc-ad)^{4/3}} + \frac{b}{ax^2\sqrt[3]{a+bx^3}}$$

Rubi [C] time = 1.29, antiderivative size = 542, normalized size of antiderivative = 2.37, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$-\frac{14c^2(a+bx^3)^{7/3}(bc-ad)}{14c^2(a+bx^3)^{7/3}(bc-ad)}$

Warning: Unable to verify antiderivative.

[In] Int[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] (28*c^4*(a + b*x^3)^2 + 168*c^3*d*x^3*(a + b*x^3)^2 + 126*c^2*d^2*x^6*(a + b*x^3)^2 - 28*c^4*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 168*c^3*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 126*c^2*d^2*x^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 15*c^2*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 42*c*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 27*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*c^2*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18*c*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(14*c^4*(b*c - a*d)*x^5*(a + b*x^3)^(7/3))

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3 (a + bx^3)^{4/3} (c + dx^3)} dx = \frac{\sqrt[3]{1 + \frac{bx^3}{a}} \int \frac{1}{x^3 \left(1 + \frac{bx^3}{a}\right)^{4/3} (c + dx^3)} dx}{a \sqrt[3]{a + bx^3}}$$

$$= \frac{28c^4 (a + bx^3)^2 + 168c^3 dx^3 (a + bx^3)^2 + 126c^2 d^2 x^6 (a + bx^3)^2 - 28c^4 (a + bx^3)^2}{144c^2 (a + bx^3) (ad - bc)}$$

Mathematica [C] time = 0.83, size = 542, normalized size = 2.37

$\frac{bc^2 d^2 (bc - ad) \sqrt[3]{a + bx^3} \operatorname{erf}\left(\frac{\sqrt[3]{a + bx^3}}{\sqrt[3]{a}}\right) + 9bc^2 d^2 (bc - ad) \sqrt[3]{a + bx^3} \operatorname{erf}\left(\frac{\sqrt[3]{a + bx^3}}{\sqrt[3]{a}}\right) + 18abcd^2 (bc - ad) \sqrt[3]{a + bx^3} \operatorname{erf}\left(\frac{\sqrt[3]{a + bx^3}}{\sqrt[3]{a}}\right) + 28c^4 (a + bx^3)^2 \operatorname{erf}\left(\frac{\sqrt[3]{a + bx^3}}{\sqrt[3]{a}}\right) - 28c^4 (a + bx^3)^2 + 168c^3 d^2 x^3 (a + bx^3)^2 - 126c^2 d^2 x^6 (a + bx^3)^2 - 28c^4 (a + bx^3)^2}{144c^2 (a + bx^3) (ad - bc)}$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] $(-28*c^4*(a + b*x^3)^2 - 168*c^3*d*x^3*(a + b*x^3)^2 - 126*c^2*d^2*x^6*(a + b*x^3)^2 + 28*c^4*(a + b*x^3)^2*\operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right] + 168*c^3*d*x^3*(a + b*x^3)^2*\operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right] + 126*c^2*d^2*x^6*(a + b*x^3)^2*\operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right] + 15*c^2*(b*c - a*d)^2*x^6*\operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right] + 42*c*d*(b*c - a*d)^2*x^9*\operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right] + 27*d^2*(b*c - a*d)^2*x^{12}*\operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right] + 9*c^2*(b*c - a*d)^2*x^6*\operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right] + 18*c*d*(b*c - a*d)^2*x^9*\operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right] + 9*d^2*(b*c - a*d)^2*x^{12}*\operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right])/(14*c^4*(-(b*c) + a*d)*x^5*(a + b*x^3)^{(7/3)})$

IntegrateAlgebraic [C] time = 3.19, size = 397, normalized size = 1.73

$\frac{-d^2 d + abc - abd x^3 + 3d^2 c x^3}{2d^2 c x^2 \sqrt[3]{a + bx^3} (ad - bc)} + \frac{(d^2 + i\sqrt{3}d^2) \log\left(2x\sqrt[3]{bc - ad} + (1 + i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a + bx^3}\right)}{6c^{5/3}(bc - ad)^{4/3}} - \frac{\sqrt[3]{-1 + i\sqrt{3}}d^2 \tan^{-1}\left(\frac{3x\sqrt[3]{bc - ad}}{\sqrt[3]{a + bx^3} - \sqrt[3]{a} - \sqrt[3]{bc - ad}}\right)}{c^{5/3}(bc - ad)^{4/3}} - \frac{i(\sqrt{3}d^2 - id^2) \log\left((\sqrt{3} + i)^{2/3}(a + bx^3)^{2/3} + \sqrt[3]{c}(-\sqrt{3}x + id)\sqrt[3]{a + bx^3}\sqrt[3]{bc - ad} - 2ix^2(bc - ad)^{2/3}\right)}{12c^{5/3}(bc - ad)^{4/3}}$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out]
$$\frac{(a*b*c - a^2*d + 3*b^2*c*x^3 - a*b*d*x^3)/(2*a^2*c*(-(b*c) + a*d)*x^2*(a + b*x^3)^{(1/3)}) - (\text{Sqrt}[-1 + I*\text{Sqrt}[3]])/6*d^2*\text{ArcTan}[(3*(b*c - a*d)^{(1/3)}*x)/(\text{Sqrt}[3]*(b*c - a*d)^{(1/3)}*x - (3*I)*c^{(1/3)}*(a + b*x^3)^{(1/3)} - \text{Sqrt}[3]*c^{(1/3)}*(a + b*x^3)^{(1/3)})]}{(c^{(5/3)}*(b*c - a*d)^{(4/3)})} + ((d^2 + I*\text{Sqrt}[3]*d^2)*\text{Log}[2*(b*c - a*d)^{(1/3)}*x + (1 + I*\text{Sqrt}[3])*c^{(1/3)}*(a + b*x^3)^{(1/3)}])/(6*c^{(5/3)}*(b*c - a*d)^{(4/3)}) - ((I/12)*((-I)*d^2 + \text{Sqrt}[3]*d^2)*\text{Log}[(-2*I)*(b*c - a*d)^{(2/3)}*x^2 + c^{(1/3)}*(b*c - a*d)^{(1/3)}*(I*x - \text{Sqrt}[3]*x)*(a + b*x^3)^{(1/3)} + (I + \text{Sqrt}[3])*c^{(2/3)}*(a + b*x^3)^{(2/3)}])/(c^{(5/3)}*(b*c - a*d)^{(4/3)})$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] int(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 (bx^3 + a)^{\frac{4}{3}} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x)

[Out] int(1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + bx^3)^{\frac{4}{3}} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)

[Out] Integral(1/(x**3*(a + b*x**3)**(4/3)*(c + d*x**3)), x)

$$3.491 \quad \int \frac{1}{x^6(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal. Leaf size=287

$$\frac{(a+bx^3)^{2/3}(6bc-ad)}{5a^2cx^5(bc-ad)} + \frac{(a+bx^3)^{2/3}(-5a^2d^2-3abcd+18b^2c^2)}{10a^3c^2x^2(bc-ad)} - \frac{d^3 \log(c+dx^3)}{6c^{8/3}(bc-ad)^{4/3}} + \frac{d^3 \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{8/3}(bc-ad)^{4/3}}$$

Rubi [C] time = 3.86, antiderivative size = 950, normalized size of antiderivative = 3.31, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

Warning: Unable to verify antiderivative.

[In] Int[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] (56*c^5*(a + b*x^3)^2 - 252*c^4*d*x^3*(a + b*x^3)^2 - 1512*c^3*d^2*x^6*(a + b*x^3)^2 - 1134*c^2*d^3*x^9*(a + b*x^3)^2 - 56*c^5*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*c^4*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1512*c^3*d^2*x^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1134*c^2*d^3*x^9*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18*c^3*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 171*c^2*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 486*c*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*d^3*(b*c - a*d)^2*x^15*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*c^3*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 216*c^2*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 351*c*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*c^3*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 81*c^2*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 81*c*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(70*c^5*(b*c - a*d)*x^8*(a + b*x^3)^(7/3))

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^6 (a + bx^3)^{4/3} (c + dx^3)} dx = \frac{\int \frac{1}{\sqrt[3]{1 + \frac{bx^3}{a}} x^6 \left(1 + \frac{bx^3}{a}\right)^{4/3} (c + dx^3)} dx}{a^3 \sqrt[3]{a + bx^3}}$$

$$= \frac{56c^5 (a + bx^3)^2 - 252c^4 dx^3 (a + bx^3)^2 - 1512c^3 d^2 x^6 (a + bx^3)^2 - 1134c^2 d^3 x^9}{\dots}$$

Mathematica [C] time = 2.31, size = 950, normalized size = 3.31

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Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -1/70*(56*c^5*(a + b*x^3)^2 - 252*c^4*d*x^3*(a + b*x^3)^2 - 1512*c^3*d^2*x^6*(a + b*x^3)^2 - 1134*c^2*d^3*x^9*(a + b*x^3)^2 - 56*c^5*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*c^4*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1512*c^3*d^2*x^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1134*c^2*d^3*x^9*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18*c^3*(b*c - a*
```

$d^2 x^6 \text{Hypergeometric2F1}\left[2, 7/3, 10/3, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right]$
 $+ 171*c^2*d*(b*c - a*d)^2*x^9 \text{Hypergeometric2F1}\left[2, 7/3, 10/3, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right]$
 $+ 486*c*d^2*(b*c - a*d)^2*x^{12} \text{Hypergeometric2F1}\left[2, 7/3, 10/3, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right]$
 $+ 297*d^3*(b*c - a*d)^2*x^{15} \text{Hypergeometric2F1}\left[2, 7/3, 10/3, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right]$
 $+ 27*c^3*(b*c - a*d)^2*x^6 \text{HypergeometricPFQ}\left[\{2, 2, 7/3\}, \{1, 10/3\}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right]$
 $+ 216*c^2*d*(b*c - a*d)^2*x^9 \text{HypergeometricPFQ}\left[\{2, 2, 7/3\}, \{1, 10/3\}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right]$
 $+ 351*c*d^2*(b*c - a*d)^2*x^{12} \text{HypergeometricPFQ}\left[\{2, 2, 7/3\}, \{1, 10/3\}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right]$
 $+ 162*d^3*(b*c - a*d)^2*x^{15} \text{HypergeometricPFQ}\left[\{2, 2, 7/3\}, \{1, 10/3\}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right]$
 $+ 27*c^3*(b*c - a*d)^2*x^6 \text{HypergeometricPFQ}\left[\{2, 2, 2, 7/3\}, \{1, 1, 10/3\}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right]$
 $+ 81*c^2*d*(b*c - a*d)^2*x^9 \text{HypergeometricPFQ}\left[\{2, 2, 2, 7/3\}, \{1, 1, 10/3\}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right]$
 $+ 81*c*d^2*(b*c - a*d)^2*x^{12} \text{HypergeometricPFQ}\left[\{2, 2, 2, 7/3\}, \{1, 1, 10/3\}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right]$
 $+ 27*d^3*(b*c - a*d)^2*x^{15} \text{HypergeometricPFQ}\left[\{2, 2, 2, 7/3\}, \{1, 1, 10/3\}, \frac{(b*c - a*d)*x^3}{c*(a + b*x^3)}\right]$
 $]/(c^5*(-(b*c) + a*d)*x^8*(a + b*x^3)^{(7/3)})$

IntegrateAlgebraic [C] time = 4.82, size = 452, normalized size = 1.57

$$\frac{-2a^3cd + 5a^3d^2c^2 + 2a^2bc^2 + a^2bcd^2 + 5a^2bd^2c^2 - 6a^2c^2d^2 + 3a^2cd^2c^2 - 18b^3c^2d^2}{10a^3c^2d^2\sqrt{a+bx^3}(ad-bc)} \cdot \frac{i(\sqrt{3}d^2 - ad) \log\left(\frac{2x\sqrt{bc-ad} + (1+i\sqrt{3})\sqrt{c}\sqrt{a+bx^3}}{6c^{3/2}(bc-ad)^{3/2}}\right) + \sqrt{\frac{1}{2}(-1+i\sqrt{3})}d^2 \tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{3}\sqrt{bc-ad}\sqrt{c}\sqrt{a+bx^3} - \sqrt{3}\sqrt{bc-ad}\sqrt{c}\sqrt{a+bx^3}}\right)}{c^{3/2}(bc-ad)^{3/2}} + \frac{(d^2 + i\sqrt{3}d^2) \log\left(\frac{(\sqrt{3}+i)c^{2/3}(a+bx^3)^{2/3} + \sqrt{c}(-\sqrt{3}x+ix)\sqrt{a+bx^3}\sqrt{bc-ad} - 2ic^2(bc-ad)^{2/3}}{12c^{2/3}(bc-ad)^{2/3}}\right)}{12c^{2/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] $(2*a^2*b*c^2 - 2*a^3*c*d - 6*a*b^2*c^2*x^3 + a^2*b*c*d*x^3 + 5*a^3*d^2*x^3 - 18*b^3*c^2*x^6 + 3*a*b^2*c*d*x^6 + 5*a^2*b*d^2*x^6)/(10*a^3*c^2*(-(b*c) + a*d)*x^5*(a + b*x^3)^{(1/3)}) + (\text{Sqrt}[(-1 + I*\text{Sqrt}[3])/6]*d^3*\text{ArcTan}[(3*(b*c - a*d)^{(1/3)}*x)/(\text{Sqrt}[3]*(b*c - a*d)^{(1/3)}*x - (3*I)*c^{(1/3)}*(a + b*x^3)^{(1/3)} - \text{Sqrt}[3]*c^{(1/3)}*(a + b*x^3)^{(1/3)})]/(c^{(8/3)}*(b*c - a*d)^{(4/3)}) - (I/6)*((-I)*d^3 + \text{Sqrt}[3]*d^3)*\text{Log}[2*(b*c - a*d)^{(1/3)}*x + (1 + I*\text{Sqrt}[3])*c^{(1/3)}*(a + b*x^3)^{(1/3)}]/(c^{(8/3)}*(b*c - a*d)^{(4/3)}) + ((d^3 + I*\text{Sqrt}[3]*d^3)*\text{Log}[(-2*I)*(b*c - a*d)^{(2/3)}*x^2 + c^{(1/3)}*(b*c - a*d)^{(1/3)}*(I*x - \text{Sqrt}[3]*x)*(a + b*x^3)^{(1/3)} + (I + \text{Sqrt}[3])*c^{(2/3)}*(a + b*x^3)^{(2/3)}]/(12*c^{(8/3)}*(b*c - a*d)^{(4/3)})$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] int(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6 (bx^3 + a)^{\frac{4}{3}} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x)

[Out] int(1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 (a + bx^3)^{\frac{4}{3}} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**6/(b*x**3+a)**(4/3)/(d*x**3+c), x)

[Out] Integral(1/(x**6*(a + b*x**3)**(4/3)*(c + d*x**3)), x)

$$3.492 \quad \int \frac{1}{x^9(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal. Leaf size=351

$$\frac{(a+bx^3)^{2/3}(9bc-4ad)(ad+3bc)}{20a^3c^2x^5(bc-ad)} - \frac{(a+bx^3)^{2/3}(9bc-ad)}{8a^2cx^8(bc-ad)} - \frac{(a+bx^3)^{2/3}(-20a^3d^3-12a^2bcd^2-9ab^2c^2d+81b^3)}{40a^4c^3x^2(bc-ad)}$$

Rubi [C] time = 8.30, antiderivative size = 1486, normalized size of antiderivative = 4.23, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

result too large to display

Warning: Unable to verify antiderivative.

[In] Int[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] (280*c^6*(a + b*x^3)^2 - 672*c^5*d*x^3*(a + b*x^3)^2 + 3024*c^4*d^2*x^6*(a + b*x^3)^2 + 18144*c^3*d^3*x^9*(a + b*x^3)^2 + 13608*c^2*d^4*x^12*(a + b*x^3)^2 - 280*c^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 672*c^5*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3024*c^4*d^2*x^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18144*c^3*d^3*x^9*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 13608*c^2*d^4*x^12*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 66*c^4*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 312*c^3*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 2268*c^2*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 6696*c*d^3*(b*c - a*d)^2*x^15*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 4050*d^4*(b*c - a*d)^2*x^18*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 189*c^4*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 108*c^3*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3618*c^2*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 6156*c*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 2835*d^4*(b*c - a*d)^2*x^18*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 54*c^4*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 648*c^3*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(

$c*(a + b*x^3)] - 2268*c^2*d^2*(b*c - a*d)^2*x^{12}*HypergeometricPFQ[\{2, 2, 2, 7/3\}, \{1, 1, 10/3\}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]$
 $- 2376*c*d^3*(b*c - a*d)^2*x^{15}*HypergeometricPFQ[\{2, 2, 2, 2, 7/3\}, \{1, 1, 10/3\}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]$
 $- 810*d^4*(b*c - a*d)^2*x^{18}*HypergeometricPFQ[\{2, 2, 2, 2, 7/3\}, \{1, 1, 1, 10/3\}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]$
 $- 81*c^4*(b*c - a*d)^2*x^6*HypergeometricPFQ[\{2, 2, 2, 2, 2, 7/3\}, \{1, 1, 1, 1, 10/3\}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]$
 $- 324*c^3*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[\{2, 2, 2, 2, 2, 2, 7/3\}, \{1, 1, 1, 1, 10/3\}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]$
 $- 486*c^2*d^2*(b*c - a*d)^2*x^{12}*HypergeometricPFQ[\{2, 2, 2, 2, 2, 2, 2, 7/3\}, \{1, 1, 1, 1, 10/3\}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]$
 $- 324*c*d^3*(b*c - a*d)^2*x^{15}*HypergeometricPFQ[\{2, 2, 2, 2, 2, 2, 2, 2, 7/3\}, \{1, 1, 1, 1, 10/3\}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]$
 $- 81*d^4*(b*c - a*d)^2*x^{18}*HypergeometricPFQ[\{2, 2, 2, 2, 2, 2, 2, 2, 2, 7/3\}, \{1, 1, 1, 1, 1, 10/3\}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(560*c^6*(b*c - a*d)*x^{11}*(a + b*x^3)^{(7/3)})$

Rule 510

$Int[((e_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^{(n_))^{(p_)}*((c_) + (d_)*(x_)^{(n_))^{(q_)}, x_Symbol] :> Simp[(a^p*c^q*(e*x)^{(m+1)}*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[m, -1] \&\& NeQ[m, n - 1] \&\& (IntegerQ[p] || GtQ[a, 0]) \&\& (IntegerQ[q] || GtQ[c, 0])$

Rule 511

$Int[((e_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^{(n_))^{(p_)}*((c_) + (d_)*(x_)^{(n_))^{(q_)}, x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[m, -1] \&\& NeQ[m, n - 1] \&\& !(IntegerQ[p] || GtQ[a, 0])$

Rubi steps

$$\int \frac{1}{x^9 (a + bx^3)^{4/3} (c + dx^3)} dx = \frac{\int \frac{1}{\sqrt[3]{1 + \frac{bx^3}{a}} x^9 \left(1 + \frac{bx^3}{a}\right)^{4/3} (c + dx^3)} dx}{a \sqrt[3]{a + bx^3}}$$

$$= \frac{280c^6 (a + bx^3)^2 - 672c^5 dx^3 (a + bx^3)^2 + 3024c^4 d^2 x^6 (a + bx^3)^2 + 18144c^3 d^3 x^9}{\dots}$$

Mathematica [A] time = 5.86, size = 278, normalized size = 0.79

$$\frac{(a + bx^3)^{2/3} \left(-\frac{x^6(20a^2d^2 + 32abcd + 41b^2c^2)}{c^3} - \frac{5a^2}{c} + \frac{40b^4x^9}{(a+bx^3)(ad-bc)} + \frac{2ax^3(4ad+7bc)}{c^2} \right) + d^4 \left(\log \left(\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}} + \frac{x^2(bc-ad)^{2/3}}{(ax^3+b)^{2/3}} + c^{2/3} \right) - 2 \log \left(\sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}} + 1 \right) \right)}{40a^4x^8} + \frac{d^4}{6c^{11/3}(bc-ad)^{4/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] ((a + b*x^3)^(2/3)*((-5*a^2)/c + (2*a*(7*b*c + 4*a*d)*x^3)/c^2 - ((41*b^2*c^2 + 32*a*b*c*d + 20*a^2*d^2)*x^6)/c^3 + (40*b^4*x^9)/((-b*c) + a*d)*(a + b*x^3)))/(40*a^4*x^8) + (d^4*(2*sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))]/sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(6*c^(11/3)*(b*c - a*d)^(4/3))

IntegrateAlgebraic [C] time = 11.24, size = 528, normalized size = 1.50

$$\frac{-5a^2d^4 + 8a^4d^2c^2 - 20a^2d^2c^2 + 5a^2bc^2 + 2^2a^2b^2d^2 - 4a^2b^2d^2c^2 - 20a^2b^2d^2c^2 - 3a^2b^2d^2c^2 - 12a^2b^2d^2c^2 + 27a^2b^2d^2c^2 - 10a^2b^2d^2c^2 + 81a^2b^2c^2}{40a^4c^2d^4 + 12d^4(ad-bc)} \log \left(\frac{2x\sqrt[3]{bc-ad} + (1 + \sqrt{3})\sqrt[3]{c}\sqrt[3]{ax^3+b}}{c^{1/3}(bc-ad)^{1/3}} \right) - \frac{\sqrt{2(1 + \sqrt{3})}d^4 \tan^{-1} \left(\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}} + 1 \right)}{c^{11/3}(bc-ad)^{4/3}} + \frac{(1 + \sqrt{3})d^4 \log \left(\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}} + 1 \right)}{12c^{11/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)), x]

[Out] (5*a^3*b*c^3 - 5*a^4*c^2*d - 9*a^2*b^2*c^3*x^3 + a^3*b*c^2*d*x^3 + 8*a^4*c*d^2*x^3 + 27*a*b^3*c^3*x^6 - 3*a^2*b^2*c^2*d*x^6 - 4*a^3*b*c*d^2*x^6 - 20*a^4*d^3*x^6 + 81*b^4*c^3*x^9 - 9*a*b^3*c^2*d*x^9 - 12*a^2*b^2*c*d^2*x^9 - 20*a^3*b*d^3*x^9)/(40*a^4*c^3*(-b*c) + a*d)*x^8*(a + b*x^3)^(1/3) - (sqrt[3]((-1 + I*sqrt[3])/6)*d^4*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3)]))/(c^(11/3)*(b*c - a*d)^(4/3)) + ((d^4 + I*sqrt[3]*d^4)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*c^(11/3)*(b*c - a*d)^(4/3)) - ((I/12)*((-I)*d^4 + sqrt[3]*d^4)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(11/3)*(b*c - a*d)^(4/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] int(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^9(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x)

[Out] int(1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9 (a + bx^3)^{\frac{4}{3}} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)

[Out] Integral(1/(x**9*(a + b*x**3)**(4/3)*(c + d*x**3)), x)

$$3.493 \quad \int \frac{x^{15}}{(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=90

$$-\frac{a^3 \log(a+bx^4)}{4b^3(bc-ad)} - \frac{x^4(ad+bc)}{4b^2d^2} + \frac{c^3 \log(c+dx^4)}{4d^3(bc-ad)} + \frac{x^8}{8bd}$$

Rubi [A] time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 72}

$$-\frac{a^3 \log(a+bx^4)}{4b^3(bc-ad)} - \frac{x^4(ad+bc)}{4b^2d^2} + \frac{c^3 \log(c+dx^4)}{4d^3(bc-ad)} + \frac{x^8}{8bd}$$

Antiderivative was successfully verified.

[In] Int[x^15/((a + b*x^4)*(c + d*x^4)),x]

[Out] -((b*c + a*d)*x^4)/(4*b^2*d^2) + x^8/(8*b*d) - (a^3*Log[a + b*x^4])/(4*b^3*(b*c - a*d)) + (c^3*Log[c + d*x^4])/(4*d^3*(b*c - a*d))

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^{15}}{(a+bx^4)(c+dx^4)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^3}{(a+bx)(c+dx)} dx, x, x^4 \right) \\ &= \frac{1}{4} \text{Subst} \left(\int \left(\frac{-bc-ad}{b^2d^2} + \frac{x}{bd} - \frac{a^3}{b^2(bc-ad)(a+bx)} - \frac{c^3}{d^2(-bc+ad)(c+dx)} \right) dx, x, x^4 \right) \\ &= -\frac{(bc+ad)x^4}{4b^2d^2} + \frac{x^8}{8bd} - \frac{a^3 \log(a+bx^4)}{4b^3(bc-ad)} + \frac{c^3 \log(c+dx^4)}{4d^3(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.07, size = 92, normalized size = 1.02

$$-\frac{a^3 \log(a + bx^4)}{4b^3(bc - ad)} + \frac{x^4(-ad - bc)}{4b^2d^2} + \frac{c^3 \log(c + dx^4)}{4d^3(bc - ad)} + \frac{x^8}{8bd}$$

Antiderivative was successfully verified.

[In] Integrate[x^15/((a + b*x^4)*(c + d*x^4)), x]

[Out] ((-(b*c) - a*d)*x^4)/(4*b^2*d^2) + x^8/(8*b*d) - (a^3*Log[a + b*x^4])/(4*b^3*(b*c - a*d)) + (c^3*Log[c + d*x^4])/(4*d^3*(b*c - a*d))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{(a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^15/((a + b*x^4)*(c + d*x^4)), x]

[Out] IntegrateAlgebraic[x^15/((a + b*x^4)*(c + d*x^4)), x]

fricas [A] time = 6.24, size = 100, normalized size = 1.11

$$\frac{(b^3cd^2 - ab^2d^3)x^8 - 2a^3d^3 \log(bx^4 + a) + 2b^3c^3 \log(dx^4 + c) - 2(b^3c^2d - a^2bd^3)x^4}{8(b^4cd^3 - ab^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^15/(b*x^4+a)/(d*x^4+c), x, algorithm="fricas")

[Out] 1/8*((b^3*c*d^2 - a*b^2*d^3)*x^8 - 2*a^3*d^3*log(b*x^4 + a) + 2*b^3*c^3*log(d*x^4 + c) - 2*(b^3*c^2*d - a^2*b*d^3)*x^4)/(b^4*c*d^3 - a*b^3*d^4)

giac [A] time = 0.17, size = 88, normalized size = 0.98

$$-\frac{a^3 \log(|bx^4 + a|)}{4(b^4c - ab^3d)} + \frac{c^3 \log(|dx^4 + c|)}{4(bcd^3 - ad^4)} + \frac{bdx^8 - 2bcx^4 - 2adx^4}{8b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^15/(b*x^4+a)/(d*x^4+c), x, algorithm="giac")

[Out] -1/4*a^3*log(abs(b*x^4 + a))/(b^4*c - a*b^3*d) + 1/4*c^3*log(abs(d*x^4 + c))/(b*c*d^3 - a*d^4) + 1/8*(b*d*x^8 - 2*b*c*x^4 - 2*a*d*x^4)/(b^2*d^2)

maple [A] time = 0.06, size = 89, normalized size = 0.99

$$\frac{x^8}{8bd} - \frac{ax^4}{4b^2d} - \frac{cx^4}{4bd^2} + \frac{a^3 \ln(bx^4 + a)}{4(ad - bc)b^3} - \frac{c^3 \ln(dx^4 + c)}{4(ad - bc)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^15/(b*x^4+a)/(d*x^4+c),x)

[Out] 1/8*x^8/b/d-1/4/b^2/d*x^4*a-1/4/b/d^2*x^4*c-1/4*c^3/d^3/(a*d-b*c)*ln(d*x^4+c)+1/4*a^3/b^3/(a*d-b*c)*ln(b*x^4+a)

maxima [A] time = 0.47, size = 84, normalized size = 0.93

$$-\frac{a^3 \log(bx^4 + a)}{4(b^4c - ab^3d)} + \frac{c^3 \log(dx^4 + c)}{4(bcd^3 - ad^4)} + \frac{bdx^8 - 2(bc + ad)x^4}{8b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out] -1/4*a^3*log(b*x^4 + a)/(b^4*c - a*b^3*d) + 1/4*c^3*log(d*x^4 + c)/(b*c*d^3 - a*d^4) + 1/8*(b*d*x^8 - 2*(b*c + a*d)*x^4)/(b^2*d^2)

mupad [B] time = 5.88, size = 88, normalized size = 0.98

$$\frac{x^8}{8bd} - \frac{c^3 \ln(dx^4 + c)}{4(ad^4 - bcd^3)} - \frac{a^3 \ln(bx^4 + a)}{4(b^4c - ab^3d)} - \frac{x^4(ad + bc)}{4b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^15/((a + b*x^4)*(c + d*x^4)),x)

[Out] x^8/(8*b*d) - (c^3*log(c + d*x^4))/(4*(a*d^4 - b*c*d^3)) - (a^3*log(a + b*x^4))/(4*(b^4*c - a*b^3*d)) - (x^4*(a*d + b*c))/(4*b^2*d^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**15/(b*x**4+a)/(d*x**4+c),x)

[Out] Timed out

$$3.494 \quad \int \frac{x^{11}}{(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=70

$$\frac{a^2 \log(a+bx^4)}{4b^2(bc-ad)} - \frac{c^2 \log(c+dx^4)}{4d^2(bc-ad)} + \frac{x^4}{4bd}$$

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 72}

$$\frac{a^2 \log(a+bx^4)}{4b^2(bc-ad)} - \frac{c^2 \log(c+dx^4)}{4d^2(bc-ad)} + \frac{x^4}{4bd}$$

Antiderivative was successfully verified.

[In] Int[x^11/((a + b*x^4)*(c + d*x^4)), x]

[Out] x^4/(4*b*d) + (a^2*Log[a + b*x^4])/(4*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^4])/(4*d^2*(b*c - a*d))

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^{11}}{(a+bx^4)(c+dx^4)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^2}{(a+bx)(c+dx)} dx, x, x^4 \right) \\ &= \frac{1}{4} \text{Subst} \left(\int \left(\frac{1}{bd} + \frac{a^2}{b(bc-ad)(a+bx)} + \frac{c^2}{d(-bc+ad)(c+dx)} \right) dx, x, x^4 \right) \\ &= \frac{x^4}{4bd} + \frac{a^2 \log(a+bx^4)}{4b^2(bc-ad)} - \frac{c^2 \log(c+dx^4)}{4d^2(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.94

$$\frac{a^2 d^2 \log(a + bx^4) - b(dx^4(ad - bc) + bc^2 \log(c + dx^4))}{4b^2 d^2 (bc - ad)}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((a + b*x^4)*(c + d*x^4)),x]

[Out] (a^2*d^2*Log[a + b*x^4] - b*(d*(-(b*c) + a*d)*x^4 + b*c^2*Log[c + d*x^4]))/(4*b^2*d^2*(b*c - a*d))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^11/((a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[x^11/((a + b*x^4)*(c + d*x^4)), x]

fricas [A] time = 1.68, size = 72, normalized size = 1.03

$$\frac{(b^2 cd - abd^2)x^4 + a^2 d^2 \log(bx^4 + a) - b^2 c^2 \log(dx^4 + c)}{4(b^3 cd^2 - ab^2 d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out] 1/4*((b^2*c*d - a*b*d^2)*x^4 + a^2*d^2*log(b*x^4 + a) - b^2*c^2*log(d*x^4 + c))/(b^3*c*d^2 - a*b^2*d^3)

giac [A] time = 0.20, size = 70, normalized size = 1.00

$$\frac{x^4}{4bd} + \frac{a^2 \log(|bx^4 + a|)}{4(b^3c - ab^2d)} - \frac{c^2 \log(|dx^4 + c|)}{4(bcd^2 - ad^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] 1/4*x^4/(b*d) + 1/4*a^2*log(abs(b*x^4 + a))/(b^3*c - a*b^2*d) - 1/4*c^2*log(abs(d*x^4 + c))/(b*c*d^2 - a*d^3)

maple [A] time = 0.05, size = 65, normalized size = 0.93

$$\frac{x^4}{4bd} - \frac{a^2 \ln(bx^4 + a)}{4(ad - bc)b^2} + \frac{c^2 \ln(dx^4 + c)}{4(ad - bc)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/(b*x^4+a)/(d*x^4+c),x)

[Out] 1/4*x^4/b/d+1/4*c^2/d^2/(a*d-b*c)*ln(d*x^4+c)-1/4*a^2/b^2/(a*d-b*c)*ln(b*x^4+a)

maxima [A] time = 0.57, size = 68, normalized size = 0.97

$$\frac{x^4}{4bd} + \frac{a^2 \log(bx^4 + a)}{4(b^3c - ab^2d)} - \frac{c^2 \log(dx^4 + c)}{4(bcd^2 - ad^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out] 1/4*x^4/(b*d) + 1/4*a^2*log(b*x^4 + a)/(b^3*c - a*b^2*d) - 1/4*c^2*log(d*x^4 + c)/(b*c*d^2 - a*d^3)

mupad [B] time = 5.63, size = 68, normalized size = 0.97

$$\frac{a^2 \ln(bx^4 + a)}{4b^3c - 4ab^2d} + \frac{c^2 \ln(dx^4 + c)}{4ad^3 - 4bcd^2} + \frac{x^4}{4bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/((a + b*x^4)*(c + d*x^4)),x)

[Out] (a^2*log(a + b*x^4))/(4*b^3*c - 4*a*b^2*d) + (c^2*log(c + d*x^4))/(4*a*d^3 - 4*b*c*d^2) + x^4/(4*b*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11/(b*x**4+a)/(d*x**4+c),x)

[Out] Timed out

$$3.495 \quad \int \frac{x^7}{(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=53

$$\frac{c \log(c + dx^4)}{4d(bc - ad)} - \frac{a \log(a + bx^4)}{4b(bc - ad)}$$

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 72}

$$\frac{c \log(c + dx^4)}{4d(bc - ad)} - \frac{a \log(a + bx^4)}{4b(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[x^7/((a + b*x^4)*(c + d*x^4)),x]

[Out] -(a*Log[a + b*x^4])/(4*b*(b*c - a*d)) + (c*Log[c + d*x^4])/(4*d*(b*c - a*d))

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^7}{(a+bx^4)(c+dx^4)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x}{(a+bx)(c+dx)} dx, x, x^4 \right) \\ &= \frac{1}{4} \text{Subst} \left(\int \left(-\frac{a}{(bc-ad)(a+bx)} + \frac{c}{(bc-ad)(c+dx)} \right) dx, x, x^4 \right) \\ &= -\frac{a \log(a+bx^4)}{4b(bc-ad)} + \frac{c \log(c+dx^4)}{4d(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.81

$$\frac{ad \log(a + bx^4) - bc \log(c + dx^4)}{4b^2cd - 4abd^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/((a + b*x^4)*(c + d*x^4)), x]

[Out] -((a*d*Log[a + b*x^4] - b*c*Log[c + d*x^4])/(4*b^2*c*d - 4*a*b*d^2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/((a + b*x^4)*(c + d*x^4)), x]

[Out] IntegrateAlgebraic[x^7/((a + b*x^4)*(c + d*x^4)), x]

fricas [A] time = 0.75, size = 42, normalized size = 0.79

$$\frac{ad \log(bx^4 + a) - bc \log(dx^4 + c)}{4(b^2cd - abd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^4+a)/(d*x^4+c), x, algorithm="fricas")

[Out] -1/4*(a*d*log(b*x^4 + a) - b*c*log(d*x^4 + c))/(b^2*c*d - a*b*d^2)

giac [A] time = 0.19, size = 51, normalized size = 0.96

$$-\frac{a \log(|bx^4 + a|)}{4(b^2c - abd)} + \frac{c \log(|dx^4 + c|)}{4(bcd - ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^4+a)/(d*x^4+c), x, algorithm="giac")

[Out] -1/4*a*log(abs(b*x^4 + a))/(b^2*c - a*b*d) + 1/4*c*log(abs(d*x^4 + c))/(b*c*d - a*d^2)

maple [A] time = 0.05, size = 50, normalized size = 0.94

$$\frac{a \ln(bx^4 + a)}{4(ad - bc)b} - \frac{c \ln(dx^4 + c)}{4(ad - bc)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(b*x^4+a)/(d*x^4+c),x)

[Out] -1/4*c/(a*d-b*c)/d*ln(d*x^4+c)+1/4*a/(a*d-b*c)/b*ln(b*x^4+a)

maxima [A] time = 0.62, size = 49, normalized size = 0.92

$$-\frac{a \log(bx^4 + a)}{4(b^2c - abd)} + \frac{c \log(dx^4 + c)}{4(bcd - ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out] -1/4*a*log(b*x^4 + a)/(b^2*c - a*b*d) + 1/4*c*log(d*x^4 + c)/(b*c*d - a*d^2)

mupad [B] time = 5.10, size = 51, normalized size = 0.96

$$-\frac{a \ln(bx^4 + a)}{4b^2c - 4abd} - \frac{c \ln(dx^4 + c)}{4ad^2 - 4bcd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/((a + b*x^4)*(c + d*x^4)),x)

[Out] - (a*log(a + b*x^4))/(4*b^2*c - 4*a*b*d) - (c*log(c + d*x^4))/(4*a*d^2 - 4*b*c*d)

sympy [B] time = 60.85, size = 144, normalized size = 2.72

$$\frac{a \log\left(x^4 + \frac{\frac{a^3d^2}{b(ad-bc)} - \frac{2a^2cd}{ad-bc} + \frac{abc^2}{ad-bc} + 2ac}{ad+bc}\right)}{4b(ad-bc)} - \frac{c \log\left(x^4 + \frac{-\frac{a^2cd}{ad-bc} + \frac{2abc^2}{ad-bc} + 2ac - \frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{4d(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(b*x**4+a)/(d*x**4+c),x)

[Out] a*log(x**4 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**2/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(4*b*(a*d - b*c)) - c*log(x**4 + (-a**2*c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b*c)))/(a*d + b*c))/(4*d*(a*d - b*c))

$$3.496 \quad \int \frac{x^3}{(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=45

$$\frac{\log(a+bx^4)}{4(bc-ad)} - \frac{\log(c+dx^4)}{4(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {444, 36, 31}

$$\frac{\log(a+bx^4)}{4(bc-ad)} - \frac{\log(c+dx^4)}{4(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^3/((a + b*x^4)*(c + d*x^4)),x]

[Out] Log[a + b*x^4]/(4*(b*c - a*d)) - Log[c + d*x^4]/(4*(b*c - a*d))

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 444

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a + bx^4)(c + dx^4)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{(a + bx)(c + dx)} dx, x, x^4 \right) \\
&= \frac{b \text{Subst} \left(\int \frac{1}{a+bx} dx, x, x^4 \right) - d \text{Subst} \left(\int \frac{1}{c+dx} dx, x, x^4 \right)}{4(bc - ad)} \\
&= \frac{\log(a + bx^4)}{4(bc - ad)} - \frac{\log(c + dx^4)}{4(bc - ad)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.69

$$\frac{\log(a + bx^4) - \log(c + dx^4)}{4bc - 4ad}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((a + b*x^4)*(c + d*x^4)),x]

[Out] (Log[a + b*x^4] - Log[c + d*x^4])/(4*b*c - 4*a*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/((a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[x^3/((a + b*x^4)*(c + d*x^4)), x]

fricas [A] time = 0.42, size = 31, normalized size = 0.69

$$\frac{\log(bx^4 + a) - \log(dx^4 + c)}{4(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out] 1/4*(log(b*x^4 + a) - log(d*x^4 + c))/(b*c - a*d)

giac [A] time = 0.34, size = 51, normalized size = 1.13

$$\frac{b \log(|bx^4 + a|)}{4(b^2c - abd)} - \frac{d \log(|dx^4 + c|)}{4(bcd - ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] 1/4*b*log(abs(b*x^4 + a))/(b^2*c - a*b*d) - 1/4*d*log(abs(d*x^4 + c))/(b*c*d - a*d^2)

maple [A] time = 0.05, size = 42, normalized size = 0.93

$$-\frac{\ln(bx^4 + a)}{4(ad - bc)} + \frac{\ln(dx^4 + c)}{4ad - 4bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^4+a)/(d*x^4+c),x)

[Out] 1/4/(a*d-b*c)*ln(d*x^4+c)-1/4/(a*d-b*c)*ln(b*x^4+a)

maxima [A] time = 0.52, size = 41, normalized size = 0.91

$$\frac{\log(bx^4 + a)}{4(bc - ad)} - \frac{\log(dx^4 + c)}{4(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out] 1/4*log(b*x^4 + a)/(b*c - a*d) - 1/4*log(d*x^4 + c)/(b*c - a*d)

mupad [B] time = 4.99, size = 1012, normalized size = 22.49

$$\frac{\operatorname{atan}\left(\frac{\frac{b^2 c^2 d^2 x^4 + 4 a b c d^2 x^2 + a^2 d^2}{4 a d - 4 b c}}{\frac{b^2 c^2 d^2 x^4 + 4 a b c d^2 x^2 + a^2 d^2}{4 a d - 4 b c}}\right)}{4 a d - 4 b c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + b*x^4)*(c + d*x^4)),x)

[Out] -(atan((((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) + ((x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4

$$\begin{aligned}
& 4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + x^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) + 8*b^4*d^4*x^4)/(4*a*d - 4*b*c) - (((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) - (x^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) - (x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) - 8*b^4*d^4*x^4)/(4*a*d - 4*b*c)))/(((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) + ((x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + x^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) + 8*b^4*d^4*x^4)/(4*a*d - 4*b*c) + ((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) - (x^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) - (x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) - 8*b^4*d^4*x^4)/(4*a*d - 4*b*c)))*2i)/(4*a*d - 4*b*c)
\end{aligned}$$

sympy [B] time = 1.91, size = 138, normalized size = 3.07

$$\frac{\log\left(x^4 + \frac{-\frac{a^2d^2}{ad-bc} + \frac{2abcd}{ad-bc} + ad - \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{4(ad-bc)} - \frac{\log\left(x^4 + \frac{\frac{a^2d^2}{ad-bc} - \frac{2abcd}{ad-bc} + ad + \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{4(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x**4+a)/(d*x**4+c),x)

[Out] log(x**4 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**2/(a*d - b*c) + b*c)/(2*b*d))/(4*(a*d - b*c)) - log(x**4 + (a**2*d**2/(a*d - b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d))/(4*(a*d - b*c))

$$3.497 \quad \int \frac{1}{x(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=62

$$-\frac{b \log(a+bx^4)}{4a(bc-ad)} + \frac{d \log(c+dx^4)}{4c(bc-ad)} + \frac{\log(x)}{ac}$$

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 72}

$$-\frac{b \log(a+bx^4)}{4a(bc-ad)} + \frac{d \log(c+dx^4)}{4c(bc-ad)} + \frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^4)*(c + d*x^4)),x]

[Out] Log[x]/(a*c) - (b*Log[a + b*x^4])/(4*a*(b*c - a*d)) + (d*Log[c + d*x^4])/(4*c*(b*c - a*d))

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a+bx^4)(c+dx^4)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x(a+bx)(c+dx)} dx, x, x^4 \right) \\ &= \frac{1}{4} \text{Subst} \left(\int \left(\frac{1}{acx} + \frac{b^2}{a(-bc+ad)(a+bx)} + \frac{d^2}{c(bc-ad)(c+dx)} \right) dx, x, x^4 \right) \\ &= \frac{\log(x)}{ac} - \frac{b \log(a+bx^4)}{4a(bc-ad)} + \frac{d \log(c+dx^4)}{4c(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.87

$$\frac{-bc \log(a + bx^4) + ad \log(c + dx^4) - 4ad \log(x) + 4bc \log(x)}{4abc^2 - 4a^2cd}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^4)*(c + d*x^4)),x]

[Out] (4*b*c*Log[x] - 4*a*d*Log[x] - b*c*Log[a + b*x^4] + a*d*Log[c + d*x^4])/(4*a*b*c^2 - 4*a^2*c*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[1/(x*(a + b*x^4)*(c + d*x^4)), x]

fricas [A] time = 2.00, size = 54, normalized size = 0.87

$$\frac{bc \log(bx^4 + a) - ad \log(dx^4 + c) - 4(bc - ad) \log(x)}{4(abc^2 - a^2cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out] -1/4*(b*c*log(b*x^4 + a) - a*d*log(d*x^4 + c) - 4*(b*c - a*d)*log(x))/(a*b*c^2 - a^2*c*d)

giac [A] time = 0.18, size = 73, normalized size = 1.18

$$-\frac{b^2 \log(|bx^4 + a|)}{4(ab^2c - a^2bd)} + \frac{d^2 \log(|dx^4 + c|)}{4(bc^2d - acd^2)} + \frac{\log(x^4)}{4ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] -1/4*b^2*log(abs(b*x^4 + a))/(a*b^2*c - a^2*b*d) + 1/4*d^2*log(abs(d*x^4 + c))/(b*c^2*d - a*c*d^2) + 1/4*log(x^4)/(a*c)

maple [A] time = 0.06, size = 59, normalized size = 0.95

$$\frac{b \ln(bx^4 + a)}{4(ad - bc)a} - \frac{d \ln(dx^4 + c)}{4(ad - bc)c} + \frac{\ln(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^4+a)/(d*x^4+c), x)

[Out] -1/4*d/c/(a*d-b*c)*ln(d*x^4+c)+1/4*b/a/(a*d-b*c)*ln(b*x^4+a)+1/a/c*ln(x)

maxima [A] time = 0.71, size = 61, normalized size = 0.98

$$-\frac{b \log(bx^4 + a)}{4(abc - a^2d)} + \frac{d \log(dx^4 + c)}{4(bc^2 - acd)} + \frac{\log(x^4)}{4ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^4+a)/(d*x^4+c), x, algorithm="maxima")

[Out] -1/4*b*log(b*x^4 + a)/(a*b*c - a^2*d) + 1/4*d*log(d*x^4 + c)/(b*c^2 - a*c*d) + 1/4*log(x^4)/(a*c)

mupad [B] time = 5.49, size = 58, normalized size = 0.94

$$\frac{b \ln(bx^4 + a)}{4a^2d - 4abc} + \frac{d \ln(dx^4 + c)}{4bc^2 - 4acd} + \frac{\ln(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^4)*(c + d*x^4)), x)

[Out] (b*log(a + b*x^4))/(4*a^2*d - 4*a*b*c) + (d*log(c + d*x^4))/(4*b*c^2 - 4*a*c*d) + log(x)/(a*c)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**4+a)/(d*x**4+c), x)

[Out] Timed out

$$3.498 \quad \int \frac{1}{x^5(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=87

$$\frac{b^2 \log(a+bx^4)}{4a^2(bc-ad)} - \frac{\log(x)(ad+bc)}{a^2c^2} - \frac{d^2 \log(c+dx^4)}{4c^2(bc-ad)} - \frac{1}{4acx^4}$$

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 72}

$$\frac{b^2 \log(a+bx^4)}{4a^2(bc-ad)} - \frac{\log(x)(ad+bc)}{a^2c^2} - \frac{d^2 \log(c+dx^4)}{4c^2(bc-ad)} - \frac{1}{4acx^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]

[Out] -1/(4*a*c*x^4) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^4])/(4*a^2*(b*c - a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^5(a+bx^4)(c+dx^4)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x^2(a+bx)(c+dx)} dx, x, x^4 \right) \\ &= \frac{1}{4} \text{Subst} \left(\int \left(\frac{1}{acx^2} + \frac{-bc-ad}{a^2c^2x} - \frac{b^3}{a^2(-bc+ad)(a+bx)} - \frac{d^3}{c^2(bc-ad)(c+dx)} \right) dx, \right. \\ &= -\frac{1}{4acx^4} - \frac{(bc+ad)\log(x)}{a^2c^2} + \frac{b^2 \log(a+bx^4)}{4a^2(bc-ad)} - \frac{d^2 \log(c+dx^4)}{4c^2(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 1.01

$$-\frac{b^2 \log(a + bx^4)}{4a^2(ad - bc)} + \frac{\log(x)(-ad - bc)}{a^2c^2} - \frac{d^2 \log(c + dx^4)}{4c^2(bc - ad)} - \frac{1}{4acx^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]

[Out] -1/4*1/(a*c*x^4) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^4])/(4*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 (a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[1/(x^5*(a + b*x^4)*(c + d*x^4)), x]

fricas [A] time = 8.27, size = 99, normalized size = 1.14

$$\frac{b^2c^2x^4 \log(bx^4 + a) - a^2d^2x^4 \log(dx^4 + c) - 4(b^2c^2 - a^2d^2)x^4 \log(x) - abc^2 + a^2cd}{4(a^2bc^3 - a^3c^2d)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out] 1/4*(b^2*c^2*x^4*log(b*x^4 + a) - a^2*d^2*x^4*log(d*x^4 + c) - 4*(b^2*c^2 - a^2*d^2)*x^4*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^4)

giac [A] time = 0.19, size = 112, normalized size = 1.29

$$\frac{b^3 \log(|bx^4 + a|)}{4(a^2b^2c - a^3bd)} - \frac{d^3 \log(|dx^4 + c|)}{4(bc^3d - ac^2d^2)} - \frac{(bc + ad) \log(x^4)}{4a^2c^2} + \frac{bcx^4 + adx^4 - ac}{4a^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] 1/4*b^3*log(abs(b*x^4 + a))/(a^2*b^2*c - a^3*b*d) - 1/4*d^3*log(abs(d*x^4 + c))/(b*c^3*d - a*c^2*d^2) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) + 1/4*(b*c*x^4 + a*d*x^4 - a*c)/(a^2*c^2*x^4)

maple [A] time = 0.06, size = 87, normalized size = 1.00

$$-\frac{b^2 \ln(bx^4 + a)}{4(ad - bc)a^2} + \frac{d^2 \ln(dx^4 + c)}{4(ad - bc)c^2} - \frac{d \ln(x)}{ac^2} - \frac{b \ln(x)}{a^2c} - \frac{1}{4acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(b*x^4+a)/(d*x^4+c),x)

[Out] 1/4*d^2/c^2/(a*d-b*c)*ln(d*x^4+c)-1/4*b^2/a^2/(a*d-b*c)*ln(b*x^4+a)-1/4/a/c/x^4-1/a/c^2*ln(x)*d-1/a^2/c*ln(x)*b

maxima [A] time = 0.55, size = 87, normalized size = 1.00

$$\frac{b^2 \log(bx^4 + a)}{4(a^2bc - a^3d)} - \frac{d^2 \log(dx^4 + c)}{4(bc^3 - ac^2d)} - \frac{(bc + ad) \log(x^4)}{4a^2c^2} - \frac{1}{4acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out] 1/4*b^2*log(b*x^4 + a)/(a^2*b*c - a^3*d) - 1/4*d^2*log(d*x^4 + c)/(b*c^3 - a*c^2*d) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) - 1/4/(a*c*x^4)

mupad [B] time = 6.21, size = 87, normalized size = 1.00

$$\frac{b^2 \ln(bx^4 + a)}{4(a^3d - a^2bc)} - \frac{d^2 \ln(dx^4 + c)}{4(bc^3 - ac^2d)} - \frac{1}{4acx^4} - \frac{\ln(x)(ad + bc)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(a + b*x^4)*(c + d*x^4)),x)

[Out] -(b^2*log(a + b*x^4))/(4*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^4))/(4*(b*c^3 - a*c^2*d)) - 1/(4*a*c*x^4) - (log(x)*(a*d + b*c))/(a^2*c^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(b*x**4+a)/(d*x**4+c),x)

[Out] Timed out

$$3.499 \quad \int \frac{x^{13}}{(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=112

$$-\frac{a^{5/2} \tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)} - \frac{x^2(ad+bc)}{2b^2d^2} + \frac{c^{5/2} \tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)} + \frac{x^6}{6bd}$$

Rubi [A] time = 0.27, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {465, 479, 582, 522, 205}

$$-\frac{a^{5/2} \tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)} - \frac{x^2(ad+bc)}{2b^2d^2} + \frac{c^{5/2} \tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)} + \frac{x^6}{6bd}$$

Antiderivative was successfully verified.

[In] Int[x^13/((a + b*x^4)*(c + d*x^4)),x]

[Out] -((b*c + a*d)*x^2)/(2*b^2*d^2) + x^6/(6*b*d) - (a^(5/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*b^(5/2)*(b*c - a*d)) + (c^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*d^(5/2)*(b*c - a*d))

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 465

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 479

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG

tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 582

Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(f*g^(n - 1)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1) + 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{13}}{(a + bx^4)(c + dx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^6}{(a + bx^2)(c + dx^2)} dx, x, x^2 \right) \\
 &= \frac{x^6}{6bd} - \frac{\text{Subst} \left(\int \frac{x^2(3ac + 3(bc + ad)x^2)}{(a + bx^2)(c + dx^2)} dx, x, x^2 \right)}{6bd} \\
 &= -\frac{(bc + ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} + \frac{\text{Subst} \left(\int \frac{3ac(bc + ad) + 3(b^2c^2 + ad(bc + ad))x^2}{(a + bx^2)(c + dx^2)} dx, x, x^2 \right)}{6b^2d^2} \\
 &= -\frac{(bc + ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} - \frac{a^3 \text{Subst} \left(\int \frac{1}{a + bx^2} dx, x, x^2 \right)}{2b^2(bc - ad)} + \frac{c^3 \text{Subst} \left(\int \frac{1}{c + dx^2} dx, x, x^2 \right)}{2d^2(bc - ad)} \\
 &= -\frac{(bc + ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} - \frac{a^{5/2} \tan^{-1} \left(\frac{\sqrt{b}x^2}{\sqrt{a}} \right)}{2b^{5/2}(bc - ad)} + \frac{c^{5/2} \tan^{-1} \left(\frac{\sqrt{d}x^2}{\sqrt{c}} \right)}{2d^{5/2}(bc - ad)}
 \end{aligned}$$

Mathematica [A] time = 0.17, size = 104, normalized size = 0.93

$$\frac{1}{6} \left(\frac{3a^{5/2} \tan^{-1} \left(\frac{\sqrt{b}x^2}{\sqrt{a}} \right)}{b^{5/2}(ad - bc)} + \frac{x^2(-3ad - 3bc + bdx^4)}{b^2d^2} + \frac{3c^{5/2} \tan^{-1} \left(\frac{\sqrt{d}x^2}{\sqrt{c}} \right)}{d^{5/2}(bc - ad)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^13/((a + b*x^4)*(c + d*x^4)), x]

[Out] ((x^2*(-3*b*c - 3*a*d + b*d*x^4))/(b^2*d^2) + (3*a^(5/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(b^(5/2)*(-b*c) + a*d)) + (3*c^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(d^(5/2)*(b*c - a*d)))/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{(a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^13/((a + b*x^4)*(c + d*x^4)), x]

[Out] IntegrateAlgebraic[x^13/((a + b*x^4)*(c + d*x^4)), x]

fricas [A] time = 2.64, size = 576, normalized size = 5.14

$$\frac{2 \left((b^2 d - a b d^2) x^6 - 3 a^2 d^2 \sqrt{-a/b} \log \left(\frac{b x^4 + 2 b x^2 \sqrt{-a/b} - a}{b x^4 + a} \right) - 3 b^2 c^2 \sqrt{-c/d} \log \left(\frac{d x^4 - 2 d x^2 \sqrt{-c/d} - c}{d x^4 + c} \right) - 6 (b^2 c^2 - a^2 d^2) x^2 \right) / (b^3 c d^2 - a b^2 d^3) + 1/12 * (2 * (b^2 * c * d - a * b * d^2) * x^6 - 6 * a^2 * d^2 * \sqrt{a/b} * \arctan(b * x^2 * \sqrt{a/b} / a) - 3 * b^2 * c^2 * \sqrt{-c/d} * \log((d * x^4 - 2 * d * x^2 * \sqrt{-c/d} - c) / (d * x^4 + c)) - 6 * (b^2 * c^2 - a^2 * d^2) * x^2) / (b^3 * c * d^2 - a * b^2 * d^3), 1/12 * (2 * (b^2 * c * d - a * b * d^2) * x^6 + 6 * b^2 * c^2 * \sqrt{c/d} * \arctan(d * x^2 * \sqrt{c/d} / c) - 3 * a^2 * d^2 * \sqrt{-a/b} * \log((b * x^4 + 2 * b * x^2 * \sqrt{-a/b} - a) / (b * x^4 + a)) - 6 * (b^2 * c^2 - a^2 * d^2) * x^2) / (b^3 * c * d^2 - a * b^2 * d^3), 1/6 * ((b^2 * c * d - a * b * d^2) * x^6 - 3 * a^2 * d^2 * \sqrt{a/b} * \arctan(b * x^2 * \sqrt{a/b} / a) + 3 * b^2 * c^2 * \sqrt{c/d} * \arctan(d * x^2 * \sqrt{c/d} / c) - 3 * (b^2 * c^2 - a^2 * d^2) * x^2) / (b^3 * c * d^2 - a * b^2 * d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^13/(b*x^4+a)/(d*x^4+c), x, algorithm="fricas")

[Out] [1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 3*a^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 6*a^2*d^2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/12*(2*(b^2*c*d - a*b*d^2)*x^6 + 6*b^2*c^2*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) - 3*a^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/6*((b^2*c*d - a*b*d^2)*x^6 - 3*a^2*d^2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + 3*b^2*c^2*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) - 3*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3)]

giac [A] time = 0.19, size = 112, normalized size = 1.00

$$-\frac{a^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(b^3c - ab^2d)\sqrt{ab}} + \frac{c^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bcd^2 - ad^3)\sqrt{cd}} + \frac{b^2d^2x^6 - 3b^2cdx^2 - 3abd^2x^2}{6b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] -1/2*a^3*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b)) + 1/2*c^3*arctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt(c*d)) + 1/6*(b^2*d^2*x^6 - 3*b^2*c*d*x^2 - 3*a*b*d^2*x^2)/(b^3*d^3)

maple [A] time = 0.06, size = 105, normalized size = 0.94

$$\frac{x^6}{6bd} + \frac{a^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(ad - bc)\sqrt{ab}b^2} - \frac{c^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(ad - bc)\sqrt{cd}d^2} - \frac{ax^2}{2b^2d} - \frac{cx^2}{2bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^13/(b*x^4+a)/(d*x^4+c),x)

[Out] 1/6*x^6/b/d-1/2/b^2/d*x^2*a-1/2/b/d^2*x^2*c-1/2*c^3/d^2/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))+1/2*a^3/b^2/(a*d-b*c)/(a*b)^(1/2)*arctan(x^2*b/(a*b)^(1/2))

maxima [A] time = 1.32, size = 100, normalized size = 0.89

$$-\frac{a^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(b^3c - ab^2d)\sqrt{ab}} + \frac{c^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bcd^2 - ad^3)\sqrt{cd}} + \frac{bdx^6 - 3(bc + ad)x^2}{6b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out] -1/2*a^3*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b)) + 1/2*c^3*arctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt(c*d)) + 1/6*(b*d*x^6 - 3*(b*c + a*d)*x^2)/(b^2*d^2)

mupad [B] time = 5.70, size = 532, normalized size = 4.75

$\frac{b^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right) + c^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right) + \frac{bdx^6 - 3(bc + ad)x^2}{6b^2d^2}}{2(b^3c - ab^2d)\sqrt{ab} + 2(bcd^2 - ad^3)\sqrt{cd}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^13/((a + b*x^4)*(c + d*x^4)),x)`

[Out] $(\log(d^{10}(-a^5b^5)^{5/2}) + b^{20}c^{10}(-a^5b^5)^{1/2} - a^2b^{23}c^{10}x^2 - a^{12}b^{13}d^{10}x^2 + 2b^{10}c^5d^5(-a^5b^5)^{3/2} + 2a^7b^{18}c^5d^5x^2)(-a^5b^5)^{1/2})/(4b^6c - 4ab^5d) - (\log(d^{10}(-a^5b^5)^{5/2}) + b^{20}c^{10}(-a^5b^5)^{1/2} + a^2b^{23}c^{10}x^2 + a^{12}b^{13}d^{10}x^2 + 2b^{10}c^5d^5(-a^5b^5)^{3/2} - 2a^7b^{18}c^5d^5x^2)(-a^5b^5)^{1/2})/(4(b^6c - ab^5d)) - (\log(b^{10}(-c^5d^5)^{5/2}) + a^{10}d^{20}(-c^5d^5)^{1/2} + a^{10}c^2d^{23}x^2 + b^{10}c^{12}d^{13}x^2 + 2a^5b^5d^{10}(-c^5d^5)^{3/2} - 2a^5b^5c^7d^{18}x^2)(-c^5d^5)^{1/2})/(4(ad^6 - bcd^5)) + (\log(b^{10}(-c^5d^5)^{5/2}) + a^{10}d^{20}(-c^5d^5)^{1/2} - a^{10}c^2d^{23}x^2 - b^{10}c^{12}d^{13}x^2 + 2a^5b^5d^{10}(-c^5d^5)^{3/2} + 2a^5b^5c^7d^{18}x^2)(-c^5d^5)^{1/2})/(4ad^6 - 4bcd^5) + x^6/(6bd) - (x^2(ad + bc))/(2b^2d^2)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**13/(b*x**4+a)/(d*x**4+c),x)`

[Out] Timed out

$$3.500 \quad \int \frac{x^9}{(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=92

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)} - \frac{c^{3/2} \tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)} + \frac{x^2}{2bd}$$

Rubi [A] time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {465, 479, 522, 205}

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)} - \frac{c^{3/2} \tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)} + \frac{x^2}{2bd}$$

Antiderivative was successfully verified.

[In] Int[x^9/((a + b*x^4)*(c + d*x^4)),x]

[Out] x^2/(2*b*d) + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*b^(3/2)*(b*c - a*d)) - (c^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*d^(3/2)*(b*c - a*d))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 479

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{x^9}{(a + bx^4)(c + dx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{(a + bx^2)(c + dx^2)} dx, x, x^2 \right) \\
 &= \frac{x^2}{2bd} - \frac{\text{Subst} \left(\int \frac{ac + (bc + ad)x^2}{(a + bx^2)(c + dx^2)} dx, x, x^2 \right)}{2bd} \\
 &= \frac{x^2}{2bd} + \frac{a^2 \text{Subst} \left(\int \frac{1}{a + bx^2} dx, x, x^2 \right)}{2b(bc - ad)} - \frac{c^2 \text{Subst} \left(\int \frac{1}{c + dx^2} dx, x, x^2 \right)}{2d(bc - ad)} \\
 &= \frac{x^2}{2bd} + \frac{a^{3/2} \tan^{-1} \left(\frac{\sqrt{b}x^2}{\sqrt{a}} \right)}{2b^{3/2}(bc - ad)} - \frac{c^{3/2} \tan^{-1} \left(\frac{\sqrt{d}x^2}{\sqrt{c}} \right)}{2d^{3/2}(bc - ad)}
 \end{aligned}$$

Mathematica [A] time = 0.14, size = 82, normalized size = 0.89

$$\frac{\frac{a^{3/2} \tan^{-1} \left(\frac{\sqrt{b}x^2}{\sqrt{a}} \right)}{b^{3/2}} + x^2 \left(\frac{c}{d} - \frac{a}{b} \right) - \frac{c^{3/2} \tan^{-1} \left(\frac{\sqrt{d}x^2}{\sqrt{c}} \right)}{d^{3/2}}}{2bc - 2ad}$$

Antiderivative was successfully verified.

[In] Integrate[x^9/((a + b*x^4)*(c + d*x^4)),x]

[Out] ((-(a/b) + c/d)*x^2 + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/b^(3/2) - (c^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/d^(3/2))/(2*b*c - 2*a*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^9/((a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[x^9/((a + b*x^4)*(c + d*x^4)), x]

fricas [A] time = 0.79, size = 416, normalized size = 4.52

$$\left[\frac{ad\sqrt{\frac{a}{b}} \log\left(\frac{bx^4-2bx^2\sqrt{\frac{a}{b}}-a}{bx^4+a}\right) + bc\sqrt{\frac{c}{d}} \log\left(\frac{dx^4+2dx^2\sqrt{\frac{c}{d}}-c}{dx^4+c}\right) - 2(bc-ad)x^2}{4(b^2cd-abd^2)}, \frac{2ad\sqrt{\frac{a}{b}} \arctan\left(\frac{bx^2}{\sqrt{\frac{a}{b}}}\right) - bc\sqrt{\frac{c}{d}} \log\left(\frac{dx^4+2dx^2\sqrt{\frac{c}{d}}-c}{dx^4+c}\right) + 2(bc-ad)x^2}{4(b^2cd-abd^2)}, \frac{2bc\sqrt{\frac{c}{d}} \arctan\left(\frac{dx^2}{\sqrt{\frac{c}{d}}}\right) + ad\sqrt{\frac{a}{b}} \log\left(\frac{bx^4-2bx^2\sqrt{\frac{a}{b}}-a}{bx^4+a}\right) - 2(bc-ad)x^2}{4(b^2cd-abd^2)}, \frac{ad\sqrt{\frac{a}{b}} \arctan\left(\frac{bx^2}{\sqrt{\frac{a}{b}}}\right) - bc\sqrt{\frac{c}{d}} \arctan\left(\frac{dx^2}{\sqrt{\frac{c}{d}}}\right) + (bc-ad)x^2}{2(b^2cd-abd^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out] [-1/4*(a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) + b*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) - 2*(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/4*(2*a*d*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) - b*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) + 2*(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), -1/4*(2*b*c*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) + a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) - 2*(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/2*(a*d*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) - b*c*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) + (b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2)]

giac [A] time = 0.18, size = 80, normalized size = 0.87

$$\frac{a^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(b^2c - abd)\sqrt{ab}} - \frac{c^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bcd - ad^2)\sqrt{cd}} + \frac{x^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] 1/2*a^2*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - 1/2*c^2*arctan(d*x^2/sqrt(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + 1/2*x^2/(b*d)

maple [A] time = 0.06, size = 81, normalized size = 0.88

$$-\frac{a^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(ad - bc)\sqrt{ab}b} + \frac{c^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(ad - bc)\sqrt{cd}d} + \frac{x^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(b*x^4+a)/(d*x^4+c),x)

[Out] 1/2/b/d*x^2+1/2*c^2/d/(a*d-b*c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x^2)-1/2*a^2/b/(a*d-b*c)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^2)

maxima [A] time = 1.19, size = 80, normalized size = 0.87

$$\frac{a^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(b^2c - abd)\sqrt{ab}} - \frac{c^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bcd - ad^2)\sqrt{cd}} + \frac{x^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out] 1/2*a^2*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - 1/2*c^2*arctan(d*x^2/sqrt(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + 1/2*x^2/(b*d)

mupad [B] time = 5.72, size = 518, normalized size = 5.63

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/((a + b*x^4)*(c + d*x^4)),x)

[Out] (log(b^9*c^6*(-a^3*b^3)^(1/2) - a^3*d^6*(-a^3*b^3)^(3/2) + a*b^11*c^6*x^2 + a^7*b^5*d^6*x^2 + 2*b^3*c^3*d^3*(-a^3*b^3)^(3/2) - 2*a^4*b^8*c^3*d^3*x^2)*(-a^3*b^3)^(1/2))/(4*b^4*c - 4*a*b^3*d) - (log(a^3*d^6*(-a^3*b^3)^(3/2) - b^9*c^6*(-a^3*b^3)^(1/2) + a*b^11*c^6*x^2 + a^7*b^5*d^6*x^2 - 2*b^3*c^3*d^3*(-a^3*b^3)^(3/2) - 2*a^4*b^8*c^3*d^3*x^2)*(-a^3*b^3)^(1/2))/(4*(b^4*c - a*b^3*d)) - (log(b^6*c^3*(-c^3*d^3)^(3/2) - a^6*d^9*(-c^3*d^3)^(1/2) + a^6*c*d^11*x^2 + b^6*c^7*d^5*x^2 - 2*a^3*b^3*d^3*(-c^3*d^3)^(3/2) - 2*a^3*b^3*c^4*d^8*x^2)*(-c^3*d^3)^(1/2))/(4*(a*d^4 - b*c*d^3)) + (log(a^6*d^9*(-c^3*d^3)^(1/2) - b^6*c^3*(-c^3*d^3)^(3/2) + a^6*c*d^11*x^2 + b^6*c^7*d^5*x^2 + 2*a^3*b^3*d^3*(-c^3*d^3)^(3/2) - 2*a^3*b^3*c^4*d^8*x^2)*(-c^3*d^3)^(1/2))/(4*a*d^4 - 4*b*c*d^3) + x^2/(2*b*d)

sympy [B] time = 29.70, size = 932, normalized size = 10.13

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9/(b*x**4+a)/(d*x**4+c),x)

[Out] -sqrt(-a**3/b**3)*log(x**2 + (-a**4*d**4*sqrt(-a**3/b**3)/(a*d - b*c) - a**3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + a**2*b**4*c*d**5*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - b**4*c**4*sqrt(-a**3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**3))/(4*(a

$$\begin{aligned}
& d - b*c)) + \sqrt{-a**3/b**3}*\log(x**2 + (a**4*d**4*\sqrt{-a**3/b**3})/(a*d - \\
& b*c) + a**3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a**2*b**4*c*d**5 \\
& *(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/ \\
& (a*d - b*c)**3 + b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + b**4*c \\
& **4*\sqrt{-a**3/b**3}/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c** \\
& 3))/(4*(a*d - b*c)) - \sqrt{-c**3/d**3}*\log(x**2 + (-a**4*d**4*\sqrt{-c**3/d \\
& **3})/(a*d - b*c) - a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a**2* \\
& b**4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-c**3/d \\
& **3)**(3/2)/(a*d - b*c)**3 - b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a*d - b*c)* \\
& **3 - b**4*c**4*\sqrt{-c**3/d**3}/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + \\
& a*b**2*c**3))/(4*(a*d - b*c)) + \sqrt{-c**3/d**3}*\log(x**2 + (a**4*d**4*\sqrt{-c \\
& **3/d**3})/(a*d - b*c) + a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)* \\
& **3 - a**2*b**4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4 \\
& *(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a \\
& *d - b*c)**3 + b**4*c**4*\sqrt{-c**3/d**3}/(a*d - b*c))/(a**3*c*d**2 + a**2* \\
& b*c**2*d + a*b**2*c**3))/(4*(a*d - b*c)) + x**2/(2*b*d)
\end{aligned}$$

$$3.501 \quad \int \frac{x^5}{(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=79

$$\frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)}$$

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {465, 481, 205}

$$\frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^5/((a + b*x^4)*(c + d*x^4)),x]

[Out] -(Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[b]*(b*c - a*d)) + (Sqrt[c]*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]]/(2*Sqrt[d]*(b*c - a*d)))

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 465

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 481

Int[((e_)*(x_))^(m_)/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x], x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a+bx^4)(c+dx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(a+bx^2)(c+dx^2)} dx, x, x^2 \right) \\
&= -\frac{a \text{Subst} \left(\int \frac{1}{a+bx^2} dx, x, x^2 \right) + c \text{Subst} \left(\int \frac{1}{c+dx^2} dx, x, x^2 \right)}{2(bc-ad)} \\
&= -\frac{\sqrt{a} \tan^{-1} \left(\frac{\sqrt{b}x^2}{\sqrt{a}} \right)}{2\sqrt{b}(bc-ad)} + \frac{\sqrt{c} \tan^{-1} \left(\frac{\sqrt{d}x^2}{\sqrt{c}} \right)}{2\sqrt{d}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.84

$$\frac{\frac{\sqrt{c} \tan^{-1} \left(\frac{\sqrt{d}x^2}{\sqrt{c}} \right)}{\sqrt{d}} - \frac{\sqrt{a} \tan^{-1} \left(\frac{\sqrt{b}x^2}{\sqrt{a}} \right)}{\sqrt{b}}}{2bc - 2ad}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^4)*(c + d*x^4)), x]

[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[b]) + (Sqrt[c]*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/Sqrt[d])/(2*b*c - 2*a*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/((a + b*x^4)*(c + d*x^4)), x]

[Out] IntegrateAlgebraic[x^5/((a + b*x^4)*(c + d*x^4)), x]

fricas [A] time = 0.49, size = 325, normalized size = 4.11

$$\left[\frac{\sqrt{\frac{a}{b}} \log \left(\frac{bx^4 + 2bx^2\sqrt{\frac{a}{b}} - a}{bx^4 + a} \right) + \sqrt{\frac{c}{d}} \log \left(\frac{dx^4 - 2dx^2\sqrt{\frac{c}{d}} - c}{dx^4 + c} \right)}{4(bc-ad)}, \frac{2\sqrt{\frac{a}{b}} \arctan \left(\frac{bx^2\sqrt{\frac{a}{b}}}{a} \right) + \sqrt{\frac{c}{d}} \log \left(\frac{dx^4 - 2dx^2\sqrt{\frac{c}{d}} - c}{dx^4 + c} \right)}{4(bc-ad)}, \frac{2\sqrt{\frac{c}{d}} \arctan \left(\frac{dx^2\sqrt{\frac{c}{d}}}{c} \right) - \sqrt{\frac{a}{b}} \log \left(\frac{bx^4 + 2bx^2\sqrt{\frac{a}{b}} - a}{bx^4 + a} \right)}{4(bc-ad)}, \frac{\sqrt{\frac{a}{b}} \arctan \left(\frac{bx^2\sqrt{\frac{a}{b}}}{a} \right) - \sqrt{\frac{c}{d}} \arctan \left(\frac{dx^2\sqrt{\frac{c}{d}}}{c} \right)}{2(bc-ad)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^4+a)/(d*x^4+c), x, algorithm="fricas")

[Out] $[-1/4*(\sqrt{-a/b}*\log((b*x^4 + 2*b*x^2*\sqrt{-a/b}) - a)/(b*x^4 + a)) + \sqrt{-c/d}*\log((d*x^4 - 2*d*x^2*\sqrt{-c/d}) - c)/(d*x^4 + c))/(b*c - a*d), -1/4*(2*\sqrt{a/b}*\arctan(b*x^2*\sqrt{a/b}/a) + \sqrt{-c/d}*\log((d*x^4 - 2*d*x^2*\sqrt{-c/d}) - c)/(d*x^4 + c))/(b*c - a*d), 1/4*(2*\sqrt{c/d}*\arctan(d*x^2*\sqrt{c/d}/c) - \sqrt{-a/b}*\log((b*x^4 + 2*b*x^2*\sqrt{-a/b}) - a)/(b*x^4 + a))/(b*c - a*d), -1/2*(\sqrt{a/b}*\arctan(b*x^2*\sqrt{a/b}/a) - \sqrt{c/d}*\arctan(d*x^2*\sqrt{c/d}/c))/(b*c - a*d)]$

giac [A] time = 0.17, size = 59, normalized size = 0.75

$$-\frac{a \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} + \frac{c \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")`

[Out] $-1/2*a*\arctan(b*x^2/\sqrt{a*b})/(\sqrt{a*b}*(b*c - a*d)) + 1/2*c*\arctan(d*x^2/\sqrt{c*d})/((b*c - a*d)*\sqrt{c*d})$

maple [A] time = 0.06, size = 60, normalized size = 0.76

$$\frac{a \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(ad-bc)\sqrt{ab}} - \frac{c \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(ad-bc)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(b*x^4+a)/(d*x^4+c),x)`

[Out] $-1/2*c/(a*d-b*c)/(c*d)^{(1/2)}*\arctan(1/(c*d)^{(1/2)}*d*x^2)+1/2*a/(a*d-b*c)/(a*b)^{(1/2)}*\arctan(1/(a*b)^{(1/2)}*b*x^2)$

maxima [A] time = 1.16, size = 59, normalized size = 0.75

$$-\frac{a \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} + \frac{c \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")`

[Out] $-1/2*a*\arctan(b*x^2/\sqrt{a*b})/(\sqrt{a*b}*(b*c - a*d)) + 1/2*c*\arctan(d*x^2/\sqrt{c*d})/((b*c - a*d)*\sqrt{c*d})$

mupad [B] time = 5.34, size = 379, normalized size = 4.80

$$\frac{\ln(\frac{d^2(-ab)^{5/2} + b^4c^2\sqrt{-ab} - b^2c^2d^2 + 2b^2cd(-ab)^{3/2} - d^2b^2d^2 + 2ab^4cd^2}{4b^2c - 4abd})}{4(b^2c - abd)} - \frac{\ln(\frac{d^2(-ab)^{5/2} + b^4c^2\sqrt{-ab} + b^2c^2d^2 + 2b^2cd(-ab)^{3/2} + d^2b^2d^2 - 2ab^4cd^2}{4(b^2c - abd)})}{4(b^2c - abd)} - \frac{\ln(\frac{d^2(-cd)^{5/2} + d^2d^2\sqrt{-cd} + d^2d^2d^2 + 2abd^2(-cd)^{3/2} + b^2c^2d^2 - 2ab^2cd^2}{4(b^2c - abd)})}{4(b^2c - abd)} + \frac{\ln(\frac{d^2(-cd)^{5/2} + d^2d^2\sqrt{-cd} - d^2d^2d^2 + 2abd^2(-cd)^{3/2} - b^2c^2d^2 + 2ab^2cd^2}{4ab^2 - 4bcd})}{4ab^2 - 4bcd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((a + b*x^4)*(c + d*x^4)),x)

[Out] $(\log(d^2(-a*b)^{(5/2)} + b^4*c^2*(-a*b)^{(1/2)} - b^5*c^2*x^2 + 2*b^2*c*d*(-a*b)^{(3/2)} - a^2*b^3*d^2*x^2 + 2*a*b^4*c*d*x^2)*(-a*b)^{(1/2)})/(4*b^2*c - 4*a*b*d) - (\log(d^2(-a*b)^{(5/2)} + b^4*c^2*(-a*b)^{(1/2)} + b^5*c^2*x^2 + 2*b^2*c*d*(-a*b)^{(3/2)} + a^2*b^3*d^2*x^2 - 2*a*b^4*c*d*x^2)*(-a*b)^{(1/2)})/(4*(b^2*c - a*b*d)) - (\log(b^2*(-c*d)^{(5/2)} + a^2*d^4*(-c*d)^{(1/2)} + a^2*d^5*x^2 + 2*a*b*d^2*(-c*d)^{(3/2)} + b^2*c^2*d^3*x^2 - 2*a*b*c*d^4*x^2)*(-c*d)^{(1/2)})/(4*(a*d^2 - b*c*d)) + (\log(b^2*(-c*d)^{(5/2)} + a^2*d^4*(-c*d)^{(1/2)} - a^2*d^5*x^2 + 2*a*b*d^2*(-c*d)^{(3/2)} - b^2*c^2*d^3*x^2 + 2*a*b*c*d^4*x^2)*(-c*d)^{(1/2)})/(4*a*d^2 - 4*b*c*d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x**4+a)/(d*x**4+c),x)

[Out] Timed out

$$3.502 \quad \int \frac{x}{(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=79

$$\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)}$$

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {465, 391, 205}

$$\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x/((a + b*x^4)*(c + d*x^4)),x]

[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*Sqrt[c]*(b*c - a*d))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 391

Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a+bx^4)(c+dx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(a+bx^2)(c+dx^2)} dx, x, x^2 \right) \\
&= \frac{b \text{Subst} \left(\int \frac{1}{a+bx^2} dx, x, x^2 \right) - d \text{Subst} \left(\int \frac{1}{c+dx^2} dx, x, x^2 \right)}{2(bc-ad)} \\
&= \frac{\sqrt{b} \tan^{-1} \left(\frac{\sqrt{b}x^2}{\sqrt{a}} \right) - \sqrt{d} \tan^{-1} \left(\frac{\sqrt{d}x^2}{\sqrt{c}} \right)}{2\sqrt{a}(bc-ad) - 2\sqrt{c}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.84

$$\frac{\frac{\sqrt{b} \tan^{-1} \left(\frac{\sqrt{b}x^2}{\sqrt{a}} \right)}{\sqrt{a}} - \frac{\sqrt{d} \tan^{-1} \left(\frac{\sqrt{d}x^2}{\sqrt{c}} \right)}{\sqrt{c}}}{2bc - 2ad}$$

Antiderivative was successfully verified.

[In] Integrate[x/((a + b*x^4)*(c + d*x^4)),x]

[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/Sqrt[c])/(2*b*c - 2*a*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^4)(c+dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/((a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[x/((a + b*x^4)*(c + d*x^4)), x]

fricas [A] time = 0.50, size = 325, normalized size = 4.11

$$\left[\frac{\sqrt{\frac{b}{a}} \log \left(\frac{bx^4 - 2ax^2 \sqrt{\frac{b}{a}} - a}{bx^4 + a} \right) + \sqrt{\frac{d}{c}} \log \left(\frac{dx^4 + 2cx^2 \sqrt{\frac{d}{c}} - c}{dx^4 + c} \right)}{4(bc-ad)}, \frac{2\sqrt{\frac{d}{c}} \arctan \left(\frac{c\sqrt{\frac{d}{c}}}{dx^2} \right) - \sqrt{\frac{b}{a}} \log \left(\frac{bx^4 - 2ax^2 \sqrt{\frac{b}{a}} - a}{bx^4 + a} \right)}{4(bc-ad)}, \frac{2\sqrt{\frac{b}{a}} \arctan \left(\frac{a\sqrt{\frac{b}{a}}}{bx^2} \right) + \sqrt{\frac{d}{c}} \log \left(\frac{dx^4 + 2cx^2 \sqrt{\frac{d}{c}} - c}{dx^4 + c} \right)}{4(bc-ad)}, \frac{\sqrt{\frac{b}{a}} \arctan \left(\frac{a\sqrt{\frac{b}{a}}}{bx^2} \right) - \sqrt{\frac{d}{c}} \arctan \left(\frac{c\sqrt{\frac{d}{c}}}{dx^2} \right)}{2(bc-ad)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out] $[-1/4*(\sqrt{-b/a}*\log((b*x^4 - 2*a*x^2*\sqrt{-b/a}) - a)/(b*x^4 + a)) + \sqrt{-d/c}*\log((d*x^4 + 2*c*x^2*\sqrt{-d/c}) - c)/(d*x^4 + c))/(b*c - a*d), 1/4*(2*\sqrt{d/c}*\arctan(c*\sqrt{d/c}/(d*x^2)) - \sqrt{-b/a}*\log((b*x^4 - 2*a*x^2*\sqrt{-b/a}) - a)/(b*x^4 + a))/(b*c - a*d), -1/4*(2*\sqrt{b/a}*\arctan(a*\sqrt{b/a}/(b*x^2)) + \sqrt{-d/c}*\log((d*x^4 + 2*c*x^2*\sqrt{-d/c}) - c)/(d*x^4 + c))/(b*c - a*d), -1/2*(\sqrt{b/a}*\arctan(a*\sqrt{b/a}/(b*x^2)) - \sqrt{d/c}*\arctan(c*\sqrt{d/c}/(d*x^2)))/(b*c - a*d)]$

giac [A] time = 0.18, size = 59, normalized size = 0.75

$$\frac{b \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc - ad)} - \frac{d \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc - ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")`

[Out] $1/2*b*\arctan(b*x^2/\sqrt{a*b})/(\sqrt{a*b}*(b*c - a*d)) - 1/2*d*\arctan(d*x^2/\sqrt{c*d})/((b*c - a*d)*\sqrt{c*d})$

maple [A] time = 0.06, size = 60, normalized size = 0.76

$$-\frac{b \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(ad - bc)\sqrt{ab}} + \frac{d \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(ad - bc)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(b*x^4+a)/(d*x^4+c),x)`

[Out] $1/2*d/(a*d-b*c)/(c*d)^{(1/2)}*\arctan(1/(c*d)^{(1/2)}*d*x^2) - 1/2*b/(a*d-b*c)/(a*b)^{(1/2)}*\arctan(1/(a*b)^{(1/2)}*b*x^2)$

maxima [A] time = 1.28, size = 59, normalized size = 0.75

$$\frac{b \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc - ad)} - \frac{d \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc - ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")`

[Out] $1/2*b*\arctan(b*x^2/\sqrt{a*b})/(\sqrt{a*b}*(b*c - a*d)) - 1/2*d*\arctan(d*x^2/\sqrt{c*d})/((b*c - a*d)*\sqrt{c*d})$

mupad [B] time = 5.29, size = 399, normalized size = 5.05

$$\frac{\ln\left(\frac{d^2 d^2 (c+d)^{10} + d^2 d^2 (c+d)^{10} + 2 d^2 d^2 (c+d)^{10} - d^2 d^2 d^2 - d^2 d^2 d^2 + 2 d^2 d^2 d^2}{4 d^2 d^2 - 4 a b c}\right) \sqrt{a b} - \ln\left(\frac{d^2 d^2 (c+d)^{10} + d^2 d^2 (c+d)^{10} + 2 c d (c+d)^{10} + d^2 d^2 d^2 + d^2 d^2 d^2 - 2 d^2 d^2 d^2}{4 (d^2 d^2 - a b c)}\right) \sqrt{a b}}{\ln\left(\frac{d^2 d^2 (c+d)^{10} + d^2 d^2 (c+d)^{10} + 2 a b (c+d)^{10} + d^2 d^2 d^2 + d^2 d^2 d^2 - 2 a b d^2 d^2}{4 (b^2 d^2 - a c d)}\right) \sqrt{-d}} - \frac{\ln\left(\frac{d^2 d^2 (c+d)^{10} + d^2 d^2 (c+d)^{10} + 2 a b (c+d)^{10} - d^2 d^2 d^2 - d^2 d^2 d^2 + 2 a b d^2 d^2}{4 b^2 d^2 - 4 a c d}\right) \sqrt{-d}}{4 b^2 d^2 - 4 a c d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + b*x^4)*(c + d*x^4)),x)

[Out] $(\log(a^2 d^2 (-a b)^{5/2} + b^2 c^2 (-a b)^{5/2} + 2 c d (-a b)^{7/2} - a^2 b^5 c^2 x^2 - a^4 b^3 d^2 x^2 + 2 a^3 b^4 c d x^2) (-a b)^{1/2}) / (4 a^2 d - 4 a b c) - (\log(a^2 d^2 (-a b)^{5/2} + b^2 c^2 (-a b)^{5/2} + 2 c d (-a b)^{7/2} + a^2 b^5 c^2 x^2 + a^4 b^3 d^2 x^2 - 2 a^3 b^4 c d x^2) (-a b)^{1/2}) / (4 (a^2 d - a b c)) - (\log(a^2 d^2 (-c d)^{5/2} + b^2 c^2 (-c d)^{5/2} + 2 a b (-c d)^{7/2} + a^2 c^2 d^5 x^2 + b^2 c^4 d^3 x^2 - 2 a b c^3 d^4 x^2) (-c d)^{1/2}) / (4 (b c^2 - a c d)) + (\log(a^2 d^2 (-c d)^{5/2} + b^2 c^2 (-c d)^{5/2} + 2 a b (-c d)^{7/2} - a^2 c^2 d^5 x^2 - b^2 c^4 d^3 x^2 + 2 a b c^3 d^4 x^2) (-c d)^{1/2}) / (4 b c^2 - 4 a c d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**4+a)/(d*x**4+c),x)

[Out] Timed out

$$3.503 \quad \int \frac{1}{x^3(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=92

$$-\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)} + \frac{d^{3/2} \tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)} - \frac{1}{2acx^2}$$

Rubi [A] time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {465, 480, 522, 205}

$$-\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)} + \frac{d^{3/2} \tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)} - \frac{1}{2acx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]

[Out] -1/(2*a*c*x^2) - (b^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)) + (d^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p)*((c_) + (d_.)*(x_)^(n_.))^q, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 480

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p)*((c_) + (d_.)*(x_)^(n_.))^q, x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e^(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 (a + bx^4)(c + dx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2 (a + bx^2)(c + dx^2)} dx, x, x^2 \right) \\ &= -\frac{1}{2acx^2} + \frac{\text{Subst} \left(\int \frac{-bc - ad - bdx^2}{(a + bx^2)(c + dx^2)} dx, x, x^2 \right)}{2ac} \\ &= -\frac{1}{2acx^2} - \frac{b^2 \text{Subst} \left(\int \frac{1}{a + bx^2} dx, x, x^2 \right)}{2a(bc - ad)} + \frac{d^2 \text{Subst} \left(\int \frac{1}{c + dx^2} dx, x, x^2 \right)}{2c(bc - ad)} \\ &= -\frac{1}{2acx^2} - \frac{b^{3/2} \tan^{-1} \left(\frac{\sqrt{b}x^2}{\sqrt{a}} \right)}{2a^{3/2}(bc - ad)} + \frac{d^{3/2} \tan^{-1} \left(\frac{\sqrt{d}x^2}{\sqrt{c}} \right)}{2c^{3/2}(bc - ad)} \end{aligned}$$

Mathematica [A] time = 0.23, size = 169, normalized size = 1.84

$$\frac{-\frac{b^{3/2}x^2 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt{a}}\right)}{a^{3/2}} - \frac{b^{3/2}x^2 \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt{a}} + 1\right)}{a^{3/2}} + \frac{b}{a} + \frac{d^{3/2}x^2 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d}x}{\sqrt{c}}\right)}{c^{3/2}} + \frac{d^{3/2}x^2 \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{d}x}{\sqrt{c}} + 1\right)}{c^{3/2}} - \frac{d}{c}}{2x^2(ad - bc)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a + b*x^4)*(c + d*x^4)), x]

[Out] (b/a - d/c - (b^(3/2)*x^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2) - (b^(3/2)*x^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2) + (d^(3/2)*x^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2) + (d^(3/2)*x^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2))/(2*(-(b*c) + a*d)*x^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^4)*(c + d*x^4)), x]

fricas [A] time = 0.88, size = 432, normalized size = 4.70

$$\frac{\frac{bcx^2\sqrt{\frac{c}{d}}\log\left(\frac{bx^4+2ax^2\sqrt{\frac{c}{d}}-a}{bx^4+a}\right)+adx^2\sqrt{\frac{c}{d}}\log\left(\frac{bx^4+2ax^2\sqrt{\frac{c}{d}}-c}{dx^4+c}\right)+2bc-2ad}{4(abc^2-a^2cd)x^2}, \frac{2adx^2\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{\frac{c}{d}}}{bx^2}\right)+bcx^2\sqrt{\frac{c}{d}}\log\left(\frac{bx^4+2ax^2\sqrt{\frac{c}{d}}-a}{bx^4+a}\right)+2bc-2ad}{4(abc^2-a^2cd)x^2}, \frac{2bcx^2\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{\frac{c}{d}}}{bx^2}\right)-adx^2\sqrt{\frac{c}{d}}\log\left(\frac{bx^4+2ax^2\sqrt{\frac{c}{d}}-c}{dx^4+c}\right)-2bc+2ad}{4(abc^2-a^2cd)x^2}, \frac{bcx^2\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{\frac{c}{d}}}{bx^2}\right)-adx^2\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{\frac{c}{d}}}{dx^2}\right)-bc+ad}{2(abc^2-a^2cd)x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out] $[-1/4*(b*c*x^2*\sqrt{-b/a}*\log((b*x^4 + 2*a*x^2*\sqrt{-b/a} - a)/(b*x^4 + a)) + a*d*x^2*\sqrt{-d/c}*\log((d*x^4 - 2*c*x^2*\sqrt{-d/c} - c)/(d*x^4 + c)) + 2*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), -1/4*(2*a*d*x^2*\sqrt{d/c}*\arctan(c*\sqrt{d/c}/(d*x^2)) + b*c*x^2*\sqrt{-b/a}*\log((b*x^4 + 2*a*x^2*\sqrt{-b/a} - a)/(b*x^4 + a)) + 2*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), 1/4*(2*b*c*x^2*\sqrt{b/a}*\arctan(a*\sqrt{b/a}/(b*x^2)) - a*d*x^2*\sqrt{-d/c}*\log((d*x^4 - 2*c*x^2*\sqrt{-d/c} - c)/(d*x^4 + c)) - 2*b*c + 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), 1/2*(b*c*x^2*\sqrt{b/a}*\arctan(a*\sqrt{b/a}/(b*x^2)) - a*d*x^2*\sqrt{d/c}*\arctan(c*\sqrt{d/c}/(d*x^2)) - b*c + a*d)/((a*b*c^2 - a^2*c*d)*x^2)]$

giac [A] time = 0.21, size = 80, normalized size = 0.87

$$-\frac{b^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(abc - a^2d)\sqrt{ab}} + \frac{d^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc^2 - acd)\sqrt{cd}} - \frac{1}{2acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] $-1/2*b^2*\arctan(b*x^2/\sqrt{a*b})/((a*b*c - a^2*d)*\sqrt{a*b}) + 1/2*d^2*\arctan(d*x^2/\sqrt{c*d})/((b*c^2 - a*c*d)*\sqrt{c*d}) - 1/2/(a*c*x^2)$

maple [A] time = 0.06, size = 81, normalized size = 0.88

$$\frac{b^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(ad - bc)\sqrt{ab}a} - \frac{d^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(ad - bc)\sqrt{cd}c} - \frac{1}{2acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x^4+a)/(d*x^4+c),x)

[Out] $-1/2*d^2/c/(a*d-b*c)/(c*d)^{(1/2)}*\arctan(1/(c*d)^{(1/2)}*d*x^2)-1/2/a/c/x^2+1/2*b^2/a/(a*d-b*c)/(a*b)^{(1/2)}*\arctan(1/(a*b)^{(1/2)}*b*x^2)$

maxima [A] time = 1.20, size = 80, normalized size = 0.87

$$-\frac{b^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(abc - a^2d)\sqrt{ab}} + \frac{d^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc^2 - acd)\sqrt{cd}} - \frac{1}{2acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out] $-1/2*b^2*\arctan(b*x^2/\sqrt{a*b})/((a*b*c - a^2*d)*\sqrt{a*b}) + 1/2*d^2*\arctan(d*x^2/\sqrt{c*d})/((b*c^2 - a*c*d)*\sqrt{c*d}) - 1/2/(a*c*x^2)$

mupad [B] time = 5.35, size = 354, normalized size = 3.85

$$\frac{\ln\left(\frac{c^3x^2(-a^3b)^{3/2} - a^3b^3c^3 + a^3d^3x^2\sqrt{-a^3b^3}}{4a^3d - 4a^3bc}\right)\sqrt{-a^3b^3}}{4(a^3d - a^3bc)} - \frac{\ln\left(\frac{c^3x^2(-a^3b)^{3/2} + a^3b^3c^3 - a^3d^3x^2\sqrt{-a^3b^3}}{4(a^3d - a^3bc)}\right)\sqrt{-a^3b^3}}{4(a^3d - a^3bc)} - \frac{1}{2acx^2} - \frac{\ln\left(\frac{b^3x^2(-c^3d)^{3/2} + b^3c^3d - a^3c^3d^3 + b^3c^3x^2\sqrt{-c^3d^3}}{4(b^3c^2 - ac^2d)}\right)\sqrt{-c^3d^3}}{4(b^3c^2 - ac^2d)} + \frac{\ln\left(\frac{b^3x^2(-c^3d)^{3/2} - b^3c^3d + a^3c^3d^3 + b^3c^3x^2\sqrt{-c^3d^3}}{4b^3c^2 - 4ac^2d}\right)\sqrt{-c^3d^3}}{4b^3c^2 - 4ac^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b*x^4)*(c + d*x^4)),x)

[Out] $(\log(c^3*x^2*(-a^3*b^3)^{(3/2)} - a^8*b*d^3 + a^5*b^4*c^3 + a^6*d^3*x^2*(-a^3*b^3)^{(1/2)})*(-a^3*b^3)^{(1/2)})/(4*a^4*d - 4*a^3*b*c) - (\log(c^3*x^2*(-a^3*b^3)^{(3/2)} + a^8*b*d^3 - a^5*b^4*c^3 + a^6*d^3*x^2*(-a^3*b^3)^{(1/2)})*(-a^3*b^3)^{(1/2)})/(4*(a^4*d - a^3*b*c)) - 1/(2*a*c*x^2) - (\log(a^3*x^2*(-c^3*d^3)^{(3/2)} + b^3*c^8*d - a^3*c^5*d^4 + b^3*c^6*x^2*(-c^3*d^3)^{(1/2)})*(-c^3*d^3)^{(1/2)})/(4*(b*c^4 - a*c^3*d)) + (\log(a^3*x^2*(-c^3*d^3)^{(3/2)} - b^3*c^8*d + a^3*c^5*d^4 + b^3*c^6*x^2*(-c^3*d^3)^{(1/2)})*(-c^3*d^3)^{(1/2)})/(4*b*c^4 - 4*a*c^3*d)$

sympy [B] time = 84.89, size = 1103, normalized size = 11.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x**4+a)/(d*x**4+c),x)

[Out] $-\sqrt{-b**3/a**3}*\log(x**2 + (-a**7*c**3*d**4*(-b**3/a**3)**(3/2))/(a*d - b*c)**3 + 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2))/(a*d - b*c)**3 - 2*a**5*b**2*c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*\sqrt{-b**3/a**3}/(a*d - b*c) + 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**3*b**4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*\sqrt{-b**3/a**3}/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(4*(a*d - b*c)) + \sqrt{-b**3/a**3}*\log(x**2 + (a**7*c**3*d**4*(-b**3/a**3)**(3/2))/(a*d - b*c)**3 + 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2))/(a*d - b*c)**3 - 2*a**5*b**2*c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*\sqrt{-b**3/a**3}/(a*d - b*c) + 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**3*b**4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*\sqrt{-b**3/a**3}/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(4*(a*d - b*c))$

$$\begin{aligned}
& - b*c)^**3 - 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + 2*a**5* \\
& b**2*c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt(-b**3/a* \\
& **3)/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a \\
& **3*b**4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(-b**3/a** \\
& 3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(4*(a*d \\
& - b*c)) - sqrt(-d**3/c**3)*log(x**2 + (-a**7*c**3*d**4*(-d**3/c**3)**(3/2)/ \\
& (a*d - b*c)**3 + 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - 2* \\
& a**5*b**2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-d* \\
& **3/c**3)/(a*d - b*c) + 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d - b*c)** \\
& 3 - a**3*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-d** \\
& 3/c**3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(4* \\
& (a*d - b*c)) + sqrt(-d**3/c**3)*log(x**2 + (a**7*c**3*d**4*(-d**3/c**3)**(3 \\
& /2)/(a*d - b*c)**3 - 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 \\
& + 2*a**5*b**2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt \\
& (-d**3/c**3)/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d - b* \\
& c)**3 + a**3*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(\\
& -d**3/c**3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2)) \\
& / (4*(a*d - b*c)) - 1/(2*a*c*x**2)
\end{aligned}$$

$$3.504 \quad \int \frac{1}{x^7(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=112

$$\frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)} + \frac{ad+bc}{2a^2c^2x^2} - \frac{d^{5/2} \tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)} - \frac{1}{6acx^6}$$

Rubi [A] time = 0.22, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {465, 480, 583, 522, 205}

$$\frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)} + \frac{ad+bc}{2a^2c^2x^2} - \frac{d^{5/2} \tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)} - \frac{1}{6acx^6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]

[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(2*a^2*c^2*x^2) + (b^(5/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*a^(5/2)*(b*c - a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*c^(5/2)*(b*c - a*d))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 480

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 583

Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m+1)*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*c*g*(m+1)), x] + Dist[1/(a*c*g^n*(m+1)), Int[(g*x)^(m+n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m+1) - e*(b*c + a*d)*(m+n+1) - e*n*(b*c*p + a*d*q) - b*e*d*(m+n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^7 (a + bx^4) (c + dx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^4 (a + bx^2) (c + dx^2)} dx, x, x^2 \right) \\
 &= -\frac{1}{6acx^6} + \frac{\text{Subst} \left(\int \frac{-3(bc+ad)-3bdx^2}{x^2(a+bx^2)(c+dx^2)} dx, x, x^2 \right)}{6ac} \\
 &= -\frac{1}{6acx^6} + \frac{bc + ad}{2a^2c^2x^2} - \frac{\text{Subst} \left(\int \frac{-3(b^2c^2+abcd+a^2d^2)-3bd(bc+ad)x^2}{(a+bx^2)(c+dx^2)} dx, x, x^2 \right)}{6a^2c^2} \\
 &= -\frac{1}{6acx^6} + \frac{bc + ad}{2a^2c^2x^2} + \frac{b^3 \text{Subst} \left(\int \frac{1}{a+bx^2} dx, x, x^2 \right)}{2a^2(bc - ad)} - \frac{d^3 \text{Subst} \left(\int \frac{1}{c+dx^2} dx, x, x^2 \right)}{2c^2(bc - ad)} \\
 &= -\frac{1}{6acx^6} + \frac{bc + ad}{2a^2c^2x^2} + \frac{b^{5/2} \tan^{-1} \left(\frac{\sqrt{b}x^2}{\sqrt{a}} \right)}{2a^{5/2}(bc - ad)} - \frac{d^{5/2} \tan^{-1} \left(\frac{\sqrt{d}x^2}{\sqrt{c}} \right)}{2c^{5/2}(bc - ad)}
 \end{aligned}$$

Mathematica [A] time = 0.26, size = 193, normalized size = 1.72

$$\frac{3b^{5/2}x^6 \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}} \right)}{a^{5/2}} + \frac{3b^{5/2}x^6 \tan^{-1} \left(\frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}} + 1 \right)}{a^{5/2}} - \frac{3b^2x^4}{a^2} + \frac{b}{a} - \frac{3d^{5/2}x^6 \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{d}x}{\sqrt[4]{c}} \right)}{c^{5/2}} - \frac{3d^{5/2}x^6 \tan^{-1} \left(\frac{\sqrt{2} \sqrt[4]{d}x}{\sqrt[4]{c}} + 1 \right)}{c^{5/2}} + \frac{3d^2x^4}{c^2} - \frac{d}{c}$$

$$\frac{\hspace{10em}}{6x^6(ad - bc)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]

[Out] (b/a - d/c - (3*b^2*x^4)/a^2 + (3*d^2*x^4)/c^2 + (3*b^(5/2)*x^6*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(5/2) + (3*b^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(5/2))/(6*(-(b*c) + a*d)*x^6)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 (a + bx^4) (c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[1/(x^7*(a + b*x^4)*(c + d*x^4)), x]

fricas [A] time = 3.45, size = 592, normalized size = 5.29

$$\frac{3 \sqrt{2} b^2 \sqrt{c} \log\left(\frac{a^2 + c \sqrt{c}}{a^2 - c \sqrt{c}}\right) + 3 \sqrt{2} d^2 \sqrt{c} \log\left(\frac{a^2 + c \sqrt{c}}{a^2 - c \sqrt{c}}\right) - 6 (b^2 - a^2) \sqrt{c} + 2 a b c^2 - 2 a^2 c d - 6 \sqrt{2} b^2 \sqrt{c} \arctan\left(\frac{\sqrt{2}}{a}\right) - 3 \sqrt{2} d^2 \sqrt{c} \log\left(\frac{a^2 + c \sqrt{c}}{a^2 - c \sqrt{c}}\right) + 6 (b^2 - a^2) \sqrt{c} + 2 a b c^2 + 2 a^2 c d - 6 \sqrt{2} b^2 \sqrt{c} \arctan\left(\frac{\sqrt{2}}{a}\right) + 3 \sqrt{2} d^2 \sqrt{c} \log\left(\frac{a^2 + c \sqrt{c}}{a^2 - c \sqrt{c}}\right) - 6 (b^2 - a^2) \sqrt{c} + 2 a b c^2 - 2 a^2 c d - 3 \sqrt{2} b^2 \sqrt{c} \arctan\left(\frac{\sqrt{2}}{a}\right) - 3 \sqrt{2} d^2 \sqrt{c} \log\left(\frac{a^2 + c \sqrt{c}}{a^2 - c \sqrt{c}}\right) + a b c^2 - a^2 c d}{12 (a^2 b^2 - a^2 d^2) c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out] [-1/12*(3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a)) + 3*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^6), 1/12*(6*a^2*d^2*x^6*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - 3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a)) + 6*(b^2*c^2 - a^2*d^2)*x^4 - 2*a*b*c^2 + 2*a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^6), -1/12*(6*b^2*c^2*x^6*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) + 3*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^6), -1/6*(3*b^2*c^2*x^6*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - 3*a^2*d^2*x^6*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - 3*(b^2*c^2 - a^2*d^2)*x^4 + a*b*c^2 - a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^6)]

giac [A] time = 0.18, size = 103, normalized size = 0.92

$$\frac{b^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(a^2bc - a^3d)\sqrt{ab}} - \frac{d^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc^3 - ac^2d)\sqrt{cd}} + \frac{3bcx^4 + 3adx^4 - ac}{6a^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] $\frac{1}{2}b^3\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)/((a^2bc - a^3d)\sqrt{ab}) - \frac{1}{2}d^3\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)/((b^3c - ac^2d)\sqrt{cd}) + \frac{1}{6}(3bcx^4 + 3ad^2x^4 - ac)/(a^2c^2x^6)$

maple [A] time = 0.06, size = 105, normalized size = 0.94

$$-\frac{b^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(ad-bc)\sqrt{ab}a^2} + \frac{d^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(ad-bc)\sqrt{cd}c^2} + \frac{d}{2ac^2x^2} + \frac{b}{2a^2cx^2} - \frac{1}{6acx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^7/(b*x^4+a)/(d*x^4+c),x)

[Out] $\frac{1}{2}d^3/c^2/(ad-bc)/(cd)^{1/2}\arctan(1/(cd)^{1/2}dx^2) - \frac{1}{2}b^3/a^2/(ad-bc)/(ab)^{1/2}\arctan(1/(ab)^{1/2}bx^2) - \frac{1}{6}a/c/x^6 + \frac{1}{2}a/c^2/x^2 + \frac{1}{2}a^2/c/x^2b$

maxima [A] time = 1.51, size = 101, normalized size = 0.90

$$\frac{b^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(a^2bc - a^3d)\sqrt{ab}} - \frac{d^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc^3 - ac^2d)\sqrt{cd}} + \frac{3(bc + ad)x^4 - ac}{6a^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out] $\frac{1}{2}b^3\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)/((a^2bc - a^3d)\sqrt{ab}) - \frac{1}{2}d^3\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)/((b^3c - ac^2d)\sqrt{cd}) + \frac{1}{6}(3(b^3c + a^3d)x^4 - ac)/(a^2c^2x^6)$

mupad [B] time = 5.53, size = 535, normalized size = 4.78

$\frac{b^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(a^2bc - a^3d)\sqrt{ab}} - \frac{d^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc^3 - ac^2d)\sqrt{cd}} + \frac{3(bc + ad)x^4 - ac}{6a^2c^2x^6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^7*(a + b*x^4)*(c + d*x^4)),x)

[Out] $(\log(c^{10}(-a^5b^5)^{(5/2)} + a^{20}d^{10}(-a^5b^5)^{(1/2)} - a^{12}b^{13}c^{10}x^2 - a^{22}b^3d^{10}x^2 + 2a^{10}c^5d^5(-a^5b^5)^{(3/2)} + 2a^{17}b^8c^5d^5x^2) * (-a^5b^5)^{(1/2)}) / (4a^6d - 4a^5b^5c) - (\log(c^{10}(-a^5b^5)^{(5/2)} + a^{20}d^{10}(-a^5b^5)^{(1/2)} + a^{12}b^{13}c^{10}x^2 + a^{22}b^3d^{10}x^2 + 2a^{10}c^5d^5(-a^5b^5)^{(3/2)} - 2a^{17}b^8c^5d^5x^2) * (-a^5b^5)^{(1/2)}) / ($

$$4*(a^6*d - a^5*b*c) - (1/(6*a*c) - (x^4*(a*d + b*c))/(2*a^2*c^2))/x^6 - (\log(a^{10}*(-c^5*d^5)^{(5/2)} + b^{10}*c^{20}*(-c^5*d^5)^{(1/2)} + a^{10}*c^{12}*d^{13}*x^2 + b^{10}*c^{22}*d^3*x^2 + 2*a^5*b^5*c^{10}*(-c^5*d^5)^{(3/2)} - 2*a^5*b^5*c^{17}*d^8*x^2)*(-c^5*d^5)^{(1/2)})/(4*(b*c^6 - a*c^5*d)) + (\log(a^{10}*(-c^5*d^5)^{(5/2)} + b^{10}*c^{20}*(-c^5*d^5)^{(1/2)} - a^{10}*c^{12}*d^{13}*x^2 - b^{10}*c^{22}*d^3*x^2 + 2*a^5*b^5*c^{10}*(-c^5*d^5)^{(3/2)} + 2*a^5*b^5*c^{17}*d^8*x^2)*(-c^5*d^5)^{(1/2)})/(4*(b*c^6 - 4*a*c^5*d))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**7/(b*x**4+a)/(d*x**4+c),x)

[Out] Timed out

$$3.505 \quad \int \frac{x^8}{(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=457

$$\frac{a^{5/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} b^{5/4}(bc-ad)} + \frac{a^{5/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} b^{5/4}(bc-ad)} - \frac{a^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} b^{5/4}(bc-ad)} + \frac{a^{5/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} b^{5/4}(bc-ad)}$$

Rubi [A] time = 0.44, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {479, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{a^{5/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} b^{5/4}(bc-ad)} + \frac{a^{5/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} b^{5/4}(bc-ad)} - \frac{a^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} b^{5/4}(bc-ad)} + \frac{a^{5/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} b^{5/4}(bc-ad)} + \frac{c^{5/4} \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} d^{5/4}(bc-ad)} - \frac{c^{5/4} \log\left(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} d^{5/4}(bc-ad)} + \frac{c^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} d^{5/4}(bc-ad)} - \frac{c^{5/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} d^{5/4}(bc-ad)} + \frac{x}{bd}$$

Antiderivative was successfully verified.

[In] Int[x^8/((a + b*x^4)*(c + d*x^4)), x]

[Out] x/(b*d) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IntQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(a+bx^4)(c+dx^4)} dx &= \frac{x}{bd} - \frac{\int \frac{ac+(bc+ad)x^4}{(a+bx^4)(c+dx^4)} dx}{bd} \\
&= \frac{x}{bd} + \frac{a^2 \int \frac{1}{a+bx^4} dx}{b(bc-ad)} - \frac{c^2 \int \frac{1}{c+dx^4} dx}{d(bc-ad)} \\
&= \frac{x}{bd} + \frac{a^{3/2} \int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4} dx}{2b(bc-ad)} + \frac{a^{3/2} \int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4} dx}{2b(bc-ad)} - \frac{c^{3/2} \int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4} dx}{2d(bc-ad)} - \frac{c^{3/2} \int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4} dx}{2d(bc-ad)} \\
&= \frac{x}{bd} + \frac{a^{3/2} \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4b^{3/2}(bc-ad)} + \frac{a^{3/2} \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4b^{3/2}(bc-ad)} - \frac{a^{5/4} \int \frac{\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2} b^{5/4}(bc-ad)} \\
&= \frac{x}{bd} - \frac{a^{5/4} \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{b} x^2)}{4\sqrt{2} b^{5/4}(bc-ad)} + \frac{a^{5/4} \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{b} x^2)}{4\sqrt{2} b^{5/4}(bc-ad)} \\
&= \frac{x}{bd} - \frac{a^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} b^{5/4}(bc-ad)} + \frac{a^{5/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} b^{5/4}(bc-ad)} + \frac{c^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} d^{5/4}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 377, normalized size = 0.82

$$\frac{-\frac{\sqrt{2}a^{5/4}\log(-\sqrt{2}\sqrt[4]{b}x+\sqrt{a}+\sqrt{bx^2})}{b^{5/4}} + \frac{\sqrt{2}a^{5/4}\log(\sqrt{2}\sqrt[4]{b}x+\sqrt{a}+\sqrt{bx^2})}{b^{5/4}} - \frac{2\sqrt{2}a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{5/4}} + \frac{2\sqrt{2}a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{5/4}} - \frac{8cx}{b} + \frac{\sqrt{2}c^{5/4}\log(-\sqrt{2}\sqrt[4]{d}x+\sqrt{c}+\sqrt{dx^2})}{d^{5/4}} - \frac{\sqrt{2}c^{5/4}\log(\sqrt{2}\sqrt[4]{d}x+\sqrt{c}+\sqrt{dx^2})}{d^{5/4}} + \frac{2\sqrt{2}c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{d^{5/4}} - \frac{2\sqrt{2}c^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{d^{5/4}} + \frac{8cx}{d}}{8bc-8ad}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((a + b*x^4)*(c + d*x^4)), x]

[Out] ((-8*a*x)/b + (8*c*x)/d - (2*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(5/4) + (2*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(5/4) + (2*Sqrt[2]*c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/d^(5/4) - (2*Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/d^(5/4) - (Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(5/4) + (Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(5/4) + (Sqrt[2]*c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/d^(5/4) - (Sqrt[2]*c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/d^(5/4))/(8*b*c - 8*a*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/((a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[x^8/((a + b*x^4)*(c + d*x^4)), x]

fricas [B] time = 0.72, size = 1378, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4*(4*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^{1/4}*b*d*\arctan(((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^{3/4}*x - (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^{3/4})*\sqrt{((a^2*x^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2))*\sqrt{-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))}/a^2))/a^4) - 4*(-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^{1/4}*b*d*\arctan(((b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*(-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^{3/4}*x - (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*(-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^{3/4})*\sqrt{((c^2*x^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4))*\sqrt{-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))}/c^2))/c^4) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^{1/4}*b*d*\log(ax + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^{1/4}*(b^2*c - a*b*d)) + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^{1/4}*b*d*\log(ax - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^{1/4}*(b^2*c - a*b*d)) + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^{1/4}*b*d*\log(c*x + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^{1/4}*(b*c*d - a*d^2)) - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^{1/4}*b*d*\log(c*x - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^{1/4}*(b*c*d - a*d^2)) - 4*x)/(b*d) \end{aligned}$$

giac [A] time = 0.22, size = 469, normalized size = 1.03

$$\frac{(ab)^{\frac{1}{4}} a \arctan\left(\frac{\sqrt{2x+\sqrt{2}(x^{\frac{1}{2}})}}{2(x^{\frac{1}{2}})}\right)}{2(\sqrt{2}bc-\sqrt{2}ad^{\frac{1}{2}})} + \frac{(ab)^{\frac{1}{4}} a \arctan\left(\frac{\sqrt{2x-\sqrt{2}(x^{\frac{1}{2}})}}{2(x^{\frac{1}{2}})}\right)}{2(\sqrt{2}bc-\sqrt{2}ad^{\frac{1}{2}})} + \frac{(cd)^{\frac{1}{4}} c \arctan\left(\frac{\sqrt{2x+\sqrt{2}(x^{\frac{1}{2}})}}{2(x^{\frac{1}{2}})}\right)}{2(\sqrt{2}bcd-\sqrt{2}ad^{\frac{1}{2}})} + \frac{(cd)^{\frac{1}{4}} c \arctan\left(\frac{\sqrt{2x-\sqrt{2}(x^{\frac{1}{2}})}}{2(x^{\frac{1}{2}})}\right)}{2(\sqrt{2}bcd-\sqrt{2}ad^{\frac{1}{2}})} + \frac{(ab)^{\frac{1}{4}} a \log\left(x^2 + \sqrt{2}x(x^{\frac{1}{2}}) + \sqrt{2}\right)}{4(\sqrt{2}bc-\sqrt{2}ad^{\frac{1}{2}})} + \frac{(ab)^{\frac{1}{4}} a \log\left(x^2 - \sqrt{2}x(x^{\frac{1}{2}}) + \sqrt{2}\right)}{4(\sqrt{2}bc-\sqrt{2}ad^{\frac{1}{2}})} + \frac{(cd)^{\frac{1}{4}} c \log\left(x^2 + \sqrt{2}x(x^{\frac{1}{2}}) + \sqrt{2}\right)}{4(\sqrt{2}bcd-\sqrt{2}ad^{\frac{1}{2}})} + \frac{(cd)^{\frac{1}{4}} c \log\left(x^2 - \sqrt{2}x(x^{\frac{1}{2}}) + \sqrt{2}\right)}{4(\sqrt{2}bcd-\sqrt{2}ad^{\frac{1}{2}})} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] $\frac{1}{2}*(a*b^3)^{\frac{1}{4}}*a*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2})*(a/b)^{\frac{1}{4}})/(a/b)^{\frac{1}{4}}/(a/b)^{\frac{1}{4}}/(\sqrt{2}*b^3*c - \sqrt{2}*a*b^2*d) + \frac{1}{2}*(a*b^3)^{\frac{1}{4}}*a*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2})*(a/b)^{\frac{1}{4}})/(a/b)^{\frac{1}{4}}/(\sqrt{2}*b^3*c - \sqrt{2}*a*b^2*d) - \frac{1}{2}*(c*d^3)^{\frac{1}{4}}*c*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2})*(c/d)^{\frac{1}{4}})/(c/d)^{\frac{1}{4}}/(\sqrt{2}*b*c*d^2 - \sqrt{2}*a*d^3) - \frac{1}{2}*(c*d^3)^{\frac{1}{4}}*c*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2})*(c/d)^{\frac{1}{4}})/(c/d)^{\frac{1}{4}}/(\sqrt{2}*b*c*d^2 - \sqrt{2}*a*d^3) + \frac{1}{4}*(a*b^3)^{\frac{1}{4}}*a*\log(x^2 + \sqrt{2}x*(a/b)^{\frac{1}{4}} + \sqrt{2}*(a/b))/(\sqrt{2}*b^3*c - \sqrt{2}*a*b^2*d) - \frac{1}{4}*(a*b^3)^{\frac{1}{4}}*a*\log(x^2 - \sqrt{2}x*(a/b)^{\frac{1}{4}} + \sqrt{2}*(a/b))/(\sqrt{2}*b^3*c - \sqrt{2}*a*b^2*d) - \frac{1}{4}*(c*d^3)^{\frac{1}{4}}*c*\log(x^2 + \sqrt{2}x*(c/d)^{\frac{1}{4}} + \sqrt{2}*(c/d))/(\sqrt{2}*b*c*d^2 - \sqrt{2}*a*d^3) + \frac{1}{4}*(c*d^3)^{\frac{1}{4}}*c*\log(x^2 - \sqrt{2}x*(c/d)^{\frac{1}{4}} + \sqrt{2}*(c/d))/(\sqrt{2}*b*c*d^2 - \sqrt{2}*a*d^3) + x/(b*d)$

maple [A] time = 0.06, size = 328, normalized size = 0.72

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} a \arctan\left(\frac{\sqrt{2}x-1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4(ad-bc)b} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} a \arctan\left(\frac{\sqrt{2}x+1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4(ad-bc)b} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} a \ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8(ad-bc)b} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} c \arctan\left(\frac{\sqrt{2}x-1}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4(ad-bc)d} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} c \arctan\left(\frac{\sqrt{2}x+1}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4(ad-bc)d} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} c \ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}\right)}{8(ad-bc)d} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(b*x^4+a)/(d*x^4+c),x)

[Out] $\frac{1}{b*d*x+1/8*d*c/(a*d-b*c)*(c/d)^{\frac{1}{4}}*2^{\frac{1}{2}}*\ln((x^2+(c/d)^{\frac{1}{4}}*x*2^{\frac{1}{2}}+(c/d)^{\frac{1}{2}})/(x^2-(c/d)^{\frac{1}{4}}*x*2^{\frac{1}{2}}+(c/d)^{\frac{1}{2}}))} + \frac{1}{4*d*c/(a*d-b*c)*(c/d)^{\frac{1}{4}}*2^{\frac{1}{2}}*\arctan(2^{\frac{1}{2}}/(c/d)^{\frac{1}{4}}*x+1)} + \frac{1}{4*d*c/(a*d-b*c)*(c/d)^{\frac{1}{4}}*2^{\frac{1}{2}}*\arctan(2^{\frac{1}{2}}/(c/d)^{\frac{1}{4}}*x-1)} - \frac{1}{8*b*a/(a*d-b*c)*(a/b)^{\frac{1}{4}}*2^{\frac{1}{2}}*\ln((x^2+(a/b)^{\frac{1}{4}}*x*2^{\frac{1}{2}}+(a/b)^{\frac{1}{2}})/(x^2-(a/b)^{\frac{1}{4}}*x*2^{\frac{1}{2}}+(a/b)^{\frac{1}{2}}))} - \frac{1}{4*b*a/(a*d-b*c)*(a/b)^{\frac{1}{4}}*2^{\frac{1}{2}}*\arctan(2^{\frac{1}{2}}/(a/b)^{\frac{1}{4}}*x+1)} - \frac{1}{4*b*a/(a*d-b*c)*(a/b)^{\frac{1}{4}}*2^{\frac{1}{2}}*\arctan(2^{\frac{1}{2}}/(a/b)^{\frac{1}{4}}*x-1)}$

maxima [A] time = 1.22, size = 375, normalized size = 0.82

$$\frac{2\sqrt{2}a^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\sqrt{a}\sqrt{2x+\sqrt{2}a^{\frac{1}{2}}}}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\sqrt{2}a^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\sqrt{a}\sqrt{2x-\sqrt{2}a^{\frac{1}{2}}}}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}} + \frac{\sqrt{2}a^{\frac{3}{4}} \log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{2}}x+\sqrt{a}\right)}{b^{\frac{1}{2}}} - \frac{\sqrt{2}a^{\frac{3}{4}} \log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{2}}x+\sqrt{a}\right)}{b^{\frac{1}{2}}} + \frac{2\sqrt{2}c^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{2x+\sqrt{2}c^{\frac{1}{2}}}}{2\sqrt{c}\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}} + \frac{2\sqrt{2}c^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{2x-\sqrt{2}c^{\frac{1}{2}}}}{2\sqrt{c}\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}} + \frac{\sqrt{2}c^{\frac{3}{4}} \log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{2}}x+\sqrt{c}\right)}{d^{\frac{1}{2}}} - \frac{\sqrt{2}c^{\frac{3}{4}} \log\left(\sqrt{d}x^2-\sqrt{2}c^{\frac{1}{2}}x+\sqrt{c}\right)}{d^{\frac{1}{2}}} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out] $\frac{1}{8} \cdot (2 \sqrt{2}) \cdot a^{3/2} \cdot \arctan\left(\frac{1}{2} \sqrt{2} \cdot (2 \sqrt{b} x + \sqrt{2}) \cdot a^{1/4} \cdot b^{1/4}\right) / \sqrt{\sqrt{a} \sqrt{b}} + 2 \sqrt{2} \cdot a^{3/2} \cdot \arctan\left(\frac{1}{2} \sqrt{2} \cdot (2 \sqrt{b} x - \sqrt{2}) \cdot a^{1/4} \cdot b^{1/4}\right) / \sqrt{\sqrt{a} \sqrt{b}} + \sqrt{2} \cdot a^{5/4} \cdot \log(\sqrt{b} x^2 + \sqrt{2}) \cdot a^{1/4} \cdot b^{1/4} x + \sqrt{a} / b^{1/4} - \sqrt{2} \cdot a^{5/4} \cdot \log(\sqrt{b} x^2 - \sqrt{2}) \cdot a^{1/4} \cdot b^{1/4} x + \sqrt{a} / b^{1/4} / (b^2 c - a b d) - \frac{1}{8} \cdot (2 \sqrt{2}) \cdot c^{3/2} \cdot \arctan\left(\frac{1}{2} \sqrt{2} \cdot (2 \sqrt{d} x + \sqrt{2}) \cdot c^{1/4} \cdot d^{1/4}\right) / \sqrt{\sqrt{c} \sqrt{d}} + 2 \sqrt{2} \cdot c^{3/2} \cdot \arctan\left(\frac{1}{2} \sqrt{2} \cdot (2 \sqrt{d} x - \sqrt{2}) \cdot c^{1/4} \cdot d^{1/4}\right) / \sqrt{\sqrt{c} \sqrt{d}} + \sqrt{2} \cdot c^{5/4} \cdot \log(\sqrt{d} x^2 + \sqrt{2}) \cdot c^{1/4} \cdot d^{1/4} x + \sqrt{c} / d^{1/4} - \sqrt{2} \cdot c^{5/4} \cdot \log(\sqrt{d} x^2 - \sqrt{2}) \cdot c^{1/4} \cdot d^{1/4} x + \sqrt{c} / d^{1/4} / (b c d - a d^2) + x / (b d)$

mupad [B] time = 5.63, size = 6361, normalized size = 13.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((a + b*x^4)*(c + d*x^4)),x)

[Out] $\operatorname{atan}\left(\left(-a^5 / (256 b^9 c^4 + 256 a^4 b^5 d^4 - 1024 a^3 b^6 c d^3 + 1536 a^2 b^7 c^2 d^2 - 1024 a b^8 c^3 d)\right)^{1/4} \cdot \left(\left(16(a^3 b^6 c^9 + a^9 c^3 d^6 - a^4 b^5 c^8 d - a^8 b c^4 d^5)\right) / (b d) - (4 x \cdot (-a^5 / (256 b^9 c^4 + 256 a^4 b^5 d^4 - 1024 a^3 b^6 c d^3 + 1536 a^2 b^7 c^2 d^2 - 1024 a b^8 c^3 d)\right)^{3/4} \cdot (256 a^3 b^9 c^8 d^4 - 768 a^4 b^8 c^7 d^5 + 512 a^5 b^7 c^6 d^6 + 512 a^6 b^6 c^5 d^7 - 768 a^7 b^5 c^4 d^8 + 256 a^8 b^4 c^3 d^9)\right) / (b d)\right) \cdot (-a^5 / (256 b^9 c^4 + 256 a^4 b^5 d^4 - 1024 a^3 b^6 c d^3 + 1536 a^2 b^7 c^2 d^2 - 1024 a b^8 c^3 d)\right)^{1/4} - (4 x \cdot (a^4 b^4 c^8 + a^8 c^4 d^4)) / (b d) \cdot i - (-a^5 / (256 b^9 c^4 + 256 a^4 b^5 d^4 - 1024 a^3 b^6 c d^3 + 1536 a^2 b^7 c^2 d^2 - 1024 a b^8 c^3 d)\right)^{1/4} \cdot \left(\left(16(a^3 b^6 c^9 + a^9 c^3 d^6 - a^4 b^5 c^8 d - a^8 b c^4 d^5)\right) / (b d) + (4 x \cdot (-a^5 / (256 b^9 c^4 + 256 a^4 b^5 d^4 - 1024 a^3 b^6 c d^3 + 1536 a^2 b^7 c^2 d^2 - 1024 a b^8 c^3 d)\right)^{3/4} \cdot (256 a^3 b^9 c^8 d^4 - 768 a^4 b^8 c^7 d^5 + 512 a^5 b^7 c^6 d^6 + 512 a^6 b^6 c^5 d^7 - 768 a^7 b^5 c^4 d^8 + 256 a^8 b^4 c^3 d^9)\right) / (b d)\right) \cdot (-a^5 / (256 b^9 c^4 + 256 a^4 b^5 d^4 - 1024 a^3 b^6 c d^3 + 1536 a^2 b^7 c^2 d^2 - 1024 a b^8 c^3 d)\right)^{1/4} + (4 x \cdot (a^4 b^4 c^8 + a^8 c^4 d^4)) / (b d) \cdot i / \left(\left(-a^5 / (256 b^9 c^4 + 256 a^4 b^5 d^4 - 1024 a^3 b^6 c d^3 + 1536 a^2 b^7 c^2 d^2 - 1024 a b^8 c^3 d)\right)^{1/4} \cdot \left(\left(16(a^3 b^6 c^9 + a^9 c^3 d^6 - a^4 b^5 c^8 d - a^8 b c^4 d^5)\right) / (b d) - (4 x \cdot (-a^5 / (256 b^9 c^4 + 256 a^4 b^5 d^4 - 1024 a^3 b^6 c d^3 + 1536 a^2 b^7 c^2 d^2 - 1024 a b^8 c^3 d)\right)^{3/4} \cdot (256 a^3 b^9 c^8 d^4 - 768 a^4 b^8 c^7 d^5 + 512 a^5 b^7 c^6 d^6 + 512 a^6 b^6 c^5 d^7 - 768 a^7 b^5 c^4 d^8 + 256 a^8 b^4 c^3 d^9)\right) / (b d)\right) \cdot (-a^5 / (256 b^9 c^4 + 256 a^4 b^5 d^4 - 1024 a^3 b^6 c d^3 + 1536 a^2 b^7 c^2 d^2 - 1024 a b^8 c^3 d)\right)^{1/4} - (4 x \cdot (a^4 b^4 c^8 + a^8 c^4 d^4)) / (b d) + (-a^5 / (256 b^9 c^4 +$

$$\begin{aligned}
&5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 \\
& - 1024*a^3*b*c*d^8))^{(3/4)}*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 5 \\
& 12*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9) \\
&/ (b*d)) * (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 \\
& - 1024*a^3*b*c*d^8))^{(1/4)} - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4)) / (b*d)) * 1i - (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 \\
& + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(1/4)} * (((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5)) / (b*d) + (4*x*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(3/4)} * (256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)) / (b*d)) * (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(1/4)} + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4)) / (b*d)) * 1i) / ((-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(1/4)} * (((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5)) / (b*d) - (4*x*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(3/4)} * (256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)) / (b*d)) * (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(1/4)} - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4)) / (b*d)) + (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(1/4)} * (((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5)) / (b*d) + (4*x*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(3/4)} * (256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)) / (b*d)) * (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(1/4)} + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4)) / (b*d))))) * (-c^5 / (256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(1/4)} * 2i - 2*atan((((-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(1/4)} * (((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5)) / (b*d) - (x*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(3/4)} * (256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)) * 4i) / (b*d)) * (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(1/4)} * 1i + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4)) / (b*d)) - (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(1/4)} * (((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5)) / (b*d) + (x*(-c^5 / (256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(3/4)} * (256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)) * 4i) / (b*d)) * (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^{(1/4)} * 2i - 2*atan(
\end{aligned}$$

$$d^6 + 1536a^2b^2c^2d^7 - 1024a^3b^3cd^8)^{1/4} \cdot i - (4x(a^4b^4c^8 + a^8c^4d^4)/(b^4d^4)) / ((-c^5/(256a^4d^9 + 256b^4c^4d^5 - 1024a^3b^3c^3d^6 + 1536a^2b^2c^2d^7 - 1024a^3b^3cd^8))^{1/4} * (((16(a^3b^6c^9 + a^9c^3d^6 - a^4b^5c^8d - a^8b^3c^4d^5))/(b^4d^4)) - (x(-c^5/(256a^4d^9 + 256b^4c^4d^5 - 1024a^3b^3c^3d^6 + 1536a^2b^2c^2d^7 - 1024a^3b^3cd^8))^{3/4} * (256a^3b^9c^8d^4 - 768a^4b^8c^7d^5 + 512a^5b^7c^6d^6 + 512a^6b^6c^5d^7 - 768a^7b^5c^4d^8 + 256a^8b^4c^3d^9) * 4i) / (b^4d^4)) * (-c^5/(256a^4d^9 + 256b^4c^4d^5 - 1024a^3b^3c^3d^6 + 1536a^2b^2c^2d^7 - 1024a^3b^3cd^8))^{1/4} * i + (4x(a^4b^4c^8 + a^8c^4d^4))/(b^4d^4)) * i + (-c^5/(256a^4d^9 + 256b^4c^4d^5 - 1024a^3b^3c^3d^6 + 1536a^2b^2c^2d^7 - 1024a^3b^3cd^8))^{1/4} * (((16(a^3b^6c^9 + a^9c^3d^6 - a^4b^5c^8d - a^8b^3c^4d^5))/(b^4d^4)) + (x(-c^5/(256a^4d^9 + 256b^4c^4d^5 - 1024a^3b^3c^3d^6 + 1536a^2b^2c^2d^7 - 1024a^3b^3cd^8))^{3/4} * (256a^3b^9c^8d^4 - 768a^4b^8c^7d^5 + 512a^5b^7c^6d^6 + 512a^6b^6c^5d^7 - 768a^7b^5c^4d^8 + 256a^8b^4c^3d^9) * 4i) / (b^4d^4)) * (-c^5/(256a^4d^9 + 256b^4c^4d^5 - 1024a^3b^3c^3d^6 + 1536a^2b^2c^2d^7 - 1024a^3b^3cd^8))^{1/4} * i - (4x(a^4b^4c^8 + a^8c^4d^4))/(b^4d^4)) * i)) * (-c^5/(256a^4d^9 + 256b^4c^4d^5 - 1024a^3b^3c^3d^6 + 1536a^2b^2c^2d^7 - 1024a^3b^3cd^8))^{1/4} + x/(b^4d^4)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(b*x**4+a)/(d*x**4+c), x)

[Out] Timed out

$$3.506 \quad \int \frac{x^6}{(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=449

$$\frac{a^{3/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} b^{3/4}(bc-ad)} + \frac{a^{3/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} b^{3/4}(bc-ad)} + \frac{a^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} b^{3/4}(bc-ad)} - \frac{a^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} b^{3/4}(bc-ad)}$$

Rubi [A] time = 0.28, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {481, 297, 1162, 617, 204, 1165, 628}

$$\frac{a^{3/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} b^{3/4}(bc-ad)} + \frac{a^{3/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} b^{3/4}(bc-ad)} + \frac{a^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} b^{3/4}(bc-ad)} - \frac{a^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} b^{3/4}(bc-ad)} + \frac{c^{3/4} \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} d^{3/4}(bc-ad)} - \frac{c^{3/4} \log\left(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} d^{3/4}(bc-ad)} - \frac{c^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} d^{3/4}(bc-ad)} + \frac{c^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} d^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^6/((a + b*x^4)*(c + d*x^4)),x]

[Out] (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)) - (c^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(2*Sqrt[2]*d^(3/4)*(b*c - a*d)) + (c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(2*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(3/4)*(b*c - a*d))

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 297

Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 481


```
Int[((e_.)*(x_))^(m_.)/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)),
  x_Symbol] := -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
  x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(a+bx^4)(c+dx^4)} dx &= -\frac{a \int \frac{x^2}{a+bx^4} dx}{bc-ad} + \frac{c \int \frac{x^2}{c+dx^4} dx}{bc-ad} \\
&= \frac{a \int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4} dx}{2\sqrt{b}(bc-ad)} - \frac{a \int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4} dx}{2\sqrt{b}(bc-ad)} - \frac{c \int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4} dx}{2\sqrt{d}(bc-ad)} + \frac{c \int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4} dx}{2\sqrt{d}(bc-ad)} \\
&= -\frac{a \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt[4]{b}} + x^2} dx}{4b(bc-ad)} - \frac{a \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt[4]{b}} + x^2} dx}{4b(bc-ad)} - \frac{a^{3/4} \int \frac{\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2} b^{3/4}(bc-ad)} - \frac{a^{3/4} \int \frac{1}{-\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2} b^{3/4}(bc-ad)} \\
&= -\frac{a^{3/4} \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{b} x^2)}{4\sqrt{2} b^{3/4}(bc-ad)} + \frac{a^{3/4} \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{b} x^2)}{4\sqrt{2} b^{3/4}(bc-ad)} + \frac{c^{3/4} \log(\sqrt{c} - \sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{d} x^2)}{4\sqrt{2} d^{3/4}(bc-ad)} - \frac{c^{3/4} \log(\sqrt{c} + \sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{d} x^2)}{4\sqrt{2} d^{3/4}(bc-ad)} \\
&= \frac{a^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} b^{3/4}(bc-ad)} - \frac{a^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} b^{3/4}(bc-ad)} - \frac{c^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} d^{3/4}(bc-ad)} + \frac{c^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} d^{3/4}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 340, normalized size = 0.76

$$\frac{-a^{3/4} d^{3/4} \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2) + a^{3/4} d^{3/4} \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2) + 2a^{3/4} d^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right) - 2a^{3/4} d^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right) + b^{3/4} c^{3/4} \log(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2) - b^{3/4} c^{3/4} \log(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2) - 2b^{3/4} c^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right) + 2b^{3/4} c^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{4\sqrt{2} b^{3/4} d^{3/4} (bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/((a + b*x^4)*(c + d*x^4)),x]

[Out] (2*a^(3/4)*d^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(3/4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - a^(3/4)*d^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] - b^(3/4)*c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*b^(3/4)*d^(3/4)*(b*c - a*d))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/((a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[x^6/((a + b*x^4)*(c + d*x^4)), x]

fricas [B] time = 0.54, size = 1358, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^{1/4} * \arctan(((b^2*c - a*b*d) * (-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6 \\ & *a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^{1/4} * x - (b^2*c - a*b*d) * (-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3 \\ & *d^4))^{1/4} * \sqrt{(a*x^2 - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)) * \sqrt{(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3 \\ & *d^4))})/a) / a) + (-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^{1/4} * \arctan(((b*c*d - a*d^2) * (-c^3/(b^4*c^4*d^3 \\ & - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^{1/4} * x - (b*c*d - a*d^2) * (-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b \\ & *c*d^6 + a^4*d^7))^{1/4} * \sqrt{(c*x^2 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)) * \sqrt{(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))})/c) / c) - 1/4 * (-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2 \\ & *b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^{1/4} * \log(a^2*x + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3) * (-a^3/(b^7*c^4 - 4*a*b^6*c^3 \\ & *d^3 + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^{3/4}) + 1/4 * (-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3 \\ & *d^4))^{1/4} * \log(a^2*x - (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3) * (-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 \\ & + a^4*b^3*d^4))^{3/4}) + 1/4 * (-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^{1/4} * \log(c^2*x + (b^3*c^3*d^2 - 3 \\ & *a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5) * (-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^{3/4}) - 1/4 * (-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^{1/4} * \log(c^2*x - (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5) * (-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^{3/4}) \end{aligned}$$

giac [A] time = 0.24, size = 453, normalized size = 1.01

$$\frac{(ab)^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2+5}\left(\frac{x}{2}\right)^{\frac{1}{2}}}{z\left(\frac{x}{2}\right)^{\frac{1}{2}}}\right)}{2(\sqrt{2}b^4c - \sqrt{2}ab^2d)} - \frac{(ab)^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2+5}\sqrt{5}\left(\frac{x}{2}\right)^{\frac{1}{2}}}{z\left(\frac{x}{2}\right)^{\frac{1}{2}}}\right)}{2(\sqrt{2}b^4c - \sqrt{2}ab^2d)} + \frac{(cd)^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2+5}\sqrt{5}\left(\frac{x}{2}\right)^{\frac{1}{2}}}{z\left(\frac{x}{2}\right)^{\frac{1}{2}}}\right)}{2(\sqrt{2}bcd^3 - \sqrt{2}ad^4)} + \frac{(cd)^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2+5}\sqrt{5}\left(\frac{x}{2}\right)^{\frac{1}{2}}}{z\left(\frac{x}{2}\right)^{\frac{1}{2}}}\right)}{2(\sqrt{2}bcd^3 - \sqrt{2}ad^4)} + \frac{(ab)^{\frac{3}{2}} \log\left(x^2 + \sqrt{2}x\left(\frac{x}{2}\right)^{\frac{1}{2}} + \sqrt{5}\right)}{4(\sqrt{2}b^4c - \sqrt{2}ab^2d)} - \frac{(ab)^{\frac{3}{2}} \log\left(x^2 - \sqrt{2}x\left(\frac{x}{2}\right)^{\frac{1}{2}} + \sqrt{5}\right)}{4(\sqrt{2}b^4c - \sqrt{2}ab^2d)} - \frac{(cd)^{\frac{3}{2}} \log\left(x^2 + \sqrt{2}x\left(\frac{x}{2}\right)^{\frac{1}{2}} + \sqrt{5}\right)}{4(\sqrt{2}bcd^3 - \sqrt{2}ad^4)} + \frac{(cd)^{\frac{3}{2}} \log\left(x^2 - \sqrt{2}x\left(\frac{x}{2}\right)^{\frac{1}{2}} + \sqrt{5}\right)}{4(\sqrt{2}bcd^3 - \sqrt{2}ad^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] $-1/2*(a*b^3)^{(3/4)}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(\sqrt{2}*b^4*c - \sqrt{2}*a*b^3*d) - 1/2*(a*b^3)^{(3/4)}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(\sqrt{2}*b^4*c - \sqrt{2}*a*b^3*d) + 1/2*(c*d^3)^{(3/4)}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(\sqrt{2}*b*c*d^3 - \sqrt{2}*a*d^4) + 1/2*(c*d^3)^{(3/4)}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(\sqrt{2}*b*c*d^3 - \sqrt{2}*a*d^4) + 1/4*(a*b^3)^{(3/4)}*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*b^4*c - \sqrt{2}*a*b^3*d) - 1/4*(a*b^3)^{(3/4)}*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*b^4*c - \sqrt{2}*a*b^3*d) - 1/4*(c*d^3)^{(3/4)}*\log(x^2 + \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b*c*d^3 - \sqrt{2}*a*d^4) + 1/4*(c*d^3)^{(3/4)}*\log(x^2 - \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b*c*d^3 - \sqrt{2}*a*d^4)$

maple [A] time = 0.05, size = 320, normalized size = 0.71

$$\frac{\sqrt{2} a \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}b} + \frac{\sqrt{2} a \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}b} + \frac{\sqrt{2} a \ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}b} - \frac{\sqrt{2} c \arctan\left(\frac{\sqrt{2} x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}d} - \frac{\sqrt{2} c \arctan\left(\frac{\sqrt{2} x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}d} - \frac{\sqrt{2} c \ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}\right)}{8(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^6/(b*x^4+a)/(d*x^4+c), x)$

[Out] $-1/8*c/(a*d-b*c)/d/(c/d)^{(1/4)}*2^{(1/2)}*\ln((x^2-(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)})/(x^2+(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)}))-1/4*c/(a*d-b*c)/d/(c/d)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x+1)-1/4*c/(a*d-b*c)/d/(c/d)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x-1)+1/8*a/(a*d-b*c)/b/(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})/(x^2+(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)}))+1/4*a/(a*d-b*c)/b/(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)+1/4*a/(a*d-b*c)/b/(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)$

maxima [A] time = 1.42, size = 363, normalized size = 0.81

$$\frac{a \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2a}^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2a}^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2} \log\left(\sqrt{b}x^2+\sqrt{2a}^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(\sqrt{b}x^2-\sqrt{2a}^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} \right)}{8(bc-ad)} + \frac{c \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2c}^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2c}^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} - \frac{\sqrt{2} \log\left(\sqrt{d}x^2+\sqrt{2c}^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(\sqrt{d}x^2-\sqrt{2c}^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}} \right)}{8(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^6/(b*x^4+a)/(d*x^4+c), x, \text{algorithm}="maxima")$

[Out] $-1/8*a*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x + \sqrt{2}*a^{(1/4)}*b^{(1/4)})/\sqrt{a}*\sqrt{b}))/(\sqrt{a}*\sqrt{b}) + 2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x - \sqrt{2}*a^{(1/4)}*b^{(1/4)})/\sqrt{a}*\sqrt{b})/(\sqrt{a}*\sqrt{b}) - \sqrt{2}*\log(\sqrt{b}*x^2 + \sqrt{2}*a^{(1/4)}*b^{(1/4)}*x + \sqrt{a})/(a^{(1/4)}*b^{(3/4)}) + \sqrt{2}*\log(\sqrt{b}*x^2 - \sqrt{2}*$

$$\begin{aligned} &) * a^{1/4} * b^{1/4} * x + \sqrt{a} / (a^{1/4} * b^{3/4}) / (b * c - a * d) + 1/8 * c * (2 * \sqrt{2} * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{2} * d * x + \sqrt{2} * c^{1/4} * d^{1/4}) / \sqrt{\sqrt{c} * \sqrt{d}})) / (\sqrt{\sqrt{c} * \sqrt{d}} * \sqrt{d}) + 2 * \sqrt{2} * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{2} * d * x - \sqrt{2} * c^{1/4} * d^{1/4}) / \sqrt{\sqrt{c} * \sqrt{d}})) / (\sqrt{\sqrt{c} * \sqrt{d}} * \sqrt{d}) - \sqrt{2} * \log(\sqrt{d} * x^2 + \sqrt{2} * c^{1/4} * d^{1/4} * x + \sqrt{c}) / (c^{1/4} * d^{3/4}) + \sqrt{2} * \log(\sqrt{d} * x^2 - \sqrt{2} * c^{1/4} * d^{1/4} * x + \sqrt{c}) / (c^{1/4} * d^{3/4}) / (b * c - a * d) \end{aligned}$$

mupad [B] time = 5.72, size = 2553, normalized size = 5.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^6 / ((a + b * x^4) * (c + d * x^4)), x)$

[Out]
$$\begin{aligned} & - 2 * \text{atan}((4 * b^4 * c^3 * x * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{1/4} + 4 * a^3 * b * d^3 * x * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{1/4} + 2048 * a^4 * b^4 * d^7 * x * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{5/4} + 2048 * b^8 * c^4 * d^3 * x * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{5/4} - 8192 * a * b^7 * c^3 * d^4 * x * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{5/4} - 8192 * a^3 * b^5 * c * d^6 * x * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{5/4} + 12288 * a^2 * b^6 * c^2 * d^5 * x * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{5/4}) / (a^3 * d^2 + a * b^2 * c^2 + a^2 * b * c * d) * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{1/4} - \text{atan}((b^4 * c^3 * x * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{1/4} * 4i + a^3 * b * d^3 * x * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{1/4} * 4i + a^4 * b^4 * d^7 * x * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{5/4} * 2048i + b^8 * c^4 * d^3 * x * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{5/4} * 2048i - a * b^7 * c^3 * d^4 * x * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{5/4} * 8192i - a^3 * b^5 * c * d^6 * x * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{5/4} * 8192i + a^2 * b^6 * c^2 * d^5 * x * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{5/4} * 12288i) / (a^3 * d^2 + a * b^2 * c^2 + a^2 * b * c * d) * (-a^3 / (256 * b^7 * c^4 + 256 * a^4 * b^3 * d^4 - 1024 * a^3 * b^4 * c * d^3 + 1536 * a^2 * b^5 * c^2 * d^2 - 1024 * a * b^6 * c^3 * d))^{1/4} * 2i - 2 * \text{atan}((4 * a^3 * d^4 * x * (-c^3 / (256 * a^4 * d^7 + 256 * b^4 * c^4 * d^3 - 1024 * a^3 * b * c * d^6))^{1/4} + 2048 * b \end{aligned}$$

$$\begin{aligned}
& ^7c^4d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(5/4)} + 4*b^3*c^3*d*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(1/4)} + 2048*a^4*b^3*d^8*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(5/4)} - \\
& 8192*a*b^6*c^3*d^5*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(5/4)} - 8192*a^3*b^4*c*d^7 \\
& *x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(5/4)} + 12288*a^2*b^5*c^2*d^6*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(5/4)})/(b^2*c^3 + a^2*c*d^2 + a*b*c^2*d))*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(1/4)} - \operatorname{atan}((a^3*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(1/4)}*4i + b^7*c^4*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(5/4)}*2048i + b^3*c^3*d*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(1/4)}*4i + a^4*b^3*d^8*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(5/4)}*2048i - a*b^6*c^3*d^5*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(5/4)}*8192i - a^3*b^4*c*d^7*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(5/4)}*8192i + a^2*b^5*c^2*d^6*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(5/4)}*12288i)/(b^2*c^3 + a^2*c*d^2 + a*b*c^2*d))*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^{(1/4)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(b*x**4+a)/(d*x**4+c),x)

[Out] Timed out

$$3.507 \quad \int \frac{x^4}{(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=449

$$\frac{\sqrt[4]{a} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} \sqrt[4]{b} (bc - ad)} - \frac{\sqrt[4]{a} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} \sqrt[4]{b} (bc - ad)} - \frac{\sqrt[4]{c} \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} \sqrt[4]{d} (bc - ad)}$$

Rubi [A] time = 0.27, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 22, number of rules / integrand size = 0.318, Rules used = {481, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} \sqrt[4]{b} (bc - ad)} - \frac{\sqrt[4]{a} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} \sqrt[4]{b} (bc - ad)} - \frac{\sqrt[4]{c} \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} \sqrt[4]{d} (bc - ad)} + \frac{\sqrt[4]{c} \log\left(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} \sqrt[4]{d} (bc - ad)} + \frac{\sqrt[4]{a} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} \sqrt[4]{b} (bc - ad)} - \frac{\sqrt[4]{a} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} \sqrt[4]{b} (bc - ad)} - \frac{\sqrt[4]{c} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} \sqrt[4]{d} (bc - ad)} + \frac{\sqrt[4]{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}} + 1\right)}{2\sqrt{2} \sqrt[4]{d} (bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[x^4/((a + b*x^4)*(c + d*x^4)),x]

[Out] (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(2*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(2*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(1/4)*(b*c - a*d))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 481

```
Int[((e_.)*(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] := -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(a + bx^4)(c + dx^4)} dx &= -\frac{a \int \frac{1}{a+bx^4} dx}{bc - ad} + \frac{c \int \frac{1}{c+dx^4} dx}{bc - ad} \\
&= -\frac{\sqrt{a} \int \frac{\sqrt{a} - \sqrt{b}x^2}{a+bx^4} dx}{2(bc - ad)} - \frac{\sqrt{a} \int \frac{\sqrt{a} + \sqrt{b}x^2}{a+bx^4} dx}{2(bc - ad)} + \frac{\sqrt{c} \int \frac{\sqrt{c} - \sqrt{d}x^2}{c+dx^4} dx}{2(bc - ad)} + \frac{\sqrt{c} \int \frac{\sqrt{c} + \sqrt{d}x^2}{c+dx^4} dx}{2(bc - ad)} \\
&= -\frac{\sqrt{a} \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{b}(bc - ad)} - \frac{\sqrt{a} \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{b}(bc - ad)} + \frac{\sqrt[4]{a} \int \frac{\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2} \sqrt[4]{b}(bc - ad)} + \frac{\sqrt[4]{a} \int \frac{\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{b}} - 2x}{-\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2} \sqrt[4]{b}(bc - ad)} \\
&= \frac{\sqrt[4]{a} \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{b}x^2)}{4\sqrt{2} \sqrt[4]{b}(bc - ad)} - \frac{\sqrt[4]{a} \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{b}x^2)}{4\sqrt{2} \sqrt[4]{b}(bc - ad)} - \frac{\sqrt[4]{c} \log(\sqrt{c} - \sqrt{2} \sqrt[4]{c} \sqrt[4]{d}x + \sqrt{d}x^2)}{4\sqrt{2} \sqrt[4]{d}(bc - ad)} + \frac{\sqrt[4]{c} \log(\sqrt{c} + \sqrt{2} \sqrt[4]{c} \sqrt[4]{d}x + \sqrt{d}x^2)}{4\sqrt{2} \sqrt[4]{d}(bc - ad)} \\
&= \frac{\sqrt[4]{a} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2} \sqrt[4]{b}(bc - ad)} - \frac{\sqrt[4]{a} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2} \sqrt[4]{b}(bc - ad)} - \frac{\sqrt[4]{c} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2} \sqrt[4]{d}(bc - ad)} + \frac{\sqrt[4]{c} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2} \sqrt[4]{d}(bc - ad)}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 340, normalized size = 0.76

$$\frac{\sqrt[4]{a} \sqrt[4]{d} \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2) - \sqrt[4]{a} \sqrt[4]{d} \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2) + 2\sqrt[4]{a} \sqrt[4]{d} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right) - 2\sqrt[4]{a} \sqrt[4]{d} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \sqrt[4]{b} \sqrt[4]{c} \log(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2) + \sqrt[4]{b} \sqrt[4]{c} \log(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2) - 2\sqrt[4]{b} \sqrt[4]{c} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d}x}{\sqrt[4]{c}}\right) + 2\sqrt[4]{b} \sqrt[4]{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{d}x}{\sqrt[4]{c}} + 1\right)}{4\sqrt{2} \sqrt[4]{b} \sqrt[4]{d}(bc - ad)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((a + b*x^4)*(c + d*x^4)), x]

[Out] (2*a^(1/4)*d^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(1/4)*d^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(1/4)*c^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(1/4)*c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + a^(1/4)*d^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + b^(1/4)*c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*b^(1/4)*d^(1/4)*(b*c - a*d))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/((a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[x^4/((a + b*x^4)*(c + d*x^4)), x]

fricas [B] time = 0.47, size = 1238, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out]
$$\begin{aligned} & (-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^{1/4} * \arctan(((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) \\ & * x * (-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^{3/4} - (b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) * \text{sqrt}(x^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2) * \text{sqrt}(-a/(b^5*c^4 - 4*a*b^4*c^3*d + \\ & 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))) * (-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^{3/4}) / a) - (-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^{1/4} * \arctan(((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4) * x * (-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^{3/4} - (b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4) * \text{sqrt}(x^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2) * \text{sqrt}(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))) * (-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^{3/4}) / c) - 1/4 * (-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^{1/4} * \log((b*c - a*d) * (-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^{1/4} + x) + 1/4 * (-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^{1/4} * \log(- (b*c - a*d) * (-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^{1/4} + x) + 1/4 * (-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^{1/4} * \log((b*c - a*d) * (-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^{1/4} + x) - 1/4 * (-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^{1/4} * \log(- (b*c - a*d) * (-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^{1/4} + x) \end{aligned}$$

giac [A] time = 0.24, size = 437, normalized size = 0.97

$$\frac{(ab)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}}{2\sqrt{c}}\right)}{2(\sqrt{2}bc - \sqrt{2}abd)} - \frac{(ab)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}}{2\sqrt{c}}\right)}{2(\sqrt{2}bc - \sqrt{2}abd)} + \frac{(cd)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}}{2\sqrt{a}}\right)}{2(\sqrt{2}bcd - \sqrt{2}ad^2)} + \frac{(cd)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}}{2\sqrt{a}}\right)}{2(\sqrt{2}bcd - \sqrt{2}ad^2)} - \frac{(ab)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4(\sqrt{2}bc - \sqrt{2}abd)} + \frac{(ab)^{\frac{1}{4}} \log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4(\sqrt{2}bc - \sqrt{2}abd)} + \frac{(cd)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{4(\sqrt{2}bcd - \sqrt{2}ad^2)} - \frac{(cd)^{\frac{1}{4}} \log\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{4(\sqrt{2}bcd - \sqrt{2}ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out]
$$-1/2*(a*b^3)^{1/4}*\arctan(1/2*\text{sqrt}(2)*(2*x + \text{sqrt}(2)*(a/b)^{1/4}))/ (a/b)^{1/4} / (\text{sqrt}(2)*b^2*c - \text{sqrt}(2)*a*b*d) - 1/2*(a*b^3)^{1/4}*\arctan(1/2*\text{sqrt}(2)*$$

$$(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)}/(\sqrt{2}*b^2*c - \sqrt{2}*a*b*d) + 1/2*(c*d^3)^{(1/4)}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(\sqrt{2}*b*c*d - \sqrt{2}*a*d^2) + 1/2*(c*d^3)^{(1/4)}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(\sqrt{2}*b*c*d - \sqrt{2}*a*d^2) - 1/4*(a*b^3)^{(1/4)}*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*b^2*c - \sqrt{2}*a*b*d) + 1/4*(a*b^3)^{(1/4)}*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*b^2*c - \sqrt{2}*a*b*d) + 1/4*(c*d^3)^{(1/4)}*\log(x^2 + \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b*c*d - \sqrt{2}*a*d^2) - 1/4*(c*d^3)^{(1/4)}*\log(x^2 - \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b*c*d - \sqrt{2}*a*d^2)$$

maple [A] time = 0.06, size = 296, normalized size = 0.66

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4ad-4bc} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4ad-4bc} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4(ad-bc)} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4(ad-bc)} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8ad-8bc} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}\right)}{8(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^4+a)/(d*x^4+c), x)

[Out] $-1/8/(a*d-b*c)*(c/d)^{(1/4)}*2^{(1/2)}*\ln((x^2+(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)})/(x^2-(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)}))-1/4/(a*d-b*c)*(c/d)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x+1)-1/4/(a*d-b*c)*(c/d)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x-1)+1/8/(a*d-b*c)*(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)}))+1/4/(a*d-b*c)*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)+1/4/(a*d-b*c)*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)$

maxima [A] time = 1.39, size = 361, normalized size = 0.80

$$\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}\sqrt{2x+\sqrt{2}\sqrt{a}}}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}\sqrt{2x-\sqrt{2}\sqrt{a}}}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}} + \frac{\sqrt{2}x^{\frac{1}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}x\sqrt{a}+\sqrt{a}\right)}{b^{\frac{1}{4}}} - \frac{\sqrt{2}x^{\frac{1}{4}}\log\left(\sqrt{b}x^2-\sqrt{2}x\sqrt{a}+\sqrt{a}\right)}{b^{\frac{1}{4}}} + \frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{2x+\sqrt{2}\sqrt{c}}}{2\sqrt{c}\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}} + \frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{2x-\sqrt{2}\sqrt{c}}}{2\sqrt{c}\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}} + \frac{\sqrt{2}x^{\frac{1}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}x\sqrt{c}+\sqrt{c}\right)}{d^{\frac{1}{4}}} - \frac{\sqrt{2}x^{\frac{1}{4}}\log\left(\sqrt{d}x^2-\sqrt{2}x\sqrt{c}+\sqrt{c}\right)}{d^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^4+a)/(d*x^4+c), x, algorithm="maxima")

[Out] $-1/8*(2*\sqrt{2}*\sqrt{a}*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*x + \sqrt{2}*a^{(1/4)}*b^{(1/4)})/\sqrt{a}*\sqrt{b}))/\sqrt{a}*\sqrt{b} + 2*\sqrt{2}*\sqrt{a}*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*x - \sqrt{2}*a^{(1/4)}*b^{(1/4)})/\sqrt{a}*\sqrt{b}))/\sqrt{a}*\sqrt{b} + \sqrt{2}*a^{(1/4)}*\log(\sqrt{b}*x^2 + \sqrt{2}*a^{(1/4)}*b^{(1/4)}*x + \sqrt{a})/b^{(1/4)} - \sqrt{2}*a^{(1/4)}*\log(\sqrt{b}*x^2 - \sqrt{2}*a^{(1/4)}*b^{(1/4)}*x + \sqrt{a})/b^{(1/4)})/(b*c - a*d) + 1/8*(2*\sqrt{2}*\sqrt{c}*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*x + \sqrt{2}*c^{(1/4)}*d^{(1/4)})/\sqrt{c}*\sqrt{d}))/\sqrt{c}*\sqrt{d} + 2*\sqrt{2}*\sqrt{c}*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*x - \sqrt{2}*c^{(1/4)}*d^{(1/4)})/\sqrt{c}*\sqrt{d}))/\sqrt{c}*\sqrt{d}$

(d)) + sqrt(2)*c^(1/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/d^(1/4) - sqrt(2)*c^(1/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/d^(1/4))/(b*c - a*d)

mupad [B] time = 5.86, size = 5889, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((a + b*x^4)*(c + d*x^4)),x)

[Out] - atan((a^2*d^2*x*i + b^2*c^2*x*i - (a^6*b*d^6*x*256i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - (a*b^6*c^5*d*x*256i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) + (a^5*b^2*c*d^5*x*768i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) + (a^2*b^5*c^4*d^2*x*768i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - (a^3*b^4*c^3*d^3*x*512i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - (a^4*b^3*c^2*d^4*x*512i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))/((-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*((a*(1024*a^6*b*d^7 + 1024*b^7*c^6*d - 6144*a*b^6*c^5*d^2 - 6144*a^5*b^2*c*d^6 + 15360*a^2*b^5*c^4*d^3 - 20480*a^3*b^4*c^3*d^4 + 15360*a^4*b^3*c^2*d^5))/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - 4*b^3*c^3 - 4*a^3*d^3 + 4*a*b^2*c^2*d + 4*a^2*b*c*d^2)))*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*2i - atan((a^2*d^2*x*i + b^2*c^2*x*i - (b^6*c^6*d*x*256i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a^5*b*c*d^6*x*256i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) + (a*b^5*c^5*d^2*x*768i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a^2*b^4*c^4*d^3*x*512i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a^3*b^3*c^3*d^4*x*512i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) + (a^4*b^2*c^2*d^5*x*768i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))/((-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*((c*(1024*a^6*b*d^7 + 1024*b^7*c^6*d - 6144*a*b^6*c^5*d^2 - 6144*a^5*b^2*c*d^6 + 15360*a^2*b^5*c^4*d^3 - 20480*a^3*b^4*c^3*d^4 + 15360*a^4*b^3*c^2*d^5))/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - 4*b^3*c^3 - 4*a^3*d^3 + 4*a*b^2*c^2*d + 4*a^2*b*c*d^2)))*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*2i - 2*atan(((x*(4*a^2*b^5*c^

$$\begin{aligned}
& 4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{1/4} * ((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{1/4} * (4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i) * (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{3/4} * 1i + 16*a^2*b^6*c^5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a^5*b^3*c^2*d^6)*1i) * (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{1/4} + (x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{1/4} * ((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) + (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{1/4} * (4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i) * (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{3/4} * 1i - 16*a^2*b^6*c^5*d^3 + 16*a^3*b^5*c^4*d^4 + 16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i) * (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{1/4}) / ((x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{1/4} * ((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{1/4} * (4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i) * (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{3/4} * 1i + 16*a^2*b^6*c^5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a^5*b^3*c^2*d^6)*1i) * (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{1/4} * 1i - (x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{1/4} * ((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) + (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{1/4} * (4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i) * (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^{3/4} * 1i - 16*a^2*b^6*c^5*d^3 + 16*a^3*b^5*c^4*d^4 + 16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i) * (-a/(256*b
\end{aligned}$$

$$\begin{aligned}
& ^5c^4 + 256a^4b^4d^4 - 1024a^3b^2c^2d^3 + 1536a^2b^3c^2d^2 - 1024a \\
& *b^4c^3d))^{(1/4)} * i) * (-a / (256b^5c^4 + 256a^4b^4d^4 - 1024a^3b^2c^2d \\
& ^3 + 1536a^2b^3c^2d^2 - 1024a^2b^4c^3d))^{(1/4)} - 2 * \operatorname{atan}(((x * (4a^2b^ \\
& 5c^4d^3 + 4a^4b^3c^2d^5) - (-c / (256a^4d^5 + 256b^4c^4d - 1024a^ \\
& b^3c^3d^2 + 1536a^2b^2c^2d^3 - 1024a^3b^4c^3d^4))^{(1/4)} * ((x * (1024a^2 \\
& *b^9c^7d^4 - 3072a^3b^8c^6d^5 + 2048a^4b^7c^5d^6 + 2048a^5b^6c^ \\
& ^4d^7 - 3072a^6b^5c^3d^8 + 1024a^7b^4c^2d^9) - (-c / (256a^4d^5 + \\
& 256b^4c^4d - 1024a^3b^3c^3d^2 + 1536a^2b^2c^2d^3 - 1024a^3b^4c^3d^ \\
& 4))^{(1/4)} * (4096a^2b^10c^8d^4 - 24576a^3b^9c^7d^5 + 61440a^4b^8c^6 \\
& d^6 - 81920a^5b^7c^5d^7 + 61440a^6b^6c^4d^8 - 24576a^7b^5c^3d^9 + \\
& 4096a^8b^4c^2d^{10}) * i) * (-c / (256a^4d^5 + 256b^4c^4d - 1024a^2b \\
& ^3c^3d^2 + 1536a^2b^2c^2d^3 - 1024a^3b^4c^3d^4))^{(3/4)} * i + 16a^2b^ \\
& 6c^5d^3 - 16a^3b^5c^4d^4 - 16a^4b^4c^3d^5 + 16a^5b^3c^2d^6) * 1 \\
& i) * (-c / (256a^4d^5 + 256b^4c^4d - 1024a^2b^3c^3d^2 + 1536a^2b^2c^2 \\
& *d^3 - 1024a^3b^4c^3d^4))^{(1/4)} + (x * (4a^2b^5c^4d^3 + 4a^4b^3c^2d^5) \\
&) - (-c / (256a^4d^5 + 256b^4c^4d - 1024a^2b^3c^3d^2 + 1536a^2b^2c^ \\
& 2*d^3 - 1024a^3b^4c^3d^4))^{(1/4)} * ((x * (1024a^2b^9c^7d^4 - 3072a^3b^8c \\
& ^6d^5 + 2048a^4b^7c^5d^6 + 2048a^5b^6c^4d^7 - 3072a^6b^5c^3d^8 \\
& + 1024a^7b^4c^2d^9) + (-c / (256a^4d^5 + 256b^4c^4d - 1024a^2b^3c^ \\
& 3*d^2 + 1536a^2b^2c^2d^3 - 1024a^3b^4c^3d^4))^{(1/4)} * (4096a^2b^10c^8 \\
& d^4 - 24576a^3b^9c^7d^5 + 61440a^4b^8c^6d^6 - 81920a^5b^7c^5d^7 \\
& + 61440a^6b^6c^4d^8 - 24576a^7b^5c^3d^9 + 4096a^8b^4c^2d^{10}) * 1 \\
& i) * (-c / (256a^4d^5 + 256b^4c^4d - 1024a^2b^3c^3d^2 + 1536a^2b^2c^2 \\
& *d^3 - 1024a^3b^4c^3d^4))^{(3/4)} * i - 16a^2b^6c^5d^3 + 16a^3b^5c^4d^ \\
& 4 + 16a^4b^4c^3d^5 - 16a^5b^3c^2d^6) * 1) * (-c / (256a^4d^5 + 256b^4 \\
& *c^4d - 1024a^2b^3c^3d^2 + 1536a^2b^2c^2d^3 - 1024a^3b^4c^3d^4))^{(1/ \\
& 4)) / ((x * (4a^2b^5c^4d^3 + 4a^4b^3c^2d^5) - (-c / (256a^4d^5 + 256b^ \\
& 4*c^4d - 1024a^2b^3c^3d^2 + 1536a^2b^2c^2d^3 - 1024a^3b^4c^3d^4))^{(1 \\
& /4)} * ((x * (1024a^2b^9c^7d^4 - 3072a^3b^8c^6d^5 + 2048a^4b^7c^5d^6 \\
& + 2048a^5b^6c^4d^7 - 3072a^6b^5c^3d^8 + 1024a^7b^4c^2d^9) - (- \\
& c / (256a^4d^5 + 256b^4c^4d - 1024a^2b^3c^3d^2 + 1536a^2b^2c^2d^3 \\
& - 1024a^3b^4c^3d^4))^{(1/4)} * (4096a^2b^10c^8d^4 - 24576a^3b^9c^7d^5 + \\
& 61440a^4b^8c^6d^6 - 81920a^5b^7c^5d^7 + 61440a^6b^6c^4d^8 - 24 \\
& 576a^7b^5c^3d^9 + 4096a^8b^4c^2d^{10}) * i) * (-c / (256a^4d^5 + 256b^4 \\
& *c^4d - 1024a^2b^3c^3d^2 + 1536a^2b^2c^2d^3 - 1024a^3b^4c^3d^4))^{(3/ \\
& 4)} * i + 16a^2b^6c^5d^3 - 16a^3b^5c^4d^4 - 16a^4b^4c^3d^5 + 16a^ \\
& ^5b^3c^2d^6) * 1) * (-c / (256a^4d^5 + 256b^4c^4d - 1024a^2b^3c^3d^2 + \\
& 1536a^2b^2c^2d^3 - 1024a^3b^4c^3d^4))^{(1/4)} * i - (x * (4a^2b^5c^4d^3 \\
& + 4a^4b^3c^2d^5) - (-c / (256a^4d^5 + 256b^4c^4d - 1024a^2b^3c^3d^ \\
& ^2 + 1536a^2b^2c^2d^3 - 1024a^3b^4c^3d^4))^{(1/4)} * ((x * (1024a^2b^9c^7 \\
& d^4 - 3072a^3b^8c^6d^5 + 2048a^4b^7c^5d^6 + 2048a^5b^6c^4d^7 - \\
& 3072a^6b^5c^3d^8 + 1024a^7b^4c^2d^9) + (-c / (256a^4d^5 + 256b^4c^ \\
& ^4d - 1024a^2b^3c^3d^2 + 1536a^2b^2c^2d^3 - 1024a^3b^4c^3d^4))^{(1/4)} \\
& * (4096a^2b^10c^8d^4 - 24576a^3b^9c^7d^5 + 61440a^4b^8c^6d^6 - 8 \\
& 1920a^5b^7c^5d^7 + 61440a^6b^6c^4d^8 - 24576a^7b^5c^3d^9 + 4096
\end{aligned}$$

$$*a^8*b^4*c^2*d^{10}*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^{3/4}*1i - 16*a^2*b^6*c^5*d^3 + 16*a^3*b^5*c^4*d^4 + 16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^{1/4}*1i))*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^{1/4}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**4+a)/(d*x**4+c), x)

[Out] Timed out

$$3.508 \quad \int \frac{x^2}{(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=449

$$\frac{\sqrt[4]{b} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} \sqrt[4]{a} (bc - ad)} - \frac{\sqrt[4]{b} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} \sqrt[4]{a} (bc - ad)} - \frac{\sqrt[4]{d} \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} \sqrt[4]{c} (bc - ad)}$$

Rubi [A] time = 0.27, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {482, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{b} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} \sqrt[4]{a} (bc - ad)} - \frac{\sqrt[4]{b} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} \sqrt[4]{a} (bc - ad)} - \frac{\sqrt[4]{d} \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} \sqrt[4]{c} (bc - ad)} + \frac{\sqrt[4]{d} \log\left(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} \sqrt[4]{c} (bc - ad)} - \frac{\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt[4]{b}}\right)}{2\sqrt{2} \sqrt[4]{a} (bc - ad)} + \frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt[4]{b}} + 1\right)}{2\sqrt{2} \sqrt[4]{a} (bc - ad)} + \frac{\sqrt[4]{d} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{d}}\right)}{2\sqrt{2} \sqrt[4]{c} (bc - ad)} - \frac{\sqrt[4]{d} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{d}} + 1\right)}{2\sqrt{2} \sqrt[4]{c} (bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x^4)*(c + d*x^4)),x]

[Out] $-(b^{1/4} \text{ArcTan}[1 - (\text{Sqrt}[2] * b^{1/4} * x) / a^{1/4}]) / (2 * \text{Sqrt}[2] * a^{1/4} * (b * c - a * d)) + (b^{1/4} \text{ArcTan}[1 + (\text{Sqrt}[2] * b^{1/4} * x) / a^{1/4}]) / (2 * \text{Sqrt}[2] * a^{1/4} * (b * c - a * d)) + (d^{1/4} \text{ArcTan}[1 - (\text{Sqrt}[2] * d^{1/4} * x) / c^{1/4}]) / (2 * \text{Sqrt}[2] * c^{1/4} * (b * c - a * d)) - (d^{1/4} \text{ArcTan}[1 + (\text{Sqrt}[2] * d^{1/4} * x) / c^{1/4}]) / (2 * \text{Sqrt}[2] * c^{1/4} * (b * c - a * d)) + (b^{1/4} * \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2] * a^{1/4} * b^{1/4} * x + \text{Sqrt}[b] * x^2]) / (4 * \text{Sqrt}[2] * a^{1/4} * (b * c - a * d)) - (b^{1/4} * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{1/4} * b^{1/4} * x + \text{Sqrt}[b] * x^2]) / (4 * \text{Sqrt}[2] * a^{1/4} * (b * c - a * d)) - (d^{1/4} * \text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2] * c^{1/4} * d^{1/4} * x + \text{Sqrt}[d] * x^2]) / (4 * \text{Sqrt}[2] * c^{1/4} * (b * c - a * d)) + (d^{1/4} * \text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2] * c^{1/4} * d^{1/4} * x + \text{Sqrt}[d] * x^2]) / (4 * \text{Sqrt}[2] * c^{1/4} * (b * c - a * d))$

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 297

Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] & AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 482


```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
  x_Symbol] := Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+bx^4)(c+dx^4)} dx &= \frac{b \int \frac{x^2}{a+bx^4} dx}{bc-ad} - \frac{d \int \frac{x^2}{c+dx^4} dx}{bc-ad} \\
&= -\frac{\sqrt{b} \int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4} dx}{2(bc-ad)} + \frac{\sqrt{b} \int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4} dx}{2(bc-ad)} + \frac{\sqrt{d} \int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4} dx}{2(bc-ad)} - \frac{\sqrt{d} \int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4} dx}{2(bc-ad)} \\
&= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}}{\sqrt[4]{b}} \frac{\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4(bc-ad)} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}}{\sqrt[4]{b}} \frac{\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4(bc-ad)} - \frac{\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2}}{\sqrt[4]{d}} \frac{\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2} dx}{4(bc-ad)} - \frac{\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} + \frac{\sqrt{2}}{\sqrt[4]{d}} \frac{\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2} dx}{4(bc-ad)} \\
&= \frac{\sqrt[4]{b} \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{b} x^2)}{4\sqrt{2} \sqrt[4]{a} (bc-ad)} - \frac{\sqrt[4]{b} \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{b} x^2)}{4\sqrt{2} \sqrt[4]{a} (bc-ad)} - \frac{\sqrt[4]{d} \log(\sqrt{c} - \sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{d} x^2)}{4\sqrt{2} \sqrt[4]{c} (bc-ad)} + \frac{\sqrt[4]{d} \log(\sqrt{c} + \sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{d} x^2)}{4\sqrt{2} \sqrt[4]{c} (bc-ad)} \\
&= -\frac{\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} \sqrt[4]{a} (bc-ad)} + \frac{\sqrt[4]{b} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} \sqrt[4]{a} (bc-ad)} + \frac{\sqrt[4]{d} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} \sqrt[4]{c} (bc-ad)} - \frac{\sqrt[4]{d} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} \sqrt[4]{c} (bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 340, normalized size = 0.76

$$\frac{\sqrt[4]{b} \sqrt[4]{c} \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2) - \sqrt[4]{b} \sqrt[4]{c} \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2) - 2\sqrt[4]{b} \sqrt[4]{c} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right) + 2\sqrt[4]{b} \sqrt[4]{c} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right) - \sqrt[4]{d} \sqrt[4]{c} \log(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2) + \sqrt[4]{d} \sqrt[4]{c} \log(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2) + 2\sqrt[4]{d} \sqrt[4]{c} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right) - 2\sqrt[4]{d} \sqrt[4]{c} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{4\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} (bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + b*x^4)*(c + d*x^4)), x]

[Out] $(-2*b^{(1/4)}*c^{(1/4)}*ArcTan[1 - (Sqrt[2]*b^{(1/4)}*x)/a^{(1/4)}] + 2*b^{(1/4)}*c^{(1/4)}*ArcTan[1 + (Sqrt[2]*b^{(1/4)}*x)/a^{(1/4)}] + 2*a^{(1/4)}*d^{(1/4)}*ArcTan[1 - (Sqrt[2]*d^{(1/4)}*x)/c^{(1/4)}] - 2*a^{(1/4)}*d^{(1/4)}*ArcTan[1 + (Sqrt[2]*d^{(1/4)}*x)/c^{(1/4)}] + b^{(1/4)}*c^{(1/4)}*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*b^{(1/4)}*x + Sqrt[b]*x^2] - b^{(1/4)}*c^{(1/4)}*Log[Sqrt[a] + Sqrt[2]*a^{(1/4)}*b^{(1/4)}*x + Sqrt[b]*x^2] - a^{(1/4)}*d^{(1/4)}*Log[Sqrt[c] - Sqrt[2]*c^{(1/4)}*d^{(1/4)}*x + Sqrt[d]*x^2] + a^{(1/4)}*d^{(1/4)}*Log[Sqrt[c] + Sqrt[2]*c^{(1/4)}*d^{(1/4)}*x + Sqrt[d]*x^2])/(4*Sqrt[2]*a^{(1/4)}*c^{(1/4)}*(b*c - a*d))$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/((a + b*x^4)*(c + d*x^4)), x]

[Out] IntegrateAlgebraic[x^2/((a + b*x^4)*(c + d*x^4)), x]

fricas [B] time = 0.50, size = 1258, normalized size = 2.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^4+a)/(d*x^4+c), x, algorithm="fricas")

[Out]
$$\begin{aligned} & (-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^{1/4} * \arctan((b*c - a*d)*x * (-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^{1/4} - (b*c - a*d)*\sqrt{(b*x^2 - (a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2))*\sqrt{-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))})/b * (-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^{1/4}) - (-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^{1/4} * \arctan((b*c - a*d)*x * (-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^{1/4} - (b*c - a*d)*\sqrt{(d*x^2 - (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2))*\sqrt{-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))})/d * (-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^{1/4}) + 1/4 * (-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^{1/4} * \log(b*x + (a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3) * (-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^{3/4}) - 1/4 * (-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^{1/4} * \log(b*x - (a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3) * (-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^{3/4}) - 1/4 * (-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^{1/4} * \log(d*x + (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3) * (-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^{3/4}) + 1/4 * (-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^{1/4} * \log(d*x - (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3) * (-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^{3/4}) \end{aligned}$$

giac [A] time = 0.23, size = 477, normalized size = 1.06

$$\frac{(ab)^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2}z + \sqrt{2}(\frac{z}{2})^{\frac{1}{2}}}{z(\frac{z}{2})^{\frac{1}{2}}}\right)}{2(\sqrt{2}ab^3c - \sqrt{2}a^2b^2d)} + \frac{(ab)^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2}z - \sqrt{2}(\frac{z}{2})^{\frac{1}{2}}}{z(\frac{z}{2})^{\frac{1}{2}}}\right)}{2(\sqrt{2}ab^3c - \sqrt{2}a^2b^2d)} - \frac{(cd)^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2}z + \sqrt{2}(\frac{z}{2})^{\frac{1}{2}}}{z(\frac{z}{2})^{\frac{1}{2}}}\right)}{2(\sqrt{2}bc^2d^2 - \sqrt{2}acd^3)} - \frac{(cd)^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2}z - \sqrt{2}(\frac{z}{2})^{\frac{1}{2}}}{z(\frac{z}{2})^{\frac{1}{2}}}\right)}{2(\sqrt{2}bc^2d^2 - \sqrt{2}acd^3)} - \frac{(ab)^{\frac{3}{2}} \log\left(x^2 + \sqrt{2}x\left(\frac{z}{2}\right)^{\frac{1}{2}} + \sqrt{\frac{a}{2}}\right)}{4(\sqrt{2}ab^3c - \sqrt{2}a^2b^2d)} + \frac{(ab)^{\frac{3}{2}} \log\left(x^2 - \sqrt{2}x\left(\frac{z}{2}\right)^{\frac{1}{2}} + \sqrt{\frac{a}{2}}\right)}{4(\sqrt{2}ab^3c - \sqrt{2}a^2b^2d)} + \frac{(cd)^{\frac{3}{2}} \log\left(x^2 + \sqrt{2}x\left(\frac{z}{2}\right)^{\frac{1}{2}} + \sqrt{\frac{c}{2}}\right)}{4(\sqrt{2}bc^2d^2 - \sqrt{2}acd^3)} - \frac{(cd)^{\frac{3}{2}} \log\left(x^2 - \sqrt{2}x\left(\frac{z}{2}\right)^{\frac{1}{2}} + \sqrt{\frac{c}{2}}\right)}{4(\sqrt{2}bc^2d^2 - \sqrt{2}acd^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^4+a)/(d*x^4+c), x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/2 * (a*b^3)^{3/4} * \arctan(1/2 * \sqrt{2} * (2*x + \sqrt{2}) * (a/b)^{1/4}) / (a/b)^{1/4} \\ &) / (\sqrt{2} * a*b^3*c - \sqrt{2} * a^2*b^2*d) + 1/2 * (a*b^3)^{3/4} * \arctan(1/2 * \sqrt{2} * \sqrt{d} * (c/d)^{1/4}) / (\sqrt{2} * d * (c/d)^{1/4}) + 1/2 * (c*d)^{3/4} * \arctan(1/2 * \sqrt{2} * (d/x + \sqrt{2}) * (c/d)^{1/4}) / (d * (c/d)^{1/4}) \\ &) / (\sqrt{2} * bc^2*d^2 - \sqrt{2} * ac*d^3) + 1/2 * (c*d)^{3/4} * \arctan(1/2 * \sqrt{2} * \sqrt{d} * (c/d)^{1/4}) / (\sqrt{2} * d * (c/d)^{1/4}) + 1/2 * (c*d)^{3/4} * \arctan(1/2 * \sqrt{2} * (d/x - \sqrt{2}) * (c/d)^{1/4}) / (d * (c/d)^{1/4}) \\ &) / (\sqrt{2} * bc^2*d^2 - \sqrt{2} * ac*d^3) \end{aligned}$$

$$\begin{aligned} & t(2) * (2*x - \sqrt{2} * (a/b)^{(1/4)}) / (a/b)^{(1/4)} / (\sqrt{2} * a * b^3 * c - \sqrt{2} * a^2 * b^2 * d) - 1/2 * (c * d^3)^{(3/4)} * \arctan(1/2 * \sqrt{2} * (2*x + \sqrt{2} * (c/d)^{(1/4)}) / (c/d)^{(1/4)}) / (\sqrt{2} * b * c^2 * d^2 - \sqrt{2} * a * c * d^3) - 1/2 * (c * d^3)^{(3/4)} * \arctan(1/2 * \sqrt{2} * (2*x - \sqrt{2} * (c/d)^{(1/4)}) / (c/d)^{(1/4)}) / (\sqrt{2} * b * c^2 * d^2 - \sqrt{2} * a * c * d^3) - 1/4 * (a * b^3)^{(3/4)} * \log(x^2 + \sqrt{2} * x * (a/b)^{(1/4)} + \sqrt{2} * (a/b)) / (\sqrt{2} * a * b^3 * c - \sqrt{2} * a^2 * b^2 * d) + 1/4 * (a * b^3)^{(3/4)} * \log(x^2 - \sqrt{2} * x * (a/b)^{(1/4)} + \sqrt{2} * (a/b)) / (\sqrt{2} * a * b^3 * c - \sqrt{2} * a^2 * b^2 * d) + 1/4 * (c * d^3)^{(3/4)} * \log(x^2 + \sqrt{2} * x * (c/d)^{(1/4)} + \sqrt{2} * (c/d)) / (\sqrt{2} * b * c^2 * d^2 - \sqrt{2} * a * c * d^3) - 1/4 * (c * d^3)^{(3/4)} * \log(x^2 - \sqrt{2} * x * (c/d)^{(1/4)} + \sqrt{2} * (c/d)) / (\sqrt{2} * b * c^2 * d^2 - \sqrt{2} * a * c * d^3) \end{aligned}$$

maple [A] time = 0.06, size = 296, normalized size = 0.66

$$\begin{aligned} & -\frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}} + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}} + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}} - \frac{\sqrt{2} \ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}} + \frac{\sqrt{2} \ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}\right)}{8(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^4+a)/(d*x^4+c),x)

[Out] 1/8/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))+1/4/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/8/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))-1/4/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/4/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)

maxima [A] time = 1.21, size = 363, normalized size = 0.81

$$\begin{aligned} & \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\frac{1}{2}\sqrt{b}x+\sqrt{2}\frac{1}{4}\frac{1}{4}\right)}{2\sqrt{c}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\sqrt{b}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\frac{1}{2}\sqrt{b}x-\sqrt{2}\frac{1}{4}\frac{1}{4}\right)}{2\sqrt{c}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\sqrt{b}} - \frac{\sqrt{2} \log\left(\sqrt{b}x^2+\sqrt{2}\frac{1}{4}\frac{1}{4}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{\sqrt{2} \log\left(\sqrt{b}x^2-\sqrt{2}\frac{1}{4}\frac{1}{4}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} \right) \frac{1}{8(bc-ad)} - \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\frac{1}{2}\sqrt{d}x+\sqrt{2}\frac{1}{4}\frac{1}{4}\right)}{2\sqrt{c}\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}\sqrt{d}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\frac{1}{2}\sqrt{d}x-\sqrt{2}\frac{1}{4}\frac{1}{4}\right)}{2\sqrt{c}\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}\sqrt{d}} - \frac{\sqrt{2} \log\left(\sqrt{d}x^2+\sqrt{2}\frac{1}{4}\frac{1}{4}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}} + \frac{\sqrt{2} \log\left(\sqrt{d}x^2-\sqrt{2}\frac{1}{4}\frac{1}{4}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}} \right) \frac{1}{8(bc-ad)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out] 1/8*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b))*sqrt(b) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b))*sqrt(b) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4))/(b*c - a*d) - 1/8*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)

$$\frac{\sqrt{d}}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + 2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\frac{(2\sqrt{d}x - \sqrt{2}c^{1/4}d^{1/4})}{\sqrt{\sqrt{c}\sqrt{d}}}\right) - \frac{\sqrt{2}\log(\sqrt{d}x^2 + \sqrt{2}c^{1/4}d^{1/4}x + \sqrt{c})}{c^{1/4}d^{3/4}} + \frac{\sqrt{2}\log(\sqrt{d}x^2 - \sqrt{2}c^{1/4}d^{1/4}x + \sqrt{c})}{c^{1/4}d^{3/4}}\bigg)/(b*c - a*d)$$

mupad [B] time = 5.50, size = 6633, normalized size = 14.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2/((a + b*x^4)*(c + d*x^4)), x)$

[Out]
$$\begin{aligned} & \text{atan}\left(\frac{x(4ab^6c^2d^5 + 4a^2b^5cd^6) + (-b/(256a^5d^4 + 256ab^4c^4 - 1024a^2b^3c^3d + 1536a^3b^2c^2d^2 - 1024a^4b^2cd^3))^{3/4}}{(x(-b/(256a^5d^4 + 256ab^4c^4 - 1024a^2b^3c^3d + 1536a^3b^2c^2d^2 - 1024a^4b^2cd^3))^{1/4} * (1024ab^{10}c^7d^4 + 1024a^7b^4cd^{10} - 4096a^2b^9c^6d^5 + 7168a^3b^8c^5d^6 - 8192a^4b^7c^4d^7 + 7168a^5b^6c^3d^8 - 4096a^6b^5c^2d^9) + 256ab^9c^6d^4 + 256a^6b^4cd^9 - 768a^2b^8c^5d^5 + 512a^3b^7c^4d^6 + 512a^4b^6c^3d^7 - 768a^5b^5c^2d^8)}\right) \\ & + \frac{-b/(256a^5d^4 + 256ab^4c^4 - 1024a^2b^3c^3d + 1536a^3b^2c^2d^2 - 1024a^4b^2cd^3)}{(x(4ab^6c^2d^5 + 4a^2b^5cd^6) - (-b/(256a^5d^4 + 256ab^4c^4 - 1024a^2b^3c^3d + 1536a^3b^2c^2d^2 - 1024a^4b^2cd^3))^{3/4})} * \frac{256ab^9c^6d^4 - x(-b/(256a^5d^4 + 256ab^4c^4 - 1024a^2b^3c^3d + 1536a^3b^2c^2d^2 - 1024a^4b^2cd^3))^{1/4} * (1024ab^{10}c^7d^4 + 1024a^7b^4cd^{10} - 4096a^2b^9c^6d^5 + 7168a^3b^8c^5d^6 - 8192a^4b^7c^4d^7 + 7168a^5b^6c^3d^8 - 4096a^6b^5c^2d^9) + 256ab^9c^6d^4 + 256a^6b^4cd^9 - 768a^2b^8c^5d^5 + 512a^3b^7c^4d^6 + 512a^4b^6c^3d^7 - 768a^5b^5c^2d^8)}{(x(4ab^6c^2d^5 + 4a^2b^5cd^6) + (-b/(256a^5d^4 + 256ab^4c^4 - 1024a^2b^3c^3d + 1536a^3b^2c^2d^2 - 1024a^4b^2cd^3))^{3/4})} * \frac{256ab^9c^6d^4 - x(-b/(256a^5d^4 + 256ab^4c^4 - 1024a^2b^3c^3d + 1536a^3b^2c^2d^2 - 1024a^4b^2cd^3))^{1/4} * (1024ab^{10}c^7d^4 + 1024a^7b^4cd^{10} - 4096a^2b^9c^6d^5 + 7168a^3b^8c^5d^6 - 8192a^4b^7c^4d^7 + 7168a^5b^6c^3d^8 - 4096a^6b^5c^2d^9) + 256ab^9c^6d^4 + 256a^6b^4cd^9 - 768a^2b^8c^5d^5 + 512a^3b^7c^4d^6 + 512a^4b^6c^3d^7 - 768a^5b^5c^2d^8)}{(x(4ab^6c^2d^5 + 4a^2b^5cd^6) - (-b/(256a^5d^4 + 256ab^4c^4 - 1024a^2b^3c^3d + 1536a^3b^2c^2d^2 - 1024a^4b^2cd^3))^{3/4})} * \frac{256ab^9c^6d^4 - x(-b/(256a^5d^4 + 256ab^4c^4 - 1024a^2b^3c^3d + 1536a^3b^2c^2d^2 - 1024a^4b^2cd^3))^{1/4} * (1024ab^{10}c^7d^4 + 1024a^7b^4cd^{10} - 4096a^2b^9c^6d^5 + 7168a^3b^8c^5d^6 - 8192a^4b^7c^4d^7 + 7168a^5b^6c^3d^8 - 4096a^6b^5c^2d^9) + 256ab^9c^6d^4 + 256a^6b^4cd^9 - 768a^2b^8c^5d^5 + 512a^3b^7c^4d^6 + 512a^4b^6c^3d^7 - 768a^5b^5c^2d^8)}{(x(4ab^6c^2d^5 + 4a^2b^5cd^6) + (-b/(256a^5d^4 + 256ab^4c^4 - 1024a^2b^3c^3d + 1536a^3b^2c^2d^2 - 1024a^4b^2cd^3))^{3/4})} \end{aligned}$$

$$\begin{aligned} & d^9 - 768a^2b^8c^5d^5 + 512a^3b^7c^4d^6 + 512a^4b^6c^3d^7 - 768 \\ & a^5b^5c^2d^8) * (-b / (256a^5d^4 + 256a*b^4c^4 - 1024a^2b^3c^3d + \\ & 1536a^3b^2c^2d^2 - 1024a^4b*c*d^3))^{(1/4)} + 2*a*b^5*c*d^5) * (-b / (256* \\ & a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024* \\ & a^4*b*c*d^3))^{(1/4)} * 2i + 2*\operatorname{atan}(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (\\ & -b / (256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 \\ & - 1024*a^4*b*c*d^3))^{(3/4)} * (256*a*b^9*c^6*d^4 - x*(-b / (256*a^5*d^4 + 256*a \\ & b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^{(1/4)} * (1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 71 \\ & 68*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6 \\ & *b^5*c^2*d^9) * 1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4 \\ & *d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8) * 1i) * (-b / (256*a^5*d^4 + 2 \\ & 56*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3 \\ &))^{(1/4)} + (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b / (256*a^5*d^4 + 256* \\ & a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^{(\\ & 3/4)} * (x*(-b / (256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b \\ & ^2*c^2*d^2 - 1024*a^4*b*c*d^3))^{(1/4)} * (1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c \\ & *d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 \\ & + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) * 1i + 256*a*b^9*c^6*d^4 + 256 \\ & *a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3 \\ & *d^7 - 768*a^5*b^5*c^2*d^8) * 1i) * (-b / (256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2 \\ & *b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^{(1/4)})) / ((x*(4*a*b^6 \\ & *c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b / (256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b \\ & ^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^{(3/4)} * (x*(-b / (256*a^5* \\ & d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4* \\ & b*c*d^3))^{(1/4)} * (1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c \\ & ^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 \\ & - 4096*a^6*b^5*c^2*d^9) * 1i + 256*a*b^9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a \\ & ^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2 \\ & *d^8) * 1i) * (-b / (256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3 \\ & *b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^{(1/4)} * 1i - (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^ \\ & 5*c*d^6) + (-b / (256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3 \\ & *b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^{(3/4)} * (256*a*b^9*c^6*d^4 - x*(-b / (256*a^5 \\ & *d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4 \\ & *b*c*d^3))^{(1/4)} * (1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9* \\ & c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 \\ & - 4096*a^6*b^5*c^2*d^9) * 1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 51 \\ & 2*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8) * 1i) * (-b / (256 \\ & *a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024 \\ & *a^4*b*c*d^3))^{(1/4)} * 1i + 2*a*b^5*c*d^5) * (-b / (256*a^5*d^4 + 256*a*b^4*c^4 \\ & - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^{(1/4)} + \operatorname{at} \\ & \operatorname{an}(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-d / (256*b^4*c^5 + 256*a^4*c*d \\ & ^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^{(3/4)} * (\\ & x*(-d / (256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3* \\ & d^2 - 1024*a*b^3*c^4*d))^{(1/4)} * (1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - \end{aligned}$$

$$\begin{aligned}
& ^{10} - 4096a^2b^9c^6d^5 + 7168a^3b^8c^5d^6 - 8192a^4b^7c^4d^7 + 7168a^5b^6c^3d^8 - 4096a^6b^5c^2d^9) \cdot 1i + 256a^5b^6c^3d^8 - 4096a^6b^5c^2d^9) \cdot 1i + 256a^6b^4c^3d^9 - 768a^2b^8c^5d^5 + 512a^3b^7c^4d^6 + 512a^4b^6c^3d^7 - 768a^5b^5c^2d^8) \cdot 1i) \cdot (-d/(256b^4c^5 + 256a^4c^4d - 1024a^3b^2c^3d^2 + 1536a^2b^2c^3d^2 - 1024a^3b^2c^3d^2 - 1024a^3b^2c^3d^2 - 1024a^3b^2c^3d^2))^{(1/4)}) / ((x*(4a^6b^6c^2d^5 + 4a^2b^5c^6d^6) - (-d/(256b^4c^5 + 256a^4c^4d - 1024a^3b^2c^3d^2 + 1536a^2b^2c^3d^2 - 1024a^3b^2c^3d^2 - 1024a^3b^2c^3d^2))^{(3/4)}) \cdot (x*(-d/(256b^4c^5 + 256a^4c^4d - 1024a^3b^2c^3d^2 + 1536a^2b^2c^3d^2 - 1024a^3b^2c^3d^2 - 1024a^3b^2c^3d^2))^{(1/4)}) \cdot (1024a^6b^10c^7d^4 + 1024a^7b^4c^4d^10 - 4096a^2b^9c^6d^5 + 7168a^3b^8c^5d^6 - 8192a^4b^7c^4d^7 + 7168a^5b^6c^3d^8 - 4096a^6b^5c^2d^9) \cdot 1i + 256a^6b^4c^3d^9 - 768a^2b^8c^5d^5 + 512a^3b^7c^4d^6 + 512a^4b^6c^3d^7 - 768a^5b^5c^2d^8) \cdot 1i) \cdot (-d/(256b^4c^5 + 256a^4c^4d - 1024a^3b^2c^3d^2 + 1536a^2b^2c^3d^2 - 1024a^3b^2c^3d^2 - 1024a^3b^2c^3d^2))^{(1/4)}) \cdot 1i - (x*(4a^6b^6c^2d^5 + 4a^2b^5c^6d^6) + (-d/(256b^4c^5 + 256a^4c^4d - 1024a^3b^2c^3d^2 + 1536a^2b^2c^3d^2 - 1024a^3b^2c^3d^2 - 1024a^3b^2c^3d^2))^{(3/4)}) \cdot (256a^6b^9c^6d^4 - x*(-d/(256b^4c^5 + 256a^4c^4d - 1024a^3b^2c^3d^2 + 1536a^2b^2c^3d^2 - 1024a^3b^2c^3d^2 - 1024a^3b^2c^3d^2))^{(1/4)}) \cdot (1024a^6b^10c^7d^4 + 1024a^7b^4c^4d^10 - 4096a^2b^9c^6d^5 + 7168a^3b^8c^5d^6 - 8192a^4b^7c^4d^7 + 7168a^5b^6c^3d^8 - 4096a^6b^5c^2d^9) \cdot 1i + 256a^6b^4c^3d^9 - 768a^2b^8c^5d^5 + 512a^3b^7c^4d^6 + 512a^4b^6c^3d^7 - 768a^5b^5c^2d^8) \cdot 1i) \cdot (-d/(256b^4c^5 + 256a^4c^4d - 1024a^3b^2c^3d^2 + 1536a^2b^2c^3d^2 - 1024a^3b^2c^3d^2 - 1024a^3b^2c^3d^2))^{(1/4)}) \cdot 1i + 2a^5b^5c^4d^5) \cdot (-d/(256b^4c^5 + 256a^4c^4d - 1024a^3b^2c^3d^2 + 1536a^2b^2c^3d^2 - 1024a^3b^2c^3d^2 - 1024a^3b^2c^3d^2))^{(1/4)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**4+a)/(d*x**4+c),x)

[Out] Timed out

$$3.509 \quad \int \frac{1}{(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=449

$$\frac{b^{3/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4}(bc-ad)} + \frac{b^{3/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4}(bc-ad)} - \frac{b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4}(bc-ad)} + \frac{b^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4}(bc-ad)}$$

Rubi [A] time = 0.26, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {391, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{3/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4}(bc-ad)} + \frac{b^{3/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4}(bc-ad)} - \frac{b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4}(bc-ad)} + \frac{b^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4}(bc-ad)} + \frac{d^{3/4} \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} c^{3/4}(bc-ad)} - \frac{d^{3/4} \log\left(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} c^{3/4}(bc-ad)} + \frac{d^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} c^{3/4}(bc-ad)} - \frac{d^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^4)*(c + d*x^4)), x]

[Out] $-(b^{3/4} \text{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} x}{a^{1/4}}\right]) / (2 \sqrt{2} a^{3/4} (b^*c - a*d)) + (b^{3/4} \text{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} x}{a^{1/4}}\right]) / (2 \sqrt{2} a^{3/4} (b^*c - a*d)) + (d^{3/4} \text{ArcTan}\left[1 - \frac{\sqrt{2} d^{1/4} x}{c^{1/4}}\right]) / (2 \sqrt{2} c^{3/4} (b^*c - a*d)) - (d^{3/4} \text{ArcTan}\left[1 + \frac{\sqrt{2} d^{1/4} x}{c^{1/4}}\right]) / (2 \sqrt{2} c^{3/4} (b^*c - a*d)) - (b^{3/4} \text{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{b} x^2\right]) / (4 \sqrt{2} a^{3/4} (b^*c - a*d)) + (b^{3/4} \text{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{b} x^2\right]) / (4 \sqrt{2} a^{3/4} (b^*c - a*d)) + (d^{3/4} \text{Log}\left[\sqrt{c} - \sqrt{2} c^{1/4} d^{1/4} x + \sqrt{d} x^2\right]) / (4 \sqrt{2} c^{3/4} (b^*c - a*d)) - (d^{3/4} \text{Log}\left[\sqrt{c} + \sqrt{2} c^{1/4} d^{1/4} x + \sqrt{d} x^2\right]) / (4 \sqrt{2} c^{3/4} (b^*c - a*d))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + bx^4)(c + dx^4)} dx &= \frac{b \int \frac{1}{a+bx^4} dx}{bc - ad} - \frac{d \int \frac{1}{c+dx^4} dx}{bc - ad} \\
&= \frac{b \int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4} dx}{2\sqrt{a}(bc - ad)} + \frac{b \int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4} dx}{2\sqrt{a}(bc - ad)} - \frac{d \int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4} dx}{2\sqrt{c}(bc - ad)} - \frac{d \int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4} dx}{2\sqrt{c}(bc - ad)} \\
&= \frac{\sqrt{b} \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}(bc - ad)} + \frac{\sqrt{b} \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}(bc - ad)} - \frac{b^{3/4} \int \frac{\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2} a^{3/4}(bc - ad)} - \frac{b^{3/4} \int \frac{\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{b}} - 2x}{-\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2} a^{3/4}(bc - ad)} \\
&= -\frac{b^{3/4} \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{b} x^2)}{4\sqrt{2} a^{3/4}(bc - ad)} + \frac{b^{3/4} \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{b} x^2)}{4\sqrt{2} a^{3/4}(bc - ad)} + \frac{d^{3/4} \log(\sqrt{c} - \sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{d} x^2)}{4\sqrt{2} c^{3/4}(bc - ad)} - \frac{d^{3/4} \log(\sqrt{c} + \sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{d} x^2)}{4\sqrt{2} c^{3/4}(bc - ad)} \\
&= -\frac{b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4}(bc - ad)} + \frac{b^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4}(bc - ad)} + \frac{d^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} c^{3/4}(bc - ad)} - \frac{d^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} c^{3/4}(bc - ad)}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 340, normalized size = 0.76

$$\frac{a^{3/4} d^{3/4} \log(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2) - a^{3/4} d^{3/4} \log(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2) + 2a^{3/4} d^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right) - 2a^{3/4} d^{3/4} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}} + 1\right) - b^{3/4} c^{3/4} \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2) + b^{3/4} c^{3/4} \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2) - 2b^{3/4} c^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right) + 2b^{3/4} c^{3/4} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2} a^{3/4} c^{3/4} (bc - ad)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x^4)*(c + d*x^4)),x]

[Out] $(-2*b^{(3/4)}*c^{(3/4)}*ArcTan[1 - (Sqrt[2]*b^{(1/4)}*x)/a^{(1/4)}] + 2*b^{(3/4)}*c^{(3/4)}*ArcTan[1 + (Sqrt[2]*b^{(1/4)}*x)/a^{(1/4)}] + 2*a^{(3/4)}*d^{(3/4)}*ArcTan[1 - (Sqrt[2]*d^{(1/4)}*x)/c^{(1/4)}] - 2*a^{(3/4)}*d^{(3/4)}*ArcTan[1 + (Sqrt[2]*d^{(1/4)}*x)/c^{(1/4)}] - b^{(3/4)}*c^{(3/4)}*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*b^{(1/4)}*x + Sqrt[b]*x^2] + b^{(3/4)}*c^{(3/4)}*Log[Sqrt[a] + Sqrt[2]*a^{(1/4)}*b^{(1/4)}*x + Sqrt[b]*x^2] + a^{(3/4)}*d^{(3/4)}*Log[Sqrt[c] - Sqrt[2]*c^{(1/4)}*d^{(1/4)}*x + Sqrt[d]*x^2] - a^{(3/4)}*d^{(3/4)}*Log[Sqrt[c] + Sqrt[2]*c^{(1/4)}*d^{(1/4)}*x + Sqrt[d]*x^2])/(4*Sqrt[2]*a^{(3/4)}*c^{(3/4)}*(b*c - a*d))$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[1/((a + b*x^4)*(c + d*x^4)), x]

fricas [B] time = 0.63, size = 1356, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -(b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^{1/4} * \arctan\left(\frac{(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^{3/4} * x - (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^{3/4} * \sqrt{(b^2*x^2 + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)) * \sqrt{-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)}}}{b^2}\right) / b^2 + (-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^{1/4} * \arctan\left(\frac{(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^{3/4} * x - (b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^{3/4} * \sqrt{(d^2*x^2 + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)) * \sqrt{-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)}}}{d^2}\right) / d^2 + 1/4 * (-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^{1/4} * \log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^{1/4}) - 1/4 * (-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^{1/4} * \log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^{1/4}) - 1/4 * (-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^{1/4} * \log(d*x + (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^{1/4}) + 1/4 * (-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^{1/4} * \log(d*x - (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^{1/4}) \end{aligned}$$

giac [A] time = 0.22, size = 437, normalized size = 0.97

$$\frac{(ab)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2+2\sqrt{\frac{a}{b}}}}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2abc} - \sqrt{2a^2d})} + \frac{(ab)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2+2\sqrt{\frac{a}{b}}}}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2abc} - \sqrt{2a^2d})} - \frac{(cd)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2+2\sqrt{\frac{c}{d}}}}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2bc^2} - \sqrt{2acd})} - \frac{(cd)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2+2\sqrt{\frac{c}{d}}}}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2bc^2} - \sqrt{2acd})} + \frac{(ab)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}}\right)}{4(\sqrt{2abc} - \sqrt{2a^2d})} - \frac{(ab)^{\frac{1}{4}} \log\left(x^2 - \sqrt{2x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}}\right)}{4(\sqrt{2abc} - \sqrt{2a^2d})} - \frac{(cd)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}}\right)}{4(\sqrt{2bc^2} - \sqrt{2acd})} + \frac{(cd)^{\frac{1}{4}} \log\left(x^2 - \sqrt{2x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}}\right)}{4(\sqrt{2bc^2} - \sqrt{2acd})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] $\frac{1}{2}*(a*b^3)^{(1/4)}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(\sqrt{2}*a*b*c - \sqrt{2}*a^2*d) + \frac{1}{2}*(a*b^3)^{(1/4)}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(\sqrt{2}*a*b*c - \sqrt{2}*a^2*d) - \frac{1}{2}*(c*d^3)^{(1/4)}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(\sqrt{2}*b*c^2 - \sqrt{2}*a*c*d) - \frac{1}{2}*(c*d^3)^{(1/4)}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(\sqrt{2}*b*c^2 - \sqrt{2}*a*c*d) + \frac{1}{4}*(a*b^3)^{(1/4)}*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*a*b*c - \sqrt{2}*a^2*d) - \frac{1}{4}*(a*b^3)^{(1/4)}*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*a*b*c - \sqrt{2}*a^2*d) - \frac{1}{4}*(c*d^3)^{(1/4)}*\log(x^2 + \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b*c^2 - \sqrt{2}*a*c*d) + \frac{1}{4}*(c*d^3)^{(1/4)}*\log(x^2 - \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b*c^2 - \sqrt{2}*a*c*d)$

maple [A] time = 0.06, size = 320, normalized size = 0.71

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} b \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4(ad-bc)a} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} b \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4(ad-bc)a} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} b \ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8(ad-bc)a} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} d \arctan\left(\frac{\sqrt{2}x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4(ad-bc)c} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} d \arctan\left(\frac{\sqrt{2}x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4(ad-bc)c} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} d \ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}\right)}{8(ad-bc)c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^4+a)/(d*x^4+c),x)`

[Out] $\frac{1}{8}*\frac{d}{(a*d-b*c)}*(c/d)^{(1/4)}/c*2^{(1/2)}*\ln((x^2+(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)})/(x^2-(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)}))+\frac{1}{4}*\frac{d}{(a*d-b*c)}*(c/d)^{(1/4)}/c*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x+1)+\frac{1}{4}*\frac{d}{(a*d-b*c)}*(c/d)^{(1/4)}/c*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x-1)-\frac{1}{8}*b/(a*d-b*c)*(a/b)^{(1/4)}/a*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)}))- \frac{1}{4}*b/(a*d-b*c)*(a/b)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)-\frac{1}{4}*b/(a*d-b*c)*(a/b)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)$

maxima [A] time = 1.38, size = 365, normalized size = 0.81

$$\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left[2\sqrt{a}\sqrt{b}\sqrt{2a^{\frac{1}{4}}b^{\frac{1}{4}}+1}\right]}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left[2\sqrt{a}\sqrt{b}\sqrt{2a^{\frac{1}{4}}b^{\frac{1}{4}}-1}\right]}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}} + \frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}\sqrt{x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}}\right)}{a^{\frac{3}{4}}} - \frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}\sqrt{x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}}\right)}{a^{\frac{3}{4}}} - \frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left[2\sqrt{c}\sqrt{d}\sqrt{2c^{\frac{1}{4}}d^{\frac{1}{4}}+1}\right]}{2\sqrt{c}\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}} + \frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left[2\sqrt{c}\sqrt{d}\sqrt{2c^{\frac{1}{4}}d^{\frac{1}{4}}-1}\right]}{2\sqrt{c}\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}} + \frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}\sqrt{x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}}\right)}{c^{\frac{3}{4}}} - \frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}\sqrt{x^2-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}}\right)}{c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")`

[Out] $\frac{1}{8}*(2*\sqrt{2}*b*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*b*x + \sqrt{2}*a^{(1/4)}*b^{(1/4)})/\sqrt{a}*\sqrt{b}))/(\sqrt{a}*\sqrt{b}) + 2*\sqrt{2}*b*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*b*x - \sqrt{2}*a^{(1/4)}*b^{(1/4)})/\sqrt{a}*\sqrt{b}))/(\sqrt{a}*\sqrt{b}) + \sqrt{2}*b^{(3/4)}*\log(\sqrt{b}*x^2 + \sqrt{2}*a^{(1/4)}*b^{(1/4)}*x + \sqrt{a})/a^{(3/4)} - \sqrt{2}*b^{(3/4)}*\log(\sqrt{b}*x^2 - \sqrt{2}*a^{(1/4)}*b^{(1/4)}*x + \sqrt{a})/a^{(3/4)})/(b*c - a*d) - \frac{1}{8}*(2*\sqrt{2}*d*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*d*x + \sqrt{2}*c^{(1/4)}*d^{(1/4)})/\sqrt{c}*\sqrt{d}))/(\sqrt{c}*\sqrt{d})$

$$\frac{\sqrt{d}}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + 2\sqrt{2}d\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{2}\sqrt{d}\sqrt{x - \sqrt{2}c^{1/4}d^{1/4}}\right) / \sqrt{\sqrt{c}\sqrt{d}} + \sqrt{2}d^{3/4}\log(\sqrt{d}\sqrt{x^2 + \sqrt{2}c^{1/4}d^{1/4}x + \sqrt{c}}) / c^{3/4} - \sqrt{2}d^{3/4}\log(\sqrt{d}\sqrt{x^2 - \sqrt{2}c^{1/4}d^{1/4}x + \sqrt{c}}) / c^{3/4} / (b\sqrt{c} - a\sqrt{d})$$

mupad [B] time = 5.85, size = 6153, normalized size = 13.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x^4)*(c + d*x^4)),x)

[Out] - atan(((((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4))*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4))*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4))*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4))*i - (((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4))*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4))*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4))*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x) *((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4))*i) / ((((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4))*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4))*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4))*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)

$$\begin{aligned}
&3b^8c^4d^7 - 3072a^4b^7c^3d^8 + 6144a^5b^6c^2d^9)) - 16a^2b^6d^8 - 16b^8c^2d^6 + 32ab^7cd^7) + 8b^7d^7x)(-d^3/(256b^4c^7 + \\
&256a^4c^3d^4 - 1024a^3b^3c^4d^3 + 1536a^2b^2c^5d^2 - 1024ab^3c^6d))^{(1/4)} + ((-d^3/(256b^4c^7 + 256a^4c^3d^4 - 1024a^3b^3c^4d^3 + \\
&1536a^2b^2c^5d^2 - 1024ab^3c^6d))^{(1/4)}) * ((-d^3/(256b^4c^7 + 256a^4c^3d^4 - 1024a^3b^3c^4d^3 + 1536a^2b^2c^5d^2 - 1024ab^3c^6d)) \\
&)^{(3/4)}) * ((-d^3/(256b^4c^7 + 256a^4c^3d^4 - 1024a^3b^3c^4d^3 + 1536a^2b^2c^5d^2 - 1024ab^3c^6d))^{(1/4)}) * (4096ab^{11}c^8d^4 + 4096a^8b^4 \\
&c^4d^{11} - 20480a^2b^{10}c^7d^5 + 36864a^3b^9c^6d^6 - 20480a^4b^8c^5d^7 - 20480a^5b^7c^4d^8 + 36864a^6b^6c^3d^9 - 20480a^7b^5c^2 \\
&d^{10}) - x(1024a^7b^4d^{11} + 1024b^{11}c^7d^4 - 4096ab^{10}c^6d^5 - 4096a^6b^5c^4d^{10} + 6144a^2b^9c^5d^6 - 3072a^3b^8c^4d^7 - 3072a^4b^7 \\
&c^3d^8 + 6144a^5b^6c^2d^9)) - 16a^2b^6d^8 - 16b^8c^2d^6 + 32ab^7cd^7) - 8b^7d^7x)(-d^3/(256b^4c^7 + 256a^4c^3d^4 - 1024a^3 \\
&b^3c^4d^3 + 1536a^2b^2c^5d^2 - 1024ab^3c^6d))^{(1/4)}) * ((-d^3/(256b^4c^7 + 256a^4c^3d^4 - 1024a^3b^3c^4d^3 + 1536a^2b^2c^5d^2 - 1024ab^3 \\
&c^6d))^{(1/4)}) * 2i - \operatorname{atan}\left(\left(\frac{-b^3}{256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^3c^3d^3}\right)^{(1/4)}\right) * \left(\frac{-b^3}{256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^3c^3d^3}\right)^{(3/4)} * \left(\frac{-b^3}{256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^3c^3d^3}\right)^{(1/4)} * (4096ab^{11}c^8d^4 + 4096a^8b^4c^4d^{11} - 20480a^2b^{10}c^7d^5 + 36864a^3b^9c^6d^6 - 20480a^4b^8c^5d^7 - 20480a^5b^7c^4d^8 + 36864a^6b^6c^3d^9 - 20480a^7b^5c^2d^{10}) + x(1024a^7b^4d^{11} + 1024b^{11}c^7d^4 - 4096ab^{10}c^6d^5 - 4096a^6b^5c^4d^{10} + 6144a^2b^9c^5d^6 - 3072a^3b^8c^4d^7 - 3072a^4b^7c^3d^8 + 6144a^5b^6c^2d^9)) - 16a^2b^6d^8 - 16b^8c^2d^6 + 32ab^7cd^7) + 8b^7d^7x)(-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^3c^3d^3))^{(1/4)}) * 1i - \left(\frac{-b^3}{256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^3c^3d^3}\right)^{(1/4)} * \left(\frac{-b^3}{256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^3c^3d^3}\right)^{(3/4)} * \left(\frac{-b^3}{256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^3c^3d^3}\right)^{(1/4)} * (4096ab^{11}c^8d^4 + 4096a^8b^4c^4d^{11} - 20480a^2b^{10}c^7d^5 + 36864a^3b^9c^6d^6 - 20480a^4b^8c^5d^7 - 20480a^5b^7c^4d^8 + 36864a^6b^6c^3d^9 - 20480a^7b^5c^2d^{10}) - x(1024a^7b^4d^{11} + 1024b^{11}c^7d^4 - 4096ab^{10}c^6d^5 - 4096a^6b^5c^4d^{10} + 6144a^2b^9c^5d^6 - 3072a^3b^8c^4d^7 - 3072a^4b^7c^3d^8 + 6144a^5b^6c^2d^9)) - 16a^2b^6d^8 - 16b^8c^2d^6 + 32ab^7cd^7) - 8b^7d^7x)(-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^3c^3d^3))^{(1/4)}) * 1i) / \left(\left(\frac{-b^3}{256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^3c^3d^3}\right)^{(1/4)} * \left(\frac{-b^3}{256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^3c^3d^3}\right)^{(3/4)} * \left(\frac{-b^3}{256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^3c^3d^3}\right)^{(1/4)} * (4096ab^{11}c^8d^4 + 4096a^8b^4c^4d^{11} - 20480a^2b^{10}c^7d^5 + 36864a^3b^9c^6d^6 - 20480a^4b^8c^5d^7 - 20480a^5b^7c^4d^8 + 36864a^6b^6c^3d^9 - 20480a^7b^5c^2d^{10})\right)
\end{aligned}$$

$$\begin{aligned}
& *c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^{10} + x*(1024*a^7*b^4*d^{11} + 1024*b^{11}*c^7*d^4 - 4096*a*b^{10}*c^6*d^5 - 4096*a^6*b^5*c*d^{10} + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7 * x) * (-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^{1/4} + ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^{1/4}) * ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^{3/4}) * ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^{1/4}) * (4096*a*b^{11}*c^8*d^4 + 4096*a^8*b^4*c*d^{11} - 20480*a^2*b^{10}*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^{10}) - x*(1024*a^7*b^4*d^{11} + 1024*b^{11}*c^7*d^4 - 4096*a*b^{10}*c^6*d^5 - 4096*a^6*b^5*c*d^{10} + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7 * x) * (-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^{1/4})) * (-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^{1/4}) * 2i - 2*atan((b^3*d^3*x - (128*b^{10}*c^7*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (128*a^7*b^3*d^7*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (768*a^2*b^8*c^5*d^2*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (384*a^3*b^7*c^4*d^3*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (384*a^4*b^6*c^3*d^4*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (768*a^5*b^5*c^2*d^5*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (512*a*b^9*c^6*d*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (512*a^6*b^4*c*d^6*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3)))/((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^{1/4}) * ((b^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2560*a^7*b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*b^3*c^4*d^4 + 4608*a^6*b^2*c^3*d^5))/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + 2*a^2*b^2*d^4 + 2*b^4*c^2*d^2 - 4*a*b^3*c*d^3)) * (-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^{1/4} - 2*atan((b^3*d^3*x - (128*a^7*d^{10}*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (128*b^7*c^7*d^3*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (384*a^3*b^4*c^4*d
\end{aligned}$$

$$\begin{aligned} & ^6*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (384*a^4*b^3*c^3*d^7*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (768*a^5*b^2*c^2*d^8*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (512*a^6*b*c*d^9*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (512*a*b^6*c^6*d^4*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))/((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4))*((d^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2560*a^7*b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*b^3*c^4*d^4 + 4608*a^6*b^2*c^3*d^5))/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + 2*a^2*b^2*d^4 + 2*b^4*c^2*d^2 - 4*a*b^3*c*d^3))*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**4+a)/(d*x**4+c),x)

[Out] Timed out

$$3.510 \quad \int \frac{1}{x^2(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=460

$$\frac{b^{5/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{5/4}(bc-ad)} + \frac{b^{5/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{5/4}(bc-ad)} + \frac{b^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{5/4}(bc-ad)} - \frac{b^{5/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{5/4}(bc-ad)}$$

Rubi [A] time = 0.45, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {480, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{5/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{5/4}(bc-ad)} + \frac{b^{5/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{5/4}(bc-ad)} + \frac{b^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{5/4}(bc-ad)} - \frac{b^{5/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{5/4}(bc-ad)} + \frac{d^{5/4} \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} c^{5/4}(bc-ad)} - \frac{d^{5/4} \log\left(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} c^{5/4}(bc-ad)} - \frac{d^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} c^{5/4}(bc-ad)} + \frac{d^{5/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} c^{5/4}(bc-ad)} - \frac{1}{acx}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]

[Out] $-(1/(a*c*x)) + (b^{5/4}*ArcTan[1 - (Sqrt[2]*b^{1/4}*x)/a^{1/4}])/(2*Sqrt[2]*a^{5/4}*(b*c - a*d)) - (b^{5/4}*ArcTan[1 + (Sqrt[2]*b^{1/4}*x)/a^{1/4}])/(2*Sqrt[2]*a^{5/4}*(b*c - a*d)) - (d^{5/4}*ArcTan[1 - (Sqrt[2]*d^{1/4}*x)/c^{1/4}])/(2*Sqrt[2]*c^{5/4}*(b*c - a*d)) + (d^{5/4}*ArcTan[1 + (Sqrt[2]*d^{1/4}*x)/c^{1/4}])/(2*Sqrt[2]*c^{5/4}*(b*c - a*d)) - (b^{5/4}*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*b^{1/4}*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^{5/4}*(b*c - a*d)) + (b^{5/4}*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*b^{1/4}*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^{5/4}*(b*c - a*d)) + (d^{5/4}*Log[Sqrt[c] - Sqrt[2]*c^{1/4}*d^{1/4}*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^{5/4}*(b*c - a*d)) - (d^{5/4}*Log[Sqrt[c] + Sqrt[2]*c^{1/4}*d^{1/4}*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^{5/4}*(b*c - a*d))$

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 297

Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((g*x)^m*(a + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a + bx^4)(c + dx^4)} dx &= -\frac{1}{acx} + \frac{\int \frac{x^2(-bc-ad-bdx^4)}{(a+bx^4)(c+dx^4)} dx}{ac} \\
&= -\frac{1}{acx} + \frac{\int \left(-\frac{b^2cx^2}{(bc-ad)(a+bx^4)} - \frac{ad^2x^2}{(-bc+ad)(c+dx^4)} \right) dx}{ac} \\
&= -\frac{1}{acx} - \frac{b^2 \int \frac{x^2}{a+bx^4} dx}{a(bc-ad)} + \frac{d^2 \int \frac{x^2}{c+dx^4} dx}{c(bc-ad)} \\
&= -\frac{1}{acx} + \frac{b^{3/2} \int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4} dx}{2a(bc-ad)} - \frac{b^{3/2} \int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4} dx}{2a(bc-ad)} - \frac{d^{3/2} \int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4} dx}{2c(bc-ad)} + \frac{d^{3/2} \int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4} dx}{2c(bc-ad)} \\
&= -\frac{1}{acx} - \frac{b \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4a(bc-ad)} - \frac{b \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4a(bc-ad)} - \frac{b^{5/4} \int \frac{\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2} a^{5/4}(bc-ad)} \\
&= -\frac{1}{acx} - \frac{b^{5/4} \log\left(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{b} x^2\right)}{4\sqrt{2} a^{5/4}(bc-ad)} + \frac{b^{5/4} \log\left(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{b} x^2\right)}{4\sqrt{2} a^{5/4}(bc-ad)} \\
&= -\frac{1}{acx} + \frac{b^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{5/4}(bc-ad)} - \frac{b^{5/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{5/4}(bc-ad)} - \frac{d^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2} c^{5/4}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 385, normalized size = 0.84

$$\frac{\sqrt{2}b^{5/4}x \log\left(-\sqrt{2} \sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2\right)}{a^{5/4}} - \frac{\sqrt{2}b^{5/4}x \log\left(\sqrt{2} \sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2\right)}{a^{5/4}} - \frac{2\sqrt{2}b^{5/4}x \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/4}} + \frac{2\sqrt{2}b^{5/4}x \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/4}} + \frac{8b}{a} - \frac{\sqrt{2}d^{5/4}x \log\left(-\sqrt{2} \sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2\right)}{c^{5/4}} + \frac{\sqrt{2}d^{5/4}x \log\left(\sqrt{2} \sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2\right)}{c^{5/4}} + \frac{2\sqrt{2}d^{5/4}x \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{5/4}} - \frac{2\sqrt{2}d^{5/4}x \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{5/4}} - \frac{8d}{c}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + b*x^4)*(c + d*x^4)), x]

[Out] ((8*b)/a - (8*d)/c - (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(5/4) + (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(5/4) + (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(5/4) - (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(5/4) + (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(5/4) - (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(5/4) - (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4) + (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4)

4)*d^(1/4)*x + Sqrt[d]*x^2))/c^(5/4) + (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2))/c^(5/4))/(-8*b*c*x + 8*a*d*x)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^4)*(c + d*x^4)), x]

fricas [B] time = 0.89, size = 1395, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out]
$$-1/4*(4*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^{1/4}*a*c*x*\arctan(((-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^{1/4}*(a*b*c - a^2*d)*x - (-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^{1/4}*(a*b*c - a^2*d)*\sqrt{(b^3*x^2 - (a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2))*\sqrt{-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4)}}/b^3)/b) - 4*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^{1/4}*a*c*x*\arctan(((-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^{1/4}*(b*c^2 - a*c*d)*x - (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^{1/4}*(b*c^2 - a*c*d)*\sqrt{(d^3*x^2 - (b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2))*\sqrt{-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4)}}/d^3)/d) + (-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^{1/4}*a*c*x*\log(b^4*x + (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^{3/4}) - (-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^{1/4}*a*c*x*\log(b^4*x - (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^{3/4}) - (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^{1/4}*a*c*x*\log(d^4*x + (b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^{3/4}) + (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^{1/4}*a*c*x*\log(d^4*x - (b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^{3/4}))$$

$$2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^{(3/4)} + 4)/(a*c*x)$$

giac [A] time = 0.21, size = 488, normalized size = 1.06

$$\frac{(ab)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2} \sqrt{z+\sqrt{z^2+1}}}{z}\right)}{2(\sqrt{2}ab^2c - \sqrt{2}a^2bd)} - \frac{(ab)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2} \sqrt{z-\sqrt{z^2+1}}}{z}\right)}{2(\sqrt{2}ab^2c - \sqrt{2}a^2bd)} + \frac{(ab)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2} \sqrt{z+\sqrt{z^2+1}}}{z}\right)}{2(\sqrt{2}bc^2d - \sqrt{2}ac^2d)} + \frac{(ab)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2} \sqrt{z-\sqrt{z^2+1}}}{z}\right)}{2(\sqrt{2}bc^2d - \sqrt{2}ac^2d)} + \frac{(ab)^{\frac{1}{2}} \log\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{4(\sqrt{2}a^2b^2c - \sqrt{2}a^2bd)} - \frac{(ab)^{\frac{1}{2}} \log\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{4(\sqrt{2}a^2b^2c - \sqrt{2}a^2bd)} - \frac{(ab)^{\frac{1}{2}} \log\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{4(\sqrt{2}bc^2d - \sqrt{2}ac^2d)} + \frac{(ab)^{\frac{1}{2}} \log\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{4(\sqrt{2}bc^2d - \sqrt{2}ac^2d)} - \frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] $-1/2*(a*b^3)^{(3/4)}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a/b)^{(1/4)}/(\sqrt{2}*a^2*b^2*c - \sqrt{2}*a^3*b*d) - 1/2*(a*b^3)^{(3/4)}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a/b)^{(1/4)}/(\sqrt{2}*a^2*b^2*c - \sqrt{2}*a^3*b*d) + 1/2*(c*d^3)^{(3/4)}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(\sqrt{2}*b*c^3*d - \sqrt{2}*a*c^2*d^2) + 1/2*(c*d^3)^{(3/4)}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(\sqrt{2}*b*c^3*d - \sqrt{2}*a*c^2*d^2) + 1/4*(a*b^3)^{(3/4)}*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*a^2*b^2*c - \sqrt{2}*a^3*b*d) - 1/4*(a*b^3)^{(3/4)}*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*a^2*b^2*c - \sqrt{2}*a^3*b*d) - 1/4*(c*d^3)^{(3/4)}*\log(x^2 + \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b*c^3*d - \sqrt{2}*a*c^2*d^2) + 1/4*(c*d^3)^{(3/4)}*\log(x^2 - \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b*c^3*d - \sqrt{2}*a*c^2*d^2) - 1/(a*c*x)$

maple [A] time = 0.06, size = 331, normalized size = 0.72

$$\frac{\sqrt{2} b \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}} a} + \frac{\sqrt{2} b \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}} a} + \frac{\sqrt{2} b \ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}} a} - \frac{\sqrt{2} d \arctan\left(\frac{\sqrt{2} x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}} c} - \frac{\sqrt{2} d \arctan\left(\frac{\sqrt{2} x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}} c} - \frac{\sqrt{2} d \ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}\right)}{8(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}} c} - \frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^4+a)/(d*x^4+c),x)

[Out] $-1/8*d/c/(a*d-b*c)/(c/d)^{(1/4)}*2^{(1/2)}*\ln((x^2-(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)})/(x^2+(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)}))-1/4*d/c/(a*d-b*c)/(c/d)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x+1)-1/4*d/c/(a*d-b*c)/(c/d)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x-1)+1/8*b/a/(a*d-b*c)/(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})/(x^2+(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)}))+1/4*b/a/(a*d-b*c)/(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)+1/4*b/a/(a*d-b*c)/(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)-1/a/c/x$

maxima [A] time = 1.27, size = 384, normalized size = 0.83

$$\frac{b^2 \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{b^2+\sqrt{2}d^{\frac{1}{4}}k^{\frac{1}{4}}}}{2\sqrt{\sqrt{c}\sqrt{b}}}\right)}{\sqrt{\sqrt{c}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{b^2-\sqrt{2}d^{\frac{1}{4}}k^{\frac{1}{4}}}}{2\sqrt{\sqrt{c}\sqrt{b}}}\right)}{\sqrt{\sqrt{c}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2} \log\left(\sqrt{b^2+\sqrt{2}d^{\frac{1}{4}}k^{\frac{1}{4}}x+\sqrt{a}}\right)}{\frac{1}{a^{\frac{1}{4}}b^{\frac{1}{4}}}} + \frac{\sqrt{2} \log\left(\sqrt{b^2-\sqrt{2}d^{\frac{1}{4}}k^{\frac{1}{4}}x+\sqrt{a}}\right)}{\frac{1}{a^{\frac{1}{4}}b^{\frac{1}{4}}}} \right)}{8(abc-a^2d)} + \frac{d^2 \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{d^2+\sqrt{2}k^{\frac{1}{4}}}}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{d^2-\sqrt{2}k^{\frac{1}{4}}}}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} - \frac{\sqrt{2} \log\left(\sqrt{d^2+\sqrt{2}k^{\frac{1}{4}}x+\sqrt{c}}\right)}{\frac{1}{c^{\frac{1}{4}}d^{\frac{1}{4}}}} + \frac{\sqrt{2} \log\left(\sqrt{d^2-\sqrt{2}k^{\frac{1}{4}}x+\sqrt{c}}\right)}{\frac{1}{c^{\frac{1}{4}}d^{\frac{1}{4}}}} \right)}{8(bc^2-acd)} - \frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out]
$$-1/8*b^2*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x + \sqrt{2})*a^{(1/4)}*b^{(1/4)})/\sqrt{\sqrt{a}*\sqrt{b}})/(\sqrt{\sqrt{a}*\sqrt{b}}*\sqrt{b}) + 2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x - \sqrt{2})*a^{(1/4)}*b^{(1/4)})/\sqrt{\sqrt{a}*\sqrt{b}})/(\sqrt{\sqrt{a}*\sqrt{b}}*\sqrt{b}) - \sqrt{2}*\log(\sqrt{b}*x^2 + \sqrt{2})*a^{(1/4)}*b^{(1/4)}*x + \sqrt{a})/(a^{(1/4)}*b^{(3/4)}) + \sqrt{2}*\log(\sqrt{b}*x^2 - \sqrt{2})*a^{(1/4)}*b^{(1/4)}*x + \sqrt{a})/(a^{(1/4)}*b^{(3/4)})/(a*b*c - a^2*d) + 1/8*d^2*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*\sqrt{d}*x + \sqrt{2})*c^{(1/4)}*d^{(1/4)})/\sqrt{\sqrt{c}*\sqrt{d}})/(\sqrt{\sqrt{c}*\sqrt{d}}*\sqrt{d}) + 2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*\sqrt{d}*x - \sqrt{2})*c^{(1/4)}*d^{(1/4)})/\sqrt{\sqrt{c}*\sqrt{d}})/(\sqrt{\sqrt{c}*\sqrt{d}}*\sqrt{d}) - \sqrt{2}*\log(\sqrt{d}*x^2 + \sqrt{2})*c^{(1/4)}*d^{(1/4)}*x + \sqrt{c})/(c^{(1/4)}*d^{(3/4)}) + \sqrt{2}*\log(\sqrt{d}*x^2 - \sqrt{2})*c^{(1/4)}*d^{(1/4)}*x + \sqrt{c})/(c^{(1/4)}*d^{(3/4)})/(b*c^2 - a*c*d) - 1/(a*c*x)$$

mupad [B] time = 6.08, size = 5962, normalized size = 12.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*x^4)*(c + d*x^4)),x)

[Out]
$$2*\operatorname{atan}\left(\left(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d)\right)^{1/4}\right)*(x*(4*a^{11}*b^9*c^{12}*d^8 + 4*a^{12}*b^8*c^{11}*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{3/4})*(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{1/4}*(1024*a^{12}*b^{12}*c^{20}*d^4 - 4096*a^{13}*b^{11}*c^{19}*d^5 + 6144*a^{14}*b^{10}*c^{18}*d^6 - 4096*a^{15}*b^9*c^{17}*d^7 + 2048*a^{16}*b^8*c^{16}*d^8 - 4096*a^{17}*b^7*c^{15}*d^9 + 6144*a^{18}*b^6*c^{14}*d^{10} - 4096*a^{19}*b^5*c^{13}*d^{11} + 1024*a^{20}*b^4*c^{12}*d^{12})*1i - 256*a^{11}*b^{12}*c^{19}*d^4 + 768*a^{12}*b^{11}*c^{18}*d^5 - 768*a^{13}*b^{10}*c^{17}*d^6 + 256*a^{14}*b^9*c^{16}*d^7 + 256*a^{16}*b^7*c^{14}*d^9 - 768*a^{17}*b^6*c^{13}*d^{10} + 768*a^{18}*b^5*c^{12}*d^{11} - 256*a^{19}*b^4*c^{11}*d^{12})*1i) + (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{1/4}*(x*(4*a^{11}*b^9*c^{12}*d^8 + 4*a^{12}*b^8*c^{11}*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{3/4})*(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{1/4}*($$

$$\begin{aligned}
& 1024*a^{12}*b^{12}*c^{20}*d^4 - 4096*a^{13}*b^{11}*c^{19}*d^5 + 6144*a^{14}*b^{10}*c^{18}*d^6 \\
& - 4096*a^{15}*b^9*c^{17}*d^7 + 2048*a^{16}*b^8*c^{16}*d^8 - 4096*a^{17}*b^7*c^{15}*d^9 \\
& + 6144*a^{18}*b^6*c^{14}*d^{10} - 4096*a^{19}*b^5*c^{13}*d^{11} + 1024*a^{20}*b^4*c^{12}*d^{12} \\
& \wedge(1/4) + 256*a^{11}*b^{12}*c^{19}*d^4 - 768*a^{12}*b^{11}*c^{18}*d^5 + 768*a^{13}*b^{10}*c^{17}*d^6 \\
& - 256*a^{14}*b^9*c^{16}*d^7 - 256*a^{16}*b^7*c^{14}*d^9 + 768*a^{17}*b^6*c^{13}*d^{10} \\
& - 768*a^{18}*b^5*c^{12}*d^{11} + 256*a^{19}*b^4*c^{11}*d^{12})*1i) / ((-d^5/(256*b^4*c^9 \\
& + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{(1/4)} \\
& *(x*(4*a^{11}*b^9*c^{12}*d^8 + 4*a^{12}*b^8*c^{11}*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 \\
& - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{(3/4)} \\
& *(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 \\
& - 1024*a*b^3*c^8*d))^{(1/4)}*(1024*a^{12}*b^{12}*c^{20}*d^4 - 4096*a^{13}*b^{11}*c^{19}*d^5 \\
& + 6144*a^{14}*b^{10}*c^{18}*d^6 - 4096*a^{15}*b^9*c^{17}*d^7 + 2048*a^{16}*b^8*c^{16}*d^8 \\
& - 4096*a^{17}*b^7*c^{15}*d^9 + 6144*a^{18}*b^6*c^{14}*d^{10} - 4096*a^{19}*b^5*c^{13}*d^{11} \\
& + 1024*a^{20}*b^4*c^{12}*d^{12})*1i - 256*a^{11}*b^{12}*c^{19}*d^4 + 768*a^{12}*b^{11}*c^{18}*d^5 \\
& - 768*a^{13}*b^{10}*c^{17}*d^6 + 256*a^{14}*b^9*c^{16}*d^7 + 256*a^{16}*b^7*c^{14}*d^9 \\
& - 768*a^{17}*b^6*c^{13}*d^{10} + 768*a^{18}*b^5*c^{12}*d^{11} - 256*a^{19}*b^4*c^{11}*d^{12})*1i) * 1i \\
& - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 \\
& - 1024*a*b^3*c^8*d))^{(1/4)}*(x*(4*a^{11}*b^9*c^{12}*d^8 + 4*a^{12}*b^8*c^{11}*d^9) - (-d^5/(256*b^4*c^9 \\
& + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{(3/4)} \\
& *(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 \\
& - 1024*a*b^3*c^8*d))^{(1/4)}*(1024*a^{12}*b^{12}*c^{20}*d^4 - 4096*a^{13}*b^{11}*c^{19}*d^5 \\
& + 6144*a^{14}*b^{10}*c^{18}*d^6 - 4096*a^{15}*b^9*c^{17}*d^7 + 2048*a^{16}*b^8*c^{16}*d^8 \\
& - 4096*a^{17}*b^7*c^{15}*d^9 + 6144*a^{18}*b^6*c^{14}*d^{10} - 4096*a^{19}*b^5*c^{13}*d^{11} \\
& + 1024*a^{20}*b^4*c^{12}*d^{12})*1i + 256*a^{11}*b^{12}*c^{19}*d^4 - 768*a^{12}*b^{11}*c^{18}*d^5 \\
& + 768*a^{13}*b^{10}*c^{17}*d^6 - 256*a^{14}*b^9*c^{16}*d^7 - 256*a^{16}*b^7*c^{14}*d^9 \\
& + 768*a^{17}*b^6*c^{13}*d^{10} - 768*a^{18}*b^5*c^{12}*d^{11} + 256*a^{19}*b^4*c^{11}*d^{12})*1i) * 1i) \\
& * (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 \\
& - 1024*a*b^3*c^8*d))^{(1/4)} + \operatorname{atan}((a^{14}*c*d^8*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 \\
& - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^{(5/4)}*1024i + a^6*b^8 \\
& *c^9*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 \\
& - 1024*a^8*b*c*d^3))^{(5/4)}*1024i + a^6*b^4*d^5*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 \\
& - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^{(1/4)}*4i + a^5*b^5 \\
& *c*d^4*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 \\
& - 1024*a^8*b*c*d^3))^{(1/4)}*4i - a^7*b^7*c^8*d*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 \\
& - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^{(5/4)}*4096i - a^{13} \\
& *b*c^2*d^7*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 \\
& - 1024*a^8*b*c*d^3))^{(5/4)}*4096i + a^8*b^6*c^7*d^2*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 \\
& - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^{(5/4)}*6144i - a^9 \\
& *b^5*c^6*d^3*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 \\
& - 1024*a^8*b*c*d^3))^{(5/4)}*4096i + a^{10}*b^4*c^5*d^4*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 \\
& - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^{(5/4)}*2048i - a^
\end{aligned}$$

$$\begin{aligned}
& 11*b^3*c^4*d^5*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d \\
& + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^{(5/4)}*4096i + a^{12}*b^2*c^3*d^6* \\
& x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2* \\
& c^2*d^2 - 1024*a^8*b*c*d^3))^{(5/4)}*6144i)/(b^9*c^4 + a^4*b^5*d^4 + a^3*b^6* \\
& c*d^3 + a^2*b^7*c^2*d^2 + a*b^8*c^3*d))*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 \\
& - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^{(1/4)}*2i \\
& + \operatorname{atan}((b^5*c^6*d^4*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{(1/4)}*4i + a*b^8*c^{14}*x*(- \\
& -d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{(5/4)}*1024i + a^9*c^6*d^8*x*(-d^5/(256*b^4*c^9 + \\
& 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{(5/4)}*1024i + a*b^4*c^5*d^5*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - \\
& 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{(1/4)}*4i - a \\
& ^2*b^7*c^{13}*d*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + \\
& 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{(5/4)}*4096i - a^8*b*c^7*d^7*x*(- \\
& -d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{(5/4)}*4096i + a^3*b^6*c^{12}*d^2*x*(-d^5/(256*b^4*c^9 \\
& + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{(5/4)}*6144i - a^4*b^5*c^{11}*d^3*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5 \\
& *d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{(5/4)} \\
& *4096i + a^5*b^4*c^{10}*d^4*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3 \\
& *b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{(5/4)}*2048i - a^6*b^ \\
& 3*c^9*d^5*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 153 \\
& 6*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{(5/4)}*4096i + a^7*b^2*c^8*d^6*x*(-d^ \\
& 5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^ \\
& 2 - 1024*a*b^3*c^8*d))^{(5/4)}*6144i)/(a^4*d^9 + b^4*c^4*d^5 + a*b^3*c^3*d^6 \\
& + a^2*b^2*c^2*d^7 + a^3*b*c*d^8))*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 10 \\
& 24*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^{(1/4)}*2i + 2*a \\
& \tan(((b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2* \\
& c^2*d^2 - 1024*a^8*b*c*d^3))^{(1/4)}*(x*(4*a^{11}*b^9*c^{12}*d^8 + 4*a^{12}*b^8* \\
& c^{11}*d^9) - (-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 153 \\
& 6*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^{(3/4)}*(x*(-b^5/(256*a^9*d^4 + 256*a^ \\
& 5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^{(1/4)}*(1024*a^{12}*b^{12}*c^{20}*d^4 - 4096*a^{13}*b^{11}*c^{19}*d^5 + 6144*a^{14}*b^{10}*c \\
& ^{18}*d^6 - 4096*a^{15}*b^9*c^{17}*d^7 + 2048*a^{16}*b^8*c^{16}*d^8 - 4096*a^{17}*b^7*c \\
& ^{15}*d^9 + 6144*a^{18}*b^6*c^{14}*d^{10} - 4096*a^{19}*b^5*c^{13}*d^{11} + 1024*a^{20}*b^4 \\
& *c^{12}*d^{12})*1i - 256*a^{11}*b^{12}*c^{19}*d^4 + 768*a^{12}*b^{11}*c^{18}*d^5 - 768*a^{13} \\
& *b^{10}*c^{17}*d^6 + 256*a^{14}*b^9*c^{16}*d^7 + 256*a^{16}*b^7*c^{14}*d^9 - 768*a^{17}*b \\
& ^6*c^{13}*d^{10} + 768*a^{18}*b^5*c^{12}*d^{11} - 256*a^{19}*b^4*c^{11}*d^{12})*1i) + (-b^5 \\
& / (256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 \\
& - 1024*a^8*b*c*d^3))^{(1/4)}*(x*(4*a^{11}*b^9*c^{12}*d^8 + 4*a^{12}*b^8*c^{11}*d^9) \\
& - (-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2* \\
& c^2*d^2 - 1024*a^8*b*c*d^3))^{(3/4)}*(x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 \\
& - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^{(1/4)}*(102 \\
& 4*a^{12}*b^{12}*c^{20}*d^4 - 4096*a^{13}*b^{11}*c^{19}*d^5 + 6144*a^{14}*b^{10}*c^{18}*d^6 -
\end{aligned}$$

$$\begin{aligned}
& 4096*a^{15}*b^9*c^{17}*d^7 + 2048*a^{16}*b^8*c^{16}*d^8 - 4096*a^{17}*b^7*c^{15}*d^9 + \\
& 6144*a^{18}*b^6*c^{14}*d^{10} - 4096*a^{19}*b^5*c^{13}*d^{11} + 1024*a^{20}*b^4*c^{12}*d^{12} \\
&)*1i + 256*a^{11}*b^{12}*c^{19}*d^4 - 768*a^{12}*b^{11}*c^{18}*d^5 + 768*a^{13}*b^{10}*c^{17} \\
& *d^6 - 256*a^{14}*b^9*c^{16}*d^7 - 256*a^{16}*b^7*c^{14}*d^9 + 768*a^{17}*b^6*c^{13}*d^{10} \\
& - 768*a^{18}*b^5*c^{12}*d^{11} + 256*a^{19}*b^4*c^{11}*d^{12})*1i))/((-b^5/(256*a^9*d^4 + \\
& 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^{1/4}*(x*(4*a^{11}*b^9*c^{12}*d^8 + \\
& 4*a^{12}*b^8*c^{11}*d^9) - (-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - \\
& 1024*a^8*b*c*d^3))^{3/4}*(x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - \\
& 1024*a^8*b*c*d^3))^{1/4}*(1024*a^{12}*b^{12}*c^{20}*d^4 - 4096*a^{13}*b^{11}*c^{19}*d^5 + 6144*a^{14}*b^{10}*c^{18}*d^6 - \\
& 4096*a^{15}*b^9*c^{17}*d^7 + 2048*a^{16}*b^8*c^{16}*d^8 - 4096*a^{17}*b^7*c^{15}*d^9 + 6144*a^{18}*b^6*c^{14}*d^{10} - \\
& 4096*a^{19}*b^5*c^{13}*d^{11} + 1024*a^{20}*b^4*c^{12}*d^{12})*1i - 256*a^{11}*b^{12}*c^{19}*d^4 + 768*a^{12}*b^{11}*c^{18}*d^5 - \\
& 768*a^{13}*b^{10}*c^{17}*d^6 + 256*a^{14}*b^9*c^{16}*d^7 + 256*a^{16}*b^7*c^{14}*d^9 - 768*a^{17}*b^6*c^{13}*d^{10} + 768*a^{18}*b^5*c^{12}*d^{11} - \\
& 256*a^{19}*b^4*c^{11}*d^{12})*1i)*1i - (-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - \\
& 1024*a^8*b*c*d^3))^{1/4}*(x*(4*a^{11}*b^9*c^{12}*d^8 + 4*a^{12}*b^8*c^{11}*d^9) - (-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - \\
& 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^{3/4}*(x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - \\
& 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^{1/4}*(1024*a^{12}*b^{12}*c^{20}*d^4 - 4096*a^{13}*b^{11}*c^{19}*d^5 + \\
& 6144*a^{14}*b^{10}*c^{18}*d^6 - 4096*a^{15}*b^9*c^{17}*d^7 + 2048*a^{16}*b^8*c^{16}*d^8 - 4096*a^{17}*b^7*c^{15}*d^9 + 6144*a^{18}*b^6*c^{14}*d^{10} - \\
& 4096*a^{19}*b^5*c^{13}*d^{11} + 1024*a^{20}*b^4*c^{12}*d^{12})*1i + 256*a^{11}*b^{12}*c^{19}*d^4 - 768*a^{12}*b^{11}*c^{18}*d^5 + \\
& 768*a^{13}*b^{10}*c^{17}*d^6 - 256*a^{14}*b^9*c^{16}*d^7 - 256*a^{16}*b^7*c^{14}*d^9 + 768*a^{17}*b^6*c^{13}*d^{10} - 768*a^{18}*b^5*c^{12}*d^{11} + \\
& 256*a^{19}*b^4*c^{11}*d^{12})*1i)*1i))*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - \\
& 1024*a^8*b*c*d^3))^{1/4} - 1/(a*c*x)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**4+a)/(d*x**4+c),x)

[Out] Timed out

$$3.511 \quad \int \frac{1}{x^4(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=462

$$\frac{b^{7/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{7/4}(bc-ad)} - \frac{b^{7/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{7/4}(bc-ad)} + \frac{b^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{7/4}(bc-ad)} - \frac{b^{7/4} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{7/4}(bc-ad)}$$

Rubi [A] time = 0.43, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {480, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{7/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{7/4}(bc-ad)} - \frac{b^{7/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{7/4}(bc-ad)} + \frac{b^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{7/4}(bc-ad)} - \frac{b^{7/4} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{7/4}(bc-ad)} - \frac{d^{7/4} \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} c^{7/4}(bc-ad)} + \frac{d^{7/4} \log\left(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} c^{7/4}(bc-ad)} - \frac{d^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} c^{7/4}(bc-ad)} + \frac{d^{7/4} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}} + 1\right)}{2\sqrt{2} c^{7/4}(bc-ad)} - \frac{1}{3ac^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^4)*(c + d*x^4)), x]

[Out] $-1/(3*a*c*x^3) + (b^{(7/4)}*ArcTan[1 - (Sqrt[2]*b^{(1/4)}*x)/a^{(1/4)}])/(2*Sqrt[2]*a^{(7/4)}*(b*c - a*d)) - (b^{(7/4)}*ArcTan[1 + (Sqrt[2]*b^{(1/4)}*x)/a^{(1/4)}])/(2*Sqrt[2]*a^{(7/4)}*(b*c - a*d)) - (d^{(7/4)}*ArcTan[1 - (Sqrt[2]*d^{(1/4)}*x)/c^{(1/4)}])/(2*Sqrt[2]*c^{(7/4)}*(b*c - a*d)) + (d^{(7/4)}*ArcTan[1 + (Sqrt[2]*d^{(1/4)}*x)/c^{(1/4)}])/(2*Sqrt[2]*c^{(7/4)}*(b*c - a*d)) + (b^{(7/4)}*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*b^{(1/4)}*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^{(7/4)}*(b*c - a*d)) - (b^{(7/4)}*Log[Sqrt[a] + Sqrt[2]*a^{(1/4)}*b^{(1/4)}*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^{(7/4)}*(b*c - a*d)) - (d^{(7/4)}*Log[Sqrt[c] - Sqrt[2]*c^{(1/4)}*d^{(1/4)}*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^{(7/4)}*(b*c - a*d)) + (d^{(7/4)}*Log[Sqrt[c] + Sqrt[2]*c^{(1/4)}*d^{(1/4)}*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^{(7/4)}*(b*c - a*d))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 480

```
Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(a+bx^4)(c+dx^4)} dx &= -\frac{1}{3acx^3} + \frac{\int \frac{-3(bc+ad)-3bdx^4}{(a+bx^4)(c+dx^4)} dx}{3ac} \\
&= -\frac{1}{3acx^3} - \frac{b^2 \int \frac{1}{a+bx^4} dx}{a(bc-ad)} + \frac{d^2 \int \frac{1}{c+dx^4} dx}{c(bc-ad)} \\
&= -\frac{1}{3acx^3} - \frac{b^2 \int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4} dx}{2a^{3/2}(bc-ad)} - \frac{b^2 \int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4} dx}{2a^{3/2}(bc-ad)} + \frac{d^2 \int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4} dx}{2c^{3/2}(bc-ad)} + \frac{d^2 \int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4} dx}{2c^{3/2}(bc-ad)} \\
&= -\frac{1}{3acx^3} - \frac{b^{3/2} \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4a^{3/2}(bc-ad)} - \frac{b^{3/2} \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4a^{3/2}(bc-ad)} + \frac{b^{7/4} \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}} dx}{4\sqrt{2}a^{7/4}(bc-ad)} \\
&= -\frac{1}{3acx^3} + \frac{b^{7/4} \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)} - \frac{b^{7/4} \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)} \\
&= -\frac{1}{3acx^3} + \frac{b^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)} - \frac{b^{7/4} \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)} - \frac{d^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)} + \frac{d^{7/4} \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 406, normalized size = 0.88

$$\frac{\frac{6\sqrt{2}b^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{6\sqrt{2}b^{7/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}} + 1\right)}{a^{7/4}} - \frac{3\sqrt{2}b^{7/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2\right)}{a^{7/4}} + \frac{3\sqrt{2}b^{7/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2\right)}{a^{7/4}} + \frac{6\sqrt{2}d^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}} - \frac{6\sqrt{2}d^{7/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}} + 1\right)}{c^{7/4}} + \frac{3\sqrt{2}d^{7/4} \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2\right)}{c^{7/4}} - \frac{3\sqrt{2}d^{7/4} \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2\right)}{c^{7/4}}}{24\sqrt{2}(ad-bc)} - \frac{8d}{c}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a + b*x^4)*(c + d*x^4)), x]

[Out] ((8*b)/a - (8*d)/c - (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(7/4) - (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(7/4) - (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*d^(7/4)*x^3*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4) - (3*Sqrt[2]*d^(7/4)*x^3*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4))/(24*(-(b*c) + a*d)*x^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^4)*(c + d*x^4)), x]

fricas [B] time = 5.40, size = 1415, normalized size = 3.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out]
$$\frac{1}{12} \left(12 \left(-b^7 / (a^7 b^4 c^4 - 4 a^8 b^3 c^3 d + 6 a^9 b^2 c^2 d^2 - 4 a^{10} b c d^3 + a^{11} d^4) \right)^{1/4} a c x^3 \arctan \left(\left(a^5 b^3 c^3 - 3 a^6 b^2 c^2 d + 3 a^7 b c d^2 - a^8 d^3 \right) \left(-b^7 / (a^7 b^4 c^4 - 4 a^8 b^3 c^3 d + 6 a^9 b^2 c^2 d^2 - 4 a^{10} b c d^3 + a^{11} d^4) \right)^{3/4} x - (a^5 b^3 c^3 - 3 a^6 b^2 c^2 d + 3 a^7 b c d^2 - a^8 d^3) \left(-b^7 / (a^7 b^4 c^4 - 4 a^8 b^3 c^3 d + 6 a^9 b^2 c^2 d^2 - 4 a^{10} b c d^3 + a^{11} d^4) \right)^{3/4} \sqrt{(b^4 x^2 + (a^4 b^2 c^2 - 2 a^5 b c d + a^6 d^2)) \sqrt{-b^7 / (a^7 b^4 c^4 - 4 a^8 b^3 c^3 d + 6 a^9 b^2 c^2 d^2 - 4 a^{10} b c d^3 + a^{11} d^4)}} / b^4 \right) / b^5 - 12 \left(-d^7 / (b^4 c^{11} - 4 a b^3 c^{10} d + 6 a^2 b^2 c^9 d^2 - 4 a^3 b c^8 d^3 + a^4 c^7 d^4) \right)^{1/4} a c x^3 \arctan \left(\left(b^3 c^8 - 3 a b^2 c^7 d + 3 a^2 b c^6 d^2 - a^3 c^5 d^3 \right) \left(-d^7 / (b^4 c^{11} - 4 a b^3 c^{10} d + 6 a^2 b^2 c^9 d^2 - 4 a^3 b c^8 d^3 + a^4 c^7 d^4) \right)^{3/4} x - (b^3 c^8 - 3 a b^2 c^7 d + 3 a^2 b c^6 d^2 - a^3 c^5 d^3) \left(-d^7 / (b^4 c^{11} - 4 a b^3 c^{10} d + 6 a^2 b^2 c^9 d^2 - 4 a^3 b c^8 d^3 + a^4 c^7 d^4) \right)^{3/4} \sqrt{(d^4 x^2 + (b^2 c^6 - 2 a b c^5 d + a^2 c^4 d^2)) \sqrt{-d^7 / (b^4 c^{11} - 4 a b^3 c^{10} d + 6 a^2 b^2 c^9 d^2 - 4 a^3 b c^8 d^3 + a^4 c^7 d^4)}} / d^4 \right) / d^5 - 3 \left(-b^7 / (a^7 b^4 c^4 - 4 a^8 b^3 c^3 d + 6 a^9 b^2 c^2 d^2 - 4 a^{10} b c d^3 + a^{11} d^4) \right)^{1/4} a c x^3 \log(b^2 x + (-b^7 / (a^7 b^4 c^4 - 4 a^8 b^3 c^3 d + 6 a^9 b^2 c^2 d^2 - 4 a^{10} b c d^3 + a^{11} d^4))^{1/4} (a^2 b c - a^3 d)) + 3 \left(-b^7 / (a^7 b^4 c^4 - 4 a^8 b^3 c^3 d + 6 a^9 b^2 c^2 d^2 - 4 a^{10} b c d^3 + a^{11} d^4) \right)^{1/4} a c x^3 \log(b^2 x - (-b^7 / (a^7 b^4 c^4 - 4 a^8 b^3 c^3 d + 6 a^9 b^2 c^2 d^2 - 4 a^{10} b c d^3 + a^{11} d^4))^{1/4} (a^2 b c - a^3 d)) + 3 \left(-d^7 / (b^4 c^{11} - 4 a b^3 c^{10} d + 6 a^2 b^2 c^9 d^2 - 4 a^3 b c^8 d^3 + a^4 c^7 d^4) \right)^{1/4} a c x^3 \log(d^2 x + (-d^7 / (b^4 c^{11} - 4 a b^3 c^{10} d + 6 a^2 b^2 c^9 d^2 - 4 a^3 b c^8 d^3 + a^4 c^7 d^4))^{1/4} (b c^3 - a c^2 d)) - 3 \left(-d^7 / (b^4 c^{11} - 4 a b^3 c^{10} d + 6 a^2 b^2 c^9 d^2 - 4 a^3 b c^8 d^3 + a^4 c^7 d^4) \right)^{1/4} a c x^3 \log(d^2 x - (-d^7 / (b^4 c^{11} - 4 a b^3 c^{10} d + 6 a^2 b^2 c^9 d^2 - 4 a^3 b c^8 d^3 + a^4 c^7 d^4))^{1/4} (b c^3 - a c^2 d)) - 4 / (a c x^3)$$

giac [A] time = 0.19, size = 472, normalized size = 1.02

$$\frac{(ab)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\sqrt{1+\sqrt{2}}\left(\frac{z}{z^2}\right)^{\frac{1}{4}}}{z\left(\frac{z}{z^2}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2}a^2bc - \sqrt{2}a^2d)} + \frac{(ab)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\sqrt{1-\sqrt{2}}\left(\frac{z}{z^2}\right)^{\frac{1}{4}}}{z\left(\frac{z}{z^2}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2}a^2bc - \sqrt{2}a^2d)} + \frac{(cd)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\sqrt{1+\sqrt{2}}\left(\frac{z}{z^2}\right)^{\frac{1}{4}}}{z\left(\frac{z}{z^2}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2}bc^3 - \sqrt{2}ac^2d)} + \frac{(cd)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\sqrt{1-\sqrt{2}}\left(\frac{z}{z^2}\right)^{\frac{1}{4}}}{z\left(\frac{z}{z^2}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2}bc^3 - \sqrt{2}ac^2d)} - \frac{(ab)^{\frac{1}{4}} b \log\left(x^2 + \sqrt{2}x\left(\frac{z}{z^2}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{b}}\right)}{4(\sqrt{2}a^2bc - \sqrt{2}a^2d)} + \frac{(ab)^{\frac{1}{4}} b \log\left(x^2 - \sqrt{2}x\left(\frac{z}{z^2}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{b}}\right)}{4(\sqrt{2}a^2bc - \sqrt{2}a^2d)} + \frac{(cd)^{\frac{1}{4}} d \log\left(x^2 + \sqrt{2}x\left(\frac{z}{z^2}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{4(\sqrt{2}bc^3 - \sqrt{2}ac^2d)} - \frac{(cd)^{\frac{1}{4}} d \log\left(x^2 - \sqrt{2}x\left(\frac{z}{z^2}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{4(\sqrt{2}bc^3 - \sqrt{2}ac^2d)} - \frac{1}{3acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c), x, algorithm="giac")

[Out]
$$-1/2*(a*b^3)^{(1/4)}*b*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4))}/(\sqrt{2}*a^2*b*c - \sqrt{2}*a^3*d) - 1/2*(a*b^3)^{(1/4)}*b*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4))}/(\sqrt{2}*a^2*b*c - \sqrt{2}*a^3*d) + 1/2*(c*d^3)^{(1/4)}*d*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4))}/(\sqrt{2}*b*c^3 - \sqrt{2}*a*c^2*d) + 1/2*(c*d^3)^{(1/4)}*d*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4))}/(\sqrt{2}*b*c^3 - \sqrt{2}*a*c^2*d) - 1/4*(a*b^3)^{(1/4)}*b*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*a^2*b*c - \sqrt{2}*a^3*d) + 1/4*(a*b^3)^{(1/4)}*b*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*a^2*b*c - \sqrt{2}*a^3*d) + 1/4*(c*d^3)^{(1/4)}*d*\log(x^2 + \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b*c^3 - \sqrt{2}*a*c^2*d) - 1/4*(c*d^3)^{(1/4)}*d*\log(x^2 - \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b*c^3 - \sqrt{2}*a*c^2*d) - 1/3/(a*c*x^3)$$

maple [A] time = 0.06, size = 343, normalized size = 0.74

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} b^2 \arctan\left(\frac{\sqrt{2}x-1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4(ad-bc)a^2} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} b^2 \arctan\left(\frac{\sqrt{2}x+1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4(ad-bc)a^2} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} d^2 \ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{b}}}\right)}{8(ad-bc)a^2} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} d^2 \arctan\left(\frac{\sqrt{2}x-1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4(ad-bc)c^2} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} d^2 \arctan\left(\frac{\sqrt{2}x+1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4(ad-bc)c^2} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} d^2 \ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}\right)}{8(ad-bc)c^2} - \frac{1}{3acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x^4+a)/(d*x^4+c), x)

[Out]
$$-1/8/c^2*d^2/(a*d-b*c)*(c/d)^{(1/4)}*2^{(1/2)}*\ln((x^2+(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)})/(x^2-(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)}))-1/4/c^2*d^2/(a*d-b*c)*(c/d)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x+1)-1/4/c^2*d^2/(a*d-b*c)*(c/d)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x-1)-1/3/a/c/x^3+1/8/a^2*b^2/(a*d-b*c)*(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)}))+1/4/a^2*b^2/(a*d-b*c)*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)+1/4/a^2*b^2/(a*d-b*c)*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)$$

maxima [A] time = 1.36, size = 390, normalized size = 0.84

$$\frac{2\sqrt{2}b^2 \arctan\left(\frac{\sqrt{2}\sqrt{1+\sqrt{2}}\left(\frac{z}{z^2}\right)^{\frac{1}{4}}}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{a}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}b^2 \arctan\left(\frac{\sqrt{2}\sqrt{1-\sqrt{2}}\left(\frac{z}{z^2}\right)^{\frac{1}{4}}}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{a}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}b^{\frac{7}{4}} \log\left(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}} - \frac{\sqrt{2}b^{\frac{7}{4}} \log\left(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}} + \frac{2\sqrt{2}b^2 \arctan\left(\frac{\sqrt{2}\sqrt{1+\sqrt{2}}\left(\frac{z}{z^2}\right)^{\frac{1}{4}}}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}b^2 \arctan\left(\frac{\sqrt{2}\sqrt{1-\sqrt{2}}\left(\frac{z}{z^2}\right)^{\frac{1}{4}}}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}d^{\frac{7}{4}} \log\left(\sqrt{d}x^2 + \sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x + \sqrt{c}\right)}{c^{\frac{3}{4}}} - \frac{\sqrt{2}d^{\frac{7}{4}} \log\left(\sqrt{d}x^2 - \sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x + \sqrt{c}\right)}{c^{\frac{3}{4}}} - \frac{1}{3acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out]
$$-1/8*(2*\sqrt{2}*b^2*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x + \sqrt{2})*a^{1/4}*b^{1/4})/\sqrt{\sqrt{a}*\sqrt{b}})/(\sqrt{a}*\sqrt{\sqrt{a}*\sqrt{b}}) + 2*\sqrt{2}*b^2*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x - \sqrt{2})*a^{1/4}*b^{1/4})/\sqrt{\sqrt{a}*\sqrt{b}})/(\sqrt{a}*\sqrt{\sqrt{a}*\sqrt{b}}) + \sqrt{2}*b^{7/4}*\log(\sqrt{b}*x^2 + \sqrt{2})*a^{1/4}*b^{1/4}*x + \sqrt{a})/a^{3/4} - \sqrt{2}*b^{7/4}*\log(\sqrt{b}*x^2 - \sqrt{2})*a^{1/4}*b^{1/4}*x + \sqrt{a})/a^{3/4})/(a*b*c - a^2*d) + 1/8*(2*\sqrt{2}*d^2*\arctan(1/2*\sqrt{2}*(2*\sqrt{d}*x + \sqrt{2})*c^{1/4}*d^{1/4})/\sqrt{\sqrt{c}*\sqrt{d}})/(\sqrt{c}*\sqrt{\sqrt{c}*\sqrt{d}}) + 2*\sqrt{2}*d^2*\arctan(1/2*\sqrt{2}*(2*\sqrt{d}*x - \sqrt{2})*c^{1/4}*d^{1/4})/\sqrt{\sqrt{c}*\sqrt{d}})/(\sqrt{c}*\sqrt{\sqrt{c}*\sqrt{d}}) + \sqrt{2}*d^{7/4}*\log(\sqrt{d}*x^2 + \sqrt{2})*c^{1/4}*d^{1/4}*x + \sqrt{c})/c^{3/4} - \sqrt{2}*d^{7/4}*\log(\sqrt{d}*x^2 - \sqrt{2})*c^{1/4}*d^{1/4}*x + \sqrt{c})/c^{3/4})/(b*c^2 - a*c*d) - 1/3/(a*c*x^3)$$

mupad [B] time = 6.19, size = 7459, normalized size = 16.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a + b*x^4)*(c + d*x^4)),x)

[Out]
$$- \operatorname{atan}\left(\frac{(a^2*b^5*d^7*x*1i + b^7*c^2*d^5*x*1i - (a^2*b^16*c^11*x*256i))/(256*a^{11}*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^{10}*b*c*d^3) - (a^4*b^{14}*c^9*d^2*x*1536i)/(256*a^{11}*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^{10}*b*c*d^3) + (a^5*b^{13}*c^8*d^3*x*1024i)/(256*a^{11}*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^{10}*b*c*d^3) - (a^6*b^{12}*c^7*d^4*x*256i)/(256*a^{11}*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^{10}*b*c*d^3) - (a^7*b^{11}*c^6*d^5*x*256i)/(256*a^{11}*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^{10}*b*c*d^3) + (a^8*b^{10}*c^5*d^6*x*1024i)/(256*a^{11}*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^{10}*b*c*d^3) - (a^9*b^9*c^4*d^7*x*1536i)/(256*a^{11}*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^{10}*b*c*d^3) + (a^{10}*b^8*c^3*d^8*x*1024i)/(256*a^{11}*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^{10}*b*c*d^3) - (a^{11}*b^7*c^2*d^9*x*256i)/(256*a^{11}*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^{10}*b*c*d^3) + (a^3*b^{15}*c^{10}*d*x*1024i)/(256*a^{11}*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^{10}*b*c*d^3)}\right)^{1/4} * ((b^7*(1024*a^4*b^8*c^{12} + 1024*a^{12}*c^4*d^8 - 5120*a^5*b^7*c^{11}*d - 5120*a^{11}*b*c^5*d^7 + 10240*a^6*b^6*c^{10}*d^2 - 11264*a^7*b^5*c^9*d^3 + 10240*a^8*b^4*c^8*d^4 - 11264*a^9*b^3*c^7*d^5 + 10240*a^{10}*b^2*c^6*d^6))/(256*a^{11}*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^{10}*b*c*d^3) + 4*a^5*$$

$$\begin{aligned}
& c^7d^4 - 1024a^3b^2c^8d^3 + 1536a^2b^2c^9d^2 - 1024ab^3c^{10}d) \wedge (\\
& 1/4) * ((-d^7 / (256b^4c^{11} + 256a^4c^7d^4 - 1024a^3b^2c^8d^3 + 1536a^2 \\
& b^2c^9d^2 - 1024ab^3c^{10}d)) \wedge (3/4) * (x * (1024a^{11}b^{13}c^{20}d^4 - 4096 \\
& a^{12}b^{12}c^{19}d^5 + 6144a^{13}b^{11}c^{18}d^6 - 4096a^{14}b^{10}c^{17}d^7 + 1 \\
& 024a^{15}b^9c^{16}d^8 + 1024a^{16}b^8c^{15}d^9 - 4096a^{17}b^7c^{14}d^{10} + \\
& 6144a^{18}b^6c^{13}d^{11} - 4096a^{19}b^5c^{12}d^{12} + 1024a^{20}b^4c^{11}d^{13} \\
&) + (-d^7 / (256b^4c^{11} + 256a^4c^7d^4 - 1024a^3b^2c^8d^3 + 1536a^2b \\
& ^2c^9d^2 - 1024ab^3c^{10}d)) \wedge (1/4) * (4096a^{13}b^{12}c^{21}d^4 - 20480a^{1 \\
& 4}b^{11}c^{20}d^5 + 40960a^{15}b^{10}c^{19}d^6 - 45056a^{16}b^9c^{18}d^7 + 4096 \\
& 0a^{17}b^8c^{17}d^8 - 45056a^{18}b^7c^{16}d^9 + 40960a^{19}b^6c^{15}d^{10} - \\
& 20480a^{20}b^5c^{14}d^{11} + 4096a^{21}b^4c^{13}d^{12}) * i) * i + 16a^9b^{12}c^ \\
& 14d^7 - 16a^{10}b^{11}c^{13}d^8 - 16a^{13}b^8c^{10}d^{11} + 16a^{14}b^7c^9d^{12} \\
& 12) * i) / ((-d^7 / (256b^4c^{11} + 256a^4c^7d^4 - 1024a^3b^2c^8d^3 + 1536 \\
& a^2b^2c^9d^2 - 1024ab^3c^{10}d)) \wedge (1/4) * (x * (4a^9b^{11}c^{11}d^9 + 4a^ \\
& 11b^9c^9d^{11}) - (-d^7 / (256b^4c^{11} + 256a^4c^7d^4 - 1024a^3b^2c^8d^ \\
& ^3 + 1536a^2b^2c^9d^2 - 1024ab^3c^{10}d)) \wedge (1/4) * ((-d^7 / (256b^4c^{11} \\
& + 256a^4c^7d^4 - 1024a^3b^2c^8d^3 + 1536a^2b^2c^9d^2 - 1024ab^3c^ \\
& c^{10}d)) \wedge (3/4) * (x * (1024a^{11}b^{13}c^{20}d^4 - 4096a^{12}b^{12}c^{19}d^5 + 6144 \\
& a^{13}b^{11}c^{18}d^6 - 4096a^{14}b^{10}c^{17}d^7 + 1024a^{15}b^9c^{16}d^8 + 10 \\
& 24a^{16}b^8c^{15}d^9 - 4096a^{17}b^7c^{14}d^{10} + 6144a^{18}b^6c^{13}d^{11} - \\
& 4096a^{19}b^5c^{12}d^{12} + 1024a^{20}b^4c^{11}d^{13}) - (-d^7 / (256b^4c^{11} + \\
& 256a^4c^7d^4 - 1024a^3b^2c^8d^3 + 1536a^2b^2c^9d^2 - 1024ab^3c^ \\
& 10d)) \wedge (1/4) * (4096a^{13}b^{12}c^{21}d^4 - 20480a^{14}b^{11}c^{20}d^5 + 40960a^ \\
& 15b^{10}c^{19}d^6 - 45056a^{16}b^9c^{18}d^7 + 40960a^{17}b^8c^{17}d^8 - 4505 \\
& 6a^{18}b^7c^{16}d^9 + 40960a^{19}b^6c^{15}d^{10} - 20480a^{20}b^5c^{14}d^{11} + \\
& 4096a^{21}b^4c^{13}d^{12}) * i) * i - 16a^9b^{12}c^{14}d^7 + 16a^{10}b^{11}c^{13} \\
& d^8 + 16a^{13}b^8c^{10}d^{11} - 16a^{14}b^7c^9d^{12}) * i) * i - (-d^7 / (256b^ \\
& 4c^{11} + 256a^4c^7d^4 - 1024a^3b^2c^8d^3 + 1536a^2b^2c^9d^2 - 1024 \\
& ab^3c^{10}d)) \wedge (1/4) * (x * (4a^9b^{11}c^{11}d^9 + 4a^{11}b^9c^9d^{11}) - (-d^ \\
& 7 / (256b^4c^{11} + 256a^4c^7d^4 - 1024a^3b^2c^8d^3 + 1536a^2b^2c^9d \\
& ^2 - 1024ab^3c^{10}d)) \wedge (1/4) * ((-d^7 / (256b^4c^{11} + 256a^4c^7d^4 - 102 \\
& 4a^3b^2c^8d^3 + 1536a^2b^2c^9d^2 - 1024ab^3c^{10}d)) \wedge (3/4) * (x * (1024 \\
& a^{11}b^{13}c^{20}d^4 - 4096a^{12}b^{12}c^{19}d^5 + 6144a^{13}b^{11}c^{18}d^6 - 4 \\
& 096a^{14}b^{10}c^{17}d^7 + 1024a^{15}b^9c^{16}d^8 + 1024a^{16}b^8c^{15}d^9 - \\
& 4096a^{17}b^7c^{14}d^{10} + 6144a^{18}b^6c^{13}d^{11} - 4096a^{19}b^5c^{12}d^{12} \\
& + 1024a^{20}b^4c^{11}d^{13}) + (-d^7 / (256b^4c^{11} + 256a^4c^7d^4 - 1024a \\
& ^3b^2c^8d^3 + 1536a^2b^2c^9d^2 - 1024ab^3c^{10}d)) \wedge (1/4) * (4096a^{13} \\
& b^{12}c^{21}d^4 - 20480a^{14}b^{11}c^{20}d^5 + 40960a^{15}b^{10}c^{19}d^6 - 4505 \\
& 6a^{16}b^9c^{18}d^7 + 40960a^{17}b^8c^{17}d^8 - 45056a^{18}b^7c^{16}d^9 + 4 \\
& 0960a^{19}b^6c^{15}d^{10} - 20480a^{20}b^5c^{14}d^{11} + 4096a^{21}b^4c^{13}d^{1 \\
& 2) * i) * i + 16a^9b^{12}c^{14}d^7 - 16a^{10}b^{11}c^{13}d^8 - 16a^{13}b^8c^{10} \\
& d^{11} + 16a^{14}b^7c^9d^{12}) * i) * i) * (-d^7 / (256b^4c^{11} + 256a^4c^7d^ \\
& 4 - 1024a^3b^2c^8d^3 + 1536a^2b^2c^9d^2 - 1024ab^3c^{10}d)) \wedge (1/4) - \\
& 2 * \operatorname{atan}(-((-b^7 / (256a^{11}d^4 + 256a^7b^4c^4 - 1024a^8b^3c^3d + 1536 \\
& a^9b^2c^2d^2 - 1024a^{10}b^3c^3d^3)) \wedge (1/4) * (x * (4a^9b^{11}c^{11}d^9 + 4a^
\end{aligned}$$


```

*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*((-b^7/(256*a^11*d^4
+ 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b
*c*d^3))^(3/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d
+ 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(4096*a^13*b^12*c^21*d^4
- 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^1
8*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*
c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i - x*(102
4*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 -
4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 -
4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^1
2 + 1024*a^20*b^4*c^11*d^13))*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13
*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)*1i)*1i))*(-b^7/(256*a^
11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024
*a^10*b*c*d^3))^(1/4) - 1/(3*a*c*x^3)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**4+a)/(d*x**4+c),x)

[Out] Timed out

$$3.512 \quad \int \frac{1}{x^6(a+bx^4)(c+dx^4)} dx$$

Optimal. Leaf size=479

$$\frac{b^{9/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{9/4}(bc-ad)} - \frac{b^{9/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{9/4}(bc-ad)} - \frac{b^{9/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{9/4}(bc-ad)} + \frac{b^{9/4} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{9/4}(bc-ad)}$$

Rubi [A] time = 0.60, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 22, number of rules / integrand size = 0.409, Rules used = {480, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{9/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{9/4}(bc-ad)} - \frac{b^{9/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{9/4}(bc-ad)} - \frac{b^{9/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{9/4}(bc-ad)} + \frac{b^{9/4} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{9/4}(bc-ad)} + \frac{ad+bc}{a^2 d^2} - \frac{d^{9/4} \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} c^{9/4}(bc-ad)} + \frac{d^{9/4} \log\left(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{4\sqrt{2} c^{9/4}(bc-ad)} + \frac{d^{9/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} c^{9/4}(bc-ad)} - \frac{d^{9/4} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} c^{9/4}(bc-ad)} - \frac{1}{5acc^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]

[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(a^2*c^2*x) - (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) + (b^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) + (d^(9/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)) - (d^(9/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)) - (b^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)) - (d^(9/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)) + (d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 297

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 480

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((e_) + (f_)*(x_)^(n_)))/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((g*x)^m*(a + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*c}, simplify[(a*c)/b^2], Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
```

& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^6 (a + bx^4)(c + dx^4)} dx &= -\frac{1}{5acx^5} + \frac{\int \frac{-5(bc+ad)-5bdx^4}{x^2(a+bx^4)(c+dx^4)} dx}{5ac} \\
 &= -\frac{1}{5acx^5} + \frac{bc + ad}{a^2c^2x} - \frac{\int \frac{x^2(-5(b^2c^2+abcd+a^2d^2)-5bd(bc+ad)x^4)}{(a+bx^4)(c+dx^4)} dx}{5a^2c^2} \\
 &= -\frac{1}{5acx^5} + \frac{bc + ad}{a^2c^2x} - \frac{\int \left(-\frac{5b^3c^2x^2}{(bc-ad)(a+bx^4)} - \frac{5a^2d^3x^2}{(-bc+ad)(c+dx^4)} \right) dx}{5a^2c^2} \\
 &= -\frac{1}{5acx^5} + \frac{bc + ad}{a^2c^2x} + \frac{b^3 \int \frac{x^2}{a+bx^4} dx}{a^2(bc - ad)} - \frac{d^3 \int \frac{x^2}{c+dx^4} dx}{c^2(bc - ad)} \\
 &= -\frac{1}{5acx^5} + \frac{bc + ad}{a^2c^2x} - \frac{b^{5/2} \int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4} dx}{2a^2(bc - ad)} + \frac{b^{5/2} \int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4} dx}{2a^2(bc - ad)} + \frac{d^{5/2} \int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4} dx}{2c^2(bc - ad)} \\
 &= -\frac{1}{5acx^5} + \frac{bc + ad}{a^2c^2x} + \frac{b^2 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4a^2(bc - ad)} + \frac{b^2 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4a^2(bc - ad)} + \frac{b^{9/4} \int \frac{1}{-\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{2} a^{9/4}(bc - ad)} \\
 &= -\frac{1}{5acx^5} + \frac{bc + ad}{a^2c^2x} + \frac{b^{9/4} \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{b}x^2)}{4\sqrt{2} a^{9/4}(bc - ad)} - \frac{b^{9/4} \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{b}x^2)}{4\sqrt{2} a^{9/4}(bc - ad)} \\
 &= -\frac{1}{5acx^5} + \frac{bc + ad}{a^2c^2x} - \frac{b^{9/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{9/4}(bc - ad)} + \frac{b^{9/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{9/4}(bc - ad)} + \frac{d^{9/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2} c^{9/4}(bc - ad)} - \frac{d^{9/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2} c^{9/4}(bc - ad)}
 \end{aligned}$$

Mathematica [A] time = 0.38, size = 428, normalized size = 0.89

$$\frac{10\sqrt{2}d^{9/4}b^3 \tan^{-1}\left(1 - \frac{\sqrt{2}bx}{\sqrt{c}}\right) - 10\sqrt{2}b^{9/4}a^3 \tan^{-1}\left(\frac{\sqrt{2}bx}{\sqrt{c}} + 1\right) - 5\sqrt{2}b^{9/4}a^3 \log\left(-\sqrt{2} \sqrt{c} \sqrt{bx^2 + \sqrt{c} + \sqrt{d}x^2}\right) + 5\sqrt{2}b^{9/4}a^3 \log\left(\sqrt{2} \sqrt{c} \sqrt{bx^2 + \sqrt{c} + \sqrt{d}x^2}\right) - \frac{40b^2d^4}{c^2} + \frac{8b}{c} - \frac{10\sqrt{2}d^{9/4}b^3 \tan^{-1}\left(1 - \frac{\sqrt{2}bx}{\sqrt{c}}\right) + 10\sqrt{2}d^{9/4}b^3 \tan^{-1}\left(\frac{\sqrt{2}bx}{\sqrt{c}} + 1\right) + 5\sqrt{2}d^{9/4}b^3 \log\left(-\sqrt{2} \sqrt{c} \sqrt{bx^2 + \sqrt{c} + \sqrt{d}x^2}\right) - 5\sqrt{2}d^{9/4}b^3 \log\left(\sqrt{2} \sqrt{c} \sqrt{bx^2 + \sqrt{c} + \sqrt{d}x^2}\right) + \frac{40b^2d^4}{c^2} - \frac{8d}{c}}{40b^5(ad - bc)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]

[Out] ((8*b)/a - (8*d)/c - (40*b^2*x^4)/a^2 + (40*d^2*x^4)/c^2 + (10*Sqrt[2]*b^(9/4)*x^5*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/a^(9/4) - (10*Sqrt[2]*b^(9/4)*x^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/a^(9/4) - (10*Sqrt[2]*d^(9/4)*x^5*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/c^(9/4) + (10*Sqrt[2]*d^(9/4)*x^5*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/c^(9/4) - (5*Sqrt[2]*b^(9/4)*x^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(9/4) + (5*Sqrt[2]*b^(9/4)*x^5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(9/4) + (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(9/4) - (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(9/4))/(40*(-(b*c) + a*d)*x^5)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 (a + bx^4)(c + dx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[1/(x^6*(a + b*x^4)*(c + d*x^4)), x]

fricas [B] time = 9.20, size = 1456, normalized size = 3.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out] 1/20*(20*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*arctan(((b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*(a^2*b*c - a^3*d)*x - (-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*(a^2*b*c - a^3*d)*sqrt((b^5*x^2 - (a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2)*sqrt(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))))/b^5)/b^2) - 20*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^5*arctan(((d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*(a^2*b*c - a^3*d)*sqrt((b^5*x^2 - (a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2)*sqrt(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))))/b^5)/b^2)

$$\begin{aligned} & \left(2*c^{11}*d^2 - 4*a^3*b*c^{10}*d^3 + a^4*c^9*d^4\right)^{1/4}*(b*c^3 - a*c^2*d)*x - \\ & \left(-d^9/(b^4*c^{13} - 4*a*b^3*c^{12}*d + 6*a^2*b^2*c^{11}*d^2 - 4*a^3*b*c^{10}*d^3 + \right. \\ & \left. a^4*c^9*d^4)\right)^{1/4}*(b*c^3 - a*c^2*d)*\sqrt{(d^5*x^2 - (b^2*c^7 - 2*a*b*c^6*d + a^2*c^5*d^2)*\sqrt{(d^9/(b^4*c^{13} - 4*a*b^3*c^{12}*d + 6*a^2*b^2*c^{11}*d^2 - 4*a^3*b*c^{10}*d^3 + a^4*c^9*d^4)))/d^5))/d^2} + 5*(-b^9/(a^9*b^4*c^4 - 4*a^{10}*b^3*c^3*d + 6*a^{11}*b^2*c^2*d^2 - 4*a^{12}*b*c*d^3 + a^{13}*d^4))^{1/4}*a^2*c^2*x^5*\log(b^7*x + (a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*b*c*d^2 - a^{10}*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^{10}*b^3*c^3*d + 6*a^{11}*b^2*c^2*d^2 - 4*a^{12}*b*c*d^3 + a^{13}*d^4))^{3/4}) - 5*(-b^9/(a^9*b^4*c^4 - 4*a^{10}*b^3*c^3*d + 6*a^{11}*b^2*c^2*d^2 - 4*a^{12}*b*c*d^3 + a^{13}*d^4))^{1/4}*a^2*c^2*x^5*\log(b^7*x - (a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*b*c*d^2 - a^{10}*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^{10}*b^3*c^3*d + 6*a^{11}*b^2*c^2*d^2 - 4*a^{12}*b*c*d^3 + a^{13}*d^4))^{3/4})) - 5*(-d^9/(b^4*c^{13} - 4*a*b^3*c^{12}*d + 6*a^2*b^2*c^{11}*d^2 - 4*a^3*b*c^{10}*d^3 + a^4*c^9*d^4))^{1/4}*a^2*c^2*x^5*\log(d^7*x + (b^3*c^{10} - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4*c^{13} - 4*a*b^3*c^{12}*d + 6*a^2*b^2*c^{11}*d^2 - 4*a^3*b*c^{10}*d^3 + a^4*c^9*d^4))^{3/4})) + 5*(-d^9/(b^4*c^{13} - 4*a*b^3*c^{12}*d + 6*a^2*b^2*c^{11}*d^2 - 4*a^3*b*c^{10}*d^3 + a^4*c^9*d^4))^{1/4}*a^2*c^2*x^5*\log(d^7*x - (b^3*c^{10} - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4*c^{13} - 4*a*b^3*c^{12}*d + 6*a^2*b^2*c^{11}*d^2 - 4*a^3*b*c^{10}*d^3 + a^4*c^9*d^4))^{3/4})) + 20*(b*c + a*d)*x^4 - 4*a*c)/(a^2*c^2*x^5) \end{aligned}$$

giac [A] time = 0.24, size = 483, normalized size = 1.01

$$\frac{(ab)^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2} \sqrt{2+ab} \sqrt{\frac{d}{a}}}{2 \sqrt{\frac{d}{a}}}\right)}{2(\sqrt{2}ab^2c - \sqrt{2}a^2d)} + \frac{(ab)^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2} \sqrt{2+ab} \sqrt{\frac{d}{a}}}{2 \sqrt{\frac{d}{a}}}\right)}{2(\sqrt{2}ab^2c - \sqrt{2}a^2d)} - \frac{(cd)^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2} \sqrt{2+cd} \sqrt{\frac{d}{c}}}{2 \sqrt{\frac{d}{c}}}\right)}{2(\sqrt{2}bc^2 - \sqrt{2}ac^2d)} - \frac{(cd)^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2} \sqrt{2+cd} \sqrt{\frac{d}{c}}}{2 \sqrt{\frac{d}{c}}}\right)}{2(\sqrt{2}bc^2 - \sqrt{2}ac^2d)} - \frac{(ab)^{\frac{3}{2}} \log\left(x^2 + \sqrt{2}x\sqrt{\frac{d}{a}} + \sqrt{\frac{d}{a}}\right)}{4(\sqrt{2}ab^2c - \sqrt{2}a^2d)} + \frac{(ab)^{\frac{3}{2}} \log\left(x^2 - \sqrt{2}x\sqrt{\frac{d}{a}} + \sqrt{\frac{d}{a}}\right)}{4(\sqrt{2}ab^2c - \sqrt{2}a^2d)} + \frac{(cd)^{\frac{3}{2}} \log\left(x^2 + \sqrt{2}x\sqrt{\frac{d}{c}} + \sqrt{\frac{d}{c}}\right)}{4(\sqrt{2}bc^2 - \sqrt{2}ac^2d)} - \frac{(cd)^{\frac{3}{2}} \log\left(x^2 - \sqrt{2}x\sqrt{\frac{d}{c}} + \sqrt{\frac{d}{c}}\right)}{4(\sqrt{2}bc^2 - \sqrt{2}ac^2d)} + \frac{5bcx^4 + 5adx^4 - ac}{5a^2c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] $\frac{1}{2}*(a*b^3)^{3/4}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{1/4}))/ (a/b)^{1/4} / (\sqrt{2}*a^3*b*c - \sqrt{2}*a^4*d) + \frac{1}{2}*(a*b^3)^{3/4}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{1/4}))/ (a/b)^{1/4} / (\sqrt{2}*a^3*b*c - \sqrt{2}*a^4*d) - \frac{1}{2}*(c*d^3)^{3/4}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(c/d)^{1/4}))/ (c/d)^{1/4} / (\sqrt{2}*b*c^4 - \sqrt{2}*a*c^3*d) - \frac{1}{2}*(c*d^3)^{3/4}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(c/d)^{1/4}))/ (c/d)^{1/4} / (\sqrt{2}*b*c^4 - \sqrt{2}*a*c^3*d) - \frac{1}{4}*(a*b^3)^{3/4}*\log(x^2 + \sqrt{2}*x*(a/b)^{1/4} + \sqrt{a/b}) / (\sqrt{2}*a^3*b*c - \sqrt{2}*a^4*d) + \frac{1}{4}*(a*b^3)^{3/4}*\log(x^2 - \sqrt{2}*x*(a/b)^{1/4} + \sqrt{a/b}) / (\sqrt{2}*a^3*b*c - \sqrt{2}*a^4*d) + \frac{1}{4}*(c*d^3)^{3/4}*\log(x^2 + \sqrt{2}*x*(c/d)^{1/4} + \sqrt{c/d}) / (\sqrt{2}*b*c^4 - \sqrt{2}*a*c^3*d) - \frac{1}{4}*(c*d^3)^{3/4}*\log(x^2 - \sqrt{2}*x*(c/d)^{1/4} + \sqrt{c/d}) / (\sqrt{2}*b*c^4 - \sqrt{2}*a*c^3*d) + \frac{1}{5}*(5*b*c*x^4 + 5*a*d*x^4 - a*c) / (a^2*c^2*x^5)$

maple [A] time = 0.07, size = 365, normalized size = 0.76

$$\frac{\sqrt{2} b^2 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}} a^2} - \frac{\sqrt{2} b^2 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}} a^2} - \frac{\sqrt{2} b^2 \ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}\right)}{8(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}} a^2} + \frac{\sqrt{2} d^2 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}} c^2} + \frac{\sqrt{2} d^2 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}} c^2} + \frac{\sqrt{2} d^2 \ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}\right)}{8(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}} c^2} + \frac{d}{a^2 c^2 x} + \frac{b}{a^2 c x} - \frac{1}{5 a c x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(b*x^4+a)/(d*x^4+c), x)

[Out] $\frac{1}{8} d^2 c^2 / (a d - b^2 c) / (c/d)^{1/4} * 2^{1/2} * \ln((x^2 - (c/d)^{1/4} * 2^{1/2} * x + (c/d)^{1/2}) / (x^2 + (c/d)^{1/4} * 2^{1/2} * x + (c/d)^{1/2})) + 1/4 * d^2 c^2 / (a d - b^2 c) / (c/d)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (c/d)^{1/4} * x + 1) + 1/4 * d^2 c^2 / (a d - b^2 c) / (c/d)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (c/d)^{1/4} * x - 1) - 1/8 * b^2 a^2 / (a d - b^2 c) / (a/b)^{1/4} * 2^{1/2} * \ln((x^2 - (a/b)^{1/4} * 2^{1/2} * x + (a/b)^{1/2}) / (x^2 + (a/b)^{1/4} * 2^{1/2} * x + (a/b)^{1/2})) - 1/4 * b^2 a^2 / (a d - b^2 c) / (a/b)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/b)^{1/4} * x + 1) - 1/4 * b^2 a^2 / (a d - b^2 c) / (a/b)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/b)^{1/4} * x - 1) - 1/5 * a/c/x^5 + 1/a/c^2/x*d + 1/a^2/c/x*b$

maxima [A] time = 1.23, size = 405, normalized size = 0.85

$$\frac{b^3 \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{a^2-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}}}{2\sqrt{a^2-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}}}\right)}{\sqrt{a^2-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{a^2-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}}}{2\sqrt{a^2-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}}}\right)}{\sqrt{a^2-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}}} - \frac{\sqrt{2} \log\left(\sqrt{a^2+\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{a^2}}\right)}{a^{\frac{1}{4}}d^{\frac{1}{4}}} + \frac{\sqrt{2} \log\left(\sqrt{a^2-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{a^2}}\right)}{a^{\frac{1}{4}}d^{\frac{1}{4}}} \right)}{8(a^2bc - a^2d)} - \frac{d^3 \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{a^2-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}}}{2\sqrt{a^2-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}}}\right)}{\sqrt{a^2-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{a^2-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}}}{2\sqrt{a^2-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}}}\right)}{\sqrt{a^2-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}}} - \frac{\sqrt{2} \log\left(\sqrt{a^2+\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}}\right)}{c^{\frac{1}{4}}d^{\frac{1}{4}}} + \frac{\sqrt{2} \log\left(\sqrt{a^2-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}}\right)}{c^{\frac{1}{4}}d^{\frac{1}{4}}} \right)}{8(bc^3 - a^2d)} + \frac{5(bc+ad)x^4 - ac}{5a^2c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c), x, algorithm="maxima")

[Out] $\frac{1}{8} b^3 * (2 * \sqrt{2} * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{2} * \sqrt{b} * x + \sqrt{2} * a^{1/4} * b^{1/4})) / \sqrt{a * \sqrt{b}}) / (\sqrt{a * \sqrt{b}} * \sqrt{b}) + 2 * \sqrt{2} * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{2} * \sqrt{b} * x - \sqrt{2} * a^{1/4} * b^{1/4})) / \sqrt{a * \sqrt{b}}) / (\sqrt{a * \sqrt{b}} * \sqrt{b}) - \sqrt{2} * \log(\sqrt{b} * x^2 + \sqrt{2} * a^{1/4} * b^{1/4} * x + \sqrt{a}) / (a^{1/4} * b^{3/4}) + \sqrt{2} * \log(\sqrt{b} * x^2 - \sqrt{2} * a^{1/4} * b^{1/4} * x + \sqrt{a}) / (a^{1/4} * b^{3/4}) / (a^2 * b * c - a^3 * d) - 1/8 * d^3 * (2 * \sqrt{2} * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{2} * \sqrt{d} * x + \sqrt{2} * c^{1/4} * d^{1/4})) / \sqrt{c * \sqrt{d}}) / (\sqrt{c * \sqrt{d}} * \sqrt{d}) + 2 * \sqrt{2} * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{2} * \sqrt{d} * x - \sqrt{2} * c^{1/4} * d^{1/4})) / \sqrt{c * \sqrt{d}}) / (\sqrt{c * \sqrt{d}} * \sqrt{d}) - \sqrt{2} * \log(\sqrt{d} * x^2 + \sqrt{2} * c^{1/4} * d^{1/4} * x + \sqrt{c}) / (c^{1/4} * d^{3/4}) + \sqrt{2} * \log(\sqrt{d} * x^2 - \sqrt{2} * c^{1/4} * d^{1/4} * x + \sqrt{c}) / (c^{1/4} * d^{3/4}) / (b * c^3 - a * c^2 * d) + 1/5 * (5 * (b * c + a * d) * x^4 - a * c) / (a^2 * c^2 * x^5)$

mupad [B] time = 6.01, size = 4547, normalized size = 9.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^6*(a + b*x^4)*(c + d*x^4)),x)$

[Out] $-2*\text{atan}((1024*a^{11}*b^{10}*c^{13}*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} + 4*a^{11}*b^6*d^9*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{1/4} + 1024*a^{21}*c^3*d^{10}*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} - 4096*a^{12}*b^9*c^{12}*d*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} - 4096*a^{20}*b*c^4*d^9*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} + 4*a^8*b^9*c^3*d^6*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{1/4} + 6144*a^{13}*b^8*c^{11}*d^2*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} - 4096*a^{14}*b^7*c^{10}*d^3*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} + 1024*a^{15}*b^6*c^9*d^4*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} + 1024*a^{17}*b^4*c^7*d^6*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} - 4096*a^{18}*b^3*c^6*d^7*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} + 6144*a^{19}*b^2*c^5*d^8*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4}))/ (b^{16}*c^8 + a^8*b^8*d^8 + a^7*b^9*c*d^7 + a^2*b^{14}*c^6*d^2 + a^3*b^{13}*c^5*d^3 + a^4*b^{12}*c^4*d^4 + a^5*b^{11}*c^3*d^5 + a^6*b^{10}*c^2*d^6 + a*b^{15}*c^7*d)) * (-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{1/4} - \text{atan}((a^{11}*b^{10}*c^{13}*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} * 1024i + a^{11}*b^6*d^9*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{1/4} * 4i + a^{21}*c^3*d^{10}*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} * 1024i - a^{12}*b^9*c^{12}*d*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} * 4096i - a^{20}*b*c^4*d^9*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} * 4096i + a^8*b^9*c^3*d^6*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{1/4} * 4i + a^{13}*b^8*c^{11}*d^2*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} * 6144i - a^{14}*b^7*c^{10}*d^3*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} * 4096i + a^{15}*b^6*c^9*d^4*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} * 1024i + a^{17}*b^4*c^7*d^6*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} * 1024i - a^{18}*b^3*c^6*d^7*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} * 6144i - a^{19}*b^2*c^5*d^8*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{5/4} * 6144i))$

$$\begin{aligned}
& 3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{(5/4)}*1024i - a^{18}*b^3*c^6*d^7*x*(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{(5/4)}*4096i + a^{19}*b^2*c^5*d^8*x \\
& *(-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{(5/4)}*6144i)/(b^{16}*c^8 + a^8*b^8*d^8 + a^7*b^9*c*d^7 + a^2*b^{14}*c^6*d^2 + a^3*b^{13}*c^5*d^3 + a^4*b^{12}*c^4*d^4 + a^5*b^{11}*c^3*d^5 + a^6*b^{10}*c^2*d^6 + a*b^{15}*c^7*d)) * (-b^9/(256*a^{13}*d^4 + 256*a^9*b^4*c^4 - 1024*a^{10}*b^3*c^3*d + 1536*a^{11}*b^2*c^2*d^2 - 1024*a^{12}*b*c*d^3))^{(1/4)}*2i - 2*atan((4*b^9*c^{11}*d^6*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(1/4)} + 1024*a^3*b^{10}*c^{21}*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)} + 1024*a^{13}*c^{11}*d^{10}*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)} - 4096*a^4*b^9*c^{20}*d*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)} - 4096*a^{12}*b*c^{12}*d^9*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)} + 4*a^3*b^6*c^8*d^9*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(1/4)} + 6144*a^5*b^8*c^{19}*d^2*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)} - 4096*a^6*b^7*c^{18}*d^3*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)} + 1024*a^7*b^6*c^{17}*d^4*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)} + 1024*a^9*b^4*c^{15}*d^6*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)} - 4096*a^{10}*b^3*c^{14}*d^7*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)} + 6144*a^{11}*b^2*c^{13}*d^8*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)})/(a^8*d^{16} + b^8*c^8*d^8 + a*b^7*c^7*d^9 + a^2*b^6*c^6*d^{10} + a^3*b^5*c^5*d^{11} + a^4*b^4*c^4*d^{12} + a^5*b^3*c^3*d^{13} + a^6*b^2*c^2*d^{14} + a^7*b*c*d^{15}))*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(1/4)} - atan((b^9*c^{11}*d^6*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(1/4)}*4i + a^3*b^{10}*c^{21}*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)}*1024i + a^{13}*c^{11}*d^{10}*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)}*1024i - a^4*b^9*c^{20}*d*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)}*4096i - a^{12}*b*c^{12}*d^9*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)}*4096i + a^3*b^6*c^8*d^9*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(1/4)}*4i + a^5*b^8*c^{19}*d^2*x*(-d^9/(25
\end{aligned}$$

$$\begin{aligned}
& 6*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 \\
& - 1024*a*b^3*c^{12}*d)^{(5/4)}*6144i - a^6*b^7*c^{18}*d^3*x*(-d^9/(256*b^4*c^{13} \\
& + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)}*4096i \\
& + a^7*b^6*c^{17}*d^4*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 \\
& - 1024*a*b^3*c^{12}*d))^{(5/4)}*1024i + a^9*b^4*c^{15}*d^6*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 \\
& + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)}*1024i - a^{10}*b^3*c^{14}*d^7*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 \\
& + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(5/4)}*4096i + a^{11}*b^2*c^{13}*d^8*x*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 \\
& - 1024*a*b^3*c^{12}*d))^{(5/4)}*6144i)/(a^8*d^{16} + b^8*c^8*d^8 + a*b^7*c^7*d^9 + a^2*b^6*c^6*d^{10} + a^3*b^5*c^5*d^{11} + a^4*b^4*c^4*d^{12} \\
& + a^5*b^3*c^3*d^{13} + a^6*b^2*c^2*d^{14} + a^7*b*c*d^{15}))*(-d^9/(256*b^4*c^{13} + 256*a^4*c^9*d^4 - 1024*a^3*b*c^{10}*d^3 + 1536*a^2*b^2*c^{11}*d^2 - 1024*a*b^3*c^{12}*d))^{(1/4)}*2i \\
& - (1/(5*a*c) - (x^4*(a*d + b*c))/(a^2*c^2))/x^5
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**6/(b*x**4+a)/(d*x**4+c),x)

[Out] Timed out

$$3.513 \quad \int \frac{x^7 \sqrt{c+dx^4}}{a+bx^4} dx$$

Optimal. Leaf size=93

$$\frac{a\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}} - \frac{a\sqrt{c+dx^4}}{2b^2} + \frac{(c+dx^4)^{3/2}}{6bd}$$

Rubi [A] time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 80, 50, 63, 208}

$$-\frac{a\sqrt{c+dx^4}}{2b^2} + \frac{a\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}} + \frac{(c+dx^4)^{3/2}}{6bd}$$

Antiderivative was successfully verified.

[In] Int[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]

[Out] -(a*Sqrt[c + d*x^4])/(2*b^2) + (c + d*x^4)^(3/2)/(6*b*d) + (a*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2))

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

$n + p + 2$)), $\text{Int}[(c + d*x)^n*(e + f*x)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, n, p\}, x] \&\& \text{NeQ}[n + p + 2, 0]$

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$ $\text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b]$

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] :> \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{x^7 \sqrt{c + dx^4}}{a + bx^4} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x \sqrt{c + dx^4}}{a + bx} dx, x, x^4 \right) \\ &= \frac{(c + dx^4)^{3/2}}{6bd} - \frac{a \text{Subst} \left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^4 \right)}{4b} \\ &= -\frac{a\sqrt{c + dx^4}}{2b^2} + \frac{(c + dx^4)^{3/2}}{6bd} - \frac{(a(bc - ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^4 \right)}{4b^2} \\ &= -\frac{a\sqrt{c + dx^4}}{2b^2} + \frac{(c + dx^4)^{3/2}}{6bd} - \frac{(a(bc - ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^4} \right)}{2b^2d} \\ &= -\frac{a\sqrt{c + dx^4}}{2b^2} + \frac{(c + dx^4)^{3/2}}{6bd} + \frac{a\sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{2b^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 0.95

$$\frac{a\sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{2b^{5/2}} + \frac{\sqrt{c + dx^4} (b(c + dx^4) - 3ad)}{6b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*sqrt[c + d*x^4])/(a + b*x^4), x]

[Out] $(\sqrt{c + dx^4} * (-3ad + b(c + dx^4))) / (6b^2d) + (a\sqrt{bc - ad} * \text{ArcTanh}[(\sqrt{b} * \sqrt{c + dx^4}) / \sqrt{bc - ad}]) / (2b^{5/2})$

IntegrateAlgebraic [A] time = 0.11, size = 98, normalized size = 1.05

$$\frac{\sqrt{c + dx^4} (-3ad + bc + bdx^4)}{6b^2d} - \frac{a\sqrt{ad - bc} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{c + dx^4} \sqrt{ad - bc}}{bc - ad}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^7*sqrt(c + dx^4))/(a + b*x^4), x]

[Out] $(\sqrt{c + dx^4} * (b*c - 3*a*d + b*d*x^4)) / (6*b^2*d) - (a*\sqrt{-(b*c) + a*d} * \text{ArcTan}[(\sqrt{b} * \sqrt{-(b*c) + a*d} * \sqrt{c + dx^4}) / (b*c - a*d)]) / (2*b^{5/2})$

fricas [A] time = 0.43, size = 195, normalized size = 2.10

$$\left[\frac{3ad\sqrt{\frac{bc-ad}{b}} \log\left(\frac{bdx^4 + 2bc - ad + 2\sqrt{dx^4 + c} \sqrt{\frac{bc-ad}{b}}}{bx^4 + a}\right) + 2(bdx^4 + bc - 3ad)\sqrt{dx^4 + c}}{12b^2d}, \frac{3ad\sqrt{\frac{bc-ad}{b}} \arctan\left(-\frac{\sqrt{dx^4 + c} \sqrt{\frac{bc-ad}{b}}}{bc - ad}\right) + (bdx^4 + bc - 3ad)\sqrt{dx^4 + c}}{6b^2d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a), x, algorithm="fricas")

[Out] $[1/12*(3*a*d*\sqrt{(b*c - a*d)/b})*\log((b*d*x^4 + 2*b*c - a*d + 2*\sqrt{d*x^4 + c})*b*\sqrt{(b*c - a*d)/b})/(b*x^4 + a) + 2*(b*d*x^4 + b*c - 3*a*d)*\sqrt{d*x^4 + c})/(b^2*d), 1/6*(3*a*d*\sqrt{-(b*c - a*d)/b})*\arctan(-\sqrt{d*x^4 + c})*b*\sqrt{-(b*c - a*d)/b}/(b*c - a*d) + (b*d*x^4 + b*c - 3*a*d)*\sqrt{d*x^4 + c})/(b^2*d)]$

giac [A] time = 0.16, size = 96, normalized size = 1.03

$$-\frac{(abc - a^2d) \arctan\left(\frac{\sqrt{dx^4 + c} b}{\sqrt{-b^2c + abd}}\right)}{2\sqrt{-b^2c + abd} b^2} + \frac{(dx^4 + c)^{\frac{3}{2}} b^2 d^2 - 3\sqrt{dx^4 + c} abd^3}{6b^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a), x, algorithm="giac")

[Out] $-1/2*(a*b*c - a^2*d)*\arctan(\sqrt{d*x^4 + c}*b/\sqrt{-b^2*c + a*b*d})/(\sqrt{-b^2*c + a*b*d})*b^2 + 1/6*((d*x^4 + c)^{(3/2)}*b^2*d^2 - 3*\sqrt{d*x^4 + c}*a*b*d^3)/(b^3*d^3)$

maple [B] time = 0.27, size = 1015, normalized size = 10.91

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^7(d*x^4+c)^{(1/2)}/(b*x^4+a), x)$

[Out] $\frac{1}{6}*(d*x^4+c)^{(3/2)}/b/d-1/4*a/b^2*((x^2+(-a*b)^{(1/2)}/b)^{2*d-2}*(-a*b)^{(1/2)}/b*d*(x^2+(-a*b)^{(1/2)}/b)-(a*d-b*c)/b)^{(1/2)}+1/4*a/b^3*(-a*b)^{(1/2)*d^{(1/2)*}\ln(((-a*b)^{(1/2)}/b*d+d*(x^2+(-a*b)^{(1/2)}/b))/d^{(1/2)}+((x^2+(-a*b)^{(1/2)}/b)^{2*d-2}*(-a*b)^{(1/2)}/b*d*(x^2+(-a*b)^{(1/2)}/b)-(a*d-b*c)/b)^{(1/2)}-1/4*a^2/b^3/((-a*d-b*c)/b)^{(1/2)*}\ln((-2*(a*d-b*c)/b-2*(-a*b)^{(1/2)}/b*d*(x^2+(-a*b)^{(1/2)}/b)+2*(-(a*d-b*c)/b)^{(1/2)*}((x^2+(-a*b)^{(1/2)}/b)^{2*d-2}*(-a*b)^{(1/2)}/b*d*(x^2+(-a*b)^{(1/2)}/b)-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)}/b))*d+1/4*a/b^2/((-a*d-b*c)/b)^{(1/2)*}\ln((-2*(a*d-b*c)/b-2*(-a*b)^{(1/2)}/b*d*(x^2+(-a*b)^{(1/2)}/b)/b)+2*(-(a*d-b*c)/b)^{(1/2)*}((x^2+(-a*b)^{(1/2)}/b)^{2*d-2}*(-a*b)^{(1/2)}/b*d*(x^2+(-a*b)^{(1/2)}/b)-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)}/b))*c-1/4*a/b^2*((x^2-(-a*b)^{(1/2)}/b)^{2*d+2}*(-a*b)^{(1/2)}/b*d*(x^2-(-a*b)^{(1/2)}/b)-(a*d-b*c)/b)^{(1/2)}-1/4*a/b^3*(-a*b)^{(1/2)*d^{(1/2)*}\ln(((a*b)^{(1/2)}/b*d+d*(x^2-(-a*b)^{(1/2)}/b))/d^{(1/2)}+((x^2-(-a*b)^{(1/2)}/b)^{2*d+2}*(-a*b)^{(1/2)}/b*d*(x^2-(-a*b)^{(1/2)}/b)-(a*d-b*c)/b)^{(1/2)}-1/4*a^2/b^3/((-a*d-b*c)/b)^{(1/2)*}\ln((-2*(a*d-b*c)/b+2*(-a*b)^{(1/2)}/b*d*(x^2-(-a*b)^{(1/2)}/b)+2*(-(a*d-b*c)/b)^{(1/2)*}((x^2-(-a*b)^{(1/2)}/b)^{2*d+2}*(-a*b)^{(1/2)}/b*d*(x^2-(-a*b)^{(1/2)}/b)-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)}/b))*d+1/4*a/b^2/((-a*d-b*c)/b)^{(1/2)*}\ln((-2*(a*d-b*c)/b+2*(-a*b)^{(1/2)}/b*d*(x^2-(-a*b)^{(1/2)}/b)+2*(-(a*d-b*c)/b)^{(1/2)*}((x^2-(-a*b)^{(1/2)}/b)^{2*d+2}*(-a*b)^{(1/2)}/b*d*(x^2-(-a*b)^{(1/2)}/b)-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)}/b))*c$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^7(d*x^4+c)^{(1/2)}/(b*x^4+a), x, \text{algorithm}=\text{"maxima"})$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.69, size = 87, normalized size = 0.94

$$\frac{(dx^4+c)^{3/2}}{6bd} - \frac{a\sqrt{dx^4+c}}{2b^2} + \frac{a \operatorname{atan}\left(\frac{a\sqrt{b}\sqrt{dx^4+c}\sqrt{ad-bc}}{a^2d-abc}\right)\sqrt{ad-bc}}{2b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^7*(c + d*x^4)^(1/2))/(a + b*x^4),x)`

[Out] $(c + d*x^4)^{3/2}/(6*b*d) - (a*(c + d*x^4)^{1/2})/(2*b^2) + (a*atan((a*b^{1/2}*(c + d*x^4)^{1/2}*(a*d - b*c)^{1/2})/(a^2*d - a*b*c))*(a*d - b*c)^{1/2})/(2*b^{5/2})$

sympy [A] time = 17.44, size = 90, normalized size = 0.97

$$\frac{2 \left(-\frac{ad^2\sqrt{c+dx^4}}{4b^2} + \frac{ad^2(ad-bc) \operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^3\sqrt{\frac{ad-bc}{b}}} + \frac{d(c+dx^4)^{\frac{3}{2}}}{12b} \right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(d*x**4+c)**(1/2)/(b*x**4+a),x)`

[Out] $2*(-a*d**2*\sqrt{c + d*x**4})/(4*b**2) + a*d**2*(a*d - b*c)*atan(\sqrt{c + d*x**4}/\sqrt{(a*d - b*c)/b})/(4*b**3*\sqrt{(a*d - b*c)/b}) + d*(c + d*x**4)**(3/2)/(12*b)/d**2$

$$3.514 \quad \int \frac{x^5 \sqrt{c+dx^4}}{a+bx^4} dx$$

Optimal. Leaf size=120

$$-\frac{\sqrt{a} \sqrt{bc-ad} \tan^{-1}\left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}}\right)}{2b^2} + \frac{(bc-2ad) \tanh^{-1}\left(\frac{\sqrt{d} x^2}{\sqrt{c+dx^4}}\right)}{4b^2 \sqrt{d}} + \frac{x^2 \sqrt{c+dx^4}}{4b}$$

Rubi [A] time = 0.15, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {465, 478, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a} \sqrt{bc-ad} \tan^{-1}\left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}}\right)}{2b^2} + \frac{(bc-2ad) \tanh^{-1}\left(\frac{\sqrt{d} x^2}{\sqrt{c+dx^4}}\right)}{4b^2 \sqrt{d}} + \frac{x^2 \sqrt{c+dx^4}}{4b}$$

Antiderivative was successfully verified.

[In] Int[(x^5*Sqrt[c + d*x^4])/(a + b*x^4), x]

[Out] (x^2*Sqrt[c + d*x^4])/(4*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*b^2) + ((b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4])/(4*b^2*Sqrt[d])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b}

, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 478

Int[((e_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))
^(q_)), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x_)
^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d,
e, f, n}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^5 \sqrt{c + dx^4}}{a + bx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2 \sqrt{c + dx^2}}{a + bx^2} dx, x, x^2 \right) \\
&= \frac{x^2 \sqrt{c + dx^4}}{4b} - \frac{\text{Subst} \left(\int \frac{ac + (-bc + 2ad)x^2}{(a + bx^2) \sqrt{c + dx^2}} dx, x, x^2 \right)}{4b} \\
&= \frac{x^2 \sqrt{c + dx^4}}{4b} + \frac{(bc - 2ad) \text{Subst} \left(\int \frac{1}{\sqrt{c + dx^2}} dx, x, x^2 \right)}{4b^2} - \frac{(a(bc - ad)) \text{Subst} \left(\int \frac{1}{(a + bx^2) \sqrt{c + dx^2}} \right)}{2b^2} \\
&= \frac{x^2 \sqrt{c + dx^4}}{4b} + \frac{(bc - 2ad) \text{Subst} \left(\int \frac{1}{1 - dx^2} dx, x, \frac{x^2}{\sqrt{c + dx^4}} \right)}{4b^2} - \frac{(a(bc - ad)) \text{Subst} \left(\int \frac{1}{a - (-bc + ad)x} \right)}{2b^2} \\
&= \frac{x^2 \sqrt{c + dx^4}}{4b} - \frac{\sqrt{a} \sqrt{bc - ad} \tan^{-1} \left(\frac{\sqrt{bc - ad} x^2}{\sqrt{a} \sqrt{c + dx^4}} \right)}{2b^2} + \frac{(bc - 2ad) \tanh^{-1} \left(\frac{\sqrt{d} x^2}{\sqrt{c + dx^4}} \right)}{4b^2 \sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 114, normalized size = 0.95

$$\frac{\frac{(bc - 2ad) \log(\sqrt{d} \sqrt{c + dx^4} + dx^2)}{\sqrt{d}} - 2\sqrt{a} \sqrt{bc - ad} \tan^{-1} \left(\frac{x^2 \sqrt{bc - ad}}{\sqrt{a} \sqrt{c + dx^4}} \right) + bx^2 \sqrt{c + dx^4}}{4b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*Sqrt[c + d*x^4])/(a + b*x^4), x]

[Out] (b*x^2*Sqrt[c + d*x^4] - 2*Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])] + ((b*c - 2*a*d)*Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]])/Sqrt[d])/(4*b^2)

IntegrateAlgebraic [A] time = 0.47, size = 173, normalized size = 1.44

$$\frac{(bc - 2ad) \log(\sqrt{c + dx^4} + \sqrt{d} x^2)}{4b^2 \sqrt{d}} - \frac{\sqrt{a} \sqrt{bc - ad} \tan^{-1} \left(\frac{b\sqrt{d} x^4}{\sqrt{a} \sqrt{bc - ad}} + \frac{bx^2 \sqrt{c + dx^4}}{\sqrt{a} \sqrt{bc - ad}} + \frac{\sqrt{a} \sqrt{d}}{\sqrt{bc - ad}} \right)}{2b^2} + \frac{x^2 \sqrt{c + dx^4}}{4b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*Sqrt[c + d*x^4])/(a + b*x^4), x]

[Out] (x^2*Sqrt[c + d*x^4])/(4*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^2*S

$$\frac{\sqrt{c + dx^4}}{\sqrt{a}\sqrt{b^2c - ad}} \Big/ (2b^2) + ((b^2c - 2ad)\text{Log}[\sqrt{d}x^2 + \sqrt{c + dx^4}]) \Big/ (4b^2\sqrt{d})$$

fricas [A] time = 0.56, size = 714, normalized size = 5.95

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")

[Out] $\frac{1}{8} \cdot (2\sqrt{d}x^4 + c) \cdot b \cdot d \cdot x^2 - (b^2c - 2ad) \cdot \sqrt{d} \cdot \log(-2d^2x^4 + 2\sqrt{d}x^4 + c) \cdot \sqrt{d} \cdot x^2 - c) + \sqrt{-abc + a^2d} \cdot d \cdot \log((b^2c^2 - 8abc^2d + 8a^2d^2)x^8 - 2(3abc^2 - 4a^2cd)x^4 + a^2c^2 - 4((b^2c - 2ad)x^6 - acx^2) \cdot \sqrt{d}x^4 + c) \cdot \sqrt{-abc + a^2d}) \Big/ (b^2x^8 + 2abx^4 + a^2) \Big/ (b^2d), \frac{1}{8} \cdot (2\sqrt{d}x^4 + c) \cdot b \cdot d \cdot x^2 - 2(b^2c - 2ad) \cdot \sqrt{-d} \cdot \arctan(\sqrt{-d}x^2/\sqrt{d}x^4 + c)) + \sqrt{-abc + a^2d} \cdot d \cdot \log((b^2c^2 - 8abc^2d + 8a^2d^2)x^8 - 2(3abc^2 - 4a^2cd)x^4 + a^2c^2 - 4((b^2c - 2ad)x^6 - acx^2) \cdot \sqrt{d}x^4 + c) \cdot \sqrt{-abc + a^2d}) \Big/ (b^2x^8 + 2abx^4 + a^2) \Big/ (b^2d), \frac{1}{8} \cdot (2\sqrt{d}x^4 + c) \cdot b \cdot d \cdot x^2 - 2\sqrt{abc - a^2d} \cdot d \cdot \arctan(1/2 \cdot ((b^2c - 2ad)x^4 - ac) \cdot \sqrt{d}x^4 + c) \cdot \sqrt{abc - a^2d}) \Big/ ((abc^2d - a^2d^2)x^6 + (abc^2 - a^2cd)x^2) - (b^2c - 2ad) \cdot \sqrt{d} \cdot \log(-2d^2x^4 + 2\sqrt{d}x^4 + c) \cdot \sqrt{d} \cdot x^2 - c) \Big/ (b^2d), \frac{1}{4} \cdot (\sqrt{d}x^4 + c) \cdot b \cdot d \cdot x^2 - (b^2c - 2ad) \cdot \sqrt{-d} \cdot \arctan(\sqrt{-d}x^2/\sqrt{d}x^4 + c) - \sqrt{abc - a^2d} \cdot d \cdot \arctan(1/2 \cdot ((b^2c - 2ad)x^4 - ac) \cdot \sqrt{d}x^4 + c) \cdot \sqrt{abc - a^2d}) \Big/ ((abc^2d - a^2d^2)x^6 + (abc^2 - a^2cd)x^2) \Big/ (b^2d)]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x); OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 0.28, size = 1066, normalized size = 8.88

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x)

```
[Out] 1/4*x^2*(d*x^4+c)^(1/2)/b+1/4/b*c/d^(1/2)*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2))+1/4*a/b/(-a*b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4*a/b^2*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4*a^2/b^2/(-a*b)^(1/2)/(-a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b)*d-1/4*a/b/(-a*b)^(1/2)/(-a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b)*c-1/4*a/b/(-a*b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4*a/b^2*d^(1/2)*ln(((x^2-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4*a^2/b^2/(-a*b)^(1/2)/(-a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b)*d+1/4*a/b/(-a*b)^(1/2)/(-a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4 + cx^5}}{bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a), x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^5/(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5 \sqrt{dx^4 + c}}{bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

```
[Out] int((x^5*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \sqrt{c + dx^4}}{a + bx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**4+c)**(1/2)/(b*x**4+a), x)
```

```
[Out] Integral(x**5*sqrt(c + d*x**4)/(a + b*x**4), x)
```


$$3.515 \quad \int \frac{x^3 \sqrt{c+dx^4}}{a+bx^4} dx$$

Optimal. Leaf size=70

$$\frac{\sqrt{c+dx^4}}{2b} - \frac{\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {444, 50, 63, 208}

$$\frac{\sqrt{c+dx^4}}{2b} - \frac{\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]

[Out] Sqrt[c + d*x^4]/(2*b) - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(3/2))

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \sqrt{c + dx^4}}{a + bx^4} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{\sqrt{c + dx}}{a + bx} dx, x, x^4 \right) \\
&= \frac{\sqrt{c + dx^4}}{2b} + \frac{(bc - ad) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^4 \right)}{4b} \\
&= \frac{\sqrt{c + dx^4}}{2b} + \frac{(bc - ad) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^4} \right)}{2bd} \\
&= \frac{\sqrt{c + dx^4}}{2b} - \frac{\sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.99

$$\frac{1}{2} \left(\frac{\sqrt{c + dx^4}}{b} - \frac{\sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{b^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Sqrt[c + d*x^4])/(a + b*x^4), x]

[Out] (Sqrt[c + d*x^4]/b - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/b^(3/2))/2

IntegrateAlgebraic [A] time = 0.09, size = 80, normalized size = 1.14

$$\frac{\sqrt{ad - bc} \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4} \sqrt{ad-bc}}{bc-ad} \right)}{2b^{3/2}} + \frac{\sqrt{c + dx^4}}{2b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3*Sqrt[c + d*x^4])/(a + b*x^4), x]

[Out] Sqrt[c + d*x^4]/(2*b) + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)])/(2*b^(3/2))

fricas [A] time = 0.45, size = 156, normalized size = 2.23

$$\left[\frac{\sqrt{\frac{bc-ad}{b}} \log\left(\frac{bdx^4+2bc-ad-2\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right) + 2\sqrt{dx^4+c}}{4b}, -\frac{\sqrt{\frac{bc-ad}{b}} \arctan\left(-\frac{\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bc-ad}\right) - \sqrt{dx^4+c}}{2b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a), x, algorithm="fricas")

[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*b*sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c))/b, -1/2*(sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqrt(d*x^4 + c))/b]

giac [A] time = 0.18, size = 66, normalized size = 0.94

$$\frac{(bc - ad) \arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}b} + \frac{\sqrt{dx^4+c}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a), x, algorithm="giac")

[Out] 1/2*(b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b) + 1/2*sqrt(d*x^4 + c)/b

maple [B] time = 0.20, size = 988, normalized size = 14.11

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(d*x^4+c)^(1/2)/(b*x^4+a), x)

[Out] 1/4/b*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/b^2*(-a*b)^(1/2)*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-(a

```

*b)^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)
)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2
+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2))/((x^2+(-a*b)^(1/2)/b))*a*d-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1
/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
)/((x^2+(-a*b)^(1/2)/b))*c+1/4/b*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/b^2*(-a*b)^(1/2)*d^(1/2)*ln((
(x^2-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/b^2/(-(a*d-
b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/((x^2-(-a*b)^(1/2)/b))*a*d-1/4/b/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/((x^2-(-a*b)^(1/2)/b))*c

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.66, size = 54, normalized size = 0.77

$$\frac{\sqrt{d x^4 + c}}{2 b} - \frac{\operatorname{atan}\left(\frac{\sqrt{b} \sqrt{d x^4 + c}}{\sqrt{a d - b c}}\right) \sqrt{a d - b c}}{2 b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(c + d*x^4)^(1/2))/(a + b*x^4),x)

[Out] (c + d*x^4)^(1/2)/(2*b) - (atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2)))*(a*d - b*c)^(1/2)/(2*b^(3/2))

sympy [A] time = 8.18, size = 65, normalized size = 0.93

$$\frac{2 \left(\frac{d\sqrt{c+dx^4}}{4b} - \frac{d(ad-bc) \operatorname{atan} \left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}} \right)}{4b^2 \sqrt{\frac{ad-bc}{b}}} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(d*x**4+c)**(1/2)/(b*x**4+a), x)`

[Out] `2*(d*sqrt(c + d*x**4)/(4*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*b**2*sqrt((a*d - b*c)/b)))/d`

$$3.516 \quad \int \frac{x\sqrt{c+dx^4}}{a+bx^4} dx$$

Optimal. Leaf size=91

$$\frac{\sqrt{bc-ad} \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}b} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {465, 402, 217, 206, 377, 205}

$$\frac{\sqrt{bc-ad} \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}b} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]

[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*Sqrt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4])]/(2*b)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] := Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
- 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.),
x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{x\sqrt{c+dx^4}}{a+bx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{c+dx^2}}{a+bx^2} dx, x, x^2 \right) \\ &= \frac{d \text{Subst} \left(\int \frac{1}{\sqrt{c+dx^2}} dx, x, x^2 \right)}{2b} + \frac{(bc-ad) \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{2b} \\ &= \frac{d \text{Subst} \left(\int \frac{1}{1-dx^2} dx, x, \frac{x^2}{\sqrt{c+dx^4}} \right)}{2b} + \frac{(bc-ad) \text{Subst} \left(\int \frac{1}{a-(-bc+ad)x^2} dx, x, \frac{x^2}{\sqrt{c+dx^4}} \right)}{2b} \\ &= \frac{\sqrt{bc-ad} \tan^{-1} \left(\frac{\sqrt{bc-ad} x^2}{\sqrt{a} \sqrt{c+dx^4}} \right)}{2\sqrt{a} b} + \frac{\sqrt{d} \tanh^{-1} \left(\frac{\sqrt{d} x^2}{\sqrt{c+dx^4}} \right)}{2b} \end{aligned}$$

Mathematica [A] time = 0.07, size = 89, normalized size = 0.98

$$\frac{\frac{\sqrt{bc-ad} \tan^{-1} \left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}} \right)}{\sqrt{a}} + \sqrt{d} \log \left(\sqrt{d} \sqrt{c+dx^4} + dx^2 \right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^4])/(a + b*x^4), x]
```

```
[Out] ((Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/
Sqrt[a] + Sqrt[d]*Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]])/(2*b)
```

IntegrateAlgebraic [A] time = 0.35, size = 144, normalized size = 1.58

$$\frac{\sqrt{bc-ad} \tan^{-1}\left(\frac{b\sqrt{d}x^4}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{2\sqrt{a}b} + \frac{\sqrt{d} \log\left(\sqrt{c+dx^4} + \sqrt{d}x^2\right)}{2b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*Sqrt[c + d*x^4])/(a + b*x^4), x]

[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)/(Sqrt[a]*Sqrt[b*c - a*d]))/(2*Sqrt[a]*b) + (Sqrt[d]*Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4]])/(2*b)

fricas [A] time = 0.49, size = 612, normalized size = 6.73

$$\frac{2\sqrt{d}\log(-2dx^4 - 2\sqrt{d}\sqrt{c+dx^4}) + \sqrt{\frac{bc-ad}{a}} \log\left(\frac{(b^2c^2 - 8ab^2cd + 8a^2d^2)x^8 - 2(3abc^2 - 4a^2cd)x^4 + a^2c^2 + 4((abc - 2a^2d)x^6 - a^2cx^2)\sqrt{d}\sqrt{c+dx^4}}{(b^2x^8 + 2abx^4 + a^2)}\right)}{8b} + \frac{4\sqrt{d}\arctan\left(\frac{\sqrt{d}\sqrt{c+dx^4}}{\sqrt{a}}\right) - \sqrt{\frac{bc-ad}{a}} \log\left(\frac{(b^2c^2 - 8ab^2cd + 8a^2d^2)x^8 - 2(3abc^2 - 4a^2cd)x^4 + a^2c^2 + 4((abc - 2a^2d)x^6 - a^2cx^2)\sqrt{d}\sqrt{c+dx^4}}{(b^2x^8 + 2abx^4 + a^2)}\right)}{8b} + \frac{\sqrt{\frac{bc-ad}{a}} \arctan\left(\frac{(b^2c^2 - 8ab^2cd + 8a^2d^2)x^8 - 2(3abc^2 - 4a^2cd)x^4 + a^2c^2 + 4((abc - 2a^2d)x^6 - a^2cx^2)\sqrt{d}\sqrt{c+dx^4}}{(b^2x^8 + 2abx^4 + a^2)}\right)}{4b} + \frac{2\sqrt{d}\arctan\left(\frac{\sqrt{d}\sqrt{c+dx^4}}{\sqrt{a}}\right) - \sqrt{\frac{bc-ad}{a}} \arctan\left(\frac{(b^2c^2 - 8ab^2cd + 8a^2d^2)x^8 - 2(3abc^2 - 4a^2cd)x^4 + a^2c^2 + 4((abc - 2a^2d)x^6 - a^2cx^2)\sqrt{d}\sqrt{c+dx^4}}{(b^2x^8 + 2abx^4 + a^2)}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a), x, algorithm="fricas")

[Out] [1/8*(2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a)))/(b^2*x^8 + 2*a*b*x^4 + a^2))/b, -1/8*(4*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a)))/(b^2*x^8 + 2*a*b*x^4 + a^2))/b, 1/4*(sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a))/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)) + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/b, -1/4*(2*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a))/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2))/b]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 0.20, size = 1000, normalized size = 10.99

$$\frac{\sqrt{d x^4+c}}{b x^4+a} \int \frac{\sqrt{d x^4+c}}{b x^4+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(d*x^4+c)^(1/2)/(b*x^4+a),x)`

[Out]
$$\begin{aligned} & -1/4/(-a*b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^{2*d-2}*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}+1/4/b*d^{(1/2)}*\ln(((x^2+(-a*b)^{(1/2)}/b)*d-(-a*b)^{(1/2)}/b*d)/d^{(1/2)}+((x^2+(-a*b)^{(1/2)}/b)^{2*d-2}*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})-1/4/(-a*b)^{(1/2)}/b/(-a*d-b*c)/b)^{(1/2)}*\ln((-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^{2*d-2}*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)}/b))*a*d+1/4/(-a*b)^{(1/2)}/(-a*d-b*c)/b)^{(1/2)}*\ln((-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^{2*d-2}*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)}/b))*c+1/4/(-a*b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^{2*d+2}*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}+1/4/b*d^{(1/2)}*\ln(((x^2-(-a*b)^{(1/2)}/b)*d+(-a*b)^{(1/2)}/b*d)/d^{(1/2)}+((x^2-(-a*b)^{(1/2)}/b)^{2*d+2}*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})+1/4/(-a*b)^{(1/2)}/b/(-a*d-b*c)/b)^{(1/2)}*\ln((2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^{2*d+2}*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)}/b))*a*d-1/4/(-a*b)^{(1/2)}/(-a*d-b*c)/b)^{(1/2)}*\ln((2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^{2*d+2}*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)}/b))*c \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d x^4+c x}}{b x^4+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")`

[Out] `integrate(sqrt(d*x^4 + c)*x/(b*x^4 + a), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \sqrt{d x^4+c}}{b x^4+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] int((x*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c + dx^4}}{a + bx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x*sqrt(c + d*x**4)/(a + b*x**4), x)
```

$$3.517 \quad \int \frac{\sqrt{c+dx^4}}{x(a+bx^4)} dx$$

Optimal. Leaf size=85

$$\frac{\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 83, 63, 208}

$$\frac{\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^4]/(x*(a + b*x^4)), x]

[Out] -(Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(2*a) + (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a*Sqrt[b])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 83

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{\sqrt{c+dx}}{x(a+bx)} dx, x, x^4 \right) \\
&= \frac{c \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^4 \right) - (bc-ad) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^4 \right)}{4a} \\
&= \frac{c \text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c+dx^4} \right) - (bc-ad) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c+dx^4} \right)}{2ad} \\
&= -\frac{\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c+dx^4}}{\sqrt{c}} \right)}{2a} + \frac{\sqrt{bc-ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{2a\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{bc-ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{\sqrt{b}} - \sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c+dx^4}}{\sqrt{c}} \right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

```
[Out] (-(Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]) + (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/Sqrt[b])/(2*a)
```

IntegrateAlgebraic [A] time = 0.12, size = 95, normalized size = 1.12

$$-\frac{\sqrt{ad-bc} \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4} \sqrt{ad-bc}}{bc-ad} \right)}{2a\sqrt{b}} - \frac{\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c+dx^4}}{\sqrt{c}} \right)}{2a}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]

[Out] $-1/2*(\text{Sqrt}[-(b*c) + a*d]*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[-(b*c) + a*d]*\text{Sqrt}[c + d*x^4])/ (b*c - a*d)])/(a*\text{Sqrt}[b]) - (\text{Sqrt}[c]*\text{ArcTanh}[\text{Sqrt}[c + d*x^4]/\text{Sqrt}[c]])/(2*a)$

fricas [A] time = 0.46, size = 383, normalized size = 4.51

$$\left[\frac{\sqrt{\frac{bc-ad}{b}} \log\left(\frac{bdx^4+2bc-ad+2\sqrt{bd}x\sqrt{\frac{bc-ad}{b}}}{b^2+a}\right) + \sqrt{c} \log\left(\frac{dx^4-2\sqrt{bd}x\sqrt{c}+2c}{x^2}\right)}{4a}, \frac{2\sqrt{\frac{bc-ad}{b}} \arctan\left(\frac{\sqrt{bd}x\sqrt{\frac{bc-ad}{b}}}{b-ad}\right) + \sqrt{c} \log\left(\frac{dx^4-2\sqrt{bd}x\sqrt{c}+2c}{x^2}\right)}{4a}, \frac{2\sqrt{-c} \arctan\left(\frac{\sqrt{bd}x\sqrt{c}}{c}\right) + \sqrt{\frac{bc-ad}{b}} \log\left(\frac{bdx^4+2bc-ad+2\sqrt{bd}x\sqrt{\frac{bc-ad}{b}}}{b^2+a}\right)}{4a}, \frac{\sqrt{\frac{bc-ad}{b}} \arctan\left(\frac{\sqrt{bd}x\sqrt{\frac{bc-ad}{b}}}{b-ad}\right) + \sqrt{-c} \arctan\left(\frac{\sqrt{bd}x\sqrt{c}}{c}\right)}{2a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="fricas")

[Out] $[1/4*(\text{sqrt}((b*c - a*d)/b)*\log((b*d*x^4 + 2*b*c - a*d + 2*\text{sqrt}(d*x^4 + c)*b*\text{sqrt}((b*c - a*d)/b)))/(b*x^4 + a) + \text{sqrt}(c)*\log((d*x^4 - 2*\text{sqrt}(d*x^4 + c)*\text{sqrt}(c) + 2*c)/x^4))/a, 1/4*(2*\text{sqrt}(-(b*c - a*d)/b)*\arctan(-\text{sqrt}(d*x^4 + c)*b*\text{sqrt}(-(b*c - a*d)/b)/(b*c - a*d)) + \text{sqrt}(c)*\log((d*x^4 - 2*\text{sqrt}(d*x^4 + c)*\text{sqrt}(c) + 2*c)/x^4))/a, 1/4*(2*\text{sqrt}(-c)*\arctan(\text{sqrt}(d*x^4 + c)*\text{sqrt}(-c)/c) + \text{sqrt}((b*c - a*d)/b)*\log((b*d*x^4 + 2*b*c - a*d + 2*\text{sqrt}(d*x^4 + c)*b*\text{sqrt}((b*c - a*d)/b)))/(b*x^4 + a))/a, 1/2*(\text{sqrt}(-(b*c - a*d)/b)*\arctan(-\text{sqrt}(d*x^4 + c)*b*\text{sqrt}(-(b*c - a*d)/b)/(b*c - a*d)) + \text{sqrt}(-c)*\arctan(\text{sqrt}(d*x^4 + c)*\text{sqrt}(-c)/c))/a]$

giac [A] time = 0.18, size = 79, normalized size = 0.93

$$-\frac{(bc - ad) \arctan\left(\frac{\sqrt{dx^4+cb}}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a} + \frac{c \arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="giac")

[Out] $-1/2*(b*c - a*d)*\arctan(\text{sqrt}(d*x^4 + c)*b/\text{sqrt}(-b^2*c + a*b*d))/(\text{sqrt}(-b^2*c + a*b*d)*a) + 1/2*c*\arctan(\text{sqrt}(d*x^4 + c)/\text{sqrt}(-c))/(a*\text{sqrt}(-c))$

maple [B] time = 0.26, size = 1037, normalized size = 12.20

$$\frac{\sqrt{\frac{bc-ad}{b}} \log\left(\frac{bdx^4+2bc-ad+2\sqrt{bd}x\sqrt{\frac{bc-ad}{b}}}{b^2+a}\right) + \sqrt{c} \log\left(\frac{dx^4-2\sqrt{bd}x\sqrt{c}+2c}{x^2}\right)}{4a} + \frac{2\sqrt{\frac{bc-ad}{b}} \arctan\left(\frac{\sqrt{bd}x\sqrt{\frac{bc-ad}{b}}}{b-ad}\right) + \sqrt{c} \log\left(\frac{dx^4-2\sqrt{bd}x\sqrt{c}+2c}{x^2}\right)}{4a} + \frac{2\sqrt{-c} \arctan\left(\frac{\sqrt{bd}x\sqrt{c}}{c}\right) + \sqrt{\frac{bc-ad}{b}} \log\left(\frac{bdx^4+2bc-ad+2\sqrt{bd}x\sqrt{\frac{bc-ad}{b}}}{b^2+a}\right)}{4a} + \frac{\sqrt{\frac{bc-ad}{b}} \arctan\left(\frac{\sqrt{bd}x\sqrt{\frac{bc-ad}{b}}}{b-ad}\right) + \sqrt{-c} \arctan\left(\frac{\sqrt{bd}x\sqrt{c}}{c}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^4+c)^(1/2)/x/(b*x^4+a),x)

[Out] $-1/4/a*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/a/b*(-a*b)^(1/2)*d^(1/2)*\ln(((x^2+(-a*b)^(1/2)/b)*d-(-$

$$\begin{aligned}
& a*b)^{(1/2)}/b*d)/d^{(1/2)}+((x^2+(-a*b)^{(1/2)}/b)^{2*d}-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}-1/4/b/(-a*d-b*c)/b)^{(1/2)}*\ln((-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^{2*d}-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)}/b))*d+1/4/a/(-a*d-b*c)/b)^{(1/2)}*\ln((-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^{2*d}-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)}/b))*c-1/4/a*((x^2-(-a*b)^{(1/2)}/b)^{2*d}+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}-1/4/a/b*((a*b)^{(1/2)}*d^{(1/2)}*\ln(((x^2-(-a*b)^{(1/2)}/b)*d+(-a*b)^{(1/2)}/b*d)/d^{(1/2)}+((x^2-(-a*b)^{(1/2)}/b)^{2*d}+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}-1/4/b/(-a*d-b*c)/b)^{(1/2)}*\ln((2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^{2*d}+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)}/b))*d+1/4/a/(-a*d-b*c)/b)^{(1/2)}*\ln((2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^{2*d}+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)}/b))*c+1/2/a*(d*x^4+c)^{(1/2)}-1/2/a*c^{(1/2)}*\ln((2*c+2*c^{(1/2)}*(d*x^4+c)^{(1/2)})/x^2)
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4 + c}}{(bx^4 + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x), x)

mupad [B] time = 4.87, size = 199, normalized size = 2.34

$$\frac{\sqrt{c} \operatorname{atanh}\left(\frac{\sqrt{c} \left(\sqrt{dx^4+c} \left(\frac{a^2 b d^4}{2} - a b^2 c d^3 + b^3 c^2 d^2\right) + \frac{c(8 a^3 b^2 d^3 - 16 a^2 b^3 c d^2) \sqrt{dx^4+c}}{16 a^2}\right)}{2 a \left(\frac{b^2 c^2 d^3}{4} - \frac{a b c d^4}{4}\right)}\right)}{2 a} + \frac{\operatorname{atanh}\left(\frac{a b^2 c d^3 \sqrt{dx^4+c} \sqrt{b^2 c - a b d}}{4 \left(\frac{a b^3 c^2 d^3}{4} - \frac{a^2 b^2 c d^4}{4}\right)}\right) \sqrt{b^2 c - a b d}}{2 a b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^4)^(1/2)/(x*(a + b*x^4)),x)

[Out] (c^(1/2)*atanh((c^(1/2)*((c + d*x^4)^(1/2)*((a^2*b*d^4)/2 + b^3*c^2*d^2 - a*b^2*c*d^3) + (c*(8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)))/(16*a^2)))/(2*a*((b^2*c^2*d^3)/4 - (a*b*c*d^4)/4)))/(2*a) + (atanh((a*b^2*c*d^3*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*((a*b^3*c^2*d^3)/4 - (a^2*b^2*c*d^4)/4)))*(b^2*c - a*b*d)^(1/2))/(2*a*b)

sympy [A] time = 12.05, size = 82, normalized size = 0.96

$$2 \frac{\left(\frac{cd \operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{4a\sqrt{-c}} + \frac{d(ad-bc) \operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4ab\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**4+c)**(1/2)/x/(b*x**4+a), x)

[Out] 2*(c*d*atan(sqrt(c + d*x**4)/sqrt(-c))/(4*a*sqrt(-c)) + d*(a*d - b*c)*atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*a*b*sqrt((a*d - b*c)/b)))/d

$$3.518 \quad \int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)} dx$$

Optimal. Leaf size=76

$$-\frac{\sqrt{bc-ad} \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}} - \frac{\sqrt{c+dx^4}}{2ax^2}$$

Rubi [A] time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {465, 475, 12, 377, 205}

$$-\frac{\sqrt{bc-ad} \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}} - \frac{\sqrt{c+dx^4}}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]

[Out] -Sqrt[c + d*x^4]/(2*a*x^2) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ

[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 475

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1) + b*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{c+dx^4}}{2ax^2} + \frac{\text{Subst} \left(\int \frac{-bc+ad}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{2a} \\
 &= -\frac{\sqrt{c+dx^4}}{2ax^2} - \frac{(bc-ad) \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{2a} \\
 &= -\frac{\sqrt{c+dx^4}}{2ax^2} - \frac{(bc-ad) \text{Subst} \left(\int \frac{1}{a-(-bc+ad)x^2} dx, x, \frac{x^2}{\sqrt{c+dx^4}} \right)}{2a} \\
 &= -\frac{\sqrt{c+dx^4}}{2ax^2} - \frac{\sqrt{bc-ad} \tan^{-1} \left(\frac{\sqrt{bc-ad} x^2}{\sqrt{a} \sqrt{c+dx^4}} \right)}{2a^{3/2}}
 \end{aligned}$$

Mathematica [C] time = 0.06, size = 53, normalized size = 0.70

$$-\frac{\sqrt{c+dx^4} {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{(ad-bc)x^4}{a(dx^4+c)} \right)}{2ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)), x]

[Out] -1/2*(Sqrt[c + d*x^4]*Hypergeometric2F1[-1/2, 1, 1/2, ((-b*c) + a*d)*x^4]/(a*(c + d*x^4)))/(a*x^2)

IntegrateAlgebraic [A] time = 0.39, size = 128, normalized size = 1.68

$$-\frac{\sqrt{bc-ad} \tan^{-1}\left(\frac{b\sqrt{d}x^4}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{2a^{3/2}} - \frac{\sqrt{c+dx^4}}{2ax^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]

[Out] $-\frac{1}{2}\sqrt{c+dx^4}/(ax^2) - \frac{(\sqrt{bc-ad})\text{ArcTan}[(\sqrt{a}\sqrt{d})/\sqrt{bc-ad}]}{\sqrt{bc-ad}} + \frac{(b\sqrt{d}x^4)/(\sqrt{a}\sqrt{bc-ad}) + (bx^2\sqrt{c+dx^4})/(\sqrt{a}\sqrt{bc-ad})}{2a^{3/2}}$

fricas [A] time = 0.46, size = 281, normalized size = 3.70

$$\left[\frac{x^2 \sqrt{\frac{bc-ad}{a}} \log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^8-2(3abc^2-4a^2cd)x^4+a^2c^2-4((abc-2a^2d)x^6-a^2cx^2)\sqrt{dx^4+c}\sqrt{\frac{bc-ad}{a}}}{b^2x^8+2abx^4+a^2}\right) - 4\sqrt{dx^4+c}}{8ax^2}, - \frac{x^2 \sqrt{\frac{bc-ad}{a}} \arctan\left(\frac{((bc-2ad)x^4-ac)\sqrt{dx^4+c}\sqrt{\frac{bc-ad}{a}}}{2((bcd-ad^2)x^6+(bc^2-acd)x^2)}\right) + 2\sqrt{dx^4+c}}{4ax^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="fricas")

[Out] $\frac{1}{8}(x^2\sqrt{-(bc-ad)/a})\log\left(\frac{(b^2c^2-8abc^2d+8a^2d^2)x^8-2(3a^2bc^2-4a^2cd)x^4+a^2c^2-4((abc-2a^2d)x^6-a^2cx^2)\sqrt{dx^4+c}\sqrt{-(bc-ad)/a}}{b^2x^8+2abx^4+a^2}\right) - 4\sqrt{dx^4+c}/(ax^2) - \frac{1}{4}(x^2\sqrt{(bc-ad)/a})\arctan\left(\frac{1}{2}\frac{(bc-2ad)x^4-ac}{(bc^2d-a^2d^2)x^6+(bcd-ad^2)x^2}\right) + 2\sqrt{dx^4+c}/(ax^2)$

giac [B] time = 1.29, size = 121, normalized size = 1.59

$$\frac{(bc\sqrt{d} - ad^{\frac{3}{2}}) \arctan\left(\frac{(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 b - bc + 2ad}{2\sqrt{abcd - a^2d^2}}\right)}{2\sqrt{abcd - a^2d^2}a} + \frac{c\sqrt{d}}{\left(\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 - c\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="giac")

[Out] $\frac{1}{2}(b\sqrt{c}\sqrt{d} - a^{\frac{3}{2}}d)\arctan\left(\frac{1}{2}\frac{(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 b - bc + 2ad}{\sqrt{abcd - a^2d^2}}\right) + \frac{c\sqrt{d}}{(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 - c}$

maple [B] time = 0.26, size = 1075, normalized size = 14.14

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x^4+c)^{(1/2)}/x^3/(b*x^4+a), x)$

[Out]
$$-1/2/a/c/x^2*(d*x^4+c)^{(3/2)}+1/2/a/c*d*x^2*(d*x^4+c)^{(1/2)}+1/2/a*d^{(1/2)}*\ln(d^{(1/2)}*x^2+(d*x^4+c)^{(1/2)})+1/4/a*b/(-a*b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^{2*d-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}-1/4/a*d^{(1/2)}*\ln(((x^2+(-a*b)^{(1/2)}/b)*d-(-a*b)^{(1/2)}/b*d)/d^{(1/2)}+((x^2+(-a*b)^{(1/2)}/b)^{2*d-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})+1/4/(-a*b)^{(1/2)}/(-a*d-b*c)/b)^{(1/2)}*\ln((-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^{2*d-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)}/b))*d-1/4/a*b/(-a*b)^{(1/2)}/(-a*d-b*c)/b)^{(1/2)}*\ln((-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^{2*d-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)}/b))*c-1/4/a*b/(-a*b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^{2*d+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}-1/4/a*d^{(1/2)}*\ln(((x^2-(-a*b)^{(1/2)}/b)*d+(-a*b)^{(1/2)}/b*d)/d^{(1/2)}+((x^2-(-a*b)^{(1/2)}/b)^{2*d+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})-1/4/(-a*b)^{(1/2)}/(-a*d-b*c)/b)^{(1/2)}*\ln((2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^{2*d+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)}/b))*d+1/4/a*b/(-a*b)^{(1/2)}/(-a*d-b*c)/b)^{(1/2)}*\ln((2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^{2*d+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)}/b))*c$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4 + c}}{(bx^4 + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((d*x^4+c)^{(1/2)}/x^3/(b*x^4+a), x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}(\text{sqrt}(d*x^4 + c)/((b*x^4 + a)*x^3), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{dx^4 + c}}{x^3 (bx^4 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^4)^(1/2)/(x^3*(a + b*x^4)), x)`

[Out] `int((c + d*x^4)^(1/2)/(x^3*(a + b*x^4)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^4}}{x^3 (a + bx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**4+c)**(1/2)/x**3/(b*x**4+a), x)`

[Out] `Integral(sqrt(c + d*x**4)/(x**3*(a + b*x**4)), x)`

$$3.519 \quad \int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)} dx$$

Optimal. Leaf size=115

$$\frac{(2bc - ad) \tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} - \frac{\sqrt{b}\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2} - \frac{\sqrt{c+dx^4}}{4ax^4}$$

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc - ad) \tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} - \frac{\sqrt{b}\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2} - \frac{\sqrt{c+dx^4}}{4ax^4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)), x]

[Out] -Sqrt[c + d*x^4]/(4*a*x^4) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(4*a^2*Sqrt[c]) - (Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a^2)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 99

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)} dx, x, x^4 \right) \\
&= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(-2bc+ad) - \frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}} dx, x, x^4 \right)}{4a} \\
&= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(b(bc-ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^4 \right)}{4a^2} - \frac{(2bc-ad) \text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^4 \right)}{8a^2} \\
&= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(b(bc-ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c+dx^4} \right)}{2a^2d} - \frac{(2bc-ad) \text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c+dx^4} \right)}{4a^2d} \\
&= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(2bc-ad) \tanh^{-1} \left(\frac{\sqrt{c+dx^4}}{\sqrt{c}} \right)}{4a^2\sqrt{c}} - \frac{\sqrt{b}\sqrt{bc-ad} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 107, normalized size = 0.93

$$\frac{\frac{(2bc-ad) \tanh^{-1} \left(\frac{\sqrt{c+dx^4}}{\sqrt{c}} \right)}{\sqrt{c}} - 2\sqrt{b}\sqrt{bc-ad} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}} \right) - \frac{a\sqrt{c+dx^4}}{x^4}}{4a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)), x]

[Out] $(-(a\sqrt{c + dx^4})/x^4) + ((2bc - ad)\text{ArcTanh}[\text{Sqrt}[c + dx^4]/\text{Sqrt}[c]])/\text{Sqrt}[c] - 2\text{Sqrt}[b]\text{Sqrt}[b*c - a*d]\text{ArcTanh}[(\text{Sqrt}[b]\text{Sqrt}[c + dx^4])/\text{Sqrt}[b*c - a*d]]/(4a^2)$

IntegrateAlgebraic [A] time = 0.27, size = 125, normalized size = 1.09

$$\frac{\sqrt{b}\sqrt{ad-bc}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{2a^2} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} - \frac{\sqrt{c+dx^4}}{4ax^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)), x]

[Out] $-1/4\text{Sqrt}[c + dx^4]/(ax^4) + (\text{Sqrt}[b]\text{Sqrt}[-(b*c) + a*d]\text{ArcTan}[(\text{Sqrt}[b]\text{Sqrt}[-(b*c) + a*d]\text{Sqrt}[c + dx^4])/(b*c - a*d)]/(2a^2) + ((2bc - ad)\text{ArcTanh}[\text{Sqrt}[c + dx^4]/\text{Sqrt}[c]])/(4a^2\text{Sqrt}[c])$

fricas [A] time = 0.47, size = 513, normalized size = 4.46

$$\frac{2\sqrt{b}\sqrt{ad-bc}\log\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right) + (2bc-ad)\sqrt{c}\log\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right) - 2\sqrt{b}\sqrt{c}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right) + \sqrt{b}\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right) + \sqrt{b}\sqrt{c}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right) + \sqrt{b}\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right) - \sqrt{b}\sqrt{c}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right) - \sqrt{b}\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a), x, algorithm="fricas")

[Out] $[1/8*(2\sqrt{b^2c - a*b*d})c*x^4*\log((b*d*x^4 + 2*b*c - a*d - 2\sqrt{d*x^4 + c})\sqrt{b^2c - a*b*d})/(b*x^4 + a) - (2*b*c - a*d)\sqrt{c}*x^4*\log((d*x^4 - 2\sqrt{d*x^4 + c})\sqrt{c} + 2*c)/x^4 - 2\sqrt{d*x^4 + c}*a*c/(a^2*c*x^4), 1/8*(4\sqrt{-b^2c + a*b*d})c*x^4*\arctan(\sqrt{d*x^4 + c}\sqrt{-b^2c + a*b*d})/(b*d*x^4 + b*c) - (2*b*c - a*d)\sqrt{c}*x^4*\log((d*x^4 - 2\sqrt{d*x^4 + c})\sqrt{c} + 2*c)/x^4 - 2\sqrt{d*x^4 + c}*a*c/(a^2*c*x^4), -1/4*((2*b*c - a*d)\sqrt{-c}*x^4*\arctan(\sqrt{d*x^4 + c}\sqrt{-c}/c) - \sqrt{b^2c - a*b*d})c*x^4*\log((b*d*x^4 + 2*b*c - a*d - 2\sqrt{d*x^4 + c})\sqrt{b^2c - a*b*d})/(b*x^4 + a) + \sqrt{d*x^4 + c}*a*c/(a^2*c*x^4), 1/4*(2\sqrt{-b^2c + a*b*d})c*x^4*\arctan(\sqrt{d*x^4 + c}\sqrt{-b^2c + a*b*d})/(b*d*x^4 + b*c) - (2*b*c - a*d)\sqrt{-c}*x^4*\arctan(\sqrt{d*x^4 + c}\sqrt{-c}/c) - \sqrt{d*x^4 + c}*a*c/(a^2*c*x^4)]$

giac [A] time = 0.17, size = 107, normalized size = 0.93

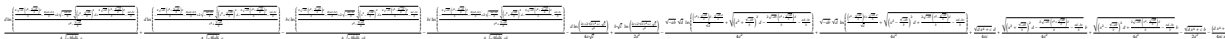
$$\frac{(b^2c - abd)\arctan\left(\frac{\sqrt{dx^4+cb}}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a^2} - \frac{(2bc - ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4a^2\sqrt{-c}} - \frac{\sqrt{dx^4+c}}{4ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c - a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) - 1/4*(2*b*c - a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/4*sqrt(d*x^4 + c)/(a*x^4)
```

maple [B] time = 0.27, size = 1107, normalized size = 9.63



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x)
```

```
[Out] -1/4/a/c/x^4*(d*x^4+c)^(3/2)-1/4/a/c^(1/2)*d*ln((2*c+2*(d*x^4+c)^(1/2)*c^(1/2))/x^2)+1/4/a/c*d*(d*x^4+c)^(1/2)+1/4/a^2*b*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/a^2*(-a*b)^(1/2)*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b)*d-1/4/a^2*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b)*c+1/4/a^2*b*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/a^2*(-a*b)^(1/2)*d^(1/2)*ln(((x^2-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/a/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b)*d-1/4/a^2*b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b)*c-1/2/a^2*b*(d*x^4+c)^(1/2)+1/2/a^2*b*c^(1/2)*ln((2*c+2*(d*x^4+c)^(1/2)*c^(1/2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4 + c}}{(bx^4 + a)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^5), x)

mupad [B] time = 5.39, size = 269, normalized size = 2.34

$$\frac{\operatorname{atanh}\left(\frac{b^3 d^4 \sqrt{d x^4+c} \sqrt{b^2 c-a b d}}{16\left(\frac{a b^3 d^5}{16}-\frac{b^4 c d^4}{16}\right)}\right) \sqrt{b^2 c-a b d}}{2 a^2}-\frac{\sqrt{d x^4+c}}{4 a x^4}-\frac{\operatorname{atanh}\left(\frac{b^4 \sqrt{c} d^4 \sqrt{d x^4+c}}{16\left(\frac{b^4 c d^4}{16}-\frac{3 a b^3 d^5}{32}+\frac{a^2 b^2 d^6}{32 c}\right)}-\frac{3 b^3 d^5 \sqrt{d x^4+c}}{32 \sqrt{c}\left(\frac{a b^2 d^6}{32 c}-\frac{3 b^3 d^5}{32}+\frac{b^4 c d^4}{16 a}\right)}+\frac{b^2 d^6 \sqrt{d x^4+c}}{32 c^{3/2}\left(\frac{b^2 d^6}{32 c}-\frac{3 b^3 d^5}{32 a}+\frac{b^4 c d^4}{16 a^2}\right)}\right)}{4 a^2 \sqrt{c}}(a d-2 b c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^4)^(1/2)/(x^5*(a + b*x^4)),x)

[Out] (atanh((b^3*d^4*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(16*((a*b^3*d^5)/16 - (b^4*c*d^4)/16)))*(b^2*c - a*b*d)^(1/2))/(2*a^2) - (c + d*x^4)^(1/2)/(4*a*x^4) - (atanh((b^4*c^(1/2)*d^4*(c + d*x^4)^(1/2))/(16*((b^4*c*d^4)/16 - (3*a*b^3*d^5)/32 + (a^2*b^2*d^6)/(32*c))) - (3*b^3*d^5*(c + d*x^4)^(1/2))/(32*c^(1/2)*((a*b^2*d^6)/(32*c) - (3*b^3*d^5)/32 + (b^4*c*d^4)/(16*a))) + (b^2*d^6*(c + d*x^4)^(1/2))/(32*c^(3/2)*((b^2*d^6)/(32*c) - (3*b^3*d^5)/(32*a) + (b^4*c*d^4)/(16*a^2))))*(a*d - 2*b*c))/(4*a^2*c^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^4}}{x^5(a + bx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**4+c)**(1/2)/x**5/(b*x**4+a),x)

[Out] Integral(sqrt(c + d*x**4)/(x**5*(a + b*x**4)), x)

$$3.520 \quad \int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)} dx$$

Optimal. Leaf size=110

$$\frac{b\sqrt{bc-ad} \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}} + \frac{\sqrt{c+dx^4}(3bc-ad)}{6a^2cx^2} - \frac{\sqrt{c+dx^4}}{6ax^6}$$

Rubi [A] time = 0.16, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 475, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^4}(3bc-ad)}{6a^2cx^2} + \frac{b\sqrt{bc-ad} \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}} - \frac{\sqrt{c+dx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

```
[Out] -Sqrt[c + d*x^4]/(6*a*x^6) + ((3*b*c - a*d)*Sqrt[c + d*x^4])/(6*a^2*c*x^2)
+ (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
```

1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 475

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 583

Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)} dx, x, x^2 \right) \\
&= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{\text{Subst} \left(\int \frac{-3bc+ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{6a} \\
&= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} - \frac{\text{Subst} \left(\int -\frac{3bc(bc-ad)}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{6a^2c} \\
&= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} + \frac{(b(bc-ad)) \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{2a^2} \\
&= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} + \frac{(b(bc-ad)) \text{Subst} \left(\int \frac{1}{a-(-bc+ad)x^2} dx, x, \frac{x^2}{\sqrt{c+dx^4}} \right)}{2a^2} \\
&= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} + \frac{b\sqrt{bc-ad} \tan^{-1} \left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}} \right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 5.33, size = 130, normalized size = 1.18

$$\frac{3bc^2x^4\sqrt{\frac{dx^4}{c}} + 1\sqrt{x^4\left(\frac{b}{a} - \frac{d}{c}\right)} \sin^{-1}\left(\frac{\sqrt{x^4\left(\frac{b}{a} - \frac{d}{c}\right)}}{\sqrt{\frac{bx^4}{a} + 1}}\right) + (c+dx^4)(3bcx^4 - a(c+dx^4))}{6a^2cx^6\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)), x]

[Out] ((c + d*x^4)*(3*b*c*x^4 - a*(c + d*x^4)) + 3*b*c^2*x^4*Sqrt[(b/a - d/c)*x^4]*Sqrt[1 + (d*x^4)/c]*ArcSin[Sqrt[(b/a - d/c)*x^4]/Sqrt[1 + (b*x^4)/a]])/(6*a^2*c*x^6*Sqrt[c + d*x^4])

IntegrateAlgebraic [A] time = 0.53, size = 151, normalized size = 1.37

$$\frac{b\sqrt{bc-ad} \tan^{-1} \left(\frac{b\sqrt{d}x^4}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}} \right)}{2a^{5/2}} + \frac{\sqrt{c+dx^4}(-ac - adx^4 + 3bcx^4)}{6a^2cx^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]

[Out] (Sqrt[c + d*x^4]*(-(a*c) + 3*b*c*x^4 - a*d*x^4))/(6*a^2*c*x^6) + (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(5/2))

fricas [A] time = 0.52, size = 329, normalized size = 2.99

$$\left[\frac{3bcx^6 \sqrt{\frac{bc-ad}{a}} \log\left(\frac{((b^2-8abcd+8a^2d^2)x^8-2(3abc^2-4a^2cd)x^4+a^2c^2+4((abc-2a^2d)x^6-a^2cx^2)\sqrt{dx^4+c}\sqrt{\frac{bc-ad}{a}})}{b^2x^8+2abx^4+a^2}\right) + 4((3bc-ad)x^4-ac)\sqrt{dx^4+c}}{24a^2cx^6}, \frac{3bcx^6 \sqrt{\frac{bc-ad}{a}} \arctan\left(\frac{((bc-2ad)x^4-ac)\sqrt{dx^4+c}\sqrt{\frac{bc-ad}{a}}}{2((bcd-ad^2)x^6+(bc^2-ad)x^2)}\right) + 2((3bc-ad)x^4-ac)\sqrt{dx^4+c}}{12a^2cx^6} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="fricas")

[Out] [1/24*(3*b*c*x^6*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*((3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6), 1/12*(3*b*c*x^6*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)) + 2*((3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6)]

giac [B] time = 1.75, size = 225, normalized size = 2.05

$$\frac{(b^2c\sqrt{d} - abd^3)\arctan\left(\frac{(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 b - bc + 2ad}{2\sqrt{abcd - a^2d^2}}\right)}{2\sqrt{abcd - a^2d^2}} - \frac{3(\sqrt{d}x^2 - \sqrt{dx^4+c})^4 bc\sqrt{d} - 3(\sqrt{d}x^2 - \sqrt{dx^4+c})^4 ad^3 - 6(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 bc^2\sqrt{d} + 3bc^3\sqrt{d} - ac^2d^3}{3\left((\sqrt{d}x^2 - \sqrt{dx^4+c})^2 - c\right)^3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="giac")

[Out] -1/2*(b^2*c*sqrt(d) - a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2) - 1/3*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b*c*sqrt(d) - 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c^2*sqrt(d) + 3*b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2 - c)^3*a^2)

maple [B] time = 0.26, size = 1116, normalized size = 10.15

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x)

[Out] $\frac{1}{2} \frac{1}{a^2} \frac{b}{c} \frac{1}{x^2} (d x^4 + c)^{3/2} - \frac{1}{2} \frac{1}{a^2} \frac{b}{c} d x^2 (d x^4 + c)^{1/2} - \frac{1}{2} \frac{1}{a^2} b^2 d^{1/2} \ln(d^{1/2} x^2 + (d x^4 + c)^{1/2}) - \frac{1}{4} \frac{1}{a^2} \frac{b^2}{(-a b)^{1/2}} ((x^2 + (-a b)^{1/2} / b)^{2d - 2} (-a b)^{1/2} (x^2 + (-a b)^{1/2} / b) / b^d - (a d - b^2 c) / b)^{1/2} + \frac{1}{4} \frac{1}{a^2} \frac{b^2}{d^{1/2}} \ln(((x^2 + (-a b)^{1/2} / b)^d - (-a b)^{1/2} / b^d) / d^{1/2} + ((x^2 + (-a b)^{1/2} / b)^{2d - 2} (-a b)^{1/2} (x^2 + (-a b)^{1/2} / b) / b^d - (a d - b^2 c) / b)^{1/2}) - \frac{1}{4} \frac{1}{a} \frac{b}{(-a b)^{1/2}} / (-a d - b^2 c) / b)^{1/2} \ln((-2 (-a b)^{1/2} (x^2 + (-a b)^{1/2} / b) / b^d - 2 (a d - b^2 c) / b + 2 (-a d - b^2 c) / b)^{1/2} ((x^2 + (-a b)^{1/2} / b)^{2d - 2} (-a b)^{1/2} (x^2 + (-a b)^{1/2} / b) / b^d - (a d - b^2 c) / b)^{1/2}) / (x^2 + (-a b)^{1/2} / b)) * d + \frac{1}{4} \frac{1}{a^2} \frac{b^2}{(-a b)^{1/2}} / (-a d - b^2 c) / b)^{1/2} \ln((-2 (-a b)^{1/2} (x^2 + (-a b)^{1/2} / b) / b^d - 2 (a d - b^2 c) / b + 2 (-a d - b^2 c) / b)^{1/2} ((x^2 + (-a b)^{1/2} / b)^{2d - 2} (-a b)^{1/2} (x^2 + (-a b)^{1/2} / b) / b^d - (a d - b^2 c) / b)^{1/2}) / (x^2 + (-a b)^{1/2} / b)) * c + \frac{1}{4} \frac{1}{a^2} \frac{b^2}{(-a b)^{1/2}} ((x^2 - (-a b)^{1/2} / b)^{2d + 2} (-a b)^{1/2} (x^2 - (-a b)^{1/2} / b) / b^d - (a d - b^2 c) / b)^{1/2} + \frac{1}{4} \frac{1}{a^2} \frac{b^2}{d^{1/2}} \ln(((x^2 - (-a b)^{1/2} / b)^d * d + (-a b)^{1/2} / b^d) / d^{1/2} + ((x^2 - (-a b)^{1/2} / b)^{2d + 2} (-a b)^{1/2} (x^2 - (-a b)^{1/2} / b) / b^d - (a d - b^2 c) / b)^{1/2}) + \frac{1}{4} \frac{1}{a} \frac{b}{(-a b)^{1/2}} / (-a d - b^2 c) / b)^{1/2} \ln((2 (-a b)^{1/2} (x^2 - (-a b)^{1/2} / b) / b^d - 2 (a d - b^2 c) / b + 2 (-a d - b^2 c) / b)^{1/2} ((x^2 - (-a b)^{1/2} / b)^{2d + 2} (-a b)^{1/2} (x^2 - (-a b)^{1/2} / b) / b^d - (a d - b^2 c) / b)^{1/2}) / (x^2 - (-a b)^{1/2} / b)) * d - \frac{1}{4} \frac{1}{a^2} \frac{b^2}{(-a b)^{1/2}} / (-a d - b^2 c) / b)^{1/2} \ln((2 (-a b)^{1/2} (x^2 - (-a b)^{1/2} / b) / b^d - 2 (a d - b^2 c) / b + 2 (-a d - b^2 c) / b)^{1/2} ((x^2 - (-a b)^{1/2} / b)^{2d + 2} (-a b)^{1/2} (x^2 - (-a b)^{1/2} / b) / b^d - (a d - b^2 c) / b)^{1/2}) / (x^2 - (-a b)^{1/2} / b)) * c - \frac{1}{6} \frac{1}{a} \frac{1}{x^6} (d x^4 + c)^{3/2} / c$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4 + c}}{(bx^4 + a)x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^7), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d x^4 + c}}{x^7 (b x^4 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^4)^(1/2)/(x^7*(a + b*x^4)),x)

[Out] int((c + d*x^4)^(1/2)/(x^7*(a + b*x^4)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^4}}{x^7 (a + bx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**4+c)**(1/2)/x**7/(b*x**4+a), x)

[Out] Integral(sqrt(c + d*x**4)/(x**7*(a + b*x**4)), x)

$$3.521 \quad \int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}} dx$$

Optimal. Leaf size=104

$$-\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^4}(ad+bc)}{2b^2d^2} + \frac{(c+dx^4)^{3/2}}{6bd^2}$$

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 88, 63, 208}

$$-\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^4}(ad+bc)}{2b^2d^2} + \frac{(c+dx^4)^{3/2}}{6bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -((b*c + a*d)*Sqrt[c + d*x^4]/(2*b^2*d^2) + (c + d*x^4)^(3/2)/(6*b*d^2) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```


Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11}}{(a + bx^4)\sqrt{c + dx^4}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^2}{(a + bx)\sqrt{c + dx}} dx, x, x^4 \right) \\
 &= \frac{1}{4} \text{Subst} \left(\int \left(\frac{-bc - ad}{b^2 d \sqrt{c + dx}} + \frac{a^2}{b^2 (a + bx) \sqrt{c + dx}} + \frac{\sqrt{c + dx}}{bd} \right) dx, x, x^4 \right) \\
 &= -\frac{(bc + ad)\sqrt{c + dx^4}}{2b^2 d^2} + \frac{(c + dx^4)^{3/2}}{6bd^2} + \frac{a^2 \text{Subst} \left(\int \frac{1}{(a + bx)\sqrt{c + dx}} dx, x, x^4 \right)}{4b^2} \\
 &= -\frac{(bc + ad)\sqrt{c + dx^4}}{2b^2 d^2} + \frac{(c + dx^4)^{3/2}}{6bd^2} + \frac{a^2 \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^4} \right)}{2b^2 d} \\
 &= -\frac{(bc + ad)\sqrt{c + dx^4}}{2b^2 d^2} + \frac{(c + dx^4)^{3/2}}{6bd^2} - \frac{a^2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^4}}{\sqrt{bc - ad}} \right)}{2b^{5/2} \sqrt{bc - ad}}
 \end{aligned}$$

Mathematica [A] time = 0.26, size = 91, normalized size = 0.88

$$\frac{\sqrt{c + dx^4} (-3ad - 2bc + bdx^4)}{6b^2 d^2} - \frac{a^2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^4}}{\sqrt{bc - ad}} \right)}{2b^{5/2} \sqrt{bc - ad}}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] (Sqrt[c + d*x^4]*(-2*b*c - 3*a*d + b*d*x^4))/(6*b^2*d^2) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[b*c - a*d])

IntegrateAlgebraic [A] time = 0.18, size = 101, normalized size = 0.97

$$\frac{\sqrt{c + dx^4} (-3ad - 2bc + bdx^4)}{6b^2 d^2} - \frac{a^2 \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^4} \sqrt{ad - bc}}{bc - ad} \right)}{2b^{5/2} \sqrt{ad - bc}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] (Sqrt[c + d*x^4]*(-2*b*c - 3*a*d + b*d*x^4))/(6*b^2*d^2) - (a^2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)])/(2*b^(5/2)*Sqrt[-(b*c) + a*d])

fricas [A] time = 0.72, size = 289, normalized size = 2.78

$$\frac{3\sqrt{b^2c - abd}a^2d^2 \log\left(\frac{bdx^4 + 2bc - ad - 2\sqrt{dx^4 + c}\sqrt{b^2c - abd}}{bx^4 + a}\right) - 2(2b^3c^2 + ab^2cd - 3a^2bd^2 - (b^3cd - ab^2d^2)x^4)\sqrt{dx^4 + c}}{12(b^4cd^2 - ab^3d^3)} + \frac{3\sqrt{-b^2c + abd}a^2d^2 \arctan\left(\frac{\sqrt{dx^4 + c}\sqrt{-b^2c + abd}}{bdx^4 + bc}\right) - (2b^3c^2 + ab^2cd - 3a^2bd^2 - (b^3cd - ab^2d^2)x^4)\sqrt{dx^4 + c}}{6(b^4cd^2 - ab^3d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [1/12*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c)/(b^4*c*d^2 - a*b^3*d^3), 1/6*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c)/(b^4*c*d^2 - a*b^3*d^3)]

giac [A] time = 0.16, size = 106, normalized size = 1.02

$$\frac{a^2 \arctan\left(\frac{\sqrt{dx^4 + c} b}{\sqrt{-b^2c + abd}}\right)}{2\sqrt{-b^2c + abd} b^2} + \frac{(dx^4 + c)^{\frac{3}{2}} b^2 d^4 - 3\sqrt{dx^4 + c} b^2 c d^4 - 3\sqrt{dx^4 + c} a b d^5}{6 b^3 d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] 1/2*a^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) + 1/6*((d*x^4 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^4 + c)*b^2*c*d^4 - 3*sqrt(d*x^4 + c)*a*b*d^5)/(b^3*d^6)

maple [B] time = 0.28, size = 378, normalized size = 3.63

$$\frac{\sqrt{dx^4 + c} x^4}{6bd} - \frac{a^2 \ln\left(\frac{2\sqrt{-ab}\left(x^2 + \frac{\sqrt{-ab}}{b}\right)d - 2abd - bc + 2\sqrt{\frac{ad-bc}{b}}\sqrt{\left(x^2 + \frac{\sqrt{-ab}}{b}\right)^2 d + \frac{ad-bc}{b}}}{x^2 + \frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{\frac{ad-bc}{b}} b^3} - \frac{a^2 \ln\left(\frac{2\sqrt{-ab}\left(x^2 + \frac{\sqrt{-ab}}{b}\right)d - 2abd - bc + 2\sqrt{\frac{ad-bc}{b}}\sqrt{\left(x^2 + \frac{\sqrt{-ab}}{b}\right)^2 d - \frac{ad-bc}{b}}}{x^2 + \frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{\frac{ad-bc}{b}} b^3} - \frac{\sqrt{dx^4 + c} a}{2b^2d} - \frac{\sqrt{dx^4 + c} c}{3bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x)

```
[Out] 1/6/b*(d*x^4+c)^(1/2)/d*x^4-1/3/b*(d*x^4+c)^(1/2)/d^2*c-1/2/b^2*a/d*(d*x^4+c)^(1/2)-1/4*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/4*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.82, size = 102, normalized size = 0.98

$$\frac{(dx^4+c)^{3/2}}{6bd^2} - \left(\frac{c}{bd^2} + \frac{2ad^3-2bcd^2}{4b^2d^4} \right) \sqrt{dx^4+c} + \frac{a^2 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^4+c}}{\sqrt{ad-bc}}\right)}{2b^{5/2}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(3/2)/(6*b*d^2) - (c/(b*d^2) + (2*a*d^3 - 2*b*c*d^2)/(4*b^2*d^4))*(c + d*x^4)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2)))/(2*b^(5/2)*(a*d - b*c)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a + bx^4)\sqrt{c + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

$$3.522 \quad \int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}} dx$$

Optimal. Leaf size=74

$$\frac{a \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^4}}{2bd}$$

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 80, 63, 208}

$$\frac{a \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^4}}{2bd}$$

Antiderivative was successfully verified.

[In] Int[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] Sqrt[c + d*x^4]/(2*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(3/2)*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^7}{(a + bx^4)\sqrt{c + dx^4}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x}{(a + bx)\sqrt{c + dx}} dx, x, x^4 \right) \\
 &= \frac{\sqrt{c + dx^4}}{2bd} - \frac{a \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^4 \right)}{4b} \\
 &= \frac{\sqrt{c + dx^4}}{2bd} - \frac{a \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^4} \right)}{2bd} \\
 &= \frac{\sqrt{c + dx^4}}{2bd} + \frac{a \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{2b^{3/2}\sqrt{bc-ad}}
 \end{aligned}$$

Mathematica [A] time = 0.08, size = 72, normalized size = 0.97

$$\frac{1}{2} \left(\frac{a \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c + dx^4}}{bd} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^7/((a + b*x^4)*Sqrt[c + d*x^4]), x]

[Out] (Sqrt[c + d*x^4]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/2

IntegrateAlgebraic [A] time = 0.10, size = 84, normalized size = 1.14

$$\frac{a \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4} \sqrt{ad-bc}}{bc-ad} \right)}{2b^{3/2}\sqrt{ad-bc}} + \frac{\sqrt{c + dx^4}}{2bd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] Sqrt[c + d*x^4]/(2*b*d) + (a*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d])*Sqrt[c + d*x^4])/(b*c - a*d)]/(2*b^(3/2)*Sqrt[-(b*c) + a*d])

fricas [A] time = 0.74, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c - abd} \operatorname{ad} \log\left(\frac{bdx^4 + 2bc - ad + 2\sqrt{dx^4 + c}\sqrt{b^2c - abd}}{bx^4 + a}\right) + 2\sqrt{dx^4 + c}(b^2c - abd)}{4(b^3cd - ab^2d^2)}, -\frac{\sqrt{-b^2c + abd} \operatorname{ad} \arctan\left(\frac{\sqrt{dx^4 + c}\sqrt{-b^2c + abd}}{bdx^4 + bc}\right) - \sqrt{dx^4 + c}(b^2c - abd)}{2(b^3cd - ab^2d^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2*c - a*b*d))/(b^3*c*d - a*b^2*d^2), -1/2*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - sqrt(d*x^4 + c)*(b^2*c - a*b*d))/(b^3*c*d - a*b^2*d^2)]

giac [A] time = 0.19, size = 64, normalized size = 0.86

$$-\frac{\operatorname{ad} \arctan\left(\frac{\sqrt{dx^4 + c}b}{\sqrt{-b^2c + abd}}\right) - \frac{\sqrt{dx^4 + c}}{b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] -1/2*(a*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b) - sqrt(d*x^4 + c)/b)/d

maple [B] time = 0.21, size = 335, normalized size = 4.53

$$a \ln \left(\frac{\frac{2\sqrt{-ab}\left(x^2 - \frac{\sqrt{-ab}}{b}\right)^d}{b} - \frac{2(ad-bc)}{b} + 2\sqrt{-\frac{ad-bc}{b}} \sqrt{\left(x^2 - \frac{\sqrt{-ab}}{b}\right)^2 + \frac{2\sqrt{-ab}\left(x^2 - \frac{\sqrt{-ab}}{b}\right)^d}{b} - \frac{ad-bc}{b}}}{x^2 - \frac{\sqrt{-ab}}{b}} \right) + a \ln \left(\frac{\frac{2\sqrt{-ab}\left(x^2 + \frac{\sqrt{-ab}}{b}\right)^d}{b} - \frac{2(ad-bc)}{b} + 2\sqrt{-\frac{ad-bc}{b}} \sqrt{\left(x^2 + \frac{\sqrt{-ab}}{b}\right)^2 + \frac{2\sqrt{-ab}\left(x^2 + \frac{\sqrt{-ab}}{b}\right)^d}{b} - \frac{ad-bc}{b}}}{x^2 + \frac{\sqrt{-ab}}{b}} \right) + \frac{\sqrt{dx^4 + c}}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x)

[Out] 1/2*(d*x^4+c)^(1/2)/b/d + 1/4*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d - 2*(a*d-b*c)/b + 2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)

$$\frac{(1/2)/b)^{2d-2}(-ab)^{(1/2)}(x^2+(-ab)^{(1/2)/b})/b^d-(ad-bc)/b)^{(1/2)}}{(x^2+(-ab)^{(1/2)/b})+1/4*a/b^2/(-(ad-bc)/b)^{(1/2)}*\ln((2*(-ab)^{(1/2)}(x^2-(-ab)^{(1/2)/b})/b^d-2*(ad-bc)/b+2*(-(ad-bc)/b)^{(1/2)}*((x^2-(-ab)^{(1/2)/b})^2d+2*(-ab)^{(1/2)}(x^2-(-ab)^{(1/2)/b})/b^d-(ad-bc)/b)^{(1/2)))/(x^2-(-ab)^{(1/2)/b})}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.73, size = 58, normalized size = 0.78

$$\frac{\sqrt{dx^4+c}}{2bd} - \frac{a \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^4+c}}{\sqrt{ad-bc}}\right)}{2b^{3/2}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/((a + b*x^4)*(c + d*x^4)^(1/2)),x)

[Out] (c + d*x^4)^(1/2)/(2*b*d) - (a*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2)))/(2*b^(3/2)*(a*d - b*c)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a + bx^4)\sqrt{c + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)

[Out] Integral(x**7/((a + b*x**4)*sqrt(c + d*x**4)), x)

$$3.523 \quad \int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}} dx$$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]]/(2*Sqrt[b]*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^4 \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{a-\frac{bc}{a}+\frac{bx^2}{d}} dx, x, \sqrt{c+dx^4} \right)}{2d} \\ &= -\frac{\tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{2\sqrt{b}\sqrt{bc-ad}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$\frac{\tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{2\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] -1/2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c - a*d])

IntegrateAlgebraic [A] time = 0.05, size = 61, normalized size = 1.20

$$\frac{\tan^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad} \right)}{2\sqrt{b}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] -1/2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)]/(Sqrt[b]*Sqrt[-(b*c) + a*d])

fricas [A] time = 0.69, size = 130, normalized size = 2.55

$$\left[\frac{\log \left(\frac{bdx^4+2bc-ad-2\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a} \right)}{4\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd} \arctan \left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc} \right)}{2(b^2c-abd)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [1/4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a))/sqrt(b^2*c - a*b*d), 1/2*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c))/(b^2*c - a*b*d)]

giac [A] time = 0.16, size = 40, normalized size = 0.78

$$\frac{\arctan\left(\frac{\sqrt{dx^4+cb}}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] 1/2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)

maple [B] time = 0.21, size = 316, normalized size = 6.20

$$\frac{\ln\left(\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d-2(ad-bc)+2\sqrt{\frac{-ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d-ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}}{4\sqrt{\frac{-ad-bc}{b}}b}\right)}{\ln\left(\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d-2(ad-bc)+2\sqrt{\frac{-ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d-ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}}{4\sqrt{\frac{-ad-bc}{b}}b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x)

[Out] -1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b)-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details) Is a*d-b*c positive or negative?

mupad [B] time = 4.80, size = 40, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{b\sqrt{dx^4+c}}{\sqrt{abd-b^2c}}\right)}{2\sqrt{abd-b^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/((a + b*x^4)*(c + d*x^4)^(1/2)),x)`

[Out] `atan((b*(c + d*x^4)^(1/2))/(a*b*d - b^2*c)^(1/2))/(2*(a*b*d - b^2*c)^(1/2))`

sympy [A] time = 14.27, size = 37, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)`

[Out] `atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(2*b*sqrt((a*d - b*c)/b))`

$$3.524 \quad \int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}} dx$$

Optimal. Leaf size=85

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}$$

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 86, 63, 208}

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] -ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/(2*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 86

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && !IntegerQ[p]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}} dx &= \frac{1}{4} \text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}} dx, x, x^4\right) \\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^4\right)}{4a} - \frac{b \text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^4\right)}{4a} \\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{a} + \frac{x^2}{d}} dx, x, \sqrt{c+dx^4}\right)}{2ad} - \frac{b \text{Subst}\left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c+dx^4}\right)}{2ad} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}} + \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}} \end{aligned}$$

Mathematica [A] time = 0.09, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]), x]

[Out] $\left(-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right]}{\sqrt{c}} + \frac{\sqrt{b} \text{ArcTanh}\left[\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right]}{\sqrt{bc-ad}}\right)/(2a)$

IntegrateAlgebraic [A] time = 0.11, size = 95, normalized size = 1.12

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{2a\sqrt{ad-bc}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)]/(2*a*Sqrt[-(b*c) + a*d]) - ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/(2*a*Sqrt[c]))

fricas [A] time = 0.73, size = 431, normalized size = 5.07

$$\left[\sqrt{\frac{b}{b^2c-ad}} \log\left(\frac{bx^4+2bx-ad+2\sqrt{b^2c-ad}\sqrt{\frac{bx^4+c}{a}}}{bx^4+a}\right) + \sqrt{c} \log\left(\frac{dx^4+2\sqrt{b^2c-ad}\sqrt{\frac{bx^4+c}{a}}}{x^4}\right), 2c\sqrt{\frac{b}{b^2c-ad}} \arctan\left(\frac{\sqrt{b^2c-ad}\sqrt{\frac{bx^4+c}{a}}}{bx^4+a}\right) + \sqrt{c} \log\left(\frac{dx^4+2\sqrt{b^2c-ad}\sqrt{\frac{bx^4+c}{a}}}{x^4}\right), c\sqrt{\frac{b}{b^2c-ad}} \log\left(\frac{bx^4+2bx-ad+2\sqrt{b^2c-ad}\sqrt{\frac{bx^4+c}{a}}}{bx^4+a}\right) + 2\sqrt{-c} \arctan\left(\frac{\sqrt{b^2c-ad}\sqrt{\frac{bx^4+c}{a}}}{c}\right), c\sqrt{\frac{b}{b^2c-ad}} \arctan\left(\frac{\sqrt{b^2c-ad}\sqrt{\frac{bx^4+c}{a}}}{bx^4+a}\right) + \sqrt{-c} \arctan\left(\frac{\sqrt{b^2c-ad}\sqrt{\frac{bx^4+c}{a}}}{c}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [1/4*(c*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(2*c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^4 + b*c)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(c*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c))/(a*c), 1/2*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^4 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c))/(a*c)]

giac [A] time = 0.17, size = 71, normalized size = 0.84

$$-\frac{b \arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a} + \frac{\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] -1/2*b*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a) + 1/2*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c))

maple [B] time = 0.29, size = 347, normalized size = 4.08

$$\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{\frac{-ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}}{4\sqrt{\frac{-ad-bc}{b}}a}\right) + \ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{\frac{-ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}}{4\sqrt{\frac{-ad-bc}{b}}a}\right) - \frac{\ln\left(\frac{2c+2\sqrt{d}x^4+c}{x^2}\sqrt{c}\right)}{2a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x)

[Out] $\frac{1}{4} \frac{1}{a} \frac{1}{(-a*d-b*c)/b} \ln\left(\frac{(-2*(-a*b))^{1/2} * (x^2+(-a*b)^{1/2}/b)}{b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b}\right)^{1/2} * \left(\frac{x^2+(-a*b)^{1/2}/b}{b*d-(a*d-b*c)/b}\right)^{1/2} / \left(\frac{x^2+(-a*b)^{1/2}/b}{b*d-2*(a*d-b*c)/b} + 1\right) + \frac{1}{4} \frac{1}{a} \frac{1}{(-a*d-b*c)/b} \ln\left(\frac{(2*(-a*b))^{1/2} * (x^2-(-a*b)^{1/2}/b)}{b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b}\right)^{1/2} * \left(\frac{x^2-(-a*b)^{1/2}/b}{b*d-(a*d-b*c)/b}\right)^{1/2} / \left(\frac{x^2-(-a*b)^{1/2}/b}{b*d-2*(a*d-b*c)/b} - 1\right) - \frac{1}{2} \frac{1}{a} \frac{1}{c^{1/2}} \ln\left(\frac{(2*c+2*(d*x^4+c))^{1/2} * c^{1/2}}{x^2}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)\sqrt{dx^4 + cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2), x, algorithm="maxima")

[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x), x)

mupad [B] time = 5.03, size = 652, normalized size = 7.67

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{dx^4+c}}{\sqrt{c}}\right)}{2a\sqrt{c}} - \frac{\operatorname{atan}\left(\frac{\frac{\sqrt{b^2c-abd}\left(2a^2b^2\beta_3\frac{(8a^3b^2d^3-16a^2b^3cd^2)\sqrt{dx^4+c}\sqrt{b^2c-abd}}{4(a^2d-abc)}\right)}{4(a^2d-abc)}} + \frac{\sqrt{b^2c-abd}\left(2a^2b^2\beta_3\frac{(8a^3b^2d^3-16a^2b^3cd^2)\sqrt{dx^4+c}\sqrt{b^2c-abd}}{4(a^2d-abc)}\right)}{4(a^2d-abc)}}{\frac{\sqrt{b^2c-abd}\left(2a^2b^2\beta_3\frac{(8a^3b^2d^3-16a^2b^3cd^2)\sqrt{dx^4+c}\sqrt{b^2c-abd}}{4(a^2d-abc)}\right)}{4(a^2d-abc)}} - \frac{\sqrt{b^2c-abd}\left(2a^2b^2\beta_3\frac{(8a^3b^2d^3-16a^2b^3cd^2)\sqrt{dx^4+c}\sqrt{b^2c-abd}}{4(a^2d-abc)}\right)}{4(a^2d-abc)}}}{2(a^2d-abc)}\right)}{\sqrt{b^2c-abd}} \operatorname{li}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^4)*(c + d*x^4)^(1/2)), x)

[Out] $-\operatorname{atanh}\left(\frac{(c + d*x^4)^{1/2}}{c^{1/2}}\right) / (2*a*c^{1/2}) - \left(\operatorname{atan}\left(\frac{((b^2*c - a*b*d))^{1/2} * (b^3*d^2*(c + d*x^4)^{1/2} - ((b^2*c - a*b*d))^{1/2} * (2*a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^{1/2} * (b^2*c - a*b*d))^{1/2}) / (4*(a^2*d - a*b*c)))}{(4*(a^2*d - a*b*c))} * 1i\right) / (4*(a^2*d - a*b*c)) + \left(\frac{(b^2*c - a*b*d)^{1/2} * (b^3*d^2*(c + d*x^4)^{1/2} + ((b^2*c - a*b*d))^{1/2} * (2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^{1/2} * (b^2*c - a*b*d))^{1/2}) / (4*(a^2*d - a*b*c)))}{(4*(a^2*d - a*b*c))} * 1i\right) / (4*(a^2*d - a*b*c))\right) / \left(\frac{(b^2*c - a*b*d)^{1/2} * (b^3*d^2*(c + d*x^4)^{1/2} - ((b^2*c - a*b*d))^{1/2} * (2*a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^{1/2} * (b^2*c - a*b*d))^{1/2}) / (4*(a^2*d - a*b*c)))}{(4*(a^2*d - a*b*c))} + \left(\frac{(b^2*c - a*b*d)^{1/2} * (b^3*d^2*(c + d*x^4)^{1/2} + ((b^2*c - a*b*d))^{1/2} * (2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^{1/2} * (b^2*c - a*b*d))^{1/2}) / (4*(a^2*d - a*b*c)))}{(4*(a^2*d - a*b*c))} * 1i\right) / (4*(a^2*d - a*b*c))\right) / (4*(a^2*d - a*b*c)) - \left(\frac{(b^2*c - a*b*d)^{1/2} * (b^3*d^2*(c + d*x^4)^{1/2} + ((b^2*c - a*b*d))^{1/2} * (2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^{1/2} * (b^2*c - a*b*d))^{1/2}) / (4*(a^2*d - a*b*c)))}{(4*(a^2*d - a*b*c))} + \left(\frac{(b^2*c - a*b*d)^{1/2} * (b^3*d^2*(c + d*x^4)^{1/2} - ((b^2*c - a*b*d))^{1/2} * (2*a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^{1/2} * (b^2*c - a*b*d))^{1/2}) / (4*(a^2*d - a*b*c)))}{(4*(a^2*d - a*b*c))} * 1i\right) / (2*(a^2*d - a*b*c))\right)$

sympy [A] time = 20.21, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2a\sqrt{\frac{ad-bc}{b}}} + \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**4+a)/(d*x**4+c)**(1/2),x)

[Out] -atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(2*a*sqrt((a*d - b*c)/b)) + atan(sqrt(c + d*x**4)/sqrt(-c))/(2*a*sqrt(-c))

$$3.525 \quad \int \frac{1}{x^5(a+bx^4)\sqrt{c+dx^4}} dx$$

Optimal. Leaf size=117

$$-\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}} + \frac{(ad+2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}} - \frac{\sqrt{c+dx^4}}{4acx^4}$$

Rubi [A] time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 103, 156, 63, 208}

$$-\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}} + \frac{(ad+2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}} - \frac{\sqrt{c+dx^4}}{4acx^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] -Sqrt[c + d*x^4]/(4*a*c*x^4) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(4*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a^2*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5 (a + bx^4) \sqrt{c + dx^4}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x^2 (a + bx) \sqrt{c + dx}} dx, x, x^4 \right) \\
&= -\frac{\sqrt{c + dx^4}}{4acx^4} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(2bc + ad) + \frac{bdx}{2}}{x(a + bx) \sqrt{c + dx}} dx, x, x^4 \right)}{4ac} \\
&= -\frac{\sqrt{c + dx^4}}{4acx^4} + \frac{b^2 \text{Subst} \left(\int \frac{1}{(a + bx) \sqrt{c + dx}} dx, x, x^4 \right)}{4a^2} - \frac{(2bc + ad) \text{Subst} \left(\int \frac{1}{x \sqrt{c + dx}} dx, x, x^4 \right)}{8a^2c} \\
&= -\frac{\sqrt{c + dx^4}}{4acx^4} + \frac{b^2 \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^4} \right)}{2a^2d} - \frac{(2bc + ad) \text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, x^4 \right)}{4a^2cd} \\
&= -\frac{\sqrt{c + dx^4}}{4acx^4} + \frac{(2bc + ad) \tanh^{-1} \left(\frac{\sqrt{c + dx^4}}{\sqrt{c}} \right)}{4a^2c^{3/2}} - \frac{b^{3/2} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^4}}{\sqrt{bc - ad}} \right)}{2a^2 \sqrt{bc - ad}}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 151, normalized size = 1.29

$$\frac{b^{3/2} \sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^4}}{\sqrt{bc - ad}} \right)}{2a^2(ad - bc)} + \frac{b \tanh^{-1} \left(\frac{\sqrt{c + dx^4}}{\sqrt{c}} \right)}{2a^2 \sqrt{c}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{c + dx^4}}{\sqrt{c}} \right)}{4ac^{3/2}} - \frac{\sqrt{c + dx^4}}{4acx^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] $-\frac{1}{4}\sqrt{c + dx^4}/(a^2cx^4) + \frac{(b\operatorname{ArcTanh}[\sqrt{c + dx^4}/\sqrt{c}])}{(2a^2\sqrt{c})} + \frac{(d\operatorname{ArcTanh}[\sqrt{c + dx^4}/\sqrt{c}])}{(4a^2c^{3/2})} + \frac{(b^{3/2}\sqrt{bc - ad}\operatorname{ArcTanh}[(\sqrt{b}\sqrt{c + dx^4})/\sqrt{bc - ad}])}{(2a^2(-bc + ad))}$

IntegrateAlgebraic [A] time = 0.25, size = 127, normalized size = 1.09

$$-\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{2a^2\sqrt{ad-bc}} + \frac{(ad+2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}} - \frac{\sqrt{c+dx^4}}{4acx^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] $-\frac{1}{4}\sqrt{c + dx^4}/(a^2cx^4) - \frac{(b^{3/2}\operatorname{ArcTan}[(\sqrt{b}\sqrt{c + dx^4})/\sqrt{bc - ad}])}{(2a^2\sqrt{bc - ad})} + \frac{((2b^2c + ad)\operatorname{ArcTanh}[\sqrt{c + dx^4}/\sqrt{c}])}{(4a^2c^{3/2})}$

fricas [A] time = 0.70, size = 565, normalized size = 4.83

$$\frac{2b^2c^2\sqrt{c}\sqrt{d}\sqrt{c+dx^4}\log\left(\frac{b^2d^2c^2+2cd^2\sqrt{c+dx^4}\sqrt{ad-bc}}{2b^2c^2}\right) + (2N+ad)\sqrt{c}\sqrt{d}\log\left(\frac{b^2d^2c^2+2cd^2\sqrt{c+dx^4}\sqrt{ad-bc}}{2b^2c^2}\right) - 2\sqrt{d}\sqrt{c}\sqrt{ad-bc} - 4b^2c^2\sqrt{c}\sqrt{d}\operatorname{arctan}\left(\frac{\sqrt{d}\sqrt{c+dx^4}\sqrt{ad-bc}}{2b^2c^2}\right) + (2N+ad)\sqrt{c}\sqrt{d}\log\left(\frac{b^2d^2c^2+2cd^2\sqrt{c+dx^4}\sqrt{ad-bc}}{2b^2c^2}\right) + 2\sqrt{d}\sqrt{c}\sqrt{ad-bc} - 4b^2c^2\sqrt{c}\sqrt{d}\operatorname{arctan}\left(\frac{b^2d^2c^2+2cd^2\sqrt{c+dx^4}\sqrt{ad-bc}}{2b^2c^2}\right) + (2N+ad)\sqrt{c}\sqrt{d}\operatorname{arctan}\left(\frac{\sqrt{d}\sqrt{c+dx^4}\sqrt{ad-bc}}{2b^2c^2}\right) + \sqrt{d}\sqrt{c}\sqrt{ad-bc}}{4c^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{8}(2b^2c^2x^4\sqrt{b/(b^2c - a^2d)})\log((b^2dx^4 + 2b^2c - a^2d - 2\sqrt{d}(dx^4 + c)(b^2c - a^2d)\sqrt{b/(b^2c - a^2d)})/(b^2x^4 + a)) + (2b^2c + a^2d)\sqrt{c}x^4\log((d^2x^4 + 2\sqrt{d}(dx^4 + c)\sqrt{c} + 2c)/x^4) - 2\sqrt{d}(dx^4 + c)a^2c/(a^2c^2x^4) - 1/8(4b^2c^2x^4\sqrt{-b/(b^2c - a^2d)})\operatorname{arctan}(-\sqrt{d}(dx^4 + c)(b^2c - a^2d)\sqrt{-b/(b^2c - a^2d)})/(b^2dx^4 + b^2c) - (2b^2c + a^2d)\sqrt{c}x^4\log((d^2x^4 + 2\sqrt{d}(dx^4 + c)\sqrt{c} + 2c)/x^4) + 2\sqrt{d}(dx^4 + c)a^2c/(a^2c^2x^4) + 1/4(b^2c^2x^4\sqrt{b/(b^2c - a^2d)})\log((b^2dx^4 + 2b^2c - a^2d - 2\sqrt{d}(dx^4 + c)(b^2c - a^2d)\sqrt{b/(b^2c - a^2d)})/(b^2x^4 + a)) - (2b^2c + a^2d)\sqrt{-c}x^4\operatorname{arctan}(\sqrt{d}(dx^4 + c)\sqrt{-c}/c) - \sqrt{d}(dx^4 + c)a^2c/(a^2c^2x^4) - 1/4(2b^2c^2x^4\sqrt{-b/(b^2c - a^2d)})\operatorname{arctan}(-\sqrt{d}(dx^4 + c)(b^2c - a^2d)\sqrt{-b/(b^2c - a^2d)})/(b^2dx^4 + b^2c) + (2b^2c + a^2d)\sqrt{-c}x^4\operatorname{arctan}(\sqrt{d}(dx^4 + c)\sqrt{-c}/c) + \sqrt{d}(dx^4 + c)a^2c/(a^2c^2x^4)$

giac [A] time = 0.17, size = 104, normalized size = 0.89

$$\frac{b^2 \arctan\left(\frac{\sqrt{dx^4+cb}}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a^2} - \frac{(2bc+ad) \arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4a^2\sqrt{-c}c} - \frac{\sqrt{dx^4+c}}{4acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] 1/2*b^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) - 1/4*(2*b*c + a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^2*sqrt(-c)*c) - 1/4*sqrt(d*x^4 + c)/(a*c*x^4)

maple [B] time = 0.29, size = 402, normalized size = 3.44

$$\frac{b \ln\left(\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b} - \frac{2(ad-bc)}{b} + 2\sqrt{\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d + \frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b} - \frac{ad-bc}{b}}\right)}{4\sqrt{\frac{ad-bc}{b}}a^2} - \frac{b \ln\left(\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b} - \frac{2(ad-bc)}{b} + 2\sqrt{\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d - \frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b} - \frac{ad-bc}{b}}\right)}{4\sqrt{\frac{ad-bc}{b}}a^2} + \frac{d \ln\left(\frac{2c+2\sqrt{dx^4+c}\sqrt{c}}{x^2}\right)}{4ac^2} + \frac{b \ln\left(\frac{2c+2\sqrt{dx^4+c}\sqrt{c}}{x^2}\right)}{2a^2\sqrt{c}} - \frac{\sqrt{dx^4+c}}{4acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x)

[Out] -1/4*(d*x^4+c)^(1/2)/a/c/x^4+1/4*a*d/c^(3/2)*ln((2*c+2*(d*x^4+c)^(1/2)*c^(1/2))/x^2)-1/4/a^2*b/(-a*d-b*c)/b^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b^(1/2))/(x^2+(-a*b)^(1/2)/b)-1/4/a^2*b/(-a*d-b*c)/b^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b^(1/2))/(x^2-(-a*b)^(1/2)/b))+1/2/a^2*b/c^(1/2)*ln((2*c+2*(d*x^4+c)^(1/2)*c^(1/2))/x^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)\sqrt{dx^4 + cx^5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^5), x)

mupad [B] time = 5.35, size = 396, normalized size = 3.38

$$\frac{\ln(\sqrt{dx^4+c}(b^4c-ab^3d)^{3/2}+b^6c^2+a^2b^4d^2-2ab^5cd)\sqrt{b^4c-ab^3d}}{4a^2d-4a^2bc} - \frac{\ln(\sqrt{dx^4+c}(b^4c-ab^3d)^{3/2}-b^6c^2-a^2b^4d^2+2ab^5cd)\sqrt{b^4c-ab^3d}}{4(a^2d-a^2bc)} - \frac{\sqrt{dx^4+c}}{4acx^4} - \frac{\operatorname{atan}\left(\frac{b^4d\sqrt{dx^4+c}}{16\sqrt{c}\left(\frac{13b^4d}{16c}+\frac{5b^3d^2}{32a^2}+\frac{d^2b^2d}{32a^2}\right)}+\frac{b^2d\sqrt{dx^4+c}}{32\sqrt{c}\left(\frac{13b^4d}{32a}+\frac{5b^3d^2}{32a}+\frac{d^2b^2d}{32a^2}\right)}+\frac{b^2d\sqrt{dx^4+c}}{32\sqrt{c}\left(\frac{13b^4d}{16a}+\frac{5b^3d^2}{32a}+\frac{d^2b^2d}{32a^2}\right)}\right)}{4a^2\sqrt{c}}}{4acx^4} (ad+2b)c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^5*(a + b*x^4)*(c + d*x^4)^(1/2)),x)`

[Out] $(\log((c + d*x^4)^{(1/2)}*(b^4*c - a*b^3*d)^{(3/2)} + b^6*c^2 + a^2*b^4*d^2 - 2*a*b^5*c*d)*(b^4*c - a*b^3*d)^{(1/2)})/(4*a^3*d - 4*a^2*b*c) - (\log((c + d*x^4)^{(1/2)}*(b^4*c - a*b^3*d)^{(3/2)} - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4*c - a*b^3*d)^{(1/2)})/(4*(a^3*d - a^2*b*c)) - (c + d*x^4)^{(1/2)}/(4*a*c*x^4) - (\operatorname{atan}((b^4*d^4*(c + d*x^4)^{(1/2)}*3i)/(16*(c^3)^{(1/2)}*((3*b^4*d^4)/(16*c) + (5*a*b^3*d^5)/(32*c^2) + (a^2*b^2*d^6)/(32*c^3)))) + (b^2*d^6*(c + d*x^4)^{(1/2)}*1i)/(32*(c^3)^{(1/2)}*((5*b^3*d^5)/(32*a) + (b^2*d^6)/(32*c) + (3*b^4*c*d^4)/(16*a^2))) + (b^3*d^5*(c + d*x^4)^{(1/2)}*5i)/(32*(c^3)^{(1/2)}*((3*b^4*d^4)/(16*a) + (5*b^3*d^5)/(32*c) + (a*b^2*d^6)/(32*c^2))))*(a*d + 2*b*c)*1i)/(4*a^2*(c^3)^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 (a + bx^4) \sqrt{c + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)`

[Out] `Integral(1/(x**5*(a + b*x**4)*sqrt(c + d*x**4)), x)`

$$3.526 \quad \int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}} dx$$

Optimal. Leaf size=123

$$\frac{a^{3/2} \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}} - \frac{(2ad+bc) \tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}} + \frac{x^2\sqrt{c+dx^4}}{4bd}$$

Rubi [A] time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {465, 479, 523, 217, 206, 377, 205}

$$\frac{a^{3/2} \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}} - \frac{(2ad+bc) \tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}} + \frac{x^2\sqrt{c+dx^4}}{4bd}$$

Antiderivative was successfully verified.

[In] Int[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] (x^2*Sqrt[c + d*x^4])/(4*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^2*d^(3/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b,

, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 479

Int[((e_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))
^(q_)), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
(m + n(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
)^(n)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{(a + bx^4)\sqrt{c + dx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{(a + bx^2)\sqrt{c + dx^2}} dx, x, x^2 \right) \\
&= \frac{x^2\sqrt{c + dx^4}}{4bd} - \frac{\text{Subst} \left(\int \frac{ac + (bc + 2ad)x^2}{(a + bx^2)\sqrt{c + dx^2}} dx, x, x^2 \right)}{4bd} \\
&= \frac{x^2\sqrt{c + dx^4}}{4bd} + \frac{a^2 \text{Subst} \left(\int \frac{1}{(a + bx^2)\sqrt{c + dx^2}} dx, x, x^2 \right)}{2b^2} - \frac{(bc + 2ad) \text{Subst} \left(\int \frac{1}{\sqrt{c + dx^2}} dx, x, x^2 \right)}{4b^2d} \\
&= \frac{x^2\sqrt{c + dx^4}}{4bd} + \frac{a^2 \text{Subst} \left(\int \frac{1}{a - (-bc + ad)x^2} dx, x, \frac{x^2}{\sqrt{c + dx^4}} \right)}{2b^2} - \frac{(bc + 2ad) \text{Subst} \left(\int \frac{1}{1 - dx^2} dx, x, \frac{x^2}{\sqrt{c + dx^4}} \right)}{4b^2d} \\
&= \frac{x^2\sqrt{c + dx^4}}{4bd} + \frac{a^{3/2} \tan^{-1} \left(\frac{\sqrt{bc - ad} x^2}{\sqrt{a} \sqrt{c + dx^4}} \right)}{2b^2\sqrt{bc - ad}} - \frac{(bc + 2ad) \tanh^{-1} \left(\frac{\sqrt{d} x^2}{\sqrt{c + dx^4}} \right)}{4b^2d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 118, normalized size = 0.96

$$\frac{2a^{3/2} \tan^{-1} \left(\frac{x^2 \sqrt{bc - ad}}{\sqrt{a} \sqrt{c + dx^4}} \right)}{\sqrt{bc - ad}} - \frac{(2ad + bc) \log \left(\sqrt{d} \sqrt{c + dx^4} + dx^2 \right)}{d^{3/2}} + \frac{bx^2 \sqrt{c + dx^4}}{d}$$

$4b^2$

Antiderivative was successfully verified.

[In] Integrate[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] ((b*x^2*Sqrt[c + d*x^4])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]])/d^(3/2))/(4*b^2)

IntegrateAlgebraic [A] time = 0.70, size = 177, normalized size = 1.44

$$\frac{a^{3/2} \tan^{-1} \left(\frac{b\sqrt{d}x^4}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}} \right)}{2b^2\sqrt{bc-ad}} + \frac{(-2ad - bc) \log \left(\sqrt{c + dx^4} + \sqrt{d} x^2 \right)}{4b^2d^{3/2}} + \frac{x^2\sqrt{c + dx^4}}{4bd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] (x^2*Sqrt[c + d*x^4])/(4*b*d) + (a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^2*Sqrt[c + d*x^4])

$\frac{1}{\sqrt{a}\sqrt{bc-ad}} \log\left(\frac{\sqrt{d}x^2 + \sqrt{c+dx^4}}{4b^2d^{3/2}}\right) + \frac{(-bc-2ad)\sqrt{bc-ad}}{2b^2\sqrt{bc-ad}}$

fricas [A] time = 0.75, size = 739, normalized size = 6.01

$$\frac{1}{8} \sqrt{d} x^2 \sqrt{bc-ad} \log\left(\frac{\sqrt{d}x^2 + \sqrt{c+dx^4}}{4b^2d^{3/2}}\right) + \frac{(-bc-2ad)\sqrt{bc-ad}}{2b^2\sqrt{bc-ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{8} (2\sqrt{d}x^4 + c) b d x^2 + a d^2 \sqrt{-a/(bc-ad)} \log\left(\frac{(b^2c^2 - 8abc d + 8a^2d^2)x^8 - 2(3abc^2 - 4a^2cd)x^4 + a^2c^2 + 4((b^2c^2 - 3abc d + 2a^2d^2)x^6 - (abc^2 - a^2cd)x^2)\sqrt{d}x^4 + c\sqrt{-a/(bc-ad)}}{(b^2x^8 + 2abx^4 + a^2)}\right) + (bc + 2ad)\sqrt{d} \log(-2dx^4 + 2\sqrt{d}x^4 + c)\sqrt{d}x^2 - c) / (b^2d^2)$, $\frac{1}{8} (2\sqrt{d}x^4 + c) b d x^2 + a d^2 \sqrt{-a/(bc-ad)} \log\left(\frac{(b^2c^2 - 8abc d + 8a^2d^2)x^8 - 2(3abc^2 - 4a^2cd)x^4 + a^2c^2 + 4((b^2c^2 - 3abc d + 2a^2d^2)x^6 - (abc^2 - a^2cd)x^2)\sqrt{d}x^4 + c\sqrt{-a/(bc-ad)}}{(b^2x^8 + 2abx^4 + a^2)}\right) + 2(bc + 2ad)\sqrt{-d} \arctan(\sqrt{-d}x^2/\sqrt{d}x^4 + c) / (b^2d^2)$, $\frac{1}{8} (2\sqrt{d}x^4 + c) b d x^2 - 2ad^2 \sqrt{a/(bc-ad)} \arctan(-1/2((bc - 2ad)x^4 - ac)\sqrt{d}x^4 + c)\sqrt{a/(bc-ad)} / (ad^2x^6 + acx^2) + (bc + 2ad)\sqrt{d} \log(-2dx^4 + 2\sqrt{d}x^4 + c)\sqrt{d}x^2 - c) / (b^2d^2)$, $\frac{1}{4} (s\sqrt{d}x^4 + c) b d x^2 - ad^2 \sqrt{a/(bc-ad)} \arctan(-1/2((bc - 2ad)x^4 - ac)\sqrt{d}x^4 + c)\sqrt{a/(bc-ad)} / (ad^2x^6 + acx^2) + (bc + 2ad)\sqrt{-d} \arctan(\sqrt{-d}x^2/\sqrt{d}x^4 + c) / (b^2d^2)$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x);;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 0.32, size = 408, normalized size = 3.32

$$\frac{a^2 \ln\left(\frac{2\sqrt{-ab}\left(x^2 - \frac{\sqrt{-ab}}{b}\right)^2 - \frac{2(ad-bc)}{b} + 2\sqrt{\frac{ad-bc}{b}} \sqrt{\left(x^2 - \frac{\sqrt{-ab}}{b}\right)^2 - \frac{2\sqrt{-ab}\left(x^2 - \frac{\sqrt{-ab}}{b}\right)^2 - \frac{ad-bc}{b}}}{x^2 - \frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}b^2} + \frac{a^2 \ln\left(\frac{2\sqrt{-ab}\left(x^2 + \frac{\sqrt{-ab}}{b}\right)^2 - \frac{2(ad-bc)}{b} + 2\sqrt{\frac{ad-bc}{b}} \sqrt{\left(x^2 + \frac{\sqrt{-ab}}{b}\right)^2 - \frac{2\sqrt{-ab}\left(x^2 + \frac{\sqrt{-ab}}{b}\right)^2 - \frac{ad-bc}{b}}}{x^2 + \frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}b^2} + \frac{\sqrt{d}x^4 + c}{4bd} - \frac{a \ln(\sqrt{d}x^2 + \sqrt{d}x^4 + c)}{2b^2\sqrt{d}} - \frac{c \ln(\sqrt{d}x^2 + \sqrt{d}x^4 + c)}{4bd^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x)`

[Out] $\frac{1}{4}x^2(d^2x^4+c)^{1/2}/bd-1/4b^2c/d^{3/2}\ln(d^{1/2}x^2+(d^2x^4+c)^{1/2})-1/2b^2a\ln(d^{1/2}x^2+(d^2x^4+c)^{1/2})/d^{1/2}+1/4a^2/b^2/(-ab)^{1/2}/(-ad-bc)/b^{1/2}\ln((-2(-ab)^{1/2}(x^2+(-ab)^{1/2}/b)/bd-2(ad-bc)/b+2(-ad-bc)/b)^{1/2}*((x^2+(-ab)^{1/2}/b)^2d-2(-ab)^{1/2}(x^2+(-ab)^{1/2}/b)/bd-(ad-bc)/b)^{1/2}/(x^2+(-ab)^{1/2}/b)-1/4a^2/b^2/(-ab)^{1/2}/(-ad-bc)/b^{1/2}\ln((2(-ab)^{1/2}(x^2-(-ab)^{1/2}/b)/bd-2(ad-bc)/b+2(-ad-bc)/b)^{1/2}*((x^2-(-ab)^{1/2}/b)^2d+2(-ab)^{1/2}(x^2-(-ab)^{1/2}/b)/bd-(ad-bc)/b)^{1/2}/(x^2-(-ab)^{1/2}/b)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(bx^4 + a)\sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^9/((b*x^4 + a)*sqrt(d*x^4 + c)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{(bx^4 + a)\sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/((a + b*x^4)*(c + d*x^4)^(1/2)),x)`

[Out] `int(x^9/((a + b*x^4)*(c + d*x^4)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(a + bx^4)\sqrt{c + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9/(b*x**4+a)/(d*x**4+c)**(1/2),x)`

[Out] `Integral(x**9/((a + b*x**4)*sqrt(c + d*x**4)), x)`

$$3.527 \quad \int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}} dx$$

Optimal. Leaf size=91

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}} - \frac{\sqrt{a} \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}$$

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 483, 217, 206, 377, 205}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}} - \frac{\sqrt{a} \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*b*Sqrt[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(2*b*Sqrt[d])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b

, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 483

Int[(((e_)*(x_))^(m_)*((c_) + (d_)*(x_)^(n_))^(q_))/((a_) + (b_)*(x_)^(
n_)), x_Symbol] := Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(a + bx^4)\sqrt{c + dx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(a + bx^2)\sqrt{c + dx^2}} dx, x, x^2 \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{\sqrt{c+dx^2}} dx, x, x^2 \right) - a \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{2b} \\ &= \frac{\text{Subst} \left(\int \frac{1}{1-dx^2} dx, x, \frac{x^2}{\sqrt{c+dx^4}} \right) - a \text{Subst} \left(\int \frac{1}{a-(-bc+ad)x^2} dx, x, \frac{x^2}{\sqrt{c+dx^4}} \right)}{2b} \\ &= -\frac{\sqrt{a} \tan^{-1} \left(\frac{\sqrt{bc-ad} x^2}{\sqrt{a} \sqrt{c+dx^4}} \right)}{2b\sqrt{bc-ad}} + \frac{\tanh^{-1} \left(\frac{\sqrt{d} x^2}{\sqrt{c+dx^4}} \right)}{2b\sqrt{d}} \end{aligned}$$

Mathematica [A] time = 0.07, size = 90, normalized size = 0.99

$$\frac{\log\left(\sqrt{d} \sqrt{c+dx^4} + dx^2\right) - \frac{\sqrt{a} \tan^{-1}\left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}}\right)}{\sqrt{bc-ad}}}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^4)*Sqrt[c + d*x^4]), x]

[Out] $(-\text{ArcTan}[\frac{\sqrt{a}\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}])/\sqrt{bc-ad} + \text{Log}[\frac{dx^2 + \sqrt{d}\sqrt{c+dx^4}}{d}]/(2b)$

IntegrateAlgebraic [A] time = 0.46, size = 144, normalized size = 1.58

$$\frac{\log\left(\sqrt{c+dx^4} + \sqrt{d}x^2\right)}{2b\sqrt{d}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{b\sqrt{d}x^4}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{2b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] $-1/2*\text{ArcTan}[\frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}] + (b*\sqrt{d}*x^4)/(\sqrt{a}\sqrt{bc-ad}) + (b*x^2*\sqrt{c+dx^4})/(\sqrt{a}\sqrt{bc-ad}) + \text{Log}[\frac{\sqrt{d}*x^2 + \sqrt{c+dx^4}}{2b*\sqrt{d}}]$

fricas [A] time = 0.67, size = 632, normalized size = 6.95

$$\frac{1}{2}\sqrt{\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abc*d+8a^2d^2)x^8-2*(3a^2bc^2-4a^2c*d)x^4+a^2c^2-4*((b^2c^2-3abc*d+2a^2d^2)x^6-(a*b*c^2-a^2c*d)x^2)*\sqrt{d*x^4+c}*\sqrt{-a/(bc-ad)}}{(b^2x^8+2a*b*x^4+a^2)}\right) + \frac{1}{2}\sqrt{d}\log\left(\frac{-2d*x^4-2*\sqrt{d*x^4+c}*\sqrt{d*x^2-c}}{b*d}\right) + 4*\sqrt{d}\arctan\left(\frac{\sqrt{d*x^4+c}}{\sqrt{d*x^2-c}}\right) + \frac{1}{4}\sqrt{\frac{a}{bc-ad}}\arctan\left(\frac{-1/2*((b*c-2*a*d)*x^4-a*c)*\sqrt{d*x^4+c}*\sqrt{a/(bc-ad)}}{a*d*x^6+a*c*x^2}\right) - 2*\sqrt{-d}\arctan\left(\frac{\sqrt{-d}*x^2/\sqrt{d*x^4+c}}{\sqrt{d*x^2-c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] $[1/8*(d*\sqrt{-a/(bc-ad)})*\log(((b^2c^2-8abc*d+8a^2d^2)x^8-2*(3a^2bc^2-4a^2c*d)x^4+a^2c^2-4*((b^2c^2-3abc*d+2a^2d^2)x^6-(a*b*c^2-a^2c*d)x^2)*\sqrt{d*x^4+c}*\sqrt{-a/(bc-ad)}))/(b^2x^8+2a*b*x^4+a^2)) + 2*\sqrt{d}*\log(-2*d*x^4-2*\sqrt{d*x^4+c}*\sqrt{d*x^2-c})/(b*d), 1/8*(d*\sqrt{-a/(bc-ad)})*\log(((b^2c^2-8abc*d+8a^2d^2)x^8-2*(3a^2bc^2-4a^2c*d)x^4+a^2c^2-4*((b^2c^2-3abc*d+2a^2d^2)x^6-(a*b*c^2-a^2c*d)x^2)*\sqrt{d*x^4+c}*\sqrt{-a/(bc-ad)}))/(b^2x^8+2a*b*x^4+a^2)) - 4*\sqrt{d}*\arctan(\sqrt{-d}*x^2/\sqrt{d*x^4+c})/(b*d), 1/4*(d*\sqrt{a/(bc-ad)})*\arctan(-1/2*((b*c-2*a*d)*x^4-a*c)*\sqrt{d*x^4+c}*\sqrt{a/(bc-ad)})/(a*d*x^6+a*c*x^2)) + \sqrt{d}*\log(-2*d*x^4-2*\sqrt{d*x^4+c}*\sqrt{d*x^2-c})/(b*d), 1/4*(d*\sqrt{a/(bc-ad)})*\arctan(-1/2*((b*c-2*a*d)*x^4-a*c)*\sqrt{d*x^4+c}*\sqrt{a/(bc-ad)})/(a*d*x^6+a*c*x^2)) - 2*\sqrt{-d}*\arctan(\sqrt{-d}*x^2/\sqrt{d*x^4+c})/(b*d)]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x);OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value

maple [B] time = 0.33, size = 356, normalized size = 3.91

$$a \ln \left(\frac{\frac{2\sqrt{-ab} \left(x^2 + \frac{\sqrt{-ab}}{b} \right) d}{b} + \frac{2(ad-bc)}{b} + 2\sqrt{\frac{-ad-bc}{b}} \sqrt{\left(x^2 + \frac{\sqrt{-ab}}{b} \right)^2 d + \frac{-ad-bc}{b}}}{x^2 + \frac{\sqrt{-ab}}{b}} \right) - a \ln \left(\frac{-\frac{2\sqrt{-ab} \left(x^2 + \frac{\sqrt{-ab}}{b} \right) d}{b} + \frac{2(ad-bc)}{b} + 2\sqrt{\frac{-ad-bc}{b}} \sqrt{\left(x^2 + \frac{\sqrt{-ab}}{b} \right)^2 d + \frac{-ad-bc}{b}}}{x^2 + \frac{\sqrt{-ab}}{b}} \right) + \frac{\ln \left(\sqrt{d} x^2 + \sqrt{d} x^4 + c \right)}{2b\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x)

[Out] $\frac{1}{2} \frac{b \ln(d^{1/2} x^2 + (d x^4 + c)^{1/2})}{d^{1/2}} - \frac{1}{4} \frac{a}{b} \frac{1}{(-a b)^{1/2}} \frac{1}{(-(a d - b c)/b)^{1/2}} \ln\left(\frac{-2(-a b)^{1/2} (x^2 + (-a b)^{1/2}/b)/b * d - 2(a d - b c)/b + 2(-a d - b c)/b}{(x^2 + (-a b)^{1/2}/b)^2 * d - 2(-a b)^{1/2} (x^2 + (-a b)^{1/2}/b)/b * d - (a d - b c)/b}\right) + \frac{1}{4} \frac{a}{b} \frac{1}{(-a b)^{1/2}} \frac{1}{(-(a d - b c)/b)^{1/2}} \ln\left(\frac{2(-a b)^{1/2} (x^2 - (-a b)^{1/2}/b)/b * d - 2(a d - b c)/b + 2(-a d - b c)/b}{(x^2 - (-a b)^{1/2}/b)^2 * d + 2(-a b)^{1/2} (x^2 - (-a b)^{1/2}/b)/b * d - (a d - b c)/b}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(b x^4 + a) \sqrt{d x^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^5/((b*x^4 + a)*sqrt(d*x^4 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{(b x^4 + a) \sqrt{d x^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((a + b*x^4)*(c + d*x^4)^(1/2)),x)

[Out] int(x^5/((a + b*x^4)*(c + d*x^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + b x^4) \sqrt{c + d x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

$$3.528 \quad \int \frac{x}{(a+bx^4)\sqrt{c+dx^4}} dx$$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {465, 377, 205}

$$\frac{\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])]/(2*Sqrt[a]*Sqrt[b*c - a*d])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a + bx^4)\sqrt{c + dx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(a + bx^2)\sqrt{c + dx^2}} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{a - (-bc + ad)x^2} dx, x, \frac{x^2}{\sqrt{c + dx^4}} \right) \\
&= \frac{\tan^{-1} \left(\frac{\sqrt{bc-ad} x^2}{\sqrt{a} \sqrt{c+dx^4}} \right)}{2\sqrt{a} \sqrt{bc - ad}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 95, normalized size = 1.76

$$\frac{x^2 \sqrt{\frac{dx^4}{c} + 1} \tanh^{-1} \left(\frac{\sqrt{\frac{dx^4}{c} - \frac{bx^4}{a}}}{\sqrt{\frac{dx^4}{c} + 1}} \right)}{2a\sqrt{c + dx^4} \sqrt{\frac{dx^4}{c} - \frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

[In] Integrate[x/((a + b*x^4)*Sqrt[c + d*x^4]), x]

[Out] (x^2*Sqrt[1 + (d*x^4)/c]*ArcTanh[Sqrt[-((b*x^4)/a) + (d*x^4)/c]/Sqrt[1 + (d*x^4)/c]])/(2*a*Sqrt[c + d*x^4]*Sqrt[-((b*x^4)/a) + (d*x^4)/c])

IntegrateAlgebraic [A] time = 0.32, size = 106, normalized size = 1.96

$$\frac{\tan^{-1} \left(\frac{b\sqrt{d}x^4}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}} \right)}{2\sqrt{a}\sqrt{bc - ad}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/((a + b*x^4)*Sqrt[c + d*x^4]), x]

[Out] ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^2*Sqrt[c + d*x^4))/(Sqrt[a]*Sqrt[b*c - a*d])]/(2*Sqrt[a]*Sqrt[b*c - a*d])

fricas [B] time = 0.51, size = 245, normalized size = 4.54

$$\left[\frac{\sqrt{-abc + a^2d} \log \left(\frac{(b^2c^2 - 8abcd + 8a^2d^2)x^8 - 2(3abc^2 - 4a^2cd)x^4 + a^2c^2 - 4((bc - 2ad)x^6 - acx^2)\sqrt{dx^4 + c}\sqrt{-abc + a^2d}}{b^2x^8 + 2abx^4 + a^2} \right)}{8(abc - a^2d)}, \frac{\arctan \left(\frac{((bc - 2ad)x^4 - ac)\sqrt{dx^4 + c}\sqrt{-abc - a^2d}}{2((abcd - a^2d^2)x^6 + (abc^2 - a^2cd)x^2)} \right)}{4\sqrt{abc - a^2d}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] $[-1/8\sqrt{-a*b*c + a^2*d}*\log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*\sqrt{(d*x^4 + c)*\sqrt{-a*b*c + a^2*d}})/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a*b*c - a^2*d), 1/4*\arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*\sqrt{(d*x^4 + c)*\sqrt{a*b*c - a^2*d}})/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2))/\sqrt{a*b*c - a^2*d}]$

giac [A] time = 0.18, size = 72, normalized size = 1.33

$$\frac{\sqrt{d} \arctan\left(\frac{(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 b - bc + 2ad}{2\sqrt{abcd - a^2d^2}}\right)}{2\sqrt{abcd - a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] $-1/2*\sqrt{d}*\arctan(1/2*((\sqrt{d}*x^2 - \sqrt{d*x^4 + c}))^2*b - b*c + 2*a*d)/\sqrt{a*b*c*d - a^2*d^2})/\sqrt{a*b*c*d - a^2*d^2}$

maple [B] time = 0.32, size = 322, normalized size = 5.96

$$\frac{\ln\left(\frac{2\sqrt{-ab}\left(x^2 - \frac{\sqrt{-ab}}{b}\right)d - \frac{2(ad-bc)}{b} + 2\sqrt{\frac{-ad-bc}{b}}\sqrt{\left(x^2 - \frac{\sqrt{-ab}}{b}\right)^2 d + \frac{2\sqrt{-ab}\left(x^2 - \frac{\sqrt{-ab}}{b}\right)d - \frac{ad-bc}{b}}}{x^2 - \frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{\frac{-ad-bc}{b}}}\right) + \ln\left(\frac{-2\sqrt{-ab}\left(x^2 + \frac{\sqrt{-ab}}{b}\right)d - \frac{2(ad-bc)}{b} + 2\sqrt{\frac{-ad-bc}{b}}\sqrt{\left(x^2 + \frac{\sqrt{-ab}}{b}\right)^2 d - \frac{2\sqrt{-ab}\left(x^2 + \frac{\sqrt{-ab}}{b}\right)d - \frac{ad-bc}{b}}}{x^2 + \frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{\frac{-ad-bc}{b}}}\right)}{4\sqrt{-ab}\sqrt{\frac{-ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^4+a)/(d*x^4+c)^(1/2),x)

[Out] $1/4/(-a*b)^(1/2)/(-a*d-b*c)/b)^(1/2)*\ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b)-1/4/(-a*b)^(1/2)/(-a*d-b*c)/b)^(1/2)*\ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^4 + a)\sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x/((b*x^4 + a)*sqrt(d*x^4 + c)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{(bx^4 + a)\sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((a + b*x^4)*(c + d*x^4)^(1/2)),x)`

[Out] `int(x/((a + b*x^4)*(c + d*x^4)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + bx^4)\sqrt{c + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**4+a)/(d*x**4+c)**(1/2),x)`

[Out] `Integral(x/((a + b*x**4)*sqrt(c + d*x**4)), x)`

$$3.529 \quad \int \frac{1}{x^3(a+bx^4)\sqrt{c+dx^4}} dx$$

Optimal. Leaf size=80

$$-\frac{b \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^4}}{2acx^2}$$

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {465, 480, 12, 377, 205}

$$-\frac{b \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^4}}{2acx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] -Sqrt[c + d*x^4]/(2*a*c*x^2) - (b*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^(3/2)*Sqrt[b*c - a*d])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ

[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 480

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 (a + bx^4) \sqrt{c + dx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2 (a + bx^2) \sqrt{c + dx^2}} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{c + dx^4}}{2acx^2} - \frac{\text{Subst} \left(\int \frac{bc}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{2ac} \\
 &= -\frac{\sqrt{c + dx^4}}{2acx^2} - \frac{b \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{2a} \\
 &= -\frac{\sqrt{c + dx^4}}{2acx^2} - \frac{b \text{Subst} \left(\int \frac{1}{a-(-bc+ad)x^2} dx, x, \frac{x^2}{\sqrt{c+dx^4}} \right)}{2a} \\
 &= -\frac{\sqrt{c + dx^4}}{2acx^2} - \frac{b \tan^{-1} \left(\frac{\sqrt{bc-ad} x^2}{\sqrt{a} \sqrt{c+dx^4}} \right)}{2a^{3/2} \sqrt{bc-ad}}
 \end{aligned}$$

Mathematica [C] time = 4.83, size = 179, normalized size = 2.24

$$\frac{\left(\frac{dx^4}{c} + 1 \right) \left(\frac{4x^4(c+dx^4)(bc-ad) {}_2F_1 \left(2, 2; \frac{5}{2}; \frac{(bc-ad)x^4}{c(bx^4+a)} \right)}{3c^2(a+bx^4)} + \frac{(c+2dx^4) \sin^{-1} \left(\sqrt{\frac{x^4(bc-ad)}{c(a+bx^4)}} \right)}{c \sqrt{\frac{ax^4(c+dx^4)(bc-ad)}{c^2(a+bx^4)^2}}} \right)}{2x^2 (a + bx^4) \sqrt{c + dx^4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] $-\frac{1}{2} \left(\frac{(1 + (d*x^4)/c) * ((c + 2*d*x^4) * \text{ArcSin}[\text{Sqrt}[(b*c - a*d)*x^4]/(c*(a + b*x^4))])}{(c*\text{Sqrt}[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a + b*x^4)^2])} + \frac{4*(b*c - a*d)*x^4*(c + d*x^4)*\text{Hypergeometric2F1}[2, 2, 5/2, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]}{(3*c^2*(a + b*x^4))} \right) / (x^2*(a + b*x^4)*\text{Sqrt}[c + d*x^4])$

IntegrateAlgebraic [A] time = 0.48, size = 142, normalized size = 1.78

$$\frac{b\sqrt{bc-ad} \tan^{-1} \left(\frac{b\sqrt{d}x^4}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}} \right)}{2a^{3/2}(ad-bc)} - \frac{\sqrt{c+dx^4}}{2acx^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] $-\frac{1}{2} \text{Sqrt}[c + d*x^4]/(a*c*x^2) + \frac{(b*\text{Sqrt}[b*c - a*d]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b*c - a*d]) + (b*\text{Sqrt}[d]*x^4)/(\text{Sqrt}[a]*\text{Sqrt}[b*c - a*d]) + (b*x^2*\text{Sqrt}[c + d*x^4])/(\text{Sqrt}[a]*\text{Sqrt}[b*c - a*d])])}{(2*a^{(3/2)}*(-(b*c) + a*d))}$

fricas [B] time = 0.53, size = 332, normalized size = 4.15

$$\left[\frac{\sqrt{-abc + a^2d} b c x^2 \log \left(\frac{(b^2c^2 - 8abcd + 8a^2d^2)x^8 - 2(3abc^2 - 4a^2cd)x^4 + a^2c^2 + 4((bc - 2ad)x^5 - acx^2)\sqrt{dx^4 + c}\sqrt{-abc + a^2d}}{b^2c^2 + 2abx^4 + a^2} \right) + 4\sqrt{dx^4 + c}(abc - a^2d)}{8(a^2bc^2 - a^3cd)x^2}, - \frac{\sqrt{abc - a^2d} b c x^2 \arctan \left(\frac{(bc - 2ad)x^4 - ac}{2((abcd - a^2d^2)x^6 + (abc^2 - a^2cd)x^2)} \right) + 2\sqrt{dx^4 + c}(abc - a^2d)}{4(a^2bc^2 - a^3cd)x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] $[-\frac{1}{8}*(\text{sqrt}(-a*b*c + a^2*d)*b*c*x^2*\log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*\text{sqrt}(d*x^4 + c)*\text{sqrt}(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*\text{sqrt}(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2), -\frac{1}{4}*(\text{sqrt}(a*b*c - a^2*d)*b*c*x^2*\arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*\text{sqrt}(d*x^4 + c)*\text{sqrt}(a*b*c - a^2*d))/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) + 2*\text{sqrt}(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2)]$

giac [A] time = 0.22, size = 116, normalized size = 1.45

$$\frac{1}{2} d^{\frac{3}{2}} \left(\frac{b \arctan \left(\frac{(\sqrt{d}x^2 - \sqrt{dx^4 + c})^2 b - bc + 2ad}{2\sqrt{abcd - a^2d^2}} \right)}{\sqrt{abcd - a^2d^2} ad} + \frac{2}{\left((\sqrt{d}x^2 - \sqrt{dx^4 + c})^2 - c \right) ad} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{2}d^{3/2}(b \arctan(1/2((\sqrt{d}x^2 - \sqrt{d^4+c})^{2b} - bc + 2ad)/\sqrt{a^2d^2 - a^2d^2}))/(\sqrt{a^2d^2 - a^2d^2}a^2d) + 2/(((\sqrt{d}x^2 - \sqrt{d^4+c})^{2b} - c)a^2d)$

maple [B] time = 0.30, size = 350, normalized size = 4.38

$$b \ln \left(\frac{\frac{2\sqrt{-ab} \left(x^2 - \frac{\sqrt{-ab}}{b} \right) d}{b} - \frac{2(ad-bc)}{b} + 2\sqrt{\frac{ad-bc}{b}} \sqrt{\left(x^2 - \frac{\sqrt{-ab}}{b} \right)^2 d + \frac{2\sqrt{-ab} \left(x^2 - \frac{\sqrt{-ab}}{b} \right) d}{b} - \frac{ad-bc}{b}}}{x^2 - \frac{\sqrt{-ab}}{b}} \right) - b \ln \left(\frac{-\frac{2\sqrt{-ab} \left(x^2 + \frac{\sqrt{-ab}}{b} \right) d}{b} - \frac{2(ad-bc)}{b} + 2\sqrt{\frac{ad-bc}{b}} \sqrt{\left(x^2 + \frac{\sqrt{-ab}}{b} \right)^2 d - \frac{2\sqrt{-ab} \left(x^2 + \frac{\sqrt{-ab}}{b} \right) d}{b} - \frac{ad-bc}{b}}}{x^2 + \frac{\sqrt{-ab}}{b}} \right) - \frac{\sqrt{d^4+c}}{2acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x)

[Out] $-1/2(d^4+c)^{1/2}/a/c/x^2 - 1/4/a*b/(-a*b)^{1/2}/(-a*d-b*c)/b)^{1/2} * \ln((-2*(-a*b)^{1/2}*(x^2+(-a*b)^{1/2}/b)/b*d - 2*(a*d-b*c)/b + 2*(-(a*d-b*c)/b)^{1/2} * ((x^2+(-a*b)^{1/2}/b)^2*d - 2*(-a*b)^{1/2}*(x^2+(-a*b)^{1/2}/b)/b*d - (a*d-b*c)/b)^{1/2} / (x^2+(-a*b)^{1/2}/b)) + 1/4/a*b/(-a*b)^{1/2}/(-a*d-b*c)/b)^{1/2} * \ln((2*(-a*b)^{1/2}*(x^2-(-a*b)^{1/2}/b)/b*d - 2*(a*d-b*c)/b + 2*(-(a*d-b*c)/b)^{1/2} * ((x^2-(-a*b)^{1/2}/b)^2*d + 2*(-a*b)^{1/2}*(x^2-(-a*b)^{1/2}/b)/b*d - (a*d-b*c)/b)^{1/2} / (x^2-(-a*b)^{1/2}/b))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)\sqrt{dx^4 + c}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 (bx^4 + a) \sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b*x^4)*(c + d*x^4)^(1/2)),x)

[Out] `int(1/(x^3*(a + b*x^4)*(c + d*x^4)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + bx^4) \sqrt{c + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(b*x**4+a)/(d*x**4+c)**(1/2), x)`

[Out] `Integral(1/(x**3*(a + b*x**4)*sqrt(c + d*x**4)), x)`

$$3.530 \quad \int \frac{1}{x^7(a+bx^4)\sqrt{c+dx^4}} dx$$

Optimal. Leaf size=115

$$\frac{b^2 \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^4}(2ad+3bc)}{6a^2c^2x^2} - \frac{\sqrt{c+dx^4}}{6acx^6}$$

Rubi [A] time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 480, 583, 12, 377, 205}

$$\frac{b^2 \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^4}(2ad+3bc)}{6a^2c^2x^2} - \frac{\sqrt{c+dx^4}}{6acx^6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] -Sqrt[c + d*x^4]/(6*a*c*x^6) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^4])/(6*a^2*c^2*x^2) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^(5/2)*Sqrt[b*c - a*d])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -

$1) * (a + b * x^{(n/k)})^p * (c + d * x^{(n/k)})^q, x], x, x^k], x] /; k \neq 1 /; \text{FreeQ} \\ [a, b, c, d, p, q], x] \&\& \text{NeQ}[b * c - a * d, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Rule 480

$\text{Int}[(e \cdot x)^m * (a + b \cdot x^n)^p * (c + d \cdot x^n)^q, x_Symbol] :> \text{Simp}[(e * x)^{m+1} * (a + b * x^n)^{p+1} * (c + d * x^n)^{q+1} / (a * c * e^{m+1}), x] - \text{Dist}[1 / (a * c * e^{m+1}), \text{Int}[(e * x)^{m+n} * (a + b * x^n)^p * (c + d * x^n)^q * \text{Simp}[(b * c + a * d) * (m + n + 1) + n * (b * c * p + a * d * q) + b * d * (m + n * (p + q + 2) + 1) * x^n, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, p, q\}, x] \&\& \text{NeQ}[b * c - a * d, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[m, -1] \&\& \text{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 583

$\text{Int}[(g \cdot x)^m * (a + b \cdot x^n)^p * (c + d \cdot x^n)^q * (e + f \cdot x^n)^r, x_Symbol] :> \text{Simp}[e * (g * x)^{m+1} * (a + b * x^n)^{p+1} * (c + d * x^n)^{q+1} / (a * c * g^{m+1}), x] + \text{Dist}[1 / (a * c * g^{m+1}), \text{Int}[(g * x)^{m+n} * (a + b * x^n)^p * (c + d * x^n)^q * \text{Simp}[a * f * c * (m + 1) - e * (b * c + a * d) * (m + n + 1) - e * n * (b * c * p + a * d * q) - b * e * d * (m + n * (p + q + 2) + 1) * x^n, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p, q\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7 (a + bx^4) \sqrt{c + dx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^4 (a + bx^2) \sqrt{c + dx^2}} dx, x, x^2 \right) \\
&= -\frac{\sqrt{c + dx^4}}{6acx^6} + \frac{\text{Subst} \left(\int \frac{-3bc - 2ad - 2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{6ac} \\
&= -\frac{\sqrt{c + dx^4}}{6acx^6} + \frac{(3bc + 2ad)\sqrt{c + dx^4}}{6a^2c^2x^2} - \frac{\text{Subst} \left(\int -\frac{3b^2c^2}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{6a^2c^2} \\
&= -\frac{\sqrt{c + dx^4}}{6acx^6} + \frac{(3bc + 2ad)\sqrt{c + dx^4}}{6a^2c^2x^2} + \frac{b^2 \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{2a^2} \\
&= -\frac{\sqrt{c + dx^4}}{6acx^6} + \frac{(3bc + 2ad)\sqrt{c + dx^4}}{6a^2c^2x^2} + \frac{b^2 \text{Subst} \left(\int \frac{1}{a - (-bc + ad)x^2} dx, x, \frac{x^2}{\sqrt{c+dx^4}} \right)}{2a^2} \\
&= -\frac{\sqrt{c + dx^4}}{6acx^6} + \frac{(3bc + 2ad)\sqrt{c + dx^4}}{6a^2c^2x^2} + \frac{b^2 \tan^{-1} \left(\frac{\sqrt{bc - ad} x^2}{\sqrt{a} \sqrt{c + dx^4}} \right)}{2a^{5/2} \sqrt{bc - ad}}
\end{aligned}$$

Mathematica [A] time = 5.55, size = 137, normalized size = 1.19

$$\frac{b^2 x^2 \sqrt{\frac{dx^4}{c} + 1} \sin^{-1} \left(\frac{\sqrt{x^4 \left(\frac{b}{a} - \frac{d}{c} \right)}}{\sqrt{\frac{bx^4}{a} + 1}} \right) + \frac{(c + dx^4)(-ac + 2adx^4 + 3bcx^4)}{3a^2c^2x^6}}{a^3 \sqrt{\frac{x^4(bc - ad)}{ac}}} \cdot \frac{1}{2\sqrt{c + dx^4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] (((c + d*x^4)*(-a*c) + 3*b*c*x^4 + 2*a*d*x^4))/(3*a^2*c^2*x^6) + (b^2*x^2*Sqrt[1 + (d*x^4)/c]*ArcSin[Sqrt[(b/a - d/c)*x^4]/Sqrt[1 + (b*x^4)/a]])/(a^3*Sqrt[((b*c - a*d)*x^4)/(a*c]))/(2*Sqrt[c + d*x^4])

IntegrateAlgebraic [A] time = 0.89, size = 163, normalized size = 1.42

$$\frac{\sqrt{c + dx^4} (-ac + 2adx^4 + 3bcx^4)}{6a^2c^2x^6} - \frac{b^2 \sqrt{bc - ad} \tan^{-1} \left(\frac{b\sqrt{d}x^4}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}} \right)}{2a^{5/2}(ad - bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] (Sqrt[c + d*x^4]*(-(a*c) + 3*b*c*x^4 + 2*a*d*x^4))/(6*a^2*c^2*x^6) - (b^2*Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(5/2)*(-(b*c) + a*d))

fricas [A] time = 0.54, size = 418, normalized size = 3.63

$$\frac{3\sqrt{-abc + a^2d}b^2c^2x^6 \log\left(\frac{(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 - 2(3ab^2c^2 - 4a^2cd)x^4 + (bc - 2ad)^2 - 4((bc - 2ad)^2 - a^2cd)\sqrt{dx^4+c}}{24(a^3bc^3 - a^4c^2d)x^6}\right) + 4(a^2bc^2 - a^3cd - (3ab^2c^2 - a^2bcd - 2a^3d^2)x^4)\sqrt{dx^4+c}}{24(a^3bc^3 - a^4c^2d)x^6} + \frac{3\sqrt{abc - a^2d}b^2c^2x^6 \arctan\left(\frac{(bc - 2ad)x^4 - a\sqrt{dx^4+c}\sqrt{abc - a^2d}}{2((abcd - a^2d^2)x^4 + (a^2bc^2 - a^3cd - (3ab^2c^2 - a^2bcd - 2a^3d^2)x^4)\sqrt{dx^4+c})}\right)}{12(a^3bc^3 - a^4c^2d)x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [-1/24*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^6*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*(a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^4)*sqrt(d*x^4 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^6), 1/12*(3*sqrt(a*b*c - a^2*d)*b^2*c^2*x^6*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d))/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) - 2*(a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^4)*sqrt(d*x^4 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^6)]

giac [B] time = 1.68, size = 205, normalized size = 1.78

$$-\frac{1}{6}d^{\frac{5}{2}}\left(\frac{3b^2\arctan\left(\frac{(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 - b - bc + 2ad}{2\sqrt{abcd - a^2d^2}}\right)}{\sqrt{abcd - a^2d^2}a^2d^2} + \frac{2\left(3\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^4 b - 6\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 bc - 6\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 ad + 3bc^2 + 2acd\right)}{\left(\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 - c\right)^3 a^2d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] -1/6*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2 - c)^3*a^2*d^2))

maple [B] time = 0.27, size = 383, normalized size = 3.33

$$\frac{b^2 \ln \left(\frac{\frac{2\sqrt{-ab} \left(x^2 - \frac{\sqrt{-ab}}{b} \right) d}{b} - \frac{2(ad-bc)}{b} + 2\sqrt{\frac{ad-bc}{b}} \sqrt{\left(x^2 - \frac{\sqrt{-ab}}{b} \right)^2 d + \frac{2\sqrt{-ab} \left(x^2 - \frac{\sqrt{-ab}}{b} \right) d}{b} - \frac{ad-bc}{b}}}{x^2 - \frac{\sqrt{-ab}}{b}} \right)}{4\sqrt{-ab} \sqrt{\frac{ad-bc}{b}} a^2} + \frac{b^2 \ln \left(\frac{-\frac{2\sqrt{-ab} \left(x^2 + \frac{\sqrt{-ab}}{b} \right) d}{b} - \frac{2(ad-bc)}{b} + 2\sqrt{\frac{ad-bc}{b}} \sqrt{\left(x^2 + \frac{\sqrt{-ab}}{b} \right)^2 d - \frac{2\sqrt{-ab} \left(x^2 + \frac{\sqrt{-ab}}{b} \right) d}{b} - \frac{ad-bc}{b}}}{x^2 + \frac{\sqrt{-ab}}{b}} \right)}{4\sqrt{-ab} \sqrt{\frac{ad-bc}{b}} a^2} + \frac{\sqrt{dx^4+c} b}{2a^2 c x^2} - \frac{\sqrt{dx^4+c} (-2dx^4+c)}{6a c^2 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2), x)`

[Out] $\frac{1}{2} \frac{1}{a^2 b} \frac{1}{x^2} (d x^4 + c)^{1/2} / c + \frac{1}{4} \frac{1}{a^2 b^2} \frac{1}{(-a b)^{1/2}} \frac{1}{(-a d - b c / b)^{1/2}} \frac{1}{2} * \ln \left(\frac{-2 * (-a b)^{1/2} * (x^2 + (-a b)^{1/2} / b) / b * d - 2 * (a d - b c) / b + 2 * (-a d - b c) / b}{(x^2 + (-a b)^{1/2} / b)^2 * d - 2 * (-a b)^{1/2} * (x^2 + (-a b)^{1/2} / b) / b * d - (a d - b c) / b} \right) / (x^2 + (-a b)^{1/2} / b) - \frac{1}{4} \frac{1}{a^2 b^2} \frac{1}{(-a b)^{1/2}} \frac{1}{(-a d - b c / b)^{1/2}} * \ln \left(\frac{2 * (-a b)^{1/2} * (x^2 - (-a b)^{1/2} / b) / b * d - 2 * (a d - b c) / b + 2 * (-a d - b c) / b}{(x^2 - (-a b)^{1/2} / b)^2 * d + 2 * (-a b)^{1/2} * (x^2 - (-a b)^{1/2} / b) / b * d - (a d - b c) / b} \right) / (x^2 - (-a b)^{1/2} / b) - \frac{1}{6} \frac{1}{a} (d x^4 + c)^{1/2} * (-2 d x^4 + c) / x^6 / c^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b x^4 + a) \sqrt{d x^4 + c} x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^7), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7 (b x^4 + a) \sqrt{d x^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^7*(a + b*x^4)*(c + d*x^4)^(1/2)), x)`

[Out] `int(1/(x^7*(a + b*x^4)*(c + d*x^4)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 (a + b x^4) \sqrt{c + d x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)/(d*x**4+c)**(1/2), x)
```

```
[Out] Integral(1/(x**7*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

$$3.531 \quad \int \frac{x^{15}}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$$

Optimal. Leaf size=175

$$\frac{a^2(6bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right) \sqrt{c+dx^4} (-15a^2d^2 - bdx^4(2bc - 5ad) + 8abcd + 4b^2c^2)}{4b^{7/2}(bc - ad)^{3/2}} - \frac{\sqrt{c+dx^4} (-15a^2d^2 - bdx^4(2bc - 5ad) + 8abcd + 4b^2c^2)}{12b^3d^2(bc - ad)} + \frac{ax^8\sqrt{c+dx^4}}{4b(a+bx^4)(bc - ad)}$$

Rubi [A] time = 0.17, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 98, 147, 63, 208}

$$\frac{\sqrt{c+dx^4} (-15a^2d^2 - bdx^4(2bc - 5ad) + 8abcd + 4b^2c^2)}{12b^3d^2(bc - ad)} - \frac{a^2(6bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc - ad)^{3/2}} + \frac{ax^8\sqrt{c+dx^4}}{4b(a+bx^4)(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]), x]

[Out] (a*x^8*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (Sqrt[c + d*x^4]*(4*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2 - b*d*(2*b*c - 5*a*d)*x^4))/(12*b^3*d^2*(b*c - a*d)) - (a^2*(6*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*b^(7/2)*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 98

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_Symbol] := -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{15}}{(a+bx^4)^2 \sqrt{c+dx^4}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^3}{(a+bx)^2 \sqrt{c+dx}} dx, x, x^4 \right) \\
&= \frac{ax^8 \sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\text{Subst} \left(\int \frac{x(2ac + \frac{1}{2}(-2bc+5ad)x)}{(a+bx)\sqrt{c+dx}} dx, x, x^4 \right)}{4b(bc-ad)} \\
&= \frac{ax^8 \sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\sqrt{c+dx^4} (4b^2c^2 + 8abcd - 15a^2d^2 - bd(2bc-5ad)x^4)}{12b^3d^2(bc-ad)} + \\
&= \frac{ax^8 \sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\sqrt{c+dx^4} (4b^2c^2 + 8abcd - 15a^2d^2 - bd(2bc-5ad)x^4)}{12b^3d^2(bc-ad)} + \\
&= \frac{ax^8 \sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\sqrt{c+dx^4} (4b^2c^2 + 8abcd - 15a^2d^2 - bd(2bc-5ad)x^4)}{12b^3d^2(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 175, normalized size = 1.00

$$\frac{a^2(5ad-6bc) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{4b^{7/2}(bc-ad)^{3/2}} - \frac{\sqrt{c+dx^4} (-15a^3d^2 + 2a^2bd(4c-5dx^4) + 2ab^2(2c^2+3cdx^4+d^2x^8) + 2b^3cx^4(2c-dx^4))}{12b^3d^2(a+bx^4)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] -1/12*(Sqrt[c + d*x^4]*(-15*a^3*d^2 + 2*a^2*b*d*(4*c - 5*d*x^4) + 2*b^3*c*x^4*(2*c - d*x^4) + 2*a*b^2*(2*c^2 + 3*c*d*x^4 + d^2*x^8)))/(b^3*d^2*(b*c - a*d)*(a + b*x^4)) + (a^2*(-6*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*b^(7/2)*(b*c - a*d)^(3/2))

IntegrateAlgebraic [A] time = 0.49, size = 198, normalized size = 1.13

$$\frac{(6a^2bc - 5a^3d) \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4} \sqrt{ad-bc}}{bc-ad} \right)}{4b^{7/2}(ad-bc)^{3/2}} - \frac{\sqrt{c+dx^4} (-15a^3d^2 + 8a^2bcd - 10a^2bd^2x^4 + 4ab^2c^2 + 6ab^2cdx^4 + 2ab^2d^2x^8 + 4b^3c^2x^4 - 2b^3cdx^8)}{12b^3d^2(a+bx^4)(bc-ad)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out]
$$-1/12*(\text{Sqrt}[c + d*x^4]*(4*a*b^2*c^2 + 8*a^2*b*c*d - 15*a^3*d^2 + 4*b^3*c^2*x^4 + 6*a*b^2*c*d*x^4 - 10*a^2*b*d^2*x^4 - 2*b^3*c*d*x^8 + 2*a*b^2*d^2*x^8))/(b^3*d^2*(b*c - a*d)*(a + b*x^4)) + ((6*a^2*b*c - 5*a^3*d)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[-(b*c) + a*d]*\text{Sqrt}[c + d*x^4])/(b*c - a*d)]/(4*b^{7/2}*(-(b*c) + a*d)^{3/2}))$$

fricas [A] time = 0.55, size = 622, normalized size = 3.55

$$\frac{(6a^2bc - 5a^3d)\sqrt{c+dx^4}\sqrt{b}\arctan\left(\frac{\sqrt{c+dx^4}\sqrt{b}}{b^2c - a^2d}\right) + (6a^2b^2c^2 + 8a^2b^2cd - 15a^3d^2 + 4b^3c^2x^4 + 6a^2b^2cdx^4 - 10a^2b^2d^2x^4 - 2b^3cdx^8 + 2a^2b^2d^2x^8)}{12b^{7/2}(b^2c - a^2d)\sqrt{c+dx^4}\sqrt{b} + 4b^{7/2}(b^2c - a^2d)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁵/(b*x⁴+a)²/(d*x⁴+c)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/24*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (6*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*x^4)* \\ & \text{sqrt}(b^2*c - a*b*d)*\log((b*d*x^4 + 2*b*c - a*d - 2*\text{sqrt}(d*x^4 + c)*\text{sqrt}(b^2 \\ & *c - a*b*d))/(b*x^4 + a)) + 2*(2*(b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b^3*d^3)* \\ & x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 - 15*a^4*b*d^3 - 2*(\\ & 2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*x^4)*\text{sqrt}(d*x^4 \\ & + c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2 - 2*a*b \\ & ^6*c*d^3 + a^2*b^5*d^4)*x^4), 1/12*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (6*a^2*b \\ & ^2*c*d^2 - 5*a^3*b*d^3)*x^4)*\text{sqrt}(-b^2*c + a*b*d)*\arctan(\text{sqrt}(d*x^4 + c)*\text{sq} \\ & \text{rt}(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + (2*(b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b \\ & ^3*d^3)*x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 - 15*a^4*b*d \\ & ^3 - 2*(2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*x^4)*\text{sq} \\ & \text{rt}(d*x^4 + c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2 \\ & - 2*a*b^6*c*d^3 + a^2*b^5*d^4)*x^4)] \end{aligned}$$

giac [A] time = 0.17, size = 180, normalized size = 1.03

$$\frac{\sqrt{dx^4 + c} a^3 d}{4(b^4c - ab^3d)((dx^4 + c)b - bc + ad)} + \frac{(6a^2bc - 5a^3d)\arctan\left(\frac{\sqrt{dx^4 + c} b}{\sqrt{-b^2c + abd}}\right)}{4(b^4c - ab^3d)\sqrt{-b^2c + abd}} + \frac{(dx^4 + c)^3 b^4 d^4 - 3\sqrt{dx^4 + c} b^4 c d^4 - 6\sqrt{dx^4 + c} ab^3 d^5}{6b^6 d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁵/(b*x⁴+a)²/(d*x⁴+c)^(1/2),x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/4*\text{sqrt}(d*x^4 + c)*a^3*d/((b^4*c - a*b^3*d)*((d*x^4 + c)*b - b*c + a*d)) + \\ & 1/4*(6*a^2*b*c - 5*a^3*d)*\arctan(\text{sqrt}(d*x^4 + c)*b/\text{sqrt}(-b^2*c + a*b*d))/ \\ & ((b^4*c - a*b^3*d)*\text{sqrt}(-b^2*c + a*b*d)) + 1/6*((d*x^4 + c)^{3/2}*b^4*d^4 - \\ & 3*\text{sqrt}(d*x^4 + c)*b^4*c*d^4 - 6*\text{sqrt}(d*x^4 + c)*a*b^3*d^5)/(b^6*d^6) \end{aligned}$$

maple [B] time = 0.25, size = 923, normalized size = 5.27

$$\frac{(6a^2bc - 5a^3d)\sqrt{c+dx^4}\sqrt{b}\arctan\left(\frac{\sqrt{c+dx^4}\sqrt{b}}{b^2c - a^2d}\right) + (6a^2b^2c^2 + 8a^2b^2cd - 15a^3d^2 + 4b^3c^2x^4 + 6a^2b^2cdx^4 - 10a^2b^2d^2x^4 - 2b^3cdx^8 + 2a^2b^2d^2x^8)}{12b^{7/2}(b^2c - a^2d)\sqrt{c+dx^4}\sqrt{b} + 4b^{7/2}(b^2c - a^2d)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{15}/(b*x^4+a)^2/(d*x^4+c)^{(1/2)}, x)$

[Out] $\frac{1}{6}b^{-2}(d*x^4+c)^{(1/2)}/d*x^4-1/3/b^2*(d*x^4+c)^{(1/2)}/d^2*c-1/b^3*a/d*(d*x^4+c)^{(1/2)}-3/4*a^2/b^4/(-(a*d-b*c)/b)^{(1/2)}*\ln(((-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^2*d-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)}/b))-3/4*a^2/b^4/(-(a*d-b*c)/b)^{(1/2)}*\ln((2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^2*d+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)}/b))-1/8*a^2/b^4*(-a*b)^{(1/2)}/(a*d-b*c)/(x^2+(-a*b)^{(1/2)}/b)*((x^2+(-a*b)^{(1/2)}/b)^2*d-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}+1/8*a^3/b^4*d/(a*d-b*c)/(-(a*d-b*c)/b)^{(1/2)}*\ln(((-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^2*d-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)}/b))+1/8*a^2/b^4*(-a*b)^{(1/2)}/(a*d-b*c)/(x^2-(-a*b)^{(1/2)}/b)*((x^2-(-a*b)^{(1/2)}/b)^2*d+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}+1/8*a^3/b^4*d/(a*d-b*c)/(-(a*d-b*c)/b)^{(1/2)}*\ln((2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^2*d+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)}/b))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{15}/(b*x^4+a)^2/(d*x^4+c)^{(1/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 5.19, size = 186, normalized size = 1.06

$$\frac{(dx^4+c)^{3/2}}{6b^2d^2} - \left(\frac{3c}{2b^2d^2} + \frac{ad-bc}{b^3d^2} \right) \sqrt{dx^4+c} + \frac{a^2 \operatorname{atan}\left(\frac{a^2\sqrt{b}\sqrt{dx^4+c}(5ad-6bc)}{\sqrt{ad-bc}(5a^3d-6a^2bc)}\right)(5ad-6bc)}{4b^{7/2}(ad-bc)^{3/2}} - \frac{a^3d\sqrt{dx^4+c}}{2(ad-bc)(2b^4(dx^4+c)-2b^4c+2ab^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{15}/((a+b*x^4)^2*(c+d*x^4)^{(1/2)}), x)$

[Out] $(c+d*x^4)^{(3/2)}/(6*b^2*d^2) - ((3*c)/(2*b^2*d^2) + (a*d - b*c)/(b^3*d^2)) * (c+d*x^4)^{(1/2)} + (a^2*\operatorname{atan}((a^2*b^{(1/2)}*(c+d*x^4)^{(1/2)}*(5*a*d - 6*b*$

$$c)/((a*d - b*c)^{(1/2)}*(5*a^3*d - 6*a^2*b*c))* (5*a*d - 6*b*c)/(4*b^{(7/2)}*(a*d - b*c)^{(3/2)}) - (a^3*d*(c + d*x^4)^{(1/2)})/(2*(a*d - b*c)*(2*b^4*(c + d*x^4) - 2*b^4*c + 2*a*b^3*d))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**15/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)

[Out] Timed out

$$3.532 \quad \int \frac{x^{11}}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$$

Optimal. Leaf size=123

$$-\frac{a^2 \sqrt{c+dx^4}}{4b^2 (a+bx^4)(bc-ad)} + \frac{a(4bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx^4}}{2b^2 d}$$

Rubi [A] time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2 \sqrt{c+dx^4}}{4b^2 (a+bx^4)(bc-ad)} + \frac{a(4bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx^4}}{2b^2 d}$$

Antiderivative was successfully verified.

[In] Int[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] Sqrt[c + d*x^4]/(2*b^2*d) - (a^2*Sqrt[c + d*x^4])/(4*b^2*(b*c - a*d)*(a + b*x^4)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*b^(5/2)*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 89

Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))

)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11}}{(a + bx^4)^2 \sqrt{c + dx^4}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^2}{(a + bx)^2 \sqrt{c + dx}} dx, x, x^4 \right) \\
 &= -\frac{a^2 \sqrt{c + dx^4}}{4b^2(bc - ad)(a + bx^4)} + \frac{\text{Subst} \left(\int \frac{-\frac{1}{2}a(2bc - ad) + b(bc - ad)x}{(a + bx)\sqrt{c + dx}} dx, x, x^4 \right)}{4b^2(bc - ad)} \\
 &= \frac{\sqrt{c + dx^4}}{2b^2d} - \frac{a^2 \sqrt{c + dx^4}}{4b^2(bc - ad)(a + bx^4)} - \frac{(a(4bc - 3ad)) \text{Subst} \left(\int \frac{1}{(a + bx)\sqrt{c + dx}} dx, x, x^4 \right)}{8b^2(bc - ad)} \\
 &= \frac{\sqrt{c + dx^4}}{2b^2d} - \frac{a^2 \sqrt{c + dx^4}}{4b^2(bc - ad)(a + bx^4)} - \frac{(a(4bc - 3ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^4} \right)}{4b^2d(bc - ad)} \\
 &= \frac{\sqrt{c + dx^4}}{2b^2d} - \frac{a^2 \sqrt{c + dx^4}}{4b^2(bc - ad)(a + bx^4)} + \frac{a(4bc - 3ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^4}}{\sqrt{bc - ad}} \right)}{4b^{5/2}(bc - ad)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.36, size = 107, normalized size = 0.87

$$\frac{1}{4} \left(\frac{\sqrt{c + dx^4} \left(\frac{a^2}{(a+bx^4)(ad-bc)} + \frac{2}{d} \right)}{b^2} + \frac{a(4bc - 3ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{b^{5/2}(bc - ad)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] ((Sqrt[c + d*x^4]*(2/d + a^2/((-b*c) + a*d)*(a + b*x^4)))/b^2 + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c - a*d)^(3/2))/4

IntegrateAlgebraic [A] time = 0.33, size = 143, normalized size = 1.16

$$\frac{(3a^2d - 4abc) \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4} \sqrt{ad-bc}}{bc-ad} \right)}{4b^{5/2}(ad - bc)^{3/2}} - \frac{\sqrt{c + dx^4} (3a^2d - 2abc + 2abdx^4 - 2b^2cx^4)}{4b^2d (a + bx^4) (bc - ad)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] -1/4*(Sqrt[c + d*x^4]*(-2*a*b*c + 3*a^2*d - 2*b^2*c*x^4 + 2*a*b*d*x^4))/(b^2*d*(b*c - a*d)*(a + b*x^4)) + ((-4*a*b*c + 3*a^2*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)])/(4*b^(5/2)*(-(b*c) + a*d)^(3/2))

fricas [B] time = 0.62, size = 475, normalized size = 3.86

$$\frac{(4a^2bd - 3a^2d^2 + (4ab^2cd - 3a^2bd^2)x^4)\sqrt{bc-ad} \log\left(\frac{bx^4+2\sqrt{bc-ad}}{2a^2d}\right) + 2(2ab^3c^2 - 5a^2b^2cd + 3a^2bd^2 + 2(b^4c^2 - 2ab^3cd + a^2b^2d^2)x^4)\sqrt{bc-ad} + c}{8(ab^2cd - 2a^2b^2cd^2 + a^2b^2d^2) + (b^2cd - 2ab^2cd^2 + a^2b^2d^2)x^4} - \frac{(4a^2bd - 3a^2d^2 + (4ab^2cd - 3a^2bd^2)x^4)\sqrt{-bc+abd} \arctan\left(\frac{\sqrt{bc-ad}\sqrt{c+dx^4}}{2a^2d}\right) - (2ab^3c^2 - 5a^2b^2cd + 3a^2bd^2 + 2(b^4c^2 - 2ab^3cd + a^2b^2d^2)x^4)\sqrt{bc-ad}}{4(ab^2cd - 2a^2b^2cd^2 + a^2b^2d^2) + (b^2cd - 2ab^2cd^2 + a^2b^2d^2)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [1/8*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4), -1/4*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - (2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b

$$\sqrt{dx^4 + c} \cdot x^4) / (a^2 b^5 c^2 d - 2 a^2 b^4 c d^2 + a^3 b^3 d^3 + (b^6 c^2 d - 2 a b^5 c d^2 + a^2 b^4 d^3) x^4]$$

giac [A] time = 0.17, size = 134, normalized size = 1.09

$$-\frac{\sqrt{dx^4 + c} a^2 d}{4(b^3 c - ab^2 d)((dx^4 + c)b - bc + ad)} - \frac{(4abc - 3a^2 d) \arctan\left(\frac{\sqrt{dx^4 + c} b}{\sqrt{-b^2 c + abd}}\right)}{4(b^3 c - ab^2 d)\sqrt{-b^2 c + abd}} + \frac{\sqrt{dx^4 + c}}{2b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(b*x⁴+a)²/(d*x⁴+c)^(1/2), x, algorithm="giac")

[Out] -1/4*sqrt(d*x⁴ + c)*a²*d/((b³*c - a*b²*d)*((d*x⁴ + c)*b - b*c + a*d)) - 1/4*(4*a*b*c - 3*a²*d)*arctan(sqrt(d*x⁴ + c)*b/sqrt(-b²*c + a*b*d))/((b³*c - a*b²*d)*sqrt(-b²*c + a*b*d)) + 1/2*sqrt(d*x⁴ + c)/(b²*d)

maple [B] time = 0.19, size = 876, normalized size = 7.12

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/(b*x⁴+a)²/(d*x⁴+c)^(1/2), x)

[Out] 1/2*(d*x⁴+c)^(1/2)/b²/d+1/2*a/b³/(-a*d-b*c)/b^(1/2)*ln((-2*(-a*b)^(1/2)*(x²+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b^(1/2)*((x²+(-a*b)^(1/2)/b)^(1/2)/b)²*d-2*(-a*b)^(1/2)*(x²+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b^(1/2))/((x²+(-a*b)^(1/2)/b)+1/2*a/b³/(-a*d-b*c)/b^(1/2)*ln((2*(-a*b)^(1/2)*(x²-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b^(1/2)*((x²-(-a*b)^(1/2)/b)^(1/2)/b)²*d+2*(-a*b)^(1/2)*(x²-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b^(1/2))/((x²-(-a*b)^(1/2)/b)+1/8*a/b³*(-a*b)^(1/2)/(a*d-b*c)/(x²+(-a*b)^(1/2)/b)*((x²+(-a*b)^(1/2)/b)²*d-2*(-a*b)^(1/2)*(x²+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b^(1/2)-1/8*a²/b³*d/(a*d-b*c)/(-a*d-b*c)/b^(1/2)*ln((-2*(-a*b)^(1/2)*(x²+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b^(1/2)*((x²+(-a*b)^(1/2)/b)^(1/2)/b)²*d-2*(-a*b)^(1/2)*(x²+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b^(1/2))/((x²+(-a*b)^(1/2)/b)-1/8*a/b³*(-a*b)^(1/2)/(a*d-b*c)/(x²-(-a*b)^(1/2)/b)*((x²-(-a*b)^(1/2)/b)^(1/2)/b)²*d+2*(-a*b)^(1/2)*(x²-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b^(1/2)-1/8*a²/b³*d/(a*d-b*c)/(-a*d-b*c)/b^(1/2)*ln((2*(-a*b)^(1/2)*(x²-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b^(1/2)*((x²-(-a*b)^(1/2)/b)^(1/2)/b)²*d+2*(-a*b)^(1/2)*(x²-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b^(1/2))/((x²-(-a*b)^(1/2)/b))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 5.12, size = 144, normalized size = 1.17

$$\frac{\sqrt{dx^4+c}}{2b^2d} - \frac{a \operatorname{atan}\left(\frac{a\sqrt{b}\sqrt{dx^4+c}(3ad-4bc)}{(3a^2d-4abc)\sqrt{ad-bc}}\right)(3ad-4bc)}{4b^{5/2}(ad-bc)^{3/2}} + \frac{a^2d\sqrt{dx^4+c}}{2(ad-bc)(2b^3(dx^4+c)-2b^3c+2ab^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)`

[Out] $(c + dx^4)^{1/2}/(2b^2d) - (a \operatorname{atan}((a \cdot b^{1/2} \cdot (c + dx^4)^{1/2} \cdot (3ad - 4bc)) / ((3a^2d - 4abc) \cdot (ad - bc)^{1/2}))) \cdot (3ad - 4bc) / (4b^{5/2} \cdot (ad - bc)^{3/2}) + (a^2d \cdot (c + dx^4)^{1/2}) / (2 \cdot (ad - bc) \cdot (2b^3 \cdot (c + dx^4) - 2b^3c + 2ab^2d))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)`

[Out] Timed out

$$3.533 \quad \int \frac{x^7}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

Rubi [A] time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] (a*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*b^(3/2)*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1))]/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 446

`Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rubi steps

$$\begin{aligned}
 \int \frac{x^7}{(a + bx^4)^2 \sqrt{c + dx^4}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x}{(a + bx)^2 \sqrt{c + dx}} dx, x, x^4 \right) \\
 &= \frac{a\sqrt{c + dx^4}}{4b(bc - ad)(a + bx^4)} + \frac{(2bc - ad) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^4 \right)}{8b(bc - ad)} \\
 &= \frac{a\sqrt{c + dx^4}}{4b(bc - ad)(a + bx^4)} + \frac{(2bc - ad) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^4} \right)}{4bd(bc - ad)} \\
 &= \frac{a\sqrt{c + dx^4}}{4b(bc - ad)(a + bx^4)} - \frac{(2bc - ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^4}}{\sqrt{bc - ad}} \right)}{4b^{3/2}(bc - ad)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.11, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^4}}{(a+bx^4)(bc-ad)} + \frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{4b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] ((a*Sqrt[b]*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + ((-2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(4*b^(3/2))

IntegrateAlgebraic [A] time = 0.17, size = 109, normalized size = 1.10

$$\frac{(2bc - ad) \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4} \sqrt{ad-bc}}{bc-ad} \right)}{4b^{3/2}(ad - bc)^{3/2}} + \frac{a\sqrt{c + dx^4}}{4b(a + bx^4)(bc - ad)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] (a*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) + ((2*b*c - a*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)]/(4*b^(3/2)*(-(b*c) + a*d)^(3/2))

fricas [A] time = 0.71, size = 348, normalized size = 3.52

$$\left[\frac{((2b^2c - abd)x^4 + 2abc - a^2d)\sqrt{b^2c - abd} \log\left(\frac{bdx^4 + 2bc - ad - 2\sqrt{dx^4 + c}\sqrt{b^2c - abd}}{bx^4 + a}\right) + 2\sqrt{dx^4 + c}(ab^2c - a^2bd)}{8(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2 + (b^5c^2 - 2ab^4cd + a^2b^3d^2)x^4)}, \frac{((2b^2c - abd)x^4 + 2abc - a^2d)\sqrt{-b^2c + abd} \arctan\left(\frac{\sqrt{dx^4 + c}\sqrt{-b^2c + abd}}{bdx^4 + bc}\right) + \sqrt{dx^4 + c}(ab^2c - a^2bd)}{4(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2 + (b^5c^2 - 2ab^4cd + a^2b^3d^2)x^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [1/8*(((2*b^2*c - a*b*d)*x^4 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^4), 1/4*(((2*b^2*c - a*b*d)*x^4 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^4)]

giac [A] time = 0.17, size = 116, normalized size = 1.17

$$\frac{\sqrt{dx^4 + c} ad^2}{(b^2c - abd)((dx^4 + c)b - bc + ad)} + \frac{(2bcd - ad^2) \arctan\left(\frac{\sqrt{dx^4 + c} b}{\sqrt{-b^2c + abd}}\right)}{(b^2c - abd)\sqrt{-b^2c + abd}}$$

4 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] 1/4*(sqrt(d*x^4 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^4 + c)*b - b*c + a*d)) + (2*b*c*d - a*d^2)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sqrt(-b^2*c + a*b*d))/d

maple [B] time = 0.20, size = 851, normalized size = 8.60

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^4+c}}{\sqrt{ad-bc}}\right)}{8(ad-bc)\sqrt{\frac{ad-bc}{b^2}}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^4+c}}{\sqrt{ad-bc}}\right)}{8(ad-bc)\sqrt{\frac{ad-bc}{b^2}}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^4+c}}{\sqrt{ad-bc}}\right)}{4\sqrt{\frac{ad-bc}{b^2}}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^4+c}}{\sqrt{ad-bc}}\right)}{4\sqrt{\frac{ad-bc}{b^2}}}\sqrt{\frac{b}{8(ad-bc)(d^2+\frac{c}{b})^2}}+\sqrt{\frac{b}{8(ad-bc)(d^2-\frac{c}{b})^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (x^7/(b*x^4+a)^2/(d*x^4+c)^{(1/2)}, x)$

[Out]
$$\begin{aligned} & -1/4/b^2/(-a*d-b*c)/b)^{(1/2)}*\ln((-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d- \\ & 2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^2*d-2*(-a*b)^{(1/2)} \\ & *(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)}/b))-1/4/b \\ & ^2/(-a*d-b*c)/b)^{(1/2)}*\ln((2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-2*(a*d- \\ & b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^2*d+2*(-a*b)^{(1/2)}*(x^2 \\ & -(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)}/b))-1/8/b^2*(-a* \\ & b)^{(1/2)}/(a*d-b*c)/(x^2+(-a*b)^{(1/2)}/b)*((x^2+(-a*b)^{(1/2)}/b)^2*d-2*(-a*b)^{(1/2)} \\ & *(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}+1/8*a/b^2*d/(a*d-b*c)/(- \\ & a*d-b*c)/b)^{(1/2)}*\ln((-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/ \\ & b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^2*d-2*(-a*b)^{(1/2)}*(x^2+(-a* \\ & b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)}/b))+1/8/b^2*(-a*b)^{(1/2)} \\ & /((a*d-b*c)/(x^2-(-a*b)^{(1/2)}/b)*((x^2-(-a*b)^{(1/2)}/b)^2*d+2*(-a*b)^{(1/2)} \\ & *(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}+1/8*a/b^2*d/(a*d-b*c)/(-a*d-b \\ & *c)/b)^{(1/2)}*\ln((2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(- \\ & (a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^2*d+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b) \\ & /b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)}/b)) \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(x^7/(b*x^4+a)^2/(d*x^4+c)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.93, size = 95, normalized size = 0.96

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^4+c}}{\sqrt{ad-bc}}\right)(ad-2bc)}{4b^{3/2}(ad-bc)^{3/2}} - \frac{ad\sqrt{dx^4+c}}{2b(ad-bc)\left(2b(dx^4+c)+2ad-2bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(4*b^(3/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^4)^(1/2))/(2*b*(a*d - b*c)*(2*b*(c + d*x^4) + 2*a*d - 2*b*c))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.534 \quad \int \frac{x^3}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$$

Optimal. Leaf size=87

$$\frac{d \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{4\sqrt{b} (bc-ad)^{3/2}} - \frac{\sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {444, 51, 63, 208}

$$\frac{d \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{4\sqrt{b} (bc-ad)^{3/2}} - \frac{\sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] -Sqrt[c + d*x^4]/(4*(b*c - a*d)*(a + b*x^4)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*Sqrt[b]*(b*c - a*d)^(3/2))

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a + bx^4)^2 \sqrt{c + dx^4}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{(a + bx)^2 \sqrt{c + dx}} dx, x, x^4 \right) \\
&= -\frac{\sqrt{c + dx^4}}{4(bc - ad)(a + bx^4)} - \frac{d \text{Subst} \left(\int \frac{1}{(a + bx) \sqrt{c + dx}} dx, x, x^4 \right)}{8(bc - ad)} \\
&= -\frac{\sqrt{c + dx^4}}{4(bc - ad)(a + bx^4)} - \frac{\text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^4} \right)}{4(bc - ad)} \\
&= -\frac{\sqrt{c + dx^4}}{4(bc - ad)(a + bx^4)} + \frac{d \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^4}}{\sqrt{bc - ad}} \right)}{4\sqrt{b}(bc - ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 85, normalized size = 0.98

$$\frac{1}{4} \left(\frac{\sqrt{c + dx^4}}{(a + bx^4)(ad - bc)} + \frac{d \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^4}}{\sqrt{ad - bc}} \right)}{\sqrt{b}(ad - bc)^{3/2}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]), x]
```

```
[Out] (Sqrt[c + d*x^4]/((-b*c) + a*d)*(a + b*x^4) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
d*x^4])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/4
```


IntegrateAlgebraic [A] time = 0.11, size = 97, normalized size = 1.11

$$\frac{\sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)} - \frac{d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{4\sqrt{b}(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] -1/4*Sqrt[c + d*x^4]/((b*c - a*d)*(a + b*x^4)) - (d*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)]/(4*Sqrt[b]*(-(b*c) + a*d)^(3/2)))

fricas [B] time = 0.72, size = 302, normalized size = 3.47

$$\left[\frac{(bdx^4 + ad)\sqrt{b^2c - abd} \log\left(\frac{bdx^4 + 2bc - ad - 2\sqrt{dx^4 + c}\sqrt{b^2c - abd}}{bx^4 + a}\right) + 2\sqrt{dx^4 + c}(b^2c - abd)}{8(ab^3c^2 - 2a^2b^2cd + a^3bd^2 + (b^4c^2 - 2ab^3cd + a^2b^2d^2)x^4)}, -\frac{(bdx^4 + ad)\sqrt{-b^2c + abd} \arctan\left(\frac{\sqrt{dx^4 + c}\sqrt{-b^2c + abd}}{bdx^4 + bc}\right) + \sqrt{dx^4 + c}(b^2c - abd)}{4(ab^3c^2 - 2a^2b^2cd + a^3bd^2 + (b^4c^2 - 2ab^3cd + a^2b^2d^2)x^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [-1/8*((b*d*x^4 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2*c - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4), -1/4*((b*d*x^4 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(b^2*c - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4)]

giac [A] time = 0.16, size = 93, normalized size = 1.07

$$\frac{d \arctan\left(\frac{\sqrt{dx^4+cb}}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^4+cd}}{4((dx^4+c)b-bc+ad)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] -1/4*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c - a*d)) - 1/4*sqrt(d*x^4 + c)*d/(((d*x^4 + c)*b - b*c + a*d)*(b*c - a*d))

maple [B] time = 0.21, size = 541, normalized size = 6.22

$$\frac{d \ln \left(\frac{2\sqrt{ab} \left(x^2 + \frac{\sqrt{ab}}{b} \right)^d}{x^2 + \frac{\sqrt{ab}}{b}} \right)^d \sqrt{\frac{ad-bc}{b}}}{8(ad-bc) \sqrt{\frac{ad-bc}{b}}} - \frac{d \ln \left(\frac{2\sqrt{ab} \left(x^2 + \frac{\sqrt{ab}}{b} \right)^d}{x^2 + \frac{\sqrt{ab}}{b}} \right)^d \sqrt{\frac{ad-bc}{b}}}{8(ad-bc) \sqrt{\frac{ad-bc}{b}}} + \frac{\sqrt{-ab} \sqrt{\left(x^2 + \frac{\sqrt{ab}}{b} \right)^2 d - \frac{ad-bc}{b}}}{8(ad-bc) \left(x^2 + \frac{\sqrt{ab}}{b} \right) ab} - \frac{\sqrt{-ab} \sqrt{\left(x^2 + \frac{\sqrt{ab}}{b} \right)^2 d + \frac{ad-bc}{b}}}{8(ad-bc) \left(x^2 + \frac{\sqrt{ab}}{b} \right) ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2), x)

[Out] $\frac{1}{8} * (-a*b)^{(1/2)} / a/b / (a*d-b*c) / (x^2+(-a*b)^{(1/2)}/b) * ((x^2+(-a*b)^{(1/2)}/b)^{-2} * d - 2*(-a*b)^{(1/2)} * (x^2+(-a*b)^{(1/2)}/b) / b * d - (a*d-b*c) / b)^{(1/2)} - 1/8 / b * d / (a*d-b*c) / (- (a*d-b*c) / b)^{(1/2)} * \ln((-2*(-a*b)^{(1/2)} * (x^2+(-a*b)^{(1/2)}/b) / b * d - 2*(a*d-b*c) / b + 2*(-(a*d-b*c) / b)^{(1/2)} * ((x^2+(-a*b)^{(1/2)}/b)^{-2} * d - 2*(-a*b)^{(1/2)} * (x^2+(-a*b)^{(1/2)}/b) / b * d - (a*d-b*c) / b)^{(1/2)}) / (x^2+(-a*b)^{(1/2)}/b)) - 1/8 * (-a*b)^{(1/2)} / a/b / (a*d-b*c) / (x^2-(-a*b)^{(1/2)}/b) * ((x^2-(-a*b)^{(1/2)}/b)^{-2} * d + 2*(-a*b)^{(1/2)} * (x^2-(-a*b)^{(1/2)}/b) / b * d - (a*d-b*c) / b)^{(1/2)} - 1/8 / b * d / (a*d-b*c) / (- (a*d-b*c) / b)^{(1/2)} * \ln((2*(-a*b)^{(1/2)} * (x^2-(-a*b)^{(1/2)}/b) / b * d - 2*(a*d-b*c) / b + 2*(-(a*d-b*c) / b)^{(1/2)} * ((x^2-(-a*b)^{(1/2)}/b)^{-2} * d + 2*(-a*b)^{(1/2)} * (x^2-(-a*b)^{(1/2)}/b) / b * d - (a*d-b*c) / b)^{(1/2)}) / (x^2-(-a*b)^{(1/2)}/b))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details) Is a*d-b*c positive or negative?

mupad [B] time = 4.85, size = 84, normalized size = 0.97

$$\frac{d \sqrt{d x^4 + c}}{2 (a d - b c) (2 b (d x^4 + c) + 2 a d - 2 b c)} + \frac{d \operatorname{atan} \left(\frac{\sqrt{b} \sqrt{d x^4 + c}}{\sqrt{a d - b c}} \right)}{4 \sqrt{b} (a d - b c)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)

[Out] $\frac{d*(c + d*x^4)^{(1/2)}}{(2*(a*d - b*c)*(2*b*(c + d*x^4) + 2*a*d - 2*b*c))} + (d*\operatorname{atan}((b^{(1/2)}*(c + d*x^4)^{(1/2)})/(a*d - b*c)^{(1/2)}))/(4*b^{(1/2)}*(a*d - b*c)^{(3/2)})$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)

[Out] Timed out

$$3.535 \quad \int \frac{1}{x(a+bx^4)^2 \sqrt{c+dx^4}} dx$$

Optimal. Leaf size=132

$$\frac{\sqrt{b}(2bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} + \frac{b\sqrt{c+dx^4}}{4a(a+bx^4)(bc-ad)}$$

Rubi [A] time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}(2bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} + \frac{b\sqrt{c+dx^4}}{4a(a+bx^4)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) - ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/(2*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*a^2*(b*c - a*d)^(3/2))

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^p, x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx^4)^2 \sqrt{c+dx^4}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x(a+bx)^2 \sqrt{c+dx}} dx, x, x^4 \right) \\
&= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} + \frac{\text{Subst} \left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}} dx, x, x^4 \right)}{4a(bc-ad)} \\
&= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} + \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^4 \right)}{4a^2} - \frac{(b(2bc-3ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^4 \right)}{8a^2(bc-ad)} \\
&= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} + \frac{\text{Subst} \left(\int \frac{1}{\frac{-c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c+dx^4} \right)}{2a^2d} - \frac{(b(2bc-3ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^4 \right)}{8a^2(bc-ad)} \\
&= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} - \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^4}}{\sqrt{c}} \right)}{2a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad) \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}} \right)}{4a^2(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^4}}{(a+bx^4)(bc-ad)} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] ((a*b*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) - (2*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(4*a^2)

IntegrateAlgebraic [A] time = 0.32, size = 146, normalized size = 1.11

$$\frac{(3a\sqrt{b}d - 2b^{3/2}c)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{4a^2(ad-bc)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} - \frac{b\sqrt{c+dx^4}}{4a(a+bx^4)(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] -1/4*(b*Sqrt[c + d*x^4])/(a*(-(b*c) + a*d)*(a + b*x^4)) + ((-2*b^(3/2)*c + 3*a*Sqrt[b]*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)]/(4*a^2*(-(b*c) + a*d)^(3/2)) - ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/(2*a^2*Sqrt[c])

fricas [A] time = 0.68, size = 862, normalized size = 6.53

1/8*(2*sqrt(d*x^4 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^4 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^4 + b*c)) + ((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [1/8*(2*sqrt(d*x^4 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^4 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^4 + b*c)) + ((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3

$*b*c*d)*x^4)$, $1/8*(2*\sqrt{d*x^4 + c})*a*b*c + 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*\sqrt{-c}*\arctan(\sqrt{d*x^4 + c}*\sqrt{-c}/c) + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*\sqrt{b/(b*c - a*d)}*\log((b*d*x^4 + 2*b*c - a*d + 2*\sqrt{d*x^4 + c}*(b*c - a*d)*\sqrt{b/(b*c - a*d)}))/(b*x^4 + a))/((a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4)$, $1/4*(\sqrt{d*x^4 + c})*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*\sqrt{-b/(b*c - a*d)}*\arctan(-\sqrt{d*x^4 + c}*(b*c - a*d)*\sqrt{-b/(b*c - a*d)})/(b*d*x^4 + b*c)) + 2*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*\sqrt{-c}*\arctan(\sqrt{d*x^4 + c}*\sqrt{-c}/c))/((a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4)]$

giac [A] time = 0.17, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^4 + c} bd}{4(abc - a^2d)((dx^4 + c)b - bc + ad)} - \frac{(2b^2c - 3abd) \arctan\left(\frac{\sqrt{dx^4 + c} b}{\sqrt{-b^2c + abd}}\right)}{4(a^2bc - a^3d)\sqrt{-b^2c + abd}} + \frac{\arctan\left(\frac{\sqrt{dx^4 + c}}{\sqrt{-c}}\right)}{2a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] $1/4*\sqrt{d*x^4 + c}*b*d/((a*b*c - a^2*d)*((d*x^4 + c)*b - b*c + a*d)) - 1/4*(2*b^2*c - 3*a*b*d)*\arctan(\sqrt{d*x^4 + c}*b/\sqrt{-b^2*c + a*b*d}))/((a^2*b*c - a^3*d)*\sqrt{-b^2*c + a*b*d}) + 1/2*\arctan(\sqrt{d*x^4 + c}/\sqrt{-c}))/((a^2*\sqrt{-c}))$

maple [B] time = 0.27, size = 880, normalized size = 6.67

$$\frac{d \ln\left(\frac{\sqrt{d x^4 + c} \sqrt{a d - b c}}{a d - b c} \sqrt{\frac{d x^4 + c}{a d - b c}}\right)}{8(a d - b c) \sqrt{d x^4 + c}} + \frac{d \ln\left(\frac{\sqrt{d x^4 + c} \sqrt{a d - b c}}{8(a d - b c) \sqrt{d x^4 + c}} \sqrt{\frac{d x^4 + c}{a d - b c}}\right)}{8(a d - b c) \sqrt{d x^4 + c}} + \frac{b \ln\left(\frac{\sqrt{d x^4 + c} \sqrt{a d - b c}}{4 \sqrt{d x^4 + c}} \sqrt{\frac{d x^4 + c}{a d - b c}}\right)}{4 \sqrt{d x^4 + c}} + \frac{b \ln\left(\frac{\sqrt{d x^4 + c} \sqrt{a d - b c}}{4 \sqrt{d x^4 + c}} \sqrt{\frac{d x^4 + c}{a d - b c}}\right)}{4 \sqrt{d x^4 + c}} + \frac{b \ln\left(\frac{\sqrt{d x^4 + c} \sqrt{a d - b c}}{2 a \sqrt{d x^4 + c}} \sqrt{\frac{d x^4 + c}{a d - b c}}\right)}{2 a \sqrt{d x^4 + c}} + \frac{\sqrt{d x^4 + c} \sqrt{a d - b c}}{8(a d - b c) \left(x^2 + \frac{a d - b c}{d}\right)^2} + \frac{\sqrt{d x^4 + c} \sqrt{a d - b c}}{8(a d - b c) \left(x^2 - \frac{a d - b c}{d}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)

[Out] $1/4/a^2/(-(a*d-b*c)/b)^(1/2)*\ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/4/a^2/(-(a*d-b*c)/b)^(1/2)*\ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/8/a^2*(-a*b)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8/a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*\ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/8/a^2*(-a*b)^(1/2)/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)$

$$\begin{aligned}
& a*d - 2*b*c) * (-b*(a*d - b*c)^3)^{(1/2)} * (64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - \\
& 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3) / (64*(a^4*d^2 + a^2*b^2*c^2 - 2 \\
& *a^3*b*c*d) * (a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) * (3* \\
& a*d - 2*b*c) * (-b*(a*d - b*c)^3)^{(1/2)} / (8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2 \\
& *c^2*d - 3*a^4*b*c*d^2)) * (3*a*d - 2*b*c) * (-b*(a*d - b*c)^3)^{(1/2)} / (8*(a^5 \\
& *d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + (((c + d*x^4)^{(1/2)} * (13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3)) / (8*(a^4*d^2 + a^2*b^2 \\
& *c^2 - 2*a^3*b*c*d)) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^3) / (a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^4)^{(1/2)} * (3*a*d - 2*b*c) * (-b*(a*d - b*c)^3)^{(1/2)} * (64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3)) / (64*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d) * (a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) * (3*a*d - 2*b*c) * (-b*(a*d - b*c)^3)^{(1/2)} / (8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) * (3*a*d - 2*b*c) * (-b*(a*d - b*c)^3)^{(1/2)} / (8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))) * (3*a*d - 2*b*c) * (-b*(a*d - b*c)^3)^{(1/2)} * i) / (4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (atan((((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^3) / (4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^4)^{(1/2)} * (64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3)) / (128*a^2*c^(1/2) * (a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))) / (4*a^2*c^(1/2)) - ((c + d*x^4)^{(1/2)} * (13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3)) / (32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))) * i) / (a^2*c^(1/2)) - (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^3) / (4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^4)^{(1/2)} * (64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3)) / (128*a^2*c^(1/2) * (a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))) / (4*a^2*c^(1/2)) + ((c + d*x^4)^{(1/2)} * (13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3)) / (32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))) * i) / (a^2*c^(1/2)) / (((3*a*b^3*d^4) / 16 - (b^4*c*d^3) / 8) / (a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^3) / (4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^4)^{(1/2)} * (64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3)) / (128*a^2*c^(1/2) * (a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))) / (4*a^2*c^(1/2)) - ((c + d*x^4)^{(1/2)} * (13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3)) / (32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))) / (a^2*c^(1/2)) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^3) / (4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^4)^{(1/2)} * (64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3)) / (128*a^2*c^(1/2) * (a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))) / (4*a^2*c^(1/2)) + ((c + d*x^4)^{(1/2)} * (13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3)) / (32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))) / (a^2*c^(1/2))))) * i) / (2*a^2*c^(1/2)) - (b*d*(c + d*x^4)^{(1/2)}) / (2*(a^2*d - a*b*c) * (2*b*(c + d*x^4) + 2*a*d - 2*b*c))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + bx^4)^2 \sqrt{c + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**4+a)**2/(d*x**4+c)**(1/2), x)

[Out] Integral(1/(x*(a + b*x**4)**2*sqrt(c + d*x**4)), x)

$$3.536 \quad \int \frac{1}{x^5(a+bx^4)^2 \sqrt{c+dx^4}} dx$$

Optimal. Leaf size=185

$$\frac{b^{3/2}(4bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc - ad)^{3/2}} + \frac{(ad + 4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}} - \frac{b\sqrt{c+dx^4}(2bc - ad)}{4a^2c(a+bx^4)(bc - ad)} - \frac{\sqrt{c+dx^4}}{4acx^4(a+bx^4)}$$

Rubi [A] time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 103, 151, 156, 63, 208}

$$\frac{b^{3/2}(4bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc - ad)^{3/2}} + \frac{(ad + 4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}} - \frac{b\sqrt{c+dx^4}(2bc - ad)}{4a^2c(a+bx^4)(bc - ad)} - \frac{\sqrt{c+dx^4}}{4acx^4(a+bx^4)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*(a + b*x^4)) - Sqrt[c + d*x^4]/(4*a*c*x^4*(a + b*x^4)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(4*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*a^3*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 151

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

```

Rule 156

```

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 446

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5 (a + bx^4)^2 \sqrt{c + dx^4}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x^2 (a + bx)^2 \sqrt{c + dx}} dx, x, x^4 \right) \\
&= -\frac{\sqrt{c + dx^4}}{4acx^4 (a + bx^4)} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(4bc+ad) + \frac{3bdx}{2}}{x(a+bx)^2 \sqrt{c+dx}} dx, x, x^4 \right)}{4ac} \\
&= -\frac{b(2bc - ad)\sqrt{c + dx^4}}{4a^2c(bc - ad)(a + bx^4)} - \frac{\sqrt{c + dx^4}}{4acx^4 (a + bx^4)} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad) + \frac{1}{2}bd(2bc-)}{x(a+bx)\sqrt{c+dx}} dx, x, x^4 \right)}{4a^2c(bc - ad)} \\
&= -\frac{b(2bc - ad)\sqrt{c + dx^4}}{4a^2c(bc - ad)(a + bx^4)} - \frac{\sqrt{c + dx^4}}{4acx^4 (a + bx^4)} + \frac{(b^2(4bc - 5ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^4 \right)}{8a^3(bc - ad)} \\
&= -\frac{b(2bc - ad)\sqrt{c + dx^4}}{4a^2c(bc - ad)(a + bx^4)} - \frac{\sqrt{c + dx^4}}{4acx^4 (a + bx^4)} + \frac{(b^2(4bc - 5ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^4}{d}} dx, x, x^4 \right)}{4a^3d(bc - ad)} \\
&= -\frac{b(2bc - ad)\sqrt{c + dx^4}}{4a^2c(bc - ad)(a + bx^4)} - \frac{\sqrt{c + dx^4}}{4acx^4 (a + bx^4)} + \frac{(4bc + ad) \tanh^{-1} \left(\frac{\sqrt{c+dx^4}}{\sqrt{c}} \right)}{4a^3c^{3/2}} - \frac{b^3}{4a^3c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.70, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^4}(a^2d+ab(dx^4-c)-2b^2cx^4)}{x^4(a+bx^4)(bc-ad)} + \frac{b^{3/2}c(5ad-4bc) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{(ad+4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^3c}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] ((a*Sqrt[c + d*x^4]*(a^2*d - 2*b^2*c*x^4 + a*b*(-c + d*x^4)))/((b*c - a*d)*x^4*(a + b*x^4)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/Sqrt[c] + (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(4*a^3*c)

IntegrateAlgebraic [A] time = 0.66, size = 187, normalized size = 1.01

$$\frac{(4b^{5/2}c - 5ab^{3/2}d) \tan^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad} \right)}{4a^3(ad - bc)^{3/2}} + \frac{(ad + 4bc) \tanh^{-1} \left(\frac{\sqrt{c+dx^4}}{\sqrt{c}} \right)}{4a^3c^{3/2}} + \frac{\sqrt{c + dx^4} (-a^2d + abc - abdx^4 + 2b^2cx^4)}{4a^2cx^4 (a + bx^4) (ad - bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] (Sqrt[c + d*x^4]*(a*b*c - a^2*d + 2*b^2*c*x^4 - a*b*d*x^4))/(4*a^2*c*(-(b*c) + a*d)*x^4*(a + b*x^4)) + ((4*b^(5/2)*c - 5*a*b^(3/2)*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)]/(4*a^3*(-(b*c) + a*d)^(3/2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(4*a^3*c^(3/2))

fricas [A] time = 0.91, size = 1236, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [1/8*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^4 + b*c)) - ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) + 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/4*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^4 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + (a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4)]

giac [A] time = 0.17, size = 257, normalized size = 1.39

$$\frac{(4b^3c - 5ab^2d) \arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right) - 2(dx^4+c)^{\frac{3}{2}}b^2cd - 2\sqrt{dx^4+c}b^2c^2d - (dx^4+c)^{\frac{3}{2}}abd^2 + 2\sqrt{dx^4+c}abcd^2 - \sqrt{dx^4+c}a^2d^3}{4(a^3bc - a^4d)\sqrt{-b^2c+abd}} - \frac{(4bc + ad) \arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4a^3\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{4} \cdot (4 \cdot b^3 \cdot c - 5 \cdot a \cdot b^2 \cdot d) \cdot \arctan\left(\frac{\sqrt{d \cdot x^4 + c} \cdot b}{\sqrt{-b^2 \cdot c + a \cdot b \cdot d}}\right) / \left((a^3 \cdot b \cdot c - a^4 \cdot d) \cdot \sqrt{-b^2 \cdot c + a \cdot b \cdot d} \right) - \frac{1}{4} \cdot (2 \cdot (d \cdot x^4 + c)^{(3/2)} \cdot b^2 \cdot c \cdot d - 2 \cdot \sqrt{d \cdot x^4 + c} \cdot b^2 \cdot c^2 \cdot d - (d \cdot x^4 + c)^{(3/2)} \cdot a \cdot b \cdot d^2 + 2 \cdot \sqrt{d \cdot x^4 + c} \cdot a \cdot b \cdot c \cdot d^2 - \sqrt{d \cdot x^4 + c} \cdot a^2 \cdot d^3) / \left((a^2 \cdot b \cdot c^2 - a^3 \cdot c \cdot d) \cdot ((d \cdot x^4 + c)^{2 \cdot b} - 2 \cdot (d \cdot x^4 + c) \cdot b \cdot c + b \cdot c^2 + (d \cdot x^4 + c) \cdot a \cdot d - a \cdot c \cdot d) \right) - \frac{1}{4} \cdot (4 \cdot b \cdot c + a \cdot d) \cdot \arctan\left(\frac{\sqrt{d \cdot x^4 + c}}{\sqrt{-c}}\right) / (a^3 \cdot \sqrt{-c} \cdot c)$

maple [B] time = 0.28, size = 938, normalized size = 5.07

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)

[Out] $-\frac{1}{4} \cdot \frac{1}{a^2 \cdot c} \cdot \frac{1}{x^4} \cdot (d \cdot x^4 + c)^{(1/2)} + \frac{1}{4} \cdot \frac{1}{a^2 \cdot d \cdot c} \cdot \ln\left(\frac{(2 \cdot c + 2 \cdot (d \cdot x^4 + c)^{(1/2)} \cdot c)^{(1/2)}}{x^2} - \frac{1}{2} \cdot \frac{1}{a^3 \cdot b} \cdot \left(\frac{-a \cdot d - b \cdot c}{b}\right)^{(1/2)} \cdot \ln\left(\frac{-2 \cdot (-a \cdot b)^{(1/2)} \cdot (x^2 + (-a \cdot b)^{(1/2)})^{(1/2)}}{b} \cdot \frac{1}{b \cdot d - 2 \cdot (a \cdot d - b \cdot c) / b + 2 \cdot (-a \cdot d - b \cdot c) / b} \cdot \left(\frac{x^2 + (-a \cdot b)^{(1/2)}}{b}\right)^{2 \cdot d - 2 \cdot (-a \cdot b)^{(1/2)} \cdot (x^2 + (-a \cdot b)^{(1/2)}) / b} \cdot \frac{1}{b \cdot d - (a \cdot d - b \cdot c) / b} \cdot \left(\frac{x^2 + (-a \cdot b)^{(1/2)}}{b}\right)\right) - \frac{1}{2} \cdot \frac{1}{a^3 \cdot b} \cdot \left(\frac{-a \cdot d - b \cdot c}{b}\right)^{(1/2)} \cdot \ln\left(\frac{2 \cdot (-a \cdot b)^{(1/2)} \cdot (x^2 - (-a \cdot b)^{(1/2)})^{(1/2)}}{b} \cdot \frac{1}{b \cdot d - 2 \cdot (a \cdot d - b \cdot c) / b + 2 \cdot (-a \cdot d - b \cdot c) / b} \cdot \left(\frac{x^2 - (-a \cdot b)^{(1/2)}}{b}\right)^{2 \cdot d + 2 \cdot (-a \cdot b)^{(1/2)} \cdot (x^2 - (-a \cdot b)^{(1/2)}) / b} \cdot \frac{1}{b \cdot d - (a \cdot d - b \cdot c) / b} \cdot \left(\frac{x^2 - (-a \cdot b)^{(1/2)}}{b}\right)\right) + \frac{1}{8} \cdot \frac{1}{a^3 \cdot b} \cdot (-a \cdot b)^{(1/2)} \cdot \frac{1}{(a \cdot d - b \cdot c)} \cdot \frac{1}{(x^2 + (-a \cdot b)^{(1/2)}) / b} \cdot \left(\frac{x^2 + (-a \cdot b)^{(1/2)}}{b}\right)^{2 \cdot d - 2 \cdot (-a \cdot b)^{(1/2)} \cdot (x^2 + (-a \cdot b)^{(1/2)}) / b} \cdot \frac{1}{b \cdot d - (a \cdot d - b \cdot c) / b} \cdot \left(\frac{x^2 + (-a \cdot b)^{(1/2)}}{b}\right) - \frac{1}{8} \cdot \frac{1}{a^3 \cdot b} \cdot (-a \cdot b)^{(1/2)} \cdot \frac{1}{(a \cdot d - b \cdot c)} \cdot \frac{1}{(x^2 - (-a \cdot b)^{(1/2)}) / b} \cdot \left(\frac{x^2 - (-a \cdot b)^{(1/2)}}{b}\right)^{2 \cdot d + 2 \cdot (-a \cdot b)^{(1/2)} \cdot (x^2 - (-a \cdot b)^{(1/2)}) / b} \cdot \frac{1}{b \cdot d - (a \cdot d - b \cdot c) / b} \cdot \left(\frac{x^2 - (-a \cdot b)^{(1/2)}}{b}\right) - \frac{1}{8} \cdot \frac{1}{a^2 \cdot b} \cdot \frac{d}{(a \cdot d - b \cdot c)} \cdot \frac{1}{(-a \cdot d - b \cdot c) / b} \cdot \ln\left(\frac{2 \cdot (-a \cdot b)^{(1/2)} \cdot (x^2 + (-a \cdot b)^{(1/2)}) / b}{b \cdot d - 2 \cdot (a \cdot d - b \cdot c) / b + 2 \cdot (-a \cdot d - b \cdot c) / b} \cdot \left(\frac{x^2 + (-a \cdot b)^{(1/2)}}{b}\right)^{2 \cdot d - 2 \cdot (-a \cdot b)^{(1/2)} \cdot (x^2 + (-a \cdot b)^{(1/2)}) / b} \cdot \frac{1}{b \cdot d - (a \cdot d - b \cdot c) / b} \cdot \left(\frac{x^2 + (-a \cdot b)^{(1/2)}}{b}\right)\right) + \frac{1}{8} \cdot \frac{1}{a^2 \cdot b} \cdot \frac{d}{(a \cdot d - b \cdot c)} \cdot \frac{1}{(-a \cdot d - b \cdot c) / b} \cdot \ln\left(\frac{2 \cdot (-a \cdot b)^{(1/2)} \cdot (x^2 - (-a \cdot b)^{(1/2)}) / b}{b \cdot d - 2 \cdot (a \cdot d - b \cdot c) / b + 2 \cdot (-a \cdot d - b \cdot c) / b} \cdot \left(\frac{x^2 - (-a \cdot b)^{(1/2)}}{b}\right)^{2 \cdot d + 2 \cdot (-a \cdot b)^{(1/2)} \cdot (x^2 - (-a \cdot b)^{(1/2)}) / b} \cdot \frac{1}{b \cdot d - (a \cdot d - b \cdot c) / b} \cdot \left(\frac{x^2 - (-a \cdot b)^{(1/2)}}{b}\right)\right) + \frac{1}{a^3 \cdot c} \cdot \ln\left(\frac{(2 \cdot c + 2 \cdot (d \cdot x^4 + c)^{(1/2)} \cdot c)^{(1/2)}}{x^2}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^2 \sqrt{dx^4 + c} x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^5), x)
```

```
mupad [B] time = 7.05, size = 3822, normalized size = 20.66
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (((c + d*x^4)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 - a*c*d)) + (b*(c + d*x^4)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/((c + d*x^4)*(2*a*d - 4*b*c) + 2*b*(c + d*x^4)^2 + 2*b*c^2 - 2*a*c*d) + (atan((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c))*((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4)))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c))*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5)))/(64*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c))*((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4)))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c))*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5)))/(64*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^6)/32 + b^7*c^3*d^3 - (3*a*b^6*c^2*d^4)/2 + (3*a^2*b^5*c*d^5)/16)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c))*((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4)))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c))*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5)))/(64*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2) + (((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c))*((c + d*x^4)^(1/2)*(a^4*b^3
```


$$\begin{aligned}
& 3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4) / (8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^{(1/2)} * (5*a*d - 4*b*c) * ((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5) / (a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^{(1/2)} * (c + d*x^4)^{(1/2)} * (5*a*d - 4*b*c) * (128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5)) / (64*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d) * (a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))) / (8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))) * (-b^3*(a*d - b*c)^3)^{(1/2)} * (5*a*d - 4*b*c) * i) / (4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan((((c + d*x^4)^{(1/2)} * (a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4) / (8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5) / (a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^4)^{(1/2)} * (a*d + 4*b*c) * (128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5)) / (64*a^3*(c^3)^{(1/2)} * (a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d))) * (a*d + 4*b*c)) / (8*a^3*(c^3)^{(1/2)})) * (a*d + 4*b*c) * i) / (8*a^3*(c^3)^{(1/2)} + (((c + d*x^4)^{(1/2)} * (a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4) / (8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5) / (a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^4)^{(1/2)} * (a*d + 4*b*c) * (128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5)) / (64*a^3*(c^3)^{(1/2)} * (a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d))) * (a*d + 4*b*c)) / (8*a^3*(c^3)^{(1/2)})) * (a*d + 4*b*c) * i) / (8*a^3*(c^3)^{(1/2)})) / (((5*a^3*b^4*d^6) / 32 + b^7*c^3*d^3 - (3*a*b^6*c^2*d^4) / 2 + (3*a^2*b^5*c*d^5) / 16) / (a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - (((c + d*x^4)^{(1/2)} * (a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4) / (8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5) / (a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^4)^{(1/2)} * (a*d + 4*b*c) * (128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5)) / (64*a^3*(c^3)^{(1/2)} * (a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d))) * (a*d + 4*b*c)) / (8*a^3*(c^3)^{(1/2)})) * (a*d + 4*b*c) * i) / (8*a^3*(c^3)^{(1/2)})) + (((c + d*x^4)^{(1/2)} * (a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4) / (8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5) / (a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^4)^{(1/2)} * (a*d + 4*b*c) * (128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5)) / (64*a^3*(c^3)^{(1/2)} * (a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d))) * (a*d + 4*b*c)) / (8*a^3*(c^3)^{(1/2)})) * (a*d + 4*b*c) * i) / (4*a^3*(c^3)^{(1/2)}))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 (a + bx^4)^2 \sqrt{c + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2), x)

[Out] Integral(1/(x**5*(a + b*x**4)**2*sqrt(c + d*x**4)), x)

$$3.537 \quad \int \frac{x^{13}}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$$

Optimal. Leaf size=191

$$\frac{a^{3/2}(5bc - 4ad) \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}} - \frac{(4ad+bc) \tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^3d^{3/2}} + \frac{x^2\sqrt{c+dx^4}(bc-2ad)}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}$$

Rubi [A] time = 0.37, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {465, 470, 582, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}(5bc - 4ad) \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}} - \frac{(4ad+bc) \tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^3d^{3/2}} + \frac{x^2\sqrt{c+dx^4}(bc-2ad)}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] ((b*c - 2*a*d)*x^2*Sqrt[c + d*x^4])/(4*b^2*d*(b*c - a*d)) + (a*x^6*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) + (a^(3/2)*(5*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*b^3*(b*c - a*d)^(3/2)) - ((b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^3*d^(3/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(f*g^(n - 1)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1) + 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13}}{(a+bx^4)^2 \sqrt{c+dx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^6}{(a+bx^2)^2 \sqrt{c+dx^2}} dx, x, x^2 \right) \\
&= \frac{ax^6 \sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\text{Subst} \left(\int \frac{x^2(3ac-2(bc-2ad)x^2)}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{4b(bc-ad)} \\
&= \frac{(bc-2ad)x^2 \sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6 \sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{\text{Subst} \left(\int \frac{-2ac(bc-2ad)-2(bc-ad)(bc+ad)}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{8b^2d(bc-ad)} \\
&= \frac{(bc-2ad)x^2 \sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6 \sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{(a^2(5bc-4ad)) \text{Subst} \left(\int \frac{1}{(a+bx^2)} dx, x, x^2 \right)}{4b^3(bc-ad)} \\
&= \frac{(bc-2ad)x^2 \sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6 \sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{(a^2(5bc-4ad)) \text{Subst} \left(\int \frac{1}{a-(-bc-x^2)} dx, x, x^2 \right)}{4b^3(bc-ad)} \\
&= \frac{(bc-2ad)x^2 \sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6 \sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{a^{3/2}(5bc-4ad) \tan^{-1} \left(\frac{\sqrt{bc-ad} x^2}{\sqrt{a} \sqrt{c+dx^4}} \right)}{4b^3(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.57, size = 150, normalized size = 0.79

$$\frac{a^{3/2}(5bc-4ad) \tan^{-1} \left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}} \right) + bx^2 \sqrt{c+dx^4} \left(\frac{a^2}{(a+bx^4)(ad-bc)} + \frac{1}{d} \right) - \frac{(4ad+bc) \log(\sqrt{a} \sqrt{c+dx^4} + dx^2)}{d^{3/2}}}{4b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] (b*x^2*Sqrt[c + d*x^4]*(d^(-1) + a^2/((-b*c) + a*d)*(a + b*x^4))) + (a^(3/2)*(5*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])]) / (b*c - a*d)^(3/2) - ((b*c + 4*a*d)*Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]])/d^(3/2)/(4*b^3)

IntegrateAlgebraic [A] time = 2.29, size = 208, normalized size = 1.09

$$\frac{(5a^{3/2}bc - 4a^{5/2}d) \tan^{-1} \left(\frac{a\sqrt{d} + bx^2\sqrt{c+dx^4} + b\sqrt{d}x^4}{\sqrt{a}\sqrt{bc-ad}} \right) - \sqrt{c+dx^4} (2a^2dx^2 - abcx^2 + abdx^6 - b^2cx^6)}{4b^3(bc-ad)^{3/2}} + \frac{(-4ad - bc) \log(\sqrt{c+dx^4} + \sqrt{d}x^2)}{4b^3d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(Sqrt[c + d*x^4]*(-(a*b*c*x^2) + 2*a^2*d*x^2 - b^2*c*x^6 + a*b*d*x^6))
/(b^2*d*(b*c - a*d)*(a + b*x^4)) + ((5*a^(3/2)*b*c - 4*a^(5/2)*d)*ArcTan[(a
*Sqrt[d] + b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])
]/(4*b^3*(b*c - a*d)^(3/2)) + ((-(b*c) - 4*a*d)*Log[Sqrt[d]*x^2 + Sqrt[c +
d*x^4]])/(4*b^3*d^(3/2))
```

fricas [A] time = 1.94, size = 1386, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*
a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) +
(5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a/(b
*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^4 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2
- a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4
+ a^2)) + 4*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*sq
rt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4), 1/
16*(4*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2
*b*d^2)*x^4)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + (5*a^2*b*c*d^
2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a/(b*c - a*d))*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^
2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)
)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*((b
^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*sqrt(d*x^4 + c))/(
a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4), -1/8*((5*a^2*b*c*
d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(a/(b*c - a*d))*ar
ctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d)))/(a*
d*x^6 + a*c*x^2)) - (a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b
^2*c*d - 4*a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)
*x^2 - c) - 2*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*s
qrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4),
1/8*(2*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^
2*b*d^2)*x^4)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - (5*a^2*b*c*d^
2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(a/(b*c - a*d))*arct
an(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d)))/(a*d*
x^6 + a*c*x^2)) + 2*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*
x^2)*sqrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*
x^4)]
```

giac [B] time = 0.59, size = 337, normalized size = 1.76

$$\frac{(5a^2bc\sqrt{d} - 4a^3d^{\frac{3}{2}}) \arctan\left(\frac{(\sqrt{d}x^2 - \sqrt{dx^4+c})^{\frac{3}{2}} b - bc + 2ad}{2\sqrt{abcd - a^2d^2}}\right) + \frac{\sqrt{dx^4+c}x^2}{4b^2d} + \frac{(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 a^2bc\sqrt{d} - 2(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 a^{\frac{3}{2}}d^{\frac{3}{2}} - a^2bc^2\sqrt{d}}{2((\sqrt{d}x^2 - \sqrt{dx^4+c})^4 b - 2(\sqrt{d}x^2 - \sqrt{dx^4+c})^3 bc + 4(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 ad + bc^2)(b^4c - ab^3d)} + \frac{(bc\sqrt{d} + 4ad^{\frac{3}{2}}) \log\left(\frac{(\sqrt{d}x^2 - \sqrt{dx^4+c})^2}{8b^3d^2}\right)}{8b^3d^2}}{4(b^4c - ab^3d)\sqrt{abcd - a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out]
$$-1/4*(5*a^2*b*c*\sqrt{d} - 4*a^3*d^{(3/2)})*\arctan(1/2*((\sqrt{d}*x^2 - \sqrt{d*x^4 + c})^2*b - b*c + 2*a*d)/\sqrt{a*b*c*d - a^2*d^2})/((b^4*c - a*b^3*d)*\sqrt{a*b*c*d - a^2*d^2}) + 1/4*\sqrt{d*x^4 + c}*x^2/(b^2*d) + 1/2*((\sqrt{d}*x^2 - \sqrt{d*x^4 + c})^2*a^2*b*c*\sqrt{d} - 2*(\sqrt{d}*x^2 - \sqrt{d*x^4 + c})^2*a^3*d^{(3/2)} - a^2*b*c^2*\sqrt{d})/(((\sqrt{d}*x^2 - \sqrt{d*x^4 + c})^4*b - 2*(\sqrt{d}*x^2 - \sqrt{d*x^4 + c})^2*b*c + 4*(\sqrt{d}*x^2 - \sqrt{d*x^4 + c})^2*a*d + b*c^2)*(b^4*c - a*b^3*d)) + 1/8*(b*c*\sqrt{d} + 4*a*d^{(3/2)})*\log((\sqrt{d}*x^2 - \sqrt{d*x^4 + c})^2/(b^3*d^2))$$

maple [B] time = 0.27, size = 953, normalized size = 4.99

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)

[Out]
$$1/4/b^2*x^2/d*(d*x^4+c)^{(1/2)} - 1/4/b^2*c/d^{(3/2)}*\ln(d^{(1/2)}*x^2+(d*x^4+c)^{(1/2)}) - 1/b^3*a*\ln(d^{(1/2)}*x^2+(d*x^4+c)^{(1/2)})/d^{(1/2)} + 5/8*a^2/b^3/(-a*b)^{(1/2)}/(-a*d-b*c)/b^{(1/2)}*\ln((-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^2*d-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}/(x^2+(-a*b)^{(1/2)}/b)) - 5/8*a^2/b^3/(-a*b)^{(1/2)}/(-a*d-b*c)/b^{(1/2)}*\ln((2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^2*d+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}/(x^2-(-a*b)^{(1/2)}/b)) + 1/8*a^2/b^3/(a*d-b*c)/(x^2-(-a*b)^{(1/2)}/b)*((x^2-(-a*b)^{(1/2)}/b)^2*d+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)} - 1/8*a^2/b^4*(-a*b)^{(1/2)}*d/(a*d-b*c)/(-a*d-b*c)/b^{(1/2)}*\ln((2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^2*d+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}/(x^2-(-a*b)^{(1/2)}/b)) + 1/8*a^2/b^3/(a*d-b*c)/(x^2+(-a*b)^{(1/2)}/b)*((x^2+(-a*b)^{(1/2)}/b)^2*d-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)} + 1/8*a^2/b^4*(-a*b)^{(1/2)}*d/(a*d-b*c)/(-a*d-b*c)/b^{(1/2)}*\ln((-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^2*d-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}/(x^2+(-a*b)^{(1/2)}/b))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{(bx^4 + a)^2 \sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^13/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13}}{(bx^4 + a)^2 \sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^13/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)

[Out] int(x^13/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{(a + bx^4)^2 \sqrt{c + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**13/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)

[Out] Integral(x**13/((a + b*x**4)**2*sqrt(c + d*x**4)), x)

$$3.538 \quad \int \frac{x^9}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$$

Optimal. Leaf size=141

$$-\frac{\sqrt{a}(3bc-2ad) \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}} + \frac{ax^2\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

Rubi [A] time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {465, 470, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}(3bc-2ad) \tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}} + \frac{ax^2\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] (a*x^2*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (Sqrt[a]*(3*b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*b^2*(b*c - a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(2*b^2*Sqrt[d])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b

, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 470

Int[((e_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))
^(q_)), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]

Rule 523

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x_)
^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d,
e, f, n}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{(a+bx^4)^2 \sqrt{c+dx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{(a+bx^2)^2 \sqrt{c+dx^2}} dx, x, x^2 \right) \\
&= \frac{ax^2 \sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\text{Subst} \left(\int \frac{ac-2(bc-ad)x^2}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{4b(bc-ad)} \\
&= \frac{ax^2 \sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{\text{Subst} \left(\int \frac{1}{\sqrt{c+dx^2}} dx, x, x^2 \right)}{2b^2} - \frac{(a(3bc-2ad)) \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{4b^2(bc-ad)} \\
&= \frac{ax^2 \sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{\text{Subst} \left(\int \frac{1}{1-dx^2} dx, x, \frac{x^2}{\sqrt{c+dx^4}} \right)}{2b^2} - \frac{(a(3bc-2ad)) \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{4b^2(bc-ad)} \\
&= \frac{ax^2 \sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\sqrt{a}(3bc-2ad) \tan^{-1} \left(\frac{\sqrt{bc-ad} x^2}{\sqrt{a} \sqrt{c+dx^4}} \right)}{4b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1} \left(\frac{\sqrt{d} x^2}{\sqrt{c+dx^4}} \right)}{2b^2 \sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 135, normalized size = 0.96

$$\frac{\frac{abx^2 \sqrt{c+dx^4}}{(a+bx^4)(bc-ad)} + \frac{\sqrt{a}(2ad-3bc) \tan^{-1} \left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}} \right)}{(bc-ad)^{3/2}} + \frac{2 \log(\sqrt{d} \sqrt{c+dx^4} + dx^2)}{\sqrt{d}}}{4b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] ((a*b*x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (Sqrt[a]*(-3*b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(b*c - a*d)^(3/2) + (2*Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]])/Sqrt[d])/(4*b^2)

IntegrateAlgebraic [A] time = 1.59, size = 166, normalized size = 1.18

$$\frac{(2a^{3/2}d - 3\sqrt{a}bc) \tan^{-1} \left(\frac{a\sqrt{d} + bx^2\sqrt{c+dx^4} + b\sqrt{d}x^4}{\sqrt{a}\sqrt{bc-ad}} \right)}{4b^2(bc-ad)^{3/2}} + \frac{ax^2 \sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)} + \frac{\log(\sqrt{c+dx^4} + \sqrt{d}x^2)}{2b^2 \sqrt{d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

```
[Out] (a*x^2*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) + ((-3*Sqrt[a]*b*c +
2*a^(3/2)*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*b^2*(b*c - a*d)^(3/2)) + Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4])/(2*b^2*Sqrt[d])
```

fricas [A] time = 0.84, size = 1077, normalized size = 7.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(d*x^4 + c)*a*b*d*x^2 + 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/16*(4*sqrt(d*x^4 + c)*a*b*d*x^2 - 8*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*a*b*d*x^2 + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + 2*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*a*b*d*x^2 - 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4)]
```

giac [B] time = 0.54, size = 298, normalized size = 2.11

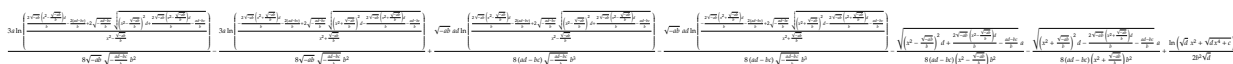
$$\frac{(3abc\sqrt{d} - 2a^2d^{\frac{3}{2}})\arctan\left(\frac{(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 b - bc + 2ad}{2\sqrt{abcd - a^2d^2}}\right)}{4(b^3c - ab^2d)\sqrt{abcd - a^2d^2}} - \frac{(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 abc\sqrt{d} - 2(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 a^2d^{\frac{3}{2}} - abc^2\sqrt{d}}{2\left((\sqrt{d}x^2 - \sqrt{dx^4+c})^4 b - 2(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 bc + 4(\sqrt{d}x^2 - \sqrt{dx^4+c})^2 ad + bc^2\right)(b^3c - ab^2d)} - \frac{\log\left(\frac{(\sqrt{d}x^2 - \sqrt{dx^4+c})^2}{4b^2\sqrt{d}}\right)}{4b^2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

[Out] $-1/4*(3*a*b*c*\sqrt{d} - 2*a^2*d^{(3/2)})*\arctan(-1/2*((\sqrt{d}*x^2 - \sqrt{d*x^4 + c})^2*b - b*c + 2*a*d)/\sqrt{a*b*c*d - a^2*d^2})/((b^3*c - a*b^2*d)*\sqrt{a*b*c*d - a^2*d^2}) - 1/2*((\sqrt{d}*x^2 - \sqrt{d*x^4 + c})^2*a*b*c*\sqrt{d} - 2*(\sqrt{d}*x^2 - \sqrt{d*x^4 + c})^2*a^2*d^{(3/2)} - a*b*c^2*\sqrt{d})/(((\sqrt{d}*x^2 - \sqrt{d*x^4 + c})^4*b - 2*(\sqrt{d}*x^2 - \sqrt{d*x^4 + c})^2*b*c + 4*(\sqrt{d}*x^2 - \sqrt{d*x^4 + c})^2*a*d + b*c^2)*(b^3*c - a*b^2*d)) - 1/4*\log((\sqrt{d}*x^2 - \sqrt{d*x^4 + c})^2)/(b^2*\sqrt{d}))$

maple [B] time = 0.24, size = 893, normalized size = 6.33



Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^9/(b*x^4+a)^2/(d*x^4+c)^{(1/2)}, x)$

[Out] $1/2/b^2*\ln(d^{(1/2)}*x^2+(d*x^4+c)^{(1/2)})/d^{(1/2)}-3/8*a/b^2/(-a*b)^{(1/2)}/(-(a*d-b*c)/b)^{(1/2)}*\ln((-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^2*d-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)}/b)+3/8*a/b^2/(-a*b)^{(1/2)}/(-(a*d-b*c)/b)^{(1/2)}*\ln((2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^2*d+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)}/b)-1/8*a/b^2/(a*d-b*c)/(x^2-(-a*b)^{(1/2)}/b)*((x^2-(-a*b)^{(1/2)}/b)^2*d+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}+1/8*a/b^3*(-a*b)^{(1/2)}*d/(a*d-b*c)/(-(a*d-b*c)/b)^{(1/2)}*\ln((2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)}/b)^2*d+2*(-a*b)^{(1/2)}*(x^2-(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)}/b)-1/8*a/b^2/(a*d-b*c)/(x^2+(-a*b)^{(1/2)}/b)*((x^2+(-a*b)^{(1/2)}/b)^2*d-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)}-1/8*a/b^3*(-a*b)^{(1/2)}*d/(a*d-b*c)/(-(a*d-b*c)/b)^{(1/2)}*\ln((-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)}/b)^2*d-2*(-a*b)^{(1/2)}*(x^2+(-a*b)^{(1/2)}/b)/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)}/b))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(bx^4 + a)^2 \sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^9/(b*x^4+a)^2/(d*x^4+c)^{(1/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}(x^9/((b*x^4 + a)^2*\sqrt{d*x^4 + c}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{(bx^4 + a)^2 \sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)

[Out] int(x^9/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(a + bx^4)^2 \sqrt{c + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9/(b*x**4+a)**2/(d*x**4+c)**(1/2), x)

[Out] Integral(x**9/((a + b*x**4)**2*sqrt(c + d*x**4)), x)

$$3.539 \quad \int \frac{x^5}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$$

Optimal. Leaf size=93

$$\frac{c \tan^{-1} \left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}} \right)}{4\sqrt{a} (bc-ad)^{3/2}} - \frac{x^2 \sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {465, 471, 12, 377, 205}

$$\frac{c \tan^{-1} \left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}} \right)}{4\sqrt{a} (bc-ad)^{3/2}} - \frac{x^2 \sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] -(x^2*Sqrt[c + d*x^4])/(4*(b*c - a*d)*(a + b*x^4)) + (c*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*Sqrt[a]*(b*c - a*d)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^(m + 1)/k -

1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 471

Int[((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{(a + bx^4)^2 \sqrt{c + dx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(a + bx^2)^2 \sqrt{c + dx^2}} dx, x, x^2 \right) \\
 &= -\frac{x^2 \sqrt{c + dx^4}}{4(bc - ad)(a + bx^4)} + \frac{\text{Subst} \left(\int \frac{c}{(a + bx^2) \sqrt{c + dx^2}} dx, x, x^2 \right)}{4(bc - ad)} \\
 &= -\frac{x^2 \sqrt{c + dx^4}}{4(bc - ad)(a + bx^4)} + \frac{c \text{Subst} \left(\int \frac{1}{(a + bx^2) \sqrt{c + dx^2}} dx, x, x^2 \right)}{4(bc - ad)} \\
 &= -\frac{x^2 \sqrt{c + dx^4}}{4(bc - ad)(a + bx^4)} + \frac{c \text{Subst} \left(\int \frac{1}{a - (-bc + ad)x^2} dx, x, \frac{x^2}{\sqrt{c + dx^4}} \right)}{4(bc - ad)} \\
 &= -\frac{x^2 \sqrt{c + dx^4}}{4(bc - ad)(a + bx^4)} + \frac{c \tan^{-1} \left(\frac{\sqrt{bc - ad} x^2}{\sqrt{a} \sqrt{c + dx^4}} \right)}{4\sqrt{a} (bc - ad)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.58, size = 124, normalized size = 1.33

$$\frac{\sqrt{c + dx^4} \left(-\frac{x^4(bc - ad)}{a + bx^4} - \frac{c \sqrt{x^4 \left(\frac{d}{c} - \frac{b}{a} \right)} \tanh^{-1} \left(\frac{\sqrt{x^4 \left(\frac{d}{c} - \frac{b}{a} \right)}}{\sqrt{\frac{dx^4}{c} + 1}} \right)}{\sqrt{\frac{dx^4}{c} + 1}} \right)}{4x^2(bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] (Sqrt[c + d*x^4]*(-(((b*c - a*d)*x^4)/(a + b*x^4)) - (c*Sqrt[(-(b/a) + d/c)*x^4]*ArcTanh[Sqrt[(-(b/a) + d/c)*x^4]/Sqrt[1 + (d*x^4)/c]])/Sqrt[1 + (d*x^4)/c]))/(4*(b*c - a*d)^2*x^2)

IntegrateAlgebraic [A] time = 1.04, size = 145, normalized size = 1.56

$$\frac{c \tan^{-1} \left(\frac{b\sqrt{d}x^4}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}} \right)}{4\sqrt{a}(bc-ad)^{3/2}} - \frac{x^2\sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] -1/4*(x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (c*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^2*Sqrt[c + d*x^4))/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*Sqrt[a]*(b*c - a*d)^(3/2))

fricas [B] time = 0.54, size = 426, normalized size = 4.58

$$\left[\frac{4\sqrt{dx^4+c}(abc-a^2d)x^2 - (bcx^4+ac)\sqrt{-abc+a^2d} \log\left(\frac{(d^2-8abcd+8a^2d^2)x^8-2(3abc^2-4a^2cd)^4+a^2c^2+4((bc-2ad)x^4-ac)\sqrt{dx^4+c}\sqrt{-abc-a^2d}}{b^2x^8+2abx^4+a^2}\right)}{16(a^2b^2c^2-2a^2bcd+a^4d^2+(ab^3c^2-2a^2b^2cd+a^3bd^2)x^4)} \right], \left[\frac{2\sqrt{dx^4+c}(abc-a^2d)x^2 - (bcx^4+ac)\sqrt{-abc+a^2d} \arctan\left(\frac{(bc-2ad)x^4-ac}{2(abcd-a^2d^2)x^4+(abc-a^2d)x^2}\right)}{8(a^2b^2c^2-2a^2bcd+a^4d^2+(ab^3c^2-2a^2b^2cd+a^3bd^2)x^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [-1/16*(4*sqrt(d*x^4 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4), -1/8*(2*sqrt(d*x^4 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4)]

giac [B] time = 1.63, size = 244, normalized size = 2.62

$$\frac{c\sqrt{d} \arctan\left(-\frac{(\sqrt{d}x^2-\sqrt{dx^4+c})^2 b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{4\sqrt{abcd-a^2d^2}(bc-ad)} + \frac{(\sqrt{d}x^2-\sqrt{dx^4+c})^2 bc\sqrt{d} - 2(\sqrt{d}x^2-\sqrt{dx^4+c})^2 ad^{\frac{3}{2}} - bc^2\sqrt{d}}{2\left((\sqrt{d}x^2-\sqrt{dx^4+c})^4 b - 2(\sqrt{d}x^2-\sqrt{dx^4+c})^2 bc + 4(\sqrt{d}x^2-\sqrt{dx^4+c})^2 ad + bc^2\right)(b^2c-abd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{4}c\sqrt{d}\arctan\left(\frac{-1/2((\sqrt{d})x^2 - \sqrt{d}x^4 + c))^2b - bc + 2ad}{\sqrt{d}\sqrt{a^2d - a^2d^2}}\right) + \frac{1}{2}\left(\frac{(\sqrt{d})x^2 - \sqrt{d}x^4 + c}{\sqrt{d}\sqrt{a^2d - a^2d^2}}\right)^2b^2c\sqrt{d} - 2\left(\frac{(\sqrt{d})x^2 - \sqrt{d}x^4 + c}{\sqrt{d}\sqrt{a^2d - a^2d^2}}\right)^2a^2d^{3/2} - b^2c^2\sqrt{d}\left(\frac{(\sqrt{d})x^2 - \sqrt{d}x^4 + c}{\sqrt{d}\sqrt{a^2d - a^2d^2}}\right)^4b - 2\left(\frac{(\sqrt{d})x^2 - \sqrt{d}x^4 + c}{\sqrt{d}\sqrt{a^2d - a^2d^2}}\right)^2b^2c + 4\left(\frac{(\sqrt{d})x^2 - \sqrt{d}x^4 + c}{\sqrt{d}\sqrt{a^2d - a^2d^2}}\right)^2a^2d + b^2c^2(b^2c - a^2bd)$

maple [B] time = 0.24, size = 861, normalized size = 9.26

$$\ln\left(\frac{(\sqrt{d})x^2 - \sqrt{d}x^4 + c}{\sqrt{d}\sqrt{a^2d - a^2d^2}}\right) + \ln\left(\frac{(\sqrt{d})x^2 - \sqrt{d}x^4 + c}{\sqrt{d}\sqrt{a^2d - a^2d^2}}\right) + \sqrt{d}\ln\left(\frac{(\sqrt{d})x^2 - \sqrt{d}x^4 + c}{\sqrt{d}\sqrt{a^2d - a^2d^2}}\right) + \sqrt{d}\ln\left(\frac{(\sqrt{d})x^2 - \sqrt{d}x^4 + c}{\sqrt{d}\sqrt{a^2d - a^2d^2}}\right) + \sqrt{\frac{d^2 - c^2}{8(ad-bc)(d^2 - \frac{c^2}{a^2})}} + \sqrt{\frac{d^2 - c^2}{8(ad-bc)(d^2 - \frac{c^2}{a^2})}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)

[Out] $\frac{1}{8}b/(-ab)^{1/2}/(-ad-bc)/b^{1/2}\ln\left(\frac{(-2(-ab)^{1/2}(x^2+(-ab)^{1/2}/b)/bd-2(ad-bc)/b+2(-ad-bc)/b^{1/2}((x^2+(-ab)^{1/2}/b)^2d-2(-ab)^{1/2}(x^2+(-ab)^{1/2}/b)/bd-(ad-bc)/b^{1/2})}{(x^2+(-ab)^{1/2}/b)}\right) - \frac{1}{8}b/(-ab)^{1/2}/(-ad-bc)/b^{1/2}\ln\left(\frac{(2(-ab)^{1/2}(x^2-(-ab)^{1/2}/b)/bd-2(ad-bc)/b+2(-ad-bc)/b^{1/2}((x^2-(-ab)^{1/2}/b)^2d+2(-ab)^{1/2}(x^2-(-ab)^{1/2}/b)/bd-(ad-bc)/b^{1/2})}{(x^2-(-ab)^{1/2}/b)}\right) + \frac{1}{8}b/(ad-bc)/(x^2-(-ab)^{1/2}/b)((x^2-(-ab)^{1/2}/b)^2d+2(-ab)^{1/2}(x^2-(-ab)^{1/2}/b)/bd-(ad-bc)/b^{1/2}) - \frac{1}{8}b^2(-ab)^{1/2}d/(ad-bc)/(-ad-bc)/b^{1/2}\ln\left(\frac{(2(-ab)^{1/2}(x^2-(-ab)^{1/2}/b)/bd-2(ad-bc)/b+2(-ad-bc)/b^{1/2}((x^2-(-ab)^{1/2}/b)^2d+2(-ab)^{1/2}(x^2-(-ab)^{1/2}/b)/bd-(ad-bc)/b^{1/2})}{(x^2-(-ab)^{1/2}/b)}\right) + \frac{1}{8}b/(ad-bc)/(x^2+(-ab)^{1/2}/b)((x^2+(-ab)^{1/2}/b)^2d-2(-ab)^{1/2}(x^2+(-ab)^{1/2}/b)/bd-(ad-bc)/b^{1/2}) + \frac{1}{8}b^2(-ab)^{1/2}d/(ad-bc)/(-ad-bc)/b^{1/2}\ln\left(\frac{(-2(-ab)^{1/2}(x^2+(-ab)^{1/2}/b)/bd-2(ad-bc)/b+2(-ad-bc)/b^{1/2}((x^2+(-ab)^{1/2}/b)^2d-2(-ab)^{1/2}(x^2+(-ab)^{1/2}/b)/bd-(ad-bc)/b^{1/2})}{(x^2+(-ab)^{1/2}/b)}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(bx^4 + a)^2 \sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^5/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{(bx^4 + a)^2 \sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)

[Out] int(x^5/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + bx^4)^2 \sqrt{c + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2), x)

[Out] Integral(x**5/((a + b*x**4)**2*sqrt(c + d*x**4)), x)

$$3.540 \quad \int \frac{x}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$$

Optimal. Leaf size=104

$$\frac{(bc - 2ad) \tan^{-1} \left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}} \right)}{4a^{3/2}(bc - ad)^{3/2}} + \frac{bx^2 \sqrt{c + dx^4}}{4a(a + bx^4)(bc - ad)}$$

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {465, 382, 377, 205}

$$\frac{(bc - 2ad) \tan^{-1} \left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}} \right)}{4a^{3/2}(bc - ad)^{3/2}} + \frac{bx^2 \sqrt{c + dx^4}}{4a(a + bx^4)(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] (b*x^2*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + ((b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(3/2)*(b*c - a*d)^(3/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 382

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[(b*c + n*(p+1)*(b*c - a*d))/(a*n*(p+1)*(b*c - a*d)], Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p+q+2)+1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

Rule 465

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
 [{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x}{(a + bx^4)^2 \sqrt{c + dx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(a + bx^2)^2 \sqrt{c + dx^2}} dx, x, x^2 \right) \\ &= \frac{bx^2 \sqrt{c + dx^4}}{4a(bc - ad)(a + bx^4)} + \frac{(bc - 2ad) \text{Subst} \left(\int \frac{1}{(a + bx^2) \sqrt{c + dx^2}} dx, x, x^2 \right)}{4a(bc - ad)} \\ &= \frac{bx^2 \sqrt{c + dx^4}}{4a(bc - ad)(a + bx^4)} + \frac{(bc - 2ad) \text{Subst} \left(\int \frac{1}{a - (-bc + ad)x^2} dx, x, \frac{x^2}{\sqrt{c + dx^4}} \right)}{4a(bc - ad)} \\ &= \frac{bx^2 \sqrt{c + dx^4}}{4a(bc - ad)(a + bx^4)} + \frac{(bc - 2ad) \tan^{-1} \left(\frac{\sqrt{bc - ad} x^2}{\sqrt{a} \sqrt{c + dx^4}} \right)}{4a^{3/2}(bc - ad)^{3/2}} \end{aligned}$$

Mathematica [C] time = 6.01, size = 407, normalized size = 3.91

$$\frac{x^2 \sqrt{c + dx^4} \left(-30dx^4 \sqrt{\frac{ax^4(c+dx^4)(bc-ad)}{c^2(a+bx^4)^2}} - 45c \sqrt{\frac{ax^4(c+dx^4)(bc-ad)}{c^2(a+bx^4)^2}} + 16dx^4 \left(\frac{x^4(bc-ad)}{c(a+bx^4)} \right)^{5/2} \sqrt{\frac{a(c+dx^4)}{c(a+bx^4)}} {}_2F_1 \left(2, 3; \frac{7}{2}; \frac{(bc-ad)x^4}{c(bx^4+a)} \right) + 16c \left(\frac{x^4(bc-ad)}{c(a+bx^4)} \right)^{5/2} \sqrt{\frac{a(c+dx^4)}{c(a+bx^4)}} {}_2F_1 \left(2, 3; \frac{7}{2}; \frac{(bc-ad)bx^4}{c(bx^4+a)} \right) + 30dx^4 \sin^{-1} \left(\frac{\sqrt{bc-ad} x^2}{\sqrt{c(a+bx^4)}} \right) + 45c \sin^{-1} \left(\frac{\sqrt{bc-ad} x^2}{\sqrt{c(a+bx^4)}} \right) \right)}{60c^2 (a + bx^4)^2 \left(\frac{x^4(bc-ad)}{c(a+bx^4)} \right)^{3/2} \sqrt{\frac{a(c+dx^4)}{c(a+bx^4)}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/((a + b*x^4)^2*Sqrt[c + d*x^4]), x]

[Out] (x^2*Sqrt[c + d*x^4]*(-45*c*Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a + b*x^4)^2)] - 30*d*x^4*Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a + b*x^4)^2)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^4)/(c*(a + b*x^4))]] + 30*d*x^4*ArcSin[Sqrt[((b*c - a*d)*x^4)/(c*(a + b*x^4))]] + 16*c*(((b*c - a*d)*x^4)/(c*(a + b*x^4)))^(5/2)*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4))]*Hypergeometric2F1[2, 3, 7/2, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 16*d*x^4*(((b*c - a*d)*x^4)/(c*(a + b*x^4)))^(5/2)*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4))]*Hypergeometric2F1[2, 3, 7/2, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(60*c^2*(((b*c - a*d)*x

$\sqrt[4]{(c*(a + b*x^4))}^{(3/2)}*(a + b*x^4)^2*\text{Sqrt}[(a*(c + d*x^4))/(c*(a + b*x^4))]$

IntegrateAlgebraic [A] time = 0.64, size = 124, normalized size = 1.19

$$\frac{(bc - 2ad) \tan^{-1} \left(\frac{a\sqrt{d} + bx^2\sqrt{c+dx^4} + b\sqrt{d}x^4}{\sqrt{a}\sqrt{bc-ad}} \right)}{4a^{3/2}(bc - ad)^{3/2}} - \frac{bx^2\sqrt{c + dx^4}}{4a(a + bx^4)(ad - bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] $-1/4*(b*x^2*\text{Sqrt}[c + d*x^4])/(a*(-(b*c) + a*d)*(a + b*x^4)) + ((b*c - 2*a*d)*\text{ArcTan}[(a*\text{Sqrt}[d] + b*\text{Sqrt}[d]*x^4 + b*x^2*\text{Sqrt}[c + d*x^4])]/(\text{Sqrt}[a]*\text{Sqrt}[b*c - a*d]))/(4*a^{(3/2)}*(b*c - a*d)^{(3/2)})$

fricas [B] time = 0.60, size = 467, normalized size = 4.49

$$\frac{4\sqrt{dx^4+c}(a^2b^2c-a^2bd)^2 - ((b^2c-2abd)x^4 + abc - 2a^2d)\sqrt{-abc+a^2d} \log\left(\frac{(b^2-8abcd+8a^2d^2)^2 - 2(3abc^2-4a^2cd)^2 + a^2c^2 - 4((bc-2ad)(a^2-ac^2)\sqrt{dx^4+c}\sqrt{-abc+a^2d})}{b^2a^2+2abcd+a^2d^2}\right)}{16(a^2b^2c^2-2a^4bcd+a^5d^2+(a^2b^3c^2-2a^3b^2cd+a^4bd^2)x^4)} + \frac{2\sqrt{dx^4+c}(a^2b^2c-a^2bd)^2 + ((b^2c-2abd)x^4 + abc - 2a^2d)\sqrt{-abc-a^2d} \arctan\left(\frac{(bc-2ad)(a^2-ac^2)\sqrt{dx^4+c}\sqrt{-abc-a^2d}}{2((abcd-a^2d^2)^2+(abc^2-a^2d)^2)}\right)}{8(a^2b^2c^2-2a^4bcd+a^5d^2+(a^2b^3c^2-2a^3b^2cd+a^4bd^2)x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] $[1/16*(4*\text{sqrt}(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 - ((b^2*c - 2*a*b*d)*x^4 + a*b*c - 2*a^2*d)*\text{sqrt}(-a*b*c + a^2*d)*\log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*\text{sqrt}(d*x^4 + c)*\text{sqrt}(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^4), 1/8*(2*\text{sqrt}(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 + ((b^2*c - 2*a*b*d)*x^4 + a*b*c - 2*a^2*d)*\text{sqrt}(a*b*c - a^2*d)*\arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*\text{sqrt}(d*x^4 + c)*\text{sqrt}(a*b*c - a^2*d))/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^4)]$

giac [B] time = 0.38, size = 237, normalized size = 2.28

$$-\frac{1}{4}d^{\frac{3}{2}} \left(\frac{(bc - 2ad) \arctan \left(\frac{(\sqrt{dx^2 - \sqrt{dx^4 + c}})^2 b - bc + 2ad}{2\sqrt{abcd - a^2d^2}} \right)}{(abcd - a^2d^2)^{\frac{3}{2}}} + \frac{2 \left((\sqrt{dx^2 - \sqrt{dx^4 + c}})^2 bc - 2(\sqrt{dx^2 - \sqrt{dx^4 + c}})^2 ad - bc^2 \right)}{\left((\sqrt{dx^2 - \sqrt{dx^4 + c}})^4 b - 2(\sqrt{dx^2 - \sqrt{dx^4 + c}})^2 bc + 4(\sqrt{dx^2 - \sqrt{dx^4 + c}})^2 ad + bc^2 \right) (abcd - a^2d^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] $-1/4*d^{(3/2)}*((b*c - 2*a*d)*\arctan(1/2*((\sqrt{d})*x^2 - \sqrt{d*x^4 + c}))^2*b - b*c + 2*a*d)/\sqrt{a*b*c*d - a^2*d^2})/(a*b*c*d - a^2*d^2)^{(3/2)} + 2*((\sqrt{d})*x^2 - \sqrt{d*x^4 + c})^2*b*c - 2*((\sqrt{d})*x^2 - \sqrt{d*x^4 + c})^2*a*d - b*c^2)/(((\sqrt{d})*x^2 - \sqrt{d*x^4 + c})^4*b - 2*((\sqrt{d})*x^2 - \sqrt{d*x^4 + c})^2*b*c + 4*((\sqrt{d})*x^2 - \sqrt{d*x^4 + c})^2*a*d + b*c^2)*(a*b*c*d - a^2*d^2))$

maple [B] time = 0.19, size = 867, normalized size = 8.34

$$\frac{\sqrt{ab} \operatorname{atan}\left(\frac{2\sqrt{d}x^2 - \sqrt{d^2x^4 + c}}{\sqrt{d}}\right)}{8(ad-bc)\sqrt{\frac{cd}{ab}}} + \frac{\sqrt{ab} \operatorname{atan}\left(\frac{2\sqrt{d}x^2 - \sqrt{d^2x^4 + c}}{\sqrt{d}}\right)}{8(ad-bc)\sqrt{\frac{cd}{ab}}} + \frac{\ln\left(\frac{2\sqrt{d}x^2 - \sqrt{d^2x^4 + c}}{\sqrt{d}}\right)}{8\sqrt{ab}\sqrt{\frac{cd}{ab}}x} + \frac{\ln\left(\frac{2\sqrt{d}x^2 - \sqrt{d^2x^4 + c}}{\sqrt{d}}\right)}{8\sqrt{ab}\sqrt{\frac{cd}{ab}}x} + \frac{\sqrt{\frac{cd}{ab}} \operatorname{atan}\left(\frac{2\sqrt{d}x^2 - \sqrt{d^2x^4 + c}}{\sqrt{d}}\right)}{8(ad-bc)\left(x^2 - \frac{cd}{ab}\right)} + \frac{\sqrt{\frac{cd}{ab}} \operatorname{atan}\left(\frac{2\sqrt{d}x^2 - \sqrt{d^2x^4 + c}}{\sqrt{d}}\right)}{8(ad-bc)\left(x^2 - \frac{cd}{ab}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}(x/(b*x^4+a)^2/(d*x^4+c)^{(1/2)}, x)$

[Out] $-1/8/a/(a*d-b*c)/(x^2-(-a*b)^{(1/2)/b})*((x^2-(-a*b)^{(1/2)/b})^2*d+2*(-a*b)^{(1/2)/b})*(x^2-(-a*b)^{(1/2)/b})/b*d-(a*d-b*c)/b)^{(1/2)}+1/8/b/a*(-a*b)^{(1/2)/b}/(a*d-b*c)/(-a*d-b*c)/b)^{(1/2)}*\ln((2*(-a*b)^{(1/2)/b})*(x^2-(-a*b)^{(1/2)/b})/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)/b})^2*d+2*(-a*b)^{(1/2)/b})*(x^2-(-a*b)^{(1/2)/b})/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)/b})+1/8/a/(-a*b)^{(1/2)/b}/(-a*d-b*c)/b)^{(1/2)}*\ln((-2*(-a*b)^{(1/2)/b})*(x^2+(-a*b)^{(1/2)/b})/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)/b})^2*d-2*(-a*b)^{(1/2)/b})*(x^2+(-a*b)^{(1/2)/b})/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)/b})-1/8/a/(-a*b)^{(1/2)/b}/(-a*d-b*c)/b)^{(1/2)}*\ln((2*(-a*b)^{(1/2)/b})*(x^2-(-a*b)^{(1/2)/b})/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2-(-a*b)^{(1/2)/b})^2*d+2*(-a*b)^{(1/2)/b})*(x^2-(-a*b)^{(1/2)/b})/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2-(-a*b)^{(1/2)/b})-1/8/a/(a*d-b*c)/(x^2+(-a*b)^{(1/2)/b})*((x^2+(-a*b)^{(1/2)/b})^2*d-2*(-a*b)^{(1/2)/b})*(x^2+(-a*b)^{(1/2)/b})/b*d-(a*d-b*c)/b)^{(1/2)}-1/8/b/a*(-a*b)^{(1/2)/b}/(a*d-b*c)/(-a*d-b*c)/b)^{(1/2)}*\ln((-2*(-a*b)^{(1/2)/b})*(x^2+(-a*b)^{(1/2)/b})/b*d-2*(a*d-b*c)/b+2*(-a*d-b*c)/b)^{(1/2)}*((x^2+(-a*b)^{(1/2)/b})^2*d-2*(-a*b)^{(1/2)/b})*(x^2+(-a*b)^{(1/2)/b})/b*d-(a*d-b*c)/b)^{(1/2)})/(x^2+(-a*b)^{(1/2)/b})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^4 + a)^2 \sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(x/(b*x^4+a)^2/(d*x^4+c)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] $\operatorname{integrate}(x/((b*x^4 + a)^2*\sqrt{d*x^4 + c}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(bx^4 + a)^2 \sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)`

[Out] `int(x/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + bx^4)^2 \sqrt{c + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)`

[Out] `Integral(x/((a + b*x**4)**2*sqrt(c + d*x**4)), x)`

$$3.541 \quad \int \frac{1}{x^3(a+bx^4)^2 \sqrt{c+dx^4}} dx$$

Optimal. Leaf size=149

$$\frac{b(3bc - 4ad) \tan^{-1}\left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}} - \frac{\sqrt{c+dx^4}(3bc-2ad)}{4a^2cx^2(bc-ad)} + \frac{b\sqrt{c+dx^4}}{4ax^2(a+bx^4)(bc-ad)}$$

Rubi [A] time = 0.20, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 472, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^4}(3bc-2ad)}{4a^2cx^2(bc-ad)} - \frac{b(3bc-4ad) \tan^{-1}\left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}} + \frac{b\sqrt{c+dx^4}}{4ax^2(a+bx^4)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*x^2) + (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x^2*(a + b*x^4)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(5/2)*(b*c - a*d)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -

1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
 [{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 472

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))
)^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)),
 Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
 b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
 ntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 583

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))
)^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
 m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 (a + bx^4)^2 \sqrt{c + dx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2 (a + bx^2)^2 \sqrt{c + dx^2}} dx, x, x^2 \right) \\
&= \frac{b\sqrt{c + dx^4}}{4a(bc - ad)x^2 (a + bx^4)} - \frac{\text{Subst} \left(\int \frac{-3bc + 2ad - 2bdx^2}{x^2(a + bx^2)\sqrt{c + dx^2}} dx, x, x^2 \right)}{4a(bc - ad)} \\
&= -\frac{(3bc - 2ad)\sqrt{c + dx^4}}{4a^2c(bc - ad)x^2} + \frac{b\sqrt{c + dx^4}}{4a(bc - ad)x^2 (a + bx^4)} - \frac{\text{Subst} \left(\int \frac{bc(3bc - 4ad)}{(a + bx^2)\sqrt{c + dx^2}} dx, \right)}{4a^2c(bc - ad)} \\
&= -\frac{(3bc - 2ad)\sqrt{c + dx^4}}{4a^2c(bc - ad)x^2} + \frac{b\sqrt{c + dx^4}}{4a(bc - ad)x^2 (a + bx^4)} - \frac{(b(3bc - 4ad)) \text{Subst} \left(\int \frac{1}{a - bx^2} dx, \right)}{4a^2(bc - ad)} \\
&= -\frac{(3bc - 2ad)\sqrt{c + dx^4}}{4a^2c(bc - ad)x^2} + \frac{b\sqrt{c + dx^4}}{4a(bc - ad)x^2 (a + bx^4)} - \frac{(b(3bc - 4ad)) \text{Subst} \left(\int \frac{1}{a - bx^2} dx, \right)}{4a^2(bc - ad)} \\
&= -\frac{(3bc - 2ad)\sqrt{c + dx^4}}{4a^2c(bc - ad)x^2} + \frac{b\sqrt{c + dx^4}}{4a(bc - ad)x^2 (a + bx^4)} - \frac{b(3bc - 4ad) \tan^{-1} \left(\frac{\sqrt{bc - ad}}{\sqrt{a} \sqrt{c + dx^4}} \right)}{4a^{5/2}(bc - ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 5.63, size = 155, normalized size = 1.04

$$\frac{a^2 (c + dx^4) \left(\frac{b^2 x^4}{(a + bx^4)(ad - bc)} - \frac{2}{c} \right) - \frac{bx^8 \sqrt{\frac{dx^4}{c} + 1} (3bc - 4ad) \sin^{-1} \left(\frac{\sqrt{x^4 \left(\frac{b}{a} - \frac{d}{c} \right)}}{\sqrt{\frac{bx^4}{a} + 1}} \right)}{c \left(\frac{x^4 (bc - ad)}{ac} \right)^{3/2}}}{4a^4 x^2 \sqrt{c + dx^4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] (a^2*(c + d*x^4)*(-2/c + (b^2*x^4)/((-(b*c) + a*d)*(a + b*x^4))) - (b*(3*b*c - 4*a*d)*x^8*Sqrt[1 + (d*x^4)/c]*ArcSin[Sqrt[(b/a - d/c)*x^4]/Sqrt[1 + (b*x^4)/a]])/(c*(((b*c - a*d)*x^4)/(a*c))^(3/2))/(4*a^4*x^2*Sqrt[c + d*x^4])

IntegrateAlgebraic [A] time = 1.37, size = 159, normalized size = 1.07

$$\frac{(4abd - 3b^2c) \tan^{-1}\left(\frac{a\sqrt{d} + bx^2\sqrt{c+dx^4} + b\sqrt{d}x^4}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{5/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx^4}(-2a^2d + 2abc - 2abdx^4 + 3b^2cx^4)}{4a^2cx^2(a+bx^4)(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(a + b*x^4)^2*sqrt[c + d*x^4]),x]

[Out] (sqrt[c + d*x^4]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^4 - 2*a*b*d*x^4))/(4*a^2*c*(-(b*c) + a*d)*x^2*(a + b*x^4)) + ((-3*b^2*c + 4*a*b*d)*ArcTan[(a*sqrt[d] + b*sqrt[d]*x^4 + b*x^2*sqrt[c + d*x^4])/(sqrt[a]*sqrt[b*c - a*d])])/(4*a^(5/2)*(b*c - a*d)^(3/2))

fricas [B] time = 0.62, size = 612, normalized size = 4.11

$$\frac{\left(\left(3b^3c - 4abd\right)^2 + \left(3a^2d^2 - 4a^2bcd + 2a^2d^2\right)\sqrt{c+ad} \log\left(\frac{\left(b^2 - 4abcd + 2a^2d^2\right)\sqrt{c+ad} + \left(3a^2d^2 - 4a^2bcd + 2a^2d^2\right)\sqrt{c+ad}}{2\sqrt{a}\sqrt{bc-ad}}\right) + 4\left(2a^2b^2c^2 - 4a^2bcd + 2a^2d^2 + \left(3a^2b^2 - 5a^2bcd + 2a^2bd^2\right)\sqrt{c+ad} + c\right)}{16\left(a^3b^3c^2 - 2a^2b^2cd + 2a^2bd^2\right)^2 + \left(a^3b^3c^2 - 2a^2b^2cd + 2a^2bd^2\right)^2} + \frac{\left(\left(3b^3c - 4abd\right)^2 + \left(3a^2d^2 - 4a^2bcd + 2a^2d^2\right)\sqrt{c+ad} \arctan\left(\frac{\left(b^2 - 4abcd + 2a^2d^2\right)\sqrt{c+ad}}{2\sqrt{a}\sqrt{bc-ad}}\right) + 2\left(2a^2b^2c^2 - 4a^2bcd + 2a^2d^2 + \left(3a^2b^2 - 5a^2bcd + 2a^2bd^2\right)\sqrt{c+ad} + c\right)\sqrt{c+ad}}{8\left(a^3b^3c^2 - 2a^2b^2cd + 2a^2bd^2\right)^2 + \left(a^3b^3c^2 - 2a^2b^2cd + 2a^2bd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [-1/16*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*(2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4)*sqrt(d*x^4 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^2), -1/8*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d))/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) + 2*(2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4)*sqrt(d*x^4 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^2)]

giac [B] time = 1.68, size = 418, normalized size = 2.81

$$\frac{1}{4} \frac{d^{\frac{5}{2}} \left(\frac{(3b^2c - 4abd) \arctan\left(\frac{\sqrt{d}x^2 - \sqrt{d}x^4 + c}{2\sqrt{abcd - a^2d^2}}\right) + 2\left(3\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^4 b^2c - 4\left(\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^4 abd - 6\left(\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^2 b^2c^2 + 14\left(\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^2 abcd - 8\left(\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^2 a^2d^2 + 3b^2c^3 - 2abc^2d \right)}{\left(a^2bcd^2 - a^3d^3\right)\sqrt{abcd - a^2d^2}} + \frac{2\left(3\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^6 b - 3\left(\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^4 bc + 4\left(\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^4 ad + 3\left(\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^2 bc^2 - 4\left(\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^2 acd - bc^3 \right)\left(a^2bcd^2 - a^3d^3\right)}{\left(\left(\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^6 b - 3\left(\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^4 bc + 4\left(\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^4 ad + 3\left(\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^2 bc^2 - 4\left(\sqrt{d}x^2 - \sqrt{d}x^4 + c\right)^2 acd - bc^3 \right)\left(a^2bcd^2 - a^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")

```
[Out] 1/4*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2)))/((a^2*b*c*d^2 - a^3*d^3)*sqrt
(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b^2*c - 4*(sq
rt(d)*x^2 - sqrt(d*x^4 + c))^4*a*b*d - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*
b^2*c^2 + 14*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b*c*d - 8*(sqrt(d)*x^2 - s
qrt(d*x^4 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/(((sqrt(d)*x^2 - sqrt(
d*x^4 + c))^6*b - 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b*c + 4*(sqrt(d)*x^2
- sqrt(d*x^4 + c))^4*a*d + 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c^2 - 4*(s
qrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3)))
```

maple [B] time = 0.27, size = 885, normalized size = 5.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2), x)
```

```
[Out] -1/2/a^2/x^2*(d*x^4+c)^(1/2)/c-3/8/a^2*b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)
^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+3/8/a^2*b/(-a*b)^(1/2)/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1/8/a^2*b/(a*d-b*c)/(x^2-(-
a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2)-1/8/a^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/
2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/
b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1/8/a^2*b/(a*d-b*c)/(x^2+(-a*b)^(
1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2)+1/8/a^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1
/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^2 \sqrt{dx^4 + c} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2), x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 (bx^4 + a)^2 \sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)`

[Out] `int(1/(x^3*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + bx^4)^2 \sqrt{c + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2), x)`

[Out] `Integral(1/(x**3*(a + b*x**4)**2*sqrt(c + d*x**4)), x)`

$$3.542 \quad \int \frac{1}{x^7(a+bx^4)^2 \sqrt{c+dx^4}} dx$$

Optimal. Leaf size=208

$$\frac{b^2(5bc - 6ad) \tan^{-1}\left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}}\right)}{4a^{7/2}(bc - ad)^{3/2}} - \frac{\sqrt{c+dx^4}(5bc - 2ad)}{12a^2cx^6(bc - ad)} + \frac{\sqrt{c+dx^4}(-4a^2d^2 - 8abcd + 15b^2c^2)}{12a^3c^2x^2(bc - ad)} + \frac{b\sqrt{c+dx^4}}{4ax^6(a+bx^4)}$$

Rubi [A] time = 0.33, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 472, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^4}(-4a^2d^2 - 8abcd + 15b^2c^2)}{12a^3c^2x^2(bc - ad)} + \frac{b^2(5bc - 6ad) \tan^{-1}\left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}}\right)}{4a^{7/2}(bc - ad)^{3/2}} - \frac{\sqrt{c+dx^4}(5bc - 2ad)}{12a^2cx^6(bc - ad)} + \frac{b\sqrt{c+dx^4}}{4ax^6(a+bx^4)(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^4])/(12*a^2*c*(b*c - a*d)*x^6) + ((15*b^2*c^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^4])/(12*a^3*c^2*(b*c - a*d)*x^2) + (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x^6*(a + b*x^4)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(7/2)*(b*c - a*d)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
  x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
  x_Symbol] := -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/
(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)),
  Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c -
a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*
((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)),
  Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0]
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7 (a + bx^4)^2 \sqrt{c + dx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^4 (a + bx^2)^2 \sqrt{c + dx^2}} dx, x, x^2 \right) \\
&= \frac{b\sqrt{c + dx^4}}{4a(bc - ad)x^6 (a + bx^4)} - \frac{\text{Subst} \left(\int \frac{-5bc + 2ad - 4bdx^2}{x^4(a+bx^2)\sqrt{c+dx^2}} dx, x, x^2 \right)}{4a(bc - ad)} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^4}}{12a^2c(bc - ad)x^6} + \frac{b\sqrt{c + dx^4}}{4a(bc - ad)x^6 (a + bx^4)} + \frac{\text{Subst} \left(\int \frac{-15b^2c^2 + 8abcd + 4a^2d}{x^2(a+bx^2)} \right)}{12a^2c(bc - ad)} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^4}}{12a^2c(bc - ad)x^6} + \frac{(15b^2c^2 - 8abcd - 4a^2d^2)\sqrt{c + dx^4}}{12a^3c^2(bc - ad)x^2} + \frac{b\sqrt{c + dx^4}}{4a(bc - ad)x^6} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^4}}{12a^2c(bc - ad)x^6} + \frac{(15b^2c^2 - 8abcd - 4a^2d^2)\sqrt{c + dx^4}}{12a^3c^2(bc - ad)x^2} + \frac{b\sqrt{c + dx^4}}{4a(bc - ad)x^6} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^4}}{12a^2c(bc - ad)x^6} + \frac{(15b^2c^2 - 8abcd - 4a^2d^2)\sqrt{c + dx^4}}{12a^3c^2(bc - ad)x^2} + \frac{b\sqrt{c + dx^4}}{4a(bc - ad)x^6} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^4}}{12a^2c(bc - ad)x^6} + \frac{(15b^2c^2 - 8abcd - 4a^2d^2)\sqrt{c + dx^4}}{12a^3c^2(bc - ad)x^2} + \frac{b\sqrt{c + dx^4}}{4a(bc - ad)x^6}
\end{aligned}$$

Mathematica [A] time = 5.87, size = 175, normalized size = 0.84

$$\frac{a^2 (c + dx^4) \left(\frac{3b^3x^8}{(a+bx^4)(bc-ad)} + \frac{4x^4(ad+3bc)}{c^2} - \frac{2a}{c} \right) + \frac{3b^2x^{12}\sqrt{\frac{dx^4}{c}+1}(5bc-6ad)\sin^{-1}\left(\frac{\sqrt{x^4\left(\frac{b}{a}-\frac{d}{c}\right)}}{\sqrt{\frac{bx^4}{a}+1}}\right)}{c\left(\frac{x^4(bc-ad)}{ac}\right)^{3/2}}}{12a^5x^6\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]), x]

[Out] (a^2*(c + d*x^4)*((-2*a)/c + (4*(3*b*c + a*d)*x^4)/c^2 + (3*b^3*x^8)/((b*c - a*d)*(a + b*x^4))) + (3*b^2*(5*b*c - 6*a*d)*x^12*Sqrt[1 + (d*x^4)/c]*ArcS

$\text{in}[\text{Sqrt}[(b/a - d/c)*x^4]/\text{Sqrt}[1 + (b*x^4)/a]]/(c*((b*c - a*d)*x^4)/(a*c))^{(3/2)})/(12*a^5*x^6*\text{Sqrt}[c + d*x^4])$

IntegrateAlgebraic [A] time = 2.28, size = 217, normalized size = 1.04

$$\frac{(5b^3c - 6ab^2d) \tan^{-1}\left(\frac{a\sqrt{d} + bx^2\sqrt{c+dx^4} + b\sqrt{d}x^4}{\sqrt{d}\sqrt{bc-ad}}\right) + \sqrt{c+dx^4}(-2a^3cd + 4a^3d^2x^4 + 2a^2bc^2 + 6a^2bcdx^4 + 4a^2bd^2x^8 - 10ab^2c^2x^4 + 8ab^2cdx^8 - 15b^3c^2x^8)}{4a^{7/2}(bc - ad)^{3/2} + 12a^3c^2x^6(a + bx^4)(ad - bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] (Sqrt[c + d*x^4]*(2*a^2*b*c^2 - 2*a^3*c*d - 10*a*b^2*c^2*x^4 + 6*a^2*b*c*d*x^4 + 4*a^3*d^2*x^4 - 15*b^3*c^2*x^8 + 8*a*b^2*c*d*x^8 + 4*a^2*b*d^2*x^8))/(12*a^3*c^2*(-(b*c) + a*d)*x^6*(a + b*x^4)) + ((5*b^3*c - 6*a*b^2*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(7/2)*(b*c - a*d)^(3/2))

fricas [A] time = 0.78, size = 760, normalized size = 3.65

$$\frac{3(5b^3c - 6ab^2d) \arctan\left(\frac{\sqrt{d}x^2 - \sqrt{dx^4+c}}{2\sqrt{abcd-a^2d^2}}\right) - \frac{6\left(\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 b^3c - 2\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 ab^2d - b^3d^2\right)}{(a^2bcd^3 - a^4d^4)\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^4 b - 2\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 bc + 4\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 ad + bc^2} - \frac{8\left(3\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^4 b - 6\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 bc - 3\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 ad + 3bc^2 + acd\right)}{\left(\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 - c\right)^3 a^3d^3}}{36(a^3c^2 - 2a^2bc + a^2d^2)(bc - ad)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [-1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^6)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^6), 1/24*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^6)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d))/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^6)]

giac [B] time = 2.04, size = 395, normalized size = 1.90

$$\frac{1}{12} \left(\frac{3(5b^3c - 6ab^2d) \arctan\left(\frac{\sqrt{d}x^2 - \sqrt{dx^4+c}}{2\sqrt{abcd-a^2d^2}}\right) - \frac{6\left(\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 b^3c - 2\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 ab^2d - b^3d^2\right)}{(a^2bcd^3 - a^4d^4)\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^4 b - 2\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 bc + 4\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 ad + bc^2} - \frac{8\left(3\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^4 b - 6\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 bc - 3\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 ad + 3bc^2 + acd\right)}{\left(\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 - c\right)^3 a^3d^3}}{36(a^3c^2 - 2a^2bc + a^2d^2)(bc - ad)^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{12}d^{7/2}(3(5b^3c - 6ab^2d)\arctan(-1/2((\sqrt{d}x^2 - \sqrt{d(x^4+c)})^2b - bc + 2ad)/\sqrt{abc d - a^2d^2}))/((a^3b^3cd^3 - a^4d^4)\sqrt{abc d - a^2d^2}) - 6((\sqrt{d}x^2 - \sqrt{d(x^4+c)})^2b^3c - 2(\sqrt{d}x^2 - \sqrt{d(x^4+c)})^2ab^2d - b^3c^2)/((a^3b^3cd^3 - a^4d^4)((\sqrt{d}x^2 - \sqrt{d(x^4+c)})^4b - 2(\sqrt{d}x^2 - \sqrt{d(x^4+c)})^2b^3c + 4(\sqrt{d}x^2 - \sqrt{d(x^4+c)})^2ad + bc^2)) - 8(3(\sqrt{d}x^2 - \sqrt{d(x^4+c)})^4b - 6(\sqrt{d}x^2 - \sqrt{d(x^4+c)})^2b^3c - 3(\sqrt{d}x^2 - \sqrt{d(x^4+c)})^2ad + 3b^3c^2 + acd)/(((\sqrt{d}x^2 - \sqrt{d(x^4+c)})^2 - c)^3a^3d^3))$

maple [B] time = 0.25, size = 923, normalized size = 4.44

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)

[Out] $\frac{b}{a^3}x^2(d x^4+c)^{1/2}/c+5/8/a^3b^2/(-ab)^{1/2}/(-(ad-bc)/b)^{1/2}*1$
 $n((-2(-ab)^{1/2}(x^2+(-ab)^{1/2}/b)/bd-2(ad-bc)/b+2(-(ad-bc)/b)^{1/2}((x^2+(-ab)^{1/2}/b)^2d-2(-ab)^{1/2}(x^2+(-ab)^{1/2}/b)/bd-(ad-bc)/b)^{1/2})/(x^2+(-ab)^{1/2}/b)-5/8/a^3b^2/(-ab)^{1/2}/(-(ad-bc)/b)^{1/2}$
 $*\ln((2(-ab)^{1/2}(x^2-(-ab)^{1/2}/b)/bd-2(ad-bc)/b+2(-(ad-bc)/b)^{1/2}((x^2-(-ab)^{1/2}/b)^2d+2(-ab)^{1/2}(x^2-(-ab)^{1/2}/b)/bd-(ad-bc)/b)^{1/2})/(x^2-(-ab)^{1/2}/b)-1/6/a^2(d x^4+c)^{1/2}(-2d x^4+c)/x^6/c^2-1/8/a^3b^2/(ad-bc)/(x^2-(-ab)^{1/2}/b)((x^2-(-ab)^{1/2}/b)^2d+2(-ab)^{1/2}(x^2-(-ab)^{1/2}/b)/bd-(ad-bc)/b)^{1/2}+1/8/a^3b^2(-ab)^{1/2}d/(ad-bc)/(-(ad-bc)/b)^{1/2}$
 $*\ln((2(-ab)^{1/2}(x^2-(-ab)^{1/2}/b)/bd-2(ad-bc)/b+2(-(ad-bc)/b)^{1/2}((x^2-(-ab)^{1/2}/b)^2d+2(-ab)^{1/2}(x^2-(-ab)^{1/2}/b)/bd-(ad-bc)/b)^{1/2})/(x^2-(-ab)^{1/2}/b)-1/8/a^3b^2/(ad-bc)/(x^2+(-ab)^{1/2}/b)((x^2+(-ab)^{1/2}/b)^2d-2(-ab)^{1/2}(x^2+(-ab)^{1/2}/b)/bd-(ad-bc)/b)^{1/2}-1/8/a^3b^2(-ab)^{1/2}d/(ad-bc)/(-(ad-bc)/b)^{1/2}$
 $*\ln((-2(-ab)^{1/2}(x^2+(-ab)^{1/2}/b)/bd-2(ad-bc)/b+2(-(ad-bc)/b)^{1/2}((x^2+(-ab)^{1/2}/b)^2d-2(-ab)^{1/2}(x^2+(-ab)^{1/2}/b)/bd-(ad-bc)/b)^{1/2})/(x^2+(-ab)^{1/2}/b))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^2 \sqrt{dx^4 + c} x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^7), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^7 (bx^4 + a)^2 \sqrt{dx^4 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^7*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)

[Out] int(1/(x^7*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 (a + bx^4)^2 \sqrt{c + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)

[Out] Integral(1/(x**7*(a + b*x**4)**2*sqrt(c + d*x**4)), x)

$$3.543 \quad \int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}} dx$$

Optimal. Leaf size=104

$$-\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^6}(ad+bc)}{3b^2d^2} + \frac{(c+dx^6)^{3/2}}{9bd^2}$$

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 88, 63, 208}

$$-\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^6}(ad+bc)}{3b^2d^2} + \frac{(c+dx^6)^{3/2}}{9bd^2}$$

Antiderivative was successfully verified.

[In] Int[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] -((b*c + a*d)*Sqrt[c + d*x^6])/(3*b^2*d^2) + (c + d*x^6)^(3/2)/(9*b*d^2) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{17}}{(a + bx^6)\sqrt{c + dx^6}} dx &= \frac{1}{6} \text{Subst} \left(\int \frac{x^2}{(a + bx)\sqrt{c + dx}} dx, x, x^6 \right) \\
 &= \frac{1}{6} \text{Subst} \left(\int \left(\frac{-bc - ad}{b^2 d \sqrt{c + dx}} + \frac{a^2}{b^2 (a + bx) \sqrt{c + dx}} + \frac{\sqrt{c + dx}}{bd} \right) dx, x, x^6 \right) \\
 &= -\frac{(bc + ad)\sqrt{c + dx^6}}{3b^2 d^2} + \frac{(c + dx^6)^{3/2}}{9bd^2} + \frac{a^2 \text{Subst} \left(\int \frac{1}{(a + bx)\sqrt{c + dx}} dx, x, x^6 \right)}{6b^2} \\
 &= -\frac{(bc + ad)\sqrt{c + dx^6}}{3b^2 d^2} + \frac{(c + dx^6)^{3/2}}{9bd^2} + \frac{a^2 \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^6} \right)}{3b^2 d} \\
 &= -\frac{(bc + ad)\sqrt{c + dx^6}}{3b^2 d^2} + \frac{(c + dx^6)^{3/2}}{9bd^2} - \frac{a^2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^6}}{\sqrt{bc - ad}} \right)}{3b^{5/2} \sqrt{bc - ad}}
 \end{aligned}$$

Mathematica [A] time = 0.28, size = 91, normalized size = 0.88

$$\frac{\sqrt{c + dx^6} (-3ad - 2bc + bdx^6)}{9b^2 d^2} - \frac{a^2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^6}}{\sqrt{bc - ad}} \right)}{3b^{5/2} \sqrt{bc - ad}}$$

Antiderivative was successfully verified.

[In] Integrate[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] (Sqrt[c + d*x^6]*(-2*b*c - 3*a*d + b*d*x^6))/(9*b^2*d^2) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])

IntegrateAlgebraic [A] time = 0.18, size = 101, normalized size = 0.97

$$\frac{\sqrt{c + dx^6} (-3ad - 2bc + bdx^6)}{9b^2 d^2} - \frac{a^2 \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^6} \sqrt{ad - bc}}{bc - ad} \right)}{3b^{5/2} \sqrt{ad - bc}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] (Sqrt[c + d*x^6]*(-2*b*c - 3*a*d + b*d*x^6))/(9*b^2*d^2) - (a^2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a*d)])/(3*b^(5/2)*Sqrt[-(b*c) + a*d])

fricas [A] time = 0.43, size = 288, normalized size = 2.77

$$\left[\frac{3\sqrt{b^2c - abd}a^2d^2 \log\left(\frac{bdx^6 + 2bc - ad - 2\sqrt{dx^6 + c}\sqrt{b^2c - abd}}{bx^6 + a}\right) + 2\left((b^3cd - ab^2d^2)x^6 - 2b^3c^2 - ab^2cd + 3a^2bd^2\right)\sqrt{dx^6 + c}}{18(b^4cd^2 - ab^3d^3)}, \frac{3\sqrt{-b^2c + abd}a^2d^2 \arctan\left(\frac{\sqrt{dx^6 + c}\sqrt{-b^2c + abd}}{bdx^6 + bc}\right) + \left((b^3cd - ab^2d^2)x^6 - 2b^3c^2 - ab^2cd + 3a^2bd^2\right)\sqrt{dx^6 + c}}{9(b^4cd^2 - ab^3d^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [1/18*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^6 - 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c)/(b^4*c*d^2 - a*b^3*d^3), 1/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^6 - 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^3*d^3)]

giac [A] time = 0.17, size = 106, normalized size = 1.02

$$\frac{a^2 \arctan\left(\frac{\sqrt{dx^6 + c}b}{\sqrt{-b^2c + abd}}\right)}{3\sqrt{-b^2c + abd}b^2} + \frac{(dx^6 + c)^{\frac{3}{2}}b^2d^4 - 3\sqrt{dx^6 + c}b^2cd^4 - 3\sqrt{dx^6 + c}abd^5}{9b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] 1/3*a^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) + 1/9*((d*x^6 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^6 + c)*b^2*c*d^4 - 3*sqrt(d*x^6 + c)*a*b*d^5)/(b^3*d^6)

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{x^{17}}{(bx^6 + a)\sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x)

[Out] `int(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.87, size = 103, normalized size = 0.99

$$\frac{(dx^6+c)^{3/2}}{9bd^2} - \left(\frac{2c}{3bd^2} + \frac{3ad^3-3bcd^2}{9b^2d^4} \right) \sqrt{dx^6+c} + \frac{a^2 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^6+c}}{\sqrt{ad-bc}}\right)}{3b^{5/2}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^17/((a+b*x^6)*(c+d*x^6)^(1/2)),x)`

[Out] $(c+d*x^6)^{3/2}/(9*b*d^2) - ((2*c)/(3*b*d^2) + (3*a*d^3 - 3*b*c*d^2)/(9*b^2*d^4))*(c+d*x^6)^{1/2} + (a^2*\operatorname{atan}((b^{1/2}*(c+d*x^6)^{1/2})/(a*d-b*c)^{1/2}))/((3*b^{5/2}*(a*d-b*c)^{1/2}))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**17/(b*x**6+a)/(d*x**6+c)**(1/2),x)`

[Out] `Integral(x**17/((a+b*x**6)*sqrt(c+d*x**6)), x)`

$$3.544 \quad \int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}} dx$$

Optimal. Leaf size=74

$$\frac{a \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^6}}{3bd}$$

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 80, 63, 208}

$$\frac{a \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^6}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] Sqrt[c + d*x^6]/(3*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{x^{11}}{(a + bx^6)\sqrt{c + dx^6}} dx &= \frac{1}{6} \text{Subst} \left(\int \frac{x}{(a + bx)\sqrt{c + dx}} dx, x, x^6 \right) \\ &= \frac{\sqrt{c + dx^6}}{3bd} - \frac{a \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^6 \right)}{6b} \\ &= \frac{\sqrt{c + dx^6}}{3bd} - \frac{a \text{Subst} \left(\int \frac{1}{a - \frac{bc}{a} + \frac{bx^2}{a}} dx, x, \sqrt{c + dx^6} \right)}{3bd} \\ &= \frac{\sqrt{c + dx^6}}{3bd} + \frac{a \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^6}}{\sqrt{bc-ad}} \right)}{3b^{3/2} \sqrt{bc - ad}} \end{aligned}$$

Mathematica [A] time = 0.13, size = 72, normalized size = 0.97

$$\frac{1}{3} \left(\frac{a \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^6}}{\sqrt{bc-ad}} \right)}{b^{3/2} \sqrt{bc - ad}} + \frac{\sqrt{c + dx^6}}{bd} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*
d]])/(b^(3/2)*Sqrt[b*c - a*d]))/3
```

IntegrateAlgebraic [A] time = 0.10, size = 84, normalized size = 1.14

$$\frac{a \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^6} \sqrt{ad-bc}}{bc-ad} \right)}{3b^{3/2} \sqrt{ad - bc}} + \frac{\sqrt{c + dx^6}}{3bd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] Sqrt[c + d*x^6]/(3*b*d) + (a*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d])*Sqrt[c + d*x^6])/(b*c - a*d)]/(3*b^(3/2)*Sqrt[-(b*c) + a*d])

fricas [A] time = 0.42, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c - abd} \operatorname{ad} \log\left(\frac{bdx^6 + 2bc - ad + 2\sqrt{dx^6 + c}\sqrt{b^2c - abd}}{bx^6 + a}\right) + 2\sqrt{dx^6 + c}(b^2c - abd)}{6(b^3cd - ab^2d^2)}, -\frac{\sqrt{-b^2c + abd} \operatorname{ad} \arctan\left(\frac{\sqrt{dx^6 + c}\sqrt{-b^2c + abd}}{bdx^6 + bc}\right) - \sqrt{dx^6 + c}(b^2c - abd)}{3(b^3cd - ab^2d^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [1/6*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^2*c - a*b*d)/(b^3*c*d - a*b^2*d^2), -1/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) - sqrt(d*x^6 + c)*(b^2*c - a*b*d)/(b^3*c*d - a*b^2*d^2)]

giac [A] time = 0.19, size = 64, normalized size = 0.86

$$-\frac{\operatorname{ad} \arctan\left(\frac{\sqrt{dx^6 + c}b}{\sqrt{-b^2c + abd}}\right) - \frac{\sqrt{dx^6 + c}}{b}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] -1/3*(a*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b) - sqrt(d*x^6 + c)/b)/d

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(bx^6 + a)\sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x)

[Out] int(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(b*x⁶+a)/(d*x⁶+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.75, size = 58, normalized size = 0.78

$$\frac{\sqrt{dx^6+c}}{3bd} - \frac{a \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^6+c}}{\sqrt{ad-bc}}\right)}{3b^{3/2}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/((a + b*x⁶)*(c + d*x⁶)^(1/2)),x)

[Out] (c + d*x⁶)^(1/2)/(3*b*d) - (a*atan((b^(1/2)*(c + d*x⁶)^(1/2))/(a*d - b*c)^(1/2)))/(3*b^(3/2)*(a*d - b*c)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a + bx^6)\sqrt{c + dx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11/(b*x**6+a)/(d*x**6+c)**(1/2),x)

[Out] Integral(x**11/((a + b*x**6)*sqrt(c + d*x**6)), x)

$$3.545 \quad \int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}} dx$$

Optimal. Leaf size=51

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {444, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/(3*Sqrt[b]*Sqrt[b*c - a*d])

Rule 63

Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 444

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}} dx &= \frac{1}{6} \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^6 \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}} dx, x, \sqrt{c+dx^6} \right)}{3d} \\ &= -\frac{\tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}} \right)}{3\sqrt{b}\sqrt{bc-ad}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$-\frac{\tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}} \right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] -1/3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c - a*d])

IntegrateAlgebraic [A] time = 0.05, size = 61, normalized size = 1.20

$$-\frac{\tan^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad} \right)}{3\sqrt{b}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] -1/3*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a*d)]/(Sqrt[b]*Sqrt[-(b*c) + a*d])

fricas [A] time = 0.41, size = 130, normalized size = 2.55

$$\left[\frac{\log \left(\frac{bdx^6+2bc-ad-2\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a} \right)}{6\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd} \arctan \left(\frac{\sqrt{dx^6+c}\sqrt{-b^2c+abd}}{bdx^6+bc} \right)}{3(b^2c-abd)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")`

[Out] $\left[\frac{1}{6} \log\left(\frac{b d x^6 + 2 b^2 c - a d - 2 \sqrt{d x^6 + c} \sqrt{b^2 c - a b d}}{b x^6 + a}\right) / \sqrt{b^2 c - a b d}, \frac{1}{3} \sqrt{b^2 c - a b d} \arctan\left(\frac{\sqrt{d x^6 + c} \sqrt{b^2 c - a b d}}{b d x^6 + b^2 c}\right) / (b^2 c - a b d) \right]$

giac [A] time = 0.17, size = 40, normalized size = 0.78

$$\frac{\arctan\left(\frac{\sqrt{d x^6 + c} b}{\sqrt{-b^2 c + a b d}}\right)}{3 \sqrt{-b^2 c + a b d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")`

[Out] $\frac{1}{3} \arctan\left(\frac{\sqrt{d x^6 + c} b}{\sqrt{-b^2 c + a b d}}\right) / \sqrt{-b^2 c + a b d}$

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(b x^6 + a) \sqrt{d x^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)`

[Out] `int(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.72, size = 40, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{b \sqrt{d x^6 + c}}{\sqrt{a b d - b^2 c}}\right)}{3 \sqrt{a b d - b^2 c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/((a + b*x^6)*(c + d*x^6)^(1/2)),x)`

[Out] `atan((b*(c + d*x^6)^(1/2))/(a*b*d - b^2*c)^(1/2))/(3*(a*b*d - b^2*c)^(1/2))`

sympy [A] time = 23.76, size = 37, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b*x**6+a)/(d*x**6+c)**(1/2),x)`

[Out] `atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b*c)/b))`

$$3.546 \quad \int \frac{1}{x(a+bx^6)\sqrt{c+dx^6}} dx$$

Optimal. Leaf size=85

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 86, 63, 208}

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] -ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/(3*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 86

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && !IntegerQ[p]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(a + bx^6)\sqrt{c + dx^6}} dx &= \frac{1}{6} \text{Subst}\left(\int \frac{1}{x(a + bx)\sqrt{c + dx}} dx, x, x^6\right) \\
 &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^6\right)}{6a} - \frac{b \text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^6\right)}{6a} \\
 &= \frac{\text{Subst}\left(\int \frac{1}{\frac{-c}{-d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx^6}\right)}{3ad} - \frac{b \text{Subst}\left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^6}\right)}{3ad} \\
 &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}} + \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}
 \end{aligned}$$

Mathematica [A] time = 0.13, size = 81, normalized size = 0.95

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right) - \tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]), x]

[Out] (-(ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/Sqrt[b*c - a*d]))/(3*a)

IntegrateAlgebraic [A] time = 0.12, size = 95, normalized size = 1.12

$$\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad}\right) - \tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a*d)]/(3*a*Sqrt[-(b*c) + a*d]) - ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/(3*a*Sqrt[c]))

fricas [A] time = 0.45, size = 431, normalized size = 5.07

$$\left[\frac{c\sqrt{\frac{c}{bc-ad}} \log\left(\frac{bd^2+2bc-ad+2\sqrt{d^2c(bc-ad)\sqrt{\frac{c}{bc-ad}}}}{bd^2+ad}\right) + \sqrt{c} \log\left(\frac{bd^2-2\sqrt{d^2c}\sqrt{c}+2c}{d^2}\right)}{6ac}, \frac{2c\sqrt{\frac{b}{bc-ad}} \arctan\left(\frac{-\sqrt{d^2c(bc-ad)\sqrt{\frac{c}{bc-ad}}}}{bd^2+bc}\right) + \sqrt{c} \log\left(\frac{bd^2-2\sqrt{d^2c}\sqrt{c}+2c}{d^2}\right)}{6ac}, \frac{c\sqrt{\frac{c}{bc-ad}} \log\left(\frac{bd^2+2bc-ad+2\sqrt{d^2c(bc-ad)\sqrt{\frac{c}{bc-ad}}}}{bd^2+ad}\right) + 2\sqrt{-c} \arctan\left(\frac{\sqrt{d^2c}\sqrt{c}}{c}\right)}{6ac}, \frac{c\sqrt{\frac{b}{bc-ad}} \arctan\left(\frac{\sqrt{d^2c(bc-ad)\sqrt{\frac{c}{bc-ad}}}}{bd^2+bc}\right) + \sqrt{-c} \arctan\left(\frac{\sqrt{d^2c}\sqrt{c}}{c}\right)}{3ac} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [1/6*(c*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(2*c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^6 + b*c)) + sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(c*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c))/(a*c), 1/3*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^6 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c))/(a*c)]

giac [A] time = 0.18, size = 71, normalized size = 0.84

$$-\frac{b \arctan\left(\frac{\sqrt{dx^6+cb}}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a} + \frac{\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] -1/3*b*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a) + 1/3*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a*sqrt(-c))

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)\sqrt{dx^6 + c}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x)

[Out] int(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)\sqrt{dx^6 + cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x), x)

mupad [B] time = 5.19, size = 652, normalized size = 7.67

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{dx^6+c}}{\sqrt{c}}\right)}{3a\sqrt{c}} - \frac{\operatorname{atan}\left(\frac{\sqrt{2c-abd}\left(\frac{2d^2y^2d^3}{27} - \frac{\sqrt{2c-abd}\left(\frac{2d^2y^2d^3}{9} - \frac{(8a^3y^2d^3-16a^2b^3cd^2)\sqrt{dx^6+c}\sqrt{2c-abd}}{36(a^2d-abc)}\right)}{6(a^2d-abc)}\right)}{\sqrt{2c-abd}\left(\frac{2b^3d^2\sqrt{dx^6+c}}{27} + \frac{\sqrt{2c-abd}\left(\frac{2d^2y^2d^3}{9} + \frac{(8a^3y^2d^3-16a^2b^3cd^2)\sqrt{dx^6+c}\sqrt{2c-abd}}{36(a^2d-abc)}\right)}{6(a^2d-abc)}\right)}\right)}{3(a^2d-abc)} \sqrt{b^2c-abd} \operatorname{li}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^6)*(c + d*x^6)^(1/2)),x)

[Out] $-\operatorname{atanh}\left(\frac{(c + d*x^6)^{1/2}}{c^{1/2}}\right)/(3*a*c^{1/2}) - \left(\operatorname{atan}\left(\frac{(b^2*c - a*b*d)^{1/2} * ((2*b^3*d^2*(c + d*x^6)^{1/2})/27 - ((b^2*c - a*b*d)^{1/2} * ((2*a^2*b^2*d^3)/9 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^{1/2} * (b^2*c - a*b*d)^{1/2}))/36*(a^2*d - a*b*c)))/6*(a^2*d - a*b*c)}\right) * 1i\right)/(a^2*d - a*b*c) + \left(\operatorname{atan}\left(\frac{(b^2*c - a*b*d)^{1/2} * ((2*b^3*d^2*(c + d*x^6)^{1/2})/27 + ((b^2*c - a*b*d)^{1/2} * ((2*a^2*b^2*d^3)/9 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^{1/2} * (b^2*c - a*b*d)^{1/2}))/36*(a^2*d - a*b*c)))/6*(a^2*d - a*b*c)}\right) * 1i\right)/(a^2*d - a*b*c) - \left(\operatorname{atan}\left(\frac{(b^2*c - a*b*d)^{1/2} * ((2*b^3*d^2*(c + d*x^6)^{1/2})/27 - ((b^2*c - a*b*d)^{1/2} * ((2*a^2*b^2*d^3)/9 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^{1/2} * (b^2*c - a*b*d)^{1/2}))/36*(a^2*d - a*b*c)))/6*(a^2*d - a*b*c)}\right) * 1i\right)/(a^2*d - a*b*c) - \left(\operatorname{atan}\left(\frac{(b^2*c - a*b*d)^{1/2} * ((2*b^3*d^2*(c + d*x^6)^{1/2})/27 + ((b^2*c - a*b*d)^{1/2} * ((2*a^2*b^2*d^3)/9 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^{1/2} * (b^2*c - a*b*d)^{1/2}))/36*(a^2*d - a*b*c)))/6*(a^2*d - a*b*c)}\right) * 1i\right)/(3*(a^2*d - a*b*c))$

sympy [A] time = 26.11, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}} + \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)) + atan(sqrt(c + d*x**6)/sqrt(-c))/(3*a*sqrt(-c))
```

$$3.547 \quad \int \frac{1}{x^7(a+bx^6)\sqrt{c+dx^6}} dx$$

Optimal. Leaf size=117

$$-\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}} + \frac{(ad+2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}} - \frac{\sqrt{c+dx^6}}{6acx^6}$$

Rubi [A] time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 103, 156, 63, 208}

$$-\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}} + \frac{(ad+2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}} - \frac{\sqrt{c+dx^6}}{6acx^6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] -Sqrt[c + d*x^6]/(6*a*c*x^6) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/(6*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/(3*a^2*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7 (a + bx^6) \sqrt{c + dx^6}} dx &= \frac{1}{6} \text{Subst} \left(\int \frac{1}{x^2 (a + bx) \sqrt{c + dx}} dx, x, x^6 \right) \\
&= -\frac{\sqrt{c + dx^6}}{6acx^6} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(2bc + ad) + \frac{bdx}{2}}{x(a + bx) \sqrt{c + dx}} dx, x, x^6 \right)}{6ac} \\
&= -\frac{\sqrt{c + dx^6}}{6acx^6} + \frac{b^2 \text{Subst} \left(\int \frac{1}{(a + bx) \sqrt{c + dx}} dx, x, x^6 \right)}{6a^2} - \frac{(2bc + ad) \text{Subst} \left(\int \frac{1}{x \sqrt{c + dx}} dx, x, x^6 \right)}{12a^2c} \\
&= -\frac{\sqrt{c + dx^6}}{6acx^6} + \frac{b^2 \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^6} \right)}{3a^2d} - \frac{(2bc + ad) \text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{dx}{d}} dx, x, \sqrt{c + dx^6} \right)}{6a^2cd} \\
&= -\frac{\sqrt{c + dx^6}}{6acx^6} + \frac{(2bc + ad) \tanh^{-1} \left(\frac{\sqrt{c + dx^6}}{\sqrt{c}} \right)}{6a^2c^{3/2}} - \frac{b^{3/2} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^6}}{\sqrt{bc - ad}} \right)}{3a^2 \sqrt{bc - ad}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 151, normalized size = 1.29

$$\frac{b^{3/2} \sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^6}}{\sqrt{bc - ad}} \right)}{3a^2(ad - bc)} + \frac{b \tanh^{-1} \left(\frac{\sqrt{c + dx^6}}{\sqrt{c}} \right)}{3a^2 \sqrt{c}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{c + dx^6}}{\sqrt{c}} \right)}{6ac^{3/2}} - \frac{\sqrt{c + dx^6}}{6acx^6}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] $-1/6*\text{Sqrt}[c + d*x^6]/(a*c*x^6) + (b*\text{ArcTanh}[\text{Sqrt}[c + d*x^6]/\text{Sqrt}[c]])/(3*a^2*\text{Sqrt}[c]) + (d*\text{ArcTanh}[\text{Sqrt}[c + d*x^6]/\text{Sqrt}[c]])/(6*a*c^{(3/2)}) + (b^{(3/2)}*\text{Sqrt}[b*c - a*d]*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[c + d*x^6])/\text{Sqrt}[b*c - a*d]])/(3*a^2*(-(b*c) + a*d))$

IntegrateAlgebraic [A] time = 0.26, size = 127, normalized size = 1.09

$$-\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{c+dx^6} \sqrt{ad-bc}}{bc-ad}\right)}{3a^2 \sqrt{ad-bc}} + \frac{(ad+2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2 c^{3/2}} - \frac{\sqrt{c+dx^6}}{6acx^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] $-1/6*\text{Sqrt}[c + d*x^6]/(a*c*x^6) - (b^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[-(b*c) + a*d])*\text{Sqrt}[c + d*x^6])/(b*c - a*d)]/(3*a^2*\text{Sqrt}[-(b*c) + a*d]) + ((2*b*c + a*d)*\text{ArcTanh}[\text{Sqrt}[c + d*x^6]/\text{Sqrt}[c]])/(6*a^2*c^{(3/2)})$

fricas [A] time = 0.47, size = 565, normalized size = 4.83

$$\frac{2\sqrt{2}\sqrt{a^2c^2}\sqrt{b}\sqrt{\frac{b^2d^2x^{12}+2\sqrt{cd}bx^6+ad^2}{a^2c^2}}}{12a^2c^2} + \frac{2\sqrt{2}\sqrt{a^2c^2}\sqrt{b}\sqrt{\frac{b^2d^2x^{12}+2\sqrt{cd}bx^6+ad^2}{a^2c^2}}}{12a^2c^2} - 2\sqrt{cd}x^6 + \frac{4b^2\sqrt{a^2c^2}\sqrt{b}\sqrt{\frac{b^2d^2x^{12}+2\sqrt{cd}bx^6+ad^2}{a^2c^2}}}{12a^2c^2} - \frac{2\sqrt{cd}x^6}{12a^2c^2} + \frac{4b^2\sqrt{a^2c^2}\sqrt{b}\sqrt{\frac{b^2d^2x^{12}+2\sqrt{cd}bx^6+ad^2}{a^2c^2}}}{12a^2c^2} + 2\sqrt{cd}x^6 + \frac{b^2\sqrt{a^2c^2}\sqrt{b}\sqrt{\frac{b^2d^2x^{12}+2\sqrt{cd}bx^6+ad^2}{a^2c^2}}}{6a^2c^2} - \frac{2\sqrt{cd}x^6}{6a^2c^2} + \frac{2b^2\sqrt{a^2c^2}\sqrt{b}\sqrt{\frac{b^2d^2x^{12}+2\sqrt{cd}bx^6+ad^2}{a^2c^2}}}{6a^2c^2} + \frac{2\sqrt{cd}x^6}{6a^2c^2} + \frac{2b^2\sqrt{a^2c^2}\sqrt{b}\sqrt{\frac{b^2d^2x^{12}+2\sqrt{cd}bx^6+ad^2}{a^2c^2}}}{6a^2c^2} + \frac{2\sqrt{cd}x^6}{6a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] $[1/12*(2*b*c^2*x^6*\text{sqrt}(b/(b*c - a*d))*\text{log}((b*d*x^6 + 2*b*c - a*d - 2*\text{sqrt}(d*x^6 + c))*(b*c - a*d)*\text{sqrt}(b/(b*c - a*d)))/(b*x^6 + a)) + (2*b*c + a*d)*\text{sqrt}(c)*x^6*\text{log}((d*x^6 + 2*\text{sqrt}(d*x^6 + c))*\text{sqrt}(c) + 2*c)/x^6) - 2*\text{sqrt}(d*x^6 + c)*a*c)/(a^2*c^2*x^6), -1/12*(4*b*c^2*x^6*\text{sqrt}(-b/(b*c - a*d))*\text{arctan}(-\text{sqrt}(d*x^6 + c)*(b*c - a*d)*\text{sqrt}(-b/(b*c - a*d)))/(b*d*x^6 + b*c)) - (2*b*c + a*d)*\text{sqrt}(c)*x^6*\text{log}((d*x^6 + 2*\text{sqrt}(d*x^6 + c))*\text{sqrt}(c) + 2*c)/x^6) + 2*\text{sqrt}(d*x^6 + c)*a*c)/(a^2*c^2*x^6), 1/6*(b*c^2*x^6*\text{sqrt}(b/(b*c - a*d))*\text{log}((b*d*x^6 + 2*b*c - a*d - 2*\text{sqrt}(d*x^6 + c))*(b*c - a*d)*\text{sqrt}(b/(b*c - a*d)))/(b*x^6 + a)) - (2*b*c + a*d)*\text{sqrt}(-c)*x^6*\text{arctan}(\text{sqrt}(d*x^6 + c)*\text{sqrt}(-c)/c) - \text{sqrt}(d*x^6 + c)*a*c)/(a^2*c^2*x^6), -1/6*(2*b*c^2*x^6*\text{sqrt}(-b/(b*c - a*d))*\text{arctan}(-\text{sqrt}(d*x^6 + c)*(b*c - a*d)*\text{sqrt}(-b/(b*c - a*d)))/(b*d*x^6 + b*c)) + (2*b*c + a*d)*\text{sqrt}(-c)*x^6*\text{arctan}(\text{sqrt}(d*x^6 + c)*\text{sqrt}(-c)/c) + \text{sqrt}(d*x^6 + c)*a*c)/(a^2*c^2*x^6)]$

giac [A] time = 0.18, size = 104, normalized size = 0.89

$$\frac{b^2 \arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2} - \frac{(2bc+ad) \arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{6a^2\sqrt{-c}c} - \frac{\sqrt{dx^6+c}}{6acx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] 1/3*b^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) - 1/6*(2*b*c + a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^2*sqrt(-c)*c) - 1/6*sqrt(d*x^6 + c)/(a*c*x^6)

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)\sqrt{dx^6 + c}x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x)

[Out] int(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)\sqrt{dx^6 + c}x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^7), x)

mupad [B] time = 5.62, size = 396, normalized size = 3.38

$$\frac{\ln\left(\frac{\sqrt{dx^6+c}(b^3c-ab^3d)^{3/2}+b^6c^2+a^2b^4d^2-2ab^5cd}{6a^3d-6a^2bc}\sqrt{b^4c-ab^3d}\right)-\ln\left(\frac{\sqrt{dx^6+c}(b^3c-ab^3d)^{3/2}-b^6c^2-a^2b^4d^2+2ab^5cd}{6(a^3d-a^2bc)}\sqrt{b^4c-ab^3d}\right)-\frac{\operatorname{atan}\left(\frac{b^4d\sqrt{dx^6+c}11}{18\sqrt{3}\left(\frac{b^4d}{18c^2}+\frac{b^4d^2}{108c^2}\right)}+\frac{b^2d\sqrt{dx^6+c}11}{108\sqrt{3}\left(\frac{b^4d}{108c^2}+\frac{b^4d^2}{108c^2}\right)}+\frac{b^3d\sqrt{dx^6+c}51}{108\sqrt{3}\left(\frac{b^4d}{108c^2}+\frac{b^4d^2}{108c^2}\right)}\right)}{6a^2\sqrt{c}}}{6a^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^7*(a + b*x^6)*(c + d*x^6)^(1/2)),x)

[Out] (log((c + d*x^6)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(6*a^3*d - 6*a^2*b*c) - (log((c + d*x^6

```

)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
*c - a*b^3*d)^(1/2))/(6*(a^3*d - a^2*b*c)) - (c + d*x^6)^(1/2)/(6*a*c*x^6)
- (atan((b^4*d^4*(c + d*x^6)^(1/2)*1i)/(18*(c^3)^(1/2)*((b^4*d^4)/(18*c) +
(5*a*b^3*d^5)/(108*c^2) + (a^2*b^2*d^6)/(108*c^3)))) + (b^2*d^6*(c + d*x^6)^(
1/2)*1i)/(108*(c^3)^(1/2)*((5*b^3*d^5)/(108*a) + (b^2*d^6)/(108*c) + (b^4*
c*d^4)/(18*a^2))) + (b^3*d^5*(c + d*x^6)^(1/2)*5i)/(108*(c^3)^(1/2)*((b^4*d
^4)/(18*a) + (5*b^3*d^5)/(108*c) + (a*b^2*d^6)/(108*c^2))))*(a*d + 2*b*c)*1
i)/(6*a^2*(c^3)^(1/2))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 (a + bx^6) \sqrt{c + dx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**7/(b*x**6+a)/(d*x**6+c)**(1/2),x)

[Out] Integral(1/(x**7*(a + b*x**6)*sqrt(c + d*x**6)), x)

$$3.548 \quad \int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}} dx$$

Optimal. Leaf size=123

$$\frac{a^{3/2} \tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}} - \frac{(2ad+bc) \tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}} + \frac{x^3\sqrt{c+dx^6}}{6bd}$$

Rubi [A] time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {465, 479, 523, 217, 206, 377, 205}

$$\frac{a^{3/2} \tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}} - \frac{(2ad+bc) \tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}} + \frac{x^3\sqrt{c+dx^6}}{6bd}$$

Antiderivative was successfully verified.

[In] Int[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] (x^3*Sqrt[c + d*x^6])/(6*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(3*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]])/(6*b^2*d^(3/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b}

, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 479

Int[((e_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))
^(q_)), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
(m + n(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x
)^(n)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^3 \right) \\
&= \frac{x^3\sqrt{c+dx^6}}{6bd} - \frac{\text{Subst} \left(\int \frac{ac+(bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^3 \right)}{6bd} \\
&= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^2 \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^3 \right)}{3b^2} - \frac{(bc+2ad) \text{Subst} \left(\int \frac{1}{\sqrt{c+dx^2}} dx, x, x^3 \right)}{6b^2d} \\
&= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^2 \text{Subst} \left(\int \frac{1}{a-(-bc+ad)x^2} dx, x, \frac{x^3}{\sqrt{c+dx^6}} \right)}{3b^2} - \frac{(bc+2ad) \text{Subst} \left(\int \frac{1}{1-dx^2} dx, x, \frac{x^3}{\sqrt{c+dx^6}} \right)}{6b^2d} \\
&= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^{3/2} \tan^{-1} \left(\frac{\sqrt{bc-ad} x^3}{\sqrt{a} \sqrt{c+dx^6}} \right)}{3b^2\sqrt{bc-ad}} - \frac{(bc+2ad) \tanh^{-1} \left(\frac{\sqrt{d} x^3}{\sqrt{c+dx^6}} \right)}{6b^2d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 118, normalized size = 0.96

$$\frac{2a^{3/2} \tan^{-1} \left(\frac{x^3 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^6}} \right)}{\sqrt{bc-ad}} - \frac{(2ad+bc) \log \left(\sqrt{d} \sqrt{c+dx^6} + dx^3 \right)}{d^{3/2}} + \frac{bx^3 \sqrt{c+dx^6}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] ((b*x^3*Sqrt[c + d*x^6])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*Log[d*x^3 + Sqrt[d]*Sqrt[c + d*x^6]])/d^(3/2))/(6*b^2)

IntegrateAlgebraic [A] time = 1.11, size = 177, normalized size = 1.44

$$\frac{a^{3/2} \tan^{-1} \left(\frac{b\sqrt{d}x^6}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}} \right)}{3b^2\sqrt{bc-ad}} + \frac{(-2ad-bc) \log \left(\sqrt{c+dx^6} + \sqrt{d}x^3 \right)}{6b^2d^{3/2}} + \frac{x^3\sqrt{c+dx^6}}{6bd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] (x^3*Sqrt[c + d*x^6])/(6*b*d) + (a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^6)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^3*Sqrt[c + d*x^6])

)/(Sqrt[a]*Sqrt[b*c - a*d]))/(3*b^2*Sqrt[b*c - a*d]) + ((-(b*c) - 2*a*d)*Log[Sqrt[d]*x^3 + Sqrt[c + d*x^6]])/(6*b^2*d^(3/2))

fricas [A] time = 0.55, size = 739, normalized size = 6.01

1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + (b*c + 2*a*d)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2), 1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*(b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2), 1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d)))/(a*d*x^9 + a*c*x^3)) + (b*c + 2*a*d)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2), 1/6*(sqrt(d*x^6 + c)*b*d*x^3 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d)))/(a*d*x^9 + a*c*x^3)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + (b*c + 2*a*d)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2), 1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*(b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2), 1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d)))/(a*d*x^9 + a*c*x^3)) + (b*c + 2*a*d)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2), 1/6*(sqrt(d*x^6 + c)*b*d*x^3 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d)))/(a*d*x^9 + a*c*x^3)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(bx^6 + a)\sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] integrate(x^14/((b*x^6 + a)*sqrt(d*x^6 + c)), x)

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(bx^6 + a)\sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x)`

[Out] `int(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(bx^6 + a)\sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^14/((b*x^6 + a)*sqrt(d*x^6 + c)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{(bx^6 + a)\sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^14/((a + b*x^6)*(c + d*x^6)^(1/2)),x)`

[Out] `int(x^14/((a + b*x^6)*(c + d*x^6)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(a + bx^6)\sqrt{c + dx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**14/(b*x**6+a)/(d*x**6+c)**(1/2),x)`

[Out] `Integral(x**14/((a + b*x**6)*sqrt(c + d*x**6)), x)`

$$3.549 \quad \int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}} dx$$

Optimal. Leaf size=91

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}} - \frac{\sqrt{a} \tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}}$$

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 483, 217, 206, 377, 205}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}} - \frac{\sqrt{a} \tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(3*b*Sqrt[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(3*b*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b,
```


, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 483

Int[(((e_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_))^(q_))/((a_) + (b_)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]

Rubi steps

$$\begin{aligned} \int \frac{x^8}{(a + bx^6)\sqrt{c + dx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(a + bx^2)\sqrt{c + dx^2}} dx, x, x^3 \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{\sqrt{c + dx^2}} dx, x, x^3 \right) - a \text{Subst} \left(\int \frac{1}{(a + bx^2)\sqrt{c + dx^2}} dx, x, x^3 \right)}{3b} \\ &= \frac{\text{Subst} \left(\int \frac{1}{1 - dx^2} dx, x, \frac{x^3}{\sqrt{c + dx^6}} \right) - a \text{Subst} \left(\int \frac{1}{a - (-bc + ad)x^2} dx, x, \frac{x^3}{\sqrt{c + dx^6}} \right)}{3b} \\ &= -\frac{\sqrt{a} \tan^{-1} \left(\frac{\sqrt{bc - ad} x^3}{\sqrt{a} \sqrt{c + dx^6}} \right)}{3b\sqrt{bc - ad}} + \frac{\tanh^{-1} \left(\frac{\sqrt{d} x^3}{\sqrt{c + dx^6}} \right)}{3b\sqrt{d}} \end{aligned}$$

Mathematica [A] time = 0.09, size = 90, normalized size = 0.99

$$\frac{\log\left(\sqrt{d}\sqrt{c+dx^6}+dx^3\right)}{\sqrt{d}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{\sqrt{bc-ad}}$$

3b

Antiderivative was successfully verified.

[In] Integrate[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] $-\left(\left(\text{Sqrt}[a] \cdot \text{ArcTan}\left[\frac{\text{Sqrt}[b \cdot c - a \cdot d] \cdot x^3}{\text{Sqrt}[a] \cdot \text{Sqrt}[c + d \cdot x^6]}\right]\right) / \text{Sqrt}[b \cdot c - a \cdot d]\right) + \text{Log}\left[\frac{d \cdot x^3 + \text{Sqrt}[d] \cdot \text{Sqrt}[c + d \cdot x^6]}{\text{Sqrt}[d]}\right] / (3 \cdot b)$

IntegrateAlgebraic [A] time = 0.62, size = 144, normalized size = 1.58

$$\frac{\log\left(\sqrt{c + dx^6} + \sqrt{d}x^3\right)}{3b\sqrt{d}} - \frac{\sqrt{a} \tan^{-1}\left(\frac{b\sqrt{d}x^6}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{3b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] $-\frac{1}{3} \cdot \left(\frac{\text{Sqrt}[a] \cdot \text{ArcTan}\left[\frac{\text{Sqrt}[a] \cdot \text{Sqrt}[d]}{\text{Sqrt}[b \cdot c - a \cdot d]}\right] / \text{Sqrt}[b \cdot c - a \cdot d] + (b \cdot \text{Sqrt}[d] \cdot x^6) / (\text{Sqrt}[a] \cdot \text{Sqrt}[b \cdot c - a \cdot d])\right) + (b \cdot x^3 \cdot \text{Sqrt}[c + d \cdot x^6]) / (\text{Sqrt}[a] \cdot \text{Sqrt}[b \cdot c - a \cdot d])\right) / (b \cdot \text{Sqrt}[b \cdot c - a \cdot d]) + \text{Log}\left[\frac{\text{Sqrt}[d] \cdot x^3 + \text{Sqrt}[c + d \cdot x^6]}{3 \cdot b \cdot \text{Sqrt}[d]}\right]$

fricas [A] time = 0.52, size = 632, normalized size = 6.95

$$\frac{1}{3} \sqrt{\frac{a}{d}} \log\left(\frac{(b^2 c^2 - 8 a b c d + 8 a^2 d^2) x^{12} - 2(3 a^2 b c^2 - 4 a^2 c^2 d) x^6 + a^2 c^2 - 4((b^2 c^2 - 3 a b c d + 2 a^2 d^2) x^9 - (a b c^2 - a^2 c d) x^3) \sqrt{d x^6 + c} \sqrt{-a/(b c - a d)}}{(b^2 x^{12} + 2 a b x^6 + a^2)}\right) + 2 \sqrt{d} \log\left(\frac{-2 d x^6 - 2 \sqrt{d x^6 + c}}{d}\right) \sqrt{\frac{a}{d}} \arctan\left(\frac{\sqrt{d} x^3}{\sqrt{d x^6 + c}}\right) + 4 \sqrt{\frac{a}{d}} \arctan\left(\frac{\sqrt{d} x^3}{\sqrt{d x^6 + c}}\right) \sqrt{\frac{a}{d}} \arctan\left(\frac{-2 a b c d + 8 a^2 d^2}{(b^2 c^2 - 8 a b c d + 8 a^2 d^2) x^{12} - 2(3 a^2 b c^2 - 4 a^2 c^2 d) x^6 + a^2 c^2 - 4((b^2 c^2 - 3 a b c d + 2 a^2 d^2) x^9 - (a b c^2 - a^2 c d) x^3) \sqrt{d x^6 + c} \sqrt{-a/(b c - a d)}}\right) - 2 \sqrt{d} \arctan\left(\frac{\sqrt{d} x^3}{\sqrt{d x^6 + c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] $\left[\frac{1}{12} \cdot (d \cdot \text{sqrt}(-a/(b \cdot c - a \cdot d))) \cdot \log\left(\frac{(b^2 \cdot c^2 - 8 \cdot a \cdot b \cdot c \cdot d + 8 \cdot a^2 \cdot d^2) \cdot x^{12} - 2 \cdot (3 \cdot a^2 \cdot b \cdot c^2 - 4 \cdot a^2 \cdot c^2 \cdot d) \cdot x^6 + a^2 \cdot c^2 - 4 \cdot ((b^2 \cdot c^2 - 3 \cdot a \cdot b \cdot c \cdot d + 2 \cdot a^2 \cdot d^2) \cdot x^9 - (a \cdot b \cdot c^2 - a^2 \cdot c \cdot d) \cdot x^3) \cdot \text{sqrt}(d \cdot x^6 + c) \cdot \text{sqrt}(-a/(b \cdot c - a \cdot d))}{(b^2 \cdot x^{12} + 2 \cdot a \cdot b \cdot x^6 + a^2)}\right) + 2 \cdot \text{sqrt}(d) \cdot \log\left(\frac{-2 \cdot d \cdot x^6 - 2 \cdot \text{sqrt}(d \cdot x^6 + c) \cdot \text{sqrt}(d) \cdot x^3 - c}{(b \cdot d)}\right), \frac{1}{12} \cdot (d \cdot \text{sqrt}(-a/(b \cdot c - a \cdot d))) \cdot \log\left(\frac{(b^2 \cdot c^2 - 8 \cdot a \cdot b \cdot c \cdot d + 8 \cdot a^2 \cdot d^2) \cdot x^{12} - 2 \cdot (3 \cdot a^2 \cdot b \cdot c^2 - 4 \cdot a^2 \cdot c^2 \cdot d) \cdot x^6 + a^2 \cdot c^2 - 4 \cdot ((b^2 \cdot c^2 - 3 \cdot a \cdot b \cdot c \cdot d + 2 \cdot a^2 \cdot d^2) \cdot x^9 - (a \cdot b \cdot c^2 - a^2 \cdot c \cdot d) \cdot x^3) \cdot \text{sqrt}(d \cdot x^6 + c) \cdot \text{sqrt}(-a/(b \cdot c - a \cdot d))}{(b^2 \cdot x^{12} + 2 \cdot a \cdot b \cdot x^6 + a^2)}\right) - 4 \cdot \text{sqrt}(-d) \cdot \arctan\left(\frac{\text{sqrt}(-d) \cdot x^3 / \text{sqrt}(d \cdot x^6 + c)}{(b \cdot d)}\right), \frac{1}{6} \cdot (d \cdot \text{sqrt}(a/(b \cdot c - a \cdot d))) \cdot \arctan\left(\frac{-1/2 \cdot (b \cdot c - 2 \cdot a \cdot d) \cdot x^6 - a \cdot c}{\text{sqrt}(d \cdot x^6 + c) \cdot \text{sqrt}(a/(b \cdot c - a \cdot d))}\right) / (a \cdot d \cdot x^9 + a \cdot c \cdot x^3) + \text{sqrt}(d) \cdot \log\left(\frac{-2 \cdot d \cdot x^6 - 2 \cdot \text{sqrt}(d \cdot x^6 + c) \cdot \text{sqrt}(d) \cdot x^3 - c}{(b \cdot d)}\right), \frac{1}{6} \cdot (d \cdot \text{sqrt}(a/(b \cdot c - a \cdot d))) \cdot \arctan\left(\frac{-1/2 \cdot (b \cdot c - 2 \cdot a \cdot d) \cdot x^6 - a \cdot c}{\text{sqrt}(d \cdot x^6 + c) \cdot \text{sqrt}(a/(b \cdot c - a \cdot d))}\right) / (a \cdot d \cdot x^9 + a \cdot c \cdot x^3) - 2 \cdot \text{sqrt}(-d) \cdot \arctan\left(\frac{\text{sqrt}(-d) \cdot x^3 / \text{sqrt}(d \cdot x^6 + c)}{(b \cdot d)}\right)\right]$

giac [B] time = 0.23, size = 156, normalized size = 1.71

$$\frac{\left(a \sqrt{-d} \arctan\left(\frac{a \sqrt{d}}{\sqrt{abc - a^2 d}}\right) - \sqrt{abc - a^2 d} \arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)\right) \text{sgn}(x)}{3 \sqrt{abc - a^2 d} b \sqrt{-d}} + \frac{a \arctan\left(\frac{a \sqrt{d + \frac{c}{x^6}}}{\sqrt{abc - a^2 d}}\right)}{3 \sqrt{abc - a^2 d} b \text{sgn}(x)} - \frac{\arctan\left(\frac{\sqrt{d + \frac{c}{x^6}}}{\sqrt{-d}}\right)}{3 b \sqrt{-d} \text{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] $-1/3*(a*\sqrt{-d}*\arctan(a*\sqrt{d}/\sqrt{a*b*c - a^2*d}) - \sqrt{a*b*c - a^2*d})*\arctan(\sqrt{d}/\sqrt{-d}))*\operatorname{sgn}(x)/(\sqrt{a*b*c - a^2*d}*b*\sqrt{-d}) + 1/3*a*\arctan(a*\sqrt{d + c/x^6}/\sqrt{a*b*c - a^2*d})/(\sqrt{a*b*c - a^2*d}*b*\operatorname{sgn}(x)) - 1/3*\arctan(\sqrt{d + c/x^6}/\sqrt{-d})/(b*\sqrt{-d}*\operatorname{sgn}(x))$

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(bx^6 + a)\sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x)

[Out] int(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(bx^6 + a)\sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^8/((b*x^6 + a)*sqrt(d*x^6 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{(bx^6 + a)\sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((a + b*x^6)*(c + d*x^6)^(1/2)),x)

[Out] int(x^8/((a + b*x^6)*(c + d*x^6)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a + bx^6)\sqrt{c + dx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

$$3.550 \quad \int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}} dx$$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {465, 377, 205}

$$\frac{\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])]/(3*Sqrt[a]*Sqrt[b*c - a*d])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \frac{1}{a-(-bc+ad)x^2} dx, x, \frac{x^3}{\sqrt{c+dx^6}} \right) \\
&= \frac{\tan^{-1} \left(\frac{\sqrt{bc-ad} x^3}{\sqrt{a} \sqrt{c+dx^6}} \right)}{3\sqrt{a} \sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 95, normalized size = 1.76

$$\frac{x^3 \sqrt{\frac{dx^6}{c} + 1} \tanh^{-1} \left(\frac{\sqrt{\frac{dx^6}{c} - \frac{bx^6}{a}}}{\sqrt{\frac{dx^6}{c} + 1}} \right)}{3a\sqrt{c+dx^6} \sqrt{\frac{dx^6}{c} - \frac{bx^6}{a}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + b*x^6)*Sqrt[c + d*x^6]), x]

[Out] (x^3*Sqrt[1 + (d*x^6)/c]*ArcTanh[Sqrt[-((b*x^6)/a) + (d*x^6)/c]/Sqrt[1 + (d*x^6)/c]])/(3*a*Sqrt[c + d*x^6]*Sqrt[-((b*x^6)/a) + (d*x^6)/c])

IntegrateAlgebraic [A] time = 0.40, size = 106, normalized size = 1.96

$$\frac{\tan^{-1} \left(\frac{b\sqrt{d}x^6}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}} \right)}{3\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((a + b*x^6)*Sqrt[c + d*x^6]), x]

[Out] ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^6)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])]/(3*Sqrt[a]*Sqrt[b*c - a*d])

fricas [B] time = 0.50, size = 245, normalized size = 4.54

$$\left[\frac{\sqrt{-abc+a^2d} \log \left(\frac{(b^2c^2-8abcd+8a^2d^2)x^{12}-2(3abc^2-4a^2cd)x^6+a^2c^2-4((bc-2ad)x^9-acx^3)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2abx^6+a^2} \right)}{12(abc-a^2d)}, \frac{\arctan \left(\frac{((bc-2ad)x^6-ac)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{2((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3)} \right)}{6\sqrt{abc-a^2d}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] $[-1/12*\sqrt{-a*b*c + a^2*d}*\log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^{12} - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*\sqrt{d*x^6 + c}*\sqrt{-a*b*c + a^2*d}))/((b^2*x^{12} + 2*a*b*x^6 + a^2)))/(a*b*c - a^2*d), 1/6*\arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*\sqrt{d*x^6 + c}*\sqrt{a*b*c - a^2*d}))/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3))/\sqrt{a*b*c - a^2*d}]$

giac [A] time = 0.25, size = 72, normalized size = 1.33

$$\frac{\sqrt{d} \arctan\left(\frac{\left(\sqrt{d}x^3 - \sqrt{dx^6+c}\right)^2 b - bc + 2ad}{2\sqrt{abcd - a^2d^2}}\right)}{3\sqrt{abcd - a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] $-1/3*\sqrt{d}*\arctan(1/2*((\sqrt{d}*x^3 - \sqrt{d*x^6 + c})^2*b - b*c + 2*a*d)/\sqrt{a*b*c*d - a^2*d^2}))/\sqrt{a*b*c*d - a^2*d^2}$

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^6 + a)\sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)

[Out] int(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^6 + a)\sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/((b*x^6 + a)*sqrt(d*x^6 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{(bx^6 + a) \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((a + b*x^6)*(c + d*x^6)^(1/2)),x)`

[Out] `int(x^2/((a + b*x^6)*(c + d*x^6)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + bx^6) \sqrt{c + dx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**6+a)/(d*x**6+c)**(1/2),x)`

[Out] `Integral(x**2/((a + b*x**6)*sqrt(c + d*x**6)), x)`

$$3.551 \quad \int \frac{1}{x^4(a+bx^6)\sqrt{c+dx^6}} dx$$

Optimal. Leaf size=80

$$-\frac{b \tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^6}}{3acx^3}$$

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {465, 480, 12, 377, 205}

$$-\frac{b \tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^6}}{3acx^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] -Sqrt[c + d*x^6]/(3*a*c*x^3) - (b*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(3*a^(3/2)*Sqrt[b*c - a*d])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ

[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 480

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^4 (a + bx^6) \sqrt{c + dx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2 (a + bx^2) \sqrt{c + dx^2}} dx, x, x^3 \right) \\
 &= -\frac{\sqrt{c + dx^6}}{3acx^3} - \frac{\text{Subst} \left(\int \frac{bc}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^3 \right)}{3ac} \\
 &= -\frac{\sqrt{c + dx^6}}{3acx^3} - \frac{b \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^3 \right)}{3a} \\
 &= -\frac{\sqrt{c + dx^6}}{3acx^3} - \frac{b \text{Subst} \left(\int \frac{1}{a - (-bc+ad)x^2} dx, x, \frac{x^3}{\sqrt{c+dx^6}} \right)}{3a} \\
 &= -\frac{\sqrt{c + dx^6}}{3acx^3} - \frac{b \tan^{-1} \left(\frac{\sqrt{bc-ad} x^3}{\sqrt{a} \sqrt{c+dx^6}} \right)}{3a^{3/2} \sqrt{bc - ad}}
 \end{aligned}$$

Mathematica [C] time = 1.17, size = 179, normalized size = 2.24

$$\frac{\left(\frac{dx^6}{c} + 1 \right) \left(\frac{4x^6(c+dx^6)(bc-ad) {}_2F_1 \left(2, 2; \frac{5}{2}; \frac{(bc-ad)x^6}{c(bx^6+a)} \right)}{3c^2(a+bx^6)} + \frac{(c+2dx^6) \sin^{-1} \left(\sqrt{\frac{x^6(bc-ad)}{c(a+bx^6)}} \right)}{c \sqrt{\frac{ax^6(c+dx^6)(bc-ad)}{c^2(a+bx^6)^2}}} \right)}{3x^3 (a + bx^6) \sqrt{c + dx^6}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out]
$$-1/3*((1 + (d*x^6)/c)*((c + 2*d*x^6)*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]])/(c*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6)^2)]) + (4*(b*c - a*d)*x^6*(c + d*x^6)*Hypergeometric2F1[2, 2, 5/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))])/(3*c^2*(a + b*x^6)))/(x^3*(a + b*x^6)*Sqrt[c + d*x^6])$$

IntegrateAlgebraic [A] time = 0.66, size = 142, normalized size = 1.78

$$\frac{b\sqrt{bc-ad} \tan^{-1}\left(\frac{b\sqrt{d}x^6}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{3a^{3/2}(ad-bc)} - \frac{\sqrt{c+dx^6}}{3acx^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out]
$$-1/3*\text{Sqrt}[c + d*x^6]/(a*c*x^3) + (b*\text{Sqrt}[b*c - a*d]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sqrt}[d])/\text{Sqrt}[b*c - a*d] + (b*\text{Sqrt}[d]*x^6)/(\text{Sqrt}[a]*\text{Sqrt}[b*c - a*d]) + (b*x^3*\text{Sqrt}[c + d*x^6])/(\text{Sqrt}[a]*\text{Sqrt}[b*c - a*d])])/(3*a^{(3/2)}*(-(b*c) + a*d))$$

fricas [B] time = 0.51, size = 332, normalized size = 4.15

$$\left[\frac{\sqrt{-abc + a^2d} b c x^3 \log\left(\frac{(b^2c^2 - 8abcd + 8a^2d^2)x^{12} - 2(3abc^2 - 4a^2cd)x^6 + a^2c^2 + 4((bc - 2ad)x^9 - acx^3)\sqrt{dx^6 + c}\sqrt{-abc + a^2d}}{b^2x^{12} + 2abx^6 + a^2}\right) + 4\sqrt{dx^6 + c}(abc - a^2d)}{12(a^2bc^2 - a^3cd)x^3}, \frac{\sqrt{abc - a^2d} b c x^3 \arctan\left(\frac{(bc - 2ad)x^6 - ac}{2\sqrt{(abcd - a^2d^2)x^9 + (abc^2 - a^2cd)x^3}}\right) + 2\sqrt{dx^6 + c}(abc - a^2d)}{6(a^2bc^2 - a^3cd)x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out]
$$[-1/12*(\text{sqrt}(-a*b*c + a^2*d)*b*c*x^3*\log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^{12} - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*\text{sqrt}(d*x^6 + c)*\text{sqrt}(-a*b*c + a^2*d))/(b^2*x^{12} + 2*a*b*x^6 + a^2)) + 4*\text{sqrt}(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3), -1/6*(\text{sqrt}(a*b*c - a^2*d)*b*c*x^3*\arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*\text{sqrt}(d*x^6 + c)*\text{sqrt}(a*b*c - a^2*d))/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*\text{sqrt}(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3)]$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)\sqrt{dx^6 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)

[Out] int(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)\sqrt{dx^6 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 (bx^6 + a) \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a + b*x^6)*(c + d*x^6)^(1/2)),x)

[Out] int(1/(x^4*(a + b*x^6)*(c + d*x^6)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + bx^6) \sqrt{c + dx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**6+a)/(d*x**6+c)**(1/2),x)

[Out] Integral(1/(x**4*(a + b*x**6)*sqrt(c + d*x**6)), x)

$$3.552 \quad \int \frac{1}{x^{10}(a+bx^6)\sqrt{c+dx^6}} dx$$

Optimal. Leaf size=115

$$\frac{b^2 \tan^{-1}\left(\frac{x^3 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^6}}\right)}{3a^{5/2} \sqrt{bc-ad}} + \frac{\sqrt{c+dx^6}(2ad+3bc)}{9a^2 c^2 x^3} - \frac{\sqrt{c+dx^6}}{9acx^9}$$

Rubi [A] time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 480, 583, 12, 377, 205}

$$\frac{b^2 \tan^{-1}\left(\frac{x^3 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^6}}\right)}{3a^{5/2} \sqrt{bc-ad}} + \frac{\sqrt{c+dx^6}(2ad+3bc)}{9a^2 c^2 x^3} - \frac{\sqrt{c+dx^6}}{9acx^9}$$

Antiderivative was successfully verified.

[In] Int[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] -Sqrt[c + d*x^6]/(9*a*c*x^9) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^6])/(9*a^2*c^2*x^3) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(3*a^(5/2)*Sqrt[b*c - a*d])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -

1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
 [{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 480

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))
)^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
 }, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
 , c, d, e, m, n, p, q, x]

Rule 583

Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))
)^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
 m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{10}(a+bx^6)\sqrt{c+dx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^4(a+bx^2)\sqrt{c+dx^2}} dx, x, x^3 \right) \\
&= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{\text{Subst} \left(\int \frac{-3bc-2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}} dx, x, x^3 \right)}{9ac} \\
&= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3} - \frac{\text{Subst} \left(\int -\frac{3b^2c^2}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^3 \right)}{9a^2c^2} \\
&= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3} + \frac{b^2 \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^3 \right)}{3a^2} \\
&= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3} + \frac{b^2 \text{Subst} \left(\int \frac{1}{a-(-bc+ad)x^2} dx, x, \frac{x^3}{\sqrt{c+dx^6}} \right)}{3a^2} \\
&= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3} + \frac{b^2 \tan^{-1} \left(\frac{\sqrt{bc-ad} x^3}{\sqrt{a} \sqrt{c+dx^6}} \right)}{3a^{5/2} \sqrt{bc-ad}}
\end{aligned}$$

Mathematica [C] time = 4.65, size = 253, normalized size = 2.20

$$\frac{\left(\frac{dx^6}{c} + 1\right) \left(-\frac{8x^6(c+dx^6)^2(bc-ad) {}_3F_2\left(2,2,2;1,\frac{5}{2};\frac{(bc-ad)x^6}{c(bx^6+a)}\right)}{a+bx^6} + \frac{3c(c^2-4cdx^6-8d^2x^{12}) \sin^{-1}\left(\sqrt{\frac{x^6(bc-ad)}{c(a+bx^6)}}\right)}{\sqrt{\frac{ax^6(c+dx^6)(bc-ad)}{c^2(a+bx^6)^2}}} + \frac{24dx^{12}(c+dx^6)(ad-bc) {}_2F_1\left(2,2;\frac{5}{2};\frac{(bc-ad)x^6}{c(bx^6+a)}\right)}{a+bx^6} \right)}{27c^3x^9(a+bx^6)\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] $-1/27*((1 + (d*x^6)/c)*((3*c*(c^2 - 4*c*d*x^6 - 8*d^2*x^{12})*\text{ArcSin}[\text{Sqrt}[(b*c - a*d)*x^6/(c*(a + b*x^6))]])/\text{Sqrt}[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6)^2]) + (24*d*(-(b*c) + a*d)*x^{12}*(c + d*x^6)*\text{Hypergeometric2F1}[2, 2, 5/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))]/(a + b*x^6) - (8*(b*c - a*d)*x^6*(c + d*x^6)^2*\text{HypergeometricPFQ}[\{2, 2, 2\}, \{1, 5/2\}, ((b*c - a*d)*x^6)/(c*(a + b*x^6))]/(a + b*x^6)))/(c^3*x^9*(a + b*x^6)*\text{Sqrt}[c + d*x^6])$

IntegrateAlgebraic [A] time = 1.30, size = 163, normalized size = 1.42

$$\frac{\sqrt{c+dx^6}(-ac+2adx^6+3bcx^6)}{9a^2c^2x^9} - \frac{b^2\sqrt{bc-ad}\tan^{-1}\left(\frac{b\sqrt{d}x^6}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{3a^{5/2}(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] (Sqrt[c + d*x^6]*(-(a*c) + 3*b*c*x^6 + 2*a*d*x^6))/(9*a^2*c^2*x^9) - (b^2*Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^6)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])])/(3*a^(5/2)*(-(b*c) + a*d))

fricas [A] time = 0.53, size = 416, normalized size = 3.62

$$\frac{3\sqrt{-abc + d^2d}b^2c^2x^9 \log\left(\frac{(b^2c^2 - 8abd + 8a^2d)^{1/2} - 2(3ab^2 - 4a^2d)^{1/2} + (bc - 2ad)^{1/2} \sqrt{bc^2 - ac^2}}{b^2c^2 + 2ab^2d}\right) - 4\left((3ab^2c^2 - a^2bcd - 2a^3d^2)^{1/2} - a^2bc^2 + a^3cd\right)\sqrt{dx^6 + c}}{36(a^3bc^3 - a^4c^2d)x^9} + \frac{3\sqrt{abc - a^2d}b^2c^2x^9 \arctan\left(\frac{(bc - 2ad)^{1/2} \sqrt{bc^2 - ac^2} \sqrt{dx^6 + c}}{2((abcd - a^2d^2)^{1/2} + (abc^2 - a^2d)^{1/2})}\right) + 2\left((3ab^2c^2 - a^2bcd - 2a^3d^2)^{1/2} - a^2bc^2 + a^3cd\right)\sqrt{dx^6 + c}}{18(a^3bc^3 - a^4c^2d)x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [-1/36*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^9*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*c*d)*sqrt(d*x^6 + c)]/((a^3*b*c^3 - a^4*c^2*d)*x^9), 1/18*(3*sqrt(a*b*c - a^2*d)*b^2*c^2*x^9*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d))/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*c*d)*sqrt(d*x^6 + c)/((a^3*b*c^3 - a^4*c^2*d)*x^9)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)\sqrt{dx^6 + c}x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x)

[Out] int(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)\sqrt{dx^6 + cx^{10}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^10), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{10} (bx^6 + a) \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^10*(a + b*x^6)*(c + d*x^6)^(1/2)),x)

[Out] int(1/(x^10*(a + b*x^6)*(c + d*x^6)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10} (a + bx^6) \sqrt{c + dx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**10/(b*x**6+a)/(d*x**6+c)**(1/2),x)

[Out] Integral(1/(x**10*(a + b*x**6)*sqrt(c + d*x**6)), x)

$$3.553 \quad \int \frac{x^{17}}{(a+bx^6)^2 \sqrt{c+dx^6}} dx$$

Optimal. Leaf size=123

$$-\frac{a^2 \sqrt{c+dx^6}}{6b^2 (a+bx^6)(bc-ad)} + \frac{a(4bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx^6}}{3b^2d}$$

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2 \sqrt{c+dx^6}}{6b^2 (a+bx^6)(bc-ad)} + \frac{a(4bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx^6}}{3b^2d}$$

Antiderivative was successfully verified.

[In] Int[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] Sqrt[c + d*x^6]/(3*b^2*d) - (a^2*Sqrt[c + d*x^6])/((6*b^2*(b*c - a*d)*(a + b*x^6)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(6*b^(5/2)*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 89

Int[((a_.) + (b_.)*(x_))^(2)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)

)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{17}}{(a + bx^6)^2 \sqrt{c + dx^6}} dx &= \frac{1}{6} \text{Subst} \left(\int \frac{x^2}{(a + bx)^2 \sqrt{c + dx}} dx, x, x^6 \right) \\
 &= -\frac{a^2 \sqrt{c + dx^6}}{6b^2(bc - ad)(a + bx^6)} + \frac{\text{Subst} \left(\int \frac{-\frac{1}{2}a(2bc - ad) + b(bc - ad)x}{(a + bx)\sqrt{c + dx}} dx, x, x^6 \right)}{6b^2(bc - ad)} \\
 &= \frac{\sqrt{c + dx^6}}{3b^2d} - \frac{a^2 \sqrt{c + dx^6}}{6b^2(bc - ad)(a + bx^6)} - \frac{(a(4bc - 3ad)) \text{Subst} \left(\int \frac{1}{(a + bx)\sqrt{c + dx}} dx, x, x^6 \right)}{12b^2(bc - ad)} \\
 &= \frac{\sqrt{c + dx^6}}{3b^2d} - \frac{a^2 \sqrt{c + dx^6}}{6b^2(bc - ad)(a + bx^6)} - \frac{(a(4bc - 3ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^6} \right)}{6b^2d(bc - ad)} \\
 &= \frac{\sqrt{c + dx^6}}{3b^2d} - \frac{a^2 \sqrt{c + dx^6}}{6b^2(bc - ad)(a + bx^6)} + \frac{a(4bc - 3ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^6}}{\sqrt{bc - ad}} \right)}{6b^{5/2}(bc - ad)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.36, size = 107, normalized size = 0.87

$$\frac{1}{6} \left(\frac{\sqrt{c + dx^6} \left(\frac{a^2}{(a+bx^6)(ad-bc)} + \frac{2}{d} \right)}{b^2} + \frac{a(4bc - 3ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^6}}{\sqrt{bc-ad}} \right)}{b^{5/2}(bc - ad)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] ((Sqrt[c + d*x^6]*(2/d + a^2/((-b*c) + a*d)*(a + b*x^6)))/b^2 + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c - a*d)^(3/2))/6

IntegrateAlgebraic [A] time = 0.33, size = 143, normalized size = 1.16

$$\frac{(3a^2d - 4abc) \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^6} \sqrt{ad-bc}}{bc-ad} \right)}{6b^{5/2}(ad - bc)^{3/2}} - \frac{\sqrt{c + dx^6} (3a^2d - 2abc + 2abdx^6 - 2b^2cx^6)}{6b^2d(a + bx^6)(bc - ad)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] -1/6*(Sqrt[c + d*x^6]*(-2*a*b*c + 3*a^2*d - 2*b^2*c*x^6 + 2*a*b*d*x^6))/(b^2*d*(b*c - a*d)*(a + b*x^6)) + ((-4*a*b*c + 3*a^2*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a*d)])/(6*b^(5/2)*(-(b*c) + a*d)^(3/2))

fricas [B] time = 0.65, size = 475, normalized size = 3.86

$$\frac{\left((4ab^2cd - 3a^2b^2d)^2 + 4a^2bcd - 3a^2b^2d \right) \sqrt{bc - ad} \log \left(\frac{ab^2c - ab^2d + 2\sqrt{bc - ad} \sqrt{c + dx^6}}{a^2 + b^2} \right) + 2 \left((b^2c^2 - 2ab^2cd + a^2b^2d)^2 + 2ab^2c^2 - 5a^2b^2cd + 3a^2b^2d \right) \sqrt{dx^6 + c}}{12(ab^2cd - 2a^2b^2cd + a^2b^2d)^2 + (b^2c^2 - 2ab^2cd + a^2b^2d)^2} - \frac{\left((4ab^2cd - 3a^2b^2d)^2 + 4a^2bcd - 3a^2b^2d \right) \sqrt{bc - ad} \arctan \left(\frac{\sqrt{bc - ad} \sqrt{c + dx^6}}{a^2 + b^2} \right) - \left((b^2c^2 - 2ab^2cd + a^2b^2d)^2 + 2ab^2c^2 - 5a^2b^2cd + 3a^2b^2d \right) \sqrt{dx^6 + c}}{6(ab^2cd - 2a^2b^2cd + a^2b^2d)^2 + (b^2c^2 - 2ab^2cd + a^2b^2d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [1/12*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^6), - 1/6*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) - (2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a*b^3*c^2 - 5*a^2*b^2*c*d +

$$3a^3b^2d^2 \sqrt{dx^6 + c} / (a^5b^2c^2d - 2a^2b^4c^2d^2 + a^3b^3d^3 + (b^6c^2d - 2a^5b^5c^2d^2 + a^2b^4d^3)x^6]$$

giac [A] time = 0.44, size = 134, normalized size = 1.09

$$\frac{\sqrt{dx^6 + c} a^2 d}{6(b^3c - ab^2d)((dx^6 + c)b - bc + ad)} - \frac{(4abc - 3a^2d) \arctan\left(\frac{\sqrt{dx^6 + c} b}{\sqrt{-b^2c + abd}}\right)}{6(b^3c - ab^2d)\sqrt{-b^2c + abd}} + \frac{\sqrt{dx^6 + c}}{3b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] -1/6*sqrt(d*x^6 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^6 + c)*b - b*c + a*d)) - 1/6*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/3*sqrt(d*x^6 + c)/(b^2*d)

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^{17}}{(bx^6 + a)^2 \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

[Out] int(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 5.09, size = 144, normalized size = 1.17

$$\frac{\sqrt{dx^6 + c}}{3b^2d} - \frac{a \operatorname{atan}\left(\frac{a\sqrt{b}\sqrt{dx^6 + c}(3ad - 4bc)}{(3a^2d - 4abc)\sqrt{ad - bc}}\right)(3ad - 4bc)}{6b^{5/2}(ad - bc)^{3/2}} + \frac{a^2d\sqrt{dx^6 + c}}{2(ad - bc)(3b^3(dx^6 + c) - 3b^3c + 3ab^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (c + d*x^6)^(1/2)/(3*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^6)^(1/2)*(3*a*d -
4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(6*b^(5/
2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^6)^(1/2))/(2*(a*d - b*c)*(3*b^3*(c
+ d*x^6) - 3*b^3*c + 3*a*b^2*d))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.554 \quad \int \frac{x^{11}}{(a+bx^6)^2 \sqrt{c+dx^6}} dx$$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^6}}{6b(a+bx^6)(bc-ad)} - \frac{(2bc-ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^6}}{6b(a+bx^6)(bc-ad)} - \frac{(2bc-ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] (a*Sqrt[c + d*x^6])/((6*b*(b*c - a*d)*(a + b*x^6)) - ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(6*b^(3/2)*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rubi steps

$$\begin{aligned} \int \frac{x^{11}}{(a + bx^6)^2 \sqrt{c + dx^6}} dx &= \frac{1}{6} \text{Subst} \left(\int \frac{x}{(a + bx)^2 \sqrt{c + dx}} dx, x, x^6 \right) \\ &= \frac{a\sqrt{c + dx^6}}{6b(bc - ad)(a + bx^6)} + \frac{(2bc - ad) \text{Subst} \left(\int \frac{1}{(a + bx)\sqrt{c + dx}} dx, x, x^6 \right)}{12b(bc - ad)} \\ &= \frac{a\sqrt{c + dx^6}}{6b(bc - ad)(a + bx^6)} + \frac{(2bc - ad) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^6} \right)}{6bd(bc - ad)} \\ &= \frac{a\sqrt{c + dx^6}}{6b(bc - ad)(a + bx^6)} - \frac{(2bc - ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^6}}{\sqrt{bc - ad}} \right)}{6b^{3/2}(bc - ad)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^6}}{(a+bx^6)(bc-ad)} + \frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{6b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]), x]

[Out] ((a*Sqrt[b]*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + ((-2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(6*b^(3/2))

IntegrateAlgebraic [A] time = 0.17, size = 109, normalized size = 1.10

$$\frac{(2bc - ad) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{c+dx^6} \sqrt{ad-bc}}{bc-ad}\right)}{6b^{3/2}(ad-bc)^{3/2}} + \frac{a\sqrt{c+dx^6}}{6b(a+bx^6)(bc-ad)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] (a*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) + ((2*b*c - a*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a*d)]/(6*b^(3/2)*(-(b*c) + a*d)^(3/2))

fricas [A] time = 0.69, size = 348, normalized size = 3.52

$$\left[\frac{\left((2b^2c - abd)x^6 + 2abc - a^2d \right) \sqrt{b^2c - abd} \log\left(\frac{bx^6 + 2bc - ad - 2\sqrt{dx^6 + c} \sqrt{b^2c - abd}}{bx^6 + a} \right) + 2\sqrt{dx^6 + c} (ab^2c - a^2bd)}{12(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2 + (b^5c^2 - 2ab^4cd + a^2b^3d^2)x^6)}, \frac{\left((2b^2c - abd)x^6 + 2abc - a^2d \right) \sqrt{-b^2c + abd} \arctan\left(\frac{\sqrt{dx^6 + c} \sqrt{-b^2c + abd}}{bbs^6 + bc} \right) + \sqrt{dx^6 + c} (ab^2c - a^2bd)}{6(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2 + (b^5c^2 - 2ab^4cd + a^2b^3d^2)x^6)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [1/12*(((2*b^2*c - a*b*d)*x^6 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^6), 1/6*(((2*b^2*c - a*b*d)*x^6 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^6)]

giac [A] time = 0.29, size = 116, normalized size = 1.17

$$\frac{\sqrt{dx^6+c} ad^2}{(b^2c-abd)((dx^6+c)b-bc+ad)} + \frac{(2bcd-ad^2) \arctan\left(\frac{\sqrt{dx^6+c} b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}$$

6d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] 1/6*(sqrt(d*x^6 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^6 + c)*b - b*c + a*d)) + (2*b*c*d - a*d^2)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sqrt(-b^2*c + a*b*d))/d

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(bx^6 + a)^2 \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/(b*x⁶+a)²/(d*x⁶+c)^(1/2),x)

[Out] int(x¹¹/(b*x⁶+a)²/(d*x⁶+c)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(b*x⁶+a)²/(d*x⁶+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.99, size = 95, normalized size = 0.96

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^6+c}}{\sqrt{ad-bc}}\right)(ad-2bc)}{6b^{3/2}(ad-bc)^{3/2}} - \frac{ad\sqrt{dx^6+c}}{2b(ad-bc)(3b(dx^6+c)+3ad-3bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/((a + b*x⁶)²*(c + d*x⁶)^(1/2)),x)

[Out] (atan((b^(1/2)*(c + d*x⁶)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(6*b^(3/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x⁶)^(1/2))/(2*b*(a*d - b*c)*(3*b*(c + d*x⁶) + 3*a*d - 3*b*c))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)

[Out] Timed out

$$3.555 \quad \int \frac{x^5}{(a+bx^6)^2 \sqrt{c+dx^6}} dx$$

Optimal. Leaf size=87

$$\frac{d \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^6}}{\sqrt{bc-ad}} \right)}{6\sqrt{b} (bc-ad)^{3/2}} - \frac{\sqrt{c+dx^6}}{6(a+bx^6)(bc-ad)}$$

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {444, 51, 63, 208}

$$\frac{d \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^6}}{\sqrt{bc-ad}} \right)}{6\sqrt{b} (bc-ad)^{3/2}} - \frac{\sqrt{c+dx^6}}{6(a+bx^6)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] -Sqrt[c + d*x^6]/(6*(b*c - a*d)*(a + b*x^6)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(6*Sqrt[b]*(b*c - a*d)^(3/2))

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 444

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]`

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{(a + bx^6)^2 \sqrt{c + dx^6}} dx &= \frac{1}{6} \text{Subst} \left(\int \frac{1}{(a + bx)^2 \sqrt{c + dx}} dx, x, x^6 \right) \\
 &= -\frac{\sqrt{c + dx^6}}{6(bc - ad)(a + bx^6)} - \frac{d \text{Subst} \left(\int \frac{1}{(a + bx) \sqrt{c + dx}} dx, x, x^6 \right)}{12(bc - ad)} \\
 &= -\frac{\sqrt{c + dx^6}}{6(bc - ad)(a + bx^6)} - \frac{\text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^6} \right)}{6(bc - ad)} \\
 &= -\frac{\sqrt{c + dx^6}}{6(bc - ad)(a + bx^6)} + \frac{d \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^6}}{\sqrt{bc - ad}} \right)}{6\sqrt{b}(bc - ad)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 85, normalized size = 0.98

$$\frac{1}{6} \left(\frac{\sqrt{c + dx^6}}{(a + bx^6)(ad - bc)} + \frac{d \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^6}}{\sqrt{ad - bc}} \right)}{\sqrt{b}(ad - bc)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] (Sqrt[c + d*x^6]/((-b*c) + a*d)*(a + b*x^6) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/6

IntegrateAlgebraic [A] time = 0.11, size = 97, normalized size = 1.11

$$\frac{\sqrt{c+dx^6}}{6(a+bx^6)(bc-ad)} - \frac{d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad}\right)}{6\sqrt{b}(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] -1/6*Sqrt[c + d*x^6]/((b*c - a*d)*(a + b*x^6)) - (d*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a*d)])/(6*Sqrt[b]*(-(b*c) + a*d)^(3/2))

fricas [B] time = 0.75, size = 302, normalized size = 3.47

$$\left[\frac{(bdx^6 + ad)\sqrt{b^2c - abd} \log\left(\frac{bdx^6 + 2bc - ad - 2\sqrt{dx^6 + c}\sqrt{b^2c - abd}}{bx^6 + a}\right) + 2\sqrt{dx^6 + c}(b^2c - abd)}{12((b^4c^2 - 2ab^3cd + a^2b^2d^2)x^6 + ab^3c^2 - 2a^2b^2cd + a^3bd^2)}, -\frac{(bdx^6 + ad)\sqrt{-b^2c + abd} \arctan\left(\frac{\sqrt{dx^6 + c}\sqrt{-b^2c + abd}}{bdx^6 + bc}\right) + \sqrt{dx^6 + c}(b^2c - abd)}{6((b^4c^2 - 2ab^3cd + a^2b^2d^2)x^6 + ab^3c^2 - 2a^2b^2cd + a^3bd^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [-1/12*((b*d*x^6 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^2*c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2), -1/6*((b*d*x^6 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(b^2*c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)]

giac [A] time = 0.30, size = 93, normalized size = 1.07

$$\frac{d \arctan\left(\frac{\sqrt{dx^6+cb}}{\sqrt{-b^2c+abd}}\right)}{6\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^6+cd}}{6((dx^6+c)b-bc+ad)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] -1/6*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c - a*d)) - 1/6*sqrt(d*x^6 + c)*d/(((d*x^6 + c)*b - b*c + a*d)*(b*c - a*d))

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(bx^6 + a)^2 \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

[Out] int(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.93, size = 84, normalized size = 0.97

$$\frac{d \sqrt{dx^6 + c}}{2(ad - bc)(3b(dx^6 + c) + 3ad - 3bc)} + \frac{d \operatorname{atan}\left(\frac{\sqrt{b} \sqrt{dx^6 + c}}{\sqrt{ad - bc}}\right)}{6\sqrt{b}(ad - bc)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)

[Out] (d*(c + d*x^6)^(1/2))/(2*(a*d - b*c)*(3*b*(c + d*x^6) + 3*a*d - 3*b*c)) + (d*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)^(1/2)))/(6*b^(1/2)*(a*d - b*c)^(3/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)

[Out] Timed out

$$3.556 \quad \int \frac{1}{x(a+bx^6)^2 \sqrt{c+dx^6}} dx$$

Optimal. Leaf size=132

$$\frac{\sqrt{b}(2bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc - ad)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{b\sqrt{c+dx^6}}{6a(a+bx^6)(bc - ad)}$$

Rubi [A] time = 0.13, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}(2bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc - ad)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{b\sqrt{c+dx^6}}{6a(a+bx^6)(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] (b*Sqrt[c + d*x^6])/((6*a*(b*c - a*d)*(a + b*x^6)) - ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/(3*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(6*a^2*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx^6)^2 \sqrt{c+dx^6}} dx &= \frac{1}{6} \text{Subst} \left(\int \frac{1}{x(a+bx)^2 \sqrt{c+dx}} dx, x, x^6 \right) \\
&= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)(a+bx^6)} + \frac{\text{Subst} \left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}} dx, x, x^6 \right)}{6a(bc-ad)} \\
&= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)(a+bx^6)} + \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^6 \right)}{6a^2} - \frac{(b(2bc-3ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^6 \right)}{12a^2(bc-ad)} \\
&= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)(a+bx^6)} + \frac{\text{Subst} \left(\int \frac{1}{-\frac{c}{a}+\frac{x^2}{a}} dx, x, \sqrt{c+dx^6} \right)}{3a^2d} - \frac{(b(2bc-3ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^6 \right)}{6a^2} \\
&= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)(a+bx^6)} - \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^6}}{\sqrt{c}} \right)}{3a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad) \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}} \right)}{6a^2(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^6}}{(a+bx^6)(bc-ad)} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{6a^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] ((a*b*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) - (2*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(6*a^2)

IntegrateAlgebraic [A] time = 0.32, size = 146, normalized size = 1.11

$$\frac{(3a\sqrt{b}d - 2b^{3/2}c)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad}\right)}{6a^2(ad-bc)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{b\sqrt{c+dx^6}}{6a(a+bx^6)(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] -1/6*(b*Sqrt[c + d*x^6])/((a*(-(b*c) + a*d)*(a + b*x^6)) + ((-2*b^(3/2)*c + 3*a*Sqrt[b]*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^6])/((b*c - a*d))])/(6*a^2*(-(b*c) + a*d)^(3/2)) - ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/(3*a^2*Sqrt[c])

fricas [A] time = 0.93, size = 862, normalized size = 6.53

Mathematica [A] time = 0.35, size = 123, normalized size = 0.93
IntegrateAlgebraic [A] time = 0.32, size = 146, normalized size = 1.11
fricas [A] time = 0.93, size = 862, normalized size = 6.53

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [1/12*(2*sqrt(d*x^6 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(sqrt(d*x^6 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^6 + b*c) + ((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d)

$\wedge 2 - a^4 * c * d$), $1/12 * (2 * \text{sqrt}(d * x^6 + c) * a * b * c + 4 * ((b^2 * c - a * b * d) * x^6 + a * b * c - a^2 * d) * \text{sqrt}(-c) * \arctan(\text{sqrt}(d * x^6 + c) * \text{sqrt}(-c) / c) + ((2 * b^2 * c^2 - 3 * a * b * c * d) * x^6 + 2 * a * b * c^2 - 3 * a^2 * c * d) * \text{sqrt}(b / (b * c - a * d)) * \log((b * d * x^6 + 2 * b * c - a * d + 2 * \text{sqrt}(d * x^6 + c) * (b * c - a * d) * \text{sqrt}(b / (b * c - a * d))) / (b * x^6 + a))) / ((a^2 * b^2 * c^2 - a^3 * b * c * d) * x^6 + a^3 * b * c^2 - a^4 * c * d)$, $1/6 * (\text{sqrt}(d * x^6 + c) * a * b * c + ((2 * b^2 * c^2 - 3 * a * b * c * d) * x^6 + 2 * a * b * c^2 - 3 * a^2 * c * d) * \text{sqrt}(-b / (b * c - a * d)) * \arctan(-\text{sqrt}(d * x^6 + c) * (b * c - a * d) * \text{sqrt}(-b / (b * c - a * d))) / (b * d * x^6 + b * c)) + 2 * ((b^2 * c - a * b * d) * x^6 + a * b * c - a^2 * d) * \text{sqrt}(-c) * \arctan(\text{sqrt}(d * x^6 + c) * \text{sqrt}(-c) / c)) / ((a^2 * b^2 * c^2 - a^3 * b * c * d) * x^6 + a^3 * b * c^2 - a^4 * c * d)]$

giac [A] time = 0.41, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^6 + c} bd}{6(abc - a^2d)((dx^6 + c)b - bc + ad)} - \frac{(2b^2c - 3abd) \arctan\left(\frac{\sqrt{dx^6 + c}}{\sqrt{-b^2c + abd}}\right)}{6(a^2bc - a^3d)\sqrt{-b^2c + abd}} + \frac{\arctan\left(\frac{\sqrt{dx^6 + c}}{\sqrt{-c}}\right)}{3a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] $1/6 * \text{sqrt}(d * x^6 + c) * b * d / ((a * b * c - a^2 * d) * ((d * x^6 + c) * b - b * c + a * d)) - 1/6 * (2 * b^2 * c - 3 * a * b * d) * \arctan(\text{sqrt}(d * x^6 + c) * b / \text{sqrt}(-b^2 * c + a * b * d)) / ((a^2 * b * c - a^3 * d) * \text{sqrt}(-b^2 * c + a * b * d)) + 1/3 * \arctan(\text{sqrt}(d * x^6 + c) / \text{sqrt}(-c)) / (a^2 * \text{sqrt}(-c))$

maple [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)^2 \sqrt{dx^6 + c} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

[Out] int(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)^2 \sqrt{dx^6 + c} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^6 + a)^2 * sqrt(d*x^6 + c) * x), x)

mupad [B] time = 6.23, size = 3025, normalized size = 22.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x*(a + b*x^6)^2*(c + d*x^6)^{(1/2)}), x)$

[Out]
$$\begin{aligned} & \left(\frac{\text{atan}\left(\frac{((c + d*x^6)^{(1/2)}*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))}{(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))} - \frac{((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)}{(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)} - \frac{((c + d*x^6)^{(1/2)}*(3*a*d - 2*b*c)*(-b*(a*d - b*c))^3}{(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3)}\right)}{(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}}{(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}}{(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}} * i) \\ & + \left(\frac{((c + d*x^6)^{(1/2)}*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))}{(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))} + \frac{((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)}{(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)} + \frac{((c + d*x^6)^{(1/2)}*(3*a*d - 2*b*c)*(-b*(a*d - b*c))^3}{(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3)}\right)}{(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}}{(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}} * i) \\ & + \frac{((a*b^3*d^4)/18 - (b^4*c*d^3)/27)}{(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)} - \frac{((c + d*x^6)^{(1/2)}*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))}{(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))} - \frac{((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)}{(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)} - \frac{((c + d*x^6)^{(1/2)}*(3*a*d - 2*b*c)*(-b*(a*d - b*c))^3}{(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3)}\right)}{(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}}{(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}} * i) \\ & + \frac{((c + d*x^6)^{(1/2)}*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))}{(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))} + \frac{((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)}{(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)} + \frac{((c + d*x^6)^{(1/2)}*(3*a*d - 2*b*c)*(-b*(a*d - b*c))^3}{(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3)}\right)}{(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}}{(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}} * i) \\ & + \frac{((c + d*x^6)^{(1/2)}*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))}{(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))} + \frac{((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)}{(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)} + \frac{((c + d*x^6)^{(1/2)}*(3*a*d - 2*b*c)*(-b*(a*d - b*c))^3}{(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3)}\right)}{(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}}{(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}} * i) \\ & + \frac{((c + d*x^6)^{(1/2)}*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))}{(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))} + \frac{((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)}{(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)} + \frac{((c + d*x^6)^{(1/2)}*(3*a*d - 2*b*c)*(-b*(a*d - b*c))^3}{(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3)}\right)}{(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}}{(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}} * i) \\ & + \frac{((c + d*x^6)^{(1/2)}*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))}{(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))} + \frac{((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)}{(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)} + \frac{((c + d*x^6)^{(1/2)}*(3*a*d - 2*b*c)*(-b*(a*d - b*c))^3}{(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3)}\right)}{(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}}{(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))} * (3*a*d - 2*b*c)*(-b*(a*d - b*c))^3)^{(1/2)}} * i) \end{aligned}$$

$$\begin{aligned} &)/(6*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (\operatorname{atan}(\left(\left(\left(\frac{4*a^6*b^2*d^5}{3} - 2*a^5*b^3*c*d^4 + \frac{2*a^4*b^4*c^2*d^3}{3}\right)/\left(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)\right) - \left((c + d*x^6)^{1/2}\right)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3)\right)/\left(648*a^2*c^{1/2}\right)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)\right))/\left(6*a^2*c^{1/2}\right) - \left((c + d*x^6)^{1/2}\right)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/\left(108*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)\right))*1i)/\left(a^2*c^{1/2}\right) - \left(\left(\left(\frac{4*a^6*b^2*d^5}{3} - 2*a^5*b^3*c*d^4 + \frac{2*a^4*b^4*c^2*d^3}{3}\right)/\left(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)\right) + \left((c + d*x^6)^{1/2}\right)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3)\right)/\left(648*a^2*c^{1/2}\right)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)\right))/\left(6*a^2*c^{1/2}\right) + \left((c + d*x^6)^{1/2}\right)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/\left(108*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)\right))*1i)/\left(a^2*c^{1/2}\right))/\left(\left(\frac{a*b^3*d^4}{18} - \frac{b^4*c*d^3}{27}\right)/\left(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d\right) + \left(\left(\frac{4*a^6*b^2*d^5}{3} - 2*a^5*b^3*c*d^4 + \frac{2*a^4*b^4*c^2*d^3}{3}\right)/\left(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)\right) - \left((c + d*x^6)^{1/2}\right)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3)\right)/\left(648*a^2*c^{1/2}\right)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)\right))/\left(6*a^2*c^{1/2}\right) - \left((c + d*x^6)^{1/2}\right)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/\left(108*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)\right))/\left(a^2*c^{1/2}\right) + \left(\left(\frac{4*a^6*b^2*d^5}{3} - 2*a^5*b^3*c*d^4 + \frac{2*a^4*b^4*c^2*d^3}{3}\right)/\left(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)\right) + \left((c + d*x^6)^{1/2}\right)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3)\right)/\left(648*a^2*c^{1/2}\right)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)\right))/\left(6*a^2*c^{1/2}\right) + \left((c + d*x^6)^{1/2}\right)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/\left(108*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)\right))/\left(a^2*c^{1/2}\right))*1i)/\left(3*a^2*c^{1/2}\right) - \left(b*d*(c + d*x^6)^{1/2}\right)/\left(2*(a^2*d - a*b*c)*(3*b*(c + d*x^6) + 3*a*d - 3*b*c)\right) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + bx^6)^2 \sqrt{c + dx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**6+a)**2/(d*x**6+c)**(1/2), x)

[Out] Integral(1/(x*(a + b*x**6)**2*sqrt(c + d*x**6)), x)

$$3.557 \quad \int \frac{1}{x^7(a+bx^6)^2 \sqrt{c+dx^6}} dx$$

Optimal. Leaf size=185

$$\frac{b^{3/2}(4bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}} + \frac{(ad + 4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}} - \frac{b\sqrt{c+dx^6}(2bc-ad)}{6a^2c(a+bx^6)(bc-ad)} - \frac{\sqrt{c+dx^6}}{6acx^6(a+bx^6)}$$

Rubi [A] time = 0.23, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 103, 151, 156, 63, 208}

$$\frac{b^{3/2}(4bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}} + \frac{(ad + 4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}} - \frac{b\sqrt{c+dx^6}(2bc-ad)}{6a^2c(a+bx^6)(bc-ad)} - \frac{\sqrt{c+dx^6}}{6acx^6(a+bx^6)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^6])/(6*a^2*c*(b*c - a*d)*(a + b*x^6)) - Sqrt[c + d*x^6]/(6*a*c*x^6*(a + b*x^6)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/(6*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(6*a^3*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 151

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

```

Rule 156

```

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 446

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7 (a + bx^6)^2 \sqrt{c + dx^6}} dx &= \frac{1}{6} \text{Subst} \left(\int \frac{1}{x^2 (a + bx)^2 \sqrt{c + dx}} dx, x, x^6 \right) \\
&= -\frac{\sqrt{c + dx^6}}{6acx^6 (a + bx^6)} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(4bc+ad) + \frac{3bdx}{2}}{x(a+bx)^2 \sqrt{c+dx}} dx, x, x^6 \right)}{6ac} \\
&= -\frac{b(2bc - ad)\sqrt{c + dx^6}}{6a^2c(bc - ad)(a + bx^6)} - \frac{\sqrt{c + dx^6}}{6acx^6 (a + bx^6)} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad) + \frac{1}{2}bd(2bc-)}{x(a+bx)\sqrt{c+dx}} dx, x, x^6 \right)}{6a^2c(bc - ad)} \\
&= -\frac{b(2bc - ad)\sqrt{c + dx^6}}{6a^2c(bc - ad)(a + bx^6)} - \frac{\sqrt{c + dx^6}}{6acx^6 (a + bx^6)} + \frac{(b^2(4bc - 5ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^6 \right)}{12a^3(bc - ad)} \\
&= -\frac{b(2bc - ad)\sqrt{c + dx^6}}{6a^2c(bc - ad)(a + bx^6)} - \frac{\sqrt{c + dx^6}}{6acx^6 (a + bx^6)} + \frac{(b^2(4bc - 5ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^6}{d}} dx, x, x^6 \right)}{6a^3d(bc - ad)} \\
&= -\frac{b(2bc - ad)\sqrt{c + dx^6}}{6a^2c(bc - ad)(a + bx^6)} - \frac{\sqrt{c + dx^6}}{6acx^6 (a + bx^6)} + \frac{(4bc + ad) \tanh^{-1} \left(\frac{\sqrt{c+dx^6}}{\sqrt{c}} \right)}{6a^3c^{3/2}} - \frac{b^3}{6a^3c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.68, size = 163, normalized size = 0.88

$$\frac{a\sqrt{c+dx^6}(a^2d+ab(dx^6-c)-2b^2cx^6)}{x^6(a+bx^6)(bc-ad)} + \frac{b^{3/2}c(5ad-4bc) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{(ad+4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}$$

$$6a^3c$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] ((a*Sqrt[c + d*x^6]*(a^2*d - 2*b^2*c*x^6 + a*b*(-c + d*x^6)))/((b*c - a*d)*x^6*(a + b*x^6)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/Sqrt[c] + (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(6*a^3*c)

IntegrateAlgebraic [A] time = 0.66, size = 187, normalized size = 1.01

$$\frac{(4b^{5/2}c - 5ab^{3/2}d) \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad}\right)}{6a^3(ad-bc)^{3/2}} + \frac{(ad+4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}} + \frac{\sqrt{c+dx^6}(-a^2d+abc-abdx^6+2b^2cx^6)}{6a^2cx^6(a+bx^6)(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] (Sqrt[c + d*x^6]*(a*b*c - a^2*d + 2*b^2*c*x^6 - a*b*d*x^6))/(6*a^2*c*(-(b*c) + a*d)*x^6*(a + b*x^6)) + ((4*b^(5/2)*c - 5*a*b^(3/2)*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a*d)]/(6*a^3*(-(b*c) + a*d)^(3/2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/(6*a^3*c^(3/2))

fricas [A] time = 0.92, size = 1236, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [1/12*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(c)*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c)]/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^6 + b*c)) - ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(c)*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c)]/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c)]/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/6*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^6 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) + ((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c)]/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6)]

giac [A] time = 0.41, size = 257, normalized size = 1.39

$$\frac{(4b^3c - 5ab^2d) \arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{6(a^3bc - a^4d)\sqrt{-b^2c+abd}} - \frac{2(dx^6+c)^{\frac{3}{2}}b^2cd - 2\sqrt{dx^6+c}b^2c^2d - (dx^6+c)^{\frac{3}{2}}abd^2 + 2\sqrt{dx^6+c}abcd^2 - \sqrt{dx^6+c}a^2d^3}{6(a^2bc^2 - a^3cd)\left((dx^6+c)^2b - 2(dx^6+c)bc + bc^2 + (dx^6+c)ad - acd\right)} - \frac{(4bc + ad) \arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{6a^3\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{6} \cdot (4 \cdot b^3 \cdot c - 5 \cdot a \cdot b^2 \cdot d) \cdot \arctan\left(\frac{\sqrt{d \cdot x^6 + c} \cdot b}{\sqrt{-b^2 \cdot c + a \cdot b \cdot d}}\right) / \left((a^3 \cdot b \cdot c - a^4 \cdot d) \cdot \sqrt{-b^2 \cdot c + a \cdot b \cdot d} \right) - \frac{1}{6} \cdot (2 \cdot (d \cdot x^6 + c)^{3/2} \cdot b^2 \cdot c \cdot d - 2 \cdot \sqrt{d \cdot x^6 + c} \cdot b^2 \cdot c^2 \cdot d - (d \cdot x^6 + c)^{3/2} \cdot a \cdot b \cdot d^2 + 2 \cdot \sqrt{d \cdot x^6 + c} \cdot a \cdot b \cdot c \cdot d^2 - \sqrt{d \cdot x^6 + c} \cdot a^2 \cdot d^3) / \left((a^2 \cdot b \cdot c^2 - a^3 \cdot c \cdot d) \cdot ((d \cdot x^6 + c)^2 \cdot b - 2 \cdot (d \cdot x^6 + c) \cdot b \cdot c + b \cdot c^2 + (d \cdot x^6 + c) \cdot a \cdot d - a \cdot c \cdot d) \right) - \frac{1}{6} \cdot (4 \cdot b \cdot c + a \cdot d) \cdot \arctan\left(\frac{\sqrt{d \cdot x^6 + c}}{\sqrt{-c}}\right) / (a^3 \cdot \sqrt{-c} \cdot c)$

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)^2 \sqrt{dx^6 + c} x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

[Out] int(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)^2 \sqrt{dx^6 + c} x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^7), x)

mupad [B] time = 7.29, size = 3860, normalized size = 20.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^7*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)

[Out] $\left((c + d \cdot x^6)^{1/2} \cdot (a^2 \cdot d^3 + 2 \cdot b^2 \cdot c^2 \cdot d - 2 \cdot a \cdot b \cdot c \cdot d^2) / (2 \cdot a^2 \cdot (b \cdot c^2 - a \cdot c \cdot d)) + (b \cdot (c + d \cdot x^6)^{3/2} \cdot (a \cdot d^2 - 2 \cdot b \cdot c \cdot d)) / (2 \cdot a^2 \cdot (b \cdot c^2 - a \cdot c \cdot d)) \right) / \left((c + d \cdot x^6) \cdot (3 \cdot a \cdot d - 6 \cdot b \cdot c) + 3 \cdot b \cdot (c + d \cdot x^6)^2 + 3 \cdot b \cdot c^2 - 3 \cdot a \cdot c \cdot d \right) + \left(\operatorname{atan}\left(\frac{(-b^3 \cdot (a \cdot d - b \cdot c)^3)^{1/2} \cdot (5 \cdot a \cdot d - 4 \cdot b \cdot c) \cdot ((c + d \cdot x^6)^{1/2} \cdot (a^4 \cdot b^3 \cdot d^6 + 32 \cdot b^7 \cdot c^4 \cdot d^2 - 64 \cdot a \cdot b^6 \cdot c^3 \cdot d^3 + 6 \cdot a^3 \cdot b^4 \cdot c \cdot d^5 + 26 \cdot a^2 \cdot b^5 \cdot c^2 \cdot d^4))}{(18 \cdot (a^4 \cdot b^2 \cdot c^4 + a^6 \cdot c^2 \cdot d^2 - 2 \cdot a^5 \cdot b \cdot c^3 \cdot d)) + ((-b^3 \cdot (a \cdot d - b \cdot c)^3)^{1/2} \cdot (5 \cdot a \cdot d - 4 \cdot b \cdot c) \cdot ((144 \cdot a^9 \cdot b^2 \cdot c \cdot d^6 + 288 \cdot a^6 \cdot b^5 \cdot c^4 \cdot d^3 - 576 \right)} \right) \right)$

$$\begin{aligned}
& *a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2 \\
& *a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^{(1/2)}*(c + d*x^6)^{(1/2)}*(5*a*d - 4*b \\
& *c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144* \\
& a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 \\
& - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/(12*(a^6*d^3 - a^3*b^3*c^3 \\
& *c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))*1i)/(12*(a^6*d^3 - a^3*b^3*c^3 + \\
& 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^{(1/2)}*(5*a*d - 4* \\
& b*c)*(((c + d*x^6)^{(1/2)}*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + \\
& 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4)))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2* \\
& a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^{(1/2)}*(5*a*d - 4*b*c)*((144*a^9*b^2*c \\
& *d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(21 \\
& 6*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^{(1/2)} \\
&)*(c + d*x^6)^{(1/2)}*(5*a*d - 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4* \\
& d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a^6*c \\
& ^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b* \\
& c*d^2)))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))*1 \\
& i)/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/(5*a^3* \\
& b^4*d^6 + 32*b^7*c^3*d^3 - 48*a*b^6*c^2*d^4 + 6*a^2*b^5*c*d^5)/(108*(a^6*b^ \\
& 2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^{(1/2)}*(5*a*d \\
& - 4*b*c)*(((c + d*x^6)^{(1/2)}*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 \\
& ^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4)))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 \\
& - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^{(1/2)}*(5*a*d - 4*b*c)*((144*a^9*b \\
& ^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5) \\
& / (216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^{(1/2)} \\
&)*(c + d*x^6)^{(1/2)}*(5*a*d - 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4* \\
& c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a \\
& ^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^ \\
& 5*b*c*d^2)))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2) \\
&))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3 \\
& *(a*d - b*c)^3)^{(1/2)}*(5*a*d - 4*b*c)*(((c + d*x^6)^{(1/2)}*(a^4*b^3*d^6 + 32 \\
& *b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(1 \\
& 8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^{(1/2)} \\
&)*(5*a*d - 4*b*c)*((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c \\
& ^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3 \\
& *d)) + ((-b^3*(a*d - b*c)^3)^{(1/2)}*(c + d*x^6)^{(1/2)}*(5*a*d - 4*b*c)*(288*a \\
& ^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^ \\
& 2*d^5))/(216*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3 \\
& *c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a \\
& ^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2 \\
& *d - 3*a^5*b*c*d^2)))*(-b^3*(a*d - b*c)^3)^{(1/2)}*(5*a*d - 4*b*c)*1i)/(6*(a \\
& ^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan((((c + d \\
& *x^6)^{(1/2)}*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c* \\
& d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) \\
& + (((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a \\
& ^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((c + d
\end{aligned}$$

$$\begin{aligned}
& *x^6)^{(1/2)}*(a*d + 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576* \\
& a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*a^3*(c^3)^{(1/2)}*(a^4*b^2*c^4 + \\
& a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(12*a^3*(c^3)^{(1/2)})))*(a*d + \\
& 4*b*c)*1i)/(12*a^3*(c^3)^{(1/2)}) + (((c + d*x^6)^{(1/2)}*(a^4*b^3*d^6 + 32*b \\
& ^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18* \\
& (a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (((144*a^9*b^2*c*d^6 + 288*a \\
& ^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c \\
& ^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) + ((c + d*x^6)^{(1/2)}*(a*d + 4*b*c)*(288* \\
& a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c \\
& ^2*d^5))/(216*a^3*(c^3)^{(1/2)}*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) \\
& *(a*d + 4*b*c))/(12*a^3*(c^3)^{(1/2)})))*(a*d + 4*b*c)*1i)/(12*a^3*(c^3)^{(1/2)} \\
&))/((5*a^3*b^4*d^6 + 32*b^7*c^3*d^3 - 48*a*b^6*c^2*d^4 + 6*a^2*b^5*c*d^5)/(\\
& 108*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - (((c + d*x^6)^{(1/2)}*(a^ \\
& 4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^ \\
& 5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((144*a^9*b \\
& ^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5) \\
& / (216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((c + d*x^6)^{(1/2)}*(a* \\
& d + 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 \\
& - 144*a^9*b^2*c^2*d^5))/(216*a^3*(c^3)^{(1/2)}*(a^4*b^2*c^4 + a^6*c^2*d^2 - \\
& 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(12*a^3*(c^3)^{(1/2)})))*(a*d + 4*b*c))/(12*a^ \\
& 3*(c^3)^{(1/2)}) + (((c + d*x^6)^{(1/2)}*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a* \\
& b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6 \\
& *c^2*d^2 - 2*a^5*b*c^3*d)) - (((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 5 \\
& 76*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - \\
& 2*a^7*b*c^3*d)) + ((c + d*x^6)^{(1/2)}*(a*d + 4*b*c)*(288*a^6*b^5*c^5*d^2 - \\
& 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*a^3* \\
& (c^3)^{(1/2)}*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(1 \\
& 2*a^3*(c^3)^{(1/2)})))*(a*d + 4*b*c))/(12*a^3*(c^3)^{(1/2)})))*(a*d + 4*b*c)*1i) \\
& / (6*a^3*(c^3)^{(1/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**7/(b*x**6+a)**2/(d*x**6+c)**(1/2), x)

[Out] Timed out

$$3.558 \quad \int \frac{x^{14}}{(a+bx^6)^2 \sqrt{c+dx^6}} dx$$

Optimal. Leaf size=141

$$-\frac{\sqrt{a}(3bc-2ad) \tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}} + \frac{ax^3\sqrt{c+dx^6}}{6b(a+bx^6)(bc-ad)} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

Rubi [A] time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {465, 470, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}(3bc-2ad) \tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}} + \frac{ax^3\sqrt{c+dx^6}}{6b(a+bx^6)(bc-ad)} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] (a*x^3*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) - (Sqrt[a]*(3*b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*b^2*(b*c - a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(3*b^2*Sqrt[d])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b,

, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))
^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]

Rule 523

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x_)
^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d,
e, f, n}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{(a+bx^6)^2 \sqrt{c+dx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^4}{(a+bx^2)^2 \sqrt{c+dx^2}} dx, x, x^3 \right) \\
&= \frac{ax^3 \sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)} - \frac{\text{Subst} \left(\int \frac{ac-2(bc-ad)x^2}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^3 \right)}{6b(bc-ad)} \\
&= \frac{ax^3 \sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)} + \frac{\text{Subst} \left(\int \frac{1}{\sqrt{c+dx^2}} dx, x, x^3 \right)}{3b^2} - \frac{(a(3bc-2ad)) \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^3 \right)}{6b^2(bc-ad)} \\
&= \frac{ax^3 \sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)} + \frac{\text{Subst} \left(\int \frac{1}{1-dx^2} dx, x, \frac{x^3}{\sqrt{c+dx^6}} \right)}{3b^2} - \frac{(a(3bc-2ad)) \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^3 \right)}{6b^2(bc-ad)} \\
&= \frac{ax^3 \sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)} - \frac{\sqrt{a}(3bc-2ad) \tan^{-1} \left(\frac{\sqrt{bc-ad} x^3}{\sqrt{a} \sqrt{c+dx^6}} \right)}{6b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1} \left(\frac{\sqrt{d} x^3}{\sqrt{c+dx^6}} \right)}{3b^2 \sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 135, normalized size = 0.96

$$\frac{\frac{abx^3 \sqrt{c+dx^6}}{(a+bx^6)(bc-ad)} + \frac{\sqrt{a}(2ad-3bc) \tan^{-1} \left(\frac{x^3 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^6}} \right)}{(bc-ad)^{3/2}} + \frac{2 \log(\sqrt{d} \sqrt{c+dx^6} + dx^3)}{\sqrt{d}}}{6b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] ((a*b*x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + (Sqrt[a]*(-3*b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(b*c - a*d)^(3/2) + (2*Log[d*x^3 + Sqrt[d]*Sqrt[c + d*x^6]])/Sqrt[d])/(6*b^2)

IntegrateAlgebraic [A] time = 2.62, size = 166, normalized size = 1.18

$$\frac{(2a^{3/2}d - 3\sqrt{a}bc) \tan^{-1} \left(\frac{a\sqrt{d} + bx^3 \sqrt{c+dx^6} + b\sqrt{d}x^6}{\sqrt{a} \sqrt{bc-ad}} \right)}{6b^2(bc-ad)^{3/2}} + \frac{ax^3 \sqrt{c+dx^6}}{6b(a+bx^6)(bc-ad)} + \frac{\log(\sqrt{c+dx^6} + \sqrt{d}x^3)}{3b^2 \sqrt{d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

```
[Out] (a*x^3*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) + ((-3*Sqrt[a]*b*c +
2*a^(3/2)*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])])/(6*b^2*(b*c - a*d)^(3/2)) + Log[Sqrt[d]*x^3 + Sqrt[c + d*x^6])/(3*b^2*Sqrt[d])
```

fricas [A] time = 1.41, size = 1077, normalized size = 7.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*sqrt(d*x^6 + c)*a*b*d*x^3 + 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)
)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c) + ((3*b^2*c*d -
2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
- 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 -
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*
x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((b^4*c*d - a
*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/24*(4*sqrt(d*x^6 + c)*a*b*d*x^3
- 8*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqr
t(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(
-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2
- 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*
b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a
*b*x^6 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/12
*(2*sqrt(d*x^6 + c)*a*b*d*x^3 + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d -
2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d
*x^6 + c)*sqrt(a/(b*c - a*d)))/(a*d*x^9 + a*c*x^3)) + 2*((b^2*c - a*b*d)*x^6
+ a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c)
)/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/12*(2*sqrt(d*x^6
+ c)*a*b*d*x^3 - 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-d)*arctan(s
qrt(-d)*x^3/sqrt(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2
*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*
x^6 + c)*sqrt(a/(b*c - a*d)))/(a*d*x^9 + a*c*x^3)))/((b^4*c*d - a*b^3*d^2)*x
^6 + a*b^3*c*d - a^2*b^2*d^2)]
```

giac [B] time = 0.51, size = 343, normalized size = 2.43

$$\frac{(3abc\sqrt{d}\arctan\left(\frac{x\sqrt{d}}{\sqrt{abc-2d}}\right) - 2a^2\sqrt{d}\arctan\left(\frac{x\sqrt{d}}{\sqrt{abc-2d}}\right) - 2\sqrt{abc-a^2d}bc\arctan\left(\frac{\sqrt{d}}{\sqrt{d}}\right) + 2\sqrt{abc-a^2d}ad\arctan\left(\frac{\sqrt{d}}{\sqrt{d}}\right) + \sqrt{abc-a^2d}a\sqrt{d}\sqrt{d}\operatorname{sgn}(x) + \frac{ac\sqrt{d} + \frac{c}{d}}{6(b^2c\operatorname{sgn}(x) - abd\operatorname{sgn}(x))(bc + d(d + \frac{c}{d}) - ad)} + \frac{(3abc - 2a^2d)\arctan\left(\frac{x\sqrt{d}}{\sqrt{abc-2d}}\right) - \arctan\left(\frac{\sqrt{d}}{\sqrt{d}}\right)}{6(b^2c\operatorname{sgn}(x) - ab^2d\operatorname{sgn}(x))\sqrt{abc-a^2d}} - \frac{1}{3b^2\sqrt{d}\operatorname{sgn}(x)}}}{6(\sqrt{abc-a^2d}b^2c\sqrt{d} - \sqrt{abc-a^2d}ab^2\sqrt{d})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(3*a*b*c*sqrt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*a^2*sqrt(-
d)*d*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*sqrt(a*b*c - a^2*d)*b*c*arct
```

$\text{an}(\sqrt{d}/\sqrt{-d}) + 2\sqrt{a*b*c - a^2*d}*a*d*\arctan(\sqrt{d}/\sqrt{-d}) +$
 $\sqrt{a*b*c - a^2*d}*a*\sqrt{-d}*\sqrt{d})*\text{sgn}(x)/(\sqrt{a*b*c - a^2*d}*b^3*c*$
 $\sqrt{-d} - \sqrt{a*b*c - a^2*d}*a*b^2*\sqrt{-d}*d) + 1/6*a*c*\sqrt{d + c/x^6}/$
 $((b^2*c*\text{sgn}(x) - a*b*d*\text{sgn}(x))*(b*c + a*(d + c/x^6) - a*d)) + 1/6*(3*a*b*c$
 $- 2*a^2*d)*\arctan(a*\sqrt{d + c/x^6}/\sqrt{a*b*c - a^2*d})/((b^3*c*\text{sgn}(x) - a$
 $*b^2*d*\text{sgn}(x))*\sqrt{a*b*c - a^2*d}) - 1/3*\arctan(\sqrt{d + c/x^6}/\sqrt{-d})/$
 $(b^2*\sqrt{-d})*\text{sgn}(x)$

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(bx^6 + a)^2 \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

[Out] int(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(bx^6 + a)^2 \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^14/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{(bx^6 + a)^2 \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^14/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)

[Out] int(x^14/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x**14/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.559 \quad \int \frac{x^8}{(a+bx^6)^2 \sqrt{c+dx^6}} dx$$

Optimal. Leaf size=93

$$\frac{c \tan^{-1} \left(\frac{x^3 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^6}} \right)}{6\sqrt{a} (bc-ad)^{3/2}} - \frac{x^3 \sqrt{c+dx^6}}{6(a+bx^6)(bc-ad)}$$

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {465, 471, 12, 377, 205}

$$\frac{c \tan^{-1} \left(\frac{x^3 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^6}} \right)}{6\sqrt{a} (bc-ad)^{3/2}} - \frac{x^3 \sqrt{c+dx^6}}{6(a+bx^6)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] -(x^3*Sqrt[c + d*x^6])/(6*(b*c - a*d)*(a + b*x^6)) + (c*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*Sqrt[a]*(b*c - a*d)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -

1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 471

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rubi steps

$$\begin{aligned}
 \int \frac{x^8}{(a + bx^6)^2 \sqrt{c + dx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(a + bx^2)^2 \sqrt{c + dx^2}} dx, x, x^3 \right) \\
 &= -\frac{x^3 \sqrt{c + dx^6}}{6(bc - ad)(a + bx^6)} + \frac{\text{Subst} \left(\int \frac{c}{(a + bx^2) \sqrt{c + dx^2}} dx, x, x^3 \right)}{6(bc - ad)} \\
 &= -\frac{x^3 \sqrt{c + dx^6}}{6(bc - ad)(a + bx^6)} + \frac{c \text{Subst} \left(\int \frac{1}{(a + bx^2) \sqrt{c + dx^2}} dx, x, x^3 \right)}{6(bc - ad)} \\
 &= -\frac{x^3 \sqrt{c + dx^6}}{6(bc - ad)(a + bx^6)} + \frac{c \text{Subst} \left(\int \frac{1}{a - (-bc + ad)x^2} dx, x, \frac{x^3}{\sqrt{c + dx^6}} \right)}{6(bc - ad)} \\
 &= -\frac{x^3 \sqrt{c + dx^6}}{6(bc - ad)(a + bx^6)} + \frac{c \tan^{-1} \left(\frac{\sqrt{bc - ad} x^3}{\sqrt{a} \sqrt{c + dx^6}} \right)}{6\sqrt{a} (bc - ad)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.58, size = 124, normalized size = 1.33

$$\frac{\sqrt{c + dx^6} \left(-\frac{x^6(bc - ad)}{a + bx^6} - \frac{c \sqrt{x^6 \left(\frac{d}{c} - \frac{b}{a} \right)} \tanh^{-1} \left(\frac{\sqrt{x^6 \left(\frac{d}{c} - \frac{b}{a} \right)}}{\sqrt{\frac{dx^6}{c} + 1}} \right)}{\sqrt{\frac{dx^6}{c} + 1}} \right)}{6x^3(bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] (Sqrt[c + d*x^6]*(-(((b*c - a*d)*x^6)/(a + b*x^6)) - (c*Sqrt[(-(b/a) + d/c)*x^6]*ArcTanh[Sqrt[(-(b/a) + d/c)*x^6]/Sqrt[1 + (d*x^6)/c]])/Sqrt[1 + (d*x^6)/c]))/(6*(b*c - a*d)^2*x^3)

IntegrateAlgebraic [A] time = 1.33, size = 145, normalized size = 1.56

$$\frac{c \tan^{-1} \left(\frac{b\sqrt{d}x^6}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}} \right)}{6\sqrt{a}(bc-ad)^{3/2}} - \frac{x^3\sqrt{c+dx^6}}{6(a+bx^6)(bc-ad)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] -1/6*(x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + (c*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^6)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])])/(6*Sqrt[a]*(b*c - a*d)^(3/2))

fricas [B] time = 0.79, size = 426, normalized size = 4.58

$$\left[\frac{4\sqrt{dx^6+c}(abc-a^2d)x^3 - (bcx^6+ac)\sqrt{-abc+a^2d} \log\left(\frac{(b^2d^2-8abcd+8a^2d^2)x^{12}-2(3abc^2-4a^2cd)x^6+a^2c^2+4((bc-2ad)x^3-ac^3)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2abx^6+a^2}\right)}{24((ab^3c^2-2a^2b^2cd+a^3bd^2)x^6+a^2b^2c^2-2a^3bcd+a^4d^2)} \right], \left[\frac{2\sqrt{dx^6+c}(abc-a^2d)x^3 - (bcx^6+ac)\sqrt{abc-a^2d} \arctan\left(\frac{(bc-2ad)x^3-ac}{2\sqrt{(abcd-a^2d^2)x^6+(abc^2-a^2cd)x^3}}\right)}{12((ab^3c^2-2a^2b^2cd+a^3bd^2)x^6+a^2b^2c^2-2a^3bcd+a^4d^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [-1/24*(4*sqrt(d*x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/12*(2*sqrt(d*x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)))/((a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(bx^6 + a)^2 \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

[Out] int(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(bx^6 + a)^2 \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^8/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{(bx^6 + a)^2 \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)

[Out] int(x^8/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)

[Out] Timed out

$$3.560 \quad \int \frac{x^2}{(a+bx^6)^2 \sqrt{c+dx^6}} dx$$

Optimal. Leaf size=104

$$\frac{(bc - 2ad) \tan^{-1} \left(\frac{x^3 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^6}} \right)}{6a^{3/2}(bc - ad)^{3/2}} + \frac{bx^3 \sqrt{c + dx^6}}{6a (a + bx^6) (bc - ad)}$$

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {465, 382, 377, 205}

$$\frac{(bc - 2ad) \tan^{-1} \left(\frac{x^3 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^6}} \right)}{6a^{3/2}(bc - ad)^{3/2}} + \frac{bx^3 \sqrt{c + dx^6}}{6a (a + bx^6) (bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] (b*x^3*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*(a + b*x^6)) + ((b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(3/2)*(b*c - a*d)^(3/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 382

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[(b*c + n*(p+1)*(b*c - a*d))/(a*n*(p+1)*(b*c - a*d)], Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p+q+2)+1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

Rule 465

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
 [{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a + bx^6)^2 \sqrt{c + dx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(a + bx^2)^2 \sqrt{c + dx^2}} dx, x, x^3 \right) \\ &= \frac{bx^3 \sqrt{c + dx^6}}{6a(bc - ad)(a + bx^6)} + \frac{(bc - 2ad) \text{Subst} \left(\int \frac{1}{(a + bx^2) \sqrt{c + dx^2}} dx, x, x^3 \right)}{6a(bc - ad)} \\ &= \frac{bx^3 \sqrt{c + dx^6}}{6a(bc - ad)(a + bx^6)} + \frac{(bc - 2ad) \text{Subst} \left(\int \frac{1}{a - (-bc + ad)x^2} dx, x, \frac{x^3}{\sqrt{c + dx^6}} \right)}{6a(bc - ad)} \\ &= \frac{bx^3 \sqrt{c + dx^6}}{6a(bc - ad)(a + bx^6)} + \frac{(bc - 2ad) \tan^{-1} \left(\frac{\sqrt{bc - ad} x^3}{\sqrt{a} \sqrt{c + dx^6}} \right)}{6a^{3/2}(bc - ad)^{3/2}} \end{aligned}$$

Mathematica [C] time = 1.00, size = 407, normalized size = 3.91

$$\frac{x^3 \sqrt{c + dx^6} \left(-30dx^6 \sqrt{\frac{ax^6(c+dx^6)(bc-ad)}{c^2(a+bx^6)^2}} - 45c \sqrt{\frac{ax^6(c+dx^6)(bc-ad)}{c^2(a+bx^6)^2}} + 16dx^6 \left(\frac{x^6(bc-ad)}{c(a+bx^6)} \right)^{5/2} \sqrt{\frac{a(c+dx^6)}{c(a+bx^6)}} {}_2F_1 \left(2, 3; \frac{7}{2}; \frac{(bc-ad)x^6}{c(bx^6+a)} \right) + 16c \left(\frac{x^6(bc-ad)}{c(a+bx^6)} \right)^{5/2} \sqrt{\frac{a(c+dx^6)}{c(a+bx^6)}} {}_2F_1 \left(2, 3; \frac{7}{2}; \frac{(bc-ad)x^6}{c(bx^6+a)} \right) + 30dx^6 \sin^{-1} \left(\frac{\sqrt{bc-ad} x^3}{\sqrt{c(a+bx^6)}} \right) + 45c \sin^{-1} \left(\frac{\sqrt{bc-ad} x^3}{\sqrt{c(a+bx^6)}} \right) \right)}{90c^2 (a + bx^6)^2 \left(\frac{x^6(bc-ad)}{c(a+bx^6)} \right)^{3/2} \sqrt{\frac{a(c+dx^6)}{c(a+bx^6)}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] (x^3*Sqrt[c + d*x^6]*(-45*c*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6)^2)] - 30*d*x^6*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6)^2)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 30*d*x^6*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 16*c*(((b*c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeometric2F1[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 16*d*x^6*(((b*c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeometric2F1[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))])/(90*c^2*(((b*c - a*d)*x

$\wedge(6)/(c*(a + b*x^6)))^{(3/2)*(a + b*x^6)^2*\text{Sqrt}[(a*(c + d*x^6))/(c*(a + b*x^6))])]$

IntegrateAlgebraic [A] time = 0.81, size = 124, normalized size = 1.19

$$\frac{(bc - 2ad) \tan^{-1} \left(\frac{a\sqrt{d} + bx^3\sqrt{c+dx^6} + b\sqrt{d}x^6}{\sqrt{a}\sqrt{bc-ad}} \right)}{6a^{3/2}(bc - ad)^{3/2}} - \frac{bx^3\sqrt{c + dx^6}}{6a(a + bx^6)(ad - bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] $-1/6*(b*x^3*\text{Sqrt}[c + d*x^6])/(a*(-(b*c) + a*d)*(a + b*x^6)) + ((b*c - 2*a*d)*\text{ArcTan}[(a*\text{Sqrt}[d] + b*\text{Sqrt}[d]*x^6 + b*x^3*\text{Sqrt}[c + d*x^6])]/(\text{Sqrt}[a]*\text{Sqrt}[b*c - a*d]))/(6*a^{(3/2)}*(b*c - a*d)^{(3/2)})$

fricas [B] time = 0.59, size = 467, normalized size = 4.49

$$\frac{4\sqrt{dx^6+c}(ab^2c-a^2bd)x^3 - ((b^2c-2abd)x^6 + abc - 2a^2d)\sqrt{-abc+a^2d} \log\left(\frac{(b^2c-8abcd+8a^2d)^2 + 2(3abc^2-4a^2cd)x^6 + a^2c^2 - 4((bc-2ad)x^3 - ac^2)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2c^2+2abcd+a^2d^2}\right)}{24(a^2b^2c^2 - 2a^2bcd + a^2d^2 + (a^2b^2c^2 - 2a^2bcd + a^2d^2)x^6)}, \frac{2\sqrt{dx^6+c}(ab^2c-a^2bd)x^3 + ((b^2c-2abd)x^6 + abc - 2a^2d)\sqrt{abc-a^2d} \arctan\left(\frac{(bc-2ad)x^3 - ac^2\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{2((abcd-a^2d^2)x^6 + (abc^2-a^2cd)^3)}\right)}{12(a^2b^2c^2 - 2a^2bcd + a^2d^2 + (a^2b^2c^2 - 2a^2bcd + a^2d^2)x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] $[1/24*(4*\text{sqrt}(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 - ((b^2*c - 2*a*b*d)*x^6 + a*b*c - 2*a^2*d)*\text{sqrt}(-a*b*c + a^2*d)*\log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^{12} - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*\text{sqrt}(d*x^6 + c)*\text{sqrt}(-a*b*c + a^2*d))/(b^2*x^{12} + 2*a*b*x^6 + a^2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^6), 1/12*(2*\text{sqrt}(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 + ((b^2*c - 2*a*b*d)*x^6 + a*b*c - 2*a^2*d)*\text{sqrt}(a*b*c - a^2*d)*\arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*\text{sqrt}(d*x^6 + c)*\text{sqrt}(a*b*c - a^2*d))/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^6)]$

giac [B] time = 0.28, size = 237, normalized size = 2.28

$$-\frac{1}{6}d^{\frac{3}{2}} \left(\frac{(bc - 2ad) \arctan\left(\frac{(\sqrt{d}x^3 - \sqrt{dx^6+c})^2 b - bc + 2ad}{2\sqrt{abcd - a^2d^2}}\right)}{(abcd - a^2d^2)^{\frac{3}{2}}} + \frac{2\left((\sqrt{d}x^3 - \sqrt{dx^6+c})^2 bc - 2(\sqrt{d}x^3 - \sqrt{dx^6+c})^2 ad - bc^2\right)}{\left((\sqrt{d}x^3 - \sqrt{dx^6+c})^4 b - 2(\sqrt{d}x^3 - \sqrt{dx^6+c})^2 bc + 4(\sqrt{d}x^3 - \sqrt{dx^6+c})^2 ad + bc^2\right)(abcd - a^2d^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")


```
[Out] -1/6*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b
- b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^3 - sqrt(d*x^6 + c))^2*b*c - 2*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^3 - sqrt(d*x^6 + c))^4*b - 2*(sqrt(d)*x^3 - sqrt(d*
x^6 + c))^2*b*c + 4*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*a*d + b*c^2)*(a*b*c*d
- a^2*d^2)))
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^6 + a)^2 \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^6 + a)^2 \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(bx^6 + a)^2 \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + bx^6)^2 \sqrt{c + dx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**6+a)**2/(d*x**6+c)**(1/2), x)
```

```
[Out] Integral(x**2/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

$$3.561 \quad \int \frac{1}{x^4(a+bx^6)^2 \sqrt{c+dx^6}} dx$$

Optimal. Leaf size=149

$$\frac{b(3bc - 4ad) \tan^{-1}\left(\frac{x^3 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}} - \frac{\sqrt{c+dx^6}(3bc-2ad)}{6a^2cx^3(bc-ad)} + \frac{b\sqrt{c+dx^6}}{6ax^3(a+bx^6)(bc-ad)}$$

Rubi [A] time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 472, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^6}(3bc-2ad)}{6a^2cx^3(bc-ad)} - \frac{b(3bc-4ad) \tan^{-1}\left(\frac{x^3 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}} + \frac{b\sqrt{c+dx^6}}{6ax^3(a+bx^6)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^6])/(6*a^2*c*(b*c - a*d)*x^3) + (b*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*x^3*(a + b*x^6)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(5/2)*(b*c - a*d)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -

$1) * (a + b * x^{(n/k)})^p * (c + d * x^{(n/k)})^q, x], x, x^k], x] /; k \neq 1 /; \text{FreeQ} \\ \{a, b, c, d, p, q\}, x] \&\& \text{NeQ}[b * c - a * d, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Rule 472

$\text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^p \cdot (c + d \cdot x^n)^q, x_Symbol] :> -\text{Simp}[b \cdot (e \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} \cdot (c + d \cdot x^n)^{q+1} / (a \cdot e \cdot n \cdot (b \cdot c - a \cdot d) \cdot (p+1)), x] + \text{Dist}[1 / (a \cdot n \cdot (b \cdot c - a \cdot d) \cdot (p+1)), \text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^{p+1} \cdot (c + d \cdot x^n)^q \cdot \text{Simp}[c \cdot b \cdot (m+1) + n \cdot (b \cdot c - a \cdot d) \cdot (p+1) + d \cdot b \cdot (m + n \cdot (p+q+2) + 1) \cdot x^n, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, q\}, x] \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 583

$\text{Int}[(g \cdot x)^m \cdot (a + b \cdot x^n)^p \cdot (c + d \cdot x^n)^q \cdot (e + f \cdot x^n), x_Symbol] :> \text{Simp}[e \cdot (g \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} \cdot (c + d \cdot x^n)^{q+1} / (a \cdot c \cdot g \cdot (m+1)), x] + \text{Dist}[1 / (a \cdot c \cdot g \cdot n \cdot (m+1)), \text{Int}[(g \cdot x)^{m+n} \cdot (a + b \cdot x^n)^p \cdot (c + d \cdot x^n)^q \cdot \text{Simp}[a \cdot f \cdot c \cdot (m+1) - e \cdot (b \cdot c + a \cdot d) \cdot (m+n+1) - e \cdot n \cdot (b \cdot c \cdot p + a \cdot d \cdot q) - b \cdot e \cdot d \cdot (m + n \cdot (p+q+2) + 1) \cdot x^n, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p, q\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (a + bx^6)^2 \sqrt{c + dx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2 (a + bx^2)^2 \sqrt{c + dx^2}} dx, x, x^3 \right) \\
&= \frac{b\sqrt{c + dx^6}}{6a(bc - ad)x^3 (a + bx^6)} - \frac{\text{Subst} \left(\int \frac{-3bc + 2ad - 2bdx^2}{x^2(a + bx^2)\sqrt{c + dx^2}} dx, x, x^3 \right)}{6a(bc - ad)} \\
&= -\frac{(3bc - 2ad)\sqrt{c + dx^6}}{6a^2c(bc - ad)x^3} + \frac{b\sqrt{c + dx^6}}{6a(bc - ad)x^3 (a + bx^6)} - \frac{\text{Subst} \left(\int \frac{bc(3bc - 4ad)}{(a + bx^2)\sqrt{c + dx^2}} dx, x, x^3 \right)}{6a^2c(bc - ad)} \\
&= -\frac{(3bc - 2ad)\sqrt{c + dx^6}}{6a^2c(bc - ad)x^3} + \frac{b\sqrt{c + dx^6}}{6a(bc - ad)x^3 (a + bx^6)} - \frac{(b(3bc - 4ad)) \text{Subst} \left(\int \frac{1}{a + bx^2} dx, x, x^3 \right)}{6a^2(bc - ad)} \\
&= -\frac{(3bc - 2ad)\sqrt{c + dx^6}}{6a^2c(bc - ad)x^3} + \frac{b\sqrt{c + dx^6}}{6a(bc - ad)x^3 (a + bx^6)} - \frac{(b(3bc - 4ad)) \text{Subst} \left(\int \frac{1}{a + bx^2} dx, x, x^3 \right)}{6a^2(bc - ad)} \\
&= -\frac{(3bc - 2ad)\sqrt{c + dx^6}}{6a^2c(bc - ad)x^3} + \frac{b\sqrt{c + dx^6}}{6a(bc - ad)x^3 (a + bx^6)} - \frac{b(3bc - 4ad) \tan^{-1} \left(\frac{\sqrt{bc - ad}}{\sqrt{a} \sqrt{c + dx^6}} \right)}{6a^{5/2}(bc - ad)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 5.40, size = 869, normalized size = 5.83

$\frac{\sqrt{c+dx^6} \operatorname{arcsin}\left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right) - 180c d x^6 \sqrt{c+dx^6} \operatorname{arcsin}\left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right) + 45c^2 \operatorname{arcsin}\left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right) + 120d^2 x^{12} \sqrt{c+dx^6} \operatorname{arcsin}\left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right) + 64c^2 \left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right) \operatorname{arcsin}\left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right) + 160c d x^6 \left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right) \operatorname{arcsin}\left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right) + 96d^2 x^{12} \left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right) \operatorname{arcsin}\left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right) + 64c^2 \left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right) \operatorname{arcsin}\left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3(a+bx^6)} - \frac{b(3bc-4ad) \tan^{-1}\left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out]
$$\begin{aligned}
& -1/90*(\text{Sqrt}[c + d*x^6]*(-45*c^2*\text{Sqrt}[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6)^2]) - 180*c*d*x^6*\text{Sqrt}[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6)^2]) - 120*d^2*x^{12}*\text{Sqrt}[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6)^2]) + 45*c^2*\text{ArcSin}[\text{Sqrt}[(b*c - a*d)*x^6/(c*(a + b*x^6))]] + 180*c*d*x^6*\text{ArcSin}[\text{Sqrt}[(b*c - a*d)*x^6/(c*(a + b*x^6))]] + 120*d^2*x^{12}*\text{ArcSin}[\text{Sqrt}[(b*c - a*d)*x^6/(c*(a + b*x^6))]] + 64*c^2*((b*c - a*d)*x^6/(c*(a + b*x^6)))^{(5/2)}*\text{Sqrt}[(a*(c + d*x^6))/(c*(a + b*x^6))]*\text{Hypergeometric2F1}[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 160*c*d*x^6*((b*c - a*d)*x^6/(c*(a + b*x^6)))^{(5/2)}*\text{Sqrt}[(a*(c + d*x^6))/(c*(a + b*x^6))]*\text{Hypergeometric2F1}[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 96*d^2*x^{12}*((b*c - a*d)*x^6/(c*(a + b*x^6)))^{(5/2)}*\text{Sqrt}[(a*(c + d*x^6))/(c*(a + b*x^6))]*\text{Hypergeometric2F1}[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))]
\end{aligned}$$

ergeometric2F1[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 32*c^2*((b*c - a*d)*x^6)/(c*(a + b*x^6))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 64*c*d*x^6*((b*c - a*d)*x^6)/(c*(a + b*x^6))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 32*d^2*x^12*((b*c - a*d)*x^6)/(c*(a + b*x^6))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^6)/(c*(a + b*x^6)))]/(c^3*x^3*((b*c - a*d)*x^6)/(c*(a + b*x^6)))^(3/2)*(a + b*x^6)^2*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6)))]

IntegrateAlgebraic [A] time = 1.63, size = 159, normalized size = 1.07

$$\frac{(4abd - 3b^2c) \tan^{-1}\left(\frac{a\sqrt{d} + bx^3\sqrt{c+dx^6} + b\sqrt{d}x^6}{\sqrt{a}\sqrt{bc-ad}}\right)}{6a^{5/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx^6}(-2a^2d + 2abc - 2abdx^6 + 3b^2cx^6)}{6a^2cx^3(a+bx^6)(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] (Sqrt[c + d*x^6]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^6 - 2*a*b*d*x^6))/(6*a^2*c*(-(b*c) + a*d)*x^3*(a + b*x^6)) + ((-3*b^2*c + 4*a*b*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])])/(6*a^(5/2)*(b*c - a*d)^(3/2))

fricas [B] time = 0.61, size = 612, normalized size = 4.11

$$\frac{\left(\frac{(3b^2c - 4abd)^2 + (3ab^2c - 4a^2bd)^2}{24(a^2b^2c^2 - 2a^2b^2cd + a^2bd^2)^2} \log\left(\frac{(b^2c^2 - 8a^2b^2cd + 8a^2d^2)x^{12} - 2(3a^2b^2c^2 - 4a^2b^2cd)x^6 + a^2c^2 + 4((b^2c - 2a^2d)x^9 - a^2cx^3)\sqrt{d^2x^6 + c}}{24(a^2b^2c^2 - 2a^2b^2cd + a^2bd^2)^2}\right) + 1}{12(a^2b^2c^2 - 2a^2b^2cd + a^2bd^2)^2}\right) \sqrt{d^2x^6 + c} + \frac{((3b^2c - 4abd)^2 + (3ab^2c - 4a^2bd)^2)\sqrt{d^2x^6 + c} \arctan\left(\frac{(b^2c - 4abd)x^6 + (3ab^2c - 4a^2bd)x^3}{2(3a^2b^2c^2 - 4a^2b^2cd)x^6 + a^2c^2 + 4((b^2c - 2a^2d)x^9 - a^2cx^3)\sqrt{d^2x^6 + c}}\right)}{12(a^2b^2c^2 - 2a^2b^2cd + a^2bd^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [-1/24*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 4*((3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sqrt(d*x^6 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^9 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^3), -1/12*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sqrt(d*x^6 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^9 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^3)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)^2 \sqrt{dx^6 + c} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

[Out] int(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)^2 \sqrt{dx^6 + c} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 (bx^6 + a)^2 \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)

[Out] int(1/(x^4*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```


$$3.562 \quad \int \frac{1}{x^{10}(a+bx^6)^2 \sqrt{c+dx^6}} dx$$

Optimal. Leaf size=208

$$\frac{b^2(5bc - 6ad) \tan^{-1}\left(\frac{x^3 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^6}}\right)}{6a^{7/2}(bc - ad)^{3/2}} - \frac{\sqrt{c+dx^6}(5bc - 2ad)}{18a^2cx^9(bc - ad)} + \frac{\sqrt{c+dx^6}(-4a^2d^2 - 8abcd + 15b^2c^2)}{18a^3c^2x^3(bc - ad)} + \frac{b\sqrt{c+dx^6}}{6ax^9(a+bx^6)}$$

Rubi [A] time = 0.31, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 472, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^6}(-4a^2d^2 - 8abcd + 15b^2c^2)}{18a^3c^2x^3(bc - ad)} + \frac{b^2(5bc - 6ad) \tan^{-1}\left(\frac{x^3 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^6}}\right)}{6a^{7/2}(bc - ad)^{3/2}} - \frac{\sqrt{c+dx^6}(5bc - 2ad)}{18a^2cx^9(bc - ad)} + \frac{b\sqrt{c+dx^6}}{6ax^9(a+bx^6)(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^6])/(18*a^2*c*(b*c - a*d)*x^9) + ((15*b^2*c^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^6])/(18*a^3*c^2*(b*c - a*d)*x^3) + (b*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*x^9*(a + b*x^6)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(7/2)*(b*c - a*d)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
  x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
  x_Symbol] := -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/
(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)),
  Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c -
a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*
((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)),
  Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0]
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{10} (a + bx^6)^2 \sqrt{c + dx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^4 (a + bx^2)^2 \sqrt{c + dx^2}} dx, x, x^3 \right) \\
&= \frac{b\sqrt{c + dx^6}}{6a(bc - ad)x^9 (a + bx^6)} - \frac{\text{Subst} \left(\int \frac{-5bc + 2ad - 4bdx^2}{x^4 (a + bx^2) \sqrt{c + dx^2}} dx, x, x^3 \right)}{6a(bc - ad)} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^6}}{18a^2c(bc - ad)x^9} + \frac{b\sqrt{c + dx^6}}{6a(bc - ad)x^9 (a + bx^6)} + \frac{\text{Subst} \left(\int \frac{-15b^2c^2 + 8abcd + 4a^2}{x^2 (a + bx^2)} dx, x, x^3 \right)}{18a^2c(bc - ad)} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^6}}{18a^2c(bc - ad)x^9} + \frac{(15b^2c^2 - 8abcd - 4a^2d^2)\sqrt{c + dx^6}}{18a^3c^2(bc - ad)x^3} + \frac{b\sqrt{c + dx^6}}{6a(bc - ad)x^9} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^6}}{18a^2c(bc - ad)x^9} + \frac{(15b^2c^2 - 8abcd - 4a^2d^2)\sqrt{c + dx^6}}{18a^3c^2(bc - ad)x^3} + \frac{b\sqrt{c + dx^6}}{6a(bc - ad)x^9} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^6}}{18a^2c(bc - ad)x^9} + \frac{(15b^2c^2 - 8abcd - 4a^2d^2)\sqrt{c + dx^6}}{18a^3c^2(bc - ad)x^3} + \frac{b\sqrt{c + dx^6}}{6a(bc - ad)x^9} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^6}}{18a^2c(bc - ad)x^9} + \frac{(15b^2c^2 - 8abcd - 4a^2d^2)\sqrt{c + dx^6}}{18a^3c^2(bc - ad)x^3} + \frac{b\sqrt{c + dx^6}}{6a(bc - ad)x^9}
\end{aligned}$$

Mathematica [A] time = 5.86, size = 175, normalized size = 0.84

$$\frac{a^2 (c + dx^6) \left(\frac{3b^3x^{12}}{(a+bx^6)(bc-ad)} + \frac{4x^6(ad+3bc)}{c^2} - \frac{2a}{c} \right) + \frac{3b^2x^{18} \sqrt{\frac{dx^6}{c} + 1} (5bc - 6ad) \sin^{-1} \left(\frac{\sqrt{x^6 \left(\frac{b}{a} - \frac{d}{c} \right)}}{\sqrt{\frac{bx^6}{a} + 1}} \right)}{c \left(\frac{x^6(bc-ad)}{ac} \right)^{3/2}}}{18a^5x^9\sqrt{c + dx^6}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]), x]

[Out] (a^2*(c + d*x^6)*((-2*a)/c + (4*(3*b*c + a*d)*x^6)/c^2 + (3*b^3*x^12)/((b*c - a*d)*(a + b*x^6))) + (3*b^2*(5*b*c - 6*a*d)*x^18*Sqrt[1 + (d*x^6)/c]*Arc

$\text{Sin}[\text{Sqrt}[(b/a - d/c)*x^6]/\text{Sqrt}[1 + (b*x^6)/a]]/(c*((b*c - a*d)*x^6)/(a*c)^{(3/2)})/(18*a^5*x^9*\text{Sqrt}[c + d*x^6])$

IntegrateAlgebraic [A] time = 2.88, size = 217, normalized size = 1.04

$$\frac{(5b^3c - 6ab^2d) \tan^{-1}\left(\frac{a\sqrt{d} + bx^3\sqrt{c+dx^6} + b\sqrt{d}x^6}{\sqrt{a}\sqrt{bc-ad}}\right) + \sqrt{c+dx^6}(-2a^3cd + 4a^3d^2x^6 + 2a^2bc^2 + 6a^2bcdx^6 + 4a^2bd^2x^{12} - 10ab^2c^2x^6 + 8ab^2cdx^{12} - 15b^3c^2x^{12})}{6a^{7/2}(bc - ad)^{3/2} + 18a^3c^2x^9(a + bx^6)(ad - bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]

[Out] $(\text{Sqrt}[c + d*x^6]*(2*a^2*b*c^2 - 2*a^3*c*d - 10*a*b^2*c^2*x^6 + 6*a^2*b*c*d*x^6 + 4*a^3*d^2*x^6 - 15*b^3*c^2*x^{12} + 8*a*b^2*c*d*x^{12} + 4*a^2*b*d^2*x^{12}))/((18*a^3*c^2*(-(b*c) + a*d)*x^9*(a + b*x^6)) + ((5*b^3*c - 6*a*b^2*d)*\text{ArcTan}[(a*\text{Sqrt}[d] + b*\text{Sqrt}[d]*x^6 + b*x^3*\text{Sqrt}[c + d*x^6])/(\text{Sqrt}[a]*\text{Sqrt}[b*c - a*d])]))/(6*a^{(7/2)}*(b*c - a*d)^{(3/2)})$

fricas [A] time = 0.77, size = 760, normalized size = 3.65

[[[...]]]]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] $[-1/72*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^{15} + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^9)*\text{sqrt}(-a*b*c + a^2*d)*\log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^{12} - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*\text{sqrt}(d*x^6 + c)*\text{sqrt}(-a*b*c + a^2*d))/(b^2*x^{12} + 2*a*b*x^6 + a^2)) - 4*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^{12} - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*\text{sqrt}(d*x^6 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^{15} + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^9), 1/36*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^{15} + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^9)*\text{sqrt}(a*b*c - a^2*d)*\text{arctan}(1/2*((b*c - 2*a*d)*x^6 - a*c)*\text{sqrt}(d*x^6 + c)*\text{sqrt}(a*b*c - a^2*d))/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^{12} - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*\text{sqrt}(d*x^6 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^{15} + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^9)]$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")`

[Out] Timed out

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)^2 \sqrt{dx^6 + c} x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)`

[Out] `int(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)^2 \sqrt{dx^6 + c} x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^10), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{10} (bx^6 + a)^2 \sqrt{dx^6 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^10*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)`

[Out] `int(1/(x^10*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**10/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)`

[Out] Timed out

$$3.563 \quad \int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}} dx$$

Optimal. Leaf size=104

$$-\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^8}(ad+bc)}{4b^2d^2} + \frac{(c+dx^8)^{3/2}}{12bd^2}$$

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 88, 63, 208}

$$-\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^8}(ad+bc)}{4b^2d^2} + \frac{(c+dx^8)^{3/2}}{12bd^2}$$

Antiderivative was successfully verified.

[In] Int[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] -((b*c + a*d)*Sqrt[c + d*x^8])/(4*b^2*d^2) + (c + d*x^8)^(3/2)/(12*b*d^2) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p *(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{23}}{(a + bx^8)\sqrt{c + dx^8}} dx &= \frac{1}{8} \text{Subst} \left(\int \frac{x^2}{(a + bx)\sqrt{c + dx}} dx, x, x^8 \right) \\
 &= \frac{1}{8} \text{Subst} \left(\int \left(\frac{-bc - ad}{b^2 d \sqrt{c + dx}} + \frac{a^2}{b^2 (a + bx) \sqrt{c + dx}} + \frac{\sqrt{c + dx}}{bd} \right) dx, x, x^8 \right) \\
 &= -\frac{(bc + ad)\sqrt{c + dx^8}}{4b^2 d^2} + \frac{(c + dx^8)^{3/2}}{12bd^2} + \frac{a^2 \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^8 \right)}{8b^2} \\
 &= -\frac{(bc + ad)\sqrt{c + dx^8}}{4b^2 d^2} + \frac{(c + dx^8)^{3/2}}{12bd^2} + \frac{a^2 \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^8} \right)}{4b^2 d} \\
 &= -\frac{(bc + ad)\sqrt{c + dx^8}}{4b^2 d^2} + \frac{(c + dx^8)^{3/2}}{12bd^2} - \frac{a^2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^8}}{\sqrt{bc-ad}} \right)}{4b^{5/2} \sqrt{bc - ad}}
 \end{aligned}$$

Mathematica [A] time = 0.22, size = 91, normalized size = 0.88

$$\frac{\sqrt{c + dx^8} (-3ad - 2bc + bdx^8)}{12b^2 d^2} - \frac{a^2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^8}}{\sqrt{bc-ad}} \right)}{4b^{5/2} \sqrt{bc - ad}}$$

Antiderivative was successfully verified.

[In] Integrate[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] (Sqrt[c + d*x^8]*(-2*b*c - 3*a*d + b*d*x^8))/(12*b^2*d^2) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[b*c - a*d])

IntegrateAlgebraic [A] time = 0.16, size = 101, normalized size = 0.97

$$\frac{\sqrt{c + dx^8} (-3ad - 2bc + bdx^8)}{12b^2 d^2} - \frac{a^2 \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^8} \sqrt{ad-bc}}{bc-ad} \right)}{4b^{5/2} \sqrt{ad - bc}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] (Sqrt[c + d*x^8]*(-2*b*c - 3*a*d + b*d*x^8))/(12*b^2*d^2) - (a^2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)])/(4*b^(5/2)*Sqrt[-(b*c) + a*d])

fricas [A] time = 0.43, size = 288, normalized size = 2.77

$$\frac{3\sqrt{b^2c - abd}a^2d^2 \log\left(\frac{bdx^8 + 2bc - ad - 2\sqrt{dx^8 + c}\sqrt{b^2c - abd}}{b^2c + a}\right) + 2\left((b^3cd - ab^2d^2)x^8 - 2b^3c^2 - ab^2cd + 3a^2bd^2\right)\sqrt{dx^8 + c} - 3\sqrt{-b^2c + abd}a^2d^2 \arctan\left(\frac{\sqrt{dx^8 + c}\sqrt{-b^2c + abd}}{bdx^8 + bc}\right) + \left((b^3cd - ab^2d^2)x^8 - 2b^3c^2 - ab^2cd + 3a^2bd^2\right)\sqrt{dx^8 + c}}{24(b^4cd^2 - ab^3d^3)}, \frac{\left((b^3cd - ab^2d^2)x^8 - 2b^3c^2 - ab^2cd + 3a^2bd^2\right)\sqrt{dx^8 + c}}{12(b^4cd^2 - ab^3d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] [1/24*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^8 - 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^3*d^3), 1/12*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^8 - 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^3*d^3)]

giac [A] time = 0.16, size = 106, normalized size = 1.02

$$\frac{a^2 \arctan\left(\frac{\sqrt{dx^8 + cb}}{\sqrt{-b^2c + abd}}\right)}{4\sqrt{-b^2c + abd}b^2} + \frac{(dx^8 + c)^{\frac{3}{2}}b^2d^4 - 3\sqrt{dx^8 + c}b^2cd^4 - 3\sqrt{dx^8 + c}abd^5}{12b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] 1/4*a^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) + 1/12*((d*x^8 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^8 + c)*b^2*c*d^4 - 3*sqrt(d*x^8 + c)*a*b*d^5)/(b^3*d^6)

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^{23}}{(bx^8 + a)\sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x)

[Out] `int(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.68, size = 103, normalized size = 0.99

$$\frac{(dx^8 + c)^{3/2}}{12bd^2} - \left(\frac{c}{2bd^2} + \frac{4ad^3 - 4bcd^2}{16b^2d^4} \right) \sqrt{dx^8 + c} + \frac{a^2 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^8+c}}{\sqrt{ad-bc}}\right)}{4b^{5/2}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^23/((a + b*x^8)*(c + d*x^8)^(1/2)),x)`

[Out] $(c + dx^8)^{3/2}/(12*b*d^2) - (c/(2*b*d^2) + (4*a*d^3 - 4*b*c*d^2)/(16*b^2*d^4))*(c + dx^8)^{1/2} + (a^2*\operatorname{atan}((b^{1/2}*(c + dx^8)^{1/2})/(a*d - b*c)^{1/2}))/ (4*b^{5/2}*(a*d - b*c)^{1/2})$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**23/(b*x**8+a)/(d*x**8+c)**(1/2),x)`

[Out] Timed out

$$3.564 \quad \int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}} dx$$

Optimal. Leaf size=74

$$\frac{a \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^8}}{4bd}$$

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 80, 63, 208}

$$\frac{a \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^8}}{4bd}$$

Antiderivative was successfully verified.

[In] Int[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] Sqrt[c + d*x^8]/(4*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*b^(3/2)*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^{15}}{(a + bx^8)\sqrt{c + dx^8}} dx &= \frac{1}{8} \text{Subst} \left(\int \frac{x}{(a + bx)\sqrt{c + dx}} dx, x, x^8 \right) \\
 &= \frac{\sqrt{c + dx^8}}{4bd} - \frac{a \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^8 \right)}{8b} \\
 &= \frac{\sqrt{c + dx^8}}{4bd} - \frac{a \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^8} \right)}{4bd} \\
 &= \frac{\sqrt{c + dx^8}}{4bd} + \frac{a \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^8}}{\sqrt{bc-ad}} \right)}{4b^{3/2}\sqrt{bc-ad}}
 \end{aligned}$$

Mathematica [A] time = 0.08, size = 72, normalized size = 0.97

$$\frac{1}{4} \left(\frac{a \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^8}}{\sqrt{bc-ad}} \right)}{b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c + dx^8}}{bd} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] (Sqrt[c + d*x^8]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d])/4

IntegrateAlgebraic [A] time = 0.10, size = 84, normalized size = 1.14

$$\frac{a \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^8} \sqrt{ad-bc}}{bc-ad} \right)}{4b^{3/2}\sqrt{ad-bc}} + \frac{\sqrt{c + dx^8}}{4bd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] Sqrt[c + d*x^8]/(4*b*d) + (a*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)])/(4*b^(3/2)*Sqrt[-(b*c) + a*d])

fricas [A] time = 0.46, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c - abd} \operatorname{ad} \log\left(\frac{bdx^8 + 2bc - ad + 2\sqrt{dx^8 + c}\sqrt{b^2c - abd}}{bx^8 + a}\right) + 2\sqrt{dx^8 + c}(b^2c - abd)}{8(b^3cd - ab^2d^2)}, \frac{\sqrt{-b^2c + abd} \operatorname{ad} \arctan\left(\frac{\sqrt{dx^8 + c}\sqrt{-b^2c + abd}}{bdx^8 + bc}\right) - \sqrt{dx^8 + c}(b^2c - abd)}{4(b^3cd - ab^2d^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] [1/8*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^2*c - a*b*d)/(b^3*c*d - a*b^2*d^2), -1/4*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) - sqrt(d*x^8 + c)*(b^2*c - a*b*d)/(b^3*c*d - a*b^2*d^2)]

giac [A] time = 0.16, size = 64, normalized size = 0.86

$$-\frac{\frac{\operatorname{ad} \arctan\left(\frac{\sqrt{dx^8 + cb}}{\sqrt{-b^2c + abd}}\right)}{\sqrt{-b^2c + abd} b} - \frac{\sqrt{dx^8 + c}}{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] -1/4*(a*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b) - sqrt(d*x^8 + c)/b)/d

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{(bx^8 + a)\sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x)

[Out] int(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.73, size = 58, normalized size = 0.78

$$\frac{\sqrt{dx^8+c}}{4bd} - \frac{a \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^8+c}}{\sqrt{ad-bc}}\right)}{4b^{3/2}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^15/((a + b*x^8)*(c + d*x^8)^(1/2)),x)`

[Out] $(c + dx^8)^{1/2}/(4*b*d) - (a*\operatorname{atan}((b^{1/2}*(c + dx^8)^{1/2})/(a*d - b*c)^{1/2}))/ (4*b^{3/2}*(a*d - b*c)^{1/2})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{(a + bx^8)\sqrt{c + dx^8}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**15/(b*x**8+a)/(d*x**8+c)**(1/2),x)`

[Out] `Integral(x**15/((a + b*x**8)*sqrt(c + d*x**8)), x)`

$$3.565 \quad \int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}} dx$$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]]/(4*Sqrt[b]*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}} dx &= \frac{1}{8} \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^8 \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}} dx, x, \sqrt{c+dx^8} \right)}{4d} \\ &= -\frac{\tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}} \right)}{4\sqrt{b}\sqrt{bc-ad}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$\frac{\tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}} \right)}{4\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] -1/4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c - a*d])

IntegrateAlgebraic [A] time = 0.05, size = 61, normalized size = 1.20

$$\frac{\tan^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad} \right)}{4\sqrt{b}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] -1/4*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)]/(Sqrt[b]*Sqrt[-(b*c) + a*d])

fricas [A] time = 0.42, size = 130, normalized size = 2.55

$$\left[\frac{\log \left(\frac{bdx^8+2bc-ad-2\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a} \right)}{8\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd} \arctan \left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc} \right)}{4(b^2c-abd)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] [1/8*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a))/sqrt(b^2*c - a*b*d), 1/4*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c))/(b^2*c - a*b*d)]

giac [A] time = 0.17, size = 40, normalized size = 0.78

$$\frac{\arctan\left(\frac{\sqrt{dx^8+cb}}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] 1/4*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(bx^8 + a)\sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)

[Out] int(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.59, size = 40, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{b\sqrt{dx^8+c}}{\sqrt{abd-b^2c}}\right)}{4\sqrt{abd-b^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/((a + b*x^8)*(c + d*x^8)^(1/2)),x)`

[Out] `atan((b*(c + d*x^8)^(1/2))/(a*b*d - b^2*c)^(1/2))/(4*(a*b*d - b^2*c)^(1/2))`

sympy [A] time = 44.57, size = 37, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(b*x**8+a)/(d*x**8+c)**(1/2),x)`

[Out] `atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(4*b*sqrt((a*d - b*c)/b))`

$$3.566 \quad \int \frac{1}{x(a+bx^8)\sqrt{c+dx^8}} dx$$

Optimal. Leaf size=85

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}$$

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 86, 63, 208}

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] -ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/(4*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*a*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 86

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && !IntegerQ[p]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(a+bx^8)\sqrt{c+dx^8}} dx &= \frac{1}{8} \text{Subst} \left(\int \frac{1}{x(a+bx)\sqrt{c+dx}} dx, x, x^8 \right) \\
 &= \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^8 \right)}{8a} - \frac{b \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^8 \right)}{8a} \\
 &= \frac{\text{Subst} \left(\int \frac{1}{\frac{-c}{-d} + \frac{x^2}{d}} dx, x, \sqrt{c+dx^8} \right)}{4ad} - \frac{b \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c+dx^8} \right)}{4ad} \\
 &= -\frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^8}}{\sqrt{c}} \right)}{4a\sqrt{c}} + \frac{\sqrt{b} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}} \right)}{4a\sqrt{bc-ad}}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{b} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}} \right)}{\sqrt{bc-ad}} - \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^8}}{\sqrt{c}} \right)}{\sqrt{c}}}{4a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]), x]

[Out] (-(ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]]/Sqrt[b*c - a*d]))/(4*a)

IntegrateAlgebraic [A] time = 0.11, size = 95, normalized size = 1.12

$$\frac{\sqrt{b} \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad} \right)}{4a\sqrt{ad-bc}} - \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^8}}{\sqrt{c}} \right)}{4a\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)]/(4*a*Sqrt[-(b*c) + a*d]) - ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/(4*a*Sqrt[c]))

fricas [A] time = 0.48, size = 431, normalized size = 5.07

$$\left[\sqrt{\frac{c}{b^2c-ad}} \log\left(\frac{bd^2+2bc-ad+2\sqrt{d^2c(bc-ad)\sqrt{\frac{c}{b^2c-ad}}}}{bd^2+ad}\right) + \sqrt{c} \log\left(\frac{d^2-2\sqrt{d^2c}\sqrt{c^2+2c}}{x^2}\right), 2c\sqrt{\frac{b}{b^2c-ad}} \arctan\left(-\frac{\sqrt{d^2c(bc-ad)\sqrt{\frac{c}{b^2c-ad}}}}{bd^2+bc}\right) + \sqrt{c} \log\left(\frac{d^2-2\sqrt{d^2c}\sqrt{c^2+2c}}{x^2}\right), c\sqrt{\frac{b}{b^2c-ad}} \log\left(\frac{bd^2+2bc-ad+2\sqrt{d^2c(bc-ad)\sqrt{\frac{c}{b^2c-ad}}}}{bd^2+ad}\right) + 2\sqrt{-c} \arctan\left(\frac{\sqrt{d^2c}\sqrt{c}}{c}\right), c\sqrt{\frac{b}{b^2c-ad}} \arctan\left(-\frac{\sqrt{d^2c(bc-ad)\sqrt{\frac{c}{b^2c-ad}}}}{bd^2+bc}\right) + \sqrt{-c} \arctan\left(\frac{\sqrt{d^2c}\sqrt{c}}{c}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] [1/8*(c*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + sqrt(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(2*c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^8 + b*c)) + sqrt(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(c*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c))/(a*c), 1/4*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^8 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c))/(a*c)]

giac [A] time = 0.16, size = 71, normalized size = 0.84

$$-\frac{b \arctan\left(\frac{\sqrt{dx^8+cb}}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}a} + \frac{\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] -1/4*b*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a) + 1/4*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a*sqrt(-c))

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x)

[Out] int(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)\sqrt{dx^8 + cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x), x)

mupad [B] time = 4.81, size = 652, normalized size = 7.67

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{dx^8+c}}{\sqrt{c}}\right)}{4a\sqrt{c}} - \frac{\operatorname{atan}\left(\frac{\frac{\sqrt{d^2c-abd}\left(\frac{b^3d^2\sqrt{dx^8+c}}{4} - \frac{\sqrt{d^2c-abd}\left(d^2b^2d^3 - \frac{(8a^3b^2d^3-16a^2b^3cd^2)\sqrt{dx^8+c}\sqrt{d^2c-abd}}{8(a^2d-abc)}\right)}{8(a^2d-abc)}\right)}{8(a^2d-abc)}\right)}{\sqrt{d^2c-abd}} + \frac{\operatorname{atan}\left(\frac{\sqrt{d^2c-abd}\left(\frac{b^3d^2\sqrt{dx^8+c}}{4} - \frac{\sqrt{d^2c-abd}\left(d^2b^2d^3 - \frac{(8a^3b^2d^3-16a^2b^3cd^2)\sqrt{dx^8+c}\sqrt{d^2c-abd}}{8(a^2d-abc)}\right)}{8(a^2d-abc)}\right)}{8(a^2d-abc)}\right)}{\sqrt{d^2c-abd}}\right)}{4(a^2d-abc)} \sqrt{b^2c-abd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^8)*(c + d*x^8)^(1/2)),x)

[Out] - atanh((c + d*x^8)^(1/2)/c^(1/2))/(4*a*c^(1/2)) - (atan((((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 - ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c)))))/(8*(a^2*d - a*b*c)))))/(8*(a^2*d - a*b*c)) + ((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 + ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c)))))/(8*(a^2*d - a*b*c)))))/(8*(a^2*d - a*b*c)))/(((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 - ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c)))))/(8*(a^2*d - a*b*c)))))/(8*(a^2*d - a*b*c)) - ((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 + ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c)))))/(8*(a^2*d - a*b*c)))))/(8*(a^2*d - a*b*c)))*((b^2*c - a*b*d)^(1/2)*1i)/(4*(a^2*d - a*b*c))

sympy [A] time = 37.86, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4a\sqrt{\frac{ad-bc}{b}}} + \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(4*a*sqrt((a*d - b*c)/b)) + atan(sqrt(c + d*x**8)/sqrt(-c))/(4*a*sqrt(-c))
```

$$3.567 \quad \int \frac{1}{x^9(a+bx^8)\sqrt{c+dx^8}} dx$$

Optimal. Leaf size=117

$$-\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}} + \frac{(ad+2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}} - \frac{\sqrt{c+dx^8}}{8acx^8}$$

Rubi [A] time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 103, 156, 63, 208}

$$-\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}} + \frac{(ad+2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}} - \frac{\sqrt{c+dx^8}}{8acx^8}$$

Antiderivative was successfully verified.

[In] Int[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] -Sqrt[c + d*x^8]/(8*a*c*x^8) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/(8*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*a^2*Sqrt[b*c - a*d])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^9 (a + bx^8) \sqrt{c + dx^8}} dx &= \frac{1}{8} \text{Subst} \left(\int \frac{1}{x^2 (a + bx) \sqrt{c + dx}} dx, x, x^8 \right) \\
&= -\frac{\sqrt{c + dx^8}}{8acx^8} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(2bc + ad) + \frac{bdx}{2}}{x(a + bx) \sqrt{c + dx}} dx, x, x^8 \right)}{8ac} \\
&= -\frac{\sqrt{c + dx^8}}{8acx^8} + \frac{b^2 \text{Subst} \left(\int \frac{1}{(a + bx) \sqrt{c + dx}} dx, x, x^8 \right)}{8a^2} - \frac{(2bc + ad) \text{Subst} \left(\int \frac{1}{x \sqrt{c + dx}} dx, x, x^8 \right)}{16a^2c} \\
&= -\frac{\sqrt{c + dx^8}}{8acx^8} + \frac{b^2 \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^8} \right)}{4a^2d} - \frac{(2bc + ad) \text{Subst} \left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, x^8 \right)}{8a^2cd} \\
&= -\frac{\sqrt{c + dx^8}}{8acx^8} + \frac{(2bc + ad) \tanh^{-1} \left(\frac{\sqrt{c + dx^8}}{\sqrt{c}} \right)}{8a^2c^{3/2}} - \frac{b^{3/2} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^8}}{\sqrt{bc - ad}} \right)}{4a^2 \sqrt{bc - ad}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 151, normalized size = 1.29

$$\frac{b^{3/2} \sqrt{bc - ad} \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^8}}{\sqrt{bc - ad}} \right)}{4a^2(ad - bc)} + \frac{b \tanh^{-1} \left(\frac{\sqrt{c + dx^8}}{\sqrt{c}} \right)}{4a^2 \sqrt{c}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{c + dx^8}}{\sqrt{c}} \right)}{8ac^{3/2}} - \frac{\sqrt{c + dx^8}}{8acx^8}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] $-\frac{1}{8}\sqrt{c + dx^8}/(a^2cx^8) + \frac{(b\operatorname{ArcTanh}[\sqrt{c + dx^8}/\sqrt{c}])}{(4a^2\sqrt{c})} + \frac{(d\operatorname{ArcTanh}[\sqrt{c + dx^8}/\sqrt{c}])}{(8a^2c^{3/2})} + \frac{(b^{3/2}\sqrt{bc - ad}\operatorname{ArcTanh}[(\sqrt{b}\sqrt{c + dx^8})/\sqrt{bc - ad}])}{(4a^2(-bc + ad))}$

IntegrateAlgebraic [A] time = 0.26, size = 127, normalized size = 1.09

$$-\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad}\right)}{4a^2\sqrt{ad-bc}} + \frac{(ad+2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}} - \frac{\sqrt{c+dx^8}}{8acx^8}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] $-\frac{1}{8}\sqrt{c + dx^8}/(a^2cx^8) - \frac{(b^{3/2}\operatorname{ArcTan}[(\sqrt{b}\sqrt{c + dx^8})/\sqrt{bc - ad}])}{(4a^2\sqrt{bc - ad})} + \frac{((2b^2c + a^2d)\operatorname{ArcTanh}[\sqrt{c + dx^8}/\sqrt{c}])}{(8a^2c^{3/2})}$

fricas [A] time = 0.47, size = 565, normalized size = 4.83

$$\frac{2b^2d^2\sqrt{c+dx^8}\log\left(\frac{b^2d^2c+ad^2\sqrt{c+dx^8}}{2b^2d^2c}\right) + (2N+ad)\sqrt{c}\log\left(\frac{b^2d^2c+ad^2\sqrt{c+dx^8}}{2b^2d^2c}\right) - 2\sqrt{d^2+c}\arctan\left(\frac{\sqrt{d^2+c}\sqrt{c+dx^8}}{bc-ad}\right) + 4b^2d^2\sqrt{\frac{bc-ad}{bc}}\operatorname{arctan}\left(\frac{\sqrt{d^2+c}\sqrt{c+dx^8}}{bc-ad}\right) + (2N+ad)\sqrt{d^2+c}\log\left(\frac{b^2d^2c+ad^2\sqrt{c+dx^8}}{2b^2d^2c}\right) + 2\sqrt{d^2+c}\arctan\left(\frac{\sqrt{d^2+c}\sqrt{c+dx^8}}{bc-ad}\right) + b^2d^2\sqrt{\frac{bc-ad}{bc}}\log\left(\frac{b^2d^2c+ad^2\sqrt{c+dx^8}}{2b^2d^2c}\right) + (2N+ad)\sqrt{d^2+c}\operatorname{arctan}\left(\frac{\sqrt{d^2+c}\sqrt{c+dx^8}}{bc-ad}\right) + \sqrt{d^2+c}\arctan\left(\frac{\sqrt{d^2+c}\sqrt{c+dx^8}}{bc-ad}\right) + (2N+ad)\sqrt{c}\log\left(\frac{b^2d^2c+ad^2\sqrt{c+dx^8}}{2b^2d^2c}\right) + \sqrt{d^2+c}\arctan\left(\frac{\sqrt{d^2+c}\sqrt{c+dx^8}}{bc-ad}\right)}{8a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{16}(2b^2c^2x^8\sqrt{b/(b^2c - a^2d)}\log((b^2dx^8 + 2b^2c - a^2d - 2\sqrt{c}(dx^8 + c)(b^2c - a^2d)\sqrt{b/(b^2c - a^2d)}))/(b^2x^8 + a) + (2b^2c + a^2d)\sqrt{c}x^8\log((dx^8 + 2\sqrt{c}(dx^8 + c)\sqrt{c} + 2c)/x^8) - 2\sqrt{c}(dx^8 + c)a^2c/(a^2c^2x^8), -\frac{1}{16}(4b^2c^2x^8\sqrt{-b/(b^2c - a^2d)}\arctan(-\sqrt{c}(dx^8 + c)(b^2c - a^2d)\sqrt{-b/(b^2c - a^2d)}))/(b^2dx^8 + b^2c) - (2b^2c + a^2d)\sqrt{c}x^8\log((dx^8 + 2\sqrt{c}(dx^8 + c)\sqrt{c} + 2c)/x^8) + 2\sqrt{c}(dx^8 + c)a^2c/(a^2c^2x^8), \frac{1}{8}(b^2c^2x^8\sqrt{b/(b^2c - a^2d)}\log((b^2dx^8 + 2b^2c - a^2d - 2\sqrt{c}(dx^8 + c)(b^2c - a^2d)\sqrt{b/(b^2c - a^2d)}))/(b^2x^8 + a) - (2b^2c + a^2d)\sqrt{-c}x^8\arctan(\sqrt{c}(dx^8 + c)\sqrt{-c}/c) - \sqrt{c}(dx^8 + c)a^2c/(a^2c^2x^8), -\frac{1}{8}(2b^2c^2x^8\sqrt{-b/(b^2c - a^2d)}\arctan(-\sqrt{c}(dx^8 + c)(b^2c - a^2d)\sqrt{-b/(b^2c - a^2d)}))/(b^2dx^8 + b^2c) + (2b^2c + a^2d)\sqrt{-c}x^8\arctan(\sqrt{c}(dx^8 + c)\sqrt{-c}/c) + \sqrt{c}(dx^8 + c)a^2c/(a^2c^2x^8)]$

giac [A] time = 0.18, size = 104, normalized size = 0.89

$$\frac{b^2 \arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}a^2} - \frac{(2bc+ad) \arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{8a^2\sqrt{-c}c} - \frac{\sqrt{dx^8+c}}{8acx^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] 1/4*b^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) - 1/8*(2*b*c + a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^2*sqrt(-c)*c) - 1/8*sqrt(d*x^8 + c)/(a*c*x^8)

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)

[Out] int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^9), x)

mupad [B] time = 5.51, size = 396, normalized size = 3.38

$$\frac{\ln\left(\frac{\sqrt{dx^8+c}(b^4c-ab^3d)^{3/2}+b^2c^2+a^2b^4d^2-2ab^3cd}{8a^3d-8a^2bc}\sqrt{b^4c-ab^3d}\right)-\ln\left(\frac{\sqrt{dx^8+c}(b^4c-ab^3d)^{3/2}-b^2c^2-a^2b^4d^2+2ab^3cd}{8(a^3d-a^2bc)}\sqrt{b^4c-ab^3d}\right)-\frac{\sqrt{dx^8+c}}{8acx^2}-\frac{\operatorname{atan}\left(\frac{b^4d^2\sqrt{dx^8+c}b}{128\sqrt{c}\left(\frac{13d^2}{128c^2}+\frac{13d^2d}{256c^2}+\frac{d^2d^2}{256c^2}\right)}+\frac{b^2d^2\sqrt{dx^8+c}b}{256\sqrt{c}\left(\frac{13d^2}{256c^2}+\frac{13d^2d}{256c^2}+\frac{d^2d^2}{128c^2}\right)}+\frac{b^4d^2\sqrt{dx^8+c}b}{256\sqrt{c}\left(\frac{13d^2}{128c^2}+\frac{13d^2d}{256c^2}+\frac{d^2d^2}{128c^2}\right)}\right)}{8a^2\sqrt{c}}}{(ad+2bc)11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^9*(a + b*x^8)*(c + d*x^8)^(1/2)),x)

[Out] (log((c + d*x^8)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(8*a^3*d - 8*a^2*b*c) - (log((c + d*x^8

$$\begin{aligned} &)^{(1/2)} * (b^4 * c - a * b^3 * d)^{(3/2)} - b^6 * c^2 - a^2 * b^4 * d^2 + 2 * a * b^5 * c * d * (b^4 \\ &* c - a * b^3 * d)^{(1/2)} / (8 * (a^3 * d - a^2 * b * c)) - (c + d * x^8)^{(1/2)} / (8 * a * c * x^8) \\ &- (\operatorname{atan}((b^4 * d^4 * (c + d * x^8)^{(1/2)} * 3i) / (128 * (c^3)^{(1/2)} * ((3 * b^4 * d^4) / (128 * c \\ &) + (5 * a * b^3 * d^5) / (256 * c^2) + (a^2 * b^2 * d^6) / (256 * c^3)))) + (b^2 * d^6 * (c + d * x \\ &^8)^{(1/2)} * 1i) / (256 * (c^3)^{(1/2)} * ((5 * b^3 * d^5) / (256 * a) + (b^2 * d^6) / (256 * c) + (\\ &3 * b^4 * c * d^4) / (128 * a^2))) + (b^3 * d^5 * (c + d * x^8)^{(1/2)} * 5i) / (256 * (c^3)^{(1/2)} * \\ &((3 * b^4 * d^4) / (128 * a) + (5 * b^3 * d^5) / (256 * c) + (a * b^2 * d^6) / (256 * c^2)))) * (a * d \\ &+ 2 * b * c) * 1i) / (8 * a^2 * (c^3)^{(1/2)}) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9 (a + bx^8) \sqrt{c + dx^8}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**9/(b*x**8+a)/(d*x**8+c)**(1/2),x)

[Out] Integral(1/(x**9*(a + b*x**8)*sqrt(c + d*x**8)), x)

$$3.568 \quad \int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}} dx$$

Optimal. Leaf size=123

$$\frac{a^{3/2} \tan^{-1}\left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}}\right)}{4b^2 \sqrt{bc-ad}} - \frac{(2ad+bc) \tanh^{-1}\left(\frac{\sqrt{d} x^4}{\sqrt{c+dx^8}}\right)}{8b^2 d^{3/2}} + \frac{x^4 \sqrt{c+dx^8}}{8bd}$$

Rubi [A] time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {465, 479, 523, 217, 206, 377, 205}

$$\frac{a^{3/2} \tan^{-1}\left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}}\right)}{4b^2 \sqrt{bc-ad}} - \frac{(2ad+bc) \tanh^{-1}\left(\frac{\sqrt{d} x^4}{\sqrt{c+dx^8}}\right)}{8b^2 d^{3/2}} + \frac{x^4 \sqrt{c+dx^8}}{8bd}$$

Antiderivative was successfully verified.

[In] Int[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] (x^4*Sqrt[c + d*x^8])/(8*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(4*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]])/(8*b^2*d^(3/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b,

, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 479

Int[((e_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))
^(q_)), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
(m + n(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
)^(n)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^{19}}{(a + bx^8)\sqrt{c + dx^8}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^4}{(a + bx^2)\sqrt{c + dx^2}} dx, x, x^4 \right) \\
&= \frac{x^4\sqrt{c + dx^8}}{8bd} - \frac{\text{Subst} \left(\int \frac{ac + (bc + 2ad)x^2}{(a + bx^2)\sqrt{c + dx^2}} dx, x, x^4 \right)}{8bd} \\
&= \frac{x^4\sqrt{c + dx^8}}{8bd} + \frac{a^2 \text{Subst} \left(\int \frac{1}{(a + bx^2)\sqrt{c + dx^2}} dx, x, x^4 \right)}{4b^2} - \frac{(bc + 2ad) \text{Subst} \left(\int \frac{1}{\sqrt{c + dx^2}} dx, x, x^4 \right)}{8b^2d} \\
&= \frac{x^4\sqrt{c + dx^8}}{8bd} + \frac{a^2 \text{Subst} \left(\int \frac{1}{a - (-bc + ad)x^2} dx, x, \frac{x^4}{\sqrt{c + dx^8}} \right)}{4b^2} - \frac{(bc + 2ad) \text{Subst} \left(\int \frac{1}{1 - dx^2} dx, x, \frac{x^4}{\sqrt{c + dx^8}} \right)}{8b^2d} \\
&= \frac{x^4\sqrt{c + dx^8}}{8bd} + \frac{a^{3/2} \tan^{-1} \left(\frac{\sqrt{bc - ad} x^4}{\sqrt{a} \sqrt{c + dx^8}} \right)}{4b^2\sqrt{bc - ad}} - \frac{(bc + 2ad) \tanh^{-1} \left(\frac{\sqrt{d} x^4}{\sqrt{c + dx^8}} \right)}{8b^2d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 118, normalized size = 0.96

$$\frac{2a^{3/2} \tan^{-1} \left(\frac{x^4 \sqrt{bc - ad}}{\sqrt{a} \sqrt{c + dx^8}} \right)}{\sqrt{bc - ad}} - \frac{(2ad + bc) \log \left(\sqrt{d} \sqrt{c + dx^8} + dx^4 \right)}{d^{3/2}} + \frac{bx^4 \sqrt{c + dx^8}}{d}$$

$8b^2$

Antiderivative was successfully verified.

[In] Integrate[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] ((b*x^4*Sqrt[c + d*x^8])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/Sqrt[b*c - a*d - ((b*c + 2*a*d)*Log[d*x^4 + Sqrt[d]*Sqrt[c + d*x^8]])/d^(3/2))/(8*b^2)

IntegrateAlgebraic [A] time = 1.90, size = 177, normalized size = 1.44

$$\frac{a^{3/2} \tan^{-1} \left(\frac{b\sqrt{d}x^8}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}} \right)}{4b^2\sqrt{bc-ad}} + \frac{(-2ad - bc) \log \left(\sqrt{c + dx^8} + \sqrt{d} x^4 \right)}{8b^2d^{3/2}} + \frac{x^4\sqrt{c + dx^8}}{8bd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] (x^4*Sqrt[c + d*x^8])/(8*b*d) + (a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^8)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^4*Sqrt[c + d*x^8])

$\left. \frac{1}{(4b^2\sqrt{bc-ad}) + ((-bc) - 2ad)L} \log[\sqrt{d}x^4 + \sqrt{c + dx^8}] \right) / (8b^2d^{3/2})$

fricas [A] time = 0.56, size = 739, normalized size = 6.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] [1/16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + (b*c + 2*a*d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b^2*d^2), 1/16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*(b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b^2*d^2), 1/16*(2*sqrt(d*x^8 + c)*b*d*x^4 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d)))/(a*d*x^12 + a*c*x^4)) + (b*c + 2*a*d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b^2*d^2), 1/8*(sqrt(d*x^8 + c)*b*d*x^4 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d)))/(a*d*x^12 + a*c*x^4)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b^2*d^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
Argument Value
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{(bx^8 + a)\sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x)`

[Out] `int(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{(bx^8 + a)\sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^19/((b*x^8 + a)*sqrt(d*x^8 + c)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{19}}{(bx^8 + a)\sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^19/((a + b*x^8)*(c + d*x^8)^(1/2)),x)`

[Out] `int(x^19/((a + b*x^8)*(c + d*x^8)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{(a + bx^8)\sqrt{c + dx^8}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**19/(b*x**8+a)/(d*x**8+c)**(1/2),x)`

[Out] `Integral(x**19/((a + b*x**8)*sqrt(c + d*x**8)), x)`

$$3.569 \quad \int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}} dx$$

Optimal. Leaf size=91

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}} - \frac{\sqrt{a} \tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}}$$

Rubi [A] time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 483, 217, 206, 377, 205}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}} - \frac{\sqrt{a} \tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(4*b*Sqrt[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(4*b*Sqrt[d])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b

, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[a, b, c, d, p, q], x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 483

Int[(((e_)*(x_)^(m_)*((c_) + (d_)*(x_)^(n_))^(q_)))/((a_) + (b_)*(x_)^(
n_)), x_Symbol] := Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]

Rubi steps

$$\begin{aligned} \int \frac{x^{11}}{(a + bx^8)\sqrt{c + dx^8}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^2}{(a + bx^2)\sqrt{c + dx^2}} dx, x, x^4 \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{\sqrt{c+dx^2}} dx, x, x^4 \right) - a \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^4 \right)}{4b} \\ &= \frac{\text{Subst} \left(\int \frac{1}{1-dx^2} dx, x, \frac{x^4}{\sqrt{c+dx^8}} \right) - a \text{Subst} \left(\int \frac{1}{a-(-bc+ad)x^2} dx, x, \frac{x^4}{\sqrt{c+dx^8}} \right)}{4b} \\ &= -\frac{\sqrt{a} \tan^{-1} \left(\frac{\sqrt{bc-ad} x^4}{\sqrt{a} \sqrt{c+dx^8}} \right)}{4b\sqrt{bc-ad}} + \frac{\tanh^{-1} \left(\frac{\sqrt{d} x^4}{\sqrt{c+dx^8}} \right)}{4b\sqrt{d}} \end{aligned}$$

Mathematica [A] time = 0.07, size = 90, normalized size = 0.99

$$\frac{\log\left(\sqrt{d} \sqrt{c+dx^8} + dx^4\right) - \frac{\sqrt{a} \tan^{-1}\left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}}\right)}{\sqrt{bc-ad}}}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] $(-\text{ArcTan}[\frac{\sqrt{a}\sqrt{b*c - a*d}*x^4}{\sqrt{a}\sqrt{c + d*x^8}}])/\sqrt{b*c - a*d} + \text{Log}[\frac{d*x^4 + \sqrt{d}\sqrt{c + d*x^8}}{\sqrt{d}}]/(4*b)$

IntegrateAlgebraic [A] time = 0.94, size = 144, normalized size = 1.58

$$\frac{\log\left(\sqrt{c + dx^8} + \sqrt{d}x^4\right)}{4b\sqrt{d}} - \frac{\sqrt{a} \tan^{-1}\left(\frac{b\sqrt{d}x^8}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{4b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] $-1/4*\text{ArcTan}[\frac{\sqrt{a}\sqrt{d}}{\sqrt{b*c - a*d}}] + (b*\sqrt{d}*x^8)/(\text{Sqrt}[a]*\text{Sqrt}[b*c - a*d]) + (b*x^4*\text{Sqrt}[c + d*x^8])/(\text{Sqrt}[a]*\text{Sqrt}[b*c - a*d]) + \text{Log}[\frac{\text{Sqrt}[d]*x^4 + \text{Sqrt}[c + d*x^8]}{4*b*\text{Sqrt}[d]}]$

fricas [A] time = 0.52, size = 632, normalized size = 6.95

$$\frac{1}{16} \sqrt{\frac{d}{a}} \log\left(\frac{(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^{16} - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^{12} - (a*b*c^2 - a^2*c*d)*x^4)*\sqrt{d*x^8 + c}}{(b^2*x^{16} + 2*a*b*x^8 + a^2)}\right) + \frac{1}{16} \sqrt{\frac{d}{a}} \log\left(\frac{(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^{16} - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^{12} - (a*b*c^2 - a^2*c*d)*x^4)*\sqrt{d*x^8 + c}}{(b^2*x^{16} + 2*a*b*x^8 + a^2)}\right) - 4*\sqrt{-d}*\arctan\left(\frac{\sqrt{-d}*x^4/\sqrt{d*x^8 + c}}{\sqrt{a}}\right) + \frac{1}{8}*\sqrt{\frac{d}{a}}*\arctan\left(\frac{-1/2*((b*c - 2*a*d)*x^8 - a*c)*\sqrt{d*x^8 + c}}{a*d*x^{12} + a*c*x^4}\right) + \sqrt{d}*\log\left(\frac{-2*d*x^8 - 2*\sqrt{d*x^8 + c}}{\sqrt{d}*x^4 - c}\right)/(b*d) + \frac{1}{8}*\sqrt{\frac{d}{a}}*\arctan\left(\frac{-1/2*((b*c - 2*a*d)*x^8 - a*c)*\sqrt{d*x^8 + c}}{a*d*x^{12} + a*c*x^4}\right) - 2*\sqrt{-d}*\arctan\left(\frac{\sqrt{-d}*x^4/\sqrt{d*x^8 + c}}{\sqrt{a}}\right)/(b*d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] $[1/16*(d*\sqrt{-a/(b*c - a*d)})*\log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^{16} - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^{12} - (a*b*c^2 - a^2*c*d)*x^4)*\sqrt{d*x^8 + c})/\sqrt{-a/(b*c - a*d)})/(b^2*x^{16} + 2*a*b*x^8 + a^2) + 2*\sqrt{d}*\log((-2*d*x^8 - 2*\sqrt{d*x^8 + c})*\sqrt{d}*x^4 - c)/(b*d), 1/16*(d*\sqrt{-a/(b*c - a*d)})*\log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^{16} - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^{12} - (a*b*c^2 - a^2*c*d)*x^4)*\sqrt{d*x^8 + c})/\sqrt{-a/(b*c - a*d)})/(b^2*x^{16} + 2*a*b*x^8 + a^2) - 4*\sqrt{-d}*\arctan(\sqrt{-d}*x^4/\sqrt{d*x^8 + c})/(b*d), 1/8*(d*\sqrt{a/(b*c - a*d)})*\arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*\sqrt{d*x^8 + c})/(a*d*x^{12} + a*c*x^4) + \sqrt{d}*\log((-2*d*x^8 - 2*\sqrt{d*x^8 + c})*\sqrt{d}*x^4 - c)/(b*d), 1/8*(d*\sqrt{a/(b*c - a*d)})*\arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*\sqrt{d*x^8 + c})/(a*d*x^{12} + a*c*x^4) - 2*\sqrt{-d}*\arctan(\sqrt{-d}*x^4/\sqrt{d*x^8 + c})/(b*d)]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x);OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(bx^8 + a)\sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x)

[Out] int(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(bx^8 + a)\sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^11/((b*x^8 + a)*sqrt(d*x^8 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{(bx^8 + a)\sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/((a + b*x^8)*(c + d*x^8)^(1/2)),x)

[Out] int(x^11/((a + b*x^8)*(c + d*x^8)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a + bx^8)\sqrt{c + dx^8}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11/(b*x**8+a)/(d*x**8+c)**(1/2),x)

[Out] Integral(x**11/((a + b*x**8)*sqrt(c + d*x**8)), x)

$$3.570 \quad \int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}} dx$$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {465, 377, 205}

$$\frac{\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])]/(4*Sqrt[a]*Sqrt[b*c - a*d])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^4 \right) \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{1}{a-(-bc+ad)x^2} dx, x, \frac{x^4}{\sqrt{c+dx^8}} \right) \\
&= \frac{\tan^{-1} \left(\frac{\sqrt{bc-ad} x^4}{\sqrt{a} \sqrt{c+dx^8}} \right)}{4\sqrt{a} \sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 95, normalized size = 1.76

$$\frac{x^4 \sqrt{\frac{dx^8}{c} + 1} \tanh^{-1} \left(\frac{\sqrt{\frac{dx^8}{c} - \frac{bx^8}{a}}}{\sqrt{\frac{dx^8}{c} + 1}} \right)}{4a\sqrt{c+dx^8} \sqrt{\frac{dx^8}{c} - \frac{bx^8}{a}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] (x^4*Sqrt[1 + (d*x^8)/c]*ArcTanh[Sqrt[-((b*x^8)/a) + (d*x^8)/c]/Sqrt[1 + (d*x^8)/c]])/(4*a*Sqrt[c + d*x^8]*Sqrt[-((b*x^8)/a) + (d*x^8)/c])

IntegrateAlgebraic [A] time = 0.49, size = 106, normalized size = 1.96

$$\frac{\tan^{-1} \left(\frac{b\sqrt{d}x^8}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}} \right)}{4\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^8)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*d])]/(4*Sqrt[a]*Sqrt[b*c - a*d])

fricas [B] time = 0.49, size = 245, normalized size = 4.54

$$\left[\frac{\sqrt{-abc+a^2d} \log \left(\frac{(b^2c^2-8abcd+8a^2d^2)x^{16}-2(3abc^2-4a^2cd)x^8+a^2c^2-4((bc-2ad)x^{12}-acx^4)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{b^2x^{16}+2abx^8+a^2} \right)}{16(abc-a^2d)}, \frac{\arctan \left(\frac{((bc-2ad)x^8-ac)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{2((abcd-a^2d^2)x^{12}+(abc^2-a^2cd)x^4)} \right)}{8\sqrt{abc-a^2d}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] [-1/16*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2))/(a*b*c - a^2*d), 1/8*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4))/sqrt(a*b*c - a^2*d)]

giac [A] time = 0.20, size = 72, normalized size = 1.33

$$\frac{\sqrt{d} \arctan\left(\frac{\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 b - bc + 2ad}{2\sqrt{abcd - a^2d^2}}\right)}{4\sqrt{abcd - a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] -1/4*sqrt(d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^8 + a)\sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)

[Out] int(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^8 + a)\sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^3/((b*x^8 + a)*sqrt(d*x^8 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{(bx^8 + a)\sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/((a + b*x^8)*(c + d*x^8)^(1/2)),x)`

[Out] `int(x^3/((a + b*x^8)*(c + d*x^8)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + bx^8)\sqrt{c + dx^8}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b*x**8+a)/(d*x**8+c)**(1/2),x)`

[Out] `Integral(x**3/((a + b*x**8)*sqrt(c + d*x**8)), x)`

$$3.571 \quad \int \frac{1}{x^5(a+bx^8)\sqrt{c+dx^8}} dx$$

Optimal. Leaf size=80

$$-\frac{b \tan^{-1}\left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}}\right)}{4a^{3/2} \sqrt{bc-ad}} - \frac{\sqrt{c+dx^8}}{4acx^4}$$

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {465, 480, 12, 377, 205}

$$-\frac{b \tan^{-1}\left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}}\right)}{4a^{3/2} \sqrt{bc-ad}} - \frac{\sqrt{c+dx^8}}{4acx^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] -Sqrt[c + d*x^8]/(4*a*c*x^4) - (b*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8]))/(4*a^(3/2)*Sqrt[b*c - a*d])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ

[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 480

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e^(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^5 (a + bx^8) \sqrt{c + dx^8}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x^2 (a + bx^2) \sqrt{c + dx^2}} dx, x, x^4 \right) \\
 &= -\frac{\sqrt{c + dx^8}}{4acx^4} - \frac{\text{Subst} \left(\int \frac{bc}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^4 \right)}{4ac} \\
 &= -\frac{\sqrt{c + dx^8}}{4acx^4} - \frac{b \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^4 \right)}{4a} \\
 &= -\frac{\sqrt{c + dx^8}}{4acx^4} - \frac{b \text{Subst} \left(\int \frac{1}{a - (-bc+ad)x^2} dx, x, \frac{x^4}{\sqrt{c+dx^8}} \right)}{4a} \\
 &= -\frac{\sqrt{c + dx^8}}{4acx^4} - \frac{b \tan^{-1} \left(\frac{\sqrt{bc-ad} x^4}{\sqrt{a} \sqrt{c+dx^8}} \right)}{4a^{3/2} \sqrt{bc-ad}}
 \end{aligned}$$

Mathematica [C] time = 0.83, size = 179, normalized size = 2.24

$$\frac{\left(\frac{dx^8}{c} + 1 \right) \left(\frac{4x^8(c+dx^8)(bc-ad) {}_2F_1 \left(2, 2; \frac{5}{2}; \frac{(bc-ad)x^8}{c(bx^8+a)} \right)}{3c^2(a+bx^8)} + \frac{(c+2dx^8) \sin^{-1} \left(\sqrt{\frac{x^8(bc-ad)}{c(a+bx^8)}} \right)}{c \sqrt{\frac{ax^8(c+dx^8)(bc-ad)}{c^2(a+bx^8)^2}} \right)}{4x^4 (a + bx^8) \sqrt{c + dx^8}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out]
$$-1/4*((1 + (d*x^8)/c)*((c + 2*d*x^8)*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]))/(c*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x^8)^2)]) + (4*(b*c - a*d)*x^8*(c + d*x^8)*Hypergeometric2F1[2, 2, 5/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))])/(3*c^2*(a + b*x^8)))/(x^4*(a + b*x^8)*Sqrt[c + d*x^8])$$

IntegrateAlgebraic [A] time = 0.80, size = 142, normalized size = 1.78

$$\frac{b\sqrt{bc-ad} \tan^{-1}\left(\frac{b\sqrt{d}x^8}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{4a^{3/2}(ad-bc)} - \frac{\sqrt{c+dx^8}}{4acx^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out]
$$-1/4*\text{Sqrt}[c + d*x^8]/(a*c*x^4) + (b*\text{Sqrt}[b*c - a*d]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sqrt}[d])/\text{Sqrt}[b*c - a*d] + (b*\text{Sqrt}[d]*x^8)/(\text{Sqrt}[a]*\text{Sqrt}[b*c - a*d]) + (b*x^4*\text{Sqrt}[c + d*x^8])/(\text{Sqrt}[a]*\text{Sqrt}[b*c - a*d])])/(4*a^(3/2)*(-b*c) + a*d)$$

fricas [B] time = 0.50, size = 332, normalized size = 4.15

$$\left[\frac{\sqrt{-abc + a^2d} b c x^4 \log\left(\frac{(b^2c^2 - 8abcd + 8a^2d^2)x^{16} - 2(3abc^2 - 4a^2cd)x^8 + a^2c^2 + 4((bc - 2ad)x^{12} - acx^4)\sqrt{dx^8 + c}\sqrt{-abc + a^2d}}{b^2x^{16} + 2abx^8 + a^2}\right) + 4\sqrt{dx^8 + c}(abc - a^2d)}{16(a^2bc^2 - a^3cd)x^4}, \frac{\sqrt{abc - a^2d} b c x^4 \arctan\left(\frac{(bc - 2ad)x^8 - ac}{2((abcd - a^2d^2)x^{12} + (abc^2 - a^2cd)x^4)}\right) + 2\sqrt{dx^8 + c}(abc - a^2d)}{8(a^2bc^2 - a^3cd)x^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out]
$$[-1/16*(\text{sqrt}(-a*b*c + a^2*d)*b*c*x^4*\log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^{16} - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^{12} - a*c*x^4)*\text{sqrt}(d*x^8 + c)*\text{sqrt}(-a*b*c + a^2*d))/(b^2*x^{16} + 2*a*b*x^8 + a^2)) + 4*\text{sqrt}(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4), -1/8*(\text{sqrt}(a*b*c - a^2*d)*b*c*x^4*\arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*\text{sqrt}(d*x^8 + c)*\text{sqrt}(a*b*c - a^2*d))/((a*b*c*d - a^2*d^2)*x^{12} + (a*b*c^2 - a^2*c*d)*x^4)) + 2*\text{sqrt}(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4)]$$

giac [A] time = 0.23, size = 116, normalized size = 1.45

$$\frac{1}{4} d^{\frac{3}{2}} \left(\frac{b \arctan\left(\frac{(\sqrt{d}x^4 - \sqrt{dx^8 + c})^2 b - bc + 2ad}{2\sqrt{abcd - a^2d^2}}\right)}{\sqrt{abcd - a^2d^2} ad} + \frac{2}{\left(\left(\sqrt{d}x^4 - \sqrt{dx^8 + c}\right)^2 - c\right) ad} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{4}d^{3/2}(b\arctan(1/2*((\sqrt{d})x^4 - \sqrt{dx^8 + c})^2b - bc + 2ad)/\sqrt{abc - a^2d^2})/(\sqrt{abc - a^2d^2}ad) + 2/(((\sqrt{d})x^4 - \sqrt{dx^8 + c})^2 - c)ad)$

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)

[Out] int(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^5), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5 (bx^8 + a) \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(a + b*x^8)*(c + d*x^8)^(1/2)),x)

[Out] int(1/(x^5*(a + b*x^8)*(c + d*x^8)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 (a + bx^8) \sqrt{c + dx^8}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

$$3.572 \quad \int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}} dx$$

Optimal. Leaf size=115

$$\frac{b^2 \tan^{-1}\left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}}\right)}{4a^{5/2} \sqrt{bc-ad}} + \frac{\sqrt{c+dx^8} (2ad+3bc)}{12a^2 c^2 x^4} - \frac{\sqrt{c+dx^8}}{12acx^{12}}$$

Rubi [A] time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 480, 583, 12, 377, 205}

$$\frac{b^2 \tan^{-1}\left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}}\right)}{4a^{5/2} \sqrt{bc-ad}} + \frac{\sqrt{c+dx^8} (2ad+3bc)}{12a^2 c^2 x^4} - \frac{\sqrt{c+dx^8}}{12acx^{12}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] -Sqrt[c + d*x^8]/(12*a*c*x^12) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^8])/(12*a^2*c^2*x^4) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(4*a^(5/2)*Sqrt[b*c - a*d])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -

1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 480

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 583

Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{13} (a + bx^8) \sqrt{c + dx^8}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x^4 (a + bx^2) \sqrt{c + dx^2}} dx, x, x^4 \right) \\
&= -\frac{\sqrt{c + dx^8}}{12acx^{12}} + \frac{\text{Subst} \left(\int \frac{-3bc - 2ad - 2bdx^2}{x^2(a + bx^2) \sqrt{c + dx^2}} dx, x, x^4 \right)}{12ac} \\
&= -\frac{\sqrt{c + dx^8}}{12acx^{12}} + \frac{(3bc + 2ad)\sqrt{c + dx^8}}{12a^2c^2x^4} - \frac{\text{Subst} \left(\int -\frac{3b^2c^2}{(a + bx^2)\sqrt{c + dx^2}} dx, x, x^4 \right)}{12a^2c^2} \\
&= -\frac{\sqrt{c + dx^8}}{12acx^{12}} + \frac{(3bc + 2ad)\sqrt{c + dx^8}}{12a^2c^2x^4} + \frac{b^2 \text{Subst} \left(\int \frac{1}{(a + bx^2)\sqrt{c + dx^2}} dx, x, x^4 \right)}{4a^2} \\
&= -\frac{\sqrt{c + dx^8}}{12acx^{12}} + \frac{(3bc + 2ad)\sqrt{c + dx^8}}{12a^2c^2x^4} + \frac{b^2 \text{Subst} \left(\int \frac{1}{a - (-bc + ad)x^2} dx, x, \frac{x^4}{\sqrt{c + dx^8}} \right)}{4a^2} \\
&= -\frac{\sqrt{c + dx^8}}{12acx^{12}} + \frac{(3bc + 2ad)\sqrt{c + dx^8}}{12a^2c^2x^4} + \frac{b^2 \tan^{-1} \left(\frac{\sqrt{bc - ad} x^4}{\sqrt{a} \sqrt{c + dx^8}} \right)}{4a^{5/2} \sqrt{bc - ad}}
\end{aligned}$$

Mathematica [C] time = 2.27, size = 253, normalized size = 2.20

$$\frac{\left(\frac{dx^8}{c} + 1 \right) \left(-\frac{8x^8(c+dx^8)^2(bc-ad) {}_3F_2 \left(2, 2, 2; 1, \frac{5}{2}; \frac{(bc-ad)x^8}{c(bx^8+a)} \right)}{a+bx^8} + \frac{3c(c^2-4cdx^8-8d^2x^{16}) \sin^{-1} \left(\sqrt{\frac{x^8(bc-ad)}{c(a+bx^8)}} \right)}{\sqrt{\frac{ax^8(c+dx^8)(bc-ad)}{c^2(a+bx^8)^2}}} + \frac{24dx^{16}(c+dx^8)(ad-bc) {}_2F_1 \left(2, 2; \frac{5}{2}; \frac{(bc-ad)x^8}{c(bx^8+a)} \right)}{a+bx^8} \right)}{36c^3x^{12} (a + bx^8) \sqrt{c + dx^8}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]), x]

[Out] $-\frac{1}{36} \left(\left(1 + \frac{d x^8}{c} \right) \left(\frac{3 c^2 (c^2 - 4 c d x^8 - 8 d^2 x^{16}) \text{ArcSin} \left[\frac{\sqrt{c - a d} x^8}{c (a + b x^8)} \right]}{\sqrt{(a (b c - a d) x^8 (c + d x^8)) (c^2 (a + b x^8)^2)}} \right) + \frac{24 d x^{16} (-b c + a d) x^{16} (c + d x^8) \text{Hypergeometric2F1} \left[2, 2, \frac{5}{2}, \frac{(b c - a d) x^8}{c (a + b x^8)} \right]}{(a + b x^8) - (8 (b c - a d) x^8 (c + d x^8)^2 \text{HypergeometricPFQ} \left[\{2, 2, 2\}, \{1, \frac{5}{2}\}, \frac{(b c - a d) x^8}{c (a + b x^8)} \right])}{(c^3 x^{12} (a + b x^8) \sqrt{c + d x^8})} \right)$

IntegrateAlgebraic [A] time = 2.18, size = 163, normalized size = 1.42

$$\frac{\sqrt{c + dx^8} (-ac + 2adx^8 + 3bcx^8)}{12a^2c^2x^{12}} - \frac{b^2 \sqrt{bc - ad} \tan^{-1} \left(\frac{b \sqrt{d} x^8}{\sqrt{a} \sqrt{bc - ad}} + \frac{bx^4 \sqrt{c + dx^8}}{\sqrt{a} \sqrt{bc - ad}} + \frac{\sqrt{a} \sqrt{d}}{\sqrt{bc - ad}} \right)}{4a^{5/2} (ad - bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] (Sqrt[c + d*x^8]*(-(a*c) + 3*b*c*x^8 + 2*a*d*x^8))/(12*a^2*c^2*x^12) - (b^2 * Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^8) / (Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(5/2)*(-(b*c) + a*d))

fricas [A] time = 0.52, size = 416, normalized size = 3.62

$$\frac{3\sqrt{-abc + d^2}b^2c^2x^{12}\log\left(\frac{(b^2-8abcd+8a^2d^2)^{1/6}2(3ab^2-4a^2cd)^{1/3}a^2c^2-4((b-2ad)^{12}-ac^4)\sqrt{d^2c-\sqrt{abc+d^2}}}{b^4a^{10}+2ab^6+a^8d^2}\right)-4\left((3ab^2c^2-a^2bcd-2a^2d^2)x^8-a^2bc^2+a^2cd\right)\sqrt{d^2c+c}}{48(a^3bc^3-a^4c^2d)x^{12}}+\frac{3\sqrt{abc-a^2d}b^2c^2x^{12}\arctan\left(\frac{(bc-2ad)^4-\sqrt{d^2c-\sqrt{abc+d^2}}}{2((abcd-a^2d^2)^{1/2}(abc-d^2d)^{1/4})}\right)+2\left((3ab^2c^2-a^2bcd-2a^2d^2)x^8-a^2bc^2+a^2cd\right)\sqrt{d^2c+c}}{24(a^3bc^3-a^4c^2d)x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] [-1/48*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^12*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^3*c*d)*sqrt(d*x^8 + c)]/((a^3*b*c^3 - a^4*c^2*d)*x^12), 1/24*(3*sqrt(a*b*c - a^2*d)*b^2*c^2*x^12*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)) + 2*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^3*c*d)*sqrt(d*x^8 + c)]/((a^3*b*c^3 - a^4*c^2*d)*x^12)]

giac [B] time = 1.69, size = 205, normalized size = 1.78

$$-\frac{1}{12}d^{\frac{5}{2}}\left(\frac{3b^2\arctan\left(\frac{(\sqrt{d}x^4-\sqrt{dx^8+c})^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a^2d^2}+\frac{2\left(3\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b-6\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2bc-6\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2ad+3bc^2+2acd\right)}{\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2-c\right)^3a^2d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] -1/12*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2 - c)^3*a^2*d^2))

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)`

[Out] `int(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c))*x^13), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{13} (bx^8 + a) \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^13*(a + b*x^8)*(c + d*x^8)^(1/2)),x)`

[Out] `int(1/(x^13*(a + b*x^8)*(c + d*x^8)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{13} (a + bx^8) \sqrt{c + dx^8}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**13/(b*x**8+a)/(d*x**8+c)**(1/2),x)`

[Out] `Integral(1/(x**13*(a + b*x**8)*sqrt(c + d*x**8)), x)`

$$3.573 \quad \int \frac{x^{23}}{(a+bx^8)^2 \sqrt{c+dx^8}} dx$$

Optimal. Leaf size=123

$$-\frac{a^2 \sqrt{c+dx^8}}{8b^2 (a+bx^8)(bc-ad)} + \frac{a(4bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx^8}}{4b^2d}$$

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2 \sqrt{c+dx^8}}{8b^2 (a+bx^8)(bc-ad)} + \frac{a(4bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx^8}}{4b^2d}$$

Antiderivative was successfully verified.

[In] Int[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] Sqrt[c + d*x^8]/(4*b^2*d) - (a^2*Sqrt[c + d*x^8])/(8*b^2*(b*c - a*d)*(a + b*x^8)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(8*b^(5/2)*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 89

Int[((a_.) + (b_.)*(x_))^(2)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))

)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{23}}{(a + bx^8)^2 \sqrt{c + dx^8}} dx &= \frac{1}{8} \text{Subst} \left(\int \frac{x^2}{(a + bx)^2 \sqrt{c + dx}} dx, x, x^8 \right) \\
 &= -\frac{a^2 \sqrt{c + dx^8}}{8b^2(bc - ad)(a + bx^8)} + \frac{\text{Subst} \left(\int \frac{-\frac{1}{2}a(2bc - ad) + b(bc - ad)x}{(a + bx)\sqrt{c + dx}} dx, x, x^8 \right)}{8b^2(bc - ad)} \\
 &= \frac{\sqrt{c + dx^8}}{4b^2d} - \frac{a^2 \sqrt{c + dx^8}}{8b^2(bc - ad)(a + bx^8)} - \frac{(a(4bc - 3ad)) \text{Subst} \left(\int \frac{1}{(a + bx)\sqrt{c + dx}} dx, x, x^8 \right)}{16b^2(bc - ad)} \\
 &= \frac{\sqrt{c + dx^8}}{4b^2d} - \frac{a^2 \sqrt{c + dx^8}}{8b^2(bc - ad)(a + bx^8)} - \frac{(a(4bc - 3ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^8} \right)}{8b^2d(bc - ad)} \\
 &= \frac{\sqrt{c + dx^8}}{4b^2d} - \frac{a^2 \sqrt{c + dx^8}}{8b^2(bc - ad)(a + bx^8)} + \frac{a(4bc - 3ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^8}}{\sqrt{bc - ad}} \right)}{8b^{5/2}(bc - ad)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.26, size = 107, normalized size = 0.87

$$\frac{1}{8} \left(\frac{\sqrt{c+dx^8} \left(\frac{a^2}{(a+bx^8)(ad-bc)} + \frac{2}{d} \right)}{b^2} + \frac{a(4bc-3ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^8}}{\sqrt{bc-ad}} \right)}{b^{5/2}(bc-ad)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] ((Sqrt[c + d*x^8]*(2/d + a^2/((-b*c) + a*d)*(a + b*x^8)))/b^2 + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c - a*d)^(3/2))/8

IntegrateAlgebraic [A] time = 0.33, size = 143, normalized size = 1.16

$$\frac{(3a^2d - 4abc) \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^8} \sqrt{ad-bc}}{bc-ad} \right)}{8b^{5/2}(ad-bc)^{3/2}} - \frac{\sqrt{c+dx^8} (3a^2d - 2abc + 2abdx^8 - 2b^2cx^8)}{8b^2d(a+bx^8)(bc-ad)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] -1/8*(Sqrt[c + d*x^8]*(-2*a*b*c + 3*a^2*d - 2*b^2*c*x^8 + 2*a*b*d*x^8))/(b^2*d*(b*c - a*d)*(a + b*x^8)) + ((-4*a*b*c + 3*a^2*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)])/(8*b^(5/2)*(-(b*c) + a*d)^(3/2))

fricas [B] time = 0.47, size = 475, normalized size = 3.86

$$\frac{\left((4a^2cd - 3a^2bd)^2 + 4a^2bcd - 3a^2d^2 \sqrt{bc-ad} \log \left(\frac{bx^8 + a + \sqrt{c+dx^8} \sqrt{bc-ad}}{bx^8 + a} \right) + 2(2(b^4c^2 - 2ab^3cd + a^2b^2d^2)^2 + 2ab^3c^2 - 5a^2b^2cd + 3a^2bd^2) \sqrt{bc-ad} + c \right) \left((4a^2cd - 3a^2bd)^2 + 4a^2bcd - 3a^2d^2 \sqrt{bc-ad} \arctan \left(\frac{\sqrt{b} \sqrt{c+dx^8}}{\sqrt{bc-ad}} \right) - (2(b^4c^2 - 2ab^3cd + a^2b^2d^2)^2 + 2ab^3c^2 - 5a^2b^2cd + 3a^2bd^2) \sqrt{bc-ad} + c \right)}{16(ab^3cd - 2ab^2bd^2 + a^2b^2d^2) \sqrt{bc-ad} + (bc^2d - 2ab^3cd + a^2b^2d^2)^2} - \frac{\sqrt{c+dx^8} (3a^2d - 2abc + 2abdx^8 - 2b^2cx^8)}{8(ab^3cd - 2ab^2bd^2 + a^2b^2d^2) \sqrt{bc-ad} + (bc^2d - 2ab^3cd + a^2b^2d^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] [1/16*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^8), - 1/8*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) - (2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a*b^3*c^2 - 5*a^2*b^2*c*d +

$3a^3bd^2\sqrt{dx^8+c}/(ab^5c^2d-2a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2ab^5cd^2+a^2b^4d^3)x^8)$

giac [A] time = 0.18, size = 134, normalized size = 1.09

$$-\frac{\sqrt{dx^8+c}a^2d}{8(b^3c-ab^2d)((dx^8+c)b-bc+ad)}-\frac{(4abc-3a^2d)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{8(b^3c-ab^2d)\sqrt{-b^2c+abd}}+\frac{\sqrt{dx^8+c}}{4b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] $-1/8\sqrt{dx^8+c}a^2d/((b^3c-a^2d)*((dx^8+c)b-bc+ad))-1/8*(4ab^3c-3a^2d)*\arctan(\sqrt{dx^8+c}b/\sqrt{-b^2c+abd})/((b^3c-a^2d)*\sqrt{-b^2c+abd})+1/4\sqrt{dx^8+c}/(b^2d)$

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^{23}}{(bx^8+a)^2\sqrt{dx^8+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

[Out] int(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 5.02, size = 144, normalized size = 1.17

$$\frac{\sqrt{dx^8+c}}{4b^2d}-\frac{a\operatorname{atan}\left(\frac{a\sqrt{b}\sqrt{dx^8+c}(3ad-4bc)}{(3a^2d-4abc)\sqrt{ad-bc}}\right)(3ad-4bc)}{8b^{5/2}(ad-bc)^{3/2}}+\frac{a^2d\sqrt{dx^8+c}}{2(ad-bc)(4b^3(dx^8+c)-4b^3c+4ab^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (c + d*x^8)^(1/2)/(4*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^8)^(1/2)*(3*a*d - 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(8*b^(5/2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^8)^(1/2))/(2*(a*d - b*c)*(4*b^3*(c + d*x^8) - 4*b^3*c + 4*a*b^2*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.574 \quad \int \frac{x^{15}}{(a+bx^8)^2 \sqrt{c+dx^8}} dx$$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^8}}{8b(a+bx^8)(bc-ad)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^8}}{8b(a+bx^8)(bc-ad)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] (a*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) - ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(8*b^(3/2)*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1))]/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 446

`Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rubi steps

$$\begin{aligned}
 \int \frac{x^{15}}{(a + bx^8)^2 \sqrt{c + dx^8}} dx &= \frac{1}{8} \text{Subst} \left(\int \frac{x}{(a + bx)^2 \sqrt{c + dx}} dx, x, x^8 \right) \\
 &= \frac{a\sqrt{c + dx^8}}{8b(bc - ad)(a + bx^8)} + \frac{(2bc - ad) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^8 \right)}{16b(bc - ad)} \\
 &= \frac{a\sqrt{c + dx^8}}{8b(bc - ad)(a + bx^8)} + \frac{(2bc - ad) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, \sqrt{c + dx^8} \right)}{8bd(bc - ad)} \\
 &= \frac{a\sqrt{c + dx^8}}{8b(bc - ad)(a + bx^8)} - \frac{(2bc - ad) \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^8}}{\sqrt{bc - ad}} \right)}{8b^{3/2}(bc - ad)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^8}}{(a+bx^8)(bc-ad)} + \frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{8b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] ((a*Sqrt[b]*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + ((-2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(8*b^(3/2))

IntegrateAlgebraic [A] time = 0.17, size = 109, normalized size = 1.10

$$\frac{(2bc - ad) \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^8} \sqrt{ad-bc}}{bc-ad} \right)}{8b^{3/2}(ad - bc)^{3/2}} + \frac{a\sqrt{c + dx^8}}{8b(a + bx^8)(bc - ad)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^15/((a + b*x^8)^2*sqrt[c + d*x^8]),x]

[Out] (a*sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) + ((2*b*c - a*d)*ArcTan[(sqrt[b]*sqrt[-(b*c) + a*d]*sqrt[c + d*x^8])/(b*c - a*d)]/(8*b^(3/2)*(-(b*c) + a*d)^(3/2))

fricas [A] time = 0.43, size = 348, normalized size = 3.52

$$\left[\frac{\left((2b^2c - abd)x^8 + 2abc - a^2d \right) \sqrt{b^2c - abd} \log \left(\frac{bdx^8 + 2bc - ad - 2\sqrt{dx^8 + c} \sqrt{b^2c - abd}}{bx^8 + a} \right) + 2\sqrt{dx^8 + c} (ab^2c - a^2bd) \left((2b^2c - abd)x^8 + 2abc - a^2d \right) \sqrt{-b^2c + abd} \arctan \left(\frac{\sqrt{dx^8 + c} \sqrt{-b^2c + abd}}{bdx^8 + bc} \right) + \sqrt{dx^8 + c} (ab^2c - a^2bd)}{16 \left((b^5c^2 - 2ab^4cd + a^2b^3d^2)x^8 + ab^4c^2 - 2a^2b^3cd + a^3b^2d^2 \right)}, \frac{\left((2b^2c - abd)x^8 + 2abc - a^2d \right) \sqrt{-b^2c + abd} \arctan \left(\frac{\sqrt{dx^8 + c} \sqrt{-b^2c + abd}}{bdx^8 + bc} \right) + \sqrt{dx^8 + c} (ab^2c - a^2bd)}{8 \left((b^5c^2 - 2ab^4cd + a^2b^3d^2)x^8 + ab^4c^2 - 2a^2b^3cd + a^3b^2d^2 \right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] [1/16*(((2*b^2*c - a*b*d)*x^8 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2), 1/8*(((2*b^2*c - a*b*d)*x^8 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)]

giac [A] time = 0.17, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^8+c} ad^2}{(b^2c-abd)((dx^8+c)b-bc+ad)} + \frac{(2bcd-ad^2) \arctan\left(\frac{\sqrt{dx^8+cb}}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] 1/8*(sqrt(d*x^8 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^8 + c)*b - b*c + a*d)) + (2*b*c*d - a*d^2)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sqrt(-b^2*c + a*b*d))/d

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{(bx^8 + a)^2 \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

[Out] int(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.84, size = 95, normalized size = 0.96

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^8+c}}{\sqrt{ad-bc}}\right)(ad-2bc)}{8b^{3/2}(ad-bc)^{3/2}} - \frac{ad\sqrt{dx^8+c}}{2b(ad-bc)(4b(dx^8+c)+4ad-4bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^15/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)

[Out] (atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(8*b^(3/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^8)^(1/2))/(2*b*(a*d - b*c)*(4*b*(c + d*x^8) + 4*a*d - 4*b*c))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**15/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)

[Out] Timed out

$$3.575 \quad \int \frac{x^7}{(a+bx^8)^2 \sqrt{c+dx^8}} dx$$

Optimal. Leaf size=87

$$\frac{d \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^8}}{\sqrt{bc-ad}} \right)}{8\sqrt{b} (bc-ad)^{3/2}} - \frac{\sqrt{c+dx^8}}{8(a+bx^8)(bc-ad)}$$

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {444, 51, 63, 208}

$$\frac{d \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx^8}}{\sqrt{bc-ad}} \right)}{8\sqrt{b} (bc-ad)^{3/2}} - \frac{\sqrt{c+dx^8}}{8(a+bx^8)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] -Sqrt[c + d*x^8]/(8*(b*c - a*d)*(a + b*x^8)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(8*Sqrt[b]*(b*c - a*d)^(3/2))

Rule 51

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 444

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]`

Rubi steps

$$\begin{aligned}
 \int \frac{x^7}{(a + bx^8)^2 \sqrt{c + dx^8}} dx &= \frac{1}{8} \text{Subst} \left(\int \frac{1}{(a + bx)^2 \sqrt{c + dx}} dx, x, x^8 \right) \\
 &= -\frac{\sqrt{c + dx^8}}{8(bc - ad)(a + bx^8)} - \frac{d \text{Subst} \left(\int \frac{1}{(a + bx)\sqrt{c + dx}} dx, x, x^8 \right)}{16(bc - ad)} \\
 &= -\frac{\sqrt{c + dx^8}}{8(bc - ad)(a + bx^8)} - \frac{\text{Subst} \left(\int \frac{1}{a - \frac{bc}{a} + \frac{bx^2}{a}} dx, x, \sqrt{c + dx^8} \right)}{8(bc - ad)} \\
 &= -\frac{\sqrt{c + dx^8}}{8(bc - ad)(a + bx^8)} + \frac{d \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^8}}{\sqrt{bc - ad}} \right)}{8\sqrt{b}(bc - ad)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 85, normalized size = 0.98

$$\frac{1}{8} \left(\frac{\sqrt{c + dx^8}}{(a + bx^8)(ad - bc)} + \frac{d \tan^{-1} \left(\frac{\sqrt{b} \sqrt{c + dx^8}}{\sqrt{ad - bc}} \right)}{\sqrt{b}(ad - bc)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] (Sqrt[c + d*x^8]/((-b*c) + a*d)*(a + b*x^8) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/8

IntegrateAlgebraic [A] time = 0.11, size = 97, normalized size = 1.11

$$-\frac{\sqrt{c+dx^8}}{8(a+bx^8)(bc-ad)} - \frac{d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad}\right)}{8\sqrt{b}(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] -1/8*Sqrt[c + d*x^8]/((b*c - a*d)*(a + b*x^8)) - (d*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)]/(8*Sqrt[b]*(-(b*c) + a*d)^(3/2)))

fricas [B] time = 0.44, size = 302, normalized size = 3.47

$$\left[\frac{(bdx^8 + ad)\sqrt{b^2c - abd} \log\left(\frac{bdx^8 + 2bc - ad - 2\sqrt{dx^8 + c}\sqrt{b^2c - abd}}{bx^8 + a}\right) + 2\sqrt{dx^8 + c}(b^2c - abd)}{16((b^4c^2 - 2ab^3cd + a^2b^2d^2)x^8 + ab^3c^2 - 2a^2b^2cd + a^3bd^2)}, \frac{(bdx^8 + ad)\sqrt{-b^2c + abd} \arctan\left(\frac{\sqrt{dx^8 + c}\sqrt{-b^2c + abd}}{bdx^8 + bc}\right) + \sqrt{dx^8 + c}(b^2c - abd)}{8((b^4c^2 - 2ab^3cd + a^2b^2d^2)x^8 + ab^3c^2 - 2a^2b^2cd + a^3bd^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] [-1/16*((b*d*x^8 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^2*c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2), -1/8*((b*d*x^8 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(b^2*c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)]

giac [A] time = 0.16, size = 93, normalized size = 1.07

$$-\frac{d \arctan\left(\frac{\sqrt{dx^8+cb}}{\sqrt{-b^2c+abd}}\right)}{8\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^8+cd}}{8((dx^8+c)b-bc+ad)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] -1/8*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c - a*d)) - 1/8*sqrt(d*x^8 + c)*d/(((d*x^8 + c)*b - b*c + a*d)*(b*c - a*d))

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(bx^8 + a)^2 \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)`

[Out] `int(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c positive or negative?

mupad [B] time = 4.80, size = 84, normalized size = 0.97

$$\frac{d \sqrt{dx^8 + c}}{2(ad - bc)(4b(dx^8 + c) + 4ad - 4bc)} + \frac{d \operatorname{atan}\left(\frac{\sqrt{b} \sqrt{dx^8 + c}}{\sqrt{ad - bc}}\right)}{8\sqrt{b}(ad - bc)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)`

[Out] `(d*(c + d*x^8)^(1/2))/(2*(a*d - b*c)*(4*b*(c + d*x^8) + 4*a*d - 4*b*c)) + (d*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(1/2)*(a*d - b*c)^(3/2))`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)`

[Out] Timed out

$$3.576 \quad \int \frac{1}{x(a+bx^8)^2 \sqrt{c+dx^8}} dx$$

Optimal. Leaf size=132

$$\frac{\sqrt{b}(2bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} + \frac{b\sqrt{c+dx^8}}{8a(a+bx^8)(bc-ad)}$$

Rubi [A] time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}(2bc-3ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} + \frac{b\sqrt{c+dx^8}}{8a(a+bx^8)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) - ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/(4*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(8*a^2*(b*c - a*d)^(3/2))

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156


```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[(((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx^8)^2 \sqrt{c+dx^8}} dx &= \frac{1}{8} \text{Subst} \left(\int \frac{1}{x(a+bx)^2 \sqrt{c+dx}} dx, x, x^8 \right) \\
&= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} + \frac{\text{Subst} \left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}} dx, x, x^8 \right)}{8a(bc-ad)} \\
&= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} + \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^8 \right)}{8a^2} - \frac{(b(2bc-3ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^8 \right)}{16a^2(bc-ad)} \\
&= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} + \frac{\text{Subst} \left(\int \frac{1}{\frac{-c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c+dx^8} \right)}{4a^2d} - \frac{(b(2bc-3ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^8 \right)}{8a^2} \\
&= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} - \frac{\tanh^{-1} \left(\frac{\sqrt{c+dx^8}}{\sqrt{c}} \right)}{4a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad) \tanh^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}} \right)}{8a^2(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^8}}{(a+bx^8)(bc-ad)} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{8a^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] ((a*b*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) - (2*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(8*a^2)

IntegrateAlgebraic [A] time = 0.30, size = 146, normalized size = 1.11

$$\frac{(3a\sqrt{b}d - 2b^{3/2}c)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad}\right)}{8a^2(ad-bc)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} - \frac{b\sqrt{c+dx^8}}{8a(a+bx^8)(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] -1/8*(b*Sqrt[c + d*x^8])/(a*(-(b*c) + a*d)*(a + b*x^8)) + ((-2*b^(3/2)*c + 3*a*Sqrt[b]*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)]/(8*a^2*(-(b*c) + a*d)^(3/2)) - ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/(4*a^2*Sqrt[c])

fricas [A] time = 0.49, size = 862, normalized size = 6.53

Mathematica [A] time = 0.36, size = 123, normalized size = 0.93
IntegrateAlgebraic [A] time = 0.30, size = 146, normalized size = 1.11
fricas [A] time = 0.49, size = 862, normalized size = 6.53

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] [1/16*(2*sqrt(d*x^8 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(sqrt(d*x^8 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^8 + b*c)) + ((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d)

$^2 - a^4*c*d$), $1/16*(2*\sqrt{d*x^8 + c})*a*b*c + 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*\sqrt{-c}*\arctan(\sqrt{d*x^8 + c}*\sqrt{-c}/c) + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*\sqrt{b/(b*c - a*d)}*\log((b*d*x^8 + 2*b*c - a*d + 2*\sqrt{d*x^8 + c})*(b*c - a*d)*\sqrt{b/(b*c - a*d)})/(b*x^8 + a)) / ((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d)$, $1/8*(\sqrt{d*x^8 + c})*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*\sqrt{-b/(b*c - a*d)}*\arctan(-\sqrt{d*x^8 + c}*(b*c - a*d)*\sqrt{-b/(b*c - a*d)})/(b*d*x^8 + b*c) + 2*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*\sqrt{-c}*\arctan(\sqrt{d*x^8 + c}*\sqrt{-c}/c) / ((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d)]$

giac [A] time = 0.21, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^8 + c} bd}{8(abc - a^2d)((dx^8 + c)b - bc + ad)} - \frac{(2b^2c - 3abd) \arctan\left(\frac{\sqrt{dx^8 + c} b}{\sqrt{-b^2c + abd}}\right)}{8(a^2bc - a^3d)\sqrt{-b^2c + abd}} + \frac{\arctan\left(\frac{\sqrt{dx^8 + c}}{\sqrt{-c}}\right)}{4a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] $1/8*\sqrt{d*x^8 + c}*b*d/((a*b*c - a^2*d)*((d*x^8 + c)*b - b*c + a*d)) - 1/8*(2*b^2*c - 3*a*b*d)*\arctan(\sqrt{d*x^8 + c}*b/\sqrt{-b^2*c + a*b*d})/((a^2*b*c - a^3*d)*\sqrt{-b^2*c + a*b*d}) + 1/4*\arctan(\sqrt{d*x^8 + c}/\sqrt{-c})/(a^2*\sqrt{-c})$

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)^2 \sqrt{dx^8 + cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

[Out] int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)^2 \sqrt{dx^8 + cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^8 + a)^2*\sqrt{d*x^8 + c})*x, x)

$$\begin{aligned}
& (a^5d^3 - a^2b^3c^3 + 3a^3b^2c^2d - 3a^4b^2cd^2) - (\operatorname{atan}(\frac{((a^6b^2d^5 - (3a^5b^3cd^4)/2 + (a^4b^4c^2d^3)/2)/(8(a^5d^2 + a^3b^2c^2 - 2a^4b^2cd)) - ((c + dx^8)^{1/2})(256a^7b^2d^5 - 1024a^6b^3cd^4 - 512a^4b^5c^3d^2 + 1280a^5b^4c^2d^3))/(2048a^2c^{1/2}(a^4d^2 + a^2b^2c^2 - 2a^3b^2cd)))/(8a^2c^{1/2}) - ((c + dx^8)^{1/2})(13a^2b^3d^4 + 8b^5c^2d^2 - 20ab^4cd^3))/(256(a^4d^2 + a^2b^2c^2 - 2a^3b^2cd)) * 1i)/(a^2c^{1/2}) - (((a^6b^2d^5 - (3a^5b^3cd^4)/2 + (a^4b^4c^2d^3)/2)/(8(a^5d^2 + a^3b^2c^2 - 2a^4b^2cd)) + ((c + dx^8)^{1/2})(256a^7b^2d^5 - 1024a^6b^3cd^4 - 512a^4b^5c^3d^2 + 1280a^5b^4c^2d^3))/(2048a^2c^{1/2}(a^4d^2 + a^2b^2c^2 - 2a^3b^2cd)))/(8a^2c^{1/2}) + ((c + dx^8)^{1/2})(13a^2b^3d^4 + 8b^5c^2d^2 - 20ab^4cd^3))/(256(a^4d^2 + a^2b^2c^2 - 2a^3b^2cd)) * 1i)/(a^2c^{1/2}))) / (((3a^3b^3d^4)/128 - (b^4cd^3)/64)/(a^5d^2 + a^3b^2c^2 - 2a^4b^2cd) + (((a^6b^2d^5 - (3a^5b^3cd^4)/2 + (a^4b^4c^2d^3)/2)/(8(a^5d^2 + a^3b^2c^2 - 2a^4b^2cd)) - ((c + dx^8)^{1/2})(256a^7b^2d^5 - 1024a^6b^3cd^4 - 512a^4b^5c^3d^2 + 1280a^5b^4c^2d^3))/(2048a^2c^{1/2}(a^4d^2 + a^2b^2c^2 - 2a^3b^2cd)))/(8a^2c^{1/2}) - ((c + dx^8)^{1/2})(13a^2b^3d^4 + 8b^5c^2d^2 - 20ab^4cd^3))/(256(a^4d^2 + a^2b^2c^2 - 2a^3b^2cd)))/(a^2c^{1/2}) + (((a^6b^2d^5 - (3a^5b^3cd^4)/2 + (a^4b^4c^2d^3)/2)/(8(a^5d^2 + a^3b^2c^2 - 2a^4b^2cd)) + ((c + dx^8)^{1/2})(256a^7b^2d^5 - 1024a^6b^3cd^4 - 512a^4b^5c^3d^2 + 1280a^5b^4c^2d^3))/(2048a^2c^{1/2}(a^4d^2 + a^2b^2c^2 - 2a^3b^2cd)))/(8a^2c^{1/2}) + ((c + dx^8)^{1/2})(13a^2b^3d^4 + 8b^5c^2d^2 - 20ab^4cd^3))/(256(a^4d^2 + a^2b^2c^2 - 2a^3b^2cd)))/(a^2c^{1/2}))) * 1i)/(4a^2c^{1/2}) - (b*d*(c + dx^8)^{1/2})/(2*(a^2d - a*b*c)*(4*b*(c + dx^8) + 4*a*d - 4*b*c))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)

[Out] Timed out

$$3.577 \quad \int \frac{1}{x^9(a+bx^8)^2 \sqrt{c+dx^8}} dx$$

Optimal. Leaf size=185

$$\frac{b^{3/2}(4bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^3(bc - ad)^{3/2}} + \frac{(ad + 4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}} - \frac{b\sqrt{c+dx^8}(2bc - ad)}{8a^2c(a+bx^8)(bc - ad)} - \frac{\sqrt{c+dx^8}}{8acx^8(a+bx^8)}$$

Rubi [A] time = 0.23, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 103, 151, 156, 63, 208}

$$\frac{b^{3/2}(4bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^3(bc - ad)^{3/2}} + \frac{(ad + 4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}} - \frac{b\sqrt{c+dx^8}(2bc - ad)}{8a^2c(a+bx^8)(bc - ad)} - \frac{\sqrt{c+dx^8}}{8acx^8(a+bx^8)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^8])/((8*a^2*c*(b*c - a*d)*(a + b*x^8)) - Sqrt[c + d*x^8]/((8*a*c*x^8*(a + b*x^8)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/(8*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(8*a^3*(b*c - a*d)^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 151

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

```

Rule 156

```

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 446

```

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^9 (a + bx^8)^2 \sqrt{c + dx^8}} dx &= \frac{1}{8} \text{Subst} \left(\int \frac{1}{x^2 (a + bx)^2 \sqrt{c + dx}} dx, x, x^8 \right) \\
&= \frac{\sqrt{c + dx^8}}{8acx^8 (a + bx^8)} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(4bc+ad) + \frac{3bdx}{2}}{x(a+bx)^2 \sqrt{c+dx}} dx, x, x^8 \right)}{8ac} \\
&= \frac{b(2bc - ad)\sqrt{c + dx^8}}{8a^2c(bc - ad)(a + bx^8)} - \frac{\sqrt{c + dx^8}}{8acx^8 (a + bx^8)} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad) + \frac{1}{2}bd(2bc-ad)}{x(a+bx)\sqrt{c+dx}} dx, x, x^8 \right)}{8a^2c(bc - ad)} \\
&= \frac{b(2bc - ad)\sqrt{c + dx^8}}{8a^2c(bc - ad)(a + bx^8)} - \frac{\sqrt{c + dx^8}}{8acx^8 (a + bx^8)} + \frac{(b^2(4bc - 5ad)) \text{Subst} \left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^8 \right)}{16a^3(bc - ad)} \\
&= \frac{b(2bc - ad)\sqrt{c + dx^8}}{8a^2c(bc - ad)(a + bx^8)} - \frac{\sqrt{c + dx^8}}{8acx^8 (a + bx^8)} + \frac{(b^2(4bc - 5ad)) \text{Subst} \left(\int \frac{1}{a - \frac{bc}{d} + \frac{bx^2}{d}} dx, x, x^8 \right)}{8a^3d(bc - ad)} \\
&= \frac{b(2bc - ad)\sqrt{c + dx^8}}{8a^2c(bc - ad)(a + bx^8)} - \frac{\sqrt{c + dx^8}}{8acx^8 (a + bx^8)} + \frac{(4bc + ad) \tanh^{-1} \left(\frac{\sqrt{c+dx^8}}{\sqrt{c}} \right)}{8a^3c^{3/2}} - \frac{b^{3/2}}{8a^3c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.60, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^8}(a^2d+ab(dx^8-c)-2b^2cx^8)}{x^8(a+bx^8)(bc-ad)} + \frac{b^{3/2}c(5ad-4bc) \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{(ad+4bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{8a^3c}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] ((a*Sqrt[c + d*x^8]*(a^2*d - 2*b^2*c*x^8 + a*b*(-c + d*x^8)))/((b*c - a*d)*x^8*(a + b*x^8)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/Sqrt[c] + (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(8*a^3*c)

IntegrateAlgebraic [A] time = 0.65, size = 187, normalized size = 1.01

$$\frac{(4b^{5/2}c - 5ab^{3/2}d) \tan^{-1} \left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad} \right)}{8a^3(ad-bc)^{3/2}} + \frac{(ad+4bc) \tanh^{-1} \left(\frac{\sqrt{c+dx^8}}{\sqrt{c}} \right)}{8a^3c^{3/2}} + \frac{\sqrt{c+dx^8}(-a^2d+abc-abdx^8+2b^2cx^8)}{8a^2cx^8(a+bx^8)(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] (Sqrt[c + d*x^8]*(a*b*c - a^2*d + 2*b^2*c*x^8 - a*b*d*x^8))/(8*a^2*c*(-(b*c) + a*d)*x^8*(a + b*x^8)) + ((4*b^(5/2)*c - 5*a*b^(3/2)*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)]/(8*a^3*(-(b*c) + a*d)^(3/2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/(8*a^3*c^(3/2))

fricas [A] time = 0.56, size = 1236, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] [1/16*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(c)*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^8 + b*c)) - ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(c)*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/8*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))/(b*d*x^8 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) + ((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8)]

giac [A] time = 0.17, size = 257, normalized size = 1.39

$$\frac{(4b^3c - 5ab^2d) \arctan\left(\frac{\sqrt{dx^8+cb}}{\sqrt{-b^2c+abd}}\right)}{8(a^3bc - a^4d)\sqrt{-b^2c+abd}} - \frac{2(dx^8+c)^{\frac{3}{2}}b^2cd - 2\sqrt{dx^8+c}b^2c^2d - (dx^8+c)^{\frac{3}{2}}abd^2 + 2\sqrt{dx^8+c}abcd^2 - \sqrt{dx^8+c}a^2d^3}{8(a^2bc^2 - a^3cd)((dx^8+c)^2b - 2(dx^8+c)bc + bc^2 + (dx^8+c)ad - acd)} - \frac{(4bc+ad) \arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{8a^3\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{8} \cdot (4b^3c - 5ab^2d) \arctan\left(\frac{\sqrt{dx^8+c} \cdot b}{\sqrt{-b^2c+abd}}\right) / \left((a^3bc - a^4d) \sqrt{-b^2c+abd} \right) - \frac{1}{8} \cdot (2(dx^8+c)^{3/2} b^2cd - 2\sqrt{dx^8+c} b^2c^2d - (dx^8+c)^{3/2} ab^2d^2 + 2\sqrt{dx^8+c} ab^2cd^2 - \sqrt{dx^8+c} a^2d^3) / \left((a^2bc^2 - a^3cd) \cdot ((dx^8+c)^{2b} - 2(dx^8+c)bc + b^2c^2 + (dx^8+c)ad - acd) \right) - \frac{1}{8} \cdot (4b^3c + abd) \arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right) / (a^3\sqrt{-c} \cdot c)$

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8+a)^2 \sqrt{dx^8+cx^9}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

[Out] int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8+a)^2 \sqrt{dx^8+cx^9}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^8+a)^2*sqrt(d*x^8+c)*x^9),x)

mupad [B] time = 7.85, size = 3832, normalized size = 20.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^9*(a+b*x^8)^2*(c+d*x^8)^(1/2)),x)

[Out] $\left(\frac{((c+dx^8)^{1/2} \cdot (a^2d^3 + 2b^2c^2d - 2ab^2cd^2)) / (2a^2 \cdot (b^2c^2 - a^2cd)) + (b \cdot (c+dx^8)^{3/2} \cdot (ad^2 - 2b^2cd)) / (2a^2 \cdot (b^2c^2 - a^2cd))}{((c+dx^8) \cdot (4ad - 8b^2c) + 4b^2 \cdot (c+dx^8)^2 + 4b^2c^2 - 4a^2cd) + (\arctan\left(\frac{(-b^3(ad-b^2c)^3)^{1/2} \cdot (5ad - 4b^2c)}{(c+dx^8)^{1/2} \cdot (a^4b^3d^6 + 32b^7c^4d^2 - 64ab^6c^3d^3 + 6a^3b^4c^2d^5 + 26a^2b^5c^2d^4)}\right)) / (32 \cdot (a^4b^2c^4 + a^6c^2d^2 - 2a^5b^2c^3d)) + ((-b^3(ad-b^2c)^3)^{1/2} \cdot (5ad - 4b^2c)) \cdot \left(\frac{(a^9b^2c^2d^6)}{2} + a^6b^5c^4d^3 - 2a^7b^2c^3d^2 \right)}{2} \right)$

$$\begin{aligned}
& 5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5) \\
& / (512*a^3*(c^3)^{(1/2)}*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) * (a*d + \\
& 4*b*c) / (16*a^3*(c^3)^{(1/2)}) * (a*d + 4*b*c) * i / (16*a^3*(c^3)^{(1/2)}) + (((c + \\
& d*x^8)^{(1/2)}*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4)) / (32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (((a^9*b^2*c*d^6) / 2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5) / 2) / (a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^8)^{(1/2)}*(a*d + 4*b*c) * (512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5)) / (512*a^3*(c^3)^{(1/2)}*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) * (a*d + 4*b*c) / (16*a^3*(c^3)^{(1/2)}) * (a*d + 4*b*c) * i / (16*a^3*(c^3)^{(1/2)}) / (((5*a^3*b^4*d^6) / 256 + (b^7*c^3*d^3) / 8 - (3*a*b^6*c^2*d^4) / 16 + (3*a^2*b^5*c*d^5) / 128) / (a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - (((c + d*x^8)^{(1/2)}*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4)) / (32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((a^9*b^2*c*d^6) / 2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5) / 2) / (a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^8)^{(1/2)}*(a*d + 4*b*c) * (512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5)) / (512*a^3*(c^3)^{(1/2)}*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) * (a*d + 4*b*c) / (16*a^3*(c^3)^{(1/2)}) * (a*d + 4*b*c) / (16*a^3*(c^3)^{(1/2)}) + (((c + d*x^8)^{(1/2)}*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4)) / (32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (((a^9*b^2*c*d^6) / 2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5) / 2) / (a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^8)^{(1/2)}*(a*d + 4*b*c) * (512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5)) / (512*a^3*(c^3)^{(1/2)}*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) * (a*d + 4*b*c) / (16*a^3*(c^3)^{(1/2)}) * (a*d + 4*b*c) / (16*a^3*(c^3)^{(1/2)}) * (a*d + 4*b*c) * i / (8*a^3*(c^3)^{(1/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**9/(b*x**8+a)**2/(d*x**8+c)**(1/2), x)

[Out] Timed out

$$3.578 \quad \int \frac{x^{19}}{(a+bx^8)^2 \sqrt{c+dx^8}} dx$$

Optimal. Leaf size=141

$$-\frac{\sqrt{a}(3bc-2ad) \tan^{-1}\left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}} + \frac{ax^4 \sqrt{c+dx^8}}{8b(a+bx^8)(bc-ad)} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

Rubi [A] time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {465, 470, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}(3bc-2ad) \tan^{-1}\left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}} + \frac{ax^4 \sqrt{c+dx^8}}{8b(a+bx^8)(bc-ad)} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] (a*x^4*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) - (Sqrt[a]*(3*b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*b^2*(b*c - a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(4*b^2*Sqrt[d])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b

, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 470

Int[((e_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))
^(q_)), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]

Rule 523

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x_)
^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d,
e, f, n}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^{19}}{(a+bx^8)^2 \sqrt{c+dx^8}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^4}{(a+bx^2)^2 \sqrt{c+dx^2}} dx, x, x^4 \right) \\
&= \frac{ax^4 \sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} - \frac{\text{Subst} \left(\int \frac{ac-2(bc-ad)x^2}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^4 \right)}{8b(bc-ad)} \\
&= \frac{ax^4 \sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} + \frac{\text{Subst} \left(\int \frac{1}{\sqrt{c+dx^2}} dx, x, x^4 \right)}{4b^2} - \frac{(a(3bc-2ad)) \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^4 \right)}{8b^2(bc-ad)} \\
&= \frac{ax^4 \sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} + \frac{\text{Subst} \left(\int \frac{1}{1-dx^2} dx, x, \frac{x^4}{\sqrt{c+dx^8}} \right)}{4b^2} - \frac{(a(3bc-2ad)) \text{Subst} \left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^4 \right)}{8b^2(bc-ad)} \\
&= \frac{ax^4 \sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} - \frac{\sqrt{a}(3bc-2ad) \tan^{-1} \left(\frac{\sqrt{bc-ad} x^4}{\sqrt{a} \sqrt{c+dx^8}} \right)}{8b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1} \left(\frac{\sqrt{d} x^4}{\sqrt{c+dx^8}} \right)}{4b^2 \sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 135, normalized size = 0.96

$$\frac{\frac{abx^4 \sqrt{c+dx^8}}{(a+bx^8)(bc-ad)} + \frac{\sqrt{a}(2ad-3bc) \tan^{-1} \left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}} \right)}{(bc-ad)^{3/2}} + \frac{2 \log(\sqrt{d} \sqrt{c+dx^8} + dx^4)}{\sqrt{d}}}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] ((a*b*x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (Sqrt[a]*(-3*b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(b*c - a*d)^(3/2) + (2*Log[d*x^4 + Sqrt[d]*Sqrt[c + d*x^8]])/Sqrt[d])/(8*b^2)

IntegrateAlgebraic [A] time = 3.67, size = 166, normalized size = 1.18

$$\frac{(2a^{3/2}d - 3\sqrt{a}bc) \tan^{-1} \left(\frac{a\sqrt{d} + bx^4\sqrt{c+dx^8} + b\sqrt{d}x^8}{\sqrt{a}\sqrt{bc-ad}} \right)}{8b^2(bc-ad)^{3/2}} + \frac{ax^4 \sqrt{c+dx^8}}{8b(a+bx^8)(bc-ad)} + \frac{\log(\sqrt{c+dx^8} + \sqrt{d}x^4)}{4b^2 \sqrt{d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

```
[Out] (a*x^4*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) + ((-3*Sqrt[a]*b*c +
2*a^(3/2)*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*b^2*(b*c - a*d)^(3/2)) + Log[Sqrt[d]*x^4 + Sqrt[c + d*x^8]]/(4*b^2*Sqrt[d])
```

fricas [A] time = 0.82, size = 1077, normalized size = 7.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(d*x^8 + c)*a*b*d*x^4 + 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/32*(4*sqrt(d*x^8 + c)*a*b*d*x^4 - 8*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/16*(2*sqrt(d*x^8 + c)*a*b*d*x^4 + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d)))/(a*d*x^12 + a*c*x^4)) + 2*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/16*(2*sqrt(d*x^8 + c)*a*b*d*x^4 - 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d)))/(a*d*x^12 + a*c*x^4)))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2)]
```

giac [B] time = 0.65, size = 298, normalized size = 2.11

$$\frac{(3abc\sqrt{d} - 2a^2d^{\frac{3}{2}})\arctan\left(\frac{(\sqrt{d}x^4 - \sqrt{dx^8+c})^2 b - bc + 2ad}{2\sqrt{abcd - a^2d^2}}\right)}{8(b^3c - ab^2d)\sqrt{abcd - a^2d^2}} - \frac{(\sqrt{d}x^4 - \sqrt{dx^8+c})^2 abc\sqrt{d} - 2(\sqrt{d}x^4 - \sqrt{dx^8+c})^2 a^2d^{\frac{3}{2}} - abc^2\sqrt{d}}{4\left((\sqrt{d}x^4 - \sqrt{dx^8+c})^4 b - 2(\sqrt{d}x^4 - \sqrt{dx^8+c})^2 bc + 4(\sqrt{d}x^4 - \sqrt{dx^8+c})^2 ad + bc^2\right)(b^3c - ab^2d)} - \frac{\log\left(\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2\right)}{8b^2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```


[Out]
$$-1/8*(3*a*b*c*\sqrt{d} - 2*a^2*d^{(3/2)})*\arctan(-1/2*((\sqrt{d})*x^4 - \sqrt{d*x^8 + c})^2*b - b*c + 2*a*d)/\sqrt{a*b*c*d - a^2*d^2})/((b^3*c - a*b^2*d)*\sqrt{a*b*c*d - a^2*d^2}) - 1/4*((\sqrt{d})*x^4 - \sqrt{d*x^8 + c})^2*a*b*c*\sqrt{d} - 2*(\sqrt{d})*x^4 - \sqrt{d*x^8 + c})^2*a^2*d^{(3/2)} - a*b*c^2*\sqrt{d})/(((\sqrt{d})*x^4 - \sqrt{d*x^8 + c})^4*b - 2*(\sqrt{d})*x^4 - \sqrt{d*x^8 + c})^2*b*c + 4*(\sqrt{d})*x^4 - \sqrt{d*x^8 + c})^2*a*d + b*c^2)*(b^3*c - a*b^2*d)) - 1/8*\log((\sqrt{d})*x^4 - \sqrt{d*x^8 + c})^2)/(b^2*\sqrt{d}))$$

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{(bx^8 + a)^2 \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)`

[Out] `int(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{(bx^8 + a)^2 \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^19/((b*x^8 + a)^2*\sqrt{d*x^8 + c}), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{19}}{(bx^8 + a)^2 \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^19/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)`

[Out] `int(x^19/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.579 \quad \int \frac{x^{11}}{(a+bx^8)^2 \sqrt{c+dx^8}} dx$$

Optimal. Leaf size=93

$$\frac{c \tan^{-1} \left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}} \right)}{8\sqrt{a} (bc-ad)^{3/2}} - \frac{x^4 \sqrt{c+dx^8}}{8(a+bx^8)(bc-ad)}$$

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {465, 471, 12, 377, 205}

$$\frac{c \tan^{-1} \left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}} \right)}{8\sqrt{a} (bc-ad)^{3/2}} - \frac{x^4 \sqrt{c+dx^8}}{8(a+bx^8)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] -(x^4*Sqrt[c + d*x^8])/(8*(b*c - a*d)*(a + b*x^8)) + (c*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*Sqrt[a]*(b*c - a*d)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -

1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
 [{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rule 471

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11}}{(a + bx^8)^2 \sqrt{c + dx^8}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^2}{(a + bx^2)^2 \sqrt{c + dx^2}} dx, x, x^4 \right) \\
 &= -\frac{x^4 \sqrt{c + dx^8}}{8(bc - ad)(a + bx^8)} + \frac{\text{Subst} \left(\int \frac{c}{(a + bx^2) \sqrt{c + dx^2}} dx, x, x^4 \right)}{8(bc - ad)} \\
 &= -\frac{x^4 \sqrt{c + dx^8}}{8(bc - ad)(a + bx^8)} + \frac{c \text{Subst} \left(\int \frac{1}{(a + bx^2) \sqrt{c + dx^2}} dx, x, x^4 \right)}{8(bc - ad)} \\
 &= -\frac{x^4 \sqrt{c + dx^8}}{8(bc - ad)(a + bx^8)} + \frac{c \text{Subst} \left(\int \frac{1}{a - (-bc + ad)x^2} dx, x, \frac{x^4}{\sqrt{c + dx^8}} \right)}{8(bc - ad)} \\
 &= -\frac{x^4 \sqrt{c + dx^8}}{8(bc - ad)(a + bx^8)} + \frac{c \tan^{-1} \left(\frac{\sqrt{bc - ad} x^4}{\sqrt{a} \sqrt{c + dx^8}} \right)}{8\sqrt{a} (bc - ad)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.55, size = 124, normalized size = 1.33

$$\frac{\sqrt{c + dx^8} \left(-\frac{x^8(bc - ad)}{a + bx^8} - \frac{c \sqrt{x^8 \left(\frac{d}{c} - \frac{b}{a} \right)} \tanh^{-1} \left(\frac{\sqrt{x^8 \left(\frac{d}{c} - \frac{b}{a} \right)}}{\sqrt{\frac{dx^8}{c} + 1}} \right)}{\sqrt{\frac{dx^8}{c} + 1}} \right)}{8x^4(bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] (Sqrt[c + d*x^8]*(-(((b*c - a*d)*x^8)/(a + b*x^8)) - (c*Sqrt[(-b/a) + d/c]*x^8)*ArcTanh[Sqrt[(-b/a) + d/c]*x^8]/Sqrt[1 + (d*x^8)/c]])/Sqrt[1 + (d*x^8)/c]))/(8*(b*c - a*d)^2*x^4)

IntegrateAlgebraic [A] time = 1.82, size = 145, normalized size = 1.56

$$\frac{c \tan^{-1} \left(\frac{b\sqrt{d}x^8}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}} \right)}{8\sqrt{a}(bc-ad)^{3/2}} - \frac{x^4\sqrt{c+dx^8}}{8(a+bx^8)(bc-ad)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] -1/8*(x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (c*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^8)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^4*Sqrt[c + d*x^8))/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*Sqrt[a]*(b*c - a*d)^(3/2))

fricas [B] time = 0.54, size = 426, normalized size = 4.58

$$\left[\frac{4\sqrt{dx^8+c}(abc-a^2d)x^4-(bcx^8+ac)\sqrt{-abc+a^2d}\log\left(\frac{(b^2d^2-8abcd+8a^2d^2)^{1/4}-2(3abc^2-4a^2cd)^{1/4}+a^2c^2+4((bc-2ad)x^8-acx^4)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{b^2x^{16}+2abx^8+a^2}\right)}{32((ab^3c^2-2a^2b^2cd+a^3bd^2)x^8+a^2b^2c^2-2a^3bcd+a^4d^2)} \right], \left[\frac{2\sqrt{dx^8+c}(abc-a^2d)x^4-(bcx^8+ac)\sqrt{-abc+a^2d}\arctan\left(\frac{(bc-2ad)x^8-ac}{2(abcd-2d^2)^{1/2}+(ab^2-d^2c)^{1/4}}\right)}{16((ab^3c^2-2a^2b^2cd+a^3bd^2)x^8+a^2b^2c^2-2a^3bcd+a^4d^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] [-1/32*(4*sqrt(d*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/16*(2*sqrt(d*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d))/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)))/((a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]

giac [B] time = 1.59, size = 244, normalized size = 2.62

$$\frac{c\sqrt{d}\arctan\left(-\frac{(\sqrt{d}x^4-\sqrt{dx^8+c})^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{8\sqrt{abcd-a^2d^2}(bc-ad)} + \frac{(\sqrt{d}x^4-\sqrt{dx^8+c})^2bc\sqrt{d}-2(\sqrt{d}x^4-\sqrt{dx^8+c})^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{4\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2bc+4\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2ad+bc^2\right)(b^2c-abd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(b*x⁸+a)²/(d*x⁸+c)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{8}c\sqrt{d}\arctan\left(\frac{-1/2\left(\sqrt{d}x^4 - \sqrt{dx^8 + c}\right)^2b - bc + 2ad}{\sqrt{abc d - a^2d^2}}\right) + \frac{1}{4}\left(\frac{\left(\sqrt{d}x^4 - \sqrt{dx^8 + c}\right)^2b c \sqrt{d} - 2\left(\sqrt{d}x^4 - \sqrt{dx^8 + c}\right)^2a d^{3/2} - b c^2 \sqrt{d}}{\left(\left(\sqrt{d}x^4 - \sqrt{dx^8 + c}\right)^4 b - 2\left(\sqrt{d}x^4 - \sqrt{dx^8 + c}\right)^2 b c + 4\left(\sqrt{d}x^4 - \sqrt{dx^8 + c}\right)^2 a d + b c^2\right)\left(b^2 c - a b d\right)}\right)$

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(bx^8 + a)^2 \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/(b*x⁸+a)²/(d*x⁸+c)^(1/2),x)

[Out] int(x¹¹/(b*x⁸+a)²/(d*x⁸+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(bx^8 + a)^2 \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(b*x⁸+a)²/(d*x⁸+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x¹¹/((b*x⁸ + a)²*sqrt(d*x⁸ + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{(bx^8 + a)^2 \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/((a + b*x⁸)²*(c + d*x⁸)^(1/2)),x)

[Out] int(x¹¹/((a + b*x⁸)²*(c + d*x⁸)^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.580 \quad \int \frac{x^3}{(a+bx^8)^2 \sqrt{c+dx^8}} dx$$

Optimal. Leaf size=104

$$\frac{(bc - 2ad) \tan^{-1} \left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}} \right)}{8a^{3/2}(bc - ad)^{3/2}} + \frac{bx^4 \sqrt{c + dx^8}}{8a (a + bx^8) (bc - ad)}$$

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {465, 382, 377, 205}

$$\frac{(bc - 2ad) \tan^{-1} \left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}} \right)}{8a^{3/2}(bc - ad)^{3/2}} + \frac{bx^4 \sqrt{c + dx^8}}{8a (a + bx^8) (bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] (b*x^4*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) + ((b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(3/2)*(b*c - a*d)^(3/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 382

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[(b*c + n*(p+1)*(b*c - a*d))/(a*n*(p+1)*(b*c - a*d)], Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p+q+2)+1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

Rule 465

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
 [{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(a + bx^8)^2 \sqrt{c + dx^8}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{(a + bx^2)^2 \sqrt{c + dx^2}} dx, x, x^4 \right) \\ &= \frac{bx^4 \sqrt{c + dx^8}}{8a(bc - ad)(a + bx^8)} + \frac{(bc - 2ad) \text{Subst} \left(\int \frac{1}{(a + bx^2) \sqrt{c + dx^2}} dx, x, x^4 \right)}{8a(bc - ad)} \\ &= \frac{bx^4 \sqrt{c + dx^8}}{8a(bc - ad)(a + bx^8)} + \frac{(bc - 2ad) \text{Subst} \left(\int \frac{1}{a - (-bc + ad)x^2} dx, x, \frac{x^4}{\sqrt{c + dx^8}} \right)}{8a(bc - ad)} \\ &= \frac{bx^4 \sqrt{c + dx^8}}{8a(bc - ad)(a + bx^8)} + \frac{(bc - 2ad) \tan^{-1} \left(\frac{\sqrt{bc - ad} x^4}{\sqrt{a} \sqrt{c + dx^8}} \right)}{8a^{3/2}(bc - ad)^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.94, size = 407, normalized size = 3.91

$$\frac{x^4 \sqrt{c + dx^8} \left(-30dx^8 \sqrt{\frac{ax^8(c+dx^8)(bc-ad)}{c^2(a+bx^8)^2}} - 45c \sqrt{\frac{ax^8(c+dx^8)(bc-ad)}{c^2(a+bx^8)^2}} + 16dx^8 \left(\frac{x^8(bc-ad)}{c(a+bx^8)} \right)^{5/2} \sqrt{\frac{a(c+dx^8)}{c(a+bx^8)}} {}_2F_1 \left(2, 3; \frac{7}{2}; \frac{(bc-ad)x^8}{c(bc^2+ad)} \right) + 16c \left(\frac{x^8(bc-ad)}{c(a+bx^8)} \right)^{5/2} \sqrt{\frac{a(c+dx^8)}{c(a+bx^8)}} {}_2F_1 \left(2, 3; \frac{7}{2}; \frac{(bc-ad)x^8}{c(bc^2+ad)} \right) + 30dx^8 \sin^{-1} \left(\sqrt{\frac{bc-ad}{c(a+bx^8)}} \right) + 45c \sin^{-1} \left(\sqrt{\frac{x^8(bc-ad)}{c(a+bx^8)}} \right) \right)}{120c^2 (a + bx^8)^2 \left(\frac{x^8(bc-ad)}{c(a+bx^8)} \right)^{3/2} \sqrt{\frac{a(c+dx^8)}{c(a+bx^8)}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] (x^4*Sqrt[c + d*x^8]*(-45*c*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x^8)^2)] - 30*d*x^8*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x^8)^2)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 30*d*x^8*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 16*c*(((b*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeometric2F1[2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 16*d*x^8*(((b*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeometric2F1[2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))])/(120*c^2*(((b*c - a*d)*

$x^8)/(c*(a + b*x^8))^{(3/2)}*(a + b*x^8)^2*\text{Sqrt}[(a*(c + d*x^8))/(c*(a + b*x^8))])$

IntegrateAlgebraic [A] time = 0.99, size = 124, normalized size = 1.19

$$\frac{(bc - 2ad) \tan^{-1} \left(\frac{a\sqrt{d} + bx^4\sqrt{c+dx^8} + b\sqrt{d}x^8}{\sqrt{a}\sqrt{bc-ad}} \right)}{8a^{3/2}(bc - ad)^{3/2}} - \frac{bx^4\sqrt{c + dx^8}}{8a(a + bx^8)(ad - bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] $-1/8*(b*x^4*\text{Sqrt}[c + d*x^8])/(a*(-(b*c) + a*d)*(a + b*x^8)) + ((b*c - 2*a*d)*\text{ArcTan}[(a*\text{Sqrt}[d] + b*\text{Sqrt}[d]*x^8 + b*x^4*\text{Sqrt}[c + d*x^8])]/(\text{Sqrt}[a]*\text{Sqrt}[b*c - a*d]))/(8*a^{(3/2)}*(b*c - a*d)^{(3/2)})$

fricas [B] time = 0.58, size = 467, normalized size = 4.49

$$\frac{4\sqrt{d^8+c}(a^2c-a^2bd)x^4 - ((b^2c-2abd)x^8 + abc - 2a^2d)\sqrt{-abc+a^2d}\log\left(\frac{(b^2-8abcd+8a^2d^2)^{1/2}-2(3ab^2-4a^2cd)x^4+a^2d^2-4((bc-2ad)^{1/2}-ac^2)\sqrt{d^8+c}\sqrt{-abc+a^2d}}{b^2c^2+2ab^2+a^2d^2}\right)}{32((a^2b^2c^2-2a^3b^2cd+a^4bd^2)x^8+a^3b^2c^2-2a^4bcd+a^5d^2)} + \frac{2\sqrt{d^8+c}(a^2c-a^2bd)x^4 + ((b^2c-2abd)x^8 + abc - 2a^2d)\sqrt{abc-a^2d}\arctan\left(\frac{(bc-2ad)^{1/2}-ac^2\sqrt{d^8+c}\sqrt{-abc+a^2d}}{2((ab^2-a^2d)^{1/2}+(ab^2-a^2d)^{1/4})}\right)}{16((a^2b^2c^2-2a^3b^2cd+a^4bd^2)x^8+a^3b^2c^2-2a^4bcd+a^5d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] $[1/32*(4*\text{sqrt}(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 - ((b^2*c - 2*a*b*d)*x^8 + a*b*c - 2*a^2*d)*\text{sqrt}(-a*b*c + a^2*d)*\log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^{16} - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^{12} - a*c*x^4)*\text{sqrt}(d*x^8 + c)*\text{sqrt}(-a*b*c + a^2*d)))/(b^2*x^{16} + 2*a*b*x^8 + a^2)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^8 + a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2), 1/16*(2*\text{sqrt}(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 + ((b^2*c - 2*a*b*d)*x^8 + a*b*c - 2*a^2*d)*\text{sqrt}(a*b*c - a^2*d)*\arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*\text{sqrt}(d*x^8 + c)*\text{sqrt}(a*b*c - a^2*d)))/((a*b*c*d - a^2*d^2)*x^{12} + (a*b*c^2 - a^2*c*d)*x^4))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^8 + a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)]$

giac [B] time = 0.39, size = 237, normalized size = 2.28

$$-\frac{1}{8}d^{\frac{3}{2}}\left(\frac{(bc - 2ad)\arctan\left(\frac{(\sqrt{d}x^4 - \sqrt{dx^8+c})^2 b - bc + 2ad}{2\sqrt{abcd-a^2d^2}}\right)}{(abcd - a^2d^2)^{\frac{3}{2}}}\right) + \frac{2\left(\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 bc - 2\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 ad - bc^2\right)}{\left(\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^4 b - 2\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 bc + 4\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 ad + bc^2\right)(abcd - a^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] $-1/8*d^{(3/2)*((b*c - 2*a*d)*\arctan(1/2*((\sqrt{d})*x^4 - \sqrt{d*x^8 + c}))^2*b - b*c + 2*a*d)/\sqrt{a*b*c*d - a^2*d^2})/(a*b*c*d - a^2*d^2)^{(3/2)} + 2*((\sqrt{d})*x^4 - \sqrt{d*x^8 + c})^2*b*c - 2*((\sqrt{d})*x^4 - \sqrt{d*x^8 + c})^2*a*d - b*c^2)/(((\sqrt{d})*x^4 - \sqrt{d*x^8 + c})^4*b - 2*((\sqrt{d})*x^4 - \sqrt{d*x^8 + c})^2*b*c + 4*((\sqrt{d})*x^4 - \sqrt{d*x^8 + c})^2*a*d + b*c^2)*(a*b*c*d - a^2*d^2))$

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^8 + a)^2 \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)`

[Out] `int(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^8 + a)^2 \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^3/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(bx^8 + a)^2 \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)`

[Out] `int(x^3/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + bx^8)^2 \sqrt{c + dx^8}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

$$3.581 \quad \int \frac{1}{x^5(a+bx^8)^2 \sqrt{c+dx^8}} dx$$

Optimal. Leaf size=149

$$\frac{b(3bc - 4ad) \tan^{-1}\left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}} - \frac{\sqrt{c+dx^8}(3bc-2ad)}{8a^2cx^4(bc-ad)} + \frac{b\sqrt{c+dx^8}}{8ax^4(a+bx^8)(bc-ad)}$$

Rubi [A] time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 472, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^8}(3bc-2ad)}{8a^2cx^4(bc-ad)} - \frac{b(3bc-4ad) \tan^{-1}\left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}} + \frac{b\sqrt{c+dx^8}}{8ax^4(a+bx^8)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]), x]

[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*x^4) + (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*x^4*(a + b*x^8)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(5/2)*(b*c - a*d)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -

$1) * (a + b * x^{(n/k)})^p * (c + d * x^{(n/k)})^q, x], x, x^k], x] /; k \neq 1 /; \text{FreeQ} \\ \{a, b, c, d, p, q\}, x] \&\& \text{NeQ}[b * c - a * d, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Rule 472

$\text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^p \cdot (c + d \cdot x^n)^q, x_Symbol] :> -\text{Simp}[b \cdot (e \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} \cdot (c + d \cdot x^n)^{q+1} / (a \cdot e \cdot n \cdot (b \cdot c - a \cdot d) \cdot (p + 1)), x] + \text{Dist}[1 / (a \cdot n \cdot (b \cdot c - a \cdot d) \cdot (p + 1)), \text{Int}[(e \cdot x)^m \cdot (a + b \cdot x^n)^{p+1} \cdot (c + d \cdot x^n)^q \cdot \text{Simp}[c \cdot b \cdot (m + 1) + n \cdot (b \cdot c - a \cdot d) \cdot (p + 1) + d \cdot b \cdot (m + n \cdot (p + q + 2) + 1) \cdot x^n, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, q\}, x] \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 583

$\text{Int}[(g \cdot x)^m \cdot (a + b \cdot x^n)^p \cdot (c + d \cdot x^n)^q \cdot (e + f \cdot x^n), x_Symbol] :> \text{Simp}[e \cdot (g \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} \cdot (c + d \cdot x^n)^{q+1} / (a \cdot c \cdot g \cdot (m + 1)), x] + \text{Dist}[1 / (a \cdot c \cdot g \cdot n \cdot (m + 1)), \text{Int}[(g \cdot x)^{m+n} \cdot (a + b \cdot x^n)^p \cdot (c + d \cdot x^n)^q \cdot \text{Simp}[a \cdot f \cdot c \cdot (m + 1) - e \cdot (b \cdot c + a \cdot d) \cdot (m + n + 1) - e \cdot n \cdot (b \cdot c \cdot p + a \cdot d \cdot q) - b \cdot e \cdot d \cdot (m + n \cdot (p + q + 2) + 1) \cdot x^n, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p, q\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[m, -1]$

Rubi steps

pergeometric2F1[2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 32*c^2*((b*c - a*d)*x^8)/(c*(a + b*x^8))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8)]]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^8)/(c*(a + b*x^8)]] + 64*c*d*x^8*((b*c - a*d)*x^8)/(c*(a + b*x^8))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8)]]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^8)/(c*(a + b*x^8)]] + 32*d^2*x^16*((b*c - a*d)*x^8)/(c*(a + b*x^8))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8)]]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^8)/(c*(a + b*x^8)]])/((c^3*x^4*((b*c - a*d)*x^8)/(c*(a + b*x^8)))^(3/2)*(a + b*x^8)^2*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8)]]])

IntegrateAlgebraic [A] time = 1.97, size = 159, normalized size = 1.07

$$\frac{(4abd - 3b^2c) \tan^{-1}\left(\frac{a\sqrt{d} + bx^4\sqrt{c+dx^8} + b\sqrt{d}x^8}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{5/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx^8}(-2a^2d + 2abc - 2abdx^8 + 3b^2cx^8)}{8a^2cx^4(a+bx^8)(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] (Sqrt[c + d*x^8]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^8 - 2*a*b*d*x^8))/(8*a^2*c*(-(b*c) + a*d)*x^4*(a + b*x^8)) + ((-3*b^2*c + 4*a*b*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*a^(5/2)*(b*c - a*d)^(3/2))

fricas [B] time = 0.61, size = 612, normalized size = 4.11

$$\frac{\left(\frac{(3d^2 - 4ad^2d) + (5ad^2 - 4d^2ad)^2\sqrt{c+dx^8} \log\left(\frac{(3d^2 - 4ad^2d) + (5ad^2 - 4d^2ad)^2\sqrt{c+dx^8} + 2(3ad^2 - 4d^2ad)\sqrt{d}x^8}{(3d^2 - 4ad^2d) + (5ad^2 - 4d^2ad)^2\sqrt{c+dx^8}}\right) + 4\left((3ad^2 - 5d^2ad) + 2d^2ad\right)^2 + 2d^2d^2 - 4d^2ad + 2d^2d\sqrt{d}x^8}{32\left((3d^2 - 4ad^2d) + (5ad^2 - 4d^2ad)^2\sqrt{c+dx^8}\right)^2 + (3d^2 - 4ad^2d)^2}\right) + \frac{\left((3d^2 - 4ad^2d) + (5ad^2 - 4d^2ad)^2\sqrt{c+dx^8} \arctan\left(\frac{(3d^2 - 4ad^2d) + (5ad^2 - 4d^2ad)^2\sqrt{c+dx^8} + 2(3ad^2 - 4d^2ad)\sqrt{d}x^8}{(3d^2 - 4ad^2d) + (5ad^2 - 4d^2ad)^2\sqrt{c+dx^8}}\right) + 2\left((3ad^2 - 5d^2ad) + 2d^2ad\right)^2 + 2d^2d^2 - 4d^2ad + 2d^2d\sqrt{d}x^8}{16\left((3d^2 - 4ad^2d) + (5ad^2 - 4d^2ad)^2\sqrt{c+dx^8}\right)^2 + (3d^2 - 4ad^2d)^2}}{16\left((3d^2 - 4ad^2d) + (5ad^2 - 4d^2ad)^2\sqrt{c+dx^8}\right)^2 + (3d^2 - 4ad^2d)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] [-1/32*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 4*((3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sqrt(d*x^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^12 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4), -1/16*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)) + 2*((3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sqrt(d*x^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^12 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4)]

giac [B] time = 1.76, size = 418, normalized size = 2.81

$$\frac{1}{8} d^{\frac{5}{2}} \left(\frac{(3b^2c - 4abd) \arctan\left(\frac{(\sqrt{dx^4 - dx^8 + c})^3 b - bc + 2ad}{2\sqrt{abcd - a^2d^2}}\right)}{(a^2bcd^2 - a^3d^3)\sqrt{abcd - a^2d^2}} + \frac{2\left(3(\sqrt{dx^4 - dx^8 + c})^4 b^2c - 4(\sqrt{dx^4 - dx^8 + c})^4 abd - 6(\sqrt{dx^4 - dx^8 + c})^2 b^2c^2 + 14(\sqrt{dx^4 - dx^8 + c})^2 abcd - 8(\sqrt{dx^4 - dx^8 + c})^2 a^2d^2 + 3b^2c^3 - 2abc^2d\right)}{\left((\sqrt{dx^4 - dx^8 + c})^6 b - 3(\sqrt{dx^4 - dx^8 + c})^4 bc + 4(\sqrt{dx^4 - dx^8 + c})^4 ad + 3(\sqrt{dx^4 - dx^8 + c})^2 bc^2 - 4(\sqrt{dx^4 - dx^8 + c})^2 acd - bc^3\right)(a^2bcd^2 - a^3d^3)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] 1/8*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2)))/((a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b^2*c - 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*a*b*d - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b^2*c^2 + 14*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b*c*d - 8*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^6*b - 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*a*d + 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c^2 - 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3))

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)^2 \sqrt{dx^8 + c} x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

[Out] int(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)^2 \sqrt{dx^8 + c} x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^5), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5 (bx^8 + a)^2 \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^5*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.582 \quad \int \frac{1}{x^{13}(a+bx^8)^2 \sqrt{c+dx^8}} dx$$

Optimal. Leaf size=208

$$\frac{b^2(5bc - 6ad) \tan^{-1}\left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}}\right)}{8a^{7/2}(bc - ad)^{3/2}} - \frac{\sqrt{c+dx^8}(5bc - 2ad)}{24a^2 cx^{12}(bc - ad)} + \frac{\sqrt{c+dx^8}(-4a^2 d^2 - 8abcd + 15b^2 c^2)}{24a^3 c^2 x^4 (bc - ad)} + \frac{b\sqrt{c+dx^8}}{8ax^{12}(a+bx^8)}$$

Rubi [A] time = 0.30, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 472, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^8}(-4a^2 d^2 - 8abcd + 15b^2 c^2)}{24a^3 c^2 x^4 (bc - ad)} + \frac{b^2(5bc - 6ad) \tan^{-1}\left(\frac{x^4 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^8}}\right)}{8a^{7/2}(bc - ad)^{3/2}} - \frac{\sqrt{c+dx^8}(5bc - 2ad)}{24a^2 cx^{12}(bc - ad)} + \frac{b\sqrt{c+dx^8}}{8ax^{12}(a+bx^8)(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]

[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^8])/(24*a^2*c*(b*c - a*d)*x^12) + ((15*b^2*c^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^8])/(24*a^3*c^2*(b*c - a*d)*x^4) + (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*x^12*(a + b*x^8)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(7/2)*(b*c - a*d)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 465

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
  x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_),
  x_Symbol] := -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/
(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)),
  Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c -
a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*
((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)),
  Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0]
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{13} (a + bx^8)^2 \sqrt{c + dx^8}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x^4 (a + bx^2)^2 \sqrt{c + dx^2}} dx, x, x^4 \right) \\
&= \frac{b\sqrt{c + dx^8}}{8a(bc - ad)x^{12} (a + bx^8)} - \frac{\text{Subst} \left(\int \frac{-5bc + 2ad - 4bdx^2}{x^4(a+bx^2)\sqrt{c+dx^2}} dx, x, x^4 \right)}{8a(bc - ad)} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^8}}{24a^2c(bc - ad)x^{12}} + \frac{b\sqrt{c + dx^8}}{8a(bc - ad)x^{12} (a + bx^8)} + \frac{\text{Subst} \left(\int \frac{-15b^2c^2 + 8abcd + 4a^2d^2}{x^2(a+bx^2)\sqrt{c+dx^2}} dx, x, x^4 \right)}{24a^2c(bc - ad)} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^8}}{24a^2c(bc - ad)x^{12}} + \frac{(15b^2c^2 - 8abcd - 4a^2d^2)\sqrt{c + dx^8}}{24a^3c^2(bc - ad)x^4} + \frac{b\sqrt{c + dx^8}}{8a(bc - ad)x^{12} (a + bx^8)} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^8}}{24a^2c(bc - ad)x^{12}} + \frac{(15b^2c^2 - 8abcd - 4a^2d^2)\sqrt{c + dx^8}}{24a^3c^2(bc - ad)x^4} + \frac{b\sqrt{c + dx^8}}{8a(bc - ad)x^{12} (a + bx^8)} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^8}}{24a^2c(bc - ad)x^{12}} + \frac{(15b^2c^2 - 8abcd - 4a^2d^2)\sqrt{c + dx^8}}{24a^3c^2(bc - ad)x^4} + \frac{b\sqrt{c + dx^8}}{8a(bc - ad)x^{12} (a + bx^8)} \\
&= -\frac{(5bc - 2ad)\sqrt{c + dx^8}}{24a^2c(bc - ad)x^{12}} + \frac{(15b^2c^2 - 8abcd - 4a^2d^2)\sqrt{c + dx^8}}{24a^3c^2(bc - ad)x^4} + \frac{b\sqrt{c + dx^8}}{8a(bc - ad)x^{12} (a + bx^8)}
\end{aligned}$$

Mathematica [A] time = 5.85, size = 175, normalized size = 0.84

$$\frac{a^2 (c + dx^8) \left(\frac{3b^3x^{16}}{(a+bx^8)(bc-ad)} + \frac{4x^8(ad+3bc)}{c^2} - \frac{2a}{c} \right) + \frac{3b^2x^{24} \sqrt{\frac{dx^8}{c} + 1} (5bc-6ad) \sin^{-1} \left(\frac{\sqrt{x^8 \left(\frac{b}{a} - \frac{d}{c} \right)}}{\sqrt{\frac{bx^8}{a} + 1}} \right)}{c \left(\frac{x^8(bc-ad)}{ac} \right)^{3/2}}}{24a^5x^{12}\sqrt{c + dx^8}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]), x]

[Out] (a^2*(c + d*x^8)*((-2*a)/c + (4*(3*b*c + a*d)*x^8)/c^2 + (3*b^3*x^16)/((b*c - a*d)*(a + b*x^8))) + (3*b^2*(5*b*c - 6*a*d)*x^24*Sqrt[1 + (d*x^8)/c]*Arc

$\text{Sin}[\text{Sqrt}[(b/a - d/c)*x^8]/\text{Sqrt}[1 + (b*x^8)/a]]/(c*((b*c - a*d)*x^8)/(a*c)^{(3/2)})/(24*a^5*x^{12}*\text{Sqrt}[c + d*x^8])$

IntegrateAlgebraic [A] time = 4.94, size = 217, normalized size = 1.04

$$\frac{(5b^3c - 6ab^2d) \tan^{-1}\left(\frac{a\sqrt{d} + bx^4\sqrt{c+dx^8} + b\sqrt{d}x^8}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{7/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx^8}(-2a^3cd + 4a^3d^2x^8 + 2a^2bc^2 + 6a^2bcdx^8 + 4a^2bd^2x^{16} - 10ab^2c^2x^8 + 8ab^2cdx^{16} - 15b^3c^2x^{16})}{24a^3c^2x^{12}(a+bx^8)(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]), x]

[Out] $(\text{Sqrt}[c + d*x^8]*(2*a^2*b*c^2 - 2*a^3*c*d - 10*a*b^2*c^2*x^8 + 6*a^2*b*c*d*x^8 + 4*a^3*d^2*x^8 - 15*b^3*c^2*x^{16} + 8*a*b^2*c*d*x^{16} + 4*a^2*b*d^2*x^{16}))/ (24*a^3*c^2*(-(b*c) + a*d)*x^{12}*(a + b*x^8)) + ((5*b^3*c - 6*a*b^2*d)*\text{ArcTan}[(a*\text{Sqrt}[d] + b*\text{Sqrt}[d]*x^8 + b*x^4*\text{Sqrt}[c + d*x^8])/(\text{Sqrt}[a]*\text{Sqrt}[b*c - a*d])])/(8*a^{(7/2)}*(b*c - a*d)^{(3/2)})$

fricas [A] time = 0.76, size = 760, normalized size = 3.65

$$\frac{3(5b^3c - 6ab^2d) \arctan\left(\frac{\sqrt{d}x^4 + \sqrt{dx^8+c}b - bc + 2ad}{2\sqrt{ad} - a^2d}\right) - \frac{6\left(\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 b^3c - 2\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 ab^2d - b^3d^2\right)}{(a^3bcd^3 - a^4d^4)\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^4 b - 2\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 bc + 4\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 ad + bc^2}}{8\left(3\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^4 b - 6\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 bc - 3\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 ad + 3bc^2 + acd\right)} - \frac{8\left(3\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^4 b - 6\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 bc - 3\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 ad + 3bc^2 + acd\right)}{\left(\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 - c\right)^3 a^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2), x, algorithm="fricas")

[Out] $[-1/96*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^{20} + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^{12})*\text{sqrt}(-a*b*c + a^2*d)*\log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^{16} - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^{12} - a*c*x^4)*\text{sqrt}(d*x^8 + c)*\text{sqrt}(-a*b*c + a^2*d))/(b^2*x^{16} + 2*a*b*x^8 + a^2)) - 4*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^{16} + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*\text{sqrt}(d*x^8 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^{20} + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^{12}), 1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^{20} + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^{12})*\text{sqrt}(a*b*c - a^2*d)*\text{arctan}(1/2*((b*c - 2*a*d)*x^8 - a*c)*\text{sqrt}(d*x^8 + c)*\text{sqrt}(a*b*c - a^2*d))/((a*b*c*d - a^2*d^2)*x^{12} + (a*b*c^2 - a^2*c*d)*x^4) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^{16} + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*\text{sqrt}(d*x^8 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^{20} + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^{12})]$

giac [B] time = 2.28, size = 395, normalized size = 1.90

$$\frac{1}{24} \left(\frac{3(5b^3c - 6ab^2d) \arctan\left(\frac{\sqrt{d}x^4 + \sqrt{dx^8+c}b - bc + 2ad}{2\sqrt{ad} - a^2d}\right)}{(a^3bcd^3 - a^4d^4)\sqrt{ad} - a^2d^2} - \frac{6\left(\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 b^3c - 2\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 ab^2d - b^3d^2\right)}{(a^3bcd^3 - a^4d^4)\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^4 b - 2\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 bc + 4\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 ad + bc^2} - \frac{8\left(3\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^4 b - 6\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 bc - 3\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 ad + 3bc^2 + acd\right)}{\left(\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 - c\right)^3 a^3 d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")`

[Out] $\frac{1}{24}d^{7/2}(3(5b^3c - 6ab^2d)\arctan(-1/2((\sqrt{d})x^4 - \sqrt{dx^8 + c})^2b - b^3c + 2ad)/\sqrt{abc d - a^2d^2})/((a^3b^3cd^3 - a^4d^4)\sqrt{abc d - a^2d^2}) - 6((\sqrt{d})x^4 - \sqrt{dx^8 + c})^2b^3c - 2(\sqrt{d})x^4 - \sqrt{dx^8 + c})^2ab^2d - b^3c^2)/((a^3b^3cd^3 - a^4d^4)((\sqrt{d})x^4 - \sqrt{dx^8 + c})^4b - 2(\sqrt{d})x^4 - \sqrt{dx^8 + c})^2b^3c + 4(\sqrt{d})x^4 - \sqrt{dx^8 + c})^2ad + b^3c^2) - 8(3(\sqrt{d})x^4 - \sqrt{dx^8 + c})^4b - 6(\sqrt{d})x^4 - \sqrt{dx^8 + c})^2b^3c - 3(\sqrt{d})x^4 - \sqrt{dx^8 + c})^2ad + 3b^3c^2 + acd)/(((\sqrt{d})x^4 - \sqrt{dx^8 + c})^2 - c)^3a^3d^3))$

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)^2 \sqrt{dx^8 + c} x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)`

[Out] `int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)^2 \sqrt{dx^8 + c} x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^13), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{13} (bx^8 + a)^2 \sqrt{dx^8 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^13*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)`

[Out] `int(1/(x^13*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**13/(b*x**8+a)**2/(d*x**8+c)**(1/2), x)

[Out] Timed out

$$3.583 \quad \int \left(a + \frac{b}{x^2} \right) \sqrt{c + \frac{d}{x^2}} x^5 dx$$

Optimal. Leaf size=123

$$\frac{d^2(2bc - ad) \tanh^{-1} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{16c^{5/2}} + \frac{dx^2 \sqrt{c + \frac{d}{x^2}} (2bc - ad)}{16c^2} + \frac{x^4 \sqrt{c + \frac{d}{x^2}} (2bc - ad)}{8c} + \frac{ax^6 \left(c + \frac{d}{x^2} \right)^{3/2}}{6c}$$

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {446, 78, 47, 51, 63, 208}

$$\frac{d^2(2bc - ad) \tanh^{-1} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{16c^{5/2}} + \frac{dx^2 \sqrt{c + \frac{d}{x^2}} (2bc - ad)}{16c^2} + \frac{x^4 \sqrt{c + \frac{d}{x^2}} (2bc - ad)}{8c} + \frac{ax^6 \left(c + \frac{d}{x^2} \right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]

[Out] (d*(2*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(16*c^2) + ((2*b*c - a*d)*Sqrt[c + d/x^2]*x^4)/(8*c) + (a*(c + d/x^2)^(3/2)*x^6)/(6*c) - (d^2*(2*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(16*c^(5/2))

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x^5 dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{(a + bx)\sqrt{c + dx}}{x^4} dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{a\left(c + \frac{d}{x^2}\right)^{3/2} x^6}{6c} - \frac{\left(3bc - \frac{3ad}{2}\right) \text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3} dx, x, \frac{1}{x^2}\right)}{6c} \\
&= \frac{(2bc - ad)\sqrt{c + \frac{d}{x^2}} x^4}{8c} + \frac{a\left(c + \frac{d}{x^2}\right)^{3/2} x^6}{6c} - \frac{(d(2bc - ad)) \text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}} dx, x, \frac{1}{x^2}\right)}{16c} \\
&= \frac{d(2bc - ad)\sqrt{c + \frac{d}{x^2}} x^2}{16c^2} + \frac{(2bc - ad)\sqrt{c + \frac{d}{x^2}} x^4}{8c} + \frac{a\left(c + \frac{d}{x^2}\right)^{3/2} x^6}{6c} + \frac{(d^2(2bc - ad))}{16c} \\
&= \frac{d(2bc - ad)\sqrt{c + \frac{d}{x^2}} x^2}{16c^2} + \frac{(2bc - ad)\sqrt{c + \frac{d}{x^2}} x^4}{8c} + \frac{a\left(c + \frac{d}{x^2}\right)^{3/2} x^6}{6c} + \frac{(d(2bc - ad))}{16c} \\
&= \frac{d(2bc - ad)\sqrt{c + \frac{d}{x^2}} x^2}{16c^2} + \frac{(2bc - ad)\sqrt{c + \frac{d}{x^2}} x^4}{8c} + \frac{a\left(c + \frac{d}{x^2}\right)^{3/2} x^6}{6c} - \frac{d^2(2bc - ad)}{16c}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 121, normalized size = 0.98

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\sqrt{c} x \sqrt{\frac{cx^2}{d} + 1} \left(a(8c^2x^4 + 2cdx^2 - 3d^2) + 6bc(2cx^2 + d) \right) + 3d^{3/2}(ad - 2bc) \sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \right)}{48c^{5/2}\sqrt{\frac{cx^2}{d} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]

[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/d]*(6*b*c*(d + 2*c*x^2) + a*(-3*d^2 + 2*c*d*x^2 + 8*c^2*x^4)) + 3*d^(3/2)*(-2*b*c + a*d)*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/(48*c^(5/2)*Sqrt[1 + (c*x^2)/d])

IntegrateAlgebraic [A] time = 0.15, size = 111, normalized size = 0.90

$$\frac{(ad^3 - 2bcd^2) \tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}} + \frac{\sqrt{\frac{cx^2+d}{x^2}} (8ac^2x^6 + 2acdx^4 - 3ad^2x^2 + 12bc^2x^4 + 6bcdx^2)}{48c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(6*b*c*d*x^2 - 3*a*d^2*x^2 + 12*b*c^2*x^4 + 2*a*c*d*x^4 + 8*a*c^2*x^6))/(48*c^2) + ((-2*b*c*d^2 + a*d^3)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/(16*c^(5/2))

fricas [A] time = 0.44, size = 242, normalized size = 1.97

$$\frac{3(2bcd^2 - ad^3)\sqrt{c} \log\left(-2cx^2 - 2\sqrt{c}x\sqrt{\frac{cx^2+d}{x^2}} - d\right) - 2(8ac^3x^6 + 2(6bc^3 + ac^2d)x^4 + 3(2bc^2d - acd^2)x^2)\sqrt{\frac{cx^2+d}{x^2}} - 3(2bcd^2 - ad^3)\sqrt{-c} \arctan\left(\frac{\sqrt{-c}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + (8ac^3x^6 + 2(6bc^3 + ac^2d)x^4 + 3(2bc^2d - acd^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{96c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] [-1/96*(3*(2*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*c^2*d - a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^3, 1/48*(3*(2*b*c*d^2 - a*d^3)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a*c^3*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*c^2*d - a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^3]

giac [A] time = 0.19, size = 143, normalized size = 1.16

$$\frac{1}{48} \left(2 \left(4ax^2\operatorname{sgn}(x) + \frac{6bc^4\operatorname{sgn}(x) + ac^3d\operatorname{sgn}(x)}{c^4} \right) x^2 + \frac{3(2bc^3d\operatorname{sgn}(x) - ac^2d^2\operatorname{sgn}(x))}{c^4} \right) \sqrt{cx^2 + dx} + \frac{(2bcd^2\operatorname{sgn}(x) - ad^3\operatorname{sgn}(x)) \log\left(-\sqrt{c}x + \sqrt{cx^2 + d}\right)}{16c^{\frac{5}{2}}} - \frac{(2bcd^2 \log(|d|) - ad^3 \log(|d|))\operatorname{sgn}(x)}{32c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] 1/48*(2*(4*a*x^2*sgn(x) + (6*b*c^4*sgn(x) + a*c^3*d*sgn(x))/c^4)*x^2 + 3*(2*b*c^3*d*sgn(x) - a*c^2*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x + 1/16*(2*b*c*d^2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(5/2) - 1/32*(2*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(5/2)

maple [A] time = 0.06, size = 162, normalized size = 1.32

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} \left(8(c x^2 + d)^{\frac{3}{2}} a c^{\frac{3}{2}} x^3 + 3a d^3 \ln\left(\sqrt{c} x + \sqrt{cx^2 + d}\right) - 6bc d^2 \ln\left(\sqrt{c} x + \sqrt{cx^2 + d}\right) + 3\sqrt{cx^2 + d} a \sqrt{c} d^2 x - 6\sqrt{cx^2 + d} b c^{\frac{3}{2}} dx - 6(c x^2 + d)^{\frac{3}{2}} a \sqrt{c} dx + 12(c x^2 + d)^{\frac{3}{2}} b c^{\frac{3}{2}} x \right)}{48\sqrt{cx^2 + d} c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x)

[Out] $\frac{1}{48} * ((c*x^2+d)/x^2)^{(1/2)} * x * (8*c^{(3/2)} * (c*x^2+d)^{(3/2)} * x^3 * a - 6*c^{(1/2)} * (c*x^2+d)^{(3/2)} * x * a * d + 12*c^{(3/2)} * (c*x^2+d)^{(3/2)} * x * b + 3*c^{(1/2)} * (c*x^2+d)^{(1/2)} * x * a * d^2 - 6*c^{(3/2)} * (c*x^2+d)^{(1/2)} * x * b * d + 3 * \ln(c^{(1/2)} * x + (c*x^2+d)^{(1/2)}) * a * d^3 - 6 * \ln(c^{(1/2)} * x + (c*x^2+d)^{(1/2)}) * b * c * d^2) / ((c*x^2+d)^{(1/2)} / c^{(5/2)})$

maxima [B] time = 1.36, size = 243, normalized size = 1.98

$$-\frac{1}{96} \left(\frac{3d^3 \log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}} + \frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3 - 8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}cd^3 - 3\sqrt{c+\frac{d}{x^2}}c^2d^3\right)}{\left(c+\frac{d}{x^2}\right)^3c^2 - 3\left(c+\frac{d}{x^2}\right)^2c^3 + 3\left(c+\frac{d}{x^2}\right)c^4 - c^5} \right) a + \frac{1}{16} \left(\frac{d^2 \log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2 + \sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c - 2\left(c+\frac{d}{x^2}\right)c^2 + c^3} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="maxima")`

[Out] $-\frac{1}{96} * (3*d^3 * \log((\sqrt{c + d/x^2}) - \sqrt{c}) / (\sqrt{c + d/x^2} + \sqrt{c})) / c^{(5/2)} + 2 * (3 * (c + d/x^2)^{(5/2)} * d^3 - 8 * (c + d/x^2)^{(3/2)} * c * d^3 - 3 * \sqrt{c + d/x^2} * c^2 * d^3) / ((c + d/x^2)^3 * c^2 - 3 * (c + d/x^2)^2 * c^3 + 3 * (c + d/x^2) * c^4 - c^5) * a + \frac{1}{16} * (d^2 * \log((\sqrt{c + d/x^2}) - \sqrt{c}) / (\sqrt{c + d/x^2} + \sqrt{c})) / c^{(3/2)} + 2 * ((c + d/x^2)^{(3/2)} * d^2 + \sqrt{c + d/x^2} * c * d^2) / ((c + d/x^2)^2 * c - 2 * (c + d/x^2) * c^2 + c^3) * b$

mupad [B] time = 5.71, size = 134, normalized size = 1.09

$$\frac{ax^6\sqrt{c+\frac{d}{x^2}}}{16} + \frac{bx^4\sqrt{c+\frac{d}{x^2}}}{8} + \frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} - \frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{16c^2} + \frac{bx^4\left(c+\frac{d}{x^2}\right)^{3/2}}{8c} - \frac{bd^2 \operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}} - \frac{ad^3 \operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}} \operatorname{li}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a + b/x^2)*(c + d/x^2)^(1/2),x)`

[Out] $(a*x^6*(c + d/x^2)^{(1/2)})/16 + (b*x^4*(c + d/x^2)^{(1/2)})/8 + (a*x^6*(c + d/x^2)^{(3/2)})/(6*c) - (a*x^6*(c + d/x^2)^{(5/2)})/(16*c^2) + (b*x^4*(c + d/x^2)^{(3/2)})/(8*c) - (a*d^3*atan(((c + d/x^2)^{(1/2)}*1i)/c^{(1/2)})*1i)/(16*c^{(5/2)}) - (b*d^2*atanh((c + d/x^2)^{(1/2)}/c^{(1/2)}))/(8*c^{(3/2)})$

sympy [B] time = 72.29, size = 226, normalized size = 1.84

$$\frac{acx^7}{6\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + \frac{5a\sqrt{d}x^5}{24\sqrt{\frac{cx^2}{d}+1}} - \frac{ad^{\frac{3}{2}}x^3}{48c\sqrt{\frac{cx^2}{d}+1}} - \frac{ad^{\frac{5}{2}}x}{16c^2\sqrt{\frac{cx^2}{d}+1}} + \frac{ad^3 \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{16c^{\frac{5}{2}}} + \frac{bcx^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + \frac{3b\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}} + \frac{bd^{\frac{3}{2}}x}{8c\sqrt{\frac{cx^2}{d}+1}} - \frac{bd^2 \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*x**5*(c+d/x**2)**(1/2),x)`

```
[Out] a*c*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 5*a*sqrt(d)*x**5/(24*sqrt(c*x**2/d + 1)) - a*d**(3/2)*x**3/(48*c*sqrt(c*x**2/d + 1)) - a*d**(5/2)*x/(16*c**2*sqrt(c*x**2/d + 1)) + a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(5/2)) + b*c*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*sqrt(d)*x**3/(8*sqrt(c*x**2/d + 1)) + b*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - b*d**2*asinh(sqrt(c)*x/sqrt(d))/(8*c**(3/2))
```

$$3.584 \quad \int \left(a + \frac{b}{x^2} \right) \sqrt{c + \frac{d}{x^2}} x^3 dx$$

Optimal. Leaf size=90

$$\frac{d(4bc - ad) \tanh^{-1} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{8c^{3/2}} + \frac{x^2 \sqrt{c + \frac{d}{x^2}} (4bc - ad)}{8c} + \frac{ax^4 \left(c + \frac{d}{x^2} \right)^{3/2}}{4c}$$

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 78, 47, 63, 208}

$$\frac{d(4bc - ad) \tanh^{-1} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{8c^{3/2}} + \frac{x^2 \sqrt{c + \frac{d}{x^2}} (4bc - ad)}{8c} + \frac{ax^4 \left(c + \frac{d}{x^2} \right)^{3/2}}{4c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]

[Out] ((4*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(8*c) + (a*(c + d/x^2)^(3/2)*x^4)/(4*c) + (d*(4*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(3/2))

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x^3 dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{(a + bx)\sqrt{c + dx}}{x^3} dx, x, \frac{1}{x^2}\right)\right) \\
 &= \frac{a\left(c + \frac{d}{x^2}\right)^{3/2} x^4}{4c} - \frac{\left(2bc - \frac{ad}{2}\right) \text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2} dx, x, \frac{1}{x^2}\right)}{4c} \\
 &= \frac{(4bc - ad)\sqrt{c + \frac{d}{x^2}} x^2}{8c} + \frac{a\left(c + \frac{d}{x^2}\right)^{3/2} x^4}{4c} - \frac{(d(4bc - ad)) \text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}} dx, x, \frac{1}{x^2}\right)}{16c} \\
 &= \frac{(4bc - ad)\sqrt{c + \frac{d}{x^2}} x^2}{8c} + \frac{a\left(c + \frac{d}{x^2}\right)^{3/2} x^4}{4c} - \frac{(4bc - ad) \text{Subst}\left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + \frac{d}{x^2}}\right)}{8c} \\
 &= \frac{(4bc - ad)\sqrt{c + \frac{d}{x^2}} x^2}{8c} + \frac{a\left(c + \frac{d}{x^2}\right)^{3/2} x^4}{4c} + \frac{d(4bc - ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.14, size = 100, normalized size = 1.11

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\sqrt{c} x \sqrt{\frac{cx^2}{d} + 1} \left(a(2cx^2 + d) + 4bc \right) + \sqrt{d}(4bc - ad) \sinh^{-1} \left(\frac{\sqrt{c}x}{\sqrt{d}} \right) \right)}{8c^{3/2} \sqrt{\frac{cx^2}{d} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]

[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/d]*(4*b*c + a*(d + 2*c*x^2)) + Sqrt[d]*(4*b*c - a*d)*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/(8*c^(3/2)*Sqrt[1 + (c*x^2)/d])

IntegrateAlgebraic [A] time = 0.15, size = 87, normalized size = 0.97

$$\frac{(4bcd - ad^2) \tanh^{-1} \left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}} \right)}{8c^{3/2}} + \frac{\sqrt{\frac{cx^2+d}{x^2}} (2acx^4 + adx^2 + 4bcx^2)}{8c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(4*b*c*x^2 + a*d*x^2 + 2*a*c*x^4))/(8*c) + ((4*b*c*d - a*d^2)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/(8*c^(3/2))

fricas [A] time = 0.44, size = 191, normalized size = 2.12

$$\left[\frac{(4bcd - ad^2)\sqrt{c} \log\left(-2cx^2 + 2\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}} - d\right) - 2(2ac^2x^4 + (4bc^2 + acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16c^2}, \frac{(4bcd - ad^2)\sqrt{-c} \arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) - (2ac^2x^4 + (4bc^2 + acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] [-1/16*((4*b*c*d - a*d^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/c^2, -1/8*((4*b*c*d - a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/c^2]

giac [A] time = 0.26, size = 105, normalized size = 1.17

$$\frac{1}{8} \left(2ax^2 \operatorname{sgn}(x) + \frac{4bc^2 \operatorname{sgn}(x) + acd \operatorname{sgn}(x)}{c^2} \right) \sqrt{cx^2 + d} - \frac{(4bcd \operatorname{sgn}(x) - ad^2 \operatorname{sgn}(x)) \log\left(\left| -\sqrt{c}x + \sqrt{cx^2 + d} \right| \right)}{8c^{\frac{3}{2}}} + \frac{(4bcd \log(|d|) - ad^2 \log(|d|)) \operatorname{sgn}(x)}{16c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] 1/8*(2*a*x^2*sgn(x) + (4*b*c^2*sgn(x) + a*c*d*sgn(x))/c^2)*sqrt(c*x^2 + d)*x - 1/8*(4*b*c*d*sgn(x) - a*d^2*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(3/2) + 1/16*(4*b*c*d*log(abs(d)) - a*d^2*log(abs(d)))*sgn(x)/c^(3/2)

maple [A] time = 0.06, size = 122, normalized size = 1.36

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} \left(-a d^2 \ln\left(\sqrt{c}x + \sqrt{cx^2+d}\right) + 4bcd \ln\left(\sqrt{c}x + \sqrt{cx^2+d}\right) - \sqrt{cx^2+d} a \sqrt{c} dx + 4\sqrt{cx^2+d} b c^{\frac{3}{2}} x + 2(c x^2 + d)^{\frac{3}{2}} a \sqrt{c} x \right) x}{8\sqrt{cx^2+d} c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x)

[Out] 1/8*((c*x^2+d)/x^2)^(1/2)*x*(2*(c*x^2+d)^(3/2)*c^(1/2)*x*a-(c*x^2+d)^(1/2)*c^(1/2)*x*a*d+4*(c*x^2+d)^(1/2)*c^(3/2)*x*b-ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*d^2+4*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c*d)/(c*x^2+d)^(1/2)/c^(3/2)

maxima [B] time = 1.24, size = 159, normalized size = 1.77

$$\frac{1}{16} \left(\frac{d^2 \log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2 + \sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c - 2\left(c+\frac{d}{x^2}\right)c^2 + c^3} \right) a + \frac{1}{4} \left(2\sqrt{c+\frac{d}{x^2}}x^2 - \frac{d \log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] 1/16*(d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2) + 2*((c + d/x^2)^(3/2)*d^2 + sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)^2*c - 2*(c + d/x^2)*c^2 + c^3))*a + 1/4*(2*sqrt(c + d/x^2)*x^2 - d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c))*b

mupad [B] time = 5.31, size = 93, normalized size = 1.03

$$\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{8} + \frac{bx^2\sqrt{c+\frac{d}{x^2}}}{2} + \frac{ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{8c} + \frac{bd\operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}} - \frac{ad^2\operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a + b/x^2)*(c + d/x^2)^(1/2), x)`

[Out] $(a*x^4*(c + d/x^2)^{(1/2)})/8 + (b*x^2*(c + d/x^2)^{(1/2)})/2 + (a*x^4*(c + d/x^2)^{(3/2)})/(8*c) + (b*d*\operatorname{atanh}((c + d/x^2)^{(1/2)}/c^{(1/2)}))/(2*c^{(1/2)}) - (a*d^2*\operatorname{atanh}((c + d/x^2)^{(1/2)}/c^{(1/2)}))/(8*c^{(3/2)})$

sympy [A] time = 59.05, size = 144, normalized size = 1.60

$$\frac{acx^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + \frac{3a\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}} + \frac{ad^{\frac{3}{2}}x}{8c\sqrt{\frac{cx^2}{d}+1}} - \frac{ad^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{3}{2}}} + \frac{b\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2} + \frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*x**3*(c+d/x**2)**(1/2), x)`

[Out] $a*c*x**5/(4*\operatorname{sqrt}(d)*\operatorname{sqrt}(c*x**2/d + 1)) + 3*a*\operatorname{sqrt}(d)*x**3/(8*\operatorname{sqrt}(c*x**2/d + 1)) + a*d**(3/2)*x/(8*c*\operatorname{sqrt}(c*x**2/d + 1)) - a*d**2*\operatorname{asinh}(\operatorname{sqrt}(c)*x/\operatorname{sqrt}(d))/(8*c**(3/2)) + b*\operatorname{sqrt}(d)*x*\operatorname{sqrt}(c*x**2/d + 1)/2 + b*d*\operatorname{asinh}(\operatorname{sqrt}(c)*x/\operatorname{sqrt}(d))/(2*\operatorname{sqrt}(c))$

$$3.585 \quad \int \left(a + \frac{b}{x^2} \right) \sqrt{c + \frac{d}{x^2}} x dx$$

Optimal. Leaf size=84

$$-\frac{\sqrt{c + \frac{d}{x^2}} (ad + 2bc)}{2c} + \frac{(ad + 2bc) \tanh^{-1} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{2\sqrt{c}} + \frac{ax^2 \left(c + \frac{d}{x^2} \right)^{3/2}}{2c}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 78, 50, 63, 208}

$$-\frac{\sqrt{c + \frac{d}{x^2}} (ad + 2bc)}{2c} + \frac{(ad + 2bc) \tanh^{-1} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{2\sqrt{c}} + \frac{ax^2 \left(c + \frac{d}{x^2} \right)^{3/2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]

[Out] -((2*b*c + a*d)*Sqrt[c + d/x^2])/(2*c) + (a*(c + d/x^2)^(3/2)*x^2)/(2*c) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*Sqrt[c])

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x \, dx &= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \frac{(a + bx)\sqrt{c + dx}}{x^2} dx, x, \frac{1}{x^2}\right)\right) \\
 &= \frac{a\left(c + \frac{d}{x^2}\right)^{3/2} x^2}{2c} - \frac{(2bc + ad) \operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x} dx, x, \frac{1}{x^2}\right)}{4c} \\
 &= -\frac{(2bc + ad)\sqrt{c + \frac{d}{x^2}}}{2c} + \frac{a\left(c + \frac{d}{x^2}\right)^{3/2} x^2}{2c} - \frac{1}{4}(2bc + ad) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{c + dx}} dx, x, \frac{1}{x}\right) \\
 &= -\frac{(2bc + ad)\sqrt{c + \frac{d}{x^2}}}{2c} + \frac{a\left(c + \frac{d}{x^2}\right)^{3/2} x^2}{2c} - \frac{(2bc + ad) \operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + \frac{d}{x^2}}\right)}{2d} \\
 &= -\frac{(2bc + ad)\sqrt{c + \frac{d}{x^2}}}{2c} + \frac{a\left(c + \frac{d}{x^2}\right)^{3/2} x^2}{2c} + \frac{(2bc + ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}
 \end{aligned}$$

Mathematica [A] time = 0.19, size = 71, normalized size = 0.85

$$\frac{1}{2} \sqrt{c + \frac{d}{x^2}} \left(\frac{x(ad + 2bc) \sinh^{-1} \left(\frac{\sqrt{c}x}{\sqrt{d}} \right)}{\sqrt{c} \sqrt{d} \sqrt{\frac{cx^2}{d} + 1}} + ax^2 - 2b \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]

[Out] (Sqrt[c + d/x^2]*(-2*b + a*x^2 + ((2*b*c + a*d)*x*ArcSinh[(Sqrt[c]*x)/Sqrt[d]]))/(Sqrt[c]*Sqrt[d]*Sqrt[1 + (c*x^2)/d]))/2

IntegrateAlgebraic [A] time = 0.11, size = 68, normalized size = 0.81

$$\frac{1}{2} (ax^2 - 2b) \sqrt{\frac{cx^2 + d}{x^2}} + \frac{(ad + 2bc) \tanh^{-1} \left(\frac{\sqrt{\frac{cx^2 + d}{x^2}}}{\sqrt{c}} \right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]

[Out] ((-2*b + a*x^2)*Sqrt[(d + c*x^2)/x^2])/2 + ((2*b*c + a*d)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/(2*Sqrt[c])

fricas [A] time = 0.43, size = 155, normalized size = 1.85

$$\left[\frac{(2bc + ad)\sqrt{c} \log\left(-2cx^2 - 2\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}} - d\right) + 2(acx^2 - 2bc)\sqrt{\frac{cx^2+d}{x^2}}}{4c}, -\frac{(2bc + ad)\sqrt{-c} \arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) - (acx^2 - 2bc)\sqrt{\frac{cx^2+d}{x^2}}}{2c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*((2*b*c + a*d)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) + 2*(a*c*x^2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c, -1/2*((2*b*c + a*d)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (a*c*x^2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c]

giac [A] time = 0.29, size = 92, normalized size = 1.10

$$\frac{1}{2} \sqrt{cx^2 + d} ax \operatorname{sgn}(x) + \frac{2b\sqrt{c}d \operatorname{sgn}(x)}{(\sqrt{c}x - \sqrt{cx^2 + d})^2 - d} - \frac{(2bc^{\frac{3}{2}} \operatorname{sgn}(x) + a\sqrt{c}d \operatorname{sgn}(x)) \log\left(\left(\sqrt{c}x - \sqrt{cx^2 + d}\right)^2\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] 1/2*sqrt(c*x^2 + d)*a*x*sgn(x) + 2*b*sqrt(c)*d*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d) - 1/4*(2*b*c^(3/2)*sgn(x) + a*sqrt(c)*d*sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)/c

maple [A] time = 0.06, size = 129, normalized size = 1.54

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} \left(-a d^2 x \ln(\sqrt{c}x + \sqrt{cx^2+d}) - 2bcdx \ln(\sqrt{c}x + \sqrt{cx^2+d}) - \sqrt{cx^2+d} a\sqrt{c} dx^2 - 2\sqrt{cx^2+d} bc^{\frac{3}{2}}x^2 + 2(cx^2+d)^{\frac{3}{2}} b\sqrt{c} \right)}{2\sqrt{cx^2+d} \sqrt{c} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*x*(c+d/x^2)^(1/2),x)

[Out] -1/2*((c*x^2+d)/x^2)^(1/2)*(-(c*x^2+d)^(1/2)*c^(1/2)*x^2*a*d-2*(c*x^2+d)^(1/2)*c^(3/2)*x^2*b+2*(c*x^2+d)^(3/2)*c^(1/2)*b-ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x*a*d^2-2*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x*b*c*d)/(c*x^2+d)^(1/2)/d/c^(1/2)

maxima [A] time = 1.21, size = 108, normalized size = 1.29

$$\frac{1}{4} \left(2 \sqrt{c + \frac{d}{x^2}} x^2 - \frac{d \log\left(\frac{\sqrt{c + \frac{d}{x^2}} - \sqrt{c}}{\sqrt{c + \frac{d}{x^2}} + \sqrt{c}}\right)}{\sqrt{c}} \right) a - \frac{1}{2} \left(\sqrt{c} \log\left(\frac{\sqrt{c + \frac{d}{x^2}} - \sqrt{c}}{\sqrt{c + \frac{d}{x^2}} + \sqrt{c}}\right) + 2 \sqrt{c + \frac{d}{x^2}} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] 1/4*(2*sqrt(c + d/x^2)*x^2 - d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c))*a - 1/2*(sqrt(c)*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))) + 2*sqrt(c + d/x^2))*b

mupad [B] time = 5.10, size = 68, normalized size = 0.81

$$\frac{a x^2 \sqrt{c + \frac{d}{x^2}}}{2} - b \sqrt{c + \frac{d}{x^2}} + b \sqrt{c} \operatorname{atanh}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right) + \frac{a d \operatorname{atanh}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{2 \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b/x^2)*(c + d/x^2)^(1/2),x)`

[Out] $(a*x^2*(c + d/x^2)^{(1/2)})/2 - b*(c + d/x^2)^{(1/2)} + b*c^{(1/2)}*\operatorname{atanh}((c + d/x^2)^{(1/2)}/c^{(1/2)}) + (a*d*\operatorname{atanh}((c + d/x^2)^{(1/2)}/c^{(1/2)}))/(2*c^{(1/2)})$

sympy [A] time = 60.43, size = 107, normalized size = 1.27

$$\frac{a\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2} + \frac{ad \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2\sqrt{c}} + b\sqrt{c} \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) - \frac{bcx}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} - \frac{b\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*x*(c+d/x**2)**(1/2),x)`

[Out] $a*\sqrt{d}*x*\sqrt{c*x**2/d + 1}/2 + a*d*\operatorname{asinh}(\sqrt{c}*x/\sqrt{d})/(2*\sqrt{c}) + b*\sqrt{c}*x*\operatorname{asinh}(\sqrt{c}*x/\sqrt{d}) - b*c*x/(\sqrt{d}*\sqrt{c*x**2/d + 1}) - b*\sqrt{d}/(x*\sqrt{c*x**2/d + 1})$

$$3.586 \quad \int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x} dx$$

Optimal. Leaf size=59

$$-a\sqrt{c + \frac{d}{x^2}} + a\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right) - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{3d}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 80, 50, 63, 208}

$$-a\sqrt{c + \frac{d}{x^2}} + a\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right) - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]

[Out] -(a*Sqrt[c + d/x^2]) - (b*(c + d/x^2)^(3/2))/(3*d) + a*Sqrt[c]*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{(a + bx)\sqrt{c + dx}}{x} dx, x, \frac{1}{x^2}\right)\right) \\
 &= -\frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{3d} - \frac{1}{2}a \text{Subst}\left(\int \frac{\sqrt{c + dx}}{x} dx, x, \frac{1}{x^2}\right) \\
 &= -a\sqrt{c + \frac{d}{x^2}} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{3d} - \frac{1}{2}(ac) \text{Subst}\left(\int \frac{1}{x\sqrt{c + dx}} dx, x, \frac{1}{x^2}\right) \\
 &= -a\sqrt{c + \frac{d}{x^2}} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{3d} - \frac{(ac) \text{Subst}\left(\int \frac{1}{\frac{-c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + \frac{d}{x^2}}\right)}{d} \\
 &= -a\sqrt{c + \frac{d}{x^2}} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{3d} + a\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)
 \end{aligned}$$

Mathematica [A] time = 0.15, size = 82, normalized size = 1.39

$$\frac{\sqrt{c + \frac{d}{x^2}} \left(\frac{3a\sqrt{c}\sqrt{d}x^3 \sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{\sqrt{\frac{cx^2}{d}+1}} - 3adx^2 - b(cx^2 + d) \right)}{3dx^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]

[Out] (Sqrt[c + d/x^2]*(-3*a*d*x^2 - b*(d + c*x^2) + (3*a*Sqrt[c]*Sqrt[d]*x^3*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/Sqrt[1 + (c*x^2)/d]))/(3*d*x^2)

IntegrateAlgebraic [A] time = 0.08, size = 74, normalized size = 1.25

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (-3adx^2 - bcx^2 - bd)}{3dx^2} + a\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-(b*d) - b*c*x^2 - 3*a*d*x^2))/(3*d*x^2) + a*Sqrt[c]*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]]

fricas [A] time = 0.44, size = 166, normalized size = 2.81

$$\left[\frac{3a\sqrt{c}dx^2 \log\left(-2cx^2 - 2\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}} - d\right) - 2((bc + 3ad)x^2 + bd)\sqrt{\frac{cx^2+d}{x^2}}}{6dx^2}, -\frac{3a\sqrt{-c}dx^2 \arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + ((bc + 3ad)x^2 + bd)\sqrt{\frac{cx^2+d}{x^2}}}{3dx^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="fricas")

[Out] [1/6*(3*a*sqrt(c)*d*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*((b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2), -1/3*(3*a*sqrt(-c)*d*x^2*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + ((b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2)]

giac [B] time = 0.49, size = 163, normalized size = 2.76

$$-\frac{1}{2}a\sqrt{c} \log\left(\left(\sqrt{c}x - \sqrt{cx^2+d}\right)^2\right) \operatorname{sgn}(x) + \frac{2\left(3\left(\sqrt{c}x - \sqrt{cx^2+d}\right)^4 bc^{\frac{3}{2}} \operatorname{sgn}(x) + 3\left(\sqrt{c}x - \sqrt{cx^2+d}\right)^4 a\sqrt{c}d \operatorname{sgn}(x) - 6\left(\sqrt{c}x - \sqrt{cx^2+d}\right)^2 a\sqrt{c}d^2 \operatorname{sgn}(x) + bc^{\frac{3}{2}}d^2 \operatorname{sgn}(x) + 3a\sqrt{c}d^3 \operatorname{sgn}(x)\right)}{3\left(\left(\sqrt{c}x - \sqrt{cx^2+d}\right)^2 - d\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="giac")

[Out] $-1/2*a*\sqrt{c}*\log((\sqrt{c}*x - \sqrt{c*x^2 + d})^2)*\text{sgn}(x) + 2/3*(3*(\sqrt{c})*x - \sqrt{c*x^2 + d})^4*b*c^{3/2}*\text{sgn}(x) + 3*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*a*\sqrt{c}*d*\text{sgn}(x) - 6*(\sqrt{c}*x - \sqrt{c*x^2 + d})^2*a*\sqrt{c}*d^2*\text{sgn}(x) + b*c^{3/2}*d^2*\text{sgn}(x) + 3*a*\sqrt{c}*d^3*\text{sgn}(x))/((\sqrt{c}*x - \sqrt{c*x^2 + d})^2 - d)^3$

maple [B] time = 0.06, size = 109, normalized size = 1.85

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} \left(-3acd x^3 \ln(\sqrt{c} x + \sqrt{cx^2+d}) - 3\sqrt{cx^2+d} a c^{\frac{3}{2}} x^4 + 3(cx^2+d)^{\frac{3}{2}} a \sqrt{c} x^2 + (cx^2+d)^{\frac{3}{2}} b \sqrt{c} \right)}{3\sqrt{cx^2+d} \sqrt{c} d x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x,x)

[Out] $-1/3*((c*x^2+d)/x^2)^{(1/2)}/x^2*(-3*(c*x^2+d)^{(1/2)}*c^{3/2}*x^4*a+3*(c*x^2+d)^{(3/2)}*c^{1/2}*x^2*a-3*\ln(c^{1/2}*x+(c*x^2+d)^{(1/2)})*x^3*a*c*d+(c*x^2+d)^{(3/2)}*b*c^{1/2})/(c*x^2+d)^{(1/2)}/d/c^{1/2}$

maxima [A] time = 1.23, size = 67, normalized size = 1.14

$$-\frac{1}{2} \left(\sqrt{c} \log \left(\frac{\sqrt{c + \frac{d}{x^2}} - \sqrt{c}}{\sqrt{c + \frac{d}{x^2}} + \sqrt{c}} \right) + 2 \sqrt{c + \frac{d}{x^2}} \right) a - \frac{b \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="maxima")

[Out] $-1/2*(\sqrt{c}*\log((\sqrt{c + d/x^2} - \sqrt{c})/(\sqrt{c + d/x^2} + \sqrt{c}))) + 2*\sqrt{c + d/x^2})*a - 1/3*b*(c + d/x^2)^{(3/2)}/d$

mupad [B] time = 5.21, size = 57, normalized size = 0.97

$$a \sqrt{c} \operatorname{atanh} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right) - a \sqrt{c + \frac{d}{x^2}} - \frac{b \sqrt{c + \frac{d}{x^2}} (cx^2 + d)}{3dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x,x)

[Out] $a*c^{(1/2)}*atanh((c + d/x^2)^{(1/2)}/c^{(1/2)}) - a*(c + d/x^2)^{(1/2)} - (b*(c + d/x^2)^{(1/2)}*(d + c*x^2))/(3*d*x^2)$

sympy [A] time = 23.44, size = 75, normalized size = 1.27

$$\frac{a \left(-\frac{2c \operatorname{atan} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{-c}} \right)}{\sqrt{-c}} - 2\sqrt{c + \frac{d}{x^2}} \right)}{2} + \frac{b \left(\begin{cases} -\frac{\sqrt{c}}{x^2} & \text{for } d = 0 \\ -\frac{2\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise} \end{cases} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x,x)`

[Out] $a*(-2*c*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) - 2*sqrt(c + d/x**2))/2 + b*Piecewise((-sqrt(c)/x**2, Eq(d, 0)), (-2*(c + d/x**2)**(3/2)/(3*d), True))/2$

$$3.587 \quad \int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^3} dx$$

Optimal. Leaf size=46

$$\frac{\left(c + \frac{d}{x^2}\right)^{3/2} (bc - ad)}{3d^2} - \frac{b \left(c + \frac{d}{x^2}\right)^{5/2}}{5d^2}$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {444, 43}

$$\frac{\left(c + \frac{d}{x^2}\right)^{3/2} (bc - ad)}{3d^2} - \frac{b \left(c + \frac{d}{x^2}\right)^{5/2}}{5d^2}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]

[Out] ((b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^2) - (b*(c + d/x^2)^(5/2))/(5*d^2)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^3} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int (a + bx) \sqrt{c + dx} dx, x, \frac{1}{x^2}\right)\right) \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \left(\frac{(-bc + ad)\sqrt{c + dx}}{d} + \frac{b(c + dx)^{3/2}}{d}\right) dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{(bc - ad)\left(c + \frac{d}{x^2}\right)^{3/2}}{3d^2} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{5d^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.02

$$\frac{\sqrt{c + \frac{d}{x^2}} (cx^2 + d) (5adx^2 - 2bcx^2 + 3bd)}{15d^2x^4}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]

[Out] -1/15*(Sqrt[c + d/x^2]*(d + c*x^2)*(3*b*d - 2*b*c*x^2 + 5*a*d*x^2))/(d^2*x^4)

IntegrateAlgebraic [A] time = 0.06, size = 66, normalized size = 1.43

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (-5acdx^4 - 5ad^2x^2 + 2bc^2x^4 - bcdx^2 - 3bd^2)}{15d^2x^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-3*b*d^2 - b*c*d*x^2 - 5*a*d^2*x^2 + 2*b*c^2*x^4 - 5*a*c*d*x^4))/(15*d^2*x^4)

fricas [A] time = 0.43, size = 60, normalized size = 1.30

$$\frac{\left((2bc^2 - 5acd)x^4 - 3bd^2 - (bcd + 5ad^2)x^2\right) \sqrt{\frac{cx^2+d}{x^2}}}{15d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="fricas")

[Out] $1/15*((2*b*c^2 - 5*a*c*d)*x^4 - 3*b*d^2 - (b*c*d + 5*a*d^2)*x^2)*\sqrt{(c*x^2 + d)/x^2}/(d^2*x^4)$

giac [B] time = 0.92, size = 250, normalized size = 5.43

$$\frac{2(15(\sqrt{cx - \sqrt{cx^2 + d}})^5 ac^2 \operatorname{sgn}(x) + 30(\sqrt{cx - \sqrt{cx^2 + d}})^6 bc^2 \operatorname{sgn}(x) - 30(\sqrt{cx - \sqrt{cx^2 + d}})^6 ac^2 d \operatorname{sgn}(x) + 10(\sqrt{cx - \sqrt{cx^2 + d}})^4 bc^2 d \operatorname{sgn}(x) + 20(\sqrt{cx - \sqrt{cx^2 + d}})^4 ac^2 d^2 \operatorname{sgn}(x) + 10(\sqrt{cx - \sqrt{cx^2 + d}})^2 bc^2 d^2 \operatorname{sgn}(x) - 10(\sqrt{cx - \sqrt{cx^2 + d}})^2 ac^2 d^3 \operatorname{sgn}(x) - 2bc^2 d^3 \operatorname{sgn}(x) + 5ac^2 d^4 \operatorname{sgn}(x))}{15((\sqrt{cx - \sqrt{cx^2 + d}})^2 - d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="giac")`

[Out] $2/15*(15*(\sqrt{c}*x - \sqrt{c*x^2 + d})^8*a*c^{(3/2)}*\operatorname{sgn}(x) + 30*(\sqrt{c}*x - \sqrt{c*x^2 + d})^6*b*c^{(5/2)}*\operatorname{sgn}(x) - 30*(\sqrt{c}*x - \sqrt{c*x^2 + d})^6*a*c^{(3/2)}*d*\operatorname{sgn}(x) + 10*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*b*c^{(5/2)}*d*\operatorname{sgn}(x) + 20*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*a*c^{(3/2)}*d^2*\operatorname{sgn}(x) + 10*(\sqrt{c}*x - \sqrt{c*x^2 + d})^2*b*c^{(5/2)}*d^2*\operatorname{sgn}(x) - 10*(\sqrt{c}*x - \sqrt{c*x^2 + d})^2*a*c^{(3/2)}*d^3*\operatorname{sgn}(x) - 2*b*c^{(5/2)}*d^3*\operatorname{sgn}(x) + 5*a*c^{(3/2)}*d^4*\operatorname{sgn}(x))/(\sqrt{c}*x - \sqrt{c*x^2 + d})^2 - d)^5$

maple [A] time = 0.05, size = 48, normalized size = 1.04

$$-\frac{\sqrt{\frac{cx^2+d}{x^2}} (5ad^2x^2 - 2bcx^2 + 3bd)(cx^2 + d)}{15d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x)`

[Out] $-1/15*((c*x^2+d)/x^2)^(1/2)*(5*a*d*x^2-2*b*c*x^2+3*b*d)*(c*x^2+d)/d^2/x^4$

maxima [A] time = 0.72, size = 49, normalized size = 1.07

$$-\frac{1}{15}b\left(\frac{3\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2} - \frac{5\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right) - \frac{a\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="maxima")`

[Out] $-1/15*b*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3/2)*c/d^2) - 1/3*a*(c + d/x^2)^(3/2)/d$

mupad [B] time = 4.82, size = 91, normalized size = 1.98

$$\frac{\sqrt{c + \frac{d}{x^2}} (bc^2 + adc)}{5d^2} - \frac{b\sqrt{c + \frac{d}{x^2}}}{5x^4} - \frac{\sqrt{c + \frac{d}{x^2}} (5ad^2 + bcd)}{15d^2x^2} - \frac{c\sqrt{c + \frac{d}{x^2}} (8ad + bc)}{15d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^3, x)

[Out] ((c + d/x^2)^(1/2)*(b*c^2 + a*c*d))/(5*d^2) - (b*(c + d/x^2)^(1/2))/(5*x^4) - ((c + d/x^2)^(1/2)*(5*a*d^2 + b*c*d))/(15*d^2*x^2) - (c*(c + d/x^2)^(1/2)*(8*a*d + b*c))/(15*d^2)

sympy [A] time = 4.02, size = 58, normalized size = 1.26

$$\frac{a \begin{pmatrix} \frac{\sqrt{c}}{x^2} & \text{for } d = 0 \\ \frac{2\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise} \end{pmatrix}}{2} - \frac{b \left(-\frac{c\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}{3} + \frac{\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}}{5} \right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**3, x)

[Out] -a*Piecewise((sqrt(c)/x**2, Eq(d, 0)), (2*(c + d/x**2)**(3/2)/(3*d), True))/2 - b*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2

$$3.588 \quad \int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^5} dx$$

Optimal. Leaf size=74

$$\frac{\left(c + \frac{d}{x^2}\right)^{5/2} (2bc - ad)}{5d^3} - \frac{c \left(c + \frac{d}{x^2}\right)^{3/2} (bc - ad)}{3d^3} - \frac{b \left(c + \frac{d}{x^2}\right)^{7/2}}{7d^3}$$

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{\left(c + \frac{d}{x^2}\right)^{5/2} (2bc - ad)}{5d^3} - \frac{c \left(c + \frac{d}{x^2}\right)^{3/2} (bc - ad)}{3d^3} - \frac{b \left(c + \frac{d}{x^2}\right)^{7/2}}{7d^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]

[Out] -(c*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^3) + ((2*b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^3) - (b*(c + d/x^2)^(7/2))/(7*d^3)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^5} dx &= -\left(\frac{1}{2} \text{Subst} \left(\int x(a + bx) \sqrt{c + dx} dx, x, \frac{1}{x^2} \right)\right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \left(\frac{c(bc - ad) \sqrt{c + dx}}{d^2} + \frac{(-2bc + ad)(c + dx)^{3/2}}{d^2} + \frac{b(c + dx)^{5/2}}{d^2} \right) dx, x, \frac{1}{x^2} \right)\right) \\
&= -\frac{c(bc - ad) \left(c + \frac{d}{x^2}\right)^{3/2}}{3d^3} + \frac{(2bc - ad) \left(c + \frac{d}{x^2}\right)^{5/2}}{5d^3} - \frac{b \left(c + \frac{d}{x^2}\right)^{7/2}}{7d^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 0.93

$$\frac{\sqrt{c + \frac{d}{x^2}} (cx^2 + d) (7adx^2 (2cx^2 - 3d) + b(-8c^2x^4 + 12cdx^2 - 15d^2))}{105d^3x^6}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]

[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(7*a*d*x^2*(-3*d + 2*c*x^2) + b*(-15*d^2 + 12*c*d*x^2 - 8*c^2*x^4)))/(105*d^3*x^6)

IntegrateAlgebraic [A] time = 0.06, size = 90, normalized size = 1.22

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (14ac^2dx^6 - 7acd^2x^4 - 21ad^3x^2 - 8bc^3x^6 + 4bc^2dx^4 - 3bcd^2x^2 - 15bd^3)}{105d^3x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-15*b*d^3 - 3*b*c*d^2*x^2 - 21*a*d^3*x^2 + 4*b*c^2*d*x^4 - 7*a*c*d^2*x^4 - 8*b*c^3*x^6 + 14*a*c^2*d*x^6))/(105*d^3*x^6)

fricas [A] time = 0.43, size = 85, normalized size = 1.15

$$\frac{(2(4bc^3 - 7ac^2d)x^6 - (4bc^2d - 7acd^2)x^4 + 15bd^3 + 3(bcd^2 + 7ad^3)x^2) \sqrt{\frac{cx^2+d}{x^2}}}{105d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="fricas")

[Out] $-1/105*(2*(4*b*c^3 - 7*a*c^2*d)*x^6 - (4*b*c^2*d - 7*a*c*d^2)*x^4 + 15*b*d^3 + 3*(b*c*d^2 + 7*a*d^3)*x^2)*\text{sqrt}((c*x^2 + d)/x^2)/(d^3*x^6)$

giac [B] time = 1.38, size = 310, normalized size = 4.19

$$\frac{4 \left(\sqrt{c x - \sqrt{c^2 + d}} \right)^{10} a^5 \operatorname{sgn}(x) + 280 \left(\sqrt{c x - \sqrt{c^2 + d}} \right)^8 b c^3 \operatorname{sgn}(x) - 175 \left(\sqrt{c x - \sqrt{c^2 + d}} \right)^6 a^5 d \operatorname{sgn}(x) + 140 \left(\sqrt{c x - \sqrt{c^2 + d}} \right)^4 b c^3 d \operatorname{sgn}(x) + 70 \left(\sqrt{c x - \sqrt{c^2 + d}} \right)^2 a^5 d^2 \operatorname{sgn}(x) + 84 \left(\sqrt{c x - \sqrt{c^2 + d}} \right)^2 b c^3 d^2 \operatorname{sgn}(x) - 42 \left(\sqrt{c x - \sqrt{c^2 + d}} \right)^2 a^5 d^3 \operatorname{sgn}(x) - 28 \left(\sqrt{c x - \sqrt{c^2 + d}} \right)^2 b c^3 d^3 \operatorname{sgn}(x) + 49 \left(\sqrt{c x - \sqrt{c^2 + d}} \right)^2 a^5 d^4 \operatorname{sgn}(x) + 4 b c^3 d^4 \operatorname{sgn}(x) - 7 a^5 d^5 \operatorname{sgn}(x)}{105 \left(\sqrt{c x - \sqrt{c^2 + d}} \right)^2 d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="giac")`

[Out] $4/105*(105*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + d))^{10}*a*c^{(5/2)}*\text{sgn}(x) + 280*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + d))^{8}*b*c^{(7/2)}*\text{sgn}(x) - 175*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + d))^{6}*a*c^{(5/2)}*d*\text{sgn}(x) + 140*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + d))^{4}*b*c^{(7/2)}*d*\text{sgn}(x) + 70*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + d))^{2}*a*c^{(5/2)}*d^2*\text{sgn}(x) + 84*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + d))^{2}*b*c^{(7/2)}*d^2*\text{sgn}(x) - 42*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + d))^{2}*a*c^{(5/2)}*d^3*\text{sgn}(x) - 28*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + d))^{2}*b*c^{(7/2)}*d^3*\text{sgn}(x) + 49*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + d))^{2}*a*c^{(5/2)}*d^4*\text{sgn}(x) + 4*b*c^{(7/2)}*d^4*\text{sgn}(x) - 7*a*c^{(5/2)}*d^5*\text{sgn}(x))/((\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + d))^{2} - d)^7$

maple [A] time = 0.05, size = 70, normalized size = 0.95

$$\frac{\sqrt{\frac{c x^2 + d}{x^2}} (14 a c d x^4 - 8 b c^2 x^4 - 21 a d^2 x^2 + 12 b c d x^2 - 15 b d^2) (c x^2 + d)}{105 d^3 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x)`

[Out] $1/105*((c*x^2+d)/x^2)^{(1/2)}*(14*a*c*d*x^4-8*b*c^2*x^4-21*a*d^2*x^2+12*b*c*d*x^2-15*b*d^2)*(c*x^2+d)/d^3/x^6$

maxima [A] time = 0.60, size = 84, normalized size = 1.14

$$-\frac{1}{105} b \left(\frac{15 \left(c + \frac{d}{x^2} \right)^{\frac{7}{2}}}{d^3} - \frac{42 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}} c}{d^3} + \frac{35 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} c^2}{d^3} \right) - \frac{1}{15} a \left(\frac{3 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}}}{d^2} - \frac{5 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} c}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="maxima")`

[Out] $-1/105*b*(15*(c + d/x^2)^{(7/2)}/d^3 - 42*(c + d/x^2)^{(5/2)}*c/d^3 + 35*(c + d/x^2)^{(3/2)}*c^2/d^3) - 1/15*a*(3*(c + d/x^2)^{(5/2)}/d^2 - 5*(c + d/x^2)^{(3/2)}*c/d^2)$

mupad [B] time = 4.92, size = 126, normalized size = 1.70

$$\frac{2ac^2\sqrt{c+\frac{d}{x^2}}}{15d^2} - \frac{b\sqrt{c+\frac{d}{x^2}}}{7x^6} - \frac{a\sqrt{c+\frac{d}{x^2}}}{5x^4} - \frac{8bc^3\sqrt{c+\frac{d}{x^2}}}{105d^3} - \frac{ac\sqrt{c+\frac{d}{x^2}}}{15dx^2} - \frac{bc\sqrt{c+\frac{d}{x^2}}}{35dx^4} + \frac{4bc^2\sqrt{c+\frac{d}{x^2}}}{105d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^5, x)`

[Out] $(2*a*c^2*(c + d/x^2)^{(1/2)})/(15*d^2) - (b*(c + d/x^2)^{(1/2)})/(7*x^6) - (a*(c + d/x^2)^{(1/2)})/(5*x^4) - (8*b*c^3*(c + d/x^2)^{(1/2)})/(105*d^3) - (a*c*(c + d/x^2)^{(1/2)})/(15*d*x^2) - (b*c*(c + d/x^2)^{(1/2)})/(35*d*x^4) + (4*b*c^2*(c + d/x^2)^{(1/2)})/(105*d^2*x^2)$

sympy [A] time = 4.56, size = 78, normalized size = 1.05

$$\frac{a\left(\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} + \frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} - \frac{b\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} - \frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5} + \frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**5, x)`

[Out] $-a*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - b*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3$

$$3.589 \quad \int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^7} dx$$

Optimal. Leaf size=104

$$\frac{c^2 \left(c + \frac{d}{x^2}\right)^{3/2} (bc - ad)}{3d^4} + \frac{\left(c + \frac{d}{x^2}\right)^{7/2} (3bc - ad)}{7d^4} - \frac{c \left(c + \frac{d}{x^2}\right)^{5/2} (3bc - 2ad)}{5d^4} - \frac{b \left(c + \frac{d}{x^2}\right)^{9/2}}{9d^4}$$

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{c^2 \left(c + \frac{d}{x^2}\right)^{3/2} (bc - ad)}{3d^4} + \frac{\left(c + \frac{d}{x^2}\right)^{7/2} (3bc - ad)}{7d^4} - \frac{c \left(c + \frac{d}{x^2}\right)^{5/2} (3bc - 2ad)}{5d^4} - \frac{b \left(c + \frac{d}{x^2}\right)^{9/2}}{9d^4}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]

[Out] (c^2*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (c*(3*b*c - 2*a*d)*(c + d/x^2)^(5/2))/(5*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^4) - (b*(c + d/x^2)^(9/2))/(9*d^4)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^7} dx &= -\left(\frac{1}{2} \text{Subst} \left(\int x^2(a + bx) \sqrt{c + dx} dx, x, \frac{1}{x^2} \right)\right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \left(-\frac{c^2(bc - ad)\sqrt{c + dx}}{d^3} + \frac{c(3bc - 2ad)(c + dx)^{3/2}}{d^3} + \frac{(-3bc + ad)(c + dx)^{5/2}}{d^3} \right) dx \right)\right) \\
&= \frac{c^2(bc - ad) \left(c + \frac{d}{x^2}\right)^{3/2}}{3d^4} - \frac{c(3bc - 2ad) \left(c + \frac{d}{x^2}\right)^{5/2}}{5d^4} + \frac{(3bc - ad) \left(c + \frac{d}{x^2}\right)^{7/2}}{7d^4} - \frac{b \left(c + \frac{d}{x^2}\right)^{9/2}}{9d^4}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 79, normalized size = 0.76

$$\frac{\sqrt{c + \frac{d}{x^2}} \left(x^2 \left(\frac{cx^2}{d} + 1 \right) (8c^2x^4 - 12cdx^2 + 15d^2) (6bc - 9ad) - 105bd^2 (cx^2 + d) \right)}{945d^3x^8}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^7, x]

[Out] (Sqrt[c + d/x^2]*(-105*b*d^2*(d + c*x^2) + (6*b*c - 9*a*d)*x^2*(1 + (c*x^2)/d)*(15*d^2 - 12*c*d*x^2 + 8*c^2*x^4)))/(945*d^3*x^8)

IntegrateAlgebraic [A] time = 0.08, size = 114, normalized size = 1.10

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (-24ac^3dx^8 + 12ac^2d^2x^6 - 9acd^3x^4 - 45ad^4x^2 + 16bc^4x^8 - 8bc^3dx^6 + 6bc^2d^2x^4 - 5bcd^3x^2 - 35bd^4)}{315d^4x^8}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*Sqrt[c + d/x^2])/x^7, x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-35*b*d^4 - 5*b*c*d^3*x^2 - 45*a*d^4*x^2 + 6*b*c^2*d^2*x^4 - 9*a*c*d^3*x^4 - 8*b*c^3*d*x^6 + 12*a*c^2*d^2*x^6 + 16*b*c^4*x^8 - 24*a*c^3*d*x^8))/(315*d^4*x^8)

fricas [A] time = 0.48, size = 109, normalized size = 1.05

$$\frac{(8(2bc^4 - 3ac^3d)x^8 - 4(2bc^3d - 3ac^2d^2)x^6 - 35bd^4 + 3(2bc^2d^2 - 3acd^3)x^4 - 5(bcd^3 + 9ad^4)x^2) \sqrt{\frac{cx^2+d}{x^2}}}{315d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7, x, algorithm="fricas")

[Out] $1/315*(8*(2*b*c^4 - 3*a*c^3*d)*x^8 - 4*(2*b*c^3*d - 3*a*c^2*d^2)*x^6 - 35*b*d^4 + 3*(2*b*c^2*d^2 - 3*a*c*d^3)*x^4 - 5*(b*c*d^3 + 9*a*d^4)*x^2)*\sqrt{(c*x^2 + d)/x^2}/(d^4*x^8)$

giac [B] time = 2.06, size = 370, normalized size = 3.56

$$\frac{16(210(\sqrt{c-x\sqrt{c^2+d}})^{10}a^2\operatorname{sgn}(c)+630(\sqrt{c-x\sqrt{c^2+d}})^{10}b^2\operatorname{sgn}(c)-315(\sqrt{c-x\sqrt{c^2+d}})^{10}a^2b\operatorname{sgn}(c)+10(210(\sqrt{c-x\sqrt{c^2+d}})^{10}a^2b\operatorname{sgn}(c)-42(\sqrt{c-x\sqrt{c^2+d}})^{10}a^2b\operatorname{sgn}(c)-72(\sqrt{c-x\sqrt{c^2+d}})^{10}a^2b\operatorname{sgn}(c)+10(210(\sqrt{c-x\sqrt{c^2+d}})^{10}a^2b\operatorname{sgn}(c)+18(\sqrt{c-x\sqrt{c^2+d}})^{10}a^2b\operatorname{sgn}(c)-27(\sqrt{c-x\sqrt{c^2+d}})^{10}a^2b\operatorname{sgn}(c))}{315((\sqrt{c-x\sqrt{c^2+d}})^2-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="giac")`

[Out] $16/315*(210*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{12}*a*c^{(7/2)}*\operatorname{sgn}(x) + 630*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{10}*b*c^{(9/2)}*\operatorname{sgn}(x) - 315*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{10}*a*c^{(7/2)}*d*\operatorname{sgn}(x) + 378*(\sqrt{c}*x - \sqrt{c*x^2 + d})^8*b*c^{(9/2)}*d*\operatorname{sgn}(x) + 63*(\sqrt{c}*x - \sqrt{c*x^2 + d})^8*a*c^{(7/2)}*d^2*\operatorname{sgn}(x) + 168*(\sqrt{c}*x - \sqrt{c*x^2 + d})^6*b*c^{(9/2)}*d^2*\operatorname{sgn}(x) - 42*(\sqrt{c}*x - \sqrt{c*x^2 + d})^6*a*c^{(7/2)}*d^3*\operatorname{sgn}(x) - 72*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*b*c^{(9/2)}*d^3*\operatorname{sgn}(x) + 108*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*a*c^{(7/2)}*d^4*\operatorname{sgn}(x) + 18*(\sqrt{c}*x - \sqrt{c*x^2 + d})^2*b*c^{(9/2)}*d^4*\operatorname{sgn}(x) - 27*(\sqrt{c}*x - \sqrt{c*x^2 + d})^2*a*c^{(7/2)}*d^5*\operatorname{sgn}(x) - 2*b*c^{(9/2)}*d^5*\operatorname{sgn}(x) + 3*a*c^{(7/2)}*d^6*\operatorname{sgn}(x))/((\sqrt{c}*x - \sqrt{c*x^2 + d})^2 - d)^9$

maple [A] time = 0.05, size = 94, normalized size = 0.90

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (24a^2cx^6 - 16bc^3x^6 - 36acd^2x^4 + 24bc^2dx^4 + 45ad^3x^2 - 30bcd^2x^2 + 35bd^3)(cx^2 + d)}{315d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x)`

[Out] $-1/315*((c*x^2+d)/x^2)^(1/2)*(24*a*c^2*d*x^6-16*b*c^3*x^6-36*a*c*d^2*x^4+24*b*c^2*d*x^4+45*a*d^3*x^2-30*b*c*d^2*x^2+35*b*d^3)*(c*x^2+d)/d^4/x^8$

maxima [A] time = 0.57, size = 118, normalized size = 1.13

$$-\frac{1}{315}b\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^4}-\frac{135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^4}+\frac{189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^4}-\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3}{d^4}\right)-\frac{1}{105}a\left(\frac{15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^3}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="maxima")`

[Out] $-1/315*b*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)^(7/2)*c/d^4 + 189*(c + d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d^4) - 1/105*a*(15*(c + d$

$(\sqrt{c + \frac{d}{x^2}})^{7/2} / d^3 - 42(c + \frac{d}{x^2})^{5/2} * c / d^3 + 35(c + \frac{d}{x^2})^{3/2} * c^2 / d^3$
)

mupad [B] time = 5.22, size = 168, normalized size = 1.62

$$\frac{16bc^4\sqrt{c+\frac{d}{x^2}}}{315d^4} - \frac{b\sqrt{c+\frac{d}{x^2}}}{9x^8} - \frac{8ac^3\sqrt{c+\frac{d}{x^2}}}{105d^3} - \frac{a\sqrt{c+\frac{d}{x^2}}}{7x^6} - \frac{ac\sqrt{c+\frac{d}{x^2}}}{35dx^4} - \frac{bc\sqrt{c+\frac{d}{x^2}}}{63dx^6} + \frac{4ac^2\sqrt{c+\frac{d}{x^2}}}{105d^2x^2} + \frac{2bc^2\sqrt{c+\frac{d}{x^2}}}{105d^2x^4} - \frac{8bc^3\sqrt{c+\frac{d}{x^2}}}{315d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^7, x)

[Out] $(16*b*c^4*(c + d/x^2)^{(1/2)})/(315*d^4) - (b*(c + d/x^2)^{(1/2)})/(9*x^8) - (8*a*c^3*(c + d/x^2)^{(1/2)})/(105*d^3) - (a*(c + d/x^2)^{(1/2)})/(7*x^6) - (a*c*(c + d/x^2)^{(1/2)})/(35*d*x^4) - (b*c*(c + d/x^2)^{(1/2)})/(63*d*x^6) + (4*a*c^2*(c + d/x^2)^{(1/2)})/(105*d^2*x^2) + (2*b*c^2*(c + d/x^2)^{(1/2)})/(105*d^2*x^4) - (8*b*c^3*(c + d/x^2)^{(1/2)})/(315*d^3*x^2)$

sympy [A] time = 5.21, size = 112, normalized size = 1.08

$$\frac{a \left(\frac{c^2 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}}}{3} - \frac{2c \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}}}{5} + \frac{\left(c + \frac{d}{x^2} \right)^{\frac{7}{2}}}{7} \right)}{d^3} - \frac{b \left(-\frac{c^3 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}}}{3} + \frac{3c^2 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}}}{5} - \frac{3c \left(c + \frac{d}{x^2} \right)^{\frac{7}{2}}}{7} + \frac{\left(c + \frac{d}{x^2} \right)^{\frac{9}{2}}}{9} \right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**7, x)

[Out] $-a*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3 - b*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4$

$$3.590 \quad \int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^9} dx$$

Optimal. Leaf size=134

$$-\frac{c^3 \left(c + \frac{d}{x^2}\right)^{3/2} (bc - ad)}{3d^5} + \frac{c^2 \left(c + \frac{d}{x^2}\right)^{5/2} (4bc - 3ad)}{5d^5} + \frac{\left(c + \frac{d}{x^2}\right)^{9/2} (4bc - ad)}{9d^5} - \frac{3c \left(c + \frac{d}{x^2}\right)^{7/2} (2bc - ad)}{7d^5} - \frac{b \left(c + \frac{d}{x^2}\right)^{11/2}}{11d^5}$$

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{c^2 \left(c + \frac{d}{x^2}\right)^{5/2} (4bc - 3ad)}{5d^5} - \frac{c^3 \left(c + \frac{d}{x^2}\right)^{3/2} (bc - ad)}{3d^5} + \frac{\left(c + \frac{d}{x^2}\right)^{9/2} (4bc - ad)}{9d^5} - \frac{3c \left(c + \frac{d}{x^2}\right)^{7/2} (2bc - ad)}{7d^5} - \frac{b \left(c + \frac{d}{x^2}\right)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]

[Out] $-(c^3(b*c - a*d)*(c + d/x^2)^{(3/2)})/(3*d^5) + (c^2*(4*b*c - 3*a*d)*(c + d/x^2)^{(5/2)})/(5*d^5) - (3*c*(2*b*c - a*d)*(c + d/x^2)^{(7/2)})/(7*d^5) + ((4*b*c - a*d)*(c + d/x^2)^{(9/2)})/(9*d^5) - (b*(c + d/x^2)^{(11/2)})/(11*d^5)$

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^9} dx &= -\left(\frac{1}{2} \text{Subst} \left(\int x^3(a + bx) \sqrt{c + dx} dx, x, \frac{1}{x^2} \right)\right) \\ &= -\left(\frac{1}{2} \text{Subst} \left(\int \left(\frac{c^3(bc - ad) \sqrt{c + dx}}{d^4} - \frac{c^2(4bc - 3ad)(c + dx)^{3/2}}{d^4} + \frac{3c(2bc - ad)(c + dx)^{5/2}}{d^4} \right. \right. \right. \\ &\quad \left. \left. \left. - \frac{c^3(bc - ad) \left(c + \frac{d}{x^2}\right)^{3/2}}{3d^5} + \frac{c^2(4bc - 3ad) \left(c + \frac{d}{x^2}\right)^{5/2}}{5d^5} - \frac{3c(2bc - ad) \left(c + \frac{d}{x^2}\right)^{7/2}}{7d^5} + \dots \right) \right) \end{aligned}$$

Mathematica [A] time = 0.08, size = 90, normalized size = 0.67

$$\frac{\sqrt{c + \frac{d}{x^2}} \left(x^2 \left(\frac{cx^2}{d} + 1 \right) (-16c^3x^6 + 24c^2dx^4 - 30cd^2x^2 + 35d^3) (8bc - 11ad) - 315bd^3 (cx^2 + d) \right)}{3465d^4x^{10}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^9, x]

[Out] (Sqrt[c + d/x^2]*(-315*b*d^3*(d + c*x^2) + (8*b*c - 11*a*d)*x^2*(1 + (c*x^2)/d)*(35*d^3 - 30*c*d^2*x^2 + 24*c^2*d*x^4 - 16*c^3*x^6)))/(3465*d^4*x^10)

IntegrateAlgebraic [A] time = 0.08, size = 138, normalized size = 1.03

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (176ac^4dx^{10} - 88ac^3d^2x^8 + 66ac^2d^3x^6 - 55acd^4x^4 - 385ad^5x^2 - 128bc^5x^{10} + 64bc^4dx^8 - 48bc^3d^2x^6 + 40bc^2d^3x^4 - 35bcd^4x^2 - 315bd^5)}{3465d^5x^{10}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*Sqrt[c + d/x^2])/x^9, x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-315*b*d^5 - 35*b*c*d^4*x^2 - 385*a*d^5*x^2 + 40*b*c^2*d^3*x^4 - 55*a*c*d^4*x^4 - 48*b*c^3*d^2*x^6 + 66*a*c^2*d^3*x^6 + 64*b*c^4*d*x^8 - 88*a*c^3*d^2*x^8 - 128*b*c^5*x^{10} + 176*a*c^4*d*x^{10}))/((3465*d^5*x^{10}))

fricas [A] time = 0.53, size = 133, normalized size = 0.99

$$\frac{(16(8bc^5 - 11ac^4d)x^{10} - 8(8bc^4d - 11ac^3d^2)x^8 + 6(8bc^3d^2 - 11ac^2d^3)x^6 + 315bd^5 - 5(8bc^2d^3 - 11acd^4)x^4 + 35(bcd^4 + 11ad^5)x^2) \sqrt{\frac{cx^2+d}{x^2}}}{3465d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9, x, algorithm="fricas")

[Out] $-1/3465*(16*(8*b*c^5 - 11*a*c^4*d)*x^{10} - 8*(8*b*c^4*d - 11*a*c^3*d^2)*x^8 + 6*(8*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 + 315*b*d^5 - 5*(8*b*c^2*d^3 - 11*a*c*d^4)*x^4 + 35*(b*c*d^4 + 11*a*d^5)*x^2)*\sqrt{(c*x^2 + d)/x^2}/(d^5*x^{10})$

giac [B] time = 2.98, size = 430, normalized size = 3.21

$\frac{35\sqrt{(c*x^2+d)/x^2}\int(16*(8*b*c^5-11*a*c^4*d)*x^{10}-8*(8*b*c^4*d-11*a*c^3*d^2)*x^8+6*(8*b*c^3*d^2-11*a*c^2*d^3)*x^6+315*b*d^5-5*(8*b*c^2*d^3-11*a*c*d^4)*x^4+35*(b*c*d^4+11*a*d^5)*x^2)\sqrt{(c*x^2+d)/x^2}/(d^5*x^{10})dx}{3465\sqrt{(c*x^2+d)/x^2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="giac")`

[Out] $32/3465*(3465*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{14}*a*c^{(9/2)}*\text{sgn}(x) + 11088*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{12}*b*c^{(11/2)}*\text{sgn}(x) - 4851*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{12}*a*c^{(9/2)}*d*\text{sgn}(x) + 7392*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{10}*b*c^{(11/2)}*d*\text{sgn}(x) + 231*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{10}*a*c^{(9/2)}*d^2*\text{sgn}(x) + 2640*(\sqrt{c}*x - \sqrt{c*x^2 + d})^8*b*c^{(11/2)}*d^2*\text{sgn}(x) - 165*(\sqrt{c}*x - \sqrt{c*x^2 + d})^8*a*c^{(9/2)}*d^3*\text{sgn}(x) - 1320*(\sqrt{c}*x - \sqrt{c*x^2 + d})^6*b*c^{(11/2)}*d^3*\text{sgn}(x) + 1815*(\sqrt{c}*x - \sqrt{c*x^2 + d})^6*a*c^{(9/2)}*d^4*\text{sgn}(x) + 440*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*b*c^{(11/2)}*d^4*\text{sgn}(x) - 605*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*a*c^{(9/2)}*d^5*\text{sgn}(x) - 88*(\sqrt{c}*x - \sqrt{c*x^2 + d})^2*b*c^{(11/2)}*d^5*\text{sgn}(x) + 121*(\sqrt{c}*x - \sqrt{c*x^2 + d})^2*a*c^{(9/2)}*d^6*\text{sgn}(x) + 8*b*c^{(11/2)}*d^6*\text{sgn}(x) - 11*a*c^{(9/2)}*d^7*\text{sgn}(x))/((\sqrt{c}*x - \sqrt{c*x^2 + d})^2 - d)^{11}$

maple [A] time = 0.05, size = 118, normalized size = 0.88

$$\frac{\sqrt{\frac{c x^2+d}{x^2}} (176 a^3 d x^8 - 128 b c^4 x^8 - 264 a c^2 d^2 x^6 + 192 b c^3 d x^6 + 330 a c d^3 x^4 - 240 b c^2 d^2 x^4 - 385 a d^4 x^2 + 280 b c d^3 x^2 - 315 b d^4) (c x^2 + d)}{3465 d^5 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x)`

[Out] $1/3465*((c*x^2+d)/x^2)^{(1/2)}*(176*a*c^3*d*x^8-128*b*c^4*x^8-264*a*c^2*d^2*x^6+192*b*c^3*d*x^6+330*a*c*d^3*x^4-240*b*c^2*d^2*x^4-385*a*d^4*x^2+280*b*c*d^3*x^2-315*b*d^4)*(c*x^2+d)/d^5/x^{10}$

maxima [A] time = 0.47, size = 152, normalized size = 1.13

$$-\frac{1}{3465} b \left(\frac{315 \left(c + \frac{d}{x^2} \right)^{\frac{11}{2}}}{d^5} - \frac{1540 \left(c + \frac{d}{x^2} \right)^{\frac{9}{2}} c}{d^5} + \frac{2970 \left(c + \frac{d}{x^2} \right)^{\frac{7}{2}} c^2}{d^5} - \frac{2772 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}} c^3}{d^5} + \frac{1155 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} c^4}{d^5} \right) - \frac{1}{315} a \left(\frac{35 \left(c + \frac{d}{x^2} \right)^{\frac{9}{2}}}{d^4} - \frac{135 \left(c + \frac{d}{x^2} \right)^{\frac{7}{2}} c}{d^4} + \frac{189 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}} c^2}{d^4} - \frac{105 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} c^3}{d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="maxima")`

[Out] $-1/3465*b*(315*(c + d/x^2)^{(11/2)}/d^5 - 1540*(c + d/x^2)^{(9/2)}*c/d^5 + 2970*(c + d/x^2)^{(7/2)}*c^2/d^5 - 2772*(c + d/x^2)^{(5/2)}*c^3/d^5 + 1155*(c + d/x^2)^{(3/2)}*c^4/d^5) - 1/315*a*(35*(c + d/x^2)^{(9/2)}/d^4 - 135*(c + d/x^2)^{(7/2)}*c/d^4 + 189*(c + d/x^2)^{(5/2)}*c^2/d^4 - 105*(c + d/x^2)^{(3/2)}*c^3/d^4)$

mupad [B] time = 5.61, size = 210, normalized size = 1.57

$$\frac{16ac^4\sqrt{c+\frac{d}{x^2}}}{315d^4} - \frac{b\sqrt{c+\frac{d}{x^2}}}{11x^{10}} - \frac{a\sqrt{c+\frac{d}{x^2}}}{9x^8} - \frac{128bc^5\sqrt{c+\frac{d}{x^2}}}{3465d^5} - \frac{ac\sqrt{c+\frac{d}{x^2}}}{63dx^6} - \frac{bc\sqrt{c+\frac{d}{x^2}}}{99dx^8} + \frac{2ac^2\sqrt{c+\frac{d}{x^2}}}{105d^2x^4} - \frac{8ac^3\sqrt{c+\frac{d}{x^2}}}{315d^3x^2} + \frac{8bc^2\sqrt{c+\frac{d}{x^2}}}{693d^2x^6} - \frac{16bc^3\sqrt{c+\frac{d}{x^2}}}{1155d^3x^4} + \frac{64bc^4\sqrt{c+\frac{d}{x^2}}}{3465d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^9, x)`

[Out] $(16*a*c^4*(c + d/x^2)^{(1/2)}/(315*d^4) - (b*(c + d/x^2)^{(1/2)})/(11*x^{10}) - (a*(c + d/x^2)^{(1/2)})/(9*x^8) - (128*b*c^5*(c + d/x^2)^{(1/2)})/(3465*d^5) - (a*c*(c + d/x^2)^{(1/2)})/(63*d*x^6) - (b*c*(c + d/x^2)^{(1/2)})/(99*d*x^8) + (2*a*c^2*(c + d/x^2)^{(1/2)})/(105*d^2*x^4) - (8*a*c^3*(c + d/x^2)^{(1/2)})/(315*d^3*x^2) + (8*b*c^2*(c + d/x^2)^{(1/2)})/(693*d^2*x^6) - (16*b*c^3*(c + d/x^2)^{(1/2)})/(1155*d^3*x^4) + (64*b*c^4*(c + d/x^2)^{(1/2)})/(3465*d^4*x^2)$

sympy [A] time = 5.86, size = 146, normalized size = 1.09

$$\frac{a \left(-\frac{c^3 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}}}{3} + \frac{3c^2 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}}}{5} - \frac{3c \left(c + \frac{d}{x^2} \right)^{\frac{7}{2}}}{7} + \frac{\left(c + \frac{d}{x^2} \right)^{\frac{9}{2}}}{9} \right)}{d^4} - \frac{b \left(\frac{c^4 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}}}{3} - \frac{4c^3 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}}}{5} + \frac{6c^2 \left(c + \frac{d}{x^2} \right)^{\frac{7}{2}}}{7} - \frac{4c \left(c + \frac{d}{x^2} \right)^{\frac{9}{2}}}{9} + \frac{\left(c + \frac{d}{x^2} \right)^{\frac{11}{2}}}{11} \right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**9, x)`

[Out] $-a*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - b*(c**4*(c + d/x**2)**(3/2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5$

$$3.591 \quad \int \left(a + \frac{b}{x^2} \right) \sqrt{c + \frac{d}{x^2}} x^{10} dx$$

Optimal. Leaf size=150

$$-\frac{16d^3x^3 \left(c + \frac{d}{x^2} \right)^{3/2} (11bc - 8ad)}{3465c^5} + \frac{8d^2x^5 \left(c + \frac{d}{x^2} \right)^{3/2} (11bc - 8ad)}{1155c^4} - \frac{2dx^7 \left(c + \frac{d}{x^2} \right)^{3/2} (11bc - 8ad)}{231c^3} + \frac{x^9 \left(c + \frac{d}{x^2} \right)^{3/2} (11bc - 8ad)}{99c^2}$$

Rubi [A] time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {453, 271, 264}

$$\frac{8d^2x^5 \left(c + \frac{d}{x^2} \right)^{3/2} (11bc - 8ad)}{1155c^4} - \frac{16d^3x^3 \left(c + \frac{d}{x^2} \right)^{3/2} (11bc - 8ad)}{3465c^5} + \frac{x^9 \left(c + \frac{d}{x^2} \right)^{3/2} (11bc - 8ad)}{99c^2} - \frac{2dx^7 \left(c + \frac{d}{x^2} \right)^{3/2} (11bc - 8ad)}{231c^3} + \frac{ax^{11} \left(c + \frac{d}{x^2} \right)^{3/2}}{11c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]

[Out] (-16*d^3*(11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^3)/(3465*c^5) + (8*d^2*(11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^5)/(1155*c^4) - (2*d*(11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^7)/(231*c^3) + ((11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^9)/(99*c^2) + (a*(c + d/x^2)^(3/2)*x^11)/(11*c)

Rule 264

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

Rule 271

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m+1)*(a + b*x^n)^(p+1))/(a*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*(m+1)), Int[x^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[(m+1)/n + p + 1] && NeQ[m, -1]

Rule 453

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m+1)*(a + b*x^n)^(p+1))/(a*e*(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (

LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x^{10} dx &= \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^{11}}{11c} + \frac{(11bc - 8ad) \int \sqrt{c + \frac{d}{x^2}} x^8 dx}{11c} \\
 &= \frac{(11bc - 8ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^9}{99c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^{11}}{11c} - \frac{(2d(11bc - 8ad)) \int \sqrt{c + \frac{d}{x^2}} x^6 dx}{33c^2} \\
 &= -\frac{2d(11bc - 8ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^7}{231c^3} + \frac{(11bc - 8ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^9}{99c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^{11}}{11c} + \dots \\
 &= \frac{8d^2(11bc - 8ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^5}{1155c^4} - \frac{2d(11bc - 8ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^7}{231c^3} + \frac{(11bc - 8ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^9}{99c^2} + \dots \\
 &= -\frac{16d^3(11bc - 8ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^3}{3465c^5} + \frac{8d^2(11bc - 8ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^5}{1155c^4} - \frac{2d(11bc - 8ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^7}{99c^2} + \dots
 \end{aligned}$$

Mathematica [A] time = 0.08, size = 108, normalized size = 0.72

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d) (a(315c^4x^8 - 280c^3dx^6 + 240c^2d^2x^4 - 192cd^3x^2 + 128d^4) + 11bc(35c^3x^6 - 30c^2dx^4 + 24cd^2x^2 - 16d^3))}{3465c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^10, x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(11*b*c*(-16*d^3 + 24*c*d^2*x^2 - 30*c^2*d*x^4 + 35*c^3*x^6) + a*(128*d^4 - 192*c*d^3*x^2 + 240*c^2*d^2*x^4 - 280*c^3*d*x^6 + 315*c^4*x^8)))/(3465*c^5)

IntegrateAlgebraic [A] time = 0.10, size = 112, normalized size = 0.75

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d) (315ac^4x^8 - 280ac^3dx^6 + 240ac^2d^2x^4 - 192acd^3x^2 + 128ad^4 + 385bc^4x^6 - 330bc^3dx^4 + 264bc^2d^2x^2 - 176bcd^3)}{3465c^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x^10, x]

[Out] $(\sqrt{c + d/x^2} * x * (d + c*x^2) * (-176*b*c*d^3 + 128*a*d^4 + 264*b*c^2*d^2*x^2 - 192*a*c*d^3*x^2 - 330*b*c^3*d*x^4 + 240*a*c^2*d^2*x^4 + 385*b*c^4*x^6 - 280*a*c^3*d*x^6 + 315*a*c^4*x^8)) / (3465*c^5)$

fricas [A] time = 0.43, size = 131, normalized size = 0.87

$$\frac{(315ac^5x^{11} + 35(11bc^5 + ac^4d)x^9 + 5(11bc^4d - 8ac^3d^2)x^7 - 6(11bc^3d^2 - 8ac^2d^3)x^5 + 8(11bc^2d^3 - 8acd^4)x^3 - 16(11bcd^4 - 8ad^5)x)\sqrt{\frac{cx^2+d}{x^2}}}{3465c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] $1/3465*(315*a*c^5*x^{11} + 35*(11*b*c^5 + a*c^4*d)*x^9 + 5*(11*b*c^4*d - 8*a*c^3*d^2)*x^7 - 6*(11*b*c^3*d^2 - 8*a*c^2*d^3)*x^5 + 8*(11*b*c^2*d^3 - 8*a*c*d^4)*x^3 - 16*(11*b*c*d^4 - 8*a*d^5)*x)*\sqrt{(c*x^2 + d)/x^2}/c^5$

giac [A] time = 0.22, size = 175, normalized size = 1.17

$$\frac{16(11bd^2 - 8ad^3)\operatorname{sgn}(x)}{3465c^5} + \frac{315(c^2 + d)^{\frac{11}{2}}\operatorname{sgn}(x) + 385(c^2 + d)^{\frac{9}{2}}bc\operatorname{sgn}(x) - 1540(c^2 + d)^{\frac{7}{2}}ad\operatorname{sgn}(x) - 1485(c^2 + d)^{\frac{5}{2}}bcd\operatorname{sgn}(x) + 2970(c^2 + d)^{\frac{3}{2}}ad^2\operatorname{sgn}(x) + 2079(c^2 + d)^{\frac{1}{2}}bcd^2\operatorname{sgn}(x) - 2772(c^2 + d)^{\frac{1}{2}}ad^3\operatorname{sgn}(x) - 1155(c^2 + d)^{\frac{1}{2}}bcd^3\operatorname{sgn}(x) + 1155(c^2 + d)^{\frac{1}{2}}ad^4\operatorname{sgn}(x)}{3465c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] $16/3465*(11*b*c*d^{(9/2)} - 8*a*d^{(11/2)})*\operatorname{sgn}(x)/c^5 + 1/3465*(315*(c*x^2 + d)^{(11/2)}*a*\operatorname{sgn}(x) + 385*(c*x^2 + d)^{(9/2)}*b*c*\operatorname{sgn}(x) - 1540*(c*x^2 + d)^{(9/2)}*a*d*\operatorname{sgn}(x) - 1485*(c*x^2 + d)^{(7/2)}*b*c*d*\operatorname{sgn}(x) + 2970*(c*x^2 + d)^{(7/2)}*a*d^2*\operatorname{sgn}(x) + 2079*(c*x^2 + d)^{(5/2)}*b*c*d^2*\operatorname{sgn}(x) - 2772*(c*x^2 + d)^{(5/2)}*a*d^3*\operatorname{sgn}(x) - 1155*(c*x^2 + d)^{(3/2)}*b*c*d^3*\operatorname{sgn}(x) + 1155*(c*x^2 + d)^{(3/2)}*a*d^4*\operatorname{sgn}(x))/c^5$

maple [A] time = 0.05, size = 113, normalized size = 0.75

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (315ax^8c^4 - 280a^3d^3x^6 + 385b^2c^4x^6 + 240a^2c^2d^2x^4 - 330b^3c^3d^3x^4 - 192ac^3d^3x^2 + 264b^2c^2d^2x^2 + 128a^4d^4 - 176bc^3d^3)(cx^2 + d)x}{3465c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x)

[Out] $1/3465*((c*x^2+d)/x^2)^(1/2)*x*(315*a*c^4*x^8-280*a*c^3*d*x^6+385*b*c^4*x^6+240*a*c^2*d^2*x^4-330*b*c^3*d*x^4-192*a*c*d^3*x^2+264*b*c^2*d^2*x^2+128*a*d^4-176*b*c*d^3)*(c*x^2+d)/c^5$

maxima [A] time = 0.64, size = 158, normalized size = 1.05

$$\frac{\left(35\left(c + \frac{d}{x^2}\right)^{\frac{9}{2}}x^9 - 135\left(c + \frac{d}{x^2}\right)^{\frac{7}{2}}dx^7 + 189\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5 - 105\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}d^3x^3\right)b}{315c^4} + \frac{\left(315\left(c + \frac{d}{x^2}\right)^{\frac{11}{2}}x^{11} - 1540\left(c + \frac{d}{x^2}\right)^{\frac{9}{2}}dx^9 + 2970\left(c + \frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7 - 2772\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5 + 1155\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}d^4x^3\right)a}{3465c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] $\frac{1}{315}*(35*(c + d/x^2)^{(9/2)}*x^9 - 135*(c + d/x^2)^{(7/2)}*d*x^7 + 189*(c + d/x^2)^{(5/2)}*d^2*x^5 - 105*(c + d/x^2)^{(3/2)}*d^3*x^3)*b/c^4 + \frac{1}{3465}*(315*(c + d/x^2)^{(11/2)}*x^{11} - 1540*(c + d/x^2)^{(9/2)}*d*x^9 + 2970*(c + d/x^2)^{(7/2)}*d^2*x^7 - 2772*(c + d/x^2)^{(5/2)}*d^3*x^5 + 1155*(c + d/x^2)^{(3/2)}*d^4*x^3)*a/c^5$

mupad [B] time = 4.59, size = 117, normalized size = 0.78

$$\sqrt{c + \frac{d}{x^2}} \left(\frac{ax^{11}}{11} + \frac{x(128ad^5 - 176bcd^4)}{3465c^5} + \frac{x^9(385bc^5 + 35ad^4)}{3465c^5} - \frac{dx^7(8ad - 11bc)}{693c^2} + \frac{2d^2x^5(8ad - 11bc)}{1155c^3} - \frac{8d^3x^3(8ad - 11bc)}{3465c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^10*(a + b/x^2)*(c + d/x^2)^(1/2),x)

[Out] $(c + d/x^2)^{(1/2)}*((a*x^{11})/11 + (x*(128*a*d^5 - 176*b*c*d^4))/(3465*c^5) + (x^9*(385*b*c^5 + 35*a*c^4*d))/(3465*c^5) - (d*x^7*(8*a*d - 11*b*c))/(693*c^2) + (2*d^2*x^5*(8*a*d - 11*b*c))/(1155*c^3) - (8*d^3*x^3*(8*a*d - 11*b*c))/(3465*c^4))$

sympy [B] time = 6.44, size = 1386, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*x**10*(c+d/x**2)**(1/2),x)

[Out] $315*a*c**9*d**(33/2)*x**18*\sqrt{c*x**2/d + 1}/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1295*a*c**8*d**(35/2)*x**16*\sqrt{c*x**2/d + 1}/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1990*a*c**7*d**(37/2)*x**14*\sqrt{c*x**2/d + 1}/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**6*d**(39/2)*x**12*\sqrt{c*x**2/d + 1}/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 343*a*c**5*d**(41/2)*x**10*\sqrt{c*x**2/d + 1}/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c**4*d**(43/2)*x**8*\sqrt{c*x**2/d + 1}/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 280*a*c**3*d**(45/2)*x**6*\sqrt{c*x**2/d + 1}/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 560*a*c**2*d**(47/2)*x**4*\sqrt{c*x**2/d + 1}/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20)$

$$\begin{aligned}
& 0*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} + 3465*c^{**5}*d^{**20}) \\
& + 448*a*c*d^{**49/2}*x^{**2}*sqrt(c*x^{**2}/d + 1)/(3465*c^{**9}*d^{**16}*x^{**8} \\
& + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} + \\
& 3465*c^{**5}*d^{**20}) + 128*a*d^{**51/2}*sqrt(c*x^{**2}/d + 1)/(3465*c^{**9}*d^{**16}*x^{**8} \\
& + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} + \\
& 3465*c^{**5}*d^{**20}) + 35*b*c^{**7}*d^{**19/2}*x^{**14}*sqrt(c*x^{**2}/d + 1)/(315*c^{**7}*d^{**9}*x^{**6} \\
& + 945*c^{**6}*d^{**10}*x^{**4} + 945*c^{**5}*d^{**11}*x^{**2} + 315*c^{**4}*d^{**12}) + 110*b*c^{**6}*d^{**21/2} \\
& *x^{**12}*sqrt(c*x^{**2}/d + 1)/(315*c^{**7}*d^{**9}*x^{**6} + 945*c^{**6}*d^{**10}*x^{**4} + 945*c^{**5}*d^{**11} \\
& *x^{**2} + 315*c^{**4}*d^{**12}) + 114*b*c^{**5}*d^{**23/2}*x^{**10}*sqrt(c*x^{**2}/d + 1)/(315*c^{**7}*d^{**9} \\
& *x^{**6} + 945*c^{**6}*d^{**10}*x^{**4} + 945*c^{**5}*d^{**11}*x^{**2} + 315*c^{**4}*d^{**12}) + 40*b*c^{**4}*d^{**25/2} \\
& *x^{**8}*sqrt(c*x^{**2}/d + 1)/(315*c^{**7}*d^{**9}*x^{**6} + 945*c^{**6}*d^{**10}*x^{**4} + 945*c^{**5}*d^{**11} \\
& *x^{**2} + 315*c^{**4}*d^{**12}) - 5*b*c^{**3}*d^{**27/2}*x^{**6}*sqrt(c*x^{**2}/d + 1)/(315*c^{**7}*d^{**9} \\
& *x^{**6} + 945*c^{**6}*d^{**10}*x^{**4} + 945*c^{**5}*d^{**11}*x^{**2} + 315*c^{**4}*d^{**12}) - 30*b*c^{**2}*d^{**29/2} \\
& *x^{**4}*sqrt(c*x^{**2}/d + 1)/(315*c^{**7}*d^{**9}*x^{**6} + 945*c^{**6}*d^{**10}*x^{**4} + 945*c^{**5}*d^{**11} \\
& *x^{**2} + 315*c^{**4}*d^{**12}) - 40*b*c*d^{**31/2}*x^{**2}*sqrt(c*x^{**2}/d + 1)/(315*c^{**7}*d^{**9} \\
& *x^{**6} + 945*c^{**6}*d^{**10}*x^{**4} + 945*c^{**5}*d^{**11}*x^{**2} + 315*c^{**4}*d^{**12}) - 16*b*d^{**33/2} \\
& *sqrt(c*x^{**2}/d + 1)/(315*c^{**7}*d^{**9}*x^{**6} + 945*c^{**6}*d^{**10}*x^{**4} + 945*c^{**5}*d^{**11} \\
& *x^{**2} + 315*c^{**4}*d^{**12})
\end{aligned}$$

$$3.592 \quad \int \left(a + \frac{b}{x^2} \right) \sqrt{c + \frac{d}{x^2}} x^8 dx$$

Optimal. Leaf size=117

$$\frac{8d^2x^3 \left(c + \frac{d}{x^2} \right)^{3/2} (3bc - 2ad)}{315c^4} - \frac{4dx^5 \left(c + \frac{d}{x^2} \right)^{3/2} (3bc - 2ad)}{105c^3} + \frac{x^7 \left(c + \frac{d}{x^2} \right)^{3/2} (3bc - 2ad)}{21c^2} + \frac{ax^9 \left(c + \frac{d}{x^2} \right)^{3/2}}{9c}$$

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {453, 271, 264}

$$\frac{8d^2x^3 \left(c + \frac{d}{x^2} \right)^{3/2} (3bc - 2ad)}{315c^4} + \frac{x^7 \left(c + \frac{d}{x^2} \right)^{3/2} (3bc - 2ad)}{21c^2} - \frac{4dx^5 \left(c + \frac{d}{x^2} \right)^{3/2} (3bc - 2ad)}{105c^3} + \frac{ax^9 \left(c + \frac{d}{x^2} \right)^{3/2}}{9c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]

[Out] (8*d^2*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(315*c^4) - (4*d*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^5)/(105*c^3) + ((3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^7)/(21*c^2) + (a*(c + d/x^2)^(3/2)*x^9)/(9*c)

Rule 264

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 271

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

Rule 453

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x^8 dx &= \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^9}{9c} + \frac{(9bc - 6ad) \int \sqrt{c + \frac{d}{x^2}} x^6 dx}{9c} \\
&= \frac{(3bc - 2ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^7}{21c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^9}{9c} - \frac{(4d(3bc - 2ad)) \int \sqrt{c + \frac{d}{x^2}} x^4 dx}{21c^2} \\
&= -\frac{4d(3bc - 2ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^5}{105c^3} + \frac{(3bc - 2ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^7}{21c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^9}{9c} + \frac{(8d^2(3bc - 2ad)) \int \sqrt{c + \frac{d}{x^2}} x^2 dx}{21c^2} \\
&= \frac{8d^2(3bc - 2ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^3}{315c^4} - \frac{4d(3bc - 2ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^5}{105c^3} + \frac{(3bc - 2ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^7}{21c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^9}{9c}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 86, normalized size = 0.74

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d) (a (35c^3x^6 - 30c^2dx^4 + 24cd^2x^2 - 16d^3) + 3bc (15c^2x^4 - 12cdx^2 + 8d^2))}{315c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(3*b*c*(8*d^2 - 12*c*d*x^2 + 15*c^2*x^4) + a*(-16*d^3 + 24*c*d^2*x^2 - 30*c^2*d*x^4 + 35*c^3*x^6)))/(315*c^4)

IntegrateAlgebraic [A] time = 0.09, size = 88, normalized size = 0.75

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d) (35ac^3x^6 - 30ac^2dx^4 + 24acd^2x^2 - 16ad^3 + 45bc^3x^4 - 36bc^2dx^2 + 24bcd^2)}{315c^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(24*b*c*d^2 - 16*a*d^3 - 36*b*c^2*d*x^2 + 24*a*c*d^2*x^2 + 45*b*c^3*x^4 - 30*a*c^2*d*x^4 + 35*a*c^3*x^6))/(315*c^4)

fricas [A] time = 0.42, size = 107, normalized size = 0.91

$$\frac{(35ac^4x^9 + 5(9bc^4 + ac^3d)x^7 + 3(3bc^3d - 2ac^2d^2)x^5 - 4(3bc^2d^2 - 2acd^3)x^3 + 8(3bcd^3 - 2ad^4)x) \sqrt{\frac{cx^2+d}{x^2}}}{315c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] 1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + a*c^3*d)*x^7 + 3*(3*b*c^3*d - 2*a*c^2*d^2)*x^5 - 4*(3*b*c^2*d^2 - 2*a*c*d^3)*x^3 + 8*(3*b*c*d^3 - 2*a*d^4)*x)*sqrt((c*x^2 + d)/x^2)/c^4

giac [A] time = 0.17, size = 140, normalized size = 1.20

$$\frac{8(3bcd^2 - 2ad^2)\operatorname{sgn}(x)}{315c^4} + \frac{35(cx^2 + d)^{\frac{9}{2}}a\operatorname{sgn}(x) + 45(cx^2 + d)^{\frac{7}{2}}bc\operatorname{sgn}(x) - 135(cx^2 + d)^{\frac{7}{2}}ad\operatorname{sgn}(x) - 126(cx^2 + d)^{\frac{5}{2}}bcd\operatorname{sgn}(x) + 189(cx^2 + d)^{\frac{5}{2}}ad^2\operatorname{sgn}(x) + 105(cx^2 + d)^{\frac{3}{2}}bcd^2\operatorname{sgn}(x) - 105(cx^2 + d)^{\frac{3}{2}}ad^3\operatorname{sgn}(x)}{315c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] -8/315*(3*b*c*d^(7/2) - 2*a*d^(9/2))*sgn(x)/c^4 + 1/315*(35*(c*x^2 + d)^(9/2)*a*sgn(x) + 45*(c*x^2 + d)^(7/2)*b*c*sgn(x) - 135*(c*x^2 + d)^(7/2)*a*d*sgn(x) - 126*(c*x^2 + d)^(5/2)*b*c*d*sgn(x) + 189*(c*x^2 + d)^(5/2)*a*d^2*sgn(x) + 105*(c*x^2 + d)^(3/2)*b*c*d^2*sgn(x) - 105*(c*x^2 + d)^(3/2)*a*d^3*sgn(x))/c^4

maple [A] time = 0.05, size = 89, normalized size = 0.76

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (35ax^6c^3 - 30ac^2dx^4 + 45bc^3x^4 + 24acd^2x^2 - 36bc^2dx^2 - 16ad^3 + 24bcd^2)(cx^2 + d)x}{315c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x)

[Out] 1/315*((c*x^2+d)/x^2)^(1/2)*x*(35*a*c^3*x^6-30*a*c^2*d*x^4+45*b*c^3*x^4+24*a*c*d^2*x^2-36*b*c^2*d*x^2-16*a*d^3+24*b*c*d^2)*(c*x^2+d)/c^4

maxima [A] time = 0.56, size = 124, normalized size = 1.06

$$\frac{\left(15\left(c + \frac{d}{x^2}\right)^{\frac{7}{2}}x^7 - 42\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}dx^5 + 35\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)b}{105c^3} + \frac{\left(35\left(c + \frac{d}{x^2}\right)^{\frac{9}{2}}x^9 - 135\left(c + \frac{d}{x^2}\right)^{\frac{7}{2}}dx^7 + 189\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5 - 105\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}d^3x^3\right)a}{315c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] 1/105*(15*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x^2)^(3/2)*d^2*x^3)*b/c^3 + 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^2)^(7/2)*d*x^7 + 189*(c + d/x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3*x^3)*a/c^4

mupad [B] time = 4.52, size = 97, normalized size = 0.83

$$\sqrt{c + \frac{d}{x^2}} \left(\frac{ax^9}{9} - \frac{x(16ad^4 - 24bcd^3)}{315c^4} + \frac{x^7(45bc^4 + 5adc^3)}{315c^4} - \frac{dx^5(2ad - 3bc)}{105c^2} + \frac{4d^2x^3(2ad - 3bc)}{315c^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(a + b/x^2)*(c + d/x^2)^(1/2),x)

[Out] (c + d/x^2)^(1/2)*((a*x^9)/9 - (x*(16*a*d^4 - 24*b*c*d^3))/(315*c^4) + (x^7*(45*b*c^4 + 5*a*c^3*d))/(315*c^4) - (d*x^5*(2*a*d - 3*b*c))/(105*c^2) + (4*d^2*x^3*(2*a*d - 3*b*c))/(315*c^3))

sympy [B] time = 4.92, size = 910, normalized size = 7.78

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*x**8*(c+d/x**2)**(1/2),x)

[Out] 35*a*c**7*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**6*d**(21/2)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**5*d**(23/2)*x**10*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 40*a*c**4*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 5*a*c**3*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 30*a*c**2*d**(29/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 40*a*c*d**(31/2)*x**2*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 16*a*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*b*c**5*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 33*b*c**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*b*c**3*d**(13/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*b*c**2*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*b*c*d**(17/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*b*d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6)

$$3.593 \quad \int \left(a + \frac{b}{x^2} \right) \sqrt{c + \frac{d}{x^2}} x^6 dx$$

Optimal. Leaf size=84

$$-\frac{2dx^3 \left(c + \frac{d}{x^2} \right)^{3/2} (7bc - 4ad)}{105c^3} + \frac{x^5 \left(c + \frac{d}{x^2} \right)^{3/2} (7bc - 4ad)}{35c^2} + \frac{ax^7 \left(c + \frac{d}{x^2} \right)^{3/2}}{7c}$$

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {453, 271, 264}

$$\frac{x^5 \left(c + \frac{d}{x^2} \right)^{3/2} (7bc - 4ad)}{35c^2} - \frac{2dx^3 \left(c + \frac{d}{x^2} \right)^{3/2} (7bc - 4ad)}{105c^3} + \frac{ax^7 \left(c + \frac{d}{x^2} \right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]

[Out] (-2*d*(7*b*c - 4*a*d)*(c + d/x^2)^(3/2)*x^3)/(105*c^3) + ((7*b*c - 4*a*d)*(c + d/x^2)^(3/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(3/2)*x^7)/(7*c)

Rule 264

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

Rule 271

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m+1)*(a+b*x^n)^(p+1))/(a*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*(m+1)), Int[x^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m+1)/n + p + 1], 0] && NeQ[m, -1]

Rule 453

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m+1)*(a+b*x^n)^(p+1))/(a*e*(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x^6 dx &= \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^7}{7c} + \frac{(7bc - 4ad) \int \sqrt{c + \frac{d}{x^2}} x^4 dx}{7c} \\
&= \frac{(7bc - 4ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^5}{35c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^7}{7c} - \frac{(2d(7bc - 4ad)) \int \sqrt{c + \frac{d}{x^2}} x^2 dx}{35c^2} \\
&= -\frac{2d(7bc - 4ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^3}{105c^3} + \frac{(7bc - 4ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^5}{35c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^7}{7c}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 64, normalized size = 0.76

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d) (a(15c^2x^4 - 12cdx^2 + 8d^2) + 7bc(3cx^2 - 2d))}{105c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(7*b*c*(-2*d + 3*c*x^2) + a*(8*d^2 - 12*c*d*x^2 + 15*c^2*x^4)))/(105*c^3)

IntegrateAlgebraic [A] time = 0.09, size = 64, normalized size = 0.76

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d) (15ac^2x^4 - 12acdx^2 + 8ad^2 + 21bc^2x^2 - 14bcd)}{105c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(-14*b*c*d + 8*a*d^2 + 21*b*c^2*x^2 - 12*a*c*d*x^2 + 15*a*c^2*x^4))/(105*c^3)

fricas [A] time = 0.42, size = 82, normalized size = 0.98

$$\frac{(15ac^3x^7 + 3(7bc^3 + ac^2d)x^5 + (7bc^2d - 4acd^2)x^3 - 2(7bcd^2 - 4ad^3)x) \sqrt{\frac{cx^2+d}{x^2}}}{105c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] 1/105*(15*a*c^3*x^7 + 3*(7*b*c^3 + a*c^2*d)*x^5 + (7*b*c^2*d - 4*a*c*d^2)*x^3 - 2*(7*b*c*d^2 - 4*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^3

giac [A] time = 0.17, size = 105, normalized size = 1.25

$$\frac{2(7bcd^5 - 4ad^7)\operatorname{sgn}(x)}{105c^3} + \frac{15(cx^2 + d)^{\frac{7}{2}}\operatorname{asgn}(x) + 21(cx^2 + d)^{\frac{5}{2}}bc\operatorname{sgn}(x) - 42(cx^2 + d)^{\frac{5}{2}}ad\operatorname{sgn}(x) - 35(cx^2 + d)^{\frac{3}{2}}bcd\operatorname{sgn}(x) + 35(cx^2 + d)^{\frac{3}{2}}ad^2\operatorname{sgn}(x)}{105c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] 2/105*(7*b*c*d^(5/2) - 4*a*d^(7/2))*sgn(x)/c^3 + 1/105*(15*(c*x^2 + d)^(7/2)*a*sgn(x) + 21*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 42*(c*x^2 + d)^(5/2)*a*d*sgn(x) - 35*(c*x^2 + d)^(3/2)*b*c*d*sgn(x) + 35*(c*x^2 + d)^(3/2)*a*d^2*sgn(x))/c^3

maple [A] time = 0.06, size = 65, normalized size = 0.77

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (15ax^4c^2 - 12acd^2x^2 + 21b^2c^2x^2 + 8ad^2 - 14bcd)(cx^2 + d)x}{105c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x)

[Out] 1/105*((c*x^2+d)/x^2)^(1/2)*x*(15*a*c^2*x^4-12*a*c*d*x^2+21*b*c^2*x^2+8*a*d^2-14*b*c*d)*(c*x^2+d)/c^3

maxima [A] time = 0.53, size = 90, normalized size = 1.07

$$\frac{\left(3\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}x^5 - 5\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)b}{15c^2} + \frac{\left(15\left(c + \frac{d}{x^2}\right)^{\frac{7}{2}}x^7 - 42\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}dx^5 + 35\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)a}{105c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] 1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3)*b/c^2 + 1/105*(15*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x^2)^(3/2)*d^2*x^3)*a/c^3

mupad [B] time = 4.49, size = 77, normalized size = 0.92

$$\sqrt{c + \frac{d}{x^2}} \left(\frac{ax^7}{7} + \frac{x(8ad^3 - 14bcd^2)}{105c^3} + \frac{x^5(21bc^3 + 3adc^2)}{105c^3} - \frac{dx^3(4ad - 7bc)}{105c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(a + b/x^2)*(c + d/x^2)^(1/2),x)`

[Out] $(c + d/x^2)^{(1/2)} * ((a*x^7)/7 + (x*(8*a*d^3 - 14*b*c*d^2))/(105*c^3) + (x^5*(21*b*c^3 + 3*a*c^2*d))/(105*c^3) - (d*x^3*(4*a*d - 7*b*c))/(105*c^2))$

sympy [B] time = 3.99, size = 422, normalized size = 5.02

$$\frac{15ac^3d^3x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^3d^4x^4+210c^4d^5x^2+105c^5d^6} + \frac{33ac^4d^2x^8\sqrt{\frac{cx^2}{d}+1}}{105c^3d^4x^4+210c^4d^5x^2+105c^5d^6} + \frac{17ac^5d^2x^6\sqrt{\frac{cx^2}{d}+1}}{105c^3d^4x^4+210c^4d^5x^2+105c^5d^6} + \frac{3ac^2d^2x^4\sqrt{\frac{cx^2}{d}+1}}{105c^3d^4x^4+210c^4d^5x^2+105c^5d^6} + \frac{12acd^2x^2\sqrt{\frac{cx^2}{d}+1}}{105c^3d^4x^4+210c^4d^5x^2+105c^5d^6} + \frac{8ad^2\sqrt{\frac{cx^2}{d}+1}}{105c^3d^4x^4+210c^4d^5x^2+105c^5d^6} + \frac{b\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5} + \frac{bd^2x^2\sqrt{\frac{cx^2}{d}+1}}{15c} - \frac{2bd^2\sqrt{\frac{cx^2}{d}+1}}{15c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*x**6*(c+d/x**2)**(1/2),x)`

[Out] $15*a*c**5*d**(9/2)*x**10*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 33*a*c**4*d**(11/2)*x**8*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**3*d**(13/2)*x**6*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*a*c**2*d**(15/2)*x**4*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c*d**(17/2)*x**2*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*d**(19/2)*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + b*\sqrt{d}*x**4*\sqrt{c*x**2/d + 1}/5 + b*d**(3/2)*x**2*\sqrt{c*x**2/d + 1}/(15*c) - 2*b*d**(5/2)*\sqrt{c*x**2/d + 1}/(15*c**2)$

$$3.594 \quad \int \left(a + \frac{b}{x^2} \right) \sqrt{c + \frac{d}{x^2}} x^4 dx$$

Optimal. Leaf size=53

$$\frac{x^3 \left(c + \frac{d}{x^2} \right)^{3/2} (5bc - 2ad)}{15c^2} + \frac{ax^5 \left(c + \frac{d}{x^2} \right)^{3/2}}{5c}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {453, 264}

$$\frac{x^3 \left(c + \frac{d}{x^2} \right)^{3/2} (5bc - 2ad)}{15c^2} + \frac{ax^5 \left(c + \frac{d}{x^2} \right)^{3/2}}{5c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]

[Out] ((5*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(15*c^2) + (a*(c + d/x^2)^(3/2)*x^5)/(5*c)

Rule 264

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c_)*x)^(m + 1)*(a + b*x^n)^(p + 1)/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 453

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x^4 dx = \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^5}{5c} + \frac{(5bc - 2ad) \int \sqrt{c + \frac{d}{x^2}} x^2 dx}{5c}$$

$$= \frac{(5bc - 2ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^3}{15c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^5}{5c}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.79

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d) (3acx^2 - 2ad + 5bc)}{15c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(5*b*c - 2*a*d + 3*a*c*x^2))/(15*c^2)

IntegrateAlgebraic [A] time = 0.07, size = 42, normalized size = 0.79

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d) (3acx^2 - 2ad + 5bc)}{15c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(5*b*c - 2*a*d + 3*a*c*x^2))/(15*c^2)

fricas [A] time = 0.42, size = 57, normalized size = 1.08

$$\frac{(3ac^2x^5 + (5bc^2 + acd)x^3 + (5bcd - 2ad^2)x) \sqrt{\frac{cx^2+d}{x^2}}}{15c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] 1/15*(3*a*c^2*x^5 + (5*b*c^2 + a*c*d)*x^3 + (5*b*c*d - 2*a*d^2)*x)*sqrt((c*x^2 + d)/x^2)/c^2

giac [A] time = 0.21, size = 72, normalized size = 1.36

$$\frac{\left(5bcd^{\frac{3}{2}} - 2ad^{\frac{5}{2}}\right)\text{sgn}(x)}{15c^2} + \frac{3\left(cx^2 + d\right)^{\frac{5}{2}}a\text{sgn}(x) + 5\left(cx^2 + d\right)^{\frac{3}{2}}bc\text{sgn}(x) - 5\left(cx^2 + d\right)^{\frac{3}{2}}ad\text{sgn}(x)}{15c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] -1/15*(5*b*c*d^(3/2) - 2*a*d^(5/2))*sgn(x)/c^2 + 1/15*(3*(c*x^2 + d)^(5/2)*a*sgn(x) + 5*(c*x^2 + d)^(3/2)*b*c*sgn(x) - 5*(c*x^2 + d)^(3/2)*a*d*sgn(x))/c^2

maple [A] time = 0.05, size = 43, normalized size = 0.81

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (3ax^2c - 2ad + 5bc)(cx^2 + d)x}{15c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x)

[Out] 1/15*((c*x^2+d)/x^2)^(1/2)*x*(3*a*c*x^2-2*a*d+5*b*c)*(c*x^2+d)/c^2

maxima [A] time = 0.66, size = 55, normalized size = 1.04

$$\frac{b\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}x^3}{3c} + \frac{\left(3\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}x^5 - 5\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)a}{15c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] 1/3*b*(c + d/x^2)^(3/2)*x^3/c + 1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3)*a/c^2

mupad [B] time = 4.44, size = 54, normalized size = 1.02

$$\sqrt{c + \frac{d}{x^2}} \left(\frac{ax^5}{5} - \frac{x(2ad^2 - 5bcd)}{15c^2} + \frac{x^3(5bc^2 + adc)}{15c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(a + b/x^2)*(c + d/x^2)^(1/2),x)

[Out] $(c + d/x^2)^{1/2} * ((a*x^5)/5 - (x*(2*a*d^2 - 5*b*c*d))/(15*c^2) + (x^3*(5*b*c^2 + a*c*d))/(15*c^2))$

sympy [B] time = 3.04, size = 119, normalized size = 2.25

$$\frac{a\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5} + \frac{ad^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c} - \frac{2ad^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2} + \frac{b\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3} + \frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*x**4*(c+d/x**2)**(1/2),x)

[Out] $a*\sqrt{d}*x**4*\sqrt{c*x**2/d + 1}/5 + a*d**(3/2)*x**2*\sqrt{c*x**2/d + 1}/(15*c) - 2*a*d**(5/2)*\sqrt{c*x**2/d + 1}/(15*c**2) + b*\sqrt{d}*x**2*\sqrt{c*x**2/d + 1}/3 + b*d**(3/2)*\sqrt{c*x**2/d + 1}/(3*c)$

$$3.595 \quad \int \left(a + \frac{b}{x^2} \right) \sqrt{c + \frac{d}{x^2}} x^2 dx$$

Optimal. Leaf size=66

$$\frac{ax^3 \left(c + \frac{d}{x^2} \right)^{3/2}}{3c} + bx \sqrt{c + \frac{d}{x^2}} - b\sqrt{d} \tanh^{-1} \left(\frac{\sqrt{d}}{x \sqrt{c + \frac{d}{x^2}}} \right)$$

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {451, 242, 277, 217, 206}

$$\frac{ax^3 \left(c + \frac{d}{x^2} \right)^{3/2}}{3c} + bx \sqrt{c + \frac{d}{x^2}} - b\sqrt{d} \tanh^{-1} \left(\frac{\sqrt{d}}{x \sqrt{c + \frac{d}{x^2}}} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]

[Out] b*Sqrt[c + d/x^2]*x + (a*(c + d/x^2)^(3/2)*x^3)/(3*c) - b*Sqrt[d]*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 242

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]

Rule 277

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*(a + b*x^n)^p/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In

t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 451

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1]))

Rubi steps

$$\begin{aligned}
 \int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} x^2 dx &= \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^3}{3c} + b \int \sqrt{c + \frac{d}{x^2}} dx \\
 &= \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^3}{3c} - b \operatorname{Subst} \left(\int \frac{\sqrt{c + dx^2}}{x^2} dx, x, \frac{1}{x} \right) \\
 &= b \sqrt{c + \frac{d}{x^2}} x + \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^3}{3c} - (bd) \operatorname{Subst} \left(\int \frac{1}{\sqrt{c + dx^2}} dx, x, \frac{1}{x} \right) \\
 &= b \sqrt{c + \frac{d}{x^2}} x + \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^3}{3c} - (bd) \operatorname{Subst} \left(\int \frac{1}{1 - dx^2} dx, x, \frac{1}{\sqrt{c + \frac{d}{x^2}} x} \right) \\
 &= b \sqrt{c + \frac{d}{x^2}} x + \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x^3}{3c} - b \sqrt{d} \tanh^{-1} \left(\frac{\sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x} \right)
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 84, normalized size = 1.27

$$\frac{x \sqrt{c + \frac{d}{x^2}} \left(\sqrt{cx^2 + d} (a(cx^2 + d) + 3bc) - 3bc \sqrt{d} \tanh^{-1} \left(\frac{\sqrt{cx^2 + d}}{\sqrt{d}} \right) \right)}{3c \sqrt{cx^2 + d}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]

[Out] (Sqrt[c + d/x^2]*x*(Sqrt[d + c*x^2]*(3*b*c + a*(d + c*x^2)) - 3*b*c*Sqrt[d]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(3*c*Sqrt[d + c*x^2])

IntegrateAlgebraic [A] time = 0.09, size = 83, normalized size = 1.26

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\frac{\sqrt{cx^2+d}(acx^2+ad+3bc)}{3c} - b\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right) \right)}{\sqrt{cx^2 + d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]

[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(3*b*c + a*d + a*c*x^2))/(3*c) - b*Sqrt[d]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/Sqrt[d + c*x^2]

fricas [A] time = 0.44, size = 156, normalized size = 2.36

$$\left[\frac{3bc\sqrt{d} \log\left(-\frac{cx^2-2\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2d}{x^2}\right) + 2(acx^3 + (3bc + ad)x)\sqrt{\frac{cx^2+d}{x^2}}}{6c}, \frac{3bc\sqrt{-d} \arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + (acx^3 + (3bc + ad)x)\sqrt{\frac{cx^2+d}{x^2}}}{3c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] [1/6*(3*b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(a*c*x^3 + (3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c, 1/3*(3*b*c*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (a*c*x^3 + (3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c]

giac [B] time = 0.19, size = 116, normalized size = 1.76

$$\frac{bd \arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right) \operatorname{sgn}(x)}{\sqrt{-d}} - \frac{\left(3bcd \arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right) + 3bc\sqrt{-d}\sqrt{d} + a\sqrt{-d}d^{\frac{3}{2}}\right) \operatorname{sgn}(x)}{3c\sqrt{-d}} + \frac{(cx^2 + d)^{\frac{3}{2}}ac^2 \operatorname{sgn}(x) + 3\sqrt{cx^2 + d}bc^3 \operatorname{sgn}(x)}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] b*d*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/3*(3*b*c*d*arctan(sqrt(d)/sqrt(-d)) + 3*b*c*sqrt(-d)*sqrt(d) + a*sqrt(-d)*d^(3/2))*sgn(x)/(c*

$\sqrt{-d}) + 1/3*((c*x^2 + d)^{(3/2)}*a*c^2*\text{sgn}(x) + 3*\sqrt{c*x^2 + d}*b*c^3*\text{sgn}(x))/c^3$

maple [A] time = 0.06, size = 83, normalized size = 1.26

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} \left(3bc\sqrt{d} \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) - 3\sqrt{cx^2+d}bc - (cx^2+d)^{\frac{3}{2}}a \right) x}{3\sqrt{cx^2+d}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x)`

[Out] $-1/3*((c*x^2+d)/x^2)^{(1/2)}*x*(3*d^{(1/2)}*\ln(2*(d^{(1/2)}*(c*x^2+d)^{(1/2)}+d)/x) *b*c-a*(c*x^2+d)^{(3/2)}-3*(c*x^2+d)^{(1/2)}*b*c)/(c*x^2+d)^{(1/2)}/c$

maxima [A] time = 1.38, size = 75, normalized size = 1.14

$$\frac{a\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}x^3}{3c} + \frac{1}{2} \left(2\sqrt{c + \frac{d}{x^2}}x + \sqrt{d} \log\left(\frac{\sqrt{c + \frac{d}{x^2}}x - \sqrt{d}}{\sqrt{c + \frac{d}{x^2}}x + \sqrt{d}}\right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="maxima")`

[Out] $1/3*a*(c + d/x^2)^{(3/2)}*x^3/c + 1/2*(2*\sqrt{c + d/x^2}*x + \sqrt{d}*\log((\sqrt{c + d/x^2}*x - \sqrt{d})/(\sqrt{c + d/x^2}*x + \sqrt{d}))) *b$

mupad [B] time = 4.72, size = 80, normalized size = 1.21

$$bx\sqrt{c + \frac{d}{x^2}} + \frac{ax\sqrt{c + \frac{d}{x^2}}(cx^2 + d)}{3c} + \frac{b\sqrt{d}\operatorname{asin}\left(\frac{\sqrt{d}1i}{\sqrt{c}x}\right)\sqrt{c + \frac{d}{x^2}}1i}{\sqrt{c}\sqrt{\frac{d}{cx^2} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b/x^2)*(c + d/x^2)^(1/2),x)`

[Out] $b*x*(c + d/x^2)^{(1/2)} + (a*x*(c + d/x^2)^{(1/2)}*(d + c*x^2))/(3*c) + (b*d^{(1/2)}*\operatorname{asin}((d^{(1/2)}*1i)/(c^{(1/2)}*x))*(c + d/x^2)^{(1/2)}*1i)/(c^{(1/2)}*(d/(c*x^2) + 1)^{(1/2)})$

sympy [A] time = 3.27, size = 107, normalized size = 1.62

$$\frac{a\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3} + \frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c} + \frac{b\sqrt{c}x}{\sqrt{1+\frac{d}{cx^2}}} - b\sqrt{d} \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right) + \frac{bd}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*x**2*(c+d/x**2)**(1/2),x)

[Out] a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + a*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c) + b*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - b*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x)) + b*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2)))

$$3.596 \quad \int \left(a + \frac{b}{x^2} \right) \sqrt{c + \frac{d}{x^2}} dx$$

Optimal. Leaf size=85

$$-\frac{\sqrt{c + \frac{d}{x^2}} (2ad + bc)}{2cx} - \frac{(2ad + bc) \tanh^{-1} \left(\frac{\sqrt{d}}{x \sqrt{c + \frac{d}{x^2}}} \right)}{2\sqrt{d}} + \frac{ax \left(c + \frac{d}{x^2} \right)^{3/2}}{c}$$

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {375, 453, 195, 217, 206}

$$-\frac{\sqrt{c + \frac{d}{x^2}} (2ad + bc)}{2cx} - \frac{(2ad + bc) \tanh^{-1} \left(\frac{\sqrt{d}}{x \sqrt{c + \frac{d}{x^2}}} \right)}{2\sqrt{d}} + \frac{ax \left(c + \frac{d}{x^2} \right)^{3/2}}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*Sqrt[c + d/x^2], x]

[Out] -((b*c + 2*a*d)*Sqrt[c + d/x^2])/(2*c*x) + (a*(c + d/x^2)^(3/2)*x)/c - ((b*c + 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*Sqrt[d])

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 375

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
  ] :-> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
  b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 453

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol]
  ] :-> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)/(a*e*(m + 1)),
  x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*
  (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] &&
  (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) &&
  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}} dx = -\text{Subst} \left(\int \frac{(a + bx^2) \sqrt{c + dx^2}}{x^2} dx, x, \frac{1}{x} \right)$$

$$= \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x}{c} + \frac{(-bc - 2ad) \text{Subst} \left(\int \sqrt{c + dx^2} dx, x, \frac{1}{x} \right)}{c}$$

$$= -\frac{(bc + 2ad) \sqrt{c + \frac{d}{x^2}}}{2cx} + \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x}{c} + \frac{1}{2}(-bc - 2ad) \text{Subst} \left(\int \frac{1}{\sqrt{c + dx^2}} dx, x, \frac{1}{x} \right)$$

$$= -\frac{(bc + 2ad) \sqrt{c + \frac{d}{x^2}}}{2cx} + \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x}{c} + \frac{1}{2}(-bc - 2ad) \text{Subst} \left(\int \frac{1}{1 - dx^2} dx, x, \frac{1}{\sqrt{c - dx^2}} \right)$$

$$= -\frac{(bc + 2ad) \sqrt{c + \frac{d}{x^2}}}{2cx} + \frac{a \left(c + \frac{d}{x^2}\right)^{3/2} x}{c} - \frac{(bc + 2ad) \tanh^{-1} \left(\frac{\sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x} \right)}{2\sqrt{d}}$$

Mathematica [A] time = 0.09, size = 75, normalized size = 0.88

$$\frac{\sqrt{c + \frac{d}{x^2}} \left(-\frac{x^2(2ad+bc) \tanh^{-1} \left(\frac{\sqrt{cx^2+d}}{\sqrt{d}} \right) + 2ax^2 - b}{\sqrt{d} \sqrt{cx^2+d}} \right)}{2x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2],x]

[Out] (Sqrt[c + d/x^2]*(-b + 2*a*x^2 - ((b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(Sqrt[d]*Sqrt[d + c*x^2]))/(2*x)

IntegrateAlgebraic [A] time = 0.15, size = 89, normalized size = 1.05

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\frac{(2ax^2 - b)\sqrt{cx^2 + d}}{2x^2} + \frac{(-2ad - bc) \tanh^{-1}\left(\frac{\sqrt{cx^2 + d}}{\sqrt{d}}\right)}{2\sqrt{d}} \right)}{\sqrt{cx^2 + d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2],x]

[Out] (Sqrt[c + d/x^2]*x*(((-b + 2*a*x^2)*Sqrt[d + c*x^2])/(2*x^2) + (((-b*c) - 2*a*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(2*Sqrt[d])))/Sqrt[d + c*x^2]

fricas [A] time = 0.45, size = 164, normalized size = 1.93

$$\left[\frac{(bc + 2ad)\sqrt{d}x \log\left(-\frac{cx^2 - 2\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2} + 2d}}{x^2}\right) + 2(2adx^2 - bd)\sqrt{\frac{cx^2+d}{x^2}}}{4dx}, \frac{(bc + 2ad)\sqrt{-d}x \arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + (2adx^2 - bd)\sqrt{\frac{cx^2+d}{x^2}}}{2dx} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x), 1/2*((b*c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x)]

giac [A] time = 0.25, size = 76, normalized size = 0.89

$$\frac{2\sqrt{cx^2 + d} \operatorname{acsgn}(x) + \frac{(bc^2 \operatorname{sgn}(x) + 2acd \operatorname{sgn}(x)) \arctan\left(\frac{\sqrt{cx^2 + d}}{\sqrt{-d}}\right) - \frac{\sqrt{cx^2 + d} \operatorname{bcsgn}(x)}{x^2}}{\sqrt{-d}}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{2} \cdot (2 \cdot \sqrt{c \cdot x^2 + d}) \cdot a \cdot c \cdot \text{sgn}(x) + (b \cdot c^2 \cdot \text{sgn}(x) + 2 \cdot a \cdot c \cdot d \cdot \text{sgn}(x)) \cdot \arctan\left(\frac{\sqrt{c \cdot x^2 + d}}{\sqrt{-d}}\right) / \sqrt{-d} - \sqrt{c \cdot x^2 + d} \cdot b \cdot c \cdot \text{sgn}(x) / x^2 / c$

maple [A] time = 0.05, size = 135, normalized size = 1.59

$$\frac{\sqrt{\frac{c x^2 + d}{x^2}} \left(2 a d^{\frac{3}{2}} x^2 \ln\left(\frac{2 d + 2 \sqrt{c x^2 + d} \sqrt{d}}{x}\right) + b c \sqrt{d} x^2 \ln\left(\frac{2 d + 2 \sqrt{c x^2 + d} \sqrt{d}}{x}\right) - 2 \sqrt{c x^2 + d} a d x^2 - \sqrt{c x^2 + d} b c x^2 + (c x^2 + d)^{\frac{3}{2}} b \right)}{2 \sqrt{c x^2 + d} d x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)*(c+d/x^2)^(1/2),x)`

[Out] $-1/2 \cdot ((c \cdot x^2 + d) / x^2)^{1/2} / x \cdot (2 \cdot d^{3/2} \cdot \ln(2 \cdot (d + (c \cdot x^2 + d)^{1/2}) \cdot d^{1/2})) / x \cdot x^2 \cdot a \cdot d^{1/2} \cdot \ln(2 \cdot (d + (c \cdot x^2 + d)^{1/2}) \cdot d^{1/2}) / x \cdot x^2 \cdot b \cdot c \cdot 2 \cdot (c \cdot x^2 + d)^{1/2} \cdot x^2 \cdot a \cdot d - (c \cdot x^2 + d)^{1/2} \cdot x^2 \cdot b \cdot c + (c \cdot x^2 + d)^{3/2} \cdot b) / (c \cdot x^2 + d)^{1/2} / d$

maxima [A] time = 1.22, size = 133, normalized size = 1.56

$$\frac{1}{2} \left(2 \sqrt{c + \frac{d}{x^2}} x + \sqrt{d} \log\left(\frac{\sqrt{c + \frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x + \sqrt{d}}\right) \right) a - \frac{1}{4} \left(\frac{2 \sqrt{c + \frac{d}{x^2}} c x}{\left(c + \frac{d}{x^2}\right) x^2 - d} - \frac{c \log\left(\frac{\sqrt{c + \frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x + \sqrt{d}}\right)}{\sqrt{d}} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{2} \cdot (2 \cdot \sqrt{c + d/x^2}) \cdot x + \sqrt{d} \cdot \log((\sqrt{c + d/x^2}) \cdot x - \sqrt{d}) / ((\sqrt{c + d/x^2}) \cdot x + \sqrt{d}) \cdot a - 1/4 \cdot (2 \cdot \sqrt{c + d/x^2}) \cdot c \cdot x / ((c + d/x^2) \cdot x^2 - d) - c \cdot \log((\sqrt{c + d/x^2}) \cdot x - \sqrt{d}) / ((\sqrt{c + d/x^2}) \cdot x + \sqrt{d}) / \sqrt{d} \cdot b$

mupad [B] time = 5.11, size = 97, normalized size = 1.14

$$a x \sqrt{c + \frac{d}{x^2}} - \frac{b \sqrt{c + \frac{d}{x^2}}}{2 x} - \frac{b c \ln\left(\sqrt{c + \frac{d}{x^2}} + \frac{\sqrt{d}}{x}\right)}{2 \sqrt{d}} + \frac{a \sqrt{d} \operatorname{asin}\left(\frac{\sqrt{d}}{\sqrt{c} x}\right) \sqrt{c + \frac{d}{x^2}}}{\sqrt{c} \sqrt{\frac{d}{c x^2} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x^2)*(c + d/x^2)^(1/2),x)`

```
[Out] a*x*(c + d/x^2)^(1/2) - (b*(c + d/x^2)^(1/2))/(2*x) - (b*c*log((c + d/x^2)^(1/2) + d^(1/2)/x))/(2*d^(1/2)) + (a*d^(1/2)*asin((d^(1/2)*1i)/(c^(1/2)*x))
*(c + d/x^2)^(1/2)*1i)/(c^(1/2)*(d/(c*x^2) + 1)^(1/2))
```

sympy [A] time = 4.48, size = 107, normalized size = 1.26

$$\frac{a\sqrt{c}x}{\sqrt{1 + \frac{d}{cx^2}}} - a\sqrt{d} \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right) + \frac{ad}{\sqrt{c}x\sqrt{1 + \frac{d}{cx^2}}} - \frac{b\sqrt{c}\sqrt{1 + \frac{d}{cx^2}}}{2x} - \frac{bc \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x)) + a
*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x)
- b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*sqrt(d))
```


$$3.597 \quad \int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^2} dx$$

Optimal. Leaf size=91

$$\frac{c(bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{8d^{3/2}} + \frac{\sqrt{c + \frac{d}{x^2}}(bc - 4ad)}{8dx} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{4dx}$$

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {459, 335, 195, 217, 206}

$$\frac{c(bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{8d^{3/2}} + \frac{\sqrt{c + \frac{d}{x^2}}(bc - 4ad)}{8dx} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{4dx}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]

[Out] ((b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d*x) - (b*(c + d/x^2)^(3/2))/(4*d*x) + (c*(b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(3/2))

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 335

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \text{ :> } -\text{Subst}[\text{Int}[(a + b/x^n)^p/x^{(m+2)}, x], x, 1/x] \text{ /; FreeQ}\{a, b, p\}, x] \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 459

$\text{Int}[(e_.)*(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}*((c_) + (d_.)*(x_)^{(n_.)}), x_Symbol] \text{ :> } \text{Simp}[(d*(e*x)^{(m+1)}*(a + b*x^n)^{(p+1)})/(b*e*(m + n*(p+1) + 1)), x] - \text{Dist}[(a*d*(m+1) - b*c*(m + n*(p+1) + 1))/(b*(m + n*(p+1) + 1)), \text{Int}[(e*x)^m*(a + b*x^n)^p, x], x] \text{ /; FreeQ}\{a, b, c, d, e, m, n, p\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[m + n*(p+1) + 1, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^2} dx &= -\frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{4dx} + \frac{(-bc + 4ad) \int \frac{\sqrt{c + \frac{d}{x^2}}}{x^2} dx}{4d} \\ &= -\frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{4dx} - \frac{(-bc + 4ad) \text{Subst}\left(\int \sqrt{c + dx^2} dx, x, \frac{1}{x}\right)}{4d} \\ &= \frac{(bc - 4ad)\sqrt{c + \frac{d}{x^2}}}{8dx} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{4dx} + \frac{(c(bc - 4ad)) \text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}} dx, x, \frac{1}{x}\right)}{8d} \\ &= \frac{(bc - 4ad)\sqrt{c + \frac{d}{x^2}}}{8dx} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{4dx} + \frac{(c(bc - 4ad)) \text{Subst}\left(\int \frac{1}{1-dx^2} dx, x, \frac{1}{\sqrt{c + \frac{d}{x^2}} x}\right)}{8d} \\ &= \frac{(bc - 4ad)\sqrt{c + \frac{d}{x^2}}}{8dx} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{4dx} + \frac{c(bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x}\right)}{8d^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.19, size = 100, normalized size = 1.10

$$\frac{\sqrt{c + \frac{d}{x^2}} \left((cx^2 + d)(4adx^2 + bcx^2 + 2bd) + cx^4 \sqrt{\frac{cx^2}{d} + 1} (4ad - bc) \tanh^{-1}\left(\sqrt{\frac{cx^2}{d} + 1}\right) \right)}{8dx^3 (cx^2 + d)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]

[Out]
$$-1/8*(\text{Sqrt}[c + d/x^2]*((d + c*x^2)*(2*b*d + b*c*x^2 + 4*a*d*x^2) + c*(-(b*c) + 4*a*d)*x^4*\text{Sqrt}[1 + (c*x^2)/d]*\text{ArcTanh}[\text{Sqrt}[1 + (c*x^2)/d]]))/(d*x^3*(d + c*x^2))$$

IntegrateAlgebraic [A] time = 0.18, size = 103, normalized size = 1.13

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\frac{(bc^2 - 4acd) \tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{8d^{3/2}} + \frac{\sqrt{cx^2+d}(-4adx^2 - bcx^2 - 2bd)}{8dx^4} \right)}{\sqrt{cx^2 + d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]

[Out]
$$(\text{Sqrt}[c + d/x^2]*x*((\text{Sqrt}[d + c*x^2]*(-2*b*d - b*c*x^2 - 4*a*d*x^2))/(8*d*x^4) + ((b*c^2 - 4*a*c*d)*\text{ArcTanh}[\text{Sqrt}[d + c*x^2]/\text{Sqrt}[d]])/(8*d^{(3/2)})))/\text{Sqrt}[d + c*x^2]$$

fricas [A] time = 0.44, size = 194, normalized size = 2.13

$$\left[\frac{(bc^2 - 4acd)\sqrt{d}x^3 \log\left(-\frac{cx^2 - 2\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}} + 2d}{x^2}\right) + 2(2bd^2 + (bcd + 4ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16d^2x^3}, \frac{(bc^2 - 4acd)\sqrt{-d}x^3 \arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + (2bd^2 + (bcd + 4ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8d^2x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="fricas")

[Out]
$$[-1/16*((b*c^2 - 4*a*c*d)*\text{sqrt}(d)*x^3*\log(-(c*x^2 - 2*\text{sqrt}(d))*x*\text{sqrt}((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*\text{sqrt}((c*x^2 + d)/x^2))/(d^2*x^3), -1/8*((b*c^2 - 4*a*c*d)*\text{sqrt}(-d)*x^3*\arctan(\text{sqrt}(-d)*x*\text{sqrt}((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*\text{sqrt}((c*x^2 + d)/x^2))/(d^2*x^3)]$$

giac [A] time = 0.24, size = 130, normalized size = 1.43

$$\frac{(bc^3\text{sgn}(x) - 4ac^2\text{dsgn}(x))\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d} + \frac{(cx^2+d)^{\frac{3}{2}}bc^3\text{sgn}(x) + 4(cx^2+d)^{\frac{3}{2}}ac^2\text{dsgn}(x) + \sqrt{cx^2+d}bc^3\text{dsgn}(x) - 4\sqrt{cx^2+d}ac^2d^2\text{sgn}(x)}{c^2dx^4}$$

8c

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="giac")

[Out]
$$-1/8*((b*c^3*\text{sgn}(x) - 4*a*c^2*d*\text{sgn}(x))*\arctan(\sqrt{c*x^2 + d}/\sqrt{-d}))/(\sqrt{-d}*d) + ((c*x^2 + d)^{(3/2)}*b*c^3*\text{sgn}(x) + 4*(c*x^2 + d)^{(3/2)}*a*c^2*d*\text{sgn}(x) + \sqrt{c*x^2 + d}*b*c^3*d*\text{sgn}(x) - 4*\sqrt{c*x^2 + d}*a*c^2*d^2*\text{sgn}(x)))/(c^2*d*x^4)/c$$

maple [B] time = 0.06, size = 175, normalized size = 1.92

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} \left(4ac d^3 x^4 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) - b c^2 \sqrt{d} x^4 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) - 4\sqrt{cx^2+d} acd x^4 + \sqrt{cx^2+d} b c^2 x^4 + 4(c x^2 + d)^{\frac{3}{2}} ad x^2 - (c x^2 + d)^{\frac{3}{2}} bc x^2 + 2(c x^2 + d)^{\frac{3}{2}} bd \right)}{8\sqrt{cx^2+d} d^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x)

[Out]
$$-1/8*((c*x^2+d)/x^2)^{(1/2)}/x^3*(4*d^{(3/2)}*\ln(2*(d+(c*x^2+d)^{(1/2)}*d^{(1/2)}))/x)*x^4*a*c-d^{(1/2)}*\ln(2*(d+(c*x^2+d)^{(1/2)}*d^{(1/2)}))/x)*x^4*b*c^2-4*(c*x^2+d)^{(1/2)}*x^4*a*c*d+(c*x^2+d)^{(1/2)}*x^4*b*c^2+4*(c*x^2+d)^{(3/2)}*x^2*a*d-(c*x^2+d)^{(3/2)}*x^2*b*c+2*(c*x^2+d)^{(3/2)}*b*d)/(c*x^2+d)^{(1/2)}/d^2$$

maxima [B] time = 1.35, size = 193, normalized size = 2.12

$$-\frac{1}{4} \left(\frac{2\sqrt{c+\frac{d}{x^2}} cx}{\left(c+\frac{d}{x^2}\right)x^2-d} - \frac{c \log\left(\frac{\sqrt{c+\frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c+\frac{d}{x^2}} x + \sqrt{d}}\right)}{\sqrt{d}} \right) a - \frac{1}{16} \left(\frac{c^2 \log\left(\frac{\sqrt{c+\frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c+\frac{d}{x^2}} x + \sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}} c^2 x^3 + \sqrt{c+\frac{d}{x^2}} c^2 dx\right)}{\left(c+\frac{d}{x^2}\right)^2 dx^4 - 2\left(c+\frac{d}{x^2}\right) d^2 x^2 + d^3} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="maxima")

[Out]
$$-1/4*(2*\sqrt{c + d/x^2}*c*x/((c + d/x^2)*x^2 - d) - c*\log((\sqrt{c + d/x^2})*x - \sqrt{d})/(\sqrt{c + d/x^2}*x + \sqrt{d}))/\sqrt{d})*a - 1/16*(c^2*\log((\sqrt{c + d/x^2})*x - \sqrt{d})/(\sqrt{c + d/x^2}*x + \sqrt{d}))/d^{(3/2)} + 2*((c + d/x^2)^{(3/2)}*c^2*x^3 + \sqrt{c + d/x^2}*c^2*d*x)/((c + d/x^2)^2*d*x^4 - 2*(c + d/x^2)*d^2*x^2 + d^3))*b$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^2, x)`

[Out] `int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^2, x)`

sympy [A] time = 6.97, size = 144, normalized size = 1.58

$$-\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2x} - \frac{ac \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2\sqrt{d}} - \frac{bc^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}} - \frac{3b\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^2 \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{3}{2}}} - \frac{bd}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**2, x)`

[Out] `-a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x) - a*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*sqrt(d)) - b*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*b*sqrt(c)/(8*x**3*sqrt(1 + d/(c*x**2))) + b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - b*d/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))`

$$3.598 \quad \int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^4} dx$$

Optimal. Leaf size=123

$$-\frac{c^2(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{16d^{5/2}} + \frac{c\sqrt{c + \frac{d}{x^2}}(bc - 2ad)}{16d^2x} + \frac{\sqrt{c + \frac{d}{x^2}}(bc - 2ad)}{8dx^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

Rubi [A] time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {459, 335, 279, 321, 217, 206}

$$-\frac{c^2(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{16d^{5/2}} + \frac{c\sqrt{c + \frac{d}{x^2}}(bc - 2ad)}{16d^2x} + \frac{\sqrt{c + \frac{d}{x^2}}(bc - 2ad)}{8dx^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]

[Out] ((b*c - 2*a*d)*Sqrt[c + d/x^2])/(8*d*x^3) - (b*(c + d/x^2)^(3/2))/(6*d*x^3) + (c*(b*c - 2*a*d)*Sqrt[c + d/x^2])/(16*d^2*x) - (c^2*(b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(16*d^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 279

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p + 1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,

p, x]

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 335

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

Rule 459

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^4} dx &= -\frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{(-3bc + 6ad) \int \frac{\sqrt{c + \frac{d}{x^2}}}{x^4} dx}{6d} \\
&= -\frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{6dx^3} - \frac{(-3bc + 6ad) \text{Subst}\left(\int x^2 \sqrt{c + dx^2} dx, x, \frac{1}{x}\right)}{6d} \\
&= \frac{(bc - 2ad) \sqrt{c + \frac{d}{x^2}}}{8dx^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{(c(bc - 2ad)) \text{Subst}\left(\int \frac{x^2}{\sqrt{c + dx^2}} dx, x, \frac{1}{x}\right)}{8d} \\
&= \frac{(bc - 2ad) \sqrt{c + \frac{d}{x^2}}}{8dx^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{c(bc - 2ad) \sqrt{c + \frac{d}{x^2}}}{16d^2x} - \frac{(c^2(bc - 2ad)) \text{Subst}\left(\int \frac{1}{\sqrt{c + dx^2}} dx, x, \frac{1}{x}\right)}{16d^2} \\
&= \frac{(bc - 2ad) \sqrt{c + \frac{d}{x^2}}}{8dx^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{c(bc - 2ad) \sqrt{c + \frac{d}{x^2}}}{16d^2x} - \frac{(c^2(bc - 2ad)) \text{Subst}\left(\int \frac{1}{\sqrt{c + dx^2}} dx, x, \frac{1}{x}\right)}{16d^2} \\
&= \frac{(bc - 2ad) \sqrt{c + \frac{d}{x^2}}}{8dx^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{c(bc - 2ad) \sqrt{c + \frac{d}{x^2}}}{16d^2x} - \frac{c^2(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c + d}}\right)}{16d^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.55

$$\frac{\sqrt{c + \frac{d}{x^2}} (cx^2 + d) \left(c^2 x^6 (bc - 2ad) {}_2F_1\left(\frac{3}{2}, 3; \frac{5}{2}; \frac{cx^2}{d} + 1\right) - bd^3 \right)}{6d^4 x^5}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]

[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(-(b*d^3) + c^2*(b*c - 2*a*d)*x^6*Hypergeometric2F1[3/2, 3, 5/2, 1 + (c*x^2)/d]))/(6*d^4*x^5)

IntegrateAlgebraic [A] time = 0.24, size = 128, normalized size = 1.04

$$\frac{x \sqrt{c + \frac{d}{x^2}} \left(\frac{\sqrt{cx^2 + d} (-6acdx^4 - 12ad^2x^2 + 3bc^2x^4 - 2bcdx^2 - 8bd^2)}{48d^2x^6} + \frac{(2ac^2d - bc^3) \tanh^{-1}\left(\frac{\sqrt{cx^2 + d}}{\sqrt{d}}\right)}{16d^{5/2}} \right)}{\sqrt{cx^2 + d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]

[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(-8*b*d^2 - 2*b*c*d*x^2 - 12*a*d^2*x^2 + 3*b*c^2*x^4 - 6*a*c*d*x^4))/(48*d^2*x^6) + ((-(b*c^3) + 2*a*c^2*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(16*d^(5/2))))/Sqrt[d + c*x^2]

fricas [A] time = 0.46, size = 244, normalized size = 1.98

$$\left| \frac{3(bc^3 - 2ac^2d)\sqrt{d}x^5 \log\left(-\frac{cx^2 + 2\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}} + 2d}{x^2}\right) - 2(3(bc^2d - 2acd^2)x^4 - 8bd^3 - 2(bcd^2 + 6ad^3)x^2)\sqrt{\frac{cx^2+d}{x^2}} - 3(bc^3 - 2ac^2d)\sqrt{-d}x^5 \arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + (3(bc^2d - 2acd^2)x^4 - 8bd^3 - 2(bcd^2 + 6ad^3)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{96d^3x^5}, \frac{3(bc^3 - 2ac^2d)\sqrt{-d}x^5 \arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + (3(bc^2d - 2acd^2)x^4 - 8bd^3 - 2(bcd^2 + 6ad^3)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{48d^3x^5} \right|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="fricas")

[Out] [-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2*(b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^5), 1/48*(3*(b*c^3 - 2*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2*(b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^5)]

giac [A] time = 0.30, size = 153, normalized size = 1.24

$$\frac{3(bc^4\operatorname{sgn}(x) - 2ac^3d\operatorname{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right) + \frac{3(cx^2+d)^{\frac{5}{2}}bc^4\operatorname{sgn}(x) - 6(cx^2+d)^{\frac{5}{2}}ac^3d\operatorname{sgn}(x) - 8(cx^2+d)^{\frac{3}{2}}bc^4d\operatorname{sgn}(x) - 3\sqrt{cx^2+d}bc^4d^2\operatorname{sgn}(x) + 6\sqrt{cx^2+d}ac^3d^3\operatorname{sgn}(x)}{c^3d^2x^6}}{\sqrt{-d}d^2}}{48c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="giac")

[Out] 1/48*(3*(b*c^4*sgn(x) - 2*a*c^3*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d^2) + (3*(c*x^2 + d)^(5/2)*b*c^4*sgn(x) - 6*(c*x^2 + d)^(5/2)*a*c^3*d*sgn(x) - 8*(c*x^2 + d)^(3/2)*b*c^4*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^4*d^2*sgn(x) + 6*sqrt(c*x^2 + d)*a*c^3*d^3*sgn(x))/(c^3*d^2*x^6))/c

maple [B] time = 0.06, size = 220, normalized size = 1.79

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} \left(6ac^2d^{\frac{3}{2}}x^6 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) - 3b^2c^3\sqrt{d}x^6 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) - 6\sqrt{cx^2+d}ac^2d^{\frac{3}{2}}x^6 + 3\sqrt{cx^2+d}bc^3x^6 + 6(cx^2+d)^{\frac{3}{2}}acd^{\frac{3}{2}}x^4 - 3(cx^2+d)^{\frac{3}{2}}bc^2x^4 - 12(cx^2+d)^{\frac{3}{2}}ad^2x^2 + 6(cx^2+d)^{\frac{3}{2}}bcdx^2 - 8(cx^2+d)^{\frac{3}{2}}bd^2 \right)}{48\sqrt{cx^2+d}d^{\frac{3}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x)

[Out] $\frac{1}{48} \left(\frac{(c*x^2+d)/x^2}{x^5} \right)^{(1/2)} / x^5 * (6*d^{(3/2)} * \ln(2*(d+(c*x^2+d)^{(1/2)} * d^{(1/2)})) / x) * x^6 * a * c^2 - 3*d^{(1/2)} * \ln(2*(d+(c*x^2+d)^{(1/2)} * d^{(1/2)})) / x) * x^6 * b * c^3 - 6*(c*x^2+d)^{(1/2)} * x^6 * a * c^2 * d + 3*(c*x^2+d)^{(1/2)} * x^6 * b * c^3 + 6*(c*x^2+d)^{(3/2)} * x^4 * a * c * d - 3*(c*x^2+d)^{(3/2)} * x^4 * b * c^2 - 12*(c*x^2+d)^{(3/2)} * x^2 * a * d^2 + 6*(c*x^2+d)^{(3/2)} * x^2 * b * c * d - 8*(c*x^2+d)^{(3/2)} * b * d^2 / (c*x^2+d)^{(1/2)} / d^3$

maxima [B] time = 1.27, size = 277, normalized size = 2.25

$$-\frac{1}{16} \left(\frac{c^2 \log \left(\frac{\sqrt{\frac{c+d}{x^2}} x - \sqrt{d}}{\sqrt{\frac{c+d}{x^2}} x + \sqrt{d}} \right)}{d^{\frac{3}{2}}} + \frac{2 \left(\left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} c^2 x^3 + \sqrt{c + \frac{d}{x^2}} c^2 dx \right)}{\left(c + \frac{d}{x^2} \right)^2 dx^4 - 2 \left(c + \frac{d}{x^2} \right) d^2 x^2 + d^3} \right) a + \frac{1}{96} \left(\frac{3 c^3 \log \left(\frac{\sqrt{\frac{c+d}{x^2}} x - \sqrt{d}}{\sqrt{\frac{c+d}{x^2}} x + \sqrt{d}} \right)}{d^{\frac{5}{2}}} + \frac{2 \left(3 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}} c^3 x^5 - 8 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} c^3 dx^3 - 3 \sqrt{c + \frac{d}{x^2}} c^3 d^2 x \right)}{\left(c + \frac{d}{x^2} \right)^3 d^2 x^6 - 3 \left(c + \frac{d}{x^2} \right)^2 d^3 x^4 + 3 \left(c + \frac{d}{x^2} \right) d^4 x^2 - d^5} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="maxima")`

[Out] $-1/16 * (c^2 * \log((\sqrt{c + d/x^2}) * x - \sqrt{d}) / (\sqrt{c + d/x^2}) * x + \sqrt{d})) / d^{(3/2)} + 2 * ((c + d/x^2)^{(3/2)} * c^2 * x^3 + \sqrt{c + d/x^2} * c^2 * d * x) / ((c + d/x^2)^2 * d * x^4 - 2 * (c + d/x^2) * d^2 * x^2 + d^3) * a + 1/96 * (3 * c^3 * \log((\sqrt{c + d/x^2}) * x - \sqrt{d}) / (\sqrt{c + d/x^2}) * x + \sqrt{d})) / d^{(5/2)} + 2 * (3 * (c + d/x^2)^{(5/2)} * c^3 * x^5 - 8 * (c + d/x^2)^{(3/2)} * c^3 * d * x^3 - 3 * \sqrt{c + d/x^2} * c^3 * d^2 * x) / ((c + d/x^2)^3 * d^2 * x^6 - 3 * (c + d/x^2)^2 * d^3 * x^4 + 3 * (c + d/x^2) * d^4 * x^2 - d^5) * b$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{x^2}\right) \sqrt{c + \frac{d}{x^2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^4,x)`

[Out] `int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^4, x)`

sympy [B] time = 11.42, size = 226, normalized size = 1.84

$$-\frac{ac^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}} - \frac{3a\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}} + \frac{ac^2 \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{cx}}\right)}{8d^{\frac{3}{2}}} - \frac{ad}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^{\frac{5}{2}}}{16d^2x\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^{\frac{3}{2}}}{48dx^3\sqrt{1+\frac{d}{cx^2}}} - \frac{5b\sqrt{c}}{24x^5\sqrt{1+\frac{d}{cx^2}}} - \frac{bc^3 \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{cx}}\right)}{16d^{\frac{5}{2}}} - \frac{bd}{6\sqrt{c}x^7\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**4,x)`

[Out] $-a*c^{(3/2)} / (8*d*x*\sqrt{1 + d/(c*x**2)}) - 3*a*\sqrt{c} / (8*x**3*\sqrt{1 + d/(c*x**2)}) + a*c**2*asinh(\sqrt{d}/(\sqrt{c}*x)) / (8*d**(3/2)) - a*d / (4*\sqrt{c})$

```
*x**5*sqrt(1 + d/(c*x**2)) + b*c**(5/2)/(16*d**2*x*sqrt(1 + d/(c*x**2))) +  
  b*c**(3/2)/(48*d*x**3*sqrt(1 + d/(c*x**2))) - 5*b*sqrt(c)/(24*x**5*sqrt(1  
+ d/(c*x**2))) - b*c**3*asinh(sqrt(d)/(sqrt(c)*x))/(16*d**(5/2)) - b*d/(6*s  
qrt(c)*x**7*sqrt(1 + d/(c*x**2)))
```

$$3.599 \quad \int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^5 dx$$

Optimal. Leaf size=123

$$\frac{d^2(6bc - ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}} + \frac{dx^2 \sqrt{c + \frac{d}{x^2}} (6bc - ad)}{16c} + \frac{x^4 \left(c + \frac{d}{x^2}\right)^{3/2} (6bc - ad)}{24c} + \frac{ax^6 \left(c + \frac{d}{x^2}\right)^{5/2}}{6c}$$

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 78, 47, 63, 208}

$$\frac{d^2(6bc - ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}} + \frac{x^4 \left(c + \frac{d}{x^2}\right)^{3/2} (6bc - ad)}{24c} + \frac{dx^2 \sqrt{c + \frac{d}{x^2}} (6bc - ad)}{16c} + \frac{ax^6 \left(c + \frac{d}{x^2}\right)^{5/2}}{6c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]

[Out] (d*(6*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(16*c) + ((6*b*c - a*d)*(c + d/x^2)^(3/2)*x^4)/(24*c) + (a*(c + d/x^2)^(5/2)*x^6)/(6*c) + (d^2*(6*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(16*c^(3/2))

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^5 dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{(a + bx)(c + dx)^{3/2}}{x^4} dx, x, \frac{1}{x^2}\right)\right) \\
 &= \frac{a\left(c + \frac{d}{x^2}\right)^{5/2} x^6}{6c} - \frac{\left(3bc - \frac{ad}{2}\right) \text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3} dx, x, \frac{1}{x^2}\right)}{6c} \\
 &= \frac{(6bc - ad)\left(c + \frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c + \frac{d}{x^2}\right)^{5/2} x^6}{6c} - \frac{(d(6bc - ad)) \text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2} dx, x, \frac{1}{x^2}\right)}{16c} \\
 &= \frac{d(6bc - ad)\sqrt{c + \frac{d}{x^2}} x^2}{16c} + \frac{(6bc - ad)\left(c + \frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c + \frac{d}{x^2}\right)^{5/2} x^6}{6c} - \frac{(d^2(6bc - ad)) \text{Subst}\left(\int \frac{1}{x} dx, x, \frac{1}{x^2}\right)}{16c} \\
 &= \frac{d(6bc - ad)\sqrt{c + \frac{d}{x^2}} x^2}{16c} + \frac{(6bc - ad)\left(c + \frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c + \frac{d}{x^2}\right)^{5/2} x^6}{6c} - \frac{(d(6bc - ad)) \text{Subst}\left(\int \frac{1}{x} dx, x, \frac{1}{x^2}\right)}{16c} \\
 &= \frac{d(6bc - ad)\sqrt{c + \frac{d}{x^2}} x^2}{16c} + \frac{(6bc - ad)\left(c + \frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c + \frac{d}{x^2}\right)^{5/2} x^6}{6c} + \frac{d^2(6bc - ad)}{16c}
 \end{aligned}$$

Mathematica [A] time = 0.14, size = 123, normalized size = 1.00

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\sqrt{c} x \sqrt{\frac{cx^2}{d} + 1} \left(a(8c^2x^4 + 14cdx^2 + 3d^2) + 6bc(2cx^2 + 5d) \right) - 3d^{3/2}(ad - 6bc) \sinh^{-1} \left(\frac{\sqrt{cx}}{\sqrt{d}} \right) \right)}{48c^{3/2} \sqrt{\frac{cx^2}{d} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]

[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/d]*(6*b*c*(5*d + 2*c*x^2) + a*(3*d^2 + 14*c*d*x^2 + 8*c^2*x^4)) - 3*d^(3/2)*(-6*b*c + a*d)*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/(48*c^(3/2)*Sqrt[1 + (c*x^2)/d])

IntegrateAlgebraic [A] time = 0.19, size = 112, normalized size = 0.91

$$\frac{(6bcd^2 - ad^3) \tanh^{-1} \left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}} \right)}{16c^{3/2}} + \frac{\sqrt{\frac{cx^2+d}{x^2}} (8ac^2x^6 + 14acdx^4 + 3ad^2x^2 + 12bc^2x^4 + 30bcdx^2)}{48c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(30*b*c*d*x^2 + 3*a*d^2*x^2 + 12*b*c^2*x^4 + 14*a*c*d*x^4 + 8*a*c^2*x^6))/(48*c) + ((6*b*c*d^2 - a*d^3)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/(16*c^(3/2))

fricas [A] time = 0.45, size = 243, normalized size = 1.98

$$\left[\frac{3(6bcd^2 - ad^3)\sqrt{c} \log\left(-2cx^2 + 2\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}} - d\right) - 2(8ac^3x^6 + 2(6bc^3 + 7ac^2d)x^4 + 3(10bc^2d + acd^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{96c^2}, \frac{3(6bcd^2 - ad^3)\sqrt{-c} \arctan\left(\frac{\sqrt{-c}\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) - (8ac^3x^6 + 2(6bc^3 + 7ac^2d)x^4 + 3(10bc^2d + acd^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{48c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="fricas")

[Out] [-1/96*(3*(6*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10*b*c^2*d + a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^2, -1/48*(3*(6*b*c*d^2 - a*d^3)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (8*a*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10*b*c^2*d + a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^2]

giac [A] time = 0.23, size = 144, normalized size = 1.17

$$\frac{1}{48} \left(2 \left(4acx^2 \operatorname{sgn}(x) + \frac{6bc^5 \operatorname{sgn}(x) + 7ac^4 d \operatorname{sgn}(x)}{c^4} \right) x^2 + \frac{3(10bc^4 d \operatorname{sgn}(x) + ac^3 d^2 \operatorname{sgn}(x))}{c^4} \right) \sqrt{cx^2 + d} - \frac{(6bcd^2 \operatorname{sgn}(x) - ad^3 \operatorname{sgn}(x)) \log\left(-\sqrt{c}x + \sqrt{cx^2 + d}\right)}{16c^{\frac{3}{2}}} + \frac{(6bcd^2 \log(|d|) - ad^3 \log(|d|)) \operatorname{sgn}(x)}{32c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="giac")

[Out] $\frac{1}{48} * (2 * (4 * a * c * x^2 * \operatorname{sgn}(x) + (6 * b * c^5 * \operatorname{sgn}(x) + 7 * a * c^4 * d * \operatorname{sgn}(x)) / c^4) * x^2 + 3 * (10 * b * c^4 * d * \operatorname{sgn}(x) + a * c^3 * d^2 * \operatorname{sgn}(x)) / c^4) * \operatorname{sqrt}(c * x^2 + d) * x - \frac{1}{16} * (6 * b * c * d^2 * \operatorname{sgn}(x) - a * d^3 * \operatorname{sgn}(x)) * \log(\operatorname{abs}(-\operatorname{sqrt}(c) * x + \operatorname{sqrt}(c * x^2 + d))) / c^{3/2} + \frac{1}{32} * (6 * b * c * d^2 * \log(\operatorname{abs}(d)) - a * d^3 * \log(\operatorname{abs}(d))) * \operatorname{sgn}(x) / c^{3/2}$

maple [A] time = 0.06, size = 162, normalized size = 1.32

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}} \left(-3ad^3 \ln(\sqrt{c}x + \sqrt{cx^2+d}) + 18bcd^2 \ln(\sqrt{c}x + \sqrt{cx^2+d}) - 3\sqrt{cx^2+d} a\sqrt{c} d^2x + 18\sqrt{cx^2+d} bc^{\frac{3}{2}} dx - 2(cx^2+d)^{\frac{3}{2}} a\sqrt{c} dx + 12(cx^2+d)^{\frac{3}{2}} bc^{\frac{3}{2}} x + 8(cx^2+d)^{\frac{5}{2}} a\sqrt{c} x\right) x^3}{48(cx^2+d)^{\frac{3}{2}} c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x)

[Out] $\frac{1}{48} * ((c * x^2 + d) / x^2)^{3/2} * x^3 * (8 * c^{1/2} * (c * x^2 + d)^{5/2} * x * a - 2 * (c * x^2 + d)^{3/2} * a * c^{1/2} * d * x + 12 * (c * x^2 + d)^{3/2} * b * c^{3/2} * x - 3 * (c * x^2 + d)^{1/2} * a * c^{1/2} * d^2 * x + 18 * (c * x^2 + d)^{1/2} * b * c^{3/2} * d * x - 3 * a * d^3 * \ln(c^{1/2} * x + (c * x^2 + d)^{1/2}) + 18 * b * c * d^2 * \ln(c^{1/2} * x + (c * x^2 + d)^{1/2})) / ((c * x^2 + d)^{3/2} / c^{3/2})$

maxima [B] time = 1.30, size = 240, normalized size = 1.95

$$\frac{1}{96} \left(\frac{3d^3 \log\left(\frac{\sqrt{\frac{c+d}{x^2}} - \sqrt{c}}{\sqrt{\frac{c+d}{x^2}} + \sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\left(3\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}} d^3 + 8\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} cd^3 - 3\sqrt{c + \frac{d}{x^2}} c^2 d^3\right)}{\left(c + \frac{d}{x^2}\right)^3 c - 3\left(c + \frac{d}{x^2}\right)^2 c^2 + 3\left(c + \frac{d}{x^2}\right) c^3 - c^4} \right) a - \frac{1}{16} \left(\frac{3d^2 \log\left(\frac{\sqrt{\frac{c+d}{x^2}} - \sqrt{c}}{\sqrt{\frac{c+d}{x^2}} + \sqrt{c}}\right)}{\sqrt{c}} - \frac{2\left(5\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} d^2 - 3\sqrt{c + \frac{d}{x^2}} cd^2\right)}{\left(c + \frac{d}{x^2}\right)^2 - 2\left(c + \frac{d}{x^2}\right) c + c^2} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="maxima")

[Out] $\frac{1}{96} * (3 * d^3 * \log((\operatorname{sqrt}(c + d/x^2) - \operatorname{sqrt}(c)) / (\operatorname{sqrt}(c + d/x^2) + \operatorname{sqrt}(c)))) / c^{3/2} + 2 * (3 * (c + d/x^2)^{5/2} * d^3 + 8 * (c + d/x^2)^{3/2} * c * d^3 - 3 * \operatorname{sqrt}(c + d/x^2) * c^2 * d^3) / ((c + d/x^2)^3 * c - 3 * (c + d/x^2)^2 * c^2 + 3 * (c + d/x^2) * c^3 - c^4) * a - \frac{1}{16} * (3 * d^2 * \log((\operatorname{sqrt}(c + d/x^2) - \operatorname{sqrt}(c)) / (\operatorname{sqrt}(c + d/x^2) + \operatorname{sqrt}(c))) / \operatorname{sqrt}(c) - 2 * (5 * (c + d/x^2)^{3/2} * d^2 - 3 * \operatorname{sqrt}(c + d/x^2) * c * d^2) / ((c + d/x^2)^2 - 2 * (c + d/x^2) * c + c^2)) * b$

mupad [B] time = 5.78, size = 130, normalized size = 1.06

$$\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{6} + \frac{5bx^4\left(c+\frac{d}{x^2}\right)^{3/2}}{8} + \frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{16c} + \frac{3bd^2\operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}} - \frac{acx^6\sqrt{c+\frac{d}{x^2}}}{16} - \frac{3bcx^4\sqrt{c+\frac{d}{x^2}}}{8} + \frac{ad^3\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)1i}{16c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a + b/x^2)*(c + d/x^2)^(3/2), x)`

[Out] $(a*x^6*(c + d/x^2)^{(3/2)})/6 + (5*b*x^4*(c + d/x^2)^{(3/2)})/8 + (a*x^6*(c + d/x^2)^{(5/2)})/(16*c) + (a*d^3*\operatorname{atan}(((c + d/x^2)^{(1/2)}*1i)/c^{(1/2)})*1i)/(16*c^{(3/2)}) + (3*b*d^2*\operatorname{atanh}((c + d/x^2)^{(1/2)}/c^{(1/2)}))/(8*c^{(1/2)}) - (a*c*x^6*(c + d/x^2)^{(1/2)})/16 - (3*b*c*x^4*(c + d/x^2)^{(1/2)})/8$

sympy [B] time = 100.22, size = 253, normalized size = 2.06

$$\frac{ac^2x^7}{6\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + \frac{11ac\sqrt{d}x^5}{24\sqrt{\frac{cx^2}{d}+1}} + \frac{17ad^2x^3}{48\sqrt{\frac{cx^2}{d}+1}} + \frac{ad^2x}{16c\sqrt{\frac{cx^2}{d}+1}} - \frac{ad^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{16c^2} + \frac{bc^2x^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + \frac{3bc\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}} + \frac{bd^2x\sqrt{\frac{cx^2}{d}+1}}{2} + \frac{bd^2x}{8\sqrt{\frac{cx^2}{d}+1}} + \frac{3bd^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**5,x)`

[Out] $a*c**2*x**7/(6*\operatorname{sqrt}(d)*\operatorname{sqrt}(c*x**2/d + 1)) + 11*a*c*\operatorname{sqrt}(d)*x**5/(24*\operatorname{sqrt}(c*x**2/d + 1)) + 17*a*d**(3/2)*x**3/(48*\operatorname{sqrt}(c*x**2/d + 1)) + a*d**(5/2)*x/(16*c*\operatorname{sqrt}(c*x**2/d + 1)) - a*d**3*\operatorname{asinh}(\operatorname{sqrt}(c)*x/\operatorname{sqrt}(d))/(16*c**(3/2)) + b*c**2*x**5/(4*\operatorname{sqrt}(d)*\operatorname{sqrt}(c*x**2/d + 1)) + 3*b*c*\operatorname{sqrt}(d)*x**3/(8*\operatorname{sqrt}(c*x**2/d + 1)) + b*d**(3/2)*x*\operatorname{sqrt}(c*x**2/d + 1)/2 + b*d**(3/2)*x/(8*\operatorname{sqrt}(c*x**2/d + 1)) + 3*b*d**2*\operatorname{asinh}(\operatorname{sqrt}(c)*x/\operatorname{sqrt}(d))/(8*\operatorname{sqrt}(c))$

$$3.600 \quad \int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^3 dx$$

Optimal. Leaf size=115

$$\frac{x^2 \left(c + \frac{d}{x^2}\right)^{3/2} (ad + 4bc)}{8c} - \frac{3d\sqrt{c + \frac{d}{x^2}} (ad + 4bc)}{8c} + \frac{3d(ad + 4bc) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}} + \frac{ax^4 \left(c + \frac{d}{x^2}\right)^{5/2}}{4c}$$

Rubi [A] time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {446, 78, 47, 50, 63, 208}

$$\frac{x^2 \left(c + \frac{d}{x^2}\right)^{3/2} (ad + 4bc)}{8c} - \frac{3d\sqrt{c + \frac{d}{x^2}} (ad + 4bc)}{8c} + \frac{3d(ad + 4bc) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}} + \frac{ax^4 \left(c + \frac{d}{x^2}\right)^{5/2}}{4c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]

[Out] (-3*d*(4*b*c + a*d)*Sqrt[c + d/x^2])/(8*c) + ((4*b*c + a*d)*(c + d/x^2)^(3/2)*x^2)/(8*c) + (a*(c + d/x^2)^(5/2)*x^4)/(4*c) + (3*d*(4*b*c + a*d)*ArcTan[h[Sqrt[c + d/x^2]/Sqrt[c]]])/(8*Sqrt[c])

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^3 dx &= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{(a+bx)(c+dx)^{3/2}}{x^3} dx, x, \frac{1}{x^2} \right)\right) \\
&= \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^4}{4c} - \frac{(4bc+ad) \text{Subst} \left(\int \frac{(c+dx)^{3/2}}{x^2} dx, x, \frac{1}{x^2} \right)}{8c} \\
&= \frac{(4bc+ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^2}{8c} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^4}{4c} - \frac{(3d(4bc+ad)) \text{Subst} \left(\int \frac{\sqrt{c+dx}}{x} dx \right)}{16c} \\
&= -\frac{3d(4bc+ad) \sqrt{c + \frac{d}{x^2}}}{8c} + \frac{(4bc+ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^2}{8c} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^4}{4c} - \frac{1}{16} (3d(4bc+ad)) \\
&= -\frac{3d(4bc+ad) \sqrt{c + \frac{d}{x^2}}}{8c} + \frac{(4bc+ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^2}{8c} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^4}{4c} - \frac{1}{8} (3(4bc+ad)) \\
&= -\frac{3d(4bc+ad) \sqrt{c + \frac{d}{x^2}}}{8c} + \frac{(4bc+ad) \left(c + \frac{d}{x^2}\right)^{3/2} x^2}{8c} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^4}{4c} + \frac{3d(4bc+ad)}{8}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 89, normalized size = 0.77

$$\frac{1}{8} \sqrt{c + \frac{d}{x^2}} \left(\frac{3\sqrt{d} x(ad + 4bc) \sinh^{-1} \left(\frac{\sqrt{c} x}{\sqrt{d}} \right)}{\sqrt{c} \sqrt{\frac{cx^2}{d} + 1}} + 2acx^4 + 5adx^2 + 4bcx^2 - 8bd \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]

[Out] (Sqrt[c + d/x^2]*(-8*b*d + 4*b*c*x^2 + 5*a*d*x^2 + 2*a*c*x^4 + (3*Sqrt[d]*(4*b*c + a*d)*x*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/(Sqrt[c]*Sqrt[1 + (c*x^2)/d]))/8

IntegrateAlgebraic [A] time = 0.14, size = 88, normalized size = 0.77

$$\frac{3(ad^2 + 4bcd) \tanh^{-1} \left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}} \right)}{8\sqrt{c}} + \frac{1}{8} \sqrt{\frac{cx^2+d}{x^2}} (2acx^4 + 5adx^2 + 4bcx^2 - 8bd)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-8*b*d + 4*b*c*x^2 + 5*a*d*x^2 + 2*a*c*x^4))/8 + (3*(4*b*c*d + a*d^2)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/(8*Sqrt[c])

fricas [A] time = 0.45, size = 203, normalized size = 1.77

$$\left[\frac{3(4bcd + ad^2)\sqrt{c} \log\left(-2cx^2 - 2\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}} - d\right) + 2(2ac^2x^4 - 8bcd + (4bc^2 + 5acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16c}, \frac{3(4bcd + ad^2)\sqrt{-c} \arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) - (2ac^2x^4 - 8bcd + (4bc^2 + 5acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="fricas")

[Out] [1/16*(3*(4*b*c*d + a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) + 2*(2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2 + 5*a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/c, -1/8*(3*(4*b*c*d + a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2 + 5*a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/c]

giac [A] time = 0.29, size = 126, normalized size = 1.10

$$\frac{2b\sqrt{c}d^2\operatorname{sgn}(x)}{(\sqrt{c}x - \sqrt{cx^2+d})^2 - d} + \frac{1}{8}\left(2acx^2\operatorname{sgn}(x) + \frac{4bc^3\operatorname{sgn}(x) + 5ac^2d\operatorname{sgn}(x)}{c^2}\right)\sqrt{cx^2+d}x - \frac{3\left(4bc^{\frac{3}{2}}d\operatorname{sgn}(x) + a\sqrt{c}d^2\operatorname{sgn}(x)\right)\log\left(\left(\sqrt{c}x - \sqrt{cx^2+d}\right)^2\right)}{16c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="giac")

[Out] 2*b*sqrt(c)*d^2*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d) + 1/8*(2*a*c*x^2*sgn(x) + (4*b*c^3*sgn(x) + 5*a*c^2*d*sgn(x))/c^2)*sqrt(c*x^2 + d)*x - 3/16*(4*b*c^(3/2)*d*sgn(x) + a*sqrt(c)*d^2*sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)/c

maple [A] time = 0.06, size = 174, normalized size = 1.51

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(3ad^3x\ln\left(\sqrt{c}x + \sqrt{cx^2+d}\right) + 12bc^2d^2x\ln\left(\sqrt{c}x + \sqrt{cx^2+d}\right) + 3\sqrt{cx^2+d}a\sqrt{c}d^2x^2 + 12\sqrt{cx^2+d}bc^{\frac{3}{2}}d^2x^2 + 2(c^2+d)^{\frac{3}{2}}a\sqrt{c}dx^2 + 8(c^2+d)^{\frac{3}{2}}bc^{\frac{3}{2}}x^2 - 8(c^2+d)^{\frac{5}{2}}b\sqrt{c}\right)x^2}{8(c^2+d)^{\frac{3}{2}}\sqrt{c}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x)

[Out] $\frac{1}{8} * ((c*x^2+d)/x^2)^{(3/2)} * x^2 * (8*c^{(3/2)} * (c*x^2+d)^{(3/2)} * x^2 * b + 12*c^{(3/2)} * (c*x^2+d)^{(1/2)} * x^2 * b * d + 2*c^{(1/2)} * (c*x^2+d)^{(3/2)} * x^2 * a * d - 8*c^{(1/2)} * (c*x^2+d)^{(5/2)} * b + 3*c^{(1/2)} * (c*x^2+d)^{(1/2)} * x^2 * a * d^2 + 3*\ln(c^{(1/2)} * x + (c*x^2+d)^{(1/2)}) * x * a * d^3 + 12*\ln(c^{(1/2)} * x + (c*x^2+d)^{(1/2)}) * x * b * c * d^2) / (c*x^2+d)^{(3/2)} / d / c^{(1/2)}$

maxima [A] time = 1.33, size = 171, normalized size = 1.49

$$-\frac{1}{16} \left[\frac{3d^2 \log\left(\frac{\sqrt{c+\frac{d}{x^2}} - \sqrt{c}}{\sqrt{c+\frac{d}{x^2}} + \sqrt{c}}\right)}{\sqrt{c}} - \frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2 - 3\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2 - 2\left(c+\frac{d}{x^2}\right)c + c^2} \right] a + \frac{1}{4} \left[2\sqrt{c+\frac{d}{x^2}}cx^2 - 3\sqrt{c}d \log\left(\frac{\sqrt{c+\frac{d}{x^2}} - \sqrt{c}}{\sqrt{c+\frac{d}{x^2}} + \sqrt{c}}\right) - 4\sqrt{c+\frac{d}{x^2}}d \right] b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="maxima")`

[Out] $-\frac{1}{16} * (3*d^2 * \log((\sqrt{c + d/x^2} - \sqrt{c}) / (\sqrt{c + d/x^2} + \sqrt{c}))) / \sqrt{c} - 2 * (5 * (c + d/x^2)^{(3/2)} * d^2 - 3 * \sqrt{c + d/x^2} * c * d^2) / ((c + d/x^2)^2 - 2 * (c + d/x^2) * c + c^2)) * a + 1/4 * (2 * \sqrt{c + d/x^2} * c * x^2 - 3 * \sqrt{c} * d * \log((\sqrt{c + d/x^2} - \sqrt{c}) / (\sqrt{c + d/x^2} + \sqrt{c}))) - 4 * \sqrt{c + d/x^2} * d) * b$

mupad [B] time = 5.70, size = 105, normalized size = 0.91

$$\frac{5ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{8} - bd\sqrt{c+\frac{d}{x^2}} + \frac{3b\sqrt{c}d \operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2} + \frac{3ad^2 \operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}} - \frac{3acx^4\sqrt{c+\frac{d}{x^2}}}{8} + \frac{bcx^2\sqrt{c+\frac{d}{x^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a + b/x^2)*(c + d/x^2)^(3/2), x)`

[Out] $(5*a*x^4*(c + d/x^2)^{(3/2)})/8 - b*d*(c + d/x^2)^{(1/2)} + (3*b*c^{(1/2)}*d*\operatorname{atanh}((c + d/x^2)^{(1/2)}/c^{(1/2)}))/2 + (3*a*d^2*\operatorname{atanh}((c + d/x^2)^{(1/2)}/c^{(1/2)}))/(8*c^{(1/2)}) - (3*a*c*x^4*(c + d/x^2)^{(1/2)})/8 + (b*c*x^2*(c + d/x^2)^{(1/2)})/2$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**3,x)`

[Out] Timed out

$$3.601 \quad \int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x dx$$

Optimal. Leaf size=110

$$-\frac{\left(c + \frac{d}{x^2}\right)^{3/2} (3ad + 2bc)}{6c} - \frac{1}{2} \sqrt{c + \frac{d}{x^2}} (3ad + 2bc) + \frac{1}{2} \sqrt{c} (3ad + 2bc) \tanh^{-1} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right) + \frac{ax^2 \left(c + \frac{d}{x^2}\right)^{5/2}}{2c}$$

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 78, 50, 63, 208}

$$-\frac{\left(c + \frac{d}{x^2}\right)^{3/2} (3ad + 2bc)}{6c} - \frac{1}{2} \sqrt{c + \frac{d}{x^2}} (3ad + 2bc) + \frac{1}{2} \sqrt{c} (3ad + 2bc) \tanh^{-1} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right) + \frac{ax^2 \left(c + \frac{d}{x^2}\right)^{5/2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]

[Out] -((2*b*c + 3*a*d)*Sqrt[c + d/x^2])/2 - ((2*b*c + 3*a*d)*(c + d/x^2)^(3/2))/(6*c) + (a*(c + d/x^2)^(5/2)*x^2)/(2*c) + (Sqrt[c]*(2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/2

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{(a + bx)(c + dx)^{3/2}}{x^2} dx, x, \frac{1}{x^2}\right)\right) \\
 &= \frac{a\left(c + \frac{d}{x^2}\right)^{5/2} x^2}{2c} - \frac{\left(bc + \frac{3ad}{2}\right) \text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x} dx, x, \frac{1}{x^2}\right)}{2c} \\
 &= -\frac{(2bc + 3ad)\left(c + \frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c + \frac{d}{x^2}\right)^{5/2} x^2}{2c} - \frac{1}{4}(2bc + 3ad) \text{Subst}\left(\int \frac{\sqrt{c + dx}}{x} dx, x, \frac{1}{x^2}\right) \\
 &= -\frac{1}{2}(2bc + 3ad)\sqrt{c + \frac{d}{x^2}} - \frac{(2bc + 3ad)\left(c + \frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c + \frac{d}{x^2}\right)^{5/2} x^2}{2c} - \frac{1}{4}(c(2bc + 3ad) \\
 &= -\frac{1}{2}(2bc + 3ad)\sqrt{c + \frac{d}{x^2}} - \frac{(2bc + 3ad)\left(c + \frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c + \frac{d}{x^2}\right)^{5/2} x^2}{2c} - \frac{c(2bc + 3ad)}{4} \\
 &= -\frac{1}{2}(2bc + 3ad)\sqrt{c + \frac{d}{x^2}} - \frac{(2bc + 3ad)\left(c + \frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c + \frac{d}{x^2}\right)^{5/2} x^2}{2c} + \frac{1}{2}\sqrt{c}(2bc + 3ad)
 \end{aligned}$$

Mathematica [C] time = 0.10, size = 78, normalized size = 0.71

$$\frac{1}{3} \sqrt{c + \frac{d}{x^2}} \left(-\frac{(3ad + 2bc) {}_2F_1\left(-\frac{3}{2}, -\frac{1}{2}; \frac{1}{2}; -\frac{cx^2}{d}\right)}{\sqrt{\frac{cx^2}{d} + 1}} - \frac{b(cx^2 + d)^2}{dx^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x, x]

[Out] (Sqrt[c + d/x^2]*(-(b*(d + c*x^2)^2)/(d*x^2)) - ((2*b*c + 3*a*d)*Hypergeometric2F1[-3/2, -1/2, 1/2, -(c*x^2)/d])/Sqrt[1 + (c*x^2)/d])/3

IntegrateAlgebraic [A] time = 0.17, size = 93, normalized size = 0.85

$$\frac{1}{2} (3a\sqrt{c}d + 2bc^{3/2}) \tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right) + \frac{\sqrt{\frac{cx^2+d}{x^2}} (3acx^4 - 6adx^2 - 8bcx^2 - 2bd)}{6x^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x, x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-2*b*d - 8*b*c*x^2 - 6*a*d*x^2 + 3*a*c*x^4))/(6*x^2) + ((2*b*c^(3/2) + 3*a*Sqrt[c]*d)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/2

fricas [A] time = 0.44, size = 195, normalized size = 1.77

$$\left[\frac{3(2bc + 3ad)\sqrt{c}x^2 \log\left(-2cx^2 - 2\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}} - d\right) + 2(3acx^4 - 2(4bc + 3ad)x^2 - 2bd)\sqrt{\frac{cx^2+d}{x^2}}}{12x^2}, -\frac{3(2bc + 3ad)\sqrt{-c}x^2 \arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) - (3acx^4 - 2(4bc + 3ad)x^2 - 2bd)\sqrt{\frac{cx^2+d}{x^2}}}{6x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x, x, algorithm="fricas")

[Out] [1/12*(3*(2*b*c + 3*a*d)*sqrt(c)*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) + 2*(3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^2 - 2*b*d)*sqrt((c*x^2 + d)/x^2))/x^2, -1/6*(3*(2*b*c + 3*a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^2 - 2*b*d)*sqrt((c*x^2 + d)/x^2))/x^2]

giac [B] time = 0.54, size = 225, normalized size = 2.05

$$\frac{1}{2} \sqrt{cx^2 + d} \operatorname{acxsgn}(x) - \frac{1}{4} (2bc^3 \operatorname{sgn}(x) + 3a\sqrt{c}d \operatorname{sgn}(x)) \log\left(\left(\sqrt{cx^2 + d}\right)^2\right) + \frac{2\left(6\left(\sqrt{cx^2 + d}\right)^4 bc^3 d \operatorname{sgn}(x) + 3\left(\sqrt{cx^2 + d}\right)^4 a\sqrt{c}d^2 \operatorname{sgn}(x) - 6\left(\sqrt{cx^2 + d}\right)^3 bc^3 d^2 \operatorname{sgn}(x) - 6\left(\sqrt{cx^2 + d}\right)^2 a\sqrt{c}d^3 \operatorname{sgn}(x) + 4bc^3 d^3 \operatorname{sgn}(x) + 3a\sqrt{c}d^4 \operatorname{sgn}(x)\right)}{3\left(\left(\sqrt{cx^2 + d}\right)^2 - d\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="giac")

[Out] $\frac{1}{2}\sqrt{c x^2 + d} a c x \operatorname{sgn}(x) - \frac{1}{4}(2 b c^{3/2} \operatorname{sgn}(x) + 3 a \sqrt{c} d \operatorname{sgn}(x)) \log((\sqrt{c} x - \sqrt{c x^2 + d})^2) + \frac{2}{3}(6(\sqrt{c} x - \sqrt{c x^2 + d})^4 b c^{3/2} d \operatorname{sgn}(x) + 3(\sqrt{c} x - \sqrt{c x^2 + d})^4 a \sqrt{c} d^2 \operatorname{sgn}(x) - 6(\sqrt{c} x - \sqrt{c x^2 + d})^2 b c^{3/2} d^2 \operatorname{sgn}(x) - 6(\sqrt{c} x - \sqrt{c x^2 + d})^2 a \sqrt{c} d^3 \operatorname{sgn}(x) + 4 b c^{3/2} d^3 \operatorname{sgn}(x) + 3 a \sqrt{c} d^4 \operatorname{sgn}(x)) / ((\sqrt{c} x - \sqrt{c x^2 + d})^2 - d)^3$

maple [B] time = 0.06, size = 216, normalized size = 1.96

$$\frac{\left(\frac{c^2+d}{x^2}\right)^{\frac{3}{2}} \left(9 a c d^3 x^3 \ln(\sqrt{c} x + \sqrt{c x^2 + d}) + 6 b c^2 d^2 x^3 \ln(\sqrt{c} x + \sqrt{c x^2 + d}) + 9 \sqrt{c x^2 + d} a c^{\frac{3}{2}} d^2 x^4 + 6 \sqrt{c x^2 + d} b c^{\frac{3}{2}} d x^4 + 6(c x^2 + d)^{\frac{3}{2}} a c^{\frac{3}{2}} d x^4 + 4(c x^2 + d)^{\frac{3}{2}} b c^{\frac{3}{2}} x^4 - 6(c x^2 + d)^{\frac{3}{2}} a \sqrt{c} d x^2 - 4(c x^2 + d)^{\frac{3}{2}} b c^{\frac{3}{2}} x^2 - 2(c x^2 + d)^{\frac{3}{2}} b \sqrt{c} d\right)}{6(c x^2 + d)^{\frac{3}{2}} \sqrt{c} d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x,x)

[Out] $\frac{1}{6}((c x^2 + d) / x^2)^{3/2} (4 c^{5/2} (c x^2 + d)^{3/2} x^4 b + 6 c^{5/2} (c x^2 + d)^{1/2} x^4 b d + 6 c^{3/2} (c x^2 + d)^{3/2} x^4 a d - 4 c^{3/2} (c x^2 + d)^{5/2} x^2 b + 9 c^{3/2} (c x^2 + d)^{1/2} x^4 a d^2 - 6 c^{1/2} (c x^2 + d)^{5/2} x^2 a d + 9 \ln(c^{1/2} x + (c x^2 + d)^{1/2}) x^3 a c d^3 + 6 \ln(c^{1/2} x + (c x^2 + d)^{1/2}) x^3 b c^2 d^2 - 2 c^{1/2} (c x^2 + d)^{5/2} b d) / (c x^2 + d)^{3/2} / d^2 / c^{1/2}$

maxima [A] time = 1.23, size = 134, normalized size = 1.22

$$\frac{1}{4} \left(2 \sqrt{c + \frac{d}{x^2}} c x^2 - 3 \sqrt{c} d \log \left(\frac{\sqrt{c + \frac{d}{x^2}} - \sqrt{c}}{\sqrt{c + \frac{d}{x^2}} + \sqrt{c}} \right) - 4 \sqrt{c + \frac{d}{x^2}} d \right) a - \frac{1}{6} \left(3 c^{\frac{3}{2}} \log \left(\frac{\sqrt{c + \frac{d}{x^2}} - \sqrt{c}}{\sqrt{c + \frac{d}{x^2}} + \sqrt{c}} \right) + 2 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} + 6 \sqrt{c + \frac{d}{x^2}} c \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="maxima")

[Out] $\frac{1}{4}(2\sqrt{c + d/x^2} c x^2 - 3\sqrt{c} d \log((\sqrt{c + d/x^2} - \sqrt{c}) / (\sqrt{c + d/x^2} + \sqrt{c}))) - 4\sqrt{c + d/x^2} d a - \frac{1}{6}(3c^{3/2} \log((\sqrt{c + d/x^2} - \sqrt{c}) / (\sqrt{c + d/x^2} + \sqrt{c}))) + 2(c + d/x^2)^{3/2} + 6\sqrt{c + d/x^2} c) b$

mupad [B] time = 5.65, size = 95, normalized size = 0.86

$$b c^{3/2} \operatorname{atanh} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right) - \frac{b \left(c + \frac{d}{x^2} \right)^{3/2}}{3} - a d \sqrt{c + \frac{d}{x^2}} - b c \sqrt{c + \frac{d}{x^2}} + \frac{3 a \sqrt{c} d \operatorname{atanh} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{2} + \frac{a c x^2 \sqrt{c + \frac{d}{x^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b/x^2)*(c + d/x^2)^(3/2),x)`

[Out] $b*c^{(3/2)}*\operatorname{atanh}((c + d/x^2)^{(1/2)}/c^{(1/2)}) - (b*(c + d/x^2)^{(3/2)})/3 - a*d*(c + d/x^2)^{(1/2)} - b*c*(c + d/x^2)^{(1/2)} + (3*a*c^{(1/2)}*d*\operatorname{atanh}((c + d/x^2)^{(1/2)}/c^{(1/2)}))/2 + (a*c*x^2*(c + d/x^2)^{(1/2)})/2$

sympy [A] time = 56.16, size = 187, normalized size = 1.70

$$\frac{3a\sqrt{c}d \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2} + \frac{ac\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2} - \frac{ac\sqrt{d}x}{\sqrt{\frac{cx^2}{d}+1}} - \frac{ad^{\frac{3}{2}}}{x\sqrt{\frac{cx^2}{d}+1}} + bc^{\frac{3}{2}} \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) - \frac{bc^2x}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} - \frac{bc\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}} + bd \begin{cases} -\frac{\sqrt{c}}{2x^2} & \text{for } d = 0 \\ -\left(\frac{c+d}{x^2}\right)^{\frac{3}{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x,x)`

[Out] $3*a*\sqrt{c}*d*\operatorname{asinh}(\sqrt{c}*x/\sqrt{d})/2 + a*c*\sqrt{d}*x*\sqrt{c*x**2/d + 1}/2 - a*c*\sqrt{d}*x/\sqrt{c*x**2/d + 1} - a*d**(3/2)/(x*\sqrt{c*x**2/d + 1}) + b*c**(3/2)*\operatorname{asinh}(\sqrt{c}*x/\sqrt{d}) - b*c**2*x/(\sqrt{d}*\sqrt{c*x**2/d + 1}) - b*c*\sqrt{d}/(x*\sqrt{c*x**2/d + 1}) + b*d*\operatorname{Piecewise}((-sqrt(c)/(2*x**2), \operatorname{Eq}(d, 0)), (-c + d/x**2)**(3/2)/(3*d), \operatorname{True}))$

$$3.602 \quad \int \frac{\left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2}}{x} dx$$

Optimal. Leaf size=76

$$ac^{3/2} \tanh^{-1} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right) - \frac{1}{3} a \left(c + \frac{d}{x^2} \right)^{3/2} - ac \sqrt{c + \frac{d}{x^2}} - \frac{b \left(c + \frac{d}{x^2} \right)^{5/2}}{5d}$$

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 80, 50, 63, 208}

$$ac^{3/2} \tanh^{-1} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right) - \frac{1}{3} a \left(c + \frac{d}{x^2} \right)^{3/2} - ac \sqrt{c + \frac{d}{x^2}} - \frac{b \left(c + \frac{d}{x^2} \right)^{5/2}}{5d}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]

[Out] -(a*c*Sqrt[c + d/x^2]) - (a*(c + d/x^2)^(3/2))/3 - (b*(c + d/x^2)^(5/2))/(5*d) + a*c^(3/2)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p

+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{\left(a + \frac{b}{x^2}\right)\left(c + \frac{d}{x^2}\right)^{3/2}}{x} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{(a + bx)(c + dx)^{3/2}}{x} dx, x, \frac{1}{x^2}\right)\right) \\
 &= -\frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}a \text{Subst}\left(\int \frac{(c + dx)^{3/2}}{x} dx, x, \frac{1}{x^2}\right) \\
 &= -\frac{1}{3}a\left(c + \frac{d}{x^2}\right)^{3/2} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}(ac) \text{Subst}\left(\int \frac{\sqrt{c + dx}}{x} dx, x, \frac{1}{x^2}\right) \\
 &= -ac\sqrt{c + \frac{d}{x^2}} - \frac{1}{3}a\left(c + \frac{d}{x^2}\right)^{3/2} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}(ac^2) \text{Subst}\left(\int \frac{1}{x\sqrt{c + dx}} dx, x, \frac{1}{x^2}\right) \\
 &= -ac\sqrt{c + \frac{d}{x^2}} - \frac{1}{3}a\left(c + \frac{d}{x^2}\right)^{3/2} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{5d} - \frac{(ac^2) \text{Subst}\left(\int \frac{1}{\frac{-c}{-d} + \frac{x^2}{d}} dx, x, \sqrt{c + \frac{d}{x^2}}\right)}{d} \\
 &= -ac\sqrt{c + \frac{d}{x^2}} - \frac{1}{3}a\left(c + \frac{d}{x^2}\right)^{3/2} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{5d} + ac^{3/2} \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)
 \end{aligned}$$

Mathematica [C] time = 0.07, size = 90, normalized size = 1.18

$$\frac{\sqrt{c + \frac{d}{x^2}} \left(5ad^2x^2 {}_2F_1\left(-\frac{3}{2}, -\frac{3}{2}; -\frac{1}{2}; -\frac{cx^2}{d}\right) + 3b(cx^2 + d)^2 \sqrt{\frac{cx^2}{d} + 1} \right)}{15dx^4 \sqrt{\frac{cx^2}{d} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x, x]

[Out] -1/15*(Sqrt[c + d/x^2]*(3*b*(d + c*x^2)^2*Sqrt[1 + (c*x^2)/d] + 5*a*d^2*x^2*Hypergeometric2F1[-3/2, -3/2, -1/2, -(c*x^2)/d]))/(d*x^4*Sqrt[1 + (c*x^2)/d])

IntegrateAlgebraic [A] time = 0.11, size = 96, normalized size = 1.26

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} \left(-20acdx^4 - 5ad^2x^2 - 3bc^2x^4 - 6bcdx^2 - 3bd^2 \right)}{15dx^4} + ac^{3/2} \tanh^{-1} \left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*(c + d/x^2)^(3/2))/x, x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-3*b*d^2 - 6*b*c*d*x^2 - 5*a*d^2*x^2 - 3*b*c^2*x^4 - 20*a*c*d*x^4))/(15*d*x^4) + a*c^(3/2)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]]

fricas [A] time = 0.43, size = 213, normalized size = 2.80

$$\left[\frac{15ac^{\frac{3}{2}}dx^4 \log\left(-2cx^2 - 2\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}} - d\right) - 2\left((3bc^2 + 20acd)x^4 + 3bd^2 + (6bcd + 5ad^2)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{30dx^4}, -\frac{15a\sqrt{-c}cdx^4 \arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + \left((3bc^2 + 20acd)x^4 + 3bd^2 + (6bcd + 5ad^2)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15dx^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x, x, algorithm="fricas")

[Out] [1/30*(15*a*c^(3/2)*d*x^4*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*((3*b*c^2 + 20*a*c*d)*x^4 + 3*b*d^2 + (6*b*c*d + 5*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2)/(d*x^4), -1/15*(15*a*sqrt(-c)*c*d*x^4*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + ((3*b*c^2 + 20*a*c*d)*x^4 + 3*b*d^2 + (6*b*c*d + 5*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2)/(d*x^4)]

giac [B] time = 1.07, size = 254, normalized size = 3.34

$$\frac{\frac{1}{2}ac^{\frac{3}{2}}\log\left(\left(\sqrt{cx - \sqrt{cx^2+d}}\right)^2\right)\operatorname{sgn}(x) + \frac{2\left(15\left(\sqrt{cx - \sqrt{cx^2+d}}\right)^6bc^{\frac{3}{2}}\operatorname{sgn}(x) + 30\left(\sqrt{cx - \sqrt{cx^2+d}}\right)^6ac^{\frac{3}{2}}d\operatorname{sgn}(x) - 90\left(\sqrt{cx - \sqrt{cx^2+d}}\right)^6ac^{\frac{3}{2}}d^2\operatorname{sgn}(x) + 30\left(\sqrt{cx - \sqrt{cx^2+d}}\right)^6bc^{\frac{3}{2}}d^3\operatorname{sgn}(x) + 110\left(\sqrt{cx - \sqrt{cx^2+d}}\right)^6ac^{\frac{3}{2}}d^4\operatorname{sgn}(x) - 70\left(\sqrt{cx - \sqrt{cx^2+d}}\right)^6ac^{\frac{3}{2}}d^5\operatorname{sgn}(x) + 3bc^{\frac{3}{2}}d^6\operatorname{sgn}(x) + 20ac^{\frac{3}{2}}d^6\operatorname{sgn}(x)\right)}{15\left(\sqrt{cx - \sqrt{cx^2+d}}\right)^2 - d}}{15dx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="giac")

[Out] $-1/2*a*c^{(3/2)}*\log((\sqrt{c}*x - \sqrt{c*x^2 + d})^2)*\text{sgn}(x) + 2/15*(15*(\sqrt{c}*x - \sqrt{c*x^2 + d})^8*b*c^{(5/2)}*\text{sgn}(x) + 30*(\sqrt{c}*x - \sqrt{c*x^2 + d})^8*a*c^{(3/2)}*d*\text{sgn}(x) - 90*(\sqrt{c}*x - \sqrt{c*x^2 + d})^6*a*c^{(3/2)}*d^2*\text{sgn}(x) + 30*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*b*c^{(5/2)}*d^2*\text{sgn}(x) + 110*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*a*c^{(3/2)}*d^3*\text{sgn}(x) - 70*(\sqrt{c}*x - \sqrt{c*x^2 + d})^2*a*c^{(3/2)}*d^4*\text{sgn}(x) + 3*b*c^{(5/2)}*d^4*\text{sgn}(x) + 20*a*c^{(3/2)}*d^5*\text{sgn}(x))/((\sqrt{c}*x - \sqrt{c*x^2 + d})^2 - d)^5$

maple [B] time = 0.07, size = 153, normalized size = 2.01

$$\frac{\left(\frac{c x^2+d}{x^2}\right)^{\frac{3}{2}}\left(15 a c^2 d^2 x^5 \ln\left(\sqrt{c} x+\sqrt{c x^2+d}\right)+15 \sqrt{c x^2+d} a c^2 d x^6+10\left(c x^2+d\right)^{\frac{3}{2}} a c^2 x^6-10\left(c x^2+d\right)^{\frac{5}{2}} a c^2 x^4-5\left(c x^2+d\right)^{\frac{5}{2}} a \sqrt{c} d x^2-3\left(c x^2+d\right)^{\frac{5}{2}} b \sqrt{c} d\right)}{15\left(c x^2+d\right)^{\frac{3}{2}} \sqrt{c} d^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x,x)

[Out] $1/15*((c*x^2+d)/x^2)^(3/2)*(10*c^(5/2)*(c*x^2+d)^(3/2)*x^6*a+15*c^(5/2)*(c*x^2+d)^(1/2)*x^6*a*d-10*c^(3/2)*(c*x^2+d)^(5/2)*x^4*a+15*\ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x^5*a*c^2*d^2-5*(c*x^2+d)^(5/2)*a*c^(1/2)*d*x^2-3*(c*x^2+d)^(5/2)*b*c^(1/2)*d)/x^2/(c*x^2+d)^(3/2)/d^2/c^(1/2)$

maxima [A] time = 1.31, size = 80, normalized size = 1.05

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5d}-\frac{1}{6}\left(3c^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}+6\sqrt{c+\frac{d}{x^2}}c\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="maxima")

[Out] $-1/5*b*(c + d/x^2)^(5/2)/d - 1/6*(3*c^(3/2)*\log((\sqrt{c + d/x^2} - \sqrt{c})/(\sqrt{c + d/x^2} + \sqrt{c})) + 2*(c + d/x^2)^(3/2) + 6*\sqrt{c + d/x^2}*c)*a$

mupad [B] time = 5.83, size = 72, normalized size = 0.95

$$a c^{3/2} \operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}}{3}-a c \sqrt{c+\frac{d}{x^2}}-\frac{b \sqrt{c+\frac{d}{x^2}}\left(c x^2+d\right)^2}{5 d x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b/x^2)*(c + d/x^2)^(3/2))/x,x)`

[Out] $a*c^{3/2}*atanh((c + d/x^2)^{1/2}/c^{1/2}) - (a*(c + d/x^2)^{3/2})/3 - a*c*(c + d/x^2)^{1/2} - (b*(c + d/x^2)^{1/2}*(d + c*x^2)^2)/(5*d*x^4)$

sympy [A] time = 54.14, size = 73, normalized size = 0.96

$$-\frac{ac^2 \operatorname{atan}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}} - ac\sqrt{c + \frac{d}{x^2}} - \frac{a\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}{3} - \frac{b\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x,x)`

[Out] $-a*c**2*atan(\operatorname{sqrt}(c + d/x**2)/\operatorname{sqrt}(-c))/\operatorname{sqrt}(-c) - a*c*\operatorname{sqrt}(c + d/x**2) - a*(c + d/x**2)**(3/2)/3 - b*(c + d/x**2)**(5/2)/(5*d)$

$$3.603 \quad \int \frac{\left(a + \frac{b}{x^2}\right)\left(c + \frac{d}{x^2}\right)^{3/2}}{x^3} dx$$

Optimal. Leaf size=46

$$\frac{\left(c + \frac{d}{x^2}\right)^{5/2} (bc - ad)}{5d^2} - \frac{b\left(c + \frac{d}{x^2}\right)^{7/2}}{7d^2}$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {444, 43}

$$\frac{\left(c + \frac{d}{x^2}\right)^{5/2} (bc - ad)}{5d^2} - \frac{b\left(c + \frac{d}{x^2}\right)^{7/2}}{7d^2}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]

[Out] ((b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^2) - (b*(c + d/x^2)^(7/2))/(7*d^2)

Rule 43

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{x^2}\right)\left(c + \frac{d}{x^2}\right)^{3/2}}{x^3} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int (a + bx)(c + dx)^{3/2} dx, x, \frac{1}{x^2}\right)\right) \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \left(\frac{(-bc + ad)(c + dx)^{3/2}}{d} + \frac{b(c + dx)^{5/2}}{d}\right) dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{(bc - ad)\left(c + \frac{d}{x^2}\right)^{5/2}}{5d^2} - \frac{b\left(c + \frac{d}{x^2}\right)^{7/2}}{7d^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.07

$$-\frac{\sqrt{c + \frac{d}{x^2}} (cx^2 + d)^2 (7adx^2 - 2bcx^2 + 5bd)}{35d^2x^6}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]

[Out] -1/35*(Sqrt[c + d/x^2]*(d + c*x^2)^2*(5*b*d - 2*b*c*x^2 + 7*a*d*x^2))/(d^2*x^6)

IntegrateAlgebraic [A] time = 0.06, size = 90, normalized size = 1.96

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (-7ac^2dx^6 - 14acd^2x^4 - 7ad^3x^2 + 2bc^3x^6 - bc^2dx^4 - 8bcd^2x^2 - 5bd^3)}{35d^2x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-5*b*d^3 - 8*b*c*d^2*x^2 - 7*a*d^3*x^2 - b*c^2*d*x^4 - 14*a*c*d^2*x^4 + 2*b*c^3*x^6 - 7*a*c^2*d*x^6))/(35*d^2*x^6)

fricas [B] time = 0.42, size = 84, normalized size = 1.83

$$\frac{\left((2bc^3 - 7ac^2d)x^6 - (bc^2d + 14acd^2)x^4 - 5bd^3 - (8bcd^2 + 7ad^3)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{35d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")

[Out] $1/35*((2*b*c^3 - 7*a*c^2*d)*x^6 - (b*c^2*d + 14*a*c*d^2)*x^4 - 5*b*d^3 - (8*b*c*d^2 + 7*a*d^3)*x^2)*\sqrt{(c*x^2 + d)/x^2}/(d^2*x^6)$

giac [B] time = 1.87, size = 370, normalized size = 8.04

$$\frac{2\left(\sqrt{c}\sqrt{c^2+d}\right)^5 a^5 \operatorname{sgn}(c) + 20\left(\sqrt{c}\sqrt{c^2+d}\right)^4 a^4 \operatorname{sgn}(c) - 70\left(\sqrt{c}\sqrt{c^2+d}\right)^3 a^3 \operatorname{sgn}(c) + 70\left(\sqrt{c}\sqrt{c^2+d}\right)^2 a^2 \operatorname{sgn}(c) + 105\left(\sqrt{c}\sqrt{c^2+d}\right) a \operatorname{sgn}(c) + 140\left(\sqrt{c}\sqrt{c^2+d}\right) a \operatorname{sgn}(c) - 140\left(\sqrt{c}\sqrt{c^2+d}\right) a \operatorname{sgn}(c) + 28\left(\sqrt{c}\sqrt{c^2+d}\right) a \operatorname{sgn}(c) + 77\left(\sqrt{c}\sqrt{c^2+d}\right) a \operatorname{sgn}(c) + 14\left(\sqrt{c}\sqrt{c^2+d}\right) a \operatorname{sgn}(c) - 14\left(\sqrt{c}\sqrt{c^2+d}\right) a \operatorname{sgn}(c) - 23a^5 \operatorname{sgn}(c) + 7a^5 \operatorname{sgn}(c)}{35\left(\sqrt{c}\sqrt{c^2+d}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")`

[Out] $2/35*(35*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{12}*a*c^{(5/2)}*\operatorname{sgn}(x) + 70*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{10}*b*c^{(7/2)}*\operatorname{sgn}(x) - 70*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{10}*a*c^{(5/2)}*d*\operatorname{sgn}(x) + 70*(\sqrt{c}*x - \sqrt{c*x^2 + d})^8*b*c^{(7/2)}*d*\operatorname{sgn}(x) + 105*(\sqrt{c}*x - \sqrt{c*x^2 + d})^8*a*c^{(5/2)}*d^2*\operatorname{sgn}(x) + 140*(\sqrt{c}*x - \sqrt{c*x^2 + d})^6*b*c^{(7/2)}*d^2*\operatorname{sgn}(x) - 140*(\sqrt{c}*x - \sqrt{c*x^2 + d})^6*a*c^{(5/2)}*d^3*\operatorname{sgn}(x) + 28*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*b*c^{(7/2)}*d^3*\operatorname{sgn}(x) + 77*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*a*c^{(5/2)}*d^4*\operatorname{sgn}(x) + 14*(\sqrt{c}*x - \sqrt{c*x^2 + d})^2*b*c^{(7/2)}*d^4*\operatorname{sgn}(x) - 14*(\sqrt{c}*x - \sqrt{c*x^2 + d})^2*a*c^{(5/2)}*d^5*\operatorname{sgn}(x) - 2*b*c^{(7/2)}*d^5*\operatorname{sgn}(x) + 7*a*c^{(5/2)}*d^6*\operatorname{sgn}(x))/((\sqrt{c}*x - \sqrt{c*x^2 + d})^2 - d)^7$

maple [A] time = 0.05, size = 48, normalized size = 1.04

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}(7adx^2 - 2bcx^2 + 5bd)(cx^2 + d)}{35d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x)`

[Out] $-1/35*((c*x^2+d)/x^2)^{(3/2)}*(7*a*d*x^2-2*b*c*x^2+5*b*d)*(c*x^2+d)/d^2/x^4$

maxima [A] time = 0.59, size = 49, normalized size = 1.07

$$-\frac{a\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}}{5d} - \frac{1}{35} \left(\frac{5\left(c + \frac{d}{x^2}\right)^{\frac{7}{2}}}{d^2} - \frac{7\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^2} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")`

[Out] $-1/5*a*(c + d/x^2)^{(5/2)}/d - 1/35*(5*(c + d/x^2)^{(7/2)}/d^2 - 7*(c + d/x^2)^{(5/2)}*c/d^2)*b$

mupad [B] time = 5.33, size = 122, normalized size = 2.65

$$\frac{2bc^3\sqrt{c+\frac{d}{x^2}}}{35d^2} - \frac{ac^2\sqrt{c+\frac{d}{x^2}}}{5d} - \frac{2ac\sqrt{c+\frac{d}{x^2}}}{5x^2} - \frac{ad\sqrt{c+\frac{d}{x^2}}}{5x^4} - \frac{8bc\sqrt{c+\frac{d}{x^2}}}{35x^4} - \frac{bd\sqrt{c+\frac{d}{x^2}}}{7x^6} - \frac{bc^2\sqrt{c+\frac{d}{x^2}}}{35dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x)`

[Out] $(2*b*c^3*(c + d/x^2)^{(1/2)})/(35*d^2) - (a*c^2*(c + d/x^2)^{(1/2)})/(5*d) - (2*a*c*(c + d/x^2)^{(1/2)})/(5*x^2) - (a*d*(c + d/x^2)^{(1/2)})/(5*x^4) - (8*b*c*(c + d/x^2)^{(1/2)})/(35*x^4) - (b*d*(c + d/x^2)^{(1/2)})/(7*x^6) - (b*c^2*(c + d/x^2)^{(1/2)})/(35*d*x^2)$

sympy [A] time = 14.16, size = 138, normalized size = 3.00

$$\frac{ac \begin{cases} \frac{\sqrt{c}}{x^2} & \text{for } d = 0 \\ \frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise} \end{cases}}{2} - \frac{a \left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} + \frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5} \right)}{d} - \frac{bc \left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} + \frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5} \right)}{d^2} - \frac{b \left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} - \frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5} + \frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7} \right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**3,x)`

[Out] $-a*c*\text{Piecewise}(\left(\text{sqrt}(c)/x**2, \text{Eq}(d, 0)\right), (2*(c + d/x**2)**(3/2))/(3*d), \text{True}))/2 - a*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d - b*c*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - b*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**2$

$$3.604 \quad \int \frac{\left(a + \frac{b}{x^2}\right)\left(c + \frac{d}{x^2}\right)^{3/2}}{x^5} dx$$

Optimal. Leaf size=74

$$\frac{\left(c + \frac{d}{x^2}\right)^{7/2} (2bc - ad)}{7d^3} - \frac{c \left(c + \frac{d}{x^2}\right)^{5/2} (bc - ad)}{5d^3} - \frac{b \left(c + \frac{d}{x^2}\right)^{9/2}}{9d^3}$$

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{\left(c + \frac{d}{x^2}\right)^{7/2} (2bc - ad)}{7d^3} - \frac{c \left(c + \frac{d}{x^2}\right)^{5/2} (bc - ad)}{5d^3} - \frac{b \left(c + \frac{d}{x^2}\right)^{9/2}}{9d^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]

[Out] -(c*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^3) + ((2*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^3) - (b*(c + d/x^2)^(9/2))/(9*d^3)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{x^2}\right)\left(c + \frac{d}{x^2}\right)^{3/2}}{x^5} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int x(a + bx)(c + dx)^{3/2} dx, x, \frac{1}{x^2}\right)\right) \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \left(\frac{c(bc - ad)(c + dx)^{3/2}}{d^2} + \frac{(-2bc + ad)(c + dx)^{5/2}}{d^2} + \frac{b(c + dx)^{7/2}}{d^2}\right) dx, x, \frac{1}{x^2}\right)\right) \\
&= -\frac{c(bc - ad)\left(c + \frac{d}{x^2}\right)^{5/2}}{5d^3} + \frac{(2bc - ad)\left(c + \frac{d}{x^2}\right)^{7/2}}{7d^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{9/2}}{9d^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 0.96

$$\frac{\sqrt{c + \frac{d}{x^2}} (cx^2 + d)^2 (9adx^2 (2cx^2 - 5d) + b(-8c^2x^4 + 20cdx^2 - 35d^2))}{315d^3x^8}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5, x]

[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(9*a*d*x^2*(-5*d + 2*c*x^2) + b*(-35*d^2 + 20*c*d*x^2 - 8*c^2*x^4)))/(315*d^3*x^8)

IntegrateAlgebraic [A] time = 0.07, size = 114, normalized size = 1.54

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (18ac^3dx^8 - 9ac^2d^2x^6 - 72acd^3x^4 - 45ad^4x^2 - 8bc^4x^8 + 4bc^3dx^6 - 3bc^2d^2x^4 - 50bcd^3x^2 - 35bd^4)}{315d^3x^8}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5, x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-35*b*d^4 - 50*b*c*d^3*x^2 - 45*a*d^4*x^2 - 3*b*c^2*d^2*x^4 - 72*a*c*d^3*x^4 + 4*b*c^3*d*x^6 - 9*a*c^2*d^2*x^6 - 8*b*c^4*x^8 + 18*a*c^3*d*x^8))/(315*d^3*x^8)

fricas [A] time = 0.48, size = 109, normalized size = 1.47

$$\frac{(2(4bc^4 - 9ac^3d)x^8 - (4bc^3d - 9ac^2d^2)x^6 + 35bd^4 + 3(bc^2d^2 + 24acd^3)x^4 + 5(10bcd^3 + 9ad^4)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{315d^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5, x, algorithm="fricas")

[Out] $-1/315*(2*(4*b*c^4 - 9*a*c^3*d)*x^8 - (4*b*c^3*d - 9*a*c^2*d^2)*x^6 + 35*b*d^4 + 3*(b*c^2*d^2 + 24*a*c*d^3)*x^4 + 5*(10*b*c*d^3 + 9*a*d^4)*x^2)*\sqrt{(c*x^2 + d)/x^2}/(d^3*x^8)$

giac [B] time = 2.69, size = 430, normalized size = 5.81

$\frac{(\sqrt{c} \sqrt{c^2 + d})^{14} \sqrt{c} \sqrt{c^2 + d} + 840 (\sqrt{c} \sqrt{c^2 + d})^{13} \sqrt{c} \sqrt{c^2 + d} + 1260 (\sqrt{c} \sqrt{c^2 + d})^{12} \sqrt{c} \sqrt{c^2 + d} + 1764 (\sqrt{c} \sqrt{c^2 + d})^{11} \sqrt{c} \sqrt{c^2 + d} + 1260 (\sqrt{c} \sqrt{c^2 + d})^{10} \sqrt{c} \sqrt{c^2 + d} + 840 (\sqrt{c} \sqrt{c^2 + d})^9 \sqrt{c} \sqrt{c^2 + d} + 315 (\sqrt{c} \sqrt{c^2 + d})^8 \sqrt{c} \sqrt{c^2 + d} + 1260 (\sqrt{c} \sqrt{c^2 + d})^7 \sqrt{c} \sqrt{c^2 + d} + 1764 (\sqrt{c} \sqrt{c^2 + d})^6 \sqrt{c} \sqrt{c^2 + d} + 1260 (\sqrt{c} \sqrt{c^2 + d})^5 \sqrt{c} \sqrt{c^2 + d} + 840 (\sqrt{c} \sqrt{c^2 + d})^4 \sqrt{c} \sqrt{c^2 + d} + 315 (\sqrt{c} \sqrt{c^2 + d})^3 \sqrt{c} \sqrt{c^2 + d} + 1260 (\sqrt{c} \sqrt{c^2 + d})^2 \sqrt{c} \sqrt{c^2 + d} + 840 (\sqrt{c} \sqrt{c^2 + d}) \sqrt{c} \sqrt{c^2 + d} + 315 \sqrt{c} \sqrt{c^2 + d}}{315 d^3 x^6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")`

[Out] $4/315*(315*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{14}*a*c^{(7/2)}*\text{sgn}(x) + 840*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{12}*b*c^{(9/2)}*\text{sgn}(x) - 315*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{10}*a*c^{(7/2)}*d*\text{sgn}(x) + 1260*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{10}*b*c^{(9/2)}*d*\text{sgn}(x) + 315*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{10}*a*c^{(7/2)}*d^2*\text{sgn}(x) + 1764*(\sqrt{c}*x - \sqrt{c*x^2 + d})^8*b*c^{(9/2)}*d^2*\text{sgn}(x) - 819*(\sqrt{c}*x - \sqrt{c*x^2 + d})^8*a*c^{(7/2)}*d^3*\text{sgn}(x) + 504*(\sqrt{c}*x - \sqrt{c*x^2 + d})^6*b*c^{(9/2)}*d^3*\text{sgn}(x) + 441*(\sqrt{c}*x - \sqrt{c*x^2 + d})^6*a*c^{(7/2)}*d^4*\text{sgn}(x) + 144*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*b*c^{(9/2)}*d^4*\text{sgn}(x) - 9*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*a*c^{(7/2)}*d^5*\text{sgn}(x) - 36*(\sqrt{c}*x - \sqrt{c*x^2 + d})^2*b*c^{(9/2)}*d^5*\text{sgn}(x) + 81*(\sqrt{c}*x - \sqrt{c*x^2 + d})^2*a*c^{(7/2)}*d^6*\text{sgn}(x) + 4*b*c^{(9/2)}*d^6*\text{sgn}(x) - 9*a*c^{(7/2)}*d^7*\text{sgn}(x))/((\sqrt{c}*x - \sqrt{c*x^2 + d})^2 - d)^9$

maple [A] time = 0.05, size = 70, normalized size = 0.95

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}} (18acd x^4 - 8b c^2 x^4 - 45a d^2 x^2 + 20bcd x^2 - 35b d^2) (cx^2 + d)}{315d^3 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x)`

[Out] $1/315*((c*x^2+d)/x^2)^{(3/2)}*(18*a*c*d*x^4-8*b*c^2*x^4-45*a*d^2*x^2+20*b*c*d*x^2-35*b*d^2)*(c*x^2+d)/d^3/x^6$

maxima [A] time = 0.64, size = 84, normalized size = 1.14

$$-\frac{1}{35} \left(\frac{5 \left(c + \frac{d}{x^2} \right)^{\frac{7}{2}}}{d^2} - \frac{7 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}} c}{d^2} \right) a - \frac{1}{315} \left(\frac{35 \left(c + \frac{d}{x^2} \right)^{\frac{9}{2}}}{d^3} - \frac{90 \left(c + \frac{d}{x^2} \right)^{\frac{7}{2}} c}{d^3} + \frac{63 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}} c^2}{d^3} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")

[Out] $-1/35*(5*(c + d/x^2)^{(7/2)}/d^2 - 7*(c + d/x^2)^{(5/2)}*c/d^2)*a - 1/315*(35*(c + d/x^2)^{(9/2)}/d^3 - 90*(c + d/x^2)^{(7/2)}*c/d^3 + 63*(c + d/x^2)^{(5/2)}*c^2/d^3)*b$

mupad [B] time = 5.76, size = 164, normalized size = 2.22

$$\frac{2ac^3\sqrt{c+\frac{d}{x^2}}}{35d^2} - \frac{8bc^4\sqrt{c+\frac{d}{x^2}}}{315d^3} - \frac{8ac\sqrt{c+\frac{d}{x^2}}}{35x^4} - \frac{ad\sqrt{c+\frac{d}{x^2}}}{7x^6} - \frac{10bc\sqrt{c+\frac{d}{x^2}}}{63x^6} - \frac{bd\sqrt{c+\frac{d}{x^2}}}{9x^8} - \frac{ac^2\sqrt{c+\frac{d}{x^2}}}{35dx^2} - \frac{bc^2\sqrt{c+\frac{d}{x^2}}}{105dx^4} + \frac{4bc^3\sqrt{c+\frac{d}{x^2}}}{315d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x)

[Out] $(2*a*c^3*(c + d/x^2)^{(1/2)})/(35*d^2) - (8*b*c^4*(c + d/x^2)^{(1/2)})/(315*d^3) - (8*a*c*(c + d/x^2)^{(1/2)})/(35*x^4) - (a*d*(c + d/x^2)^{(1/2)})/(7*x^6) - (10*b*c*(c + d/x^2)^{(1/2)})/(63*x^6) - (b*d*(c + d/x^2)^{(1/2)})/(9*x^8) - (a*c^2*(c + d/x^2)^{(1/2)})/(35*d*x^2) - (b*c^2*(c + d/x^2)^{(1/2)})/(105*d*x^4) + (4*b*c^3*(c + d/x^2)^{(1/2)})/(315*d^2*x^2)$

sympy [B] time = 15.61, size = 194, normalized size = 2.62

$$\frac{ac\left(-\frac{c\left(\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} + \frac{\left(\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} - \frac{a\left(\frac{c^2\left(\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} - \frac{2c\left(\frac{d}{x^2}\right)^{\frac{5}{2}}}{5} + \frac{\left(\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^2} - \frac{bc\left(\frac{c^2\left(\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} - \frac{2c\left(\frac{d}{x^2}\right)^{\frac{5}{2}}}{5} + \frac{\left(\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} - \frac{b\left(-\frac{c^3\left(\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} + \frac{3c^2\left(\frac{d}{x^2}\right)^{\frac{5}{2}}}{5} - \frac{3c\left(\frac{d}{x^2}\right)^{\frac{7}{2}}}{7} + \frac{\left(\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**5,x)

[Out] $-a*c*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - a*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**2 - b*c*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3 - b*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**3$

$$3.605 \quad \int \frac{\left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2}}{x^7} dx$$

Optimal. Leaf size=104

$$\frac{c^2 \left(c + \frac{d}{x^2}\right)^{5/2} (bc - ad)}{5d^4} + \frac{\left(c + \frac{d}{x^2}\right)^{9/2} (3bc - ad)}{9d^4} - \frac{c \left(c + \frac{d}{x^2}\right)^{7/2} (3bc - 2ad)}{7d^4} - \frac{b \left(c + \frac{d}{x^2}\right)^{11/2}}{11d^4}$$

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{c^2 \left(c + \frac{d}{x^2}\right)^{5/2} (bc - ad)}{5d^4} + \frac{\left(c + \frac{d}{x^2}\right)^{9/2} (3bc - ad)}{9d^4} - \frac{c \left(c + \frac{d}{x^2}\right)^{7/2} (3bc - 2ad)}{7d^4} - \frac{b \left(c + \frac{d}{x^2}\right)^{11/2}}{11d^4}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7, x]

[Out] (c^2*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (c*(3*b*c - 2*a*d)*(c + d/x^2)^(7/2))/(7*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(9/2))/(9*d^4) - (b*(c + d/x^2)^(11/2))/(11*d^4)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{x^2}\right)\left(c + \frac{d}{x^2}\right)^{3/2}}{x^7} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int x^2(a + bx)(c + dx)^{3/2} dx, x, \frac{1}{x^2}\right)\right) \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \left(-\frac{c^2(bc - ad)(c + dx)^{3/2}}{d^3} + \frac{c(3bc - 2ad)(c + dx)^{5/2}}{d^3} + \frac{(-3bc + ad)(c + dx)^{7/2}}{d^3}\right) dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{c^2(bc - ad)\left(c + \frac{d}{x^2}\right)^{5/2}}{5d^4} - \frac{c(3bc - 2ad)\left(c + \frac{d}{x^2}\right)^{7/2}}{7d^4} + \frac{(3bc - ad)\left(c + \frac{d}{x^2}\right)^{9/2}}{9d^4} - \frac{b\left(c + \frac{d}{x^2}\right)^{11/2}}{11d^4}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 94, normalized size = 0.90

$$\frac{\sqrt{c + \frac{d}{x^2}} (cx^2 + d)^2 (-11adx^2 (8c^2x^4 - 20cdx^2 + 35d^2) - 3b (-16c^3x^6 + 40c^2dx^4 - 70cd^2x^2 + 105d^3))}{3465d^4x^{10}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7, x]

[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(-11*a*d*x^2*(35*d^2 - 20*c*d*x^2 + 8*c^2*x^4) - 3*b*(105*d^3 - 70*c*d^2*x^2 + 40*c^2*d*x^4 - 16*c^3*x^6)))/(3465*d^4*x^10)

IntegrateAlgebraic [A] time = 0.08, size = 138, normalized size = 1.33

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (-88ac^4dx^{10} + 44ac^3d^2x^8 - 33ac^2d^3x^6 - 550acd^4x^4 - 385ad^5x^2 + 48bc^5x^{10} - 24bc^4dx^8 + 18bc^3d^2x^6 - 15bc^2d^3x^4 - 420bcd^4x^2 - 315bd^5)}{3465d^4x^{10}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7, x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-315*b*d^5 - 420*b*c*d^4*x^2 - 385*a*d^5*x^2 - 15*b*c^2*d^3*x^4 - 550*a*c*d^4*x^4 + 18*b*c^3*d^2*x^6 - 33*a*c^2*d^3*x^6 - 24*b*c^4*d*x^8 + 44*a*c^3*d^2*x^8 + 48*b*c^5*x^10 - 88*a*c^4*d*x^10))/(3465*d^4*x^10)

fricas [A] time = 0.54, size = 134, normalized size = 1.29

$$\frac{(8(6bc^5 - 11ac^4d)x^{10} - 4(6bc^4d - 11ac^3d^2)x^8 + 3(6bc^3d^2 - 11ac^2d^3)x^6 - 315bd^5 - 5(3bc^2d^3 + 110acd^4)x^4 - 35(12bcd^4 + 11ad^5)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{3465d^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")

[Out] 1/3465*(8*(6*b*c^5 - 11*a*c^4*d)*x^10 - 4*(6*b*c^4*d - 11*a*c^3*d^2)*x^8 + 3*(6*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 - 315*b*d^5 - 5*(3*b*c^2*d^3 + 110*a*c*d^4)*x^4 - 35*(12*b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^10)

giac [B] time = 3.28, size = 490, normalized size = 4.71

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")

[Out] 16/3465*(2310*(sqrt(c)*x - sqrt(c*x^2 + d))^16*a*c^(9/2)*sgn(x) + 6930*(sqrt(c)*x - sqrt(c*x^2 + d))^14*b*c^(11/2)*sgn(x) - 1155*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(9/2)*d*sgn(x) + 12474*(sqrt(c)*x - sqrt(c*x^2 + d))^12*b*c^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(9/2)*d^2*sgn(x) + 15246*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(11/2)*d^2*sgn(x) - 4851*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^3*sgn(x) + 4950*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(11/2)*d^3*sgn(x) + 2475*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(9/2)*d^4*sgn(x) + 990*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(11/2)*d^4*sgn(x) + 495*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(9/2)*d^5*sgn(x) - 330*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*d^5*sgn(x) + 605*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(9/2)*d^6*sgn(x) + 66*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(11/2)*d^6*sgn(x) - 121*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(9/2)*d^7*sgn(x) - 6*b*c^(11/2)*d^7*sgn(x) + 11*a*c^(9/2)*d^8*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^11

maple [A] time = 0.05, size = 94, normalized size = 0.90

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}} \left(88a^2cdx^6 - 48bc^3x^6 - 220acd^2x^4 + 120b^2c^2dx^4 + 385ad^3x^2 - 210bcd^2x^2 + 315bd^3\right) (cx^2 + d)}{3465d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x)

[Out] -1/3465*((c*x^2+d)/x^2)^(3/2)*(88*a*c^2*d*x^6-48*b*c^3*x^6-220*a*c*d^2*x^4+120*b*c^2*d*x^4+385*a*d^3*x^2-210*b*c*d^2*x^2+315*b*d^3)*(c*x^2+d)/d^4/x^8

maxima [A] time = 0.58, size = 118, normalized size = 1.13

$$-\frac{1}{315} \left(\frac{35 \left(c + \frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3} - \frac{90 \left(c + \frac{d}{x^2}\right)^{\frac{7}{2}} c}{d^3} + \frac{63 \left(c + \frac{d}{x^2}\right)^{\frac{5}{2}} c^2}{d^3} \right) a - \frac{1}{1155} \left(\frac{105 \left(c + \frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4} - \frac{385 \left(c + \frac{d}{x^2}\right)^{\frac{9}{2}} c}{d^4} + \frac{495 \left(c + \frac{d}{x^2}\right)^{\frac{7}{2}} c^2}{d^4} - \frac{231 \left(c + \frac{d}{x^2}\right)^{\frac{5}{2}} c^3}{d^4} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")

[Out] $-1/315*(35*(c + d/x^2)^{(9/2)}/d^3 - 90*(c + d/x^2)^{(7/2)}*c/d^3 + 63*(c + d/x^2)^{(5/2)}*c^2/d^3)*a - 1/1155*(105*(c + d/x^2)^{(11/2)}/d^4 - 385*(c + d/x^2)^{(9/2)}*c/d^4 + 495*(c + d/x^2)^{(7/2)}*c^2/d^4 - 231*(c + d/x^2)^{(5/2)}*c^3/d^4)*b$

mupad [B] time = 6.31, size = 206, normalized size = 1.98

$$\frac{16bc^5\sqrt{c+\frac{d}{x^2}}}{1155d^4} - \frac{8ac^4\sqrt{c+\frac{d}{x^2}}}{315d^3} - \frac{10ac\sqrt{c+\frac{d}{x^2}}}{63x^6} - \frac{ad\sqrt{c+\frac{d}{x^2}}}{9x^8} - \frac{4bc\sqrt{c+\frac{d}{x^2}}}{33x^8} - \frac{bd\sqrt{c+\frac{d}{x^2}}}{11x^{10}} - \frac{ac^2\sqrt{c+\frac{d}{x^2}}}{105dx^4} + \frac{4ac^3\sqrt{c+\frac{d}{x^2}}}{315d^2x^2} - \frac{bc^2\sqrt{c+\frac{d}{x^2}}}{231dx^6} + \frac{2bc^3\sqrt{c+\frac{d}{x^2}}}{385d^2x^4} - \frac{8bc^4\sqrt{c+\frac{d}{x^2}}}{1155d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x)

[Out] $(16*b*c^5*(c + d/x^2)^{(1/2)})/(1155*d^4) - (8*a*c^4*(c + d/x^2)^{(1/2)})/(315*d^3) - (10*a*c*(c + d/x^2)^{(1/2)})/(63*x^6) - (a*d*(c + d/x^2)^{(1/2)})/(9*x^8) - (4*b*c*(c + d/x^2)^{(1/2)})/(33*x^8) - (b*d*(c + d/x^2)^{(1/2)})/(11*x^{10}) - (a*c^2*(c + d/x^2)^{(1/2)})/(105*d*x^4) + (4*a*c^3*(c + d/x^2)^{(1/2)})/(315*d^2*x^2) - (b*c^2*(c + d/x^2)^{(1/2)})/(231*d*x^6) + (2*b*c^3*(c + d/x^2)^{(1/2)})/(385*d^2*x^4) - (8*b*c^4*(c + d/x^2)^{(1/2)})/(1155*d^3*x^2)$

sympy [B] time = 17.96, size = 262, normalized size = 2.52

$$\frac{ac\left(\frac{c^3\left(\frac{c+d}{x^2}\right)^{\frac{3}{2}}}{3} - \frac{2c\left(\frac{c+d}{x^2}\right)^{\frac{5}{2}}}{5} + \frac{\left(\frac{c+d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} - \frac{a\left(\frac{c^3\left(\frac{c+d}{x^2}\right)^{\frac{3}{2}}}{3} + \frac{3c^2\left(\frac{c+d}{x^2}\right)^{\frac{5}{2}}}{5} - \frac{3c\left(\frac{c+d}{x^2}\right)^{\frac{7}{2}}}{7} + \frac{\left(\frac{c+d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^3} - \frac{bc\left(\frac{c^3\left(\frac{c+d}{x^2}\right)^{\frac{3}{2}}}{3} + \frac{3c^2\left(\frac{c+d}{x^2}\right)^{\frac{5}{2}}}{5} - \frac{3c\left(\frac{c+d}{x^2}\right)^{\frac{7}{2}}}{7} + \frac{\left(\frac{c+d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} - \frac{b\left(\frac{c^4\left(\frac{c+d}{x^2}\right)^{\frac{3}{2}}}{3} - \frac{4c^3\left(\frac{c+d}{x^2}\right)^{\frac{5}{2}}}{5} + \frac{6c^2\left(\frac{c+d}{x^2}\right)^{\frac{7}{2}}}{7} - \frac{4c\left(\frac{c+d}{x^2}\right)^{\frac{9}{2}}}{9} + \frac{\left(\frac{c+d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**7,x)

[Out] $-a*c*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3 - a*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**3 - b*c*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - b*(c**4*(c + d/x**2)**(3/2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**4$

$$3.606 \quad \int \frac{\left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2}}{x^9} dx$$

Optimal. Leaf size=134

$$\frac{c^3 \left(c + \frac{d}{x^2}\right)^{5/2} (bc - ad)}{5d^5} + \frac{c^2 \left(c + \frac{d}{x^2}\right)^{7/2} (4bc - 3ad)}{7d^5} + \frac{\left(c + \frac{d}{x^2}\right)^{11/2} (4bc - ad)}{11d^5} - \frac{c \left(c + \frac{d}{x^2}\right)^{9/2} (2bc - ad)}{3d^5} - \frac{b \left(c + \frac{d}{x^2}\right)^{13/2}}{13d^5}$$

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{c^2 \left(c + \frac{d}{x^2}\right)^{7/2} (4bc - 3ad)}{7d^5} - \frac{c^3 \left(c + \frac{d}{x^2}\right)^{5/2} (bc - ad)}{5d^5} + \frac{\left(c + \frac{d}{x^2}\right)^{11/2} (4bc - ad)}{11d^5} - \frac{c \left(c + \frac{d}{x^2}\right)^{9/2} (2bc - ad)}{3d^5} - \frac{b \left(c + \frac{d}{x^2}\right)^{13/2}}{13d^5}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]

[Out] $-(c^3(b*c - a*d)*(c + d/x^2)^{(5/2)})/(5*d^5) + (c^2*(4*b*c - 3*a*d)*(c + d/x^2)^{(7/2)})/(7*d^5) - (c*(2*b*c - a*d)*(c + d/x^2)^{(9/2)})/(3*d^5) + ((4*b*c - a*d)*(c + d/x^2)^{(11/2)})/(11*d^5) - (b*(c + d/x^2)^{(13/2)})/(13*d^5)$

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{x^2}\right)\left(c + \frac{d}{x^2}\right)^{3/2}}{x^9} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int x^3(a + bx)(c + dx)^{3/2} dx, x, \frac{1}{x^2}\right)\right) \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \left(\frac{c^3(bc - ad)(c + dx)^{3/2}}{d^4} - \frac{c^2(4bc - 3ad)(c + dx)^{5/2}}{d^4} + \frac{3c(2bc - ad)(c + dx)^{7/2}}{d^4}\right) dx, x, \frac{1}{x^2}\right)\right) \\
&= -\frac{c^3(bc - ad)\left(c + \frac{d}{x^2}\right)^{5/2}}{5d^5} + \frac{c^2(4bc - 3ad)\left(c + \frac{d}{x^2}\right)^{7/2}}{7d^5} - \frac{c(2bc - ad)\left(c + \frac{d}{x^2}\right)^{9/2}}{3d^5} + \dots
\end{aligned}$$

Mathematica [A] time = 0.06, size = 115, normalized size = 0.86

$$\frac{\sqrt{c + \frac{d}{x^2}} (cx^2 + d)^2 (13adx^2(16c^3x^6 - 40c^2dx^4 + 70cd^2x^2 - 105d^3) + b(-128c^4x^8 + 320c^3dx^6 - 560c^2d^2x^4 + 840cd^3x^2 - 1155d^4))}{15015d^5x^{12}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9, x]

[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(13*a*d*x^2*(-105*d^3 + 70*c*d^2*x^2 - 40*c^2*d*x^4 + 16*c^3*x^6) + b*(-1155*d^4 + 840*c*d^3*x^2 - 560*c^2*d^2*x^4 + 320*c^3*d*x^6 - 128*c^4*x^8)))/(15015*d^5*x^12)

IntegrateAlgebraic [A] time = 0.09, size = 162, normalized size = 1.21

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (208ac^5dx^{12} - 104ac^4d^2x^{10} + 78ac^3d^3x^8 - 65ac^2d^4x^6 - 1820acd^5x^4 - 1365ad^6x^2 - 128bc^6x^{12} + 64bc^5dx^{10} - 48bc^4d^2x^8 + 40bc^3d^3x^6 - 35bc^2d^4x^4 - 1470bcd^5x^2 - 1155bd^6)}{15015d^5x^{12}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9, x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-1155*b*d^6 - 1470*b*c*d^5*x^2 - 1365*a*d^6*x^2 - 35*b*c^2*d^4*x^4 - 1820*a*c*d^5*x^4 + 40*b*c^3*d^3*x^6 - 65*a*c^2*d^4*x^6 - 48*b*c^4*d^2*x^8 + 78*a*c^3*d^3*x^8 + 64*b*c^5*d*x^10 - 104*a*c^4*d^2*x^10 - 128*b*c^6*x^12 + 208*a*c^5*d*x^12))/(15015*d^5*x^12)

fricas [A] time = 0.61, size = 157, normalized size = 1.17

$$\frac{(16(8bc^6 - 13ac^5d)x^{12} - 8(8bc^5d - 13ac^4d^2)x^{10} + 6(8bc^4d^2 - 13ac^3d^3)x^8 + 1155bd^6 - 5(8bc^3d^3 - 13ac^2d^4)x^6 + 35(bc^2d^4 + 52acd^5)x^4 + 105(14bcd^5 + 13ad^6)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{15015d^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9, x, algorithm="fricas")

[Out] $-1/15015*(16*(8*b*c^6 - 13*a*c^5*d)*x^{12} - 8*(8*b*c^5*d - 13*a*c^4*d^2)*x^{10} + 6*(8*b*c^4*d^2 - 13*a*c^3*d^3)*x^8 + 1155*b*d^6 - 5*(8*b*c^3*d^3 - 13*a*c^2*d^4)*x^6 + 35*(b*c^2*d^4 + 52*a*c*d^5)*x^4 + 105*(14*b*c*d^5 + 13*a*d^6)*x^2)*\sqrt{(c*x^2 + d)/x^2}/(d^5*x^{12})$

giac [B] time = 4.63, size = 550, normalized size = 4.10

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")`

[Out] $32/15015*(15015*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{18}*a*c^{(11/2)}*\text{sgn}(x) + 48048*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{16}*b*c^{(13/2)}*\text{sgn}(x) - 3003*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{16}*a*c^{(11/2)}*d*\text{sgn}(x) + 96096*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{14}*b*c^{(13/2)}*d*\text{sgn}(x) - 6006*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{14}*a*c^{(11/2)}*d^2*\text{sgn}(x) + 109824*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{12}*b*c^{(13/2)}*d^2*\text{sgn}(x) - 28314*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{12}*a*c^{(11/2)}*d^3*\text{sgn}(x) + 37752*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{10}*b*c^{(13/2)}*d^3*\text{sgn}(x) + 13728*(\sqrt{c}*x - \sqrt{c*x^2 + d})^{10}*a*c^{(11/2)}*d^4*\text{sgn}(x) + 5720*(\sqrt{c}*x - \sqrt{c*x^2 + d})^8*b*c^{(13/2)}*d^4*\text{sgn}(x) + 5720*(\sqrt{c}*x - \sqrt{c*x^2 + d})^8*a*c^{(11/2)}*d^5*\text{sgn}(x) - 2288*(\sqrt{c}*x - \sqrt{c*x^2 + d})^6*b*c^{(13/2)}*d^5*\text{sgn}(x) + 3718*(\sqrt{c}*x - \sqrt{c*x^2 + d})^6*a*c^{(11/2)}*d^6*\text{sgn}(x) + 624*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*b*c^{(13/2)}*d^6*\text{sgn}(x) - 1014*(\sqrt{c}*x - \sqrt{c*x^2 + d})^4*a*c^{(11/2)}*d^7*\text{sgn}(x) - 104*(\sqrt{c}*x - \sqrt{c*x^2 + d})^2*b*c^{(13/2)}*d^7*\text{sgn}(x) + 169*(\sqrt{c}*x - \sqrt{c*x^2 + d})^2*a*c^{(11/2)}*d^8*\text{sgn}(x) + 8*b*c^{(13/2)}*d^8*\text{sgn}(x) - 13*a*c^{(11/2)}*d^9*\text{sgn}(x))/((\sqrt{c}*x - \sqrt{c*x^2 + d})^2 - d)^{13}$

maple [A] time = 0.05, size = 118, normalized size = 0.88

$$\frac{\left(\frac{c x^2+d}{x^2}\right)^{\frac{3}{2}}\left(208 a^3 c^3 d x^8-128 b c^4 x^8-520 a c^2 d^2 x^6+320 b c^3 d x^6+910 a c d^3 x^4-560 b c^2 d^2 x^4-1365 a d^4 x^2+840 b c d^3 x^2-1155 b d^4\right)\left(c x^2+d\right)}{15015 d^5 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x)`

[Out] $1/15015*((c*x^2+d)/x^2)^(3/2)*(208*a*c^3*d*x^8-128*b*c^4*x^8-520*a*c^2*d^2*x^6+320*b*c^3*d*x^6+910*a*c*d^3*x^4-560*b*c^2*d^2*x^4-1365*a*d^4*x^2+840*b*c*d^3*x^2-1155*b*d^4)*(c*x^2+d)/d^5/x^{10}$

maxima [A] time = 0.63, size = 152, normalized size = 1.13

$$-\frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4}-\frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4}+\frac{495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^4}-\frac{231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^4}\right)a-\frac{1}{15015}\left(\frac{1155\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}}{d^5}-\frac{5460\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}c}{d^5}+\frac{10010\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c^2}{d^5}-\frac{8580\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^3}{d^5}+\frac{3003\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^4}{d^5}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")

[Out] $-1/1155*(105*(c + d/x^2)^{(11/2)}/d^4 - 385*(c + d/x^2)^{(9/2)}*c/d^4 + 495*(c + d/x^2)^{(7/2)}*c^2/d^4 - 231*(c + d/x^2)^{(5/2)}*c^3/d^4)*a - 1/15015*(1155*(c + d/x^2)^{(13/2)}/d^5 - 5460*(c + d/x^2)^{(11/2)}*c/d^5 + 10010*(c + d/x^2)^{(9/2)}*c^2/d^5 - 8580*(c + d/x^2)^{(7/2)}*c^3/d^5 + 3003*(c + d/x^2)^{(5/2)}*c^4/d^5)*b$

mupad [B] time = 6.81, size = 248, normalized size = 1.85

$$\frac{16ac^5\sqrt{c+\frac{d}{x^2}}}{1155d^4} - \frac{128b^6\sqrt{c+\frac{d}{x^2}}}{15015d^5} - \frac{4ac\sqrt{c+\frac{d}{x^2}}}{33x^8} - \frac{ad\sqrt{c+\frac{d}{x^2}}}{11x^{10}} - \frac{14bc\sqrt{c+\frac{d}{x^2}}}{143x^{10}} - \frac{bd\sqrt{c+\frac{d}{x^2}}}{13x^{12}} - \frac{a^2\sqrt{c+\frac{d}{x^2}}}{231dx^6} + \frac{2ac^3\sqrt{c+\frac{d}{x^2}}}{385d^2x^4} - \frac{8ac^4\sqrt{c+\frac{d}{x^2}}}{1155d^3x^2} - \frac{bc^2\sqrt{c+\frac{d}{x^2}}}{429dx^8} + \frac{8bc^3\sqrt{c+\frac{d}{x^2}}}{3003d^2x^6} - \frac{16bc^4\sqrt{c+\frac{d}{x^2}}}{5005d^3x^4} + \frac{64bc^5\sqrt{c+\frac{d}{x^2}}}{15015d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x)

[Out] $(16*a*c^5*(c + d/x^2)^{(1/2)})/(1155*d^4) - (128*b*c^6*(c + d/x^2)^{(1/2)})/(15015*d^5) - (4*a*c*(c + d/x^2)^{(1/2)})/(33*x^8) - (a*d*(c + d/x^2)^{(1/2)})/(11*x^{10}) - (14*b*c*(c + d/x^2)^{(1/2)})/(143*x^{10}) - (b*d*(c + d/x^2)^{(1/2)})/(13*x^{12}) - (a*c^2*(c + d/x^2)^{(1/2)})/(231*d*x^6) + (2*a*c^3*(c + d/x^2)^{(1/2)})/(385*d^2*x^4) - (8*a*c^4*(c + d/x^2)^{(1/2)})/(1155*d^3*x^2) - (b*c^2*(c + d/x^2)^{(1/2)})/(429*d*x^8) + (8*b*c^3*(c + d/x^2)^{(1/2)})/(3003*d^2*x^6) - (16*b*c^4*(c + d/x^2)^{(1/2)})/(5005*d^3*x^4) + (64*b*c^5*(c + d/x^2)^{(1/2)})/(15015*d^4*x^2)$

sympy [B] time = 19.45, size = 326, normalized size = 2.43

$$ac \left(-\frac{c^{\frac{5}{2}}(c+\frac{d}{x^2})^{\frac{5}{2}}}{3} + \frac{3c^{\frac{3}{2}}(c+\frac{d}{x^2})^{\frac{3}{2}}}{5} - \frac{3c^{\frac{1}{2}}(c+\frac{d}{x^2})^{\frac{1}{2}}}{7} + \frac{(c+\frac{d}{x^2})^{\frac{3}{2}}}{9} \right) \frac{1}{d^4} - a \left(\frac{c^{\frac{5}{2}}(c+\frac{d}{x^2})^{\frac{5}{2}}}{3} - \frac{4c^{\frac{3}{2}}(c+\frac{d}{x^2})^{\frac{3}{2}}}{5} + \frac{6c^{\frac{1}{2}}(c+\frac{d}{x^2})^{\frac{1}{2}}}{7} - \frac{4c^{\frac{1}{2}}(c+\frac{d}{x^2})^{\frac{1}{2}}}{9} + \frac{(c+\frac{d}{x^2})^{\frac{3}{2}}}{11} \right) \frac{1}{d^4} - bc \left(\frac{c^{\frac{5}{2}}(c+\frac{d}{x^2})^{\frac{5}{2}}}{3} - \frac{4c^{\frac{3}{2}}(c+\frac{d}{x^2})^{\frac{3}{2}}}{5} + \frac{6c^{\frac{1}{2}}(c+\frac{d}{x^2})^{\frac{1}{2}}}{7} - \frac{4c^{\frac{1}{2}}(c+\frac{d}{x^2})^{\frac{1}{2}}}{9} + \frac{(c+\frac{d}{x^2})^{\frac{3}{2}}}{11} \right) \frac{1}{d^6} - b \left(-\frac{c^{\frac{5}{2}}(c+\frac{d}{x^2})^{\frac{5}{2}}}{3} + c^4 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}} - \frac{10c^{\frac{3}{2}}(c+\frac{d}{x^2})^{\frac{3}{2}}}{7} + \frac{10c^{\frac{1}{2}}(c+\frac{d}{x^2})^{\frac{1}{2}}}{9} - \frac{5c^{\frac{1}{2}}(c+\frac{d}{x^2})^{\frac{1}{2}}}{11} + \frac{(c+\frac{d}{x^2})^{\frac{3}{2}}}{13} \right) \frac{1}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**9,x)

[Out] $-a*c*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - a*(c**4*(c + d/x**2)**(3/2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**4 - b*c*(c**4*(c + d/x**2)**(3/2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5 - b*(-c**5*(c + d/x**2)**(3/2)/3 + c**4*(c + d/x**2)**(5/2) - 10*c**3*(c + d/x**2)**(7/2)/7 + 10*c**2*(c + d/x**2)**(9/2)/9 - 5*c*(c + d/x**2)**(11/2)/11 + (c + d/x**2)**(13/2)/13)/d**5$

$$3.607 \quad \int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^{12} dx$$

Optimal. Leaf size=150

$$-\frac{16d^3x^5 \left(c + \frac{d}{x^2}\right)^{5/2} (13bc - 8ad)}{15015c^5} + \frac{8d^2x^7 \left(c + \frac{d}{x^2}\right)^{5/2} (13bc - 8ad)}{3003c^4} - \frac{2dx^9 \left(c + \frac{d}{x^2}\right)^{5/2} (13bc - 8ad)}{429c^3} + \frac{x^{11} \left(c + \frac{d}{x^2}\right)^{5/2} (13bc - 8ad)}{143c^2}$$

Rubi [A] time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {453, 271, 264}

$$\frac{8d^2x^7 \left(c + \frac{d}{x^2}\right)^{5/2} (13bc - 8ad)}{3003c^4} - \frac{16d^3x^5 \left(c + \frac{d}{x^2}\right)^{5/2} (13bc - 8ad)}{15015c^5} + \frac{x^{11} \left(c + \frac{d}{x^2}\right)^{5/2} (13bc - 8ad)}{143c^2} - \frac{2dx^9 \left(c + \frac{d}{x^2}\right)^{5/2} (13bc - 8ad)}{429c^3} + \frac{ax^{13} \left(c + \frac{d}{x^2}\right)^{5/2}}{13c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]

[Out] (-16*d^3*(13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^5)/(15015*c^5) + (8*d^2*(13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^7)/(3003*c^4) - (2*d*(13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^9)/(429*c^3) + ((13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^11)/(143*c^2) + (a*(c + d/x^2)^(5/2)*x^13)/(13*c)

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 271

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (

LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^{12} dx &= \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^{13}}{13c} + \frac{(13bc - 8ad) \int \left(c + \frac{d}{x^2}\right)^{3/2} x^{10} dx}{13c} \\
 &= \frac{(13bc - 8ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^{11}}{143c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^{13}}{13c} - \frac{(6d(13bc - 8ad)) \int \left(c + \frac{d}{x^2}\right)^{3/2} x^8 dx}{143c^2} \\
 &= -\frac{2d(13bc - 8ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^9}{429c^3} + \frac{(13bc - 8ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^{11}}{143c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^{13}}{13c} \\
 &= \frac{8d^2(13bc - 8ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^7}{3003c^4} - \frac{2d(13bc - 8ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^9}{429c^3} + \frac{(13bc - 8ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^{11}}{143c^2} \\
 &= -\frac{16d^3(13bc - 8ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^5}{15015c^5} + \frac{8d^2(13bc - 8ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^7}{3003c^4} - \frac{2d(13bc - 8ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^9}{143c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^{13}}{13c}
 \end{aligned}$$

Mathematica [A] time = 0.11, size = 110, normalized size = 0.73

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d)^2 (a(1155c^4x^8 - 840c^3dx^6 + 560c^2d^2x^4 - 320cd^3x^2 + 128d^4) + 13bc(105c^3x^6 - 70c^2dx^4 + 40cd^2x^2 - 16d^3))}{15015c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(13*b*c*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*d*x^4 + 105*c^3*x^6) + a*(128*d^4 - 320*c*d^3*x^2 + 560*c^2*d^2*x^4 - 840*c^3*d*x^6 + 1155*c^4*x^8)))/(15015*c^5)

IntegrateAlgebraic [A] time = 0.13, size = 114, normalized size = 0.76

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d)^2 (1155ac^4x^8 - 840ac^3dx^6 + 560ac^2d^2x^4 - 320acd^3x^2 + 128ad^4 + 1365bc^4x^6 - 910bc^3dx^4 + 520bc^2d^2x^2 - 208bcd^3)}{15015c^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]

[Out] $(\sqrt{c + d/x^2}) * x * (d + c*x^2)^2 * (-208*b*c*d^3 + 128*a*d^4 + 520*b*c^2*d^2 * x^2 - 320*a*c*d^3*x^2 - 910*b*c^3*d*x^4 + 560*a*c^2*d^2*x^4 + 1365*b*c^4*x^6 - 840*a*c^3*d*x^6 + 1155*a*c^4*x^8) / (15015*c^5)$

fricas [A] time = 0.42, size = 155, normalized size = 1.03

$$\frac{(1155 a^6 c^{13} + 105 (13 b c^6 + 14 a c^5 d) x^{11} + 35 (52 b c^5 d + a c^4 d^2) x^9 + 5 (13 b c^4 d^2 - 8 a c^3 d^3) x^7 - 6 (13 b c^3 d^3 - 8 a c^2 d^4) x^5 + 8 (13 b c^2 d^4 - 8 a c d^5) x^3 - 16 (13 b c d^5 - 8 a d^6) x) \sqrt{\frac{c x^2 + d}{x^2}}}{15015 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="fricas")`

[Out] $1/15015 * (1155 * a * c^6 * x^{13} + 105 * (13 * b * c^6 + 14 * a * c^5 * d) * x^{11} + 35 * (52 * b * c^5 * d + a * c^4 * d^2) * x^9 + 5 * (13 * b * c^4 * d^2 - 8 * a * c^3 * d^3) * x^7 - 6 * (13 * b * c^3 * d^3 - 8 * a * c^2 * d^4) * x^5 + 8 * (13 * b * c^2 * d^4 - 8 * a * c * d^5) * x^3 - 16 * (13 * b * c * d^5 - 8 * a * d^6) * x) * \text{sqrt}((c * x^2 + d) / x^2) / c^5$

giac [A] time = 0.18, size = 175, normalized size = 1.17

$$\frac{16 \left(\frac{13 b c d^2 - 8 a d^3}{15015 c^2} \right) \text{sgn}(x) + 1155 (c^2 + d)^{\frac{11}{2}} \text{sgn}(x) + 1365 (c^2 + d)^{\frac{9}{2}} b c \text{sgn}(x) - 5460 (c^2 + d)^{\frac{7}{2}} a d \text{sgn}(x) - 5005 (c^2 + d)^{\frac{5}{2}} b c d \text{sgn}(x) + 10010 (c^2 + d)^{\frac{3}{2}} a d^2 \text{sgn}(x) + 6435 (c^2 + d)^{\frac{1}{2}} b c d^2 \text{sgn}(x) - 8580 (c^2 + d)^{\frac{1}{2}} a d^3 \text{sgn}(x) - 3003 (c^2 + d)^{\frac{1}{2}} b c d^3 \text{sgn}(x) + 3003 (c^2 + d)^{\frac{1}{2}} a d^4 \text{sgn}(x)}{15015 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="giac")`

[Out] $16/15015 * (13 * b * c * d^{(11/2)} - 8 * a * d^{(13/2)}) * \text{sgn}(x) / c^5 + 1/15015 * (1155 * (c * x^2 + d)^{(13/2)} * a * \text{sgn}(x) + 1365 * (c * x^2 + d)^{(11/2)} * b * c * \text{sgn}(x) - 5460 * (c * x^2 + d)^{(9/2)} * a * d * \text{sgn}(x) - 5005 * (c * x^2 + d)^{(7/2)} * b * c * d * \text{sgn}(x) + 10010 * (c * x^2 + d)^{(5/2)} * a * d^2 * \text{sgn}(x) + 6435 * (c * x^2 + d)^{(3/2)} * b * c * d^2 * \text{sgn}(x) - 8580 * (c * x^2 + d)^{(1/2)} * a * d^3 * \text{sgn}(x) - 3003 * (c * x^2 + d)^{(1/2)} * b * c * d^3 * \text{sgn}(x) + 3003 * (c * x^2 + d)^{(1/2)} * a * d^4 * \text{sgn}(x)) / c^5$

maple [A] time = 0.05, size = 115, normalized size = 0.77

$$\frac{\left(\frac{c x^2 + d}{x^2} \right)^{\frac{3}{2}} (1155 a x^8 c^4 - 840 a c^3 d x^6 + 1365 b c^4 x^6 + 560 a c^2 d^2 x^4 - 910 b c^3 d x^4 - 320 a c d^3 x^2 + 520 b c^2 d^2 x^2 + 128 a d^4 - 208 b c d^3) (c x^2 + d) x^3}{15015 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x)`

[Out] $1/15015 * ((c * x^2 + d) / x^2)^{(3/2)} * x^3 * (1155 * a * c^4 * x^8 - 840 * a * c^3 * d * x^6 + 1365 * b * c^4 * x^6 + 560 * a * c^2 * d^2 * x^4 - 910 * b * c^3 * d * x^4 - 320 * a * c * d^3 * x^2 + 520 * b * c^2 * d^2 * x^2 + 128 * a * d^4 - 208 * b * c * d^3) * (c * x^2 + d) / c^5$

maxima [A] time = 0.63, size = 158, normalized size = 1.05

$$\frac{\left(105 \left(c + \frac{d}{x^2} \right)^{\frac{11}{2}} x^{11} - 385 \left(c + \frac{d}{x^2} \right)^{\frac{9}{2}} d x^9 + 495 \left(c + \frac{d}{x^2} \right)^{\frac{7}{2}} d^2 x^7 - 231 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}} d^3 x^5 \right) b}{1155 c^4} + \frac{\left(1155 \left(c + \frac{d}{x^2} \right)^{\frac{13}{2}} x^{13} - 5460 \left(c + \frac{d}{x^2} \right)^{\frac{11}{2}} d x^{11} + 10010 \left(c + \frac{d}{x^2} \right)^{\frac{9}{2}} d^2 x^9 - 8580 \left(c + \frac{d}{x^2} \right)^{\frac{7}{2}} d^3 x^7 + 3003 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}} d^4 x^5 \right) a}{15015 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="maxima")

[Out] 1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c + d/x^2)^(9/2)*d*x^9 + 495*(c + d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)*d^3*x^5)*b/c^4 + 1/15015*(1155*(c + d/x^2)^(13/2)*x^13 - 5460*(c + d/x^2)^(11/2)*d*x^11 + 10010*(c + d/x^2)^(9/2)*d^2*x^9 - 8580*(c + d/x^2)^(7/2)*d^3*x^7 + 3003*(c + d/x^2)^(5/2)*d^4*x^5)*a/c^5

mupad [B] time = 4.66, size = 137, normalized size = 0.91

$$\sqrt{c + \frac{d}{x^2}} \left(\frac{x (128 a d^6 - 208 b c d^5)}{15015 c^5} + \frac{x^{11} (1365 b c^6 + 1470 a d c^5)}{15015 c^5} + \frac{a c x^{13}}{13} + \frac{d x^9 (a d + 52 b c)}{429 c} - \frac{d^2 x^7 (8 a d - 13 b c)}{3003 c^2} + \frac{2 d^3 x^5 (8 a d - 13 b c)}{5005 c^3} - \frac{8 d^4 x^3 (8 a d - 13 b c)}{15015 c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^12*(a + b/x^2)*(c + d/x^2)^(3/2),x)

[Out] (c + d/x^2)^(1/2)*((x*(128*a*d^6 - 208*b*c*d^5))/(15015*c^5) + (x^11*(1365*b*c^6 + 1470*a*c^5*d))/(15015*c^5) + (a*c*x^13)/13 + (d*x^9*(a*d + 52*b*c))/(429*c) - (d^2*x^7*(8*a*d - 13*b*c))/(3003*c^2) + (2*d^3*x^5*(8*a*d - 13*b*c))/(5005*c^3) - (8*d^4*x^3*(8*a*d - 13*b*c))/(15015*c^4))

sympy [B] time = 12.60, size = 3351, normalized size = 22.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**12,x)

[Out] 693*a*c**12*d**(51/2)*x**22*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 3528*a*c**11*d**(53/2)*x**20*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 7175*a*c**10*d**(55/2)*x**18*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 7290*a*c**9*d**(57/2)*x**16*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 315*a*c**9*d**(35/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 3699*a*c**8*d**(59/2)*x**14*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 1295*a*c**8*d**(37/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6

$$\begin{aligned}
& + 20790*c^{**7}*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} + 3465*c^{**5}*d^{**20}) + 756*a \\
& *c^{**7}*d^{**61/2}*x^{**12}*sqrt(c*x^{**2}/d + 1)/(9009*c^{**11}*d^{**25}*x^{**10} + 45045*c^{** \\
& *10*d^{**26}*x^{**8} + 90090*c^{**9}*d^{**27}*x^{**6} + 90090*c^{**8}*d^{**28}*x^{**4} + 45045*c^{**7} \\
& *d^{**29}*x^{**2} + 9009*c^{**6}*d^{**30}) + 1990*a*c^{**7}*d^{**39/2}*x^{**14}*sqrt(c*x^{**2}/d \\
& + 1)/(3465*c^{**9}*d^{**16}*x^{**8} + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**4} \\
& + 13860*c^{**6}*d^{**19}*x^{**2} + 3465*c^{**5}*d^{**20}) - 63*a*c^{**6}*d^{**63/2}*x^{**10}*sqrt \\
& (c*x^{**2}/d + 1)/(9009*c^{**11}*d^{**25}*x^{**10} + 45045*c^{**10}*d^{**26}*x^{**8} + 90090*c^{**9} \\
& *d^{**27}*x^{**6} + 90090*c^{**8}*d^{**28}*x^{**4} + 45045*c^{**7}*d^{**29}*x^{**2} + 9009*c^{**6}*d^{** \\
& *30) + 1358*a*c^{**6}*d^{**41/2}*x^{**12}*sqrt(c*x^{**2}/d + 1)/(3465*c^{**9}*d^{**16}*x^{**8} \\
& + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} + \\
& 3465*c^{**5}*d^{**20}) - 630*a*c^{**5}*d^{**65/2}*x^{**8}*sqrt(c*x^{**2}/d + 1)/(9009*c^{**11} \\
& *d^{**25}*x^{**10} + 45045*c^{**10}*d^{**26}*x^{**8} + 90090*c^{**9}*d^{**27}*x^{**6} + 90090*c^{**8}* \\
& d^{**28}*x^{**4} + 45045*c^{**7}*d^{**29}*x^{**2} + 9009*c^{**6}*d^{**30}) + 343*a*c^{**5}*d^{**43/2} \\
&)*x^{**10}*sqrt(c*x^{**2}/d + 1)/(3465*c^{**9}*d^{**16}*x^{**8} + 13860*c^{**8}*d^{**17}*x^{**6} + \\
& 20790*c^{**7}*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} + 3465*c^{**5}*d^{**20}) - 1680*a*c \\
& **4*d^{**67/2}*x^{**6}*sqrt(c*x^{**2}/d + 1)/(9009*c^{**11}*d^{**25}*x^{**10} + 45045*c^{**10} \\
& *d^{**26}*x^{**8} + 90090*c^{**9}*d^{**27}*x^{**6} + 90090*c^{**8}*d^{**28}*x^{**4} + 45045*c^{**7}*d^{** \\
& *29}*x^{**2} + 9009*c^{**6}*d^{**30}) + 35*a*c^{**4}*d^{**45/2}*x^{**8}*sqrt(c*x^{**2}/d + 1)/(\\
& 3465*c^{**9}*d^{**16}*x^{**8} + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**4} + 1386 \\
& 0*c^{**6}*d^{**19}*x^{**2} + 3465*c^{**5}*d^{**20}) - 2016*a*c^{**3}*d^{**69/2}*x^{**4}*sqrt(c*x* \\
& **2/d + 1)/(9009*c^{**11}*d^{**25}*x^{**10} + 45045*c^{**10}*d^{**26}*x^{**8} + 90090*c^{**9}*d^{** \\
& 27}*x^{**6} + 90090*c^{**8}*d^{**28}*x^{**4} + 45045*c^{**7}*d^{**29}*x^{**2} + 9009*c^{**6}*d^{**30}) \\
& + 280*a*c^{**3}*d^{**47/2}*x^{**6}*sqrt(c*x^{**2}/d + 1)/(3465*c^{**9}*d^{**16}*x^{**8} + 1386 \\
& 0*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} + 3465*c* \\
& **5*d^{**20}) - 1152*a*c^{**2}*d^{**71/2}*x^{**2}*sqrt(c*x^{**2}/d + 1)/(9009*c^{**11}*d^{**25} \\
& *x^{**10} + 45045*c^{**10}*d^{**26}*x^{**8} + 90090*c^{**9}*d^{**27}*x^{**6} + 90090*c^{**8}*d^{**28}* \\
& x^{**4} + 45045*c^{**7}*d^{**29}*x^{**2} + 9009*c^{**6}*d^{**30}) + 560*a*c^{**2}*d^{**49/2}*x^{**4} \\
& *sqrt(c*x^{**2}/d + 1)/(3465*c^{**9}*d^{**16}*x^{**8} + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c \\
& **7*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} + 3465*c^{**5}*d^{**20}) - 256*a*c*d^{**73/ \\
& 2}*sqrt(c*x^{**2}/d + 1)/(9009*c^{**11}*d^{**25}*x^{**10} + 45045*c^{**10}*d^{**26}*x^{**8} + 90 \\
& 090*c^{**9}*d^{**27}*x^{**6} + 90090*c^{**8}*d^{**28}*x^{**4} + 45045*c^{**7}*d^{**29}*x^{**2} + 9009* \\
& c^{**6}*d^{**30}) + 448*a*c*d^{**51/2}*x^{**2}*sqrt(c*x^{**2}/d + 1)/(3465*c^{**9}*d^{**16}*x* \\
& **8 + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} \\
& + 3465*c^{**5}*d^{**20}) + 128*a*d^{**53/2}*sqrt(c*x^{**2}/d + 1)/(3465*c^{**9}*d^{**16}*x* \\
& **8 + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} \\
& + 3465*c^{**5}*d^{**20}) + 315*b*c^{**10}*d^{**33/2}*x^{**18}*sqrt(c*x^{**2}/d + 1)/(3465*c \\
& **9*d^{**16}*x^{**8} + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**4} + 13860*c^{**6} \\
& *d^{**19}*x^{**2} + 3465*c^{**5}*d^{**20}) + 1295*b*c^{**9}*d^{**35/2}*x^{**16}*sqrt(c*x^{**2}/d \\
& + 1)/(3465*c^{**9}*d^{**16}*x^{**8} + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**4} \\
& + 13860*c^{**6}*d^{**19}*x^{**2} + 3465*c^{**5}*d^{**20}) + 1990*b*c^{**8}*d^{**37/2}*x^{**14}*sq \\
& rt(c*x^{**2}/d + 1)/(3465*c^{**9}*d^{**16}*x^{**8} + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7} \\
& *d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} + 3465*c^{**5}*d^{**20}) + 1358*b*c^{**7}*d^{**39 \\
& /2}*x^{**12}*sqrt(c*x^{**2}/d + 1)/(3465*c^{**9}*d^{**16}*x^{**8} + 13860*c^{**8}*d^{**17}*x^{**6} \\
& + 20790*c^{**7}*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} + 3465*c^{**5}*d^{**20}) + 35*b*c \\
& **7*d^{**21/2}*x^{**14}*sqrt(c*x^{**2}/d + 1)/(315*c^{**7}*d^{**9}*x^{**6} + 945*c^{**6}*d^{**10}
\end{aligned}$$

$$\begin{aligned}
& *x^{**4} + 945*c^{**5}*d^{**11}*x^{**2} + 315*c^{**4}*d^{**12}) + 343*b*c^{**6}*d^{**}(41/2)*x^{**10}* \\
& \text{sqrt}(c*x^{**2}/d + 1)/(3465*c^{**9}*d^{**16}*x^{**8} + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**} \\
& *7*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} + 3465*c^{**5}*d^{**20}) + 110*b*c^{**6}*d^{**}(2 \\
& 3/2)*x^{**12}*\text{sqrt}(c*x^{**2}/d + 1)/(315*c^{**7}*d^{**9}*x^{**6} + 945*c^{**6}*d^{**10}*x^{**4} + 9 \\
& 45*c^{**5}*d^{**11}*x^{**2} + 315*c^{**4}*d^{**12}) + 35*b*c^{**5}*d^{**}(43/2)*x^{**8}*\text{sqrt}(c*x^{**2} \\
& /d + 1)/(3465*c^{**9}*d^{**16}*x^{**8} + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**} \\
& *4 + 13860*c^{**6}*d^{**19}*x^{**2} + 3465*c^{**5}*d^{**20}) + 114*b*c^{**5}*d^{**}(25/2)*x^{**10}* \\
& \text{sqrt}(c*x^{**2}/d + 1)/(315*c^{**7}*d^{**9}*x^{**6} + 945*c^{**6}*d^{**10}*x^{**4} + 945*c^{**5}*d^{**} \\
& 11*x^{**2} + 315*c^{**4}*d^{**12}) + 280*b*c^{**4}*d^{**}(45/2)*x^{**6}*\text{sqrt}(c*x^{**2}/d + 1)/(3 \\
& 465*c^{**9}*d^{**16}*x^{**8} + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**4} + 13860 \\
& *c^{**6}*d^{**19}*x^{**2} + 3465*c^{**5}*d^{**20}) + 40*b*c^{**4}*d^{**}(27/2)*x^{**8}*\text{sqrt}(c*x^{**2}/ \\
& d + 1)/(315*c^{**7}*d^{**9}*x^{**6} + 945*c^{**6}*d^{**10}*x^{**4} + 945*c^{**5}*d^{**11}*x^{**2} + 31 \\
& 5*c^{**4}*d^{**12}) + 560*b*c^{**3}*d^{**}(47/2)*x^{**4}*\text{sqrt}(c*x^{**2}/d + 1)/(3465*c^{**9}*d^{**} \\
& 16*x^{**8} + 13860*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}* \\
& x^{**2} + 3465*c^{**5}*d^{**20}) - 5*b*c^{**3}*d^{**}(29/2)*x^{**6}*\text{sqrt}(c*x^{**2}/d + 1)/(315*c \\
& **7*d^{**9}*x^{**6} + 945*c^{**6}*d^{**10}*x^{**4} + 945*c^{**5}*d^{**11}*x^{**2} + 315*c^{**4}*d^{**12}) \\
& + 448*b*c^{**2}*d^{**}(49/2)*x^{**2}*\text{sqrt}(c*x^{**2}/d + 1)/(3465*c^{**9}*d^{**16}*x^{**8} + 138 \\
& 60*c^{**8}*d^{**17}*x^{**6} + 20790*c^{**7}*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} + 3465*c \\
& **5*d^{**20}) - 30*b*c^{**2}*d^{**}(31/2)*x^{**4}*\text{sqrt}(c*x^{**2}/d + 1)/(315*c^{**7}*d^{**9}*x^{**} \\
& 6 + 945*c^{**6}*d^{**10}*x^{**4} + 945*c^{**5}*d^{**11}*x^{**2} + 315*c^{**4}*d^{**12}) + 128*b*c*d \\
& ***(51/2)*\text{sqrt}(c*x^{**2}/d + 1)/(3465*c^{**9}*d^{**16}*x^{**8} + 13860*c^{**8}*d^{**17}*x^{**6} + \\
& 20790*c^{**7}*d^{**18}*x^{**4} + 13860*c^{**6}*d^{**19}*x^{**2} + 3465*c^{**5}*d^{**20}) - 40*b*c* \\
& d^{**}(33/2)*x^{**2}*\text{sqrt}(c*x^{**2}/d + 1)/(315*c^{**7}*d^{**9}*x^{**6} + 945*c^{**6}*d^{**10}*x^{**4} \\
& + 945*c^{**5}*d^{**11}*x^{**2} + 315*c^{**4}*d^{**12}) - 16*b*d^{**}(35/2)*\text{sqrt}(c*x^{**2}/d + 1 \\
&)/(315*c^{**7}*d^{**9}*x^{**6} + 945*c^{**6}*d^{**10}*x^{**4} + 945*c^{**5}*d^{**11}*x^{**2} + 315*c^{**} \\
& 4*d^{**12})
\end{aligned}$$

$$3.608 \quad \int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^{10} dx$$

Optimal. Leaf size=117

$$\frac{8d^2x^5 \left(c + \frac{d}{x^2}\right)^{5/2} (11bc - 6ad)}{3465c^4} - \frac{4dx^7 \left(c + \frac{d}{x^2}\right)^{5/2} (11bc - 6ad)}{693c^3} + \frac{x^9 \left(c + \frac{d}{x^2}\right)^{5/2} (11bc - 6ad)}{99c^2} + \frac{ax^{11} \left(c + \frac{d}{x^2}\right)^{5/2}}{11c}$$

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {453, 271, 264}

$$\frac{8d^2x^5 \left(c + \frac{d}{x^2}\right)^{5/2} (11bc - 6ad)}{3465c^4} + \frac{x^9 \left(c + \frac{d}{x^2}\right)^{5/2} (11bc - 6ad)}{99c^2} - \frac{4dx^7 \left(c + \frac{d}{x^2}\right)^{5/2} (11bc - 6ad)}{693c^3} + \frac{ax^{11} \left(c + \frac{d}{x^2}\right)^{5/2}}{11c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]

[Out] (8*d^2*(11*b*c - 6*a*d)*(c + d/x^2)^(5/2)*x^5)/(3465*c^4) - (4*d*(11*b*c - 6*a*d)*(c + d/x^2)^(5/2)*x^7)/(693*c^3) + ((11*b*c - 6*a*d)*(c + d/x^2)^(5/2)*x^9)/(99*c^2) + (a*(c + d/x^2)^(5/2)*x^11)/(11*c)

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 271

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^{10} dx &= \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^{11}}{11c} + \frac{(11bc - 6ad) \int \left(c + \frac{d}{x^2}\right)^{3/2} x^8 dx}{11c} \\
&= \frac{(11bc - 6ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^9}{99c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^{11}}{11c} - \frac{(4d(11bc - 6ad)) \int \left(c + \frac{d}{x^2}\right)^{3/2}}{99c^2} \\
&= -\frac{4d(11bc - 6ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^7}{693c^3} + \frac{(11bc - 6ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^9}{99c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^{11}}{11c} \\
&= \frac{8d^2(11bc - 6ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^5}{3465c^4} - \frac{4d(11bc - 6ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^7}{693c^3} + \frac{(11bc - 6ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^9}{99c^2}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 89, normalized size = 0.76

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d)^2 (3a(105c^3x^6 - 70c^2dx^4 + 40cd^2x^2 - 16d^3) + 11bc(35c^2x^4 - 20cdx^2 + 8d^2))}{3465c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10, x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(11*b*c*(8*d^2 - 20*c*d*x^2 + 35*c^2*x^4) + 3*a*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*d*x^4 + 105*c^3*x^6)))/(3465*c^4)

IntegrateAlgebraic [A] time = 0.10, size = 90, normalized size = 0.77

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d)^2 (315ac^3x^6 - 210ac^2dx^4 + 120acd^2x^2 - 48ad^3 + 385bc^3x^4 - 220bc^2dx^2 + 88bcd^2)}{3465c^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10, x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(88*b*c*d^2 - 48*a*d^3 - 220*b*c^2*d*x^2 + 120*a*c*d^2*x^2 + 385*b*c^3*x^4 - 210*a*c^2*d*x^4 + 315*a*c^3*x^6))/(3465*c^4)

fricas [A] time = 0.44, size = 132, normalized size = 1.13

$$\frac{(315ac^5x^{11} + 35(11bc^5 + 12ac^4d)x^9 + 5(110bc^4d + 3ac^3d^2)x^7 + 3(11bc^3d^2 - 6ac^2d^3)x^5 - 4(11bc^2d^3 - 6acd^4)x^3 + 8(11bcd^4 - 6ad^5)x) \sqrt{\frac{cx^2+d}{x^2}}}{3465c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="fricas")

[Out] 1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + 12*a*c^4*d)*x^9 + 5*(110*b*c^4*d + 3*a*c^3*d^2)*x^7 + 3*(11*b*c^3*d^2 - 6*a*c^2*d^3)*x^5 - 4*(11*b*c^2*d^3 - 6*a*c*d^4)*x^3 + 8*(11*b*c*d^4 - 6*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^4

giac [A] time = 0.18, size = 140, normalized size = 1.20

$$\frac{8(11bcd^2 - 6ad^2)sgn(x)}{3465c^4} + \frac{315(cx^2 + d)^{\frac{11}{2}}asgn(x) + 385(cx^2 + d)^{\frac{9}{2}}bcsgn(x) - 1155(cx^2 + d)^{\frac{7}{2}}adsgn(x) - 990(cx^2 + d)^{\frac{5}{2}}bcdsgn(x) + 1485(cx^2 + d)^{\frac{3}{2}}ad^2sgn(x) + 693(cx^2 + d)^{\frac{1}{2}}bcd^2sgn(x) - 693(cx^2 + d)^{\frac{1}{2}}ad^3sgn(x)}{3465c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="giac")

[Out] -8/3465*(11*b*c*d^(9/2) - 6*a*d^(11/2))*sgn(x)/c^4 + 1/3465*(315*(c*x^2 + d)^(11/2)*a*sgn(x) + 385*(c*x^2 + d)^(9/2)*b*c*sgn(x) - 1155*(c*x^2 + d)^(9/2)*a*d*sgn(x) - 990*(c*x^2 + d)^(7/2)*b*c*d*sgn(x) + 1485*(c*x^2 + d)^(7/2)*a*d^2*sgn(x) + 693*(c*x^2 + d)^(5/2)*b*c*d^2*sgn(x) - 693*(c*x^2 + d)^(5/2)*a*d^3*sgn(x))/c^4

maple [A] time = 0.05, size = 91, normalized size = 0.78

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}} (315ax^6c^3 - 210ac^2dx^4 + 385b^3c^3x^4 + 120acd^2x^2 - 220bc^2dx^2 - 48ad^3 + 88bcd^2)(cx^2 + d)x^3}{3465c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x)

[Out] 1/3465*((c*x^2+d)/x^2)^(3/2)*x^3*(315*a*c^3*x^6-210*a*c^2*d*x^4+385*b*c^3*x^4+120*a*c*d^2*x^2-220*b*c^2*d*x^2-48*a*d^3+88*b*c*d^2)*(c*x^2+d)/c^4

maxima [A] time = 0.49, size = 124, normalized size = 1.06

$$\frac{\left(35\left(c + \frac{d}{x^2}\right)^{\frac{9}{2}}x^9 - 90\left(c + \frac{d}{x^2}\right)^{\frac{7}{2}}dx^7 + 63\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)b}{315c^3} + \frac{\left(105\left(c + \frac{d}{x^2}\right)^{\frac{11}{2}}x^{11} - 385\left(c + \frac{d}{x^2}\right)^{\frac{9}{2}}dx^9 + 495\left(c + \frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7 - 231\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5\right)a}{1155c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="maxima")

[Out] 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x^2)^(5/2)*d^2*x^5)*b/c^3 + 1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c + d/x^2)^(9/2)*d*x^9 + 495*(c + d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)*d^3*x^5)*a/c^4

mupad [B] time = 4.57, size = 118, normalized size = 1.01

$$\sqrt{c + \frac{d}{x^2}} \left(\frac{x^9 (385bc^5 + 420adc^4)}{3465c^4} - \frac{x(48ad^5 - 88bcd^4)}{3465c^4} + \frac{acx^{11}}{11} + \frac{dx^7(3ad + 110bc)}{693c} - \frac{d^2x^5(6ad - 11bc)}{1155c^2} + \frac{4d^3x^3(6ad - 11bc)}{3465c^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^10*(a + b/x^2)*(c + d/x^2)^(3/2),x)

[Out] (c + d/x^2)^(1/2)*((x^9*(385*b*c^5 + 420*a*c^4*d))/(3465*c^4) - (x*(48*a*d^5 - 88*b*c*d^4))/(3465*c^4) + (a*c*x^11)/11 + (d*x^7*(3*a*d + 110*b*c))/(693*c) - (d^2*x^5*(6*a*d - 11*b*c))/(1155*c^2) + (4*d^3*x^3*(6*a*d - 11*b*c))/(3465*c^3))

sympy [B] time = 9.65, size = 2304, normalized size = 19.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**10,x)

[Out] 315*a*c**10*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1295*a*c**9*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1990*a*c**8*d**(37/2)*x**14*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**7*d**(39/2)*x**12*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c**7*d**(21/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 343*a*c**6*d**(41/2)*x**10*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 110*a*c**6*d**(23/2)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 35*a*c**5*d**(43/2)*x**8*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 114*a*c**5*d**(25/2)*x**10*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 280*a*c**4*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 40*a*c**4*d**(27/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 560*a*c**3*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 5*a*c**3*d**(29/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c

$$\begin{aligned}
& **6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 448*a*c**2*d**(49/ \\
& 2)*x**2*\sqrt{c*x**2/d + 1}/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + \\
& 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 30*a*c** \\
& 2*d**(31/2)*x**4*\sqrt{c*x**2/d + 1}/(315*c**7*d**9*x**6 + 945*c**6*d**10*x** \\
& *4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 128*a*c*d**(51/2)*\sqrt{c*x**2/ \\
& d + 1}/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x** \\
& 4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 40*a*c*d**(33/2)*x**2*\sqrt{c \\
& *x**2/d + 1}/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x** \\
& 2 + 315*c**4*d**12) - 16*a*d**(35/2)*\sqrt{c*x**2/d + 1}/(315*c**7*d**9*x**6 \\
& + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 35*b*c**8* \\
& d**(19/2)*x**14*\sqrt{c*x**2/d + 1}/(315*c**7*d**9*x**6 + 945*c**6*d**10*x** \\
& 4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*b*c**7*d**(21/2)*x**12*\sqrt{ \\
& (c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x \\
& **2 + 315*c**4*d**12) + 114*b*c**6*d**(23/2)*x**10*\sqrt{c*x**2/d + 1}/(315* \\
& c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12 \\
&) + 40*b*c**5*d**(25/2)*x**8*\sqrt{c*x**2/d + 1}/(315*c**7*d**9*x**6 + 945*c \\
& **6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*b*c**5*d**(11/2 \\
&)*x**10*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c \\
& **3*d**6) - 5*b*c**4*d**(27/2)*x**6*\sqrt{c*x**2/d + 1}/(315*c**7*d**9*x**6 \\
& + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 33*b*c**4*d \\
& ***(13/2)*x**8*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + \\
& 105*c**3*d**6) - 30*b*c**3*d**(29/2)*x**4*\sqrt{c*x**2/d + 1}/(315*c**7*d** \\
& 9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 17*b \\
& *c**3*d**(15/2)*x**6*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5 \\
& *x**2 + 105*c**3*d**6) - 40*b*c**2*d**(31/2)*x**2*\sqrt{c*x**2/d + 1}/(315*c \\
& **7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) \\
& + 3*b*c**2*d**(17/2)*x**4*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c** \\
& 4*d**5*x**2 + 105*c**3*d**6) - 16*b*c*d**(33/2)*\sqrt{c*x**2/d + 1}/(315*c** \\
& 7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + \\
& 12*b*c*d**(19/2)*x**2*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d \\
& **5*x**2 + 105*c**3*d**6) + 8*b*d**(21/2)*\sqrt{c*x**2/d + 1}/(105*c**5*d**4* \\
& x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6)
\end{aligned}$$

$$3.609 \quad \int \left(a + \frac{b}{x^2} \right) \left(c + \frac{d}{x^2} \right)^{3/2} x^8 dx$$

Optimal. Leaf size=84

$$-\frac{2dx^5 \left(c + \frac{d}{x^2} \right)^{5/2} (9bc - 4ad)}{315c^3} + \frac{x^7 \left(c + \frac{d}{x^2} \right)^{5/2} (9bc - 4ad)}{63c^2} + \frac{ax^9 \left(c + \frac{d}{x^2} \right)^{5/2}}{9c}$$

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {453, 271, 264}

$$\frac{x^7 \left(c + \frac{d}{x^2} \right)^{5/2} (9bc - 4ad)}{63c^2} - \frac{2dx^5 \left(c + \frac{d}{x^2} \right)^{5/2} (9bc - 4ad)}{315c^3} + \frac{ax^9 \left(c + \frac{d}{x^2} \right)^{5/2}}{9c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]

[Out] (-2*d*(9*b*c - 4*a*d)*(c + d/x^2)^(5/2)*x^5)/(315*c^3) + ((9*b*c - 4*a*d)*(c + d/x^2)^(5/2)*x^7)/(63*c^2) + (a*(c + d/x^2)^(5/2)*x^9)/(9*c)

Rule 264

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

Rule 271

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m+1)*(a+b*x^n)^(p+1))/(a*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*(m+1)), Int[x^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m+1)/n + p + 1], 0] && NeQ[m, -1]

Rule 453

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m+1)*(a+b*x^n)^(p+1))/(a*e*(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^8 dx &= \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^9}{9c} + \frac{(9bc - 4ad) \int \left(c + \frac{d}{x^2}\right)^{3/2} x^6 dx}{9c} \\
&= \frac{(9bc - 4ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^7}{63c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^9}{9c} - \frac{(2d(9bc - 4ad)) \int \left(c + \frac{d}{x^2}\right)^{3/2} x^4 dx}{63c^2} \\
&= -\frac{2d(9bc - 4ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^5}{315c^3} + \frac{(9bc - 4ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^7}{63c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^9}{9c}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.79

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d)^2 (a(35c^2x^4 - 20cdx^2 + 8d^2) + 9bc(5cx^2 - 2d))}{315c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(9*b*c*(-2*d + 5*c*x^2) + a*(8*d^2 - 20*c*d*x^2 + 35*c^2*x^4)))/(315*c^3)

IntegrateAlgebraic [A] time = 0.09, size = 66, normalized size = 0.79

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d)^2 (35ac^2x^4 - 20acdx^2 + 8ad^2 + 45bc^2x^2 - 18bcd)}{315c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(-18*b*c*d + 8*a*d^2 + 45*b*c^2*x^2 - 20*a*c*d*x^2 + 35*a*c^2*x^4)))/(315*c^3)

fricas [A] time = 0.43, size = 106, normalized size = 1.26

$$\frac{(35ac^4x^9 + 5(9bc^4 + 10ac^3d)x^7 + 3(24bc^3d + ac^2d^2)x^5 + (9bc^2d^2 - 4acd^3)x^3 - 2(9bcd^3 - 4ad^4)x)\sqrt{\frac{cx^2+d}{x^2}}}{315c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="fricas")

[Out] 1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + 10*a*c^3*d)*x^7 + 3*(24*b*c^3*d + a*c^2*d^2)*x^5 + (9*b*c^2*d^2 - 4*a*c*d^3)*x^3 - 2*(9*b*c*d^3 - 4*a*d^4)*x)*sqrt((c*x^2 + d)/x^2)/c^3

giac [A] time = 0.17, size = 105, normalized size = 1.25

$$\frac{2(9bcd^7 - 4ad^9)\operatorname{sgn}(x)}{315c^3} + \frac{35(cx^2 + d)^{\frac{9}{2}}a\operatorname{sgn}(x) + 45(cx^2 + d)^{\frac{7}{2}}bc\operatorname{sgn}(x) - 90(cx^2 + d)^{\frac{7}{2}}ad\operatorname{sgn}(x) - 63(cx^2 + d)^{\frac{5}{2}}bcd\operatorname{sgn}(x) + 63(cx^2 + d)^{\frac{5}{2}}ad^2\operatorname{sgn}(x)}{315c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="giac")

[Out] 2/315*(9*b*c*d^(7/2) - 4*a*d^(9/2))*sgn(x)/c^3 + 1/315*(35*(c*x^2 + d)^(9/2)*a*sgn(x) + 45*(c*x^2 + d)^(7/2)*b*c*sgn(x) - 90*(c*x^2 + d)^(7/2)*a*d*sgn(x) - 63*(c*x^2 + d)^(5/2)*b*c*d*sgn(x) + 63*(c*x^2 + d)^(5/2)*a*d^2*sgn(x))/c^3

maple [A] time = 0.06, size = 67, normalized size = 0.80

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}(35ax^4c^2 - 20acd^2x^2 + 45b^2c^2x^2 + 8ad^2 - 18bcd)(cx^2 + d)x^3}{315c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x)

[Out] 1/315*((c*x^2+d)/x^2)^(3/2)*x^3*(35*a*c^2*x^4-20*a*c*d*x^2+45*b*c^2*x^2+8*a*d^2-18*b*c*d)*(c*x^2+d)/c^3

maxima [A] time = 0.76, size = 90, normalized size = 1.07

$$\frac{\left(5\left(c + \frac{d}{x^2}\right)^{\frac{7}{2}}x^7 - 7\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)b}{35c^2} + \frac{\left(35\left(c + \frac{d}{x^2}\right)^{\frac{9}{2}}x^9 - 90\left(c + \frac{d}{x^2}\right)^{\frac{7}{2}}dx^7 + 63\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)a}{315c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="maxima")

[Out] 1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x^2)^(5/2)*d*x^5)*b/c^2 + 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x^2)^(5/2)*d^2*x^5)*a/c^3

mupad [B] time = 4.55, size = 97, normalized size = 1.15

$$\sqrt{c + \frac{d}{x^2}} \left(\frac{x(8ad^4 - 18bcd^3)}{315c^3} + \frac{x^7(45bc^4 + 50adc^3)}{315c^3} + \frac{acx^9}{9} + \frac{dx^5(ad + 24bc)}{105c} - \frac{d^2x^3(4ad - 9bc)}{315c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*(a + b/x^2)*(c + d/x^2)^(3/2),x)`

[Out] $(c + d/x^2)^{(1/2)} * ((x*(8*a*d^4 - 18*b*c*d^3))/(315*c^3) + (x^7*(45*b*c^4 + 50*a*c^3*d))/(315*c^3) + (a*c*x^9)/9 + (d*x^5*(a*d + 24*b*c))/(105*c) - (d^2*x^3*(4*a*d - 9*b*c))/(315*c^2))$

sympy [B] time = 7.64, size = 1340, normalized size = 15.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**8,x)`

[Out] $35*a*c**8*d**(19/2)*x**14*\sqrt{c*x**2/d + 1}/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**7*d**(21/2)*x**12*\sqrt{c*x**2/d + 1}/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**6*d**(23/2)*x**10*\sqrt{c*x**2/d + 1}/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 40*a*c**5*d**(25/2)*x**8*\sqrt{c*x**2/d + 1}/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*a*c**5*d**(11/2)*x**10*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) - 5*a*c**4*d**(27/2)*x**6*\sqrt{c*x**2/d + 1}/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 33*a*c**4*d**(13/2)*x**8*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) - 30*a*c**3*d**(29/2)*x**4*\sqrt{c*x**2/d + 1}/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 17*a*c**3*d**(15/2)*x**6*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) - 40*a*c**2*d**(31/2)*x**2*\sqrt{c*x**2/d + 1}/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 3*a*c**2*d**(17/2)*x**4*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) - 16*a*c*d**(33/2)*\sqrt{c*x**2/d + 1}/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 12*a*c*d**(19/2)*x**2*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*d**(21/2)*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 15*b*c**6*d**(9/2)*x**10*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 33*b*c**5*d**(11/2)*x**8*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*b*c**4*d**(13/2)*x**6*\sqrt{c*x**2/d + 1}/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*b*c**3$

$$\begin{aligned}
& d^{15/2} x^4 \sqrt{c x^2/d + 1} / (105 c^5 d^4 x^4 + 210 c^4 d^5 x^2 \\
& + 105 c^3 d^6) + 12 b c^2 d^{17/2} x^2 \sqrt{c x^2/d + 1} / (105 c^5 d \\
& x^4 + 210 c^4 d^5 x^2 + 105 c^3 d^6) + 8 b c d^{19/2} \sqrt{c x^2/d + 1} / (105 c^5 d^4 x^4 + 210 c^4 d^5 x^2 + 105 c^3 d^6) + b d^{3/2} x^4 \sqrt{c x^2/d + 1} / 5 + b d^{5/2} x^2 \sqrt{c x^2/d + 1} / (15 c) \\
& - 2 b d^{7/2} \sqrt{c x^2/d + 1} / (15 c^2)
\end{aligned}$$

$$3.610 \quad \int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^6 dx$$

Optimal. Leaf size=53

$$\frac{x^5 \left(c + \frac{d}{x^2}\right)^{5/2} (7bc - 2ad)}{35c^2} + \frac{ax^7 \left(c + \frac{d}{x^2}\right)^{5/2}}{7c}$$

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {453, 264}

$$\frac{x^5 \left(c + \frac{d}{x^2}\right)^{5/2} (7bc - 2ad)}{35c^2} + \frac{ax^7 \left(c + \frac{d}{x^2}\right)^{5/2}}{7c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]

[Out] ((7*b*c - 2*a*d)*(c + d/x^2)^(5/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(5/2)*x^7)/(7*c)

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^6 dx = \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^7}{7c} + \frac{(7bc - 2ad) \int \left(c + \frac{d}{x^2}\right)^{3/2} x^4 dx}{7c}$$

$$= \frac{(7bc - 2ad) \left(c + \frac{d}{x^2}\right)^{5/2} x^5}{35c^2} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^7}{7c}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.83

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d)^2 (5acx^2 - 2ad + 7bc)}{35c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(7*b*c - 2*a*d + 5*a*c*x^2))/(35*c^2)

IntegrateAlgebraic [A] time = 0.08, size = 44, normalized size = 0.83

$$\frac{x \sqrt{c + \frac{d}{x^2}} (cx^2 + d)^2 (5acx^2 - 2ad + 7bc)}{35c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(7*b*c - 2*a*d + 5*a*c*x^2))/(35*c^2)

fricas [A] time = 0.41, size = 80, normalized size = 1.51

$$\frac{(5ac^3x^7 + (7bc^3 + 8ac^2d)x^5 + (14bc^2d + acd^2)x^3 + (7bcd^2 - 2ad^3)x) \sqrt{\frac{cx^2+d}{x^2}}}{35c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="fricas")

[Out] 1/35*(5*a*c^3*x^7 + (7*b*c^3 + 8*a*c^2*d)*x^5 + (14*b*c^2*d + a*c*d^2)*x^3 + (7*b*c*d^2 - 2*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^2

giac [A] time = 0.18, size = 72, normalized size = 1.36

$$-\frac{\left(7bcd^{\frac{5}{2}} - 2ad^{\frac{7}{2}}\right)\operatorname{sgn}(x)}{35c^2} + \frac{5\left(cx^2 + d\right)^{\frac{7}{2}}a\operatorname{sgn}(x) + 7\left(cx^2 + d\right)^{\frac{5}{2}}bc\operatorname{sgn}(x) - 7\left(cx^2 + d\right)^{\frac{5}{2}}ad\operatorname{sgn}(x)}{35c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="giac")

[Out] -1/35*(7*b*c*d^(5/2) - 2*a*d^(7/2))*sgn(x)/c^2 + 1/35*(5*(c*x^2 + d)^(7/2)*a*sgn(x) + 7*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 7*(c*x^2 + d)^(5/2)*a*d*sgn(x))/c^2

maple [A] time = 0.04, size = 45, normalized size = 0.85

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(5ax^2c - 2ad + 7bc\right)\left(cx^2 + d\right)x^3}{35c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x)

[Out] 1/35*((c*x^2+d)/x^2)^(3/2)*x^3*(5*a*c*x^2-2*a*d+7*b*c)*(c*x^2+d)/c^2

maxima [A] time = 0.65, size = 55, normalized size = 1.04

$$\frac{b\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}x^5}{5c} + \frac{\left(5\left(c + \frac{d}{x^2}\right)^{\frac{7}{2}}x^7 - 7\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)a}{35c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="maxima")

[Out] 1/5*b*(c + d/x^2)^(5/2)*x^5/c + 1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x^2)^(5/2)*d*x^5)*a/c^2

mupad [B] time = 4.63, size = 77, normalized size = 1.45

$$\sqrt{c + \frac{d}{x^2}} \left(\frac{x^5 (7bc^3 + 8adc^2)}{35c^2} - \frac{x (2ad^3 - 7bcd^2)}{35c^2} + \frac{acx^7}{7} + \frac{dx^3 (ad + 14bc)}{35c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(a + b/x^2)*(c + d/x^2)^(3/2),x)`

[Out] $(c + d/x^2)^{(1/2)} * ((x^5 * (7 * b * c^3 + 8 * a * c^2 * d)) / (35 * c^2) - (x * (2 * a * d^3 - 7 * b * c * d^2)) / (35 * c^2) + (a * c * x^7) / 7 + (d * x^3 * (a * d + 14 * b * c)) / (35 * c))$

sympy [B] time = 5.95, size = 498, normalized size = 9.40

$$\frac{15ac^3d^2x^{10}\sqrt{\frac{d}{x}+1}}{105c^5d^4+210c^4d^5+105c^3d^6} + \frac{33ac^3d^2x^8\sqrt{\frac{d}{x}+1}}{105c^5d^4+210c^4d^5+105c^3d^6} + \frac{17ac^4d^2x^6\sqrt{\frac{d}{x}+1}}{105c^5d^4+210c^4d^5+105c^3d^6} + \frac{3ac^3d^2x^4\sqrt{\frac{d}{x}+1}}{105c^5d^4+210c^4d^5+105c^3d^6} + \frac{12ac^2d^2x^2\sqrt{\frac{d}{x}+1}}{105c^5d^4+210c^4d^5+105c^3d^6} + \frac{8ad^2\sqrt{\frac{d}{x}+1}}{105c^5d^4+210c^4d^5+105c^3d^6} + \frac{ad^3x^4\sqrt{\frac{d}{x}+1}}{5} + \frac{ad^2x^2\sqrt{\frac{d}{x}+1}}{15c} - \frac{2ad^2\sqrt{\frac{d}{x}+1}}{15c^2} + \frac{bc\sqrt{d}\sqrt{\frac{d}{x}+1}}{5} + \frac{2bd^2\sqrt{\frac{d}{x}+1}}{5} + \frac{bd^3\sqrt{\frac{d}{x}+1}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**6,x)`

[Out] $15*a*c**6*d**(9/2)*x**10*\text{sqrt}(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 33*a*c**5*d**(11/2)*x**8*\text{sqrt}(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**4*d**(13/2)*x**6*\text{sqrt}(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*a*c**3*d**(15/2)*x**4*\text{sqrt}(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c**2*d**(17/2)*x**2*\text{sqrt}(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*c*d**(19/2)*\text{sqrt}(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + a*d**(3/2)*x**4*\text{sqrt}(c*x**2/d + 1)/5 + a*d**(5/2)*x**2*\text{sqrt}(c*x**2/d + 1)/(15*c) - 2*a*d**(7/2)*\text{sqrt}(c*x**2/d + 1)/(15*c**2) + b*c*\text{sqrt}(d)*x**4*\text{sqrt}(c*x**2/d + 1)/5 + 2*b*d**(3/2)*x**2*\text{sqrt}(c*x**2/d + 1)/5 + b*d**(5/2)*\text{sqrt}(c*x**2/d + 1)/(5*c)$

$$3.611 \quad \int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^4 dx$$

Optimal. Leaf size=86

$$\frac{ax^5 \left(c + \frac{d}{x^2}\right)^{5/2}}{5c} - bd^{3/2} \tanh^{-1} \left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}} \right) + bdx\sqrt{c + \frac{d}{x^2}} + \frac{1}{3}bx^3 \left(c + \frac{d}{x^2}\right)^{3/2}$$

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {451, 335, 277, 217, 206}

$$\frac{ax^5 \left(c + \frac{d}{x^2}\right)^{5/2}}{5c} - bd^{3/2} \tanh^{-1} \left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}} \right) + \frac{1}{3}bx^3 \left(c + \frac{d}{x^2}\right)^{3/2} + bdx\sqrt{c + \frac{d}{x^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]

[Out] b*d*Sqrt[c + d/x^2]*x + (b*(c + d/x^2)^(3/2)*x^3)/3 + (a*(c + d/x^2)^(5/2)*x^5)/(5*c) - b*d^(3/2)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 277

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 335

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

Rule 451

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1]))

Rubi steps

$$\begin{aligned}
 \int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^4 dx &= \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^5}{5c} + b \int \left(c + \frac{d}{x^2}\right)^{3/2} x^2 dx \\
 &= \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^5}{5c} - b \operatorname{Subst} \left(\int \frac{\left(c + dx^2\right)^{3/2}}{x^4} dx, x, \frac{1}{x} \right) \\
 &= \frac{1}{3} b \left(c + \frac{d}{x^2}\right)^{3/2} x^3 + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^5}{5c} - (bd) \operatorname{Subst} \left(\int \frac{\sqrt{c + dx^2}}{x^2} dx, x, \frac{1}{x} \right) \\
 &= bd \sqrt{c + \frac{d}{x^2}} x + \frac{1}{3} b \left(c + \frac{d}{x^2}\right)^{3/2} x^3 + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^5}{5c} - (bd^2) \operatorname{Subst} \left(\int \frac{1}{\sqrt{c + dx^2}} dx, x, \frac{1}{x} \right) \\
 &= bd \sqrt{c + \frac{d}{x^2}} x + \frac{1}{3} b \left(c + \frac{d}{x^2}\right)^{3/2} x^3 + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^5}{5c} - (bd^2) \operatorname{Subst} \left(\int \frac{1}{1 - dx^2} dx, x, \frac{1}{x} \right) \\
 &= bd \sqrt{c + \frac{d}{x^2}} x + \frac{1}{3} b \left(c + \frac{d}{x^2}\right)^{3/2} x^3 + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^5}{5c} - bd^{3/2} \tanh^{-1} \left(\frac{\sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x} \right)
 \end{aligned}$$

Mathematica [A] time = 0.15, size = 81, normalized size = 0.94

$$\frac{1}{15}x\sqrt{c + \frac{d}{x^2}} \left(\frac{3a(cx^2 + d)^2}{c} - \frac{15bd^{3/2} \tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{\sqrt{cx^2+d}} + 5b(cx^2 + 4d) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]

[Out] (Sqrt[c + d/x^2]*x*((3*a*(d + c*x^2)^2)/c + 5*b*(4*d + c*x^2) - (15*b*d^(3/2)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/Sqrt[d + c*x^2))/15

IntegrateAlgebraic [A] time = 0.12, size = 107, normalized size = 1.24

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\frac{\sqrt{cx^2+d}(3ac^2x^4+6acd^2+3ad^2+5bc^2x^2+20bcd)}{15c} - bd^{3/2} \tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right) \right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]

[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(20*b*c*d + 3*a*d^2 + 5*b*c^2*x^2 + 6*a*c*d*x^2 + 3*a*c^2*x^4))/(15*c) - b*d^(3/2)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/Sqrt[d + c*x^2]

fricas [A] time = 0.45, size = 203, normalized size = 2.36

$$\left[\frac{15bcd^{\frac{3}{2}} \log\left(\frac{cx^2-2\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2d}{x^2}\right)}{30c} + 2(3ac^2x^5 + (5bc^2 + 6acd)x^3 + (20bcd + 3ad^2)x)\sqrt{\frac{cx^2+d}{x^2}}}{15c}, \frac{15bc\sqrt{-d} \arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + (3ac^2x^5 + (5bc^2 + 6acd)x^3 + (20bcd + 3ad^2)x)\sqrt{\frac{cx^2+d}{x^2}}}{15c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="fricas")

[Out] [1/30*(15*b*c*d^(3/2)*log(-(c*x^2 - 2*sqrt(d))*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20*b*c*d + 3*a*d^2)*x)*sqrt((c*x^2 + d)/x^2))/c, 1/15*(15*b*c*sqrt(-d)*d*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20*b*c*d + 3*a*d^2)*x)*sqrt((c*x^2 + d)/x^2))/c]

giac [A] time = 0.20, size = 140, normalized size = 1.63

$$\frac{bd^2 \arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right) \operatorname{sgn}(x)}{\sqrt{-d}} - \frac{(15bcd^2 \arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right) + 20bc\sqrt{-d}d^{\frac{3}{2}} + 3a\sqrt{-d}d^{\frac{5}{2}}) \operatorname{sgn}(x)}{15c\sqrt{-d}} + \frac{3(cx^2 + d)^{\frac{5}{2}}ac^4 \operatorname{sgn}(x) + 5(cx^2 + d)^{\frac{3}{2}}bc^5 \operatorname{sgn}(x) + 15\sqrt{cx^2 + d}bc^5d \operatorname{sgn}(x)}{15c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="giac")

[Out] b*d^2*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/15*(15*b*c*d^2*arctan(sqrt(d)/sqrt(-d)) + 20*b*c*sqrt(-d)*d^(3/2) + 3*a*sqrt(-d)*d^(5/2))*sgn(x)/(c*sqrt(-d)) + 1/15*(3*(c*x^2 + d)^(5/2)*a*c^4*sgn(x) + 5*(c*x^2 + d)^(3/2)*b*c^5*sgn(x) + 15*sqrt(c*x^2 + d)*b*c^5*d*sgn(x))/c^5

maple [A] time = 0.06, size = 99, normalized size = 1.15

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}} \left(-15bcd^3 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) + 15\sqrt{cx^2+d}bcd + 5(cx^2+d)^{\frac{3}{2}}bc + 3(cx^2+d)^{\frac{5}{2}}a\right)x^3}{15(cx^2+d)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x)

[Out] 1/15*((c*x^2+d)/x^2)^(3/2)*x^3*(3*a*(c*x^2+d)^(5/2)+5*(c*x^2+d)^(3/2)*b*c-15*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*b*c+15*(c*x^2+d)^(1/2)*b*c*d)/(c*x^2+d)^(3/2)/c

maxima [A] time = 1.35, size = 91, normalized size = 1.06

$$\frac{a\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}x^5}{5c} + \frac{1}{6} \left(2\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}x^3 + 6\sqrt{c + \frac{d}{x^2}}dx + 3d^{\frac{3}{2}} \log\left(\frac{\sqrt{c + \frac{d}{x^2}}x - \sqrt{d}}{\sqrt{c + \frac{d}{x^2}}x + \sqrt{d}}\right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="maxima")

[Out] 1/5*a*(c + d/x^2)^(5/2)*x^5/c + 1/6*(2*(c + d/x^2)^(3/2)*x^3 + 6*sqrt(c + d/x^2)*d*x + 3*d^(3/2)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*b

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(a + b/x^2)*(c + d/x^2)^(3/2),x)

[Out] $\text{int}(x^4*(a + b/x^2)*(c + d/x^2)^{(3/2)}, x)$

sympy [B] time = 5.25, size = 184, normalized size = 2.14

$$\frac{ac\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5} + \frac{2ad^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5} + \frac{ad^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c} + \frac{b\sqrt{c}dx}{\sqrt{1+\frac{d}{cx^2}}} + \frac{bc\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3} + \frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3} - bd^{\frac{3}{2}}\text{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right) + \frac{bd^2}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**4,x)`

[Out] `a*c*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*a*d**(3/2)*x**2*sqrt(c*x**2/d + 1)/5 + a*d**(5/2)*sqrt(c*x**2/d + 1)/(5*c) + b*sqrt(c)*d*x/sqrt(1 + d/(c*x**2)) + b*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d + 1)/3 - b*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x)) + b*d**2/(sqrt(c)*x*sqrt(1 + d/(c*x**2)))`

$$3.612 \quad \int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^2 dx$$

Optimal. Leaf size=121

$$\frac{x \left(c + \frac{d}{x^2}\right)^{3/2} (2ad + 3bc)}{3c} - \frac{d \sqrt{c + \frac{d}{x^2}} (2ad + 3bc)}{2cx} - \frac{1}{2} \sqrt{d} (2ad + 3bc) \tanh^{-1} \left(\frac{\sqrt{d}}{x \sqrt{c + \frac{d}{x^2}}} \right) + \frac{ax^3 \left(c + \frac{d}{x^2}\right)^{5/2}}{3c}$$

Rubi [A] time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {453, 242, 277, 195, 217, 206}

$$\frac{x \left(c + \frac{d}{x^2}\right)^{3/2} (2ad + 3bc)}{3c} - \frac{d \sqrt{c + \frac{d}{x^2}} (2ad + 3bc)}{2cx} - \frac{1}{2} \sqrt{d} (2ad + 3bc) \tanh^{-1} \left(\frac{\sqrt{d}}{x \sqrt{c + \frac{d}{x^2}}} \right) + \frac{ax^3 \left(c + \frac{d}{x^2}\right)^{5/2}}{3c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]

[Out] -(d*(3*b*c + 2*a*d)*Sqrt[c + d/x^2])/(2*c*x) + ((3*b*c + 2*a*d)*(c + d/x^2)^(3/2)*x)/(3*c) + (a*(c + d/x^2)^(5/2)*x^3)/(3*c) - (Sqrt[d]*(3*b*c + 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)))/2

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} x^2 dx &= \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^3}{3c} + \frac{(3bc + 2ad) \int \left(c + \frac{d}{x^2}\right)^{3/2} dx}{3c} \\
&= \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^3}{3c} - \frac{(3bc + 2ad) \text{Subst} \left(\int \frac{(c+dx^2)^{3/2}}{x^2} dx, x, \frac{1}{x} \right)}{3c} \\
&= \frac{(3bc + 2ad) \left(c + \frac{d}{x^2}\right)^{3/2} x}{3c} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^3}{3c} - \frac{(d(3bc + 2ad)) \text{Subst} \left(\int \sqrt{c + dx^2} dx, x, \frac{1}{x} \right)}{3c} \\
&= -\frac{d(3bc + 2ad) \sqrt{c + \frac{d}{x^2}}}{2cx} + \frac{(3bc + 2ad) \left(c + \frac{d}{x^2}\right)^{3/2} x}{3c} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^3}{3c} - \frac{1}{2} (d(3bc + 2ad)) \sqrt{c + dx^2} \\
&= -\frac{d(3bc + 2ad) \sqrt{c + \frac{d}{x^2}}}{2cx} + \frac{(3bc + 2ad) \left(c + \frac{d}{x^2}\right)^{3/2} x}{3c} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^3}{3c} - \frac{1}{2} (d(3bc + 2ad)) \sqrt{c + dx^2} \\
&= -\frac{d(3bc + 2ad) \sqrt{c + \frac{d}{x^2}}}{2cx} + \frac{(3bc + 2ad) \left(c + \frac{d}{x^2}\right)^{3/2} x}{3c} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x^3}{3c} - \frac{1}{2} \sqrt{d} (3bc + 2ad) \sqrt{c + dx^2}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 105, normalized size = 0.87

$$\frac{\sqrt{c + \frac{d}{x^2}} \left(\sqrt{cx^2 + d} (2acx^4 + 8adx^2 + 6bcx^2 - 3bd) - 3\sqrt{d} x^2 (2ad + 3bc) \tanh^{-1} \left(\frac{\sqrt{cx^2 + d}}{\sqrt{d}} \right) \right)}{6x\sqrt{cx^2 + d}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]

[Out] (Sqrt[c + d/x^2]*(Sqrt[d + c*x^2]*(-3*b*d + 6*b*c*x^2 + 8*a*d*x^2 + 2*a*c*x^4) - 3*Sqrt[d]*(3*b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(6*x*Sqrt[d + c*x^2])

IntegrateAlgebraic [A] time = 0.18, size = 109, normalized size = 0.90

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\frac{1}{2} (-2ad^{3/2} - 3bc\sqrt{d}) \tanh^{-1} \left(\frac{\sqrt{cx^2 + d}}{\sqrt{d}} \right) + \frac{\sqrt{cx^2 + d} (2acx^4 + 8adx^2 + 6bcx^2 - 3bd)}{6x^2} \right)}{\sqrt{cx^2 + d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]

[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(-3*b*d + 6*b*c*x^2 + 8*a*d*x^2 + 2*a*c*x^4))/(6*x^2) + ((-3*b*c*Sqrt[d] - 2*a*d^(3/2))*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/2))/Sqrt[d + c*x^2]

fricas [A] time = 0.44, size = 190, normalized size = 1.57

$$\frac{3(3bc + 2ad)\sqrt{d}x \log\left(-\frac{cx^2 - 2\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}} + 2d}{x^2}\right) + 2(2acx^4 + 2(3bc + 4ad)x^2 - 3bd)\sqrt{\frac{cx^2+d}{x^2}}}{12x} + \frac{3(3bc + 2ad)\sqrt{-d}x \arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + (2acx^4 + 2(3bc + 4ad)x^2 - 3bd)\sqrt{\frac{cx^2+d}{x^2}}}{6x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="fricas")

[Out] [1/12*(3*(3*b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d)*sqrt((c*x^2 + d)/x^2))/x, 1/6*(3*(3*b*c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d)*sqrt((c*x^2 + d)/x^2))/x]

giac [A] time = 0.27, size = 115, normalized size = 0.95

$$\frac{2(cx^2 + d)^{\frac{3}{2}}ac\operatorname{sgn}(x) + 6\sqrt{cx^2 + d}bc^2\operatorname{sgn}(x) + 6\sqrt{cx^2 + d}acd\operatorname{sgn}(x) - \frac{3\sqrt{cx^2 + d}bcd\operatorname{sgn}(x)}{x^2} + \frac{3(3bc^2d\operatorname{sgn}(x) + 2acd^2\operatorname{sgn}(x))\arctan\left(\frac{\sqrt{cx^2 + d}}{\sqrt{-d}}\right)}{\sqrt{-d}}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="giac")

[Out] 1/6*(2*(c*x^2 + d)^(3/2)*a*c*sgn(x) + 6*sqrt(c*x^2 + d)*b*c^2*sgn(x) + 6*sqrt(c*x^2 + d)*a*c*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c*d*sgn(x)/x^2 + 3*(3*b*c^2*d*sgn(x) + 2*a*c*d^2*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d)/c

maple [A] time = 0.06, size = 170, normalized size = 1.40

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(6ad^{\frac{5}{2}}x^2\ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) + 9bc d^{\frac{3}{2}}x^2\ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) - 6\sqrt{cx^2+d}ad^2x^2 - 9\sqrt{cx^2+d}bcdx^2 - 2(cx^2+d)^{\frac{3}{2}}adx^2 - 3(cx^2+d)^{\frac{3}{2}}bcx^2 + 3(cx^2+d)^{\frac{5}{2}}b\right)x}{6(cx^2+d)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x)

[Out] $-1/6*((c*x^2+d)/x^2)^{(3/2)}*x*(-2*(c*x^2+d)^{(3/2)}*a*d*x^2-3*(c*x^2+d)^{(3/2)}*b*c*x^2+6*d^{(5/2)}*\ln(2*(d+(c*x^2+d)^{(1/2)}*d^{(1/2)})/x)*x^2*a+9*d^{(3/2)}*\ln(2*(d+(c*x^2+d)^{(1/2)}*d^{(1/2)})/x)*x^2*b*c+3*(c*x^2+d)^{(5/2)}*b-6*(c*x^2+d)^{(1/2)}*x^2*a*d^2-9*(c*x^2+d)^{(1/2)}*x^2*b*c*d)/(c*x^2+d)^{(3/2)}/d$

maxima [A] time = 1.36, size = 163, normalized size = 1.35

$$\frac{1}{6} \left(2 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} x^3 + 6 \sqrt{c + \frac{d}{x^2}} dx + 3 d^{\frac{3}{2}} \log \left(\frac{\sqrt{c + \frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x + \sqrt{d}} \right) \right) a + \frac{1}{4} \left(4 \sqrt{c + \frac{d}{x^2}} cx - \frac{2 \sqrt{c + \frac{d}{x^2}} c dx}{\left(c + \frac{d}{x^2} \right) x^2 - d} + 3 c \sqrt{d} \log \left(\frac{\sqrt{c + \frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x + \sqrt{d}} \right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="maxima")`

[Out] $1/6*(2*(c + d/x^2)^{(3/2)}*x^3 + 6*\sqrt{c + d/x^2}*d*x + 3*d^{(3/2)}*\log((\sqrt{c + d/x^2}*x - \sqrt{d})/(\sqrt{c + d/x^2}*x + \sqrt{d}))) * a + 1/4*(4*\sqrt{c + d/x^2}*c*x - 2*\sqrt{c + d/x^2}*c*d*x/((c + d/x^2)*x^2 - d) + 3*c*\sqrt{d}*\log((\sqrt{c + d/x^2}*x - \sqrt{d})/(\sqrt{c + d/x^2}*x + \sqrt{d}))) * b$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a + \frac{b}{x^2} \right) \left(c + \frac{d}{x^2} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b/x^2)*(c + d/x^2)^(3/2), x)`

[Out] `int(x^2*(a + b/x^2)*(c + d/x^2)^(3/2), x)`

sympy [A] time = 7.65, size = 202, normalized size = 1.67

$$\frac{a\sqrt{c}dx}{\sqrt{1+\frac{d}{cx^2}}} + \frac{ac\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3} + \frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3} - ad^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right) + \frac{ad^2}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}} - \frac{b\sqrt{c}d\sqrt{1+\frac{d}{cx^2}}}{2x} + \frac{b\sqrt{c}d}{x\sqrt{1+\frac{d}{cx^2}}} - \frac{3bc\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**2,x)`

[Out] $a*\sqrt{c}*d*x/\sqrt{1 + d/(c*x**2)} + a*c*\sqrt{d}*x**2*\sqrt{c*x**2/d + 1}/3 + a*d**(3/2)*\sqrt{c*x**2/d + 1}/3 - a*d**(3/2)*\operatorname{asinh}(\sqrt{d}/(\sqrt{c}*x)) + a*d**2/(\sqrt{c}*x*\sqrt{1 + d/(c*x**2)}) + b*c**(3/2)*x/\sqrt{1 + d/(c*x**2)} - b*\sqrt{c}*d*\sqrt{1 + d/(c*x**2)}/(2*x) + b*\sqrt{c}*d/(x*\sqrt{1 + d/(c*x**2)}) - 3*b*c*\sqrt{d}*\operatorname{asinh}(\sqrt{d}/(\sqrt{c}*x))/2$

$$3.613 \quad \int \left(a + \frac{b}{x^2} \right) \left(c + \frac{d}{x^2} \right)^{3/2} dx$$

Optimal. Leaf size=112

$$\frac{\left(c + \frac{d}{x^2} \right)^{3/2} (4ad + bc)}{4cx} - \frac{3\sqrt{c + \frac{d}{x^2}} (4ad + bc)}{8x} - \frac{3c(4ad + bc) \tanh^{-1} \left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}} \right)}{8\sqrt{d}} + \frac{ax \left(c + \frac{d}{x^2} \right)^{5/2}}{c}$$

Rubi [A] time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {375, 453, 195, 217, 206}

$$\frac{\left(c + \frac{d}{x^2} \right)^{3/2} (4ad + bc)}{4cx} - \frac{3\sqrt{c + \frac{d}{x^2}} (4ad + bc)}{8x} - \frac{3c(4ad + bc) \tanh^{-1} \left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}} \right)}{8\sqrt{d}} + \frac{ax \left(c + \frac{d}{x^2} \right)^{5/2}}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2), x]

[Out] (-3*(b*c + 4*a*d)*Sqrt[c + d/x^2])/(8*x) - ((b*c + 4*a*d)*(c + d/x^2)^(3/2))/(4*c*x) + (a*(c + d/x^2)^(5/2)*x)/c - (3*c*(b*c + 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]/(8*Sqrt[d])

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 375

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
  ] :-> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
  b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 453

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol]
  ] :-> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x]
  + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)
  *(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
  && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0]
  && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
 \int \left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2} dx &= -\text{Subst} \left(\int \frac{(a + bx^2)(c + dx^2)^{3/2}}{x^2} dx, x, \frac{1}{x} \right) \\
 &= \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x}{c} + \frac{(-bc - 4ad) \text{Subst} \left(\int (c + dx^2)^{3/2} dx, x, \frac{1}{x} \right)}{c} \\
 &= -\frac{(bc + 4ad) \left(c + \frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x}{c} - \frac{1}{4} (3(bc + 4ad)) \text{Subst} \left(\int \sqrt{c + dx^2} dx \right) \\
 &= -\frac{3(bc + 4ad) \sqrt{c + \frac{d}{x^2}}}{8x} - \frac{(bc + 4ad) \left(c + \frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x}{c} - \frac{1}{8} (3c(bc + 4ad)) \\
 &= -\frac{3(bc + 4ad) \sqrt{c + \frac{d}{x^2}}}{8x} - \frac{(bc + 4ad) \left(c + \frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x}{c} - \frac{1}{8} (3c(bc + 4ad)) \\
 &= -\frac{3(bc + 4ad) \sqrt{c + \frac{d}{x^2}}}{8x} - \frac{(bc + 4ad) \left(c + \frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a \left(c + \frac{d}{x^2}\right)^{5/2} x}{c} - \frac{3c(bc + 4ad) \text{ta}}{8\sqrt{c + dx^2}}
 \end{aligned}$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.61

$$\frac{\sqrt{c + \frac{d}{x^2}} (cx^2 + d)^2 \left(cx^4(4ad + bc) {}_2F_1\left(2, \frac{5}{2}; \frac{7}{2}; \frac{cx^2}{d} + 1\right) - 5bd^2 \right)}{20d^3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2), x]

[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(-5*b*d^2 + c*(b*c + 4*a*d)*x^4*Hypergeometric2F1[2, 5/2, 7/2, 1 + (c*x^2)/d]))/(20*d^3*x^3)

IntegrateAlgebraic [A] time = 0.22, size = 107, normalized size = 0.96

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\frac{\sqrt{cx^2+d}(8acx^4-4adx^2-5bcx^2-2bd)}{8x^4} - \frac{3(4acd+bc^2)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{8\sqrt{d}} \right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2), x]

[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(-2*b*d - 5*b*c*x^2 - 4*a*d*x^2 + 8*a*c*x^4))/(8*x^4) - (3*(b*c^2 + 4*a*c*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(8*Sqrt[d])))/Sqrt[d + c*x^2]

fricas [A] time = 0.44, size = 216, normalized size = 1.93

$$\left[\frac{3(bc^2 + 4acd)\sqrt{d}x^3 \log\left(-\frac{cx^2 - 2\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}} + 2d}{x^2}\right) + 2(8acdx^4 - 2bd^2 - (5bcd + 4ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16dx^3}, \frac{3(bc^2 + 4acd)\sqrt{-d}x^3 \arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + (8acdx^4 - 2bd^2 - (5bcd + 4ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8dx^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2), x, algorithm="fricas")

[Out] [1/16*(3*(b*c^2 + 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(8*a*c*d*x^4 - 2*b*d^2 - (5*b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^3), 1/8*(3*(b*c^2 + 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a*c*d*x^4 - 2*b*d^2 - (5*b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^3)]

giac [A] time = 0.26, size = 145, normalized size = 1.29

$$\frac{8\sqrt{cx^2+d}ac^2\text{sgn}(x) + \frac{3(bc^3\text{sgn}(x)+4ac^2d\text{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}} - \frac{5(cx^2+d)^{\frac{3}{2}}bc^3\text{sgn}(x)+4(cx^2+d)^{\frac{3}{2}}ac^2d\text{sgn}(x)-3\sqrt{cx^2+d}bc^3d\text{sgn}(x)-4\sqrt{cx^2+d}ac^2d^2\text{sgn}(x)}{c^2x^4}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="giac")

[Out] $\frac{1}{8} * (8 * \sqrt{c * x^2 + d} * a * c^2 * \operatorname{sgn}(x) + 3 * (b * c^3 * \operatorname{sgn}(x) + 4 * a * c^2 * d * \operatorname{sgn}(x))) * \operatorname{arctan}(\sqrt{c * x^2 + d} / \sqrt{-d}) / \sqrt{-d} - (5 * (c * x^2 + d)^{(3/2)} * b * c^3 * \operatorname{sgn}(x) + 4 * (c * x^2 + d)^{(3/2)} * a * c^2 * d * \operatorname{sgn}(x) - 3 * \sqrt{c * x^2 + d} * b * c^3 * d * \operatorname{sgn}(x) - 4 * \sqrt{c * x^2 + d} * a * c^2 * d^2 * \operatorname{sgn}(x)) / (c^2 * x^4) / c$

maple [B] time = 0.06, size = 213, normalized size = 1.90

$$\frac{\left(\frac{c x^2+d}{x^2}\right)^{\frac{3}{2}} \left(12 a c d^{\frac{5}{2}} x^4 \ln\left(\frac{2 d+2 \sqrt{c x^2+d} \sqrt{d}}{x}\right)+3 b c^2 d^{\frac{3}{2}} x^4 \ln\left(\frac{2 d+2 \sqrt{c x^2+d} \sqrt{d}}{x}\right)-12 \sqrt{c x^2+d} a c d^2 x^4-3 \sqrt{c x^2+d} b c^2 d x^4-4\left(c x^2+d\right)^{\frac{3}{2}} a c d x^4-\left(c x^2+d\right)^{\frac{3}{2}} b c^2 x^4+4\left(c x^2+d\right)^{\frac{5}{2}} a d x^2+\left(c x^2+d\right)^{\frac{5}{2}} b c x^2+2\left(c x^2+d\right)^{\frac{5}{2}} b d\right)}{8\left(c x^2+d\right)^{\frac{3}{2}} d^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(3/2),x)

[Out] $-1/8 * ((c * x^2 + d) / x^2)^{(3/2)} / x * (12 * d^{(5/2)} * \ln(2 * (d + (c * x^2 + d)^{(1/2)} * d^{(1/2)})) / x) * x^4 * a * c + 3 * d^{(3/2)} * \ln(2 * (d + (c * x^2 + d)^{(1/2)} * d^{(1/2)})) / x * x^4 * b * c^2 - 4 * (c * x^2 + d)^{(3/2)} * a * c * d * x^4 - (c * x^2 + d)^{(3/2)} * b * c^2 * x^4 + 4 * (c * x^2 + d)^{(5/2)} * x^2 * a * d + (c * x^2 + d)^{(5/2)} * x^2 * b * c - 12 * (c * x^2 + d)^{(1/2)} * x^4 * a * c * d^2 - 3 * (c * x^2 + d)^{(1/2)} * x^4 * b * c^2 * d + 2 * (c * x^2 + d)^{(5/2)} * b * d) / (c * x^2 + d)^{(3/2)} / d^2$

maxima [B] time = 1.42, size = 207, normalized size = 1.85

$$\frac{1}{4} \left(4 \sqrt{c + \frac{d}{x^2}} c x - \frac{2 \sqrt{c + \frac{d}{x^2}} c d x}{\left(c + \frac{d}{x^2}\right) x^2 - d} + 3 c \sqrt{d} \log\left(\frac{\sqrt{c + \frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x + \sqrt{d}}\right) \right) a + \frac{1}{16} \left(\frac{3 c^2 \log\left(\frac{\sqrt{c + \frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x + \sqrt{d}}\right)}{\sqrt{d}} - \frac{2 \left(5 \left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} c^2 x^3 - 3 \sqrt{c + \frac{d}{x^2}} c^2 d x\right)}{\left(c + \frac{d}{x^2}\right)^2 x^4 - 2 \left(c + \frac{d}{x^2}\right) d x^2 + d^2} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{4} * (4 * \sqrt{c + d / x^2} * c * x - 2 * \sqrt{c + d / x^2} * c * d * x / ((c + d / x^2) * x^2 - d) + 3 * c * \sqrt{d} * \log((\sqrt{c + d / x^2} * x - \sqrt{d}) / (\sqrt{c + d / x^2} * x + \sqrt{d}))) * a + 1 / 16 * (3 * c^2 * \log((\sqrt{c + d / x^2} * x - \sqrt{d}) / (\sqrt{c + d / x^2} * x + \sqrt{d})) / \sqrt{d} - 2 * (5 * (c + d / x^2)^{(3/2)} * c^2 * x^3 - 3 * \sqrt{c + d / x^2} * c^2 * d * x) / ((c + d / x^2)^2 * x^4 - 2 * (c + d / x^2) * d * x^2 + d^2)) * b$

mupad [B] time = 5.86, size = 78, normalized size = 0.70

$$\frac{a x (c x^2 + d)^{3/2} {}_2F_1\left(-\frac{3}{2}, -\frac{1}{2}; \frac{1}{2}; -\frac{d}{c x^2}\right)}{\left(\frac{d}{c} + x^2\right)^{3/2}} - \frac{b (c x^2 + d)^{3/2} {}_2F_1\left(-\frac{3}{2}, \frac{1}{2}; \frac{3}{2}; -\frac{d}{c x^2}\right)}{x \left(\frac{d}{c} + x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x^2)*(c + d/x^2)^(3/2),x)`

[Out] $(a*x*(d + c*x^2)^{(3/2)}*\text{hypergeom}([-3/2, -1/2], 1/2, -d/(c*x^2)))/(d/c + x^2)^{(3/2)} - (b*(d + c*x^2)^{(3/2)}*\text{hypergeom}([-3/2, 1/2], 3/2, -d/(c*x^2)))/(x*(d/c + x^2)^{(3/2)})$

sympy [B] time = 11.78, size = 216, normalized size = 1.93

$$\frac{ac^3x}{\sqrt{1+\frac{d}{cx^2}}} - \frac{a\sqrt{c}d\sqrt{1+\frac{d}{cx^2}}}{2x} + \frac{a\sqrt{c}d}{x\sqrt{1+\frac{d}{cx^2}}} - \frac{3ac\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2} - \frac{bc^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x} - \frac{bc^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}} - \frac{3b\sqrt{c}d}{8x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{3bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8\sqrt{d}} - \frac{bd^2}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(3/2),x)`

[Out] $a*c^{(3/2)}*x/\sqrt{1 + d/(c*x**2)} - a*\sqrt{c}*d*\sqrt{1 + d/(c*x**2)}/(2*x) + a*\sqrt{c}*d/(x*\sqrt{1 + d/(c*x**2)}) - 3*a*c*\sqrt{d}*asinh(\sqrt{d}/(\sqrt{c}*x))/2 - b*c^{(3/2)}*\sqrt{1 + d/(c*x**2)}/(2*x) - b*c^{(3/2)}/(8*x*\sqrt{1 + d/(c*x**2)}) - 3*b*\sqrt{c}*d/(8*x**3*\sqrt{1 + d/(c*x**2)}) - 3*b*c**2*asinh(\sqrt{d}/(\sqrt{c}*x))/(8*\sqrt{d}) - b*d**2/(4*\sqrt{c}*x**5*\sqrt{1 + d/(c*x**2)})$

$$3.614 \quad \int \frac{\left(a + \frac{b}{x^2}\right)\left(c + \frac{d}{x^2}\right)^{3/2}}{x^2} dx$$

Optimal. Leaf size=123

$$\frac{c^2(bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{16d^{3/2}} + \frac{\left(c + \frac{d}{x^2}\right)^{3/2} (bc - 6ad)}{24dx} + \frac{c\sqrt{c + \frac{d}{x^2}} (bc - 6ad)}{16dx} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{6dx}$$

Rubi [A] time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {459, 335, 195, 217, 206}

$$\frac{c^2(bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{16d^{3/2}} + \frac{\left(c + \frac{d}{x^2}\right)^{3/2} (bc - 6ad)}{24dx} + \frac{c\sqrt{c + \frac{d}{x^2}} (bc - 6ad)}{16dx} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{6dx}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]

[Out] (c*(b*c - 6*a*d)*Sqrt[c + d/x^2])/(16*d*x) + ((b*c - 6*a*d)*(c + d/x^2)^(3/2))/(24*d*x) - (b*(c + d/x^2)^(5/2))/(6*d*x) + (c^2*(b*c - 6*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)))/(16*d^(3/2))

Rule 195

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 335

$\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.) * (x_)^{(n_)})^{(p_)}, x_Symbol] := -\text{Subst}[\text{Int}[(a + b/x^n)^p / x^{(m+2)}, x], x, 1/x] /; \text{FreeQ}\{a, b, p\}, x] \&\& \text{ILtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Rule 459

$\text{Int}[(e_.) * (x_)^{(m_.)} * ((a_) + (b_.) * (x_)^{(n_)})^{(p_.)} * ((c_) + (d_.) * (x_)^{(n_.)}), x_Symbol] := \text{Simp}[(d * (e * x)^{(m+1}) * (a + b * x^n)^{(p+1)}) / (b * e * (m + n * (p + 1) + 1)), x] - \text{Dist}[(a * d * (m + 1) - b * c * (m + n * (p + 1) + 1)) / (b * (m + n * (p + 1) + 1)), \text{Int}[(e * x)^m * (a + b * x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& \text{NeQ}[b * c - a * d, 0] \&\& \text{NeQ}[m + n * (p + 1) + 1, 0]$

Rubi steps

$$\begin{aligned}
 \int \frac{\left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2}}{x^2} dx &= -\frac{b \left(c + \frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{(-bc + 6ad) \int \frac{\left(c + \frac{d}{x^2}\right)^{3/2}}{x^2} dx}{6d} \\
 &= -\frac{b \left(c + \frac{d}{x^2}\right)^{5/2}}{6dx} - \frac{(-bc + 6ad) \text{Subst}\left(\int (c + dx^2)^{3/2} dx, x, \frac{1}{x}\right)}{6d} \\
 &= \frac{(bc - 6ad) \left(c + \frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b \left(c + \frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{(c(bc - 6ad)) \text{Subst}\left(\int \sqrt{c + dx^2} dx, x, \frac{1}{x}\right)}{8d} \\
 &= \frac{c(bc - 6ad) \sqrt{c + \frac{d}{x^2}}}{16dx} + \frac{(bc - 6ad) \left(c + \frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b \left(c + \frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{(c^2(bc - 6ad)) \text{Subst}}{16d} \\
 &= \frac{c(bc - 6ad) \sqrt{c + \frac{d}{x^2}}}{16dx} + \frac{(bc - 6ad) \left(c + \frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b \left(c + \frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{(c^2(bc - 6ad)) \text{Subst}}{16d} \\
 &= \frac{c(bc - 6ad) \sqrt{c + \frac{d}{x^2}}}{16dx} + \frac{(bc - 6ad) \left(c + \frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b \left(c + \frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{c^2(bc - 6ad) \tanh^{-1}}{16d^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.24, size = 126, normalized size = 1.02

$$\frac{\sqrt{c + \frac{d}{x^2}} \left((cx^2 + d) (6adx^2 (5cx^2 + 2d) + b (3c^2x^4 + 14cdx^2 + 8d^2)) + 3c^2x^6 \sqrt{\frac{cx^2}{d} + 1} (6ad - bc) \tanh^{-1} \left(\sqrt{\frac{cx^2}{d} + 1} \right) \right)}{48dx^5 (cx^2 + d)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]

[Out] -1/48*(Sqrt[c + d/x^2]*((d + c*x^2)*(6*a*d*x^2*(2*d + 5*c*x^2) + b*(8*d^2 + 14*c*d*x^2 + 3*c^2*x^4)) + 3*c^2*(-(b*c) + 6*a*d)*x^6*Sqrt[1 + (c*x^2)/d]*ArcTanh[Sqrt[1 + (c*x^2)/d]]))/(d*x^5*(d + c*x^2))

IntegrateAlgebraic [A] time = 0.25, size = 127, normalized size = 1.03

$$\frac{x \sqrt{c + \frac{d}{x^2}} \left(\frac{\sqrt{cx^2+d} (-30acdx^4 - 12ad^2x^2 - 3bc^2x^4 - 14bcdx^2 - 8bd^2)}{48dx^6} + \frac{(bc^3 - 6ac^2d) \tanh^{-1} \left(\frac{\sqrt{cx^2+d}}{\sqrt{d}} \right)}{16d^{3/2}} \right)}{\sqrt{cx^2 + d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]

[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(-8*b*d^2 - 14*b*c*d*x^2 - 12*a*d^2*x^2 - 3*b*c^2*x^4 - 30*a*c*d*x^4))/(48*d*x^6) + ((b*c^3 - 6*a*c^2*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(16*d^(3/2))))/Sqrt[d + c*x^2]

fricas [A] time = 0.46, size = 246, normalized size = 2.00

$$\frac{3(bc^3 - 6ac^2d)\sqrt{d}x^5 \log\left(\frac{cx^2 - 2\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2} + 2d}}{x^2}\right) + 2(3(bc^2d + 10acd^2)x^4 + 8bd^3 + 2(7bcd^2 + 6ad^3)x^2)\sqrt{\frac{cx^2+d}{x^2}} - 3(bc^3 - 6ac^2d)\sqrt{-d}x^5 \arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + (3(bc^2d + 10acd^2)x^4 + 8bd^3 + 2(7bcd^2 + 6ad^3)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{96d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")

[Out] [-1/96*(3*(b*c^3 - 6*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 - 2*sqrt(d))*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 + 2*(7*b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5), -1/48*(3*(b*c^3 - 6*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 + 2*(7*b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5)]

giac [A] time = 0.30, size = 173, normalized size = 1.41

$$\frac{3(bc^4 \operatorname{sgn}(x) - 6ac^3 d \operatorname{sgn}(x)) \arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right) + 3(cx^2+d)^{\frac{5}{2}} bc^4 \operatorname{sgn}(x) + 30(cx^2+d)^{\frac{5}{2}} ac^3 d \operatorname{sgn}(x) + 8(cx^2+d)^{\frac{3}{2}} bc^4 d \operatorname{sgn}(x) - 48(cx^2+d)^{\frac{3}{2}} ac^3 d^2 \operatorname{sgn}(x) - 3\sqrt{cx^2+d} bc^4 d^2 \operatorname{sgn}(x) + 18\sqrt{cx^2+d} ac^3 d^3 \operatorname{sgn}(x)}{\sqrt{-d}} + \frac{48c}{c^3 dx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")

[Out]
$$-1/48*(3*(b*c^4*\operatorname{sgn}(x) - 6*a*c^3*d*\operatorname{sgn}(x))*\arctan(\operatorname{sqrt}(c*x^2 + d)/\operatorname{sqrt}(-d)) /(\operatorname{sqrt}(-d)*d) + (3*(c*x^2 + d)^{(5/2)}*b*c^4*\operatorname{sgn}(x) + 30*(c*x^2 + d)^{(5/2)}*a*c^3*d*\operatorname{sgn}(x) + 8*(c*x^2 + d)^{(3/2)}*b*c^4*d*\operatorname{sgn}(x) - 48*(c*x^2 + d)^{(3/2)}*a*c^3*d^2*\operatorname{sgn}(x) - 3*\operatorname{sqrt}(c*x^2 + d)*b*c^4*d^2*\operatorname{sgn}(x) + 18*\operatorname{sqrt}(c*x^2 + d)*a*c^3*d^3*\operatorname{sgn}(x))/(c^3*d*x^6)/c$$

maple [B] time = 0.07, size = 259, normalized size = 2.11

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}} \left(18a^2 d^{\frac{5}{2}} x^6 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) - 3b^2 c^{\frac{3}{2}} d^{\frac{3}{2}} x^6 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) - 18\sqrt{cx^2+d} a^2 d^2 x^6 + 3\sqrt{cx^2+d} b^2 d^2 x^6 - 6(c^2+d)^{\frac{3}{2}} a^2 d x^6 + (c^2+d)^{\frac{3}{2}} b^2 c^3 x^6 + 6(c^2+d)^{\frac{5}{2}} a c d x^4 - (c^2+d)^{\frac{5}{2}} b^2 c^2 x^4 + 12(c^2+d)^{\frac{5}{2}} a d^2 x^2 - 2(c^2+d)^{\frac{5}{2}} b c d x^2 + 8(c^2+d)^{\frac{3}{2}} b d^2\right)}{48(c^2+d)^{\frac{3}{2}} d^3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x)

[Out]
$$-1/48*((c*x^2+d)/x^2)^{(3/2)}/x^3*(18*d^{(5/2)}*\ln(2*(d+(c*x^2+d)^{(1/2)})*d^{(1/2)})/x)*x^6*a*c^2-3*d^{(3/2)}*\ln(2*(d+(c*x^2+d)^{(1/2)})*d^{(1/2)})/x)*x^6*b*c^3-6*(c*x^2+d)^{(3/2)}*x^6*a*c^2*d+(c*x^2+d)^{(3/2)}*x^6*b*c^3+6*(c*x^2+d)^{(5/2)}*x^4*a*c*d-(c*x^2+d)^{(5/2)}*x^4*b*c^2-18*(c*x^2+d)^{(1/2)}*x^6*a*c^2*d^2+3*(c*x^2+d)^{(1/2)}*x^6*b*c^3*d+12*(c*x^2+d)^{(5/2)}*x^2*a*d^2-2*(c*x^2+d)^{(5/2)}*x^2*b*c*d+8*(c*x^2+d)^{(5/2)}*b*d^2)/(c*x^2+d)^{(3/2)}/d^3$$

maxima [B] time = 1.38, size = 275, normalized size = 2.24

$$\frac{1}{16} \left(\frac{3c^2 \log\left(\frac{\sqrt{\frac{c+d}{x^2}x-\sqrt{d}}}{\sqrt{\frac{c+d}{x^2}x+\sqrt{d}}}\right)}{\sqrt{d}} - \frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3-3\sqrt{c+\frac{d}{x^2}}c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2x^4-2\left(c+\frac{d}{x^2}\right)dx^2+d^2} \right) a - \frac{1}{96} \left(\frac{3c^3 \log\left(\frac{\sqrt{\frac{c+d}{x^2}x-\sqrt{d}}}{\sqrt{\frac{c+d}{x^2}x+\sqrt{d}}}\right)}{d^{\frac{3}{2}}} + \frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5+8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3-3\sqrt{c+\frac{d}{x^2}}c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3dx^6-3\left(c+\frac{d}{x^2}\right)^2d^2x^4+3\left(c+\frac{d}{x^2}\right)d^3x^2-d^4} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")

[Out]
$$1/16*(3*c^2*\log((\operatorname{sqrt}(c + d/x^2)*x - \operatorname{sqrt}(d))/(\operatorname{sqrt}(c + d/x^2)*x + \operatorname{sqrt}(d)))/\operatorname{sqrt}(d) - 2*(5*(c + d/x^2)^{(3/2)}*c^2*x^3 - 3*\operatorname{sqrt}(c + d/x^2)*c^2*d*x)/((c + d/x^2)^2*x^4 - 2*(c + d/x^2)*d*x^2 + d^2))*a - 1/96*(3*c^3*\log((\operatorname{sqrt}(c + d/x^2)*x - \operatorname{sqrt}(d))/(\operatorname{sqrt}(c + d/x^2)*x + \operatorname{sqrt}(d))))/d^{(3/2)} + 2*(3*(c + d/x^2)^{(5/2)}*c^3*x^5 + 8*(c + d/x^2)^{(3/2)}*c^3*d*x^3 - 3*\operatorname{sqrt}(c + d/x^2)*c^3*d^2*x)/d^3$$

$\frac{d^2 x}{((c + d/x^2)^3 d x^6 - 3(c + d/x^2)^2 d^2 x^4 + 3(c + d/x^2) d^3 x^2 - d^4))} * b$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^2, x)

[Out] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^2, x)

sympy [B] time = 18.95, size = 253, normalized size = 2.06

$$-\frac{ac^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x} - \frac{ac^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}} - \frac{3a\sqrt{c}d}{8x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{3ac^2 \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{cx}}\right)}{8\sqrt{d}} - \frac{ad^2}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}} - \frac{bc^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}} - \frac{17bc^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{11b\sqrt{c}d}{24x^5\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^3 \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{cx}}\right)}{16d^{\frac{3}{2}}} - \frac{bd^2}{6\sqrt{c}x^7\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**2, x)

[Out] $-a*c^{3/2}*\sqrt{1 + d/(c*x**2)}/(2*x) - a*c^{3/2}/(8*x*\sqrt{1 + d/(c*x**2)}) - 3*a*\sqrt{c}*d/(8*x**3*\sqrt{1 + d/(c*x**2)}) - 3*a*c^{3/2}*\operatorname{asinh}(\sqrt{d}/(\sqrt{c}*x))/(8*\sqrt{d}) - a*d**2/(4*\sqrt{c}*x**5*\sqrt{1 + d/(c*x**2)}) - b*c^{5/2}/(16*d*x*\sqrt{1 + d/(c*x**2)}) - 17*b*c^{3/2}/(48*x**3*\sqrt{1 + d/(c*x**2)}) - 11*b*\sqrt{c}*d/(24*x**5*\sqrt{1 + d/(c*x**2)}) + b*c^{3/2}*\operatorname{asinh}(\sqrt{d}/(\sqrt{c}*x))/(16*d^{3/2}) - b*d**2/(6*\sqrt{c}*x**7*\sqrt{1 + d/(c*x**2)})$

$$3.615 \quad \int \frac{\left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2}}{x^4} dx$$

Optimal. Leaf size=159

$$-\frac{c^3(3bc - 8ad) \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{128d^{5/2}} + \frac{c^2\sqrt{c + \frac{d}{x^2}}(3bc - 8ad)}{128d^2x} + \frac{c\sqrt{c + \frac{d}{x^2}}(3bc - 8ad)}{64dx^3} + \frac{\left(c + \frac{d}{x^2}\right)^{3/2}(3bc - 8ad)}{48dx^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

Rubi [A] time = 0.09, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {459, 335, 279, 321, 217, 206}

$$\frac{c^2\sqrt{c + \frac{d}{x^2}}(3bc - 8ad)}{128d^2x} - \frac{c^3(3bc - 8ad) \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{128d^{5/2}} + \frac{c\sqrt{c + \frac{d}{x^2}}(3bc - 8ad)}{64dx^3} + \frac{\left(c + \frac{d}{x^2}\right)^{3/2}(3bc - 8ad)}{48dx^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]

[Out] (c*(3*b*c - 8*a*d)*Sqrt[c + d/x^2])/(64*d*x^3) + ((3*b*c - 8*a*d)*(c + d/x^2)^(3/2))/(48*d*x^3) - (b*(c + d/x^2)^(5/2))/(8*d*x^3) + (c^2*(3*b*c - 8*a*d)*Sqrt[c + d/x^2])/(128*d^2*x) - (c^3*(3*b*c - 8*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(128*d^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 279

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p + 1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,

p, x]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 335

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

Rule 459

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{x^2}\right)\left(c + \frac{d}{x^2}\right)^{3/2}}{x^4} dx &= -\frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{(-3bc + 8ad) \int \frac{\left(c + \frac{d}{x^2}\right)^{3/2}}{x^4} dx}{8d} \\
&= -\frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{8dx^3} - \frac{(-3bc + 8ad) \text{Subst}\left(\int x^2\left(c + dx^2\right)^{3/2} dx, x, \frac{1}{x}\right)}{8d} \\
&= \frac{(3bc - 8ad)\left(c + \frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{(c(3bc - 8ad)) \text{Subst}\left(\int x^2\sqrt{c + dx^2} dx, x, \frac{1}{x}\right)}{16d} \\
&= \frac{c(3bc - 8ad)\sqrt{c + \frac{d}{x^2}}}{64dx^3} + \frac{(3bc - 8ad)\left(c + \frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{(c^2(3bc - 8ad)) \text{Subst}\left(\int \sqrt{c + dx^2} dx, x, \frac{1}{x}\right)}{16d} \\
&= \frac{c(3bc - 8ad)\sqrt{c + \frac{d}{x^2}}}{64dx^3} + \frac{(3bc - 8ad)\left(c + \frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{c^2(3bc - 8ad)\sqrt{c + \frac{d}{x^2}}}{128d^2x} \\
&= \frac{c(3bc - 8ad)\sqrt{c + \frac{d}{x^2}}}{64dx^3} + \frac{(3bc - 8ad)\left(c + \frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{c^2(3bc - 8ad)\sqrt{c + \frac{d}{x^2}}}{128d^2x} \\
&= \frac{c(3bc - 8ad)\sqrt{c + \frac{d}{x^2}}}{64dx^3} + \frac{(3bc - 8ad)\left(c + \frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{c^2(3bc - 8ad)\sqrt{c + \frac{d}{x^2}}}{128d^2x}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 71, normalized size = 0.45

$$\frac{\sqrt{c + \frac{d}{x^2}} (cx^2 + d)^2 \left(c^3 x^8 (8ad - 3bc) {}_2F_1\left(\frac{5}{2}, 4; \frac{7}{2}; \frac{cx^2}{d} + 1\right) - 5bd^4\right)}{40d^5 x^7}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]

[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(-5*b*d^4 + c^3*(-3*b*c + 8*a*d)*x^8*Hypergeometric2F1[5/2, 4, 7/2, 1 + (c*x^2)/d]))/(40*d^5*x^7)

IntegrateAlgebraic [A] time = 0.31, size = 152, normalized size = 0.96

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\frac{(8ac^3d - 3bc^4) \tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{128d^{5/2}} + \frac{\sqrt{cx^2+d}(-24ac^2dx^6 - 112acd^2x^4 - 64ad^3x^2 + 9bc^3x^6 - 6bc^2dx^4 - 72bcd^2x^2 - 48bd^3)}{384d^2x^8} \right)}{\sqrt{cx^2 + d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4, x]

[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(-48*b*d^3 - 72*b*c*d^2*x^2 - 64*a*d^3*x^2 - 6*b*c^2*d*x^4 - 112*a*c*d^2*x^4 + 9*b*c^3*x^6 - 24*a*c^2*d*x^6))/(384*d^2*x^8) + ((-3*b*c^4 + 8*a*c^3*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(128*d^(5/2))))/Sqrt[d + c*x^2]

fricas [A] time = 0.47, size = 298, normalized size = 1.87

$$\frac{3(3bc^4 - 8ac^2d)\sqrt{d}x^7 \log\left(-\frac{c^2+2\sqrt{d}\sqrt{\frac{cx^2+d}{x^2}}}{x^2}\right) - 2(3(3bc^2d - 8ac^2d^2)x^4 - 48bd^4 - 2(3bc^2d^2 + 56acd^2)x^4 - 8(9bcd^3 + 8ad^4)x^2)\sqrt{\frac{cx^2+d}{x^2}} - 3(3bc^4 - 8ac^2d)\sqrt{-d}x^7 \arctan\left(\frac{\sqrt{-d}\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + (3(3bc^3d - 8ac^2d^2)x^4 - 48bd^4 - 2(3bc^2d^2 + 56acd^2)x^4 - 8(9bcd^3 + 8ad^4)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{768d^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4, x, algorithm="fricas")

[Out] [-1/768*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(d)*x^7*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(3*b*c^3*d - 8*a*c^2*d^2)*x^6 - 48*b*d^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*a*d^4)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^7), 1/384*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(-d)*x^7*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*(3*b*c^3*d - 8*a*c^2*d^2)*x^6 - 48*b*d^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*a*d^4)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^7)]

giac [A] time = 0.37, size = 214, normalized size = 1.35

$$\frac{3(3bc^5\operatorname{sgn}(x) - 8ac^4d\operatorname{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right) + 9(cx^2+d)^{\frac{7}{2}}bc^5\operatorname{sgn}(x) - 24(cx^2+d)^{\frac{7}{2}}ac^4d\operatorname{sgn}(x) - 33(cx^2+d)^{\frac{5}{2}}bc^5d\operatorname{sgn}(x) - 40(cx^2+d)^{\frac{5}{2}}ac^4d^2\operatorname{sgn}(x) - 33(cx^2+d)^{\frac{3}{2}}bc^5d^2\operatorname{sgn}(x) + 88(cx^2+d)^{\frac{3}{2}}ac^4d^3\operatorname{sgn}(x) + 9\sqrt{cx^2+d}bc^5d^3\operatorname{sgn}(x) - 24\sqrt{cx^2+d}ac^4d^4\operatorname{sgn}(x)}{384c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4, x, algorithm="giac")

[Out] 1/384*(3*(3*b*c^5*sgn(x) - 8*a*c^4*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d^2) + (9*(c*x^2 + d)^(7/2)*b*c^5*sgn(x) - 24*(c*x^2 + d)^(7/2)*a*c^4*d*sgn(x) - 33*(c*x^2 + d)^(5/2)*b*c^5*d*sgn(x) - 40*(c*x^2 + d)^(5/2)*a*c^4*d^2*sgn(x) - 33*(c*x^2 + d)^(3/2)*b*c^5*d^2*sgn(x) + 88*(c*x^2 + d

$(3/2)*a*c^4*d^3*\text{sgn}(x) + 9*\text{sqrt}(c*x^2 + d)*b*c^5*d^3*\text{sgn}(x) - 24*\text{sqrt}(c*x^2 + d)*a*c^4*d^4*\text{sgn}(x))/(c^4*d^2*x^8)/c$

maple [B] time = 0.07, size = 302, normalized size = 1.90

$$\frac{\left(\frac{c^2+d}{x^2}\right)^{\frac{3}{2}} \left(24a c^4 d^4 x^8 \ln\left(\frac{24a c^4 d^4 x^8 + \sqrt{c^2+d} \sqrt{d}}{c}\right) - 9b c^5 d^3 x^8 \ln\left(\frac{24a c^4 d^4 x^8 + \sqrt{c^2+d} \sqrt{d}}{c}\right) - 24\sqrt{c^2+d} a c^4 d^4 x^8 + 9\sqrt{c^2+d} b c^5 d^3 x^8 - 8(c^2+d)^{\frac{3}{2}} a c^4 d^4 x^8 + 3(c^2+d)^{\frac{3}{2}} b c^5 d^3 x^8 + 8(c^2+d)^{\frac{3}{2}} a c^4 d^4 x^8 - 3(c^2+d)^{\frac{3}{2}} b c^5 d^3 x^8 + 16(c^2+d)^{\frac{3}{2}} a c^4 d^4 x^8 - 6(c^2+d)^{\frac{3}{2}} b c^5 d^3 x^8 - 64(c^2+d)^{\frac{3}{2}} a d^4 x^8 + 24(c^2+d)^{\frac{3}{2}} b c^5 d^3 x^8 - 48(c^2+d)^{\frac{3}{2}} b d^6\right)}{384(c^2+d)^{\frac{3}{2}} d^4 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x)

[Out] $\frac{1}{384} * ((c*x^2+d)/x^2)^{(3/2)} / x^5 * (24*d^{(5/2)} * \ln(2*(d+(c*x^2+d))^{(1/2)} * d^{(1/2)}) / x) * x^8 * a * c^3 - 9*d^{(3/2)} * \ln(2*(d+(c*x^2+d))^{(1/2)} * d^{(1/2)}) / x) * x^8 * b * c^4 - 8*(c*x^2+d)^{(3/2)} * x^8 * a * c^3 * d + 3*(c*x^2+d)^{(3/2)} * x^8 * b * c^4 + 8*(c*x^2+d)^{(5/2)} * x^6 * a * c^2 * d - 3*(c*x^2+d)^{(5/2)} * x^6 * b * c^3 - 24*(c*x^2+d)^{(1/2)} * x^8 * a * c^3 * d^2 + 9*(c*x^2+d)^{(1/2)} * x^8 * b * c^4 * d + 16*(c*x^2+d)^{(5/2)} * x^4 * a * c * d^2 - 6*(c*x^2+d)^{(5/2)} * x^4 * b * c^2 * d - 64*(c*x^2+d)^{(5/2)} * x^2 * a * d^3 + 24*(c*x^2+d)^{(5/2)} * x^2 * b * c * d^2 - 48*(c*x^2+d)^{(5/2)} * b * d^3) / (c*x^2+d)^{(3/2)} / d^4$

maxima [B] time = 1.57, size = 354, normalized size = 2.23

$$-\frac{1}{96} \left(\frac{3c^3 \log\left(\frac{\sqrt{\frac{c+d}{x^2}} x - \sqrt{d}}{\sqrt{\frac{c+d}{x^2}} x + \sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{2\left(3\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}} c^3 x^5 + 8\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} c^3 d x^3 - 3\sqrt{c + \frac{d}{x^2}} c^3 d^2 x\right)}{\left(c + \frac{d}{x^2}\right)^3 d x^6 - 3\left(c + \frac{d}{x^2}\right)^2 d^2 x^4 + 3\left(c + \frac{d}{x^2}\right) d^3 x^2 - d^4} \right) + \frac{1}{256} \left(\frac{3c^4 \log\left(\frac{\sqrt{\frac{c+d}{x^2}} x - \sqrt{d}}{\sqrt{\frac{c+d}{x^2}} x + \sqrt{d}}\right)}{d^{\frac{5}{2}}} + \frac{2\left(3\left(c + \frac{d}{x^2}\right)^{\frac{7}{2}} c^4 x^7 - 11\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}} c^4 d x^5 - 11\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} c^4 d^2 x^3 + 3\sqrt{c + \frac{d}{x^2}} c^4 d^3 x\right)}{\left(c + \frac{d}{x^2}\right)^4 d^2 x^8 - 4\left(c + \frac{d}{x^2}\right)^3 d^3 x^6 + 6\left(c + \frac{d}{x^2}\right)^2 d^4 x^4 - 4\left(c + \frac{d}{x^2}\right) d^5 x^2 + d^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")

[Out] $-1/96 * (3*c^3 * \log((\text{sqrt}(c + d/x^2)*x - \text{sqrt}(d))/(\text{sqrt}(c + d/x^2)*x + \text{sqrt}(d))))/d^{(3/2)} + 2*(3*(c + d/x^2)^{(5/2)} * c^3 * x^5 + 8*(c + d/x^2)^{(3/2)} * c^3 * d * x^3 - 3*\text{sqrt}(c + d/x^2) * c^3 * d^2 * x) / ((c + d/x^2)^3 * d * x^6 - 3*(c + d/x^2)^2 * d^2 * x^4 + 3*(c + d/x^2) * d^3 * x^2 - d^4) * a + 1/256 * (3*c^4 * \log((\text{sqrt}(c + d/x^2)*x - \text{sqrt}(d))/(\text{sqrt}(c + d/x^2)*x + \text{sqrt}(d))))/d^{(5/2)} + 2*(3*(c + d/x^2)^{(7/2)} * c^4 * x^7 - 11*(c + d/x^2)^{(5/2)} * c^4 * d * x^5 - 11*(c + d/x^2)^{(3/2)} * c^4 * d^2 * x^3 + 3*\text{sqrt}(c + d/x^2) * c^4 * d^3 * x) / ((c + d/x^2)^4 * d^2 * x^8 - 4*(c + d/x^2)^3 * d^3 * x^6 + 6*(c + d/x^2)^2 * d^4 * x^4 - 4*(c + d/x^2) * d^5 * x^2 + d^6) * b$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{x^2}\right) \left(c + \frac{d}{x^2}\right)^{3/2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x)

[Out] $\int \left((a + b/x^2)(c + d/x^2)^{3/2} \right) / x^4, x$

sympy [B] time = 29.16, size = 287, normalized size = 1.81

$$-\frac{ac^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{c^2}}} - \frac{17ac^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{c^2}}} - \frac{11a\sqrt{cd}}{24x^5\sqrt{1+\frac{d}{c^2}}} + \frac{ac^3 \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{cx}}\right)}{16d^{\frac{3}{2}}} - \frac{ad^2}{6\sqrt{c}x^7\sqrt{1+\frac{d}{c^2}}} + \frac{3bc^{\frac{7}{2}}}{128d^2x\sqrt{1+\frac{d}{c^2}}} + \frac{bc^{\frac{5}{2}}}{128dx^3\sqrt{1+\frac{d}{c^2}}} - \frac{13bc^{\frac{3}{2}}}{64x^5\sqrt{1+\frac{d}{c^2}}} - \frac{5b\sqrt{cd}}{16x^7\sqrt{1+\frac{d}{c^2}}} - \frac{3bc^4 \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{cx}}\right)}{128d^{\frac{5}{2}}} - \frac{bd^2}{8\sqrt{c}x^9\sqrt{1+\frac{d}{c^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**4,x)`

[Out] $-a*c^{5/2}/(16*d*x*\sqrt{1+d/(c*x^2)}) - 17*a*c^{3/2}/(48*x^3*\sqrt{1+d/(c*x^2)}) - 11*a*\sqrt{c}*d/(24*x^5*\sqrt{1+d/(c*x^2)}) + a*c^3*\operatorname{asin}(\sqrt{d}/(\sqrt{c}*x))/(16*d^{3/2}) - a*d^2/(6*\sqrt{c}*x^7*\sqrt{1+d/(c*x^2)}) + 3*b*c^{7/2}/(128*d^2*x*\sqrt{1+d/(c*x^2)}) + b*c^{5/2}/(128*d*x^3*\sqrt{1+d/(c*x^2)}) - 13*b*c^{3/2}/(64*x^5*\sqrt{1+d/(c*x^2)}) - 5*b*\sqrt{c}*d/(16*x^7*\sqrt{1+d/(c*x^2)}) - 3*b*c^4*\operatorname{asinh}(\sqrt{d}/(\sqrt{c}*x))/(128*d^{5/2}) - b*d^2/(8*\sqrt{c}*x^9*\sqrt{1+d/(c*x^2)})$

$$3.616 \quad \int \frac{\left(a + \frac{b}{x^2}\right)x^3}{\sqrt{c + \frac{d}{x^2}}} dx$$

Optimal. Leaf size=90

$$-\frac{d(4bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}} + \frac{x^2 \sqrt{c + \frac{d}{x^2}} (4bc - 3ad)}{8c^2} + \frac{ax^4 \sqrt{c + \frac{d}{x^2}}}{4c}$$

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 78, 51, 63, 208}

$$\frac{x^2 \sqrt{c + \frac{d}{x^2}} (4bc - 3ad)}{8c^2} - \frac{d(4bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}} + \frac{ax^4 \sqrt{c + \frac{d}{x^2}}}{4c}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*x^3)/Sqrt[c + d/x^2], x]

[Out] ((4*b*c - 3*a*d)*Sqrt[c + d/x^2]*x^2)/(8*c^2) + (a*Sqrt[c + d/x^2]*x^4)/(4*c) - (d*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(5/2))

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{\left(a + \frac{b}{x^2}\right) x^3}{\sqrt{c + \frac{d}{x^2}}} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{a + bx}{x^3 \sqrt{c + dx}} dx, x, \frac{1}{x^2}\right)\right) \\
 &= \frac{a\sqrt{c + \frac{d}{x^2}} x^4}{4c} - \frac{\left(2bc - \frac{3ad}{2}\right) \text{Subst}\left(\int \frac{1}{x^2 \sqrt{c + dx}} dx, x, \frac{1}{x^2}\right)}{4c} \\
 &= \frac{(4bc - 3ad)\sqrt{c + \frac{d}{x^2}} x^2}{8c^2} + \frac{a\sqrt{c + \frac{d}{x^2}} x^4}{4c} + \frac{(d(4bc - 3ad)) \text{Subst}\left(\int \frac{1}{x \sqrt{c + dx}} dx, x, \frac{1}{x^2}\right)}{16c^2} \\
 &= \frac{(4bc - 3ad)\sqrt{c + \frac{d}{x^2}} x^2}{8c^2} + \frac{a\sqrt{c + \frac{d}{x^2}} x^4}{4c} + \frac{(4bc - 3ad) \text{Subst}\left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + \frac{d}{x^2}}\right)}{8c^2} \\
 &= \frac{(4bc - 3ad)\sqrt{c + \frac{d}{x^2}} x^2}{8c^2} + \frac{a\sqrt{c + \frac{d}{x^2}} x^4}{4c} - \frac{d(4bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 0.11, size = 95, normalized size = 1.06

$$\frac{\sqrt{c} x (c x^2 + d) (2 a c x^2 - 3 a d + 4 b c) + d \sqrt{c x^2 + d} (3 a d - 4 b c) \tanh^{-1} \left(\frac{\sqrt{c} x}{\sqrt{c x^2 + d}} \right)}{8 c^{5/2} x \sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*x^3)/Sqrt[c + d/x^2], x]

[Out] (Sqrt[c]*x*(d + c*x^2)*(4*b*c - 3*a*d + 2*a*c*x^2) + d*(-4*b*c + 3*a*d)*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[d + c*x^2]])/(8*c^(5/2)*Sqrt[c + d/x^2]*x)

IntegrateAlgebraic [A] time = 0.14, size = 88, normalized size = 0.98

$$\frac{(3 a d^2 - 4 b c d) \tanh^{-1} \left(\frac{\sqrt{\frac{c x^2 + d}{x^2}}}{\sqrt{c}} \right)}{8 c^{5/2}} + \frac{\sqrt{\frac{c x^2 + d}{x^2}} (2 a c x^4 - 3 a d x^2 + 4 b c x^2)}{8 c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*x^3)/Sqrt[c + d/x^2], x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(4*b*c*x^2 - 3*a*d*x^2 + 2*a*c*x^4))/(8*c^2) + ((-4*b*c*d + 3*a*d^2)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/(8*c^(5/2))

fricas [A] time = 0.44, size = 192, normalized size = 2.13

$$\left[\frac{(4 b c d - 3 a d^2) \sqrt{c} \log \left(-2 c x^2 - 2 \sqrt{c} x^2 \sqrt{\frac{c x^2 + d}{x^2}} - d \right) - 2 (2 a c^2 x^4 + (4 b c^2 - 3 a c d) x^2) \sqrt{\frac{c x^2 + d}{x^2}}}{16 c^3}, \frac{(4 b c d - 3 a d^2) \sqrt{-c} \arctan \left(\frac{\sqrt{-c} x^2 \sqrt{\frac{c x^2 + d}{x^2}}}{c x^2 + d} \right) + (2 a c^2 x^4 + (4 b c^2 - 3 a c d) x^2) \sqrt{\frac{c x^2 + d}{x^2}}}{8 c^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2), x, algorithm="fricas")

[Out] [-1/16*((4*b*c*d - 3*a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/c^3, 1/8*((4*b*c*d - 3*a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/c^3]

giac [A] time = 0.26, size = 113, normalized size = 1.26

$$\frac{1}{8} \sqrt{c x^4 + d x^2} \left(\frac{2 a x^2}{c} + \frac{4 b c - 3 a d}{c^2} \right) + \frac{(4 b c d - 3 a d^2) \log \left(\left| -2 \left(\sqrt{c} x^2 - \sqrt{c x^4 + d x^2} \right) \sqrt{c} - d \right| \right)}{16 c^{\frac{5}{2}}} - \frac{4 b c d \log(|d|) - 3 a d^2 \log(|d|)}{16 c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{8}\sqrt{c x^4 + d x^2} \left(2 a x^2 / c + (4 b c - 3 a d) / c^2 \right) + \frac{1}{16} \left(4 b c d - 3 a d^2 \right) \log \left(\frac{\sqrt{c} x^2 - \sqrt{c x^4 + d x^2}}{\sqrt{c} x^2 + \sqrt{c x^4 + d x^2}} \right) / c^{5/2} - \frac{1}{16} \left(4 b c d \log(\sqrt{d}) - 3 a d^2 \log(\sqrt{d}) \right) / c^{5/2}$

maple [A] time = 0.06, size = 129, normalized size = 1.43

$$\frac{\sqrt{c x^2 + d} \left(2 \sqrt{c x^2 + d} a c^{\frac{5}{2}} x^3 + 3 a c d^2 \ln \left(\frac{\sqrt{c} x + \sqrt{c x^2 + d}}{\sqrt{c} x - \sqrt{c x^2 + d}} \right) - 4 b c^2 d \ln \left(\frac{\sqrt{c} x + \sqrt{c x^2 + d}}{\sqrt{c} x - \sqrt{c x^2 + d}} \right) - 3 \sqrt{c x^2 + d} a c^{\frac{3}{2}} d x + 4 \sqrt{c x^2 + d} b c^{\frac{5}{2}} x \right)}{8 \sqrt{\frac{c x^2 + d}{x^2}} c^{\frac{7}{2}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x)

[Out] $\frac{1}{8} (c x^2 + d)^{1/2} \left(2 (c x^2 + d)^{1/2} c^{5/2} x^3 a - 3 (c x^2 + d)^{1/2} c^{3/2} x^2 a d + 4 (c x^2 + d)^{1/2} c^{5/2} x b + 3 \ln(c^{1/2} x + (c x^2 + d)^{1/2}) a c d^2 - 4 \ln(c^{1/2} x + (c x^2 + d)^{1/2}) b c^2 d \right) / \left((c x^2 + d)^{1/2} / x / c^{7/2} \right)$

maxima [B] time = 1.17, size = 178, normalized size = 1.98

$$\frac{1}{4} b \left(\frac{2 \sqrt{c + \frac{d}{x^2}} d}{\left(c + \frac{d}{x^2}\right) c - c^2} + \frac{d \log \left(\frac{\sqrt{c + \frac{d}{x^2}} - \sqrt{c}}{\sqrt{c + \frac{d}{x^2}} + \sqrt{c}} \right)}{c^{\frac{3}{2}}} \right) - \frac{1}{16} a \left(\frac{3 d^2 \log \left(\frac{\sqrt{c + \frac{d}{x^2}} - \sqrt{c}}{\sqrt{c + \frac{d}{x^2}} + \sqrt{c}} \right)}{c^{\frac{5}{2}}} + \frac{2 \left(3 \left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} d^2 - 5 \sqrt{c + \frac{d}{x^2}} c d^2 \right)}{\left(c + \frac{d}{x^2}\right)^2 c^2 - 2 \left(c + \frac{d}{x^2}\right) c^3 + c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] $\frac{1}{4} b \left(2 \sqrt{c + d/x^2} d / \left((c + d/x^2) c - c^2 \right) + d \log \left(\frac{\sqrt{c + d/x^2} - \sqrt{c}}{\sqrt{c + d/x^2} + \sqrt{c}} \right) / c^{3/2} \right) - \frac{1}{16} a \left(3 d^2 \log \left(\frac{\sqrt{c + d/x^2} - \sqrt{c}}{\sqrt{c + d/x^2} + \sqrt{c}} \right) / c^{5/2} + 2 \left(3 \left(c + d/x^2\right)^{3/2} d^2 - 5 \sqrt{c + d/x^2} c d^2 \right) / \left(\left(c + d/x^2\right)^2 c^2 - 2 \left(c + d/x^2\right) c^3 + c^4 \right) \right)$

mupad [B] time = 5.35, size = 99, normalized size = 1.10

$$\frac{5 a x^4 \sqrt{c + \frac{d}{x^2}}}{8 c} - \frac{3 a x^4 \left(c + \frac{d}{x^2}\right)^{3/2}}{8 c^2} + \frac{b x^2 \sqrt{c + \frac{d}{x^2}}}{2 c} - \frac{b d \operatorname{atanh} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{2 c^{3/2}} + \frac{3 a d^2 \operatorname{atanh} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{8 c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(a + b/x^2))/(c + d/x^2)^(1/2), x)`

[Out] $(5*a*x^4*(c + d/x^2)^{(1/2)})/(8*c) - (3*a*x^4*(c + d/x^2)^{(3/2)})/(8*c^2) + (b*x^2*(c + d/x^2)^{(1/2)})/(2*c) - (b*d*atanh((c + d/x^2)^{(1/2)}/c^{(1/2)}))/(2*c^{(3/2)}) + (3*a*d^2*atanh((c + d/x^2)^{(1/2)}/c^{(1/2)}))/(8*c^{(5/2)})$

sympy [A] time = 71.06, size = 150, normalized size = 1.67

$$\frac{ax^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} - \frac{a\sqrt{d}x^3}{8c\sqrt{\frac{cx^2}{d}+1}} - \frac{3ad^{\frac{3}{2}}x}{8c^2\sqrt{\frac{cx^2}{d}+1}} + \frac{3ad^2 \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{5}{2}}} + \frac{b\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2c} - \frac{bd \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*x**3/(c+d/x**2)**(1/2), x)`

[Out] $a*x^{**5}/(4*\sqrt{d}*\sqrt{c*x^{**2}/d + 1}) - a*\sqrt{d}*x^{**3}/(8*c*\sqrt{c*x^{**2}/d + 1}) - 3*a*d^{**3/2}*x/(8*c^{**2}*\sqrt{c*x^{**2}/d + 1}) + 3*a*d^{**2}*\operatorname{asinh}(\sqrt{c}*x/\sqrt{d})/(8*c^{**5/2}) + b*\sqrt{d}*x*\sqrt{c*x^{**2}/d + 1}/(2*c) - b*d*\operatorname{asinh}(\sqrt{c}*x/\sqrt{d})/(2*c^{**3/2})$

$$3.617 \quad \int \frac{\left(a + \frac{b}{x^2}\right)x}{\sqrt{c + \frac{d}{x^2}}} dx$$

Optimal. Leaf size=59

$$\frac{(2bc - ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}} + \frac{ax^2 \sqrt{c + \frac{d}{x^2}}}{2c}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {446, 78, 63, 208}

$$\frac{(2bc - ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}} + \frac{ax^2 \sqrt{c + \frac{d}{x^2}}}{2c}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*x)/Sqrt[c + d/x^2], x]

[Out] (a*Sqrt[c + d/x^2]*x^2)/(2*c) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*c^(3/2))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{\left(a + \frac{b}{x^2}\right)x}{\sqrt{c + \frac{d}{x^2}}} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{a + bx}{x^2\sqrt{c + dx}} dx, x, \frac{1}{x^2}\right)\right) \\
 &= \frac{a\sqrt{c + \frac{d}{x^2}} x^2}{2c} - \frac{\left(bc - \frac{ad}{2}\right) \text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}} dx, x, \frac{1}{x^2}\right)}{2c} \\
 &= \frac{a\sqrt{c + \frac{d}{x^2}} x^2}{2c} - \frac{\left(bc - \frac{ad}{2}\right) \text{Subst}\left(\int \frac{1}{\frac{-c}{-d} + \frac{x^2}{d}} dx, x, \sqrt{c + \frac{d}{x^2}}\right)}{cd} \\
 &= \frac{a\sqrt{c + \frac{d}{x^2}} x^2}{2c} + \frac{(2bc - ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 79, normalized size = 1.34

$$\frac{\sqrt{cx^2 + d} (2bc - ad) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2 + d}}\right) + a\sqrt{c}x (cx^2 + d)}{2c^{3/2}x\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*x)/Sqrt[c + d/x^2], x]

[Out] (a*Sqrt[c]*x*(d + c*x^2) + (2*b*c - a*d)*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[d + c*x^2]])/(2*c^(3/2)*Sqrt[c + d/x^2]*x)

IntegrateAlgebraic [A] time = 0.10, size = 67, normalized size = 1.14

$$\frac{(2bc - ad) \tanh^{-1} \left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}} \right)}{2c^{3/2}} + \frac{ax^2 \sqrt{\frac{cx^2+d}{x^2}}}{2c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]

[Out] (a*x^2*Sqrt[(d + c*x^2)/x^2])/(2*c) + ((2*b*c - a*d)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/(2*c^(3/2))

fricas [A] time = 0.44, size = 146, normalized size = 2.47

$$\left[\frac{2acx^2 \sqrt{\frac{cx^2+d}{x^2}} - (2bc - ad)\sqrt{c} \log\left(-2cx^2 + 2\sqrt{c}x^2 \sqrt{\frac{cx^2+d}{x^2}} - d\right)}{4c^2}, \frac{acx^2 \sqrt{\frac{cx^2+d}{x^2}} - (2bc - ad)\sqrt{-c} \arctan\left(\frac{\sqrt{-c}x^2 \sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{2c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*(2*a*c*x^2*sqrt((c*x^2 + d)/x^2) - (2*b*c - a*d)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/c^2, 1/2*(a*c*x^2*sqrt((c*x^2 + d)/x^2) - (2*b*c - a*d)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/c^2]

giac [A] time = 0.26, size = 88, normalized size = 1.49

$$\frac{\sqrt{cx^4 + dx^2} a}{2c} - \frac{(2bc - ad) \log\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + dx^2}\right)\sqrt{c} - d\right|\right)}{4c^{\frac{3}{2}}} + \frac{2bc \log(|d|) - ad \log(|d|)}{4c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] 1/2*sqrt(c*x^4 + d*x^2)*a/c - 1/4*(2*b*c - a*d)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))*sqrt(c) - d))/c^(3/2) + 1/4*(2*b*c*log(abs(d)) - a*d*log(abs(d)))/c^(3/2)

maple [A] time = 0.05, size = 90, normalized size = 1.53

$$\frac{\sqrt{cx^2+d} \left(-acd \ln\left(\sqrt{c}x + \sqrt{cx^2+d}\right) + 2bc^2 \ln\left(\sqrt{c}x + \sqrt{cx^2+d}\right) + \sqrt{cx^2+d} ac^{\frac{3}{2}}x \right)}{2\sqrt{\frac{cx^2+d}{x^2}} c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*x/(c+d/x^2)^(1/2),x)

[Out] 1/2*(c*x^2+d)^(1/2)*(c^(3/2)*(c*x^2+d)^(1/2)*x*a+2*b*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*c^2-ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c*d)/((c*x^2+d)/x^2)^(1/2)/x/c^(5/2)

maxima [B] time = 1.20, size = 109, normalized size = 1.85

$$\frac{1}{4}a \left(\frac{2\sqrt{c+\frac{d}{x^2}}d}{\left(c+\frac{d}{x^2}\right)c-c^2} + \frac{d \log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}} \right) - \frac{b \log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] 1/4*a*(2*sqrt(c + d/x^2)*d/((c + d/x^2)*c - c^2) + d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2)) - 1/2*b*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c)

mupad [B] time = 5.08, size = 59, normalized size = 1.00

$$\frac{b \operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{ax^2\sqrt{c+\frac{d}{x^2}}}{2c} - \frac{ad \operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b/x^2))/(c + d/x^2)^(1/2),x)

[Out] (b*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(1/2) + (a*x^2*(c + d/x^2)^(1/2))/(2*c) - (a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(3/2))

sympy [A] time = 84.41, size = 66, normalized size = 1.12

$$\frac{a\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2c} - \frac{ad \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}} + \frac{b \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*x/(c+d/x**2)**(1/2),x)

[Out] a*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - a*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**
3/2) + b*asinh(sqrt(c)*x/sqrt(d))/sqrt(c)

$$3.618 \quad \int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}}} dx$$

Optimal. Leaf size=43

$$\frac{a \tanh^{-1} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{\sqrt{c}} - \frac{b \sqrt{c + \frac{d}{x^2}}}{d}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {446, 80, 63, 208}

$$\frac{a \tanh^{-1} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{\sqrt{c}} - \frac{b \sqrt{c + \frac{d}{x^2}}}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x), x]

[Out] -((b*Sqrt[c + d/x^2])/d) + (a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/Sqrt[c]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208


```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{a + bx}{x\sqrt{c + dx}} dx, x, \frac{1}{x^2}\right)\right) \\
 &= -\frac{b\sqrt{c + \frac{d}{x^2}}}{d} - \frac{1}{2}a \text{Subst}\left(\int \frac{1}{x\sqrt{c + dx}} dx, x, \frac{1}{x^2}\right) \\
 &= -\frac{b\sqrt{c + \frac{d}{x^2}}}{d} - \frac{a \text{Subst}\left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + \frac{d}{x^2}}\right)}{d} \\
 &= -\frac{b\sqrt{c + \frac{d}{x^2}}}{d} + \frac{a \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 73, normalized size = 1.70

$$\frac{adx\sqrt{cx^2 + d} \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2 + d}}\right) - b\sqrt{c}(cx^2 + d)}{\sqrt{c} dx^2 \sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x), x]
```

```
[Out] (- (b*Sqrt[c]*(d + c*x^2)) + a*d*x*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[
d + c*x^2]])/(Sqrt[c]*d*Sqrt[c + d/x^2]*x^2)
```

IntegrateAlgebraic [A] time = 0.07, size = 51, normalized size = 1.19

$$\frac{a \tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b\sqrt{\frac{cx^2+d}{x^2}}}{d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/(Sqrt[c + d/x^2]*x), x]

[Out] -((b*Sqrt[(d + c*x^2)/x^2])/d) + (a*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/Sqrt[c]

fricas [A] time = 0.44, size = 130, normalized size = 3.02

$$\left[\frac{a\sqrt{c}d \log\left(-2cx^2 - 2\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}} - d\right) - 2bc\sqrt{\frac{cx^2+d}{x^2}}}{2cd}, -\frac{a\sqrt{-c}d \arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + bc\sqrt{\frac{cx^2+d}{x^2}}}{cd} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2), x, algorithm="fricas")

[Out] [1/2*(a*sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*b*c*sqrt((c*x^2 + d)/x^2))/(c*d), -(a*sqrt(-c)*d*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*c*sqrt((c*x^2 + d)/x^2))/(c*d)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep)]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [86,-97,-82]Warning, choosing root of [1,0,%%{-2,[1,0,1]%%}+%%{-4,[0

,1,0]]],0,[[1,[2,0,2]]] at parameters values [-64,61.7937478349,70]Sign error ({{d,0}}+{{-2*sqrt(c)*sqrt(d),1/2}}+{{2*c,1}}+{{-c*sqrt(c)*sqrt(d)/d,3/2}}+{{c^2*sqrt(c)*sqrt(d)/(4*d^2),5/2}}+{{undef,7/2}})Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [A] time = 0.05, size = 70, normalized size = 1.63

$$\frac{\sqrt{cx^2+d} \left(-adx \ln \left(\sqrt{c} x + \sqrt{cx^2+d} \right) + \sqrt{cx^2+d} b\sqrt{c} \right)}{\sqrt{\frac{cx^2+d}{x^2}} \sqrt{c} dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/x/(c+d/x^2)^(1/2),x)

[Out] -(c*x^2+d)^(1/2)*(-a*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*d*x+b*(c*x^2+d)^(1/2)*c^(1/2))/((c*x^2+d)/x^2)^(1/2)/x^2/c^(1/2)/d

maxima [A] time = 1.10, size = 54, normalized size = 1.26

$$\frac{a \log \left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}} \right)}{2\sqrt{c}} - \frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] -1/2*a*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c) - b*sqrt(c + d/x^2)/d

mupad [B] time = 4.85, size = 35, normalized size = 0.81

$$\frac{a \operatorname{atanh} \left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}} \right)}{\sqrt{c}} - \frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x^2)/(x*(c + d/x^2)^(1/2)),x)

[Out] (a*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(1/2) - (b*(c + d/x^2)^(1/2))/d

sympy [A] time = 22.34, size = 63, normalized size = 1.47

$$-\frac{a \operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{c}} \sqrt{c + \frac{d}{x^2}}}\right)}{c \sqrt{-\frac{1}{c}}} + \frac{b \begin{cases} -\frac{1}{\sqrt{c} x^2} & \text{for } d = 0 \\ 2\sqrt{c + \frac{d}{x^2}} & \\ -\frac{d}{2} & \text{otherwise} \end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)/x/(c+d/x**2)**(1/2),x)

[Out] -a*atan(1/(sqrt(-1/c)*sqrt(c + d/x**2)))/(c*sqrt(-1/c)) + b*Piecewise((-1/(sqrt(c)*x**2), Eq(d, 0)), (-2*sqrt(c + d/x**2)/d, True))/2

$$3.619 \quad \int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^3} dx$$

Optimal. Leaf size=43

$$\frac{\sqrt{c + \frac{d}{x^2}} (bc - ad)}{d^2} - \frac{b \left(c + \frac{d}{x^2}\right)^{3/2}}{3d^2}$$

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {444, 43}

$$\frac{\sqrt{c + \frac{d}{x^2}} (bc - ad)}{d^2} - \frac{b \left(c + \frac{d}{x^2}\right)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]

[Out] ((b*c - a*d)*Sqrt[c + d/x^2])/d^2 - (b*(c + d/x^2)^(3/2))/(3*d^2)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^3} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{a + bx}{\sqrt{c + dx}} dx, x, \frac{1}{x^2}\right)\right) \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \left(\frac{-bc + ad}{d\sqrt{c + dx}} + \frac{b\sqrt{c + dx}}{d}\right) dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{(bc - ad)\sqrt{c + \frac{d}{x^2}}}{d^2} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{3d^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 0.91

$$-\frac{\sqrt{c + \frac{d}{x^2}} (3adx^2 + b(d - 2cx^2))}{3d^2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3), x]

[Out] -1/3*(Sqrt[c + d/x^2]*(3*a*d*x^2 + b*(d - 2*c*x^2)))/(d^2*x^2)

IntegrateAlgebraic [A] time = 0.05, size = 44, normalized size = 1.02

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (-3adx^2 + 2bcx^2 - bd)}{3d^2x^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3), x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-(b*d) + 2*b*c*x^2 - 3*a*d*x^2))/(3*d^2*x^2)

fricas [A] time = 0.41, size = 39, normalized size = 0.91

$$\frac{((2bc - 3ad)x^2 - bd)\sqrt{\frac{cx^2+d}{x^2}}}{3d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2), x, algorithm="fricas")

[Out] 1/3*((2*b*c - 3*a*d)*x^2 - b*d)*sqrt((c*x^2 + d)/x^2)/(d^2*x^2)

giac [B] time = 0.41, size = 88, normalized size = 2.05

$$\frac{3\left(\sqrt{c}x^2 - \sqrt{cx^4 + dx^2}\right)^2 a + 3\left(\sqrt{c}x^2 - \sqrt{cx^4 + dx^2}\right)b\sqrt{c} + bd}{3\left(\sqrt{c}x^2 - \sqrt{cx^4 + dx^2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] 1/3*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^2*a + 3*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))*b*sqrt(c) + b*d)/(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^3

maple [A] time = 0.05, size = 47, normalized size = 1.09

$$\frac{(3ad x^2 - 2bc x^2 + bd)(c x^2 + d)}{3\sqrt{\frac{cx^2+d}{x^2}} d^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x)

[Out] -1/3*(3*a*d*x^2-2*b*c*x^2+b*d)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/d^2/x^4

maxima [A] time = 0.53, size = 48, normalized size = 1.12

$$-\frac{1}{3}b\left(\frac{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2} - \frac{3\sqrt{c + \frac{d}{x^2}}c}{d^2}\right) - \frac{a\sqrt{c + \frac{d}{x^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] -1/3*b*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2) - a*sqrt(c + d/x^2)/d

mupad [B] time = 4.56, size = 35, normalized size = 0.81

$$\frac{\sqrt{c + \frac{d}{x^2}}(bd + 3ad x^2 - 2bc x^2)}{3d^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x^2)/(x^3*(c + d/x^2)^(1/2)),x)`

[Out] `-((c + d/x^2)^(1/2)*(b*d + 3*a*d*x^2 - 2*b*c*x^2))/(3*d^2*x^2)`

sympy [A] time = 7.05, size = 138, normalized size = 3.21

$$\left\{ \begin{array}{ll} \frac{\frac{a}{x^2} - \frac{b}{2x^4}}{\sqrt{c}} & \text{for } d = 0 \\ \frac{\frac{2ac}{\sqrt{c+\frac{d}{x^2}}} + 2a \left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}} - \sqrt{c+\frac{d}{x^2}} \right) + \frac{2bc \left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}} - \sqrt{c+\frac{d}{x^2}} \right)}{d} + \frac{2b \left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}} + 2c \sqrt{c+\frac{d}{x^2}} - \frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} \right)}{d}}{d} & \text{otherwise} \end{array} \right.$$

2

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)/x**3/(c+d/x**2)**(1/2),x)`

[Out] `Piecewise((((-a/x**2 - b/(2*x**4))/sqrt(c), Eq(d, 0)), ((2*a*c/sqrt(c + d/x**2) + 2*a*(-c/sqrt(c + d/x**2) - sqrt(c + d/x**2)) + 2*b*c*(-c/sqrt(c + d/x**2) - sqrt(c + d/x**2))/d + 2*b*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(c + d/x**2) - (c + d/x**2)**(3/2)/3)/d)/d, True))/2`

$$3.620 \quad \int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^5} dx$$

Optimal. Leaf size=72

$$\frac{\left(c + \frac{d}{x^2}\right)^{3/2} (2bc - ad)}{3d^3} - \frac{c\sqrt{c + \frac{d}{x^2}} (bc - ad)}{d^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{5d^3}$$

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{\left(c + \frac{d}{x^2}\right)^{3/2} (2bc - ad)}{3d^3} - \frac{c\sqrt{c + \frac{d}{x^2}} (bc - ad)}{d^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{5d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5), x]

[Out] -((c*(b*c - a*d)*Sqrt[c + d/x^2])/d^3) + ((2*b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^3) - (b*(c + d/x^2)^(5/2))/(5*d^3)

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^5} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{x(a + bx)}{\sqrt{c + dx}} dx, x, \frac{1}{x^2}\right)\right) \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \left(\frac{c(bc - ad)}{d^2 \sqrt{c + dx}} + \frac{(-2bc + ad)\sqrt{c + dx}}{d^2} + \frac{b(c + dx)^{3/2}}{d^2}\right) dx, x, \frac{1}{x^2}\right)\right) \\
&= -\frac{c(bc - ad)\sqrt{c + \frac{d}{x^2}}}{d^3} + \frac{(2bc - ad)\left(c + \frac{d}{x^2}\right)^{3/2}}{3d^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{5d^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.83

$$\frac{\sqrt{c + \frac{d}{x^2}} \left(b(-8c^2x^4 + 4cdx^2 - 3d^2) - 5adx^2(d - 2cx^2) \right)}{15d^3x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5), x]

[Out] (Sqrt[c + d/x^2]*(-5*a*d*x^2*(d - 2*c*x^2) + b*(-3*d^2 + 4*c*d*x^2 - 8*c^2*x^4)))/(15*d^3*x^4)

IntegrateAlgebraic [A] time = 0.07, size = 66, normalized size = 0.92

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} \left(10acdx^4 - 5ad^2x^2 - 8bc^2x^4 + 4bcdx^2 - 3bd^2 \right)}{15d^3x^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5), x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-3*b*d^2 + 4*b*c*d*x^2 - 5*a*d^2*x^2 - 8*b*c^2*x^4 + 10*a*c*d*x^4))/(15*d^3*x^4)

fricas [A] time = 0.43, size = 62, normalized size = 0.86

$$\frac{\left(2(4bc^2 - 5acd)x^4 + 3bd^2 - (4bcd - 5ad^2)x^2 \right) \sqrt{\frac{cx^2+d}{x^2}}}{15d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2), x, algorithm="fricas")

[Out] $-1/15*(2*(4*b*c^2 - 5*a*c*d)*x^4 + 3*b*d^2 - (4*b*c*d - 5*a*d^2)*x^2)*\sqrt{(c*x^2 + d)/x^2}/(d^3*x^4)$

giac [B] time = 0.50, size = 153, normalized size = 2.12

$$\frac{15\left(\sqrt{c}x^2 - \sqrt{cx^4 + dx^2}\right)^3 a\sqrt{c} + 20\left(\sqrt{c}x^2 - \sqrt{cx^4 + dx^2}\right)^2 bc + 5\left(\sqrt{c}x^2 - \sqrt{cx^4 + dx^2}\right)^2 ad + 15\left(\sqrt{c}x^2 - \sqrt{cx^4 + dx^2}\right)b\sqrt{c}d + 3bd^2}{15\left(\sqrt{c}x^2 - \sqrt{cx^4 + dx^2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="giac")`

[Out] $1/15*(15*(\sqrt{c}*x^2 - \sqrt{c*x^4 + d*x^2})^3*a*\sqrt{c} + 20*(\sqrt{c}*x^2 - \sqrt{c*x^4 + d*x^2})^2*b*c + 5*(\sqrt{c}*x^2 - \sqrt{c*x^4 + d*x^2})^2*a*d + 15*(\sqrt{c}*x^2 - \sqrt{c*x^4 + d*x^2})*b*\sqrt{c}*d + 3*b*d^2)/(\sqrt{c}*x^2 - \sqrt{c*x^4 + d*x^2})^5$

maple [A] time = 0.05, size = 70, normalized size = 0.97

$$\frac{(10acd x^4 - 8b c^2 x^4 - 5a d^2 x^2 + 4bcd x^2 - 3b d^2)(c x^2 + d)}{15\sqrt{\frac{cx^2+d}{x^2}} d^3 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x)`

[Out] $1/15*(10*a*c*d*x^4-8*b*c^2*x^4-5*a*d^2*x^2+4*b*c*d*x^2-3*b*d^2)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/d^3/x^6$

maxima [A] time = 0.49, size = 83, normalized size = 1.15

$$-\frac{1}{15}b\left(\frac{3\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3} - \frac{10\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3} + \frac{15\sqrt{c + \frac{d}{x^2}}c^2}{d^3}\right) - \frac{1}{3}a\left(\frac{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2} - \frac{3\sqrt{c + \frac{d}{x^2}}c}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="maxima")`

[Out] $-1/15*b*(3*(c + d/x^2)^(5/2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*\sqrt{c + d/x^2}*c^2/d^3) - 1/3*a*((c + d/x^2)^(3/2)/d^2 - 3*\sqrt{c + d/x^2}*c/d^2)$

mupad [B] time = 4.68, size = 58, normalized size = 0.81

$$\frac{\sqrt{c + \frac{d}{x^2}} (8bc^2x^4 - 10acd^2x^4 - 4bcdx^2 + 5ad^2x^2 + 3bd^2)}{15d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x^2)/(x^5*(c + d/x^2)^(1/2)),x)

[Out] -((c + d/x^2)^(1/2)*(3*b*d^2 + 5*a*d^2*x^2 + 8*b*c^2*x^4 - 10*a*c*d*x^4 - 4*b*c*d*x^2))/(15*d^3*x^4)

sympy [A] time = 13.17, size = 204, normalized size = 2.83

$$\left\{ \begin{array}{ll} \frac{-\frac{a}{2x^4} - \frac{b}{3x^6}}{\sqrt{c}} & \text{for } d = 0 \\ \frac{2ac \left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}} - \sqrt{c+\frac{d}{x^2}} \right) + 2a \left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}} + 2c\sqrt{c+\frac{d}{x^2}} - \frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} \right) + 2bc \left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}} + 2c\sqrt{c+\frac{d}{x^2}} - \frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} \right) + 2b \left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}} - 3c^2\sqrt{c+\frac{d}{x^2}} + c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}} - \frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5} \right)}{d^2} & \text{otherwise} \end{array} \right.$$

2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)/x**5/(c+d/x**2)**(1/2),x)

[Out] Piecewise(((-a/(2*x**4) - b/(3*x**6))/sqrt(c), Eq(d, 0)), ((2*a*c*(-c/sqrt(c + d/x**2) - sqrt(c + d/x**2))/d + 2*a*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(c + d/x**2) - (c + d/x**2)**(3/2)/3)/d + 2*b*c*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(c + d/x**2) - (c + d/x**2)**(3/2)/3)/d**2 + 2*b*(-c**3/sqrt(c + d/x**2) - 3*c**2*sqrt(c + d/x**2) + c*(c + d/x**2)**(3/2) - (c + d/x**2)**(5/2)/5)/d**2)/d, True))/2

$$3.621 \quad \int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^7} dx$$

Optimal. Leaf size=101

$$\frac{c^2 \sqrt{c + \frac{d}{x^2}} (bc - ad)}{d^4} + \frac{\left(c + \frac{d}{x^2}\right)^{5/2} (3bc - ad)}{5d^4} - \frac{c \left(c + \frac{d}{x^2}\right)^{3/2} (3bc - 2ad)}{3d^4} - \frac{b \left(c + \frac{d}{x^2}\right)^{7/2}}{7d^4}$$

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{c^2 \sqrt{c + \frac{d}{x^2}} (bc - ad)}{d^4} + \frac{\left(c + \frac{d}{x^2}\right)^{5/2} (3bc - ad)}{5d^4} - \frac{c \left(c + \frac{d}{x^2}\right)^{3/2} (3bc - 2ad)}{3d^4} - \frac{b \left(c + \frac{d}{x^2}\right)^{7/2}}{7d^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7), x]

[Out] (c^2*(b*c - a*d)*Sqrt[c + d/x^2])/d^4 - (c*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2))/(3*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (b*(c + d/x^2)^(7/2))/(7*d^4)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^7} dx &= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{x^2(a + bx)}{\sqrt{c + dx}} dx, x, \frac{1}{x^2} \right)\right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \left(-\frac{c^2(bc - ad)}{d^3 \sqrt{c + dx}} + \frac{c(3bc - 2ad)\sqrt{c + dx}}{d^3} + \frac{(-3bc + ad)(c + dx)^{3/2}}{d^3} + \frac{b(c + dx)^{5/2}}{d^3} \right) dx \right)\right) \\
&= \frac{c^2(bc - ad)\sqrt{c + \frac{d}{x^2}}}{d^4} - \frac{c(3bc - 2ad)\left(c + \frac{d}{x^2}\right)^{3/2}}{3d^4} + \frac{(3bc - ad)\left(c + \frac{d}{x^2}\right)^{5/2}}{5d^4} - \frac{b\left(c + \frac{d}{x^2}\right)^{7/2}}{7d^4}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.90

$$\frac{\left(\frac{cx^2}{d} + 1\right)(8c^2x^4 - 4cdx^2 + 3d^2)(6bc - 7ad)}{105d^3x^6\sqrt{c + \frac{d}{x^2}}} - \frac{b(cx^2 + d)}{7dx^8\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7), x]

[Out] -1/7*(b*(d + c*x^2))/(d*Sqrt[c + d/x^2]*x^8) + ((6*b*c - 7*a*d)*(1 + (c*x^2)/d)*(3*d^2 - 4*c*d*x^2 + 8*c^2*x^4))/(105*d^3*Sqrt[c + d/x^2]*x^6)

IntegrateAlgebraic [A] time = 0.07, size = 90, normalized size = 0.89

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} \left(-56ac^2dx^6 + 28acd^2x^4 - 21ad^3x^2 + 48bc^3x^6 - 24bc^2dx^4 + 18bcd^2x^2 - 15bd^3\right)}{105d^4x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7), x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-15*b*d^3 + 18*b*c*d^2*x^2 - 21*a*d^3*x^2 - 24*b*c^2*d*x^4 + 28*a*c*d^2*x^4 + 48*b*c^3*x^6 - 56*a*c^2*d*x^6))/(105*d^4*x^6)

fricas [A] time = 0.43, size = 86, normalized size = 0.85

$$\frac{\left(8\left(6bc^3 - 7ac^2d\right)x^6 - 4\left(6bc^2d - 7acd^2\right)x^4 - 15bd^3 + 3\left(6bcd^2 - 7ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{105d^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{105} \cdot (8 \cdot (6 \cdot b \cdot c^3 - 7 \cdot a \cdot c^2 \cdot d) \cdot x^6 - 4 \cdot (6 \cdot b \cdot c^2 \cdot d - 7 \cdot a \cdot c \cdot d^2) \cdot x^4 - 15 \cdot b \cdot d^3 + 3 \cdot (6 \cdot b \cdot c \cdot d^2 - 7 \cdot a \cdot d^3) \cdot x^2) \cdot \sqrt{(c \cdot x^2 + d) / x^2} / (d^4 \cdot x^6)$

giac [B] time = 0.62, size = 219, normalized size = 2.17

$$\frac{140 \left(\sqrt{c x^2 - \sqrt{c x^4 + d x^2}} \right)^4 a c + 210 \left(\sqrt{c x^2 - \sqrt{c x^4 + d x^2}} \right)^3 b c^{\frac{3}{2}} + 105 \left(\sqrt{c x^2 - \sqrt{c x^4 + d x^2}} \right)^3 a \sqrt{c} d + 252 \left(\sqrt{c x^2 - \sqrt{c x^4 + d x^2}} \right)^2 b c d + 21 \left(\sqrt{c x^2 - \sqrt{c x^4 + d x^2}} \right)^2 a d^2 + 105 \left(\sqrt{c x^2 - \sqrt{c x^4 + d x^2}} \right) b \sqrt{c} d^2 + 15 b d^3}{105 \left(\sqrt{c x^2 - \sqrt{c x^4 + d x^2}} \right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{105} \cdot (140 \cdot (\sqrt{c} \cdot x^2 - \sqrt{c \cdot x^4 + d \cdot x^2})^4 \cdot a \cdot c + 210 \cdot (\sqrt{c} \cdot x^2 - \sqrt{c \cdot x^4 + d \cdot x^2})^3 \cdot b \cdot c^{\frac{3}{2}} + 105 \cdot (\sqrt{c} \cdot x^2 - \sqrt{c \cdot x^4 + d \cdot x^2})^3 \cdot a \cdot \sqrt{c} \cdot d + 252 \cdot (\sqrt{c} \cdot x^2 - \sqrt{c \cdot x^4 + d \cdot x^2})^2 \cdot b \cdot c \cdot d + 21 \cdot (\sqrt{c} \cdot x^2 - \sqrt{c \cdot x^4 + d \cdot x^2})^2 \cdot a \cdot d^2 + 105 \cdot (\sqrt{c} \cdot x^2 - \sqrt{c \cdot x^4 + d \cdot x^2}) \cdot b \cdot \sqrt{c} \cdot d^2 + 15 \cdot b \cdot d^3) / (\sqrt{c} \cdot x^2 - \sqrt{c \cdot x^4 + d \cdot x^2})^7$

maple [A] time = 0.05, size = 94, normalized size = 0.93

$$\frac{(56 a c^2 d x^6 - 48 b c^3 x^6 - 28 a c d^2 x^4 + 24 b c^2 d x^4 + 21 a d^3 x^2 - 18 b c d^2 x^2 + 15 b d^3) (c x^2 + d)}{105 \sqrt{\frac{c x^2 + d}{x^2}} d^4 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x)

[Out] $\frac{-1}{105} \cdot (56 \cdot a \cdot c^2 \cdot d \cdot x^6 - 48 \cdot b \cdot c^3 \cdot x^6 - 28 \cdot a \cdot c \cdot d^2 \cdot x^4 + 24 \cdot b \cdot c^2 \cdot d \cdot x^4 + 21 \cdot a \cdot d^3 \cdot x^2 - 18 \cdot b \cdot c \cdot d^2 \cdot x^2 + 15 \cdot b \cdot d^3) \cdot (c \cdot x^2 + d) / ((c \cdot x^2 + d) / x^2)^{\frac{1}{2}} / d^4 / x^8$

maxima [A] time = 0.58, size = 118, normalized size = 1.17

$$\frac{1}{35} b \left(\frac{5 \left(c + \frac{d}{x^2} \right)^{\frac{7}{2}}}{d^4} - \frac{21 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}} c}{d^4} + \frac{35 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} c^2}{d^4} - \frac{35 \sqrt{c + \frac{d}{x^2}} c^3}{d^4} \right) - \frac{1}{15} a \left(\frac{3 \left(c + \frac{d}{x^2} \right)^{\frac{5}{2}}}{d^3} - \frac{10 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} c}{d^3} + \frac{15 \sqrt{c + \frac{d}{x^2}} c^2}{d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] $\frac{-1}{35} \cdot b \cdot (5 \cdot (c + d/x^2)^{\frac{7}{2}} / d^4 - 21 \cdot (c + d/x^2)^{\frac{5}{2}} \cdot c / d^4 + 35 \cdot (c + d/x^2)^{\frac{3}{2}} \cdot c^2 / d^4 - 35 \cdot \sqrt{c + d/x^2} \cdot c^3 / d^4) - \frac{1}{15} \cdot a \cdot (3 \cdot (c + d/x^2)^{\frac{5}{2}} / d^3 - 10 \cdot (c + d/x^2)^{\frac{3}{2}} \cdot c / d^3 + 15 \cdot \sqrt{c + d/x^2} \cdot c^2 / d^3)$

mupad [B] time = 4.72, size = 102, normalized size = 1.01

$$\frac{\sqrt{c + \frac{d}{x^2}} (48 b c^3 - 56 a c^2 d)}{105 d^4} - \frac{b \sqrt{c + \frac{d}{x^2}}}{7 d x^6} - \frac{\sqrt{c + \frac{d}{x^2}} (24 b c^2 - 28 a c d)}{105 d^3 x^2} - \frac{\sqrt{c + \frac{d}{x^2}} (7 a d - 6 b c)}{35 d^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x^2)/(x^7*(c + d/x^2)^(1/2)),x)

[Out] ((c + d/x^2)^(1/2)*(48*b*c^3 - 56*a*c^2*d))/(105*d^4) - (b*(c + d/x^2)^(1/2))/(7*d*x^6) - ((c + d/x^2)^(1/2)*(24*b*c^2 - 28*a*c*d))/(105*d^3*x^2) - ((c + d/x^2)^(1/2)*(7*a*d - 6*b*c))/(35*d^2*x^4)

sympy [A] time = 18.00, size = 269, normalized size = 2.66

$$\left\{ \begin{array}{ll} \frac{-\frac{a}{3x^6} - \frac{b}{4x^8}}{\sqrt{c}} & \text{for } d = 0 \\ \frac{2ac \left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}} + 2c \sqrt{c+\frac{d}{x^2}} - \frac{(c+\frac{d}{x^2})^{\frac{3}{2}}}{3} \right)}{d^2} + \frac{2a \left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}} - 3c^2 \sqrt{c+\frac{d}{x^2}} + c \left(c+\frac{d}{x^2} \right)^{\frac{3}{2}} - \frac{(c+\frac{d}{x^2})^{\frac{5}{2}}}{5} \right)}{d^2} + \frac{2bc \left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}} - 3c^2 \sqrt{c+\frac{d}{x^2}} + c \left(c+\frac{d}{x^2} \right)^{\frac{3}{2}} - \frac{(c+\frac{d}{x^2})^{\frac{5}{2}}}{5} \right)}{d} + \frac{2b \left(\frac{c^4}{\sqrt{c+\frac{d}{x^2}}} + 4c^3 \sqrt{c+\frac{d}{x^2}} - 2c^2 \left(c+\frac{d}{x^2} \right)^{\frac{3}{2}} + \frac{4c \left(c+\frac{d}{x^2} \right)^{\frac{5}{2}}}{5} - \frac{(c+\frac{d}{x^2})^{\frac{7}{2}}}{7} \right)}{d^3} & \text{otherwise} \end{array} \right.$$

2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)/x**7/(c+d/x**2)**(1/2),x)

[Out] Piecewise(((-a/(3*x**6) - b/(4*x**8))/sqrt(c), Eq(d, 0)), ((2*a*c*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(c + d/x**2) - (c + d/x**2)**(3/2)/3)/d**2 + 2*a*(-c**3/sqrt(c + d/x**2) - 3*c**2*sqrt(c + d/x**2) + c*(c + d/x**2)**(3/2) - (c + d/x**2)**(5/2)/5)/d**2 + 2*b*c*(-c**3/sqrt(c + d/x**2) - 3*c**2*sqrt(c + d/x**2) + c*(c + d/x**2)**(3/2) - (c + d/x**2)**(5/2)/5)/d**3 + 2*b*(c**4/sqrt(c + d/x**2) + 4*c**3*sqrt(c + d/x**2) - 2*c**2*(c + d/x**2)**(3/2) + 4*c*(c + d/x**2)**(5/2)/5 - (c + d/x**2)**(7/2)/7)/d, True))/2

$$3.622 \quad \int \frac{\left(a + \frac{b}{x^2}\right)x^4}{\sqrt{c + \frac{d}{x^2}}} dx$$

Optimal. Leaf size=82

$$-\frac{2dx\sqrt{c + \frac{d}{x^2}}(5bc - 4ad)}{15c^3} + \frac{x^3\sqrt{c + \frac{d}{x^2}}(5bc - 4ad)}{15c^2} + \frac{ax^5\sqrt{c + \frac{d}{x^2}}}{5c}$$

Rubi [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {453, 271, 191}

$$\frac{x^3\sqrt{c + \frac{d}{x^2}}(5bc - 4ad)}{15c^2} - \frac{2dx\sqrt{c + \frac{d}{x^2}}(5bc - 4ad)}{15c^3} + \frac{ax^5\sqrt{c + \frac{d}{x^2}}}{5c}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*x^4)/Sqrt[c + d/x^2], x]

[Out] (-2*d*(5*b*c - 4*a*d)*Sqrt[c + d/x^2]*x)/(15*c^3) + ((5*b*c - 4*a*d)*Sqrt[c + d/x^2]*x^3)/(15*c^2) + (a*Sqrt[c + d/x^2]*x^5)/(5*c)

Rule 191

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 271

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

Rule 453

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{x^2}\right)x^4}{\sqrt{c + \frac{d}{x^2}}} dx &= \frac{a\sqrt{c + \frac{d}{x^2}}x^5}{5c} + \frac{(5bc - 4ad) \int \frac{x^2}{\sqrt{c + \frac{d}{x^2}}} dx}{5c} \\
&= \frac{(5bc - 4ad)\sqrt{c + \frac{d}{x^2}}x^3}{15c^2} + \frac{a\sqrt{c + \frac{d}{x^2}}x^5}{5c} - \frac{(2d(5bc - 4ad)) \int \frac{1}{\sqrt{c + \frac{d}{x^2}}} dx}{15c^2} \\
&= -\frac{2d(5bc - 4ad)\sqrt{c + \frac{d}{x^2}}x}{15c^3} + \frac{(5bc - 4ad)\sqrt{c + \frac{d}{x^2}}x^3}{15c^2} + \frac{a\sqrt{c + \frac{d}{x^2}}x^5}{5c}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 56, normalized size = 0.68

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(a(3c^2x^4 - 4cdx^2 + 8d^2) + 5bc(cx^2 - 2d) \right)}{15c^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*x^4)/Sqrt[c + d/x^2], x]

[Out] (Sqrt[c + d/x^2]*x*(5*b*c*(-2*d + c*x^2) + a*(8*d^2 - 4*c*d*x^2 + 3*c^2*x^4)))/(15*c^3)

IntegrateAlgebraic [A] time = 0.08, size = 57, normalized size = 0.70

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(3ac^2x^4 - 4acdx^2 + 8ad^2 + 5bc^2x^2 - 10bcd \right)}{15c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*x^4)/Sqrt[c + d/x^2], x]

[Out] (Sqrt[c + d/x^2]*x*(-10*b*c*d + 8*a*d^2 + 5*b*c^2*x^2 - 4*a*c*d*x^2 + 3*a*c^2*x^4))/(15*c^3)

fricas [A] time = 0.41, size = 59, normalized size = 0.72

$$\frac{\left(3ac^2x^5 + (5bc^2 - 4acd)x^3 - 2(5bcd - 4ad^2)x \right) \sqrt{\frac{cx^2+d}{x^2}}}{15c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] 1/15*(3*a*c^2*x^5 + (5*b*c^2 - 4*a*c*d)*x^3 - 2*(5*b*c*d - 4*a*d^2)*x)*sqrt((c*x^2 + d)/x^2)/c^3

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sqrt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,constant vecteur & l) Error: Bad Argument Value

maple [A] time = 0.05, size = 67, normalized size = 0.82

$$\frac{(3ax^4c^2 - 4acd^2x^2 + 5b^2c^2x^2 + 8ad^2 - 10bcd)(cx^2 + d)}{15\sqrt{\frac{cx^2+d}{x^2}}c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x)

[Out] 1/15/x*(3*a*c^2*x^4-4*a*c*d*x^2+5*b*c^2*x^2+8*a*d^2-10*b*c*d)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/c^3

maxima [A] time = 0.49, size = 85, normalized size = 1.04

$$\frac{\left(\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}x^3 - 3\sqrt{c + \frac{d}{x^2}}dx\right)b}{3c^2} + \frac{\left(3\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}x^5 - 10\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}dx^3 + 15\sqrt{c + \frac{d}{x^2}}d^2x\right)a}{15c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] $\frac{1}{3}((c + d/x^2)^{3/2} * x^3 - 3 * \sqrt{c + d/x^2} * d * x) * b/c^2 + \frac{1}{15} * (3 * (c + d/x^2)^{5/2} * x^5 - 10 * (c + d/x^2)^{3/2} * d * x^3 + 15 * \sqrt{c + d/x^2} * d^2 * x) * a/c^3$

mupad [B] time = 5.31, size = 53, normalized size = 0.65

$$\frac{x \sqrt{c + \frac{d}{x^2}} (3 a c^2 x^4 + 5 b c^2 x^2 - 4 a c d x^2 - 10 b c d + 8 a d^2)}{15 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4*(a + b/x^2))/(c + d/x^2)^(1/2), x)`

[Out] $(x * (c + d/x^2)^{1/2} * (8 * a * d^2 + 3 * a * c^2 * x^4 + 5 * b * c^2 * x^2 - 10 * b * c * d - 4 * a * c * d * x^2)) / (15 * c^3)$

sympy [B] time = 3.55, size = 338, normalized size = 4.12

$$\frac{3ac^4d^9x^8\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^3x^2+15c^3d^6} + \frac{2ac^3d^{11}x^6\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^3x^2+15c^3d^6} + \frac{3ac^2d^{13}x^4\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^3x^2+15c^3d^6} + \frac{12acd^{15}x^2\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^3x^2+15c^3d^6} + \frac{8ad^{17}\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^3x^2+15c^3d^6} + \frac{b\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3c} - \frac{2bd^3\sqrt{\frac{cx^2}{d}+1}}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*x**4/(c+d/x**2)**(1/2), x)`

[Out] $3 * a * c^{**4} * d^{**9} * x^{**8} * \sqrt{c * x^{**2} / d + 1} / (15 * c^{**5} * d^{**4} * x^{**4} + 30 * c^{**4} * d^{**3} * x^{**2} + 15 * c^{**3} * d^{**6}) + 2 * a * c^{**3} * d^{**11} * x^{**6} * \sqrt{c * x^{**2} / d + 1} / (15 * c^{**5} * d^{**4} * x^{**4} + 30 * c^{**4} * d^{**3} * x^{**2} + 15 * c^{**3} * d^{**6}) + 3 * a * c^{**2} * d^{**13} * x^{**4} * \sqrt{c * x^{**2} / d + 1} / (15 * c^{**5} * d^{**4} * x^{**4} + 30 * c^{**4} * d^{**3} * x^{**2} + 15 * c^{**3} * d^{**6}) + 12 * a * c * d^{**15} * x^{**2} * \sqrt{c * x^{**2} / d + 1} / (15 * c^{**5} * d^{**4} * x^{**4} + 30 * c^{**4} * d^{**3} * x^{**2} + 15 * c^{**3} * d^{**6}) + 8 * a * d^{**17} * \sqrt{c * x^{**2} / d + 1} / (15 * c^{**5} * d^{**4} * x^{**4} + 30 * c^{**4} * d^{**3} * x^{**2} + 15 * c^{**3} * d^{**6}) + b * \sqrt{d} * x^2 * \sqrt{c * x^{**2} / d + 1} / (3 * c) - 2 * b * d^{**3} * \sqrt{c * x^{**2} / d + 1} / (3 * c^2)$

$$3.623 \quad \int \frac{\left(a + \frac{b}{x^2}\right)x^2}{\sqrt{c + \frac{d}{x^2}}} dx$$

Optimal. Leaf size=51

$$\frac{x\sqrt{c + \frac{d}{x^2}}(3bc - 2ad)}{3c^2} + \frac{ax^3\sqrt{c + \frac{d}{x^2}}}{3c}$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {453, 191}

$$\frac{x\sqrt{c + \frac{d}{x^2}}(3bc - 2ad)}{3c^2} + \frac{ax^3\sqrt{c + \frac{d}{x^2}}}{3c}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*x^2)/Sqrt[c + d/x^2], x]

[Out] ((3*b*c - 2*a*d)*Sqrt[c + d/x^2]*x)/(3*c^2) + (a*Sqrt[c + d/x^2]*x^3)/(3*c)

Rule 191

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 453

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\int \frac{\left(a + \frac{b}{x^2}\right)x^2}{\sqrt{c + \frac{d}{x^2}}} dx = \frac{a\sqrt{c + \frac{d}{x^2}}x^3}{3c} + \frac{(3bc - 2ad) \int \frac{1}{\sqrt{c + \frac{d}{x^2}}} dx}{3c}$$

$$= \frac{(3bc - 2ad)\sqrt{c + \frac{d}{x^2}}x}{3c^2} + \frac{a\sqrt{c + \frac{d}{x^2}}x^3}{3c}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 0.67

$$\frac{x\sqrt{c + \frac{d}{x^2}}(acx^2 - 2ad + 3bc)}{3c^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*x^2)/Sqrt[c + d/x^2], x]

[Out] (Sqrt[c + d/x^2]*x*(3*b*c - 2*a*d + a*c*x^2))/(3*c^2)

IntegrateAlgebraic [A] time = 0.06, size = 34, normalized size = 0.67

$$\frac{x\sqrt{c + \frac{d}{x^2}}(acx^2 - 2ad + 3bc)}{3c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*x^2)/Sqrt[c + d/x^2], x]

[Out] (Sqrt[c + d/x^2]*x*(3*b*c - 2*a*d + a*c*x^2))/(3*c^2)

fricas [A] time = 0.42, size = 36, normalized size = 0.71

$$\frac{(acx^3 + (3bc - 2ad)x)\sqrt{\frac{cx^2+d}{x^2}}}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2), x, algorithm="fricas")

[Out] 1/3*(a*c*x^3 + (3*b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/c^2

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="giac")`

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(t_n
 ostep)]Warning, integration of abs or sign assumes constant sign by interva
 ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
 rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
 t vecteur & l) Error: Bad Argument Value

maple [A] time = 0.04, size = 44, normalized size = 0.86

$$\frac{(ax^2c - 2ad + 3bc)(cx^2 + d)}{3\sqrt{\frac{cx^2+d}{x^2}}c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x)`

[Out] `1/3/x*(a*c*x^2-2*a*d+3*b*c)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/c^2`

maxima [A] time = 0.59, size = 49, normalized size = 0.96

$$\frac{b\sqrt{c + \frac{d}{x^2}}x}{c} + \frac{\left(\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}x^3 - 3\sqrt{c + \frac{d}{x^2}}dx\right)a}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")`

[Out] `b*sqrt(c + d/x^2)*x/c + 1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x)
 *a/c^2`

mupad [B] time = 4.92, size = 67, normalized size = 1.31

$$\frac{ax^3\sqrt{c + \frac{d}{x^2}}\left(c - \frac{2d}{x^2}\right)}{3c^2} + \frac{bx\sqrt{\frac{cx^2}{d} + 1}}{\sqrt{c + \frac{d}{x^2}}\left(\sqrt{\frac{cx^2}{d} + 1} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(a + b/x^2))/(c + d/x^2)^(1/2),x)`

[Out] $(a*x^3*(c + d/x^2)^{(1/2)}*(c - (2*d)/x^2))/(3*c^2) + (b*x*((c*x^2)/d + 1)^{(1/2)))/((c + d/x^2)^{(1/2)}*((c*x^2)/d + 1)^{(1/2)} + 1))$

sympy [A] time = 2.76, size = 70, normalized size = 1.37

$$\frac{a\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3c} - \frac{2ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^2} + \frac{b\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*x**2/(c+d/x**2)**(1/2),x)

[Out] $a*\sqrt{d}*x**2*\sqrt{c*x**2/d + 1}/(3*c) - 2*a*d**(3/2)*\sqrt{c*x**2/d + 1}/(3*c**2) + b*\sqrt{d}*\sqrt{c*x**2/d + 1}/c$

$$3.624 \quad \int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}}} dx$$

Optimal. Leaf size=47

$$\frac{ax\sqrt{c + \frac{d}{x^2}}}{c} - \frac{b \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{\sqrt{d}}$$

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {375, 451, 217, 206}

$$\frac{ax\sqrt{c + \frac{d}{x^2}}}{c} - \frac{b \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/Sqrt[c + d/x^2], x]

[Out] (a*Sqrt[c + d/x^2]*x)/c - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)))/Sqrt[d]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 375

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1]))
```

Rubi steps

$$\begin{aligned} \int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}}} dx &= -\text{Subst} \left(\int \frac{a + bx^2}{x^2 \sqrt{c + dx^2}} dx, x, \frac{1}{x} \right) \\ &= \frac{a\sqrt{c + \frac{d}{x^2}}}{c} - b \text{Subst} \left(\int \frac{1}{\sqrt{c + dx^2}} dx, x, \frac{1}{x} \right) \\ &= \frac{a\sqrt{c + \frac{d}{x^2}}}{c} - b \text{Subst} \left(\int \frac{1}{1 - dx^2} dx, x, \frac{1}{\sqrt{c + \frac{d}{x^2}}} \right) \\ &= \frac{a\sqrt{c + \frac{d}{x^2}}}{c} - \frac{b \tanh^{-1} \left(\frac{\sqrt{d}}{\sqrt{c + \frac{d}{x^2}}} \right)}{\sqrt{d}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 1.51

$$\frac{a\sqrt{d}(cx^2 + d) - bc\sqrt{cx^2 + d} \tanh^{-1} \left(\frac{\sqrt{cx^2 + d}}{\sqrt{d}} \right)}{c\sqrt{d}x\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/Sqrt[c + d/x^2],x]

[Out] (a*Sqrt[d]*(d + c*x^2) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(c*Sqrt[d]*Sqrt[c + d/x^2]*x)

IntegrateAlgebraic [A] time = 0.08, size = 67, normalized size = 1.43

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\frac{a\sqrt{cx^2+d}}{c} - \frac{b \tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{\sqrt{d}} \right)}{\sqrt{cx^2 + d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/Sqrt[c + d/x^2],x]

[Out] (Sqrt[c + d/x^2]*x*((a*Sqrt[d + c*x^2])/c - (b*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/Sqrt[d])/Sqrt[d + c*x^2]

fricas [A] time = 0.44, size = 131, normalized size = 2.79

$$\left[\frac{2 adx\sqrt{\frac{cx^2+d}{x^2}} + bc\sqrt{d} \log\left(-\frac{cx^2-2\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2d}{x^2}\right)}{2 cd}, \frac{adx\sqrt{\frac{cx^2+d}{x^2}} + bc\sqrt{-d} \arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{cd} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] [1/2*(2*a*d*x*sqrt((c*x^2 + d)/x^2) + b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c*d), (a*d*x*sqrt((c*x^2 + d)/x^2) + b*c*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c*d)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sqrt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.05, size = 73, normalized size = 1.55

$$\frac{\sqrt{cx^2+d} \left(-bc \ln \left(\frac{2d+2\sqrt{cx^2+d} \sqrt{d}}{x} \right) + \sqrt{cx^2+d} a\sqrt{d} \right)}{\sqrt{\frac{cx^2+d}{x^2}} c\sqrt{d} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)/(c+d/x^2)^(1/2),x)`

[Out] `(c*x^2+d)^(1/2)*(a*(c*x^2+d)^(1/2)*d^(1/2)-b*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*c)/((c*x^2+d)/x^2)^(1/2)/x/c/d^(1/2)`

maxima [A] time = 1.28, size = 58, normalized size = 1.23

$$\frac{a\sqrt{c+\frac{d}{x^2}}x}{c} + \frac{b \log \left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}} \right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="maxima")`

[Out] `a*sqrt(c+d/x^2)*x/c+1/2*b*log((sqrt(c+d/x^2)*x-sqrt(d))/(sqrt(c+d/x^2)*x+sqrt(d)))/sqrt(d)`

mupad [B] time = 5.00, size = 65, normalized size = 1.38

$$\frac{ax\sqrt{\frac{cx^2}{d}+1}}{\sqrt{c+\frac{d}{x^2}} \left(\sqrt{\frac{cx^2}{d}+1} + 1 \right)} - \frac{b \ln \left(\sqrt{c+\frac{d}{x^2}} + \frac{\sqrt{d}}{x} \right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)/(c+d/x^2)^(1/2),x)`

[Out] `(a*x*((c*x^2)/d+1)^(1/2))/((c+d/x^2)^(1/2)*(((c*x^2)/d+1)^(1/2)+1))- (b*log((c+d/x^2)^(1/2)+d^(1/2)/x))/d^(1/2)`

sympy [A] time = 2.86, size = 39, normalized size = 0.83

$$\frac{a\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}{c} - \frac{b \operatorname{asinh} \left(\frac{\sqrt{d}}{\sqrt{cx}} \right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*sqrt(c*x**2/d + 1)/c - b*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d)
```

$$3.625 \quad \int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}}} dx$$

Optimal. Leaf size=61

$$\frac{(bc - 2ad) \tanh^{-1} \left(\frac{\sqrt{d}}{x \sqrt{c + \frac{d}{x^2}}} \right)}{2d^{3/2}} - \frac{b \sqrt{c + \frac{d}{x^2}}}{2dx}$$

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {459, 335, 217, 206}

$$\frac{(bc - 2ad) \tanh^{-1} \left(\frac{\sqrt{d}}{x \sqrt{c + \frac{d}{x^2}}} \right)}{2d^{3/2}} - \frac{b \sqrt{c + \frac{d}{x^2}}}{2dx}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]

[Out] -(b*Sqrt[c + d/x^2])/(2*d*x) + ((b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)))/(2*d^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 335

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

Rule 459

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^2} dx &= -\frac{b\sqrt{c + \frac{d}{x^2}}}{2dx} + \frac{(-bc + 2ad) \int \frac{1}{\sqrt{c + \frac{d}{x^2}} x^2} dx}{2d} \\
 &= -\frac{b\sqrt{c + \frac{d}{x^2}}}{2dx} - \frac{(-bc + 2ad) \text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}} dx, x, \frac{1}{x}\right)}{2d} \\
 &= -\frac{b\sqrt{c + \frac{d}{x^2}}}{2dx} - \frac{(-bc + 2ad) \text{Subst}\left(\int \frac{1}{1-dx^2} dx, x, \frac{1}{\sqrt{c + \frac{d}{x^2}} x}\right)}{2d} \\
 &= -\frac{b\sqrt{c + \frac{d}{x^2}}}{2dx} + \frac{(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x}\right)}{2d^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 1.31

$$\frac{x^2 \sqrt{cx^2 + d} (bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{cx^2 + d}}{\sqrt{d}}\right) - b\sqrt{d} (cx^2 + d)}{2d^{3/2} x^3 \sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2), x]

[Out] (-(b*Sqrt[d]*(d + c*x^2)) + (b*c - 2*a*d)*x^2*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(2*d^(3/2)*Sqrt[c + d/x^2]*x^3)

IntegrateAlgebraic [A] time = 0.12, size = 82, normalized size = 1.34

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\frac{(bc-2ad) \tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{2d^{3/2}} - \frac{b\sqrt{cx^2+d}}{2dx^2} \right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]

[Out] (Sqrt[c + d/x^2]*x*(-1/2*(b*Sqrt[d + c*x^2])/(d*x^2) + ((b*c - 2*a*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(2*d^(3/2))))/Sqrt[d + c*x^2]

fricas [A] time = 0.42, size = 144, normalized size = 2.36

$$\left[\frac{(bc-2ad)\sqrt{d}x \log\left(-\frac{cx^2-2\sqrt{d}x\sqrt{cx^2+d}+2d}{x^2}\right) + 2bd\sqrt{\frac{cx^2+d}{x^2}}}{4d^2x}, \frac{(bc-2ad)\sqrt{-d}x \arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + bd\sqrt{\frac{cx^2+d}{x^2}}}{2d^2x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] [-1/4*((b*c - 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*b*d*sqrt((c*x^2 + d)/x^2))/(d^2*x), -1/2*((b*c - 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*d*sqrt((c*x^2 + d)/x^2))/(d^2*x)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sqrt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,constant vecteur & l) Error: Bad Argument Value

maple [B] time = 0.06, size = 105, normalized size = 1.72

$$\frac{\sqrt{cx^2+d} \left(2ad^2x^2 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) - bcdx^2 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) + \sqrt{cx^2+d}bd^{\frac{3}{2}} \right)}{2\sqrt{\frac{cx^2+d}{x^2}}d^{\frac{5}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x)

[Out] $-1/2*(c*x^2+d)^{(1/2)}*(2*a*\ln(2*(d+(c*x^2+d)^{(1/2)}*d^{(1/2))}/x)*x^2*d^2-\ln(2*(d+(c*x^2+d)^{(1/2)}*d^{(1/2))}/x)*x^2*b*c*d+(c*x^2+d)^{(1/2)}*d^{(3/2)}*b)/((c*x^2+d)/x^2)^{(1/2)}/x^3/d^{(5/2)}$

maxima [B] time = 1.18, size = 121, normalized size = 1.98

$$-\frac{1}{4} \left(\frac{2\sqrt{c+\frac{d}{x^2}}cx}{\left(c+\frac{d}{x^2}\right)dx^2-d^2} + \frac{c \log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}} \right) b + \frac{a \log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] $-1/4*(2*\text{sqrt}(c+d/x^2)*c*x/((c+d/x^2)*d*x^2-d^2)+c*\log((\text{sqrt}(c+d/x^2)*x-\text{sqrt}(d))/(\text{sqrt}(c+d/x^2)*x+\text{sqrt}(d)))/d^{(3/2)})*b+1/2*a*\log((\text{sqrt}(c+d/x^2)*x-\text{sqrt}(d))/(\text{sqrt}(c+d/x^2)*x+\text{sqrt}(d)))/\text{sqrt}(d)$

mupad [B] time = 5.53, size = 94, normalized size = 1.54

$$\begin{cases} -\frac{3ax^2+b}{3\sqrt{c}x^3} & \text{if } d=0 \\ \frac{bc \ln\left(2\sqrt{c+\frac{d}{x^2}}+\frac{2\sqrt{d}}{x}\right)}{2d^{3/2}} - \frac{b\sqrt{c+\frac{d}{x^2}}}{2dx} - \frac{a \ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{\sqrt{d}} & \text{if } d \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/(x^2*(c+d/x^2)^(1/2)),x)

[Out] $\text{piecewise}(d==0, -(b+3*a*x^2)/(3*c^{(1/2)}*x^3), d \neq 0, -(a*\log((c+d/x^2)^{(1/2)}+d^{(1/2)}/x))/d^{(1/2)}-(b*(c+d/x^2)^{(1/2)})/(2*d*x)+(b*c*\log(2*(c+d/x^2)^{(1/2)}+(2*d^{(1/2)}/x))/(2*d^{(3/2)}))$

sympy [A] time = 4.92, size = 66, normalized size = 1.08

$$-\frac{a \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{\sqrt{d}} - \frac{b\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2dx} + \frac{bc \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)/x**2/(c+d/x**2)**(1/2),x)

[Out] -a*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*d*x) + b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(3/2))

$$3.626 \quad \int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^4} dx$$

Optimal. Leaf size=93

$$-\frac{c(3bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{8d^{5/2}} + \frac{\sqrt{c + \frac{d}{x^2}}(3bc - 4ad)}{8d^2x} - \frac{b\sqrt{c + \frac{d}{x^2}}}{4dx^3}$$

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {459, 335, 321, 217, 206}

$$\frac{\sqrt{c + \frac{d}{x^2}}(3bc - 4ad)}{8d^2x} - \frac{c(3bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{8d^{5/2}} - \frac{b\sqrt{c + \frac{d}{x^2}}}{4dx^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4), x]

[Out] -(b*Sqrt[c + d/x^2])/(4*d*x^3) + ((3*b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d^2*x) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]/(8*d^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 335

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

Rule 459

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^4} dx &= -\frac{b\sqrt{c + \frac{d}{x^2}}}{4dx^3} + \frac{(-3bc + 4ad) \int \frac{1}{\sqrt{c + \frac{d}{x^2}} x^4} dx}{4d} \\
 &= -\frac{b\sqrt{c + \frac{d}{x^2}}}{4dx^3} - \frac{(-3bc + 4ad) \text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}} dx, x, \frac{1}{x}\right)}{4d} \\
 &= -\frac{b\sqrt{c + \frac{d}{x^2}}}{4dx^3} + \frac{(3bc - 4ad)\sqrt{c + \frac{d}{x^2}}}{8d^2x} - \frac{(c(3bc - 4ad)) \text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}} dx, x, \frac{1}{x}\right)}{8d^2} \\
 &= -\frac{b\sqrt{c + \frac{d}{x^2}}}{4dx^3} + \frac{(3bc - 4ad)\sqrt{c + \frac{d}{x^2}}}{8d^2x} - \frac{(c(3bc - 4ad)) \text{Subst}\left(\int \frac{1}{1-dx^2} dx, x, \frac{1}{\sqrt{c + \frac{d}{x^2}} x}\right)}{8d^2} \\
 &= -\frac{b\sqrt{c + \frac{d}{x^2}}}{4dx^3} + \frac{(3bc - 4ad)\sqrt{c + \frac{d}{x^2}}}{8d^2x} - \frac{c(3bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x}\right)}{8d^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 0.18, size = 107, normalized size = 1.15

$$\frac{(cx^2 + d) \left(d \sqrt{\frac{cx^2}{d} + 1} (4adx^2 - 3bcx^2 + 2bd) + cx^4 (3bc - 4ad) \tanh^{-1} \left(\sqrt{\frac{cx^2}{d} + 1} \right) \right)}{8d^3 x^5 \sqrt{c + \frac{d}{x^2}} \sqrt{\frac{cx^2}{d} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4), x]

[Out] $-1/8*((d + c*x^2)*(d*Sqrt[1 + (c*x^2)/d]*(2*b*d - 3*b*c*x^2 + 4*a*d*x^2) + c*(3*b*c - 4*a*d)*x^4*ArcTanh[Sqrt[1 + (c*x^2)/d]]))/(d^3*Sqrt[c + d/x^2]*x^5*Sqrt[1 + (c*x^2)/d])$

IntegrateAlgebraic [A] time = 0.20, size = 104, normalized size = 1.12

$$\frac{x \sqrt{c + \frac{d}{x^2}} \left(\frac{(4acd - 3bc^2) \tanh^{-1} \left(\frac{\sqrt{cx^2 + d}}{\sqrt{d}} \right)}{8d^{5/2}} + \frac{\sqrt{cx^2 + d} (-4adx^2 + 3bcx^2 - 2bd)}{8d^2 x^4} \right)}{\sqrt{cx^2 + d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4), x]

[Out] $(Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(-2*b*d + 3*b*c*x^2 - 4*a*d*x^2))/(8*d^2*x^4) + ((-3*b*c^2 + 4*a*c*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(8*d^(5/2))))/Sqrt[d + c*x^2]$

fricas [A] time = 0.45, size = 201, normalized size = 2.16

$$\left[\frac{(3bc^2 - 4acd)\sqrt{d}x^3 \log\left(-\frac{cx^2 + 2\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2} + 2d}}{x^2}\right) + 2(2bd^2 - (3bcd - 4ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16d^3x^3}, \frac{(3bc^2 - 4acd)\sqrt{-d}x^3 \arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) - (2bd^2 - (3bcd - 4ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8d^3x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2), x, algorithm="fricas")

[Out] $[-1/16*((3*b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 + 2*sqrt(d))*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^3), 1/8*((3*b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^3)]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value

maple [A] time = 0.06, size = 146, normalized size = 1.57

$$\frac{\sqrt{cx^2+d} \left(-4ac d^2 x^4 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) + 3bc^2 d x^4 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) + 4\sqrt{cx^2+d} a d^5 x^2 - 3\sqrt{cx^2+d} bc d^3 x^2 + 2\sqrt{cx^2+d} b d^5 \right)}{8\sqrt{\frac{cx^2+d}{x^2}} d^7 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x)

[Out] $-1/8*(c*x^2+d)^{(1/2)}*(-4*\ln(2*(d+(c*x^2+d)^{(1/2)}*d^{(1/2)})/x)*x^4*a*c*d^2+3*\ln(2*(d+(c*x^2+d)^{(1/2)}*d^{(1/2)})/x)*x^4*b*c^2*d+4*(c*x^2+d)^{(1/2)}*d^{(5/2)}*x^2*a-3*(c*x^2+d)^{(1/2)}*d^{(3/2)}*x^2*b*c+2*(c*x^2+d)^{(1/2)}*d^{(5/2)}*b)/((c*x^2+d)/x^2)^{(1/2)}/x^5/d^{(7/2)}$

maxima [B] time = 1.23, size = 200, normalized size = 2.15

$$-\frac{1}{4} \left(\frac{2\sqrt{c+\frac{d}{x^2}} cx}{\left(c+\frac{d}{x^2}\right) dx^2 - d^2} + \frac{c \log\left(\frac{\sqrt{c+\frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c+\frac{d}{x^2}} x + \sqrt{d}}\right)}{d^{\frac{3}{2}}}\right) a + \frac{1}{16} b \left(\frac{3c^2 \log\left(\frac{\sqrt{c+\frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c+\frac{d}{x^2}} x + \sqrt{d}}\right)}{d^{\frac{5}{2}}} + \frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}} c^2 x^3 - 5\sqrt{c+\frac{d}{x^2}} c^2 dx\right)}{\left(c+\frac{d}{x^2}\right)^2 d^2 x^4 - 2\left(c+\frac{d}{x^2}\right) d^3 x^2 + d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] $-1/4*(2*\sqrt{c+d/x^2}*c*x/((c+d/x^2)*d*x^2-d^2)+c*\log((\sqrt{c+d/x^2}*x-\sqrt{d})/(\sqrt{c+d/x^2}*x+\sqrt{d}))/d^{(3/2)})*a+1/16*b*(3*c^2*\log((\sqrt{c+d/x^2}*x-\sqrt{d})/(\sqrt{c+d/x^2}*x+\sqrt{d}))/d^{(5/2)}+$

$2*(3*(c + d/x^2)^{(3/2)}*c^2*x^3 - 5*\sqrt{c + d/x^2}*c^2*d*x)/((c + d/x^2)^2*d^2*x^4 - 2*(c + d/x^2)*d^3*x^2 + d^4)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + \frac{b}{x^2}}{x^4 \sqrt{c + \frac{d}{x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x^2)/(x^4*(c + d/x^2)^(1/2)), x)`

[Out] `int((a + b/x^2)/(x^4*(c + d/x^2)^(1/2)), x)`

sympy [A] time = 8.46, size = 150, normalized size = 1.61

$$-\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2dx} + \frac{ac \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{3}{2}}} + \frac{3bc^{\frac{3}{2}}}{8d^2x\sqrt{1+\frac{d}{cx^2}}} + \frac{b\sqrt{c}}{8dx^3\sqrt{1+\frac{d}{cx^2}}} - \frac{3bc^2 \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{5}{2}}} - \frac{b}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)/x**4/(c+d/x**2)**(1/2), x)`

[Out] `-a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*d*x) + a*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(3/2)) + 3*b*c**(3/2)/(8*d**2*x*sqrt(1 + d/(c*x**2))) + b*sqrt(c)/(8*d*x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(5/2)) - b/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))`

$$3.627 \quad \int \frac{\left(a + \frac{b}{x^2}\right)x^3}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$$

Optimal. Leaf size=118

$$-\frac{3d(4bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}} + \frac{3d(4bc - 5ad)}{8c^3\sqrt{c + \frac{d}{x^2}}} + \frac{x^2(4bc - 5ad)}{8c^2\sqrt{c + \frac{d}{x^2}}} + \frac{ax^4}{4c\sqrt{c + \frac{d}{x^2}}}$$

Rubi [A] time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 78, 51, 63, 208}

$$\frac{3x^2\sqrt{c + \frac{d}{x^2}}(4bc - 5ad)}{8c^3} - \frac{x^2(4bc - 5ad)}{4c^2\sqrt{c + \frac{d}{x^2}}} - \frac{3d(4bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}} + \frac{ax^4}{4c\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2), x]

[Out] -((4*b*c - 5*a*d)*x^2)/(4*c^2*Sqrt[c + d/x^2]) + (3*(4*b*c - 5*a*d)*Sqrt[c + d/x^2]*x^2)/(8*c^3) + (a*x^4)/(4*c*Sqrt[c + d/x^2]) - (3*d*(4*b*c - 5*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(7/2))

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{x^2}\right)x^3}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{a + bx}{x^3(c + dx)^{3/2}} dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{ax^4}{4c\sqrt{c + \frac{d}{x^2}}} - \frac{\left(2bc - \frac{5ad}{2}\right) \text{Subst}\left(\int \frac{1}{x^2(c+dx)^{3/2}} dx, x, \frac{1}{x^2}\right)}{4c} \\
&= -\frac{(4bc - 5ad)x^2}{4c^2\sqrt{c + \frac{d}{x^2}}} + \frac{ax^4}{4c\sqrt{c + \frac{d}{x^2}}} - \frac{(3(4bc - 5ad)) \text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}} dx, x, \frac{1}{x^2}\right)}{8c^2} \\
&= -\frac{(4bc - 5ad)x^2}{4c^2\sqrt{c + \frac{d}{x^2}}} + \frac{3(4bc - 5ad)\sqrt{c + \frac{d}{x^2}}x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c + \frac{d}{x^2}}} + \frac{(3d(4bc - 5ad)) \text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}} dx\right)}{16c^3} \\
&= -\frac{(4bc - 5ad)x^2}{4c^2\sqrt{c + \frac{d}{x^2}}} + \frac{3(4bc - 5ad)\sqrt{c + \frac{d}{x^2}}x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c + \frac{d}{x^2}}} + \frac{(3(4bc - 5ad)) \text{Subst}\left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx\right)}{8c^3} \\
&= -\frac{(4bc - 5ad)x^2}{4c^2\sqrt{c + \frac{d}{x^2}}} + \frac{3(4bc - 5ad)\sqrt{c + \frac{d}{x^2}}x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c + \frac{d}{x^2}}} - \frac{3d(4bc - 5ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 111, normalized size = 0.94

$$\frac{\sqrt{c}x\left(a\left(2c^2x^4 - 5cdx^2 - 15d^2\right) + 4bc\left(cx^2 + 3d\right)\right) + 3d^{3/2}\sqrt{\frac{cx^2}{d}} + 1\left(5ad - 4bc\right)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{7/2}x\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2), x]

[Out] (Sqrt[c]*x*(4*b*c*(3*d + c*x^2) + a*(-15*d^2 - 5*c*d*x^2 + 2*c^2*x^4)) + 3*d^(3/2)*(-4*b*c + 5*a*d)*Sqrt[1 + (c*x^2)/d]*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/(8*c^(7/2)*Sqrt[c + d/x^2]*x)

IntegrateAlgebraic [A] time = 0.18, size = 119, normalized size = 1.01

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (2ac^2x^6 - 5acdx^4 - 15ad^2x^2 + 4bc^2x^4 + 12bcdx^2)}{8c^3(cx^2 + d)} - \frac{3(4bcd - 5ad^2) \tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2), x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(12*b*c*d*x^2 - 15*a*d^2*x^2 + 4*b*c^2*x^4 - 5*a*c*d*x^4 + 2*a*c^2*x^6))/(8*c^3*(d + c*x^2)) - (3*(4*b*c*d - 5*a*d^2)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/(8*c^(7/2))

fricas [A] time = 0.46, size = 304, normalized size = 2.58

$$\frac{3(4bcd^2 - 5ad^3 + (4bc^2d - 5acd^2)x^2)\sqrt{c} \log\left(-2cx^2 - 2\sqrt{c}\sqrt{\frac{cx^2+d}{x^2}} - d\right) - 2(2ac^3x^6 + (4bc^3 - 5ac^2d)x^4 + 3(4bc^2d - 5acd^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16(c^5x^2 + c^4d)} - \frac{3(4bcd^2 - 5ad^3 + (4bc^2d - 5acd^2)x^2)\sqrt{-c} \arctan\left(\frac{\sqrt{-c}\sqrt{\frac{cx^2+d}{x^2}}}{c^2+d}\right) + (2ac^3x^6 + (4bc^3 - 5ac^2d)x^4 + 3(4bc^2d - 5acd^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8(c^5x^2 + c^4d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2), x, algorithm="fricas")

[Out] [-1/16*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(2*a*c^3*x^6 + (4*b*c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2)]/(c^5*x^2 + c^4*d), 1/8*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c^3*x^6 + (4*b*c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2)]/(c^5*x^2 + c^4*d)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x);;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep)]Unable to divide, perhaps due to rounding error%%{%%{-2, [1]%%}, [2, 1, 2]%%}+%%{%%{4, 0} : [1, 0, %%{-1, [1]%%}]%%}, [1, 1, 3]%%}+%%{-2, [0, 1, 4]%%} / %%{%%{1, [2]%%}, [2, 0, 0]%%}+%%{%%{[-2, [1]%%}, 0] : [1, 0, %%{-1, [1

]]]]], [1, 0, 1]]]] +]]]]{]]]{1, [1]]]], [0, 0, 2]]]] Error: Bad Argument Value

maple [A] time = 0.06, size = 140, normalized size = 1.19

$$\frac{(cx^2 + d) \left(2ac^2x^5 - 5a^2c^2dx^3 + 4bc^2x^3 - 15ac^2d^2x + 12bc^2dx + 15\sqrt{cx^2 + d} acd^2 \ln(\sqrt{c}x + \sqrt{cx^2 + d}) - 12\sqrt{cx^2 + d} bc^2d \ln(\sqrt{c}x + \sqrt{cx^2 + d}) \right)}{8 \left(\frac{cx^2 + d}{x^2} \right)^{\frac{3}{2}} c^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*x^3/(c+d/x^2)^(3/2), x)

[Out] $\frac{1}{8} (cx^2 + d) (2c^{7/2} x^5 a - 5c^{5/2} x^3 a d + 4c^{7/2} x^3 b - 15c^{3/2} x a d^2 + 12c^{5/2} x b d + 15(cx^2 + d)^{1/2} \ln(c^{1/2} x + (cx^2 + d)^{1/2}) a c d^2 - 12(cx^2 + d)^{1/2} \ln(c^{1/2} x + (cx^2 + d)^{1/2}) b c^2 d) / ((cx^2 + d)/x^2)^{3/2} / x^3 / c^{9/2}$

maxima [B] time = 1.38, size = 215, normalized size = 1.82

$$-\frac{1}{16} a \left(\frac{2 \left(15 \left(c + \frac{d}{x^2} \right)^2 d^2 - 25 \left(c + \frac{d}{x^2} \right) c d^2 + 8 c^2 d^2 \right)}{\left(c + \frac{d}{x^2} \right)^{\frac{5}{2}} c^3 - 2 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} c^4 + \sqrt{c + \frac{d}{x^2}} c^5} + \frac{15 d^2 \log \left(\frac{\sqrt{c + \frac{d}{x^2}} - \sqrt{c}}{\sqrt{c + \frac{d}{x^2}} + \sqrt{c}} \right)}{c^{\frac{7}{2}}} \right) + \frac{1}{4} b \left(\frac{2 \left(3 \left(c + \frac{d}{x^2} \right) d - 2 c d \right)}{\left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} c^2 - \sqrt{c + \frac{d}{x^2}} c^3} + \frac{3 d \log \left(\frac{\sqrt{c + \frac{d}{x^2}} - \sqrt{c}}{\sqrt{c + \frac{d}{x^2}} + \sqrt{c}} \right)}{c^{\frac{5}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2), x, algorithm="maxima")

[Out] $-1/16 * a * (2 * (15 * (c + d/x^2)^2 * d^2 - 25 * (c + d/x^2) * c * d^2 + 8 * c^2 * d^2) / ((c + d/x^2)^{5/2} * c^3 - 2 * (c + d/x^2)^{3/2} * c^4 + \text{sqrt}(c + d/x^2) * c^5) + 15 * d^2 * \log((\text{sqrt}(c + d/x^2) - \text{sqrt}(c)) / (\text{sqrt}(c + d/x^2) + \text{sqrt}(c))) / c^{7/2}) + 1/4 * b * (2 * (3 * (c + d/x^2) * d - 2 * c * d) / ((c + d/x^2)^{3/2} * c^2 - \text{sqrt}(c + d/x^2) * c^3) + 3 * d * \log((\text{sqrt}(c + d/x^2) - \text{sqrt}(c)) / (\text{sqrt}(c + d/x^2) + \text{sqrt}(c))) / c^{5/2}))$

mapad [B] time = 6.34, size = 134, normalized size = 1.14

$$\frac{ax^4}{4c\sqrt{c + \frac{d}{x^2}}} - \frac{15ad^2}{8c^3\sqrt{c + \frac{d}{x^2}}} + \frac{bx^2}{2c\sqrt{c + \frac{d}{x^2}}} - \frac{3bd \operatorname{atanh}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}} + \frac{15ad^2 \operatorname{atanh}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}} + \frac{3bd}{2c^2\sqrt{c + \frac{d}{x^2}}} - \frac{5adx^2}{8c^2\sqrt{c + \frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(a + b/x^2))/(c + d/x^2)^(3/2), x)

[Out] $(a*x^4)/(4*c*(c + d/x^2)^{(1/2)}) - (15*a*d^2)/(8*c^3*(c + d/x^2)^{(1/2)}) + (b*x^2)/(2*c*(c + d/x^2)^{(1/2)}) - (3*b*d*atanh((c + d/x^2)^{(1/2)}/c^{(1/2)}))/(2*c^{(5/2)}) + (15*a*d^2*atanh((c + d/x^2)^{(1/2)}/c^{(1/2)}))/(8*c^{(7/2)}) + (3*b*d)/(2*c^2*(c + d/x^2)^{(1/2)}) - (5*a*d*x^2)/(8*c^2*(c + d/x^2)^{(1/2)})$

sympy [A] time = 103.90, size = 177, normalized size = 1.50

$$a \left(\frac{x^5}{4c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} - \frac{5\sqrt{d}x^3}{8c^2\sqrt{\frac{cx^2}{d}+1}} - \frac{15d^{\frac{3}{2}}x}{8c^3\sqrt{\frac{cx^2}{d}+1}} + \frac{15d^2 \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{7}{2}}} \right) + b \left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + \frac{3\sqrt{d}x}{2c^2\sqrt{\frac{cx^2}{d}+1}} - \frac{3d \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{5}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*x**3/(c+d/x**2)**(3/2), x)`

[Out] $a*(x**5/(4*c*\sqrt{d}*\sqrt{c*x**2/d + 1})) - 5*\sqrt{d}*x**3/(8*c**2*\sqrt{c*x**2/d + 1}) - 15*d**(3/2)*x/(8*c**3*\sqrt{c*x**2/d + 1}) + 15*d**2*\operatorname{asinh}(\sqrt{c}*x/\sqrt{d})/(8*c**(7/2)) + b*(x**3/(2*c*\sqrt{d}*\sqrt{c*x**2/d + 1}) + 3*\sqrt{d}*x/(2*c**2*\sqrt{c*x**2/d + 1}) - 3*d*\operatorname{asinh}(\sqrt{c}*x/\sqrt{d})/(2*c*(5/2)))$

$$3.628 \quad \int \frac{\left(a + \frac{b}{x^2}\right)x}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$$

Optimal. Leaf size=86

$$\frac{(2bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}} - \frac{2bc - 3ad}{2c^2 \sqrt{c + \frac{d}{x^2}}} + \frac{ax^2}{2c \sqrt{c + \frac{d}{x^2}}}$$

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 78, 51, 63, 208}

$$-\frac{2bc - 3ad}{2c^2 \sqrt{c + \frac{d}{x^2}}} + \frac{(2bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}} + \frac{ax^2}{2c \sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*x)/(c + d/x^2)^(3/2), x]

[Out] -(2*b*c - 3*a*d)/(2*c^2*Sqrt[c + d/x^2]) + (a*x^2)/(2*c*Sqrt[c + d/x^2]) + ((2*b*c - 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*c^(5/2))

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{x^2}\right)x}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{a + bx}{x^2(c + dx)^{3/2}} dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{ax^2}{2c\sqrt{c + \frac{d}{x^2}}} - \frac{\left(bc - \frac{3ad}{2}\right) \text{Subst}\left(\int \frac{1}{x(c+dx)^{3/2}} dx, x, \frac{1}{x^2}\right)}{2c} \\
&= -\frac{2bc - 3ad}{2c^2\sqrt{c + \frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c + \frac{d}{x^2}}} - \frac{(2bc - 3ad) \text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}} dx, x, \frac{1}{x^2}\right)}{4c^2} \\
&= -\frac{2bc - 3ad}{2c^2\sqrt{c + \frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c + \frac{d}{x^2}}} - \frac{(2bc - 3ad) \text{Subst}\left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + \frac{d}{x^2}}\right)}{2c^2d} \\
&= -\frac{2bc - 3ad}{2c^2\sqrt{c + \frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c + \frac{d}{x^2}}} + \frac{(2bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 89, normalized size = 1.03

$$\frac{\sqrt{c}x(acx^2 + 3ad - 2bc) - \sqrt{d}\sqrt{\frac{cx^2}{d} + 1}(3ad - 2bc)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{5/2}x\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*x)/(c + d/x^2)^(3/2), x]

[Out] (Sqrt[c]*x*(-2*b*c + 3*a*d + a*c*x^2) - Sqrt[d]*(-2*b*c + 3*a*d)*Sqrt[1 + (c*x^2)/d]*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/(2*c^(5/2)*Sqrt[c + d/x^2]*x)

IntegrateAlgebraic [A] time = 0.18, size = 93, normalized size = 1.08

$$\frac{(2bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}} + \frac{\sqrt{\frac{cx^2+d}{x^2}}(acx^4 + 3adx^2 - 2bcx^2)}{2c^2(cx^2 + d)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*x)/(c + d/x^2)^(3/2), x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-2*b*c*x^2 + 3*a*d*x^2 + a*c*x^4))/(2*c^2*(d + c*x^2)) + ((2*b*c - 3*a*d)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/(2*c^(5/2))

fricas [A] time = 0.44, size = 249, normalized size = 2.90

$$\frac{(2bcd - 3ad^2 + (2bc^2 - 3acd)x^2)\sqrt{c} \log\left(-2cx^2 + 2\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}} - d\right) - 2(ac^2x^4 - (2bc^2 - 3acd)x^2)\sqrt{\frac{cx^2+d}{x^2}} - (2bcd - 3ad^2 + (2bc^2 - 3acd)x^2)\sqrt{c} \arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) - (ac^2x^4 - (2bc^2 - 3acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{4(c^4x^2 + c^3d) \sqrt{c^4x^2 + c^3d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2), x, algorithm="fricas")

[Out] [-1/4*((2*b*c*d - 3*a*d^2 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(a*c^2*x^4 - (2*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2)]/(c^4*x^2 + c^3*d), -1/2*((2*b*c*d - 3*a*d^2 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (a*c^2*x^4 - (2*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2)]/(c^4*x^2 + c^3*d)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep)]Unable to divide, perhaps due to rounding error%%{%%{-2, [1]%%}, [2, 1, 2]%%}+%%{%%{4, 0]: [1, 0, %%{-1, [1]%%}]}%%}, [1, 1, 3]%%}+%%{-2, [0, 1, 4]%%} / %%{%%{1, [2]%%}, [2, 0, 0]%%}+%%{%%{[-2, [1]%%}, 0]: [1, 0, %%{-1, [1]%%}]}%%}, [1, 0, 1]%%}+%%{%%{1, [1]%%}, [0, 0, 2]%%} Error: Bad Argument Value

maple [A] time = 0.06, size = 115, normalized size = 1.34

$$\frac{(cx^2 + d)\left(-ac^2x^3 - 3ac^2dx + 2bc^2x + 3\sqrt{cx^2 + d} \operatorname{acn}(\sqrt{c}x + \sqrt{cx^2 + d}) - 2\sqrt{cx^2 + d}bc^2 \operatorname{ln}(\sqrt{c}x + \sqrt{cx^2 + d})\right)}{2\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}c^{\frac{7}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)*x/(c+d/x^2)^(3/2),x)`

[Out] $-1/2*(c*x^2+d)*(-c^{(5/2)}*x^3*a-3*c^{(3/2)}*x*a*d+2*c^{(5/2)}*x*b+3*(c*x^2+d)^{(1/2)}*\ln(c^{(1/2)}*x+(c*x^2+d)^{(1/2)})*a*c*d-2*(c*x^2+d)^{(1/2)}*\ln(c^{(1/2)}*x+(c*x^2+d)^{(1/2)})*b*c^2)/((c*x^2+d)/x^2)^{(3/2)}/x^3/c^{(7/2)}$

maxima [B] time = 1.20, size = 144, normalized size = 1.67

$$\frac{1}{4}a \left(\frac{2 \left(3 \left(c + \frac{d}{x^2} \right) d - 2cd \right)}{\left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} c^2 - \sqrt{c + \frac{d}{x^2}} c^3} + \frac{3d \log \left(\frac{\sqrt{c + \frac{d}{x^2}} - \sqrt{c}}{\sqrt{c + \frac{d}{x^2}} + \sqrt{c}} \right)}{c^{\frac{5}{2}}} \right) - \frac{1}{2}b \left(\frac{\log \left(\frac{\sqrt{c + \frac{d}{x^2}} - \sqrt{c}}{\sqrt{c + \frac{d}{x^2}} + \sqrt{c}} \right)}{c^{\frac{3}{2}}} + \frac{2}{\sqrt{c + \frac{d}{x^2}} c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="maxima")`

[Out] $1/4*a*(2*(3*(c + d/x^2)*d - 2*c*d)/((c + d/x^2)^(3/2)*c^2 - \sqrt{c + d/x^2}*c^3) + 3*d*\log((\sqrt{c + d/x^2} - \sqrt{c})/(\sqrt{c + d/x^2} + \sqrt{c}))/c^{(5/2)} - 1/2*b*(\log((\sqrt{c + d/x^2} - \sqrt{c})/(\sqrt{c + d/x^2} + \sqrt{c}))/c^{(3/2)} + 2/(\sqrt{c + d/x^2}*c))$

mupad [B] time = 5.61, size = 90, normalized size = 1.05

$$\frac{b \operatorname{atanh} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{c^{3/2}} - \frac{b}{c \sqrt{c + \frac{d}{x^2}}} + \frac{a x^2}{2c \sqrt{c + \frac{d}{x^2}}} - \frac{3ad \operatorname{atanh} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{2c^{5/2}} + \frac{3ad}{2c^2 \sqrt{c + \frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(a + b/x^2))/(c + d/x^2)^(3/2),x)`

[Out] $(b*\operatorname{atanh}((c + d/x^2)^{(1/2)}/c^{(1/2)}))/c^{(3/2)} - b/(c*(c + d/x^2)^{(1/2)}) + (a*x^2)/(2*c*(c + d/x^2)^{(1/2)}) - (3*a*d*\operatorname{atanh}((c + d/x^2)^{(1/2)}/c^{(1/2)}))/(2*c^{(5/2)}) + (3*a*d)/(2*c^2*(c + d/x^2)^{(1/2)})$

sympy [B] time = 52.42, size = 264, normalized size = 3.07

$$a \left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + \frac{3\sqrt{d}x}{2c^2\sqrt{\frac{cx^2}{d}+1}} - \frac{3d \operatorname{asinh} \left(\frac{\sqrt{c}x}{\sqrt{d}} \right)}{2c^2} \right) + b \left(-\frac{2c^3x^2\sqrt{1+\frac{d}{cx^2}}}{2c^2x^2+2c^2d} - \frac{c^3x^2 \log \left(\frac{d}{cx^2} \right)}{2c^2x^2+2c^2d} + \frac{2c^3x^2 \log \left(\sqrt{1+\frac{d}{cx^2}}+1 \right)}{2c^2x^2+2c^2d} - \frac{c^2d \log \left(\frac{d}{cx^2} \right)}{2c^2x^2+2c^2d} + \frac{2c^2d \log \left(\sqrt{1+\frac{d}{cx^2}}+1 \right)}{2c^2x^2+2c^2d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*x/(c+d/x**2)**(3/2),x)`

[Out] $a \cdot \frac{x^3}{2c\sqrt{d}\sqrt{cx^2/d + 1}} + 3\sqrt{d}x / (2c^2\sqrt{cx^2/d + 1}) - 3d \operatorname{asinh}(\sqrt{c}x/\sqrt{d}) / (2c^{5/2}) + b \cdot \frac{-2c^3x^2\sqrt{1 + d/(cx^2)}}{(2c^{9/2}x^2 + 2c^{7/2}d) - c^3x^2\log(d/(cx^2))} / (2c^{9/2}x^2 + 2c^{7/2}d) + 2c^3x^2\log(\sqrt{1 + d/(cx^2)}) / (2c^{9/2}x^2 + 2c^{7/2}d) - c^2d\log(d/(cx^2)) / (2c^{9/2}x^2 + 2c^{7/2}d) + 2c^2d\log(\sqrt{1 + d/(cx^2)}) / (2c^{9/2}x^2 + 2c^{7/2}d)$

$$3.629 \quad \int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x} dx$$

Optimal. Leaf size=52

$$\frac{bc - ad}{cd\sqrt{c + \frac{d}{x^2}}} + \frac{a \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {446, 78, 63, 208}

$$\frac{bc - ad}{cd\sqrt{c + \frac{d}{x^2}}} + \frac{a \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]

[Out] (b*c - a*d)/(c*d*Sqrt[c + d/x^2]) + (a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/c^(3/2)

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
```

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{a + bx}{x(c + dx)^{3/2}} dx, x, \frac{1}{x^2}\right)\right) \\
 &= \frac{bc - ad}{cd\sqrt{c + \frac{d}{x^2}}} - \frac{a \text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}} dx, x, \frac{1}{x^2}\right)}{2c} \\
 &= \frac{bc - ad}{cd\sqrt{c + \frac{d}{x^2}}} - \frac{a \text{Subst}\left(\int \frac{1}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + \frac{d}{x^2}}\right)}{cd} \\
 &= \frac{bc - ad}{cd\sqrt{c + \frac{d}{x^2}}} + \frac{a \tanh^{-1}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 73, normalized size = 1.40

$$\frac{\sqrt{c} x(bc - ad) + ad^{3/2} \sqrt{\frac{cx^2}{d} + 1} \sinh^{-1}\left(\frac{\sqrt{c} x}{\sqrt{d}}\right)}{c^{3/2} dx \sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x), x]

[Out] $(\sqrt{c}*(b*c - a*d)*x + a*d^{(3/2)}*\sqrt{1 + (c*x^2)/d}*\text{ArcSinh}[(\sqrt{c}*x)/\sqrt{d}])/(c^{(3/2)}*d*\sqrt{c + d/x^2})*x$

IntegrateAlgebraic [A] time = 0.11, size = 73, normalized size = 1.40

$$\frac{a \tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}} - \frac{x^2 \sqrt{\frac{cx^2+d}{x^2}} (ad - bc)}{cd(cx^2 + d)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x), x]

[Out] $-\left(\frac{-(b*c) + a*d}{c}\right)*x^2*\sqrt{\frac{d + c*x^2}{x^2}}/(c*d*(d + c*x^2)) + (a*\text{ArcTan}[\sqrt{\frac{d + c*x^2}{x^2}}/\sqrt{c}])/c^{(3/2)}$

fricas [B] time = 0.46, size = 200, normalized size = 3.85

$$\left[\frac{2(bc^2 - acd)x^2 \sqrt{\frac{cx^2+d}{x^2}} + (acd x^2 + ad^2)\sqrt{c} \log\left(-2cx^2 - 2\sqrt{c}x^2 \sqrt{\frac{cx^2+d}{x^2}} - d\right)}{2(c^3 dx^2 + c^2 d^2)}, \frac{(bc^2 - acd)x^2 \sqrt{\frac{cx^2+d}{x^2}} - (acd x^2 + ad^2)\sqrt{-c} \arctan\left(\frac{\sqrt{-c}x^2 \sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{c^3 dx^2 + c^2 d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="fricas")

[Out] $[1/2*(2*(b*c^2 - a*c*d)*x^2*\sqrt{(c*x^2 + d)/x^2} + (a*c*d*x^2 + a*d^2)*\sqrt{c}*\log(-2*c*x^2 - 2*\sqrt{c}*x^2*\sqrt{(c*x^2 + d)/x^2} - d))/(c^3*d*x^2 + c^2*d^2), ((b*c^2 - a*c*d)*x^2*\sqrt{(c*x^2 + d)/x^2} - (a*c*d*x^2 + a*d^2)*\sqrt{-c}*\arctan(\sqrt{-c}*x^2*\sqrt{(c*x^2 + d)/x^2}/(c*x^2 + d)))/(c^3*d*x^2 + c^2*d^2)]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep)]Unable to divide, perhaps due to rounding error%%{%%{-2,[1]%%},[2

, 1, 2]%%}+%%%{%%{[4, 0] : [1, 0, %%{-1, [1]%%}]}%%}, [1, 1, 3]%%}+%%%{-2, [0, 1, 4]%%} / %%%{%%%{1, [2]%%}%, [2, 0, 0]%%}+%%%{%%%{[%%%{-2, [1]%%}%, 0] : [1, 0, %%{-1, [1]%%}]}%%}, [1, 0, 1]%%}+%%%{%%%{1, [1]%%}%, [0, 0, 2]%%} Error: Bad Argument Value

maple [A] time = 0.06, size = 75, normalized size = 1.44

$$\frac{(cx^2 + d) \left(-ac^{\frac{3}{2}} dx + bc^{\frac{5}{2}} x + \sqrt{cx^2 + d} acd \ln \left(\sqrt{c} x + \sqrt{cx^2 + d} \right) \right)}{\left(\frac{cx^2 + d}{x^2} \right)^{\frac{3}{2}} c^{\frac{5}{2}} dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x,x)

[Out] (c*x^2+d)*(-a*c^(3/2)*d*x+b*c^(5/2)*x+(c*x^2+d)^(1/2)*a*c*d*ln(c^(1/2)*x+(c*x^2+d)^(1/2)))/((c*x^2+d)/x^2)^(3/2)/x^3/c^(5/2)/d

maxima [A] time = 1.26, size = 69, normalized size = 1.33

$$-\frac{1}{2} a \left(\frac{\log \left(\frac{\sqrt{c + \frac{d}{x^2}} - \sqrt{c}}{\sqrt{c + \frac{d}{x^2}} + \sqrt{c}} \right)}{c^{\frac{3}{2}}} + \frac{2}{\sqrt{c + \frac{d}{x^2}} c} \right) + \frac{b}{\sqrt{c + \frac{d}{x^2}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="maxima")

[Out] -1/2*a*(log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2) + 2/(sqrt(c + d/x^2)*c)) + b/(sqrt(c + d/x^2)*d)

mupad [B] time = 5.06, size = 54, normalized size = 1.04

$$\frac{a \operatorname{atanh} \left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{c^{3/2}} - \frac{a}{c \sqrt{c + \frac{d}{x^2}}} + \frac{b \sqrt{x^2}}{d \sqrt{cx^2 + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x^2)/(x*(c + d/x^2)^(3/2)),x)

[Out] $(a \operatorname{atanh}((c + d/x^2)^{1/2}/c^{1/2}))/c^{3/2} - a/(c*(c + d/x^2)^{1/2}) + (b*(x^2)^{1/2})/(d*(d + c*x^2)^{1/2})$

sympy [A] time = 20.33, size = 49, normalized size = 0.94

$$-\frac{a \operatorname{atan}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{-c}}\right)}{c\sqrt{-c}} - \frac{ad - bc}{cd\sqrt{c + \frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x,x)`

[Out] `-a*atan(sqrt(c + d/x**2)/sqrt(-c))/(c*sqrt(-c)) - (a*d - b*c)/(c*d*sqrt(c + d/x**2))`

$$3.630 \quad \int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^3} dx$$

Optimal. Leaf size=42

$$-\frac{bc - ad}{d^2 \sqrt{c + \frac{d}{x^2}}} - \frac{b \sqrt{c + \frac{d}{x^2}}}{d^2}$$

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {444, 43}

$$-\frac{bc - ad}{d^2 \sqrt{c + \frac{d}{x^2}}} - \frac{b \sqrt{c + \frac{d}{x^2}}}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3), x]

[Out] -((b*c - a*d)/(d^2*Sqrt[c + d/x^2])) - (b*Sqrt[c + d/x^2])/d^2

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^3} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{a + bx}{(c + dx)^{3/2}} dx, x, \frac{1}{x^2}\right)\right) \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \left(\frac{-bc + ad}{d(c + dx)^{3/2}} + \frac{b}{d\sqrt{c + dx}}\right) dx, x, \frac{1}{x^2}\right)\right) \\
&= -\frac{bc - ad}{d^2\sqrt{c + \frac{d}{x^2}}} - \frac{b\sqrt{c + \frac{d}{x^2}}}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.86

$$\frac{adx^2 - b(2cx^2 + d)}{d^2x^2\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3), x]

[Out] (a*d*x^2 - b*(d + 2*c*x^2))/(d^2*Sqrt[c + d/x^2]*x^2)

IntegrateAlgebraic [A] time = 0.06, size = 46, normalized size = 1.10

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (adx^2 - 2bcx^2 - bd)}{d^2 (cx^2 + d)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3), x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-(b*d) - 2*b*c*x^2 + a*d*x^2))/(d^2*(d + c*x^2))

fricas [A] time = 0.42, size = 46, normalized size = 1.10

$$-\frac{((2bc - ad)x^2 + bd)\sqrt{\frac{cx^2+d}{x^2}}}{cd^2x^2 + d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")

[Out] $-\left((2bc - ad)x^2 + b^2\right)\sqrt{\frac{cx^2 + d}{x^2}} / (cd^2x^2 + d^3)$

giac [A] time = 0.26, size = 37, normalized size = 0.88

$$-\frac{\frac{(2bc-ad)x^2}{d^2} + \frac{b}{d}}{\sqrt{cx^4 + dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")`

[Out] $-\left((2bc - ad)x^2/d^2 + b/d\right)/\sqrt{cx^4 + dx^2}$

maple [A] time = 0.05, size = 46, normalized size = 1.10

$$\frac{(adx^2 - 2bcx^2 - bd)(cx^2 + d)}{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}} d^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x)`

[Out] $(a dx^2 - 2bcx^2 - bd)(cx^2 + d) / \left(\frac{cx^2+d}{x^2}\right)^{3/2} / d^2 / x^4$

maxima [A] time = 0.46, size = 46, normalized size = 1.10

$$-b \left(\frac{\sqrt{c + \frac{d}{x^2}}}{d^2} + \frac{c}{\sqrt{c + \frac{d}{x^2}} d^2} \right) + \frac{a}{\sqrt{c + \frac{d}{x^2}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")`

[Out] $-b(\sqrt{c + d/x^2}/d^2 + c/(\sqrt{c + d/x^2}*d^2)) + a/(\sqrt{c + d/x^2}*d)$

mupad [B] time = 4.52, size = 46, normalized size = 1.10

$$\frac{x \sqrt{c + \frac{d}{x^2}} \left(x^2 \left(\frac{a}{d} - \frac{2bc}{d^2} \right) - \frac{b}{d} \right)}{cx^3 + dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x^2)/(x^3*(c + d/x^2)^(3/2)),x)`

[Out] $(x*(c + d/x^2)^{(1/2)}*(x^2*(a/d - (2*b*c)/d^2) - b/d))/(d*x + c*x^3)$

sympy [A] time = 3.42, size = 68, normalized size = 1.62

$$\begin{cases} \frac{a}{d\sqrt{c+\frac{d}{x^2}}} - \frac{2bc}{d^2\sqrt{c+\frac{d}{x^2}}} - \frac{b}{dx^2\sqrt{c+\frac{d}{x^2}}} & \text{for } d \neq 0 \\ \frac{-\frac{a}{2x^2} - \frac{b}{4x^4}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**3,x)

[Out] Piecewise((a/(d*sqrt(c + d/x**2)) - 2*b*c/(d**2*sqrt(c + d/x**2)) - b/(d*x**2*sqrt(c + d/x**2)), Ne(d, 0)), ((-a/(2*x**2) - b/(4*x**4))/c**(3/2), True))

$$3.631 \quad \int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^5} dx$$

Optimal. Leaf size=68

$$\frac{\sqrt{c + \frac{d}{x^2}} (2bc - ad)}{d^3} + \frac{c(bc - ad)}{d^3 \sqrt{c + \frac{d}{x^2}}} - \frac{b \left(c + \frac{d}{x^2}\right)^{3/2}}{3d^3}$$

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{\sqrt{c + \frac{d}{x^2}} (2bc - ad)}{d^3} + \frac{c(bc - ad)}{d^3 \sqrt{c + \frac{d}{x^2}}} - \frac{b \left(c + \frac{d}{x^2}\right)^{3/2}}{3d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5), x]

[Out] (c*(b*c - a*d))/(d^3*Sqrt[c + d/x^2]) + ((2*b*c - a*d)*Sqrt[c + d/x^2])/d^3 - (b*(c + d/x^2)^(3/2))/(3*d^3)

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^5} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{x(a + bx)}{(c + dx)^{3/2}} dx, x, \frac{1}{x^2}\right)\right) \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \left(\frac{c(bc - ad)}{d^2(c + dx)^{3/2}} + \frac{-2bc + ad}{d^2\sqrt{c + dx}} + \frac{b\sqrt{c + dx}}{d^2}\right) dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{c(bc - ad)}{d^3\sqrt{c + \frac{d}{x^2}}} + \frac{(2bc - ad)\sqrt{c + \frac{d}{x^2}}}{d^3} - \frac{b\left(c + \frac{d}{x^2}\right)^{3/2}}{3d^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.88

$$\frac{b(8c^2x^4 + 4cdx^2 - d^2) - 3adx^2(2cx^2 + d)}{3d^3x^4\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5), x]

[Out] (-3*a*d*x^2*(d + 2*c*x^2) + b*(-d^2 + 4*c*d*x^2 + 8*c^2*x^4))/(3*d^3*Sqrt[c + d/x^2]*x^4)

IntegrateAlgebraic [A] time = 0.08, size = 75, normalized size = 1.10

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}(-6acdx^4 - 3ad^2x^2 + 8bc^2x^4 + 4bcdx^2 - bd^2)}{3d^3x^2(cx^2 + d)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5), x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-(b*d^2) + 4*b*c*d*x^2 - 3*a*d^2*x^2 + 8*b*c^2*x^4 - 6*a*c*d*x^4))/(3*d^3*x^2*(d + c*x^2))

fricas [A] time = 0.42, size = 73, normalized size = 1.07

$$\frac{(2(4bc^2 - 3acd)x^4 - bd^2 + (4bcd - 3ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{3(cd^3x^4 + d^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")

[Out] $\frac{1}{3} * (2 * (4 * b * c^2 - 3 * a * c * d) * x^4 - b * d^2 + (4 * b * c * d - 3 * a * d^2) * x^2) * \text{sqrt}((c * x^2 + d) / x^2) / (c * d^3 * x^4 + d^4 * x^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")

[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^5), x)

maple [A] time = 0.05, size = 69, normalized size = 1.01

$$\frac{(6acd x^4 - 8b c^2 x^4 + 3a d^2 x^2 - 4bcd x^2 + b d^2)(c x^2 + d)}{3 \left(\frac{c x^2 + d}{x^2}\right)^{\frac{3}{2}} d^3 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x)

[Out] $-\frac{1}{3} * (6 * a * c * d * x^4 - 8 * b * c^2 * x^4 + 3 * a * d^2 * x^2 - 4 * b * c * d * x^2 + b * d^2) * (c * x^2 + d) / ((c * x^2 + d) / x^2)^{(3/2)} / d^3 / x^6$

maxima [A] time = 0.61, size = 81, normalized size = 1.19

$$-\frac{1}{3} b \left(\frac{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3} - \frac{6 \sqrt{c + \frac{d}{x^2}} c}{d^3} - \frac{3 c^2}{\sqrt{c + \frac{d}{x^2}} d^3} \right) - a \left(\frac{\sqrt{c + \frac{d}{x^2}}}{d^2} + \frac{c}{\sqrt{c + \frac{d}{x^2}} d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")

[Out] $-\frac{1}{3} * b * ((c + d/x^2)^{(3/2)} / d^3 - 6 * \text{sqrt}(c + d/x^2) * c / d^3 - 3 * c^2 / (\text{sqrt}(c + d/x^2) * d^3)) - a * (\text{sqrt}(c + d/x^2) / d^2 + c / (\text{sqrt}(c + d/x^2) * d^2))$

mupad [B] time = 4.64, size = 66, normalized size = 0.97

$$\frac{\sqrt{c + \frac{d}{x^2}} \left(-8bc^2x^4 + 6acdx^4 - 4bcdx^2 + 3ad^2x^2 + bd^2 \right)}{3d^3x^2(cx^2 + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x^2)/(x^5*(c + d/x^2)^(3/2)),x)`

[Out] `-((c + d/x^2)^(1/2)*(b*d^2 + 3*a*d^2*x^2 - 8*b*c^2*x^4 + 6*a*c*d*x^4 - 4*b*c*d*x^2))/(3*d^3*x^2*(d + c*x^2))`

sympy [A] time = 11.07, size = 61, normalized size = 0.90

$$-\frac{b\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}{3d^3} - \frac{c(ad - bc)}{d^3\sqrt{c + \frac{d}{x^2}}} - \frac{\sqrt{c + \frac{d}{x^2}}(ad - 2bc)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**5,x)`

[Out] `-b*(c + d/x**2)**(3/2)/(3*d**3) - c*(a*d - b*c)/(d**3*sqrt(c + d/x**2)) - sqrt(c + d/x**2)*(a*d - 2*b*c)/d**3`

$$3.632 \quad \int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^7} dx$$

Optimal. Leaf size=100

$$-\frac{c^2(bc - ad)}{d^4 \sqrt{c + \frac{d}{x^2}}} + \frac{\left(c + \frac{d}{x^2}\right)^{3/2} (3bc - ad)}{3d^4} - \frac{c \sqrt{c + \frac{d}{x^2}} (3bc - 2ad)}{d^4} - \frac{b \left(c + \frac{d}{x^2}\right)^{5/2}}{5d^4}$$

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$-\frac{c^2(bc - ad)}{d^4 \sqrt{c + \frac{d}{x^2}}} + \frac{\left(c + \frac{d}{x^2}\right)^{3/2} (3bc - ad)}{3d^4} - \frac{c \sqrt{c + \frac{d}{x^2}} (3bc - 2ad)}{d^4} - \frac{b \left(c + \frac{d}{x^2}\right)^{5/2}}{5d^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7), x]

[Out] -((c^2*(b*c - a*d))/(d^4*Sqrt[c + d/x^2])) - (c*(3*b*c - 2*a*d)*Sqrt[c + d/x^2])/d^4 + ((3*b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (b*(c + d/x^2)^(5/2))/(5*d^4)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^7} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{x^2(a + bx)}{(c + dx)^{3/2}} dx, x, \frac{1}{x^2}\right)\right) \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \left(-\frac{c^2(bc - ad)}{d^3(c + dx)^{3/2}} + \frac{c(3bc - 2ad)}{d^3\sqrt{c + dx}} + \frac{(-3bc + ad)\sqrt{c + dx}}{d^3} + \frac{b(c + dx)^{3/2}}{d^3}\right) dx,\right.\right. \\
&= -\frac{c^2(bc - ad)}{d^4\sqrt{c + \frac{d}{x^2}}} - \frac{c(3bc - 2ad)\sqrt{c + \frac{d}{x^2}}}{d^4} + \frac{(3bc - ad)\left(c + \frac{d}{x^2}\right)^{3/2}}{3d^4} - \frac{b\left(c + \frac{d}{x^2}\right)^{5/2}}{5d^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 0.81

$$\frac{-5adx^2(-8c^2x^4 - 4cdx^2 + d^2) - 3b(16c^3x^6 + 8c^2dx^4 - 2cd^2x^2 + d^3)}{15d^4x^6\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7), x]

[Out] (-5*a*d*x^2*(d^2 - 4*c*d*x^2 - 8*c^2*x^4) - 3*b*(d^3 - 2*c*d^2*x^2 + 8*c^2*d*x^4 + 16*c^3*x^6))/(15*d^4*sqrt[c + d/x^2]*x^6)

IntegrateAlgebraic [A] time = 0.10, size = 99, normalized size = 0.99

$$\frac{\sqrt{\frac{cx^2+d}{x^2}} (40ac^2dx^6 + 20acd^2x^4 - 5ad^3x^2 - 48bc^3x^6 - 24bc^2dx^4 + 6bcd^2x^2 - 3bd^3)}{15d^4x^4(cx^2 + d)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7), x]

[Out] (sqrt[(d + c*x^2)/x^2]*(-3*b*d^3 + 6*b*c*d^2*x^2 - 5*a*d^3*x^2 - 24*b*c^2*d*x^4 + 20*a*c*d^2*x^4 - 48*b*c^3*x^6 + 40*a*c^2*d*x^6))/(15*d^4*x^4*(d + c*x^2))

fricas [A] time = 0.43, size = 98, normalized size = 0.98

$$\frac{(8(6bc^3 - 5ac^2d)x^6 + 4(6bc^2d - 5acd^2)x^4 + 3bd^3 - (6bcd^2 - 5ad^3)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{15(cd^4x^6 + d^5x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")

[Out] $-1/15*(8*(6*b*c^3 - 5*a*c^2*d)*x^6 + 4*(6*b*c^2*d - 5*a*c*d^2)*x^4 + 3*b*d^3 - (6*b*c*d^2 - 5*a*d^3)*x^2)*\sqrt{(c*x^2 + d)/x^2}/(c*d^4*x^6 + d^5*x^4)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")

[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^7), x)

maple [A] time = 0.05, size = 94, normalized size = 0.94

$$\frac{(40a^2c^2d^2x^6 - 48b^2c^3x^6 + 20ac^2d^2x^4 - 24b^2c^2d^2x^4 - 5ad^3x^2 + 6bc^2d^2x^2 - 3bd^3)(cx^2 + d)}{15\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x)

[Out] $1/15*(40*a*c^2*d*x^6-48*b*c^3*x^6+20*a*c*d^2*x^4-24*b*c^2*d*x^4-5*a*d^3*x^2+6*b*c*d^2*x^2-3*b*d^3)*(c*x^2+d)/((c*x^2+d)/x^2)^{(3/2)}/d^4/x^8$

maxima [A] time = 0.62, size = 116, normalized size = 1.16

$$-\frac{1}{5}b\left(\frac{\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4} - \frac{5\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4} + \frac{15\sqrt{c + \frac{d}{x^2}}c^2}{d^4} + \frac{5c^3}{\sqrt{c + \frac{d}{x^2}}d^4}\right) - \frac{1}{3}a\left(\frac{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3} - \frac{6\sqrt{c + \frac{d}{x^2}}c}{d^3} - \frac{3c^2}{\sqrt{c + \frac{d}{x^2}}d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")

[Out] $-1/5*b*((c + d/x^2)^{(5/2)}/d^4 - 5*(c + d/x^2)^{(3/2)}*c/d^4 + 15*\sqrt{c + d/x^2}*c^2/d^4 + 5*c^3/(\sqrt{c + d/x^2}*d^4)) - 1/3*a*((c + d/x^2)^{(3/2)}/d^3 - 6*\sqrt{c + d/x^2}*c/d^3 - 3*c^2/(\sqrt{c + d/x^2}*d^3))$

mupad [B] time = 4.84, size = 91, normalized size = 0.91

$$\frac{\sqrt{c + \frac{d}{x^2}} (48 b c^3 x^6 - 40 a c^2 d x^6 + 24 b c^2 d x^4 - 20 a c d^2 x^4 - 6 b c d^2 x^2 + 5 a d^3 x^2 + 3 b d^3)}{15 d^4 x^4 (c x^2 + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x^2)/(x^7*(c + d/x^2)^(3/2)),x)

[Out] -((c + d/x^2)^(1/2)*(3*b*d^3 + 5*a*d^3*x^2 + 48*b*c^3*x^6 - 20*a*c*d^2*x^4 - 40*a*c^2*d*x^6 - 6*b*c*d^2*x^2 + 24*b*c^2*d*x^4))/(15*d^4*x^4*(d + c*x^2))

sympy [A] time = 13.28, size = 90, normalized size = 0.90

$$-\frac{b \left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}}{5d^4} + \frac{c^2(ad - bc)}{d^4 \sqrt{c + \frac{d}{x^2}}} - \frac{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}(ad - 3bc)}{3d^4} - \frac{\sqrt{c + \frac{d}{x^2}}(-2acd + 3bc^2)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**7,x)

[Out] -b*(c + d/x**2)**(5/2)/(5*d**4) + c**2*(a*d - b*c)/(d**4*sqrt(c + d/x**2)) - (c + d/x**2)**(3/2)*(a*d - 3*b*c)/(3*d**4) - sqrt(c + d/x**2)*(-2*a*c*d + 3*b*c**2)/d**4

$$3.633 \quad \int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^9} dx$$

Optimal. Leaf size=126

$$\frac{c^3(bc - ad)}{d^5 \sqrt{c + \frac{d}{x^2}}} + \frac{c^2 \sqrt{c + \frac{d}{x^2}} (4bc - 3ad)}{d^5} + \frac{\left(c + \frac{d}{x^2}\right)^{5/2} (4bc - ad)}{5d^5} - \frac{c \left(c + \frac{d}{x^2}\right)^{3/2} (2bc - ad)}{d^5} - \frac{b \left(c + \frac{d}{x^2}\right)^{7/2}}{7d^5}$$

Rubi [A] time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{c^2 \sqrt{c + \frac{d}{x^2}} (4bc - 3ad)}{d^5} + \frac{c^3(bc - ad)}{d^5 \sqrt{c + \frac{d}{x^2}}} + \frac{\left(c + \frac{d}{x^2}\right)^{5/2} (4bc - ad)}{5d^5} - \frac{c \left(c + \frac{d}{x^2}\right)^{3/2} (2bc - ad)}{d^5} - \frac{b \left(c + \frac{d}{x^2}\right)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9), x]

[Out] (c^3*(b*c - a*d))/(d^5*Sqrt[c + d/x^2]) + (c^2*(4*b*c - 3*a*d)*Sqrt[c + d/x^2])/d^5 - (c*(2*b*c - a*d)*(c + d/x^2)^(3/2))/d^5 + ((4*b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^5) - (b*(c + d/x^2)^(7/2))/(7*d^5)

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^9} dx = -\left(\frac{1}{2} \text{Subst}\left(\int \frac{x^3(a + bx)}{(c + dx)^{3/2}} dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2} \text{Subst}\left(\int \left(\frac{c^3(bc - ad)}{d^4(c + dx)^{3/2}} - \frac{c^2(4bc - 3ad)}{d^4\sqrt{c + dx}} + \frac{3c(2bc - ad)\sqrt{c + dx}}{d^4} + \frac{(-4bc + ad)(c + dx)}{d^4}\right) dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{c^3(bc - ad)}{d^5\sqrt{c + \frac{d}{x^2}}} + \frac{c^2(4bc - 3ad)\sqrt{c + \frac{d}{x^2}}}{d^5} - \frac{c(2bc - ad)\left(c + \frac{d}{x^2}\right)^{3/2}}{d^5} + \frac{(4bc - ad)\left(c + \frac{d}{x^2}\right)^{5/2}}{5d^5} - \frac{b}{5d^5}$$

Mathematica [A] time = 0.04, size = 104, normalized size = 0.83

$$\frac{b(128c^4x^8 + 64c^3dx^6 - 16c^2d^2x^4 + 8cd^3x^2 - 5d^4) - 7adx^2(16c^3x^6 + 8c^2dx^4 - 2cd^2x^2 + d^3)}{35d^5x^8\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9), x]

[Out] (-7*a*d*x^2*(d^3 - 2*c*d^2*x^2 + 8*c^2*d*x^4 + 16*c^3*x^6) + b*(-5*d^4 + 8*c*d^3*x^2 - 16*c^2*d^2*x^4 + 64*c^3*d*x^6 + 128*c^4*x^8))/(35*d^5*sqrt[c + d/x^2]*x^8)

IntegrateAlgebraic [A] time = 0.11, size = 123, normalized size = 0.98

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}(-112ac^3dx^8 - 56ac^2d^2x^6 + 14acd^3x^4 - 7ad^4x^2 + 128bc^4x^8 + 64bc^3dx^6 - 16bc^2d^2x^4 + 8bcd^3x^2 - 5bd^4)}{35d^5x^6(cx^2 + d)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9), x]

[Out] (sqrt[(d + c*x^2)/x^2]*(-5*b*d^4 + 8*b*c*d^3*x^2 - 7*a*d^4*x^2 - 16*b*c^2*d^2*x^4 + 14*a*c*d^3*x^4 + 64*b*c^3*d*x^6 - 56*a*c^2*d^2*x^6 + 128*b*c^4*x^8 - 112*a*c^3*d*x^8))/(35*d^5*x^6*(d + c*x^2))

fricas [A] time = 0.45, size = 121, normalized size = 0.96

$$\frac{(16(8bc^4 - 7ac^3d)x^8 + 8(8bc^3d - 7ac^2d^2)x^6 - 5bd^4 - 2(8bc^2d^2 - 7acd^3)x^4 + (8bcd^3 - 7ad^4)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{35(cd^5x^8 + d^6x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")

[Out] 1/35*(16*(8*b*c^4 - 7*a*c^3*d)*x^8 + 8*(8*b*c^3*d - 7*a*c^2*d^2)*x^6 - 5*b*d^4 - 2*(8*b*c^2*d^2 - 7*a*c*d^3)*x^4 + (8*b*c*d^3 - 7*a*d^4)*x^2)*sqrt((c*x^2 + d)/x^2)/(c*d^5*x^8 + d^6*x^6)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")

[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^9), x)

maple [A] time = 0.05, size = 118, normalized size = 0.94

$$\frac{(112ac^3dx^8 - 128bc^4x^8 + 56a^2c^2d^2x^6 - 64bc^3dx^6 - 14ac^2d^3x^4 + 16bc^2d^2x^4 + 7ad^4x^2 - 8bcd^3x^2 + 5bd^4)(cx^2 + d)}{35\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x)

[Out] -1/35*(112*a*c^3*d*x^8-128*b*c^4*x^8+56*a*c^2*d^2*x^6-64*b*c^3*d*x^6-14*a*c*d^3*x^4+16*b*c^2*d^2*x^4+7*a*d^4*x^2-8*b*c*d^3*x^2+5*b*d^4)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^5/x^10

maxima [A] time = 0.55, size = 151, normalized size = 1.20

$$-\frac{1}{35}b\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^5}-\frac{28\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^5}+\frac{70\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^5}-\frac{140\sqrt{c+\frac{d}{x^2}}c^3}{d^5}-\frac{35c^4}{\sqrt{c+\frac{d}{x^2}}d^5}\right)-\frac{1}{5}a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4}+\frac{15\sqrt{c+\frac{d}{x^2}}c^2}{d^4}+\frac{5c^3}{\sqrt{c+\frac{d}{x^2}}d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")

[Out] -1/35*b*(5*(c + d/x^2)^(7/2)/d^5 - 28*(c + d/x^2)^(5/2)*c/d^5 + 70*(c + d/x^2)^(3/2)*c^2/d^5 - 140*sqrt(c + d/x^2)*c^3/d^5 - 35*c^4/(sqrt(c + d/x^2)*d

$\wedge 5)) - 1/5*a*((c + d/x^2)^{(5/2)}/d^4 - 5*(c + d/x^2)^{(3/2)}*c/d^4 + 15*\sqrt{c + d/x^2}*c^2/d^4 + 5*c^3/(\sqrt{c + d/x^2})*d^4))$

mupad [B] time = 4.92, size = 154, normalized size = 1.22

$$\frac{c\sqrt{c+\frac{d}{x^2}}(21ad-29bc)}{35d^4x^2} - \frac{b\sqrt{c+\frac{d}{x^2}}}{7d^2x^6} - \frac{\sqrt{c+\frac{d}{x^2}}(7ad^2-13bcd)}{35d^4x^4} - \frac{\sqrt{c+\frac{d}{x^2}}\left(x^2\left(\frac{58bc^4-42ac^3d}{35d^5} + \frac{2c^3(77ad-93bc)}{35d^5}\right) + \frac{c^2(77ad-93bc)}{35d^4}\right)}{cx^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x^2)/(x^9*(c + d/x^2)^(3/2)),x)`

[Out] $(c*(c + d/x^2)^{(1/2)}*(21*a*d - 29*b*c))/(35*d^4*x^2) - (b*(c + d/x^2)^{(1/2)})/(7*d^2*x^6) - ((c + d/x^2)^{(1/2)}*(7*a*d^2 - 13*b*c*d))/(35*d^4*x^4) - ((c + d/x^2)^{(1/2)}*(x^2*((58*b*c^4 - 42*a*c^3*d)/(35*d^5) + (2*c^3*(77*a*d - 93*b*c))/(35*d^5)) + (c^2*(77*a*d - 93*b*c))/(35*d^4)))/(d + c*x^2)$

sympy [A] time = 16.14, size = 122, normalized size = 0.97

$$\frac{b\left(c + \frac{d}{x^2}\right)^{\frac{7}{2}}}{7d^5} - \frac{c^3(ad-bc)}{d^5\sqrt{c + \frac{d}{x^2}}} - \frac{\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}(ad-4bc)}{5d^5} - \frac{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}(-3acd + 6bc^2)}{3d^5} - \frac{\sqrt{c + \frac{d}{x^2}}(3ac^2d - 4bc^3)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**9,x)`

[Out] $-b*(c + d/x**2)**(7/2)/(7*d**5) - c**3*(a*d - b*c)/(d**5*\sqrt{c + d/x**2}) - (c + d/x**2)**(5/2)*(a*d - 4*b*c)/(5*d**5) - (c + d/x**2)**(3/2)*(-3*a*c*d + 6*b*c**2)/(3*d**5) - \sqrt{c + d/x**2}*(3*a*c**2*d - 4*b*c**3)/d**5$

$$3.634 \quad \int \frac{\left(a + \frac{b}{x^2}\right)x^4}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$$

Optimal. Leaf size=111

$$-\frac{8dx\sqrt{c + \frac{d}{x^2}}(5bc - 6ad)}{15c^4} + \frac{4dx(5bc - 6ad)}{15c^3\sqrt{c + \frac{d}{x^2}}} + \frac{x^3(5bc - 6ad)}{15c^2\sqrt{c + \frac{d}{x^2}}} + \frac{ax^5}{5c\sqrt{c + \frac{d}{x^2}}}$$

Rubi [A] time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {453, 271, 192, 191}

$$\frac{x^3(5bc - 6ad)}{15c^2\sqrt{c + \frac{d}{x^2}}} - \frac{8dx\sqrt{c + \frac{d}{x^2}}(5bc - 6ad)}{15c^4} + \frac{4dx(5bc - 6ad)}{15c^3\sqrt{c + \frac{d}{x^2}}} + \frac{ax^5}{5c\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2), x]

[Out] (4*d*(5*b*c - 6*a*d)*x)/(15*c^3*Sqrt[c + d/x^2]) - (8*d*(5*b*c - 6*a*d)*Sqrt[c + d/x^2]*x)/(15*c^4) + ((5*b*c - 6*a*d)*x^3)/(15*c^2*Sqrt[c + d/x^2]) + (a*x^5)/(5*c*Sqrt[c + d/x^2])

Rule 191

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 192

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rule 271

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e^(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\left(a + \frac{b}{x^2}\right) x^4}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx &= \frac{ax^5}{5c\sqrt{c + \frac{d}{x^2}}} + \frac{(5bc - 6ad) \int \frac{x^2}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx}{5c} \\ &= \frac{(5bc - 6ad)x^3}{15c^2\sqrt{c + \frac{d}{x^2}}} + \frac{ax^5}{5c\sqrt{c + \frac{d}{x^2}}} - \frac{(4d(5bc - 6ad)) \int \frac{1}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx}{15c^2} \\ &= \frac{4d(5bc - 6ad)x}{15c^3\sqrt{c + \frac{d}{x^2}}} + \frac{(5bc - 6ad)x^3}{15c^2\sqrt{c + \frac{d}{x^2}}} + \frac{ax^5}{5c\sqrt{c + \frac{d}{x^2}}} - \frac{(8d(5bc - 6ad)) \int \frac{1}{\sqrt{c + \frac{d}{x^2}}} dx}{15c^3} \\ &= \frac{4d(5bc - 6ad)x}{15c^3\sqrt{c + \frac{d}{x^2}}} - \frac{8d(5bc - 6ad)\sqrt{c + \frac{d}{x^2}}x}{15c^4} + \frac{(5bc - 6ad)x^3}{15c^2\sqrt{c + \frac{d}{x^2}}} + \frac{ax^5}{5c\sqrt{c + \frac{d}{x^2}}} \end{aligned}$$

Mathematica [A] time = 0.08, size = 80, normalized size = 0.72

$$\frac{3a(c^3x^6 - 2c^2dx^4 + 8cd^2x^2 + 16d^3) + 5bc(c^2x^4 - 4cdx^2 - 8d^2)}{15c^4x\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2), x]
```

```
[Out] (5*b*c*(-8*d^2 - 4*c*d*x^2 + c^2*x^4) + 3*a*(16*d^3 + 8*c*d^2*x^2 - 2*c^2*d*x^4 + c^3*x^6))/(15*c^4*Sqrt[c + d/x^2]*x)
```

IntegrateAlgebraic [A] time = 0.10, size = 90, normalized size = 0.81

$$\frac{x\sqrt{c + \frac{d}{x^2}} (3ac^3x^6 - 6ac^2dx^4 + 24acd^2x^2 + 48ad^3 + 5bc^3x^4 - 20bc^2dx^2 - 40bcd^2)}{15c^4(cx^2 + d)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]

[Out] (Sqrt[c + d/x^2]*x*(-40*b*c*d^2 + 48*a*d^3 - 20*b*c^2*d*x^2 + 24*a*c*d^2*x^2 + 5*b*c^3*x^4 - 6*a*c^2*d*x^4 + 3*a*c^3*x^6))/(15*c^4*(d + c*x^2))

fricas [A] time = 0.43, size = 95, normalized size = 0.86

$$\frac{(3ac^3x^7 + (5bc^3 - 6ac^2d)x^5 - 4(5bc^2d - 6acd^2)x^3 - 8(5bcd^2 - 6ad^3)x)\sqrt{\frac{cx^2+d}{x^2}}}{15(c^5x^2 + c^4d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="fricas")

[Out] 1/15*(3*a*c^3*x^7 + (5*b*c^3 - 6*a*c^2*d)*x^5 - 4*(5*b*c^2*d - 6*a*c*d^2)*x^3 - 8*(5*b*c*d^2 - 6*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/(c^5*x^2 + c^4*d)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sqrt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.06, size = 91, normalized size = 0.82

$$\frac{(3ax^6c^3 - 6ac^2dx^4 + 5bc^3x^4 + 24acd^2x^2 - 20bc^2dx^2 + 48ad^3 - 40bcd^2)(cx^2 + d)}{15\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x)`

[Out] $\frac{1}{15}*(3*a*c^3*x^6-6*a*c^2*d*x^4+5*b*c^3*x^4+24*a*c*d^2*x^2-20*b*c^2*d*x^2+48*a*d^3-40*b*c*d^2)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^4$

maxima [A] time = 0.56, size = 128, normalized size = 1.15

$$\frac{1}{3}b \left(\frac{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} x^3 - 6\sqrt{c + \frac{d}{x^2}} dx}{c^3} - \frac{3d^2}{\sqrt{c + \frac{d}{x^2}} c^3 x} \right) + \frac{1}{5}a \left(\frac{5d^3}{\sqrt{c + \frac{d}{x^2}} c^4 x} + \frac{\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}} x^5 - 5\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} dx^3 + 15\sqrt{c + \frac{d}{x^2}} d^2 x}{c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="maxima")`

[Out] $\frac{1}{3}b*\left(\left(c + d/x^2\right)^{(3/2)}*x^3 - 6*\text{sqrt}(c + d/x^2)*d*x\right)/c^3 - 3*d^2/(\text{sqrt}(c + d/x^2)*c^3*x) + 1/5*a*(5*d^3/(\text{sqrt}(c + d/x^2)*c^4*x) + \left(c + d/x^2\right)^{(5/2)}*x^5 - 5*(c + d/x^2)^{(3/2)}*d*x^3 + 15*\text{sqrt}(c + d/x^2)*d^2*x)/c^4$

mupad [B] time = 5.77, size = 79, normalized size = 0.71

$$\frac{3ac^3x^6 + 5bc^3x^4 - 6ac^2dx^4 - 20bc^2dx^2 + 24acd^2x^2 - 40bcd^2 + 48ad^3}{15c^4x\sqrt{c + \frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4*(a + b/x^2))/(c + d/x^2)^(3/2),x)`

[Out] $\frac{(48*a*d^3 + 3*a*c^3*x^6 + 5*b*c^3*x^4 - 40*b*c*d^2 + 24*a*c*d^2*x^2 - 6*a*c^2*d*x^4 - 20*b*c^2*d*x^2)/(15*c^4*x*(c + d/x^2)^(1/2))$

sympy [B] time = 7.83, size = 561, normalized size = 5.05

$$\left(\frac{c^{\frac{3}{2}}d^3\sqrt{\frac{d}{x^2}+1}}{5c^2d^3+15c^2d^2x^2+5c^2d^2} + \frac{5c^{\frac{3}{2}}d^2x^4\sqrt{\frac{d}{x^2}+1}}{5c^2d^3+15c^2d^2x^2+5c^2d^2} + \frac{30c^{\frac{3}{2}}d^2x^4\sqrt{\frac{d}{x^2}+1}}{5c^2d^3+15c^2d^2x^2+5c^2d^2} + \frac{40d^{\frac{3}{2}}x^2\sqrt{\frac{d}{x^2}+1}}{5c^2d^3+15c^2d^2x^2+5c^2d^2} + \frac{16d^{\frac{3}{2}}\sqrt{\frac{d}{x^2}+1}}{5c^2d^3+15c^2d^2x^2+5c^2d^2} \right) + b \left(\frac{c^{\frac{3}{2}}d^3\sqrt{\frac{d}{x^2}+1}}{3c^2d^3+6c^2d^2+3c^2d} - \frac{3c^{\frac{3}{2}}d^2x^4\sqrt{\frac{d}{x^2}+1}}{3c^2d^3+6c^2d^2+3c^2d} - \frac{12d^{\frac{3}{2}}x^2\sqrt{\frac{d}{x^2}+1}}{3c^2d^3+6c^2d^2+3c^2d} - \frac{8d^{\frac{3}{2}}\sqrt{\frac{d}{x^2}+1}}{3c^2d^3+6c^2d^2+3c^2d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)*x**4/(c+d/x**2)**(3/2),x)`

[Out] $a*(c**5*d**(19/2)*x**10*\text{sqrt}(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 5*c**3*d**(23/2)*x**6*\text{sqrt}(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 30*c**2*d**(25/2)*x**4*\text{sqrt}(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 40*c*d**(27/2)*$

$$\begin{aligned}
& x^{**2}*\text{sqrt}(c*x^{**2}/d + 1)/(5*c^{**7}*d^{**9}*x^{**6} + 15*c^{**6}*d^{**10}*x^{**4} + 15*c^{**5}*d^{**11}*x^{**2} + 5*c^{**4}*d^{**12}) + 16*d^{**29/2}*\text{sqrt}(c*x^{**2}/d + 1)/(5*c^{**7}*d^{**9}*x^{**6} + 15*c^{**6}*d^{**10}*x^{**4} + 15*c^{**5}*d^{**11}*x^{**2} + 5*c^{**4}*d^{**12}) + b*(c^{**3}*d^{**9/2}*x^{**6}*\text{sqrt}(c*x^{**2}/d + 1)/(3*c^{**5}*d^{**4}*x^{**4} + 6*c^{**4}*d^{**5}*x^{**2} + 3*c^{**3}*d^{**6}) - 3*c^{**2}*d^{**11/2}*x^{**4}*\text{sqrt}(c*x^{**2}/d + 1)/(3*c^{**5}*d^{**4}*x^{**4} + 6*c^{**4}*d^{**5}*x^{**2} + 3*c^{**3}*d^{**6}) - 12*c*d^{**13/2}*x^{**2}*\text{sqrt}(c*x^{**2}/d + 1)/(3*c^{**5}*d^{**4}*x^{**4} + 6*c^{**4}*d^{**5}*x^{**2} + 3*c^{**3}*d^{**6}) - 8*d^{**15/2}*\text{sqrt}(c*x^{**2}/d + 1)/(3*c^{**5}*d^{**4}*x^{**4} + 6*c^{**4}*d^{**5}*x^{**2} + 3*c^{**3}*d^{**6}))
\end{aligned}$$

$$3.635 \quad \int \frac{\left(a + \frac{b}{x^2}\right)x^2}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$$

Optimal. Leaf size=79

$$\frac{2x\sqrt{c + \frac{d}{x^2}}(3bc - 4ad)}{3c^3} - \frac{x(3bc - 4ad)}{3c^2\sqrt{c + \frac{d}{x^2}}} + \frac{ax^3}{3c\sqrt{c + \frac{d}{x^2}}}$$

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {453, 192, 191}

$$\frac{2x\sqrt{c + \frac{d}{x^2}}(3bc - 4ad)}{3c^3} - \frac{x(3bc - 4ad)}{3c^2\sqrt{c + \frac{d}{x^2}}} + \frac{ax^3}{3c\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2), x]

[Out] -((3*b*c - 4*a*d)*x)/(3*c^2*Sqrt[c + d/x^2]) + (2*(3*b*c - 4*a*d)*Sqrt[c + d/x^2]*x)/(3*c^3) + (a*x^3)/(3*c*Sqrt[c + d/x^2])

Rule 191

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 192

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rule 453

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (

LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{\left(a + \frac{b}{x^2}\right)x^2}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx &= \frac{ax^3}{3c\sqrt{c + \frac{d}{x^2}}} + \frac{(3bc - 4ad) \int \frac{1}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx}{3c} \\ &= -\frac{(3bc - 4ad)x}{3c^2\sqrt{c + \frac{d}{x^2}}} + \frac{ax^3}{3c\sqrt{c + \frac{d}{x^2}}} + \frac{(2(3bc - 4ad)) \int \frac{1}{\sqrt{c + \frac{d}{x^2}}} dx}{3c^2} \\ &= -\frac{(3bc - 4ad)x}{3c^2\sqrt{c + \frac{d}{x^2}}} + \frac{2(3bc - 4ad)\sqrt{c + \frac{d}{x^2}}x}{3c^3} + \frac{ax^3}{3c\sqrt{c + \frac{d}{x^2}}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.72

$$\frac{a(c^2x^4 - 4cdx^2 - 8d^2) + 3bc(cx^2 + 2d)}{3c^3x\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2), x]

[Out] (3*b*c*(2*d + c*x^2) + a*(-8*d^2 - 4*c*d*x^2 + c^2*x^4))/(3*c^3*Sqrt[c + d/x^2]*x)

IntegrateAlgebraic [A] time = 0.09, size = 65, normalized size = 0.82

$$\frac{x\sqrt{c + \frac{d}{x^2}}(ac^2x^4 - 4acd^2 - 8ad^2 + 3bc^2x^2 + 6bcd)}{3c^3(cx^2 + d)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2), x]

[Out] (Sqrt[c + d/x^2]*x*(6*b*c*d - 8*a*d^2 + 3*b*c^2*x^2 - 4*a*c*d*x^2 + a*c^2*x^4))/(3*c^3*(d + c*x^2))

fricas [A] time = 0.43, size = 70, normalized size = 0.89

$$\frac{(ac^2x^5 + (3bc^2 - 4acd)x^3 + 2(3bcd - 4ad^2)x)\sqrt{\frac{cx^2+d}{x^2}}}{3(c^4x^2 + c^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="fricas")

[Out] 1/3*(a*c^2*x^5 + (3*b*c^2 - 4*a*c*d)*x^3 + 2*(3*b*c*d - 4*a*d^2)*x)*sqrt((c*x^2 + d)/x^2)/(c^4*x^2 + c^3*d)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sqrt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.06, size = 66, normalized size = 0.84

$$\frac{(ax^4c^2 - 4acd x^2 + 3bc^2x^2 - 8ad^2 + 6bcd)(cx^2 + d)}{3\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x)

[Out] 1/3*(a*c^2*x^4-4*a*c*d*x^2+3*b*c^2*x^2-8*a*d^2+6*b*c*d)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/c^3/x^3

maxima [A] time = 0.53, size = 90, normalized size = 1.14

$$b\left(\frac{\sqrt{c+\frac{d}{x^2}}x}{c^2} + \frac{d}{\sqrt{c+\frac{d}{x^2}}c^2x}\right) + \frac{1}{3}a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3 - 6\sqrt{c+\frac{d}{x^2}}dx}{c^3} - \frac{3d^2}{\sqrt{c+\frac{d}{x^2}}c^3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="maxima")

[Out] b*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) + 1/3*a*(((c + d/x^2)^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c + d/x^2)*c^3*x))

mupad [B] time = 5.20, size = 81, normalized size = 1.03

$$\frac{b c^2 x^4 + 3 b c d x^2 + 2 b d^2}{c^2 x^3 \left(c + \frac{d}{x^2}\right)^{3/2}} - \frac{-a c^2 x^4 + 4 a c d x^2 + 8 a d^2}{3 c^3 x \sqrt{c + \frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a + b/x^2))/(c + d/x^2)^(3/2),x)

[Out] (2*b*d^2 + b*c^2*x^4 + 3*b*c*d*x^2)/(c^2*x^3*(c + d/x^2)^(3/2)) - (8*a*d^2 - a*c^2*x^4 + 4*a*c*d*x^2)/(3*c^3*x*(c + d/x^2)^(1/2))

sympy [B] time = 7.31, size = 267, normalized size = 3.38

$$a \left(\frac{c^3 d^2 x^6 \sqrt{\frac{c x^2}{d} + 1}}{3 c^5 d^4 x^4 + 6 c^4 d^5 x^2 + 3 c^3 d^6} - \frac{3 c^2 d^{11} x^4 \sqrt{\frac{c x^2}{d} + 1}}{3 c^5 d^4 x^4 + 6 c^4 d^5 x^2 + 3 c^3 d^6} - \frac{12 c d^{13} x^2 \sqrt{\frac{c x^2}{d} + 1}}{3 c^5 d^4 x^4 + 6 c^4 d^5 x^2 + 3 c^3 d^6} - \frac{8 d^{15} \sqrt{\frac{c x^2}{d} + 1}}{3 c^5 d^4 x^4 + 6 c^4 d^5 x^2 + 3 c^3 d^6} \right) + b \left(\frac{x^2}{c \sqrt{d} \sqrt{\frac{c x^2}{d} + 1}} + \frac{2 \sqrt{d}}{c^2 \sqrt{\frac{c x^2}{d} + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*x**2/(c+d/x**2)**(3/2),x)

[Out] a*(c**3*d**(9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6)) + b*(x**2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1)))

$$3.636 \quad \int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$$

Optimal. Leaf size=45

$$\frac{ax}{c\sqrt{c + \frac{d}{x^2}}} - \frac{bc - 2ad}{c^2x\sqrt{c + \frac{d}{x^2}}}$$

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {375, 453, 191}

$$\frac{ax}{c\sqrt{c + \frac{d}{x^2}}} - \frac{bc - 2ad}{c^2x\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/(c + d/x^2)^(3/2), x]

[Out] -((b*c - 2*a*d)/(c^2*sqrt[c + d/x^2]*x)) + (a*x)/(c*sqrt[c + d/x^2])

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 375

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx &= -\text{Subst} \left(\int \frac{a + bx^2}{x^2 (c + dx^2)^{3/2}} dx, x, \frac{1}{x} \right) \\
&= \frac{ax}{c\sqrt{c + \frac{d}{x^2}}} + \frac{(-bc + 2ad) \text{Subst} \left(\int \frac{1}{(c+dx^2)^{3/2}} dx, x, \frac{1}{x} \right)}{c} \\
&= -\frac{bc - 2ad}{c^2\sqrt{c + \frac{d}{x^2}} x} + \frac{ax}{c\sqrt{c + \frac{d}{x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.73

$$\frac{acx^2 + 2ad - bc}{c^2x\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/(c + d/x^2)^(3/2), x]

[Out] (-b*c) + 2*a*d + a*c*x^2)/(c^2*Sqrt[c + d/x^2]*x)

IntegrateAlgebraic [A] time = 0.08, size = 40, normalized size = 0.89

$$\frac{x\sqrt{c + \frac{d}{x^2}} (acx^2 + 2ad - bc)}{c^2 (cx^2 + d)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/(c + d/x^2)^(3/2), x]

[Out] (Sqrt[c + d/x^2]*x*(-b*c) + 2*a*d + a*c*x^2)/(c^2*(d + c*x^2))

fricas [A] time = 0.48, size = 47, normalized size = 1.04

$$\frac{(acx^3 - (bc - 2ad)x)\sqrt{\frac{cx^2+d}{x^2}}}{c^3x^2 + c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="fricas")

[Out] (a*c*x^3 - (b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/(c^3*x^2 + c^2*d)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sqrt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.04, size = 43, normalized size = 0.96

$$\frac{(ax^2c + 2ad - bc)(cx^2 + d)}{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}} c^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/(c+d/x^2)^(3/2),x)

[Out] (a*c*x^2+2*a*d-b*c)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^2

maxima [A] time = 0.68, size = 53, normalized size = 1.18

$$a \left(\frac{\sqrt{c + \frac{d}{x^2}} x}{c^2} + \frac{d}{\sqrt{c + \frac{d}{x^2}} c^2 x} \right) - \frac{b}{\sqrt{c + \frac{d}{x^2}} cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="maxima")

[Out] a*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) - b/(sqrt(c + d/x^2)*c*x)

mupad [B] time = 4.90, size = 38, normalized size = 0.84

$$\frac{(cx^2 + d)(acx^2 + 2ad - bc)}{c^2 x^3 \left(c + \frac{d}{x^2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x^2)/(c + d/x^2)^(3/2), x)

[Out] ((d + c*x^2)*(2*a*d - b*c + a*c*x^2))/(c^2*x^3*(c + d/x^2)^(3/2))

sympy [A] time = 7.51, size = 65, normalized size = 1.44

$$a \left(\frac{x^2}{c\sqrt{d}\sqrt{\frac{cx^2}{d} + 1}} + \frac{2\sqrt{d}}{c^2\sqrt{\frac{cx^2}{d} + 1}} \right) - \frac{b}{c\sqrt{d}\sqrt{\frac{cx^2}{d} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2), x)

[Out] a*(x**2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1)) - b/(c*sqrt(d)*sqrt(c*x**2/d + 1))

$$3.637 \quad \int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^2} dx$$

Optimal. Leaf size=59

$$\frac{bc - ad}{cdx\sqrt{c + \frac{d}{x^2}}} - \frac{b \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{d^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {452, 335, 217, 206}

$$\frac{bc - ad}{cdx\sqrt{c + \frac{d}{x^2}}} - \frac{b \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2), x]

[Out] (b*c - a*d)/(c*d*sqrt[c + d/x^2]*x) - (b*ArcTanh[Sqrt[d]/(sqrt[c + d/x^2]*x)])/d^(3/2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 335

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

Rule 452

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^2} dx &= \frac{bc - ad}{cd\sqrt{c + \frac{d}{x^2}} x} + \frac{b \int \frac{1}{\sqrt{c + \frac{d}{x^2}} x^2} dx}{d} \\
 &= \frac{bc - ad}{cd\sqrt{c + \frac{d}{x^2}} x} - \frac{b \operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}} dx, x, \frac{1}{x}\right)}{d} \\
 &= \frac{bc - ad}{cd\sqrt{c + \frac{d}{x^2}} x} - \frac{b \operatorname{Subst}\left(\int \frac{1}{1-dx^2} dx, x, \frac{1}{\sqrt{c + \frac{d}{x^2}} x}\right)}{d} \\
 &= \frac{bc - ad}{cd\sqrt{c + \frac{d}{x^2}} x} - \frac{b \tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x}\right)}{d^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 1.20

$$\frac{\sqrt{d}(bc - ad) - bc\sqrt{cx^2 + d} \tanh^{-1}\left(\frac{\sqrt{cx^2 + d}}{\sqrt{d}}\right)}{cd^{3/2}x\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2), x]

[Out] (Sqrt[d]*(b*c - a*d) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(c*d^(3/2)*Sqrt[c + d/x^2]*x)

IntegrateAlgebraic [A] time = 0.12, size = 77, normalized size = 1.31

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\frac{bc-ad}{cd\sqrt{cx^2+d}} - \frac{b \tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{d^{3/2}} \right)}{\sqrt{cx^2 + d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2), x]

[Out] (Sqrt[c + d/x^2]*x*((b*c - a*d)/(c*d*Sqrt[d + c*x^2]) - (b*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/d^(3/2))/Sqrt[d + c*x^2]

fricas [A] time = 0.46, size = 195, normalized size = 3.31

$$\left[\frac{2(bcd - ad^2)x\sqrt{\frac{cx^2+d}{x^2}} + (bc^2x^2 + bcd)\sqrt{d} \log\left(-\frac{cx^2 - 2\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}} + 2d}{x^2}\right)}{2(c^2d^2x^2 + cd^3)}, \frac{(bcd - ad^2)x\sqrt{\frac{cx^2+d}{x^2}} + (bc^2x^2 + bcd)\sqrt{-d} \arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{c^2d^2x^2 + cd^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")

[Out] [1/2*(2*(b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*d)*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c^2*d^2*x^2 + c*d^3), ((b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*d)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c^2*d^2*x^2 + c*d^3)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [sign(t_n ostep),sign(t_n ostep+sqrt(d)/c*sign(t_n ostep))]sym2poly/r2sym(const gen & e,const index_m & i,constant vecteur & l) Error: Bad Argument Value

maple [A] time = 0.06, size = 79, normalized size = 1.34

$$\frac{(cx^2 + d) \left(\sqrt{cx^2 + d} bcd \ln \left(\frac{2d + 2\sqrt{cx^2 + d} \sqrt{d}}{x} \right) + ad^{\frac{5}{2}} - bcd^{\frac{3}{2}} \right)}{\left(\frac{cx^2 + d}{x^2} \right)^{\frac{3}{2}} cd^{\frac{5}{2}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x)`

[Out] `-(c*x^2+d)*((c*x^2+d)^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*b*c*d+a*d^(5/2)-d^(3/2)*b*c)/((c*x^2+d)/x^2)^(3/2)/x^3/c/d^(5/2)`

maxima [A] time = 1.21, size = 80, normalized size = 1.36

$$\frac{1}{2} b \left(\frac{\log \left(\frac{\sqrt{c + \frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x + \sqrt{d}} \right)}{d^{\frac{3}{2}}} + \frac{2}{\sqrt{c + \frac{d}{x^2}} dx} \right) - \frac{a}{\sqrt{c + \frac{d}{x^2}} cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")`

[Out] `1/2*b*(log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2/(sqrt(c + d/x^2)*d*x)) - a/(sqrt(c + d/x^2)*c*x)`

mupad [B] time = 5.11, size = 60, normalized size = 1.02

$$\frac{b}{dx \sqrt{c + \frac{d}{x^2}}} - \frac{a}{cx \sqrt{c + \frac{d}{x^2}}} - \frac{b \ln \left(\sqrt{c + \frac{d}{x^2}} + \frac{\sqrt{d}}{x} \right)}{d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x^2)/(x^2*(c + d/x^2)^(3/2)),x)`

[Out] `b/(d*x*(c + d/x^2)^(1/2)) - a/(c*x*(c + d/x^2)^(1/2)) - (b*log((c + d/x^2)^(1/2) + d^(1/2)/x))/d^(3/2)`

sympy [B] time = 11.71, size = 206, normalized size = 3.49

$$-\frac{a}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + b \left(\frac{cd^2x^2 \log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2 + 2d^{\frac{9}{2}}} - \frac{2cd^2x^2 \log\left(\sqrt{\frac{cx^2}{d}+1} + 1\right)}{2cd^{\frac{7}{2}}x^2 + 2d^{\frac{9}{2}}} + \frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2 + 2d^{\frac{9}{2}}} + \frac{d^3 \log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2 + 2d^{\frac{9}{2}}} - \frac{2d^3 \log\left(\sqrt{\frac{cx^2}{d}+1} + 1\right)}{2cd^{\frac{7}{2}}x^2 + 2d^{\frac{9}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**2,x)

[Out] -a/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + b*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x**2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*sqrt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + d**3*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d**3*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)))

$$3.638 \quad \int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^4} dx$$

Optimal. Leaf size=92

$$\frac{(3bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{2d^{5/2}} - \frac{3bc - 2ad}{2d^2 x \sqrt{c + \frac{d}{x^2}}} - \frac{b}{2dx^3 \sqrt{c + \frac{d}{x^2}}}$$

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {459, 335, 288, 217, 206}

$$-\frac{3bc - 2ad}{2d^2 x \sqrt{c + \frac{d}{x^2}}} + \frac{(3bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{2d^{5/2}} - \frac{b}{2dx^3 \sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4), x]

[Out] -b/(2*d*Sqrt[c + d/x^2]*x^3) - (3*b*c - 2*a*d)/(2*d^2*Sqrt[c + d/x^2]*x) + ((3*b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]/(2*d^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]

```

/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

```

Rule 335

```

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]

```

Rule 459

```

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
+ 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2}} x^4 dx &= -\frac{b}{2d\sqrt{c + \frac{d}{x^2}} x^3} + \frac{(-3bc + 2ad) \int \frac{1}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx}{2d} \\
&= -\frac{b}{2d\sqrt{c + \frac{d}{x^2}} x^3} - \frac{(-3bc + 2ad) \text{Subst}\left(\int \frac{x^2}{(c+dx^2)^{3/2}} dx, x, \frac{1}{x}\right)}{2d} \\
&= -\frac{b}{2d\sqrt{c + \frac{d}{x^2}} x^3} - \frac{3bc - 2ad}{2d^2\sqrt{c + \frac{d}{x^2}} x} + \frac{(3bc - 2ad) \text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}} dx, x, \frac{1}{x}\right)}{2d^2} \\
&= -\frac{b}{2d\sqrt{c + \frac{d}{x^2}} x^3} - \frac{3bc - 2ad}{2d^2\sqrt{c + \frac{d}{x^2}} x} + \frac{(3bc - 2ad) \text{Subst}\left(\int \frac{1}{1-dx^2} dx, x, \frac{1}{\sqrt{c + \frac{d}{x^2}} x}\right)}{2d^2} \\
&= -\frac{b}{2d\sqrt{c + \frac{d}{x^2}} x^3} - \frac{3bc - 2ad}{2d^2\sqrt{c + \frac{d}{x^2}} x} + \frac{(3bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x}\right)}{2d^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.62

$$\frac{x^2(2ad - 3bc) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{cx^2}{d} + 1\right) - bd}{2d^2x^3\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4), x]

[Out] (-b*d) + (-3*b*c + 2*a*d)*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/d]/(2*d^2*Sqrt[c + d/x^2]*x^3)

IntegrateAlgebraic [A] time = 0.24, size = 101, normalized size = 1.10

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\frac{(3bc-2ad) \tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{2adx^2-3bcx^2-bd}{2d^2x^2\sqrt{cx^2+d}} \right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4), x]

[Out] (Sqrt[c + d/x^2]*x*((-b*d) - 3*b*c*x^2 + 2*a*d*x^2)/(2*d^2*x^2*Sqrt[d + c*x^2]) + ((3*b*c - 2*a*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]/(2*d^(5/2)))/Sqrt[d + c*x^2]

fricas [A] time = 0.44, size = 248, normalized size = 2.70

$$\frac{\left((3bc^2 - 2acd)x^3 + (3bcd - 2ad^2)x \right) \sqrt{d} \log \left(-\frac{cx^2 - 2\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}} + d}{x^2} \right) + 2(bd^2 + (3bcd - 2ad^2)x^2) \sqrt{\frac{cx^2+d}{x^2}} - \left((3bc^2 - 2acd)x^3 + (3bcd - 2ad^2)x \right) \sqrt{-d} \arctan \left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d} \right) + (bd^2 + (3bcd - 2ad^2)x^2) \sqrt{\frac{cx^2+d}{x^2}}}{4(cd^3x^3 + d^4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")

[Out] [-1/4*(((3*b*c^2 - 2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(b*d^2 + (3*b*c*d - 2*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^3*x^3 + d^4*x), -1/2*(((3*b*c^2 - 2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (b*d^2 + (3*b*c*d - 2*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^3*x^3 + d^4*x)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [sign(t_n ostep),sign(t_n ostep+sqrt(d)/c*sign(t_n ostep))]sym2poly/r2sym(const gen & e,const index_m & i,constant vecteur & l) Error: Bad Argument Value

maple [A] time = 0.06, size = 132, normalized size = 1.43

$$\frac{(cx^2 + d) \left(-2\sqrt{cx^2 + d} a d^2 x^2 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) + 3\sqrt{cx^2 + d} bcd x^2 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) + 2a d^{\frac{5}{2}} x^2 - 3bc d^{\frac{3}{2}} x^2 - b d^{\frac{5}{2}} \right)}{2 \left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}} d^{\frac{7}{2}} x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x)

[Out] $\frac{1}{2} * (c * x^2 + d) * (2 * d^{(5/2)} * x^2 * a - 3 * d^{(3/2)} * x^2 * b * c - 2 * \ln(2 * (d + (c * x^2 + d)^{(1/2)} * d^{(1/2)})) / x) * (c * x^2 + d)^{(1/2)} * x^2 * a * d^2 + 3 * \ln(2 * (d + (c * x^2 + d)^{(1/2)} * d^{(1/2)})) / x) * (c * x^2 + d)^{(1/2)} * x^2 * b * c * d - d^{(5/2)} * b) / ((c * x^2 + d) / x^2)^{(3/2)} / x^5 / d^{(7/2)}$

maxima [B] time = 1.22, size = 162, normalized size = 1.76

$$-\frac{1}{4} b \left(\frac{2 \left(3 \left(c + \frac{d}{x^2}\right) c x^2 - 2 c d\right)}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} d^2 x^3 - \sqrt{c + \frac{d}{x^2}} d^3 x} + \frac{3 c \log\left(\frac{\sqrt{c + \frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x + \sqrt{d}}\right)}{d^{\frac{5}{2}}}\right) + \frac{1}{2} a \left(\frac{\log\left(\frac{\sqrt{c + \frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x + \sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{2}{\sqrt{c + \frac{d}{x^2}} dx} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")

[Out] $-1/4 * b * (2 * (3 * (c + d/x^2) * c * x^2 - 2 * c * d) / ((c + d/x^2)^{(3/2)} * d^2 * x^3 - \text{sqrt}(c + d/x^2) * d^3 * x) + 3 * c * \log((\text{sqrt}(c + d/x^2) * x - \text{sqrt}(d)) / (\text{sqrt}(c + d/x^2) * x + \text{sqrt}(d)))) / d^{(5/2)}) + 1/2 * a * (\log((\text{sqrt}(c + d/x^2) * x - \text{sqrt}(d)) / (\text{sqrt}(c + d/x^2) * x + \text{sqrt}(d)))) / d^{(3/2)} + 2 / (\text{sqrt}(c + d/x^2) * d * x))$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + \frac{b}{x^2}}{x^4 \left(c + \frac{d}{x^2}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x^2)/(x^4*(c + d/x^2)^(3/2)),x)

[Out] int((a + b/x^2)/(x^4*(c + d/x^2)^(3/2)), x)

sympy [B] time = 18.44, size = 262, normalized size = 2.85

$$a \left(\frac{cd^2x^2 \log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2 + 2d^{\frac{9}{2}}} - \frac{2cd^2x^2 \log\left(\sqrt{\frac{cx^2}{d} + 1} + 1\right)}{2cd^{\frac{7}{2}}x^2 + 2d^{\frac{9}{2}}} + \frac{2d^3\sqrt{\frac{cx^2}{d} + 1}}{2cd^{\frac{7}{2}}x^2 + 2d^{\frac{9}{2}}} + \frac{d^3 \log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2 + 2d^{\frac{9}{2}}} - \frac{2d^3 \log\left(\sqrt{\frac{cx^2}{d} + 1} + 1\right)}{2cd^{\frac{7}{2}}x^2 + 2d^{\frac{9}{2}}} \right) + b \left(-\frac{3\sqrt{c}}{2d^2x\sqrt{1 + \frac{d}{cx^2}}} + \frac{3c \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{cx}}\right)}{2d^{\frac{5}{2}}} - \frac{1}{2\sqrt{c}dx^3\sqrt{1 + \frac{d}{cx^2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**4,x)

[Out] a*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x**2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*sqrt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + d**3*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d**3*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2))) + b*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2))) + 3*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqrt(1 + d/(c*x**2))))

$$3.639 \quad \int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^6} dx$$

Optimal. Leaf size=123

$$-\frac{3c(5bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{8d^{7/2}} + \frac{3\sqrt{c + \frac{d}{x^2}}(5bc - 4ad)}{8d^3x} - \frac{5bc - 4ad}{4d^2x^3\sqrt{c + \frac{d}{x^2}}} - \frac{b}{4dx^5\sqrt{c + \frac{d}{x^2}}}$$

Rubi [A] time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {459, 335, 288, 321, 217, 206}

$$\frac{3\sqrt{c + \frac{d}{x^2}}(5bc - 4ad)}{8d^3x} - \frac{5bc - 4ad}{4d^2x^3\sqrt{c + \frac{d}{x^2}}} - \frac{3c(5bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{8d^{7/2}} - \frac{b}{4dx^5\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6), x]

[Out] -b/(4*d*Sqrt[c + d/x^2]*x^5) - (5*b*c - 4*a*d)/(4*d^2*Sqrt[c + d/x^2]*x^3) + (3*(5*b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d^3*x) - (3*c*(5*b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(7/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 288

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
+ 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2}} x^6 dx &= -\frac{b}{4d\sqrt{c + \frac{d}{x^2}} x^5} + \frac{(-5bc + 4ad) \int \frac{1}{\left(c + \frac{d}{x^2}\right)^{3/2}} x^6 dx}{4d} \\
&= -\frac{b}{4d\sqrt{c + \frac{d}{x^2}} x^5} - \frac{(-5bc + 4ad) \text{Subst}\left(\int \frac{x^4}{(c+dx^2)^{3/2}} dx, x, \frac{1}{x}\right)}{4d} \\
&= -\frac{b}{4d\sqrt{c + \frac{d}{x^2}} x^5} - \frac{5bc - 4ad}{4d^2\sqrt{c + \frac{d}{x^2}} x^3} + \frac{(3(5bc - 4ad)) \text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}} dx, x, \frac{1}{x}\right)}{4d^2} \\
&= -\frac{b}{4d\sqrt{c + \frac{d}{x^2}} x^5} - \frac{5bc - 4ad}{4d^2\sqrt{c + \frac{d}{x^2}} x^3} + \frac{3(5bc - 4ad)\sqrt{c + \frac{d}{x^2}}}{8d^3x} - \frac{(3c(5bc - 4ad)) \text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}} dx, x, \frac{1}{x}\right)}{8d^3} \\
&= -\frac{b}{4d\sqrt{c + \frac{d}{x^2}} x^5} - \frac{5bc - 4ad}{4d^2\sqrt{c + \frac{d}{x^2}} x^3} + \frac{3(5bc - 4ad)\sqrt{c + \frac{d}{x^2}}}{8d^3x} - \frac{(3c(5bc - 4ad)) \text{Subst}\left(\int \frac{1}{1-d} dx, x, \frac{1}{x}\right)}{8d^3} \\
&= -\frac{b}{4d\sqrt{c + \frac{d}{x^2}} x^5} - \frac{5bc - 4ad}{4d^2\sqrt{c + \frac{d}{x^2}} x^3} + \frac{3(5bc - 4ad)\sqrt{c + \frac{d}{x^2}}}{8d^3x} - \frac{3c(5bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c + \frac{d}{x^2}}}\right)}{8d^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 60, normalized size = 0.49

$$\frac{cx^4(5bc - 4ad) {}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; \frac{cx^2}{d} + 1\right) - bd^2}{4d^3x^5\sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6), x]

[Out] $(-(b*d^2) + c*(5*b*c - 4*a*d)*x^4*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (c*x^2)/d])/(4*d^3*sqrt[c + d/x^2]*x^5)$

IntegrateAlgebraic [A] time = 0.25, size = 126, normalized size = 1.02

$$\frac{x\sqrt{c + \frac{d}{x^2}} \left(\frac{-12acd x^4 - 4ad^2 x^2 + 15bc^2 x^4 + 5bcdx^2 - 2bd^2}{8d^3 x^4 \sqrt{cx^2 + d}} - \frac{3(5bc^2 - 4acd) \tanh^{-1}\left(\frac{\sqrt{cx^2 + d}}{\sqrt{d}}\right)}{8d^{7/2}} \right)}{\sqrt{cx^2 + d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6), x]

[Out] (Sqrt[c + d/x^2]*x*((-2*b*d^2 + 5*b*c*d*x^2 - 4*a*d^2*x^2 + 15*b*c^2*x^4 - 12*a*c*d*x^4)/(8*d^3*x^4*Sqrt[d + c*x^2]) - (3*(5*b*c^2 - 4*a*c*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(8*d^(7/2))))/Sqrt[d + c*x^2]

fricas [A] time = 0.45, size = 314, normalized size = 2.55

$$\frac{3((5bc^3 - 4ac^2d)x^5 + (5bc^2d - 4acd^2)x^4) \sqrt{d} \log\left(\frac{cx^2 + 2\sqrt{d}\sqrt{\frac{cx^2 + d}{d}}}{d}\right) - 2(3(5bc^2d - 4acd^2)x^4 - 2bd^3 + (5bcd - 4ad^2)x^2) \sqrt{\frac{cx^2 + d}{d}} + 3((5bc^3 - 4ac^2d)x^5 + (5bc^2d - 4acd^2)x^4 - 2bd^3 + (5bcd - 4ad^2)x^2) \sqrt{\frac{cx^2 + d}{d}} \arctan\left(\frac{\sqrt{d}\sqrt{\frac{cx^2 + d}{d}}}{cx^2 + d}\right) + (3(5bc^2d - 4acd^2)x^4 - 2bd^3 + (5bcd - 4ad^2)x^2) \sqrt{\frac{cx^2 + d}{d}}}{16(cd^4x^5 + d^5x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="fricas")

[Out] [-1/16*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b*c^2*d - 4*a*c*d^2)*x^3)*sqrt(d)*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(5*b*c^2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^4*x^5 + d^5*x^3), 1/8*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b*c^2*d - 4*a*c*d^2)*x^3)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*(5*b*c^2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^4*x^5 + d^5*x^3)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq

rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value

maple [A] time = 0.06, size = 157, normalized size = 1.28

$$\frac{(cx^2 + d) \left(12\sqrt{cx^2 + d} ac d^2 x^4 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) - 15\sqrt{cx^2 + d} b c^2 d x^4 \ln\left(\frac{2d+2\sqrt{cx^2+d}\sqrt{d}}{x}\right) - 12ac d^{\frac{5}{2}} x^4 + 15b c^2 d^{\frac{3}{2}} x^4 - 4a d^{\frac{7}{2}} x^2 + 5bc d^{\frac{5}{2}} x^2 - 2b d^{\frac{7}{2}} \right)}{8 \left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}} d^{\frac{9}{2}} x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x)

[Out] 1/8*(c*x^2+d)*(12*(c*x^2+d)^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^4*a
*c*d^2-15*(c*x^2+d)^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^4*b*c^2*d-1
2*d^(5/2)*x^4*a*c+15*d^(3/2)*x^4*b*c^2-4*d^(7/2)*x^2*a+5*d^(5/2)*x^2*b*c-2*
d^(7/2)*b)/((c*x^2+d)/x^2)^(3/2)/x^7/d^(9/2)

maxima [B] time = 1.28, size = 243, normalized size = 1.98

$$\frac{1}{16} b \left(\frac{2 \left(15 \left(c + \frac{d}{x^2} \right)^2 c^2 x^4 - 25 \left(c + \frac{d}{x^2} \right) c^2 d x^2 + 8 c^2 d^2 \right)}{\left(c + \frac{d}{x^2} \right)^{\frac{5}{2}} d^3 x^5 - 2 \left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} d^4 x^3 + \sqrt{c + \frac{d}{x^2}} d^5 x} + \frac{15 c^2 \log \left(\frac{\sqrt{c + \frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x + \sqrt{d}} \right)}{d^{\frac{7}{2}}} \right) - \frac{1}{4} a \left(\frac{2 \left(3 \left(c + \frac{d}{x^2} \right) c x^2 - 2 c d \right)}{\left(c + \frac{d}{x^2} \right)^{\frac{3}{2}} d^2 x^3 - \sqrt{c + \frac{d}{x^2}} d^3 x} + \frac{3 c \log \left(\frac{\sqrt{c + \frac{d}{x^2}} x - \sqrt{d}}{\sqrt{c + \frac{d}{x^2}} x + \sqrt{d}} \right)}{d^{\frac{5}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="maxima")

[Out] 1/16*b*(2*(15*(c + d/x^2)^2*c^2*x^4 - 25*(c + d/x^2)*c^2*d*x^2 + 8*c^2*d^2)
/((c + d/x^2)^(5/2)*d^3*x^5 - 2*(c + d/x^2)^(3/2)*d^4*x^3 + sqrt(c + d/x^2)
*d^5*x) + 15*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt
t(d)))/d^(7/2)) - 1/4*a*(2*(3*(c + d/x^2)*c*x^2 - 2*c*d)/((c + d/x^2)^(3/2)
*d^2*x^3 - sqrt(c + d/x^2)*d^3*x) + 3*c*log((sqrt(c + d/x^2)*x - sqrt(d))/(
sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + \frac{b}{x^2}}{x^6 \left(c + \frac{d}{x^2} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x^2)/(x^6*(c + d/x^2)^(3/2)),x)

[Out] $\text{int}((a + b/x^2)/(x^6*(c + d/x^2)^{(3/2)}), x)$

sympy [A] time = 29.81, size = 180, normalized size = 1.46

$$a \left(-\frac{3\sqrt{c}}{2d^2x\sqrt{1+\frac{d}{cx^2}}} + \frac{3c \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{5}{2}}} - \frac{1}{2\sqrt{c}dx^3\sqrt{1+\frac{d}{cx^2}}} \right) + b \left(\frac{15c^{\frac{3}{2}}}{8d^3x\sqrt{1+\frac{d}{cx^2}}} + \frac{5\sqrt{c}}{8d^2x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{15c^2 \operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{7}{2}}} - \frac{1}{4\sqrt{c}dx^5\sqrt{1+\frac{d}{cx^2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a+b/x^{**2})/(c+d/x^{**2})^{**}(3/2)/x^{**6},x)$

[Out] $a*(-3*\sqrt{c}/(2*d^{**2}*x*\sqrt{1+d/(c*x^{**2})}) + 3*c*\operatorname{asinh}(\sqrt{d}/(\sqrt{c}*x))/(2*d^{**}(5/2)) - 1/(2*\sqrt{c}*d*x^{**3}*\sqrt{1+d/(c*x^{**2})})) + b*(15*c^{**}(3/2)/(8*d^{**3}*x*\sqrt{1+d/(c*x^{**2})}) + 5*\sqrt{c}/(8*d^{**2}*x^{**3}*\sqrt{1+d/(c*x^{**2})}) - 15*c^{**2}*\operatorname{asinh}(\sqrt{d}/(\sqrt{c}*x))/(8*d^{**}(7/2)) - 1/(4*\sqrt{c}*d*x^{**5}*\sqrt{1+d/(c*x^{**2})}))$

$$3.640 \quad \int \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2} dx$$

Optimal. Leaf size=135

$$\frac{1}{4} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{7/2} - \frac{1}{24} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{5/2} - \frac{5}{96} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{3/2} - \frac{5}{64} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} \sqrt{x} - \frac{5}{64} \cosh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {280, 323, 330, 52}

$$\frac{1}{4} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{7/2} - \frac{1}{24} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{5/2} - \frac{5}{96} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{3/2} - \frac{5}{64} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} \sqrt{x} - \frac{5}{64} \cosh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2), x]

[Out] (-5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/64 - (5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2))/96 - (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2))/24 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2))/4 - (5*ArcCosh[Sqrt[x]])/64

Rule 52

Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] := Simp[ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

Rule 280

Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)^(m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1)), x], x] /; FreeQ[{a1, b1, a2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && NeQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rule 323

Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,

0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rule 330

Int[((c_.)*(x_)^(m_))*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a1 + (b1*x^(k*n)))/c^n]^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rubi steps

$$\begin{aligned}
 \int \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2} dx &= \frac{1}{4} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{7/2} - \frac{1}{8} \int \frac{x^{5/2}}{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}} dx \\
 &= -\frac{1}{24} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2} + \frac{1}{4} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{7/2} - \frac{5}{48} \int \frac{x}{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}} dx \\
 &= -\frac{5}{96} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} - \frac{1}{24} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2} + \frac{1}{4} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{7/2} \\
 &= -\frac{5}{64} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} - \frac{5}{96} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} - \frac{1}{24} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2} \\
 &= -\frac{5}{64} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} - \frac{5}{96} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} - \frac{1}{24} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2} \\
 &= -\frac{5}{64} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} - \frac{5}{96} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} - \frac{1}{24} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 111, normalized size = 0.82

$$\frac{\sqrt{\sqrt{x} + 1} \sqrt{x} (48x^{7/2} - 8x^{5/2} - 10x^{3/2} - 48x^3 + 8x^2 + 10x - 15\sqrt{x} + 15) + 30\sqrt{1 - \sqrt{x}} \sin^{-1}\left(\frac{\sqrt{1 - \sqrt{x}}}{\sqrt{2}}\right)}{192\sqrt{\sqrt{x} - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2), x]

[Out] (Sqrt[1 + Sqrt[x]]*Sqrt[x]*(15 - 15*Sqrt[x] + 10*x - 10*x^(3/2) + 8*x^2 - 8*x^(5/2) - 48*x^3 + 48*x^(7/2)) + 30*Sqrt[1 - Sqrt[x]]*ArcSin[Sqrt[1 - Sqrt[x]]/Sqrt[2]])/(192*Sqrt[-1 + Sqrt[x]])

IntegrateAlgebraic [A] time = 2.44, size = 216, normalized size = 1.60

$$\frac{\left(\frac{15(\sqrt{x}-1)^7}{(\sqrt{x}+1)^7} + \frac{397(\sqrt{x}-1)^6}{(\sqrt{x}+1)^6} + \frac{895(\sqrt{x}-1)^5}{(\sqrt{x}+1)^5} + \frac{1765(\sqrt{x}-1)^4}{(\sqrt{x}+1)^4} + \frac{1765(\sqrt{x}-1)^3}{(\sqrt{x}+1)^3} + \frac{895(\sqrt{x}-1)^2}{(\sqrt{x}+1)^2} + \frac{397(\sqrt{x}-1)}{\sqrt{x}+1} + 15\right)\sqrt{\sqrt{x}-1}}{96\left(\frac{\sqrt{x}-1}{\sqrt{x}+1} - 1\right)^8\sqrt{\sqrt{x}+1}} - \frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2), x]

[Out] ((15 + (15*(-1 + Sqrt[x])^7)/(1 + Sqrt[x])^7 + (397*(-1 + Sqrt[x])^6)/(1 + Sqrt[x])^6 + (895*(-1 + Sqrt[x])^5)/(1 + Sqrt[x])^5 + (1765*(-1 + Sqrt[x])^4)/(1 + Sqrt[x])^4 + (1765*(-1 + Sqrt[x])^3)/(1 + Sqrt[x])^3 + (895*(-1 + Sqrt[x])^2)/(1 + Sqrt[x])^2 + (397*(-1 + Sqrt[x]))/(1 + Sqrt[x]))*Sqrt[-1 + Sqrt[x]])/(96*(-1 + (-1 + Sqrt[x]))/(1 + Sqrt[x]))^8*Sqrt[1 + Sqrt[x]]) - (5 * ArcTanh[Sqrt[-1 + Sqrt[x]]/Sqrt[1 + Sqrt[x]])]/32

fricas [A] time = 0.39, size = 62, normalized size = 0.46

$$\frac{1}{192}(48x^3 - 8x^2 - 10x - 15)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{5}{128}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - 2x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2), x, algorithm="fricas")

[Out] 1/192*(48*x^3 - 8*x^2 - 10*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 5/128*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)

giac [A] time = 0.29, size = 162, normalized size = 1.20

$$\frac{1}{6720}\left((2\left(4\left(5\left(6\left(7\sqrt{x}-50\right)\left(\sqrt{x}+1\right)+1219\right)\left(\sqrt{x}+1\right)-12463\right)\left(\sqrt{x}+1\right)+64233\right)\left(\sqrt{x}+1\right)-53963\right)\left(\sqrt{x}+1\right)+59465\right)\left(\sqrt{x}+1\right)-23205\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{840}\left(2\left(4\left(5\left(6\left(7\sqrt{x}-37\right)\left(\sqrt{x}+1\right)+661\right)\left(\sqrt{x}+1\right)-4551\right)\left(\sqrt{x}+1\right)+4781\right)\left(\sqrt{x}+1\right)-6335\right)\left(\sqrt{x}+1\right)+2835\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{5}{32}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2), x, algorithm="giac")

[Out] 1/6720*((2*((4*(5*(6*(7*sqrt(x) - 50)*(sqrt(x) + 1) + 1219)*(sqrt(x) + 1) - 12463)*(sqrt(x) + 1) + 64233)*(sqrt(x) + 1) - 53963)*(sqrt(x) + 1) + 59465)*(sqrt(x) + 1) - 23205)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/840*((2*((4*(5*(6*(7*sqrt(x) - 37)*(sqrt(x) + 1) + 661)*(sqrt(x) + 1) - 4551)*(sqrt(x) + 1) + 4781)*(sqrt(x) + 1) - 6335)*(sqrt(x) + 1) + 2835)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 5/32*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))))

1) + 4781)*(sqrt(x) + 1) - 6335)*(sqrt(x) + 1) + 2835)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 5/32*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))

maple [A] time = 0.05, size = 75, normalized size = 0.56

$$\frac{\sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} \left(-48\sqrt{x-1} x^{\frac{7}{2}} + 8\sqrt{x-1} x^{\frac{5}{2}} + 10\sqrt{x-1} x^{\frac{3}{2}} + 15 \ln(\sqrt{x} + \sqrt{x-1}) + 15\sqrt{x-1} \sqrt{x} \right)}{192\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x)

[Out] -1/192*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-48*x^(7/2)*(x-1)^(1/2)+8*x^(5/2)*(x-1)^(1/2)+10*x^(3/2)*(x-1)^(1/2)+15*(x-1)^(1/2)*x^(1/2)+15*ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)

maxima [A] time = 0.55, size = 57, normalized size = 0.42

$$\frac{1}{4} (x-1)^{\frac{3}{2}} x^{\frac{5}{2}} + \frac{5}{24} (x-1)^{\frac{3}{2}} x^{\frac{3}{2}} + \frac{5}{32} (x-1)^{\frac{3}{2}} \sqrt{x} + \frac{5}{64} \sqrt{x-1} \sqrt{x} - \frac{5}{64} \log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima")

[Out] 1/4*(x - 1)^(3/2)*x^(5/2) + 5/24*(x - 1)^(3/2)*x^(3/2) + 5/32*(x - 1)^(3/2)*sqrt(x) + 5/64*sqrt(x - 1)*sqrt(x) - 5/64*log(2*sqrt(x - 1) + 2*sqrt(x))

mupad [B] time = 52.03, size = 831, normalized size = 6.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)

[Out] ((1723*((x^(1/2) - 1)^(1/2) - 1i)^5)/(48*((x^(1/2) + 1)^(1/2) - 1)^5) - (235*((x^(1/2) - 1)^(1/2) - 1i)^3)/(48*((x^(1/2) + 1)^(1/2) - 1)^3) + (72283*((x^(1/2) - 1)^(1/2) - 1i)^7)/(16*((x^(1/2) + 1)^(1/2) - 1)^7) + (848801*((x^(1/2) - 1)^(1/2) - 1i)^9)/(16*((x^(1/2) + 1)^(1/2) - 1)^9) + (4181067*((x^(1/2) - 1)^(1/2) - 1i)^11)/(16*((x^(1/2) + 1)^(1/2) - 1)^11) + (10994181*((x^(1/2) - 1)^(1/2) - 1i)^13)/(16*((x^(1/2) + 1)^(1/2) - 1)^13) + (17457599*((x^(1/2) - 1)^(1/2) - 1i)^15)/(16*((x^(1/2) + 1)^(1/2) - 1)^15) + (17457599*((x^(1/2) - 1)^(1/2) - 1i)^17)/(16*((x^(1/2) + 1)^(1/2) - 1)^17) + (10994181*((x^(1/2) - 1)^(1/2) - 1i)^19)/(16*((x^(1/2) + 1)^(1/2) - 1)^19) + (4181067*((x^(1/2) - 1)^(1/2) - 1i)^21)/(16*((x^(1/2) + 1)^(1/2) - 1)^21) + (848801*((x^(1/2) - 1)^(1/2) - 1i)^23)/(16*((x^(1/2) + 1)^(1/2) - 1)^23) + (72

$$\begin{aligned}
& 283*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{25}/(16*((x^{(1/2)} + 1)^{(1/2)} - 1)^{25}) + (172 \\
& 3*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{27}/(48*((x^{(1/2)} + 1)^{(1/2)} - 1)^{27}) - (235*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{29}/(48*((x^{(1/2)} + 1)^{(1/2)} - 1)^{29}) + (5*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{31}/(16*((x^{(1/2)} + 1)^{(1/2)} - 1)^{31}) + (5*((x^{(1/2)} - 1)^{(1/2)} - 1i))/((16*((x^{(1/2)} + 1)^{(1/2)} - 1))) \\
& /((120*((x^{(1/2)} - 1)^{(1/2)} - 1i)^4)/((x^{(1/2)} + 1)^{(1/2)} - 1)^4 - (16*((x^{(1/2)} - 1)^{(1/2)} - 1i)^2)/((x^{(1/2)} + 1)^{(1/2)} - 1)^2 - (560*((x^{(1/2)} - 1)^{(1/2)} - 1i)^6)/((x^{(1/2)} + 1)^{(1/2)} - 1)^6 + (1820*((x^{(1/2)} - 1)^{(1/2)} - 1i)^8)/((x^{(1/2)} + 1)^{(1/2)} - 1)^8 - (4368*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{10})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{10} + (8008*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{12})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{12} - (11440*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{14})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{14} + (12870*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{16})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{16} - (11440*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{18})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{18} + (8008*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{20})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{20} - (4368*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{22})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{22} + (1820*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{24})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{24} - (560*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{26})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{26} + (120*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{28})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{28} - (16*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{30})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{30} + ((x^{(1/2)} - 1)^{(1/2)} - 1i)^{32}/((x^{(1/2)} + 1)^{(1/2)} - 1)^{32} + 1) - (5 * atanh(((x^{(1/2)} - 1)^{(1/2)} - 1i)/((x^{(1/2)} + 1)^{(1/2)} - 1)))/16
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}} \sqrt{\sqrt{x} - 1} \sqrt{\sqrt{x} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(5/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2), x)

[Out] Integral(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)

$$3.641 \quad \int \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} dx$$

Optimal. Leaf size=104

$$\frac{1}{3} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{5/2} - \frac{1}{12} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{3/2} - \frac{1}{8} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} \sqrt{x} - \frac{1}{8} \cosh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {280, 323, 330, 52}

$$\frac{1}{3} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{5/2} - \frac{1}{12} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{3/2} - \frac{1}{8} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} \sqrt{x} - \frac{1}{8} \cosh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2), x]

[Out] -(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/8 - (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2))/12 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2))/3 - ArcCosh[Sqrt[x]]/8

Rule 52

Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

Rule 280

Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && NeQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rule 323

Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,

0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rule 330

Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a1 + (b1*x^(k*n)))/c^n]^p*(a2 + (b2*x^(k*n)))/c^n]^p, x], (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rubi steps

$$\begin{aligned}
 \int \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} dx &= \frac{1}{3} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2} - \frac{1}{6} \int \frac{x^{3/2}}{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}} dx \\
 &= -\frac{1}{12} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} + \frac{1}{3} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2} - \frac{1}{8} \int \frac{1}{\sqrt{-1 + \sqrt{x}}} dx \\
 &= -\frac{1}{8} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} - \frac{1}{12} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} + \frac{1}{3} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2} \\
 &= -\frac{1}{8} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} - \frac{1}{12} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} + \frac{1}{3} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2} \\
 &= -\frac{1}{8} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} - \frac{1}{12} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} + \frac{1}{3} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2}
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 99, normalized size = 0.95

$$\frac{\sqrt{\sqrt{x} + 1} \sqrt{x} (8x^{5/2} - 2x^{3/2} - 8x^2 + 2x - 3\sqrt{x} + 3) + 6\sqrt{1 - \sqrt{x}} \sin^{-1}\left(\frac{\sqrt{1 - \sqrt{x}}}{\sqrt{2}}\right)}{24\sqrt{\sqrt{x} - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2), x]

[Out] (Sqrt[1 + Sqrt[x]]*Sqrt[x]*(3 - 3*Sqrt[x] + 2*x - 2*x^(3/2) - 8*x^2 + 8*x^(5/2)) + 6*Sqrt[1 - Sqrt[x]]*ArcSin[Sqrt[1 - Sqrt[x]]/Sqrt[2]])/(24*Sqrt[-1 + Sqrt[x]])

IntegrateAlgebraic [A] time = 1.30, size = 176, normalized size = 1.69

$$\frac{\left(\frac{3(\sqrt{x}-1)^5}{(\sqrt{x}+1)^5} + \frac{47(\sqrt{x}-1)^4}{(\sqrt{x}+1)^4} + \frac{78(\sqrt{x}-1)^3}{(\sqrt{x}+1)^3} + \frac{78(\sqrt{x}-1)^2}{(\sqrt{x}+1)^2} + \frac{47(\sqrt{x}-1)}{\sqrt{x}+1} + 3\right)\sqrt{\sqrt{x}-1}}{12\left(\frac{\sqrt{x}-1}{\sqrt{x}+1} - 1\right)^6\sqrt{\sqrt{x}+1}} - \frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2), x]

[Out] ((3 + (3*(-1 + Sqrt[x]))^5)/(1 + Sqrt[x])^5 + (47*(-1 + Sqrt[x])^4)/(1 + Sqrt[x])^4 + (78*(-1 + Sqrt[x])^3)/(1 + Sqrt[x])^3 + (78*(-1 + Sqrt[x])^2)/(1 + Sqrt[x])^2 + (47*(-1 + Sqrt[x]))/(1 + Sqrt[x]))*Sqrt[-1 + Sqrt[x]]/(12*(-1 + (-1 + Sqrt[x]))/(1 + Sqrt[x]))^6*Sqrt[1 + Sqrt[x]]) - ArcTanh[Sqrt[-1 + Sqrt[x]]/Sqrt[1 + Sqrt[x]]]/4

fricas [A] time = 0.41, size = 57, normalized size = 0.55

$$\frac{1}{24}(8x^2 - 2x - 3)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{16}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - 2x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2), x, algorithm="fricas")

[Out] 1/24*(8*x^2 - 2*x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/16*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)

giac [A] time = 0.22, size = 127, normalized size = 1.22

$$\frac{1}{120}\left((2((4(5\sqrt{x}-26)(\sqrt{x}+1)+321)(\sqrt{x}+1)-451)(\sqrt{x}+1)+745)(\sqrt{x}+1)-405\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{60}\left((2(3(4\sqrt{x}-17)(\sqrt{x}+1)+133)(\sqrt{x}+1)-295)(\sqrt{x}+1)+195\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{4}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2), x, algorithm="giac")

[Out] 1/120*((2*((4*(5*sqrt(x) - 26)*(sqrt(x) + 1) + 321)*(sqrt(x) + 1) - 451)*(sqrt(x) + 1) + 745)*(sqrt(x) + 1) - 405)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/60*((2*(3*(4*sqrt(x) - 17)*(sqrt(x) + 1) + 133)*(sqrt(x) + 1) - 295)*(sqrt(x) + 1) + 195)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/4*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1)))

maple [A] time = 0.05, size = 65, normalized size = 0.62

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\left(-8\sqrt{x-1}x^{\frac{5}{2}}+2\sqrt{x-1}x^{\frac{3}{2}}+3\ln(\sqrt{x}+\sqrt{x-1})+3\sqrt{x-1}\sqrt{x}\right)}{24\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{3/2}*(x^{1/2}-1)^{1/2}*(x^{1/2}+1)^{1/2}, x)$

[Out] $-1/24*(x^{1/2}-1)^{1/2}*(x^{1/2}+1)^{1/2}*(-8*(x-1)^{1/2}*x^{5/2}+2*(x-1)^{1/2}*x^{3/2}+3*(x-1)^{1/2}*x^{1/2}+3*\ln(x^{1/2}+(x-1)^{1/2}))/((x-1)^{1/2})$

maxima [A] time = 0.49, size = 47, normalized size = 0.45

$$\frac{1}{3}(x-1)^{\frac{3}{2}}x^{\frac{3}{2}} + \frac{1}{4}(x-1)^{\frac{3}{2}}\sqrt{x} + \frac{1}{8}\sqrt{x-1}\sqrt{x} - \frac{1}{8}\log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{3/2}*(-1+x^{1/2})^{1/2}*(1+x^{1/2})^{1/2}, x, \text{algorithm}=\text{"maxima"})$

[Out] $1/3*(x-1)^{3/2}*x^{3/2} + 1/4*(x-1)^{3/2}*\text{sqrt}(x) + 1/8*\text{sqrt}(x-1)*\text{sqrt}(x) - 1/8*\log(2*\text{sqrt}(x-1) + 2*\text{sqrt}(x))$

mupad [B] time = 31.39, size = 632, normalized size = 6.08

$$\frac{\text{atanh}\left(\frac{\sqrt{x-1}}{\sqrt{x+1}}\right)}{2} - \frac{35(\sqrt{x-1})^3}{6(\sqrt{x+1})^3} + \frac{757(\sqrt{x-1})^5}{2(\sqrt{x+1})^5} + \frac{7339(\sqrt{x-1})^7}{2(\sqrt{x+1})^7} + \frac{41929(\sqrt{x-1})^9}{3(\sqrt{x+1})^9} + \frac{25661(\sqrt{x-1})^{11}}{(\sqrt{x+1})^{11}} + \frac{25661(\sqrt{x-1})^{13}}{(\sqrt{x+1})^{13}} + \frac{41929(\sqrt{x-1})^{15}}{3(\sqrt{x+1})^{15}} + \frac{7339(\sqrt{x-1})^{17}}{2(\sqrt{x+1})^{17}} + \frac{757(\sqrt{x-1})^{19}}{2(\sqrt{x+1})^{19}} + \frac{35(\sqrt{x-1})^{21}}{6(\sqrt{x+1})^{21}} - \frac{(\sqrt{x-1})^{23}}{2(\sqrt{x+1})^{23}} - \frac{\sqrt{x-1}}{2(\sqrt{x+1})^2} - \frac{66(\sqrt{x-1})^4}{(\sqrt{x+1})^4} - \frac{220(\sqrt{x-1})^6}{(\sqrt{x+1})^6} + \frac{495(\sqrt{x-1})^8}{(\sqrt{x+1})^8} - \frac{792(\sqrt{x-1})^{10}}{(\sqrt{x+1})^{10}} + \frac{924(\sqrt{x-1})^{12}}{(\sqrt{x+1})^{12}} - \frac{792(\sqrt{x-1})^{14}}{(\sqrt{x+1})^{14}} + \frac{495(\sqrt{x-1})^{16}}{(\sqrt{x+1})^{16}} - \frac{220(\sqrt{x-1})^{18}}{(\sqrt{x+1})^{18}} + \frac{66(\sqrt{x-1})^{20}}{(\sqrt{x+1})^{20}} - \frac{12(\sqrt{x-1})^{22}}{(\sqrt{x+1})^{22}} + \frac{(\sqrt{x-1})^{24}}{(\sqrt{x+1})^{24}} - \frac{12(\sqrt{x-1})^2}{(\sqrt{x+1})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{3/2}*(x^{1/2}-1)^{1/2}*(x^{1/2}+1)^{1/2}, x)$

[Out] $-\text{atanh}\left(\frac{(x^{1/2}-1)^{1/2}-1i}{(x^{1/2}+1)^{1/2}-1}\right)/2 - \left(\frac{35*((x^{1/2}-1)^{1/2}-1i)^3}{6*((x^{1/2}+1)^{1/2}-1)^3} + \frac{757*((x^{1/2}-1)^{1/2}-1i)^5}{2*((x^{1/2}+1)^{1/2}-1)^5} + \frac{7339*((x^{1/2}-1)^{1/2}-1i)^7}{2*((x^{1/2}+1)^{1/2}-1)^7} + \frac{41929*((x^{1/2}-1)^{1/2}-1i)^9}{3*((x^{1/2}+1)^{1/2}-1)^9} + \frac{25661*((x^{1/2}-1)^{1/2}-1i)^{11}}{((x^{1/2}+1)^{1/2}-1)^{11}} + \frac{25661*((x^{1/2}-1)^{1/2}-1i)^{13}}{((x^{1/2}+1)^{1/2}-1)^{13}} + \frac{41929*((x^{1/2}-1)^{1/2}-1i)^{15}}{3*((x^{1/2}+1)^{1/2}-1)^{15}} + \frac{7339*((x^{1/2}-1)^{1/2}-1i)^{17}}{2*((x^{1/2}+1)^{1/2}-1)^{17}} + \frac{757*((x^{1/2}-1)^{1/2}-1i)^{19}}{2*((x^{1/2}+1)^{1/2}-1)^{19}} + \frac{35*((x^{1/2}-1)^{1/2}-1i)^{21}}{6*((x^{1/2}+1)^{1/2}-1)^{21}} - \frac{((x^{1/2}-1)^{1/2}-1i)^{23}}{2*((x^{1/2}+1)^{1/2}-1)^{23}} - \frac{((x^{1/2}-1)^{1/2}-1i)}{2*((x^{1/2}+1)^{1/2}-1)}\right)/\left(\frac{66*((x^{1/2}-1)^{1/2}-1i)^4}{((x^{1/2}+1)^{1/2}-1)^4} - \frac{12*((x^{1/2}-1)^{1/2}-1i)^2}{((x^{1/2}+1)^{1/2}-1)^2} - \frac{220*((x^{1/2}-1)^{1/2}-1i)^6}{((x^{1/2}+1)^{1/2}-1)^6} + \frac{495*((x^{1/2}-1)^{1/2}-1i)^8}{((x^{1/2}+1)^{1/2}-1)^8} - \frac{792*((x^{1/2}-1)^{1/2}-1i)^{10}}{((x^{1/2}+1)^{1/2}-1)^{10}} + \frac{924*((x^{1/2}-1)^{1/2}-1i)^{12}}{((x^{1/2}+1)^{1/2}-1)^{12}} - \frac{792*((x^{1/2}-1)^{1/2}-1i)^{14}}{((x^{1/2}+1)^{1/2}-1)^{14}} + \frac{495*((x^{1/2}-1)^{1/2}-1i)^{16}}{((x^{1/2}+1)^{1/2}-1)^{16}} - \frac{220*((x^{1/2}-1)^{1/2}-1i)^{18}}{((x^{1/2}+1)^{1/2}-1)^{18}} + \frac{66*((x^{1/2}-1)^{1/2}-1i)^{20}}{((x^{1/2}+1)^{1/2}-1)^{20}} - \frac{12*((x^{1/2}-1)^{1/2}-1i)^{22}}{((x^{1/2}+1)^{1/2}-1)^{22}} + \frac{((x^{1/2}-1)^{1/2}-1i)^{24}}{((x^{1/2}+1)^{1/2}-1)^{24}} - \frac{12*((x^{1/2}-1)^{1/2}-1i)^2}{((x^{1/2}+1)^{1/2}-1)^2}\right)$

$$\begin{aligned}
 & - 1)^{12} - (792*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{14})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{14} \\
 & + (495*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{16})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{16} - (220*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{18})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{18} + (66*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{20})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{20} - (12*((x^{(1/2)} - 1)^{(1/2)} - 1i)^{22})/((x^{(1/2)} + 1)^{(1/2)} - 1)^{22} + ((x^{(1/2)} - 1)^{(1/2)} - 1i)^{24}/((x^{(1/2)} + 1)^{(1/2)} - 1)^{24} + 1)
 \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \sqrt{\sqrt{x} - 1} \sqrt{\sqrt{x} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2), x)

[Out] Integral(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)

$$3.642 \quad \int \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} dx$$

Optimal. Leaf size=73

$$\frac{1}{2} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{3/2} - \frac{1}{4} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} \sqrt{x} - \frac{1}{4} \cosh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {280, 323, 330, 52}

$$\frac{1}{2} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{3/2} - \frac{1}{4} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} \sqrt{x} - \frac{1}{4} \cosh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x], x]

[Out] -(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/4 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2))/2 - ArcCosh[Sqrt[x]]/4

Rule 52

Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] := Simp[ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

Rule 280

Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)^(m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1)), x], x] /; FreeQ[{a1, b1, a2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && NeQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rule 323

Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a1 + (b1*x^(k*n)))/c^n]^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rubi steps

$$\begin{aligned} \int \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} dx &= \frac{1}{2} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} - \frac{1}{4} \int \frac{\sqrt{x}}{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}} dx \\ &= -\frac{1}{4} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} + \frac{1}{2} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} - \frac{1}{8} \int \frac{1}{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}} dx \\ &= -\frac{1}{4} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} + \frac{1}{2} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{\sqrt{-1 + u}} du, \sqrt{x} \right) \\ &= -\frac{1}{4} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} + \frac{1}{2} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2} - \frac{1}{4} \cosh^{-1}(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.08, size = 87, normalized size = 1.19

$$\frac{\sqrt{\sqrt{x} + 1} \sqrt{x} (2x^{3/2} - 2x - \sqrt{x} + 1) + 2\sqrt{1 - \sqrt{x}} \sin^{-1} \left(\frac{\sqrt{1 - \sqrt{x}}}{\sqrt{2}} \right)}{4\sqrt{\sqrt{x} - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x], x]

[Out] (Sqrt[1 + Sqrt[x]]*Sqrt[x]*(1 - Sqrt[x] - 2*x + 2*x^(3/2)) + 2*Sqrt[1 - Sqrt[x]]*ArcSin[Sqrt[1 - Sqrt[x]]/Sqrt[2]])/(4*Sqrt[-1 + Sqrt[x]])

IntegrateAlgebraic [B] time = 1.48, size = 404, normalized size = 5.53

$$\frac{-4\sqrt{x} + 1(-1136x^{11} - 1096x^{10} + 17296x^9 - 4752x^8 + 7248x^7 + 55360x + 28224\sqrt{x} - 18816) - 4\sqrt{x-1}\sqrt{x+1}(-1944x^{11} - 6079x^{10} + 6992x^9 - 3120x^8 - 104x^7 + 38632x + 74488\sqrt{x} + 32592) + \sqrt{x}\left(-4\sqrt{x-1}\left(656x^{11} + 3832x^{10} - 30928x^9 + 3488x^8 - 1192x^7 - 41472x - 52416\sqrt{x} - 18816\right) - 4\left(1800x^{11} - 1148x^{10} - 23268x^9 + 112x^8 + 3416x^7 - 6676x^6 - 41440x - 10872\sqrt{x} + 10864\right)\right) + \text{tanh}^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{x+1}}\right)}{24960x^{11} + \sqrt{x}\sqrt{x+1}(-5248x^{10} - 22816x - 11264\sqrt{x} + 7168) + \sqrt{x-1}\left(10880x^{10} + \sqrt{x}\sqrt{x+1}(-896x^{10} - 14400x - 28672\sqrt{x} - 1248) + 47040x + 69168\sqrt{x} + 21504\right) + 1352x^8 + 49408x + 13312\sqrt{x} - 1248}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]

[Out] $(-4\sqrt{1 + \sqrt{x}})(-18816 + 28224\sqrt{x} + 55360x + 17296x^{3/2} + 7240x^2 - 1096x^{5/2} - 4752x^3 - 1136x^{7/2}) - 4\sqrt{-1 + \sqrt{x}}\sqrt{1 + \sqrt{x}}(32592 + 74488\sqrt{x} + 38632x + 6992x^{3/2} - 104x^2 - 6079x^{5/2} - 3120x^3 - 194x^{7/2}) + \sqrt{3}(-4\sqrt{-1 + \sqrt{x}})(-18816 - 52416\sqrt{x} - 41472x - 10928x^{3/2} - 1192x^2 + 3832x^{5/2} + 3408x^3 + 656x^{7/2}) - 4(10864 - 10872\sqrt{x} - 41440x - 23268x^{3/2} - 6678x^2 - 1148x^{5/2} + 3416x^3 + 1800x^{7/2} + 112x^4)))/(-12416 + 13312\sqrt{x} + 49408x + 24960x^{3/2} + 1552x^2 + \sqrt{3}\sqrt{1 + \sqrt{x}}\sqrt{1 + \sqrt{x}}(7168 - 11264\sqrt{x} - 22016x - 5248x^{3/2}) + \sqrt{-1 + \sqrt{x}}\sqrt{1 + \sqrt{x}}(21504 + 60416\sqrt{x} + 47104x + 9088x^{3/2}) + \sqrt{3}\sqrt{1 + \sqrt{x}}\sqrt{1 + \sqrt{x}}(-12416 - 28672\sqrt{x} - 14400x - 896x^{3/2}))) + \text{ArcTanh}((-1 + \sqrt{-1 + \sqrt{x}})/(\sqrt{3} - \sqrt{1 + \sqrt{x}}))$

fricas [A] time = 0.40, size = 52, normalized size = 0.71

$$\frac{1}{4}(2x-1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{8}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - 2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas")

[Out] $1/4*(2*x - 1)*\text{sqrt}(x)*\text{sqrt}(\text{sqrt}(x) + 1)*\text{sqrt}(\text{sqrt}(x) - 1) + 1/8*\log(2*\text{sqrt}(x)*\text{sqrt}(\text{sqrt}(x) + 1)*\text{sqrt}(\text{sqrt}(x) - 1) - 2*x + 1)$

giac [B] time = 0.19, size = 92, normalized size = 1.26

$$\frac{1}{12}((2(3\sqrt{x}-10)(\sqrt{x}+1)+43)(\sqrt{x}+1)-39)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{3}((2\sqrt{x}-5)(\sqrt{x}+1)+9)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{2}\log(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")

[Out] $1/12*((2*(3*\text{sqrt}(x) - 10)*(\text{sqrt}(x) + 1) + 43)*(\text{sqrt}(x) + 1) - 39)*\text{sqrt}(\text{sqrt}(x) + 1)*\text{sqrt}(\text{sqrt}(x) - 1) + 1/3*((2*\text{sqrt}(x) - 5)*(\text{sqrt}(x) + 1) + 9)*\text{sqrt}(\text{sqrt}(x) + 1)*\text{sqrt}(\text{sqrt}(x) - 1) + 1/2*\log(\text{sqrt}(\text{sqrt}(x) + 1) - \text{sqrt}(\text{sqrt}(x) - 1))$

maple [A] time = 0.05, size = 52, normalized size = 0.71

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\left(-2\sqrt{x-1}x^{\frac{3}{2}} + \ln(\sqrt{x} + \sqrt{x-1}) + \sqrt{x-1}\sqrt{x}\right)}{4\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x)`

[Out] $-1/4*(x^{1/2}-1)^{1/2}*(x^{1/2}+1)^{1/2}*(-2*(x-1)^{1/2}*x^{3/2}+(x-1)^{1/2})*x^{1/2}+\ln(x^{1/2}+(x-1)^{1/2}))/((x-1)^{1/2})$

maxima [A] time = 0.58, size = 37, normalized size = 0.51

$$\frac{1}{2}(x-1)^{\frac{3}{2}}\sqrt{x} + \frac{1}{4}\sqrt{x-1}\sqrt{x} - \frac{1}{4}\log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima")`

[Out] $1/2*(x-1)^{3/2}*\text{sqrt}(x) + 1/4*\text{sqrt}(x-1)*\text{sqrt}(x) - 1/4*\log(2*\text{sqrt}(x-1) + 2*\text{sqrt}(x))$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x)`

[Out] `int(x^(1/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)`

[Out] `Integral(sqrt(x)*sqrt(sqrt(x)-1)*sqrt(sqrt(x)+1),x)`

$$3.643 \quad \int \frac{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}}{\sqrt{x}} dx$$

Optimal. Leaf size=37

$$\sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} \sqrt{x} - \cosh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {280, 330, 52}

$$\sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} \sqrt{x} - \cosh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x], x]

[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - ArcCosh[Sqrt[x]]

Rule 52

Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] := Simp[ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

Rule 280

Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)^(m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && NeQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rule 330

Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}}{\sqrt{x}} dx &= \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} - \frac{1}{2} \int \frac{1}{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x}} dx \\
&= \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} - \text{Subst} \left(\int \frac{1}{\sqrt{-1 + x} \sqrt{1 + x}} dx, x, \sqrt{x} \right) \\
&= \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} - \cosh^{-1}(\sqrt{x})
\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 1.95

$$\frac{\sqrt{\sqrt{x} + 1} \sqrt{x} (\sqrt{x} - 1) + 2\sqrt{1 - \sqrt{x}} \sin^{-1} \left(\frac{\sqrt{1 - \sqrt{x}}}{\sqrt{2}} \right)}{\sqrt{\sqrt{x} - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x], x]

[Out] ((-1 + Sqrt[x])*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*Sqrt[1 - Sqrt[x]]*ArcSin[Sqrt[1 - Sqrt[x]]/Sqrt[2]])/Sqrt[-1 + Sqrt[x]]

IntegrateAlgebraic [B] time = 1.17, size = 268, normalized size = 7.24

$$\frac{-4\sqrt{\sqrt{x} + 1} (-20x^{3/2} - 4x + 96\sqrt{x} + 48) - 4\sqrt{\sqrt{x} - 1} \sqrt{\sqrt{x} + 1} (-7x^{3/2} - 28x + 10\sqrt{x} + 84) + \sqrt{3} (-4\sqrt{\sqrt{x} - 1} (12x^{3/2} + 20x - 32\sqrt{x} - 48) - 4(14x^{3/2} + 4x^2 - 18x - 70\sqrt{x} - 28))}{\sqrt{3}\sqrt{\sqrt{x} + 1} (-48\sqrt{x} - 32) + \sqrt{\sqrt{x} - 1} (\sqrt{3}\sqrt{\sqrt{x} + 1} (-16\sqrt{x} - 56) + 80\sqrt{x} + 96)} + 4 \tanh^{-1} \left(\frac{\sqrt{\sqrt{x} - 1} - 1}{\sqrt{3} - \sqrt{\sqrt{x} + 1}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x], x]

[Out] (-4*Sqrt[1 + Sqrt[x]]*(48 + 96*Sqrt[x] - 4*x - 20*x^(3/2)) - 4*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*(84 + 10*Sqrt[x] - 28*x - 7*x^(3/2)) + Sqrt[3]*(-4*Sqrt[-1 + Sqrt[x]]*(-48 - 32*Sqrt[x] + 20*x + 12*x^(3/2)) - 4*(-28 - 70*Sqrt[x] - 18*x + 14*x^(3/2) + 4*x^2)))/(56 + Sqrt[3]*(-32 - 48*Sqrt[x])*Sqrt[1 + Sqrt[x]] + Sqrt[-1 + Sqrt[x]]*(96 + Sqrt[3]*(-56 - 16*Sqrt[x])*Sqrt[1 + Sqrt[x]] + 80*Sqrt[x] + 112*Sqrt[x] + 28*x) + 4*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]

fricas [A] time = 0.41, size = 46, normalized size = 1.24

$$\sqrt{x} \sqrt{\sqrt{x} + 1} \sqrt{\sqrt{x} - 1} + \frac{1}{2} \log \left(2 \sqrt{x} \sqrt{\sqrt{x} + 1} \sqrt{\sqrt{x} - 1} - 2x + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="fricas")

[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/2*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)

giac [B] time = 0.18, size = 57, normalized size = 1.54

$$\sqrt{\sqrt{x} + 1} \sqrt{\sqrt{x} - 1} (\sqrt{x} - 2) + 2 \sqrt{\sqrt{x} + 1} \sqrt{\sqrt{x} - 1} + 2 \log \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="giac")

[Out] sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*(sqrt(x) - 2) + 2*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))

maple [B] time = 0.05, size = 72, normalized size = 1.95

$$-\frac{\sqrt{(\sqrt{x} - 1)(\sqrt{x} + 1)} \ln(\sqrt{x} + \sqrt{x - 1})}{\sqrt{\sqrt{x} + 1} \sqrt{\sqrt{x} - 1}} + \sqrt{\sqrt{x} - 1} (\sqrt{x} + 1)^{\frac{3}{2}} - \sqrt{\sqrt{x} - 1} \sqrt{\sqrt{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(1/2),x)

[Out] (x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(3/2)-(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)-((x^(1/2)-1)*(x^(1/2)+1))^(1/2)/(x^(1/2)+1)^(1/2)/(x^(1/2)-1)^(1/2)*ln(x^(1/2)+(x-1)^(1/2))

maxima [A] time = 0.49, size = 26, normalized size = 0.70

$$\sqrt{x-1} \sqrt{x} - \log \left(2 \sqrt{x-1} + 2 \sqrt{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="maxima")

[Out] sqrt(x - 1)*sqrt(x) - log(2*sqrt(x - 1) + 2*sqrt(x))

mupad [B] time = 5.07, size = 41, normalized size = 1.11

$$\sqrt{x} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} - \ln\left(\sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} + \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(1/2), x)`

[Out] `x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2) - log((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2) + x^(1/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1}}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(1/2), x)`

[Out] `Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/sqrt(x), x)`

$$3.644 \quad \int \frac{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}}{x^{3/2}} dx$$

Optimal. Leaf size=67

$$\frac{2(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{\sqrt{x}} - 2\sqrt{\sqrt{x}-1}\sqrt{x}\sqrt{\sqrt{x}+1} + 2\cosh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {327, 280, 330, 52}

$$\frac{2(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{\sqrt{x}} - 2\sqrt{\sqrt{x}-1}\sqrt{x}\sqrt{\sqrt{x}+1} + 2\cosh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2), x]

[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*ArcCosh[Sqrt[x]]

Rule 52

Int[1/(Sqrt[(a_) + (b_.)*(x_)])*Sqrt[(c_) + (d_.)*(x_)]], x_Symbol] := Simp[ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

Rule 280

Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)^(m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && NeQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rule 327

Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*c^(2*n)*(m + 1)), Int[(c*x)^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IG

tQ[2*n, 0] && LtQ[m, -1] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rule 330

Int[((c_.)*(x_)^(m_))*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a1 + (b1*x^(k*n)))/c^n]^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}}{x^{3/2}} dx &= \frac{2(-1 + \sqrt{x})^{3/2} (1 + \sqrt{x})^{3/2}}{\sqrt{x}} - 2 \int \frac{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}}{\sqrt{x}} dx \\
 &= \frac{2(-1 + \sqrt{x})^{3/2} (1 + \sqrt{x})^{3/2}}{\sqrt{x}} - 2\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} + \int \frac{1}{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}} dx \\
 &= \frac{2(-1 + \sqrt{x})^{3/2} (1 + \sqrt{x})^{3/2}}{\sqrt{x}} - 2\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} + 2 \operatorname{Subst} \left(\int \frac{1}{\sqrt{-1 + x}} dx, \sqrt{x} \right) \\
 &= \frac{2(-1 + \sqrt{x})^{3/2} (1 + \sqrt{x})^{3/2}}{\sqrt{x}} - 2\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} \sqrt{x} + 2 \cosh^{-1}(\sqrt{x})
 \end{aligned}$$

Mathematica [A] time = 0.08, size = 74, normalized size = 1.10

$$\frac{2 \left(-\frac{\sqrt{\sqrt{x}+1}(\sqrt{x}-1)}{\sqrt{x}} - 2\sqrt{1-\sqrt{x}} \sin^{-1} \left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}} \right) \right)}{\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2), x]

[Out] (2*(-(((-1 + Sqrt[x]) * Sqrt[1 + Sqrt[x]]) / Sqrt[x]) - 2*Sqrt[1 - Sqrt[x]] * ArcSin[Sqrt[1 - Sqrt[x]] / Sqrt[2]])) / Sqrt[-1 + Sqrt[x]]

IntegrateAlgebraic [B] time = 1.13, size = 184, normalized size = 2.75

$$\frac{\left(\sqrt{\sqrt{x}-1}-1\right)\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)\left(\sqrt{\sqrt{x}-1}+\sqrt{3}\sqrt{\sqrt{x}+1}-\sqrt{x}-2\right)}{\left(\sqrt{3}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\sqrt{\sqrt{x}-1}+2\sqrt{3}\sqrt{\sqrt{x}+1}-2\sqrt{x}-3\right)\sqrt{x}}-8 \tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}-1}{\sqrt{3}-\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]

[Out] ((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + Sqrt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x]))/((-3 - 2*Sqrt[-1 + Sqrt[x]] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]] - 2*Sqrt[x])*Sqrt[x]) - 8*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]

fricas [A] time = 0.40, size = 55, normalized size = 0.82

$$\frac{x \log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)+2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="fricas")

[Out] -(x*log(2*sqrt(x)*sqrt(sqrt(x)+1)*sqrt(sqrt(x)-1)-2*x+1)+2*sqrt(x)*sqrt(sqrt(x)+1)*sqrt(sqrt(x)-1)+2*x)/x

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.05, size = 47, normalized size = 0.70

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\left(\sqrt{x}\ln(\sqrt{x}+\sqrt{x-1})-\sqrt{x-1}\right)}{\sqrt{x-1}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(3/2),x)`

[Out] $2*(x^{(1/2)}-1)^{(1/2)}*(x^{(1/2)}+1)^{(1/2)}*(\ln(x^{(1/2)}+(x-1)^{(1/2)})*x^{(1/2)}-(x-1)^{(1/2)})/(x-1)^{(1/2)}/x^{(1/2)}$

maxima [A] time = 1.28, size = 27, normalized size = 0.40

$$-\frac{2\sqrt{x-1}}{\sqrt{x}} + 2 \log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="maxima")`

[Out] $-2*\text{sqrt}(x - 1)/\text{sqrt}(x) + 2*\log(2*\text{sqrt}(x - 1) + 2*\text{sqrt}(x))$

mupad [B] time = 6.25, size = 129, normalized size = 1.93

$$8 \operatorname{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-i}{\sqrt{\sqrt{x}+1}-1}\right) - \frac{\frac{5\left(\sqrt{\sqrt{x}-1}-i\right)^2}{2\left(\sqrt{\sqrt{x}+1}-1\right)^2} + \frac{1}{2}}{\frac{\left(\sqrt{\sqrt{x}-1}-i\right)^3}{\left(\sqrt{\sqrt{x}+1}-1\right)^3} + \frac{\sqrt{\sqrt{x}-1}-i}{\sqrt{\sqrt{x}+1}-1}} - \frac{\sqrt{\sqrt{x}-1}-i}{2\left(\sqrt{\sqrt{x}+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2))/x^(3/2),x)`

[Out] $8*\operatorname{atanh}\left(\frac{(x^{(1/2)}-1)^{(1/2)}-1i}{(x^{(1/2)}+1)^{(1/2)}-1}\right) - \left(\frac{5*((x^{(1/2)}-1)^{(1/2)}-1i)^2}{2*((x^{(1/2)}+1)^{(1/2)}-1)^2} + \frac{1}{2}\right) / \left(\frac{(x^{(1/2)}-1)^{(1/2)}-1i}{(x^{(1/2)}+1)^{(1/2)}-1}\right)^3 + \frac{(x^{(1/2)}-1)^{(1/2)}-1i}{\sqrt{(x^{(1/2)}+1)^{(1/2)}-1}} - \frac{(x^{(1/2)}-1)^{(1/2)}-1i}{2*((x^{(1/2)}+1)^{(1/2)}-1)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(3/2), x)
```

$$3.645 \quad \int \frac{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}}{x^{5/2}} dx$$

Optimal. Leaf size=31

$$\frac{2(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{3x^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {265}

$$\frac{2(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]

[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(3*x^(3/2))

Rule 265

```
Int[((c_.)*(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}}{x^{5/2}} dx = \frac{2(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{3x^{3/2}}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.00

$$\frac{2(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]

[Out] $(2*(-1 + \text{Sqrt}[x])^{3/2}*(1 + \text{Sqrt}[x])^{3/2})/(3*x^{3/2})$

IntegrateAlgebraic [B] time = 2.38, size = 512, normalized size = 16.52

$$\frac{\left(\frac{(\sqrt{x-1})^2}{(\sqrt{3}-\sqrt{x+1})^2}+1\right)\left(\frac{9(\sqrt{x-1})^8}{(\sqrt{3}-\sqrt{x+1})^3}+\frac{42\sqrt{3}(\sqrt{x-1})^7}{(\sqrt{3}-\sqrt{x+1})^7}+\frac{256(\sqrt{x-1})^6}{(\sqrt{3}-\sqrt{x+1})^6}+\frac{294\sqrt{3}(\sqrt{x-1})^5}{(\sqrt{3}-\sqrt{x+1})^5}+\frac{638(\sqrt{x-1})^4}{(\sqrt{3}-\sqrt{x+1})^4}+\frac{294\sqrt{3}(\sqrt{x-1})^3}{(\sqrt{3}-\sqrt{x+1})^3}+\frac{256(\sqrt{x-1})^2}{(\sqrt{3}-\sqrt{x+1})^2}+\frac{42\sqrt{3}(\sqrt{x-1})}{\sqrt{3}-\sqrt{x+1}}+9\right)\left(\frac{1}{384}-\frac{\sqrt{x-1}}{384}\right)(\sqrt{3}-\sqrt{x+1})^{12}}{(\sqrt{x+1}-\sqrt{3})\left(-2x-2\sqrt{x-1}\sqrt{x}+\sqrt{3}\sqrt{x-1}\sqrt{x+1}\sqrt{x}+2\sqrt{3}\sqrt{x+1}\sqrt{x}-3\sqrt{x}\right)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]

[Out] $((1 + (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^2/(\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^2)*(9 + (9*(-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^8)/(\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^8 + (42*\text{Sqrt}[3]*(-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^7)/(\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^7 + (256*(-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^6)/(\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^6 + (294*\text{Sqrt}[3]*(-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^5)/(\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^5 + (638*(-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^4)/(\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^4 + (294*\text{Sqrt}[3]*(-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^3)/(\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^3 + (256*(-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^2)/(\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^2 + (42*\text{Sqrt}[3]*(-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]]))/(\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])))/(384 - \text{Sqrt}[-1 + \text{Sqrt}[x]])*(\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^{12}/((- \text{Sqrt}[3] + \text{Sqrt}[1 + \text{Sqrt}[x]])*(-3*\text{Sqrt}[x] - 2*\text{Sqrt}[-1 + \text{Sqrt}[x]]*\text{Sqrt}[x] + 2*\text{Sqrt}[3]*\text{Sqrt}[1 + \text{Sqrt}[x]]*\text{Sqrt}[x] + \text{Sqrt}[3]*\text{Sqrt}[-1 + \text{Sqrt}[x]]*\text{Sqrt}[1 + \text{Sqrt}[x]]*\text{Sqrt}[x] - 2*x)^3)$

fricas [A] time = 0.42, size = 30, normalized size = 0.97

$$\frac{2\left((x-1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+x^2\right)}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="fricas")

[Out] $2/3*((x-1)*\text{sqrt}(x)*\text{sqrt}(\text{sqrt}(x)+1)*\text{sqrt}(\text{sqrt}(x)-1)+x^2)/x^2$

giac [B] time = 0.23, size = 48, normalized size = 1.55

$$\frac{16\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^8+16\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="giac")

[Out] 16/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 16)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)^3

maple [A] time = 0.05, size = 23, normalized size = 0.74

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}(x-1)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(5/2),x)

[Out] 2/3*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)/x^(3/2)

maxima [A] time = 1.32, size = 10, normalized size = 0.32

$$\frac{2(x-1)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="maxima")

[Out] 2/3*(x - 1)^(3/2)/x^(3/2)

mupad [B] time = 5.26, size = 31, normalized size = 1.00

$$\frac{\sqrt{\sqrt{x}-1}\left(\frac{2x\sqrt{\sqrt{x}+1}}{3}-\frac{2\sqrt{\sqrt{x}+1}}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(5/2),x)

[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/3 - (2*(x^(1/2) + 1)^(1/2))/3))/x^(3/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(5/2), x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(5/2), x)
```

$$3.646 \quad \int \frac{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}}{x^{7/2}} dx$$

Optimal. Leaf size=63

$$\frac{4(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{15x^{3/2}} + \frac{2(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{5x^{5/2}}$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {272, 265}

$$\frac{4(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{15x^{3/2}} + \frac{2(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2), x]

[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(5*x^(5/2)) + (4*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(15*x^(3/2))

Rule 265

```
Int[((c_.)*(x_)^(m_.)*((a1_.) + (b1_.)*(x_)^(n_.))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a1_.) + (b1_.)*(x_)^(n_.))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1)), Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}}{x^{7/2}} dx = \frac{2(-1 + \sqrt{x})^{3/2} (1 + \sqrt{x})^{3/2}}{5x^{5/2}} + \frac{2}{5} \int \frac{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}}{x^{5/2}} dx$$

$$= \frac{2(-1 + \sqrt{x})^{3/2} (1 + \sqrt{x})^{3/2}}{5x^{5/2}} + \frac{4(-1 + \sqrt{x})^{3/2} (1 + \sqrt{x})^{3/2}}{15x^{3/2}}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.57

$$\frac{2(\sqrt{x} - 1)^{3/2} (\sqrt{x} + 1)^{3/2} (2x + 3)}{15x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2), x]

[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(3 + 2*x))/(15*x^(5/2))

IntegrateAlgebraic [B] time = 10.09, size = 836, normalized size = 13.27

$$\frac{\left(\frac{(\sqrt{x-1})^{15}}{(\sqrt{x-1})^{15}} + \frac{20(\sqrt{x-1})^{14}}{(\sqrt{x-1})^{14}} + \frac{200(\sqrt{x-1})^{13}}{(\sqrt{x-1})^{13}} + \frac{1000(\sqrt{x-1})^{12}}{(\sqrt{x-1})^{12}} + \frac{3000(\sqrt{x-1})^{11}}{(\sqrt{x-1})^{11}} + \frac{6000(\sqrt{x-1})^{10}}{(\sqrt{x-1})^{10}} + \frac{8000(\sqrt{x-1})^9}{(\sqrt{x-1})^9} + \frac{7000(\sqrt{x-1})^8}{(\sqrt{x-1})^8} + \frac{4500(\sqrt{x-1})^7}{(\sqrt{x-1})^7} + \frac{2000(\sqrt{x-1})^6}{(\sqrt{x-1})^6} + \frac{500(\sqrt{x-1})^5}{(\sqrt{x-1})^5} + \frac{100(\sqrt{x-1})^4}{(\sqrt{x-1})^4} + \frac{10(\sqrt{x-1})^3}{(\sqrt{x-1})^3} + \frac{1(\sqrt{x-1})^2}{(\sqrt{x-1})^2} + \frac{1(\sqrt{x-1})}{(\sqrt{x-1})} + \frac{1}{(\sqrt{x-1})} \right) \left(\frac{1}{\sqrt{x+1}} \right) \left(\sqrt{x-1} \right)^{15}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2), x]

[Out] ((1 + (-1 + Sqrt[-1 + Sqrt[x]])^2/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2)*(45 + (45*(-1 + Sqrt[-1 + Sqrt[x]])^16)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^16 + (300*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^15)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^15 + (2960*(-1 + Sqrt[-1 + Sqrt[x]])^14)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^14 + (6620*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^13)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^13 + (34252*(-1 + Sqrt[-1 + Sqrt[x]])^12)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^12 + (46980*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^11)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^11 + (152688*(-1 + Sqrt[-1 + Sqrt[x]])^10)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^10 + (129140*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^9)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^9 + (254222*(-1 + Sqrt[-1 + Sqrt[x]])^8)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^8 + (129140*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^7)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^7 + (152688*(-1 + Sqrt[-1 + Sqrt[x]])^6)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^6 + (46980*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^5)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^5 + (34252*(-1 + Sqrt[-1 + Sqrt[x]])^4)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^4 + (6620*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^3)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^3 + (2960*(-1 + Sqrt[-1 + Sqrt[x]])^2)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2 + (300*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]]))/(Sqrt[3] - Sqrt[1 + Sqrt[x]]))*(1/122880 - Sqr

t[-1 + Sqrt[x]]/122880)*(Sqrt[3] - Sqrt[1 + Sqrt[x]])^20)/((-Sqrt[3] + Sqrt[1 + Sqrt[x]])*(-3*Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]*Sqrt[x] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - 2*x)^5)

fricas [A] time = 0.41, size = 37, normalized size = 0.59

$$\frac{2 \left(2x^3 + (2x^2 + x - 3)\sqrt{x} \sqrt{\sqrt{x} + 1} \sqrt{\sqrt{x} - 1} \right)}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="fricas")

[Out] 2/15*(2*x^3 + (2*x^2 + x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1))/x^3

giac [B] time = 0.28, size = 90, normalized size = 1.43

$$\frac{128 \left(15 \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^{12} - 20 \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^8 + 80 \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^4 + 64 \right)}{15 \left(\left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^4 + 4 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="giac")

[Out] 128/15*(15*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^12 - 20*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 80*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 64)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)^5

maple [A] time = 0.05, size = 28, normalized size = 0.44

$$\frac{2\sqrt{\sqrt{x} - 1} \sqrt{\sqrt{x} + 1} (x - 1)(2x + 3)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(7/2),x)

[Out] 2/15*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)*(2*x+3)/x^(5/2)

maxima [A] time = 1.31, size = 21, normalized size = 0.33

$$\frac{4(x-1)^{\frac{3}{2}}}{15x^{\frac{3}{2}}} + \frac{2(x-1)^{\frac{3}{2}}}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="maxima")

[Out] 4/15*(x - 1)^(3/2)/x^(3/2) + 2/5*(x - 1)^(3/2)/x^(5/2)

mupad [B] time = 5.07, size = 43, normalized size = 0.68

$$\frac{\sqrt{\sqrt{x}-1} \left(\frac{2x\sqrt{\sqrt{x}+1}}{15} - \frac{2\sqrt{\sqrt{x}+1}}{5} + \frac{4x^2\sqrt{\sqrt{x}+1}}{15} \right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(7/2),x)

[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/15 - (2*(x^(1/2) + 1)^(1/2))/5 + (4*x^2*(x^(1/2) + 1)^(1/2))/15))/x^(5/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(7/2),x)

[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(7/2), x)

$$3.647 \quad \int \frac{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}}{x^{9/2}} dx$$

Optimal. Leaf size=94

$$\frac{16(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{105x^{3/2}} + \frac{8(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{35x^{5/2}} + \frac{2(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{7x^{7/2}}$$

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {272, 265}

$$\frac{16(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{105x^{3/2}} + \frac{8(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{35x^{5/2}} + \frac{2(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2), x]

[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(7*x^(7/2)) + (8*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(35*x^(5/2)) + (16*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(105*x^(3/2))

Rule 265

```
Int[((c_.)*(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1)), Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}} dx &= \frac{2(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{7x^{7/2}} + \frac{4}{7} \int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}} dx \\
&= \frac{2(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{7x^{7/2}} + \frac{8(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{35x^{5/2}} + \frac{8}{35} \int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}} dx \\
&= \frac{2(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{7x^{7/2}} + \frac{8(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{35x^{5/2}} + \frac{16(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{105x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.44

$$\frac{2(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}(8x^2+12x+15)}{105x^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2), x]

[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(15 + 12*x + 8*x^2))/(105*x^(7/2))

IntegrateAlgebraic [B] time = 36.64, size = 1160, normalized size = 12.34

$$\frac{\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}} dx}{\left(\sqrt{3}-1\right)\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(-1+\sqrt{1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2), x]

[Out] ((1 + (-1 + Sqrt[-1 + Sqrt[x]])^2/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2)*(315 + (315*(-1 + Sqrt[-1 + Sqrt[x]])^24)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^24 + (2730*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^23)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^23 + (37520*(-1 + Sqrt[-1 + Sqrt[x]])^22)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^22 + (123550*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^21)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^21 + (965902*(-1 + Sqrt[-1 + Sqrt[x]])^20)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^20 + (2042838*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^19)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^19 + (10643536*(-1 + Sqrt[-1 + Sqrt[x]])^18)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^18 + (15378258*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^17)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^17 + (56109493*(-1 + Sqrt[-1 + Sqrt[x]])^16)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^16 + (57833188*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^15)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^15 + (151811360*(-1 + Sqrt[-1 + Sqrt[x]])^14)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^14 + (112621740*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^13)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^13)

```

rt[3] - Sqrt[1 + Sqrt[x]]^13 + (212123652*(-1 + Sqrt[-1 + Sqrt[x]])^12)/(S
qrt[3] - Sqrt[1 + Sqrt[x]])^12 + (112621740*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]
])^11)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^11 + (151811360*(-1 + Sqrt[-1 + Sqrt[x]
]))^10)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^10 + (57833188*Sqrt[3]*(-1 + Sqrt[-1
+ Sqrt[x]])^9)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^9 + (56109493*(-1 + Sqrt[-1 +
Sqrt[x]])^8)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^8 + (15378258*Sqrt[3]*(-1 + Sqrt
[-1 + Sqrt[x]])^7)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^7 + (10643536*(-1 + Sqrt[-
1 + Sqrt[x]])^6)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^6 + (2042838*Sqrt[3]*(-1 + S
qrt[-1 + Sqrt[x]])^5)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^5 + (965902*(-1 + Sqrt[
-1 + Sqrt[x]])^4)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^4 + (123550*Sqrt[3]*(-1 + S
qrt[-1 + Sqrt[x]])^3)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^3 + (37520*(-1 + Sqrt[-
1 + Sqrt[x]])^2)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2 + (2730*Sqrt[3]*(-1 + Sqrt
[-1 + Sqrt[x]]))/((Sqrt[3] - Sqrt[1 + Sqrt[x]]))*(1/55050240 - Sqrt[-1 + Sqr
t[x]]/55050240)*(Sqrt[3] - Sqrt[1 + Sqrt[x]])^28)/((-Sqrt[3] + Sqrt[1 + Sqr
t[x]])*(-3*Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]*Sqrt[x] + 2*Sqrt[3]*Sqrt[1 + Sqrt
[x]]*Sqrt[x] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - 2*x)^
7)

```

fricas [A] time = 0.40, size = 44, normalized size = 0.47

$$\frac{2 \left(8x^4 + (8x^3 + 4x^2 + 3x - 15)\sqrt{x} \sqrt{\sqrt{x} + 1} \sqrt{\sqrt{x} - 1} \right)}{105x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(8*x^4 + (8*x^3 + 4*x^2 + 3*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1))/x^4
```

giac [A] time = 0.29, size = 111, normalized size = 1.18

$$\frac{4096 \left(35 \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^{16} - 70 \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^{12} + 168 \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^8 + 224 \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^4 + 128 \right)}{105 \left(\left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^4 + 4 \right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 4096/105*(35*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^16 - 70*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^12 + 168*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 224*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 128)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)^7
```


maple [A] time = 0.05, size = 33, normalized size = 0.35

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}(x-1)(8x^2+12x+15)}{105x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(9/2), x)

[Out] 2/105*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)*(8*x^2+12*x+15)/x^(7/2)

maxima [A] time = 1.55, size = 31, normalized size = 0.33

$$\frac{16(x-1)^{\frac{3}{2}}}{105x^{\frac{3}{2}}} + \frac{8(x-1)^{\frac{3}{2}}}{35x^{\frac{5}{2}}} + \frac{2(x-1)^{\frac{3}{2}}}{7x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2), x, algorithm="maxima")

[Out] 16/105*(x - 1)^(3/2)/x^(3/2) + 8/35*(x - 1)^(3/2)/x^(5/2) + 2/7*(x - 1)^(3/2)/x^(7/2)

mupad [B] time = 5.04, size = 55, normalized size = 0.59

$$\frac{\sqrt{\sqrt{x}-1}\left(\frac{2x\sqrt{\sqrt{x}+1}}{35} - \frac{2\sqrt{\sqrt{x}+1}}{7} + \frac{8x^2\sqrt{\sqrt{x}+1}}{105} + \frac{16x^3\sqrt{\sqrt{x}+1}}{105}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(9/2), x)

[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/35 - (2*(x^(1/2) + 1)^(1/2))/7 + (8*x^2*(x^(1/2) + 1)^(1/2))/105 + (16*x^3*(x^(1/2) + 1)^(1/2))/105))/x^(7/2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(9/2), x)

[Out] Timed out

$$3.648 \quad \int \frac{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}}{x^{11/2}} dx$$

Optimal. Leaf size=125

$$\frac{32(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{315x^{3/2}} + \frac{16(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{105x^{5/2}} + \frac{4(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{21x^{7/2}} + \frac{2(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{9x^{9/2}}$$

Rubi [A] time = 0.04, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {272, 265}

$$\frac{32(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{315x^{3/2}} + \frac{16(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{105x^{5/2}} + \frac{4(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{21x^{7/2}} + \frac{2(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}}{9x^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2), x]

[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(9*x^(9/2)) + (4*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(21*x^(7/2)) + (16*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(105*x^(5/2)) + (32*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(315*x^(3/2))

Rule 265

```
Int[((c_.)*(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1)), Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}} dx &= \frac{2(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{9x^{9/2}} + \frac{2}{3} \int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}} dx \\
&= \frac{2(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{9x^{9/2}} + \frac{4(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{21x^{7/2}} + \frac{8}{21} \int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}} dx \\
&= \frac{2(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{9x^{9/2}} + \frac{4(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{21x^{7/2}} + \frac{16(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{105x^{5/2}} \\
&= \frac{2(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{9x^{9/2}} + \frac{4(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{21x^{7/2}} + \frac{16(-1+\sqrt{x})^{3/2}(1+\sqrt{x})^{3/2}}{105x^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.37

$$\frac{2(\sqrt{x}-1)^{3/2}(\sqrt{x}+1)^{3/2}(16x^3+24x^2+30x+35)}{315x^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2), x]

[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(35 + 30*x + 24*x^2 + 16*x^3))/(315*x^(9/2))

IntegrateAlgebraic [B] time = 111.25, size = 1484, normalized size = 11.87

result too large to display

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2), x]

[Out] ((1 + (-1 + Sqrt[-1 + Sqrt[x]])^2/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2)*(945 + (945*(-1 + Sqrt[-1 + Sqrt[x]])^32)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^32 + (10080*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^31)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^31 + (178080*(-1 + Sqrt[-1 + Sqrt[x]])^30)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^30 + (771120*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^29)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^29 + (7981176*(-1 + Sqrt[-1 + Sqrt[x]])^28)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^28 + (22539888*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^27)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^27 + (159375840*(-1 + Sqrt[-1 + Sqrt[x]])^26)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^26 + (319049280*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^25)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^25 + (1649481052*(-1 + Sqrt[-1 + Sqrt[x]])^24)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^24 + (2469292416*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^23)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^23 + (9691565216*(-1 + Sqrt[-1 + Sqrt[x]])^22)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^22)/((Sqrt[3] - Sqrt[1 + Sqrt[x]])^2)

$$\begin{aligned} & \text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]]^22 + (11128795920 * \text{Sqrt}[3] * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^21) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^21 + (33755226952 * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^20) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^20 + (30109229136 * \text{Sqrt}[3] * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^19) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^19 + (71150262752 * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^18) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^18 + (49500527328 * \text{Sqrt}[3] * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^17) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^17 + (91239315046 * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^16) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^16 + (49500527328 * \text{Sqrt}[3] * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^15) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^15 + (71150262752 * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^14) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^14 + (30109229136 * \text{Sqrt}[3] * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^13) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^13 + (33755226952 * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^12) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^12 + (11128795920 * \text{Sqrt}[3] * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^11) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^11 + (9691565216 * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^10) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^10 + (2469292416 * \text{Sqrt}[3] * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^9) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^9 + (1649481052 * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^8) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^8 + (319049280 * \text{Sqrt}[3] * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^7) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^7 + (159375840 * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^6) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^6 + (22539888 * \text{Sqrt}[3] * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^5) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^5 + (7981176 * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^4) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^4 + (771120 * \text{Sqrt}[3] * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^3) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^3 + (178080 * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])^2) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^2 + (10080 * \text{Sqrt}[3] * (-1 + \text{Sqrt}[-1 + \text{Sqrt}[x]])) / (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]]) * (1/10569646080 - \text{Sqrt}[-1 + \text{Sqrt}[x]]/10569646080) * (\text{Sqrt}[3] - \text{Sqrt}[1 + \text{Sqrt}[x]])^36 / ((-\text{Sqrt}[3] + \text{Sqrt}[1 + \text{Sqrt}[x]]) * (-3 * \text{Sqrt}[x] - 2 * \text{Sqrt}[-1 + \text{Sqrt}[x]] * \text{Sqrt}[x] + 2 * \text{Sqrt}[3] * \text{Sqrt}[1 + \text{Sqrt}[x]] * \text{Sqrt}[x] + \text{Sqrt}[3] * \text{Sqrt}[-1 + \text{Sqrt}[x]] * \text{Sqrt}[1 + \text{Sqrt}[x]] * \text{Sqrt}[x] - 2 * x)^9) \end{aligned}$$

fricas [A] time = 0.40, size = 49, normalized size = 0.39

$$\frac{2 \left(16x^5 + (16x^4 + 8x^3 + 6x^2 + 5x - 35)\sqrt{x}\sqrt{\sqrt{x} + 1}\sqrt{\sqrt{x} - 1} \right)}{315x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="fricas")

[Out] 2/315*(16*x^5 + (16*x^4 + 8*x^3 + 6*x^2 + 5*x - 35)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1))/x^5

giac [A] time = 0.43, size = 132, normalized size = 1.06

$$\frac{16384 \left(315 \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^{20} - 756 \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^{16} + 1344 \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^{12} + 2304 \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^8 + 2304 \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^4 + 1024 \right)}{315 \left(\left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^4 + 4 \right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="giac")

[Out] 16384/315*(315*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^20 - 756*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^16 + 1344*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^12 + 2304*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 2304*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 1024)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)^9

maple [A] time = 0.06, size = 38, normalized size = 0.30

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}(x-1)(16x^3+24x^2+30x+35)}{315x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(11/2),x)

[Out] 2/315*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)*(16*x^3+24*x^2+30*x+35)/x^(9/2)

maxima [A] time = 1.22, size = 41, normalized size = 0.33

$$\frac{32(x-1)^{\frac{3}{2}}}{315x^{\frac{3}{2}}} + \frac{16(x-1)^{\frac{3}{2}}}{105x^{\frac{5}{2}}} + \frac{4(x-1)^{\frac{3}{2}}}{21x^{\frac{7}{2}}} + \frac{2(x-1)^{\frac{3}{2}}}{9x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="maxima")

[Out] 32/315*(x - 1)^(3/2)/x^(3/2) + 16/105*(x - 1)^(3/2)/x^(5/2) + 4/21*(x - 1)^(3/2)/x^(7/2) + 2/9*(x - 1)^(3/2)/x^(9/2)

mupad [B] time = 5.02, size = 67, normalized size = 0.54

$$\frac{\sqrt{\sqrt{x}-1}\left(\frac{2x\sqrt{\sqrt{x}+1}}{63} - \frac{2\sqrt{\sqrt{x}+1}}{9} + \frac{4x^2\sqrt{\sqrt{x}+1}}{105} + \frac{16x^3\sqrt{\sqrt{x}+1}}{315} + \frac{32x^4\sqrt{\sqrt{x}+1}}{315}\right)}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(11/2),x)

[Out]
$$\left((x^{1/2} - 1)^{1/2} \cdot \left(\frac{2x(x^{1/2} + 1)^{1/2}}{63} - \frac{2(x^{1/2} + 1)^{1/2}}{9} + \frac{4x^2(x^{1/2} + 1)^{1/2}}{105} + \frac{16x^3(x^{1/2} + 1)^{1/2}}{315} + \frac{32x^4(x^{1/2} + 1)^{1/2}}{315} \right) \right) / x^{9/2}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(11/2),x)`

[Out] Timed out

$$3.649 \quad \int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}} dx$$

Optimal. Leaf size=104

$$\frac{1}{3} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{5/2} + \frac{5}{12} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{3/2} + \frac{5}{8} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} \sqrt{x} + \frac{5}{8} \cosh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {323, 330, 52}

$$\frac{1}{3} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{5/2} + \frac{5}{12} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} x^{3/2} + \frac{5}{8} \sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} \sqrt{x} + \frac{5}{8} \cosh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]

[Out] (5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/8 + (5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2))/12 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2))/3 + (5*ArcCosh[Sqrt[x]])/8

Rule 52

Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] := Simp[ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

Rule 323

Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rule 330

Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a1 + (b1*x^(k*n)))/c^n]^p*(a2 + (b2*x^(k*n)))/c^n^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m]

, p, x]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}} dx &= \frac{1}{3} \sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{5/2} + \frac{5}{6} \int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}} dx \\
 &= \frac{5}{12} \sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{3/2} + \frac{1}{3} \sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{5/2} + \frac{5}{8} \int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}} dx \\
 &= \frac{5}{8} \sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} \sqrt{x} + \frac{5}{12} \sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{3/2} + \frac{1}{3} \sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{5/2} \\
 &= \frac{5}{8} \sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} \sqrt{x} + \frac{5}{12} \sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{3/2} + \frac{1}{3} \sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{5/2} \\
 &= \frac{5}{8} \sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} \sqrt{x} + \frac{5}{12} \sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{3/2} + \frac{1}{3} \sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{5/2}
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.64

$$\frac{1}{24} \left(\sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} \sqrt{x} (8x^2 + 10x + 15) + 30 \tanh^{-1} \left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]

[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(15 + 10*x + 8*x^2) + 30*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/24

IntegrateAlgebraic [A] time = 1.42, size = 176, normalized size = 1.69

$$\frac{\sqrt{\sqrt{x}-1} \left(\frac{33(\sqrt{x}-1)^5}{(\sqrt{x}+1)^5} + \frac{5(\sqrt{x}-1)^4}{(\sqrt{x}+1)^4} + \frac{90(\sqrt{x}-1)^3}{(\sqrt{x}+1)^3} + \frac{90(\sqrt{x}-1)^2}{(\sqrt{x}+1)^2} + \frac{5(\sqrt{x}-1)}{\sqrt{x}+1} + 33 \right)}{12 \left(\frac{\sqrt{x}-1}{\sqrt{x}+1} - 1 \right)^6 \sqrt{\sqrt{x}+1}} + \frac{5}{4} \tanh^{-1} \left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]

[Out] $((33 + (33*(-1 + \sqrt{x}))^5)/(1 + \sqrt{x})^5 + (5*(-1 + \sqrt{x}))^4/(1 + \sqrt{x})^4 + (90*(-1 + \sqrt{x})^3)/(1 + \sqrt{x})^3 + (90*(-1 + \sqrt{x})^2)/(1 + \sqrt{x})^2 + (5*(-1 + \sqrt{x}))/((1 + \sqrt{x}))*\sqrt{-1 + \sqrt{x}})/(12*(-1 + (-1 + \sqrt{x})/(1 + \sqrt{x}))^6*\sqrt{1 + \sqrt{x}}) + (5*\text{ArcTanh}[\sqrt{-1 + \sqrt{x}}]/\sqrt{1 + \sqrt{x}}))/4$

fricas [A] time = 0.41, size = 57, normalized size = 0.55

$$\frac{1}{24} (8x^2 + 10x + 15) \sqrt{x} \sqrt{\sqrt{x} + 1} \sqrt{\sqrt{x} - 1} - \frac{5}{16} \log \left(2 \sqrt{x} \sqrt{\sqrt{x} + 1} \sqrt{\sqrt{x} - 1} - 2x + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas")`

[Out] $1/24*(8*x^2 + 10*x + 15)*\text{sqrt}(x)*\text{sqrt}(\text{sqrt}(x) + 1)*\text{sqrt}(\text{sqrt}(x) - 1) - 5/16*\log(2*\text{sqrt}(x)*\text{sqrt}(\text{sqrt}(x) + 1)*\text{sqrt}(\text{sqrt}(x) - 1) - 2*x + 1)$

giac [A] time = 0.23, size = 76, normalized size = 0.73

$$\frac{1}{24} ((2((4(\sqrt{x} + 1)(\sqrt{x} - 4) + 45)(\sqrt{x} + 1) - 55)(\sqrt{x} + 1) + 85)(\sqrt{x} + 1) - 33)\sqrt{\sqrt{x} + 1}\sqrt{\sqrt{x} - 1} - \frac{5}{4} \log(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")`

[Out] $1/24*((2*((4*(\text{sqrt}(x) + 1)*(\text{sqrt}(x) - 4) + 45)*(\text{sqrt}(x) + 1) - 55)*(\text{sqrt}(x) + 1) + 85)*(\text{sqrt}(x) + 1) - 33)*\text{sqrt}(\text{sqrt}(x) + 1)*\text{sqrt}(\text{sqrt}(x) - 1) - 5/4*\log(\text{sqrt}(\text{sqrt}(x) + 1) - \text{sqrt}(\text{sqrt}(x) - 1))$

maple [A] time = 0.06, size = 65, normalized size = 0.62

$$\frac{\sqrt{\sqrt{x} - 1} \sqrt{\sqrt{x} + 1} \left(8\sqrt{x - 1} x^{\frac{5}{2}} + 10\sqrt{x - 1} x^{\frac{3}{2}} + 15 \ln(\sqrt{x} + \sqrt{x - 1}) + 15\sqrt{x - 1} \sqrt{x} \right)}{24\sqrt{x - 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(5/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)`

[Out] $1/24*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(8*(x-1)^(1/2)*x^(5/2)+10*(x-1)^(1/2)*x^(3/2)+15*(x-1)^(1/2)*x^(1/2)+15*\ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)$

maxima [A] time = 0.61, size = 47, normalized size = 0.45

$$\frac{1}{3} \sqrt{x - 1} x^{\frac{5}{2}} + \frac{5}{12} \sqrt{x - 1} x^{\frac{3}{2}} + \frac{5}{8} \sqrt{x - 1} \sqrt{x} + \frac{5}{8} \log(2 \sqrt{x - 1} + 2 \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima")

[Out] 1/3*sqrt(x - 1)*x^(5/2) + 5/12*sqrt(x - 1)*x^(3/2) + 5/8*sqrt(x - 1)*sqrt(x) + 5/8*log(2*sqrt(x - 1) + 2*sqrt(x))

mupad [B] time = 27.09, size = 632, normalized size = 6.08

$$\frac{5 \operatorname{atanh}\left(\frac{\sqrt{x-1}}{\sqrt{x+1}}\right)}{2} - \frac{\frac{175(\sqrt{x-1})^3}{6(\sqrt{x+1})^3} + \frac{311(\sqrt{x-1})^5}{2(\sqrt{x+1})^5} + \frac{8361(\sqrt{x-1})^7}{2(\sqrt{x+1})^7} + \frac{42259(\sqrt{x-1})^9}{3(\sqrt{x+1})^9} + \frac{25295(\sqrt{x-1})^{11}}{(\sqrt{x+1})^{11}} + \frac{25295(\sqrt{x-1})^{13}}{(\sqrt{x+1})^{13}} + \frac{42259(\sqrt{x-1})^{15}}{3(\sqrt{x+1})^{15}} + \frac{8361(\sqrt{x-1})^{17}}{2(\sqrt{x+1})^{17}} + \frac{311(\sqrt{x-1})^{19}}{2(\sqrt{x+1})^{19}} - \frac{175(\sqrt{x-1})^{21}}{6(\sqrt{x+1})^{21}} + \frac{5(\sqrt{x-1})^{23}}{2(\sqrt{x+1})^{23}} + \frac{5(\sqrt{x-1})^5}{2(\sqrt{x+1})^5}}{1 + \frac{66(\sqrt{x-1})^4}{(\sqrt{x+1})^4} - \frac{220(\sqrt{x-1})^6}{(\sqrt{x+1})^6} + \frac{495(\sqrt{x-1})^8}{(\sqrt{x+1})^8} - \frac{792(\sqrt{x-1})^{10}}{(\sqrt{x+1})^{10}} + \frac{924(\sqrt{x-1})^{12}}{(\sqrt{x+1})^{12}} - \frac{792(\sqrt{x-1})^{14}}{(\sqrt{x+1})^{14}} + \frac{495(\sqrt{x-1})^{16}}{(\sqrt{x+1})^{16}} - \frac{220(\sqrt{x-1})^{18}}{(\sqrt{x+1})^{18}} + \frac{66(\sqrt{x-1})^{20}}{(\sqrt{x+1})^{20}} - \frac{12(\sqrt{x-1})^{22}}{(\sqrt{x+1})^{22}} + \frac{(\sqrt{x-1})^{24}}{(\sqrt{x+1})^{24}} - \frac{12(\sqrt{x-1})^2}{(\sqrt{x+1})^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)

[Out] (5*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1i)))/2 - ((311*((x^(1/2) - 1)^(1/2) - 1i)^5)/(2*((x^(1/2) + 1)^(1/2) - 1)^5) - (175*((x^(1/2) - 1)^(1/2) - 1i)^3)/(6*((x^(1/2) + 1)^(1/2) - 1)^3) + (8361*((x^(1/2) - 1)^(1/2) - 1i)^7)/(2*((x^(1/2) + 1)^(1/2) - 1)^7) + (42259*((x^(1/2) - 1)^(1/2) - 1i)^9)/(3*((x^(1/2) + 1)^(1/2) - 1)^9) + (25295*((x^(1/2) - 1)^(1/2) - 1i)^11)/((x^(1/2) + 1)^(1/2) - 1)^11 + (25295*((x^(1/2) - 1)^(1/2) - 1i)^13)/((x^(1/2) + 1)^(1/2) - 1)^13 + (42259*((x^(1/2) - 1)^(1/2) - 1i)^15)/(3*((x^(1/2) + 1)^(1/2) - 1)^15) + (8361*((x^(1/2) - 1)^(1/2) - 1i)^17)/(2*((x^(1/2) + 1)^(1/2) - 1)^17) + (311*((x^(1/2) - 1)^(1/2) - 1i)^19)/(2*((x^(1/2) + 1)^(1/2) - 1)^19) - (175*((x^(1/2) - 1)^(1/2) - 1i)^21)/(6*((x^(1/2) + 1)^(1/2) - 1)^21) + (5*((x^(1/2) - 1)^(1/2) - 1i)^23)/(2*((x^(1/2) + 1)^(1/2) - 1)^23) + (5*((x^(1/2) - 1)^(1/2) - 1i))/((2*((x^(1/2) + 1)^(1/2) - 1))) / ((66*((x^(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (12*((x^(1/2) - 1)^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (220*((x^(1/2) - 1)^(1/2) - 1i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (495*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^(1/2) + 1)^(1/2) - 1)^8 - (792*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1)^(1/2) - 1)^10 + (924*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2) - 1)^12 - (792*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14 + (495*((x^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/2) - 1)^16 - (220*((x^(1/2) - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) - 1)^18 + (66*((x^(1/2) - 1)^(1/2) - 1i)^20)/((x^(1/2) + 1)^(1/2) - 1)^20 - (12*((x^(1/2) - 1)^(1/2) - 1i)^22)/((x^(1/2) + 1)^(1/2) - 1)^22 + ((x^(1/2) - 1)^(1/2) - 1i)^24/((x^(1/2) + 1)^(1/2) - 1)^24 + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2), x)
```

```
[Out] Integral(x**(5/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

$$3.650 \quad \int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}} dx$$

Optimal. Leaf size=73

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2} + \frac{3}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x} + \frac{3}{4}\cosh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {323, 330, 52}

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2} + \frac{3}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x} + \frac{3}{4}\cosh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]

[Out] (3*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/4 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2))/2 + (3*ArcCosh[Sqrt[x]])/4

Rule 52

Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] := Simp[ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

Rule 323

Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rule 330

Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a1 + (b1*x^(k*n)))/c^n]^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m

, p, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}} dx &= \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}} dx \\
&= \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{3}{8}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}} dx \\
&= \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}} dx\right) \\
&= \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{3}{4}\cosh^{-1}(\sqrt{x})
\end{aligned}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.85

$$\frac{1}{4}\left(\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}(2x+3)+6\tanh^{-1}\left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]), x]

[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(3 + 2*x) + 6*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/4

IntegrateAlgebraic [A] time = 1.32, size = 136, normalized size = 1.86

$$\frac{\sqrt{\sqrt{x}-1}\left(\frac{5(\sqrt{x}-1)^3}{(\sqrt{x}+1)^3}+\frac{3(\sqrt{x}-1)^2}{(\sqrt{x}+1)^2}+\frac{3(\sqrt{x}-1)}{\sqrt{x}+1}+5\right)}{2\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}-1\right)^4\sqrt{\sqrt{x}+1}}+\frac{3}{2}\tanh^{-1}\left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]), x]

[Out] ((5 + (5*(-1 + Sqrt[x])^3)/(1 + Sqrt[x])^3 + (3*(-1 + Sqrt[x])^2)/(1 + Sqrt[x])^2 + (3*(-1 + Sqrt[x]))/(1 + Sqrt[x]))*Sqrt[-1 + Sqrt[x]])/(2*(-1 + (-1

+ Sqrt[x]]/(1 + Sqrt[x]))^4*Sqrt[1 + Sqrt[x]]) + (3*ArcTanh[Sqrt[-1 + Sqrt[x]]]/Sqrt[1 + Sqrt[x]]))/2

fricas [A] time = 0.40, size = 52, normalized size = 0.71

$$\frac{1}{4}(2x+3)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{3}{8}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - 2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas")

[Out] 1/4*(2*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 3/8*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)

giac [A] time = 0.23, size = 59, normalized size = 0.81

$$\frac{1}{4}\left(\left(2(\sqrt{x}+1)(\sqrt{x}-2)+9\right)(\sqrt{x}+1)-5\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{3}{2}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")

[Out] 1/4*((2*(sqrt(x) + 1)*(sqrt(x) - 2) + 9)*(sqrt(x) + 1) - 5)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 3/2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))

maple [A] time = 0.06, size = 55, normalized size = 0.75

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\left(2\sqrt{x-1}x^{\frac{3}{2}}+3\ln(\sqrt{x}+\sqrt{x-1})+3\sqrt{x-1}\sqrt{x}\right)}{4\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)

[Out] 1/4*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(2*(x-1)^(1/2)*x^(3/2)+3*(x-1)^(1/2)*x^(1/2)+3*ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)

maxima [A] time = 0.46, size = 37, normalized size = 0.51

$$\frac{1}{2}\sqrt{x-1}x^{\frac{3}{2}} + \frac{3}{4}\sqrt{x-1}\sqrt{x} + \frac{3}{4}\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(x - 1)*x^(3/2) + 3/4*sqrt(x - 1)*sqrt(x) + 3/4*log(2*sqrt(x - 1) + 2*sqrt(x))

mupad [B] time = 18.76, size = 429, normalized size = 5.88

$$3 \operatorname{atanh}\left(\frac{\sqrt{\sqrt{x}-1-i}}{\sqrt{\sqrt{x}+1-1}}\right) + \frac{\frac{23(\sqrt{\sqrt{x}-1-i})^3}{(\sqrt{\sqrt{x}+1-1})^3} + \frac{333(\sqrt{\sqrt{x}-1-i})^5}{(\sqrt{\sqrt{x}+1-1})^5} + \frac{671(\sqrt{\sqrt{x}-1-i})^7}{(\sqrt{\sqrt{x}+1-1})^7} + \frac{671(\sqrt{\sqrt{x}-1-i})^9}{(\sqrt{\sqrt{x}+1-1})^9} + \frac{333(\sqrt{\sqrt{x}-1-i})^{11}}{(\sqrt{\sqrt{x}+1-1})^{11}} + \frac{23(\sqrt{\sqrt{x}-1-i})^{13}}{(\sqrt{\sqrt{x}+1-1})^{13}} - \frac{3(\sqrt{\sqrt{x}-1-i})^{15}}{(\sqrt{\sqrt{x}+1-1})^{15}} - \frac{3(\sqrt{\sqrt{x}-1-i})}{\sqrt{\sqrt{x}+1-1}}}{1 + \frac{28(\sqrt{\sqrt{x}-1-i})^4}{(\sqrt{\sqrt{x}+1-1})^4} - \frac{56(\sqrt{\sqrt{x}-1-i})^6}{(\sqrt{\sqrt{x}+1-1})^6} + \frac{70(\sqrt{\sqrt{x}-1-i})^8}{(\sqrt{\sqrt{x}+1-1})^8} - \frac{56(\sqrt{\sqrt{x}-1-i})^{10}}{(\sqrt{\sqrt{x}+1-1})^{10}} + \frac{28(\sqrt{\sqrt{x}-1-i})^{12}}{(\sqrt{\sqrt{x}+1-1})^{12}} - \frac{8(\sqrt{\sqrt{x}-1-i})^{14}}{(\sqrt{\sqrt{x}+1-1})^{14}} + \frac{(\sqrt{\sqrt{x}-1-i})^{16}}{(\sqrt{\sqrt{x}+1-1})^{16}} - \frac{8(\sqrt{\sqrt{x}-1-i})^2}{(\sqrt{\sqrt{x}+1-1})^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)

[Out] 3*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)) + ((23*((x^(1/2) - 1)^(1/2) - 1i)^3)/((x^(1/2) + 1)^(1/2) - 1)^3 + (333*((x^(1/2) - 1)^(1/2) - 1i)^5)/((x^(1/2) + 1)^(1/2) - 1)^5 + (671*((x^(1/2) - 1)^(1/2) - 1i)^7)/((x^(1/2) + 1)^(1/2) - 1)^7 + (671*((x^(1/2) - 1)^(1/2) - 1i)^9)/((x^(1/2) + 1)^(1/2) - 1)^9 + (333*((x^(1/2) - 1)^(1/2) - 1i)^11)/((x^(1/2) + 1)^(1/2) - 1)^11 + (23*((x^(1/2) - 1)^(1/2) - 1i)^13)/((x^(1/2) + 1)^(1/2) - 1)^13 - (3*((x^(1/2) - 1)^(1/2) - 1i)^15)/((x^(1/2) + 1)^(1/2) - 1)^15 - (3*((x^(1/2) - 1)^(1/2) - 1i))/((x^(1/2) + 1)^(1/2) - 1))/((28*((x^(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (8*((x^(1/2) - 1)^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (56*((x^(1/2) - 1)^(1/2) - 1i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (70*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^(1/2) + 1)^(1/2) - 1)^8 - (56*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1)^(1/2) - 1)^10 + (28*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2) - 1)^12 - (8*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14 + ((x^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/2) - 1)^16 + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)

[Out] Integral(x**(3/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)

$$3.651 \quad \int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}} dx$$

Optimal. Leaf size=35

$$\sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} \sqrt{x} + \cosh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {323, 330, 52}

$$\sqrt{\sqrt{x}-1} \sqrt{\sqrt{x}+1} \sqrt{x} + \cosh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]

[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sqrt[x]]

Rule 52

Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

Rule 323

Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rule 330

Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a1 + (b1*x^(k*n)))/c^n]^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}} dx &= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2} \int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}} dx \\
&= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}} dx, x, \sqrt{x}\right) \\
&= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \cosh^{-1}(\sqrt{x})
\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.57

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x} + 2 \tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]), x]

[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*ArcTanh[Sqrt[-1 + Sqrt[x]]/Sqrt[1 + Sqrt[x]]]

IntegrateAlgebraic [B] time = 1.27, size = 268, normalized size = 7.66

$$\frac{-4\sqrt{\sqrt{x}+1}(-20x^{3/2}-4x+96\sqrt{x}+48)-4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}(-7x^{3/2}-28x+10\sqrt{x}+84)+\sqrt{3}\left(-4\sqrt{\sqrt{x}-1}(12x^{3/2}+20x-32\sqrt{x}-48)-4(14x^{3/2}+4x^2-18x-70\sqrt{x}-28)\right)}{\sqrt{3}\sqrt{\sqrt{x}+1}(-48\sqrt{x}-32)+\sqrt{\sqrt{x}-1}\left(\sqrt{3}\sqrt{\sqrt{x}+1}(-16\sqrt{x}-56)+80\sqrt{x}+96\right)+28x+112\sqrt{x}+56}-4 \tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}-1}{\sqrt{3}-\sqrt{\sqrt{x}+1}}\right)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]), x]

[Out] (-4*Sqrt[1 + Sqrt[x]]*(48 + 96*Sqrt[x] - 4*x - 20*x^(3/2)) - 4*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*(84 + 10*Sqrt[x] - 28*x - 7*x^(3/2)) + Sqrt[3]*(-4*Sqrt[-1 + Sqrt[x]]*(-48 - 32*Sqrt[x] + 20*x + 12*x^(3/2)) - 4*(-28 - 70*Sqrt[x] - 18*x + 14*x^(3/2) + 4*x^2)))/(56 + Sqrt[3]*(-32 - 48*Sqrt[x])*Sqrt[1 + Sqrt[x]] + Sqrt[-1 + Sqrt[x]]*(96 + Sqrt[3]*(-56 - 16*Sqrt[x])*Sqrt[1 + Sqrt[x]] + 80*Sqrt[x] + 112*Sqrt[x] + 28*x) - 4*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]

fricas [A] time = 0.41, size = 46, normalized size = 1.31

$$\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{1}{2} \log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - 2x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas")

[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 1/2*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)

giac [A] time = 0.22, size = 39, normalized size = 1.11

$$\sqrt{x} \sqrt{\sqrt{x} + 1} \sqrt{\sqrt{x} - 1} - 2 \log \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")

[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))

maple [A] time = 0.06, size = 41, normalized size = 1.17

$$\frac{\sqrt{\sqrt{x} - 1} \sqrt{\sqrt{x} + 1} \left(\ln(\sqrt{x} + \sqrt{x - 1}) + \sqrt{x - 1} \sqrt{x} \right)}{\sqrt{x - 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)

[Out] (x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*((x-1)^(1/2)*x^(1/2)+ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)

maxima [A] time = 0.45, size = 24, normalized size = 0.69

$$\sqrt{x - 1} \sqrt{x} + \log \left(2 \sqrt{x - 1} + 2 \sqrt{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima")

[Out] sqrt(x - 1)*sqrt(x) + log(2*sqrt(x - 1) + 2*sqrt(x))

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{x}}{\sqrt{\sqrt{x} - 1} \sqrt{\sqrt{x} + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)), x)`

[Out] `int(x^(1/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)), x)`

sympy [C] time = 27.91, size = 83, normalized size = 2.37

$$\frac{G_{6,6}^{6,2} \left(\begin{matrix} -\frac{3}{4}, -\frac{1}{4} & -\frac{1}{2}, -\frac{1}{2}, 0, 1 \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \middle| \frac{1}{x} \right)}{2\pi^{\frac{3}{2}}} - \frac{iG_{6,6}^{2,6} \left(\begin{matrix} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} & -\frac{3}{2}, -1, -1, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{x} \right)}{2\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2), x)`

[Out] `meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0),
()), 1/x)/(2*pi**(3/2)) - I*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()),
((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_polar(2*I*pi)/x)/(2*pi**(3/2))`

$$3.652 \quad \int \frac{1}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} \sqrt{x}} dx$$

Optimal. Leaf size=8

$$2 \cosh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {330, 52}

$$2 \cosh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]

[Out] 2*ArcCosh[Sqrt[x]]

Rule 52

Int[1/(Sqrt[(a_) + (b_)*(x_)]*Sqrt[(c_) + (d_)*(x_)]), x_Symbol] :> Simp[ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

Rule 330

Int[((c_)*(x_))^(m_)*((a1_) + (b1_)*(x_)^(n_))^(p_)*((a2_) + (b2_)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a1 + (b1*x^(k*n)))/c^n]^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} \sqrt{x}} dx &= 2 \text{Subst} \left(\int \frac{1}{\sqrt{-1+x} \sqrt{1+x}} dx, x, \sqrt{x} \right) \\ &= 2 \cosh^{-1}(\sqrt{x}) \end{aligned}$$

Mathematica [B] time = 0.01, size = 26, normalized size = 3.25

$$4 \tanh^{-1} \left(\frac{\sqrt{\sqrt{x} - 1}}{\sqrt{\sqrt{x} + 1}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]

[Out] 4*ArcTanh[Sqrt[-1 + Sqrt[x]]/Sqrt[1 + Sqrt[x]]]

IntegrateAlgebraic [B] time = 1.02, size = 38, normalized size = 4.75

$$-8 \tanh^{-1} \left(\frac{\sqrt{\sqrt{x} - 1} - 1}{\sqrt{3} - \sqrt{\sqrt{x} + 1}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]

[Out] -8*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]

fricas [B] time = 0.41, size = 27, normalized size = 3.38

$$-\log \left(2 \sqrt{x} \sqrt{\sqrt{x} + 1} \sqrt{\sqrt{x} - 1} - 2x + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas")

[Out] -log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)

giac [B] time = 0.20, size = 20, normalized size = 2.50

$$-4 \log \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")

[Out] -4*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))

maple [B] time = 0.05, size = 40, normalized size = 5.00

$$\frac{2\sqrt{(\sqrt{x}-1)(\sqrt{x}+1)} \ln(\sqrt{x}+\sqrt{x-1})}{\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(1/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)

[Out] 2*((x^(1/2)-1)*(x^(1/2)+1))^(1/2)/(x^(1/2)+1)^(1/2)/(x^(1/2)-1)^(1/2)*ln(x^(1/2)+(x-1)^(1/2))

maxima [B] time = 0.65, size = 16, normalized size = 2.00

$$2 \log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima")

[Out] 2*log(2*sqrt(x - 1) + 2*sqrt(x))

mupad [B] time = 5.29, size = 6, normalized size = 0.75

$$2 \operatorname{acosh}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)),x)

[Out] 2*acosh(x^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)

[Out] Integral(1/(sqrt(x)*sqrt(sqrt(x)-1)*sqrt(sqrt(x)+1)), x)

$$3.653 \quad \int \frac{1}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{3/2}} dx$$

Optimal. Leaf size=29

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {265}

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]

[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x]

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_.) + (b1_.)*(x_)^(n_.))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{3/2}} dx = \frac{2\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}}{\sqrt{x}}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]

[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x]

IntegrateAlgebraic [B] time = 1.14, size = 188, normalized size = 6.48

$$\frac{\left(\frac{(\sqrt{\sqrt{x}-1}-1)^3}{2(\sqrt{3}-\sqrt{\sqrt{x}+1})^3} + \frac{\sqrt{\sqrt{x}-1}-1}{2(\sqrt{3}-\sqrt{\sqrt{x}+1})} \right) (\sqrt{3} - \sqrt{\sqrt{x}+1})^4}{-2x - 2\sqrt{\sqrt{x}-1}\sqrt{x} + \sqrt{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x} + 2\sqrt{3}\sqrt{\sqrt{x}+1}\sqrt{x} - 3\sqrt{x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]

[Out] (((-1 + Sqrt[-1 + Sqrt[x]])^3/(2*(Sqrt[3] - Sqrt[1 + Sqrt[x]]))^3) + (-1 + Sqrt[-1 + Sqrt[x]])/(2*(Sqrt[3] - Sqrt[1 + Sqrt[x]])))*(Sqrt[3] - Sqrt[1 + Sqrt[x]])^4)/(-3*Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]*Sqrt[x] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - 2*x)

fricas [A] time = 0.42, size = 25, normalized size = 0.86

$$\frac{2\left(\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+x\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas")

[Out] 2*(sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x)/x

giac [A] time = 0.22, size = 25, normalized size = 0.86

$$\frac{16}{\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")

[Out] 16/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)

maple [A] time = 0.06, size = 20, normalized size = 0.69

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^(3/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)`

[Out] `2*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(1/2)`

maxima [A] time = 1.21, size = 10, normalized size = 0.34

$$\frac{2\sqrt{x-1}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima")`

[Out] `2*sqrt(x - 1)/sqrt(x)`

mupad [B] time = 5.56, size = 19, normalized size = 0.66

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(3/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)`

[Out] `(2*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(1/2)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)`

[Out] `Integral(1/(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)`

$$3.654 \quad \int \frac{1}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{5/2}} dx$$

Optimal. Leaf size=63

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}} + \frac{4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {272, 265}

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}} + \frac{4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]

[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(3*x^(3/2)) + (4*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(3*Sqrt[x])

Rule 265

```
Int[((c_.)*(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x]
;/; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(a1*a2*(m + 1)), x]
- Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1)), Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x]
;/; FreeQ[{a1, b1, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{5/2}} dx = \frac{2\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}}{3x^{3/2}} + \frac{2}{3} \int \frac{1}{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}} x^{3/2}} dx$$

$$= \frac{2\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}}{3x^{3/2}} + \frac{4\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}}{3\sqrt{x}}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.57

$$\frac{2\sqrt{\sqrt{x} - 1} \sqrt{\sqrt{x} + 1} (2x + 1)}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)), x]

[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*(1 + 2*x))/(3*x^(3/2))

IntegrateAlgebraic [B] time = 2.75, size = 512, normalized size = 8.13

$$\frac{\left(\frac{(\sqrt{\sqrt{x}-1})^2}{(\sqrt{\sqrt{x}-1})^2+1} \right) \left(\frac{3(\sqrt{\sqrt{x}-1})^8}{(\sqrt{\sqrt{x}-1})^8} + \frac{6\sqrt{3}(\sqrt{\sqrt{x}-1})^7}{(\sqrt{\sqrt{x}-1})^7} + \frac{32(\sqrt{\sqrt{x}-1})^6}{(\sqrt{\sqrt{x}-1})^6} + \frac{42\sqrt{3}(\sqrt{\sqrt{x}-1})^5}{(\sqrt{\sqrt{x}-1})^5} + \frac{106(\sqrt{\sqrt{x}-1})^4}{(\sqrt{\sqrt{x}-1})^4} + \frac{42\sqrt{3}(\sqrt{\sqrt{x}-1})^3}{(\sqrt{\sqrt{x}-1})^3} + \frac{32(\sqrt{\sqrt{x}-1})^2}{(\sqrt{\sqrt{x}-1})^2} + \frac{6\sqrt{3}(\sqrt{\sqrt{x}-1})}{\sqrt{\sqrt{x}-1}} + 3 \right) \left(\frac{1}{384} - \frac{\sqrt{\sqrt{x}-1}}{384} \right) (\sqrt{3} - \sqrt{\sqrt{x}+1})^{12}}{(\sqrt{\sqrt{x}+1} - \sqrt{3}) \left(-2x - 2\sqrt{\sqrt{x}-1}\sqrt{x} + \sqrt{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x} + 2\sqrt{3}\sqrt{\sqrt{x}+1}\sqrt{x} - 3\sqrt{x} \right)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)), x]

[Out] ((1 + (-1 + Sqrt[-1 + Sqrt[x]])^2/(Sqrt[3] - Sqrt[1 + Sqrt[x]]))^2*(3 + (3*(-1 + Sqrt[-1 + Sqrt[x]])^8)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^8 + (6*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^7)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^7 + (32*(-1 + Sqrt[-1 + Sqrt[x]])^6)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^6 + (42*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^5)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^5 + (106*(-1 + Sqrt[-1 + Sqrt[x]])^4)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^4 + (42*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^3)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^3 + (32*(-1 + Sqrt[-1 + Sqrt[x]])^2)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2 + (6*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]]))/(Sqrt[3] - Sqrt[1 + Sqrt[x]]))*(1/384 - Sqrt[-1 + Sqrt[x]]/384)*(Sqrt[3] - Sqrt[1 + Sqrt[x]])^12)/((-Sqrt[3] + Sqrt[1 + Sqrt[x]])*(-3*Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]*Sqrt[x] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - 2*x)^3)

fricas [A] time = 0.40, size = 34, normalized size = 0.54

$$\frac{2 \left((2x+1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + 2x^2 \right)}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas")

[Out] 2/3*((2*x + 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x^2)/x^2

giac [A] time = 0.22, size = 48, normalized size = 0.76

$$\frac{128 \left(3 \left(\sqrt{\sqrt{x}+1} - \sqrt{\sqrt{x}-1} \right)^4 + 4 \right)}{3 \left(\left(\sqrt{\sqrt{x}+1} - \sqrt{\sqrt{x}-1} \right)^4 + 4 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")

[Out] 128/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)^3

maple [A] time = 0.06, size = 25, normalized size = 0.40

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}(2x+1)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)

[Out] 2/3*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(2*x+1)/x^(3/2)

maxima [A] time = 1.20, size = 21, normalized size = 0.33

$$\frac{4\sqrt{x-1}}{3\sqrt{x}} + \frac{2\sqrt{x-1}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima")`

[Out] $4/3*\sqrt{x - 1}/\sqrt{x} + 2/3*\sqrt{x - 1}/x^{3/2}$

mupad [B] time = 5.50, size = 33, normalized size = 0.52

$$\frac{\sqrt{\sqrt{x} - 1} \left(\frac{4x}{3} + \frac{2\sqrt{x}}{3} + \frac{4x^{3/2}}{3} + \frac{2}{3} \right)}{x^{3/2} \sqrt{\sqrt{x} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(5/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)`

[Out] $((x^{1/2} - 1)^{1/2} * ((4*x)/3 + (2*x^{1/2})/3 + (4*x^{3/2})/3 + 2/3)) / (x^{3/2} * (x^{1/2} + 1)^{1/2})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}} \sqrt{\sqrt{x} - 1} \sqrt{\sqrt{x} + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)`

[Out] `Integral(1/(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)`

$$3.655 \quad \int \frac{1}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{7/2}} dx$$

Optimal. Leaf size=94

$$\frac{8\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{15x^{3/2}} + \frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{5x^{5/2}} + \frac{16\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{15\sqrt{x}}$$

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {272, 265}

$$\frac{8\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{15x^{3/2}} + \frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{5x^{5/2}} + \frac{16\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{15\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]

[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(5*x^(5/2)) + (8*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(15*x^(3/2)) + (16*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(15*Sqrt[x])

Rule 265

```
Int[((c_.)*(x_)^(m_.)*((a1_.) + (b1_.)*(x_)^(n_.))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a1_.) + (b1_.)*(x_)^(n_.))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1)), Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{7/2}} dx &= \frac{2\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}}{5x^{5/2}} + \frac{4}{5} \int \frac{1}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} x^{5/2}} dx \\
&= \frac{2\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}}{5x^{5/2}} + \frac{8\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}}{15x^{3/2}} + \frac{8}{15} \int \frac{1}{\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}} dx \\
&= \frac{2\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}}{5x^{5/2}} + \frac{8\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}}{15x^{3/2}} + \frac{16\sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}}}{15\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.44

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}(8x^2+4x+3)}{15x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]

[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*(3 + 4*x + 8*x^2))/(15*x^(5/2))

IntegrateAlgebraic [A] time = 1.37, size = 127, normalized size = 1.35

$$\frac{4\left(\frac{15(\sqrt{x}-1)^4}{(\sqrt{x}+1)^4} + \frac{20(\sqrt{x}-1)^3}{(\sqrt{x}+1)^3} + \frac{58(\sqrt{x}-1)^2}{(\sqrt{x}+1)^2} + \frac{20(\sqrt{x}-1)}{\sqrt{x}+1} + 15\right)\sqrt{\sqrt{x}-1}}{15\left(\frac{\sqrt{x}-1}{\sqrt{x}+1} + 1\right)^5\sqrt{\sqrt{x}+1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]

[Out] (4*(15 + (15*(-1 + Sqrt[x])^4)/(1 + Sqrt[x])^4 + (20*(-1 + Sqrt[x])^3)/(1 + Sqrt[x])^3 + (58*(-1 + Sqrt[x])^2)/(1 + Sqrt[x])^2 + (20*(-1 + Sqrt[x]))/(1 + Sqrt[x]))*Sqrt[-1 + Sqrt[x]])/(15*(1 + (-1 + Sqrt[x])/(1 + Sqrt[x]))^5*Sqrt[1 + Sqrt[x]])

fricas [A] time = 0.41, size = 39, normalized size = 0.41

$$\frac{2\left(8x^3 + (8x^2 + 4x + 3)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas")

[Out] 2/15*(8*x^3 + (8*x^2 + 4*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1))/x^3

giac [A] time = 0.26, size = 69, normalized size = 0.73

$$\frac{4096 \left(5 \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^8 + 10 \left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^4 + 8 \right)}{15 \left(\left(\sqrt{\sqrt{x} + 1} - \sqrt{\sqrt{x} - 1} \right)^4 + 4 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")

[Out] 4096/15*(5*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 10*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 8)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)^5

maple [A] time = 0.06, size = 30, normalized size = 0.32

$$\frac{2\sqrt{\sqrt{x} - 1} \sqrt{\sqrt{x} + 1} (8x^2 + 4x + 3)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(7/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)

[Out] 2/15*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(8*x^2+4*x+3)/x^(5/2)

maxima [A] time = 1.30, size = 31, normalized size = 0.33

$$\frac{16\sqrt{x-1}}{15\sqrt{x}} + \frac{8\sqrt{x-1}}{15x^{\frac{3}{2}}} + \frac{2\sqrt{x-1}}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima")

[Out] $16/15*\sqrt{x - 1}/\sqrt{x} + 8/15*\sqrt{x - 1}/x^{(3/2)} + 2/5*\sqrt{x - 1}/x^{(5/2)}$

mupad [B] time = 5.66, size = 43, normalized size = 0.46

$$\frac{\sqrt{\sqrt{x} - 1} \left(\frac{8x}{15} + \frac{16x^2}{15} + \frac{2\sqrt{x}}{5} + \frac{8x^{3/2}}{15} + \frac{16x^{5/2}}{15} + \frac{2}{5} \right)}{x^{5/2} \sqrt{\sqrt{x} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(7/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)), x)`

[Out] $((x^{(1/2)} - 1)^{(1/2)}*((8*x)/15 + (16*x^2)/15 + (2*x^{(1/2)})/5 + (8*x^{(3/2)})/15 + (16*x^{(5/2)})/15 + 2/5))/(x^{(5/2)}*(x^{(1/2)} + 1)^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7/2} \sqrt{\sqrt{x} - 1} \sqrt{\sqrt{x} + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(7/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2), x)`

[Out] `Integral(1/(x**(7/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)`

$$3.656 \quad \int \frac{1+x^6}{x(1-x^6)} dx$$

Optimal. Leaf size=15

$$\log(x) - \frac{1}{3} \log(1 - x^6)$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {446, 72}

$$\log(x) - \frac{1}{3} \log(1 - x^6)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^6)/(x*(1 - x^6)),x]

[Out] Log[x] - Log[1 - x^6]/3

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1+x^6}{x(1-x^6)} dx &= \frac{1}{6} \text{Subst} \left(\int \frac{1+x}{(1-x)x} dx, x, x^6 \right) \\ &= \frac{1}{6} \text{Subst} \left(\int \left(-\frac{2}{-1+x} + \frac{1}{x} \right) dx, x, x^6 \right) \\ &= \log(x) - \frac{1}{3} \log(1 - x^6) \end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\log(x) - \frac{1}{3} \log(1 - x^6)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^6)/(x*(1 - x^6)), x]

[Out] Log[x] - Log[1 - x^6]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + x^6}{x(1 - x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^6)/(x*(1 - x^6)), x]

[Out] IntegrateAlgebraic[(1 + x^6)/(x*(1 - x^6)), x]

fricas [A] time = 0.40, size = 11, normalized size = 0.73

$$-\frac{1}{3} \log(x^6 - 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+1)/x/(-x^6+1), x, algorithm="fricas")

[Out] -1/3*log(x^6 - 1) + log(x)

giac [A] time = 0.19, size = 16, normalized size = 1.07

$$\frac{1}{6} \log(x^6) - \frac{1}{3} \log(|x^6 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+1)/x/(-x^6+1), x, algorithm="giac")

[Out] 1/6*log(x^6) - 1/3*log(abs(x^6 - 1))

maple [B] time = 0.05, size = 36, normalized size = 2.40

$$\ln(x) - \frac{\ln(x-1)}{3} - \frac{\ln(x+1)}{3} - \frac{\ln(x^2-x+1)}{3} - \frac{\ln(x^2+x+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^6+1)/x/(-x^6+1),x)`

[Out] `-1/3*ln(x-1)-1/3*ln(x^2+x+1)-1/3*ln(x+1)+ln(x)-1/3*ln(x^2-x+1)`

maxima [A] time = 0.46, size = 15, normalized size = 1.00

$$-\frac{1}{3} \log(x^6 - 1) + \frac{1}{6} \log(x^6)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^6+1)/x/(-x^6+1),x, algorithm="maxima")`

[Out] `-1/3*log(x^6 - 1) + 1/6*log(x^6)`

mupad [B] time = 0.09, size = 11, normalized size = 0.73

$$\ln(x) - \frac{\ln(x^6 - 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^6 + 1)/(x*(x^6 - 1)),x)`

[Out] `log(x) - log(x^6 - 1)/3`

sympy [A] time = 0.12, size = 10, normalized size = 0.67

$$\log(x) - \frac{\log(x^6 - 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**6+1)/x/(-x**6+1),x)`

[Out] `log(x) - log(x**6 - 1)/3`

$$3.657 \quad \int (ex)^m (a + bx^n)^p (a(1 + m) + b(1 + m + n + np)x^n) dx$$

Optimal. Leaf size=22

$$\frac{(ex)^{m+1} (a + bx^n)^{p+1}}{e}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {449}

$$\frac{(ex)^{m+1} (a + bx^n)^{p+1}}{e}$$

Antiderivative was successfully verified.

[In] Int[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n), x]

[Out] ((e*x)^(1 + m)*(a + b*x^n)^(1 + p))/e

Rule 449

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]

Rubi steps

$$\int (ex)^m (a + bx^n)^p (a(1 + m) + b(1 + m + n + np)x^n) dx = \frac{(ex)^{1+m} (a + bx^n)^{1+p}}{e}$$

Mathematica [C] time = 0.17, size = 110, normalized size = 5.00

$$x(ex)^m (a + bx^n)^p \left(\frac{bx^n}{a} + 1 \right)^{-p} \left(\frac{bx^n(m + np + n + 1) {}_2F_1\left(\frac{m+n+1}{n}, -p; \frac{m+2n+1}{n}; -\frac{bx^n}{a}\right)}{m+n+1} + a {}_2F_1\left(\frac{m+1}{n}, -p; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n), x]

[Out] (x*(e*x)^m*(a + b*x^n)^p*(a*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n, -(b*x^n)/a] + (b*(1 + m + n + n*p)*x^n*Hypergeometric2F1[(1 + m + n)/n, -p, (1 + m + 2*n)/n, -(b*x^n)/a]))/(1 + m + n))/(1 + (b*x^n)/a)^p

IntegrateAlgebraic [F] time = 0.26, size = 0, normalized size = 0.00

$$\int (ex)^m (a + bx^n)^p (a(1 + m) + b(1 + m + n + np)x^n) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n), x]

[Out] Defer[IntegrateAlgebraic] [(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n), x]

fricas [A] time = 0.41, size = 40, normalized size = 1.82

$$\left(bxx^n e^{(m \log(e) + m \log(x))} + axe^{(m \log(e) + m \log(x))} \right) (bx^n + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n), x, algorithm="fricas")

[Out] (b*x*x^n*e^(m*log(e) + m*log(x)) + a*x*e^(m*log(e) + m*log(x)))*(b*x^n + a)^p

giac [A] time = 0.29, size = 38, normalized size = 1.73

$$(bx^n + a)^p bxx^m x^n e^m + (bx^n + a)^p axx^m e^m$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n), x, algorithm="giac")

[Out] (b*x^n + a)^p b*x*x^m*x^n*e^m + (b*x^n + a)^p a*x*x^m*e^m

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \left((np + m + n + 1) b x^n + (m + 1) a \right) (ex)^m (b x^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^m*(b*x^n+a)^p*(a*(m+1)+b*(n*p+m+n+1)*x^n), x)

[Out] int((e*x)^m*(b*x^n+a)^p*(a*(m+1)+b*(n*p+m+n+1)*x^n), x)

maxima [A] time = 0.95, size = 36, normalized size = 1.64

$$\left(a e^m x x^m + b e^m x e^{(m \log(x) + n \log(x))} \right) (b x^n + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="maxima")

[Out] (a*e^m*x*x^m + b*e^m*x*e^(m*log(x) + n*log(x)))*(b*x^n + a)^p

mupad [B] time = 4.94, size = 31, normalized size = 1.41

$$\left(a x (e x)^m + b x^{n+1} (e x)^m \right) (a + b x^n)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^m*(a*(m + 1) + b*x^n*(m + n + n*p + 1))*(a + b*x^n)^p,x)

[Out] (a*x*(e*x)^m + b*x^(n + 1)*(e*x)^m)*(a + b*x^n)^p

sympy [B] time = 8.43, size = 39, normalized size = 1.77

$$a e^m x x^m (a + b x^n)^p + b e^m x x^m x^n (a + b x^n)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**m*(a+b*x**n)**p*(a*(1+m)+b*(n*p+m+n+1)*x**n),x)

[Out] a*e**m*x*x**m*(a + b*x**n)**p + b*e**m*x*x**m*x**n*(a + b*x**n)**p

$$3.658 \quad \int \frac{1}{x(a+bx^n)(c+dx^n)} dx$$

Optimal. Leaf size=63

$$-\frac{b \log(a+bx^n)}{an(bc-ad)} + \frac{d \log(c+dx^n)}{cn(bc-ad)} + \frac{\log(x)}{ac}$$

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 72}

$$-\frac{b \log(a+bx^n)}{an(bc-ad)} + \frac{d \log(c+dx^n)}{cn(bc-ad)} + \frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^n)*(c + d*x^n)),x]

[Out] Log[x]/(a*c) - (b*Log[a + b*x^n])/(a*(b*c - a*d)*n) + (d*Log[c + d*x^n])/(c*(b*c - a*d)*n)

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a+bx^n)(c+dx^n)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)} dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{acx} + \frac{b^2}{a(-bc+ad)(a+bx)} + \frac{d^2}{c(bc-ad)(c+dx)}\right) dx, x, x^n\right)}{n} \\ &= \frac{\log(x)}{ac} - \frac{b \log(a+bx^n)}{a(bc-ad)n} + \frac{d \log(c+dx^n)}{c(bc-ad)n} \end{aligned}$$

Mathematica [A] time = 0.06, size = 56, normalized size = 0.89

$$\frac{-bc \log(a + bx^n) + ad \log(c + dx^n) - adn \log(x) + bcn \log(x)}{abc^2n - a^2cdn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^n)*(c + d*x^n)), x]

[Out] (b*c*n*Log[x] - a*d*n*Log[x] - b*c*Log[a + b*x^n] + a*d*Log[c + d*x^n])/(a*b*c^2*n - a^2*c*d*n)

IntegrateAlgebraic [A] time = 0.07, size = 67, normalized size = 1.06

$$\frac{b \log(a + bx^n)}{an(ad - bc)} + \frac{d \log(c + dx^n)}{cn(bc - ad)} + \frac{\log(x^n)}{acn}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^n)*(c + d*x^n)), x]

[Out] Log[x^n]/(a*c*n) + (b*Log[a + b*x^n])/(a*(-(b*c) + a*d)*n) + (d*Log[c + d*x^n])/(c*(b*c - a*d)*n)

fricas [A] time = 0.45, size = 58, normalized size = 0.92

$$\frac{bc \log(bx^n + a) - ad \log(dx^n + c) - (bc - ad)n \log(x)}{(abc^2 - a^2cd)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n)/(c+d*x^n), x, algorithm="fricas")

[Out] -(b*c*log(b*x^n + a) - a*d*log(d*x^n + c) - (b*c - a*d)*n*log(x))/((a*b*c^2 - a^2*c*d)*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n)/(c+d*x^n), x, algorithm="giac")

[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x), x)

maple [A] time = 0.10, size = 69, normalized size = 1.10

$$\frac{b \ln(bx^n + a)}{(ad - bc)an} - \frac{d \ln(dx^n + c)}{(ad - bc)cn} + \frac{\ln(x^n)}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^n+a)/(d*x^n+c),x)

[Out] -1/n/c*d/(a*d-b*c)*ln(d*x^n+c)+1/n/a*b/(a*d-b*c)*ln(b*x^n+a)+1/n/c/a*ln(x^n)

maxima [A] time = 0.57, size = 69, normalized size = 1.10

$$-\frac{b \log\left(\frac{bx^n+a}{b}\right)}{abcn - a^2dn} + \frac{d \log\left(\frac{dx^n+c}{d}\right)}{bc^2n - acdn} + \frac{\log(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")

[Out] -b*log((b*x^n + a)/b)/(a*b*c*n - a^2*d*n) + d*log((d*x^n + c)/d)/(b*c^2*n - a*c*d*n) + log(x)/(a*c)

mupad [B] time = 5.72, size = 162, normalized size = 2.57

$$\frac{b \ln\left(-\frac{1}{bdx} - \frac{2acn+adnx^n+bcnx^n}{dx(a^2dn-abcn)}\right)}{a^2dn - abc n} + \frac{d \ln\left(-\frac{1}{bdx} - \frac{2acn+adnx^n+bcnx^n}{bx(bc^2n-acdn)}\right)}{bc^2n - acdn} + \frac{\ln(x)(n-1)}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^n)*(c + d*x^n)),x)

[Out] (b*log(- 1/(b*d*x) - (2*a*c*n + a*d*n*x^n + b*c*n*x^n)/(d*x*(a^2*d*n - a*b*c*n))))/(a^2*d*n - a*b*c*n) + (d*log(- 1/(b*d*x) - (2*a*c*n + a*d*n*x^n + b*c*n*x^n)/(b*x*(b*c^2*n - a*c*d*n))))/(b*c^2*n - a*c*d*n) + (log(x)*(n - 1))/(a*c*n)

sympy [A] time = 4.02, size = 332, normalized size = 5.27

$$\left\{ \begin{array}{l}
 \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b} + x^n\right)}{an} \\
 \frac{c}{c} \\
 \frac{-\frac{x^{-n}}{cn} + \frac{d \log\left(x^{-n} + \frac{d}{c}\right)}{c^2n}}{b} \\
 \frac{cn \log(x)}{ac^2n+acdnx^n} - \frac{c \log\left(\frac{c}{d} + x^n\right)}{ac^2n+acdnx^n} + \frac{c}{ac^2n+acdnx^n} + \frac{dnx^n \log(x)}{ac^2n+acdnx^n} - \frac{dx^n \log\left(\frac{c}{d} + x^n\right)}{ac^2n+acdnx^n} \\
 \frac{-\frac{x^{-n}}{an} + \frac{b \log\left(x^{-n} + \frac{b}{a}\right)}{a^2n}}{d} \\
 \frac{\log(x)}{(a+b)(c+d)} \\
 \frac{\log(x)}{c} - \frac{\log\left(\frac{c}{d} + x^n\right)}{cn} \\
 \frac{a}{a} \\
 \frac{adn \log(x)}{a^2cdn-abc^2n} - \frac{ad \log\left(\frac{c}{d} + x^n\right)}{a^2cdn-abc^2n} - \frac{bcn \log(x)}{a^2cdn-abc^2n} + \frac{bc \log\left(\frac{a}{b} + x^n\right)}{a^2cdn-abc^2n}
 \end{array} \right. \begin{array}{l}
 \text{for } d = 0 \\
 \\
 \text{for } a = 0 \\
 \text{for } b = \frac{ad}{c} \\
 \text{for } c = 0 \\
 \text{for } n = 0 \\
 \text{for } b = 0 \\
 \text{otherwise}
 \end{array}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x**n)/(c+d*x**n), x)

[Out] Piecewise(((log(x)/a - log(a/b + x**n)/(a*n))/c, Eq(d, 0)), ((-x**(-n)/(c*n) + d*log(x**(-n) + d/c)/(c**2*n))/b, Eq(a, 0)), (c*n*log(x)/(a*c**2*n + a*c*d*n*x**n) - c*log(c/d + x**n)/(a*c**2*n + a*c*d*n*x**n) + c/(a*c**2*n + a*c*d*n*x**n) + d*n*x**n*log(x)/(a*c**2*n + a*c*d*n*x**n) - d*x**n*log(c/d + x**n)/(a*c**2*n + a*c*d*n*x**n), Eq(b, a*d/c)), ((-x**(-n)/(a*n) + b*log(x**(-n) + b/a)/(a**2*n))/d, Eq(c, 0)), (log(x)/((a + b)*(c + d)), Eq(n, 0)), ((log(x)/c - log(c/d + x**n)/(c*n))/a, Eq(b, 0)), (a*d*n*log(x)/(a**2*c*d*n - a*b*c**2*n) - a*d*log(c/d + x**n)/(a**2*c*d*n - a*b*c**2*n) - b*c*n*log(x)/(a**2*c*d*n - a*b*c**2*n) + b*c*log(a/b + x**n)/(a**2*c*d*n - a*b*c**2*n), True))

$$3.659 \quad \int \frac{1}{x(a+bx^n)^2(c+dx^n)} dx$$

Optimal. Leaf size=101

$$-\frac{b(bc-2ad)\log(a+bx^n)}{a^2n(bc-ad)^2} + \frac{\log(x)}{a^2c} - \frac{d^2\log(c+dx^n)}{cn(bc-ad)^2} + \frac{b}{an(bc-ad)(a+bx^n)}$$

Rubi [A] time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 72}

$$-\frac{b(bc-2ad)\log(a+bx^n)}{a^2n(bc-ad)^2} + \frac{\log(x)}{a^2c} - \frac{d^2\log(c+dx^n)}{cn(bc-ad)^2} + \frac{b}{an(bc-ad)(a+bx^n)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^n)^2*(c + d*x^n)), x]

[Out] b/(a*(b*c - a*d)*n*(a + b*x^n)) + Log[x]/(a^2*c) - (b*(b*c - 2*a*d)*Log[a + b*x^n])/(a^2*(b*c - a*d)^2*n) - (d^2*Log[c + d*x^n])/(c*(b*c - a*d)^2*n)

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a+bx^n)^2(c+dx^n)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)} dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2cx} + \frac{b^2}{a(-bc+ad)(a+bx)^2} + \frac{b^2(-bc+2ad)}{a^2(-bc+ad)^2(a+bx)} - \frac{d^3}{c(bc-ad)^2(c+dx)}\right) dx, x, x^n\right)}{n} \\ &= \frac{b}{a(bc-ad)n(a+bx^n)} + \frac{\log(x)}{a^2c} - \frac{b(bc-2ad)\log(a+bx^n)}{a^2(bc-ad)^2n} - \frac{d^2\log(c+dx^n)}{c(bc-ad)^2n} \end{aligned}$$

Mathematica [A] time = 0.33, size = 97, normalized size = 0.96

$$\frac{-\frac{b(bc-2ad)\log(ax^n)}{a^2(bc-ad)^2} + \frac{n\log(x)}{a^2c} - \frac{d^2\log(c+dx^n)}{c(bc-ad)^2} + \frac{b}{a(bc-ad)(a+bx^n)}}{n}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^n)^2*(c + d*x^n)), x]

[Out] (b/(a*(b*c - a*d)*(a + b*x^n)) + (n*Log[x])/(a^2*c) - (b*(b*c - 2*a*d)*Log[a + b*x^n])/(a^2*(b*c - a*d)^2) - (d^2*Log[c + d*x^n])/(c*(b*c - a*d)^2))/n

IntegrateAlgebraic [A] time = 0.13, size = 109, normalized size = 1.08

$$\frac{(2abd - b^2c)\log(a + bx^n)}{a^2n(ad - bc)^2} + \frac{\log(x^n)}{a^2cn} - \frac{d^2\log(c + dx^n)}{cn(bc - ad)^2} - \frac{b}{an(ad - bc)(a + bx^n)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^n)^2*(c + d*x^n)), x]

[Out] -(b/(a*(-(b*c) + a*d)*n*(a + b*x^n))) + Log[x^n]/(a^2*c*n) + (((-b^2*c) + 2*a*b*d)*Log[a + b*x^n])/(a^2*(-(b*c) + a*d)^2*n) - (d^2*Log[c + d*x^n])/(c*(b*c - a*d)^2*n)

fricas [B] time = 0.47, size = 224, normalized size = 2.22

$$\frac{ab^2c^2 - a^2bcd + (b^3c^2 - 2ab^2cd + a^2bd^2)nx^n\log(x) + (ab^2c^2 - 2a^2bcd + a^3d^2)n\log(x) - (ab^2c^2 - 2a^2bcd + (b^3c^2 - 2ab^2cd)x^n)\log(bx^n + a) - (a^2bd^2x^n + a^3d^2)\log(dx^n + c)}{(a^2b^3c^3 - 2a^3b^2c^2d + a^4bcd^2)nx^n + (a^3b^2c^3 - 2a^4bc^2d + a^5cd^2)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n), x, algorithm="fricas")

[Out] (a*b^2*c^2 - a^2*b*c*d + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*n*x^n*log(x) + (a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*n*log(x) - (a*b^2*c^2 - 2*a^2*b*c*d + (b^3*c^2 - 2*a*b^2*c*d)*x^n)*log(b*x^n + a) - (a^2*b*d^2*x^n + a^3*d^2)*log(d*x^n + c))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*n*x^n + (a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")

[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)*x), x)

maple [A] time = 0.10, size = 131, normalized size = 1.30

$$\frac{2bd \ln(bx^n + a)}{(ad - bc)^2 an} - \frac{b^2c \ln(bx^n + a)}{(ad - bc)^2 a^2n} - \frac{d^2 \ln(dx^n + c)}{(ad - bc)^2 cn} - \frac{b}{(ad - bc)(bx^n + a)an} + \frac{\ln(x^n)}{a^2cn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^n+a)^2/(d*x^n+c),x)

[Out] -1/n/(a*d-b*c)^2/c*d^2*ln(d*x^n+c)-1/n/a*b/(a*d-b*c)/(b*x^n+a)+2/n*b/(a*d-b*c)^2/a*ln(b*x^n+a)*d-1/n*b^2/(a*d-b*c)^2/a^2*ln(b*x^n+a)*c+1/n/c/a^2*ln(x^n)

maxima [A] time = 0.67, size = 151, normalized size = 1.50

$$\frac{d^2 \log\left(\frac{dx^n+c}{d}\right)}{b^2c^3n - 2abc^2dn + a^2cd^2n} - \frac{(b^2c - 2abd) \log\left(\frac{bx^n+a}{b}\right)}{a^2b^2c^2n - 2a^3bcdn + a^4d^2n} + \frac{b}{a^2bcn - a^3dn + (ab^2cn - a^2bdn)x^n} + \frac{\log(x)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")

[Out] -d^2*log((d*x^n + c)/d)/(b^2*c^3*n - 2*a*b*c^2*d*n + a^2*c*d^2*n) - (b^2*c - 2*a*b*d)*log((b*x^n + a)/b)/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n) + b/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) + log(x)/(a^2*c)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x(a + bx^n)^2(c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^n)^2*(c + d*x^n)),x)

[Out] int(1/(x*(a + b*x^n)^2*(c + d*x^n)), x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x**n)**2/(c+d*x**n),x)

[Out] Exception raised: HeuristicGCDFailed

$$3.660 \quad \int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n} dx$$

Optimal. Leaf size=130

$$\frac{bx^{2n}(3a^2d^2 - 3abcd + b^2c^2)}{2d^3n} - \frac{b^2x^{3n}(bc - 3ad)}{3d^2n} + \frac{c(bc - ad)^3 \log(c + dx^n)}{d^5n} - \frac{x^n(bc - ad)^3}{d^4n} + \frac{b^3x^{4n}}{4dn}$$

Rubi [A] time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {446, 77}

$$\frac{bx^{2n}(3a^2d^2 - 3abcd + b^2c^2)}{2d^3n} - \frac{b^2x^{3n}(bc - 3ad)}{3d^2n} - \frac{x^n(bc - ad)^3}{d^4n} + \frac{c(bc - ad)^3 \log(c + dx^n)}{d^5n} + \frac{b^3x^{4n}}{4dn}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n), x]

[Out] -(((b*c - a*d)^3*x^n)/(d^4*n)) + (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(2*n))/(2*d^3*n) - (b^2*(b*c - 3*a*d)*x^(3*n))/(3*d^2*n) + (b^3*x^(4*n))/(4*d*n) + (c*(b*c - a*d)^3*Log[c + d*x^n])/(d^5*n)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n} (a + bx^n)^3}{c + dx^n} dx = \frac{\text{Subst} \left(\int \frac{x^{(a+bx)^3}}{c+dx} dx, x, x^n \right)}{n}$$

$$= \frac{\text{Subst} \left(\int \left(\frac{(-bc+ad)^3}{d^4} + \frac{b(b^2c^2-3abcd+3a^2d^2)x}{d^3} - \frac{b^2(bc-3ad)x^2}{d^2} + \frac{b^3x^3}{d} + \frac{c(bc-ad)^3}{d^4(c+dx)} \right) dx, x, x^n \right)}{n}$$

$$= -\frac{(bc-ad)^3 x^n}{d^4 n} + \frac{b(b^2c^2-3abcd+3a^2d^2)x^{2n}}{2d^3 n} - \frac{b^2(bc-3ad)x^{3n}}{3d^2 n} + \frac{b^3x^{4n}}{4dn} + \frac{c(bc-ad)}{d^4(c+dx)}$$

Mathematica [A] time = 0.35, size = 115, normalized size = 0.88

$$\frac{6bd^2x^{2n}(3a^2d^2-3abcd+b^2c^2)-4b^2d^3x^{3n}(bc-3ad)+12dx^n(ad-bc)^3+12c(bc-ad)^3\log(c+dx^n)+3b^3d^4x^{4n}}{12d^5n}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + 2*n))*(a + b*x^n)^3]/(c + d*x^n), x]

[Out] (12*d*(-(b*c) + a*d)^3*x^n + 6*b*d^2*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(2*n) - 4*b^2*d^3*(b*c - 3*a*d)*x^(3*n) + 3*b^3*d^4*x^(4*n) + 12*c*(b*c - a*d)^3*Log[c + d*x^n])/(12*d^5*n)

IntegrateAlgebraic [A] time = 0.11, size = 155, normalized size = 1.19

$$\frac{x^n(12a^3d^3-36a^2bcd^2+18a^2bd^3x^n+36ab^2c^2d-18ab^2cd^2x^n+12ab^2d^3x^{2n}-12b^3c^3+6b^3c^2dx^n-4b^3cd^2x^{2n}+3b^3d^3x^{3n})}{12d^4n} + \frac{c(bc-ad)^3\log(c+dx^n)}{d^5n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(-1 + 2*n))*(a + b*x^n)^3]/(c + d*x^n), x]

[Out] (x^n*(-12*b^3*c^3 + 36*a*b^2*c^2*d - 36*a^2*b*c*d^2 + 12*a^3*d^3 + 6*b^3*c^2*d*x^n - 18*a*b^2*c*d^2*x^n + 18*a^2*b*d^3*x^n - 4*b^3*c*d^2*x^(2*n) + 12*a*b^2*d^3*x^(2*n) + 3*b^3*d^3*x^(3*n)))/(12*d^4*n) + (c*(b*c - a*d)^3*Log[c + d*x^n])/(d^5*n)

fricas [A] time = 0.45, size = 177, normalized size = 1.36

$$\frac{3b^3d^4x^{4n}-4(b^3cd^3-3ab^2d^4)x^{3n}+6(b^3c^2d^2-3ab^2cd^3+3a^2bd^4)x^{2n}-12(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3-a^3d^4)x^n+12(b^3c^4-3ab^2c^3d+3a^2bc^2d^2-a^3cd^3)\log(dx^n+c)}{12d^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n), x, algorithm="fricas")

[Out] $1/12*(3*b^3*d^4*x^{(4*n)} - 4*(b^3*c*d^3 - 3*a*b^2*d^4)*x^{(3*n)} + 6*(b^3*c^2*d^2 - 3*a*b^2*c*d^3 + 3*a^2*b*d^4)*x^{(2*n)} - 12*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*x^n + 12*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*\log(dx^n + c)/(d^5*n)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^3 x^{2n-1}}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")`

[Out] `integrate((b*x^n + a)^3*x^(2*n - 1)/(d*x^n + c), x)`

maple [B] time = 0.08, size = 284, normalized size = 2.18

$$-\frac{a^2 c \ln(d e^{n \ln(x)} + c)}{d^2 n} + \frac{a^3 e^{n \ln(x)}}{d n} + \frac{3 a^2 b c^2 \ln(d e^{n \ln(x)} + c)}{d^3 n} - \frac{3 a^2 b c e^{n \ln(x)}}{d^2 n} + \frac{3 a^2 b e^{2 n \ln(x)}}{2 d n} - \frac{3 a b^2 c^3 \ln(d e^{n \ln(x)} + c)}{d^4 n} + \frac{3 a b^2 c^2 e^{n \ln(x)}}{d^3 n} - \frac{3 a b^2 c e^{2 n \ln(x)}}{2 d^2 n} + \frac{a b^2 e^{3 n \ln(x)}}{d n} + \frac{b^2 c^4 \ln(d e^{n \ln(x)} + c)}{d^5 n} - \frac{b^3 c^3 e^{n \ln(x)}}{d^4 n} + \frac{b^3 c^2 e^{2 n \ln(x)}}{2 d^3 n} - \frac{b^3 c e^{3 n \ln(x)}}{3 d^2 n} + \frac{b^3 e^{4 n \ln(x)}}{4 d n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2*n-1)*(b*x^n+a)^3/(d*x^n+c),x)`

[Out] $1/d/n*\exp(n*\ln(x))*a^3-3/d^2/n*\exp(n*\ln(x))*a^2*b*c+3/d^3/n*\exp(n*\ln(x))*a*b^2*c^2-1/d^4/n*\exp(n*\ln(x))*b^3*c^3+1/4*b^3/d/n*\exp(n*\ln(x))^4+3/2*b/d/n*\exp(n*\ln(x))^2*a^2-3/2*b^2/d^2/n*\exp(n*\ln(x))^2*a*c+1/2*b^3/d^3/n*\exp(n*\ln(x))^2*c^2+b^2/d/n*\exp(n*\ln(x))^3*a-1/3*b^3/d^2/n*\exp(n*\ln(x))^3*c-c/d^2/n*\ln(d*\exp(n*\ln(x))+c)*a^3+3*c^2/d^3/n*\ln(d*\exp(n*\ln(x))+c)*a^2*b-3*c^3/d^4/n*\ln(d*\exp(n*\ln(x))+c)*a*b^2+c^4/d^5/n*\ln(d*\exp(n*\ln(x))+c)*b^3$

maxima [A] time = 0.53, size = 231, normalized size = 1.78

$$a^3 \left(\frac{x^n}{d n} - \frac{c \log\left(\frac{dx^n+c}{d}\right)}{d^2 n} \right) + \frac{1}{12} b^3 \left(\frac{12 c^4 \log\left(\frac{dx^n+c}{d}\right)}{d^5 n} + \frac{3 d^3 x^4 n - 4 c d^2 x^3 n + 6 c^2 d x^2 n - 12 c^3 x^n}{d^4 n} \right) - \frac{1}{2} a b^2 \left(\frac{6 c^3 \log\left(\frac{dx^n+c}{d}\right)}{d^4 n} - \frac{2 d^2 x^3 n - 3 c d x^2 n + 6 c^2 x^n}{d^3 n} \right) + \frac{3}{2} a^2 b \left(\frac{2 c^2 \log\left(\frac{dx^n+c}{d}\right)}{d^3 n} + \frac{d x^2 n - 2 c x^n}{d^2 n} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")`

[Out] $a^3*(x^n/(d*n) - c*\log((d*x^n + c)/d)/(d^2*n)) + 1/12*b^3*(12*c^4*\log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3*n) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/2*a*b^2*(6*c^3*\log((d*x^n + c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 3/2*a^2*b*(2*c^2*\log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1} (a + b x^n)^3}{c + d x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(2*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)`

[Out] `int((x^(2*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)`

sympy [A] time = 135.87, size = 320, normalized size = 2.46

$$\begin{cases} \frac{(a+b)^3 \log(x)}{c} & \text{for } d = 0 \wedge n = 0 \\ \frac{a^3 2^n + \frac{a^2 b n^{2n}}{2n} + \frac{3 a b^2 4^n}{4n} + \frac{b^3 5^n}{5n}}{c} & \text{for } d = 0 \\ \frac{(a+b)^3 \log(x)}{c+d} & \text{for } n = 0 \\ -\frac{a^3 c \log\left(\frac{c}{d} + x^n\right)}{d^2 n} + \frac{a^3 x^n}{d n} + \frac{3 a^2 b c^2 \log\left(\frac{c}{d} + x^n\right)}{d^3 n} - \frac{3 a^2 b c x^n}{d^2 n} + \frac{3 a^2 b x^{2n}}{2 d n} - \frac{3 a b^2 c^3 \log\left(\frac{c}{d} + x^n\right)}{d^4 n} + \frac{3 a b^2 c^2 x^n}{d^3 n} - \frac{3 a b^2 c x^{2n}}{2 d^2 n} + \frac{a b^2 x^{3n}}{d n} + \frac{b^3 c^4 \log\left(\frac{c}{d} + x^n\right)}{d^5 n} - \frac{b^3 c^3 x^n}{d^4 n} + \frac{b^3 c^2 x^{2n}}{2 d^3 n} - \frac{b^3 c x^{3n}}{3 d^2 n} + \frac{b^3 x^{4n}}{4 d n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+2*n)*(a+b*x**n)**3/(c+d*x**n), x)`

[Out] `Piecewise(((a + b)**3*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**3*x**(2*n)/(2*n) + a**2*b*x**(3*n)/n + 3*a*b**2*x**(4*n)/(4*n) + b**3*x**(5*n)/(5*n))/c, Eq(d, 0)), ((a + b)**3*log(x)/(c + d), Eq(n, 0)), (-a**3*c*log(c/d + x**n)/(d**2*n) + a**3*x**n/(d*n) + 3*a**2*b*c**2*log(c/d + x**n)/(d**3*n) - 3*a**2*b*c*x**n/(d**2*n) + 3*a**2*b*x**(2*n)/(2*d*n) - 3*a*b**2*c**3*log(c/d + x**n)/(d**4*n) + 3*a*b**2*c**2*x**n/(d**3*n) - 3*a*b**2*c*x**(2*n)/(2*d**2*n) + a*b**2*x**(3*n)/(d*n) + b**3*c**4*log(c/d + x**n)/(d**5*n) - b**3*c**3*x**n/(d**4*n) + b**3*c**2*x**(2*n)/(2*d**3*n) - b**3*c*x**(3*n)/(3*d**2*n) + b**3*x**(4*n)/(4*d*n), True))`

$$3.661 \quad \int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n} dx$$

Optimal. Leaf size=90

$$-\frac{c(bc-ad)^2 \log(c+dx^n)}{d^4n} + \frac{x^n(bc-ad)^2}{d^3n} - \frac{bx^{2n}(bc-2ad)}{2d^2n} + \frac{b^2x^{3n}}{3dn}$$

Rubi [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {446, 77}

$$\frac{x^n(bc-ad)^2}{d^3n} - \frac{bx^{2n}(bc-2ad)}{2d^2n} - \frac{c(bc-ad)^2 \log(c+dx^n)}{d^4n} + \frac{b^2x^{3n}}{3dn}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n), x]

[Out] ((b*c - a*d)^2*x^n)/(d^3*n) - (b*(b*c - 2*a*d)*x^(2*n))/(2*d^2*n) + (b^2*x^(3*n))/(3*d*n) - (c*(b*c - a*d)^2*Log[c + d*x^n])/(d^4*n)

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\int \frac{x^{-1+2n} (a + bx^n)^2}{c + dx^n} dx = \frac{\text{Subst} \left(\int \frac{x(a+bx)^2}{c+dx} dx, x, x^n \right)}{n}$$

$$= \frac{\text{Subst} \left(\int \left(\frac{(-bc+ad)^2}{d^3} - \frac{b(bc-2ad)x}{d^2} + \frac{b^2x^2}{d} - \frac{c(bc-ad)^2}{d^3(c+dx)} \right) dx, x, x^n \right)}{n}$$

$$= \frac{(bc-ad)^2x^n}{d^3n} - \frac{b(bc-2ad)x^{2n}}{2d^2n} + \frac{b^2x^{3n}}{3dn} - \frac{c(bc-ad)^2 \log(c+dx^n)}{d^4n}$$

Mathematica [A] time = 0.14, size = 82, normalized size = 0.91

$$\frac{-\frac{c(bc-ad)^2 \log(c+dx^n)}{d^4} + \frac{x^n(bc-ad)^2}{d^3} - \frac{bx^{2n}(bc-2ad)}{2d^2} + \frac{b^2x^{3n}}{3d}}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n), x]

[Out] (((b*c - a*d)^2*x^n)/d^3 - (b*(b*c - 2*a*d)*x^(2*n))/(2*d^2) + (b^2*x^(3*n))/(3*d) - (c*(b*c - a*d)^2*Log[c + d*x^n])/d^4)/n

IntegrateAlgebraic [A] time = 0.08, size = 97, normalized size = 1.08

$$\frac{x^n (6a^2d^2 - 12abcd + 6abd^2x^n + 6b^2c^2 - 3b^2cdx^n + 2b^2d^2x^{2n})}{6d^3n} - \frac{c(bc-ad)^2 \log(c+dx^n)}{d^4n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n), x]

[Out] (x^n*(6*b^2*c^2 - 12*a*b*c*d + 6*a^2*d^2 - 3*b^2*c*d*x^n + 6*a*b*d^2*x^n + 2*b^2*d^2*x^(2*n)))/(6*d^3*n) - (c*(b*c - a*d)^2*Log[c + d*x^n])/d^4*n

fricas [A] time = 0.44, size = 108, normalized size = 1.20

$$\frac{2b^2d^3x^{3n} - 3(b^2cd^2 - 2abd^3)x^{2n} + 6(b^2c^2d - 2abcd^2 + a^2d^3)x^n - 6(b^2c^3 - 2abc^2d + a^2cd^2) \log(dx^n + c)}{6d^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n), x, algorithm="fricas")

[Out] 1/6*(2*b^2*d^3*x^(3*n) - 3*(b^2*c*d^2 - 2*a*b*d^3)*x^(2*n) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^n - 6*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d*x^n + c))/(d^4*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2 x^{2n-1}}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*xⁿ)²/(c+d*xⁿ),x, algorithm="giac")

[Out] integrate((b*xⁿ + a)²*x^(2*n - 1)/(d*xⁿ + c), x)

maple [A] time = 0.08, size = 173, normalized size = 1.92

$$-\frac{a^2 c \ln(d e^{n \ln(x)} + c)}{d^2 n} + \frac{a^2 e^{n \ln(x)}}{d n} + \frac{2 a b c^2 \ln(d e^{n \ln(x)} + c)}{d^3 n} - \frac{2 a b c e^{n \ln(x)}}{d^2 n} + \frac{a b e^{2 n \ln(x)}}{d n} - \frac{b^2 c^3 \ln(d e^{n \ln(x)} + c)}{d^4 n} + \frac{b^2 c^2 e^{n \ln(x)}}{d^3 n} - \frac{b^2 c e^{2 n \ln(x)}}{2 d^2 n} + \frac{b^2 e^{3 n \ln(x)}}{3 d n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n-1)*(b*xⁿ+a)²/(d*xⁿ+c),x)

[Out] 1/d/n*exp(n*ln(x))*a²-2/d²/n*exp(n*ln(x))*a*b*c+1/d³/n*exp(n*ln(x))*b²*c²+1/3*b²/d/n*exp(n*ln(x))³+b/d/n*exp(n*ln(x))²*a-1/2*b²/d²/n*exp(n*ln(x))²*c-c/d²/n*ln(d*exp(n*ln(x))+c)*a²+2*c²/d³/n*ln(d*exp(n*ln(x))+c)*a*b-c³/d⁴/n*ln(d*exp(n*ln(x))+c)*b²

maxima [A] time = 0.70, size = 150, normalized size = 1.67

$$a^2 \left(\frac{x^n}{d n} - \frac{c \log\left(\frac{dx^n+c}{d}\right)}{d^2 n} \right) - \frac{1}{6} b^2 \left(\frac{6 c^3 \log\left(\frac{dx^n+c}{d}\right)}{d^4 n} - \frac{2 d^2 x^{3 n} - 3 c d x^{2 n} + 6 c^2 x^n}{d^3 n} \right) + a b \left(\frac{2 c^2 \log\left(\frac{dx^n+c}{d}\right)}{d^3 n} + \frac{d x^{2 n} - 2 c x^n}{d^2 n} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*xⁿ)²/(c+d*xⁿ),x, algorithm="maxima")

[Out] a²*(xⁿ/(d*n) - c*log((d*xⁿ + c)/d)/(d²*n)) - 1/6*b²*(6*c³*log((d*xⁿ + c)/d)/(d⁴*n) - (2*d²*x^(3*n) - 3*c*d*x^(2*n) + 6*c²*xⁿ)/(d³*n)) + a*b*(2*c²*log((d*xⁿ + c)/d)/(d³*n) + (d*x^(2*n) - 2*c*xⁿ)/(d²*n))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1} (a + b x^n)^2}{c + d x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(2*n - 1)*(a + b*xⁿ)²)/(c + d*xⁿ),x)

[Out] $\int ((x^{2n-1})(a + bx^n)^2)/(c + dx^n), x$

sympy [A] time = 63.51, size = 202, normalized size = 2.24

$$\begin{cases} \frac{(a+b)^2 \log(x)}{c} & \text{for } d = 0 \wedge n = 0 \\ \frac{\frac{a^2 x^{2n}}{2n} + \frac{2abx^{3n}}{3n} + \frac{b^2 x^{4n}}{4n}}{c} & \text{for } d = 0 \\ \frac{(a+b)^2 \log(x)}{c+d} & \text{for } n = 0 \\ -\frac{a^2 c \log\left(\frac{c}{d} + x^n\right)}{d^2 n} + \frac{a^2 x^n}{dn} + \frac{2abc^2 \log\left(\frac{c}{d} + x^n\right)}{d^3 n} - \frac{2abcx^n}{d^2 n} + \frac{abx^{2n}}{dn} - \frac{b^2 c^3 \log\left(\frac{c}{d} + x^n\right)}{d^4 n} + \frac{b^2 c^2 x^n}{d^3 n} - \frac{b^2 cx^{2n}}{2d^2 n} + \frac{b^2 x^{3n}}{3dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{(-1+2*n)}*(a+b*x^n)**2/(c+d*x^n), x)$

[Out] $\text{Piecewise}(((a + b)**2*\log(x)/c, \text{Eq}(d, 0) \ \& \ \text{Eq}(n, 0)), ((a**2*x**(2*n)/(2*n) + 2*a*b*x**(3*n)/(3*n) + b**2*x**(4*n)/(4*n))/c, \text{Eq}(d, 0)), ((a + b)**2*\log(x)/(c + d), \text{Eq}(n, 0)), (-a**2*c*\log(c/d + x^n)/(d**2*n) + a**2*x^n/(d*n) + 2*a*b*c**2*\log(c/d + x^n)/(d**3*n) - 2*a*b*c*x^n/(d**2*n) + a*b*x**(2*n)/(d*n) - b**2*c**3*\log(c/d + x^n)/(d**4*n) + b**2*c**2*x^n/(d**3*n) - b**2*c*x**(2*n)/(2*d**2*n) + b**2*x**(3*n)/(3*d*n), \text{True}))$

$$3.662 \quad \int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n} dx$$

Optimal. Leaf size=60

$$\frac{c(bc-ad)\log(c+dx^n)}{d^3n} - \frac{x^n(bc-ad)}{d^2n} + \frac{bx^{2n}}{2dn}$$

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {446, 77}

$$-\frac{x^n(bc-ad)}{d^2n} + \frac{c(bc-ad)\log(c+dx^n)}{d^3n} + \frac{bx^{2n}}{2dn}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n), x]

[Out] -(((b*c - a*d)*x^n)/(d^2*n)) + (b*x^(2*n))/(2*d*n) + (c*(b*c - a*d)*Log[c + d*x^n])/(d^3*n)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n} dx &= \frac{\text{Subst}\left(\int \frac{x^{(a+bx)}}{c+dx} dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{-bc+ad}{d^2} + \frac{bx}{d} + \frac{c(bc-ad)}{d^2(c+dx)}\right) dx, x, x^n\right)}{n} \\ &= -\frac{(bc-ad)x^n}{d^2n} + \frac{bx^{2n}}{2dn} + \frac{c(bc-ad)\log(c+dx^n)}{d^3n} \end{aligned}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.83

$$\frac{dx^n(2ad - 2bc + bdx^n) + 2c(bc - ad)\log(c + dx^n)}{2d^3n}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n), x]

[Out] (d*x^n*(-2*b*c + 2*a*d + b*d*x^n) + 2*c*(b*c - a*d)*Log[c + d*x^n])/(2*d^3*n)

IntegrateAlgebraic [A] time = 0.05, size = 55, normalized size = 0.92

$$\frac{(bc^2 - acd)\log(c + dx^n)}{d^3n} + \frac{x^n(2ad - 2bc + bdx^n)}{2d^2n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n), x]

[Out] (x^n*(-2*b*c + 2*a*d + b*d*x^n))/(2*d^2*n) + ((b*c^2 - a*c*d)*Log[c + d*x^n])/ (d^3*n)

fricas [A] time = 0.43, size = 56, normalized size = 0.93

$$\frac{bd^2x^{2n} - 2(bcd - ad^2)x^n + 2(bc^2 - acd)\log(dx^n + c)}{2d^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n), x, algorithm="fricas")

[Out] 1/2*(b*d^2*x^(2*n) - 2*(b*c*d - a*d^2)*x^n + 2*(b*c^2 - a*c*d)*log(d*x^n + c))/(d^3*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)x^{2n-1}}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*xⁿ)/(c+d*xⁿ), x, algorithm="giac")

[Out] integrate((b*xⁿ + a)*x^(2*n - 1)/(d*xⁿ + c), x)

maple [A] time = 0.07, size = 87, normalized size = 1.45

$$-\frac{ac \ln(d e^{n \ln(x)} + c)}{d^2 n} + \frac{a e^{n \ln(x)}}{dn} + \frac{b c^2 \ln(d e^{n \ln(x)} + c)}{d^3 n} - \frac{bc e^{n \ln(x)}}{d^2 n} + \frac{b e^{2n \ln(x)}}{2dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n-1)*(b*xⁿ+a)/(d*xⁿ+c), x)

[Out] 1/d/n*exp(n*ln(x))*a-1/d²/n*exp(n*ln(x))*b*c+1/2*b/d/n*exp(n*ln(x))²-c/d²/n*ln(d*exp(n*ln(x))+c)*a+c²/d³/n*ln(d*exp(n*ln(x))+c)*b

maxima [A] time = 0.70, size = 83, normalized size = 1.38

$$a \left(\frac{x^n}{dn} - \frac{c \log\left(\frac{dx^n+c}{d}\right)}{d^2 n} \right) + \frac{1}{2} b \left(\frac{2c^2 \log\left(\frac{dx^n+c}{d}\right)}{d^3 n} + \frac{dx^{2n} - 2cx^n}{d^2 n} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*xⁿ)/(c+d*xⁿ), x, algorithm="maxima")

[Out] a*(xⁿ/(d*n) - c*log((d*xⁿ + c)/d)/(d²*n)) + 1/2*b*(2*c²*log((d*xⁿ + c)/d)/(d³*n) + (d*x^(2*n) - 2*c*xⁿ)/(d²*n))

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{2n-1} (a + b x^n)}{c + d x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(2*n - 1)*(a + b*xⁿ))/(c + d*xⁿ), x)

[Out] int((x^(2*n - 1)*(a + b*xⁿ))/(c + d*xⁿ), x)

sympy [A] time = 26.97, size = 105, normalized size = 1.75

$$\begin{cases} \frac{(a+b)\log(x)}{c} & \text{for } d = 0 \wedge n = 0 \\ \frac{\frac{ax^{2n}}{2n} + \frac{bx^{3n}}{3n}}{c} & \text{for } d = 0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n = 0 \\ -\frac{ac\log\left(\frac{c}{d}+x^n\right)}{d^2n} + \frac{ax^n}{dn} + \frac{bc^2\log\left(\frac{c}{d}+x^n\right)}{d^3n} - \frac{bcx^n}{d^2n} + \frac{bx^{2n}}{2dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+2*n)*(a+b*x**n)/(c+d*x**n),x)

[Out] Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a*x**(2*n)/(2*n) + b*x**(3*n)/(3*n))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq(n, 0)), (-a*c*log(c/d + x**n)/(d**2*n) + a*x**n/(d*n) + b*c**2*log(c/d + x**n)/(d**3*n) - b*c*x**n/(d**2*n) + b*x**(2*n)/(2*d*n), True))

$$3.663 \quad \int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)} dx$$

Optimal. Leaf size=54

$$\frac{c \log(c + dx^n)}{dn(bc - ad)} - \frac{a \log(a + bx^n)}{bn(bc - ad)}$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {446, 72}

$$\frac{c \log(c + dx^n)}{dn(bc - ad)} - \frac{a \log(a + bx^n)}{bn(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)), x]

[Out] -((a*Log[a + b*x^n])/(b*(b*c - a*d)*n)) + (c*Log[c + d*x^n])/(d*(b*c - a*d)*n)

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)} dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)} dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)} + \frac{c}{(bc-ad)(c+dx)}\right) dx, x, x^n\right)}{n} \\ &= -\frac{a \log(a + bx^n)}{b(bc - ad)n} + \frac{c \log(c + dx^n)}{d(bc - ad)n} \end{aligned}$$

Mathematica [A] time = 0.05, size = 44, normalized size = 0.81

$$\frac{ad \log(a + bx^n) - bc \log(c + dx^n)}{b^2cdn - abd^2n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)/((a + b*xⁿ)*(c + d*xⁿ)), x]

[Out] -((a*d*Log[a + b*xⁿ] - b*c*Log[c + d*xⁿ])/(b²*c*d*n - a*b*d²*n))

IntegrateAlgebraic [A] time = 0.06, size = 55, normalized size = 1.02

$$\frac{a \log(a + bx^n)}{bn(bc - ad)} - \frac{c \log(c + dx^n)}{dn(ad - bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 2*n)/((a + b*xⁿ)*(c + d*xⁿ)), x]

[Out] -((a*Log[a + b*xⁿ])/(b*(b*c - a*d)*n)) - (c*Log[c + d*xⁿ])/(d*(-(b*c) + a*d)*n)

fricas [A] time = 0.44, size = 45, normalized size = 0.83

$$\frac{ad \log(bx^n + a) - bc \log(dx^n + c)}{(b^2cd - abd^2)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*xⁿ)/(c+d*xⁿ), x, algorithm="fricas")

[Out] -(a*d*log(b*xⁿ + a) - b*c*log(d*xⁿ + c))/((b²*c*d - a*b*d²)*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n + a)(dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*xⁿ)/(c+d*xⁿ), x, algorithm="giac")

[Out] integrate(x^(2*n - 1)/((b*xⁿ + a)*(d*xⁿ + c)), x)

maple [A] time = 0.07, size = 59, normalized size = 1.09

$$\frac{a \ln(b e^{n \ln(x)} + a)}{(ad - bc) bn} - \frac{c \ln(d e^{n \ln(x)} + c)}{(ad - bc) dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2*n-1)/(b*x^n+a)/(d*x^n+c),x)`

[Out] `a/(a*d-b*c)/b/n*ln(b*exp(n*ln(x))+a)-c/d/n/(a*d-b*c)*ln(d*exp(n*ln(x))+c)`

maxima [A] time = 0.59, size = 60, normalized size = 1.11

$$-\frac{a \log\left(\frac{bx^n+a}{b}\right)}{b^2cn - abdn} + \frac{c \log\left(\frac{dx^n+c}{d}\right)}{bcdn - ad^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")`

[Out] `-a*log((b*x^n + a)/b)/(b^2*c*n - a*b*d*n) + c*log((d*x^n + c)/d)/(b*c*d*n - a*d^2*n)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{2n-1}}{(a + b x^n)(c + d x^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2*n - 1)/((a + b*x^n)*(c + d*x^n)),x)`

[Out] `int(x^(2*n - 1)/((a + b*x^n)*(c + d*x^n)), x)`

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+2*n)/(a+b*x**n)/(c+d*x**n),x)`

[Out] Exception raised: HeuristicGCDFailed

$$3.664 \quad \int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)} dx$$

Optimal. Leaf size=75

$$\frac{a}{bn(bc-ad)(a+bx^n)} + \frac{c \log(a+bx^n)}{n(bc-ad)^2} - \frac{c \log(c+dx^n)}{n(bc-ad)^2}$$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {446, 77}

$$\frac{a}{bn(bc-ad)(a+bx^n)} + \frac{c \log(a+bx^n)}{n(bc-ad)^2} - \frac{c \log(c+dx^n)}{n(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]

[Out] a/(b*(b*c - a*d)*n*(a + b*x^n)) + (c*Log[a + b*x^n])/((b*c - a*d)^2*n) - (c*Log[c + d*x^n])/((b*c - a*d)^2*n)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)} dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)} dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^2} + \frac{bc}{(bc-ad)^2(a+bx)} - \frac{cd}{(bc-ad)^2(c+dx)}\right) dx, x, x^n\right)}{n}$$

$$= \frac{a}{b(bc-ad)n(a+bx^n)} + \frac{c \log(a+bx^n)}{(bc-ad)^2n} - \frac{c \log(c+dx^n)}{(bc-ad)^2n}$$

Mathematica [A] time = 0.14, size = 58, normalized size = 0.77

$$\frac{\frac{a(bc-ad)}{b(a+bx^n)} + c \log(a+bx^n) - c \log(c+dx^n)}{n(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]

[Out] ((a*(b*c - a*d))/(b*(a + b*x^n)) + c*Log[a + b*x^n] - c*Log[c + d*x^n])/((b*c - a*d)^2*n)

IntegrateAlgebraic [A] time = 0.09, size = 75, normalized size = 1.00

$$\frac{a}{bn(bc-ad)(a+bx^n)} + \frac{c \log(a+bx^n)}{n(bc-ad)^2} - \frac{c \log(c+dx^n)}{n(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]

[Out] a/(b*(b*c - a*d)*n*(a + b*x^n)) + (c*Log[a + b*x^n])/((b*c - a*d)^2*n) - (c*Log[c + d*x^n])/((b*c - a*d)^2*n)

fricas [A] time = 0.45, size = 120, normalized size = 1.60

$$\frac{abc - a^2d + (b^2cx^n + abc) \log(bx^n + a) - (b^2cx^n + abc) \log(dx^n + c)}{(b^4c^2 - 2ab^3cd + a^2b^2d^2)nx^n + (ab^3c^2 - 2a^2b^2cd + a^3bd^2)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")

[Out] $(a*b*c - a^2*d + (b^2*c*x^n + a*b*c)*\log(b*x^n + a) - (b^2*c*x^n + a*b*c)*\log(d*x^n + c))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^n + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*n)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n + a)^2(dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")

[Out] integrate(x^(2*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)

maple [A] time = 0.09, size = 109, normalized size = 1.45

$$\frac{c \ln(b e^{n \ln(x)} + a)}{(a^2 d^2 - 2abcd + b^2 c^2) n} - \frac{c \ln(d e^{n \ln(x)} + c)}{(a^2 d^2 - 2abcd + b^2 c^2) n} + \frac{e^{n \ln(x)}}{(ad - bc)(b e^{n \ln(x)} + a) n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n-1)/(b*x^n+a)^2/(d*x^n+c),x)

[Out] $1/(a*d-b*c)/n*\exp(n*\ln(x))/(b*\exp(n*\ln(x))+a)+c/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*\ln(b*\exp(n*\ln(x))+a)-c/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*\ln(d*\exp(n*\ln(x))+c)$

maxima [A] time = 0.52, size = 121, normalized size = 1.61

$$\frac{c \log\left(\frac{bx^n+a}{b}\right)}{b^2c^2n - 2abcdn + a^2d^2n} - \frac{c \log\left(\frac{dx^n+c}{d}\right)}{b^2c^2n - 2abcdn + a^2d^2n} + \frac{a}{ab^2cn - a^2bdn + (b^3cn - ab^2dn)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")

[Out] $c*\log((b*x^n + a)/b)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) - c*\log((d*x^n + c)/d)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) + a/(a*b^2*c*n - a^2*b*d*n + (b^3*c*n - a*b^2*d*n)*x^n)$

mapad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{(a + b x^n)^2 (c + d x^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

```
[Out] int(x^(2*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

```
sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**2/(c+d*x**n), x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

$$3.665 \quad \int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)} dx$$

Optimal. Leaf size=105

$$\frac{a}{2bn(bc-ad)(a+bx^n)^2} - \frac{c}{n(bc-ad)^2(a+bx^n)} - \frac{cd \log(a+bx^n)}{n(bc-ad)^3} + \frac{cd \log(c+dx^n)}{n(bc-ad)^3}$$

Rubi [A] time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {446, 77}

$$\frac{a}{2bn(bc-ad)(a+bx^n)^2} - \frac{c}{n(bc-ad)^2(a+bx^n)} - \frac{cd \log(a+bx^n)}{n(bc-ad)^3} + \frac{cd \log(c+dx^n)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]

[Out] a/(2*b*(b*c - a*d)*n*(a + b*x^n)^2) - c/((b*c - a*d)^2*n*(a + b*x^n)) - (c*d*Log[a + b*x^n])/((b*c - a*d)^3*n) + (c*d*Log[c + d*x^n])/((b*c - a*d)^3*n)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)} dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^3(c+dx)} dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^3} + \frac{bc}{(bc-ad)^2(a+bx)^2} - \frac{bcd}{(bc-ad)^3(a+bx)} + \frac{cd^2}{(bc-ad)^3(c+dx)}\right) dx, x, x^n\right)}{n}$$

$$= \frac{a}{2b(bc-ad)n(a+bx^n)^2} - \frac{c}{(bc-ad)^2n(a+bx^n)} - \frac{cd \log(a+bx^n)}{(bc-ad)^3n} + \frac{cd \log(c+dx^n)}{(bc-ad)^3n}$$

Mathematica [A] time = 0.17, size = 97, normalized size = 0.92

$$\frac{\frac{a}{2b(bc-ad)(a+bx^n)^2} - \frac{c}{(bc-ad)^2(a+bx^n)} - \frac{cd \log(a+bx^n)}{(bc-ad)^3} + \frac{cd \log(c+dx^n)}{(bc-ad)^3}}{n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)), x]

[Out] (a/(2*b*(b*c - a*d)*(a + b*x^n)^2) - c/((b*c - a*d)^2*(a + b*x^n)) - (c*d*Log[a + b*x^n])/(b*c - a*d)^3 + (c*d*Log[c + d*x^n])/(b*c - a*d)^3)/n

IntegrateAlgebraic [A] time = 0.12, size = 100, normalized size = 0.95

$$\frac{-a^2d - abc - 2b^2cx^n}{2bn(bc-ad)^2(a+bx^n)^2} - \frac{cd \log(a+bx^n)}{n(bc-ad)^3} + \frac{cd \log(c+dx^n)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)), x]

[Out] (-a*b*c) - a^2*d - 2*b^2*c*x^n)/(2*b*(b*c - a*d)^2*n*(a + b*x^n)^2) - (c*d*Log[a + b*x^n])/(b*c - a*d)^3*n + (c*d*Log[c + d*x^n])/(b*c - a*d)^3*n

fricas [B] time = 0.46, size = 267, normalized size = 2.54

$$\frac{ab^2c^2 - a^3d^2 + 2(b^3c^2 - ab^2cd)x^n + 2(b^3cdx^{2n} + 2ab^2cdx^n + a^2bcd) \log(bx^n + a) - 2(b^3cdx^{2n} + 2ab^2cdx^n + a^2bcd) \log(dx^n + c)}{2((b^6c^3 - 3ab^5c^2d + 3a^2b^4cd^2 - a^3b^3d^3)nx^{2n} + 2(ab^5c^3 - 3a^2b^4c^2d + 3a^3b^3cd^2 - a^4b^2d^3)nx^n + (a^2b^4c^3 - 3a^3b^3c^2d + 3a^4b^2cd^2 - a^5bd^3)n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n), x, algorithm="fricas")

[Out] -1/2*(a*b^2*c^2 - a^3*d^2 + 2*(b^3*c^2 - a*b^2*c*d)*x^n + 2*(b^3*c*d*x^(2*n)) + 2*a*b^2*c*d*x^n + a^2*b*c*d)*log(b*x^n + a) - 2*(b^3*c*d*x^(2*n) + 2*a*

$$b^2*c*d*x^n + a^2*b*c*d)*\log(d*x^n + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*n*x^(2*n) + 2*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*n*x^n + (a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*n)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n + a)^3(dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")

[Out] integrate(x^(2*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)

maple [A] time = 0.11, size = 203, normalized size = 1.93

$$\frac{cd \ln(b e^{n \ln(x)} + a)}{(a^3 d^3 - 3a^2 b c d^2 + 3a b^2 c^2 d - b^3 c^3)n} - \frac{cd \ln(d e^{n \ln(x)} + c)}{(a^3 d^3 - 3a^2 b c d^2 + 3a b^2 c^2 d - b^3 c^3)n} + \frac{\frac{bc e^{n \ln(x)}}{(a^2 d^2 - 2abcd + b^2 c^2)n} + \frac{(-abd - b^2 c)a}{2(a^2 d^2 - 2abcd + b^2 c^2)b^2 n}}{(b e^{n \ln(x)} + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n-1)/(b*x^n+a)^3/(d*x^n+c),x)

[Out] (-b*c/(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*exp(n*ln(x))+1/2*a*(-a*b*d-b^2*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/n)/(b*exp(n*ln(x))+a)^2+c*d/n/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(b*exp(n*ln(x))+a)-c*d/n/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(d*exp(n*ln(x))+c)

maxima [B] time = 0.66, size = 243, normalized size = 2.31

$$\frac{cd \log\left(\frac{bx+a}{b}\right)}{b^3 c^3 n - 3 a b^2 c^2 d n + 3 a^2 b c d^2 n - a^3 d^3 n} + \frac{cd \log\left(\frac{dx+c}{d}\right)}{b^3 c^3 n - 3 a b^2 c^2 d n + 3 a^2 b c d^2 n - a^3 d^3 n} - \frac{2 b^2 c x^n + a b c + a^2 d}{2(a^2 b^3 c^2 n - 2 a^3 b^2 c d n + a^4 b d^2 n + (b^5 c^2 n - 2 a b^4 c d n + a^2 b^3 d^2 n)x^2 + 2(a b^4 c^2 n - 2 a^2 b^3 c d n + a^3 b^2 d^2 n)x^n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")

[Out] -c*d*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^3*d^3*n) + c*d*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^3*d^3*n) - 1/2*(2*b^2*c*x^n + a*b*c + a^2*d)/(a^2*b^3*c^2*n - 2*a^3*b^2*c*d*n + a^4*b*d^2*n + (b^5*c^2*n - 2*a*b^4*c*d*n + a^2*b^3*d^2*n)*x^(2*n) + 2*(a*b^4*c^2*n - 2*a^2*b^3*c*d*n + a^3*b^2*d^2*n)*x^n)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{(a + b x^n)^3 (c + d x^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)
```

```
[Out] int(x^(2*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)
```

```
sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**3/(c+d*x**n), x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

$$3.666 \quad \int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n} dx$$

Optimal. Leaf size=158

$$\frac{bx^{3n}(3a^2d^2 - 3abcd + b^2c^2)}{3d^3n} - \frac{b^2x^{4n}(bc - 3ad)}{4d^2n} - \frac{c^2(bc - ad)^3 \log(c + dx^n)}{d^6n} + \frac{cx^n(bc - ad)^3}{d^5n} - \frac{x^{2n}(bc - ad)^3}{2d^4n} + \frac{b^3x^{5n}}{5dn}$$

Rubi [A] time = 0.15, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {446, 88}

$$\frac{bx^{3n}(3a^2d^2 - 3abcd + b^2c^2)}{3d^3n} - \frac{b^2x^{4n}(bc - 3ad)}{4d^2n} - \frac{c^2(bc - ad)^3 \log(c + dx^n)}{d^6n} + \frac{cx^n(bc - ad)^3}{d^5n} - \frac{x^{2n}(bc - ad)^3}{2d^4n} + \frac{b^3x^{5n}}{5dn}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + 3*n))*(a + b*x^n)^3]/(c + d*x^n), x]

[Out] (c*(b*c - a*d)^3*x^n)/(d^5*n) - ((b*c - a*d)^3*x^(2*n))/(2*d^4*n) + (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(3*n))/(3*d^3*n) - (b^2*(b*c - 3*a*d)*x^(4*n))/(4*d^2*n) + (b^3*x^(5*n))/(5*d*n) - (c^2*(b*c - a*d)^3*Log[c + d*x^n])/(d^6*n)

Rule 88

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegerQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\int \frac{x^{-1+3n} (a + bx^n)^3}{c + dx^n} dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^3}{c+dx} dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{c(bc-ad)^3}{d^5} + \frac{(-bc+ad)^3x}{d^4} + \frac{b(b^2c^2-3abcd+3a^2d^2)x^2}{d^3} - \frac{b^2(bc-3ad)x^3}{d^2} + \frac{b^3x^4}{d} - \frac{c^2(bc-ad)^3}{d^5(c+dx)}\right) dx, x, x^n\right)}{n}$$

$$= \frac{c(bc-ad)^3x^n}{d^5n} - \frac{(bc-ad)^3x^{2n}}{2d^4n} + \frac{b(b^2c^2-3abcd+3a^2d^2)x^{3n}}{3d^3n} - \frac{b^2(bc-3ad)x^{4n}}{4d^2n} + \frac{b^3x^{5n}}{5d} - \frac{c^2(bc-ad)^3}{d^5(c+dx)}$$

Mathematica [A] time = 0.31, size = 138, normalized size = 0.87

$$\frac{20bd^3x^{3n}(3a^2d^2 - 3abcd + b^2c^2) - 15b^2d^4x^{4n}(bc - 3ad) - 60c^2(bc - ad)^3 \log(c + dx^n) + 30d^2x^{2n}(ad - bc)^3 + 60cdx^n(bc - ad)^3 + 12b^3d^5x^{5n}}{60d^6n}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n), x]

[Out] (60*c*d*(b*c - a*d)^3*x^n + 30*d^2*(-(b*c) + a*d)^3*x^(2*n) + 20*b*d^3*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(3*n) - 15*b^2*d^4*(b*c - 3*a*d)*x^(4*n) + 12*b^3*d^5*x^(5*n) - 60*c^2*(b*c - a*d)^3*Log[c + d*x^n])/(60*d^6*n)

IntegrateAlgebraic [A] time = 0.13, size = 220, normalized size = 1.39

$$\frac{x^n(-60a^3cd^3 + 30a^3d^4x^n + 180a^2bc^2d^2 - 90a^2bcd^3x^n + 60a^2bd^4x^{2n} - 180ab^2c^2d + 90ab^2c^2d^2x^n - 60ab^2cd^3x^{2n} + 45ab^2d^4x^{3n} + 60b^3c^4 - 30b^3c^3dx^n + 20b^3c^2d^2x^{2n} - 15b^3cd^3x^{3n} + 12b^3d^4x^{4n}) - c^2(bc - ad)^3 \log(c + dx^n)}{60d^6n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n), x]

[Out] (x^n*(60*b^3*c^4 - 180*a*b^2*c^3*d + 180*a^2*b*c^2*d^2 - 60*a^3*c*d^3 - 30*b^3*c^3*d*x^n + 90*a*b^2*c^2*d^2*x^n - 90*a^2*b*c*d^3*x^n + 30*a^3*d^4*x^n + 20*b^3*c^2*d^2*x^(2*n) - 60*a*b^2*c*d^3*x^(2*n) + 60*a^2*b*d^4*x^(2*n) - 15*b^3*c*d^3*x^(3*n) + 45*a*b^2*d^4*x^(3*n) + 12*b^3*d^4*x^(4*n)))/(60*d^5*n) - (c^2*(b*c - a*d)^3*Log[c + d*x^n])/(d^6*n)

fricas [A] time = 0.49, size = 230, normalized size = 1.46

$$\frac{12b^3d^5x^{5n} - 15(b^3cd^4 - 3ab^2d^5)x^{4n} + 20(b^3c^2d^2 - 3ab^2cd^3 + 3a^2bd^4)x^{3n} - 30(b^3c^3d^2 - 3ab^2c^2d^3 + 3a^2bcd^4 - a^3d^5)x^{2n} + 60(b^3c^4d - 3ab^2c^3d^2 + 3a^2bc^2d^3 - a^3cd^4)x^n - 60(b^3c^5 - 3ab^2c^4d + 3a^2bc^3d^2 - a^3c^2d^3) \log(dx^n + c)}{60d^6n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n), x, algorithm="fricas")

[Out] $\frac{1}{60}*(12*b^3*d^5*x^{(5*n)} - 15*(b^3*c*d^4 - 3*a*b^2*d^5)*x^{(4*n)} + 20*(b^3*c^2*d^3 - 3*a*b^2*c*d^4 + 3*a^2*b*d^5)*x^{(3*n)} - 30*(b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^{(2*n)} + 60*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x^n - 60*(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*\log(d*x^n + c))/(d^6*n)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^3 x^{3n-1}}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")`

[Out] `integrate((b*x^n + a)^3*x^(3*n - 1)/(d*x^n + c), x)`

maple [B] time = 0.07, size = 342, normalized size = 2.16

$$\frac{a^3 c^2 \ln\left(\frac{x^n + \frac{c}{d}}{d}\right)}{d^3 n} - \frac{a^3 c x^n}{d^2 n} + \frac{a^3 x^{2n}}{2dn} - \frac{3a^2 b c^3 \ln\left(\frac{x^n + \frac{c}{d}}{d}\right)}{d^3 n} + \frac{3a^2 b c^2 x^n}{d^3 n} - \frac{3a^2 b c x^{2n}}{2d^2 n} + \frac{a^2 b x^{3n}}{dn} + \frac{3a b^2 c^4 \ln\left(\frac{x^n + \frac{c}{d}}{d}\right)}{d^3 n} - \frac{3a b^2 c^3 x^n}{d^4 n} + \frac{3a b^2 c^2 x^{2n}}{2d^2 n} - \frac{a b^2 c x^{3n}}{d^2 n} + \frac{3a b^2 x^{4n}}{4dn} - \frac{b^3 c^5 \ln\left(\frac{x^n + \frac{c}{d}}{d}\right)}{d^3 n} + \frac{b^3 c^4 x^n}{d^3 n} - \frac{b^3 c^3 x^{2n}}{2d^4 n} + \frac{b^3 c^2 x^{3n}}{3d^3 n} - \frac{b^3 c x^{4n}}{4d^2 n} + \frac{b^3 x^{5n}}{5dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3*n-1)*(b*x^n+a)^3/(d*x^n+c),x)`

[Out] $\frac{1}{5}b^3/d/n*(x^n)^5 + \frac{3}{4}b^2/d/n*(x^n)^4*a - \frac{1}{4}b^3/d^2/n*(x^n)^4*c + b/d/n*(x^n)^3*a^2 - b^2/d^2/n*(x^n)^3*a*c + \frac{1}{3}b^3/d^3/n*(x^n)^3*c^2 + \frac{1}{2}d/n*(x^n)^2*a^3 - \frac{3}{2}d^2/n*(x^n)^2*a^2*b*c + \frac{3}{2}d^3/n*(x^n)^2*a*b^2*c^2 - \frac{1}{2}d^4/n*(x^n)^2*b^3*c^3 - c/d^2/n*x^n*a^3 + \frac{3}{2}d^3/n*x^n*a^2*b - \frac{3}{2}d^4/n*x^n*a*b^2 + c^4/d^5/n*x^n*b^3 + c^2/d^3/n*\ln(x^n+c/d)*a^3 - \frac{3}{2}d^4/n*\ln(x^n+c/d)*a^2*b + \frac{3}{2}d^4/d^5/n*\ln(x^n+c/d)*a*b^2 - c^5/d^6/n*\ln(x^n+c/d)*b^3$

maxima [A] time = 0.66, size = 286, normalized size = 1.81

$$\frac{1}{60}b^3 \left(\frac{60c^5 \log\left(\frac{dx^n+c}{d}\right)}{d^6 n} - \frac{12d^4 x^5 n - 15cd^3 x^4 n + 20c^2 d^2 x^3 n - 30c^3 dx^2 n + 60c^4 x^n}{d^6 n} \right) + \frac{1}{4}d^2 \left(\frac{12c^4 \log\left(\frac{dx^n+c}{d}\right)}{d^6 n} + \frac{3d^3 x^{4n} - 4cd^2 x^3 n + 6c^2 dx^2 n - 12c^3 x^n}{d^6 n} \right) - \frac{1}{2}d^2 b \left(\frac{6c^3 \log\left(\frac{dx^n+c}{d}\right)}{d^4 n} - \frac{2d^2 x^3 n - 3cdx^2 n + 6c^2 x^n}{d^4 n} \right) + \frac{1}{2}d^3 \left(\frac{2c^2 \log\left(\frac{dx^n+c}{d}\right)}{d^3 n} + \frac{dx^2 n - 2cx^n}{d^3 n} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")`

[Out] $-1/60*b^3*(60*c^5*\log((d*x^n + c)/d)/(d^6*n) - (12*d^4*x^{(5*n)} - 15*c*d^3*x^{(4*n)} + 20*c^2*d^2*x^{(3*n)} - 30*c^3*d*x^{(2*n)} + 60*c^4*x^n)/(d^5*n)) + 1/4*a*b^2*(12*c^4*\log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^{(4*n)} - 4*c*d^2*x^{(3*n)} + 6*c^2*d*x^{(2*n)} - 12*c^3*x^n)/(d^4*n)) - 1/2*a^2*b*(6*c^3*\log((d*x^n + c)/d)/(d^4*n) - (2*d^2*x^{(3*n)} - 3*c*d*x^{(2*n)} + 6*c^2*x^n)/(d^3*n)) + 1/2*a^3*(2*c^2*\log((d*x^n + c)/d)/(d^3*n) + (d*x^{(2*n)} - 2*c*x^n)/(d^2*n))$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1} (a + b x^n)^3}{c + d x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(3*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)

[Out] int((x^(3*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+3*n)*(a+b*x**n)**3/(c+d*x**n), x)

[Out] Timed out

$$3.667 \quad \int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n} dx$$

Optimal. Leaf size=118

$$\frac{c^2(bc-ad)^2 \log(c+dx^n)}{d^5n} - \frac{cx^n(bc-ad)^2}{d^4n} + \frac{x^{2n}(bc-ad)^2}{2d^3n} - \frac{bx^{3n}(bc-2ad)}{3d^2n} + \frac{b^2x^{4n}}{4dn}$$

Rubi [A] time = 0.12, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {446, 88}

$$\frac{c^2(bc-ad)^2 \log(c+dx^n)}{d^5n} - \frac{cx^n(bc-ad)^2}{d^4n} + \frac{x^{2n}(bc-ad)^2}{2d^3n} - \frac{bx^{3n}(bc-2ad)}{3d^2n} + \frac{b^2x^{4n}}{4dn}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + 3*n))*(a + b*x^n)^2]/(c + d*x^n), x]

[Out] -((c*(b*c - a*d)^2*x^n)/(d^4*n)) + ((b*c - a*d)^2*x^(2*n))/(2*d^3*n) - (b*(b*c - 2*a*d)*x^(3*n))/(3*d^2*n) + (b^2*x^(4*n))/(4*d*n) + (c^2*(b*c - a*d)^2*Log[c + d*x^n])/(d^5*n)

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\int \frac{x^{-1+3n} (a + bx^n)^2}{c + dx^n} dx = \frac{\text{Subst} \left(\int \frac{x^{2(a+bx)^2}}{c+dx} dx, x, x^n \right)}{n}$$

$$= \frac{\text{Subst} \left(\int \left(-\frac{c(bc-ad)^2}{d^4} + \frac{(-bc+ad)^2 x}{d^3} - \frac{b(bc-2ad)x^2}{d^2} + \frac{b^2 x^3}{d} + \frac{c^2(bc-ad)^2}{d^4(c+dx)} \right) dx, x, x^n \right)}{n}$$

$$= -\frac{c(bc-ad)^2 x^n}{d^4 n} + \frac{(bc-ad)^2 x^{2n}}{2d^3 n} - \frac{b(bc-2ad)x^{3n}}{3d^2 n} + \frac{b^2 x^{4n}}{4dn} + \frac{c^2(bc-ad)^2 \log(c+dx^n)}{d^5 n}$$

Mathematica [A] time = 0.27, size = 103, normalized size = 0.87

$$\frac{12c^2(bc-ad)^2 \log(c+dx^n) - 4bd^3 x^{3n}(bc-2ad) + 6d^2 x^{2n}(bc-ad)^2 - 12cdx^n(bc-ad)^2 + 3b^2 d^4 x^{4n}}{12d^5 n}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n), x]

[Out] (-12*c*d*(b*c - a*d)^2*x^n + 6*d^2*(b*c - a*d)^2*x^(2*n) - 4*b*d^3*(b*c - 2*a*d)*x^(3*n) + 3*b^2*d^4*x^(4*n) + 12*c^2*(b*c - a*d)^2*Log[c + d*x^n])/(12*d^5*n)

IntegrateAlgebraic [A] time = 0.10, size = 141, normalized size = 1.19

$$\frac{x^n (-12a^2cd^2 + 6a^2d^3x^n + 24abcd - 12abcd^2x^n + 8abd^3x^{2n} - 12b^2c^3 + 6b^2c^2dx^n - 4b^2cd^2x^{2n} + 3b^2d^3x^{3n})}{12d^4n} + \frac{c^2(bc-ad)^2 \log(c+dx^n)}{d^5n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n), x]

[Out] (x^n*(-12*b^2*c^3 + 24*a*b*c^2*d - 12*a^2*c*d^2 + 6*b^2*c^2*d*x^n - 12*a*b*c*d^2*x^n + 6*a^2*d^3*x^n - 4*b^2*c*d^2*x^(2*n) + 8*a*b*d^3*x^(2*n) + 3*b^2*d^3*x^(3*n)))/(12*d^4*n) + (c^2*(b*c - a*d)^2*Log[c + d*x^n])/(d^5*n)

fricas [A] time = 0.48, size = 146, normalized size = 1.24

$$\frac{3b^2d^4x^{4n} - 4(b^2cd^3 - 2abd^4)x^{3n} + 6(b^2c^2d^2 - 2abcd^3 + a^2d^4)x^{2n} - 12(b^2c^3d - 2abc^2d^2 + a^2cd^3)x^n + 12(b^2c^4 - 2abc^3d + a^2c^2d^2) \log(dx^n + c)}{12d^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n), x, algorithm="fricas")

[Out] 1/12*(3*b^2*d^4*x^(4*n) - 4*(b^2*c*d^3 - 2*a*b*d^4)*x^(3*n) + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^(2*n) - 12*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^n + 12*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*log(d*x^n + c))/(d^5*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2 x^{3n-1}}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")

[Out] integrate((b*x^n + a)^2*x^(3*n - 1)/(d*x^n + c), x)

maple [B] time = 0.07, size = 236, normalized size = 2.00

$$\frac{a^2 c^2 \ln(d e^{n \ln(x)} + c)}{d^3 n} - \frac{a^2 c e^{n \ln(x)}}{d^2 n} + \frac{a^2 e^{2n \ln(x)}}{2dn} - \frac{2ab c^3 \ln(d e^{n \ln(x)} + c)}{d^4 n} + \frac{2ab c^2 e^{n \ln(x)}}{d^3 n} - \frac{abc e^{2n \ln(x)}}{d^2 n} + \frac{2ab e^{3n \ln(x)}}{3dn} + \frac{b^2 c^4 \ln(d e^{n \ln(x)} + c)}{d^5 n} - \frac{b^2 c^3 e^{n \ln(x)}}{d^4 n} + \frac{b^2 c^2 e^{2n \ln(x)}}{2d^3 n} - \frac{b^2 c e^{3n \ln(x)}}{3d^2 n} + \frac{b^2 e^{4n \ln(x)}}{4dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3*n-1)*(b*x^n+a)^2/(d*x^n+c),x)

[Out] $\frac{1}{4} b^2 \frac{1}{d} \frac{1}{n} \exp(n \ln(x))^4 + \frac{1}{2} \frac{1}{d} \frac{1}{n} \exp(n \ln(x))^2 a^2 - \frac{1}{d^2} \frac{1}{n} \exp(n \ln(x))^2 2 a b c + \frac{1}{2} \frac{1}{d^3} \frac{1}{n} \exp(n \ln(x))^2 b^2 c^2 + \frac{2}{3} \frac{1}{b} \frac{1}{d} \frac{1}{n} \exp(n \ln(x))^3 a - \frac{1}{3} \frac{1}{b^2} \frac{1}{d^2} \frac{1}{n} \exp(n \ln(x))^3 c - \frac{c}{d^2} \frac{1}{n} \exp(n \ln(x)) a^2 + \frac{2}{d^3} \frac{1}{n} \exp(n \ln(x)) a b - \frac{c^3}{d^4} \frac{1}{n} \exp(n \ln(x)) b^2 + \frac{c^2}{d^3} \frac{1}{n} \ln(d \exp(n \ln(x)) + c) a^2 - \frac{2}{d^4} \frac{1}{n} \ln(d \exp(n \ln(x)) + c) a b + \frac{c^4}{d^5} \frac{1}{n} \ln(d \exp(n \ln(x)) + c) b^2$

maxima [A] time = 0.54, size = 192, normalized size = 1.63

$$\frac{1}{12} b^2 \left(\frac{12 c^4 \log\left(\frac{dx^n+c}{d}\right)}{d^5 n} + \frac{3 d^3 x^4 n - 4 c d^2 x^3 n + 6 c^2 d x^2 n - 12 c^3 x n}{d^4 n} \right) - \frac{1}{3} a b \left(\frac{6 c^3 \log\left(\frac{dx^n+c}{d}\right)}{d^4 n} - \frac{2 d^2 x^3 n - 3 c d x^2 n + 6 c^2 x n}{d^3 n} \right) + \frac{1}{2} a^2 \left(\frac{2 c^2 \log\left(\frac{dx^n+c}{d}\right)}{d^3 n} + \frac{d x^2 n - 2 c x n}{d^2 n} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")

[Out] $\frac{1}{12} b^2 \left(\frac{12 c^4 \log\left(\frac{d x^n + c}{d}\right)}{d^5 n} + \frac{3 d^3 x^4 n - 4 c d^2 x^3 n + 6 c^2 d x^2 n - 12 c^3 x n}{d^4 n} \right) - \frac{1}{3} a b \left(\frac{6 c^3 \log\left(\frac{d x^n + c}{d}\right)}{d^4 n} - \frac{2 d^2 x^3 n - 3 c d x^2 n + 6 c^2 x n}{d^3 n} \right) + \frac{1}{2} a^2 \left(\frac{2 c^2 \log\left(\frac{d x^n + c}{d}\right)}{d^3 n} + \frac{d x^2 n - 2 c x n}{d^2 n} \right)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1} (a + b x^n)^2}{c + d x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(3*n - 1)*(a + b*x^n)^2)/(c + d*x^n),x)

```
[Out] int((x^(3*n - 1)*(a + b*x^n)^2)/(c + d*x^n), x)
```

```
sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**2/(c+d*x**n), x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

$$3.668 \quad \int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n} dx$$

Optimal. Leaf size=86

$$-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4n} + \frac{cx^n(bc-ad)}{d^3n} - \frac{x^{2n}(bc-ad)}{2d^2n} + \frac{bx^{3n}}{3dn}$$

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {446, 77}

$$-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4n} + \frac{cx^n(bc-ad)}{d^3n} - \frac{x^{2n}(bc-ad)}{2d^2n} + \frac{bx^{3n}}{3dn}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n), x]

[Out] (c*(b*c - a*d)*x^n)/(d^3*n) - ((b*c - a*d)*x^(2*n))/(2*d^2*n) + (b*x^(3*n))/(3*d*n) - (c^2*(b*c - a*d)*Log[c + d*x^n])/(d^4*n)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x]
/; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n} dx = \frac{\text{Subst}\left(\int \frac{x^{2(a+bx)}}{c+dx} dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{c(bc-ad)}{d^3} + \frac{(-bc+ad)x}{d^2} + \frac{bx^2}{d} - \frac{c^2(bc-ad)}{d^3(c+dx)}\right) dx, x, x^n\right)}{n}$$

$$= \frac{c(bc-ad)x^n}{d^3n} - \frac{(bc-ad)x^{2n}}{2d^2n} + \frac{bx^{3n}}{3dn} - \frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}$$

Mathematica [A] time = 0.10, size = 78, normalized size = 0.91

$$\frac{-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4} + \frac{cx^n(bc-ad)}{d^3} - \frac{x^{2n}(bc-ad)}{2d^2} + \frac{bx^{3n}}{3d}}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n), x]

[Out] ((c*(b*c - a*d)*x^n)/d^3 - ((b*c - a*d)*x^(2*n))/(2*d^2) + (b*x^(3*n))/(3*d) - (c^2*(b*c - a*d)*Log[c + d*x^n])/d^4)/n

IntegrateAlgebraic [A] time = 0.07, size = 82, normalized size = 0.95

$$\frac{x^n(-6acd + 3ad^2x^n + 6bc^2 - 3bcdx^n + 2bd^2x^{2n})}{6d^3n} + \frac{(ac^2d - bc^3)\log(c + dx^n)}{d^4n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n), x]

[Out] (x^n*(6*b*c^2 - 6*a*c*d - 3*b*c*d*x^n + 3*a*d^2*x^n + 2*b*d^2*x^(2*n)))/(6*d^3*n) + ((-(b*c^3) + a*c^2*d)*Log[c + d*x^n])/(d^4*n)

fricas [A] time = 0.43, size = 82, normalized size = 0.95

$$\frac{2bd^3x^{3n} - 3(bcd^2 - ad^3)x^{2n} + 6(bc^2d - acd^2)x^n - 6(bc^3 - ac^2d)\log(dx^n + c)}{6d^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n), x, algorithm="fricas")

[Out] 1/6*(2*b*d^3*x^(3*n) - 3*(b*c*d^2 - a*d^3)*x^(2*n) + 6*(b*c^2*d - a*c*d^2)*x^n - 6*(b*c^3 - a*c^2*d)*log(d*x^n + c))/(d^4*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)x^{3n-1}}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)*(a+b*xⁿ)/(c+d*xⁿ),x, algorithm="giac")

[Out] integrate((b*xⁿ + a)*x^(3*n - 1)/(d*xⁿ + c), x)

maple [A] time = 0.07, size = 125, normalized size = 1.45

$$\frac{a c^2 \ln(d e^{n \ln(x)} + c)}{d^3 n} - \frac{a c e^{n \ln(x)}}{d^2 n} + \frac{a e^{2 n \ln(x)}}{2 d n} - \frac{b c^3 \ln(d e^{n \ln(x)} + c)}{d^4 n} + \frac{b c^2 e^{n \ln(x)}}{d^3 n} - \frac{b c e^{2 n \ln(x)}}{2 d^2 n} + \frac{b e^{3 n \ln(x)}}{3 d n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3*n-1)*(b*xⁿ+a)/(d*xⁿ+c),x)

[Out] 1/3*b/d/n*exp(n*ln(x))^{3+1/2/d/n}*exp(n*ln(x))^{2*a-1/2/d^2/n}*exp(n*ln(x))^{2*b*c-c/d^2/n}*exp(n*ln(x))*a+c^{2/d^3/n}*exp(n*ln(x))*b+c^{2/d^3/n}*ln(d*exp(n*ln(x))+c)*a-c^{3/d^4/n}*ln(d*exp(n*ln(x))+c)*b

maxima [A] time = 0.53, size = 112, normalized size = 1.30

$$-\frac{1}{6} b \left(\frac{6 c^3 \log\left(\frac{dx^n+c}{d}\right)}{d^4 n} - \frac{2 d^2 x^{3n} - 3 c d x^{2n} + 6 c^2 x^n}{d^3 n} \right) + \frac{1}{2} a \left(\frac{2 c^2 \log\left(\frac{dx^n+c}{d}\right)}{d^3 n} + \frac{dx^{2n} - 2 c x^n}{d^2 n} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)*(a+b*xⁿ)/(c+d*xⁿ),x, algorithm="maxima")

[Out] -1/6*b*(6*c³*log((d*xⁿ + c)/d)/(d⁴*n) - (2*d²*x^(3*n) - 3*c*d*x^(2*n) + 6*c²*xⁿ)/(d³*n) + 1/2*a*(2*c²*log((d*xⁿ + c)/d)/(d³*n) + (d*x^(2*n) - 2*c*xⁿ)/(d²*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1} (a + b x^n)}{c + d x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(3*n - 1)*(a + b*xⁿ))/(c + d*xⁿ),x)

[Out] `int((x^(3*n - 1)*(a + b*x^n))/(c + d*x^n), x)`

sympy [A] time = 57.94, size = 139, normalized size = 1.62

$$\begin{cases} \frac{(a+b)\log(x)}{c} & \text{for } d = 0 \wedge n = 0 \\ \frac{\frac{ax^{3n}}{3n} + \frac{bx^{4n}}{4n}}{c} & \text{for } d = 0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n = 0 \\ \frac{ac^2 \log\left(\frac{c}{d} + x^n\right)}{d^3 n} - \frac{acx^n}{d^2 n} + \frac{ax^{2n}}{2dn} - \frac{bc^3 \log\left(\frac{c}{d} + x^n\right)}{d^4 n} + \frac{bc^2 x^n}{d^3 n} - \frac{bcx^{2n}}{2d^2 n} + \frac{bx^{3n}}{3dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+3*n)*(a+b*x**n)/(c+d*x**n), x)`

[Out] `Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a*x**(3*n)/(3*n) + b*x**(4*n)/(4*n))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq(n, 0)), (a*c**2*log(c/d + x**n)/(d**3*n) - a*c*x**n/(d**2*n) + a*x**(2*n)/(2*d*n) - b*c**3*log(c/d + x**n)/(d**4*n) + b*c**2*x**n/(d**3*n) - b*c*x**(2*n)/(2*d**2*n) + b*x**(3*n)/(3*d*n), True))`

$$3.669 \quad \int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)} dx$$

Optimal. Leaf size=71

$$\frac{a^2 \log(a+bx^n)}{b^2n(bc-ad)} - \frac{c^2 \log(c+dx^n)}{d^2n(bc-ad)} + \frac{x^n}{bdn}$$

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {446, 72}

$$\frac{a^2 \log(a+bx^n)}{b^2n(bc-ad)} - \frac{c^2 \log(c+dx^n)}{d^2n(bc-ad)} + \frac{x^n}{bdn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)), x]

[Out] x^n/(b*d*n) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)*n) - (c^2*Log[c + d*x^n])/(d^2*(b*c - a*d)*n)

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)} dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{bd} + \frac{a^2}{b(bc-ad)(a+bx)} + \frac{c^2}{d(-bc+ad)(c+dx)}\right) dx, x, x^n\right)}{n} \\ &= \frac{x^n}{bdn} + \frac{a^2 \log(a+bx^n)}{b^2(bc-ad)n} - \frac{c^2 \log(c+dx^n)}{d^2(bc-ad)n} \end{aligned}$$

Mathematica [A] time = 0.11, size = 66, normalized size = 0.93

$$\frac{\frac{a^2 \log(ax^n) - c^2 \log(cx^n) + \frac{x^n}{bd}}{b^2(bc-ad)} - \frac{c^2 \log(cx^n) + \frac{x^n}{bd}}{d^2(bc-ad)}}{n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)), x]

[Out] (x^n/(b*d) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x^n])/(d^2*(b*c - a*d)))/n

IntegrateAlgebraic [A] time = 0.07, size = 70, normalized size = 0.99

$$\frac{a^2 \log(a + bx^n)}{b^2 n (bc - ad)} + \frac{c^2 \log(c + dx^n)}{d^2 n (ad - bc)} + \frac{x^n}{bdn}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)), x]

[Out] x^n/(b*d*n) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)*n) + (c^2*Log[c + d*x^n])/(d^2*(-(b*c) + a*d)*n)

fricas [A] time = 0.46, size = 74, normalized size = 1.04

$$\frac{a^2 d^2 \log(bx^n + a) - b^2 c^2 \log(dx^n + c) + (b^2 cd - abd^2)x^n}{(b^3 cd^2 - ab^2 d^3)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n), x, algorithm="fricas")

[Out] (a^2*d^2*log(b*x^n + a) - b^2*c^2*log(d*x^n + c) + (b^2*c*d - a*b*d^2)*x^n)/((b^3*c*d^2 - a*b^2*d^3)*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(bx^n + a)(dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n), x, algorithm="giac")

[Out] integrate(x^(3*n - 1)/((b*x^n + a)*(d*x^n + c)), x)

maple [A] time = 0.10, size = 78, normalized size = 1.10

$$-\frac{a^2 \ln(b e^{n \ln(x)} + a)}{(ad - bc) b^2 n} + \frac{c^2 \ln(d e^{n \ln(x)} + c)}{(ad - bc) d^2 n} + \frac{e^{n \ln(x)}}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3*n-1)/(b*x^n+a)/(d*x^n+c),x)

[Out] 1/b/d/n*exp(n*ln(x))+c^2/d^2/n/(a*d-b*c)*ln(d*exp(n*ln(x))+c)-a^2/(a*d-b*c)/b^2/n*ln(b*exp(n*ln(x))+a)

maxima [A] time = 0.63, size = 81, normalized size = 1.14

$$\frac{a^2 \log\left(\frac{bx^n+a}{b}\right)}{b^3cn - ab^2dn} - \frac{c^2 \log\left(\frac{dx^n+c}{d}\right)}{bcd^2n - ad^3n} + \frac{x^n}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")

[Out] a^2*log((b*x^n + a)/b)/(b^3*c*n - a*b^2*d*n) - c^2*log((d*x^n + c)/d)/(b*c*d^2*n - a*d^3*n) + x^n/(b*d*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1}}{(a + b x^n)(c + d x^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3*n - 1)/((a + b*x^n)*(c + d*x^n)),x)

[Out] int(x^(3*n - 1)/((a + b*x^n)*(c + d*x^n)), x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+3*n)/(a+b*x**n)/(c+d*x**n),x)

[Out] Exception raised: HeuristicGCDFailed

$$3.670 \quad \int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)} dx$$

Optimal. Leaf size=95

$$-\frac{a^2}{b^2n(bc-ad)(a+bx^n)} - \frac{a(2bc-ad)\log(a+bx^n)}{b^2n(bc-ad)^2} + \frac{c^2\log(c+dx^n)}{dn(bc-ad)^2}$$

Rubi [A] time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {446, 88}

$$-\frac{a^2}{b^2n(bc-ad)(a+bx^n)} - \frac{a(2bc-ad)\log(a+bx^n)}{b^2n(bc-ad)^2} + \frac{c^2\log(c+dx^n)}{dn(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]

[Out] -(a^2/(b^2*(b*c - a*d)*n*(a + b*x^n))) - (a*(2*b*c - a*d)*Log[a + b*x^n])/ (b^2*(b*c - a*d)^2*n) + (c^2*Log[c + d*x^n])/(d*(b*c - a*d)^2*n)

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)} dx = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^2(c+dx)} dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^2} + \frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)} + \frac{c^2}{(bc-ad)^2(c+dx)}\right) dx, x, x^n\right)}{n}$$

$$= -\frac{a^2}{b^2(bc-ad)n(a+bx^n)} - \frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2n} + \frac{c^2\log(c+dx^n)}{d(bc-ad)^2n}$$

Mathematica [A] time = 0.11, size = 90, normalized size = 0.95

$$\frac{-\frac{a^2}{b^2(bc-ad)(a+bx^n)} - \frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2} + \frac{c^2\log(c+dx^n)}{d(bc-ad)^2}}{n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)), x]

[Out] $-(a^2/(b^2*(b*c - a*d)*(a + b*x^n))) - (a*(2*b*c - a*d)*\text{Log}[a + b*x^n])/(b^2*(b*c - a*d)^2) + (c^2*\text{Log}[c + d*x^n])/(d*(b*c - a*d)^2)/n$

IntegrateAlgebraic [A] time = 0.11, size = 95, normalized size = 1.00

$$-\frac{a^2}{b^2n(bc-ad)(a+bx^n)} + \frac{(a^2d - 2abc)\log(a+bx^n)}{b^2n(bc-ad)^2} + \frac{c^2\log(c+dx^n)}{dn(ad-bc)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)), x]

[Out] $-(a^2/(b^2*(b*c - a*d)*n*(a + b*x^n))) + ((-2*a*b*c + a^2*d)*\text{Log}[a + b*x^n])/(b^2*(b*c - a*d)^2*n) + (c^2*\text{Log}[c + d*x^n])/(d*(-(b*c) + a*d)^2*n)$

fricas [A] time = 0.45, size = 166, normalized size = 1.75

$$\frac{a^2bcd - a^3d^2 + (2a^2bcd - a^3d^2 + (2ab^2cd - a^2bd^2)x^n)\log(bx^n + a) - (b^3c^2x^n + ab^2c^2)\log(dx^n + c)}{(b^5c^2d - 2ab^4cd^2 + a^2b^3d^3)nx^n + (ab^4c^2d - 2a^2b^3cd^2 + a^3b^2d^3)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n), x, algorithm="fricas")

[Out] $-(a^2*b*c*d - a^3*d^2 + (2*a^2*b*c*d - a^3*d^2 + (2*a*b^2*c*d - a^2*b*d^2)*x^n)*\log(b*x^n + a) - (b^3*c^2*x^n + a*b^2*c^2)*\log(d*x^n + c))/((b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*n*x^n + (a*b^4*c^2*d - 2*a^2*b^3*c*d^2 + a^3*b^2*d^3)*n)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(bx^n + a)^2(dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")`

[Out] `integrate(x^(3*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)`

maple [A] time = 0.08, size = 163, normalized size = 1.72

$$\frac{a^2 d \ln(b e^{n \ln(x)} + a)}{(a^2 d^2 - 2abcd + b^2 c^2) b^2 n} - \frac{2ac \ln(b e^{n \ln(x)} + a)}{(a^2 d^2 - 2abcd + b^2 c^2) b n} + \frac{c^2 \ln(d e^{n \ln(x)} + c)}{(a^2 d^2 - 2abcd + b^2 c^2) d n} + \frac{a^2}{(ad - bc)(b e^{n \ln(x)} + a) b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3*n-1)/(b*x^n+a)^2/(d*x^n+c),x)`

[Out] $a^2/(a*d-b*c)/b^2/n/(b*\exp(n*\ln(x))+a)+c^2/d/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*\ln(d*\exp(n*\ln(x))+c)+a^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/n*\ln(b*\exp(n*\ln(x))+a)*d-2*a/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/n*\ln(b*\exp(n*\ln(x))+a)*c$

maxima [A] time = 0.58, size = 147, normalized size = 1.55

$$\frac{c^2 \log\left(\frac{dx^n+c}{d}\right)}{b^2 c^2 d n - 2 a b c d^2 n + a^2 d^3 n} - \frac{a^2}{a b^3 c n - a^2 b^2 d n + (b^4 c n - a b^3 d n) x^n} - \frac{(2 a b c - a^2 d) \log\left(\frac{b x^n + a}{b}\right)}{b^4 c^2 n - 2 a b^3 c d n + a^2 b^2 d^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")`

[Out] $c^2*\log((d*x^n + c)/d)/(b^2*c^2*d*n - 2*a*b*c*d^2*n + a^2*d^3*n) - a^2/(a*b^3*c*n - a^2*b^2*d*n + (b^4*c*n - a*b^3*d*n)*x^n) - (2*a*b*c - a^2*d)*\log((b*x^n + a)/b)/(b^4*c^2*n - 2*a*b^3*c*d*n + a^2*b^2*d^2*n)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1}}{(a + b x^n)^2 (c + d x^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

```
sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**2/(c+d*x**n), x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```


$$3.671 \quad \int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)} dx$$

Optimal. Leaf size=120

$$-\frac{a^2}{2b^2n(bc-ad)(a+bx^n)^2} + \frac{a(2bc-ad)}{b^2n(bc-ad)^2(a+bx^n)} + \frac{c^2 \log(a+bx^n)}{n(bc-ad)^3} - \frac{c^2 \log(c+dx^n)}{n(bc-ad)^3}$$

Rubi [A] time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {446, 88}

$$-\frac{a^2}{2b^2n(bc-ad)(a+bx^n)^2} + \frac{a(2bc-ad)}{b^2n(bc-ad)^2(a+bx^n)} + \frac{c^2 \log(a+bx^n)}{n(bc-ad)^3} - \frac{c^2 \log(c+dx^n)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]

[Out] -a^2/(2*b^2*(b*c - a*d)*n*(a + b*x^n)^2) + (a*(2*b*c - a*d))/(b^2*(b*c - a*d)^2*n*(a + b*x^n)) + (c^2*Log[a + b*x^n])/((b*c - a*d)^3*n) - (c^2*Log[c + d*x^n])/((b*c - a*d)^3*n)

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)} dx = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^3(c+dx)} dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^3} + \frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)^2} + \frac{bc^2}{(bc-ad)^3(a+bx)} - \frac{c^2d}{(bc-ad)^3(c+dx)}\right) dx, x, x^n\right)}{n}$$

$$= -\frac{a^2}{2b^2(bc-ad)n(a+bx^n)^2} + \frac{a(2bc-ad)}{b^2(bc-ad)^2n(a+bx^n)} + \frac{c^2 \log(a+bx^n)}{(bc-ad)^3n} - \frac{c^2 \log(c+dx^n)}{(bc-ad)^3n}$$

Mathematica [A] time = 0.27, size = 112, normalized size = 0.93

$$-\frac{a^2}{2b^2(bc-ad)(a+bx^n)^2} + \frac{a(2bc-ad)}{b^2(bc-ad)^2(a+bx^n)} + \frac{c^2 \log(a+bx^n)}{(bc-ad)^3} - \frac{c^2 \log(c+dx^n)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)), x]

[Out] (-1/2*a^2/(b^2*(b*c - a*d)*(a + b*x^n)^2) + (a*(2*b*c - a*d))/(b^2*(b*c - a*d)^2*(a + b*x^n)) + (c^2*Log[a + b*x^n])/(b*c - a*d)^3 - (c^2*Log[c + d*x^n])/(b*c - a*d)^3)/n

IntegrateAlgebraic [A] time = 0.15, size = 110, normalized size = 0.92

$$-\frac{a(a^2d - 3abc + 2abdx^n - 4b^2cx^n)}{2b^2n(bc-ad)^2(a+bx^n)^2} + \frac{c^2 \log(a+bx^n)}{n(bc-ad)^3} - \frac{c^2 \log(c+dx^n)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)), x]

[Out] -1/2*(a*(-3*a*b*c + a^2*d - 4*b^2*c*x^n + 2*a*b*d*x^n))/(b^2*(b*c - a*d)^2*n*(a + b*x^n)^2) + (c^2*Log[a + b*x^n])/((b*c - a*d)^3*n) - (c^2*Log[c + d*x^n])/((b*c - a*d)^3*n)

fricas [B] time = 0.45, size = 301, normalized size = 2.51

$$\frac{3a^2b^2c^2 - 4a^3bcd + a^4d^2 + 2(2a^2b^3c^2 - 3a^2b^2cd + a^3bd^2)x^n + 2(b^4c^2x^{2n} + 2ab^3c^2x^n + a^2b^2c^2)\log(bx^n + a) - 2(b^4c^2x^{2n} + 2ab^3c^2x^n + a^2b^2c^2)\log(dx^n + c)}{2((b^7c^3 - 3ab^6c^2d + 3a^2b^5cd^2 - a^3b^4d^3)nx^{2n} + 2(ab^6c^3 - 3a^2b^5c^2d + 3a^3b^4cd^2 - a^4b^3d^3)nx^n + (a^2b^5c^3 - 3a^3b^4c^2d + 3a^4b^3cd^2 - a^5b^2d^3)n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n), x, algorithm="fricas")

[Out] $\frac{1}{2}*(3*a^2*b^2*c^2 - 4*a^3*b*c*d + a^4*d^2 + 2*(2*a*b^3*c^2 - 3*a^2*b^2*c*d + a^3*b*d^2)*x^n + 2*(b^4*c^2*x^{(2*n)} + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*\log(b*x^n + a) - 2*(b^4*c^2*x^{(2*n)} + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*\log(d*x^n + c))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*n*x^{(2*n)} + 2*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*n*x^n + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*n)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(bx^n + a)^3(dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")`

[Out] `integrate(x^(3*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)`

maple [A] time = 0.12, size = 214, normalized size = 1.78

$$-\frac{c^2 \ln(b e^{n \ln(x)} + a)}{(a^3 d^3 - 3a^2 b c d^2 + 3a b^2 c^2 d - b^3 c^3) n} + \frac{c^2 \ln(d e^{n \ln(x)} + c)}{(a^3 d^3 - 3a^2 b c d^2 + 3a b^2 c^2 d - b^3 c^3) n} + \frac{\frac{(-ad+2bc)a e^{n \ln(x)}}{(a^2 d^2 - 2abcd + b^2 c^2) b n} + \frac{(-ad+3bc)a^2}{2(a^2 d^2 - 2abcd + b^2 c^2) b^2 n}}{(b e^{n \ln(x)} + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3*n-1)/(b*x^n+a)^3/(d*x^n+c),x)`

[Out] $((-a*d+2*b*c)*a/n/b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*\exp(n*\ln(x))+1/2*a^2*(-a*d+3*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/n)/(b*\exp(n*\ln(x))+a)^2+c^2/n/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*\ln(d*\exp(n*\ln(x))+c)-c^2/n/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*\ln(b*\exp(n*\ln(x))+a)$

maxima [B] time = 0.68, size = 262, normalized size = 2.18

$$\frac{c^2 \log\left(\frac{bx^n+a}{b}\right)}{b^3 c^3 n - 3ab^2 c^2 d n + 3a^2 b c d^2 n - a^3 d^3 n} - \frac{c^2 \log\left(\frac{dx^n+c}{d}\right)}{b^3 c^3 n - 3ab^2 c^2 d n + 3a^2 b c d^2 n - a^3 d^3 n} + \frac{3a^2 bc - a^3 d + 2(2ab^2 c - a^2 b d)x^n}{2(a^2 b^4 c^2 n - 2a^3 b^3 c d n + a^4 b^2 d^2 n + (b^6 c^2 n - 2ab^5 c d n + a^2 b^4 d^2 n)x^{2n} + 2(ab^5 c^2 n - 2a^2 b^4 c d n + a^3 b^3 d^2 n)x^n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")`

[Out] $c^2*\log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^3*d^3*n) - c^2*\log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^3*d^3*n) + 1/2*(3*a^2*b*c - a^3*d + 2*(2*a*b^2*c - a^2*b*d)*x^n)/(a^2*b^4*c^2*n - 2*a^3*b^3*c*d*n + a^4*b^2*d^2*n + (b^6*c^2*n - 2*a*b^5*c*d*n + a^2*b^4*d^2*n)*x^{(2*n)} + 2*(a*b^5*c^2*n - 2*a^2*b^4*c*d*n + a^3*b^3*d^2*n)*x^n)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1}}{(a + b x^n)^3 (c + d x^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)

[Out] int(x^(3*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+3*n)/(a+b*x**n)**3/(c+d*x**n), x)

[Out] Exception raised: HeuristicGCDFailed

$$3.672 \quad \int x^{13}(b + cx)^{13}(b + 2cx) dx$$

Optimal. Leaf size=14

$$\frac{1}{14}x^{14}(b + cx)^{14}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {74}

$$\frac{1}{14}x^{14}(b + cx)^{14}$$

Antiderivative was successfully verified.

[In] Int[x¹³*(b + c*x)¹³*(b + 2*c*x), x]

[Out] (x¹⁴*(b + c*x)¹⁴)/14

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rubi steps

$$\int x^{13}(b + cx)^{13}(b + 2cx) dx = \frac{1}{14}x^{14}(b + cx)^{14}$$

Mathematica [B] time = 0.01, size = 172, normalized size = 12.29

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13}{2}b^{12}c^2x^{16} + 26b^{11}c^3x^{17} + \frac{143}{2}b^{10}c^4x^{18} + 143b^9c^5x^{19} + \frac{429}{2}b^8c^6x^{20} + \frac{1716}{7}b^7c^7x^{21} + \frac{429}{2}b^6c^8x^{22} + 143b^5c^9x^{23} + \frac{143}{2}b^4c^{10}x^{24} + 26b^3c^{11}x^{25} + \frac{13}{2}b^2c^{12}x^{26} + bc^{13}x^{27} + \frac{c^{14}x^{28}}{14}$$

Antiderivative was successfully verified.

[In] Integrate[x¹³*(b + c*x)¹³*(b + 2*c*x), x]

[Out] (b¹⁴*x¹⁴)/14 + b¹³*c*x¹⁵ + (13*b¹²*c²*x¹⁶)/2 + 26*b¹¹*c³*x¹⁷ + (143*b¹⁰*c⁴*x¹⁸)/2 + 143*b⁹*c⁵*x¹⁹ + (429*b⁸*c⁶*x²⁰)/2 + (1716*b⁷*c⁷*x²¹)/7 + (429*b⁶*c⁸*x²²)/2 + 143*b⁵*c⁹*x²³ + (143*b⁴*c¹⁰*x²⁴)/2 + 26*b³*c¹¹*x²⁵ + (13*b²*c¹²*x²⁶)/2 + b*c¹³*x²⁷ + (c¹⁴*x²⁸)/14

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{13}(b + cx)^{13}(b + 2cx) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^13*(b + c*x)^13*(b + 2*c*x), x]

[Out] IntegrateAlgebraic[x^13*(b + c*x)^13*(b + 2*c*x), x]

fricas [B] time = 0.35, size = 154, normalized size = 11.00

$$\frac{1}{14}x^{28}c^{14} + x^{27}c^{13}b + \frac{13}{2}x^{26}c^{12}b^2 + 26x^{25}c^{11}b^3 + \frac{143}{2}x^{24}c^{10}b^4 + 143x^{23}c^9b^5 + \frac{429}{2}x^{22}c^8b^6 + \frac{1716}{7}x^{21}c^7b^7 + \frac{429}{2}x^{20}c^6b^8 + 143x^{19}c^5b^9 + \frac{143}{2}x^{18}c^4b^{10} + 26x^{17}c^3b^{11} + \frac{13}{2}x^{16}c^2b^{12} + x^{15}cb^{13} + \frac{1}{14}x^{14}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^13*(c*x+b)^13*(2*c*x+b), x, algorithm="fricas")

[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*b^3 + 143/2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6 + 1716/7*x^21*c^7*b^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 + 143/2*x^18*c^4*b^10 + 26*x^17*c^3*b^11 + 13/2*x^16*c^2*b^12 + x^15*c*b^13 + 1/14*x^14*b^14

giac [A] time = 0.15, size = 13, normalized size = 0.93

$$\frac{1}{14} (cx^2 + bx)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^13*(c*x+b)^13*(2*c*x+b), x, algorithm="giac")

[Out] 1/14*(c*x^2 + b*x)^14

maple [B] time = 0.04, size = 155, normalized size = 11.07

$$\frac{1}{14}c^{14}x^{28} + b c^{13}x^{27} + \frac{13}{2}b^2c^{12}x^{26} + 26b^3c^{11}x^{25} + \frac{143}{2}b^4c^{10}x^{24} + 143b^5c^9x^{23} + \frac{429}{2}b^6c^8x^{22} + \frac{1716}{7}b^7c^7x^{21} + \frac{429}{2}b^8c^6x^{20} + 143b^9c^5x^{19} + \frac{143}{2}b^{10}c^4x^{18} + 26b^{11}c^3x^{17} + \frac{13}{2}b^{12}c^2x^{16} + b^{13}cx^{15} + \frac{1}{14}b^{14}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^13*(c*x+b)^13*(2*c*x+b), x)

[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+143/2*b^4*c^10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*c^7*x^21+429/2*b^8*c^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+26*b^11*c^3*x^17+13/2*b^12*c^2*x^16+b^13*c*x^15+1/14*b^14*x^14

maxima [B] time = 0.57, size = 154, normalized size = 11.00

$$\frac{1}{14}c^{14}x^{28} + bc^{13}x^{27} + \frac{13}{2}b^2c^{12}x^{26} + 26b^3c^{11}x^{25} + \frac{143}{2}b^4c^{10}x^{24} + 143b^5c^9x^{23} + \frac{429}{2}b^6c^8x^{22} + \frac{1716}{7}b^7c^7x^{21} + \frac{429}{2}b^8c^6x^{20} + 143b^9c^5x^{19} + \frac{143}{2}b^{10}c^4x^{18} + 26b^{11}c^3x^{17} + \frac{13}{2}b^{12}c^2x^{16} + b^{13}cx^{15} + \frac{1}{14}b^{14}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^13*(c*x+b)^13*(2*c*x+b), x, algorithm="maxima")

$$\begin{aligned} \text{[Out]} & \frac{1}{14}c^{14}x^{28} + b*c^{13}x^{27} + \frac{13}{2}b^2c^{12}x^{26} + 26*b^3c^{11}x^{25} + \frac{143}{2}b^4c^{10}x^{24} + 143*b^5c^9x^{23} + \frac{429}{2}b^6c^8x^{22} + \frac{1716}{7}b^7c^7x^{21} \\ & + 429/2*b^8c^6x^{20} + 143*b^9c^5x^{19} + 143/2*b^{10}c^4x^{18} + 26*b^{11}c^3x^{17} + 13/2*b^{12}c^2x^{16} + b^{13}cx^{15} + 1/14*b^{14}x^{14} \end{aligned}$$

mupad [B] time = 0.16, size = 154, normalized size = 11.00

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} + 143b^5c^9x^{23} + \frac{143b^4c^{10}x^{24}}{2} + 26b^3c^{11}x^{25} + \frac{13b^2c^{12}x^{26}}{2} + bc^{13}x^{27} + \frac{c^{14}x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^13*(b + c*x)^13*(b + 2*c*x), x)

$$\begin{aligned} \text{[Out]} & (b^{14}x^{14})/14 + (c^{14}x^{28})/14 + b^{13}c*x^{15} + b*c^{13}x^{27} + (13*b^{12}c^2*x^{16})/2 + 26*b^{11}c^3*x^{17} \\ & + (143*b^{10}c^4*x^{18})/2 + 143*b^9c^5*x^{19} + (429*b^8c^6*x^{20})/2 + (1716*b^7c^7*x^{21})/7 + (429*b^6c^8*x^{22})/2 + 143*b^5c^9*x^{23} \\ & + (143*b^4c^{10}x^{24})/2 + 26*b^3c^{11}x^{25} + (13*b^2c^{12}x^{26})/2 \end{aligned}$$

sympy [B] time = 0.12, size = 175, normalized size = 12.50

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} + 143b^5c^9x^{23} + \frac{143b^4c^{10}x^{24}}{2} + 26b^3c^{11}x^{25} + \frac{13b^2c^{12}x^{26}}{2} + bc^{13}x^{27} + \frac{c^{14}x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**13*(c*x+b)**13*(2*c*x+b), x)

$$\begin{aligned} \text{[Out]} & b^{14}x^{14}/14 + b^{13}c*x^{15} + 13*b^{12}c^2*x^{16}/2 + 26*b^{11}c^3*x^{17} \\ & + 143*b^{10}c^4*x^{18}/2 + 143*b^9c^5*x^{19} + 429*b^8c^6*x^{20}/2 + 1716*b^7c^7*x^{21}/7 + 429*b^6c^8*x^{22}/2 + 143*b^5c^9*x^{23} + 143*b^4c^{10}x^{24}/2 \\ & + 26*b^3c^{11}x^{25} + 13*b^2c^{12}x^{26}/2 + b*c^{13}x^{27} + c^{14}x^{28}/14 \end{aligned}$$

$$3.673 \quad \int x^{27} (b + cx^2)^{13} (b + 2cx^2) dx$$

Optimal. Leaf size=16

$$\frac{1}{28}x^{28}(b + cx^2)^{14}$$

Rubi [A] time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {446, 74}

$$\frac{1}{28}x^{28}(b + cx^2)^{14}$$

Antiderivative was successfully verified.

[In] Int[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]

[Out] (x^28*(b + c*x^2)^14)/28

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int x^{27} (b + cx^2)^{13} (b + 2cx^2) dx &= \frac{1}{2} \text{Subst} \left(\int x^{13} (b + cx)^{13} (b + 2cx) dx, x, x^2 \right) \\ &= \frac{1}{28}x^{28}(b + cx^2)^{14} \end{aligned}$$

Mathematica [B] time = 0.01, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28} + \frac{1}{2}b^{13}cx^{30} + \frac{13}{4}b^{12}c^2x^{32} + 13b^{11}c^3x^{34} + \frac{143}{4}b^{10}c^4x^{36} + \frac{143}{2}b^9c^5x^{38} + \frac{429}{4}b^8c^6x^{40} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^6c^8x^{44} + \frac{143}{2}b^5c^9x^{46} + \frac{143}{4}b^4c^{10}x^{48} + 13b^3c^{11}x^{50} + \frac{13}{4}b^2c^{12}x^{52} + \frac{1}{2}bc^{13}x^{54} + \frac{c^{14}x^{56}}{28}$$

Antiderivative was successfully verified.

[In] Integrate[x²⁷*(b + c*x²)¹³*(b + 2*c*x²), x]

[Out] (b¹⁴*x²⁸)/28 + (b¹³*c*x³⁰)/2 + (13*b¹²*c²*x³²)/4 + 13*b¹¹*c³*x³⁴ + (143*b¹⁰*c⁴*x³⁶)/4 + (143*b⁹*c⁵*x³⁸)/2 + (429*b⁸*c⁶*x⁴⁰)/4 + (858*b⁷*c⁷*x⁴²)/7 + (429*b⁶*c⁸*x⁴⁴)/4 + (143*b⁵*c⁹*x⁴⁶)/2 + (143*b⁴*c¹⁰*x⁴⁸)/4 + 13*b³*c¹¹*x⁵⁰ + (13*b²*c¹²*x⁵²)/4 + (b*c¹³*x⁵⁴)/2 + (c¹⁴*x⁵⁶)/28

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{27} (b + cx^2)^{13} (b + 2cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x²⁷*(b + c*x²)¹³*(b + 2*c*x²), x]

[Out] IntegrateAlgebraic[x²⁷*(b + c*x²)¹³*(b + 2*c*x²), x]

fricas [B] time = 0.36, size = 156, normalized size = 9.75

$$\frac{1}{28}x^{56}c^{14} + \frac{1}{2}x^{54}c^{13}b + \frac{13}{4}x^{52}c^{12}b^2 + 13x^{50}c^{11}b^3 + \frac{143}{4}x^{48}c^{10}b^4 + \frac{143}{2}x^{46}c^9b^5 + \frac{429}{4}x^{44}c^8b^6 + \frac{858}{7}x^{42}c^7b^7 + \frac{429}{4}x^{40}c^6b^8 + \frac{143}{2}x^{38}c^5b^9 + \frac{143}{4}x^{36}c^4b^{10} + 13x^{34}c^3b^{11} + \frac{13}{4}x^{32}c^2b^{12} + \frac{1}{2}x^{30}cb^{13} + \frac{1}{28}x^{28}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x²⁷*(c*x²+b)¹³*(2*c*x²+b), x, algorithm="fricas")

[Out] 1/28*x⁵⁶*c¹⁴ + 1/2*x⁵⁴*c¹³*b + 13/4*x⁵²*c¹²*b² + 13*x⁵⁰*c¹¹*b³ + 143/4*x⁴⁸*c¹⁰*b⁴ + 143/2*x⁴⁶*c⁹*b⁵ + 429/4*x⁴⁴*c⁸*b⁶ + 858/7*x⁴²*c⁷*b⁷ + 429/4*x⁴⁰*c⁶*b⁸ + 143/2*x³⁸*c⁵*b⁹ + 143/4*x³⁶*c⁴*b¹⁰ + 13*x³⁴*c³*b¹¹ + 13/4*x³²*c²*b¹² + 1/2*x³⁰*c*b¹³ + 1/28*x²⁸*b¹⁴

giac [B] time = 0.19, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x²⁷*(c*x²+b)¹³*(2*c*x²+b), x, algorithm="giac")

[Out] 1/28*c¹⁴*x⁵⁶ + 1/2*b*c¹³*x⁵⁴ + 13/4*b²*c¹²*x⁵² + 13*b³*c¹¹*x⁵⁰ + 143/4*b⁴*c¹⁰*x⁴⁸ + 143/2*b⁵*c⁹*x⁴⁶ + 429/4*b⁶*c⁸*x⁴⁴ + 858/7*b⁷*c⁷*x⁴² + 429/4*b⁸*c⁶*x⁴⁰ + 143/2*b⁹*c⁵*x³⁸ + 143/4*b¹⁰*c⁴*x³⁶ + 13*b¹¹*c³*x³⁴ + 13/4*b¹²*c²*x³² + 1/2*b¹³*c*x³⁰ + 1/28*b¹⁴*x²⁸

maple [B] time = 0.04, size = 157, normalized size = 9.81

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^27*(c*x^2+b)^13*(2*c*x^2+b),x)`

[Out] $\frac{1}{28}c^{14}x^{56} + \frac{1}{2}b^2c^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$

maxima [B] time = 0.73, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}b^2c^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="maxima")`

[Out] $\frac{1}{28}c^{14}x^{56} + \frac{1}{2}b^2c^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$

mpad [B] time = 4.93, size = 156, normalized size = 9.75

$$\frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} + \frac{143b^5c^9x^{46}}{2} + \frac{143b^4c^{10}x^{48}}{4} + 13b^3c^{11}x^{50} + \frac{13b^2c^{12}x^{52}}{4} + \frac{b^{13}cx^{30}}{2} + \frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^27*(b + c*x^2)^13*(b + 2*c*x^2),x)`

[Out] $\frac{(b^{14}x^{28})}{28} + \frac{(c^{14}x^{56})}{28} + \frac{(b^{13}cx^{30})}{2} + \frac{(b^2c^{13}x^{54})}{2} + \frac{(13b^{12}c^2x^{32})}{4} + \frac{13b^{11}c^3x^{34}}{2} + \frac{(143b^{10}c^4x^{36})}{4} + \frac{(143b^9c^5x^{38})}{2} + \frac{(429b^8c^6x^{40})}{4} + \frac{(858b^7c^7x^{42})}{7} + \frac{(429b^6c^8x^{44})}{4} + \frac{(143b^5c^9x^{46})}{2} + \frac{(143b^4c^{10}x^{48})}{4} + 13b^3c^{11}x^{50} + \frac{(13b^2c^{12}x^{52})}{4}$

sympy [B] time = 0.12, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} + \frac{143b^5c^9x^{46}}{2} + \frac{143b^4c^{10}x^{48}}{4} + 13b^3c^{11}x^{50} + \frac{13b^2c^{12}x^{52}}{4} + \frac{b^{13}cx^{30}}{2} + \frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**27*(c*x**2+b)**13*(2*c*x**2+b),x)`

[Out] $b^{14}x^{28}/28 + b^{13}cx^{30}/2 + 13b^{12}c^2x^{32}/4 + 13b^{11}c^3x^{34}/2 + 143b^{10}c^4x^{36}/4 + 143b^9c^5x^{38}/2 + 429b^8c^6x^{40}/4 + 858b^7c^7x^{42}/7 + 429b^6c^8x^{44}/4 + 143b^5c^9x^{46}/2 + 143b^4c^{10}x^{48}/4 + 13b^3c^{11}x^{50} + 13b^2c^{12}x^{52}/4 + b^{13}cx^{30}/2 + c^{14}x^{56}/28$

$$3.674 \quad \int x^{41} (b + cx^3)^{13} (b + 2cx^3) dx$$

Optimal. Leaf size=16

$$\frac{1}{42} x^{42} (b + cx^3)^{14}$$

Rubi [A] time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {446, 74}

$$\frac{1}{42} x^{42} (b + cx^3)^{14}$$

Antiderivative was successfully verified.

[In] Int[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]

[Out] (x^42*(b + c*x^3)^14)/42

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int x^{41} (b + cx^3)^{13} (b + 2cx^3) dx &= \frac{1}{3} \text{Subst} \left(\int x^{13} (b + cx)^{13} (b + 2cx) dx, x, x^3 \right) \\ &= \frac{1}{42} x^{42} (b + cx^3)^{14} \end{aligned}$$

Mathematica [B] time = 0.02, size = 186, normalized size = 11.62

$$\frac{b^{14}x^{42}}{42} + \frac{1}{3}b^{13}cx^{45} + \frac{13}{6}b^{12}c^2x^{48} + \frac{26}{3}b^{11}c^3x^{51} + \frac{143}{6}b^{10}c^4x^{54} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{2}b^8c^6x^{60} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^6c^8x^{66} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{6}b^4c^{10}x^{72} + \frac{26}{3}b^3c^{11}x^{75} + \frac{13}{6}b^2c^{12}x^{78} + \frac{1}{3}bc^{13}x^{81} + \frac{c^{14}x^{84}}{42}$$

Antiderivative was successfully verified.

[In] Integrate[x⁴¹*(b + c*x³)¹³*(b + 2*c*x³),x]

[Out] (b¹⁴*x⁴²)/42 + (b¹³*c*x⁴⁵)/3 + (13*b¹²*c²*x⁴⁸)/6 + (26*b¹¹*c³*x⁵¹)/3 + (143*b¹⁰*c⁴*x⁵⁴)/6 + (143*b⁹*c⁵*x⁵⁷)/3 + (143*b⁸*c⁶*x⁶⁰)/2 + (572*b⁷*c⁷*x⁶³)/7 + (143*b⁶*c⁸*x⁶⁶)/2 + (143*b⁵*c⁹*x⁶⁹)/3 + (143*b⁴*c¹⁰*x⁷²)/6 + (26*b³*c¹¹*x⁷⁵)/3 + (13*b²*c¹²*x⁷⁸)/6 + (b*c¹³*x⁸¹)/3 + (c¹⁴*x⁸⁴)/42

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{41} (b + cx^3)^{13} (b + 2cx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x⁴¹*(b + c*x³)¹³*(b + 2*c*x³),x]

[Out] IntegrateAlgebraic[x⁴¹*(b + c*x³)¹³*(b + 2*c*x³), x]

fricas [B] time = 0.36, size = 156, normalized size = 9.75

$$\frac{1}{42}x^{84}c^{14} + \frac{1}{3}x^{81}c^{13}b + \frac{13}{6}x^{78}c^{12}b^2 + \frac{26}{3}x^{75}c^{11}b^3 + \frac{143}{6}x^{72}c^{10}b^4 + \frac{143}{3}x^{69}c^9b^5 + \frac{143}{2}x^{66}c^8b^6 + \frac{572}{7}x^{63}c^7b^7 + \frac{143}{2}x^{60}c^6b^8 + \frac{143}{3}x^{57}c^5b^9 + \frac{143}{6}x^{54}c^4b^{10} + \frac{26}{3}x^{51}c^3b^{11} + \frac{13}{6}x^{48}c^2b^{12} + \frac{1}{3}x^{45}cb^{13} + \frac{1}{42}x^{42}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁴¹*(c*x³+b)¹³*(2*c*x³+b),x, algorithm="fricas")

[Out] 1/42*x⁸⁴*c¹⁴ + 1/3*x⁸¹*c¹³*b + 13/6*x⁷⁸*c¹²*b² + 26/3*x⁷⁵*c¹¹*b³ + 143/6*x⁷²*c¹⁰*b⁴ + 143/3*x⁶⁹*c⁹*b⁵ + 143/2*x⁶⁶*c⁸*b⁶ + 572/7*x⁶³*c⁷*b⁷ + 143/2*x⁶⁰*c⁶*b⁸ + 143/3*x⁵⁷*c⁵*b⁹ + 143/6*x⁵⁴*c⁴*b¹⁰ + 26/3*x⁵¹*c³*b¹¹ + 13/6*x⁴⁸*c²*b¹² + 1/3*x⁴⁵*c*b¹³ + 1/42*x⁴²*b¹⁴

giac [B] time = 0.16, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{6}b^{10}c^4x^{54} + \frac{26}{3}b^{11}c^3x^{51} + \frac{13}{6}b^{12}c^2x^{48} + \frac{1}{3}b^{13}cx^{45} + \frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁴¹*(c*x³+b)¹³*(2*c*x³+b),x, algorithm="giac")

[Out] 1/42*c¹⁴*x⁸⁴ + 1/3*b*c¹³*x⁸¹ + 13/6*b²*c¹²*x⁷⁸ + 26/3*b³*c¹¹*x⁷⁵ + 143/6*b⁴*c¹⁰*x⁷² + 143/3*b⁵*c⁹*x⁶⁹ + 143/2*b⁶*c⁸*x⁶⁶ + 572/7*b⁷*c⁷*x⁶³ + 143/2*b⁸*c⁶*x⁶⁰ + 143/3*b⁹*c⁵*x⁵⁷ + 143/6*b¹⁰*c⁴*x⁵⁴ + 26/3*b¹¹*c³*x⁵¹ + 13/6*b¹²*c²*x⁴⁸ + 1/3*b¹³*c*x⁴⁵ + 1/42*b¹⁴*x⁴²

maple [B] time = 0.04, size = 157, normalized size = 9.81

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{6}b^{10}c^4x^{54} + \frac{26}{3}b^{11}c^3x^{51} + \frac{13}{6}b^{12}c^2x^{48} + \frac{1}{3}b^{13}cx^{45} + \frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{41}*(c*x^3+b)^{13}*(2*c*x^3+b), x)$

[Out] $\frac{1}{42}c^{14}x^{84} + \frac{1}{3}b^3c^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{6}b^{10}c^4x^{54} + \frac{26}{3}b^{11}c^3x^{51} + \frac{13}{6}b^{12}c^2x^{48} + \frac{1}{3}b^{13}cx^{45} + \frac{1}{42}b^{14}x^{42}$

maxima [B] time = 0.62, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}b^3c^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{6}b^{10}c^4x^{54} + \frac{26}{3}b^{11}c^3x^{51} + \frac{13}{6}b^{12}c^2x^{48} + \frac{1}{3}b^{13}cx^{45} + \frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{41}*(c*x^3+b)^{13}*(2*c*x^3+b), x, \text{algorithm}="maxima")$

[Out] $\frac{1}{42}c^{14}x^{84} + \frac{1}{3}b^3c^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{6}b^{10}c^4x^{54} + \frac{26}{3}b^{11}c^3x^{51} + \frac{13}{6}b^{12}c^2x^{48} + \frac{1}{3}b^{13}cx^{45} + \frac{1}{42}b^{14}x^{42}$

mupad [B] time = 4.77, size = 156, normalized size = 9.75

$$\frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^2x^{48}}{6} + \frac{26b^{11}c^3x^{51}}{3} + \frac{143b^{10}c^4x^{54}}{6} + \frac{143b^9c^5x^{57}}{3} + \frac{143b^8c^6x^{60}}{2} + \frac{572b^7c^7x^{63}}{7} + \frac{143b^6c^8x^{66}}{2} + \frac{143b^5c^9x^{69}}{3} + \frac{143b^4c^{10}x^{72}}{6} + \frac{26b^3c^{11}x^{75}}{3} + \frac{13b^2c^{12}x^{78}}{6} + \frac{b^{13}cx^{81}}{3} + \frac{c^{14}x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{41}*(b + c*x^3)^{13}*(b + 2*c*x^3), x)$

[Out] $\frac{b^{14}x^{42}}{42} + \frac{c^{14}x^{84}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{b^{12}c^2x^{48}}{6} + \frac{b^{11}c^3x^{51}}{3} + \frac{b^{10}c^4x^{54}}{6} + \frac{b^9c^5x^{57}}{3} + \frac{b^8c^6x^{60}}{2} + \frac{b^7c^7x^{63}}{7} + \frac{b^6c^8x^{66}}{2} + \frac{b^5c^9x^{69}}{3} + \frac{b^4c^{10}x^{72}}{6} + \frac{b^3c^{11}x^{75}}{3} + \frac{b^2c^{12}x^{78}}{6} + \frac{bc^{13}x^{81}}{3} + \frac{c^{14}x^{84}}{42}$

sympy [B] time = 0.12, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^2x^{48}}{6} + \frac{26b^{11}c^3x^{51}}{3} + \frac{143b^{10}c^4x^{54}}{6} + \frac{143b^9c^5x^{57}}{3} + \frac{143b^8c^6x^{60}}{2} + \frac{572b^7c^7x^{63}}{7} + \frac{143b^6c^8x^{66}}{2} + \frac{143b^5c^9x^{69}}{3} + \frac{143b^4c^{10}x^{72}}{6} + \frac{26b^3c^{11}x^{75}}{3} + \frac{13b^2c^{12}x^{78}}{6} + \frac{bc^{13}x^{81}}{3} + \frac{c^{14}x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{41}*(c*x^3+b)^{13}*(2*c*x^3+b), x)$

[Out] $\frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^2x^{48}}{6} + \frac{26b^{11}c^3x^{51}}{3} + \frac{143b^{10}c^4x^{54}}{6} + \frac{143b^9c^5x^{57}}{3} + \frac{143b^8c^6x^{60}}{2} + \frac{572b^7c^7x^{63}}{7} + \frac{143b^6c^8x^{66}}{2} + \frac{143b^5c^9x^{69}}{3} + \frac{143b^4c^{10}x^{72}}{6} + \frac{26b^3c^{11}x^{75}}{3} + \frac{13b^2c^{12}x^{78}}{6} + \frac{bc^{13}x^{81}}{3} + \frac{c^{14}x^{84}}{42}$

$$3.675 \quad \int x^{-1+14n} (b + cx^n)^{13} (b + 2cx^n) dx$$

Optimal. Leaf size=21

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {446, 74}

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n), x]

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int x^{-1+14n} (b + cx^n)^{13} (b + 2cx^n) dx &= \frac{\text{Subst}\left(\int x^{13}(b + cx)^{13}(b + 2cx) dx, x, x^n\right)}{n} \\ &= \frac{x^{14n} (b + cx^n)^{14}}{14n} \end{aligned}$$

Mathematica [A] time = 0.19, size = 21, normalized size = 1.00

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n), x]

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)

IntegrateAlgebraic [A] time = 0.04, size = 21, normalized size = 1.00

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n), x]

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)

fricas [B] time = 0.46, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28n} + 14bc^{13}x^{27n} + 91b^2c^{12}x^{26n} + 364b^3c^{11}x^{25n} + 1001b^4c^{10}x^{24n} + 2002b^5c^9x^{23n} + 3003b^6c^8x^{22n} + 3432b^7c^7x^{21n} + 3003b^8c^6x^{20n} + 2002b^9c^5x^{19n} + 1001b^{10}c^4x^{18n} + 364b^{11}c^3x^{17n} + 91b^{12}c^2x^{16n} + 14b^{13}cx^{15n} + b^{14}x^{14n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n), x, algorithm="fricas")

[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(16*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n

giac [B] time = 0.57, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28n} + 14bc^{13}x^{27n} + 91b^2c^{12}x^{26n} + 364b^3c^{11}x^{25n} + 1001b^4c^{10}x^{24n} + 2002b^5c^9x^{23n} + 3003b^6c^8x^{22n} + 3432b^7c^7x^{21n} + 3003b^8c^6x^{20n} + 2002b^9c^5x^{19n} + 1001b^{10}c^4x^{18n} + 364b^{11}c^3x^{17n} + 91b^{12}c^2x^{16n} + 14b^{13}cx^{15n} + b^{14}x^{14n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n), x, algorithm="giac")

[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(16*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n

maple [B] time = 0.08, size = 230, normalized size = 10.95

$$\frac{b^{14}x^{14n}}{14n} + \frac{b^{13}cx^{15n}}{n} + \frac{13b^{12}c^2x^{16n}}{2n} + \frac{26b^{11}c^3x^{17n}}{n} + \frac{143b^{10}c^4x^{18n}}{2n} + \frac{143b^9c^5x^{19n}}{n} + \frac{429b^8c^6x^{20n}}{2n} + \frac{1716b^7c^7x^{21n}}{7n} + \frac{429b^6c^8x^{22n}}{2n} + \frac{143b^5c^9x^{23n}}{n} + \frac{143b^4c^{10}x^{24n}}{2n} + \frac{26b^3c^{11}x^{25n}}{n} + \frac{13b^2c^{12}x^{26n}}{2n} + \frac{bc^{13}x^{27n}}{n} + \frac{c^{14}x^{28n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{(-1+14*n)}*(b+c*x^n)^{13}*(b+2*c*x^n), x)$

[Out] $\frac{1}{14}c^{14}/n*(x^n)^{28}+b*c^{13}/n*(x^n)^{27}+13/2*b^2*c^{12}/n*(x^n)^{26}+26*b^3*c^{11}/n*(x^n)^{25}+143/2*b^4*c^{10}/n*(x^n)^{24}+143*b^5*c^9/n*(x^n)^{23}+429/2*b^6*c^8/n*(x^n)^{22}+1716/7*b^7*c^7/n*(x^n)^{21}+429/2*b^8*c^6/n*(x^n)^{20}+143*b^9*c^5/n*(x^n)^{19}+143/2*b^{10}*c^4/n*(x^n)^{18}+26*b^{11}*c^3/n*(x^n)^{17}+13/2*b^{12}*c^2/n*(x^n)^{16}+b^{13}*c/n*(x^n)^{15}+1/14*b^{14}/n*(x^n)^{14}$

maxima [B] time = 0.64, size = 229, normalized size = 10.90

$$\frac{c^{14}x^{28}}{14n} + \frac{bc^{13}x^{27}}{n} + \frac{13b^2c^{12}x^{26}}{2n} + \frac{26b^3c^{11}x^{25}}{n} + \frac{143b^4c^{10}x^{24}}{2n} + \frac{143b^5c^9x^{23}}{n} + \frac{429b^6c^8x^{22}}{2n} + \frac{1716b^7c^7x^{21}}{7n} + \frac{429b^8c^6x^{20}}{2n} + \frac{143b^9c^5x^{19}}{n} + \frac{143b^{10}c^4x^{18}}{2n} + \frac{26b^{11}c^3x^{17}}{n} + \frac{13b^{12}c^2x^{16}}{2n} + \frac{b^{13}cx^{15}}{n} + \frac{b^{14}x^{14}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{(-1+14*n)}*(b+c*x^n)^{13}*(b+2*c*x^n), x, \text{algorithm}="maxima")$

[Out] $\frac{1}{14}c^{14}x^{(28*n)}/n + b*c^{13}x^{(27*n)}/n + 13/2*b^2*c^{12}x^{(26*n)}/n + 26*b^3*c^{11}x^{(25*n)}/n + 143/2*b^4*c^{10}x^{(24*n)}/n + 143*b^5*c^9*x^{(23*n)}/n + 429/2*b^6*c^8*x^{(22*n)}/n + 1716/7*b^7*c^7*x^{(21*n)}/n + 429/2*b^8*c^6*x^{(20*n)}/n + 143*b^9*c^5*x^{(19*n)}/n + 143/2*b^{10}*c^4*x^{(18*n)}/n + 26*b^{11}*c^3*x^{(17*n)}/n + 13/2*b^{12}*c^2*x^{(16*n)}/n + b^{13}*c*x^{(15*n)}/n + 1/14*b^{14}x^{(14*n)}/n$

mupad [B] time = 5.21, size = 229, normalized size = 10.90

$$\frac{b^{14}x^{14}}{14n} + \frac{c^{14}x^{28}}{14n} + \frac{13b^{12}c^2x^{16}}{2n} + \frac{26b^{11}c^3x^{17}}{n} + \frac{143b^{10}c^4x^{18}}{2n} + \frac{143b^9c^5x^{19}}{n} + \frac{429b^8c^6x^{20}}{2n} + \frac{1716b^7c^7x^{21}}{7n} + \frac{429b^6c^8x^{22}}{2n} + \frac{143b^5c^9x^{23}}{n} + \frac{143b^4c^{10}x^{24}}{2n} + \frac{26b^3c^{11}x^{25}}{n} + \frac{13b^2c^{12}x^{26}}{2n} + \frac{b^{13}cx^{15}}{n} + \frac{b^{14}x^{27}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{(14*n - 1)}*(b + c*x^n)^{13}*(b + 2*c*x^n), x)$

[Out] $\frac{(b^{14}x^{(14*n)})}{(14*n)} + \frac{(c^{14}x^{(28*n)})}{(14*n)} + \frac{(13*b^{12}*c^2*x^{(16*n)})}{(2*n)} + \frac{(26*b^{11}*c^3*x^{(17*n)})}{n} + \frac{(143*b^{10}*c^4*x^{(18*n)})}{(2*n)} + \frac{(143*b^9*c^5*x^{(19*n)})}{n} + \frac{(429*b^8*c^6*x^{(20*n)})}{(2*n)} + \frac{(1716*b^7*c^7*x^{(21*n)})}{(7*n)} + \frac{(429*b^6*c^8*x^{(22*n)})}{(2*n)} + \frac{(143*b^5*c^9*x^{(23*n)})}{n} + \frac{(143*b^4*c^{10}*x^{(24*n)})}{(2*n)} + \frac{(26*b^3*c^{11}*x^{(25*n)})}{n} + \frac{(13*b^2*c^{12}*x^{(26*n)})}{(2*n)} + \frac{(b^{13}*c*x^{(15*n)})}{n} + \frac{(b*c^{13}*x^{(27*n)})}{n}$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{**(-1+14*n)}*(b+c*x**n)**13*(b+2*c*x**n), x)$

[Out] Timed out

$$3.676 \quad \int x^{-1+m} (a + bx^n)^{-1+p} (am + b(m + np)x^n) dx$$

Optimal. Leaf size=13

$$x^m (a + bx^n)^p$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {449}

$$x^m (a + bx^n)^p$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n), x]

[Out] x^m*(a + b*x^n)^p

Rule 449

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]

Rubi steps

$$\int x^{-1+m} (a + bx^n)^{-1+p} (am + b(m + np)x^n) dx = x^m (a + bx^n)^p$$

Mathematica [C] time = 0.18, size = 107, normalized size = 8.23

$$\frac{x^m (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \left(bx^n(m + np) {}_2F_1\left(\frac{m+n}{n}, 1-p; \frac{m}{n} + 2; -\frac{bx^n}{a}\right) + a(m+n) {}_2F_1\left(\frac{m}{n}, 1-p; \frac{m+n}{n}; -\frac{bx^n}{a}\right)\right)}{a(m+n)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n), x]

[Out] (x^m*(a + b*x^n)^p*(a*(m + n)*Hypergeometric2F1[m/n, 1 - p, (m + n)/n, -(b*x^n)/a] + b*(m + n*p)*x^n*Hypergeometric2F1[(m + n)/n, 1 - p, 2 + m/n, -(b*x^n)/a]))/(a*(m + n)*(1 + (b*x^n)/a)^p)

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int x^{-1+m} (a + bx^n)^{-1+p} (am + b(m + np)x^n) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 + m)*(a + b*xⁿ)^(-1 + p)*(a*m + b*(m + n*p)*xⁿ), x]

[Out] Defer[IntegrateAlgebraic][x^(-1 + m)*(a + b*xⁿ)^(-1 + p)*(a*m + b*(m + n*p)*xⁿ), x]

fricas [B] time = 0.43, size = 32, normalized size = 2.46

$$(bxx^{m-1}x^n + axx^{m-1})(bx^n + a)^{p-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+m)*(a+b*xⁿ)^(-1+p)*(a*m+b*(n*p+m)*xⁿ),x, algorithm="fricas")

[Out] (b*x*x^(m - 1)*xⁿ + a*x*x^(m - 1))*(b*xⁿ + a)^(p - 1)

giac [B] time = 0.28, size = 70, normalized size = 5.38

$$bxx^n e^{(p \log(bx^n+a)+m \log(x)-\log(bx^n+a)-\log(x))} + ax e^{(p \log(bx^n+a)+m \log(x)-\log(bx^n+a)-\log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+m)*(a+b*xⁿ)^(-1+p)*(a*m+b*(n*p+m)*xⁿ),x, algorithm="giac")

[Out] b*x*xⁿ*e^{(p*log(b*xⁿ + a) + m*log(x) - log(b*xⁿ + a) - log(x))} + a*x*e^{(p*log(b*xⁿ + a) + m*log(x) - log(b*xⁿ + a) - log(x))}

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int (am + (np + m)bx^n)x^{m-1}(bx^n + a)^{p-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(m-1)*(b*xⁿ+a)^(p-1)*(a*m+b*(n*p+m)*xⁿ), x)

[Out] int(x^(m-1)*(b*xⁿ+a)^(p-1)*(a*m+b*(n*p+m)*xⁿ), x)

maxima [A] time = 0.84, size = 16, normalized size = 1.23

$$e^{(p \log(bx^n+a)+m \log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+m)*(a+b*xⁿ)^(-1+p)*(a*m+b*(n*p+m)*xⁿ),x, algorithm="maxima")

[Out] e^{(p*log(b*xⁿ + a) + m*log(x))}

mupad [B] time = 4.81, size = 25, normalized size = 1.92

$$(a x^m + b x^{m+n}) (a + b x^n)^{p-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(m - 1)*(a*m + b*xⁿ*(m + n*p))*(a + b*xⁿ)^(p - 1),x)

[Out] (a*x^m + b*x^(m + n))*(a + b*xⁿ)^(p - 1)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+m)*(a+b*xⁿ)^(-1+p)*(a*m+b*(n*p+m)*xⁿ),x)

[Out] Timed out

$$3.677 \quad \int \frac{b+2cx}{x(b+cx)} dx$$

Optimal. Leaf size=8

$$\log(x(b+cx))$$

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {72}

$$\log(b+cx) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(x*(b + c*x)),x]

[Out] Log[x] + Log[b + c*x]

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{b+2cx}{x(b+cx)} dx &= \int \left(\frac{1}{x} + \frac{c}{b+cx} \right) dx \\ &= \log(x) + \log(b+cx) \end{aligned}$$

Mathematica [A] time = 0.02, size = 9, normalized size = 1.12

$$\log(b+cx) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)/(x*(b + c*x)),x]

[Out] Log[x] + Log[b + c*x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx}{x(b+cx)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x)/(x*(b + c*x)),x]

[Out] IntegrateAlgebraic[(b + 2*c*x)/(x*(b + c*x)), x]

fricas [A] time = 0.40, size = 10, normalized size = 1.25

$$\log(cx^2 + bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="fricas")

[Out] log(c*x^2 + b*x)

giac [A] time = 0.15, size = 11, normalized size = 1.38

$$\log(|cx + b|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="giac")

[Out] log(abs(c*x + b)) + log(abs(x))

maple [A] time = 0.04, size = 9, normalized size = 1.12

$$\ln((cx + b)x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)/x/(c*x+b),x)

[Out] ln(x*(c*x+b))

maxima [A] time = 0.48, size = 9, normalized size = 1.12

$$\log(cx + b) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="maxima")

[Out] log(c*x + b) + log(x)

mupad [B] time = 4.66, size = 8, normalized size = 1.00

$$\ln(x(b + cx))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(x*(b + c*x)),x)
```

```
[Out] log(x*(b + c*x))
```

sympy [A] time = 0.12, size = 8, normalized size = 1.00

$$\log\left(bx + cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x)
```

```
[Out] log(b*x + c*x**2)
```

$$3.678 \quad \int \frac{b+2cx^2}{x(b+cx^2)} dx$$

Optimal. Leaf size=15

$$\frac{1}{2} \log(b + cx^2) + \log(x)$$

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {446, 72}

$$\frac{1}{2} \log(b + cx^2) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^2)/(x*(b + c*x^2)),x]

[Out] Log[x] + Log[b + c*x^2]/2

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{b+2cx^2}{x(b+cx^2)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{b+2cx}{x(b+cx)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{x} + \frac{c}{b+cx} \right) dx, x, x^2 \right) \\ &= \log(x) + \frac{1}{2} \log(b + cx^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{2} \log(b + cx^2) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^2)/(x*(b + c*x^2)),x]

[Out] Log[x] + Log[b + c*x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + 2cx^2}{x(b + cx^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x^2)/(x*(b + c*x^2)),x]

[Out] IntegrateAlgebraic[(b + 2*c*x^2)/(x*(b + c*x^2)), x]

fricas [A] time = 0.40, size = 13, normalized size = 0.87

$$\frac{1}{2} \log(cx^2 + b) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="fricas")

[Out] 1/2*log(c*x^2 + b) + log(x)

giac [A] time = 0.15, size = 18, normalized size = 1.20

$$\frac{1}{2} \log(x^2) + \frac{1}{2} \log(|cx^2 + b|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="giac")

[Out] 1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))

maple [A] time = 0.05, size = 14, normalized size = 0.93

$$\ln(x) + \frac{\ln(cx^2 + b)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*c*x^2+b)/x/(c*x^2+b),x)`

[Out] `ln(x)+1/2*ln(c*x^2+b)`

maxima [A] time = 0.49, size = 17, normalized size = 1.13

$$\frac{1}{2} \log(cx^2 + b) + \frac{1}{2} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="maxima")`

[Out] `1/2*log(c*x^2 + b) + 1/2*log(x^2)`

mupad [B] time = 0.06, size = 13, normalized size = 0.87

$$\frac{\ln(cx^2 + b)}{2} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b + 2*c*x^2)/(x*(b + c*x^2)),x)`

[Out] `log(b + c*x^2)/2 + log(x)`

sympy [A] time = 0.18, size = 12, normalized size = 0.80

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x**2+b)/x/(c*x**2+b),x)`

[Out] `log(x) + log(b/c + x**2)/2`

$$3.679 \quad \int \frac{b+2cx^3}{x(b+cx^3)} dx$$

Optimal. Leaf size=15

$$\frac{1}{3} \log(b + cx^3) + \log(x)$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {446, 72}

$$\frac{1}{3} \log(b + cx^3) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^3)/(x*(b + c*x^3)),x]

[Out] Log[x] + Log[b + c*x^3]/3

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{b+2cx^3}{x(b+cx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{b+2cx}{x(b+cx)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{x} + \frac{c}{b+cx} \right) dx, x, x^3 \right) \\ &= \log(x) + \frac{1}{3} \log(b + cx^3) \end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{1}{3} \log(b + cx^3) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^3)/(x*(b + c*x^3)), x]

[Out] Log[x] + Log[b + c*x^3]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + 2cx^3}{x(b + cx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x^3)/(x*(b + c*x^3)), x]

[Out] IntegrateAlgebraic[(b + 2*c*x^3)/(x*(b + c*x^3)), x]

fricas [A] time = 0.40, size = 13, normalized size = 0.87

$$\frac{1}{3} \log(cx^3 + b) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/x/(c*x^3+b), x, algorithm="fricas")

[Out] 1/3*log(c*x^3 + b) + log(x)

giac [A] time = 0.16, size = 15, normalized size = 1.00

$$\frac{1}{3} \log(|cx^3 + b|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/x/(c*x^3+b), x, algorithm="giac")

[Out] 1/3*log(abs(c*x^3 + b)) + log(abs(x))

maple [A] time = 0.05, size = 14, normalized size = 0.93

$$\ln(x) + \frac{\ln(cx^3 + b)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*c*x^3+b)/x/(c*x^3+b),x)`

[Out] `ln(x)+1/3*ln(c*x^3+b)`

maxima [A] time = 0.55, size = 17, normalized size = 1.13

$$\frac{1}{3} \log(cx^3 + b) + \frac{1}{3} \log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="maxima")`

[Out] `1/3*log(c*x^3 + b) + 1/3*log(x^3)`

mupad [B] time = 4.63, size = 13, normalized size = 0.87

$$\frac{\ln(cx^3 + b)}{3} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b + 2*c*x^3)/(x*(b + c*x^3)),x)`

[Out] `log(b + c*x^3)/3 + log(x)`

sympy [A] time = 0.19, size = 12, normalized size = 0.80

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x**3+b)/x/(c*x**3+b),x)`

[Out] `log(x) + log(b/c + x**3)/3`

$$3.680 \quad \int \frac{b+2cx^n}{x(b+cx^n)} dx$$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{n} + \log(x)$$

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {446, 72}

$$\frac{\log(b+cx^n)}{n} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^n)/(x*(b + c*x^n)), x]

[Out] Log[x] + Log[b + c*x^n]/n

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{b+2cx^n}{x(b+cx^n)} dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)} dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x} + \frac{c}{b+cx}\right) dx, x, x^n\right)}{n} \\ &= \log(x) + \frac{\log(b+cx^n)}{n} \end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{\log(b + cx^n)}{n} + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^n)/(x*(b + c*x^n)), x]

[Out] Log[x] + Log[b + c*x^n]/n

IntegrateAlgebraic [A] time = 0.03, size = 24, normalized size = 1.60

$$\frac{\log(bn + cnx^n)}{n} + \frac{\log(x^n)}{n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(b + 2*c*x^n)/(x*(b + c*x^n)), x]

[Out] Log[x^n]/n + Log[b*n + c*n*x^n]/n

fricas [A] time = 0.44, size = 17, normalized size = 1.13

$$\frac{n \log(x) + \log(cx^n + b)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x^n)/x/(b+c*x^n), x, algorithm="fricas")

[Out] (n*log(x) + log(c*x^n + b))/n

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2cx^n + b}{(cx^n + b)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x^n)/x/(b+c*x^n), x, algorithm="giac")

[Out] integrate((2*c*x^n + b)/((c*x^n + b)*x), x)

maple [A] time = 0.05, size = 17, normalized size = 1.13

$$\frac{\ln((cx^n + b)x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b+2*c*x^n)/x/(b+c*x^n),x)`

[Out] $1/n*\ln(x^n*(b+c*x^n))$

maxima [B] time = 0.49, size = 47, normalized size = 3.13

$$b \left(\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn} \right) + \frac{2 \log\left(\frac{cx^n+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="maxima")`

[Out] $b*(\log(x)/b - \log((c*x^n + b)/c)/(b*n)) + 2*\log((c*x^n + b)/c)/n$

mupad [B] time = 4.72, size = 15, normalized size = 1.00

$$\ln(x) + \frac{\ln(b + c x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b + 2*c*x^n)/(x*(b + c*x^n)),x)`

[Out] $\log(x) + \log(b + c*x^n)/n$

sympy [A] time = 0.62, size = 29, normalized size = 1.93

$$\begin{cases} \log(x) & \text{for } c = 0 \wedge n = 0 \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n = 0 \\ \log(x) & \text{for } c = 0 \\ \log(x) + \frac{\log\left(\frac{b}{c} + x^n\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b+2*c*x**n)/x/(b+c*x**n),x)`

[Out] `Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), ((b + 2*c)*log(x)/(b + c), Eq(n, 0)), (log(x), Eq(c, 0)), (log(x) + log(b/c + x**n)/n, True))`

$$3.681 \quad \int \frac{b+2cx}{x^8(b+cx)^8} dx$$

Optimal. Leaf size=14

$$-\frac{1}{7x^7(b+cx)^7}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {74}

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(x^8*(b + c*x)^8), x]

[Out] -1/(7*x^7*(b + c*x)^7)

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rubi steps

$$\int \frac{b+2cx}{x^8(b+cx)^8} dx = -\frac{1}{7x^7(b+cx)^7}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 1.00

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)/(x^8*(b + c*x)^8), x]

[Out] -1/7*1/(x^7*(b + c*x)^7)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + 2cx}{x^8(b + cx)^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x)/(x^8*(b + c*x)^8), x]

[Out] IntegrateAlgebraic[(b + 2*c*x)/(x^8*(b + c*x)^8), x]

fricas [B] time = 0.42, size = 81, normalized size = 5.79

$$\frac{1}{7(c^7x^{14} + 7bc^6x^{13} + 21b^2c^5x^{12} + 35b^3c^4x^{11} + 35b^4c^3x^{10} + 21b^5c^2x^9 + 7b^6cx^8 + b^7x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="fricas")

[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)

giac [A] time = 0.17, size = 13, normalized size = 0.93

$$-\frac{1}{7(cx^2 + bx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="giac")

[Out] -1/7/(c*x^2 + b*x)^7

maple [B] time = 0.06, size = 177, normalized size = 12.64

$$\frac{c^7}{7(cx+b)^7b^7} + \frac{c^7}{(cx+b)^6b^8} + \frac{4c^7}{(cx+b)^5b^9} + \frac{12c^7}{(cx+b)^4b^{10}} + \frac{30c^7}{(cx+b)^3b^{11}} + \frac{66c^7}{(cx+b)^2b^{12}} + \frac{132c^7}{(cx+b)b^{13}} - \frac{132c^6}{b^{13}x} + \frac{66c^5}{b^{12}x^2} - \frac{30c^4}{b^{11}x^3} + \frac{12c^3}{b^{10}x^4} - \frac{4c^2}{b^9x^5} + \frac{c}{b^8x^6} - \frac{1}{7b^7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)/x^8/(c*x+b)^8,x)

[Out] 132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^11*c^7/(c*x+b)^3+12/b^10*c^7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+c^7/b^8/(c*x+b)^6+1/7*c^7/b^7/(c*x+b)^7-1/7/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4-4/b^9*c^2/x^5+1/b^8*c/x^6

maxima [B] time = 0.61, size = 81, normalized size = 5.79

$$\frac{1}{7(c^7x^{14} + 7bc^6x^{13} + 21b^2c^5x^{12} + 35b^3c^4x^{11} + 35b^4c^3x^{10} + 21b^5c^2x^9 + 7b^6cx^8 + b^7x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="maxima")

[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)

mupad [B] time = 7.09, size = 12, normalized size = 0.86

$$\frac{1}{7x^7(b+cx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x)/(x^8*(b + c*x)^8),x)

[Out] -1/(7*x^7*(b + c*x)^7)

sympy [B] time = 0.89, size = 87, normalized size = 6.21

$$\frac{1}{7b^7x^7 + 49b^6cx^8 + 147b^5c^2x^9 + 245b^4c^3x^{10} + 245b^3c^4x^{11} + 147b^2c^5x^{12} + 49bc^6x^{13} + 7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/x**8/(c*x+b)**8,x)

[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**14)

$$3.682 \quad \int \frac{b+2cx^2}{x^{15}(b+cx^2)^8} dx$$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {446, 74}

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8), x]

[Out] -1/(14*x^14*(b + c*x^2)^7)

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{b+2cx^2}{x^{15}(b+cx^2)^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{b+2cx}{x^8(b+cx)^8} dx, x, x^2 \right) \\ &= -\frac{1}{14x^{14}(b+cx^2)^7} \end{aligned}$$

Mathematica [A] time = 0.05, size = 16, normalized size = 1.00

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8), x]

[Out] -1/14*1/(x^14*(b + c*x^2)^7)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + 2cx^2}{x^{15}(b + cx^2)^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8), x]

[Out] IntegrateAlgebraic[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8), x]

fricas [B] time = 0.41, size = 81, normalized size = 5.06

$$\frac{1}{14(c^7x^{28} + 7bc^6x^{26} + 21b^2c^5x^{24} + 35b^3c^4x^{22} + 35b^4c^3x^{20} + 21b^5c^2x^{18} + 7b^6cx^{16} + b^7x^{14})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="fricas")

[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)

giac [A] time = 0.15, size = 15, normalized size = 0.94

$$-\frac{1}{14(cx^4 + bx^2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="giac")

[Out] -1/14/(c*x^4 + b*x^2)^7

maple [B] time = 0.06, size = 197, normalized size = 12.31

$$\left(\frac{b^6}{7(c^2x+b)^7c} - \frac{b^5}{(c^2x+b)^6c} - \frac{4b^4}{(c^2x+b)^5c} - \frac{12b^3}{(c^2x+b)^4c} - \frac{30b^2}{(c^2x+b)^3c} - \frac{66b}{(c^2x+b)^2c} - \frac{132}{(c^2x+b)c} \right) c^8 - \frac{66c^6}{b^{13}x^2} + \frac{33c^5}{b^{12}x^4} - \frac{15c^4}{b^{11}x^6} + \frac{6c^3}{b^{10}x^8} - \frac{2c^2}{b^9x^{10}} + \frac{c}{2b^8x^{12}} - \frac{1}{14b^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x^2+b)/x^15/(c*x^2+b)^8,x)

[Out] $-1/2*c^8/b^{13}*(-4*b^4/c/(c*x^2+b)^5-66*b/c/(c*x^2+b)^2-1/7*b^6/c/(c*x^2+b)^7-b^5/c/(c*x^2+b)^6-12*b^3/c/(c*x^2+b)^4-132/c/(c*x^2+b)-30*b^2/c/(c*x^2+b)^3)-1/14/b^7/x^{14}-66/b^{13}*c^6/x^2+33/b^{12}*c^5/x^4-15/b^{11}*c^4/x^6+6/b^{10}*c^3/x^8-2/b^9*c^2/x^{10}+1/2/b^8*c/x^{12}$

maxima [B] time = 0.64, size = 81, normalized size = 5.06

$$\frac{1}{14(c^7x^{28} + 7bc^6x^{26} + 21b^2c^5x^{24} + 35b^3c^4x^{22} + 35b^4c^3x^{20} + 21b^5c^2x^{18} + 7b^6cx^{16} + b^7x^{14})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="maxima")

[Out] $-1/14/(c^7*x^{28} + 7*b*c^6*x^{26} + 21*b^2*c^5*x^{24} + 35*b^3*c^4*x^{22} + 35*b^4*c^3*x^{20} + 21*b^5*c^2*x^{18} + 7*b^6*c*x^{16} + b^7*x^{14})$

mupad [B] time = 2.32, size = 14, normalized size = 0.88

$$-\frac{1}{14x^{14}(cx^2+b)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x)

[Out] $-1/(14*x^{14}*(b + c*x^2)^7)$

sympy [B] time = 1.36, size = 87, normalized size = 5.44

$$\frac{1}{14b^7x^{14} + 98b^6cx^{16} + 294b^5c^2x^{18} + 490b^4c^3x^{20} + 490b^3c^4x^{22} + 294b^2c^5x^{24} + 98bc^6x^{26} + 14c^7x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x**2+b)/x**15/(c*x**2+b)**8,x)

[Out] $-1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**7*x**28)$

$$3.683 \quad \int \frac{b+2cx^3}{x^{22}(b+cx^3)^8} dx$$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}(b+cx^3)^7}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {446, 74}

$$-\frac{1}{21x^{21}(b+cx^3)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]

[Out] -1/(21*x^21*(b + c*x^3)^7)

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{b+2cx^3}{x^{22}(b+cx^3)^8} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{b+2cx}{x^8(b+cx)^8} dx, x, x^3 \right) \\ &= -\frac{1}{21x^{21}(b+cx^3)^7} \end{aligned}$$

Mathematica [A] time = 0.05, size = 16, normalized size = 1.00

$$-\frac{1}{21x^{21}(b+cx^3)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8), x]

[Out] -1/21*1/(x^21*(b + c*x^3)^7)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + 2cx^3}{x^{22}(b + cx^3)^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8), x]

[Out] IntegrateAlgebraic[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8), x]

fricas [B] time = 0.39, size = 81, normalized size = 5.06

$$-\frac{1}{21(c^7x^{42} + 7bc^6x^{39} + 21b^2c^5x^{36} + 35b^3c^4x^{33} + 35b^4c^3x^{30} + 21b^5c^2x^{27} + 7b^6cx^{24} + b^7x^{21})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="fricas")

[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)

giac [A] time = 0.17, size = 15, normalized size = 0.94

$$-\frac{1}{21(cx^6 + bx^3)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="giac")

[Out] -1/21/(c*x^6 + b*x^3)^7

maple [B] time = 0.06, size = 197, normalized size = 12.31

$$\frac{\left(-\frac{b^6}{7(c x^3+b)^7 c}-\frac{b^5}{(c x^3+b)^6 c}-\frac{4 b^4}{(c x^3+b)^5 c}-\frac{12 b^3}{(c x^3+b)^4 c}-\frac{30 b^2}{(c x^3+b)^3 c}-\frac{66 b}{(c x^3+b)^2 c}-\frac{132}{(c x^3+b) c}\right) c^8}{3 b^{13}}-\frac{44 c^6}{b^{13} x^3}+\frac{22 c^5}{b^{12} x^6}-\frac{10 c^4}{b^{11} x^9}+\frac{4 c^3}{b^{10} x^{12}}-\frac{4 c^2}{3 b^9 x^{15}}+\frac{c}{3 b^8 x^{18}}-\frac{1}{21 b^7 x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x^3+b)/x^22/(c*x^3+b)^8,x)

[Out] $-1/3*c^8/b^{13}*(-4*b^4/c/(c*x^3+b)^5-66*b/c/(c*x^3+b)^2-1/7*b^6/c/(c*x^3+b)^7-b^5/c/(c*x^3+b)^6-12*b^3/c/(c*x^3+b)^4-132/c/(c*x^3+b)^3-30*b^2/c/(c*x^3+b)^2)-1/21/b^7/x^{21}-44/b^{13}*c^6/x^3+22/b^{12}*c^5/x^6-10/b^{11}*c^4/x^9+4/b^{10}*c^3/x^{12}-4/3/b^9*c^2/x^{15}+1/3/b^8*c/x^{18}$

maxima [B] time = 0.61, size = 81, normalized size = 5.06

$$\frac{1}{21 \left(c^7 x^{42} + 7 b c^6 x^{39} + 21 b^2 c^5 x^{36} + 35 b^3 c^4 x^{33} + 35 b^4 c^3 x^{30} + 21 b^5 c^2 x^{27} + 7 b^6 c x^{24} + b^7 x^{21} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="maxima")

[Out] $-1/21/(c^7*x^{42} + 7*b*c^6*x^{39} + 21*b^2*c^5*x^{36} + 35*b^3*c^4*x^{33} + 35*b^4*c^3*x^{30} + 21*b^5*c^2*x^{27} + 7*b^6*c*x^{24} + b^7*x^{21})$

mupad [B] time = 9.98, size = 14, normalized size = 0.88

$$-\frac{1}{21 x^{21} (c x^3 + b)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x)

[Out] $-1/(21*x^{21}*(b + c*x^3)^7)$

sympy [B] time = 1.91, size = 87, normalized size = 5.44

$$-\frac{1}{21 b^7 x^{21} + 147 b^6 c x^{24} + 441 b^5 c^2 x^{27} + 735 b^4 c^3 x^{30} + 735 b^3 c^4 x^{33} + 441 b^2 c^5 x^{36} + 147 b c^6 x^{39} + 21 c^7 x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x**3+b)/x**22/(c*x**3+b)**8,x)

[Out] $-1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c**7*x**42)$

$$3.684 \quad \int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8} dx$$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {446, 74}

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]

[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8} dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8} dx, x, x^n\right)}{n} \\ &= -\frac{x^{-7n}}{7n(b+cx^n)^7} \end{aligned}$$

Mathematica [A] time = 0.25, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]

[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)

IntegrateAlgebraic [A] time = 0.10, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]

[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)

fricas [B] time = 0.48, size = 105, normalized size = 5.00

$$\frac{1}{7(c^7nx^{14n} + 7bc^6nx^{13n} + 21b^2c^5nx^{12n} + 35b^3c^4nx^{11n} + 35b^4c^3nx^{10n} + 21b^5c^2nx^9n + 7b^6c^1nx^8n + b^7nx^7n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="fricas")

[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3*c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n*x^(8*n) + b^7*n*x^(7*n))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2cx^n + b)x^{-7n-1}}{(cx^n + b)^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="giac")

[Out] integrate((2*c*x^n + b)*x^(-7*n - 1)/(c*x^n + b)^8, x)

maple [B] time = 0.10, size = 203, normalized size = 9.67

$$\frac{x^{-7n}}{7b^{7n}} + \frac{cx^{-6n}}{b^{6n}} - \frac{4c^2x^{-5n}}{b^{9n}} + \frac{12c^3x^{-4n}}{b^{10n}} - \frac{30c^4x^{-3n}}{b^{11n}} + \frac{66c^5x^{-2n}}{b^{12n}} + \frac{(9009b^5cx^n + 20020b^4c^2x^{2n} + 24024b^3c^3x^{3n} + 16380b^2c^4x^{4n} + 6006bc^5x^{5n} + 924c^6x^{6n} + 1716b^6)c^7}{7(cx^n + b)^7 b^{13n}} - \frac{132c^6x^{-n}}{b^{13n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-7*n)*(b+2*c*x^n)/(c*x^n+b)^8,x)

[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(c*x^n+b)^7

maxima [B] time = 0.79, size = 612, normalized size = 29.14

$$\frac{1}{28} \left(\frac{360360c^{13}x^{13n} + 2342340b^3c^{12}x^{12n} + 6426420b^2c^{11}x^{11n} + 9579570b^3c^{10}x^{10n} + 8270262b^4c^9x^{9n} + 4018014b^5c^8x^{8n} + 934362b^6c^7x^{7n} + 45045b^7c^6x^{6n} - 5005b^8c^5x^{5n} + 1001b^9c^4x^{4n} - 273b^{10}c^3x^{3n} + 91b^{11}c^2x^{2n} - 35b^{12}cx^n + 15b^{13}}{b^{14}c^7n x^{14n} + 7b^{15}c^6n x^{13n} + 21b^{16}c^5n x^{12n} + 35b^{17}c^4n x^{11n} + 35b^{18}c^3n x^{10n} + 21b^{19}c^2n x^{9n} + 7b^{20}cn x^{8n} + b^{21}n x^{7n}} \right) + \frac{360360c^7 \log(x)}{b^{15}} - \frac{360360c^7 \log((cx^n + b)/c)}{b^{15n}} + \frac{1}{105} c \left(\frac{360360c^{12}x^{12n} + 2342340b^3c^{11}x^{11n} + 6426420b^2c^{10}x^{10n} + 9579570b^3c^9x^{9n} + 8270262b^4c^8x^{8n} + 4018014b^5c^7x^{7n} + 934362b^6c^6x^{6n} + 45045b^7c^5x^{5n} - 5005b^8c^4x^{4n} + 1001b^9c^3x^{3n} - 273b^{10}c^2x^{2n} + 91b^{11}cx^n - 35b^{12}}{b^{13}c^7n x^{13n} + 7b^{14}c^6n x^{12n} + 21b^{15}c^5n x^{11n} + 35b^{16}c^4n x^{10n} + 35b^{17}c^3n x^{9n} + 21b^{18}c^2n x^{8n} + 7b^{19}cn x^{7n} + b^{20}n x^{6n}} \right) + \frac{360360c^6 \log(x)}{b^{14}} - \frac{360360c^6 \log((cx^n + b)/c)}{b^{14n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="maxima")

[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^11*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n) + 1/105*c*((360360*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 9579570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) + 934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 1001*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n + b)/c)/(b^14*n))

mupad [B] time = 4.99, size = 105, normalized size = 5.00

$$\frac{1}{7x^{7n} (b^7n + c^7nx^{7n} + 7b^6cnx^{6n} + 7bc^6nx^{6n} + 21b^5c^2nx^{2n} + 35b^4c^3nx^{3n} + 35b^3c^4nx^{4n} + 21b^2c^5nx^{5n})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x^n)/(x^(7*n + 1)*(b + c*x^n)^8),x)

```
[Out] -1/(7*x^(7*n)*(b^7*n + c^7*n*x^(7*n) + 7*b^6*c*n*x^n + 7*b*c^6*n*x^(6*n) +  
21*b^5*c^2*n*x^(2*n) + 35*b^4*c^3*n*x^(3*n) + 35*b^3*c^4*n*x^(4*n) + 21*b^2  
*c^5*n*x^(5*n)))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-7*n)*(b+2*c*x**n)/(b+c*x**n)**8,x)
```

```
[Out] Timed out
```

$$3.685 \quad \int \frac{x^{31} \sqrt{1+x^{16}}}{1-x^{16}} dx$$

Optimal. Leaf size=52

$$-\frac{1}{24} (x^{16} + 1)^{3/2} - \frac{\sqrt{x^{16} + 1}}{8} + \frac{\tanh^{-1}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 80, 50, 63, 206}

$$-\frac{1}{24} (x^{16} + 1)^{3/2} - \frac{\sqrt{x^{16} + 1}}{8} + \frac{\tanh^{-1}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(x^31*Sqrt[1 + x^16])/(1 - x^16), x]

[Out] -Sqrt[1 + x^16]/8 - (1 + x^16)^(3/2)/24 + ArcTanh[Sqrt[1 + x^16]/Sqrt[2]]/(4*Sqrt[2])

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p

+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}} dx &= \frac{1}{16} \text{Subst} \left(\int \frac{x\sqrt{1+x}}{1-x} dx, x, x^{16} \right) \\
 &= -\frac{1}{24} (1+x^{16})^{3/2} + \frac{1}{16} \text{Subst} \left(\int \frac{\sqrt{1+x}}{1-x} dx, x, x^{16} \right) \\
 &= -\frac{1}{8} \sqrt{1+x^{16}} - \frac{1}{24} (1+x^{16})^{3/2} + \frac{1}{8} \text{Subst} \left(\int \frac{1}{(1-x)\sqrt{1+x}} dx, x, x^{16} \right) \\
 &= -\frac{1}{8} \sqrt{1+x^{16}} - \frac{1}{24} (1+x^{16})^{3/2} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{2-x^2} dx, x, \sqrt{1+x^{16}} \right) \\
 &= -\frac{1}{8} \sqrt{1+x^{16}} - \frac{1}{24} (1+x^{16})^{3/2} + \frac{\tanh^{-1} \left(\frac{\sqrt{1+x^{16}}}{\sqrt{2}} \right)}{4\sqrt{2}}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.85

$$\frac{1}{24} \left(3\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}} \right) - \sqrt{x^{16}+1} (x^{16}+4) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^31*sqrt[1 + x^16])/(1 - x^16), x]

[Out] $(-(\text{Sqrt}[1 + x^{16}](4 + x^{16})) + 3*\text{Sqrt}[2]*\text{ArcTanh}[\text{Sqrt}[1 + x^{16}]/\text{Sqrt}[2]])/24$

IntegrateAlgebraic [A] time = 0.07, size = 46, normalized size = 0.88

$$\frac{1}{24} \sqrt{x^{16} + 1} (-x^{16} - 4) + \frac{\tanh^{-1}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^31*Sqrt[1 + x^16])/(1 - x^16), x]

[Out] $((-4 - x^{16})*\text{Sqrt}[1 + x^{16}])/24 + \text{ArcTanh}[\text{Sqrt}[1 + x^{16}]/\text{Sqrt}[2]]/(4*\text{Sqrt}[2])$

fricas [A] time = 0.42, size = 46, normalized size = 0.88

$$-\frac{1}{24} (x^{16} + 4) \sqrt{x^{16} + 1} + \frac{1}{16} \sqrt{2} \log\left(\frac{x^{16} + 2\sqrt{2}\sqrt{x^{16} + 1} + 3}{x^{16} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1), x, algorithm="fricas")

[Out] $-1/24*(x^{16} + 4)*\text{sqrt}(x^{16} + 1) + 1/16*\text{sqrt}(2)*\log((x^{16} + 2*\text{sqrt}(2)*\text{sqrt}(x^{16} + 1) + 3)/(x^{16} - 1))$

giac [A] time = 0.15, size = 56, normalized size = 1.08

$$-\frac{1}{24} (x^{16} + 1)^{\frac{3}{2}} - \frac{1}{16} \sqrt{2} \log\left(\frac{|-2\sqrt{2} + 2\sqrt{x^{16} + 1}|}{2(\sqrt{2} + \sqrt{x^{16} + 1})}\right) - \frac{1}{8} \sqrt{x^{16} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1), x, algorithm="giac")

[Out] $-1/24*(x^{16} + 1)^{(3/2)} - 1/16*\text{sqrt}(2)*\log(1/2*\text{abs}(-2*\text{sqrt}(2) + 2*\text{sqrt}(x^{16} + 1))/(\text{sqrt}(2) + \text{sqrt}(x^{16} + 1))) - 1/8*\text{sqrt}(x^{16} + 1)$

maple [C] time = 1.15, size = 86, normalized size = 1.65

$$\frac{\text{RootOf}(-Z^2 - 2) \ln\left(-\frac{x^{16} \text{RootOf}(-Z^2 - 2) + 3 \text{RootOf}(-Z^2 - 2) + 4 \sqrt{x^{16} + 1}}{(x-1)(x+1)(x^2+1)(x^4+1)(x^8+1)}\right)}{16} - \frac{(x^{16} + 4) \sqrt{x^{16} + 1}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^31*(x^16+1)^(1/2)/(-x^16+1),x)`

[Out] $-1/24*(x^{16}+4)*(x^{16}+1)^{(1/2)}+1/16*\text{RootOf}(_Z^2-2)*\ln(-(\text{RootOf}(_Z^2-2)*x^{16}+4*(x^{16}+1)^{(1/2)}+3*\text{RootOf}(_Z^2-2)))/(x-1)/(x+1)/(x^2+1)/(x^4+1)/(x^8+1))$

maxima [A] time = 1.25, size = 53, normalized size = 1.02

$$-\frac{1}{24}(x^{16}+1)^{\frac{3}{2}}-\frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^{16}+1}}{\sqrt{2}+\sqrt{x^{16}+1}}\right)-\frac{1}{8}\sqrt{x^{16}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="maxima")`

[Out] $-1/24*(x^{16}+1)^{(3/2)}-1/16*\text{sqrt}(2)*\log(-(\text{sqrt}(2)-\text{sqrt}(x^{16}+1)))/(\text{sqrt}(2)+\text{sqrt}(x^{16}+1)))-1/8*\text{sqrt}(x^{16}+1)$

mupad [B] time = 4.82, size = 37, normalized size = 0.71

$$\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{x^{16}+1}}{2}\right)}{8}-\frac{\sqrt{x^{16}+1}}{8}-\frac{(x^{16}+1)^{3/2}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^31*(x^16+1)^(1/2))/(x^16-1),x)`

[Out] $(2^{(1/2)}*\operatorname{atanh}((2^{(1/2)}*(x^{16}+1)^{(1/2}))/2))/8-(x^{16}+1)^{(1/2)}/8-(x^{16}+1)^{(3/2)}/24$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**31*(x**16+1)**(1/2)/(-x**16+1),x)`

[Out] Timed out

$$3.686 \quad \int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}} dx$$

Optimal. Leaf size=93

$$\frac{2\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{2\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {446, 105, 63, 217, 206, 93, 208}

$$\frac{2\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{2\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d/x]/(Sqrt[a + b/x]*x), x]

[Out] (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[a] - (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])])/Sqrt[b]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)/((e_.) + (f_.)*(x
_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dist[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m, 0] || (!RationalQ[m] && (SumSimplerQ[m, -1] || !SumSimplerQ[n, -1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c + \frac{d}{x}}}{\sqrt{a + \frac{b}{x}}} dx &= -\text{Subst} \left(\int \frac{\sqrt{c + dx}}{x\sqrt{a + bx}} dx, x, \frac{1}{x} \right) \\
&= - \left(c \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx}\sqrt{c + dx}} dx, x, \frac{1}{x} \right) \right) - d \text{Subst} \left(\int \frac{1}{\sqrt{a + bx}\sqrt{c + dx}} dx, x, \frac{1}{x} \right) \\
&= - \left((2c) \text{Subst} \left(\int \frac{1}{-a + cx^2} dx, x, \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}} \right) \right) - \frac{(2d) \text{Subst} \left(\int \frac{1}{\sqrt{c - \frac{ad}{b} + \frac{dx^2}{b}}} dx, x, \sqrt{a + \frac{b}{x}} \right)}{b} \\
&= \frac{2\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a + \frac{b}{x}}}{\sqrt{a} \sqrt{c + \frac{d}{x}}} \right)}{\sqrt{a}} - \frac{(2d) \text{Subst} \left(\int \frac{1}{1 - \frac{dx^2}{b}} dx, x, \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}} \right)}{b} \\
&= \frac{2\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a + \frac{b}{x}}}{\sqrt{a} \sqrt{c + \frac{d}{x}}} \right)}{\sqrt{a}} - \frac{2\sqrt{d} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{b} \sqrt{c + \frac{d}{x}}} \right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.68, size = 142, normalized size = 1.53

$$\frac{2\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a + \frac{b}{x}}}{\sqrt{a} \sqrt{c + \frac{d}{x}}} \right)}{\sqrt{a}} - \frac{2\sqrt{d} x \sqrt{c + \frac{d}{x}} \sqrt{bc - ad} \sqrt{\frac{b(cx+d)}{x(bc-ad)}} \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{bc-ad}} \right)}{bcx + bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d/x]/(Sqrt[a + b/x]*x), x]

[Out] (-2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[c + d/x]*x*Sqrt[(b*(d + c*x))/((b*c - a*d)*x)]*ArcSinh[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]]/(b*d + b*c*x) + (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/Sqrt[a]*Sqrt[c + d/x]])/Sqrt[a]

IntegrateAlgebraic [B] time = 2.67, size = 442, normalized size = 4.75

$$\frac{c\sqrt{\frac{a}{c}}\sqrt{ax+b}\sqrt{c+\frac{d}{x}}\left(\sqrt{ax+b}-\sqrt{\frac{a}{c}}\sqrt{cx+d}\right)\left(-2c\sqrt{\frac{a}{c}}\sqrt{ax+b}\sqrt{cx+d}+a(2cx+d)+bc\right)}{\sqrt{a+\frac{b}{x}}\sqrt{cx+d}\left(a\left(4c^2x\sqrt{\frac{a}{c}}\sqrt{ax+b}+3cd\sqrt{\frac{a}{c}}\sqrt{ax+b}-4acx\sqrt{cx+d}-ad\sqrt{cx+d}\right)+bc\left(c\sqrt{\frac{a}{c}}\sqrt{ax+b}-3a\sqrt{cx+d}\right)\right)} \left(-\frac{2c\sqrt{\frac{a}{c}}\log\left(\frac{\sqrt{a(cx+d)}-\frac{ad}{c}+b-\sqrt{\frac{a}{c}}\sqrt{cx+d}}{a}\right)}{a} - \frac{2\sqrt{c}\sqrt{d}\tanh^{-1}\left(-\frac{\sqrt{d}(cx+d)}{\sqrt{b}\sqrt{c}}+\frac{\sqrt{c}\sqrt{\frac{a}{c}}\sqrt{cx+d}\sqrt{\frac{a(cx+d)}{c}-\frac{ad}{c}+b}}{\sqrt{a}\sqrt{b}\sqrt{d}}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}\right)}{\sqrt{a}\sqrt{b}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]

[Out] (Sqrt[a/c]*c*Sqrt[c + d/x]*Sqrt[b + a*x]*(Sqrt[b + a*x] - Sqrt[a/c]*Sqrt[d + c*x])*(b*c - 2*Sqrt[a/c]*c*Sqrt[b + a*x]*Sqrt[d + c*x] + a*(d + 2*c*x))*(-2*Sqrt[a/c]*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]) - (Sqrt[a]*(d + c*x))/(Sqrt[b]*Sqrt[c]*Sqrt[d]) + (Sqrt[a/c]*Sqrt[c]*Sqrt[d + c*x]*Sqrt[b - (a*d)/c + (a*(d + c*x))/c])/(Sqrt[a]*Sqrt[b]*Sqrt[d])]/(Sqrt[a]*Sqrt[b]) - (2*Sqrt[a/c]*c*Log[-(Sqrt[a/c]*Sqrt[d + c*x]) + Sqrt[b - (a*d)/c + (a*(d + c*x))/c]])/a)/(Sqrt[a + b/x]*Sqrt[d + c*x]*(b*c*(Sqrt[a/c]*c*Sqrt[b + a*x] - 3*a*Sqrt[d + c*x]) + a*(3*Sqrt[a/c]*c*d*Sqrt[b + a*x] + 4*Sqrt[a/c]*c^2*x*Sqrt[b + a*x] - a*d*Sqrt[d + c*x] - 4*a*c*x*Sqrt[d + c*x])))

fricas [B] time = 0.71, size = 757, normalized size = 8.14



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a^2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x) + 1/2*sqrt(d/b)*log(-8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x/x^2), -sqrt(-c/a)*arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)) + 1/2*sqrt(d/b)*log(-8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x/x^2), sqrt(-d/b)*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(-d/b)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*c*d*x^2 + b*d^2 + (b*c*d + a*d^2)*x)) + 1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a^2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x), -sqrt(-c/a)*arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)) + sqrt(-d/b)*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(-d/b)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*c*d*x^2 + b*d^2 + (b*c*d + a*d^2)*x))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + \frac{d}{x}}}{\sqrt{a + \frac{b}{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)

maple [B] time = 0.10, size = 143, normalized size = 1.54

$$\frac{\sqrt{\frac{ax+b}{x}} \sqrt{\frac{cx+d}{x}} \left(-\sqrt{bd} c \ln \left(\frac{2acx+ad+bc+2\sqrt{(cx+d)(ax+b)} \sqrt{ac}}{2\sqrt{ac}} \right) + \sqrt{ac} d \ln \left(\frac{adx+bcx+2bd+2\sqrt{bd} \sqrt{(cx+d)(ax+b)}}{x} \right) \right)}{\sqrt{(cx+d)(ax+b)} \sqrt{bd} \sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^(1/2)/x/(a+b/x)^(1/2), x)

[Out] -((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(ln((a*d*x+b*c*x+2*(b*d)^(1/2)*((c*x+d)*(a*x+b))^(1/2)+2*b*d)/x)*d*(a*c)^(1/2)-ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*c*(b*d)^(1/2))/((c*x+d)*(a*x+b))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + \frac{d}{x}}}{\sqrt{a + \frac{b}{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c + \frac{d}{x}}}{x \sqrt{a + \frac{b}{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/x)^(1/2)/(x*(a + b/x)^(1/2)), x)

[Out] int((c + d/x)^(1/2)/(x*(a + b/x)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + \frac{d}{x}}}{x \sqrt{a + \frac{b}{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**(1/2)/x/(a+b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d/x)/(x*sqrt(a + b/x)), x)
```

$$3.687 \quad \int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}} dx$$

Optimal. Leaf size=252

$$\frac{5(bc-ad)^3(ad+7bc) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{3/2}d^{9/2}n} - \frac{5(bc-ad)^2(ad+7bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(ad+7bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{96bd^3n}$$

Rubi [A] time = 0.23, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {446, 80, 50, 63, 217, 206}

$$\frac{5(bc-ad)^3(ad+7bc) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{3/2}d^{9/2}n} - \frac{(ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24bd^2n} + \frac{5(bc-ad)(ad+7bc)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{5(bc-ad)^2(ad+7bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bdn}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n], x]

[Out] (-5*(b*c - a*d)^2*(7*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(64*b*d^4*n) + (5*(b*c - a*d)*(7*b*c + a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(96*b*d^3*n) - ((7*b*c + a*d)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(24*b*d^2*n) + ((a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(4*b*d*n) + (5*(b*c - a*d)^3*(7*b*c + a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(64*b^(3/2)*d^(9/2)*n)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n} (a + bx^n)^{5/2}}{\sqrt{c + dx^n}} dx &= \frac{\text{Subst} \left(\int \frac{x^{(a+bx)^{5/2}}}{\sqrt{c+dx}} dx, x, x^n \right)}{n} \\
&= \frac{(a + bx^n)^{7/2} \sqrt{c + dx^n}}{4bdn} - \frac{(7bc + ad) \text{Subst} \left(\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}} dx, x, x^n \right)}{8bdn} \\
&= -\frac{(7bc + ad) (a + bx^n)^{5/2} \sqrt{c + dx^n}}{24bd^2n} + \frac{(a + bx^n)^{7/2} \sqrt{c + dx^n}}{4bdn} + \frac{(5(bc - ad)(7bc + ad))}{96bd^3n} \\
&= \frac{5(bc - ad)(7bc + ad) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{96bd^3n} - \frac{(7bc + ad) (a + bx^n)^{5/2} \sqrt{c + dx^n}}{24bd^2n} + \frac{(a + bx^n)^{7/2} \sqrt{c + dx^n}}{4bdn} \\
&= -\frac{5(bc - ad)^2(7bc + ad) \sqrt{a + bx^n} \sqrt{c + dx^n}}{64bd^4n} + \frac{5(bc - ad)(7bc + ad) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{96bd^3n} \\
&= -\frac{5(bc - ad)^2(7bc + ad) \sqrt{a + bx^n} \sqrt{c + dx^n}}{64bd^4n} + \frac{5(bc - ad)(7bc + ad) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{96bd^3n} \\
&= -\frac{5(bc - ad)^2(7bc + ad) \sqrt{a + bx^n} \sqrt{c + dx^n}}{64bd^4n} + \frac{5(bc - ad)(7bc + ad) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{96bd^3n} \\
&= -\frac{5(bc - ad)^2(7bc + ad) \sqrt{a + bx^n} \sqrt{c + dx^n}}{64bd^4n} + \frac{5(bc - ad)(7bc + ad) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{96bd^3n}
\end{aligned}$$

Mathematica [A] time = 1.09, size = 223, normalized size = 0.88

$$\frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)(15a^3d^3+a^2bd^2(118dx^n-191c)+ab^2d(265c^2-172cdx^n+136d^2x^{2n})+b^3(-105c^3+70c^2dx^n-56cd^2x^{2n}+48d^3x^{3n}))+15(ad+7bc)(bc-ad)^{7/2}\sqrt{\frac{bc+dx^n}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{192b^2d^{9/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n], x]

[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(15*a^3*d^3 + a^2*b*d^2*(-191*c + 118*d*x^n) + a*b^2*d*(265*c^2 - 172*c*d*x^n + 136*d^2*x^(2*n)) + b^3*(-105*c^3 + 70*c^2*d*x^n - 56*c*d^2*x^(2*n) + 48*d^3*x^(3*n))) + 15*(b*c - a*d)^(7/2)*(7*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(192*b^2*d^(9/2)*n*Sqrt[c + d*x^n])

IntegrateAlgebraic [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n} (a + bx^n)^{5/2}}{\sqrt{c + dx^n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^(-1 + 2*n))*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n], x]

[Out] Defer[IntegrateAlgebraic] [(x^(-1 + 2*n))*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n], x]

fricas [A] time = 0.54, size = 607, normalized size = 2.41

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2), x, algorithm="fricas")

[Out] [-1/768*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^5*n), -1/384*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^5*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}} x^{2n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2), x, algorithm="giac")

[Out] integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)

maple [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}} x^{2n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n-1)*(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2), x)

[Out] int(x^(2*n-1)*(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}} x^{2n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2), x, algorithm="maxima")

[Out] integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{2n-1} (a + bx^n)^{5/2}}{\sqrt{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(2*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2), x)

[Out] int((x^(2*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+2*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2), x)

[Out] Timed out

$$3.688 \quad \int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}} dx$$

Optimal. Leaf size=199

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{3/2}d^{7/2}n} + \frac{(bc-ad)(ad+5bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(ad+5bc)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n}$$

Rubi [A] time = 0.16, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {446, 80, 50, 63, 217, 206}

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{3/2}d^{7/2}n} - \frac{(ad+5bc)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(bc-ad)(ad+5bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} + \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n], x]

[Out] ((b*c - a*d)*(5*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(8*b*d^3*n) - ((5*b*c + a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(12*b*d^2*n) + ((a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(3*b*d*n) - ((b*c - a*d)^2*(5*b*c + a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(8*b^(3/2)*d^(7/2)*n)

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n} (a + bx^n)^{3/2}}{\sqrt{c + dx^n}} dx &= \frac{\text{Subst} \left(\int \frac{x^{(a+bx)^{3/2}}}{\sqrt{c+dx}} dx, x, x^n \right)}{n} \\
&= \frac{(a + bx^n)^{5/2} \sqrt{c + dx^n}}{3bdn} - \frac{(5bc + ad) \text{Subst} \left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}} dx, x, x^n \right)}{6bdn} \\
&= -\frac{(5bc + ad) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{12bd^2n} + \frac{(a + bx^n)^{5/2} \sqrt{c + dx^n}}{3bdn} + \frac{((bc - ad)(5bc + ad)) S}{8b} \\
&= \frac{(bc - ad)(5bc + ad) \sqrt{a + bx^n} \sqrt{c + dx^n}}{8bd^3n} - \frac{(5bc + ad) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{12bd^2n} + \frac{(a + bx)}{8b} \\
&= \frac{(bc - ad)(5bc + ad) \sqrt{a + bx^n} \sqrt{c + dx^n}}{8bd^3n} - \frac{(5bc + ad) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{12bd^2n} + \frac{(a + bx)}{8b} \\
&= \frac{(bc - ad)(5bc + ad) \sqrt{a + bx^n} \sqrt{c + dx^n}}{8bd^3n} - \frac{(5bc + ad) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{12bd^2n} + \frac{(a + bx)}{8b} \\
&= \frac{(bc - ad)(5bc + ad) \sqrt{a + bx^n} \sqrt{c + dx^n}}{8bd^3n} - \frac{(5bc + ad) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{12bd^2n} + \frac{(a + bx)}{8b}
\end{aligned}$$

Mathematica [A] time = 0.67, size = 178, normalized size = 0.89

$$\frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)(3a^2d^2+2abd(7dx^n-11c)+b^2(15c^2-10cdx^n+8d^2x^{2n}))-3(bc-ad)^{5/2}(ad+5bc)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{24b^2d^{7/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n], x]

[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(3*a^2*d^2 + 2*a*b*d*(-11*c + 7*d*x^n) + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(5/2)*(5*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(24*b^2*d^(7/2)*n*Sqrt[c + d*x^n])

IntegrateAlgebraic [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n} (a + bx^n)^{3/2}}{\sqrt{c + dx^n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^(-1 + 2*n))*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n], x]

[Out] Defer[IntegrateAlgebraic][(x^(-1 + 2*n))*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n], x]

fricas [A] time = 0.50, size = 469, normalized size = 2.36

$$\frac{3(5b^2c^2 - 9ab^2d + 3a^2d^2)\sqrt{b} \log\left(\frac{(b^2c^2 + d^2)\sqrt{bx^n + a} + b^2cd + a^2d^2 - 4(2\sqrt{bd}c + db + ad)\sqrt{bx^n + a} + 8(b^2d + ad^2)c}{2\sqrt{bd}c + db + ad}\right) + 4(8b^2c^2 + 15b^2d - 22abd + 3a^2d - 2(5b^2c - 7abd)c)\sqrt{bx^n + a} + 2(5b^2c^2 - 9ab^2d + 3a^2d^2)\sqrt{bd} \arctan\left(\frac{(b^2c^2 + d^2)\sqrt{bx^n + a} + b^2cd + a^2d^2 - 4(2\sqrt{bd}c + db + ad)\sqrt{bx^n + a} + 8(b^2d + ad^2)c}{2\sqrt{bd}c + db + ad}\right) + 2(8b^2c^2 + 15b^2d - 22abd + 3a^2d - 2(5b^2c - 7abd)c)\sqrt{bx^n + a} + 4(8b^2c^2 - 9ab^2d + 3a^2d^2)\sqrt{bd} \arctan\left(\frac{(b^2c^2 + d^2)\sqrt{bx^n + a} + b^2cd + a^2d^2 - 4(2\sqrt{bd}c + db + ad)\sqrt{bx^n + a} + 8(b^2d + ad^2)c}{2\sqrt{bd}c + db + ad}\right)}{2\sqrt{bd}c + db + ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2), x, algorithm="fricas")

[Out] [1/96*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^4*n), 1/48*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^4*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}} x^{2n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2), x, algorithm="giac")

[Out] integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}} x^{2n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n-1)*(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2), x)

[Out] `int(x^(2*n-1)*(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}} x^{2n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1} (a + bx^n)^{3/2}}{\sqrt{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(2*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2),x)`

[Out] `int((x^(2*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+2*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)`

[Out] Timed out

$$3.689 \quad \int \frac{x^{-1+2n} \sqrt{a+bx^n}}{\sqrt{c+dx^n}} dx$$

Optimal. Leaf size=146

$$\frac{(bc - ad)(ad + 3bc) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{b} \sqrt{c+dx^n}} \right)}{4b^{3/2}d^{5/2}n} - \frac{(ad + 3bc)\sqrt{a + bx^n} \sqrt{c + dx^n}}{4bd^2n} + \frac{(a + bx^n)^{3/2} \sqrt{c + dx^n}}{2bdn}$$

Rubi [A] time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {446, 80, 50, 63, 217, 206}

$$\frac{(bc - ad)(ad + 3bc) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{b} \sqrt{c+dx^n}} \right)}{4b^{3/2}d^{5/2}n} - \frac{(ad + 3bc)\sqrt{a + bx^n} \sqrt{c + dx^n}}{4bd^2n} + \frac{(a + bx^n)^{3/2} \sqrt{c + dx^n}}{2bdn}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n], x]

[Out] -((3*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n]/(4*b*d^2*n) + ((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]/(2*b*d*n) + ((b*c - a*d)*(3*b*c + a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(4*b^(3/2)*d^(5/2)*n)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p

+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}} dx &= \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\sqrt{c+dx}} dx, x, x^n\right)}{n} \\
&= \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} - \frac{(3bc+ad)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}} dx, x, x^n\right)}{4bdn} \\
&= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{((bc-ad)(3bc+ad))\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}} dx, x, x^n\right)}{8bd^2n} \\
&= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{((bc-ad)(3bc+ad))\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}} dx, x, x^n\right)}{8bd^2n} \\
&= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{((bc-ad)(3bc+ad))\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}} dx, x, x^n\right)}{4b^2c} \\
&= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{(bc-ad)(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{4b^{3/2}d^{5/2}n}
\end{aligned}$$

Mathematica [A] time = 0.51, size = 141, normalized size = 0.97

$$\frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)(ad-3bc+2bdx^n) + (ad+3bc)(bc-ad)^{3/2}\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{4b^2d^{5/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n], x]

[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c + a*d + 2*b*d*x^n) + (b*c - a*d)^(3/2)*(3*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^2*d^(5/2)*n*Sqrt[c + d*x^n])

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^(-1 + 2*n))*Sqrt[a + b*x^n])/Sqrt[c + d*x^n], x]

[Out] Defer[IntegrateAlgebraic] [(x^(-1 + 2*n))*Sqrt[a + b*x^n])/Sqrt[c + d*x^n], x]

fricas [A] time = 0.46, size = 359, normalized size = 2.46

$$\frac{(3b^2c^2 - 2abcd - a^2d^2)\sqrt{bd} \log(8b^2d^2x^{2n} + b^2c^2 + 6abcd + a^2d^2 - 4(2\sqrt{bd}bdx^n + (bc + ad)\sqrt{bd})\sqrt{bx^n + a}\sqrt{dx^n + c} + 8(b^2cd + abd^2)x^n) - 4(2b^2d^2x^n - 3b^2cd + abd^2)\sqrt{bx^n + a}\sqrt{dx^n + c}}{16b^2d^3n} - \frac{(3b^2c^2 - 2abcd - a^2d^2)\sqrt{-bd} \arctan\left(\frac{(2\sqrt{-bd}bdx^n + ab\sqrt{-bd})\sqrt{bx^n + a}\sqrt{dx^n + c}}{2(b^2d^2x^n + abd^2)x^n}\right) - 2(2b^2d^2x^n - 3b^2cd + abd^2)\sqrt{bx^n + a}\sqrt{dx^n + c}}{8b^2d^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2), x, algorithm="fricas")

[Out] [-1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n - 4*(2*b^2*d^2*x^n - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c)))/(b^2*d^3*n), -1/8*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n) - 2*(2*b^2*d^2*x^n - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^3*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + a} x^{2n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)

maple [F] time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + a} x^{2n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n-1)*(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2), x)

[Out] int(x^(2*n-1)*(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + a} x^{2n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*xⁿ)^(1/2)/(c+d*xⁿ)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*xⁿ + a)*x^(2*n - 1)/sqrt(d*xⁿ + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1} \sqrt{a + b x^n}}{\sqrt{c + d x^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(2*n - 1)*(a + b*xⁿ)^(1/2))/(c + d*xⁿ)^(1/2),x)

[Out] int((x^(2*n - 1)*(a + b*xⁿ)^(1/2))/(c + d*xⁿ)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1} \sqrt{a + b x^n}}{\sqrt{c + d x^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{**(-1+2*n)}*(a+b*x^{**n})^{** (1/2)}/(c+d*x^{**n})^{** (1/2)},x)

[Out] Integral(x^{** (2*n - 1)}*sqrt(a + b*x^{**n})/sqrt(c + d*x^{**n}), x)

$$3.690 \quad \int \frac{x^{-1+2n}}{\sqrt{a+bx^n} \sqrt{c+dx^n}} dx$$

Optimal. Leaf size=89

$$\frac{\sqrt{a+bx^n} \sqrt{c+dx^n}}{bdn} - \frac{(ad+bc) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 80, 63, 217, 206}

$$\frac{\sqrt{a+bx^n} \sqrt{c+dx^n}}{bdn} - \frac{(ad+bc) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]

[Out] (Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(b*d*n) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*d^(3/2)*n)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}} dx &= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\sqrt{c+dx}} dx, x, x^n\right)}{n} \\
 &= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}} dx, x, x^n\right)}{2bdn} \\
 &= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}} dx, x, \sqrt{a+bx^n}\right)}{b^2dn} \\
 &= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}} dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^2dn} \\
 &= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}
 \end{aligned}$$

Mathematica [A] time = 0.32, size = 123, normalized size = 1.38

$$\frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n) - \sqrt{bc-ad}(ad+bc)\sqrt{\frac{b(c+dx^n)}{bc-ad}} \sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^2d^{3/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x]

[Out] $(b\sqrt{d}\sqrt{a + bx^n})(c + dx^n) - \sqrt{bc - ad}(bc + a)d\sqrt{((b(c + dx^n))/(bc - ad))\text{ArcSinh}(\sqrt{d}\sqrt{a + bx^n})/\sqrt{bc - ad}})/(b^2d^{3/2}n\sqrt{c + dx^n})$

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}}{\sqrt{a + bx^n} \sqrt{c + dx^n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 + 2*n)/(Sqrt[a + b*xⁿ]*Sqrt[c + d*xⁿ]), x]

[Out] Defer[IntegrateAlgebraic][x^(-1 + 2*n)/(Sqrt[a + b*xⁿ]*Sqrt[c + d*xⁿ]), x]

fricas [A] time = 0.52, size = 281, normalized size = 3.16

$$\left[\frac{4\sqrt{bx^n + a}\sqrt{dx^n + c}bd + (bc + ad)\sqrt{bd} \log(8b^2d^2x^{2n} + b^2c^2 + 6abcd + a^2d^2 - 4(2\sqrt{bd}bdx^n + (bc + ad)\sqrt{bd})\sqrt{bx^n + a}\sqrt{dx^n + c} + 8(b^2cd + abd^2)x^n)}{4b^2d^2n}, \frac{2\sqrt{bx^n + a}\sqrt{dx^n + c}bd + (bc + ad)\sqrt{bd} \arctan\left(\frac{(2\sqrt{bd}bdx^n + (bc + ad)\sqrt{bd})\sqrt{bx^n + a}\sqrt{dx^n + c}}{2(b^2d^2x^{2n} + abcd + (b^2cd + abd^2)x^n)}\right)}{2b^2d^2n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*xⁿ)^(1/2)/(c+d*xⁿ)^(1/2), x, algorithm="fricas")

[Out] $[1/4*(4*\text{sqrt}(b*x^n + a)*\text{sqrt}(d*x^n + c)*b*d + (b*c + a*d)*\text{sqrt}(b*d)*\log(8*b^2*d^2*x^{2*n} + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*\text{sqrt}(b*d)*b*d*x^n + (b*c + a*d)*\text{sqrt}(b*d))*\text{sqrt}(b*x^n + a)*\text{sqrt}(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/(b^2*d^2*n), 1/2*(2*\text{sqrt}(b*x^n + a)*\text{sqrt}(d*x^n + c)*b*d + (b*c + a*d)*\text{sqrt}(-b*d)*\arctan(1/2*(2*\text{sqrt}(-b*d)*b*d*x^n + (b*c + a*d)*\text{sqrt}(-b*d))*\text{sqrt}(b*x^n + a)*\text{sqrt}(d*x^n + c))/(b^2*d^2*x^{2*n} + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/(b^2*d^2*n)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*xⁿ)^(1/2)/(c+d*xⁿ)^(1/2), x, algorithm="giac")

[Out] sage0*x

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{bx^n + a} \sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2*n-1)/(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)`

[Out] `int(x^(2*n-1)/(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{bx^n + a} \sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^(2*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{\sqrt{a + bx^n} \sqrt{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)),x)`

[Out] `int(x^(2*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{a + bx^n} \sqrt{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+2*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)`

[Out] `Integral(x**(2*n - 1)/(sqrt(a + b*x**n)*sqrt(c + d*x**n)), x)`

$$3.691 \quad \int \frac{x^{-1+2n}}{(a+bx^n)^{3/2} \sqrt{c+dx^n}} dx$$

Optimal. Leaf size=91

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{b} \sqrt{c+dx^n}} \right)}{b^{3/2} \sqrt{d} n} + \frac{2a \sqrt{c+dx^n}}{bn(bc-ad) \sqrt{a+bx^n}}$$

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 78, 63, 217, 206}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{b} \sqrt{c+dx^n}} \right)}{b^{3/2} \sqrt{d} n} + \frac{2a \sqrt{c+dx^n}}{bn(bc-ad) \sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]

[Out] (2*a*Sqrt[c + d*x^n])/(b*(b*c - a*d)*n*Sqrt[a + b*x^n]) + (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*Sqrt[d]*n)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{-1+2n}}{(a + bx^n)^{3/2} \sqrt{c + dx^n}} dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2} \sqrt{c+dx}} dx, x, x^n\right)}{n} \\
 &= \frac{2a\sqrt{c + dx^n}}{b(bc - ad)n\sqrt{a + bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx} \sqrt{c+dx}} dx, x, x^n\right)}{bn} \\
 &= \frac{2a\sqrt{c + dx^n}}{b(bc - ad)n\sqrt{a + bx^n}} + \frac{2 \text{Subst}\left(\int \frac{1}{\sqrt{c - \frac{ad}{b} + \frac{dx^2}{b}}} dx, x, \sqrt{a + bx^n}\right)}{b^2n} \\
 &= \frac{2a\sqrt{c + dx^n}}{b(bc - ad)n\sqrt{a + bx^n}} + \frac{2 \text{Subst}\left(\int \frac{1}{1 - \frac{dx^2}{b}} dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^2n} \\
 &= \frac{2a\sqrt{c + dx^n}}{b(bc - ad)n\sqrt{a + bx^n}} + \frac{2 \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{b} \sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d} n}
 \end{aligned}$$

Mathematica [A] time = 0.90, size = 122, normalized size = 1.34

$$\frac{2 \left(\frac{ab(c+dx^n)}{(bc-ad)\sqrt{a+bx^n}} + \frac{\sqrt{bc-ad} \sqrt{\frac{b(c+dx^n)}{bc-ad}} \sinh^{-1}\left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{\sqrt{d}} \right)}{b^2n\sqrt{c + dx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]

[Out] (2*((a*b*(c + d*x^n))/((b*c - a*d)*Sqrt[a + b*x^n]) + (Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/Sqrt[d]))/(b^2*n*Sqrt[c + d*x^n])

IntegrateAlgebraic [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}}{(a + bx^n)^{3/2} \sqrt{c + dx^n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]

[Out] Defer[IntegrateAlgebraic][x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]

fricas [B] time = 0.60, size = 408, normalized size = 4.48

$$\frac{4\sqrt{bx^n+a}\sqrt{dx^n+c}\operatorname{arctan}\left(\frac{(b^2c-ad)\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\sqrt{bx^n+a}\sqrt{dx^n+c}}\right) + (abc-a^2d)\sqrt{bd}\log\left(\frac{8b^2d^2x^{2n}+b^2c^2+6abcd+a^2d^2+4(2\sqrt{bd}bdx^n+(bc+ad)\sqrt{bd})\sqrt{bx^n+a}\sqrt{dx^n+c}+8(b^2cd+abd^2)x^n}{2((b^2cd-ab^2d^2)x^n+(ab^2d-a^2b^2d^2)n)}\right)}{2((b^2cd-ab^2d^2)x^n+(ab^2d-a^2b^2d^2)n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")

[Out] [1/2*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*a*b*d + ((b^2*c - a*b*d)*sqrt(b*d)*x^n + (a*b*c - a^2*d)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n)/((b^4*c*d - a*b^3*d^2)*n*x^n + (a*b^3*c*d - a^2*b^2*d^2)*n), (2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*a*b*d - ((b^2*c - a*b*d)*sqrt(-b*d)*x^n + (a*b*c - a^2*d)*sqrt(-b*d))*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n))/((b^4*c*d - a*b^3*d^2)*n*x^n + (a*b^3*c*d - a^2*b^2*d^2)*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n + a)^{\frac{3}{2}} \sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n + a)^{\frac{3}{2}} \sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n-1)/(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2), x)

[Out] int(x^(2*n-1)/(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n + a)^{\frac{3}{2}} \sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2), x, algorithm="maxima")

[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{(a + bx^n)^{3/2} \sqrt{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)

[Out] int(x^(2*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+2*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2), x)

[Out] Timed out

$$3.692 \quad \int \frac{x^{-1+2n}}{(a+bx^n)^{5/2} \sqrt{c+dx^n}} dx$$

Optimal. Leaf size=95

$$\frac{2a\sqrt{c+dx^n}}{3bn(bc-ad)(a+bx^n)^{3/2}} - \frac{2(3bc-ad)\sqrt{c+dx^n}}{3bn(bc-ad)^2\sqrt{a+bx^n}}$$

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 78, 37}

$$\frac{2a\sqrt{c+dx^n}}{3bn(bc-ad)(a+bx^n)^{3/2}} - \frac{2(3bc-ad)\sqrt{c+dx^n}}{3bn(bc-ad)^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]), x]

[Out] (2*a*Sqrt[c + d*x^n])/((3*b*(b*c - a*d)*n*(a + b*x^n)^(3/2)) - (2*(3*b*c - a*d)*Sqrt[c + d*x^n])/((3*b*(b*c - a*d)^2*n*Sqrt[a + b*x^n]))

Rule 37

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+2n}}{(a+bx^n)^{5/2} \sqrt{c+dx^n}} dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^{5/2} \sqrt{c+dx}} dx, x, x^n\right)}{n} \\ &= \frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n(a+bx^n)^{3/2}} + \frac{(3bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)^{3/2} \sqrt{c+dx}} dx, x, x^n\right)}{3b(bc-ad)n} \\ &= \frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n(a+bx^n)^{3/2}} - \frac{2(3bc-ad)\sqrt{c+dx^n}}{3b(bc-ad)^2 n \sqrt{a+bx^n}} \end{aligned}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 0.60

$$\frac{2\sqrt{c+dx^n}(-2ac+adx^n-3bcx^n)}{3n(bc-ad)^2(a+bx^n)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]), x]

[Out] (2*Sqrt[c + d*x^n]*(-2*a*c - 3*b*c*x^n + a*d*x^n))/(3*(b*c - a*d)^2*n*(a + b*x^n)^(3/2))

IntegrateAlgebraic [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2} \sqrt{c+dx^n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]), x]

[Out] Defer[IntegrateAlgebraic][x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]), x]

fricas [A] time = 0.74, size = 135, normalized size = 1.42

$$\frac{2(2ac + (3bc - ad)x^n)\sqrt{bx^n + a}\sqrt{dx^n + c}}{3\left(\left(b^4c^2 - 2ab^3cd + a^2b^2d^2\right)nx^{2n} + 2\left(ab^3c^2 - 2a^2b^2cd + a^3bd^2\right)nx^n + \left(a^2b^2c^2 - 2a^3bcd + a^4d^2\right)n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*xⁿ)^(5/2)/(c+d*xⁿ)^(1/2),x, algorithm="fricas")

[Out]
$$-2/3*(2*a*c + (3*b*c - a*d)*x^n)*\sqrt{b*x^n + a}*\sqrt{d*x^n + c}/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^{(2*n)} + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*n*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*n)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n + a)^{\frac{5}{2}} \sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*xⁿ)^(5/2)/(c+d*xⁿ)^(1/2),x, algorithm="giac")

[Out] integrate(x^(2*n - 1)/((b*xⁿ + a)^(5/2)*sqrt(d*xⁿ + c)), x)

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n + a)^{\frac{5}{2}} \sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n-1)/(b*xⁿ+a)^(5/2)/(d*xⁿ+c)^(1/2),x)

[Out] int(x^(2*n-1)/(b*xⁿ+a)^(5/2)/(d*xⁿ+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n + a)^{\frac{5}{2}} \sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*xⁿ)^(5/2)/(c+d*xⁿ)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(2*n - 1)/((b*xⁿ + a)^(5/2)*sqrt(d*xⁿ + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{(a + bx^n)^{5/2} \sqrt{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n - 1)/((a + b*xⁿ)^(5/2)*(c + d*xⁿ)^(1/2)),x)


```
[Out] int(x^(2*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)), x)
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2), x)
```

```
[Out] Timed out
```

$$3.693 \quad \int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}} dx$$

Optimal. Leaf size=358

$$\frac{(bc-ad)^2(3a^2d^2+14abcd+63b^2c^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)(3a^2d^2+14abcd+63b^2c^2)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n}$$

Rubi [A] time = 0.40, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{(3a^2d^2+14abcd+63b^2c^2)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n} - \frac{(bc-ad)(3a^2d^2+14abcd+63b^2c^2)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n} + \frac{(bc-ad)^2(3a^2d^2+14abcd+63b^2c^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)^3(3a^2d^2+14abcd+63b^2c^2)\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{128b^2d^{11/2}n} - \frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{40b^2d^3n} + \frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bdn}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n], x]

[Out] ((b*c - a*d)^2*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/((128*b^2*d^5*n) - ((b*c - a*d)*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(192*b^2*d^4*n) + ((63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(240*b^2*d^3*n) - (3*(3*b*c + a*d)*(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(40*b^2*d^2*n) + (x^n*(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(5*b*d*n) - ((b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(128*b^(5/2)*d^(11/2)*n)

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n} (a + bx^n)^{5/2}}{\sqrt{c + dx^n}} dx &= \frac{\text{Subst} \left(\int \frac{x^2 (a+bx)^{5/2}}{\sqrt{c+dx}} dx, x, x^n \right)}{n} \\
&= \frac{x^n (a + bx^n)^{7/2} \sqrt{c + dx^n}}{5bdn} + \frac{\text{Subst} \left(\int \frac{(a+bx)^{5/2} \left(-ac - \frac{3}{2}(3bc+ad)x \right)}{\sqrt{c+dx}} dx, x, x^n \right)}{5bdn} \\
&= -\frac{3(3bc + ad) (a + bx^n)^{7/2} \sqrt{c + dx^n}}{40b^2d^2n} + \frac{x^n (a + bx^n)^{7/2} \sqrt{c + dx^n}}{5bdn} + \frac{(63b^2c^2 + 14abcd - 3a^2d^2) (a + bx^n)^{5/2} \sqrt{c + dx^n}}{240b^2d^3n} \\
&= \frac{(63b^2c^2 + 14abcd + 3a^2d^2) (a + bx^n)^{5/2} \sqrt{c + dx^n}}{240b^2d^3n} - \frac{3(3bc + ad) (a + bx^n)^{7/2} \sqrt{c + dx^n}}{40b^2d^2n} \\
&= -\frac{(bc - ad) (63b^2c^2 + 14abcd + 3a^2d^2) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{192b^2d^4n} + \frac{(63b^2c^2 + 14abcd + 3a^2d^2) (a + bx^n)^{5/2} \sqrt{c + dx^n}}{240b^2d^3n} \\
&= \frac{(bc - ad)^2 (63b^2c^2 + 14abcd + 3a^2d^2) \sqrt{a + bx^n} \sqrt{c + dx^n}}{128b^2d^5n} - \frac{(bc - ad) (63b^2c^2 + 14abcd + 3a^2d^2) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{192b^2d^4n} \\
&= \frac{(bc - ad)^2 (63b^2c^2 + 14abcd + 3a^2d^2) \sqrt{a + bx^n} \sqrt{c + dx^n}}{128b^2d^5n} - \frac{(bc - ad) (63b^2c^2 + 14abcd + 3a^2d^2) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{192b^2d^4n} \\
&= \frac{(bc - ad)^2 (63b^2c^2 + 14abcd + 3a^2d^2) \sqrt{a + bx^n} \sqrt{c + dx^n}}{128b^2d^5n} - \frac{(bc - ad) (63b^2c^2 + 14abcd + 3a^2d^2) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{192b^2d^4n} \\
&= \frac{(bc - ad)^2 (63b^2c^2 + 14abcd + 3a^2d^2) \sqrt{a + bx^n} \sqrt{c + dx^n}}{128b^2d^5n} - \frac{(bc - ad) (63b^2c^2 + 14abcd + 3a^2d^2) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{192b^2d^4n} \\
&= \frac{(bc - ad)^2 (63b^2c^2 + 14abcd + 3a^2d^2) \sqrt{a + bx^n} \sqrt{c + dx^n}}{128b^2d^5n} - \frac{(bc - ad) (63b^2c^2 + 14abcd + 3a^2d^2) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{192b^2d^4n}
\end{aligned}$$

Mathematica [A] time = 2.02, size = 274, normalized size = 0.77

$$\frac{\sqrt{c + dx^n} \left(\frac{5(bc-ad)^3(3a^2d^2+14abcd+63b^2c^2) \left(\frac{16a^3(a+bx^n)^3}{15(ad-bc)^3} - \frac{4d^2(a+bx^n)^2}{3(bc-ad)^2} - \frac{2d(a+bx^n)}{ad-bc} - \frac{2\sqrt{d}\sqrt{a+bx^n} \sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}}\right)}{4bd^5} - \frac{24(ad+3bc)(a+bx^n)^4}{bd} + 64x^n(a+bx^n)^4 \right)}{320bdn\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n], x]

```
[Out] (Sqrt[c + d*x^n]*((-24*(3*b*c + a*d)*(a + b*x^n)^4)/(b*d) + 64*x^n*(a + b*x^n)^4 + (5*(b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*((-2*d*(a + b*x^n))/(-b*c) + a*d) - (4*d^2*(a + b*x^n)^2)/(3*(b*c - a*d)^2) - (16*d^3*(a + b*x^n)^3)/(15*(-b*c) + a*d)^3) - (2*Sqrt[d]*Sqrt[a + b*x^n]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n))/(b*c - a*d)])))/(4*b*d^5))/(320*b*d*n*Sqrt[a + b*x^n])
```

IntegrateAlgebraic [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n} (a + bx^n)^{5/2}}{\sqrt{c + dx^n}} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n], x]
```

```
[Out] Defer[IntegrateAlgebraic] [(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n], x]
```

fricas [A] time = 0.58, size = 771, normalized size = 2.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2), x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2) + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(384*b^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^(3*n) + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^(2*n) - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^6*n), 1/3840*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n) + 2*(384*b^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^(3*n) + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^(2*n) - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^6*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}} x^{3n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x[^](-1+3*n)*(a+b*x[^]n)^{^(5/2)}/(c+d*x[^]n)^{^(1/2)},x, algorithm="giac")

[Out] integrate((b*x[^]n + a)^{^(5/2)}*x^{^(3*n - 1)}/sqrt(d*x[^]n + c), x)

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}} x^{3n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^{^(3*n-1)}*(b*x[^]n+a)^{^(5/2)}/(d*x[^]n+c)^{^(1/2)},x)

[Out] int(x^{^(3*n-1)}*(b*x[^]n+a)^{^(5/2)}/(d*x[^]n+c)^{^(1/2)},x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}} x^{3n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x[^](-1+3*n)*(a+b*x[^]n)^{^(5/2)}/(c+d*x[^]n)^{^(1/2)},x, algorithm="maxima")

[Out] integrate((b*x[^]n + a)^{^(5/2)}*x^{^(3*n - 1)}/sqrt(d*x[^]n + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3n-1} (a + bx^n)^{5/2}}{\sqrt{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^{^(3*n - 1)}*(a + b*x[^]n)^{^(5/2)})/(c + d*x[^]n)^{^(1/2)},x)

[Out] int((x^{^(3*n - 1)}*(a + b*x[^]n)^{^(5/2)})/(c + d*x[^]n)^{^(1/2)}, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

$$3.694 \quad \int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}} dx$$

Optimal. Leaf size=291

$$-\frac{(bc-ad)(3a^2d^2+10abcd+35b^2c^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{(3a^2d^2+10abcd+35b^2c^2)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n} + \dots$$

Rubi [A] time = 0.32, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{(3a^2d^2+10abcd+35b^2c^2)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n} - \frac{(bc-ad)(3a^2d^2+10abcd+35b^2c^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{(bc-ad)^2(3a^2d^2+10abcd+35b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{5/2}d^{7/2}n} - \frac{(3ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n], x]

[Out] -((b*c - a*d)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(64*b^2*d^4*n) + ((35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(96*b^2*d^3*n) - ((7*b*c + 3*a*d)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(24*b^2*d^2*n) + (x^n*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(4*b*d*n) + ((b*c - a*d)^2*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[Sqrt[d]*Sqrt[a + b*x^n]/(Sqrt[b]*Sqrt[c + d*x^n])]/(64*b^(5/2)*d^(9/2)*n)

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80


```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n} (a + bx^n)^{3/2}}{\sqrt{c + dx^n}} dx &= \frac{\text{Subst} \left(\int \frac{x^{2(a+bx)^{3/2}}}{\sqrt{c+dx}} dx, x, x^n \right)}{n} \\
&= \frac{x^n (a + bx^n)^{5/2} \sqrt{c + dx^n}}{4bdn} + \frac{\text{Subst} \left(\int \frac{(a+bx)^{3/2} \left(-ac - \frac{1}{2}(7bc+3ad)x \right)}{\sqrt{c+dx}} dx, x, x^n \right)}{4bdn} \\
&= -\frac{(7bc + 3ad) (a + bx^n)^{5/2} \sqrt{c + dx^n}}{24b^2d^2n} + \frac{x^n (a + bx^n)^{5/2} \sqrt{c + dx^n}}{4bdn} + \frac{(35b^2c^2 + 10abcd - 3a^2d^2)}{96b^2d^3n} \\
&= \frac{(35b^2c^2 + 10abcd + 3a^2d^2) (a + bx^n)^{3/2} \sqrt{c + dx^n}}{96b^2d^3n} - \frac{(7bc + 3ad) (a + bx^n)^{5/2} \sqrt{c + dx^n}}{24b^2d^2n} \\
&= -\frac{(bc - ad) (35b^2c^2 + 10abcd + 3a^2d^2) \sqrt{a + bx^n} \sqrt{c + dx^n}}{64b^2d^4n} + \frac{(35b^2c^2 + 10abcd + 3a^2d^2)}{96b^2d^3n} \\
&= -\frac{(bc - ad) (35b^2c^2 + 10abcd + 3a^2d^2) \sqrt{a + bx^n} \sqrt{c + dx^n}}{64b^2d^4n} + \frac{(35b^2c^2 + 10abcd + 3a^2d^2)}{96b^2d^3n} \\
&= -\frac{(bc - ad) (35b^2c^2 + 10abcd + 3a^2d^2) \sqrt{a + bx^n} \sqrt{c + dx^n}}{64b^2d^4n} + \frac{(35b^2c^2 + 10abcd + 3a^2d^2)}{96b^2d^3n} \\
&= -\frac{(bc - ad) (35b^2c^2 + 10abcd + 3a^2d^2) \sqrt{a + bx^n} \sqrt{c + dx^n}}{64b^2d^4n} + \frac{(35b^2c^2 + 10abcd + 3a^2d^2)}{96b^2d^3n}
\end{aligned}$$

Mathematica [A] time = 1.01, size = 241, normalized size = 0.83

$$\frac{3(bc - ad)^{3/2} (3a^2d^2 + 10abcd + 35b^2c^2) \sqrt{\frac{bc+dx^n}{bc-ad}} \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{bc-ad}} \right) - b\sqrt{d} \sqrt{a + bx^n} (c + dx^n) (9a^3d^3 + 3a^2bd^2 (5c - 2dx^n) - ab^2d (145c^2 - 92cdx^n + 72d^2x^{2n}) + b^3 (105c^3 - 70c^2dx^n + 56cd^2x^{2n} - 48d^3x^{3n}))}{192b^3d^{9/2}n\sqrt{c + dx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n], x]

[Out] $(-(b*\text{Sqrt}[d]*\text{Sqrt}[a + b*x^n]*(c + d*x^n)*(9*a^3*d^3 + 3*a^2*b*d^2*(5*c - 2*d*x^n) - a*b^2*d*(145*c^2 - 92*c*d*x^n + 72*d^2*x^(2*n)) + b^3*(105*c^3 - 70*c^2*d*x^n + 56*c*d^2*x^(2*n) - 48*d^3*x^(3*n)))) + 3*(b*c - a*d)^(5/2)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*\text{Sqrt}[(b*(c + d*x^n))/(b*c - a*d)]*\text{ArcSinh}[(\text{Sqrt}[d]*\text{Sqrt}[a + b*x^n])/ \text{Sqrt}[b*c - a*d]])/(192*b^3*d^(9/2)*n*\text{Sqrt}[c + d*x^n])$

IntegrateAlgebraic [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n} (a + bx^n)^{3/2}}{\sqrt{c + dx^n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^(-1 + 3*n))*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n], x]

[Out] Defer[IntegrateAlgebraic][(x^(-1 + 3*n))*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n], x]

fricas [A] time = 0.54, size = 607, normalized size = 2.09

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2), x, algorithm="fricas")

[Out] [1/768*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^5*n), -1/384*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^5*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2 x^{3n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2), x, algorithm="giac")

[Out] integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)

maple [F] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}} x^{3n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3*n-1)*(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2), x)

[Out] int(x^(3*n-1)*(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}} x^{3n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2), x, algorithm="maxima")

[Out] integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3n-1} (a + bx^n)^{3/2}}{\sqrt{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(3*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)

[Out] int((x^(3*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+3*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2), x)

[Out] Timed out

$$3.695 \quad \int \frac{x^{-1+3n} \sqrt{a+bx^n}}{\sqrt{c+dx^n}} dx$$

Optimal. Leaf size=221

$$\frac{(a^2d^2 + 2abcd + 5b^2c^2) \sqrt{a+bx^n} \sqrt{c+dx^n}}{8b^2d^3n} - \frac{(bc-ad)(a^2d^2 + 2abcd + 5b^2c^2) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{5/2}d^{7/2}n} - \frac{(3ad + 5bc)}{8b^{5/2}d^{7/2}n}$$

Rubi [A] time = 0.25, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{(a^2d^2 + 2abcd + 5b^2c^2) \sqrt{a+bx^n} \sqrt{c+dx^n}}{8b^2d^3n} - \frac{(bc-ad)(a^2d^2 + 2abcd + 5b^2c^2) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{5/2}d^{7/2}n} - \frac{(3ad + 5bc)(a+bx^n)^{3/2} \sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/2} \sqrt{c+dx^n}}{3bdn}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n], x]

[Out] ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(8*b^2*d^3*n) - ((5*b*c + 3*a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(12*b^2*d^2*n) + (x^n*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(3*b*d*n) - ((b*c - a*d)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(8*b^(5/2)*d^(7/2)*n)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(2*((c_.) + (d_.)*(x_))^(n_.))*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n} \sqrt{a+bx^n}}{\sqrt{c+dx^n}} dx &= \frac{\text{Subst}\left(\int \frac{x^2 \sqrt{a+bx}}{\sqrt{c+dx}} dx, x, x^n\right)}{n} \\
&= \frac{x^n (a+bx^n)^{3/2} \sqrt{c+dx^n}}{3bdn} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx} \left(-ac - \frac{1}{2}(5bc+3ad)x\right)}{\sqrt{c+dx}} dx, x, x^n\right)}{3bdn} \\
&= -\frac{(5bc+3ad)(a+bx^n)^{3/2} \sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n (a+bx^n)^{3/2} \sqrt{c+dx^n}}{3bdn} + \frac{(5b^2c^2+2abcd+a^2d^2) \sqrt{a+bx^n} \sqrt{c+dx^n}}{8b^2d^3n} \\
&= \frac{(5b^2c^2+2abcd+a^2d^2) \sqrt{a+bx^n} \sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2} \sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n (a+bx^n)^{3/2} \sqrt{c+dx^n}}{3bdn} \\
&= \frac{(5b^2c^2+2abcd+a^2d^2) \sqrt{a+bx^n} \sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2} \sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n (a+bx^n)^{3/2} \sqrt{c+dx^n}}{3bdn} \\
&= \frac{(5b^2c^2+2abcd+a^2d^2) \sqrt{a+bx^n} \sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2} \sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n (a+bx^n)^{3/2} \sqrt{c+dx^n}}{3bdn} \\
&= \frac{(5b^2c^2+2abcd+a^2d^2) \sqrt{a+bx^n} \sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2} \sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n (a+bx^n)^{3/2} \sqrt{c+dx^n}}{3bdn}
\end{aligned}$$

Mathematica [A] time = 0.62, size = 191, normalized size = 0.86

$$\frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)(-3a^2d^2+2abd(dx^n-2c)+b^2(15c^2-10cdx^n+8d^2x^{2n}))-3(bc-ad)^{3/2}(a^2d^2+2abcd+5b^2c^2)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{24b^3d^{7/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1+3*n)*Sqrt[a+b*x^n])/Sqrt[c+d*x^n],x]

[Out] (b*Sqrt[d]*Sqrt[a+b*x^n]*(c+d*x^n)*(-3*a^2*d^2+2*a*b*d*(-2*c+d*x^n)+b^2*(15*c^2-10*c*d*x^n+8*d^2*x^(2*n)))-3*(b*c-a*d)^(3/2)*(5*b^2*c^2+2*a*b*c*d+a^2*d^2)*Sqrt[(b*(c+d*x^n))/(b*c-a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a+b*x^n])/Sqrt[b*c-a*d]])/(24*b^3*d^(7/2)*n*Sqrt[c+d*x^n])

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n} \sqrt{a+bx^n}}{\sqrt{c+dx^n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n], x]

[Out] Defer[IntegrateAlgebraic] [(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n], x]

fricas [A] time = 0.51, size = 471, normalized size = 2.13

$$\frac{3(5b^3c^3 - 3ab^2c^2d - a^2b^2c^2d - a^3d^3)\sqrt{bd}\log\left(\frac{(b^2c^2d^2x^{2n} + b^2c^2d^2 + 6ab^2c^2d + a^2d^2 + 4(2\sqrt{bd})\sqrt{bd} + 8(b^2c^2d + ab^2c^2))\sqrt{bd} + a\sqrt{bd} + 8(b^2c^2d + ab^2c^2)}{2(5b^3c^3 - 3ab^2c^2d - a^2b^2c^2d - a^3d^3)\sqrt{bd}}\right) + 2(5b^3c^3 + 15b^3c^2d - 4ab^2c^2d - 3a^2b^2c^2d - 2(5b^3c^3 - 3ab^2c^2d - a^2b^2c^2d - a^3d^3))\sqrt{bd} + a\sqrt{bd}}{48b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2), x, algorithm="fricas")

[Out] [-1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b^2*c^2*d - a^3*d^3)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^4*n), 1/48*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b^2*c^2*d - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^4*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + a} x^{3n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)

maple [F] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + a} x^{3n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3*n-1)*(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2), x)

[Out] $\text{int}(x^{(3*n-1)}*(b*x^n+a)^{(1/2)}/(d*x^n+c)^{(1/2)}, x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + a} x^{3n-1}}{\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{(-1+3*n)}*(a+b*x^n)^{(1/2)}/(c+d*x^n)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(\text{sqrt}(b*x^n + a)*x^{(3*n - 1)}/\text{sqrt}(d*x^n + c), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3n-1} \sqrt{a + bx^n}}{\sqrt{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^{(3*n - 1)}*(a + b*x^n)^{(1/2)})/(c + d*x^n)^{(1/2)}, x)$

[Out] $\text{int}((x^{(3*n - 1)}*(a + b*x^n)^{(1/2)})/(c + d*x^n)^{(1/2)}, x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1} \sqrt{a + bx^n}}{\sqrt{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{*(-1+3*n)}*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2), x)$

[Out] $\text{Integral}(x^{(3*n - 1)}*\text{sqrt}(a + b*x**n)/\text{sqrt}(c + d*x**n), x)$

$$3.696 \quad \int \frac{x^{-1+3n}}{\sqrt{a+bx^n} \sqrt{c+dx^n}} dx$$

Optimal. Leaf size=150

$$\frac{(4abcd - 3(ad + bc)^2) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{b} \sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n} - \frac{3(ad + bc)\sqrt{a + bx^n} \sqrt{c + dx^n}}{4b^2d^2n} + \frac{x^n \sqrt{a + bx^n} \sqrt{c + dx^n}}{2bdn}$$

Rubi [A] time = 0.15, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {446, 90, 80, 63, 217, 206}

$$-\frac{3(ad + bc)\sqrt{a + bx^n} \sqrt{c + dx^n}}{4b^2d^2n} - \frac{(4abcd - 3(ad + bc)^2) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{b} \sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n} + \frac{x^n \sqrt{a + bx^n} \sqrt{c + dx^n}}{2bdn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x]

[Out] (-3*(b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(4*b^2*d^2*n) + (x^n*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(2*b*d*n) - ((4*a*b*c*d - 3*(b*c + a*d)^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(4*b^(5/2)*d^(5/2)*n)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 90

Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)

```

^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]

```

Rule 206

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rule 217

```

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

```

Rule 446

```

Int[(x_)^(m_.)*((a_) + (b_)*(x_)^(n_))^(p_.)*((c_) + (d_)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}\sqrt{c+dx}} dx, x, x^n\right)}{n} \\
&= \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} + \frac{\text{Subst}\left(\int \frac{-ac-\frac{3}{2}(bc+ad)x}{\sqrt{a+bx}\sqrt{c+dx}} dx, x, x^n\right)}{2bdn} \\
&= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{(4abcd-3(bc+ad)^2)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}} dx, x, x^n\right)}{8bd^2n} \\
&= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{(4abcd-3(bc+ad)^2)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}} dx, x, x^n\right)}{4bd^2n} \\
&= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{(4abcd-3(bc+ad)^2)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}} dx, x, x^n\right)}{4bd^2n} \\
&= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{(4abcd-3(bc+ad)^2)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}} dx, x, x^n\right)}{4b^5/2d^5/2n}
\end{aligned}$$

Mathematica [A] time = 0.46, size = 157, normalized size = 1.05

$$\frac{\sqrt{bc-ad} (3a^2d^2 + 2abcd + 3b^2c^2) \sqrt{\frac{b(c+dx^n)}{bc-ad}} \sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right) + b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)(-3ad-3bc+2bdx^n)}{4b^3d^{5/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x]

[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c - 3*a*d + 2*b*d*x^n) + Sqrt[b*c - a*d]*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^3*d^(5/2)*n*Sqrt[c + d*x^n])

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]

[Out] Defer[IntegrateAlgebraic][x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x]

fricas [A] time = 0.50, size = 361, normalized size = 2.41

$$\frac{(3b^2d^2 + 2abcd + 3a^2d^2)\sqrt{bd}\log(8b^2d^2x^{2n} + b^2d^2 + 6abcd + a^2d^2 + 4(2\sqrt{bd}bdx^n + (bc + ad)\sqrt{bd})\sqrt{bx^n + a}\sqrt{dx^n + c} + 8(b^2cd + abd^2)x^n) + 4(2b^2d^2x^n - 3b^2cd - 3abd^2)\sqrt{bx^n + a}\sqrt{dx^n + c}}{16b^3d^3n} \cdot \frac{(3b^2d^2 + 2abcd + 3a^2d^2)\sqrt{-bd}\arctan\left(\frac{(2\sqrt{3bd}bdx^n + bc + ad)\sqrt{bd}\sqrt{bx^n + a}\sqrt{dx^n + c}}{2(b^2d^2x^n + abd^2 + b^2cd)}\right) - 2(2b^2d^2x^n - 3b^2cd - 3abd^2)\sqrt{bx^n + a}\sqrt{dx^n + c}}{8b^3d^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")

[Out] [1/16*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(2*b^2*d^2*x^n - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^3*n), -1/8*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n) - 2*(2*b^2*d^2*x^n - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^3*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")

[Out] sage0*x

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{\sqrt{bx^n + a}\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3*n-1)/(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)

[Out] int(x^(3*n-1)/(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{\sqrt{bx^n + a}\sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*xⁿ)^(1/2)/(c+d*xⁿ)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(3*n - 1)/(sqrt(b*xⁿ + a)*sqrt(d*xⁿ + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1}}{\sqrt{a + bx^n} \sqrt{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3*n - 1)/((a + b*xⁿ)^(1/2)*(c + d*xⁿ)^(1/2)),x)

[Out] int(x^(3*n - 1)/((a + b*xⁿ)^(1/2)*(c + d*xⁿ)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{\sqrt{a + bx^n} \sqrt{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{**(-1+3*n)}/(a+b*x^{**n})^{** (1/2)}/(c+d*x^{**n})^{** (1/2)},x)

[Out] Integral(x^{** (3*n - 1)}/(sqrt(a + b*x^{**n})*sqrt(c + d*x^{**n})), x)

$$3.697 \quad \int \frac{x^{-1+3n}}{(a+bx^n)^{3/2} \sqrt{c+dx^n}} dx$$

Optimal. Leaf size=133

$$-\frac{2a^2 \sqrt{c+dx^n}}{b^2 n (bc-ad) \sqrt{a+bx^n}} - \frac{(3ad+bc) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{b} \sqrt{c+dx^n}} \right)}{b^{5/2} d^{3/2} n} + \frac{\sqrt{a+bx^n} \sqrt{c+dx^n}}{b^2 dn}$$

Rubi [A] time = 0.16, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {446, 89, 80, 63, 217, 206}

$$-\frac{2a^2 \sqrt{c+dx^n}}{b^2 n (bc-ad) \sqrt{a+bx^n}} - \frac{(3ad+bc) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{b} \sqrt{c+dx^n}} \right)}{b^{5/2} d^{3/2} n} + \frac{\sqrt{a+bx^n} \sqrt{c+dx^n}}{b^2 dn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]

[Out] (-2*a^2*Sqrt[c + d*x^n])/(b^2*(b*c - a*d)*n*Sqrt[a + b*x^n]) + (Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(b^2*d*n) - ((b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(5/2)*d^(3/2)*n)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^(n)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 89

Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c

```
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
+ 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2} \sqrt{c+dx^n}} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2} \sqrt{c+dx}} dx, x, x^n\right)}{n} \\
&= -\frac{2a^2 \sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{2 \text{Subst}\left(\int \frac{-\frac{1}{2}a(bc-ad) + \frac{1}{2}b(bc-ad)x}{\sqrt{a+bx} \sqrt{c+dx}} dx, x, x^n\right)}{b^2(bc-ad)n} \\
&= -\frac{2a^2 \sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n} \sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx} \sqrt{c+dx}} dx, x, x^n\right)}{2b^2dn} \\
&= -\frac{2a^2 \sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n} \sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad) \text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b} + \frac{dx^2}{b}}} dx, x, x^n\right)}{b^3dn} \\
&= -\frac{2a^2 \sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n} \sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad) \text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}} dx, x, x^n\right)}{b^3dn} \\
&= -\frac{2a^2 \sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n} \sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{b} \sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}
\end{aligned}$$

Mathematica [A] time = 0.52, size = 185, normalized size = 1.39

$$\frac{\sqrt{bc-ad} \left(-3a^2d^2 + 2abcd + b^2c^2\right) \sqrt{a+bx^n} \sqrt{\frac{b(c+dx^n)}{bc-ad}} \sinh^{-1}\left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{bc-ad}}\right) - b\sqrt{d} (c+dx^n) \left(-3a^2d + ab(c-dx^n) + b^2cx^n\right)}{b^3d^{3/2}n(ad-bc)\sqrt{a+bx^n} \sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]), x]

[Out] $(-(b\sqrt{d}(c+dx^n)(-3a^2d + b^2c^2 + a^2d^2) + \sqrt{b^2c^2 - a^2d}(b^2c^2 + 2a^2b^2cd - 3a^2d^2)\sqrt{a+bx^n}\sqrt{(b(c+dx^n))/(b^2c - a^2d)}\text{ArcSinh}[(\sqrt{d}\sqrt{a+bx^n})/\sqrt{b^2c - a^2d}]))/(b^3d^{3/2}n(-b^2c + a^2d)n\sqrt{a+bx^n}\sqrt{c+dx^n})$

IntegrateAlgebraic [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2} \sqrt{c+dx^n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]

[Out] Defer[IntegrateAlgebraic][x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]), x]

fricas [B] time = 0.57, size = 540, normalized size = 4.06

$$\frac{\left(\frac{4 \left(a^2 d - 3 a b c + (b^2 d - a^2 c) \sqrt{c + d x^n} + ((b^2 + 2 a b d - 3 a^2 c) \sqrt{c + d x^n} + (a b^2 + 2 a^2 d - 3 a^2 c) \sqrt{c + d x^n}) \log(8 b^2 d^2 x^{2 n} + b^2 c^2 + 6 a b c d + a^2 d^2 - 4 (2 \sqrt{b d} + b c + a d) \sqrt{c + d x^n} + 8 (b c + a d) \sqrt{c + d x^n}) \right)}{4 \left((b^2 d - a^2 c) \sqrt{c + d x^n} + (a b^2 + 2 a^2 d - 3 a^2 c) \sqrt{c + d x^n} \right)} \right)^2 \sqrt{c + d x^n} + (b^2 d - 3 a b c + (b^2 + 2 a b d - 3 a^2 c) \sqrt{c + d x^n} + ((b^2 + 2 a b d - 3 a^2 c) \sqrt{c + d x^n} + (a b^2 + 2 a^2 d - 3 a^2 c) \sqrt{c + d x^n}) \log(8 b^2 d^2 x^{2 n} + b^2 c^2 + 6 a b c d + a^2 d^2 - 4 (2 \sqrt{b d} + b c + a d) \sqrt{c + d x^n} + 8 (b c + a d) \sqrt{c + d x^n})}{2 \left((b^2 d - a^2 c) \sqrt{c + d x^n} + (a b^2 + 2 a^2 d - 3 a^2 c) \sqrt{c + d x^n} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")

[Out] [1/4*(4*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) + ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(b*d)*x^n + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^5*c*d^2 - a*b^4*d^3)*n*x^n + (a*b^4*c*d^2 - a^2*b^3*d^3)*n), 1/2*(2*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) + ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(-b*d)*x^n + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b*d))*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^5*c*d^2 - a*b^4*d^3)*n*x^n + (a*b^4*c*d^2 - a^2*b^3*d^3)*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(bx^n + a)^2 \sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(bx^n + a)^2 \sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3*n-1)/(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)

[Out] `int(x^(3*n-1)/(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(bx^n + a)^{\frac{3}{2}} \sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1}}{(a + b x^n)^{3/2} \sqrt{c + d x^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)),x)`

[Out] `int(x^(3*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+3*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)`

[Out] Timed out

$$3.698 \quad \int \frac{x^{-1+3n}}{(a+bx^n)^{5/2} \sqrt{c+dx^n}} dx$$

Optimal. Leaf size=147

$$-\frac{2a^2\sqrt{c+dx^n}}{3b^2n(bc-ad)(a+bx^n)^{3/2}} + \frac{2 \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}n} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2n(bc-ad)^2\sqrt{a+bx^n}}$$

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {446, 89, 78, 63, 217, 206}

$$-\frac{2a^2\sqrt{c+dx^n}}{3b^2n(bc-ad)(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2n(bc-ad)^2\sqrt{a+bx^n}} + \frac{2 \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}n}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]), x]

[Out] (-2*a^2*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)*n*(a + b*x^n)^(3/2)) + (4*a*(3*b*c - 2*a*d)*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)^2*n*Sqrt[a + b*x^n]) + (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(5/2)*Sqrt[d]*n)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 89

```

Int[((a_.) + (b_.)*(x_))2*((c_.) + (d_.)*(x_))(n_.)*((e_.) + (f_.)*(x_))(p_.), x_Symbol] := Simp[((b*c - a*d)2*(c + d*x)(n + 1)*(e + f*x)(p + 1))/(d2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d2*(d*e - c*f)*(n + 1)), Int[(c + d*x)(n + 1)*(e + f*x)p*Simp[a2*d2*f*(n + p + 2) + b2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

```

Rule 206

```

Int[((a_) + (b_.)*(x_)2)(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

```

Rule 217

```

Int[1/Sqrt[(a_) + (b_.)*(x_)2], x_Symbol] := Subst[Int[1/(1 - b*x2), x], x, x/Sqrt[a + b*x2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

```

Rule 446

```

Int[(x_)(m_.)*((a_) + (b_.)*(x_)(n_.))(p_.)*((c_) + (d_.)*(x_)(n_.))(q_.), x_Symbol] := Dist[1/n, Subst[Int[x(Simplify[(m + 1)/n] - 1)*(a + b*x)p*(c + d*x)q, x], x, xn], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2} \sqrt{c+dx^n}} dx &= \frac{\text{Subst} \left(\int \frac{x^2}{(a+bx)^{5/2} \sqrt{c+dx}} dx, x, x^n \right)}{n} \\
&= -\frac{2a^2 \sqrt{c+dx^n}}{3b^2(bc-ad)n(a+bx^n)^{3/2}} + \frac{2 \text{Subst} \left(\int \frac{-\frac{1}{2}a(3bc-ad) + \frac{3}{2}b(bc-ad)x}{(a+bx)^{3/2} \sqrt{c+dx}} dx, x, x^n \right)}{3b^2(bc-ad)n} \\
&= -\frac{2a^2 \sqrt{c+dx^n}}{3b^2(bc-ad)n(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2 n \sqrt{a+bx^n}} + \frac{\text{Subst} \left(\int \frac{1}{\sqrt{a+bx} \sqrt{c+dx}} dx, x, x^n \right)}{b^2 n} \\
&= -\frac{2a^2 \sqrt{c+dx^n}}{3b^2(bc-ad)n(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2 n \sqrt{a+bx^n}} + \frac{2 \text{Subst} \left(\int \frac{1}{\sqrt{c-\frac{ad}{b} + \frac{dx^2}{b}}} dx, x, x^n \right)}{b^3 n} \\
&= -\frac{2a^2 \sqrt{c+dx^n}}{3b^2(bc-ad)n(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2 n \sqrt{a+bx^n}} + \frac{2 \text{Subst} \left(\int \frac{1}{1-\frac{dx^2}{b}} dx, x, x^n \right)}{b^3 n} \\
&= -\frac{2a^2 \sqrt{c+dx^n}}{3b^2(bc-ad)n(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2 n \sqrt{a+bx^n}} + \frac{2 \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{b} \sqrt{c+dx^n}} \right)}{b^{5/2} \sqrt{d} n}
\end{aligned}$$

Mathematica [A] time = 0.81, size = 217, normalized size = 1.48

$$\frac{2\sqrt{c+dx^n} \left(\frac{(3b^2c^2 - a^2d^2)(a+bx^n)}{d(bc-ad)^2} + \frac{a^2}{ad-bc} - \frac{3(a+bx^n) \left(\sqrt{bc-ad} \sqrt{\frac{b(c+dx^n)}{bc-ad}} - \sqrt{d} \sqrt{a+bx^n} \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^n}}{\sqrt{bc-ad}} \right) \right)}{d\sqrt{bc-ad} \sqrt{\frac{b(c+dx^n)}{bc-ad}}} \right)}{3b^2n(a+bx^n)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]), x]

[Out] (2*Sqrt[c + d*x^n]*(a^2/(-(b*c) + a*d) + ((3*b^2*c^2 - a^2*d^2)*(a + b*x^n))/(d*(b*c - a*d)^2) - (3*(a + b*x^n)*(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n))/(b*c - a*d]) - Sqrt[d]*Sqrt[a + b*x^n]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]]))/(d*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n))/(b*c - a*d])))/(3*b^2*n*(a + b*x^n)^(3/2))

IntegrateAlgebraic [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n}}{(a + bx^n)^{5/2} \sqrt{c + dx^n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]

[Out] Defer[IntegrateAlgebraic][x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]

fricas [B] time = 0.79, size = 769, normalized size = 5.23

([1/6*(4*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*sqrt(b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(b*d)*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n)/((b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^2*b^5*c*d^2 + a^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^3)*n), 1/3*(2*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) - 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*sqrt(-b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(-b*d)*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b*d))*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^2*b^5*c*d^2 + a^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^3)*n)]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")

[Out] [1/6*(4*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*sqrt(b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(b*d)*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n)/((b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^2*b^5*c*d^2 + a^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^3)*n), 1/3*(2*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) - 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*sqrt(-b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(-b*d)*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b*d))*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^2*b^5*c*d^2 + a^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^3)*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(bx^n + a)^{5/2} \sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")

[Out] integrate($x^{(3*n - 1)/((b*x^n + a)^{(5/2)*sqrt(d*x^n + c))}$), x)

maple [F] time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(bx^n + a)^{\frac{5}{2}} \sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int($x^{(3*n-1)/(b*x^n+a)^{(5/2)/(d*x^n+c)^{(1/2)}$), x)

[Out] int($x^{(3*n-1)/(b*x^n+a)^{(5/2)/(d*x^n+c)^{(1/2)}$), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(bx^n + a)^{\frac{5}{2}} \sqrt{dx^n + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x^{(-1+3*n)/(a+b*x^n)^{(5/2)/(c+d*x^n)^{(1/2)}$, x, algorithm="maxima")

[Out] integrate($x^{(3*n - 1)/((b*x^n + a)^{(5/2)*sqrt(d*x^n + c))}$), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1}}{(a + bx^n)^{5/2} \sqrt{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int($x^{(3*n - 1)/((a + b*x^n)^{(5/2)*(c + d*x^n)^{(1/2)}$), x)

[Out] int($x^{(3*n - 1)/((a + b*x^n)^{(5/2)*(c + d*x^n)^{(1/2)}$), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x^{*(-1+3*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2)}$, x)

[Out] Timed out

$$3.699 \quad \int x^p (b + cx)^p (b + 2cx) dx$$

Optimal. Leaf size=20

$$\frac{x^{p+1}(b + cx)^{p+1}}{p + 1}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {74}

$$\frac{x^{p+1}(b + cx)^{p+1}}{p + 1}$$

Antiderivative was successfully verified.

[In] Int[x^p*(b + c*x)^p*(b + 2*c*x), x]

[Out] (x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rubi steps

$$\int x^p (b + cx)^p (b + 2cx) dx = \frac{x^{1+p} (b + cx)^{1+p}}{1 + p}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{x^{p+1}(b + cx)^{p+1}}{p + 1}$$

Antiderivative was successfully verified.

[In] Integrate[x^p*(b + c*x)^p*(b + 2*c*x), x]

[Out] (x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^p (b + cx)^p (b + 2cx) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^p*(b + c*x)^p*(b + 2*c*x), x]

[Out] Defer[IntegrateAlgebraic][x^p*(b + c*x)^p*(b + 2*c*x), x]

fricas [A] time = 0.42, size = 25, normalized size = 1.25

$$\frac{(cx^2 + bx)(cx + b)^p x^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^p*(c*x+b)^p*(2*c*x+b), x, algorithm="fricas")

[Out] (c*x^2 + b*x)*(c*x + b)^p*x^p/(p + 1)

giac [A] time = 0.16, size = 35, normalized size = 1.75

$$\frac{(cx + b)^p cx^2 x^p + (cx + b)^p bxx^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^p*(c*x+b)^p*(2*c*x+b), x, algorithm="giac")

[Out] ((c*x + b)^p*c*x^2*x^p + (c*x + b)^p*b*x*x^p)/(p + 1)

maple [A] time = 0.05, size = 21, normalized size = 1.05

$$\frac{x^{p+1} (cx + b)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^p*(c*x+b)^p*(2*c*x+b), x)

[Out] x^(p+1)*(c*x+b)^(p+1)/(p+1)

maxima [A] time = 0.82, size = 29, normalized size = 1.45

$$\frac{(cx^2 + bx)e^{(p \log(cx+b) + p \log(x))}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="maxima")`

[Out] $(c*x^2 + b*x)*e^{(p*\log(c*x + b) + p*\log(x))/(p + 1)}$

mupad [B] time = 4.80, size = 22, normalized size = 1.10

$$\frac{x x^p (b + c x)^p (b + c x)}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^p*(b + c*x)^p*(b + 2*c*x),x)`

[Out] $(x*x^p*(b + c*x)^p*(b + c*x))/(p + 1)$

sympy [A] time = 3.10, size = 46, normalized size = 2.30

$$\begin{cases} \frac{b x x^p (b + c x)^p}{p + 1} + \frac{c x^2 x^p (b + c x)^p}{p + 1} & \text{for } p \neq -1 \\ \log(x) + \log\left(\frac{b}{c} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**p*(c*x+b)**p*(2*c*x+b),x)`

[Out] `Piecewise((b*x*x**p*(b + c*x)**p/(p + 1) + c*x**2*x**p*(b + c*x)**p/(p + 1), Ne(p, -1)), (log(x) + log(b/c + x), True))`

$$3.700 \quad \int x^{-1+2(1+p)} (b + cx^2)^p (b + 2cx^2) dx$$

Optimal. Leaf size=27

$$\frac{x^{2(p+1)} (b + cx^2)^{p+1}}{2(p+1)}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {449}

$$\frac{x^{2(p+1)} (b + cx^2)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^{(-1 + 2*(1 + p))}*(b + c*x²)^p*(b + 2*c*x²), x]

[Out] (x^{(2*(1 + p))}*(b + c*x²)^(1 + p))/(2*(1 + p))

Rule 449

Int[((e_.)*(x_))^{(m_.)*((a_) + (b_.)*(x_)^(n_))^{(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*xⁿ)^(p + 1)/(a*e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]}}

Rubi steps

$$\int x^{-1+2(1+p)} (b + cx^2)^p (b + 2cx^2) dx = \frac{x^{2(1+p)} (b + cx^2)^{1+p}}{2(1+p)}$$

Mathematica [C] time = 0.13, size = 97, normalized size = 3.59

$$\frac{x^{2p+2} (b + cx^2)^p \left(\frac{cx^2}{b} + 1\right)^{-p} \left(2c(p+1)x^2 {}_2F_1\left(-p, p+2; p+3; -\frac{cx^2}{b}\right) + b(p+2) {}_2F_1\left(-p, p+1; p+2; -\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^{(-1 + 2*(1 + p))}*(b + c*x²)^p*(b + 2*c*x²), x]

[Out] $(x^{(2+2p)}(b+cx^2)^p(b*(2+p)*\text{Hypergeometric2F1}[-p, 1+p, 2+p, -(cx^2/b)] + 2*c*(1+p)*x^2*\text{Hypergeometric2F1}[-p, 2+p, 3+p, -(cx^2/b)])) / (2*(1+p)*(2+p)*(1+(cx^2/b)^p)$

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^{-1+2(1+p)} (b+cx^2)^p (b+2cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^{(-1+2*(1+p))}*(b+cx²)^p*(b+2*cx²),x]

[Out] Defer[IntegrateAlgebraic][x^{(-1+2*(1+p))}*(b+cx²)^p*(b+2*cx²), x]

fricas [A] time = 0.43, size = 32, normalized size = 1.19

$$\frac{(cx^3+bx)(cx^2+b)^p x^{2p+1}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+2*p)*(c*x²+b)^p*(2*c*x²+b),x, algorithm="fricas")

[Out] 1/2*(c*x³+b*x)*(c*x²+b)^p*x^(2*p+1)/(p+1)

giac [B] time = 0.19, size = 52, normalized size = 1.93

$$\frac{(cx^2+b)^p cx^3 e^{(2p \log(x) + \log(x))} + (cx^2+b)^p b x e^{(2p \log(x) + \log(x))}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+2*p)*(c*x²+b)^p*(2*c*x²+b),x, algorithm="giac")

[Out] 1/2*((c*x²+b)^p*c*x³*e^{(2*p*log(x)+log(x))}+(c*x²+b)^p*b*x*e^{(2*p*log(x)+log(x))})/(p+1)

maple [A] time = 0.04, size = 26, normalized size = 0.96

$$\frac{x^{2p+2} (cx^2+b)^{p+1}}{2p+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2*p+1)*(c*x^2+b)^p*(2*c*x^2+b),x)`

[Out] $\frac{1}{2}x^{(2p+2)}(cx^2+b)^{(p+1)}/(p+1)$

maxima [A] time = 0.89, size = 35, normalized size = 1.30

$$\frac{(cx^4 + bx^2)e^{(p \log(cx^2+b) + 2p \log(x))}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="maxima")`

[Out] $\frac{1}{2}(cx^4 + bx^2)e^{(p \log(cx^2 + b) + 2p \log(x))}/(p + 1)$

mupad [B] time = 4.88, size = 47, normalized size = 1.74

$$(cx^2 + b)^p \left(\frac{cx^{2p+1}x^3}{2p+2} + \frac{bx^{2p+1}}{2p+2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2*p + 1)*(b + c*x^2)^p*(b + 2*c*x^2),x)`

[Out] $(b + cx^2)^p((cx^{(2p+1)}x^3)/(2p+2) + (b*x*x^{(2p+1)})/(2p+2))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1+2*p)*(c*x**2+b)**p*(2*c*x**2+b),x)`

[Out] Timed out

$$3.701 \quad \int x^{-1+3(1+p)} (b + cx^3)^p (b + 2cx^3) dx$$

Optimal. Leaf size=27

$$\frac{x^{3(p+1)} (b + cx^3)^{p+1}}{3(p+1)}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {449}

$$\frac{x^{3(p+1)} (b + cx^3)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3), x]

[Out] (x^(3*(1 + p))*(b + c*x^3)^(1 + p))/(3*(1 + p))

Rule 449

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]

Rubi steps

$$\int x^{-1+3(1+p)} (b + cx^3)^p (b + 2cx^3) dx = \frac{x^{3(1+p)} (b + cx^3)^{1+p}}{3(1+p)}$$

Mathematica [C] time = 0.10, size = 97, normalized size = 3.59

$$\frac{x^{3p+3} (b + cx^3)^p \left(\frac{cx^3}{b} + 1\right)^{-p} \left(2c(p+1)x^3 {}_2F_1\left(-p, p+2; p+3; -\frac{cx^3}{b}\right) + b(p+2) {}_2F_1\left(-p, p+1; p+2; -\frac{cx^3}{b}\right)\right)}{3(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3), x]

[Out] $(x^{(3+3p)}(b+cx^3)^p(b*(2+p)*\text{Hypergeometric2F1}[-p, 1+p, 2+p, -((c*x^3)/b)] + 2*c*(1+p)*x^3*\text{Hypergeometric2F1}[-p, 2+p, 3+p, -((c*x^3)/b)]))/((3*(1+p)*(2+p)*(1+(c*x^3)/b)^p)$

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^{-1+3(1+p)} (b+cx^3)^p (b+2cx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^{(-1+3*(1+p))}*(b+c*x³)^p*(b+2*c*x³),x]

[Out] Defer[IntegrateAlgebraic][x^{(-1+3*(1+p))}*(b+c*x³)^p*(b+2*c*x³), x]

fricas [A] time = 0.43, size = 32, normalized size = 1.19

$$\frac{(cx^4+bx)(cx^3+b)^p x^{3p+2}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+3*p)*(c*x³+b)^p*(2*c*x³+b),x, algorithm="fricas")

[Out] 1/3*(c*x⁴+b*x)*(c*x³+b)^p*x^(3*p+2)/(p+1)

giac [B] time = 0.26, size = 56, normalized size = 2.07

$$\frac{(cx^3+b)^p cx^4 e^{(3p \log(x)+2 \log(x))} + (cx^3+b)^p bxe^{(3p \log(x)+2 \log(x))}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+3*p)*(c*x³+b)^p*(2*c*x³+b),x, algorithm="giac")

[Out] 1/3*((c*x³+b)^p*c*x⁴*e^{(3*p*log(x)+2*log(x))}+(c*x³+b)^p*b*x*e^{(3*p*log(x)+2*log(x))})/(p+1)

maple [A] time = 0.04, size = 26, normalized size = 0.96

$$\frac{x^{3p+3} (cx^3+b)^{p+1}}{3p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x)`

[Out] $\frac{1}{3}x^{(3+3p)}(cx^3+b)^{(p+1)}/(p+1)$

maxima [A] time = 0.69, size = 35, normalized size = 1.30

$$\frac{(cx^6 + bx^3)e^{(p \log(cx^3+b) + 3p \log(x))}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="maxima")`

[Out] $\frac{1}{3}(cx^6 + bx^3)e^{(p \log(cx^3 + b) + 3p \log(x))}/(p + 1)$

mupad [B] time = 4.90, size = 47, normalized size = 1.74

$$(cx^3 + b)^p \left(\frac{cx^{3p+2}x^4}{3p+3} + \frac{bx^{3p+2}}{3p+3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3*p + 2)*(b + c*x^3)^p*(b + 2*c*x^3),x)`

[Out] $(b + cx^3)^p \left(\frac{(cx^3)^{(3p+2)}x^4}{(3p+3)} + \frac{bxx^{(3p+2)}}{(3p+3)} \right)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(2+3*p)*(c*x**3+b)**p*(2*c*x**3+b),x)`

[Out] Timed out

$$3.702 \quad \int x^{-1+n(1+p)} (b + cx^n)^p (b + 2cx^n) dx$$

Optimal. Leaf size=27

$$\frac{x^{n(p+1)} (b + cx^n)^{p+1}}{n(p+1)}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {449}

$$\frac{x^{n(p+1)} (b + cx^n)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n), x]

[Out] (x^(n*(1 + p))*(b + c*x^n)^(1 + p))/(n*(1 + p))

Rule 449

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]

Rubi steps

$$\int x^{-1+n(1+p)} (b + cx^n)^p (b + 2cx^n) dx = \frac{x^{n(1+p)} (b + cx^n)^{1+p}}{n(1+p)}$$

Mathematica [C] time = 0.16, size = 101, normalized size = 3.74

$$\frac{(b + cx^n)^p \left(\frac{cx^n}{b} + 1\right)^{-p} \left(b(p+2)x^{n(p+1)} {}_2F_1\left(-p, p+1; p+2; -\frac{cx^n}{b}\right) + 2c(p+1)x^{n(p+2)} {}_2F_1\left(-p, p+2; p+3; -\frac{cx^n}{b}\right)\right)}{n(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n), x]

[Out] ((b + c*x^n)^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p, 2 + p, -(c*x^n)/b] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 + p, 3 + p, -(c*x^n)/b])/((n*(1 + p)*(2 + p)*(1 + (c*x^n)/b)^p)

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int x^{-1+n(1+p)} (b + cx^n)^p (b + 2cx^n) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x[^](-1 + n*(1 + p))*(b + c*x[^]n)[^]p*(b + 2*c*x[^]n),x]

[Out] Defer[IntegrateAlgebraic][x[^](-1 + n*(1 + p))*(b + c*x[^]n)[^]p*(b + 2*c*x[^]n), x]

fricas [A] time = 0.45, size = 35, normalized size = 1.30

$$\frac{(c x x^n + b x)(c x^n + b)^p x^{np+n-1}}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x[^](-1+n*(1+p))*(b+c*x[^]n)[^]p*(b+2*c*x[^]n),x, algorithm="fricas")

[Out] (c*x*x[^]n + b*x)*(c*x[^]n + b)[^]p*x[^](n*p + n - 1)/(n*p + n)

giac [B] time = 0.25, size = 66, normalized size = 2.44

$$\frac{(c x^n + b)^p c x x^n e^{(np \log(x) + n \log(x) - \log(x))} + (c x^n + b)^p b x e^{(np \log(x) + n \log(x) - \log(x))}}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x[^](-1+n*(1+p))*(b+c*x[^]n)[^]p*(b+2*c*x[^]n),x, algorithm="giac")

[Out] ((c*x[^]n + b)[^]p*c*x*x[^]n*e[^](n*p*log(x) + n*log(x) - log(x)) + (c*x[^]n + b)[^]p*b*x*e[^](n*p*log(x) + n*log(x) - log(x)))/(n*p + n)

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int (2c x^n + b) x^{(p+1)n-1} (c x^n + b)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x[^](-1+n*(p+1))*(c*x[^]n+b)[^]p*(b+2*c*x[^]n),x)

[Out] int(x[^](-1+n*(p+1))*(c*x[^]n+b)[^]p*(b+2*c*x[^]n),x)

maxima [A] time = 0.96, size = 39, normalized size = 1.44

$$\frac{(cx^{2n} + bx^n)e^{(np \log(x) + p \log(cx^n + b))}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{(-1+n*(1+p))}*(b+c*xⁿ)^p*(b+2*c*xⁿ), x, algorithm="maxima")

[Out] (c*x^(2*n) + b*xⁿ)*e^{(n*p*log(x) + p*log(c*xⁿ + b))}/(n*(p + 1))

mupad [B] time = 4.86, size = 54, normalized size = 2.00

$$\left(\frac{bx x^{n(p+1)-1}}{n(p+1)} + \frac{cx x^n x^{n(p+1)-1}}{n(p+1)} \right) (b + cx^n)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^{(n*(p + 1) - 1)}*(b + c*xⁿ)^p*(b + 2*c*xⁿ), x)

[Out] ((b*x*x^{(n*(p + 1) - 1)})/(n*(p + 1)) + (c*x*xⁿ*x^{(n*(p + 1) - 1)})/(n*(p + 1)))* (b + c*xⁿ)^p

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{(-1+n*(1+p))}*(b+c*xⁿ)^p*(b+2*c*xⁿ), x)

[Out] Timed out

Chapter 4

Appendix

Local contents

- 4.1 Download section3666
- 4.2 Listing of Grading functions3666

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
```

```

If[ExpnType[result]<=ExpnType[optimal],
  If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
    If[LeafCount[result]<=2*LeafCount[optimal],
      "A",
      "B"],
    "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],

```

```

    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
    If[Head[expn]===RootSum,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
    If[Head[expn]===Integrate || Head[expn]===Int,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
    9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1}, func]

```

4.2.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```



```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
            ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                 asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                 asinh,acosh,atanh,acoth,asech,acsch
                 ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
                 fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                 gamma,loggamma,digamma,zeta,polylog,LambertW,
                 elliptic_f,elliptic_e,elliptic_pi,exp_polar
                 ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
    ))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.2.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```



```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(6,m1) #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```